Title: Gravitational self-force effect on the periapsis advance in Schwarzschild spacetime

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Abstract: TBA

Gravitational self-force effect on the periapsis advance in Schwarzschild spacetime

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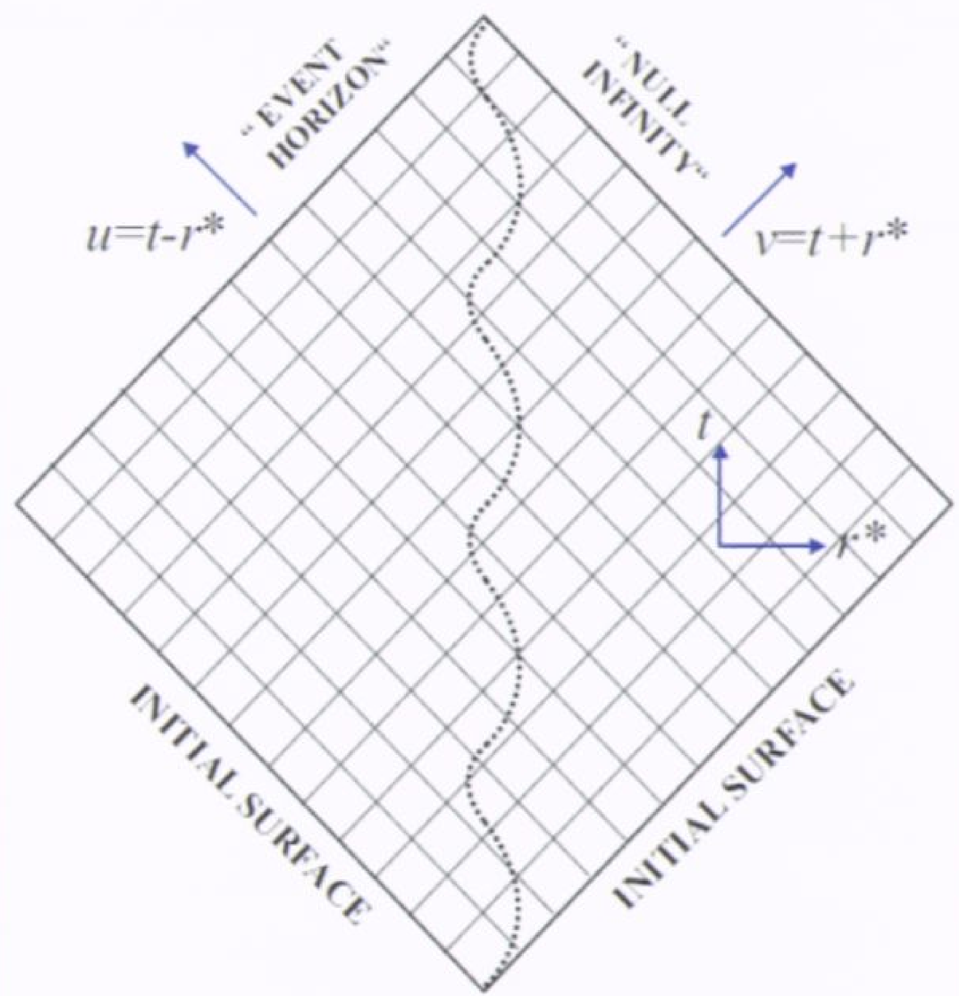
Perimeter Institute

Outline of this talk

- Self-force calculation code
- Periapsis advance (test particle case)
- Conservative SF correction in periapsis advance
- Dissipative effect on periapsis advance
- Summary & Future work

Self-force calculation code

[Barack and NS, 2007, 2010]



Orbits

- Eccentric orbits

Metric perturbation

- Lorenz gauge
- (1+1)-dim. time domain
- Double null coordinates
- Uniform gridspace
- 4th order scheme

