Title: Non-singular sources for self-force calculations in numerical relativity

Date: Jun 21, 2010 09:30 AM

URL: http://pirsa.org/10060044

Abstract: Motivated by the goal of self-consistently calculating self-force-corrected orbits and waveforms with (3+1) evolution codes we derive a covariant expression for a non-singular representation of a scalar point charge moving along a geodesic of an arbitrary spacetime. This differs from previous representations that were anchored to the use a particular locally-inertial coordinate system.

Review of the (3+1) approach

- "Regularization before evolution"
- Solve the following equations simultaneously for ψ^{R} and u^α:

$$\Box\psi^{\mathsf{R}} = S(x^\alpha, u^\alpha)$$
$$u^\beta\nabla_\beta u^\alpha = g^{\alpha\beta}\nabla_\beta\psi^{\mathsf{R}}$$

- Advantages:
  - Non-post-processing approach to self-consistent evolution
  - Does not rely on the underlying symmetries of the spacetime
  - Uses existing numerical relativity infrastructure
- Disadvantages:
  - Lower accuracy – (could be a deal breaker)
  - Inherits the difficulties that confront any (3+1) calculation

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Effective Source

Away from the worldline, the effective source is defined by

$$S = -\Box(W\tilde{\psi}^S) = -W\Box\tilde{\psi}^{\mathsf{S}} - 2\nabla^\alpha W \nabla_\alpha \tilde{\psi}^{\mathsf{S}} - \tilde{\psi}^{\mathsf{S}}\Box W$$

where

- $\tilde{\psi}^{\mathsf{S}}$ is an accurate-enough (explicit) approximation to the exact Detweiler-Whiting singular field, and
- W is an appropriately chosen window function, defined so that
  - $\lim_{x\to\gamma} \nabla_\alpha \psi^{\mathsf{R}} = F_\alpha$,
  - $\psi^{\mathsf{R}} \to \psi^{\mathsf{ret}}$ away from the source,
  - S_{eff} is regular on the worldline.

Example of a singular field approximation:

$$\psi^{\mathsf{S}} = q/\rho + O(\rho^3),$$

where $\rho = \sqrt{x^2 + y^2 + z^2}$ in THZ coordinates (t, x, y, z)

"New" covariant starting point

Covariant singular field (Haas and Poisson, 2006)

$$\psi^{\mathsf{S}} = \frac{q}{2r} + \frac{q}{2r_{\mathsf{adv}}} + O(\epsilon^3)$$

$$\psi^{\mathsf{S}} = \frac{q}{s}\left(1 + \frac{\bar{r}^2 - s^2}{6s^2}R_{u\sigma u\sigma} + \frac{\bar{r}(\bar{r}^2 - 3s^2)}{24s^2}R_{u\sigma u\sigma|u} - \frac{(\bar{r}^2 - s^2)}{24s^2}R_{u\sigma u\sigma|\sigma}\right) + O(\epsilon^3)$$

Five scalar functions

$\{s, \bar{r}, R_{u\sigma u\sigma}, R_{u\sigma u\sigma|u}, R_{u\sigma u\sigma|\sigma}\}$.

$$s^2 = (g^{\bar{\alpha}\bar{\beta}} + u^{\bar{\alpha}}u^{\bar{\beta}})\sigma_{\bar{\alpha}}\sigma_{\bar{\beta}}$$

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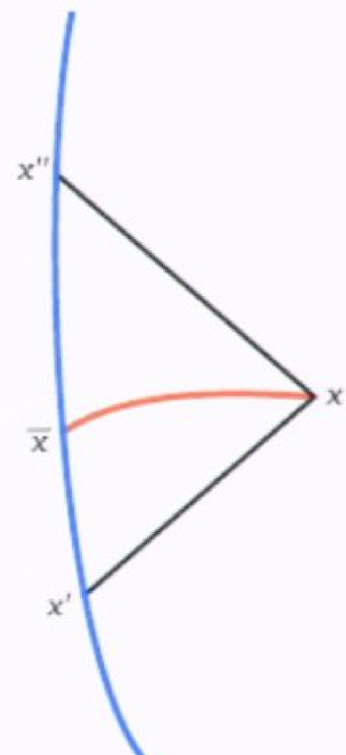

