Title: Holographic Singularity Resolution

Date: Jun 17, 2010 05:15 PM

URL: http://pirsa.org/10060026

Abstract: TBA

Holographic Singularity Resolution

B. Craps, T. Hertog, NT (2009);
NT (in prep);
M. Smolkin, NT (in prep)

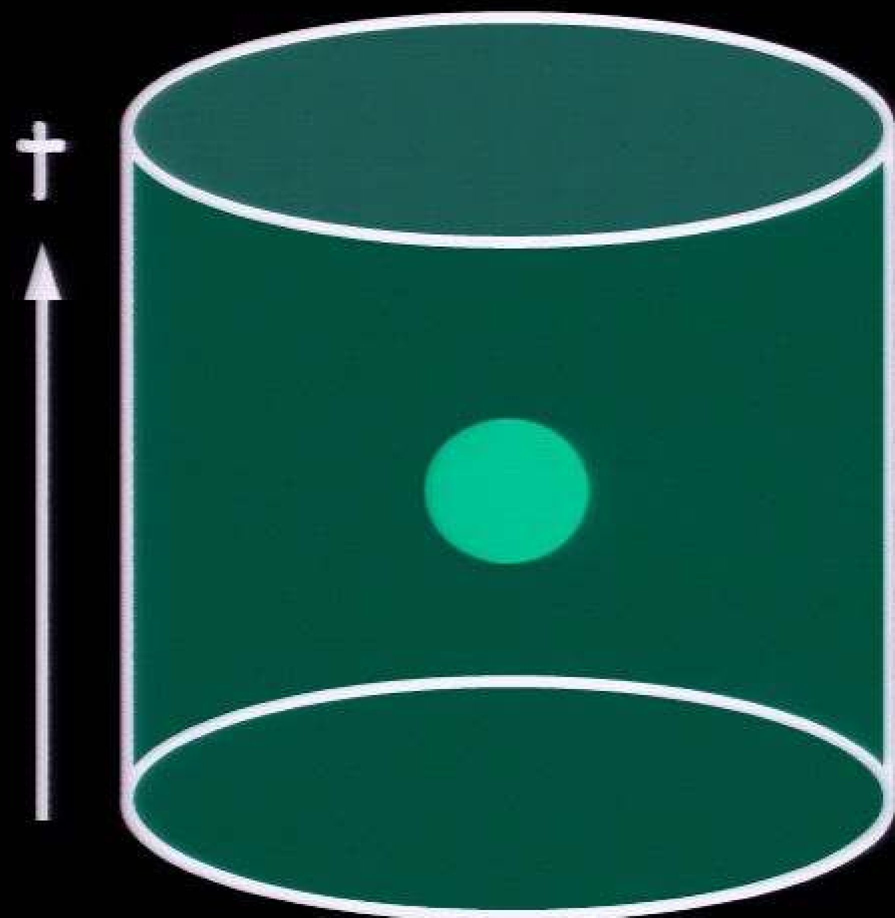
on crime and disease

is the singularity the key?

was it the gateway from
a previous universe?

will there be another
in the future?

holography



a theory with gravity

"in a box" is

dual to

a theory with no gravity
living on the boundary!

- a conformal field theory

Holography:

Quantum Gravity = CFT on fixed background!

Cosmology -> dual CFT has H unbounded
below (on flat space)

-> need to make sense of new class of CFTs

Guiding Principle:

