

Title: Scalar field self-force for eccentric orbits in the equatorial plane of a Kerr black hole.

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Abstract: TBA

Self-Force for a Scalar Particle in Kerr Spacetime: Eccentric Equatorial Orbits

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1 Circular orbits

- Method
- Results

2 Eccentric orbits

- Method of extended homogeneous solutions
- Results

3 Future Work

Circular equatorial orbits

Frequency domain

Wave equation

The minimally coupled Klein-Gordon equation with source T

$$\square\Phi \equiv \Phi_{;\alpha}^{\alpha} = -4\pi T$$

Method

- Decompose field Φ into **spheroidal** harmonic and frequency modes

$$\Phi = \sum_{\hat{l}m} R_{\hat{l}m}(r) S_{\hat{l}m}(\theta) e^{im\phi} e^{i\omega t}$$

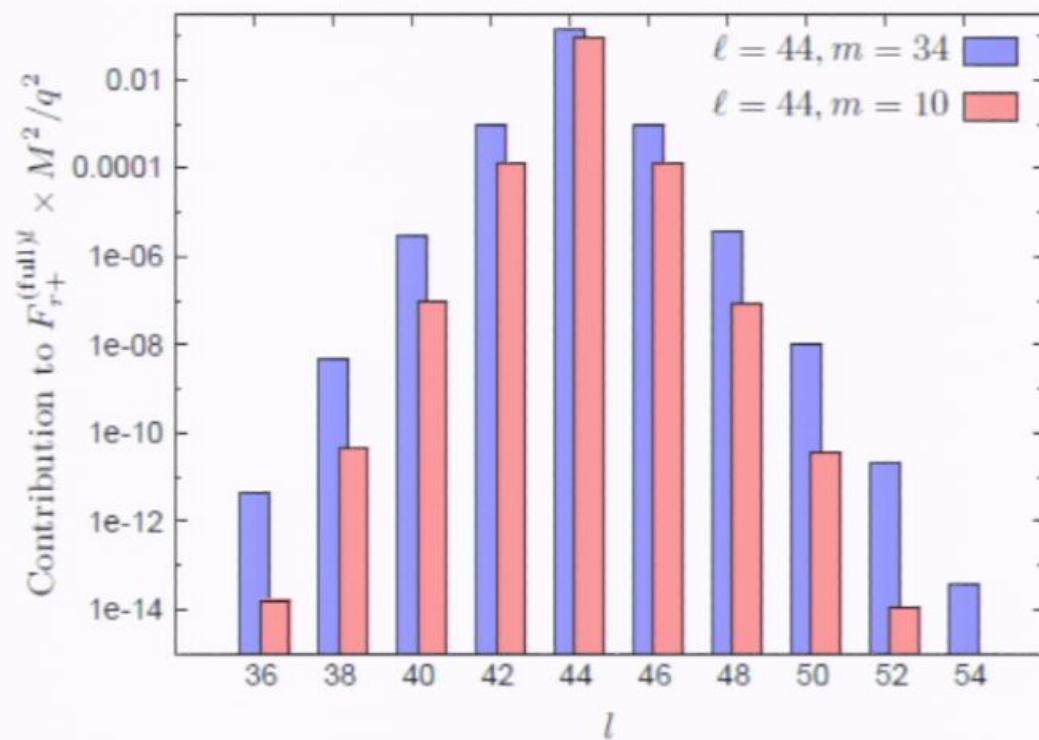
- Numerically solve the radial equation $R_{\hat{l}m}(r)$ for each mode
- Regularize using mode-sum scheme

Spheroidal to spherical decomposition

Spherical decomposition

Mode-sum scheme requires
spherical harmonic modes as
input

$$S_{lm}^c(\theta) = \sum_{l=0}^{\infty} b_{lm}^{\dagger} Y_{lm}(\theta)$$

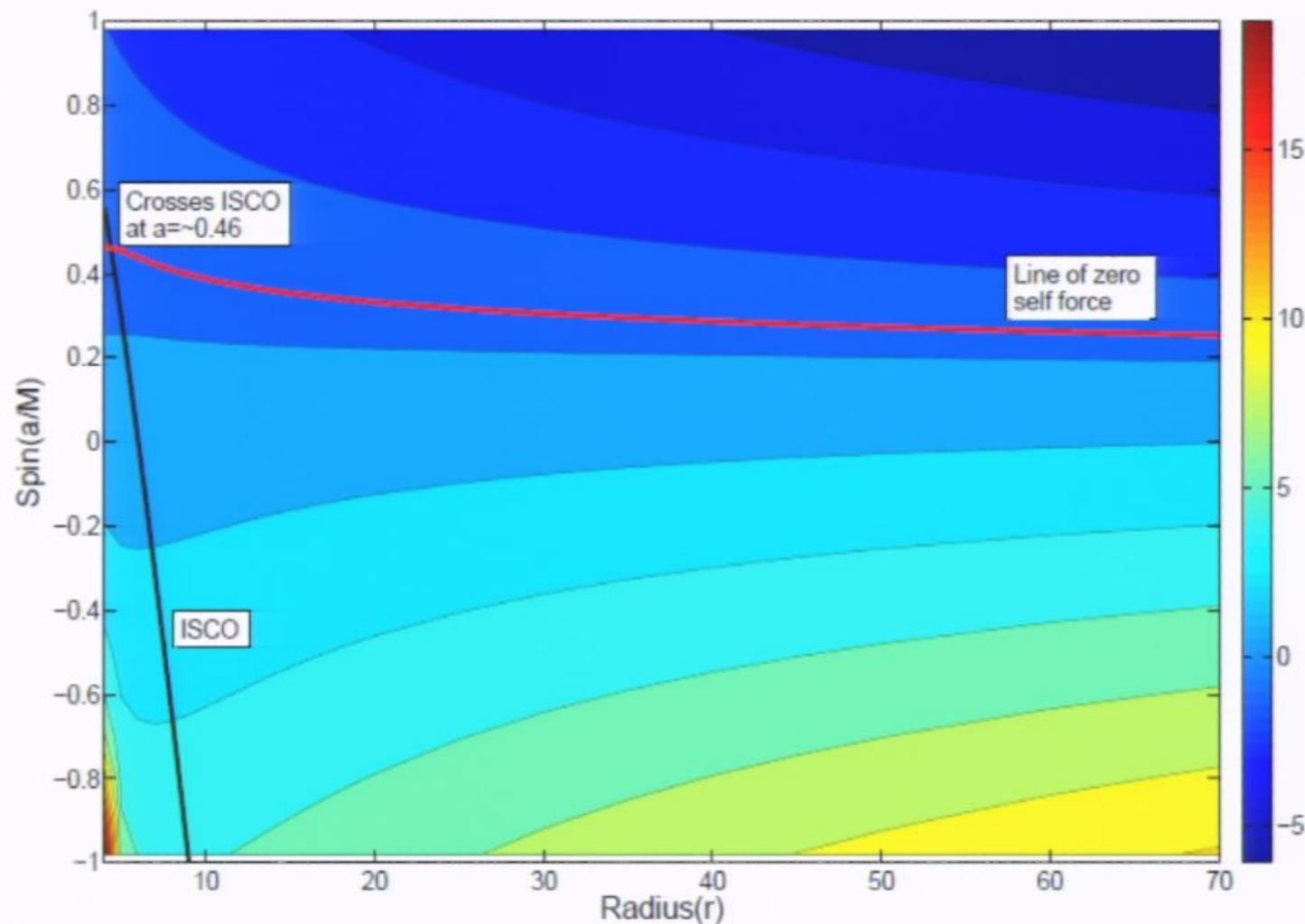


$$F_{\alpha}^{(\text{full})l}(x) = q \nabla_{\alpha} \sum_{\hat{l}=0}^{\infty} \sum_{m=-\hat{l}}^{\hat{l}} b_{lm}^{\dagger} R_{lm}(r) Y_{lm}(\theta, \phi) e^{-i\omega_m t}$$