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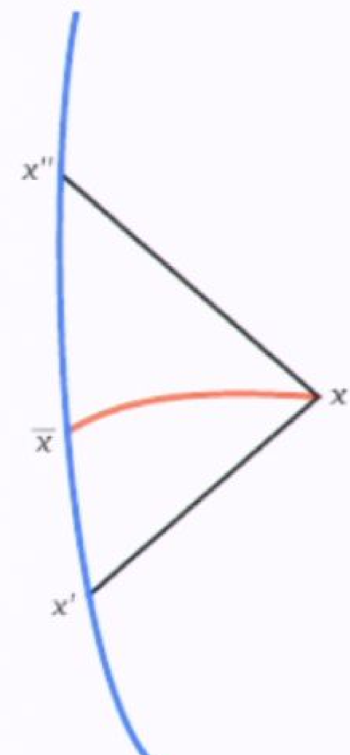

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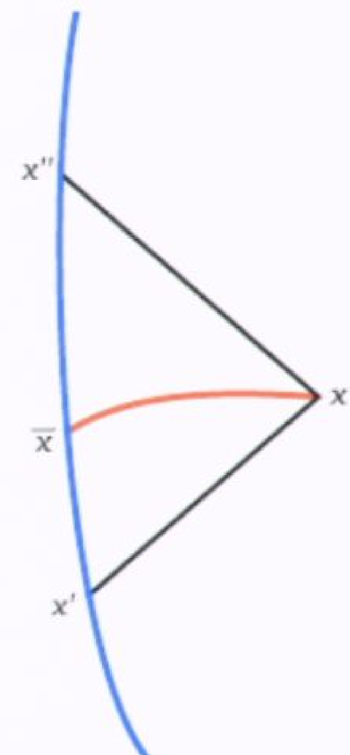

Background coordinates

$\sigma_{\bar\alpha}$ can be expressed in terms of an expansion in the coordinate displacement $w^\alpha = (x^\alpha - \bar{x}^\alpha)$:

$$-\sigma_{\bar\alpha}(x,\bar{x}) = g_{\alpha\beta}w^\beta + A_{\alpha\beta\gamma}w^\beta w^\gamma + A_{\alpha\beta\gamma\delta}w^\beta w^\gamma w^\delta + A_{\alpha\beta\gamma\delta\epsilon}w^\beta w^\gamma w^\delta w^\epsilon + \ldots$$

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are all quantities evaluated on γ.

Here then we have a perfectly legitimate (generic) singular field approximation in terms of the background coordinates.

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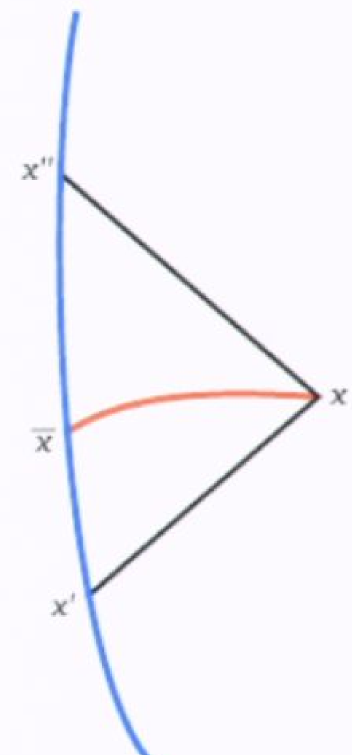

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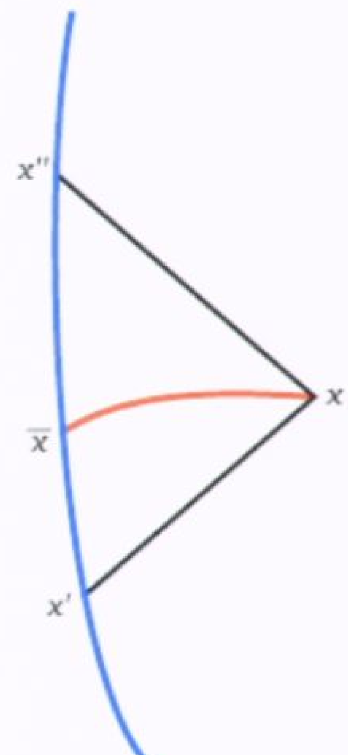

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A potential issue with the covariant approach?

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- Derive covariant expressions for $\Box\tilde{\psi}^{\mathsf{S}}$ and $\nabla_\alpha\tilde{\psi}^{\mathsf{S}}$ from Eric's covariant approximation of the singular field.
- This requires truncations of higher-order terms.
- Truncation introduces inconsistencies in the effective source that may ruin the key relation $\psi^{\mathsf{R}} = \psi^{\mathsf{ret}} - W\tilde{\psi}^{\mathsf{S}}$
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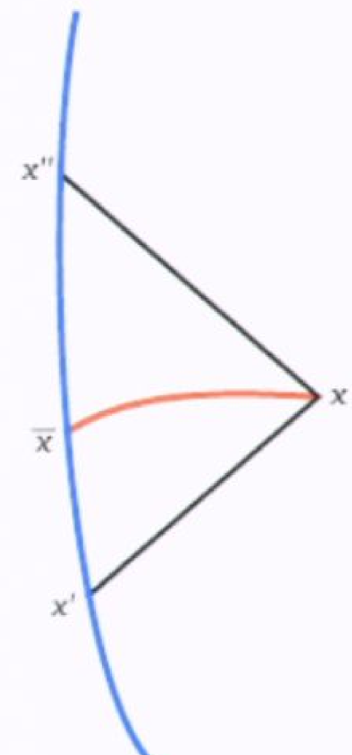

