Title: 3+1 approach to self-force computations

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Abstract: TBA

(3+1) approach to self-force

Ian Vega[1]

Steve Detweiler[2] Peter Diener[3] Wolfgang Tichy[4] Barry Wardell[5]

[1]University of Guelph
[2]University of Florida
[3]Louisiana State University
[4]Florida Atlantic University
[5]Albert Einstein Institute

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Outline

- Background
- Test results
- New developments from Capra 13
- Prospects and Challenges

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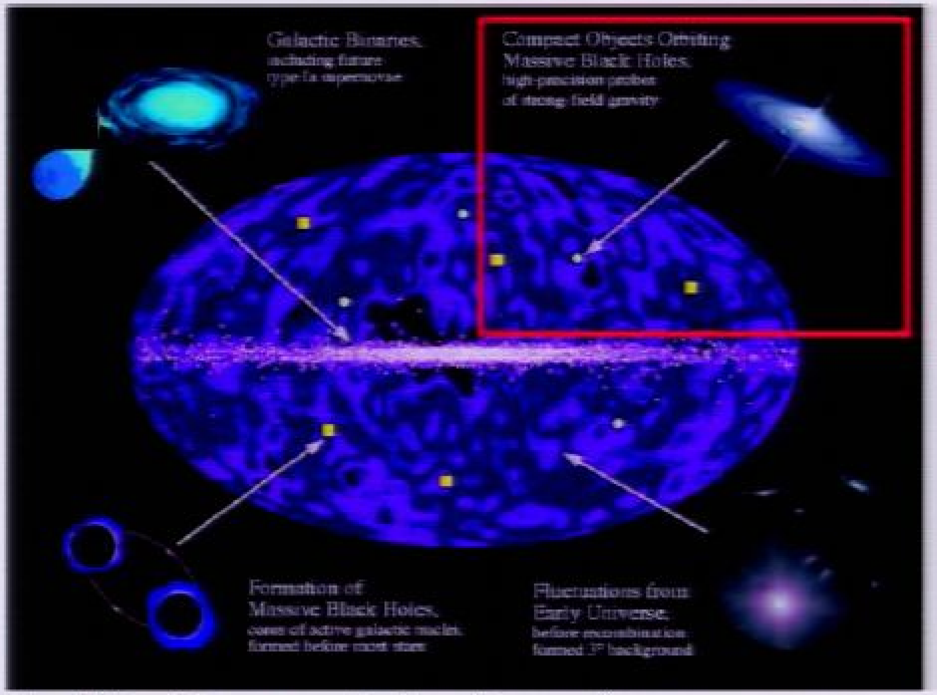
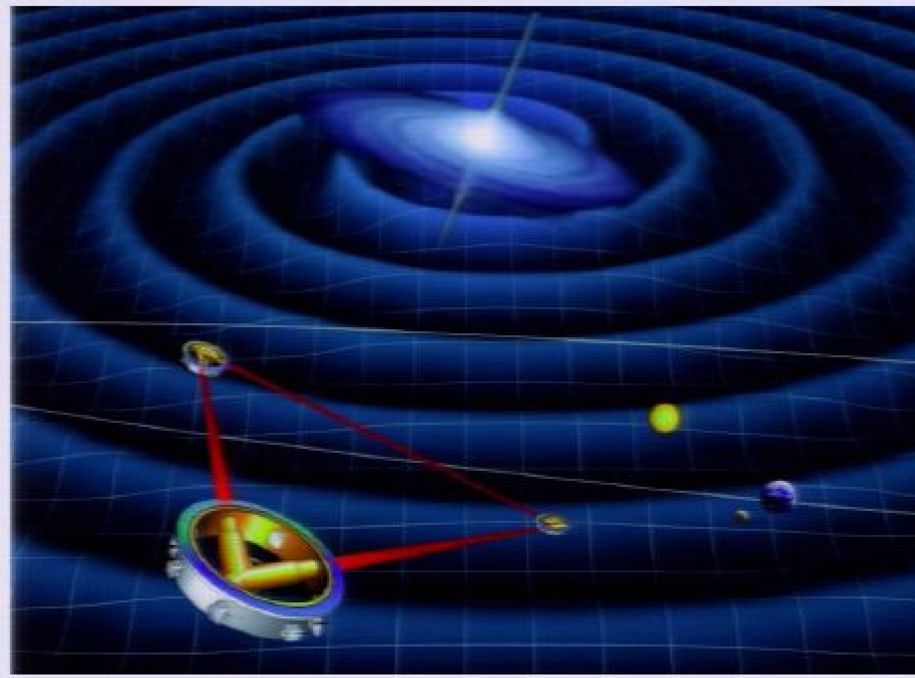
Background

Motivation

Extreme-mass-ratio inspirals (EMRIs) for LISA





(http://lisa.jpl.nasa.gov/gallery/images/)

These sources are conveniently modeled as point masses moving in a black hole spacetime. This motion is best characterized as a secular departure away from geodesic motion that is driven by a backreacting self-force.

An elementary self-force calculation

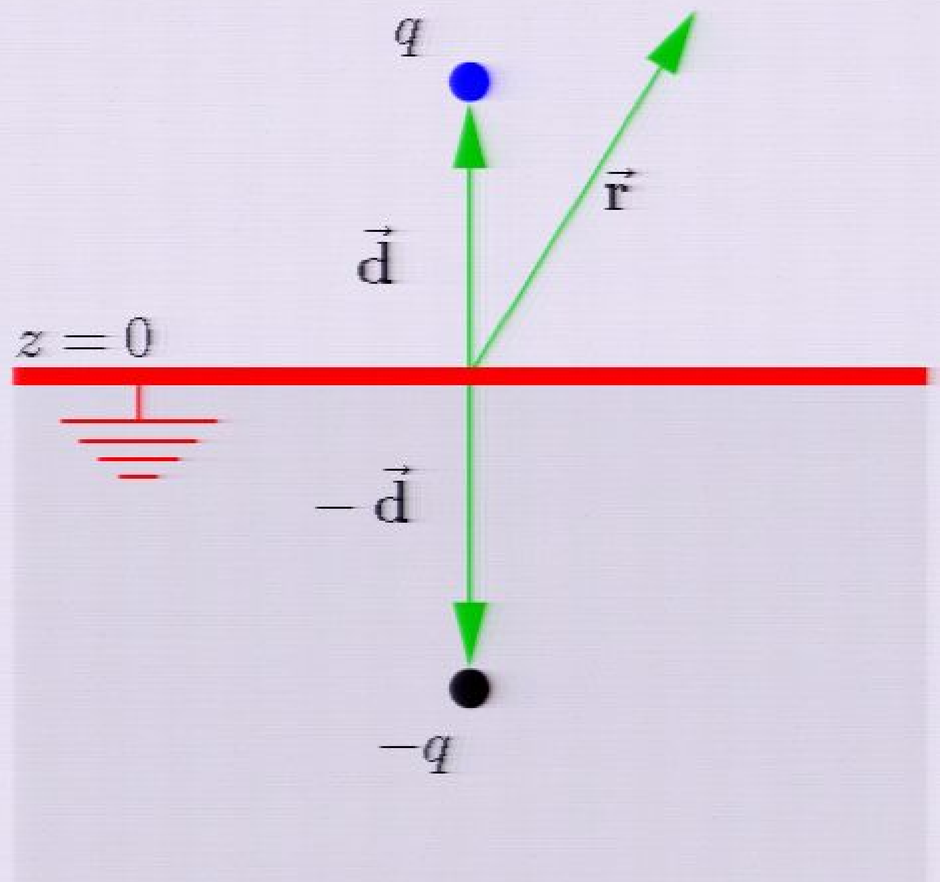
$V = \frac{q}{|\mathbf{r}-\mathbf{d}|} - \frac{q}{|\mathbf{r}+\mathbf{d}|}$

$\mathbf{E} = -\vec{\nabla}V = q\frac{\mathbf{r}-\mathbf{d}}{|\mathbf{r}-\mathbf{d}|^3} - q\frac{\mathbf{r}+\mathbf{d}}{|\mathbf{r}+\mathbf{d}|^3}$

Identification: $\mathbf{E}^{\mathsf{S}} = q\frac{\mathbf{r}-\mathbf{d}}{|\mathbf{r}-\mathbf{d}|^3}$

