Title: Gauge-Independent Self-force Formula

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Abstract: TBA

Self-force and Gauge

A simple thought:

> Since the gravitational self-force is pure gauge, shouldn't there be a "pure gauge" derivation?

Related:

> Shouldn't there be a way of writing the force that holds for all (allowed) gauge choices?

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> Spoiler: the result is that the self-force in any allowed gauge is simply the angle-average of the bare self-force

Advantages of the Gauge-independent Approach

The gauge-independent derivation

- is incredibly simple, involving essentially no computation

- never loses sight of the gauge-dependent nature of the force

- has a great shot of generalizing to second-order self-force (whereas the harmonic gauge approach would be a giant mess)

The gauge-independent result

- doesn't prefer humanity's favorite gauge choice

- comes in a "regularization" form suited to numerical computation

- tells you how to compute self-force in a wider class of gauges (such as the one used in Milwaukee?)

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Steps in a self-force Derivation

1. Make assumptions about your spacetime (really, one-parameter family).

   - I will use the assumptions of Gralla&Wald 2008

2. Define the "position" or representative worldline of the particle

   - I will use a more general definition of position than used in GW2008

3. Compute an equation for the worldline

   - I will take a new, "gauge-independent" computational approach

Note that there are some errors in the gauge calculations in the appendix of GW2008. This work corrects them (although the computations are done differently).

Step 1. Assumpions

(review of our formalism)

To treat a "small body," consider a one-parameter-family of spacetimes g(λ) containing a body that shrinks to zero size. The body must shrink to zero mass too, since point particles are not allowed in GR.

We accomplish this by assuming the existence of a second, "scaled" limit designed to hold any shrinking body at fixed size.

Rescale coordinates and metric: $\bar{t} = \frac{t - t_0}{\lambda} \qquad \bar{x}^i = \frac{x^i}{\lambda} \qquad \bar{g}_{\mu\nu} \equiv \lambda^{-2} g_{\mu\nu}$

Let λ → 0 $\quad \bar{g}^{(0)}_{\bar{\mu}\bar{\nu}}(t_0) \equiv \lim_{\lambda \to 0} \bar{g}_{\bar{\mu}\bar{\nu}}(\lambda; t_0),$

This limit zooms in on the body at some time t_0, giving a stationary, asysmptotically flat spacetime characterizing the body at time t_0.
I will call this spacetime the "body metric".

*We also assume a "uniformity condition" which is like the "region

Example Family

Consider the Schwarzschild-deSitter metric of mass λ*M,

$$ds^2(\lambda) = -\left(1 - \frac{2M_0\lambda}{r} - C_0 r^2\right) dt^2 + \left(1 - \frac{2M_0\lambda}{r} - C_0 r^2\right)^{-1} dr^2 + r^2 d\Omega^2, r > \lambda R_0$$

As λ→0, the body disappears and we are left with a deSitter background.

But rescaling the metric and coordinates,

$$\bar{g}_{\bar{\mu}\bar{\nu}} \equiv \lambda^{-2} g_{\bar{\mu}\bar{\nu}}. \quad \bar{t} \equiv t/\lambda \quad \bar{r} \equiv r/\lambda$$

we have

$$d\bar{s}^2(\lambda) = \left(1 - \frac{2M_0}{\bar{r}} - C_0\lambda^2\bar{r}^2\right) d\bar{t}^2 + \left(1 - \frac{2M_0}{\bar{r}} - C_0\lambda^2\bar{r}^2\right)^{-1} d\bar{r}^2 + \bar{r}^2 d\Omega^2, \bar{r} > R_0.$$

Now, as λ→0 the *background* disappears and we are left with the Schwarzschild metric. The scaled limit zooms in on the body.

Step 2. Definitions

The lowest-order motion is given by the worldline "left behind" by the shrinking body, which we proved must be geodesic.

The mass, spin, and displacement from geodesic motion are defined to be the mass, angular momentum, and center of mass of the body metric at time t_0.