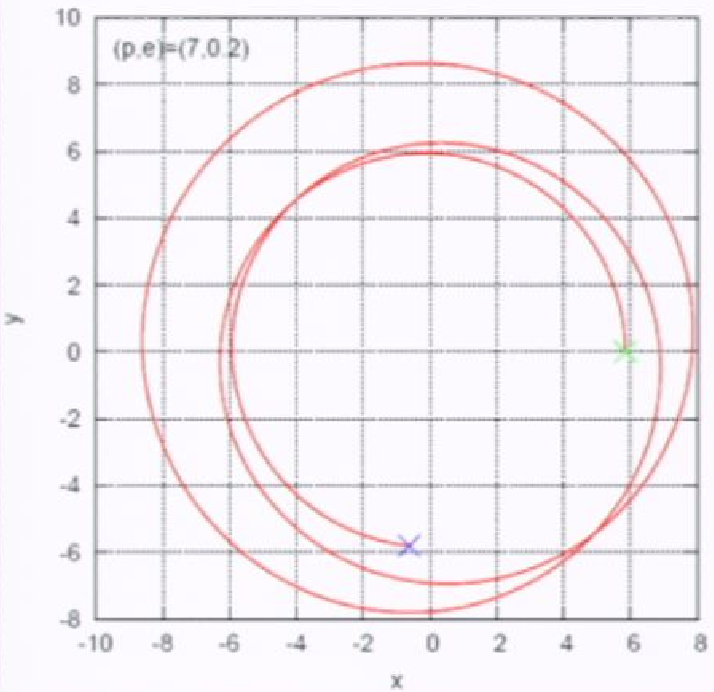
SF calculation

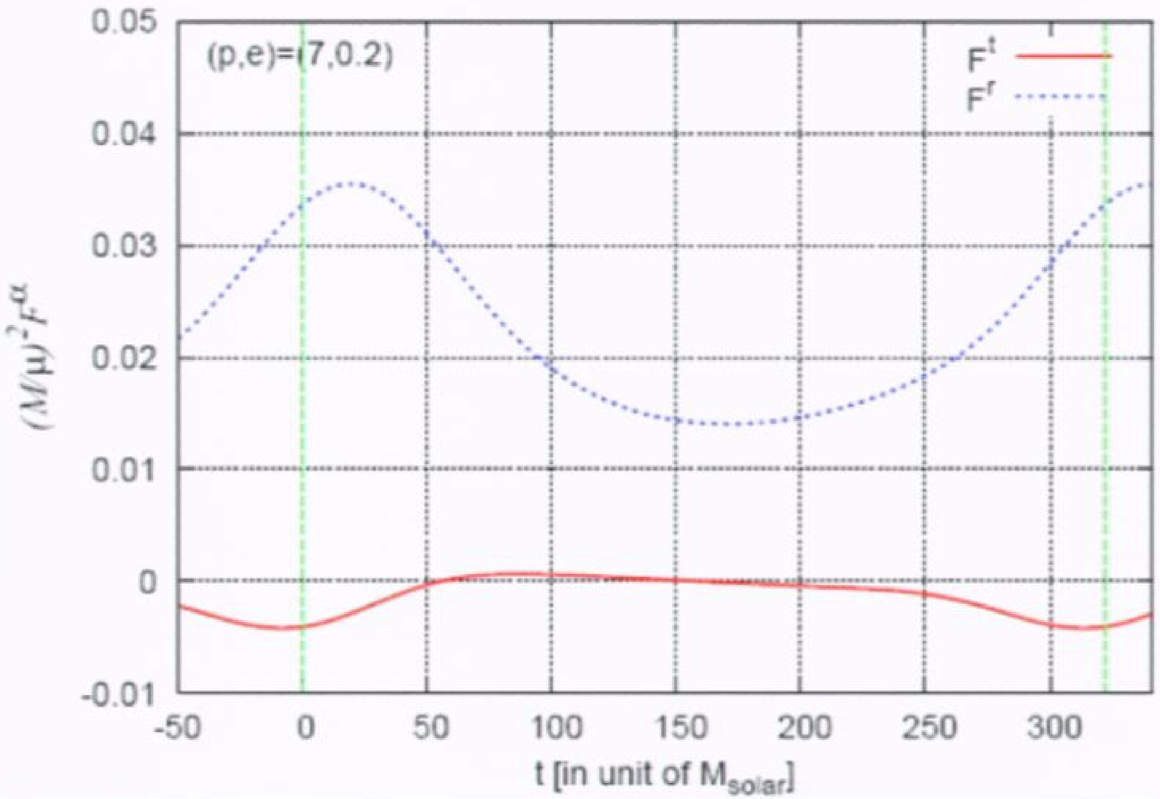
- Mode-sum scheme

$$F^{\alpha}(\tau) = \sum \left[F_{\text{full}}^{\alpha,l}(x_p) - A^{\alpha} L - B^{\alpha} \right]$$