Subtraction: $\mathbf{E}^{\mathsf{R}} = \mathbf{E} - \mathbf{E}^{\mathsf{S}}$

$\mathbf{F} = q\mathbf{E}^{\mathsf{R}}|_{\mathbf{r}=\mathbf{d}} = -\frac{q^2}{4d^2}\hat{\mathbf{z}}$



Comparing methods

Standard

- Compute for the retarded field of a wave equation that's sourced by a delta function:
  - frequency-domain
  - (1+1) time-domain
- Regularize the retarded field with so-called "regularization parameters".
- "Evolve first, regularize later"

(3+1) approach

- Work with a regular effective point particle source.
- Compute for the regularized field and self-force with a (3+1) code.
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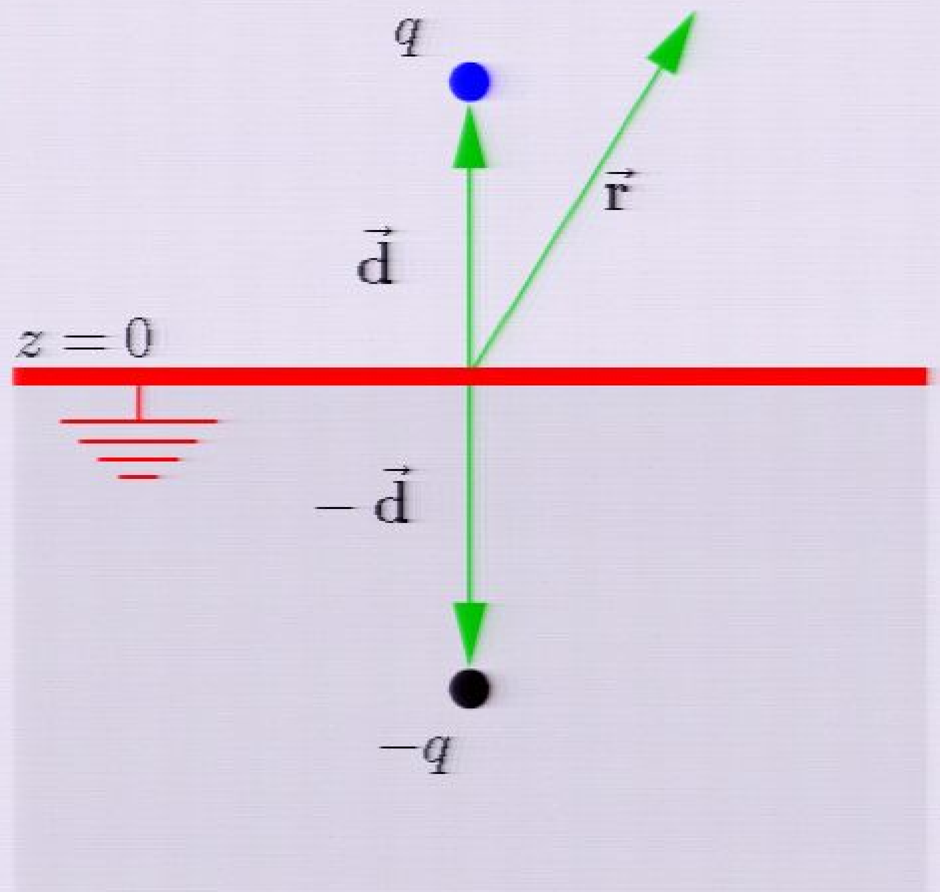

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Constructing a regular "effective" source

Detweiler-Whiting decomposition

$$\psi^{\mathrm{ret}} = \psi^{\mathrm{R}} + \psi^{\mathrm{S}}$$

$$F_a^{(\mathrm{self})} = \lim_{x \to z} \nabla_a \psi^{\mathrm{R}} \text{ and } \Box \psi^{\mathrm{S}} = -q\delta$$

Let $\psi^{\mathrm{ret}} = \psi^R + W\tilde{\psi}^{\mathrm{S}}$, where $\tilde{\psi}^{\mathrm{S}} \equiv q/\rho$ (an approximation to the singular field), and W is a suitably chosen "window function"

$$\Box \psi^{\mathrm{ret}} = -q\delta$$

$$\Longrightarrow \Box \psi^{\mathrm{R}} = -\Box(W\tilde{\psi}^{\mathrm{S}}) - q\delta$$

$$\Longrightarrow \Box \psi^{\mathrm{R}} = S_{\mathrm{eff}}(x^a, z^a(\tau), u^a(\tau))$$

Our simple prescription

Simultaneously integrate the following equations with a (3+1) evolution code and ODE solver:

$$\Box\psi^{\mathrm{R}} = S_{\mathrm{eff}}(x^a, z^a(\tau), u^a(\tau))$$

$$\frac{\mathrm{d}^2 z^a}{\mathrm{d}\tau^2} = \frac{q}{m}(g^{ab} + u^a u^b)(\nabla_b \psi^{\mathrm{R}})|_{z^a}.$$

Main requirements

- Code that efficiently evaluates S_{eff} → We provide this!
- An accurate (3+1) code for the wave equation → NR has this!
- A good ODE solver → easy!

Some comments

- $\tilde{\psi}^S$ is not the exact singular field. Therefore, S_{eff} is only of finite differentiability at the location of the charge.
- The window function W is an arbitrary construct, except for the following conditions:
  1. $W \to 1$ sufficiently fast as one approaches the particle,
  2. $\nabla_a W \to 0$ sufficiently fast as one approaches the particle, and
  3. $W = 0$ outside a compact region R surrounding the particle.

These conditions guarantee that
(a) $\nabla_a \psi^{\mathrm{R}}|_{\text{point charge}}$ gives the self-force.
(b) ψ^{R} gives fluxes in the wavezone.

Pros and cons of this approach

Why (3+1)?

- Non-post-processing approach to self-consistent evolution.
- Does not rely on the underlying symmetries of the spacetime.

- No delta functions, just extended sources of finite differentiability
- The evolved field, ψ^{R}, becomes the retarded field in the wavezone.
- The self-force is very easy to compute from ψ^{R}.

Why not (3+1)?

- Lesser accuracy compared to more developed methods
- Inherits all the difficulties of any (3+1) calculation

Some test results

Test application: scalar charge in a circular orbit of Schwarzschild

- In Schwarzschild coordinates, F_α has only two unknown independent components.

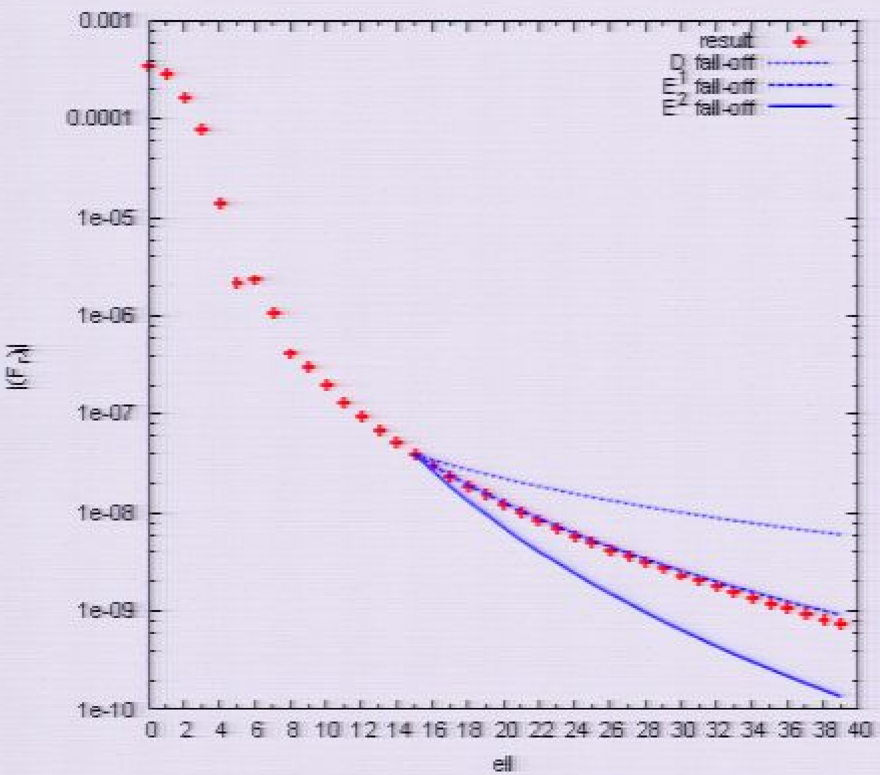
$$F_t = ?, \; F_r = ?, \; F_\theta = 0, \; F_\phi = -\frac{1}{\Omega} F_t$$