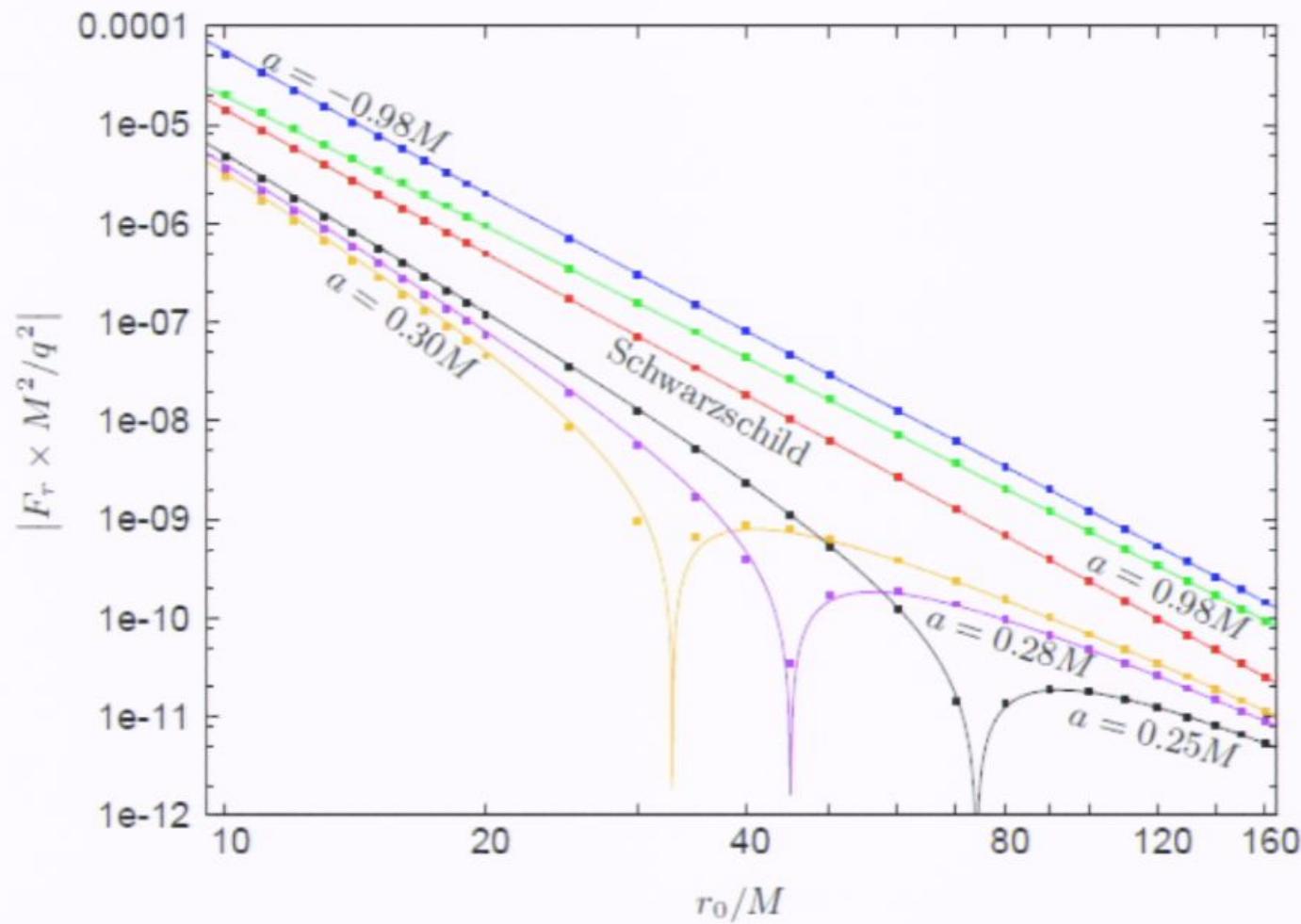
Results

Phys. Rev. D. 81. 084039 (2010)

Results - $r^5 F_r$



PN Fit



Eccentric equatorial orbits

Method of extended homogeneous solutions

Eccentric orbits

Spectrum now bi-periodic

$$\omega = m\Omega_\phi + n\Omega_r$$

$$\Phi_{lm} = \sum_n \phi_{lm\omega}$$

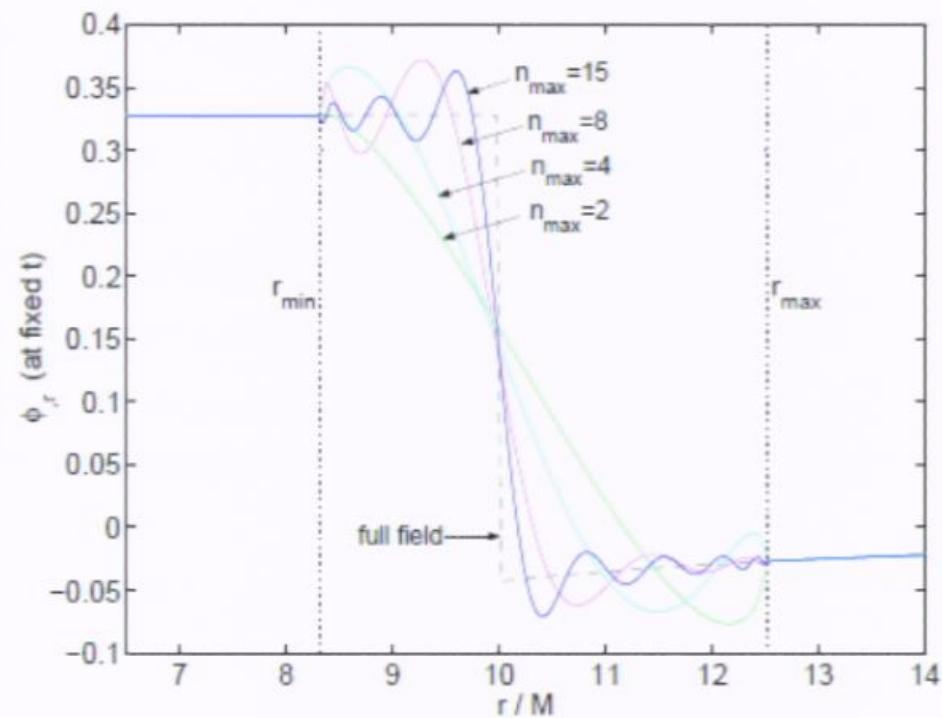
Method of extended homogeneous solutions

Eccentric orbits

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$$\Phi_{lm} = \sum_n \phi_{lmn}$$



Credit: Barack, Ori and Sago

Method of extended homogeneous solutions

Eccentric orbits

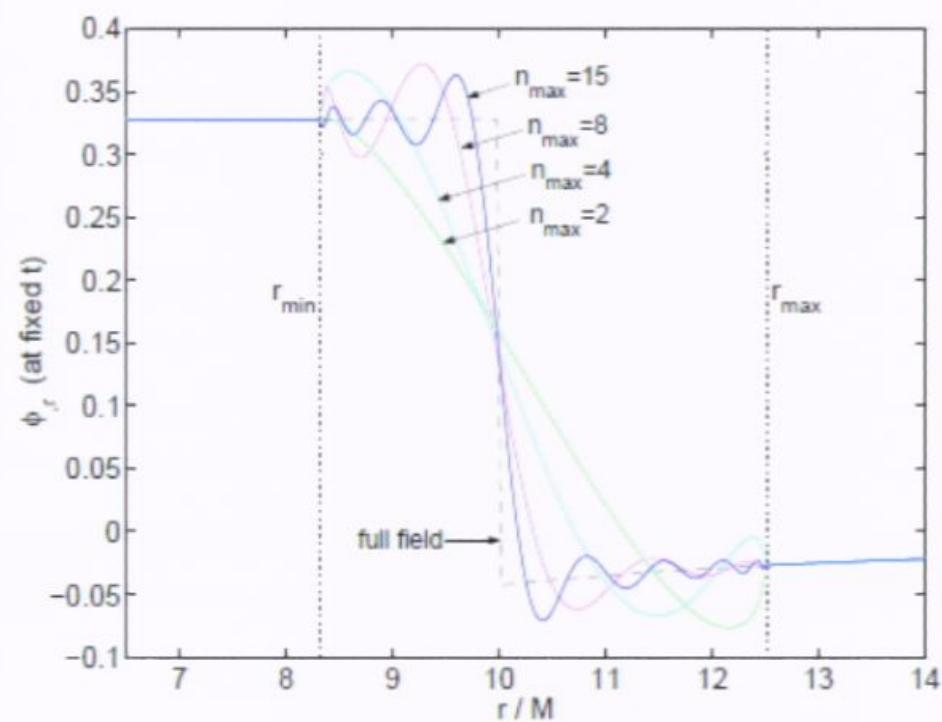
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Extended homogeneous solutions

