Title: Self Force Differences

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Abstract: TBA

Motivation for Self-force Differences

Self-force difference

-Consider a particle in two different spacetimes which differ only far away from the particle's position and compute the difference in the self-force.

$$f_{\text{diff}} = f_{(\mathcal{M}_1, g_1)} - f_{(\mathcal{M}_2, g_2)}$$

- We do calculations using the Schwarzschild exterior with different interiors.

Advantages

- Easy to calculate, no need for regularization.

-Once the self-force for Schwarzschild is known, it is very simple to get the force in spacetimes with different interiors.

-This difference can be used to probe the locality of the force.

Locality

- Self-force is an integral over the entire infinite past of the particle.

$$f^{\mu} = q^2(\delta_{\nu}{}^{\mu} + u^{\mu}u_{\nu}) \int_{-\infty}^{\tau^-} \nabla^{\nu} G_+(z(\tau), z'(\tau)) d\tau'$$

- How far back in the past history of the particle do we really need to go in order to evaluate the self-force?

- Or more broadly, in what situations can the self-force be written as a local expression?

Practical use for Locality?

- If the tail is moderately local, one could replace the particle's worldline with anything we want in the region before the tail starts to matter, making calculation far easier.

-For example a zoom-whirl orbit could be replaced by a circular orbit.

-This type of trick could make frequency-domain techniques viable for even very complicated orbits.

Schwarzschild Scalar Wave Equation

The scalar field equation for a non-minimally coupled scalar field is

$$[\Box - \xi R]\,\Phi = -4\pi\rho(x^\mu)$$

Solving the field equation is done using Green's functions. It satisfies:

$$(\Box - \xi R)\,G(x,x') = \frac{-4\pi}{\sqrt{-g}}\delta^4(x-x') \quad .$$

The Green's function is decomposed into a mode sum,

$$G(x,x') = \int d\omega \sum_{l=0}^{\infty}\sum_{m=-l}^{l} Y^{lm}(\theta,\phi)Y^{lm*}(\theta',\phi')g_{l\omega}(r,r')e^{-i\omega t}e^{i\omega t'}$$

After decomposing we get:

$$g''_{l\omega}(r) + \frac{2(r-M)}{r(r-2M)}g'_{l\omega}(r) - \left(\frac{l(l+1)}{r(r-2M)} - \frac{\omega^2}{(1-\frac{2M}{r})^2}\right)g_{l\omega}(r) = -\frac{4\pi}{r(r-2M)}\delta(r-r') \quad .$$

Choose the solutions to correspond to the retarded Green's function.
R_{lw}^{H}, purely ingoing radiation at horizon, and R_{lw}^{∞}, purely outgoing radiation at infinity.

$$g_{l\omega} = \frac{4\pi}{M}\frac{1}{\Delta W(R^H_{l\omega}(r'), R^\infty_{l\omega}(r'))}R^H_{l\omega}(r_<)R^\infty_{l\omega}(r_>)$$

$(r_</r_>$ for $r, r')$

$W(R_{lw}^{H}, R_{lw}^{\infty})$ is the Wronskian
Delta is r(r-2M)

General Interior Spacetime

Arbitrary static spherically symmetric interior metric with a matter distribution confined $r<r_0$

$$ds^2 = \begin{cases} -e^{2\Phi}d\tilde{t}^2 + e^{2\Lambda}d\tilde{r}^2 + \tilde{r}^2 d\Omega^2 & r < r_0 \\ -(1-\frac{2M}{r})dt^2 + (1-\frac{2M}{r})^{-1}dr^2 + r^2 d\Omega^2 & r > r_0 \end{cases}$$

Following the same method as before, we get:

$$g''_{l\omega}(\tilde{r}) + \left(-\Lambda'(\tilde{r}) + \frac{2}{\tilde{r}} - \Phi'(\tilde{r})\right) g'_{l\omega}(\tilde{r}) - e^{2\Lambda(\tilde{r})}\left(\frac{l(l+1)}{\tilde{r}^2} + e^{-2\Phi(\tilde{r})}\omega^2 + \xi R\right) g_{l\omega}(\tilde{r}) = 0 \qquad r < r_0$$

$$g''_{l\omega}(r) + \frac{2(r-M)}{r(r-2M)} g'_{l\omega}(r) - \left(\frac{l(l+1)}{r(r-2M)} - \frac{\omega^2}{(1-\frac{2M}{r})^2}\right) g_{l\omega}(r) = -\frac{4\pi}{r(r-2M)}\delta(r-r') \qquad r > r_0$$

Instead of ingoing radiation at the horizon, we want a regular solution at the origin. It can be proven that there is only one interior regular solution at the origin,

$$g^{in}_{l\omega} = A_l I_{l\omega}(r) .$$

Outside of the matter, the delta function is canceled off by the Schwarzschild solution. The general solution has a smooth correction determined by matching at the boundary r_0.