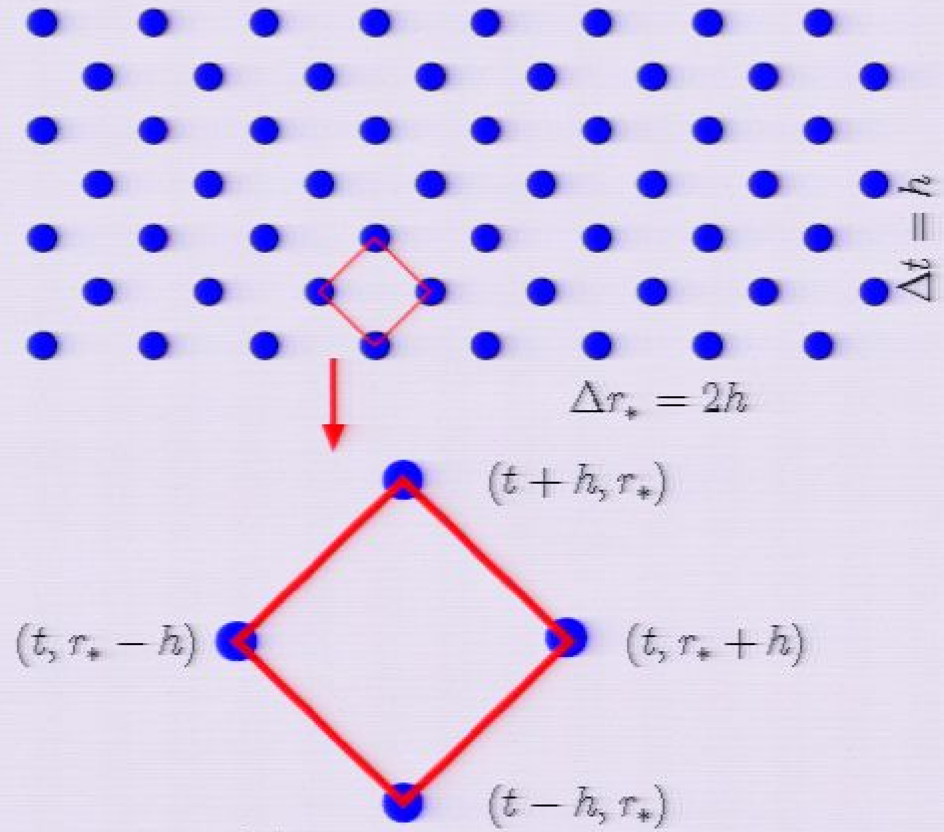
- The fluxes through the event horizon and infinity are related to F_t:

$$\left.\frac{\mathrm{d}E}{\mathrm{d}t}\right|_{r=2M} + \left.\frac{\mathrm{d}E}{\mathrm{d}t}\right|_{r=\infty} = -\sqrt{1-\frac{3M}{R}}F_t.$$

Awkward (1+1) implementation

(IV and Detweiler, PRD 2008)





We achieved self-force accuracies to within $\sim 1\%$ and matched the retarded field in the wavezone to within relative error of $\sim 10^{-6}$.

(3+1) Codes

- Multiblock code (with P. Diener (LSU)) - gr-qc/0602104
  - high-order finite differencing code
- SGRID code (with W. Tichy (FAU)) - gr-qc/0609087
  - pseudospectral code: Fourier expansion in angular directions, Chebyshev polynomials in the radial direction

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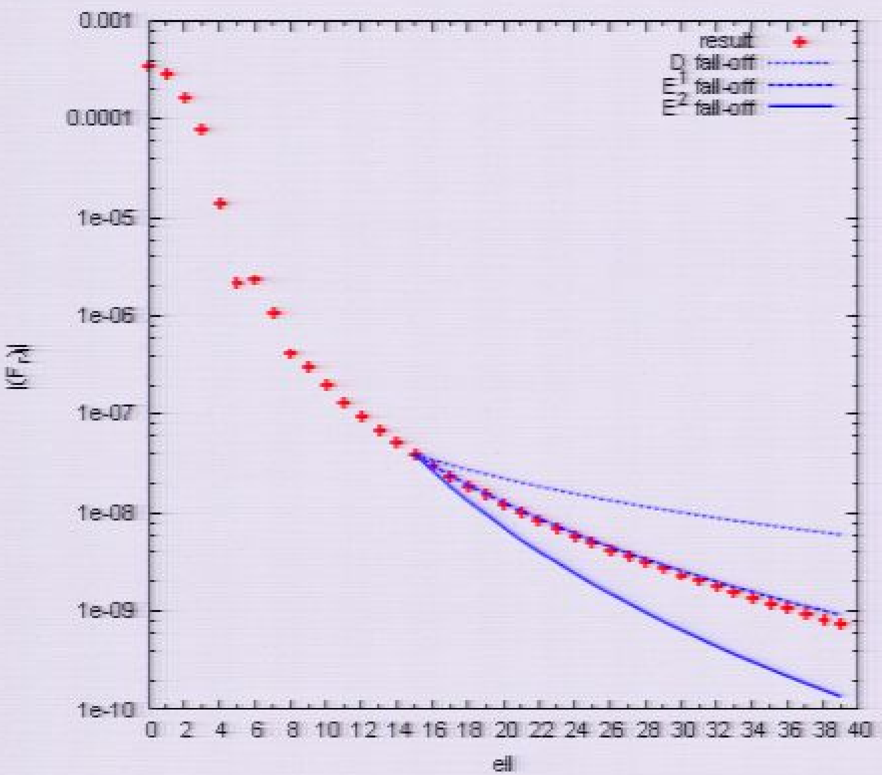