- Avoids Gibbs phenomenon
- Only need to integrate once for each $lm\omega$ mode



Method of extended homogeneous solutions

Eccentric orbits

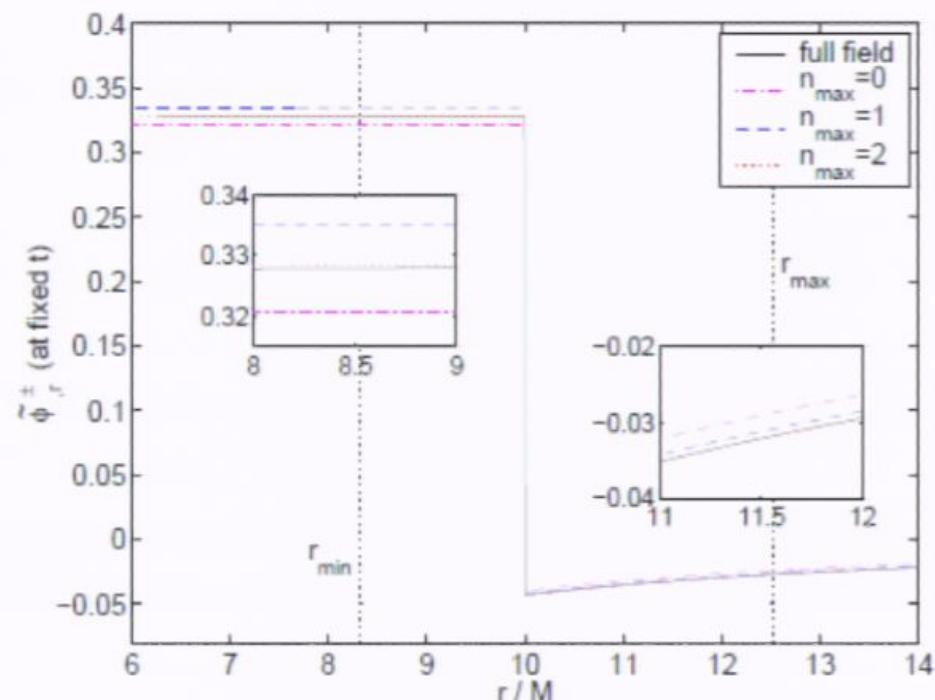
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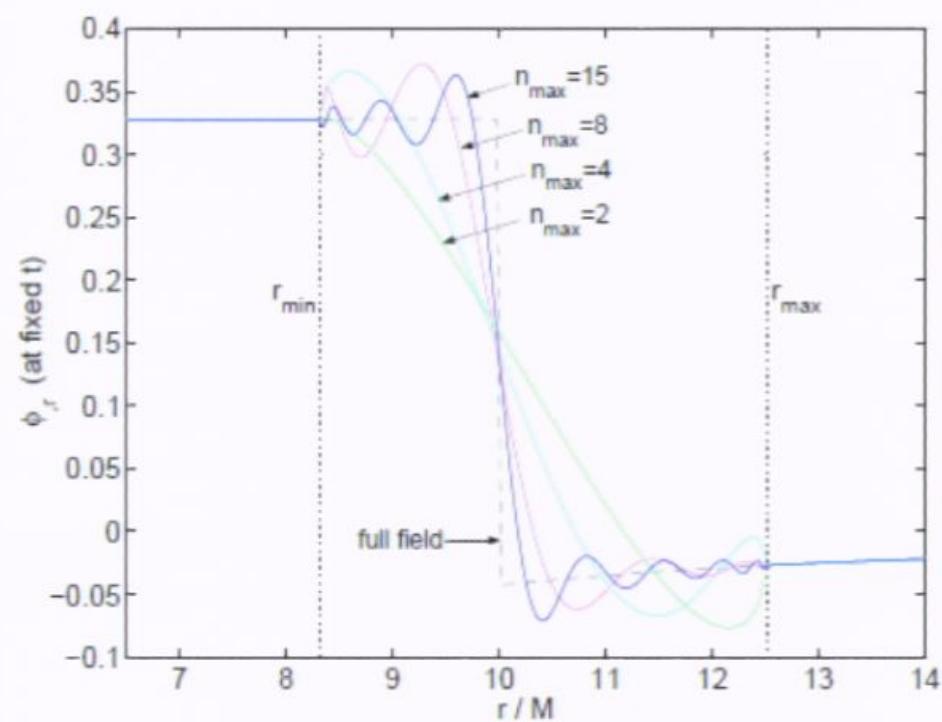
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Method of extended homogeneous solutions

Eccentric orbits

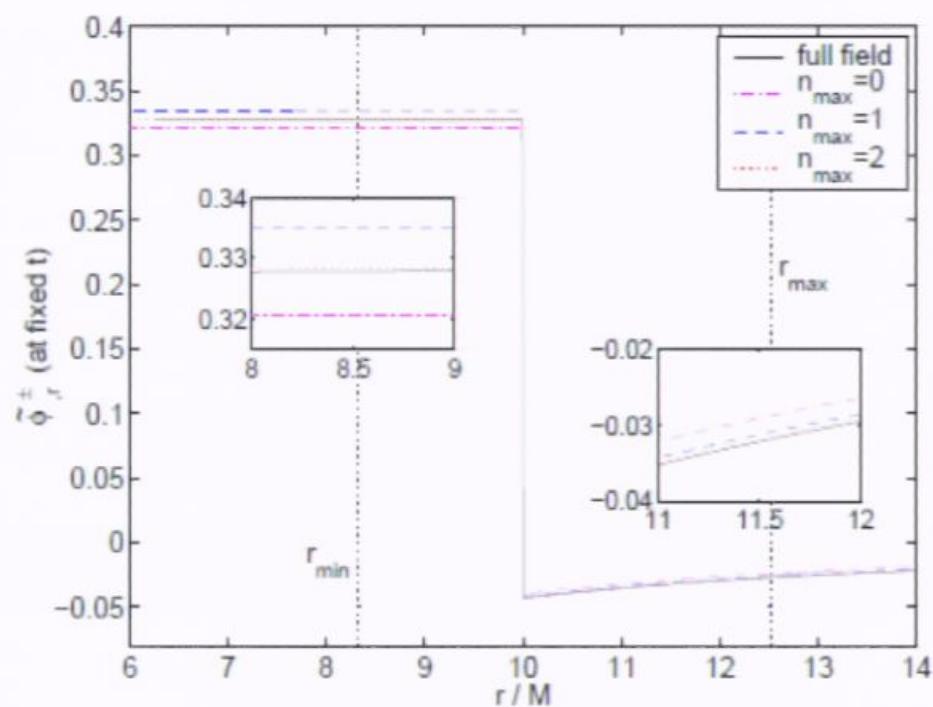
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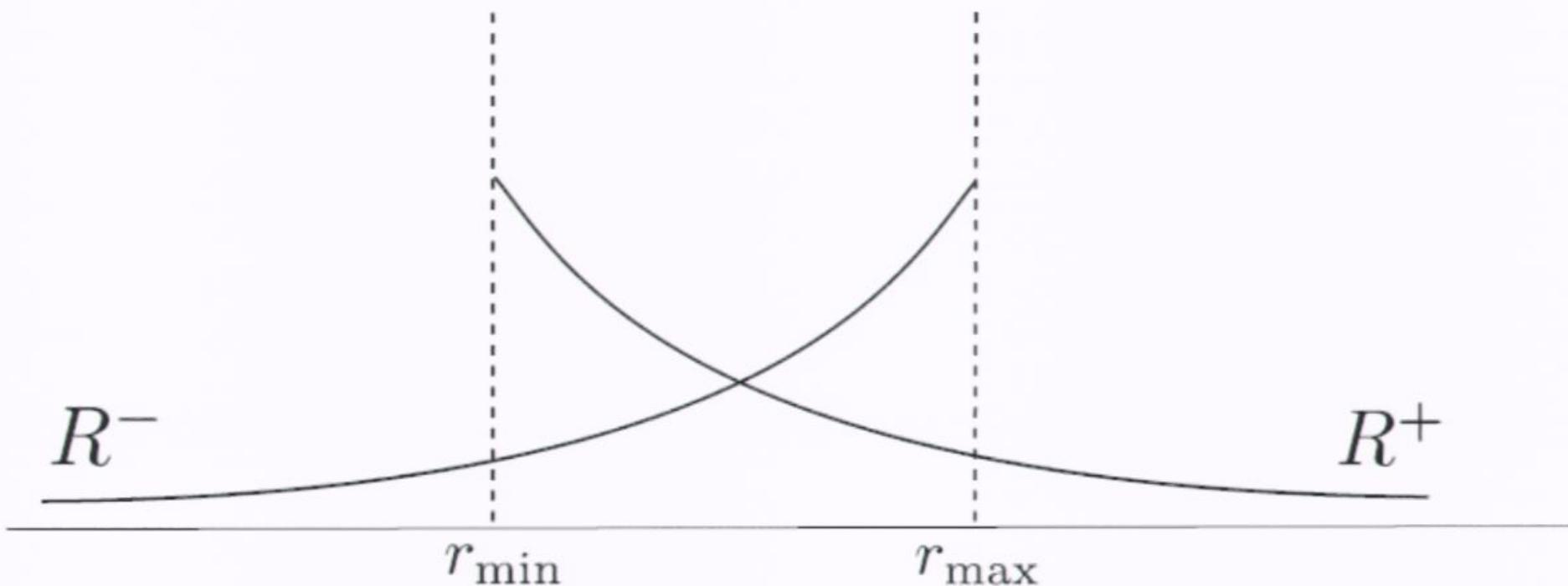
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Method of extended homogeneous solutions

