

Title: Computational aspects of the self-force

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Abstract: TBA

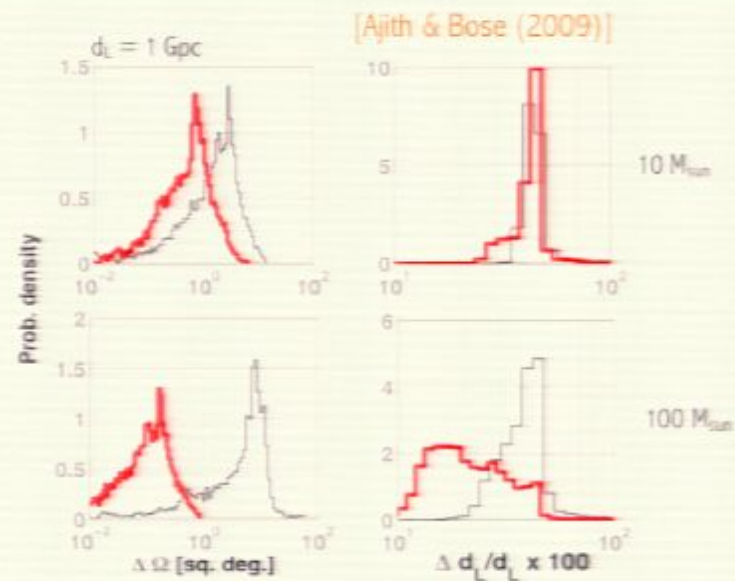
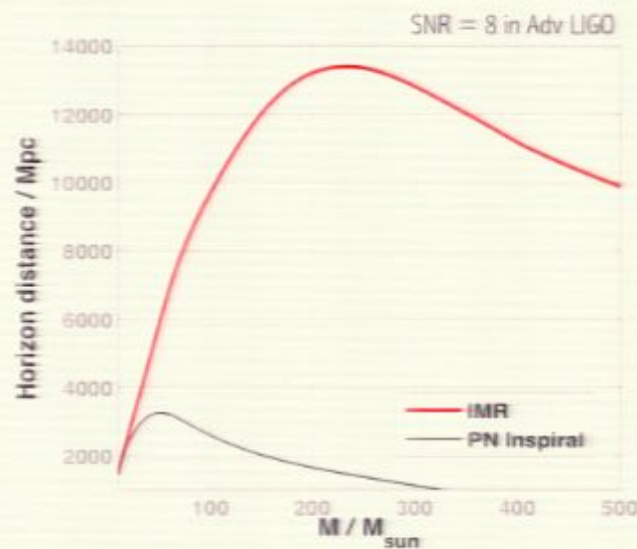


Interfacing numerical- and analytical relativity for GW astronomy: Status and prospects

P. Ajith California Institute of Technology

Theory Meets Data Analysis at Comparable and Extreme Mass Ratios **Capra + NRDA Meeting**
Perimeter Institute, Waterloo, Canada
24 June 2010

Why NR for GW astronomy?



- Significant improvements in the “distance reach” and parameter-estimation accuracies for “high-mass” binaries.
- NR of sources other than compact binaries, provide useful information for fine tuning the searches.

Constructing analytical IMR waveforms tuned to NR

- Based on the effective-one-body formalism
 - Map the two-body dynamics into one-body dynamics in the presence of an effective (“deformed”) metric.
 - Several free parameters can be introduced. Propose some ansatz for the mass-ratio (and spin) dependence of the free parameters, and tune them against NR.
 - Match the calibrated EOB inspiral-plunge waveforms with several QNM modes.

EOB-dynamics adjustable parameters	EOB-waveform adjustable parameters
$a_T(\nu)$	$t_{\text{merge}}^{(m)}(\nu)$
$v_{\text{cycle}}(\nu)$	$\Delta t_{\text{match}}^{(m)}(\nu)$
$a_{\text{RR}}^{\mathcal{F}}(\nu)$ or $a_{\text{RR}}^{\mathcal{F}_0}(\nu)$	$a_i^{h_m}(\nu); i = 1, \dots, 4$
A_{q}	

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EOB-dynamics adjustable parameters	EOB-waveform adjustable parameters
$a_1(\nu)$	$t_{\text{match}}^{\text{EOB}}(\nu)$
$v_{\text{merge}}(\nu)$	$\Delta t_{\text{match}}^{\text{EOB}}(\nu)$
$a_{\text{QNM}}^{\ell m}(\nu)$ or $a_{\text{QNM}}^{\ell o}(\nu)$	$a_i^{\ell m}(\nu); i = 1, \dots, 4$
A_2	

- Phenomenological approaches connecting PN with NR
 - Construct hybrid waveforms by matching PN and NR in region where both calculations are assumed to be valid.
 - Parametrize the hybrid waveforms using some appropriate fitting functions. Modelling or inspiral and ring down is motivated by PN and BH perturbation theory.

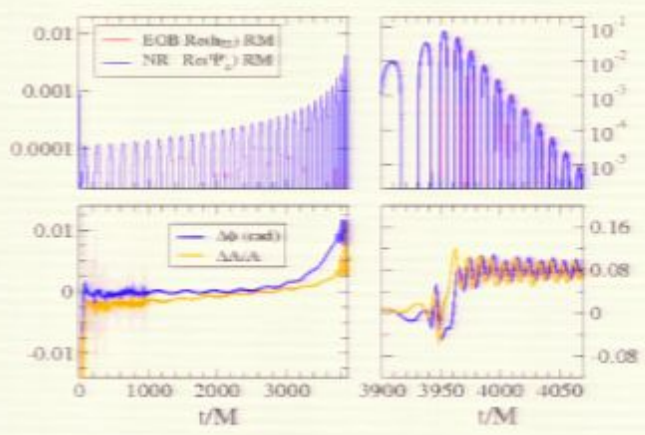
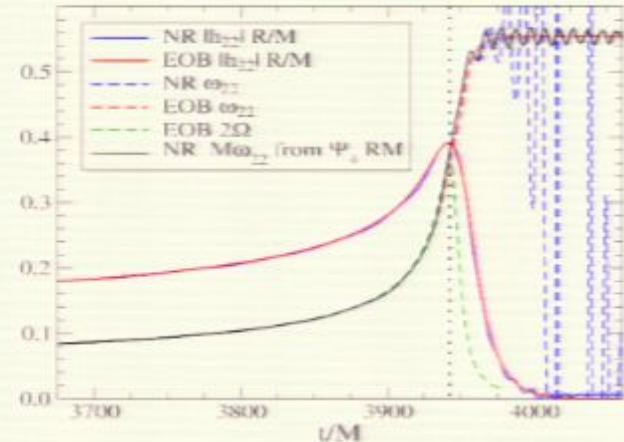
$$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128\eta v^5} \left(1 + \sum_{k=2}^7 v^k \psi_k \right)$$