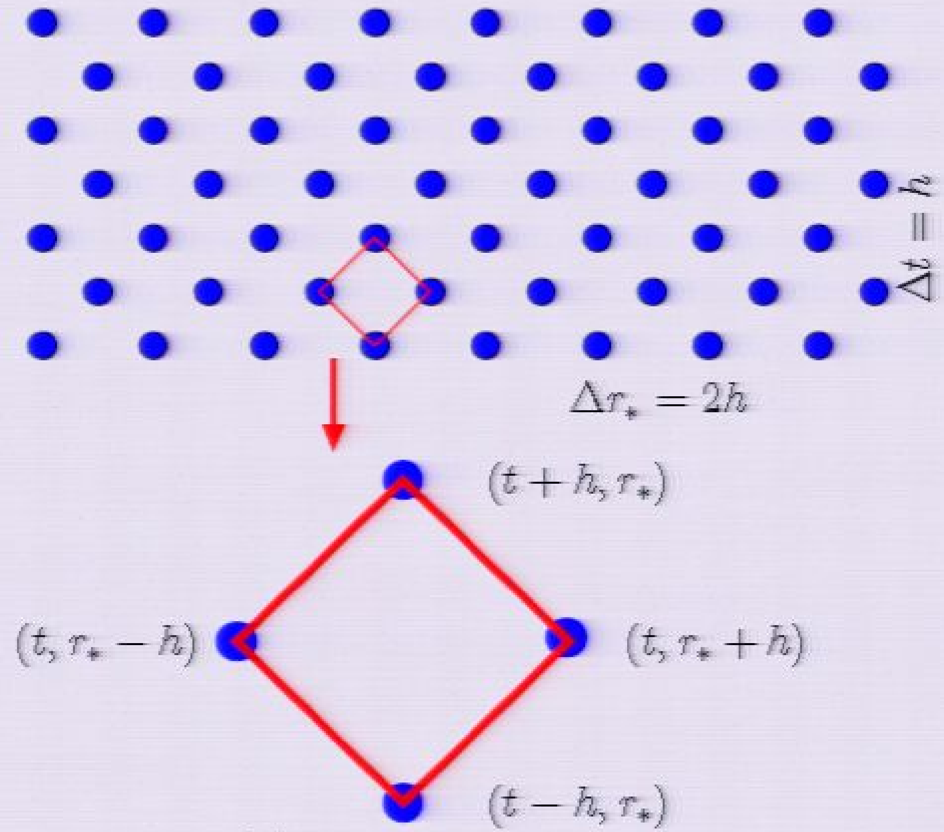

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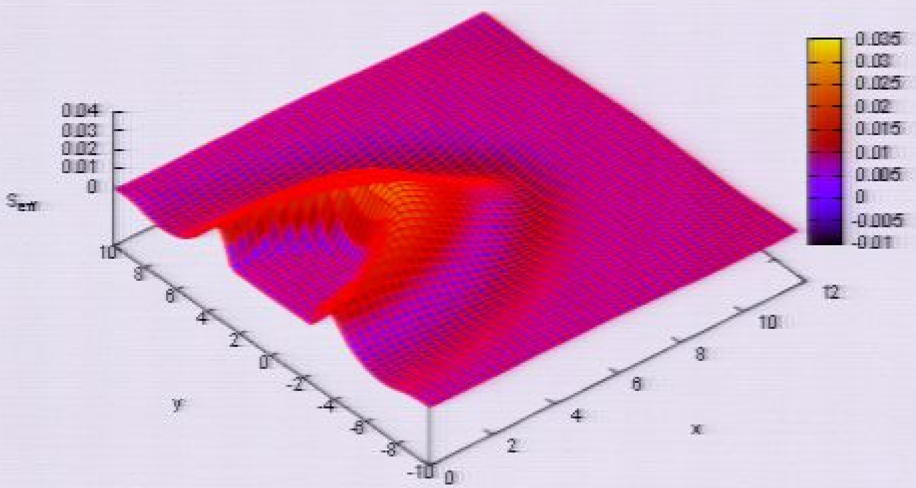
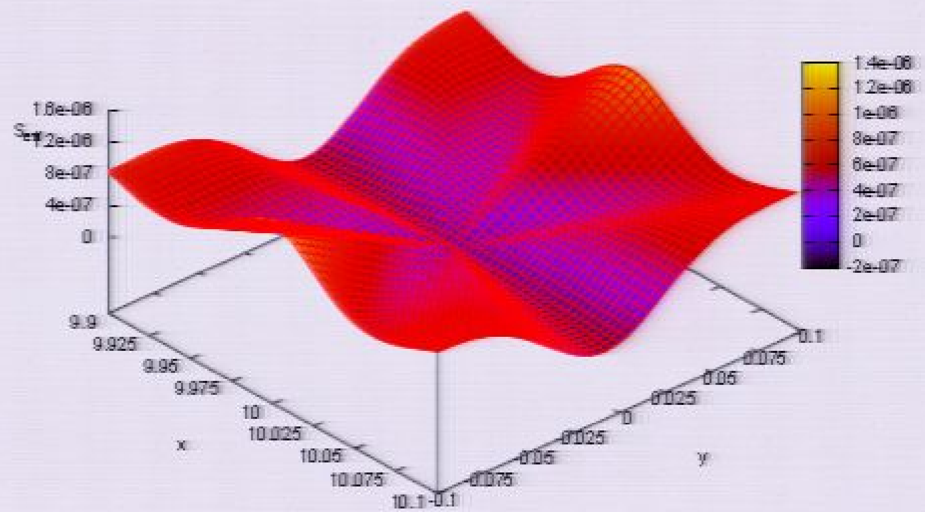
Effective source

(IV, Diener, Tichy, Detweiler, PRD 2009)





Magnitude of S_{eff} across a surface of constant Kerr-Schild z. The effective source is C^0 at the location of the charge, and C^∞ everywhere else.

Window function

Consider the transition function f depending on 4 parameters: $\{r_0, w, q, s\}$. (Yunes, Tichy, Owen, Brugmann, 2006)

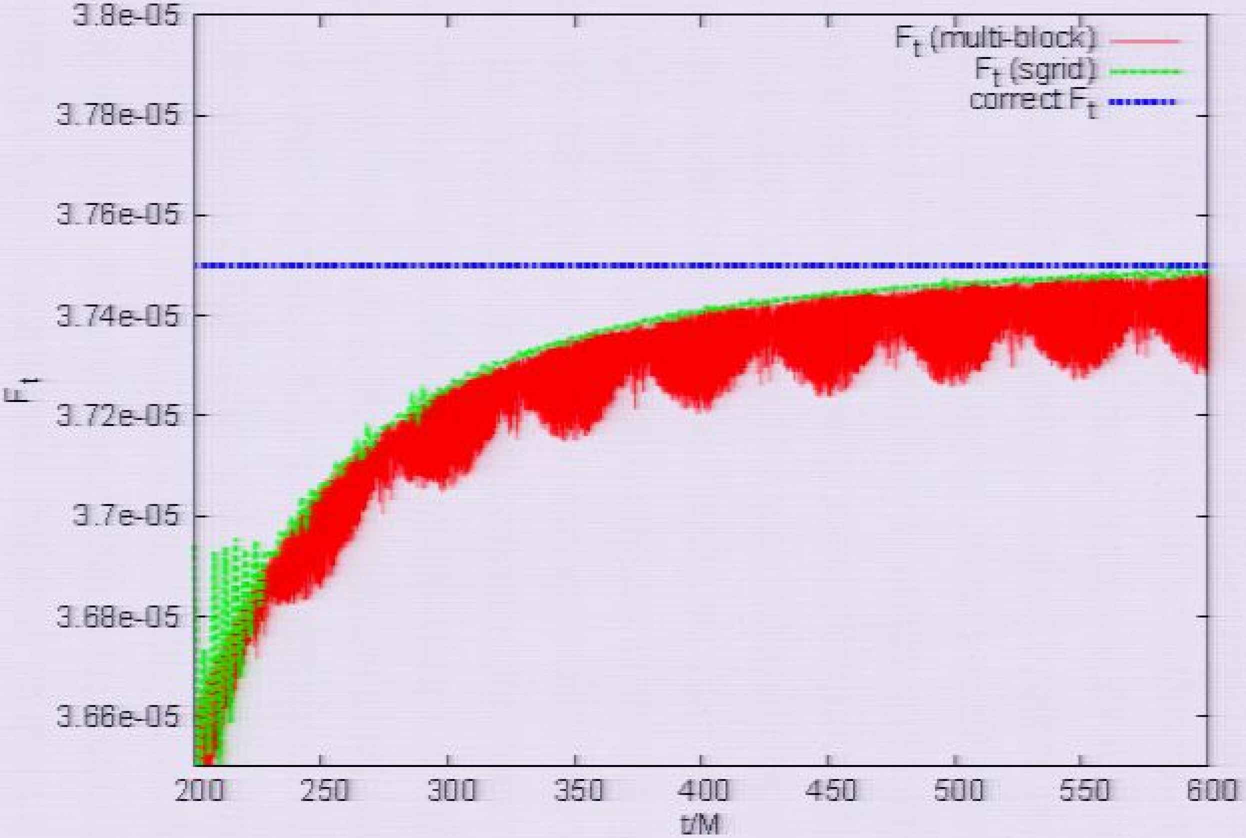
$$f(r|r_0, w, q, s) = \begin{cases} 0, & r \leq r_0 \\ \frac{1}{2} + \frac{1}{2}\tanh\left[\frac{s}{\pi}\left(\tan\left(\frac{\pi}{2w}(r-r_0)\right) - \frac{q^2}{\tan\left(\frac{\pi}{2w}(r-r_0)\right)}\right)\right], & r_0 < r < r_0 + w \\ 1, & r \geq r_0 + w. \end{cases}$$

With this, we construct the window

$$W(r) = \begin{cases} f(r|(R-\delta_1-w_1), w_1, q_1, s_1) & r \leq R \\ 1 - f(r|(R+\delta_2), w_2, q_2, s_2) & r > R \end{cases}$$