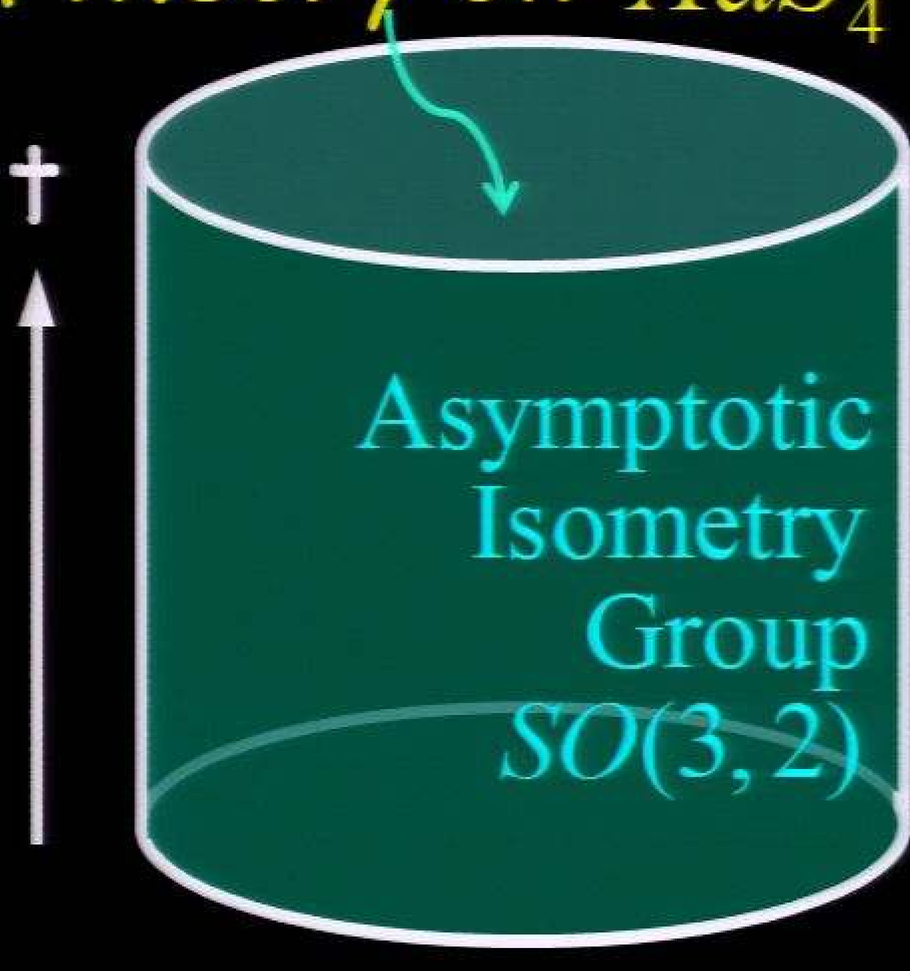
Conformal Symmetry <-> UV-completeness

CHT 2009

Setup:

M theory on $AdS_4 \times (S_7 / \mathbb{Z}_k)$

- fibration over $\mathbb{C}P_3$



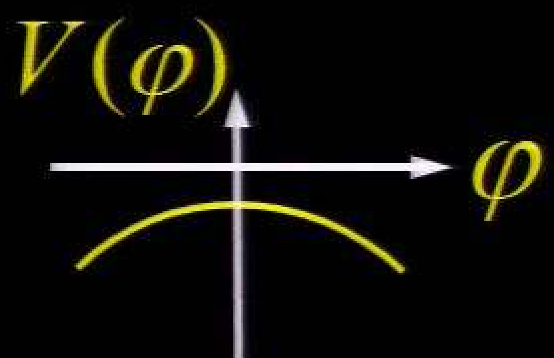
CFT_3 on $S_2 \times \mathbb{R}$

Conformal Group $SO(3,2)$

For purposes of choosing AdS-invariant bcs, truncate bulk theory to gravity + scalar

$$S_{bulk} = \int_{bulk} \left(\tfrac{1}{2} R - \tfrac{1}{2} (\partial\varphi)^2 + R_{AdS}^{-2} (3\cosh\sqrt{\tfrac{2}{3}}\varphi) \right.$$

quadrupole of S_7 traceless bilinear under $SO(8)$



eg $O(6)\times O(2)$ invariant

$$\varphi = 0 : m_\varphi^2 = -2R_{AdS}^{-2} > -\tfrac{9}{4} R_{AdS}^{-2} \equiv m_{BF}^2$$

-> stable with usual SUSY bcs

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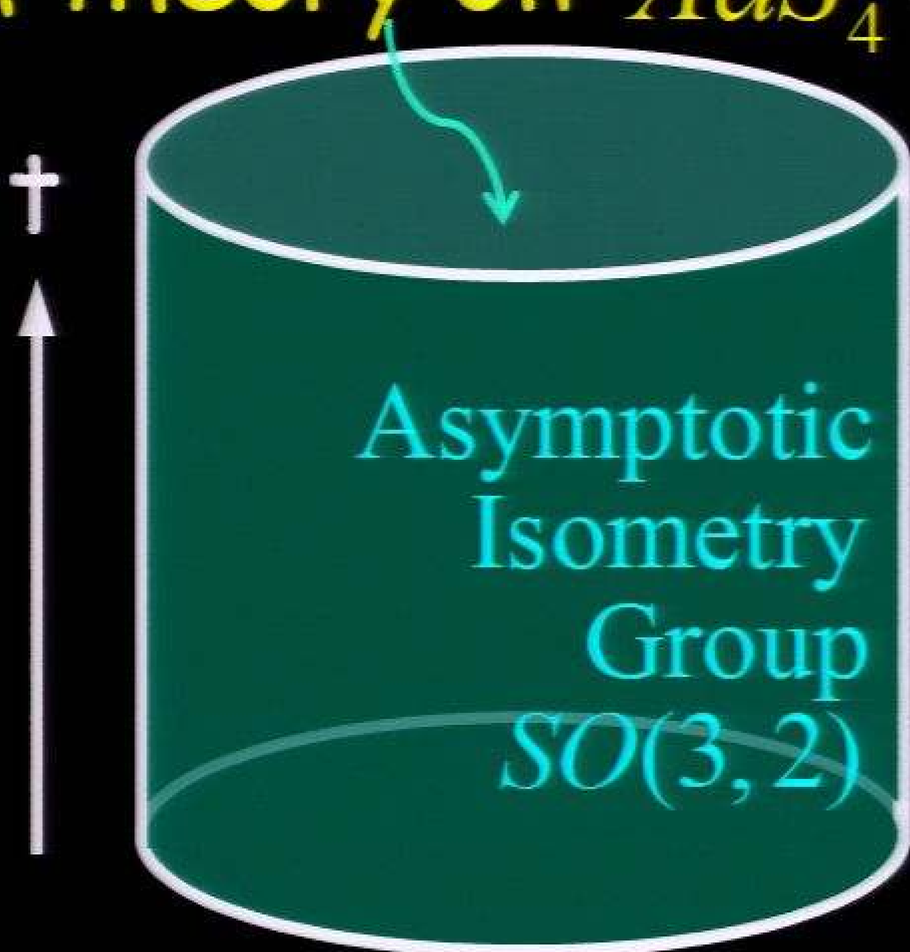