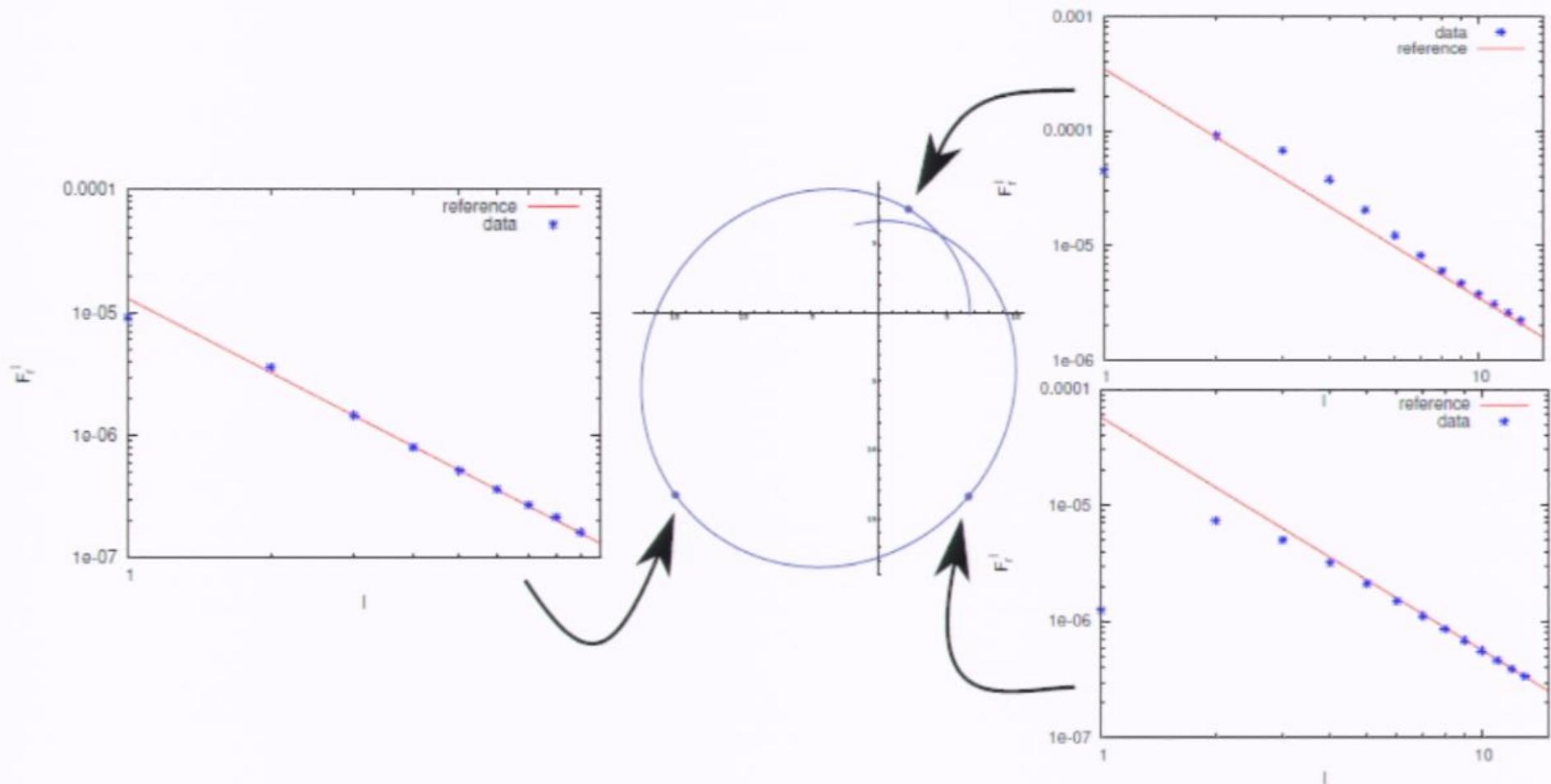


$$\phi_{lm}^\pm = \sum_n C_{lm\omega}^\pm R_{lm\omega}^\pm(r) e^{-i\omega t} \quad C_{lm\omega}^\pm = \int_{r_{\min}}^{r_{\max}} \frac{R_{lm\omega}^\mp(r) Z_{lm\omega}(r)}{f(r)} dr$$

