Title: Foundational aspects of the self-force

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Abstract: TBA

Foundations of the self-force problem

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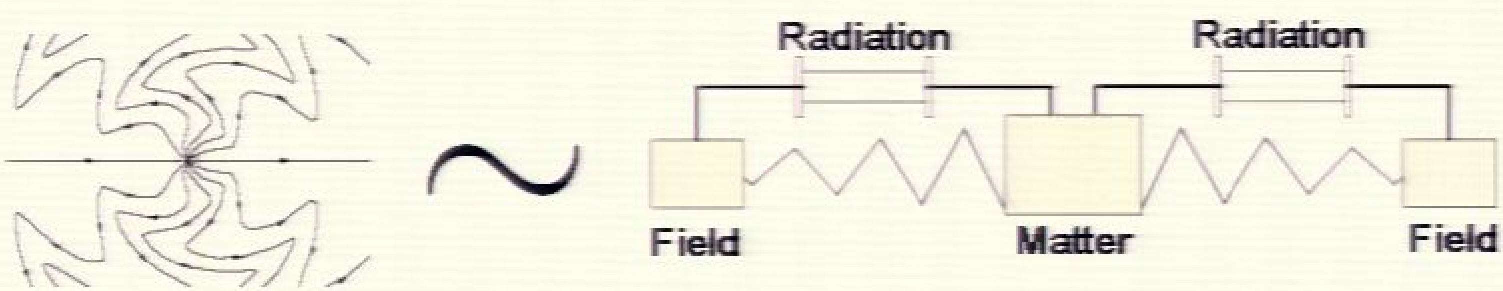
Outline

- What is the self-force?
  - What does it do?
  - What is a worldline?
- What is known and why?
  - "Singular" self-field and its effects
  - MiSaTaQuWa etc.
- Comments and open problems

The self-force problem

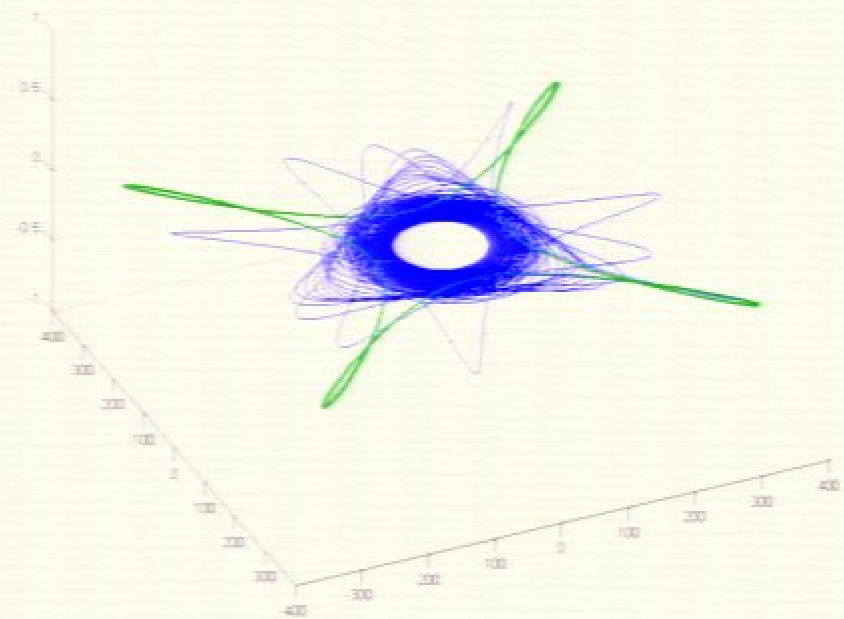
- Compact objects coupled to long-range fields "carry" some of these fields with them as they move. How does this affect their motion?
- The fields radiate energy and momentum away from the system. They also store energy and momentum, and therefore have inertia. Self-force is not just radiation reaction.
- Mass and spring analogy (Miller and Robiscoe [1995]):



Typical consequences

For a sufficiently small object nearly in internal equilibrium:

- Trajectories usually circularize and decay.
- Orbital frequencies can shift due to conservative effects.
- Linear and angular momentum are shifted (along with higher multipole moments of the stress-energy tensor).