$$g_l^{out} = \frac{4\pi}{M} \frac{1}{\Delta W(R_l^H(r'), R_l^\infty(r'))} R_l^H(\bar{r}_<) R_l^\infty(\bar{r}_>) + C_l R_l^\infty(\bar{r}) \qquad C_l = R^\infty_{l\omega}(r_q) \frac{I'_{l\omega}(r_0) R^H_{l\omega}(r_0) - I_{l\omega}(r_0) R^{H\prime}_{l\omega}(r_0)}{I'_{l\omega}(r_0) R^\infty_{l\omega}(r_0) - I_{l\omega}(r_0) R^{\infty\prime}_{l\omega}(r_0)}$$

"Schwarzschild part" smooth correction

Difference in Fields

$$g^{out}_{l\omega} = \frac{4\pi}{M}\frac{1}{\Delta \mathrm{W}(R^H_{l\omega}(r'), R^\infty_{l\omega}(r'))} R^H_{l\omega}(r_<)R^\infty_{l\omega}(r_>) + C_l R^\infty_{l\omega}(r)$$

Arbitrary Interior

$$g_{l\omega} = \frac{4\pi}{M}\frac{1}{\Delta \mathrm{W}(R^H_{l\omega}(r'), R^\infty_{l\omega}(r'))} R^H_{l\omega}(r_<)R^\infty_{l\omega}(r_>)$$

Schwarzschild

So, if we are to look only at the difference between the self-forces, then we only need to worry about evaluating the smooth correction term, which unlike the solution to Schwarzschild does not diverge and is finite. The part of the Green's function contributing to the self force difference is given by:

$$g^{\mathrm{diff}}_{l\omega} = R^\infty_{l\omega}(r_q)R^\infty_{l\omega}(r)\frac{I'_{l\omega}(r_0)R^H_{l\omega}(r_0) - I_{l\omega}(r_0)R^{H\prime}_{l\omega}(r_0)}{I'_{l\omega}(r_0)R^\infty_{l\omega}(r_0) - I_{l\omega}(r_0)R^{\infty\prime}_{l\omega}(r_0)}$$

Next step is to look at specific cases.

Static Charge Case, Arbitrary Interior

In the static case, the radial equation for Schwarzschild becomes the familiar Legendre equation with the solution regular at the horizon being P_l and the solution regular at infinity being Q_l. Wiseman has shown that the self-force in Schwarzschild vanishes for the static case (Wiseman 2000 therefore the self force difference is the self-force so

$$f_r = q^2\sqrt{1-\frac{2M}{r_q}}\sum_{l=0}^{\infty}\frac{(2l+1)}{M^2}Q_l(\bar{r}_q)Q_l'(\bar{r}_q)\left.\frac{W(I_l,P_l)}{W(I_l,Q_l)}\right|_{r_0}.$$

It is a property that for large r_q, the Legendre polynomials have an $r_q^{-(2l+3)}$ dependence. Therefore, the leading far-field behavior comes from $l=0$, so we have

$$f_r = \frac{q^2 M}{r_q^3}\left.\frac{W(I_0,P_0)}{W(I_0,Q_0)}\right|_{r_0} + O\left(\frac{q^2M^3}{r_q^5}\right).$$

Notice that if the unique interior solution is a constant then the Wronskian vanishes. The far field self force is independent of the distribution of matter if and only if the interior wave equation admits a constant solution.

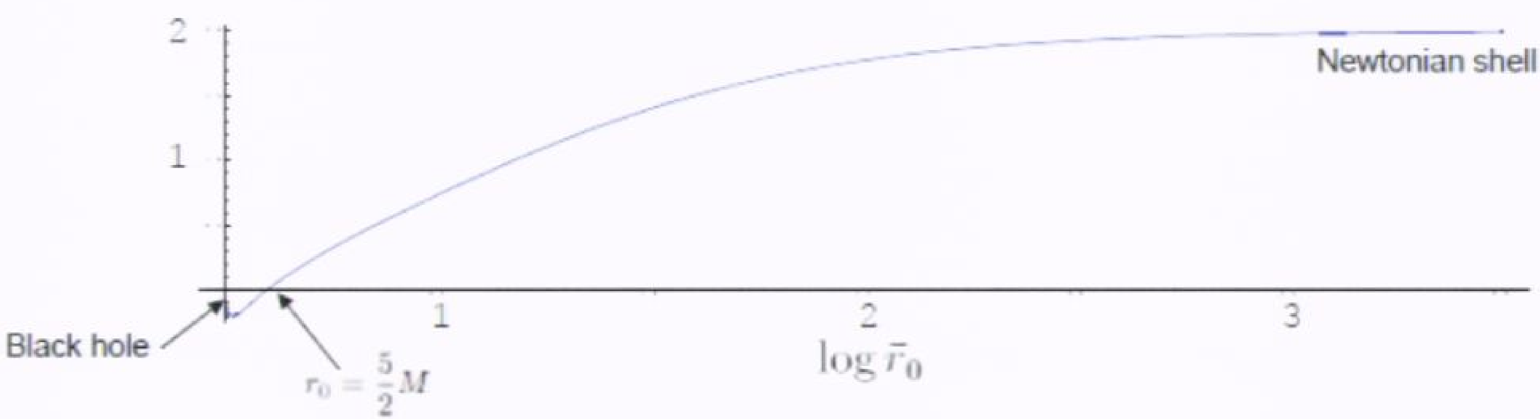
The wave equation admits a constant solution if and only if $\xi = 0$ (minimally coupled); therefore the leading order self-force is proportional to ξ.