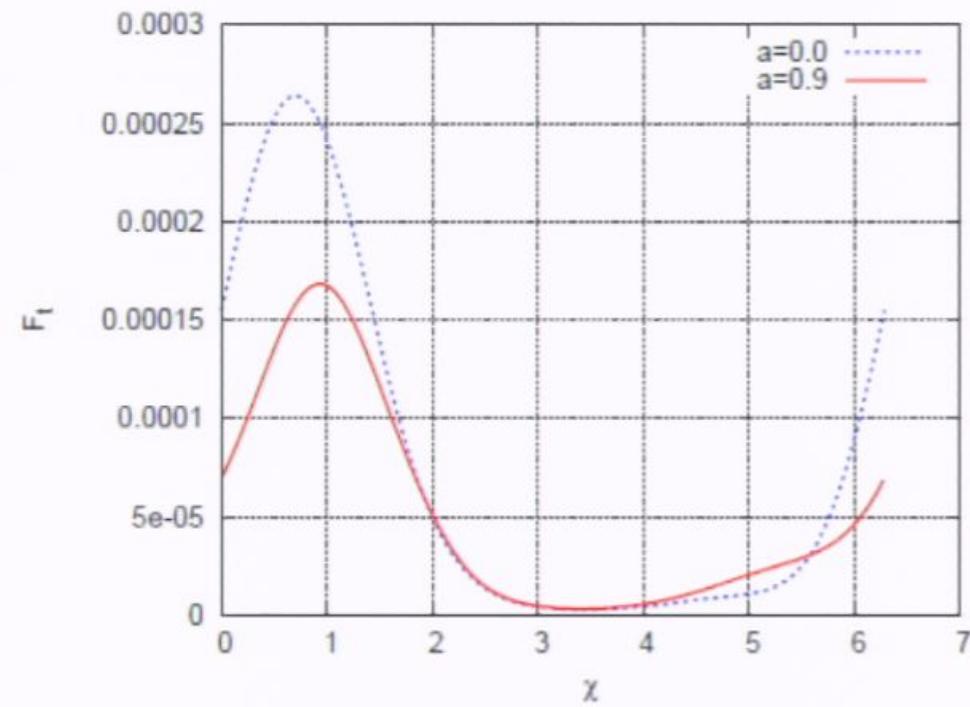
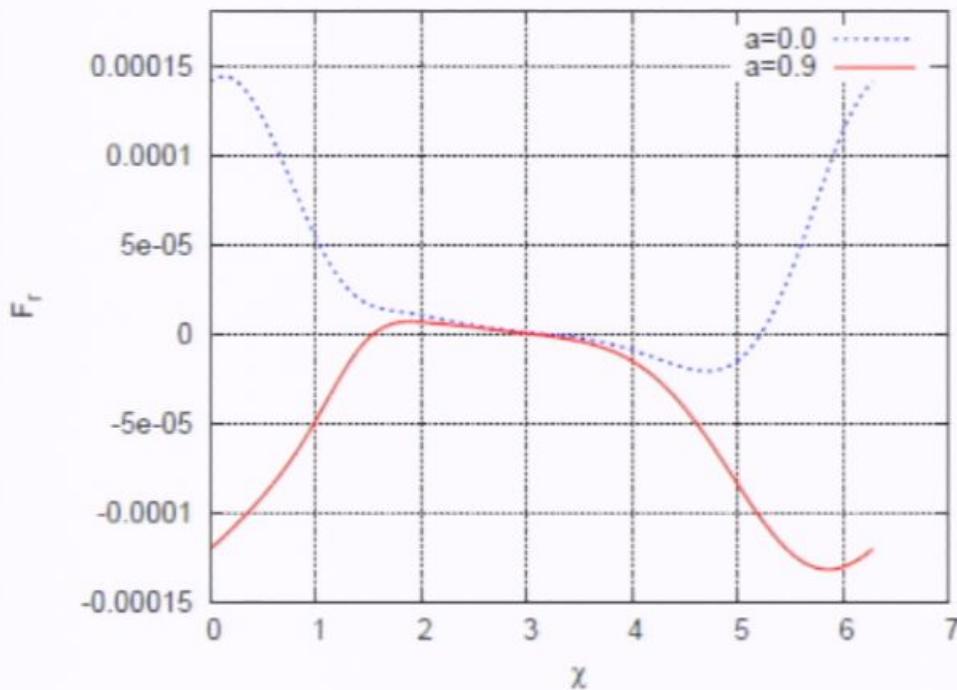
Results