Small charged particle orbiting a large spinning charge in flat spacetime with (blue) and without (green) self-force corrections

Why?

Various motivations over the last 100+ years:

- ~~Models for elementary particles~~
- ~~Guidance to fix infinities in QFT~~
- ~~Origin of inertia~~
- Structure and limits of classical field theories and mechanics
- Just one more step towards understanding motion in certain systems (e.g. EMRIs for gravitational wave astronomy)

etc.

What's so hard?

- Multiple length scales
- Point particles (as typically defined) don't work
- Explicit calculations often not useful for demonstrating universal behavior
- What is a self-field?
- What should be computed anyway?

What is motion?

- The full theory uses PDEs to evolve data off of an initial hypersurface (and imposes constraints on that data).
- This detail is often unnecessary and obscures general trends.
- Point particles are incompatible with GR (Geroch and Traschen [1987]) or EM, yet people are still interested in worldlines and “point-like” limits.
- Axiomatize point particles or try to abstract real objects (star, black hole, etc.) to worldline + multipole moments.

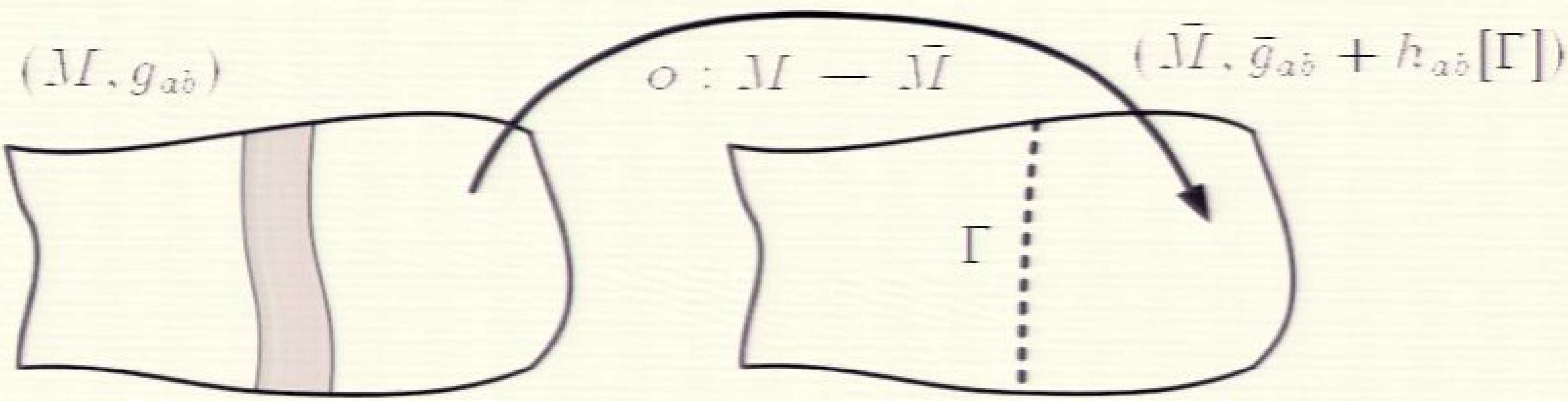
Motion II

- Which worldline?
- All definitions are nonlocal and coupled to fields with infinite degrees of freedom.
- Their exact causality properties are very unlikely to be similar to those found in Newtonian particle mechanics.
- Its dynamical equations form an effective theory. Don't take it too seriously.

Perturbative worldlines

Intrinsically perturbative notions of worldline are popular in gravitational problems:

Where would the far-field metric perturbations "come from" if they were produced by, e.g., a point source coupled to the linearized (Lorenz-reduced) Einstein equation?



Choose Γ such that $g_{ab} \approx \phi_*(\bar{g} + h[\Gamma])_{ab}$ in some "buffer region" $N \subset M$.

Also limiting worldtubes, etc. See Pound [2010].