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s truncation allowed in going from $\tilde{\psi}^{\mathsf{S}}$ to $\Box\tilde{\psi}^{\mathsf{S}}$?

Recall the wave equation with the effective source:

$$\Box\psi^{\mathsf{R}} = -\tilde{\psi}^{\mathsf{S}}\Box W - 2\nabla^{\alpha}W\nabla_{\alpha}\tilde{\psi}^{\mathsf{S}} - W\Box\tilde{\psi}^{\mathsf{S}}$$

Determine a nice covariant expression for $\Box\tilde{\psi}^{\mathsf{S}}$; call it $F(\sigma)$.

$$\Box\tilde{\psi}^{\mathsf{S}} = F(\sigma) + R\sigma^{n}.$$

Jse this truncated version in the effective source:

$$\Box\bar{\psi}^{\mathsf{R}} = -\tilde{\psi}^{\mathsf{S}}\Box W - 2\nabla^{\alpha}W\nabla_{\alpha}\tilde{\psi}^{\mathsf{S}} - WF(\sigma)$$

s there any crucial difference between ψ^{R} and $\bar{\psi}^{R}$?

$$\Box(\psi^{\mathsf{R}} - \bar{\psi}^{\mathsf{R}}) = WR\sigma^{n}$$

Does this imply that $(\psi^{R} - \bar{\psi}^{R})$ vanishes on γ?

Choosing the approximate singular field

1) Re-package the singular field as

$$\tilde{\psi}^{\mathsf{S}} = \frac{q}{s^3} K$$

where

$$K := s^2 + \frac{(\bar{r}^2 - s^2)}{6} R_{u\sigma u\sigma} + \frac{\bar{r}(\bar{r}^2 - 3s^2)}{24} R_{u\sigma u\sigma|u} - \frac{(\bar{r}^2 - s^2)}{24} R_{u\sigma u\sigma|\sigma}.$$

2) Truncate s^2 and F at the appropriate order.

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Write the approximate singular field and effective source in terms of 8 polynomials in w^α and their (partial) derivatives

Coding up the effective source

- Following the original THZ code, I wanted the effective source to be computed entirely through evaluations of the helping polynomials and their explicit (partial) derivatives.
- 72 C/C++ functions
- These helping polynomials are generally VERY long and nasty.
- Many many thanks to Maple and its code generation facility!
- ... and to "grOptionDefaultSimp := 0" in grtensor. (Whoever laughs at this is a true pro!)

C/C++ code for effective scalar charge moving along a generic geodesic in Schwarzschild → DONE!

Since nothing said so far is specific to Schwarzschild, Kerr should now be trivial.