Static Case, Thin Shell

For a thin-shell spacetime in the case of a static scalar charge, we can evaluate the mode functions analytically. Once again, in the far-field limit, only the leading order contributes. Some results in far field limit are:

Doesn't Depend on Central Object $$f_r = 2\xi q^2 \frac{M}{r_q^3}$$ large, Newtonian shell
agrees with (Pfenning & Poisson, 2001)

Depends on Central Object $$f_r = -2\xi q^2 \frac{M}{r_q^3}\sqrt{\frac{r_0}{2M} - 1}$$ small, highly relativistic shell



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Dynamic Case: Conservative and Dissipative Self-forces

Symmetric and Antisymmetric Parts under retarded ↔ advanced

$$\Phi_{ret} = \frac{1}{2}(\Phi_{ret} + \Phi_{adv}) + \frac{1}{2}(\Phi_{ret} - \Phi_{adv}) = \Phi_{cons} + \Phi_{diss}$$

Conservative part of field (difference) and force (difference)

$$\Phi_{cons} = \frac{1}{2}(\Phi_{ret} + \Phi_{adv}) = \frac{2q}{u^t}\sum_{l=1}^{\infty}\sum_{m=1}^{l} Re[g_{l\omega}(r)](2l+1)\frac{(l-m)!}{(l+m)!}P_l^m(\frac{\pi}{2})P_l^m(\theta)cos[m(\phi - \Omega t)]$$

$$\nabla_r\Phi_{cons} = \frac{2q}{u^t}\sum_{l=1}^{\infty}\sum_{m=1}^{l} Re[g'_{l\omega}(r)](2l+1)\frac{(l-m)!}{(l+m)!}P_l^m(\frac{\pi}{2})^2$$

Dissipative part of field (difference) and force (difference)

$$\Phi_{diss} = \frac{1}{2}(\Phi_{ret} - \Phi_{adv}) = \frac{2q}{u^t}\sum_{l=1}^{\infty}\sum_{m=1}^{l} Im[g_{l\omega}(r)](2l+1)\frac{(l-m)!}{(l+m)!}P_l^m(\frac{\pi}{2})P_l^m(\theta)sin[m(\phi - \Omega t)]$$