$$\psi_k \text{ and } \mu_k \equiv \{f_1, f_2, \sigma, f_3\}$$

EOB calibrated to NR: nonspin equal-mass waveforms

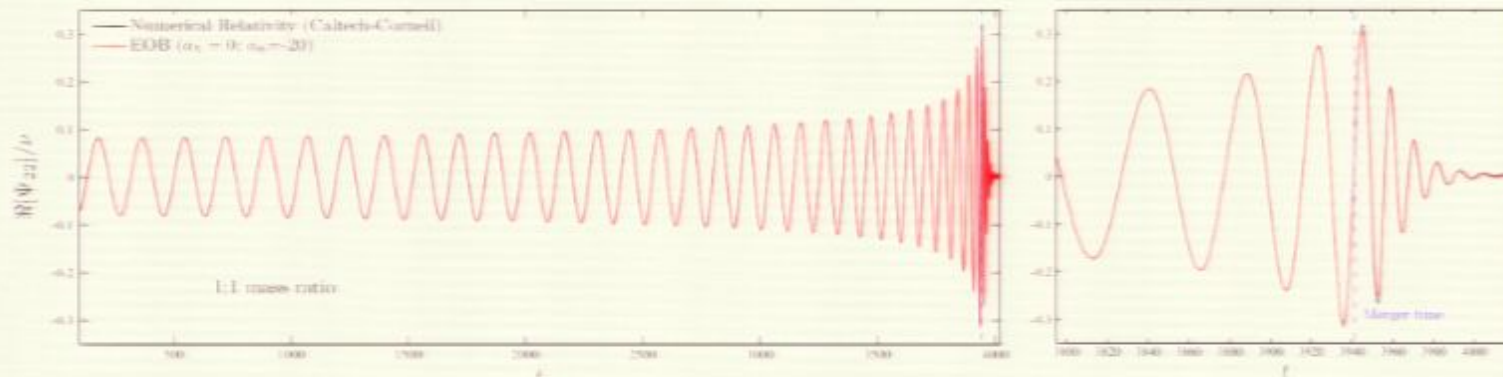
EOB-dynamics adjustable parameters	EOB-waveform adjustable parameter:
$a_2(\nu)$ $v_{\text{pole}}(\nu)$ $a_{\text{RR}}^{\text{cal}}(\nu)$ or $a_{\text{RR}}^{\text{Fid}}(\nu)$ A_0	$t_{\text{match}}^{\text{cal}}(\nu)$ $\Delta t_{\text{match}}^{\text{cal}}(\nu)$ $a_i^{\text{NR}}(\nu); i = 1, \dots, 4$

[Buonanno et al. 2009]



- Phase and amplitude difference within numerical errors. Faithfulness > 0.9999 (w.r.t. the non-spinning, equal-mass SpEC NR waveform).
- “Full” (with non-quadrupole modes) non-spin EOBNR being calibrated to NR [Talk by Y. Pan].

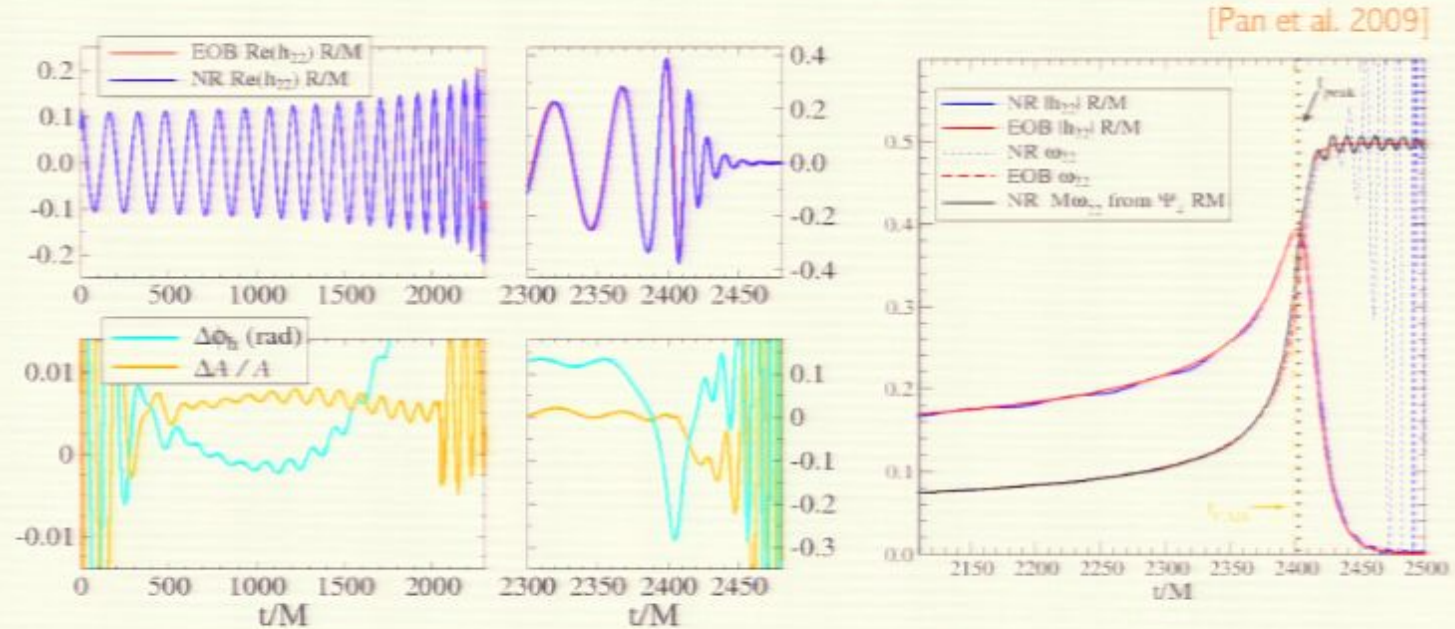
Alternative way of calibrating EOB with NR



[Damour & Nagar (2009)]

- Two free parameters in the EOB potential: $A(u; a_5, a_6, \nu) \equiv P_5^1[A^{3\text{PN}}(u) + \nu a_5 u^5 + \nu a_6 u^6]$
- “Improved resummation” of the RR force: $\mathcal{F}_\varphi = -\frac{1}{8\pi\Omega} \sum_{\ell=2}^{\ell_{\text{max}}} \sum_{m=1}^{\ell} (m\Omega)^2 |Rh_{\ell m}^{(\ell)}|^2$
- “Next to quasi-circular corrections” to the waveforms.
- NR calibration of the maximum GW amplitude.

EOB calibrated to NR: non-precessing equal-mass equal-spin waveforms



- Calibrated to SpEC simulation of an equal-mass binary with equal-amplitude anti aligned spins: $\chi_1 = \chi_2 = 0.43757$
- Improvements made in modeling spin Hamiltonian and radiation reaction [Talks by E. Barausse and Y. Pan]

A data-analysis-based approach to spinning IMR waveforms

- 8 dimensional template bank – large **false alarms**. [e.g., Van Den Broeck et al (2009)]
- Requires systematic and accurate exploration of the **large parameter space** using NR simulations – might not be possible in the near future.
- Several technical difficulties of implementing a search using generic spinning templates.
- A sub-family: **non-precessing spins**: Making use of the degeneracy of the parameter space, try to model the binaries using just **one extra parameter** in the waveform.

