

Title: Self force integrated orbits around a Schwarzschild black hole and Lorenz gauge calculations of the self force

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Abstract: TBA

# Self force integrated orbits around a Schwarzschild black hole and Lorenz gauge calculations of the self force

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Work with Kristen Lackeos and Gaurav Khanna

# How does the orbit evolve under the self force effect?

$$\frac{dx^\mu}{d\tau} = \left. \frac{dx^\mu}{d\tau} \right|_{\text{geodesic}} + \int f_{\text{RR}}^\mu(\tau) d\tau$$

Actual integration of the orbit has to be more subtle than that:  
For quasi-circular orbits in Schwarzschild - non-zero  $u_r$   
even if  $f_r = 0$

$$u_r = u_r^{\text{geod}} + u_r^{(1)} + u_r^{(2)}$$

$u_r^{(1)}$ : from the dissipation effect of  $f_t$  and  $f_{\text{phi}}$  - find  $u_t$  and  $u_{\text{phi}}$ , and use the normalization condition

$u_r^{(2)}$ : from  $f_r$

**We need the self-force at every evaluation point along the orbit**

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# Barack and Sago, 2007

$r_0/M$	$F^* \times (M/\mu)^2$	Error from disagreement		Error from non-stationarity		
		$[F^*]_{- \leftrightarrow} [F^*]_{-}$				
6.0	$2.44661 \times 10^{-2}$	$[9 \times 10^{-4}]$	$1.20 \times 10^{-7}$	$[5 \times 10^{-6}]$	$1.83 \times 10^{-8}$	$[7 \times 10^{-7}]$
6.2	$2.39651 \times 10^{-2}$	$[9 \times 10^{-4}]$	$5.01 \times 10^{-8}$	$[2 \times 10^{-7}]$	$5.92 \times 10^{-8}$	$[2 \times 10^{-6}]$
6.4	$2.33954 \times 10^{-2}$	$[8 \times 10^{-4}]$	$8.54 \times 10^{-8}$	$[4 \times 10^{-6}]$	$3.91 \times 10^{-8}$	$[2 \times 10^{-6}]$
6.6	$2.27829 \times 10^{-2}$	$[7 \times 10^{-4}]$	$1.56 \times 10^{-7}$	$[7 \times 10^{-6}]$	$4.24 \times 10^{-8}$	$[2 \times 10^{-6}]$
6.8	$2.21462 \times 10^{-2}$	$[7 \times 10^{-4}]$	$2.11 \times 10^{-7}$	$[1 \times 10^{-5}]$	$3.21 \times 10^{-8}$	$[1 \times 10^{-6}]$
7.0	$2.14989 \times 10^{-2}$	$[6 \times 10^{-4}]$	$2.42 \times 10^{-7}$	$[1 \times 10^{-5}]$	$2.64 \times 10^{-8}$	$[1 \times 10^{-6}]$
7.2	$2.08504 \times 10^{-2}$	$[6 \times 10^{-4}]$	$2.75 \times 10^{-7}$	$[1 \times 10^{-5}]$	$2.24 \times 10^{-8}$	$[1 \times 10^{-6}]$
7.4	$2.02078 \times 10^{-2}$	$[6 \times 10^{-4}]$	$3.02 \times 10^{-7}$	$[1 \times 10^{-5}]$	$1.93 \times 10^{-8}$	$[1 \times 10^{-6}]$
7.6	$1.95761 \times 10^{-2}$	$[5 \times 10^{-4}]$	$3.18 \times 10^{-7}$	$[2 \times 10^{-5}]$	$1.59 \times 10^{-8}$	$[8 \times 10^{-7}]$
7.8	$1.89586 \times 10^{-2}$	$[5 \times 10^{-4}]$	$3.28 \times 10^{-7}$	$[2 \times 10^{-5}]$	$1.32 \times 10^{-8}$	$[7 \times 10^{-7}]$