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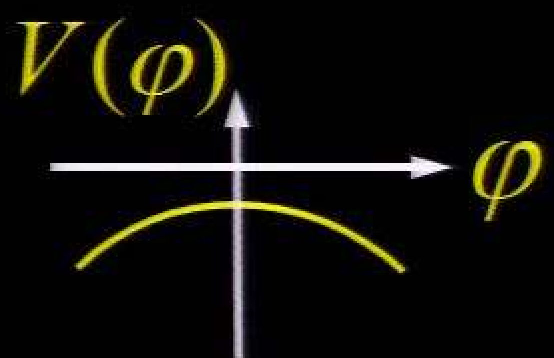

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generalised AdS-invart cosmological bcs

$$\beta = h\alpha^2$$

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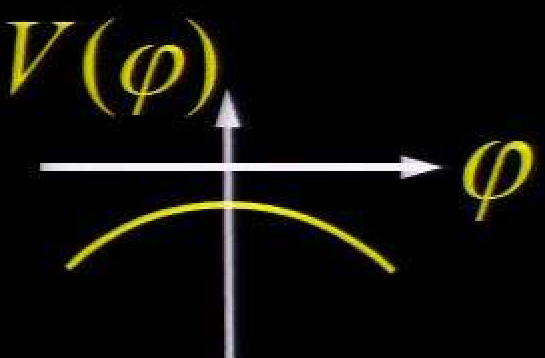

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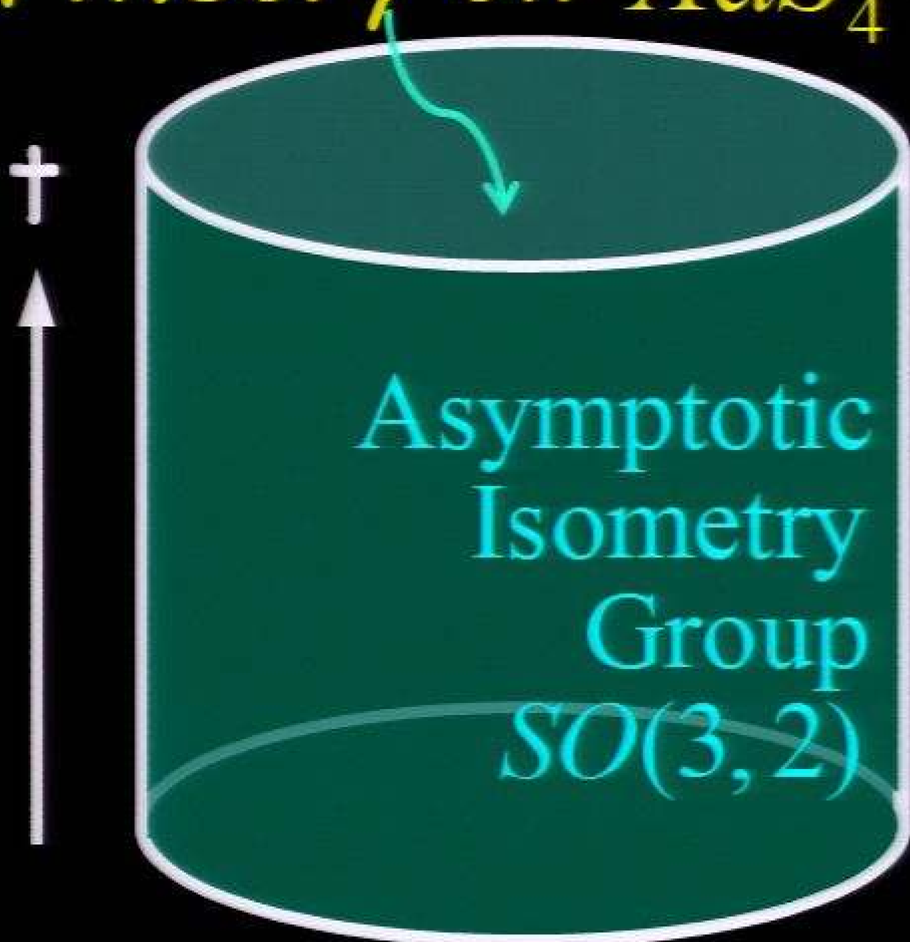

