Title: Time-domain computations of the self-force on a scalar charged particle in eccentric orbits

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Abstract: TBA

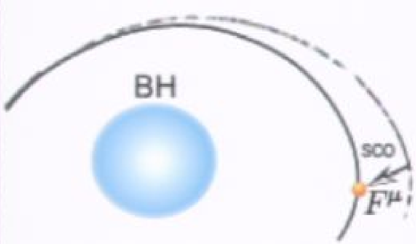
Outlook

- Review of the Extreme−Mass Ratio Inspiral (EMRI) problem.
- The Particle−without−Particle scheme
- Results from the simulations.
- Conclusions and future work.

The EMRI problem

path to EMRIs in 7 points

The inspiral of a stellar compact object (SCO), $m \sim 1 - 50\ M_\odot$, into a massive black hole (BH), $M \sim 10^4 - 10^7\ M_\odot$, will be one of the main sources of gravitational waves for the Laser Interferometer Space Antenna (LISA).



2. Due to their extreme mass-ratio, $m/M \sim 10^{-7} - 10^{-3}$,EMRIs can be treated in the frame of perturbation theory where the back-reaction is pictured as the action of a local self-force.

3. An analogous EMRI problem consists in a scalar point particle q orbiting in a geodesic of a non-rotating MBH spacetime (Schwarzschild). This framework provides us of a testbed for numerical codes to compute the gravitational self-force

$$-\rho 4\pi = g^{\mu\nu}\nabla_\mu\nabla_\nu\Phi^{ret} = \Box\Phi^{ret} \quad \longrightarrow \quad F^\mu = m\frac{du^\mu}{d\tau} = q(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu\Phi^{ret}$$

$$\rho = -4\pi q \int \delta_4[x - z(\tau)] d\tau \qquad u^\mu = \frac{dz^\mu}{\ldots}$$

..A path to EMRIs in 7 points

4. Due to the spherical symmetry of this system the retarded field can be decomposed in spherical harmonics that decouples as:

$$\Phi^{ret} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \Phi^{lm}(t,r) Y^{lm}(\theta,\varphi)$$

5. The point–like character of the particle leads to singularities in the retarded field and by extension in the self–force. Thus, the self–force must be regularized and we use the mode-sum regularization scheme[1]:

$$\Box\Phi^{ret} = \Box(\Phi^S + \Phi^R) \begin{cases} \Box\Phi^S = -4\pi q \delta(z) \\ \Box\Phi^R = 0 \end{cases} \longrightarrow \mathcal{F}_\alpha = q(\nabla_\alpha \Phi^{ret} - \nabla_\alpha \Phi^S) = q\nabla_\alpha \Phi^R$$

This scheme provides an analytic expression for the ℓ–modes of the singular field.

6. We need a numerical method to compute the full retarded field and by applying the mode–sum regularization scheme obtain the self–force.

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7.

- We have developed a multidomain numerical code which avoids working with the spatial scale associated with the q (SCO).
- Due that it is a time-domain method, we can deal easily with eccentric orbits.

Solving the Field Equation:

The Particle-without-Particle Scheme

We perform a division of the spatial computational domain into two disjoints regions or subdomains, one at the left of the particle $r^* < r^*_p$ and other at the right of the particle $r^* > r^*_p$ ($r^* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$):

- Locating the particle at the interface between subdomains:
  - ★ We avoid the problems associated with the singularity of the source term.
  - ★ We evolve independent homogeneous wave equations inside each region.
- The solutions have to be matched across the boundaries.

Looking for the matching conditions

- Since discontinuities in hyperbolic equations can only appear along characteristics, we adopt a hyperbolic formulation of the field equation by defining the retarded field variables:

$$\psi^{\ell m} = r\,\Phi^{\ell m}$$
$$\phi^{\ell m} = \partial_t \psi^{\ell m} \qquad \longrightarrow \qquad \mathcal{U} = (\psi^{\ell m}, \phi^{\ell m}, \varphi^{\ell m})$$
$$\varphi^{\ell m} = \partial_{r^*} \psi^{\ell m}$$