8.0	$1.83577 \times 10^{-2}$	$[5 \times 10^{-4}]$	$3.34 \times 10^{-7}$	$[2 \times 10^{-5}]$	$1.13 \times 10^{-8}$	$[6 \times 10^{-7}]$
9.0	$1.56369 \times 10^{-2}$	$[4 \times 10^{-4}]$	$3.23 \times 10^{-7}$	$[2 \times 10^{-5}]$	$5.70 \times 10^{-9}$	$[4 \times 10^{-7}]$
10.0	$1.33895 \times 10^{-2}$	$[8 \times 10^{-5}]$	$1.00 \times 10^{-9}$	$[7 \times 10^{-6}]$	$2.89 \times 10^{-9}$	$[2 \times 10^{-7}]$
11.0	$1.15518 \times 10^{-2}$	$[6 \times 10^{-5}]$	$1.55 \times 10^{-9}$	$[1 \times 10^{-7}]$	$1.49 \times 10^{-11}$	$[1 \times 10^{-9}]$
12.0	$1.00463 \times 10^{-2}$	$[5 \times 10^{-5}]$	$9.79 \times 10^{-10}$	$[1 \times 10^{-7}]$	$7.84 \times 10^{-12}$	$[8 \times 10^{-10}]$
13.0	$8.80489 \times 10^{-3}$	$[4 \times 10^{-5}]$	$3.38 \times 10^{-9}$	$[4 \times 10^{-7}]$	$1.96 \times 10^{-9}$	$[2 \times 10^{-7}]$
14.0	$7.77307 \times 10^{-3}$	$[1 \times 10^{-5}]$	$1.50 \times 10^{-9}$	$[2 \times 10^{-7}]$	$1.62 \times 10^{-9}$	$[2 \times 10^{-7}]$
15.0	$6.90815 \times 10^{-3}$	$[3 \times 10^{-5}]$	$1.17 \times 10^{-9}$	$[2 \times 10^{-7}]$	$1.25 \times 10^{-9}$	$[2 \times 10^{-7}]$
20.0	$4.15706 \times 10^{-3}$	$[1 \times 10^{-5}]$	$1.76 \times 10^{-10}$	$[4 \times 10^{-8}]$	$7.11 \times 10^{-10}$	$[2 \times 10^{-7}]$
30.0	$1.96982 \times 10^{-3}$	$[5 \times 10^{-6}]$	$8.39 \times 10^{-11}$	$[4 \times 10^{-8}]$	$2.86 \times 10^{-10}$	$[1 \times 10^{-7}]$
40.0	$1.14288 \times 10^{-3}$	$[2 \times 10^{-6}]$	$1.65 \times 10^{-11}$	$[1 \times 10^{-8}]$	$8.78 \times 10^{-11}$	$[8 \times 10^{-8}]$
50.0	$7.44949 \times 10^{-4}$	$[1 \times 10^{-6}]$	$3.03 \times 10^{-12}$	$[4 \times 10^{-9}]$	$4.98 \times 10^{-11}$	$[7 \times 10^{-8}]$
60.0	$5.23613 \times 10^{-4}$	$[2 \times 10^{-6}]$	$4.86 \times 10^{-10}$	$[9 \times 10^{-7}]$	$1.57 \times 10^{-11}$	$[3 \times 10^{-8}]$
70.0	$3.88010 \times 10^{-4}$	$[1 \times 10^{-6}]$	$2.47 \times 10^{-10}$	$[6 \times 10^{-7}]$	$1.79 \times 10^{-12}$	$[5 \times 10^{-9}]$
80.0	$2.98979 \times 10^{-4}$	$[8 \times 10^{-6}]$	$1.36 \times 10^{-10}$	$[5 \times 10^{-7}]$	$1.39 \times 10^{-11}$	$[5 \times 10^{-8}]$
90.0	$2.37406 \times 10^{-4}$	$[7 \times 10^{-6}]$	$8.01 \times 10^{-11}$	$[3 \times 10^{-7}]$	$1.58 \times 10^{-11}$	$[7 \times 10^{-8}]$
100.0	$1.93063 \times 10^{-4}$	$[5 \times 10^{-6}]$	$4.90 \times 10^{-11}$	$[3 \times 10^{-7}]$	$1.53 \times 10^{-11}$	$[8 \times 10^{-8}]$
120.0	$1.34868 \times 10^{-4}$	$[4 \times 10^{-6}]$	$2.17 \times 10^{-11}$	$[2 \times 10^{-7}]$	$1.22 \times 10^{-12}$	$[9 \times 10^{-9}]$
150.0	$8.68274 \times 10^{-5}$	$[2 \times 10^{-6}]$	$7.62 \times 10^{-12}$	$[9 \times 10^{-8}]$	$1.48 \times 10^{-11}$	$[2 \times 10^{-7}]$