Self-force calculation code

example: $(p,e)=(7,0.2)$

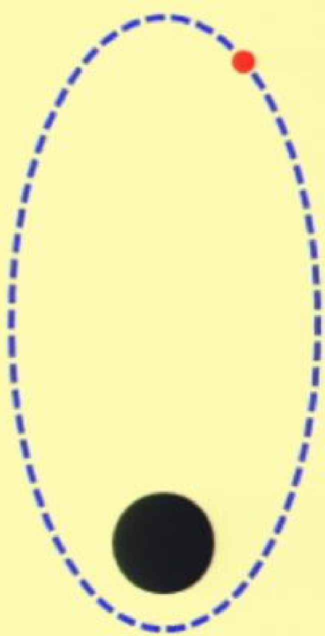




Main topic - SF correction in periapsis advance -

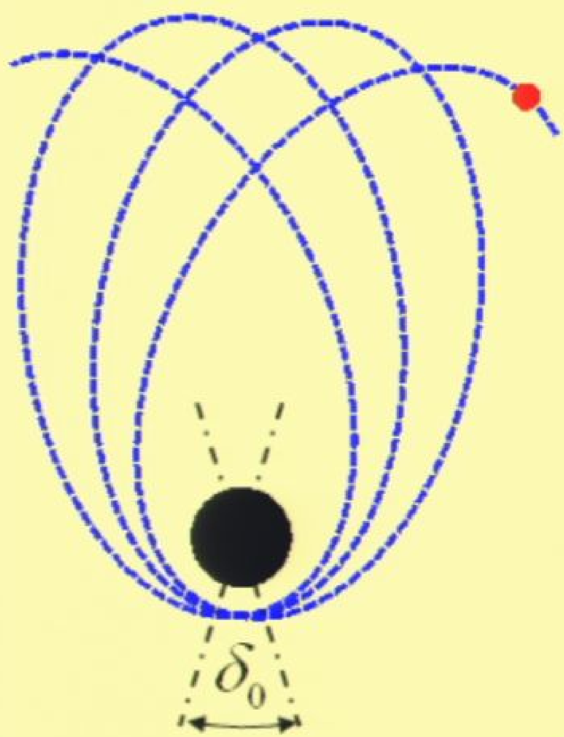
<u>Newtonian</u>

$$\frac{\Omega_\varphi}{\Omega_r} = 1$$



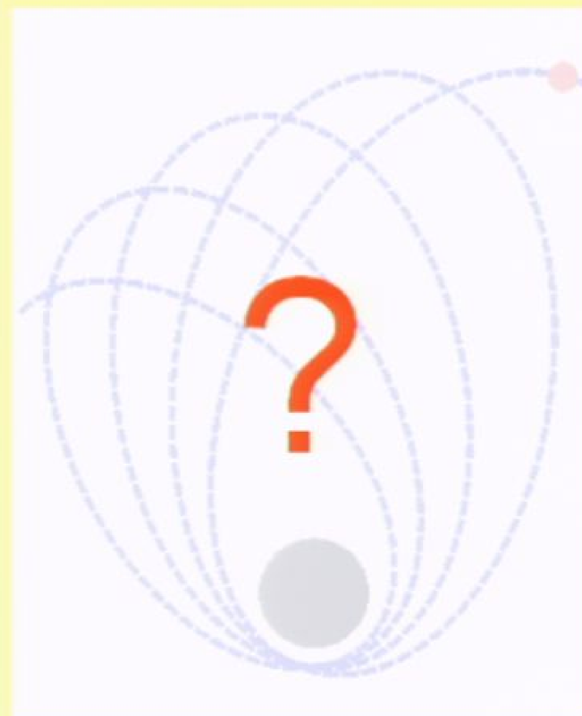
<u>GR (geodesic)</u>

$$\frac{\Omega_\varphi}{\Omega_r} = 1 + \frac{\delta_0}{2\pi}$$



<u>GR (SF included)</u>

$$\frac{\Omega_\varphi}{\Omega_r} = 1 + \frac{\delta_0 + \delta_{SF}}{2\pi}$$

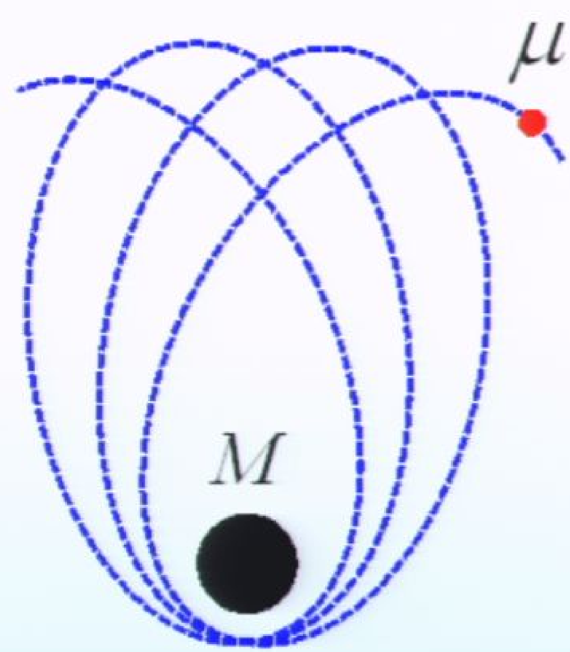


Geodesic equations for eccentric orbit

Radial phase parameter χ

$$r_p(\chi) = \frac{pM}{1 + e\cos\chi} \qquad (\chi = 0 \text{ at periapsis})$$

$0 \le \chi \le 2\pi$ corresponds to one radial period.