Thus, introducing them in the master equation, we obtain a system of PDEs

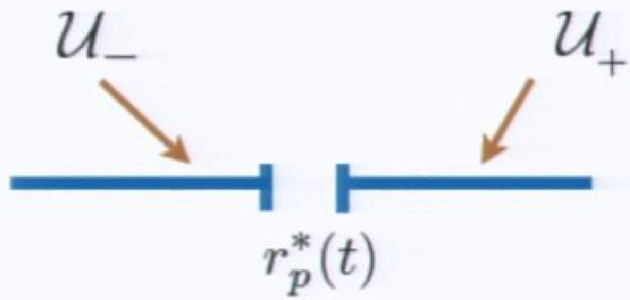
$$\partial_t \mathcal{U} = \mathbb{A}\cdot\partial_{r^*}\mathcal{U} + \mathbb{B}\cdot\mathcal{U} + \mathcal{S}$$

$$\mathbb{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \mathbb{B} = \begin{pmatrix} 0 & 1 & 0 \\ -V_\ell & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \mathcal{S} = \left(0, -\frac{S^{\ell m}}{f(r_p)}\delta(r^* - r^*_p(t)), 0\right).$$

This satisfied at each subdomain independently.

..Looking for the matching conditions

- We split the field variables into two contributions, one at the left and other at the right of the particle:



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$$\partial_t \mathcal{U} = \mathbb{A} \cdot \partial_{r^*} \mathcal{U} + \mathbb{B} \cdot \mathcal{U} + \mathcal{S}$$

- We obtain the homogeneous wave equations at each side of the particle:

$$\partial_t \mathcal{U}_\pm = \mathbb{A} \cdot \partial_{r^*} \mathcal{U}_\pm + \mathbb{B} \cdot \mathcal{U}_\pm$$

and the matching conditions at the particle location:

$$[\mathcal{U}] = \lim_{r^* \to r^*_p} \mathcal{U}_+ - \lim_{r^* \to r^*_p} \mathcal{U}_-$$

...Looking for the matching conditions

- We found that the matching conditions at the particle location are given by[2]:

Circular Orbit[3]: $r_p^* = constant$

$$[\psi^{\ell m}]_p = 0\,,$$

$$\left[\partial_t \psi^{\ell m}\right]_p = 0$$

$$\left[\partial_{r^*} \psi^{\ell m}\right]_p = \frac{S^{\ell m}}{f(r_p)}$$

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2 C. F. Sopuerta & P. Laguna '06; 3 P. Canizares & C. F. Sopuerta '09; 4 P. Canizares, C. F. Sopuerta, J. L. Jaramillo '10

Implementation of the PwP scheme:
The multidomain pseudospectral method

- Solving the set of PDEs numerically:
  - The pseudospectral collocation (PSC) method to discretize in space.
  - We use a Runge–Kutta method for the time evolution.

- With **PSC methods** the solutions are approximated by an expansion in a basis of Chebyshev polynomials $\{T_n(X)\}$:

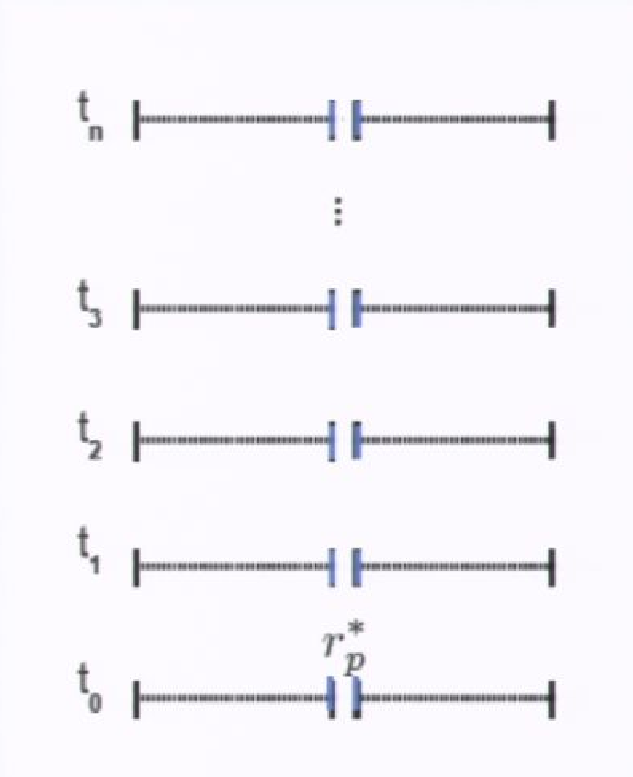
$$\mathcal{U}_N(t, r^*) = \sum_n^N a_n(t) T_n(r^*)$$

where $a_n(t)$ are the spectral coefficients.