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nterpolation

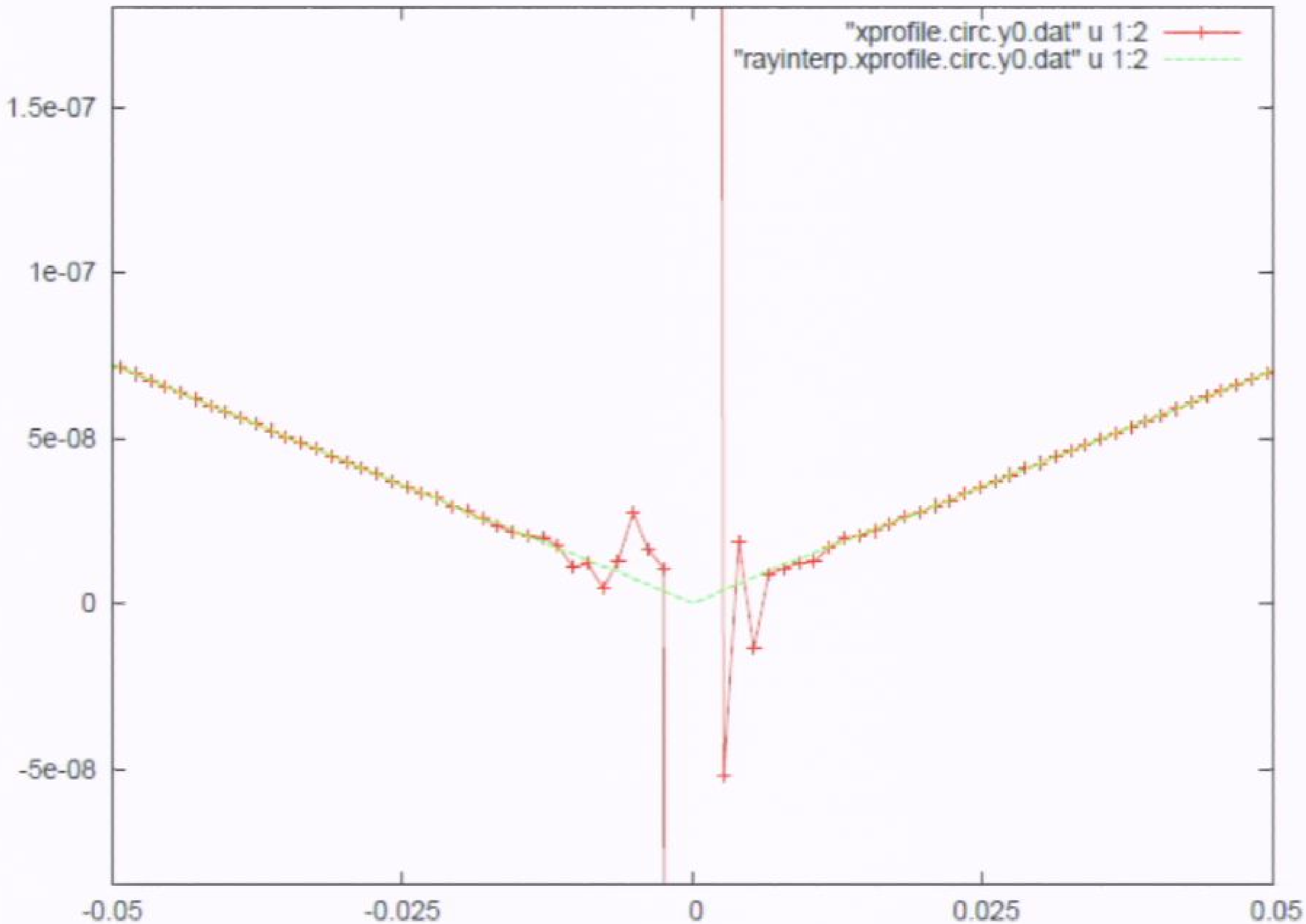
\ problem that arises in calculating the effective source is the subtraction ›f very large numbers occuring close to the particle.

Two important facts:
1) $S = 0$ on γ (formally).
2) S is smooth everywhere except on γ, where it is just C^0.

Consider $S(\lambda) := S(\vec{x}(\lambda))$ as a function of λ, along the coordinate ray, $\vec{x} = \vec{x}_0 + \lambda\hat{r}$, where $\hat{r} := \frac{\vec{x}-\vec{x}_0}{|\vec{x}-\vec{x}_0|}$.