$r_0/M$	$(M/\mu)^2 F^*$			$(M/\mu)^2 F_t/u_0^t$	$(M/\mu)^2 \dot{E}_{total}$	rel. diff.
6.0	$-1.99476 \times 10^{-3}$	$[7 \times 10^{-5}]$	$[6 \times 10^{-6}]$	$9.40338 \times 10^{-4}$	$9.40190 \times 10^{-4}$	$1.6 \times 10^{-4}$
6.2	$-1.60515 \times 10^{-3}$	$[8 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$7.81183 \times 10^{-4}$	$7.81064 \times 10^{-4}$	$1.5 \times 10^{-4}$
6.4	$-1.30550 \times 10^{-3}$	$[7 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$6.54180 \times 10^{-4}$	$6.54101 \times 10^{-4}$	$1.2 \times 10^{-4}$
6.6	$-1.07197 \times 10^{-3}$	$[7 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$5.51794 \times 10^{-4}$	$5.51723 \times 10^{-4}$	$1.3 \times 10^{-4}$
6.8	$-8.87844 \times 10^{-4}$	$[6 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$4.68497 \times 10^{-4}$	$4.68411 \times 10^{-4}$	$1.8 \times 10^{-4}$
7.0	$-7.41101 \times 10^{-4}$	$[7 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$4.00157 \times 10^{-4}$	$4.00117 \times 10^{-4}$	$1.0 \times 10^{-4}$
7.2	$-6.23065 \times 10^{-4}$	$[7 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$3.43687 \times 10^{-4}$	$3.43627 \times 10^{-4}$	$1.7 \times 10^{-4}$
7.4	$-5.27271 \times 10^{-4}$	$[6 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$2.96692 \times 10^{-4}$	$2.96645 \times 10^{-4}$	$1.6 \times 10^{-4}$
7.6	$-4.48905 \times 10^{-4}$	$[6 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$2.57336 \times 10^{-4}$	$2.57288 \times 10^{-4}$	$1.9 \times 10^{-4}$
7.8	$-3.84324 \times 10^{-4}$	$[5 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$2.24184 \times 10^{-4}$	$2.24148 \times 10^{-4}$	$1.6 \times 10^{-4}$
8.0	$-3.30740 \times 10^{-4}$	$[5 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$1.96105 \times 10^{-4}$	$1.96066 \times 10^{-4}$	$2.0 \times 10^{-4}$
9.0	$-1.66810 \times 10^{-4}$	$[5 \times 10^{-5}]$	$[1 \times 10^{-5}]$	$1.03933 \times 10^{-4}$	$1.03908 \times 10^{-4}$	$2.4 \times 10^{-4}$
10.0	$-9.19067 \times 10^{-5}$	$[3 \times 10^{-5}]$	$[9 \times 10^{-6}]$	$6.15158 \times 10^{-5}$	$6.15047 \times 10^{-5}$	$1.8 \times 10^{-4}$
11.0	$-5.41605 \times 10^{-5}$	$[3 \times 10^{-5}]$	$[2 \times 10^{-5}]$	$3.77904 \times 10^{-5}$	$3.77856 \times 10^{-5}$	$1.3 \times 10^{-4}$
12.0	$-3.36587 \times 10^{-5}$	$[2 \times 10^{-5}]$	$[2 \times 10^{-5}]$	$2.42911 \times 10^{-5}$	$2.42857 \times 10^{-5}$	$2.2 \times 10^{-4}$
13.0	$-2.18388 \times 10^{-5}$	$[2 \times 10^{-5}]$	$[2 \times 10^{-5}]$	$1.62071 \times 10^{-5}$	$1.62022 \times 10^{-5}$	$3.1 \times 10^{-4}$