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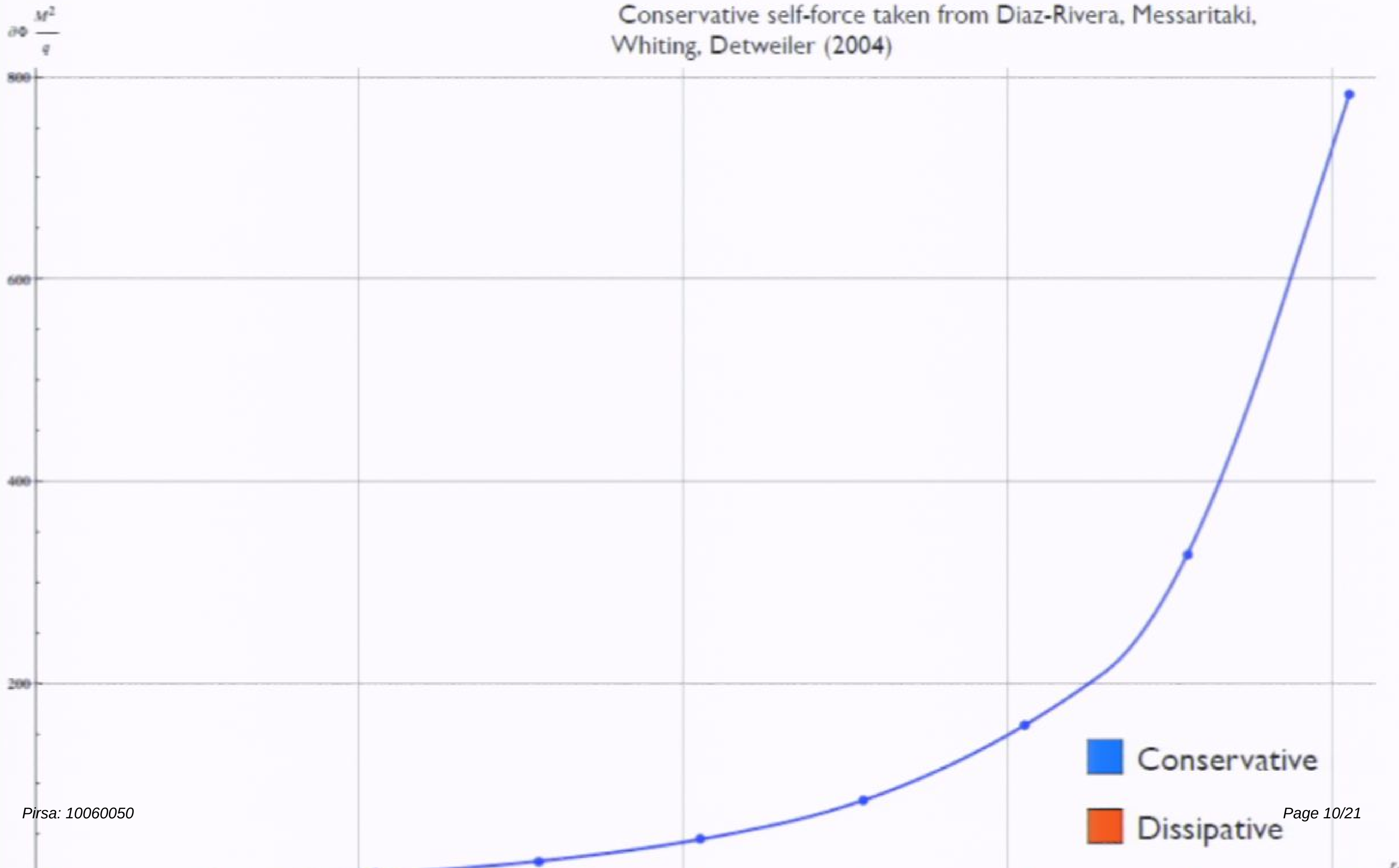
Circular Orbit: Fractional Self-Force Difference

r_q=14, varying shell radius, ξ = 0

Conservative self-force taken from Diaz-Rivera, Messaritaki, Whiting, Detweiler (2004)



Circular Orbit: Checks

-Integration Check

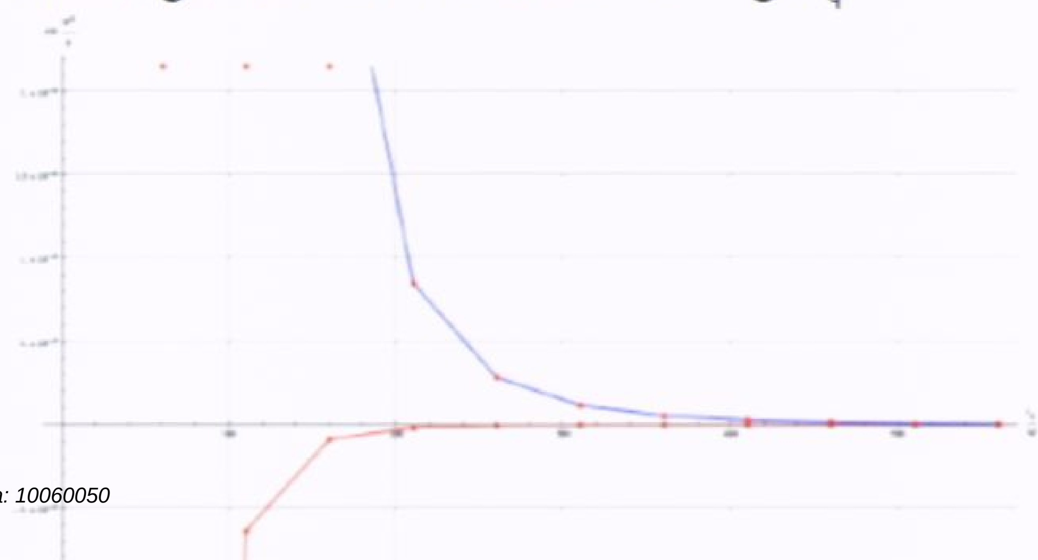
- -Constant Wronskian Check
- -Results agree starting integration at different points

-Flux check

- -dissipative self-force matches energy flux

-Results for dissipative self-force without shell in agreement with known results (Gralla, Friedman, Wiseman, 2005)

-Vanishing of self-force difference for large r_q



r_0=150

r_q= [160, 600]

ξ = 1

Conservative

Dissipative

Conclusions

- Self-force differences are easy to compute, requiring no renormalization

- Once you know the self-force in Schwarzschild, it is easy to get the self-force for any other interior (boson star?)