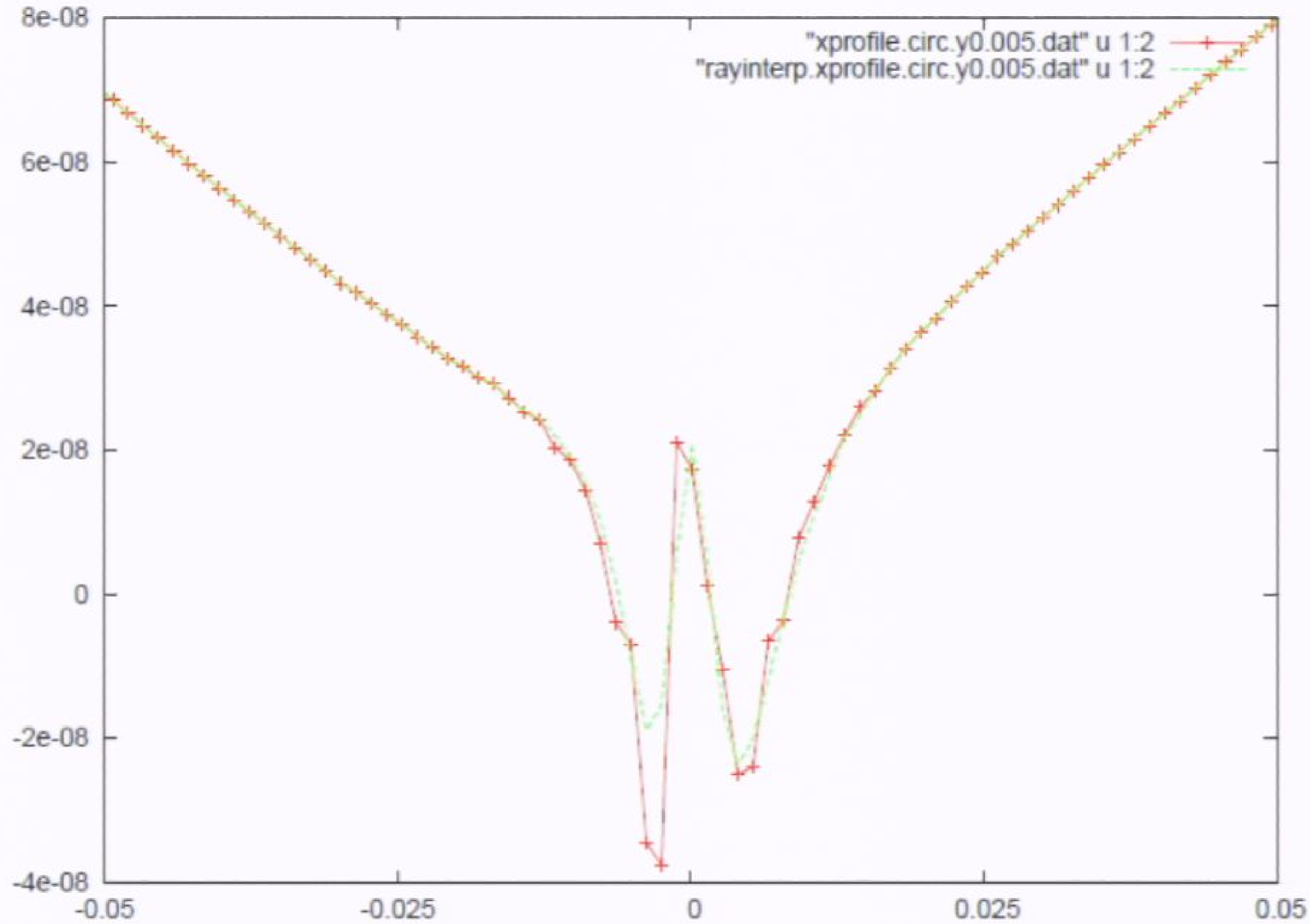
To compute S at a point very close to γ, instead use (Lagrange) nterpolation with $S(\lambda = 0) = 0$ and a few other evaluations of S along he coordinate ray.

Results after interpolation



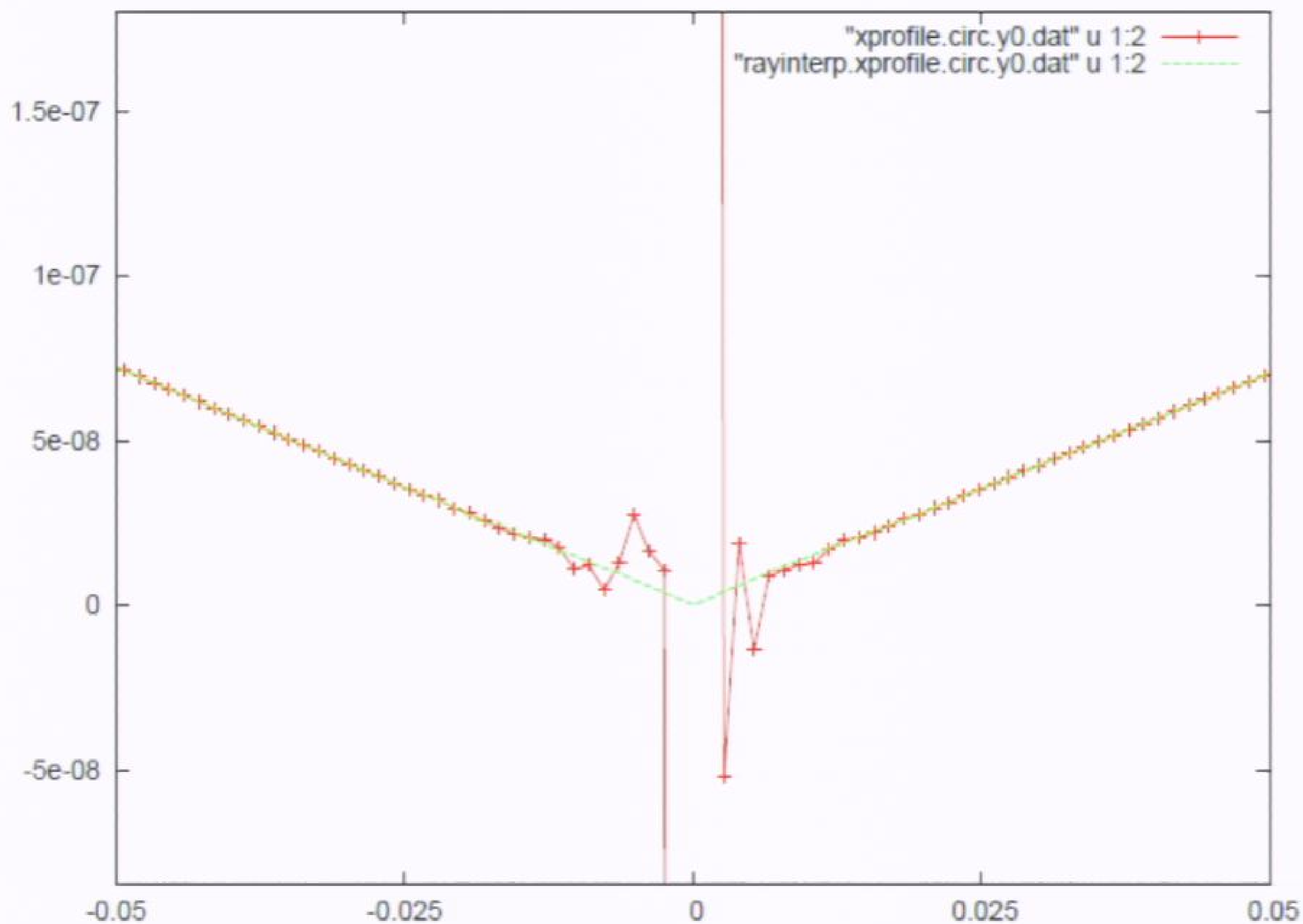
The effective source along the ray $(x = 0, y = 0)$, before and after interpolation