14.0	$-1.46851 \times 10^{-5}$	$[1 \times 10^{-4}]$	$[2 \times 10^{-5}]$	$1.11574 \times 10^{-5}$	$1.11564 \times 10^{-5}$	$8.5 \times 10^{-5}$
15.0	$-1.01772 \times 10^{-5}$	$[1 \times 10^{-4}]$	$[2 \times 10^{-5}]$	$7.88904 \times 10^{-6}$	$7.88597 \times 10^{-6}$	$3.9 \times 10^{-4}$
20.0	$-2.25549 \times 10^{-6}$	$[6 \times 10^{-5}]$	$[6 \times 10^{-6}]$	$1.87151 \times 10^{-6}$	$1.87111 \times 10^{-6}$	$2.2 \times 10^{-4}$
30.0	$-2.80813 \times 10^{-7}$	$[4 \times 10^{-5}]$	$[4 \times 10^{-6}]$	$2.48643 \times 10^{-7}$	$2.48600 \times 10^{-7}$	$1.7 \times 10^{-4}$
40.0	$-6.51219 \times 10^{-8}$	$[3 \times 10^{-5}]$	$[3 \times 10^{-6}]$	$5.95007 \times 10^{-8}$	$5.94897 \times 10^{-8}$	$1.8 \times 10^{-4}$
50.0	$-2.10849 \times 10^{-8}$	$[2 \times 10^{-5}]$	$[4 \times 10^{-6}]$	$1.96249 \times 10^{-8}$	$1.96203 \times 10^{-8}$	$2.3 \times 10^{-4}$
60.0	$-8.41306 \times 10^{-9}$	$[9 \times 10^{-5}]$	$[3 \times 10^{-6}]$	$7.92670 \times 10^{-9}$	$7.92424 \times 10^{-9}$	$3.1 \times 10^{-4}$
70.0	$-3.87411 \times 10^{-9}$	$[8 \times 10^{-5}]$	$[4 \times 10^{-6}]$	$3.68189 \times 10^{-9}$	$3.68086 \times 10^{-9}$	$2.8 \times 10^{-4}$
80.0	$-1.98069 \times 10^{-9}$	$[7 \times 10^{-5}]$	$[8 \times 10^{-6}]$	$1.89462 \times 10^{-9}$	$1.89360 \times 10^{-9}$	$5.4 \times 10^{-4}$
90.0	$-1.09654 \times 10^{-9}$	$[6 \times 10^{-5}]$	$[6 \times 10^{-6}]$	$1.05415 \times 10^{-9}$	$1.05365 \times 10^{-9}$	$4.8 \times 10^{-4}$
100.0	$-6.46305 \times 10^{-10}$	$[6 \times 10^{-5}]$	$[4 \times 10^{-6}]$	$6.23806 \times 10^{-10}$	$6.23628 \times 10^{-10}$	$2.9 \times 10^{-4}$
120.0	$-2.59096 \times 10^{-10}$	$[5 \times 10^{-5}]$	$[3 \times 10^{-6}]$	$2.51573 \times 10^{-10}$	$2.51496 \times 10^{-10}$	$3.1 \times 10^{-4}$
150.0	$-8.47172 \times 10^{-11}$	$[4 \times 10^{-5}]$	$[6 \times 10^{-6}]$	$8.27475 \times 10^{-11}$	$8.27279 \times 10^{-11}$	$2.4 \times 10^{-4}$

Find a continuous fit function for the self force:

$r < 8M$ :

$$f_r = \left[ i + j \left( 1 - \frac{6M}{r} \right) + k \left( 1 - \frac{6M}{r} \right)^2 + l \left( 1 - \frac{6M}{r} \right)^3 \right] \left( \frac{\mu}{r} \right)^2$$

$$i = 1.32120$$

$$j = 1.2391$$

$$k = -1.297$$

$$l = 1.07$$

$r > 8M:$

$$f_r = \left[ q + u \frac{M}{r} + v \left( \frac{M}{r} \right)^2 + w \left( \frac{M}{r} \right)^3 \right] \left( \frac{\mu}{r} \right)^2 \frac{1}{(1 - 2\frac{M}{r})}$$

$$q = 1.999991$$

$$u = -6.9969$$

$$v = 6.29$$

$$w = -24.6$$