Leading term in the spin-orbit coupling

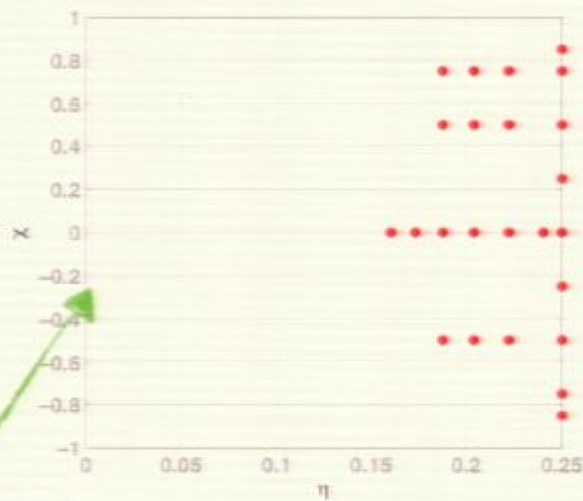
$$\chi \equiv \chi_s + \delta \chi_a$$

sum of spins mass asymmetry difference in spins

The diagram illustrates the decomposition of the spin parameter χ . It shows the equation $\chi \equiv \chi_s + \delta \chi_a$. An orange arrow points from the text "Leading term in the spin-orbit coupling" to the χ_s term. Three green arrows point from the labels "sum of spins", "mass asymmetry", and "difference in spins" to the χ_s , δ , and χ_a terms respectively.

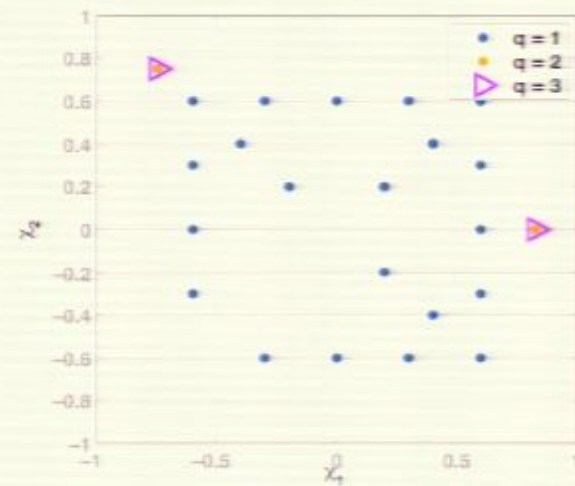
NR Simulations

Used 24 BAM equal-spin simulations ($|\chi| \leq 0.85$, $1 \leq q \leq 4$) to construct the template family.



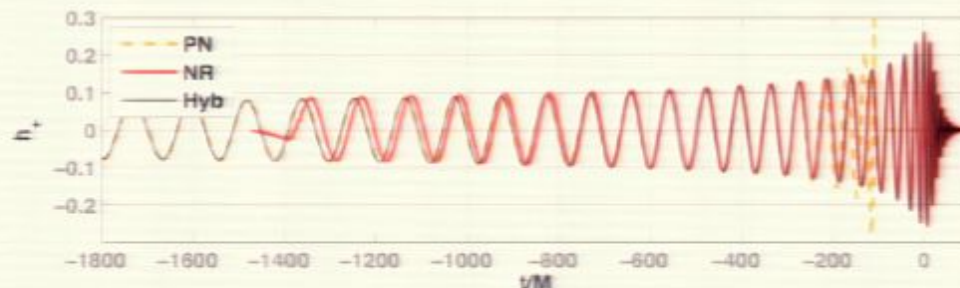
Inputs from BH perturbation

Used more than 30 (BAM, CCATIE, LLAMA, SpeC) different simulations (including unequal spins and precession) to test the template family.



Parametrizing the hybrid waveforms

- Construct NR-PN hybrids:



- Parametrisation:

$$A(f) \equiv C f_1^{-7/6} \begin{cases} f^{t-7/6} (1 + \sum_{i=2}^3 \alpha_i v^i) & \text{if } f < f_1 \\ w_m f^{t-2/3} (1 + \sum_{i=1}^2 \varepsilon_i v^i) & \text{if } f_1 \leq f < f_2 \\ w_r \mathcal{L}(f, f_2, \sigma) & \text{if } f_2 \leq f < f_3, \end{cases}$$

$$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128\eta v^5} \left(1 + \sum_{k=2}^7 v^k \psi_k \right). \quad (1)$$

Parametrizing the hybrid waveforms

- The phenomenological parameters are written in terms of the physical parameters as:

$$\psi_k = \sum_{i=1}^3 \sum_{j=0}^N x_k^{(ij)} \eta^i \chi^j + \psi_k^0, \quad \mu_k = \sum_{i=1}^3 \sum_{j=0}^N \frac{y_k^{(ij)} \eta^i \chi^j}{\pi M} + \mu_k^0,$$

↑ estimated from the hybrid waveforms

- such that, in the test-mass ($\eta \rightarrow 0$) limit:

LSO frequency
of a Kerr BH with
apprpr. mass &
spin

Freq & quality
factor of the
dominant ring down
mode

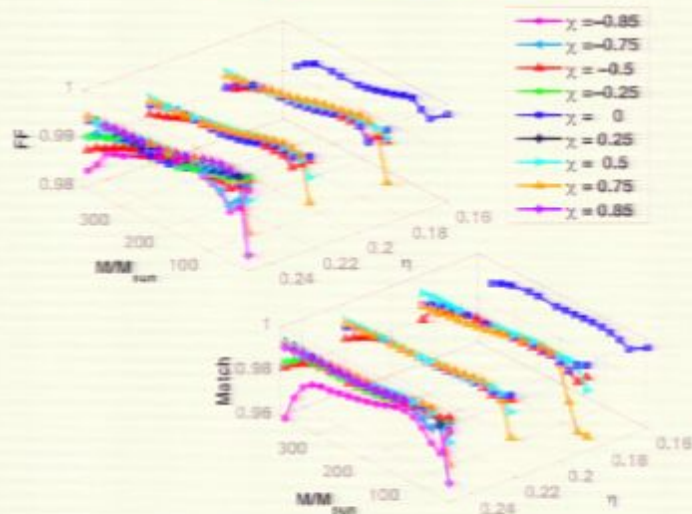
2PN phasing
coefficients in the
test-mass limit

$$f_1 \rightarrow f_{\text{LSO}}^0, \quad f_2 \rightarrow f_{\text{QNM}}^0, \quad \sigma \rightarrow f_{\text{QNM}}^0 / Q^0, \quad \psi_k \rightarrow \psi_k^0,$$

Testing the analytical waveforms

[P. Ajith et al (2009)]

Using BAM Simulations

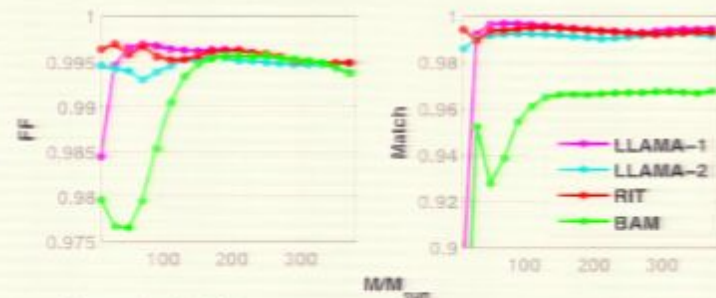
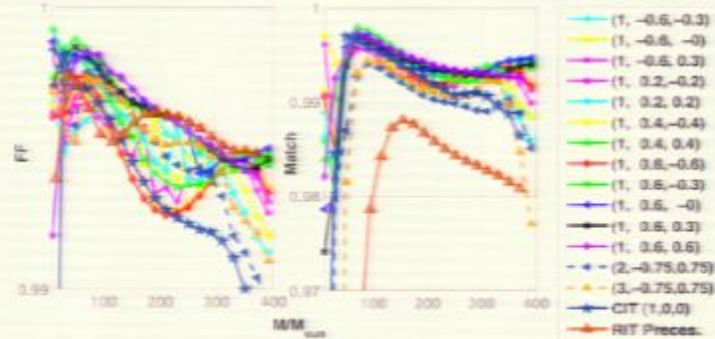