- We considered a non-minimally coupled scalar field.

  -The leading order far field self-force (difference) on a static charge outside an arbitrary (SS) interior is proportional to ξ.

  -Explicitly computed the far field static self-force (difference) for a thin shell spacetime.

  -Numerically computed self-force differences for a scalar charge in circular orbit around a thin shell.

    -Conservative self-force difference much larger than the Dissipative self-force difference

    -The conservative self-force is very big when the charge is near the shell!

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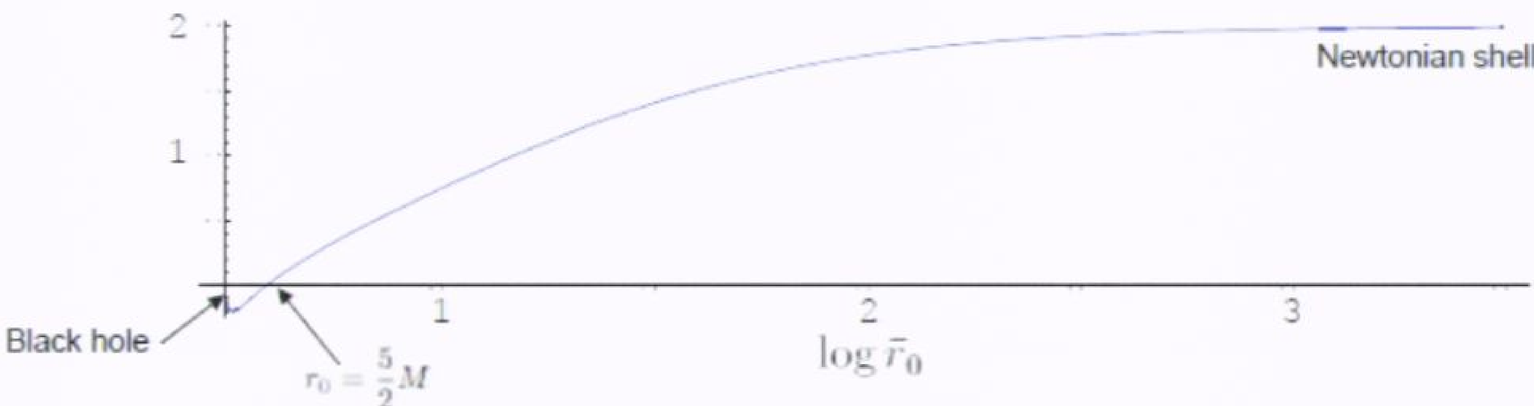

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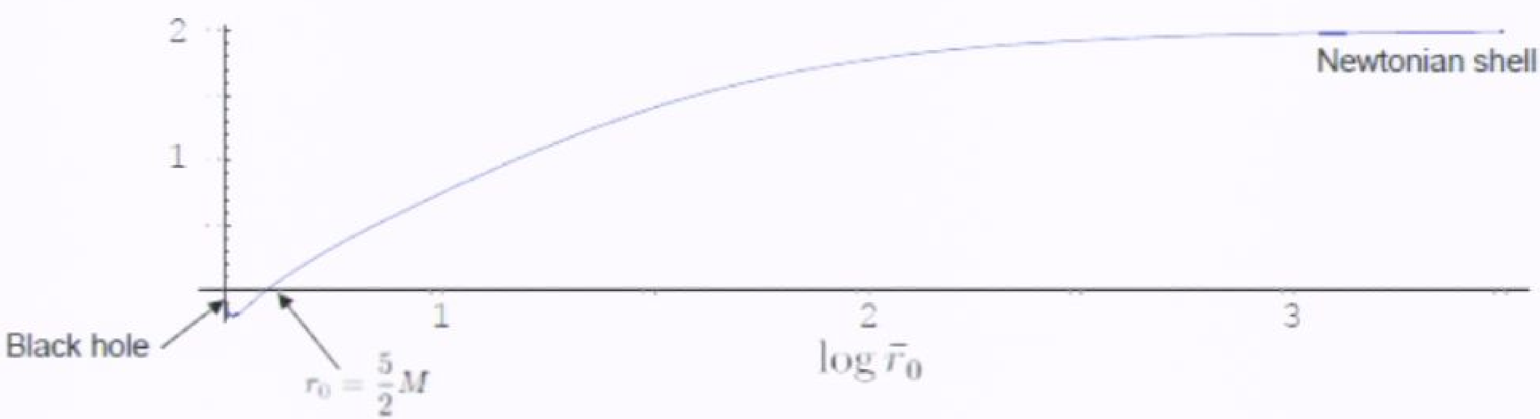