Results after interpolation



The effective source along the ray $(x = 0, y = 0.005M)$, before and after

Results after interpolation



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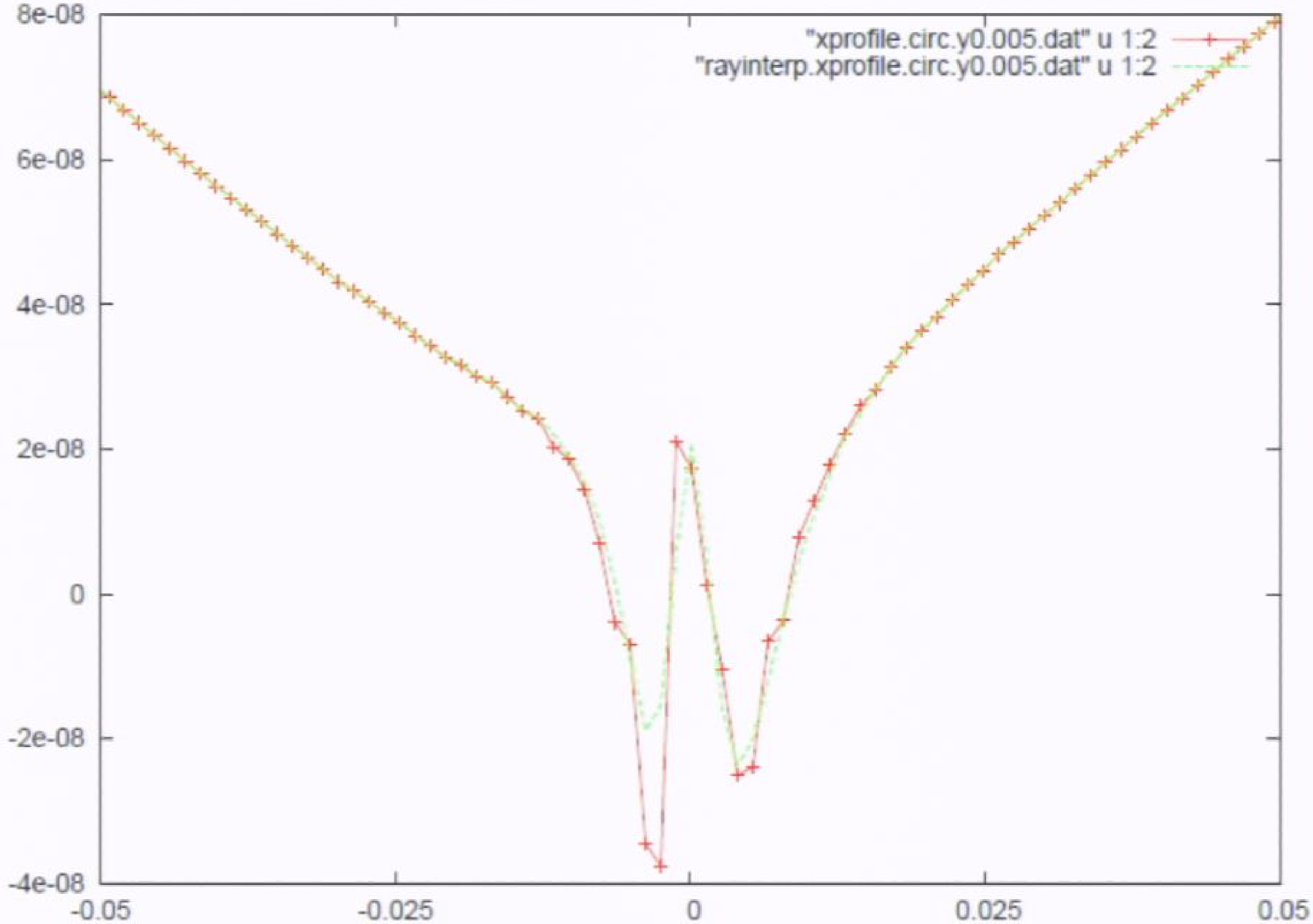