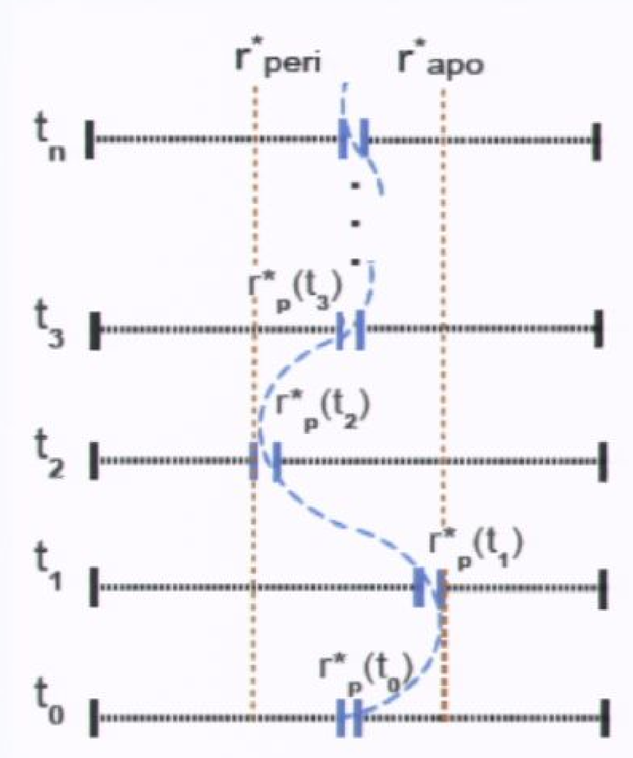
★ A property of the PSC method provides **exponential convergence with N for smooth functions.**

★ The key point of the method is to keep the particle at the interface between subomains

Circular Orbit



Eccentric Orbit



For eccentric orbits, we maintain the particle at the interface by using a time dependent mapping between the physical and spectral domains.

he multidomain pseudospectral method consist in:

- The spatial computational domain is divided in several subdomains.
- The solutions are expanded in a basis of Chebishev polynomials at each subdomain independently.
- The particle is set at the interface between two subdomains.
- We solve homogeneous waves equations inside each subdomain.
- The solutions are communicated across the boundaries of the subdomains imposing the jump conditions.

... The multidomain pseudospectral method:

- The solutions are communicated across the subdomains using two different methods:
  - The penalty method: the system is dynamically driven to fulfill a set of additional conditions. That is, we introduce constant terms which are proportional to the junction conditions
  - The direct communication of the characteristic fields: The subdomains are communicated by passing the value of the characteristic fields.

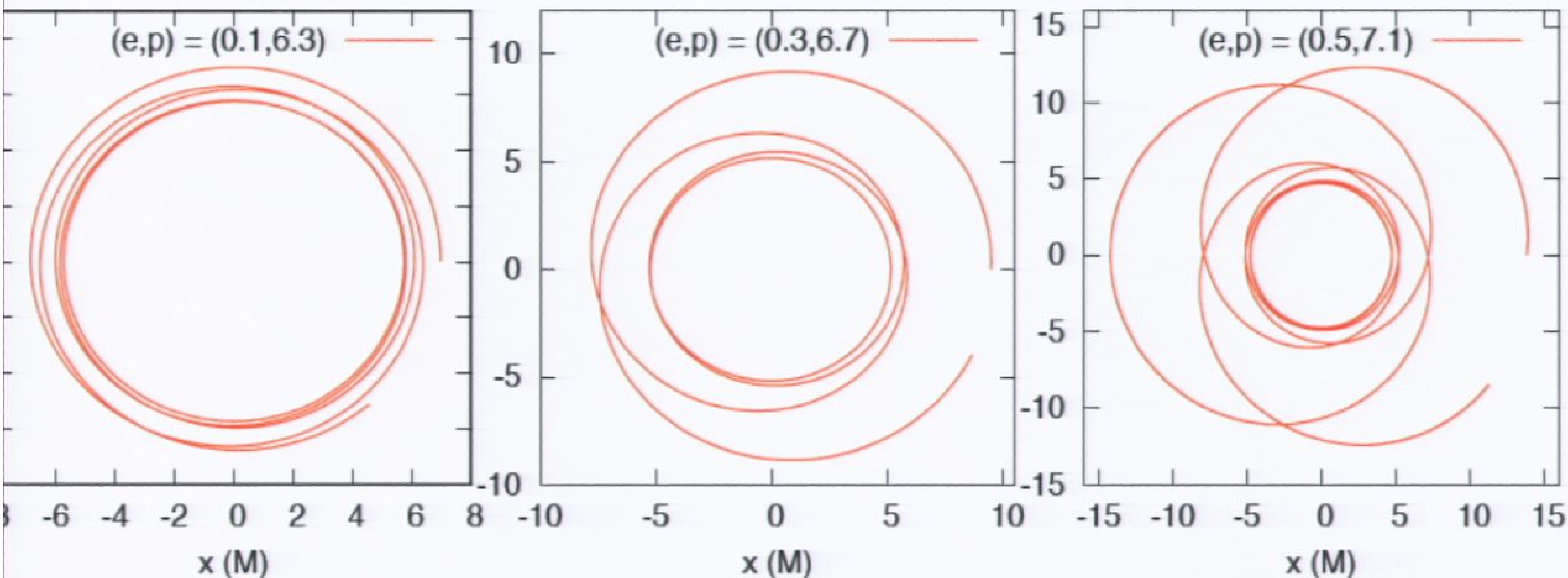
One advantage of the multidomain pseudospectral method is that by performing further divisions on the spatial computational domain, the resolution of our solutions are improved