“Physical” worldlines

Non-perturbatively choose a collection of points inside a matter distribution.

- Look for something with nice properties and argue that it’s “representative.”
- Usually consider points satisfying $p^a(z,\Sigma)S_{ab}(z,\Sigma) = 0.$
- This requires defining linear and angular momentum.
- Relatively simple to interpret, but not useful for black holes.
- Used in scalar and EM SF problems. Partially implemented in the gravitational case .

Ignorable self-fields

Most of the field near an object is large and highly non-uniform, but does not contribute any net force (other than stress-energy renormalization). Newtonian self-force:

$$\vec{F}_{\rm self} \equiv -\int \rho \vec{\nabla} \phi_{\rm S} \mathrm{d}V = 0$$

$$\phi_{\rm S}(x) \equiv -\int \frac{\rho(x')}{|x - x'|} \mathrm{d}V'$$

Useful to identify this field at the outset and subtract it out:

$$\vec{F} = -\int \rho \vec{\nabla} \phi_{\rm H} \mathrm{d}V$$

$$\phi_{\rm H} \equiv \phi - \phi_{\rm S} \quad \Rightarrow \quad (\nabla^2 \phi_{\rm H})|_{B} = 0$$

Ignorable self-fields II

ϕ_{H} is…

- homogeneous near the body: $(\nabla^2 \phi_{\mathrm{H}})|_B = 0$.

- relatively simple to compute: basically a type of angle-averaged field (roughly analogous to what appears in the Quinn-Wald axioms, etc.):

$$\phi_{\mathrm{H}}(x) = \frac{1}{4\pi} \oint [\vec{\nabla}' \phi(x') G_{\mathrm{S}}(x, x') - \phi(x') \vec{\nabla}' G_{\mathrm{S}}(x, x')] \cdot \mathrm{d}\vec{S}'$$

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$$G_{\mathrm{S}}(x, x') = \frac{1}{|x - x'|}$$

- slowly varying and well-behaved even for a point particle: $\vec{F} \approx -m \vec{\nabla} \phi_{\mathrm{H}}$.

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Why does this work? Newton's 3rd, background symmetries, ...

$$\begin{aligned}\vec{F}_{\rm self} &= -\int dV \int dV' \rho(x)\rho(x')\vec{\nabla} G_S(x,x') \\ &= -\frac{1}{2}\int dV \int dV' \rho(x)\rho(x')(\vec{\nabla}+\vec{\nabla}')G_S(x,x')\end{aligned}$$

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More interesting S-fields

- Yes, there are generalizations (Detweiler and Whiting [2003], Poisson [2004], ...). Define a special Green function G_S.

- Two new effects for relativistic fields:
  - ϕ_S now contributes an effective momentum.
  - ϕ_S is no longer the full self-field. There's also, e.g., radiation.

- This was originally an axiomatic approach developed to get reasonable answers using point particle methods (see also Dirac [1938], Quinn and Wald [1997], ...).

- Non-perturbative versions now stated and proven for arbitrary
  - extended objects coupled to scalar or electromagnetic fields on fixed backgrounds: AIH [2008, 2009].
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Electromagnetism in a curved background (first in DeWitt and Brehme [1960] and Hobbs [1968]):

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Essentially a geodesic in a “smoothed” metric:

$$\frac{\mathrm{D_H}}{\mathrm{d}s} u^a \approx \frac{\bar{\mathrm{D}}}{\mathrm{d}s} u^a + \frac{1}{2} p^{ab} (2\bar{\nabla}_d h^{\mathrm{H}}_{bc} - \bar{\nabla}_b h^{\mathrm{H}}_{cd}) u^c u^d \approx (\mathrm{spin}) + O(h^2, \dots)$$