Equation of motion

$$\frac{dt_p}{d\chi}(r_p; E_0, L_0) = \frac{E_0}{f_p}\left[E_0^2 - f_p\left(1 + \frac{L_0^2}{r_p^2}\right)\right]^{-1/2}\left(\frac{dr_p}{d\chi}\right)$$

$$\frac{d\varphi_p}{d\chi}(r_p; E_0, L_0) = \frac{L_0}{r_p^2}\left[E_0^2 - f_p\left(1 + \frac{L_0^2}{r_p^2}\right)\right]^{-1/2}\left(\frac{dr_p}{d\chi}\right)$$

μ : Mass of particle

M : Mass of central BH

p : Semi-latus rectum

e : Eccentricity

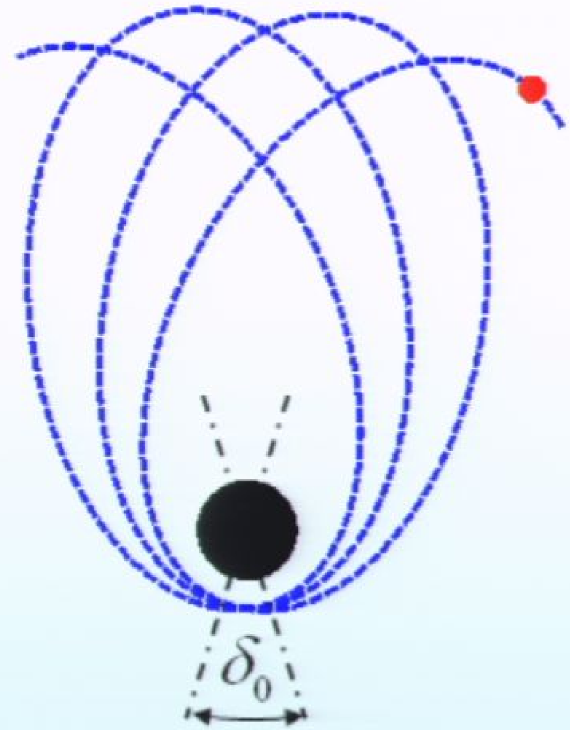
where E_0 and L_0 are the specific energy and angular momentum:

Periapsis advance (test particle case)

Periapsis advance

$$T_r \equiv \int_0^{2\pi} \left(\frac{dt_p}{d\chi} \right) (\chi; E_0, L_0) d\chi$$

$$\Delta\varphi \equiv \int_0^{2\pi} \left(\frac{d\varphi_p}{d\chi} \right) (\chi; E_0, L_0) d\chi$$



The periapsis advance can be obtained by:

$$\delta_0(p, e) = \Delta\varphi - 2\pi$$

Asymptotic form

For large p limit

$$\delta_0 = \frac{\pi}{p}\left[6 + \left(27 + \frac{3}{2} e^2 \right) \frac{1}{p} + O(p^{-2}) \right]$$

For small e limit

$$\delta_0 = 2\pi \left[\sqrt{\frac{p}{p-6}} - 1 \right] + O(e^2)$$

Conservative part of self-force

[Hinderer-Flanagan '08]

Here we define the conservative piece of SF as that derived from the symmetric field (half retarded + half advanced):

$$F^{\alpha}_{\rm cons} = \tfrac{1}{2}\left[F^{\alpha}_{\rm ret} + F^{\alpha}_{\rm adv}\right]$$

Using the symmetry of geodesics with respect to the transformation of $(\chi, t, r, \theta, \varphi) \to (-\chi, -t, r, \theta, -\varphi)$, we can obtain the relation between the retarded and advanced SF as

$$F^{r}_{\rm ret}(-\chi) = F^{r}_{\rm adv}(\chi)\,,\ \ F^{t/\varphi}_{\rm ret}(-\chi) = -F^{t/\varphi}_{\rm adv}(\chi)$$

$$F^{r}_{\rm cons}(\chi) \equiv \tfrac{1}{2}\left[F^{r}(\chi) + F^{r}(-\chi)\right]$$

$$F^{t/\varphi}(\chi) \equiv \tfrac{1}{2}\left[F^{t/\varphi}(\chi) - F^{t/\varphi}(-\chi)\right]$$