Equal-spin hybrids (used to construct the template family)

$FF > 0.98$ for $q \leq 4$, $|\chi| \leq 0.85$ in Initial/Enhanced LIGO

Testing the analytical waveforms

Unequal-spin hybrids Using CCATIE, BAM, SpeC Simulations



Precessing hybrids

- **LLAMA-1** $q = 1$, $\chi = [0.42, 0, 0.42], [0, 0, 0]$
- **LLAMA-2** $q = 1$, $\chi = [0.15, 0, 0], [0, 0, 0]$
- **RIT** $q = 1.25$, $\chi = [-0.2, -0.14, 0.32], [-0.09, 0.48, 0.35]$
- **BAM** $q = 3$, $\chi = [0, 0, 0], [0.75, 0, 0]$

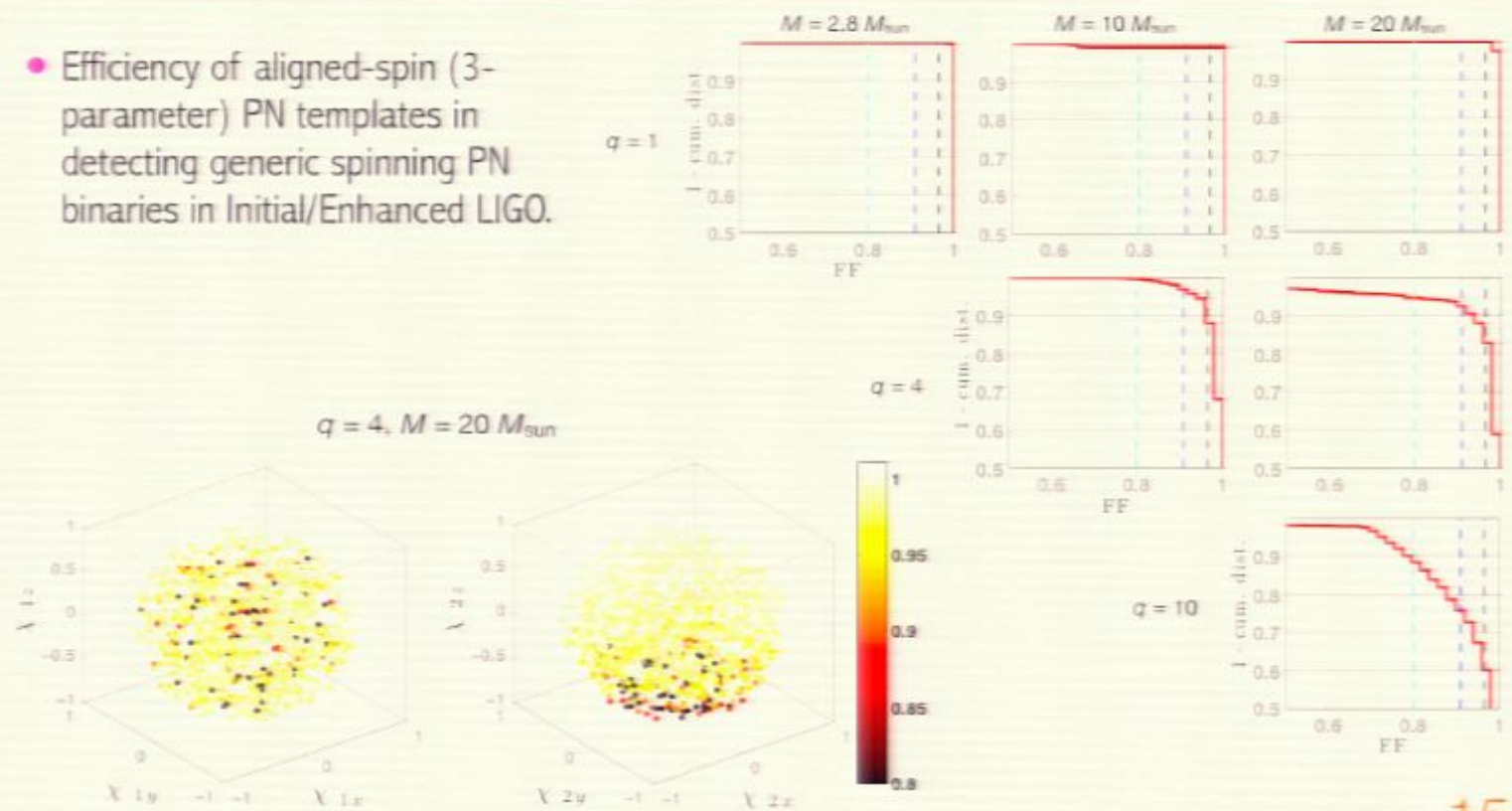
- The template family is able capture the spin-orbit coupling.
- Not enough NR waveforms to make general statements about the efficiency of the template family in detecting generic spinning binaries.
- But, we might be able to get some useful insights from PN.

Monte-Carlo simulation of generic spin PN binaries

- Generate generic spinning PN binaries with arbitrary spins and test the efficiency of “aligned-spin” PN templates (where the spins are represented by a single spin parameter) in detecting them.