Find h^{H}_{ab} by subtracting h^{S}_{ab} from h_{ab} or use a surface integral “average:”

$$\begin{aligned} h^{\mathrm{H}}_{ab} &\equiv h_{ab} - h^{\mathrm{S}}_{ab} \\ &= \frac{1}{4\pi} (\delta^c_a \delta^d_b - \frac{1}{2} \bar{g}_{ab} \bar{g}^{cd}) \oint_{\partial E} (G^{\mathrm{S}}_{cda'b'} \bar{\nabla}^{c'} \gamma^{a'b'} - \bar{\nabla}^{c'} G^{\mathrm{S}}_{cda'b'} \gamma^{a'b'}) \mathrm{d}\bar{S}_{c'} + O(h^2) \end{aligned}$$

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MiSaTaQuWa equation (GR)

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Some comments

- MiSaTaQuWa has now been rigorously derived in various ways:
  - Variants of matched asymptotic expansions: Gralla and Wald [2008], Pound [2010]
  - An extended-body approach using (mostly) physical worldlines: AIH [in preparation]
- Be careful about definitions and approximations.
- Dipole and spin effects can be comparable to the self-force.
- MiSaTaQuWa is a general equation. Better methods may exist for special applications (symmetric backgrounds, quasicircular orbits in specific spacetimes...). See, e.g., Hinderer and Flanagan [2008].

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Ignorable self-fields III

Why does this work? Newton's 3rd, background symmetries, ...

$$\begin{aligned}\vec{F}_{\rm self} &= -\int \mathrm{d}V \int \mathrm{d}V' \rho(x)\rho(x')\vec{\nabla} G_S(x,x') \\ &= -\frac{1}{2}\int \mathrm{d}V \int \mathrm{d}V' \rho(x)\rho(x')\overbrace{(\vec{\nabla}+\vec{\nabla}')G_S(x,x')}\end{aligned}$$

$$\mathcal{L}_\xi G_S(x,x') = (\xi^a \nabla_a + \xi^{a'} \nabla_{a'}) G_S = 0$$

Translation invariance: $\vec{F}_{\rm self} = 0$

Rotational invariance: $\vec{N}_{\rm self} = 0$

Can this be generalized to relativistic systems (and generic spacetimes)?

"Physical" worldlines

Non-perturbatively choose a collection of points inside a matter distribution.

- Look for something with nice properties and argue that it's "representative."
- Usually consider points satisfying $p^a(z,\Sigma)S_{ab}(z,\Sigma) = 0$.
- This requires defining linear and angular momentum.
- Relatively simple to interpret, but not useful for black holes.
- Used in scalar and EM SF problems. Partially implemented in the gravitational case .

Self-force in general

Dixon's [1974] linear and angular momenta can be shown to satisfy, e.g.,

$$\frac{\mathrm{d}}{\mathrm{d}s}\tilde{p}^a = \mathcal{F}^a_{\mathrm{test}}[\tilde{T}, \rho, \ldots ; \bar{g}, \ldots, o_{\mathrm{H}}, \ldots]$$

$$o_{\mathrm{S}} \equiv \int \rho(x') G_{\mathrm{S}}(x, x') \mathrm{d}V'$$

$$o_{\mathrm{H}} \equiv o - o_{\mathrm{S}} \quad \Rightarrow \quad (\Box o_{\mathrm{H}})|_B = 0$$

The *only* effect of o_{S} is to (finitely) renormalize all multipole moments of T^{ab}: AIH [2010]. Otherwise only test body-type effects due to o_{H} :

$$\tilde{p}^a \equiv p^a + \delta p^a_{\mathrm{S}}, \qquad \tilde{S}^{ab} \equiv S^{ab} + \delta S^{ab}_{\mathrm{S}}, \qquad \tilde{J}^{abcd} \equiv J^{abcd} + \delta J^{abcd}_{\mathrm{S}}, \ldots$$

End of slide show, click to exit.

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Open problems

- Gravitational self-force in the presence of strong non-gravitational forces
  - Comparable EM and gravitational SF?
- Higher order gravitational self-force is not understood.
  - Notions of "self-field" become much less clear (and may not be useful).
  - Universality isn't known (black holes vs. neutron star?).
  - Is it still possible to introduce fictitious worldlines? Is it useful?
- Sharp long-term error estimates
- Practical methods

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