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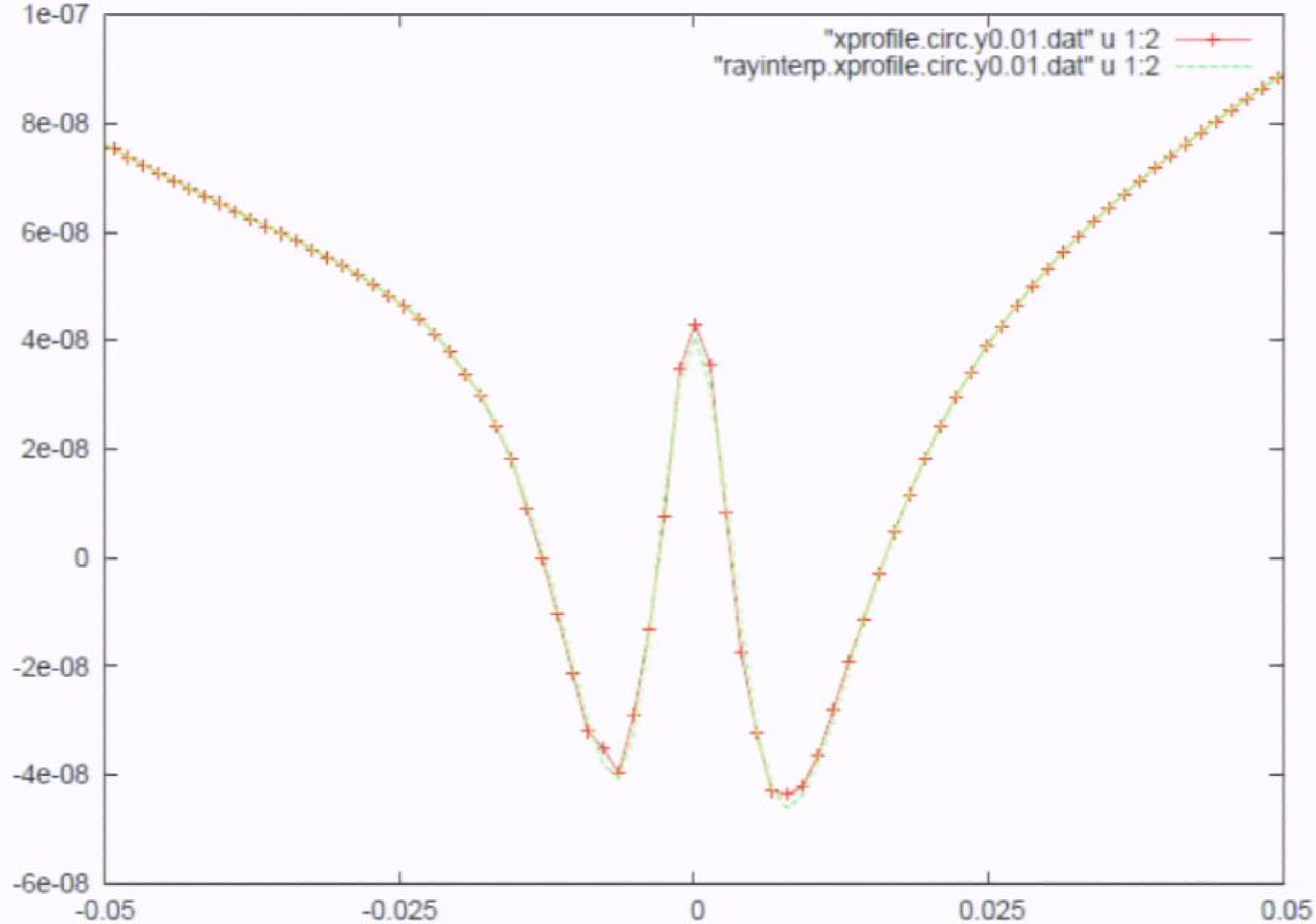
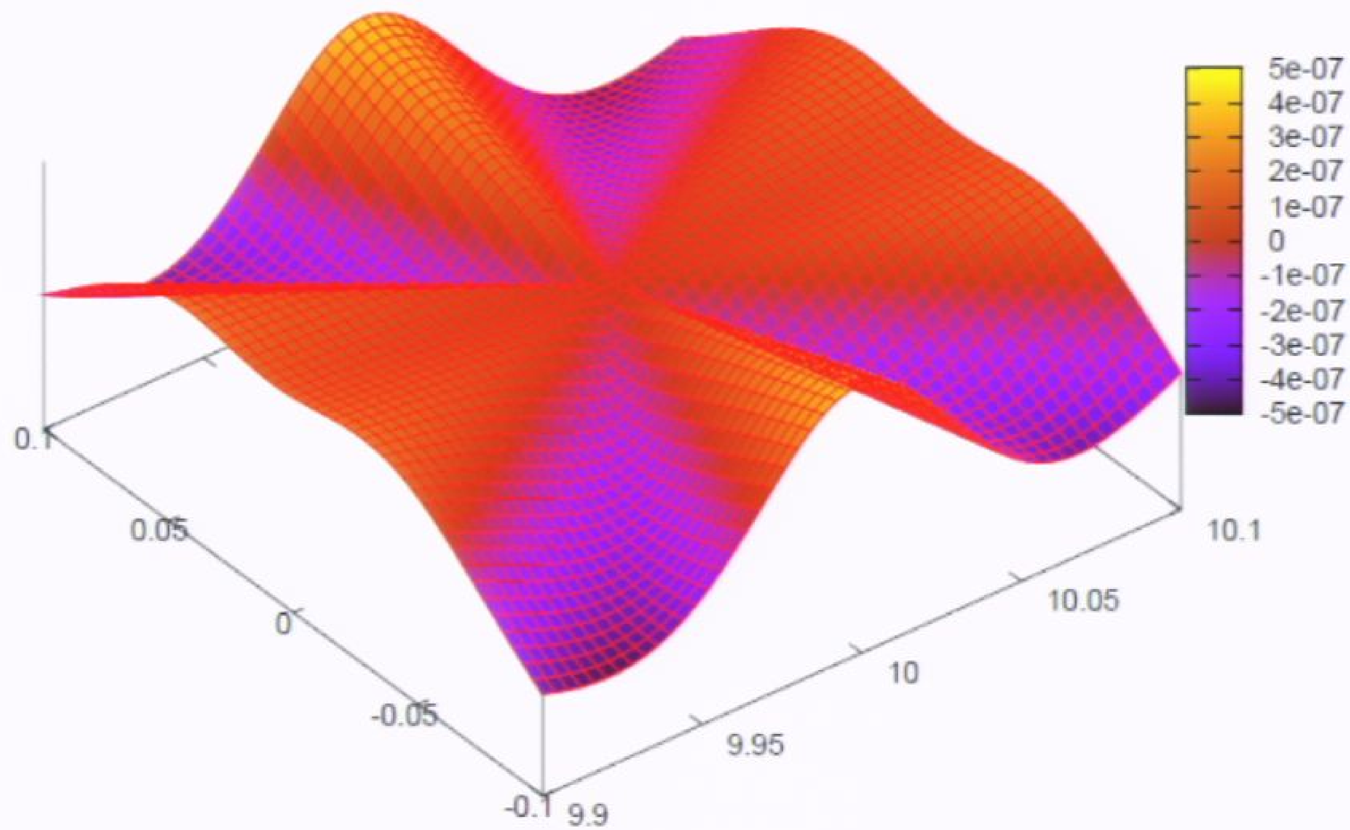


The effective source along the ray $(x = 0, y = 0.01M)$, before and after

Effective source for a circular orbit in Schwarzschild



The effective source along the equatorial plane. (This is for a circular orbit, but it

ake home message

- C/C++ code for an effective scalar charge moving along generic geodesics of Schwarzschild . (Kerr is to follow quickly).
- Interpolation effectively handles the errors arising from the delicate cancellations that occur close to the particle.
- With Peter Diener's (3+1) finite difference code, self-consistent evolution for a scalar point charge is now possible.