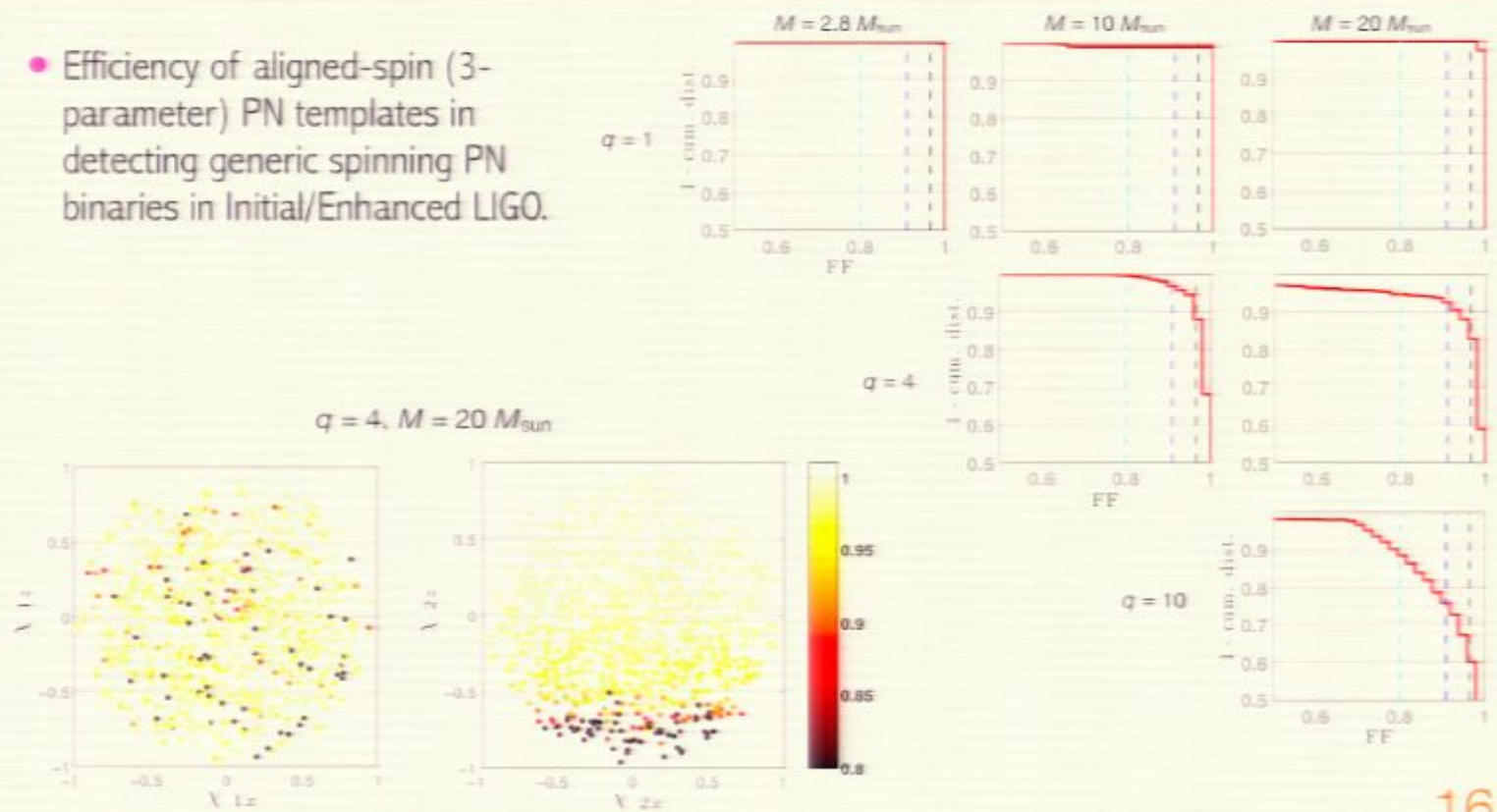
Monte-Carlo simulation of generic spin PN binaries

- Efficiency of aligned-spin (3-parameter) PN templates in detecting generic spinning PN binaries in Initial/Enhanced LIGO.



Monte-Carlo simulation of generic spin PN binaries

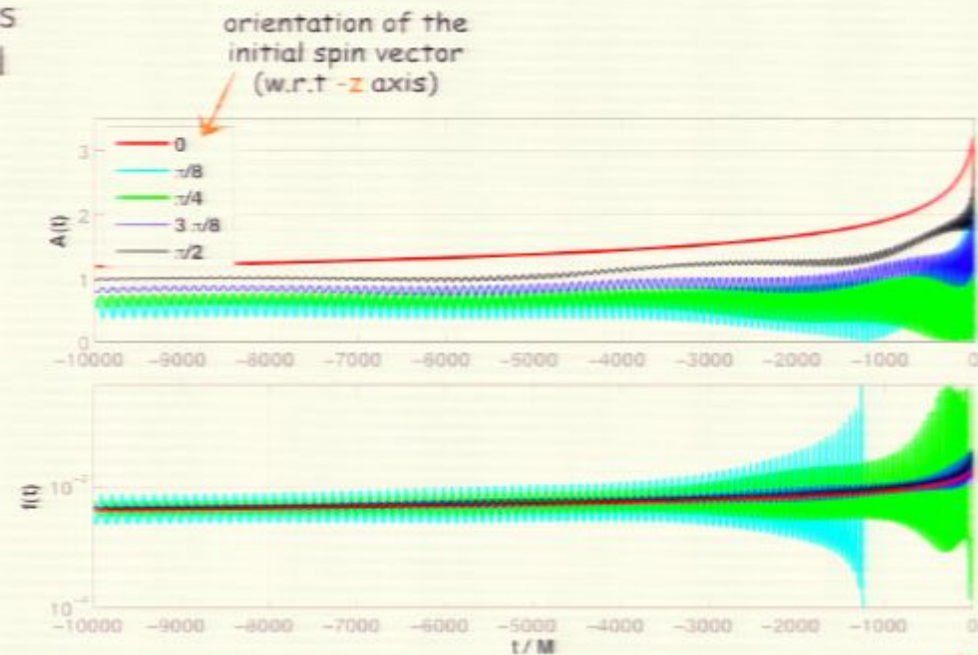
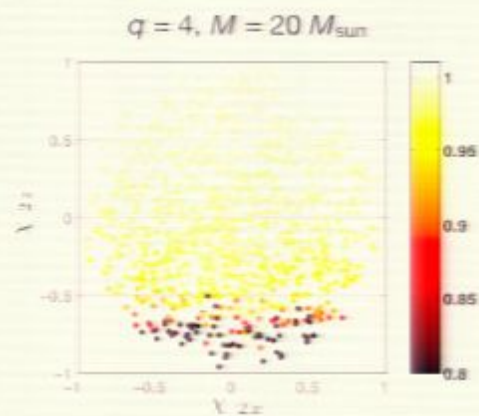
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Monte-Carlo simulation of generic spin PN binaries

[P. Ajith et al, in Prep (2010)]

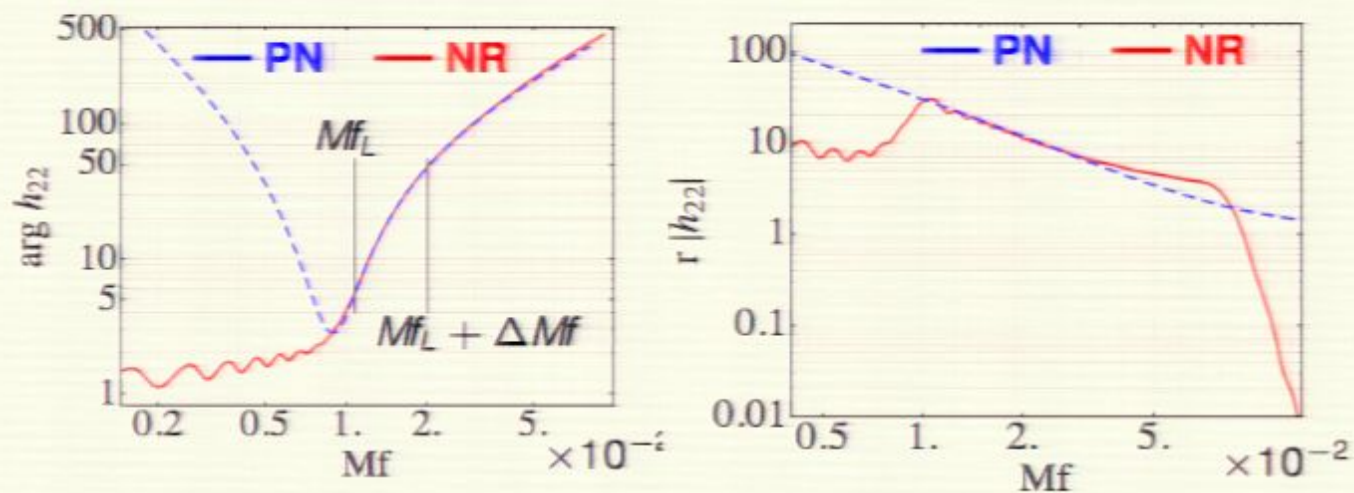
- Aligned-spin templates are less efficient in detecting certain high-precession configurations (involving “flips” of the orbital plane).