Conservative SF correction in periapsis advance

t- and φ- components of EOM including the conservative SF

$$\frac{d}{d\tau}E = -F_t^{\mathrm{cons}}\ ,\ \frac{d}{d\tau}L = -F_\varphi^{\mathrm{cons}}$$

Integrating these equations, we obtain:

$$E_0 \to E(\chi) = E_0 + \delta E(0) - \int_0^\chi F_t^{\mathrm{cons}}(\chi')(d\tau/d\chi)d\chi'$$

$$L_0 \to L(\chi) = L_0 + \delta L(0) + \int_0^\chi F_\phi^{\mathrm{cons}}(\chi')(d\tau/d\chi)d\chi'$$

E,L are not constant along the orbit, but periodic.

$\delta E(0)$: correction at χ=0

$\delta L(0)$ (in terms of SF)

Conservative SF correction in periapsis advance

Correction of periapsis advance

$$\Delta\varphi = \int_0^{2\pi} \left(\frac{d\varphi_p}{d\chi}\right)(\chi; E, L)\, d\chi$$

$$= \int_0^{2\pi} \left[\left(\frac{d\varphi_p}{d\chi}\right)(\chi; E_0, L_0) + \delta\left(\frac{d\varphi_p}{d\chi}\right)(\chi; E_0, L_0) \right] d\chi$$