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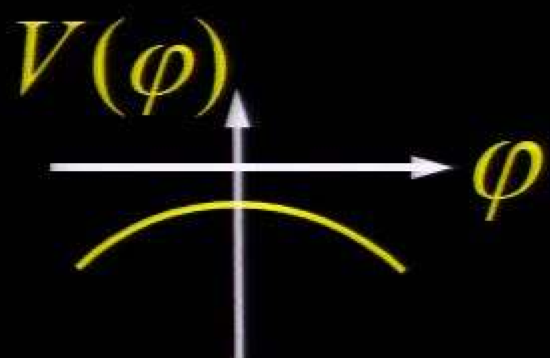

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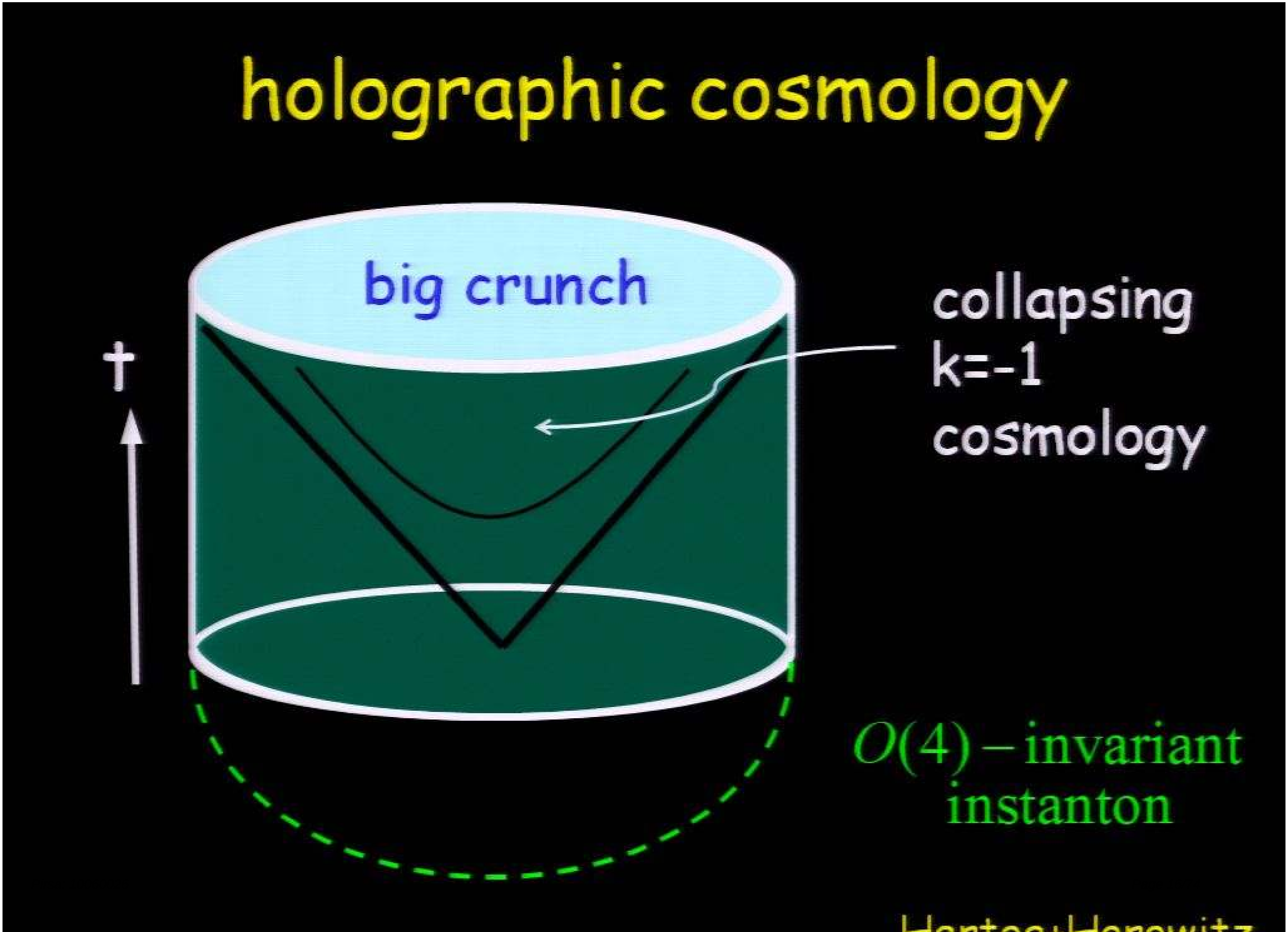
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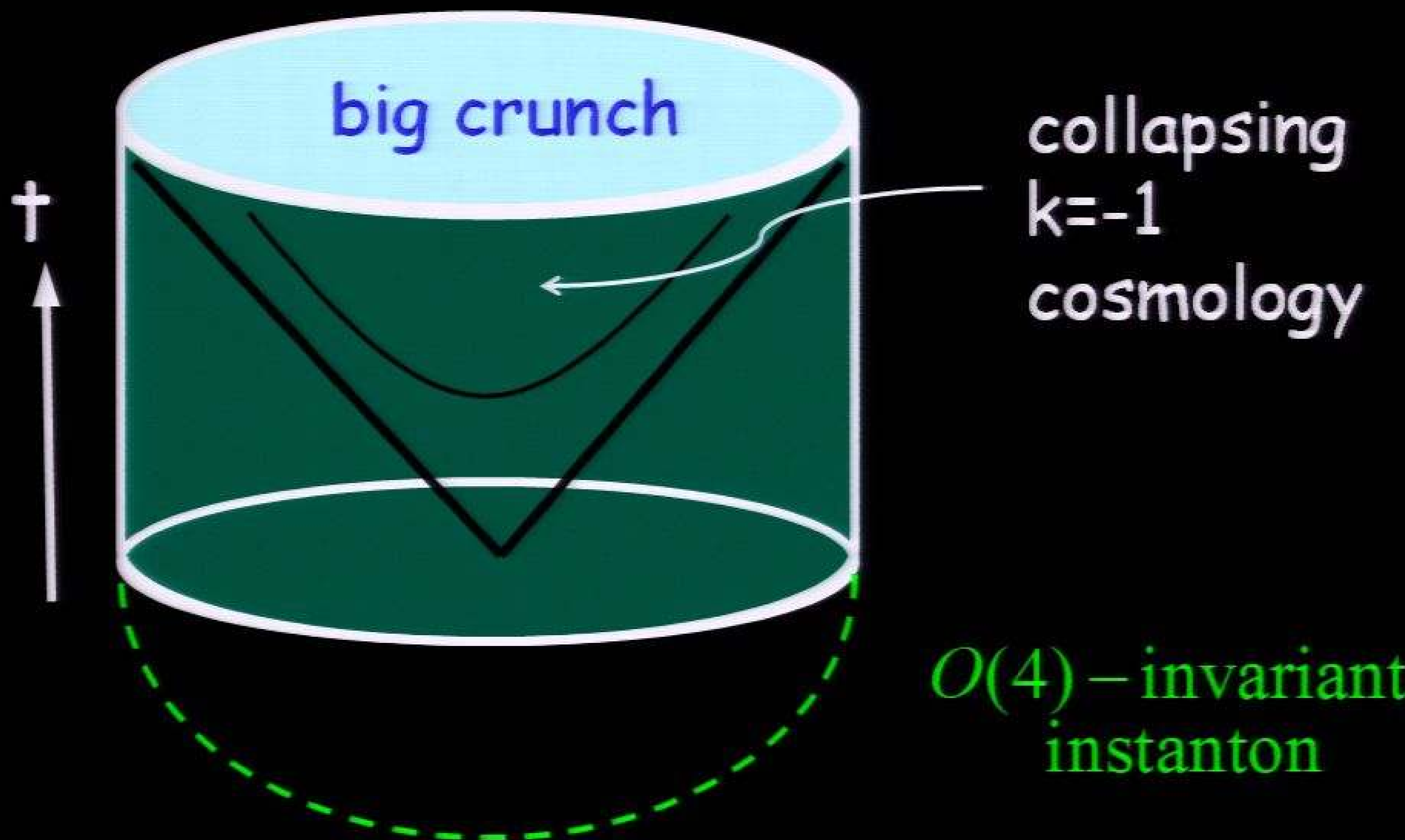