Self-force and ISCO shift

Validation: regularization

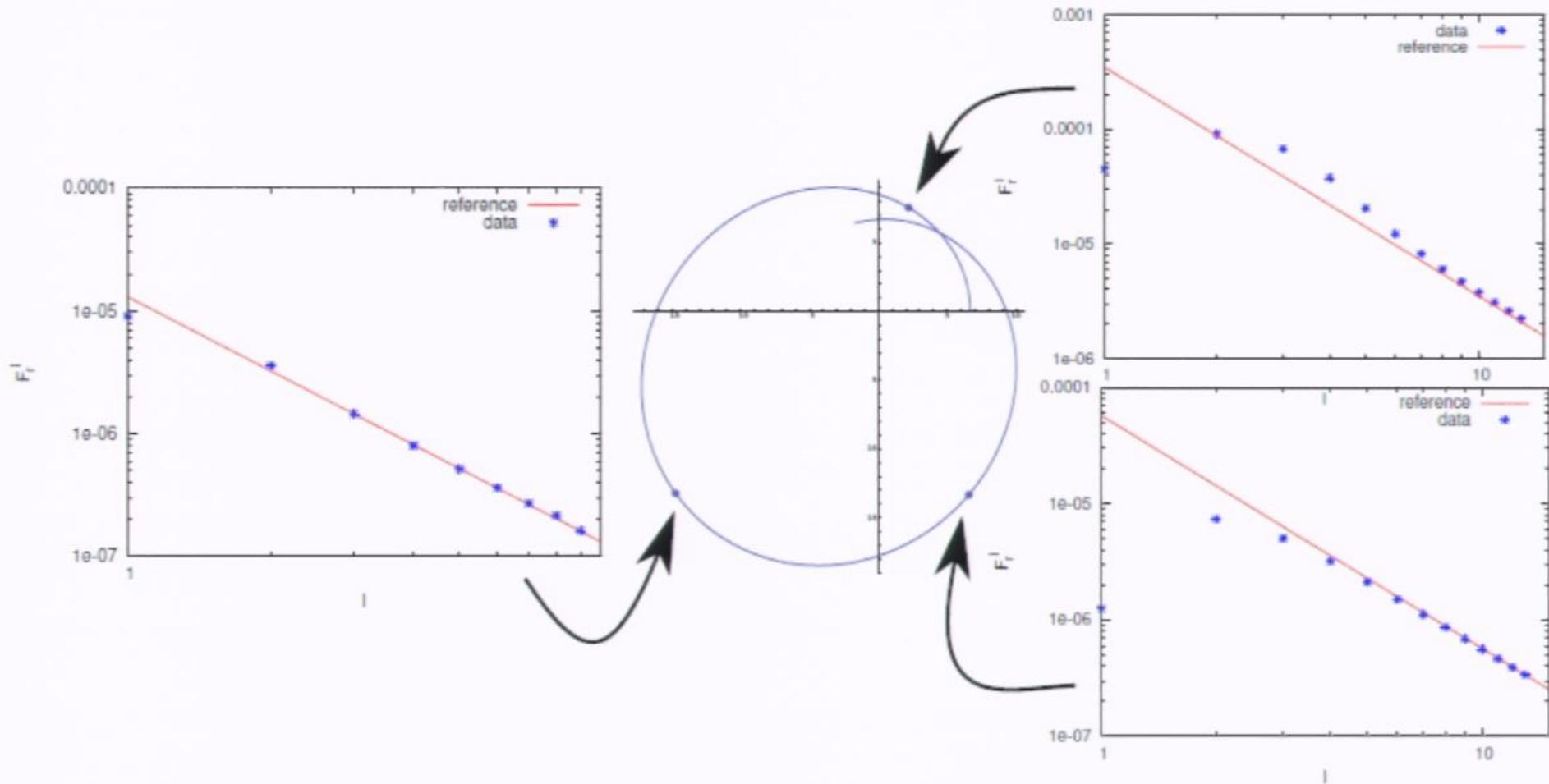


Sample results

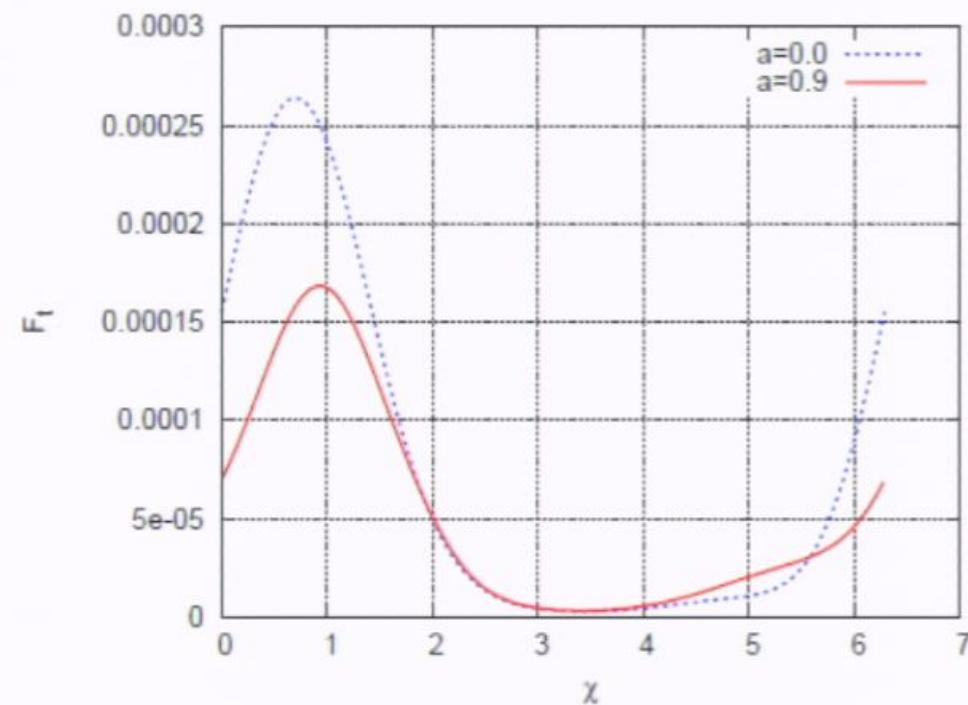
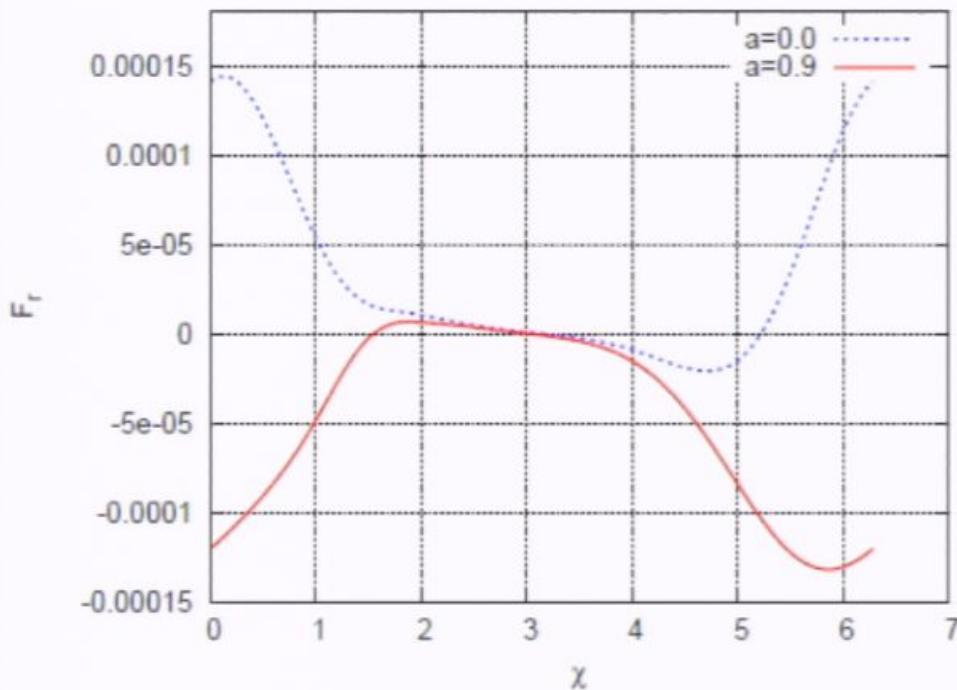


$$(p, e) = (10, 0.5)$$

Validation: regularization



Sample results



$$(p, e) = (10, 0.5)$$

Schwarzschild ISCO shift

- Self-force corrections shift the location of the ISCO
- Formula for the radial shift given by

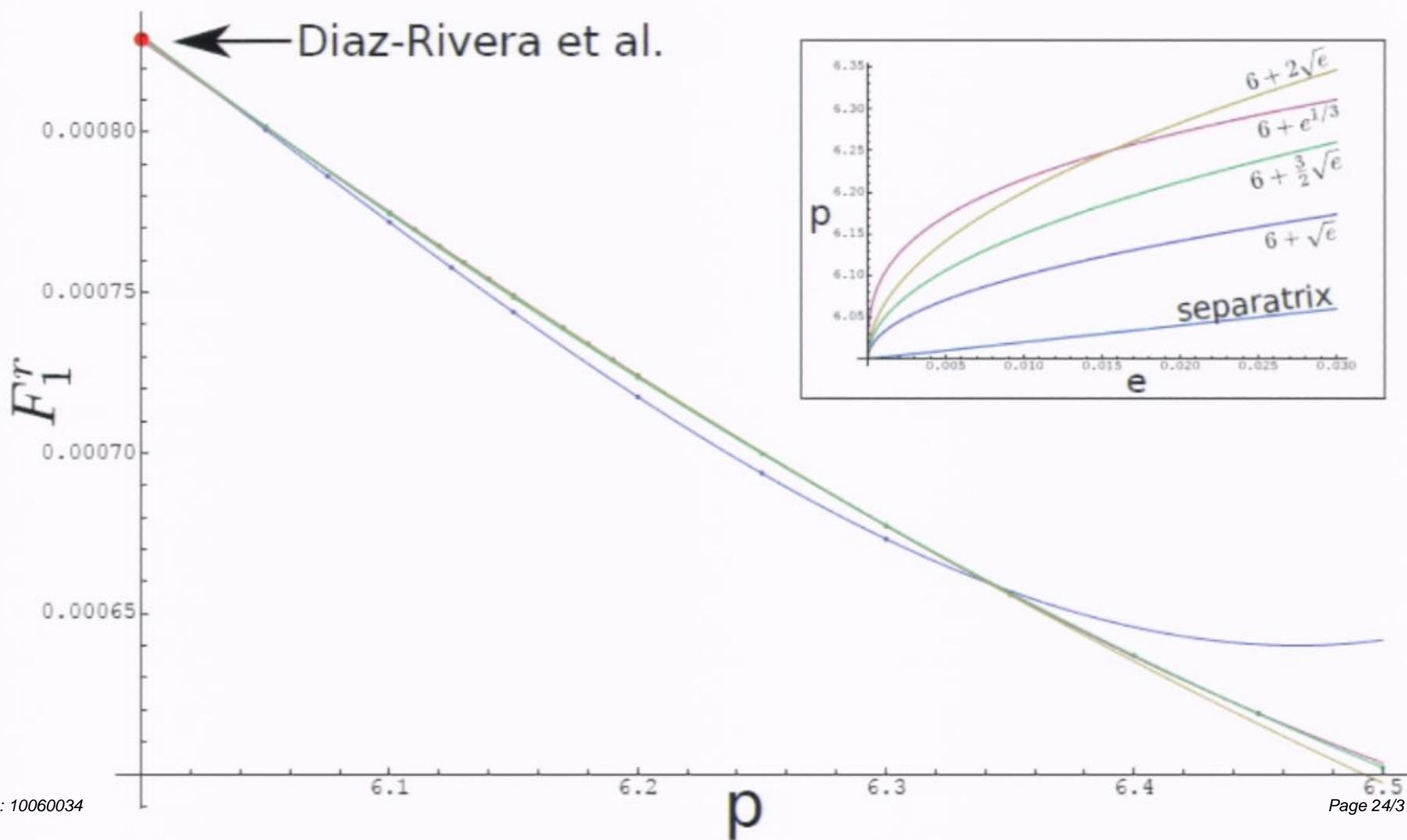
$$\Delta r_{\text{isco}} = 216F_0^r - 72F_1^r + 6\sqrt{2}F_t^1 + \frac{4}{\sqrt{3}}F_\phi^1$$

where e.g.