★ In order to achieve a determined degree of resolution in our results, it is computationally cheaper to introduce further subdomains with a relatively small N than increase N in a subdomain.

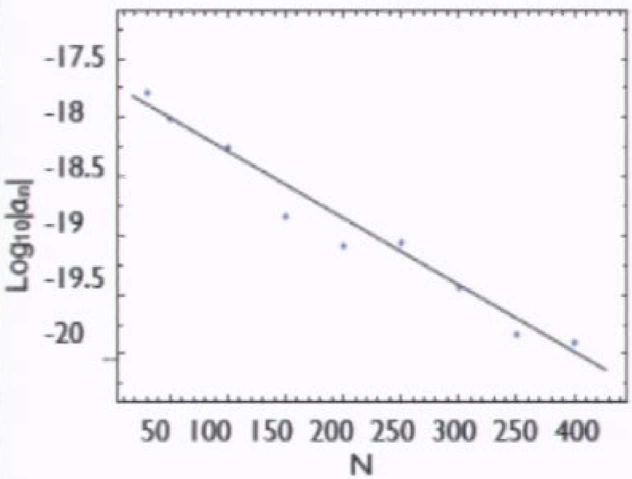
Results from the simulations

- We have computed the derivatives of the regular field for eccentric orbits with different eccentricity (e) and semilatus rectum (p)