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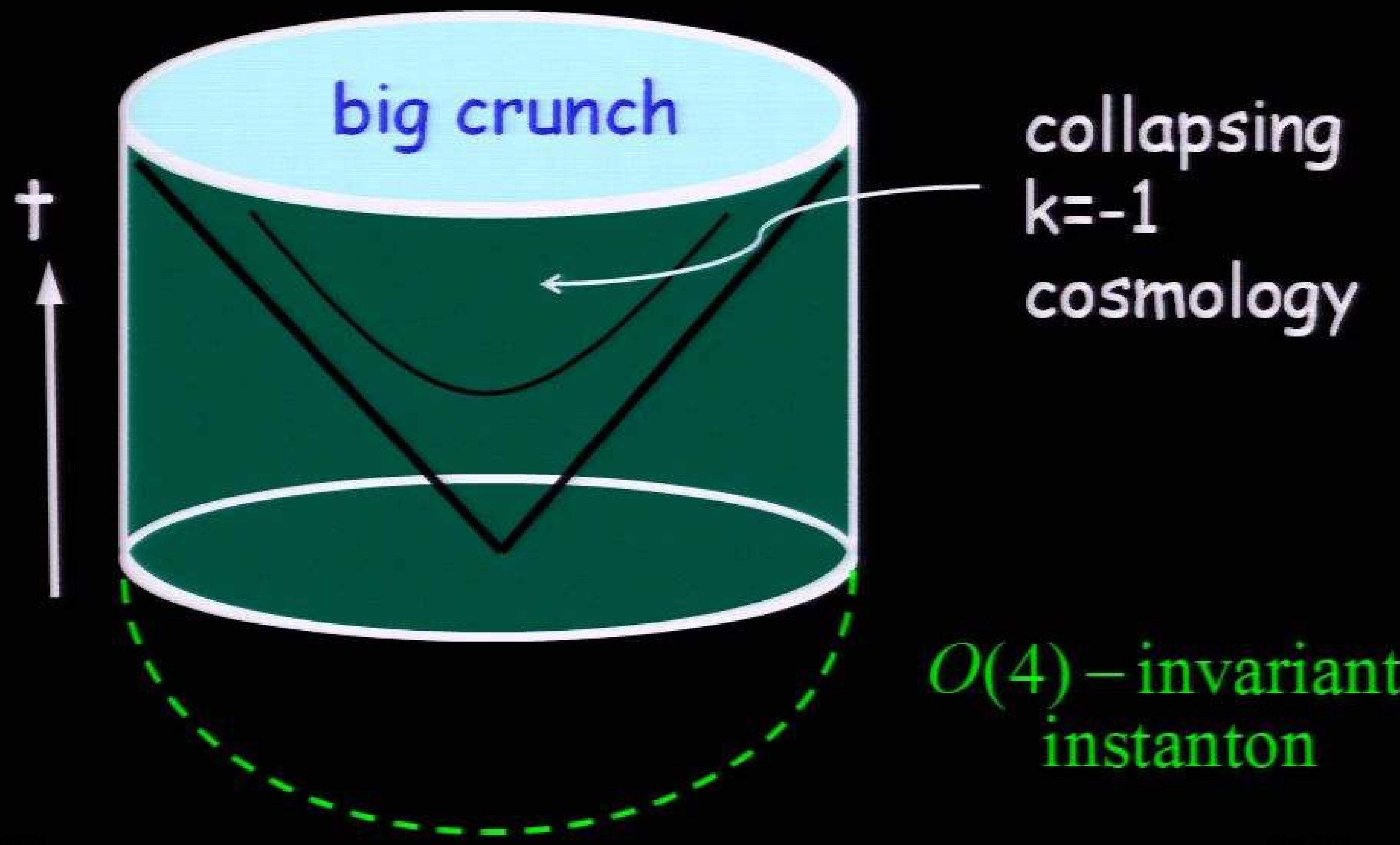