Systematics in the construction of hybrid waveforms

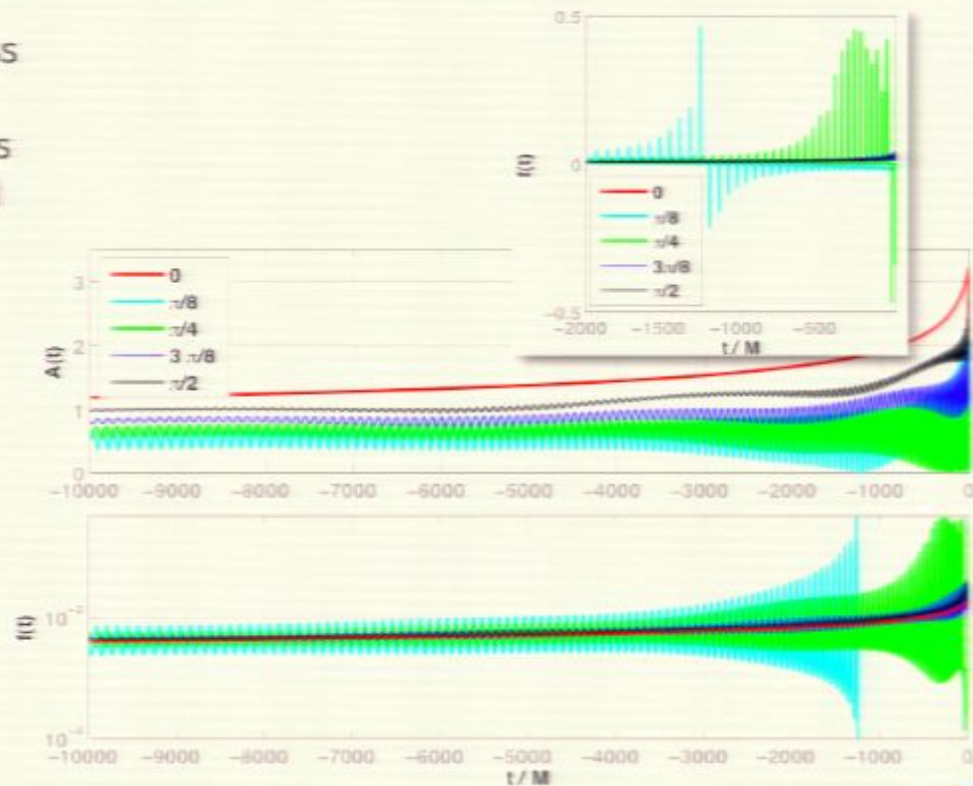
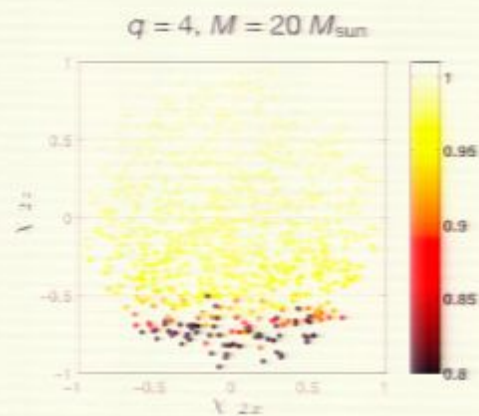
[Santamaria, Ohme et al 2010]

- Alternative ways of constructing hybrid waveforms (e.g., in frequency domain)
 - **PN** 3.5PN TaylorF2 phase, 3PN amplitude
 - **NR** BAM, LLAMA, CCATIE, SpEC waveforms, $q \leq 4$, non-precessing spins



Monte-Carlo simulation of generic spin PN binaries

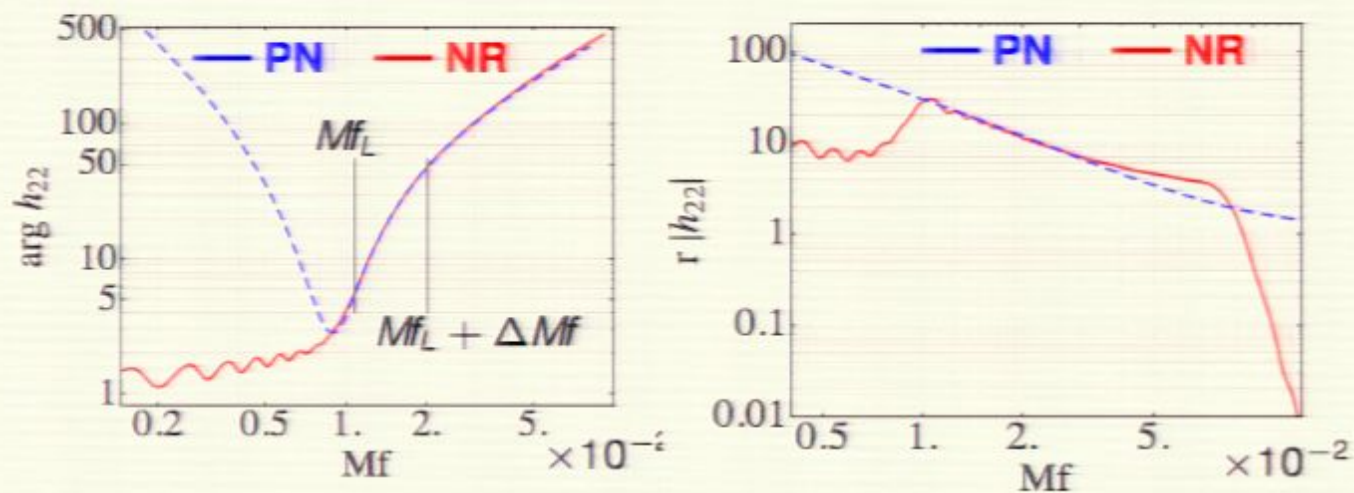
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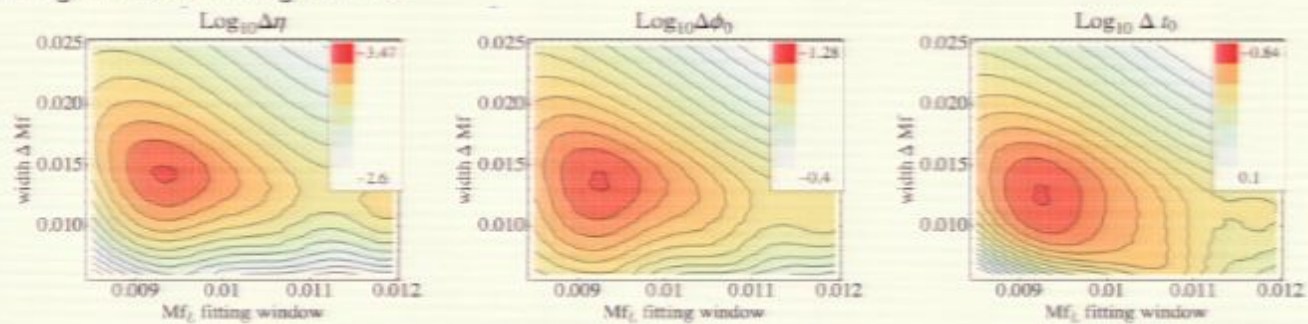
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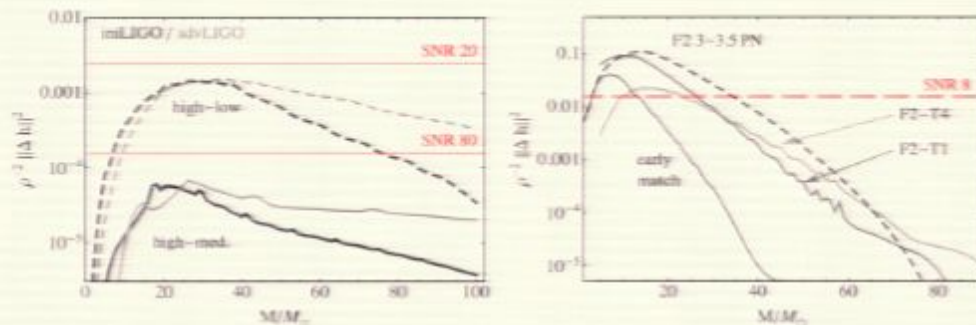