SCO Orbits around the MBH

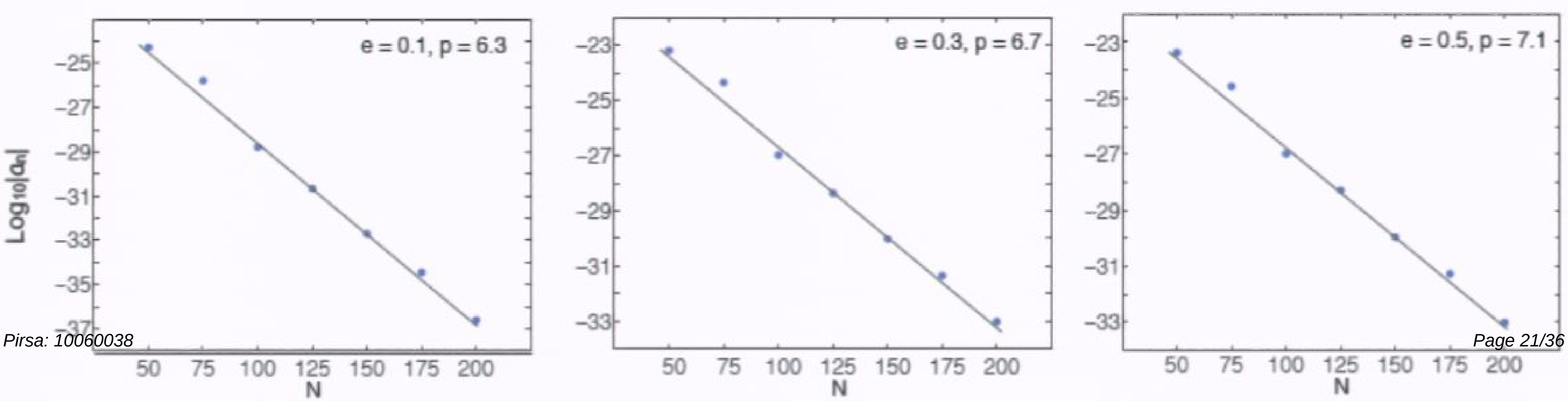


Circular Orbit

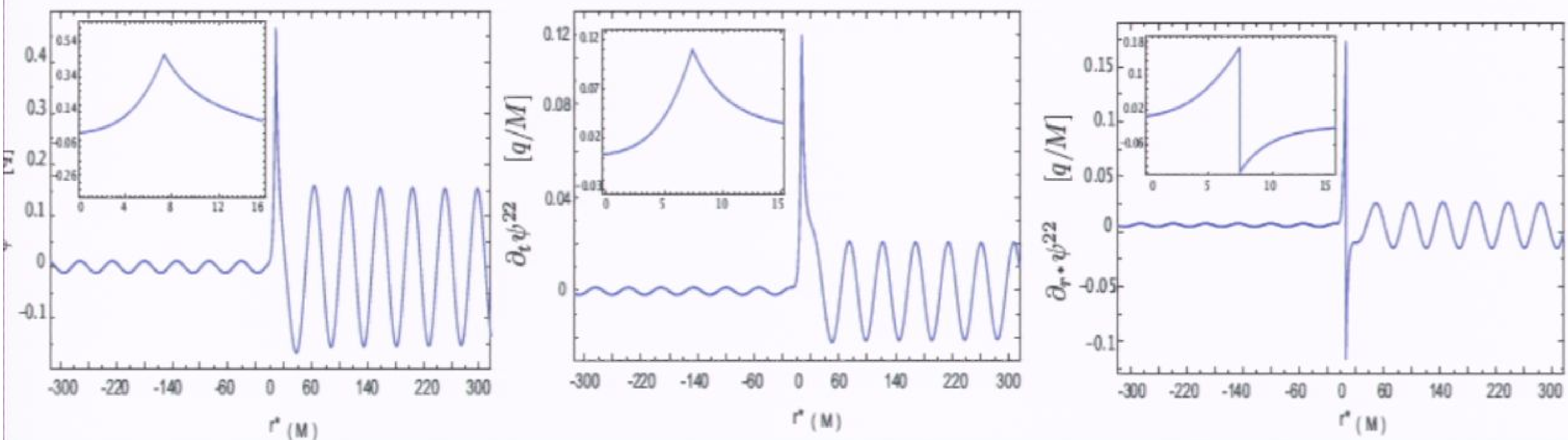


The dependence of the truncation error ($\sim |a_N|$) with respect increasing numbers of collocation points, N, give us an estimation of the exponential convergence of the code: e^{-N}

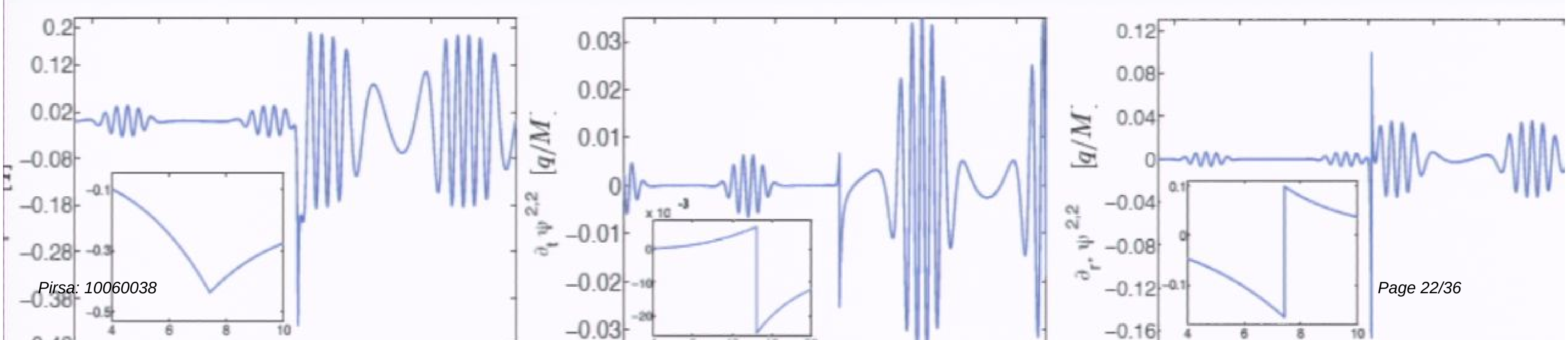
Eccentric Orbit



Snapshots from the Circular case (D=12, N=50)