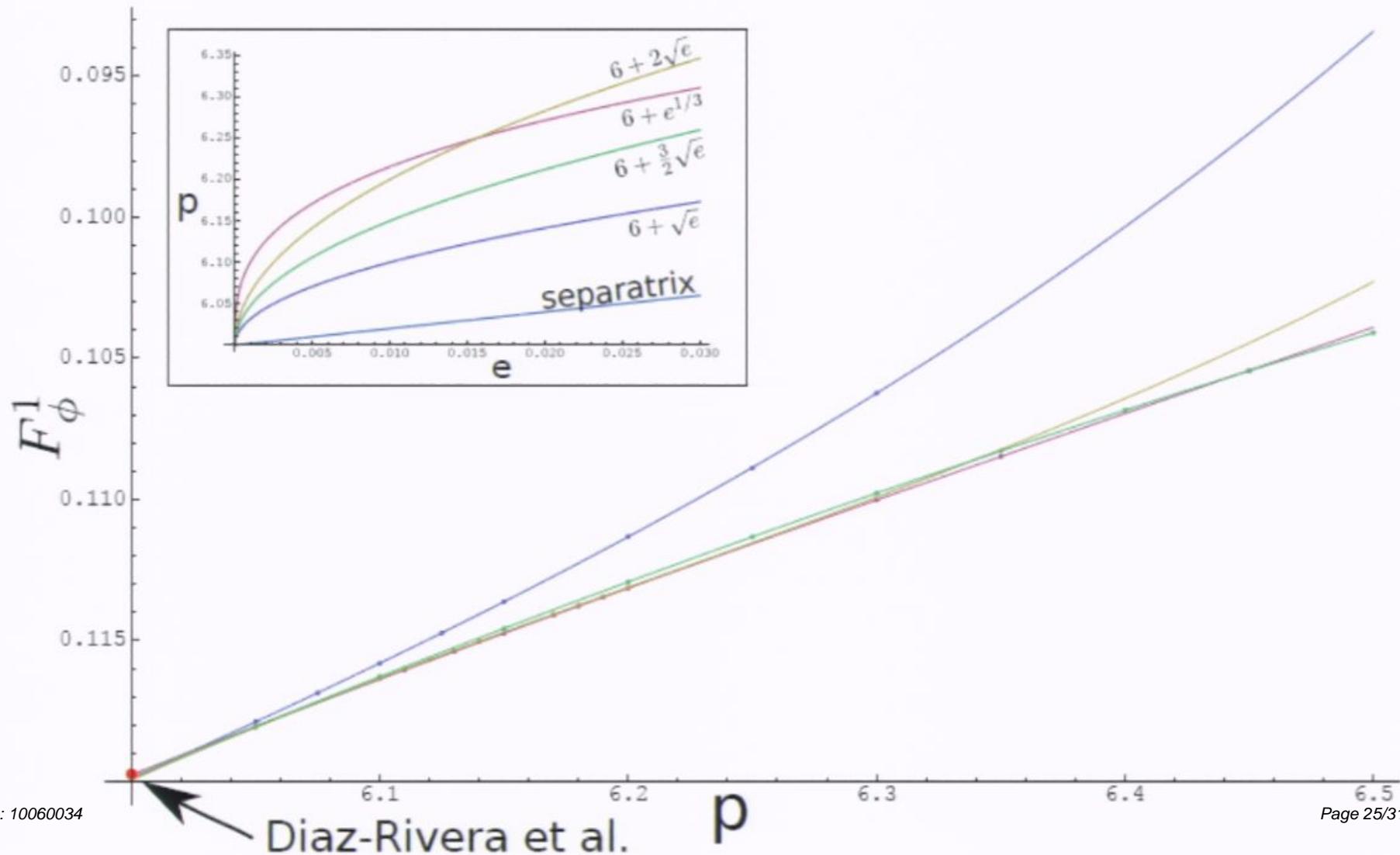
$$F^r = F_0^r + eF_1^r \cos(\omega\tau)$$

- Calculate the (conservative) self-force and F_1^r , F_ϕ^1 and F_t^1 for slightly eccentric orbits and then extrapolate to the ISCO