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$$S_{CFT} \sim \int_3 \left(k(A\wedge F + A\wedge A\wedge A) - |DY|^2 - k^{-2}(Y^*Y)^3 + fermions \right)$$

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$$M, k \to \infty, \text{ at fixed } g_t \equiv M/k; \quad \frac{R_{AdS}}{l_{Pl}} \sim (Mk)^{1/4}, \quad \frac{R_{AdS}}{l_s} \sim \left(\frac{M}{k}\right)^{1/4}$$

Cosmological deformation

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$$O = Tr(Y^1 Y_1^\dagger - Y^2 Y_2^\dagger - Y^3 Y_3^\dagger - Y^4 Y_4^\dagger)$$

Each $Y^I_{a\bar{a}} \to 2M^2$ real fields

$\Rightarrow O(N)\times O(3N)$ model, with $N = 2M^2$

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Big question:

does QFT with an unbounded negative potential make sense?

* no ground state or equilibrium

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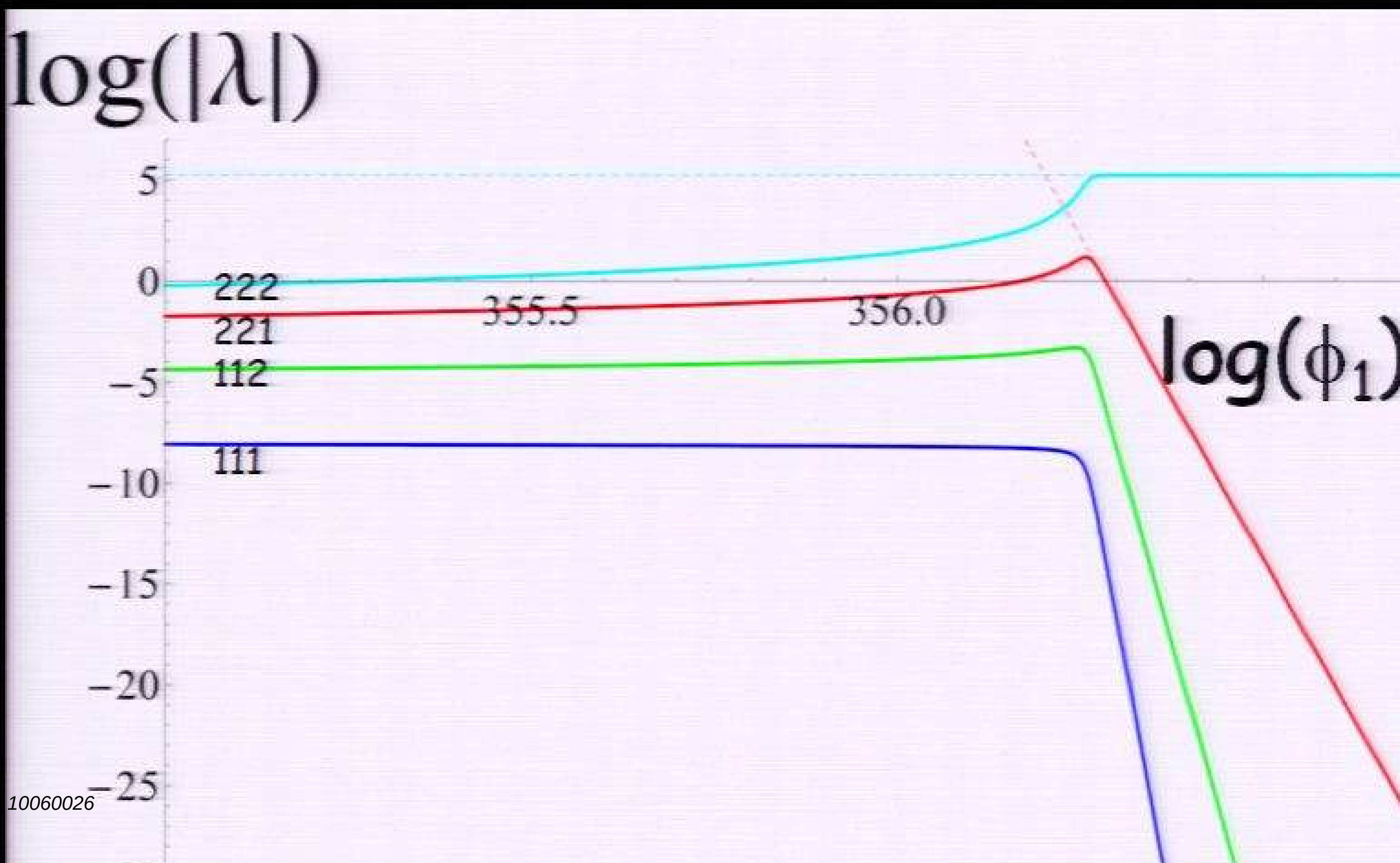

10060026

UV fixed p

$\lambda_{222} = 192$

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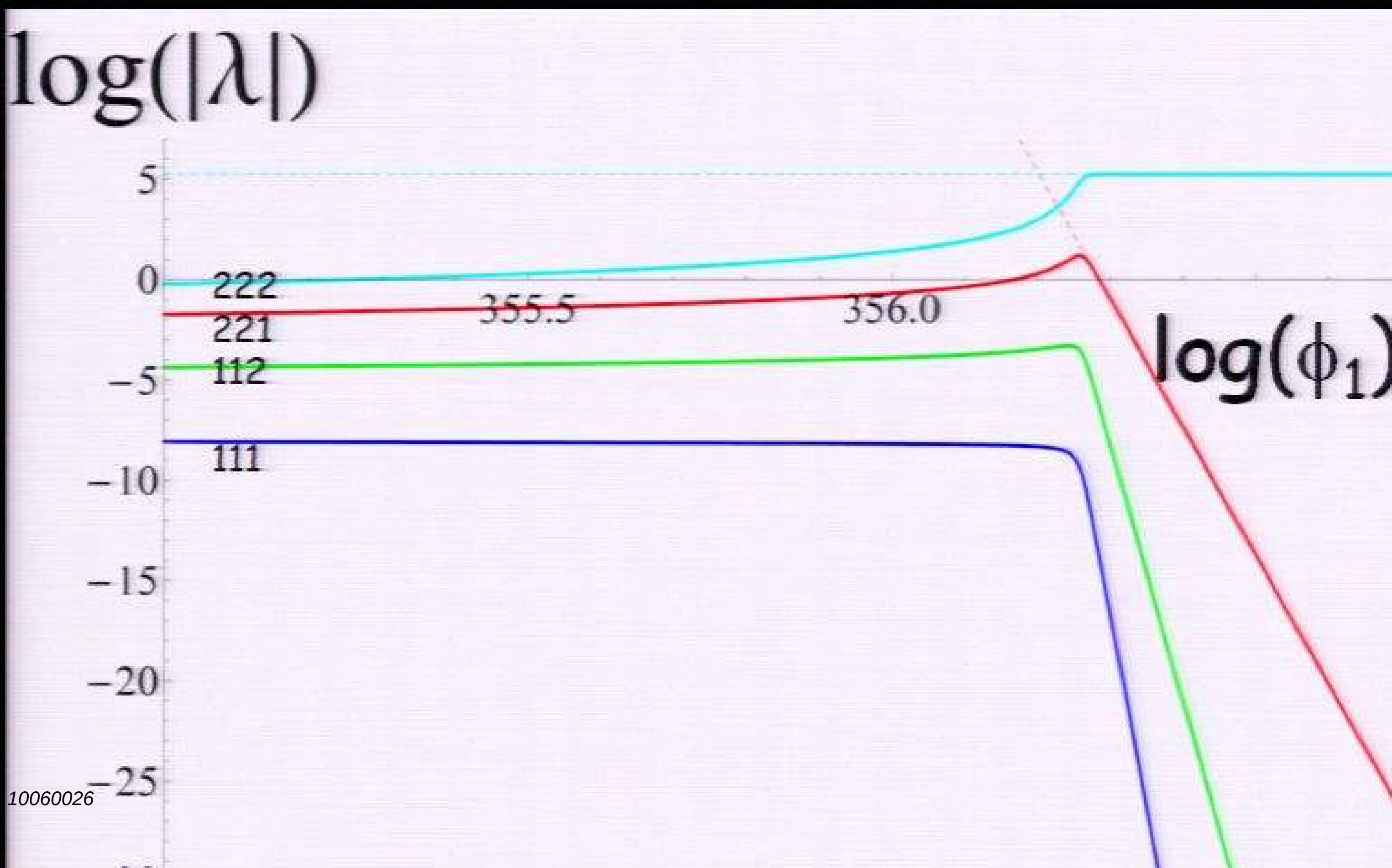