The center of mass of an SAF metric is given by (Regge & Teitelboim 1974)

$$D^I = -\frac{1}{16\pi} \lim_{r\to\infty} \int r^2 d\Omega\, n^j \left[x^I \left(\partial_k g_{jk} - \partial_j g_{kk}\right) - g_{jI} + g_{kk}\delta_{jI}\right]$$

Some properties:

1. The CM is only defined when the metric is even parity at O(1/r)
2. The CM depends only the l=1, O(1/r^2) part of the metric
3. If the coordinates are (super)translated, the CM changes by

Step 3. Computation (1)

Under an allowed change of gauge,

$$\xi^{\mu} = F^{\mu}(t,\theta,\phi) + O(r) \quad \rightarrow \quad \bar{\xi}^{\bar{\mu}} = F^{\mu}(t_0,\theta,\phi)$$

even parity

the deviation from geodesic motion changes by

$$\delta X^I = \frac{1}{4\pi}\int F^I d\Omega = \frac{1}{4\pi}\lim_{r\to 0}\int \xi^I d\Omega \equiv \langle \xi^I \rangle_{r\to 0}$$

and hence the acceleration changes by (working in Fermi coordinates)

vanish by parity

Ricci Identity

$$\begin{aligned}
\delta \ddot{X}^i &= \langle \partial_0 \partial_0 \xi^i \rangle_{r\to 0} \\
&= \langle \nabla_0 \nabla_0 \xi_i + \Gamma^k_{00}\partial_k \xi_i + \nabla_0 \nabla_i \xi_0 - \nabla_i \nabla_0 \xi_0 + R_{i00}{}^k \xi_k \rangle_{r\to 0} \\
&= \langle \nabla_0 \delta g^{(1)}_{0i} - \frac{1}{2}\nabla_i \delta g^{(1)}_{00} \rangle_{r\to 0} - R_{0i0k}\langle \xi^k \rangle_{r\to 0} \\
&= \delta \langle f^i_{\text{bare}} \rangle_{r\to 0} - R_{0i0k}\delta X^k
\end{aligned}$$

gauge invariant correction term

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Step 3. Computation (2)

We have

$$\ddot{X}^i = \langle f^i_{\mathrm{bare}} \rangle_{r\to 0} - R_{0j0}{}^i X^j + \mathcal{F}^i$$

It only remains to determine the gauge-invariant contribution $\mathcal{F}^i$ by picking a gauge and computing the other terms. Harmonic gauge would be hard work. But there is a very convenient gauge…

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The gauge in which there is no deviation from geodesic motion!

$X^i(t) = 0$ → $\mathcal{F}^i = -\langle f^i_{\rm bare} \rangle_{r\to 0}$ in this gauge

Only l=1 angular dependence in the metric perturbations can survive the angle-average. But the relevant terms actually satisfy the linearized Einstein equation about Schwarzschild ("in the near zone"). The fact that l=1 (electric) perturbations are pure gauge allows us to show immediately

Result

$$\ddot{X}^i = \langle f^i_{\mathrm{bare}} \rangle_{r\to 0} - R_{0j0}{}^i X^j$$

self-force geodesic deviation

$$f^i_{\mathrm{bare}} = \nabla_0 g^{(1)}_{0i} - \frac{1}{2}\nabla_i g^{(1)}_{00}$$

This is the equation for the deviation from geodesic motion in any allowed gauge. The SF agrees with the standard result in harmonic gauge.

Ideology

1. This form of the force is more fundamental than the "tail integral" form because it holds in any allowed gauge.

2. This form of the force is more fundamental than the "singular field subtraction" form because it holds in any allowed gauge. Note that the concept of a singular field never arises in this derivation.

3. What's so great about a singular field anyway? Angle-averaging seems to best reflect the basic physics: only asymmetric self-fields can contribute an overall force.

Summary and Conclusions

- The self-force in any allowed gauge is given by the angle average of the bare self-force in that gauge.

- The allowed class of gauges are those for which the "ADM center of mass" of the body metric (near-zone background metric) is finite. These gauges are the ones in which the metric perturbations diverge as 1/r and have even parity to 1/r. This extends the self-force to a larger class of gauges.

- The derivation involves essentially no computation, which gives some hope for a straightforward generalization to second-order. This would produce a "regularization" (rather than "tail integral") form more suited to numerical computation.

- It also may be possible to generalize the results to an arbitrary diffeomorphism-covariant theory of tensor fields (matter fields, Einstein-Maxwell, modified gravity, etc.) without much additional computation.