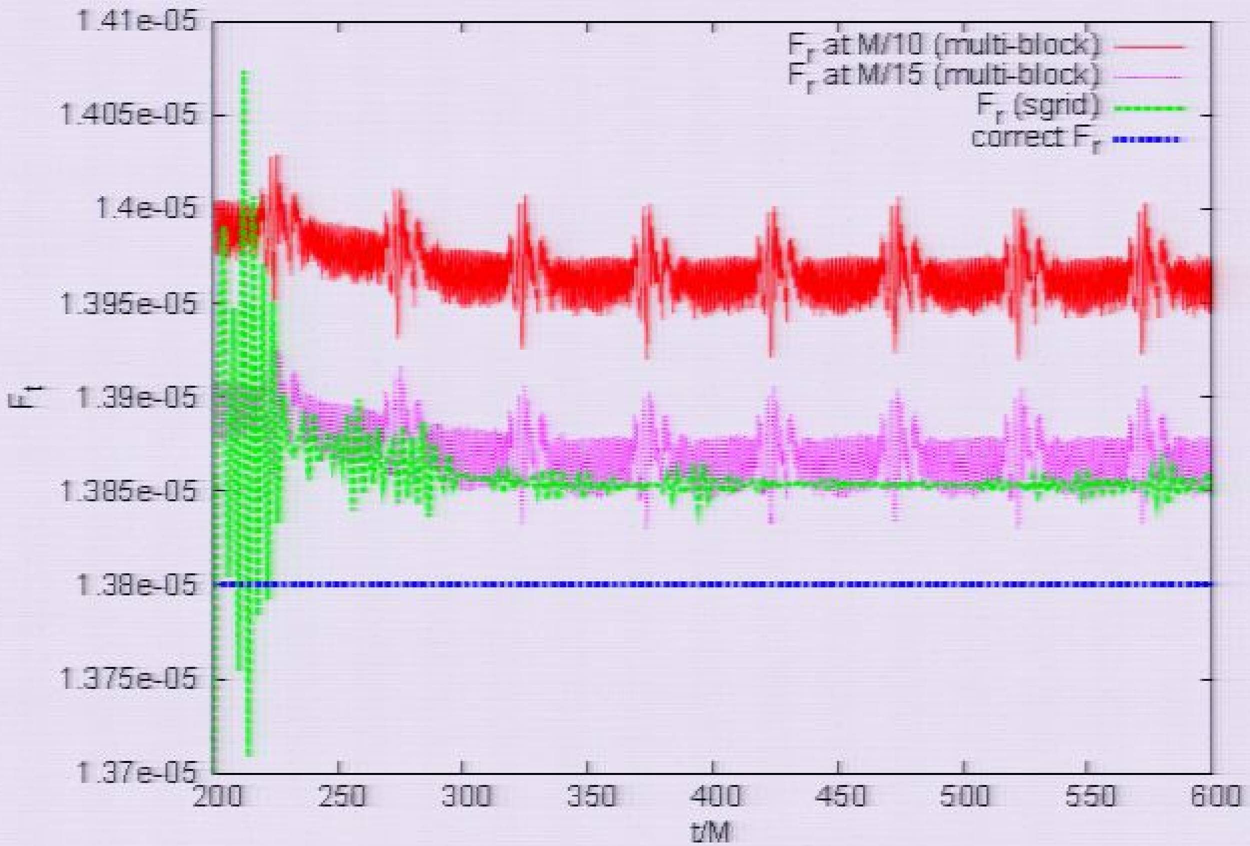
which depends on 8 parameters: 2 sets of $\{\delta, w, q, s\}$.

Results: dissipative piece, F_t



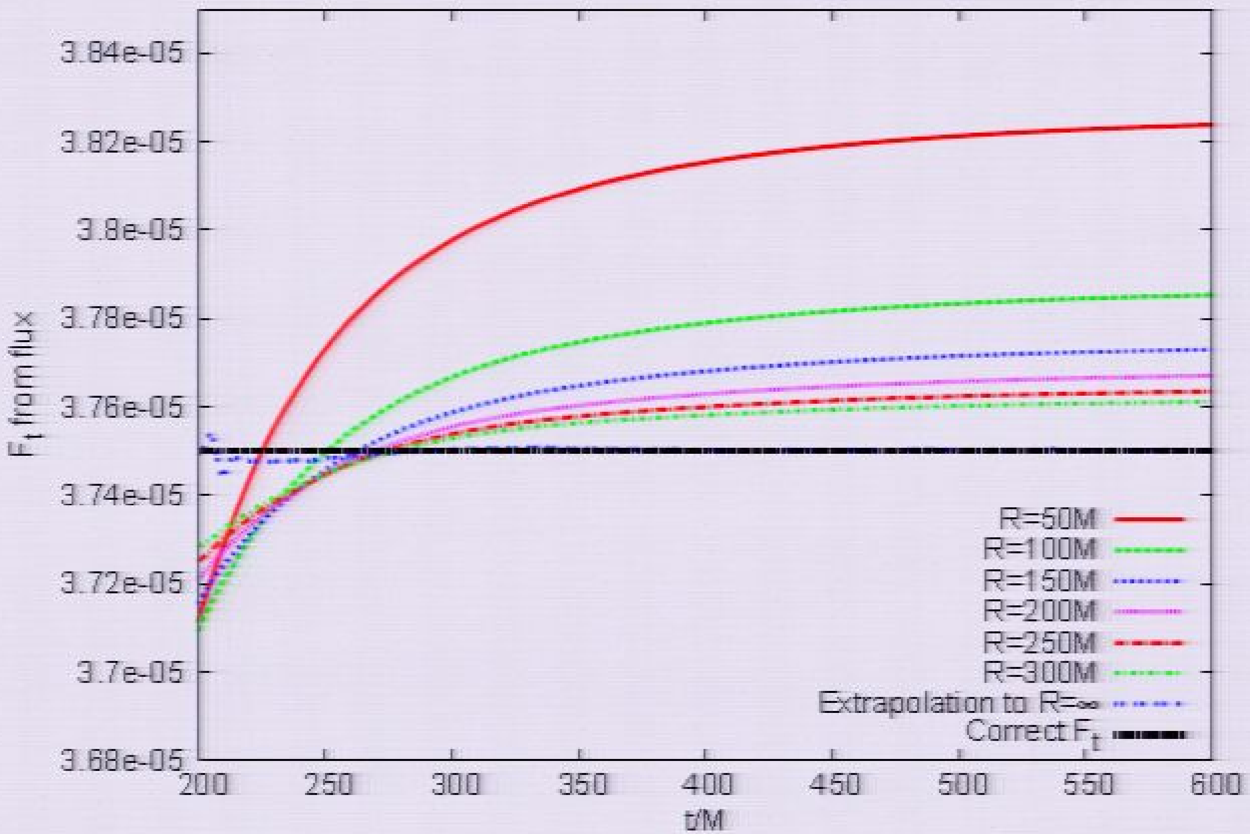
Time-component of the self-force as a function of Kerr-Schild time. In both codes, it approaches the correct constant value as the scalar field achieves helical symmetry.

Results: conservative piece, F_r



Radial component of the self-force as a function of Kerr-Schild time. This plot also shows covergence in the multi-block code with increasing radial resolution.

Results: Energy flux



Energy flux as a function of Kerr-Schild time. Shown here is the dependence of the flux on outer extraction radii, and the extrapolation to infinite outer extraction radius.

Representative (3+1) results

	Code	Result	Error
F_t	mb	$(3.728 - 3.748) \times 10^{-5}$	0.05%-0.6%
F_t	sgrid	$(3.7481 - 3.7487) \times 10^{-5}$	0.05%
F_r	mb	$(1.384 - 1.389) \times 10^{-5}$	0.4%-0.8%
F_r	sgrid	$(1.384 - 1.386) \times 10^{-5}$	0.4%-0.5%

	Code	Result	Error
$\dot{E}(R = 150)$	mb	3.773×10^{-5}	0.6%
$\dot{E}(R = 150)$	sgrid	3.771×10^{-5}	0.6%
$\dot{E}(R = 300)$	mb	3.761×10^{-5}	0.2%
$\dot{E}(R = \infty)$	mb	3.7502×10^{-5}	0.0005%

Extraction time: t=600M. Error is determined by a comparison with an accurate frequency domain calculation giving $F_t = 3.750227 \times 10^{-5}$ and $F_r = 1.378448 \times 10^{-5}$.