Systematics in the construction of hybrid waveforms

- Matching window, fitting errors



- PN/NR uncertainties



Difference between high and med. resolution indistinguishable for SNR 80

Difference between hybrids constructed using different PN approximants

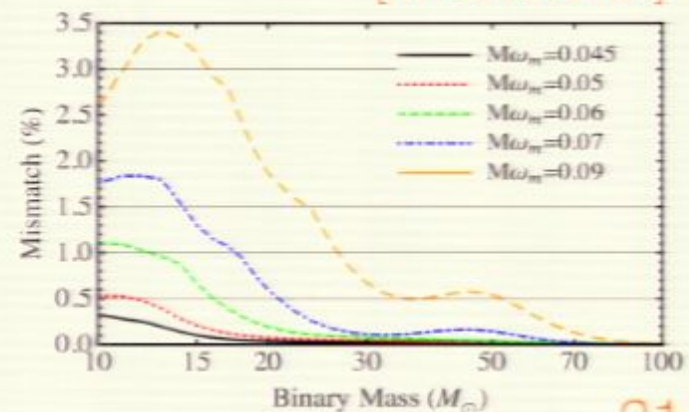
[F. Ohme's Talk]

PN/NR comparison, minimum length of NR waveforms ...

- PN-NR Comparison
 - Equal-mass, non-spinning & non-precessing spins.
 - TaylorT4 extremely accurate in the non-spinning case.
 - TaylorT1 is the most robust in the spin case.
 - PN amplitude accuracy $< 4\%$.
- Minimum length for NR waveforms for detection.
 - Compare NR+PN hybrids by varying the matching frequency ω_m .
 - In the equal-mass, non-spinning case, 4 orbits are sufficient to produce $FF > 0.98$ between different hybrids.

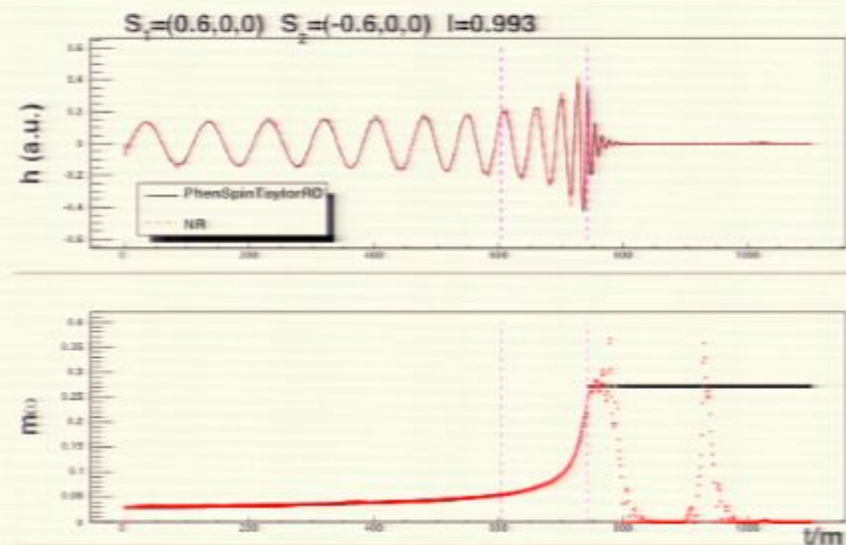
[H. Pfeiffer's Poster]

[M. Hannam's Talk]



Phenomenological IMR waveforms for generic spinning BBHs

- **Inspiral** “Restricted” PN waveforms computed in the TaylorT4 approximation, up to a phenomenological “merger” frequency f_m , which is determined empirically.
- **Merger** Phenomenological model. Free parameters are constrained by imposing continuity on the phase and its derivatives & comparing with NR.
- **Ring down** Attach ring down (with overtones) at f_{RD} . Motivated by the EOBNR approach.
- Phenomenological parameters are currently estimated from equal-mass simulations by GeorgiaTech.
- Overlaps of 0.95-0.99 with NR waveforms.



[R. Sturani's Talk]

NR-AR Collaboration

- **Goal**

- Produce accurate NR simulations spanning a large region of the binary parameter space.
- Develop in time for the searches of GWs the best-calibrated template families covering a large parameter space.

- **Resources**

- NSF has made available 11M CPU-hrs on the Teragrid machine Kraken (April - Summer 2010). NR groups were also encouraged to use their own resources.

- **Current status**

- 14 NR groups have joined the NR-AR Collaboration.
- First simulations started at the beginning of April. Several runs are in progress.

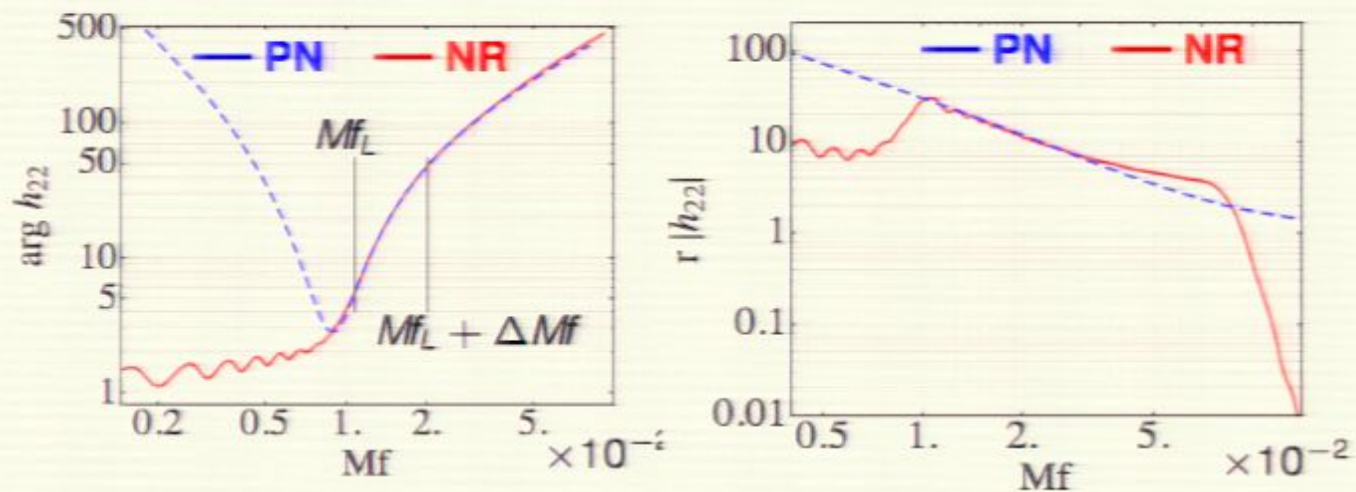
Summary

- Steady progress in the construction of analytical IMR waveforms inspired by NR.
- More NR simulations, calculation of higher order PN terms (especially involving spins) are in order.
- The NR-AR effort will provide a “zoo” of NR waveforms. Useful for the construction/calibration of analytical templates as well as for NINJA.
- Lot of efforts to quantify the systematics in the PN-NR matching, hybrid construction, minimum length of NR simulations etc.