$$\delta_0(p,e) + 2\pi$$

Leading (geodesic) term

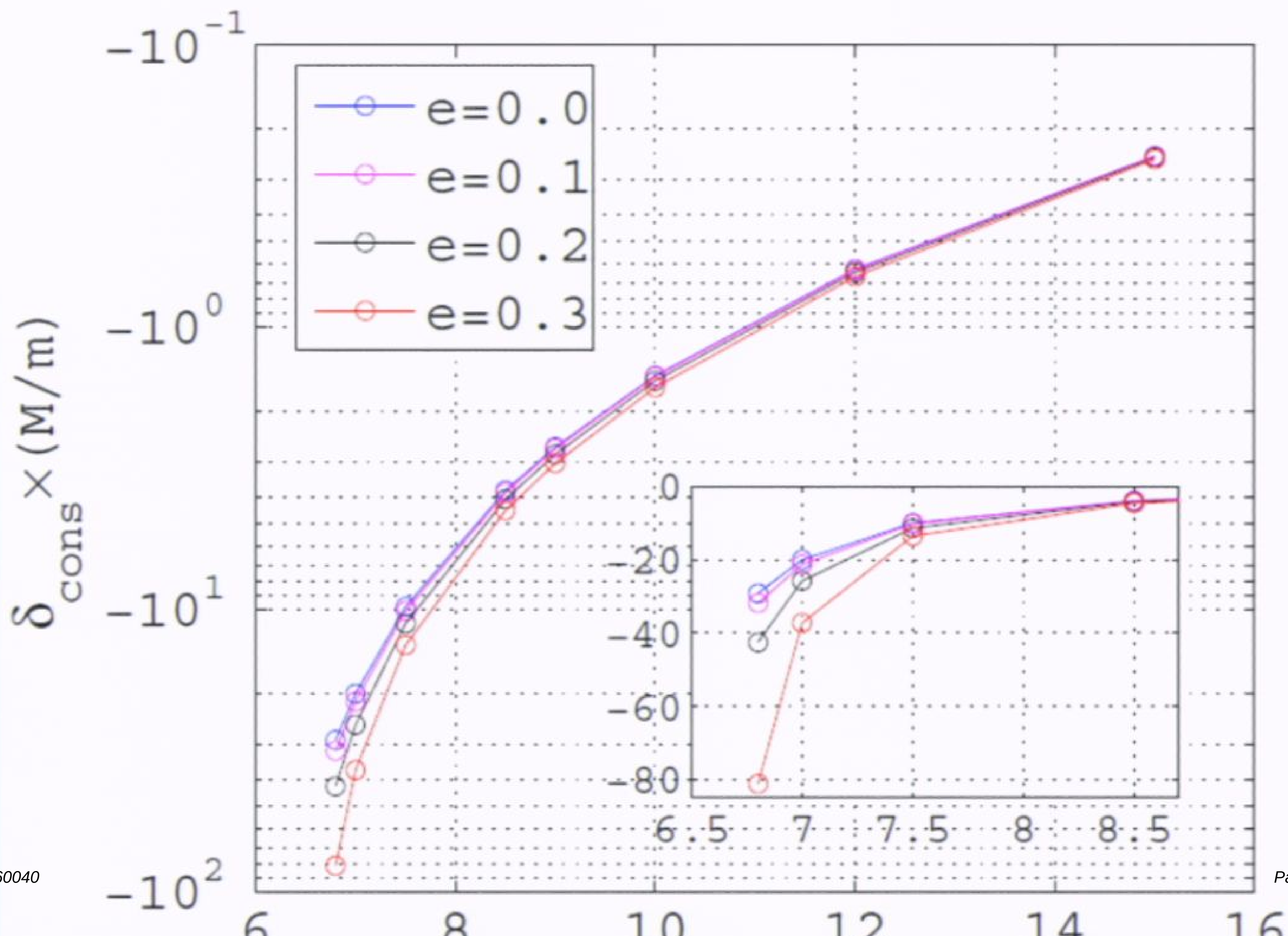
$$\delta_{\rm cons}(p,e)$$

Conservative correction term

$$\delta\left(\frac{d\varphi_p}{d\chi}\right) \equiv \frac{\partial}{\partial E}\left(\frac{d\varphi_p}{d\chi}\right)\Bigg|_0 \left[\delta E(0) - \int_0^{\chi} F_t^{\rm cons}(\chi')(d\tau/d\chi)d\chi'\right] + \frac{\partial}{\partial L}\left(\frac{d\varphi_p}{d\chi}\right)\Bigg|_0 \left[\delta L(0) + \int_0^{\chi} F_\phi^{\rm cons}(\chi')(d\tau/d\chi)d\chi'\right]$$

The conservative correction can be described in terms double

Conservative SF correction in periapsis advance



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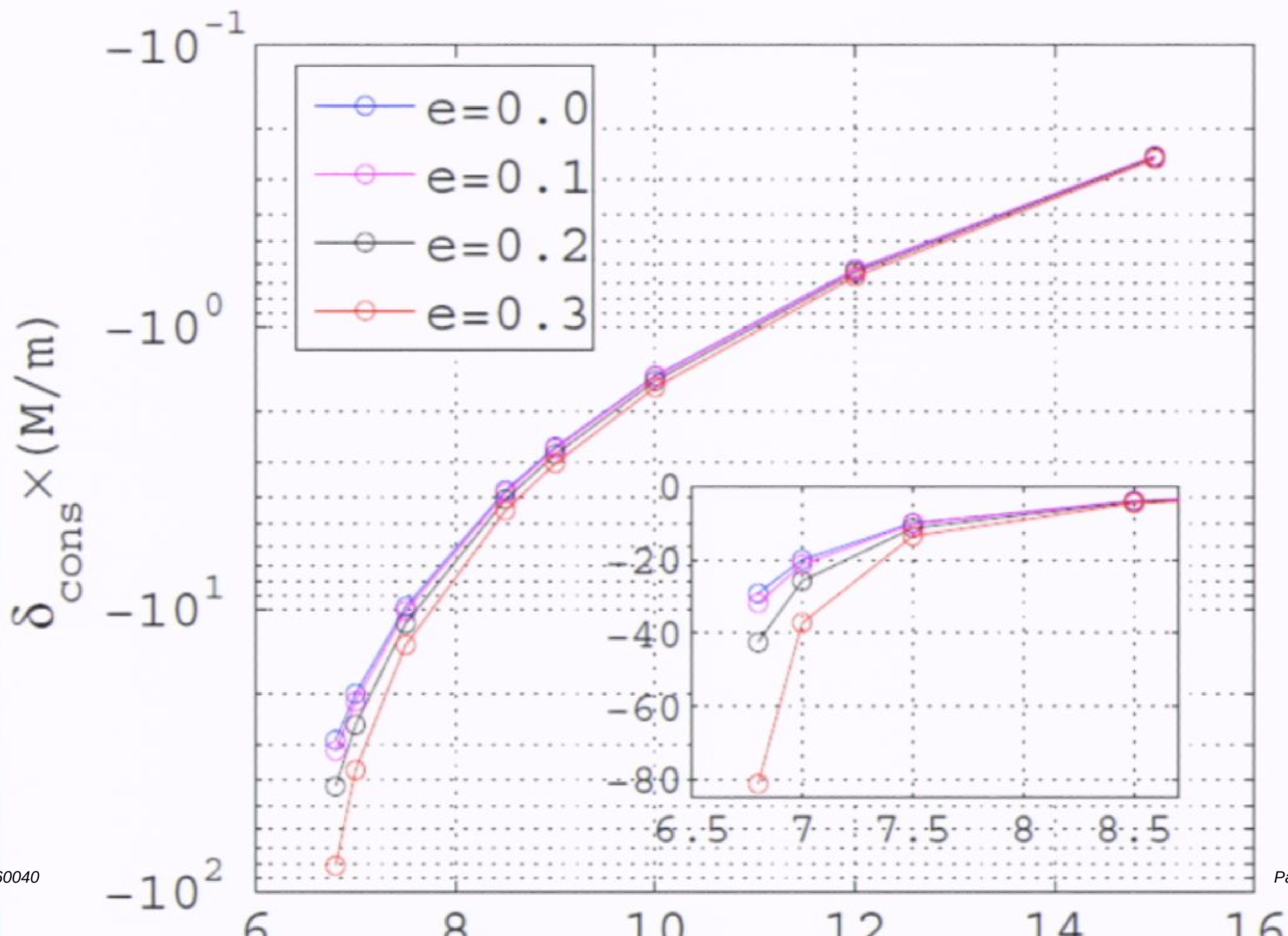
Conservative SF correction in periapsis advance