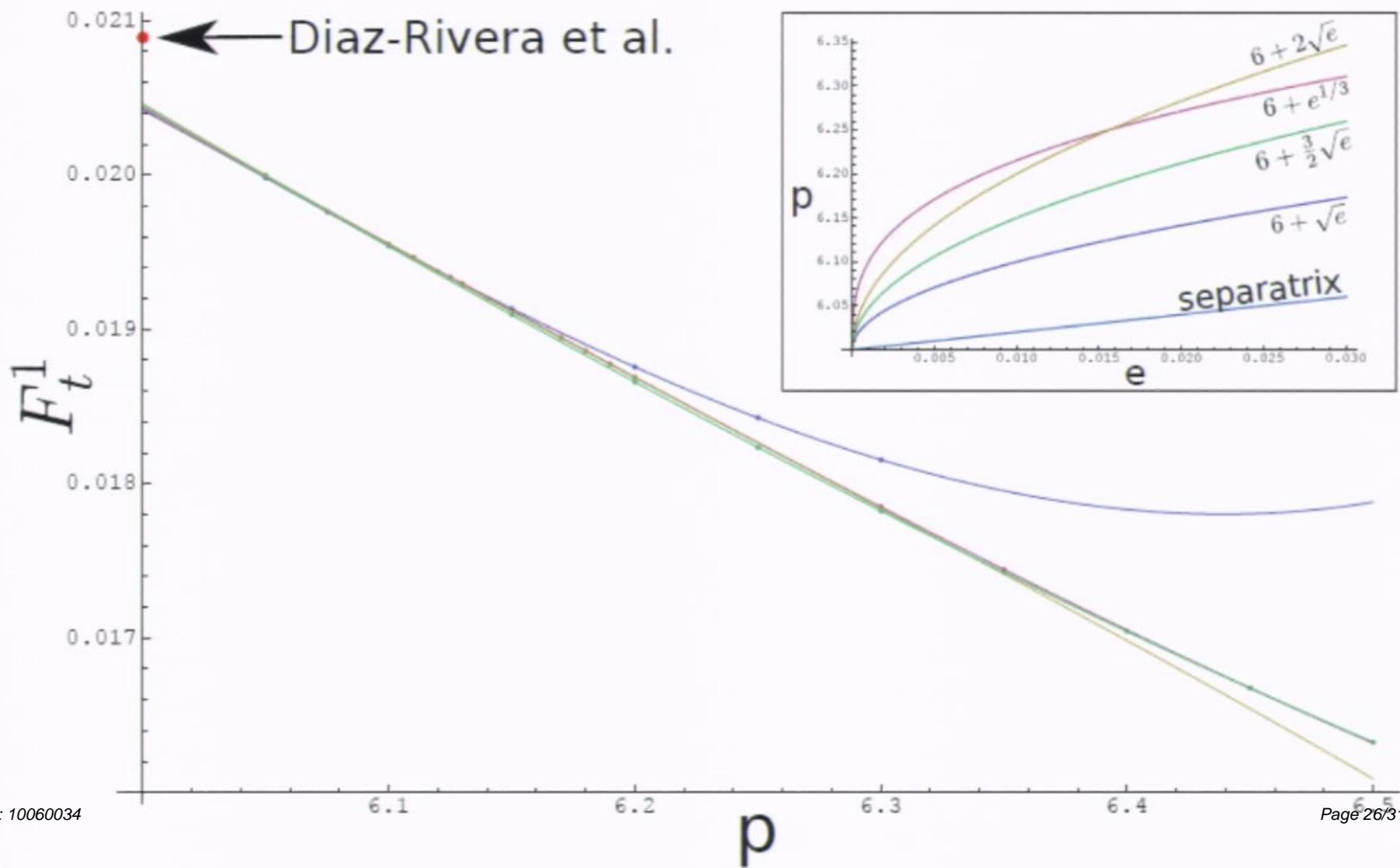
F_1^r



F_ϕ^1

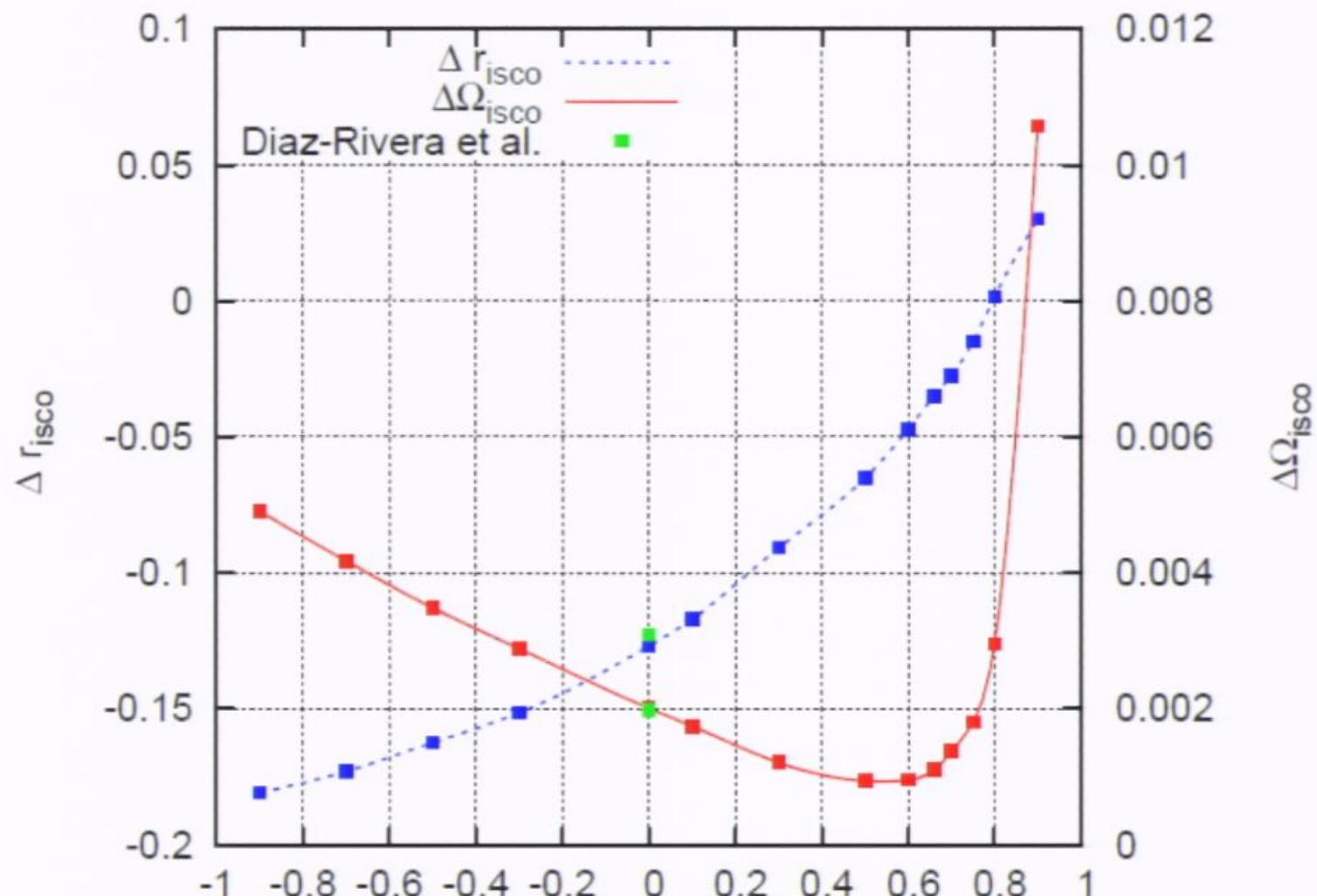


F_t^1

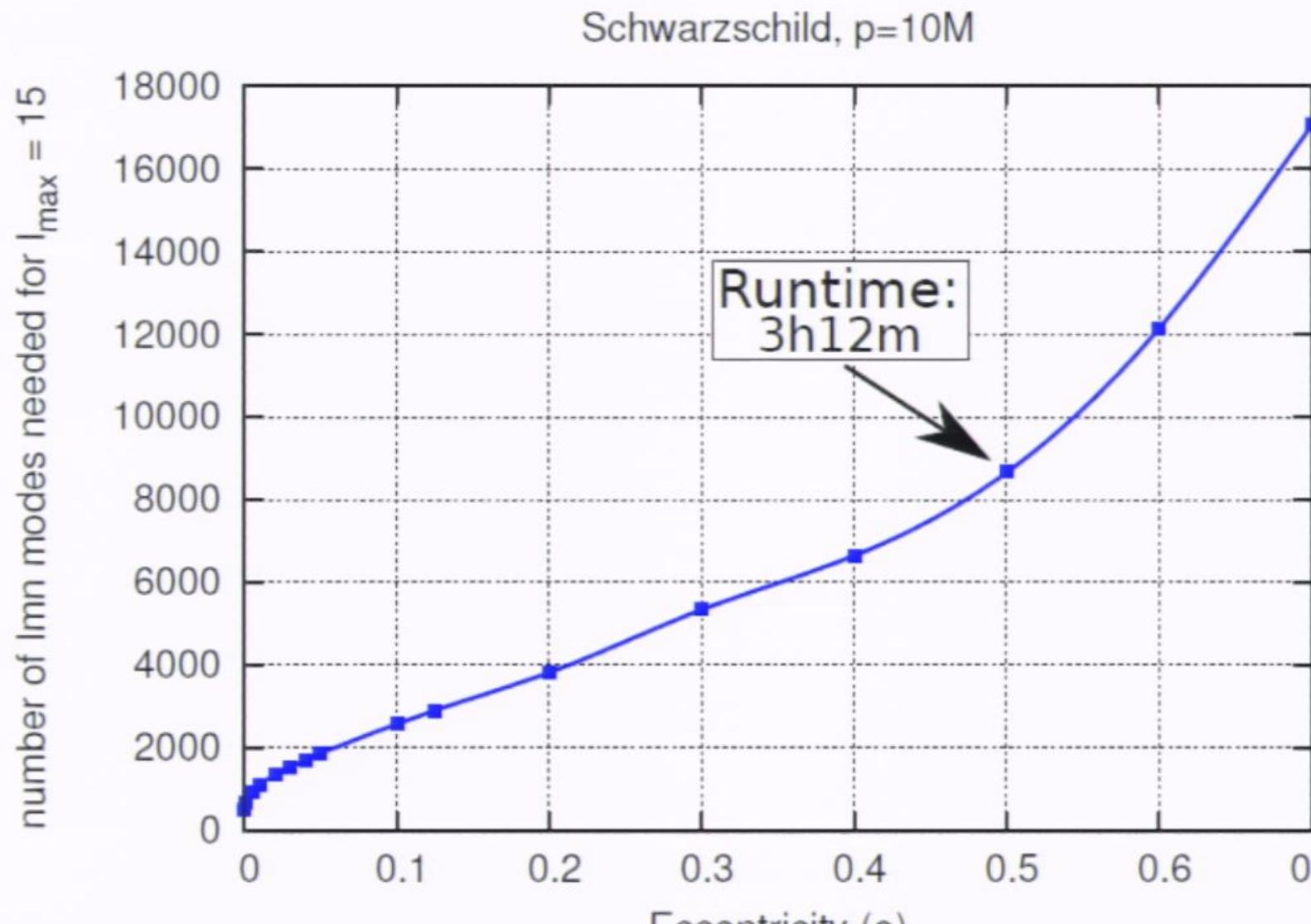


Kerr ISCO shift

Similar procedure can be applied to Kerr



Efficiency of the method



Future Work

- Mass change

$$F_{\text{self}}^{\alpha} = u^{\beta} \nabla_{\beta} (\mu u^{\alpha}) = u^{\beta} (\mu \nabla_{\beta} u^{\alpha} + u^{\alpha} \nabla_{\beta} \mu)$$

- Circular inclined orbits

$$\omega = m\Omega_{\phi} + k\Omega_{\theta}$$

- Limits of method

$$\text{Spheroidicity} = -a^2\omega^2$$

Summary

- Mode-sum calculation in Kerr
- Scalar self-force for eccentric equatorial orbits
- Kerr ISCO shift

Kerr ISCO shift

Similar procedure can be applied to Kerr