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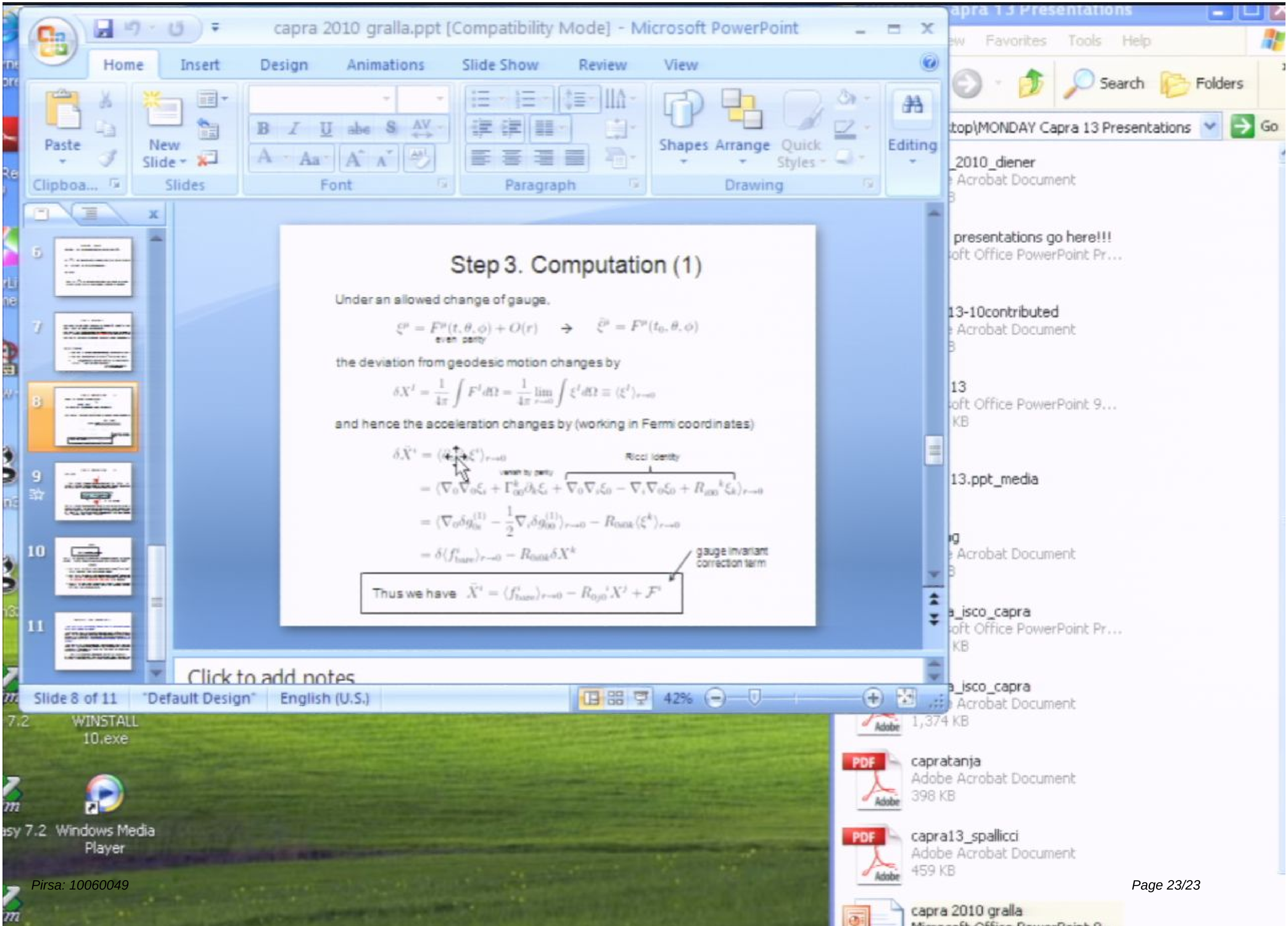
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Thus we have $\ddot{X}^i = \langle f^i_{bare} \rangle_{r\to 0} - R_{0j0}{}^i X^j + \mathcal{F}^i$
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