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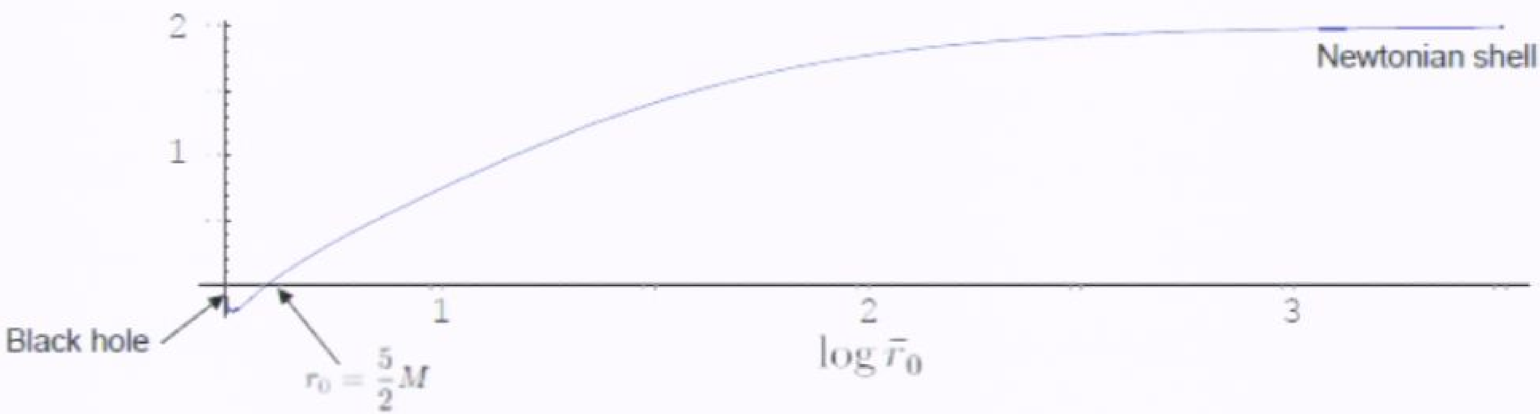

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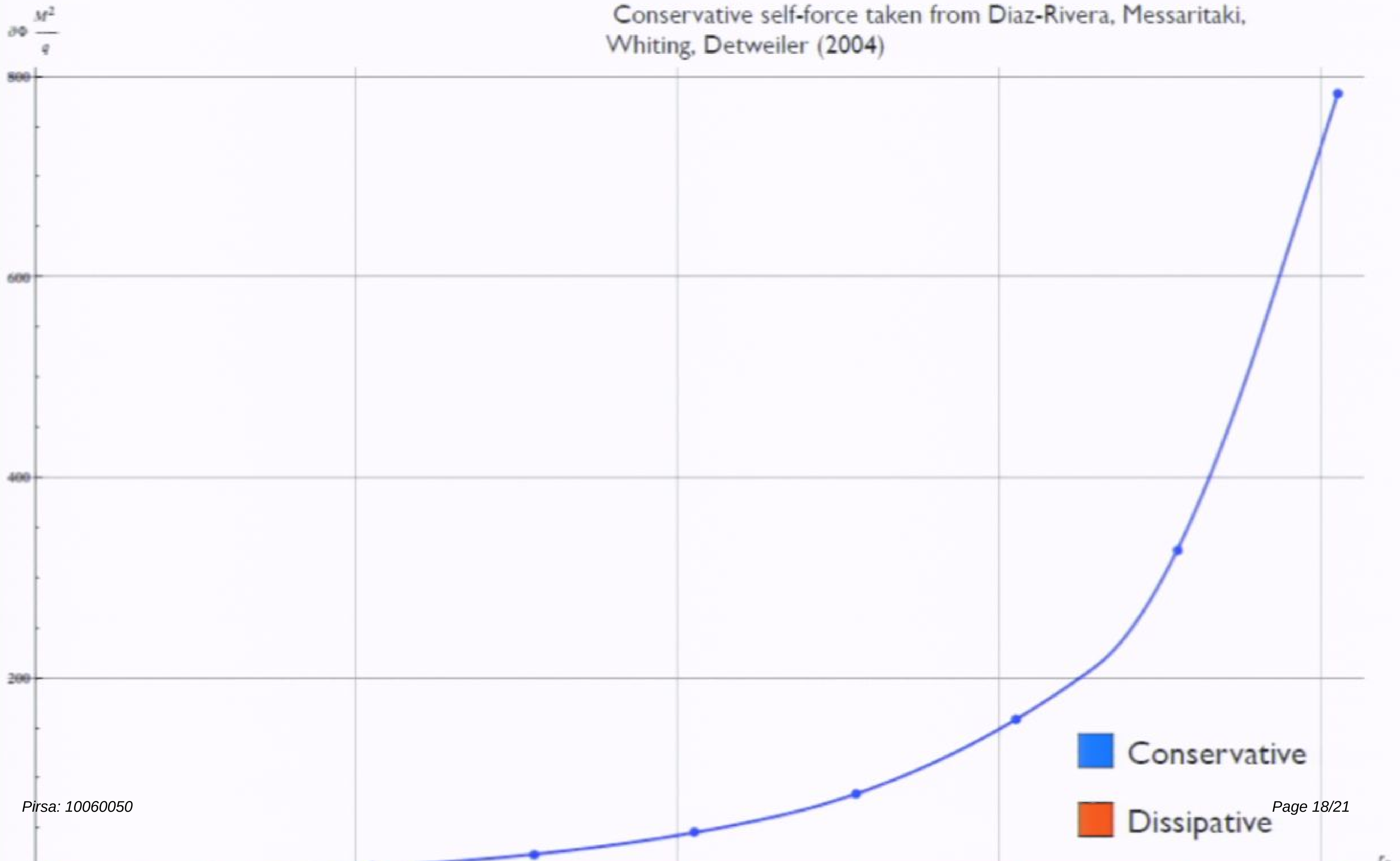