Systematics in the construction of hybrid waveforms

[Santamaria, Ohme et al 2010]

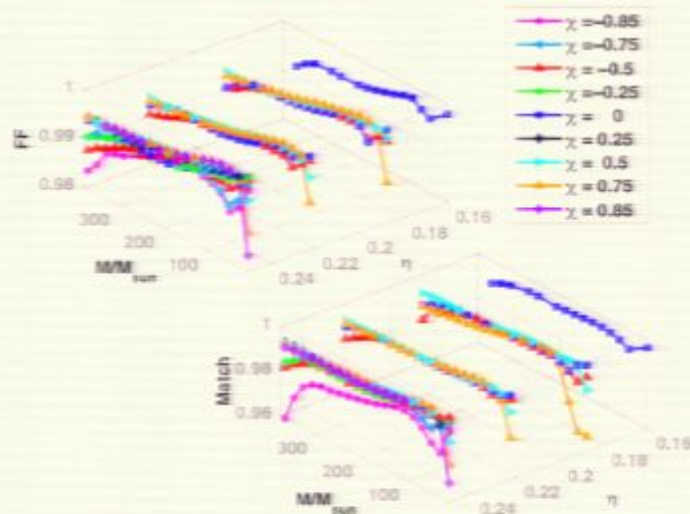
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Testing the analytical waveforms

[P. Ajith et al (2009)]

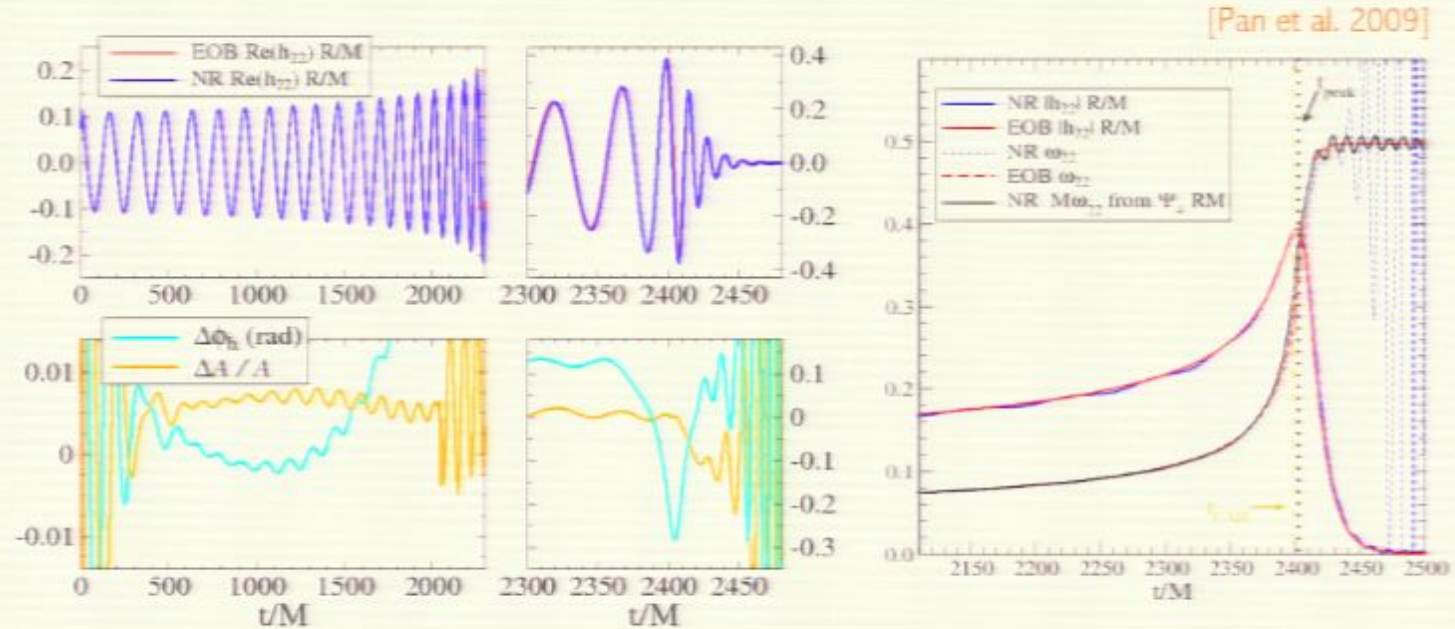
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EOB calibrated to NR: non-precessing equal-mass equal-spin waveforms

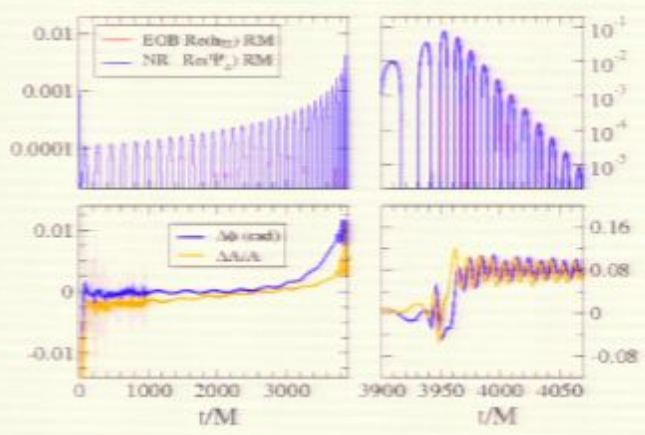
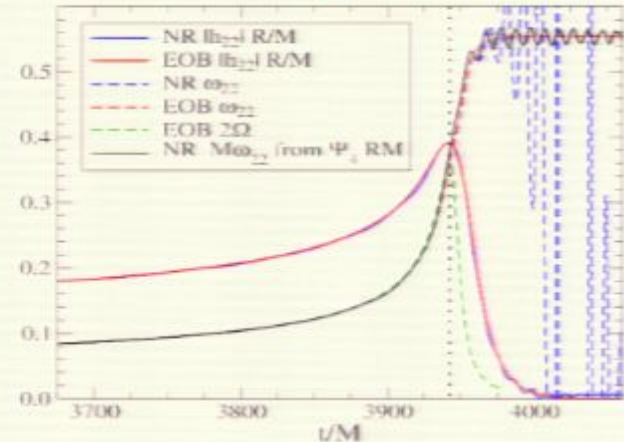


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