New developments from Capra 13

Effective scalar source for generic orbits

Covariant singular field (Haas and Poisson, PRD 2006)

$$\psi^{\mathrm{S}} = \frac{q}{2r} + \frac{q}{2r_{\mathrm{adv}}} + O(\epsilon^3)$$

$$\psi^{\mathrm{S}} = \frac{q}{s}\left(1 + \frac{\bar{r}^2 - s^2}{6s^2} R_{u\sigma u\sigma} + \frac{\bar{r}(\bar{r}^2 - 3s^2)}{24s^2} R_{u\sigma u\sigma|u} - \frac{(\bar{r}^2 - s^2)}{24s^2} R_{u\sigma u\sigma|\sigma}\right) + O(\epsilon^3)$$

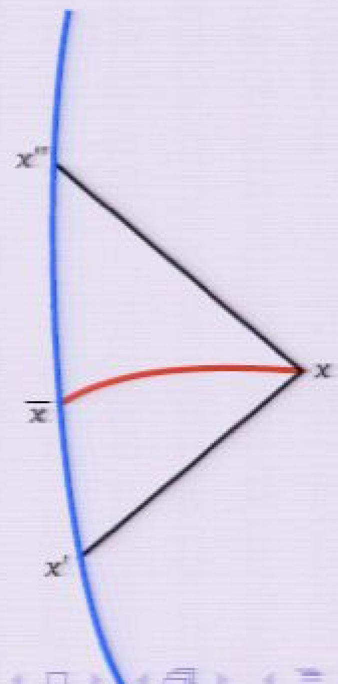
Five scalar functions

$\{s, \bar{r}, R_{u\sigma u\sigma}, R_{u\sigma u\sigma|u}, R_{u\sigma u\sigma|\sigma}\}$.

$$s^2 = (g^{\bar{\alpha}\bar{\beta}} + u^{\bar{\alpha}} u^{\bar{\beta}})\sigma_{\bar{\alpha}}\sigma_{\bar{\beta}}$$

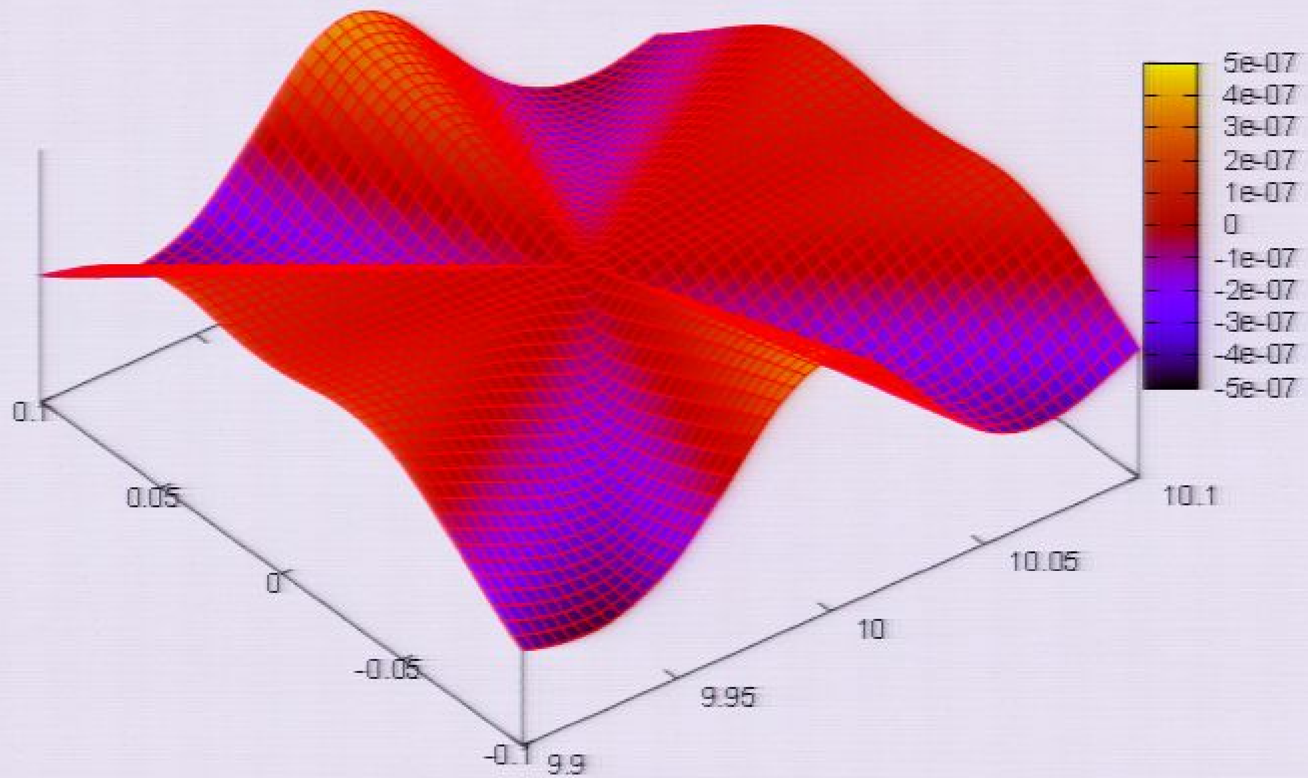
$$\bar{r} = u^{\bar{\alpha}}\sigma_{\bar{\beta}}$$

$$R_{u\sigma u\sigma|u} = R_{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta};\bar{\epsilon}} u^{\bar{\alpha}}\sigma^{\bar{\beta}} u^{\bar{\gamma}}\sigma^{\bar{\delta}} u^{\bar{\epsilon}}$$



Effective scalar source for generic orbits

(independent work by Wardell and IV)



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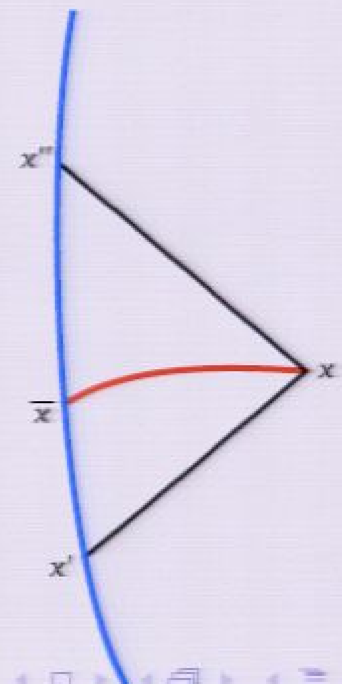

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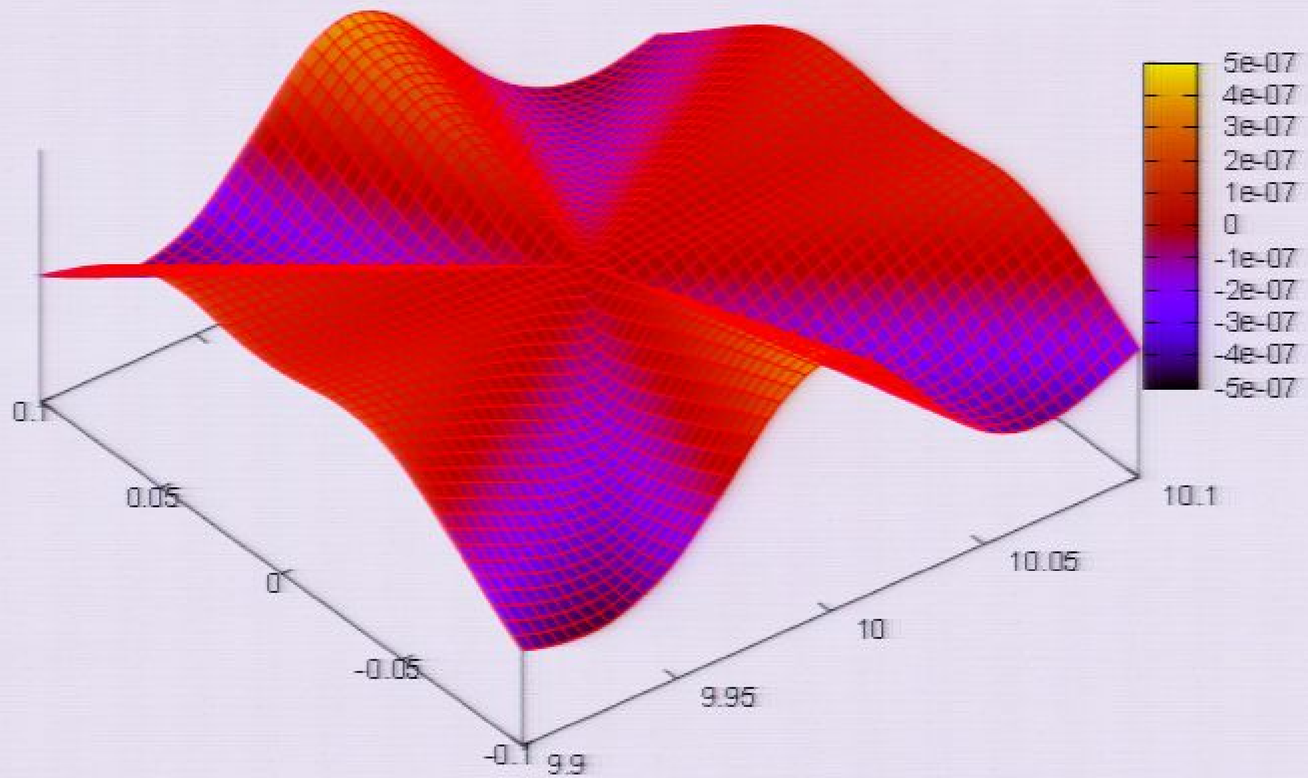

Hyperboloidal slicing for long term evolutions

One problem identified in previous work was the huge impact of the spurious reflection one gets from the outer boundary. This is due mainly to the fact that the self-force is a really small quantity.