Snapshots from the Eccentric (e=0.5, p= 7.1) case (D=10, N= 100)



- Results for the retarded field derivatives obtained using the penalty method: D = 12, N = 50

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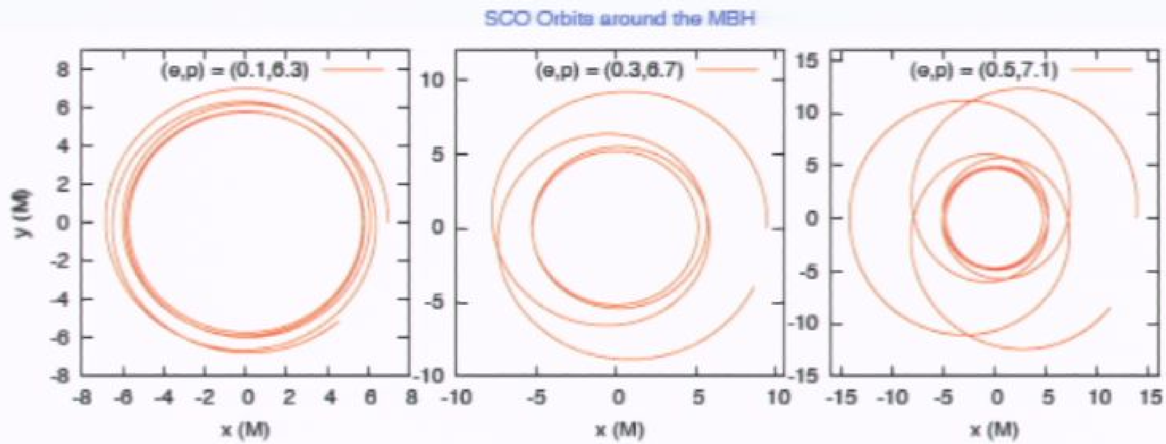
Circular Orbit

M)	Component of Φ^{R}_{α}	Estimation using the PSC Method	Estimation from Frequency-domain (a,b)	Error relative to Frequency-domain (a,b)	Error relative to Time-domain (c)
	$(\Phi^{R,-}_{t}, \Phi^{R,+}_{t})$	$(3.60777, 3.60778) \cdot 10^{-4}$	$3.609072 \cdot 10^{-4}$	$(0.03, 0.03)\%$	$(0.12, 0.12)\%$
	$(\Phi^{R,-}_{r}, \Phi^{R,+}_{r})$	$(1.67364, 1.67362) \cdot 10^{-4}$	$1.67728 \cdot 10^{-4}$	$(0.2, 0.2)\%$	$(0.18, 0.18)\ \%$
	$(\Phi^{R,-}_{\varphi}, \Phi^{R,+}_{\varphi})$	$(-5.3042, -5.3044) \cdot 10^{-3}$	$-5.304231 \cdot 10^{-3}$	$(4 \cdot 10^{-4}, 10^{-3})\ \%$	$(6 \cdot 10^{-4}, 10^{-3})\%$

(a) [Diaz-Rivera et al. PRD 70, 124018 (2004)] , (b) [Haas, Poisson. PRD 74, 044009 (2006)] (c) [Hass. PRD 75, 124011 (2007)]

Values of the retarded field obtained near the pericenter and obtained using the direct communication of the characteristic fields: D = 80, N = 50



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	$(e,p) = (0.0, 6.0)^a$	$(e,p) = (0.1, 6.3)$	$(e,p) = (0.3, 6.7)$	$(e,p) = (0.5, 7.1)$
$r_p\ (M)^c$	6.0	5.7272925	5.1538801	4.7333989
$\frac{M^2}{q}\,\Phi^R_t$	$3.609072 \cdot 10^{-4}$	$4.5171 \cdot 10^{-4}$	$7.6980 \cdot 10^{-4}$	$1.2330 \cdot 10^{-3}$
$\frac{M^2}{q}\,\Phi^R_r$	$1.67728 \cdot 10^{-4}$	$2.1250 \cdot 10^{-4}$	$3.6339 \cdot 10^{-4}$	$5.6122 \cdot 10^{-4}$
$\frac{M}{q}\,\Phi^R_\varphi$	$-5.304231 \cdot 10^{-3}$	$-6.2040 \cdot 10^{-3}$	$-9.0402 \cdot 10^{-3}$	$-1.2685 \cdot 10^{-2}$