Comparison with post-Newtonian (circular limit)

Consider the ratio of two frequency at the circular limit:

Ω_φ : azimuthal frequency

Ω_r : radial frequency

$$W \equiv \left(\frac{\Omega_r}{\Omega_\varphi}\right)^2$$

The SF correction in 'W' can be given by

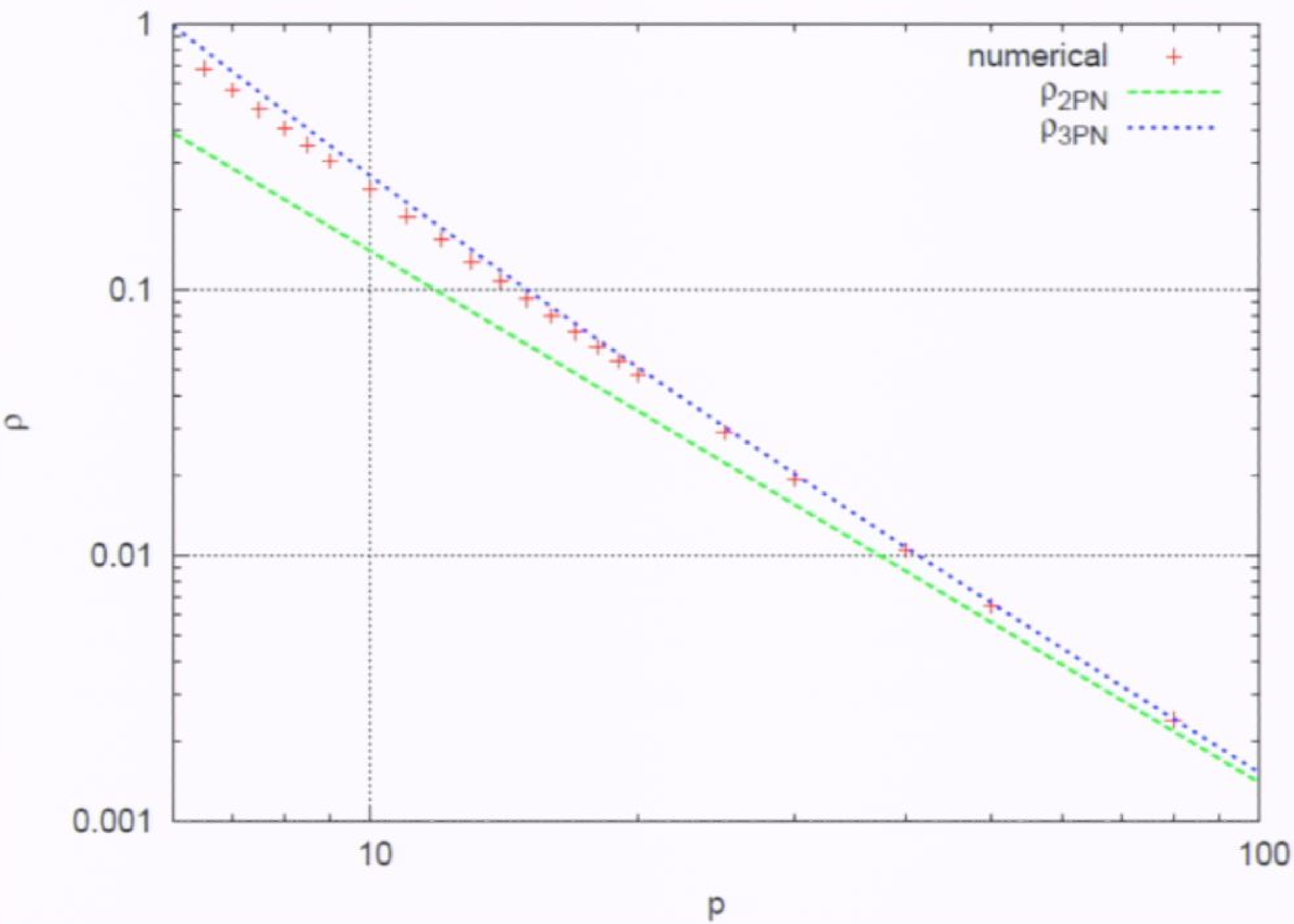
$$\rho \equiv \Delta_{SF} W = -\frac{\delta_{SF}}{\pi}\left(1 - \frac{6}{p}\right)^{3/2}$$