It appeared then that for longer evolutions we would necessarily have to move outer boundary further outward to further delay it getting into causal contact with the charge.

An idea (from Zenginoğlu & Tiglio, PRD 2009) to compactify in the radial direction and transform the time coordinate so that the spatial slices asymptotes to $\mathscr{I}^+$ appears to be a good strategy for the outer boundary reflection.

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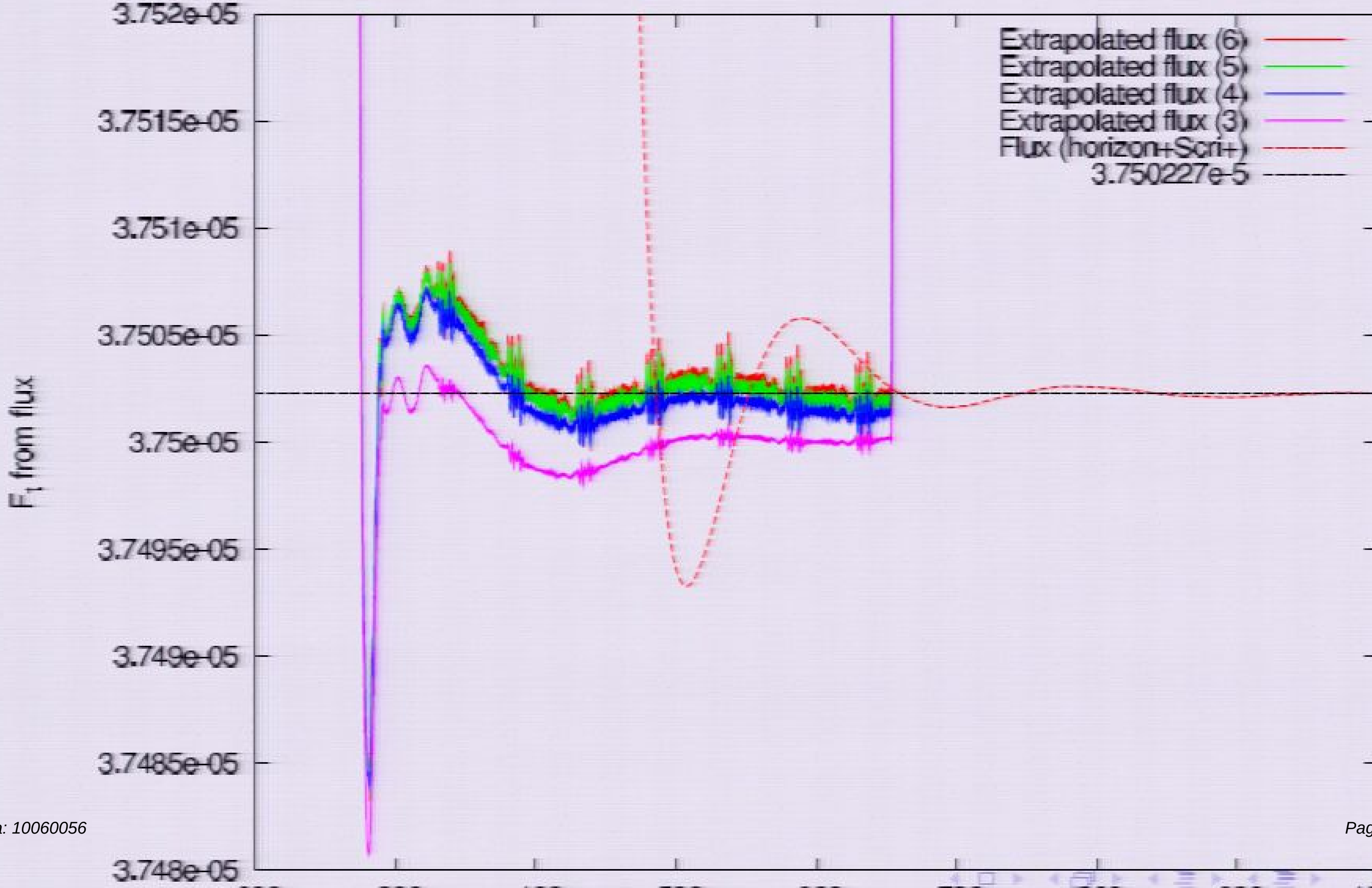

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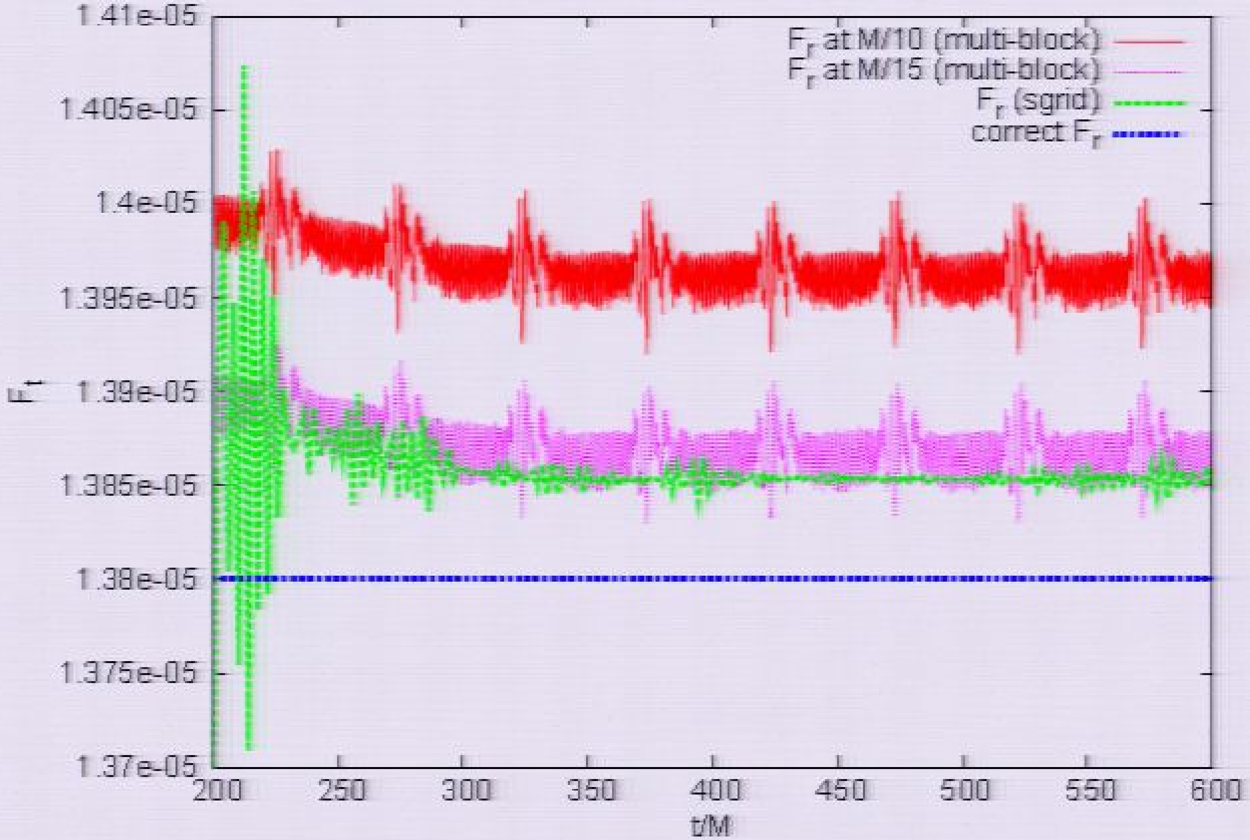