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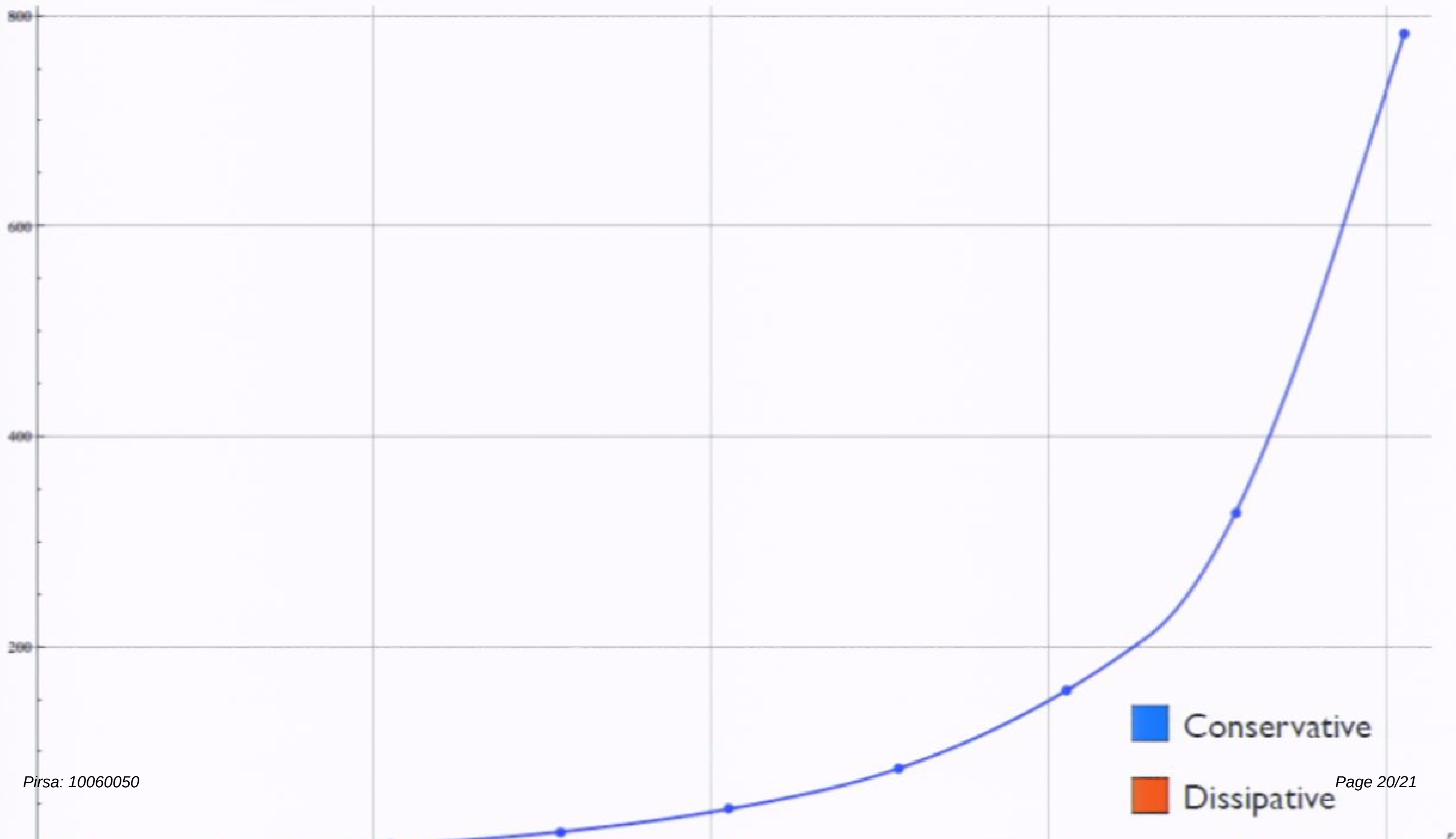