<u>Post-Newtonian formula of ρ for circular limit:</u> [Damour '09]

$$\rho_{2\mathrm{PN}} = \frac{14}{p^2}$$

$$\rho_{\cdots} = \frac{14}{\cdots} + \left(\frac{397}{\cdots} - \frac{123}{\cdots}\pi^2\right)\frac{1}{\cdots}$$

Comparison with post-Newtonian (circular limit)



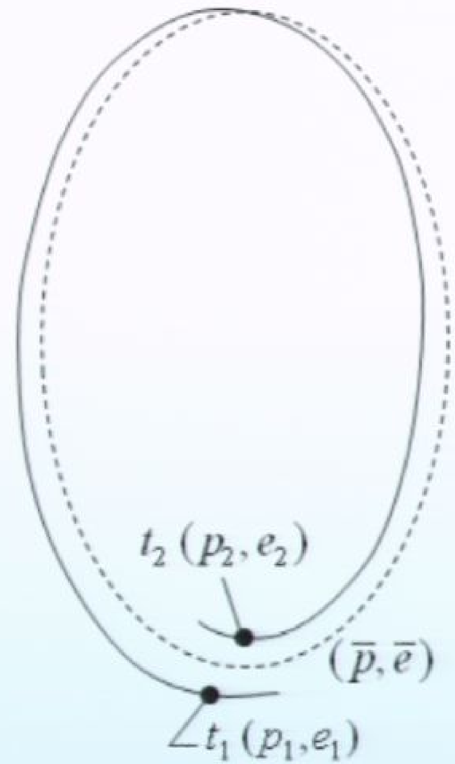
Dissipative effect on periapsis advance

In reality, the parameters (p,e) changes in time due to the dissipation. Even in this case, we can define the periapsis advance:

$$\Delta\varphi_{\rm true}(t_1;t_2) = \int_{t_1}^{t_2} \dot\varphi(p(t),e(t))dt$$

$$= \int_{t_1}^{t_2} \left[\dot\varphi(\bar p,\bar e) + \partial_{\bar p}\dot\varphi(\bar p,\bar e)(p(t)-\bar p) + \partial_{\bar e}\dot\varphi(\bar p,\bar e)(e(t)-\bar e)\right]dt$$

t_1, t_2 : consecutive radial turning points with $\dot u^r > 0$



$\bar p = \frac{1}{2}[p_1 + p_2]$

$\bar e = \frac{1}{2}[e_1 + e_2]$

$[\text{1st term}] = \Delta\varphi_{\rm cons}(\bar p,\bar e) + O(\mu^2)$

$[\text{2nd, 3rd term}] = O(\mu^2)$

$$\Delta\varphi_{\rm true}(t_1;t_2) = 2\pi + \delta_0(\bar p,\bar e) + \delta_{\rm cons}(\bar p,\bar e) + O(\mu^2)$$

The periapsis advance is not affected by dissipation explicitly

Summary & Future work

This work

Derive the SF correction in the periapsis advance

$$\Delta\varphi_{\text{true}}(t_1;t_2) = 2\pi + \delta_0(\bar{p},\bar{e}) + \delta_{\text{cons}}(\bar{p},\bar{e}) + O(\mu^2)$$

- Conservative effect is derived in terms of integrals of SF
- Dissipative effect is at the higher order $O(\mu^2)$.

Future work

Comparison to other methods (NR, PN, EOB, …)
For this, we need some gauge invariant quantities.

- Circular limit [Barack, Damour, NS]
  Described by only one orbital parameter (p).
  We can use the angular frequency instead of p.
- Eccentric orbit [Barack and NS]
  Described by two parameters (p, e).