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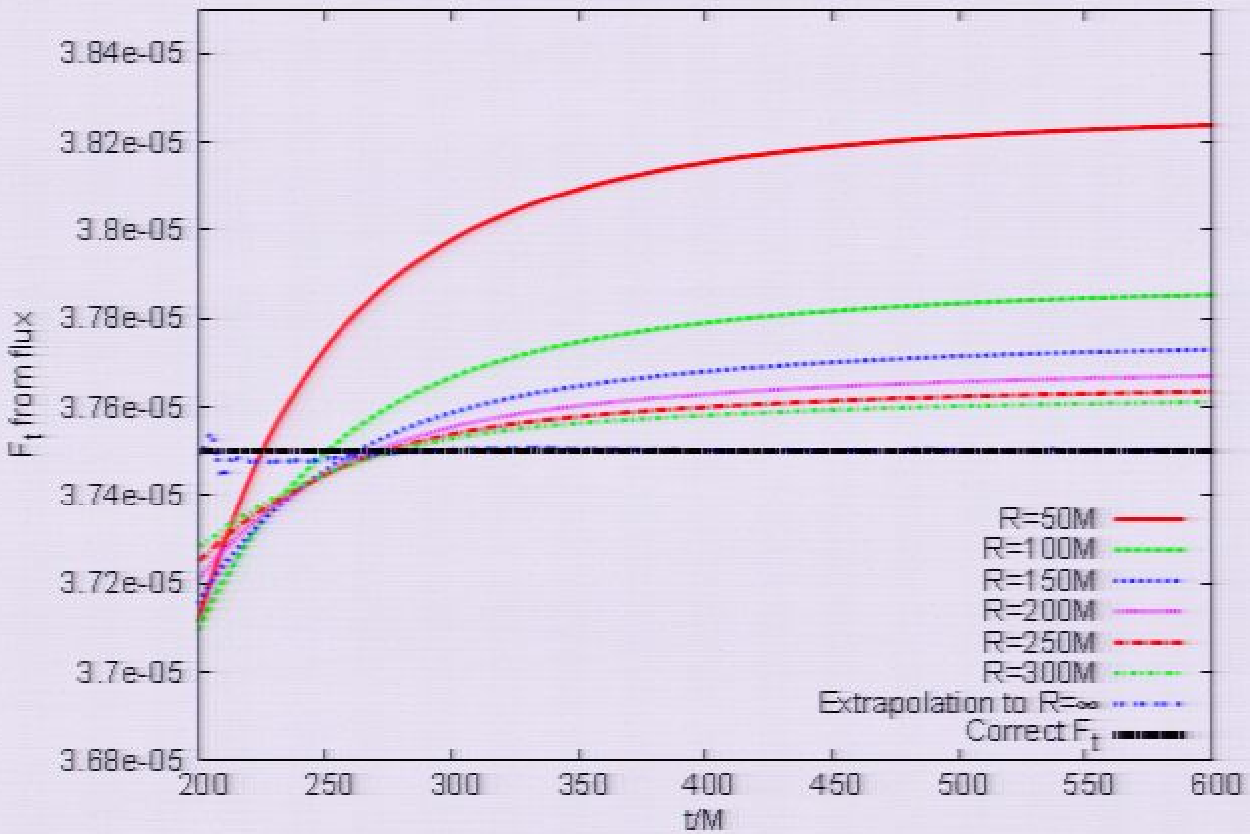

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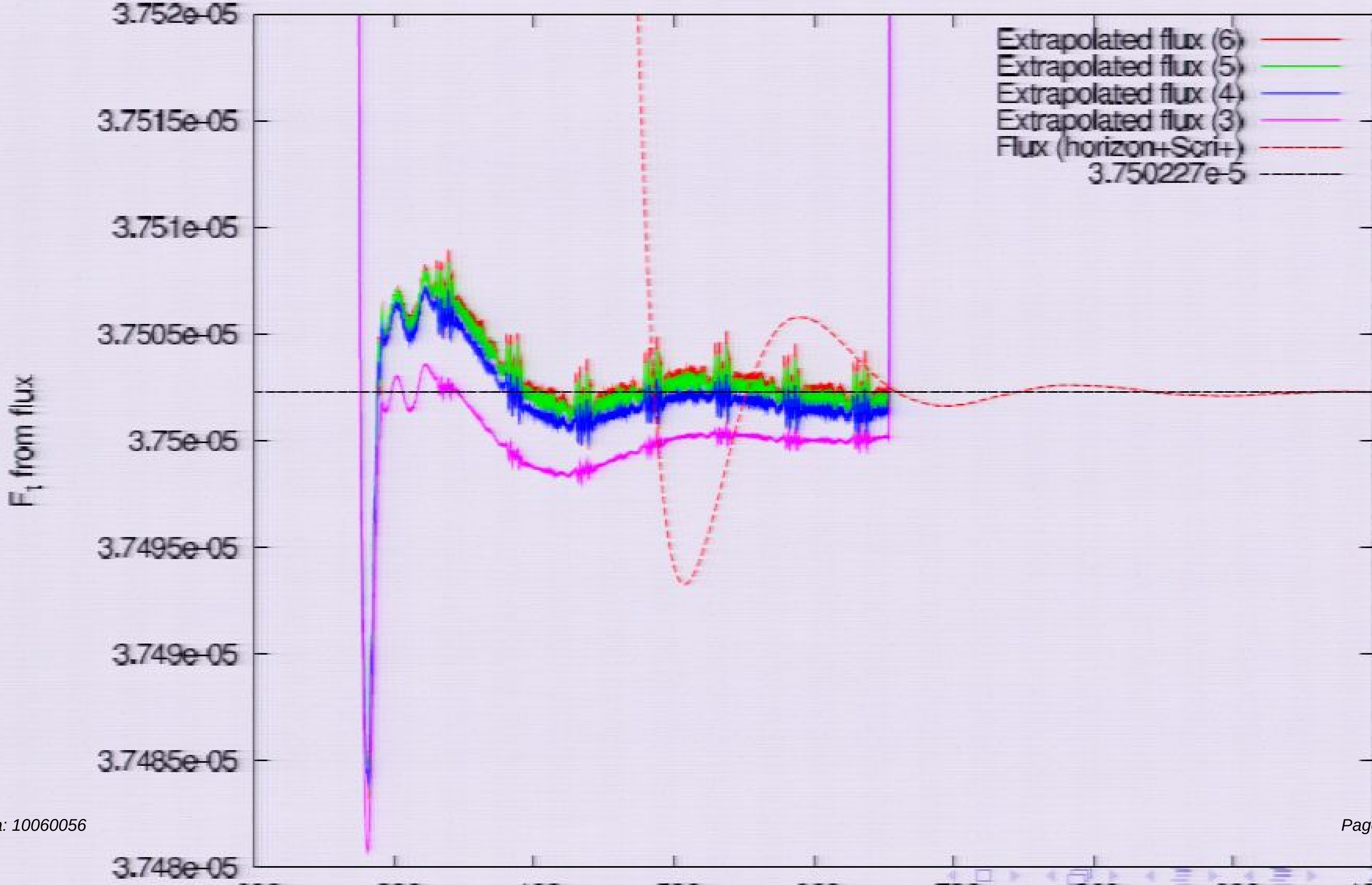

Looking ahead

More on the scalar case

- We need strategies to handle the C^0 nature of the effective source.
- We need to reduce the noise in the local self-force calculation to make self-consistent evolution for a scalar charge in a Kerr background possible.

Extension to gravity

- Effective source for a point mass should be straightforward.
- We need a (3+1) evolution code for the linearized Einstein equation in Lorenz gauge.
- We need a clear prescription for how to handle the Lorenz gauge condition.

Summary

- There exists a (3+1) approach to the self-force programme.
- Initial tests indicate that this approach is promising.
- We are now well in position to performing fully self-consistent evolution for the scalar charge.
- From a purely numerical point-of-view, the main challenge appears to be that of reducing the noise in the self-force calculation.
- To extend to gravity, we still need several things:
  - (3+1) code to solve the linearized Einstein equations (in Lorenz gauge)
  - A prescription for handling the gauge condition.