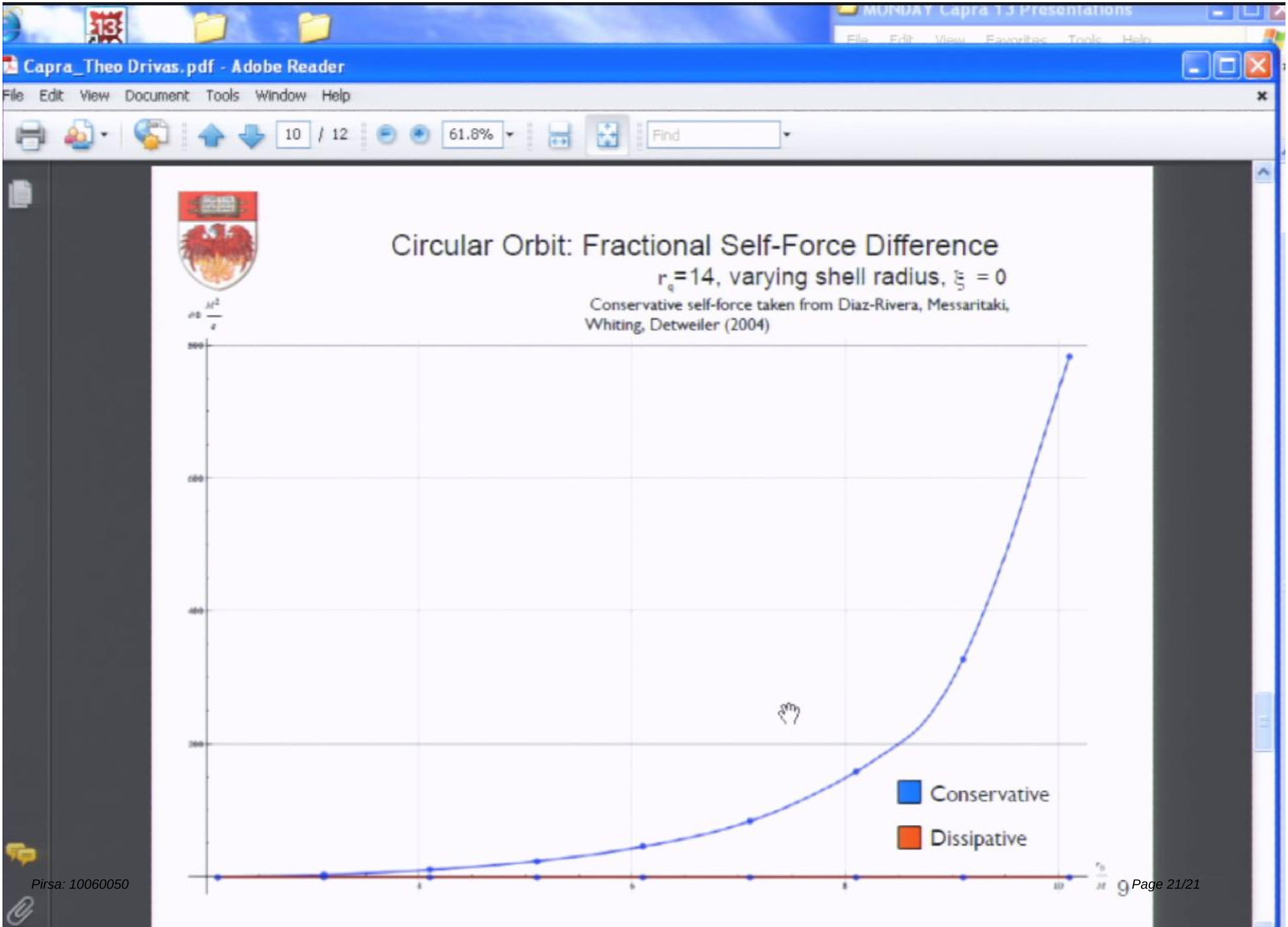
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