Conclusions & Future work

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- We have developed a new time-domain technique for the simulations of eccentric EMRIs:
  - Avoids the introduction of a small scale in our code, and provides precise determination of the field and its derivatives near an on the SCO.
  - It is an efficient method to make time-domain computations of the self-force because preserves the properties of the PSC method.
- We would like to apply these techniques to the gravitational and Kerr cases.

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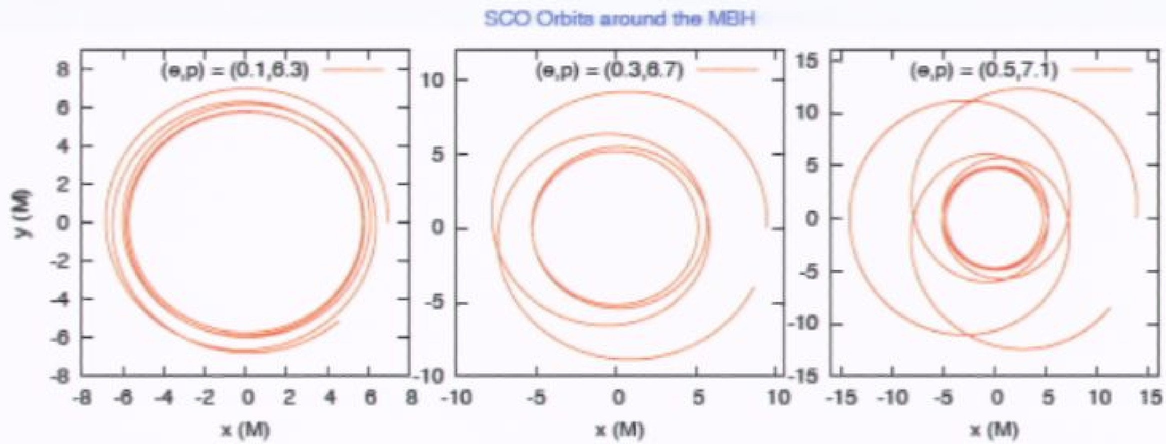

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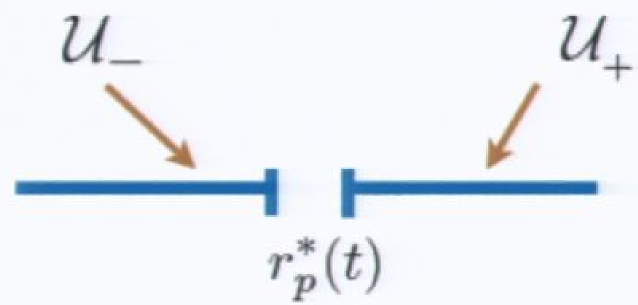

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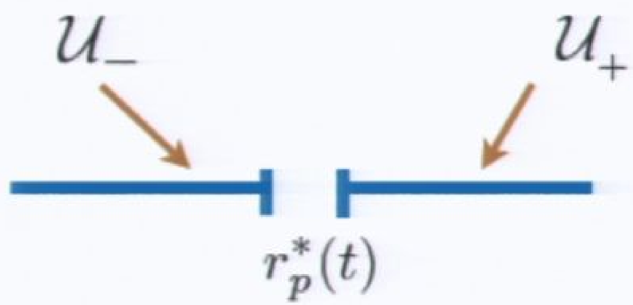

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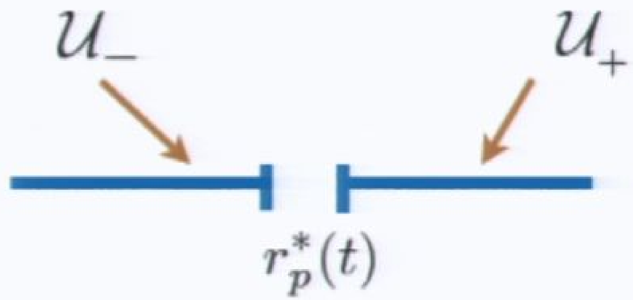

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