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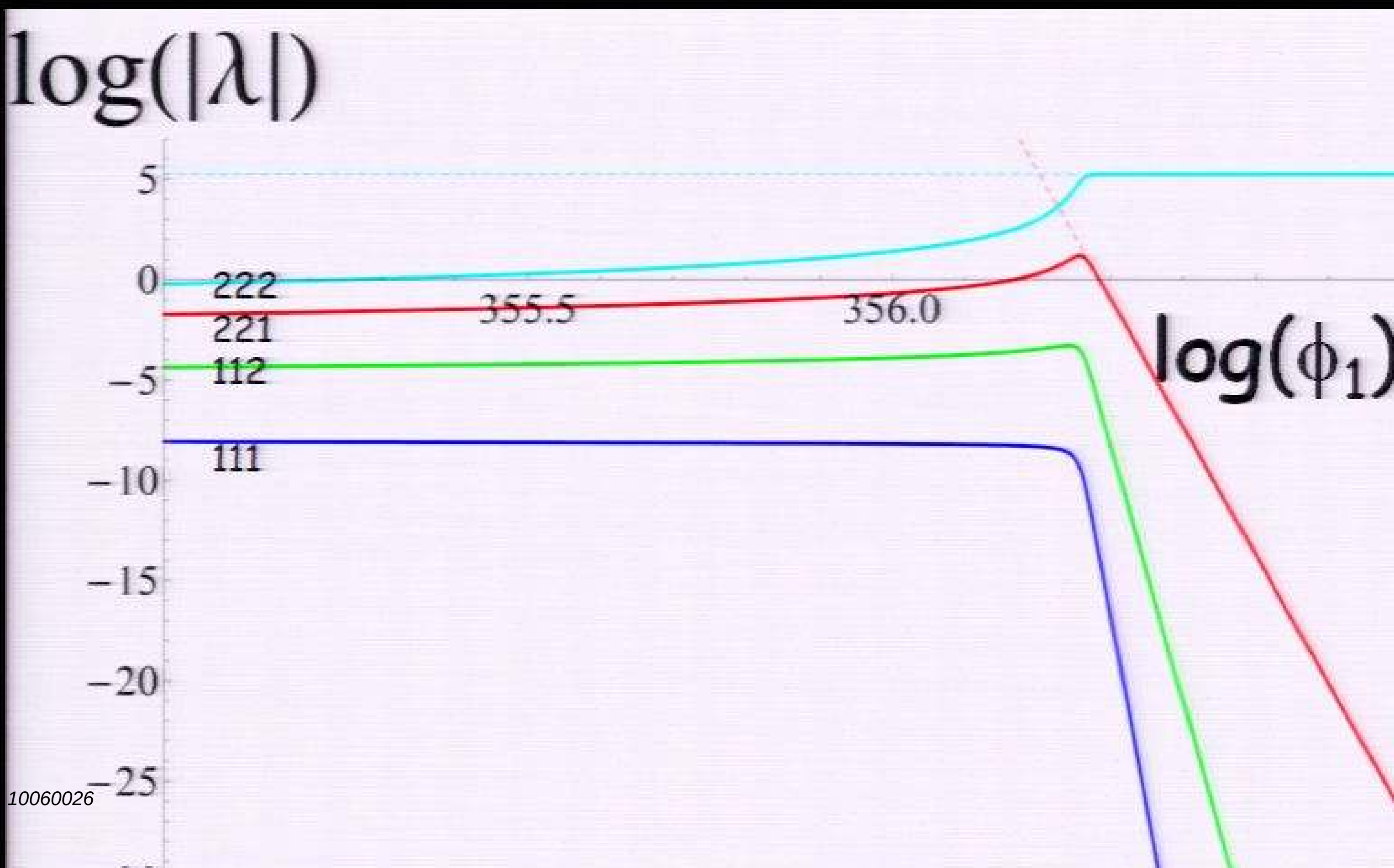

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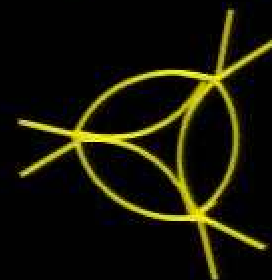

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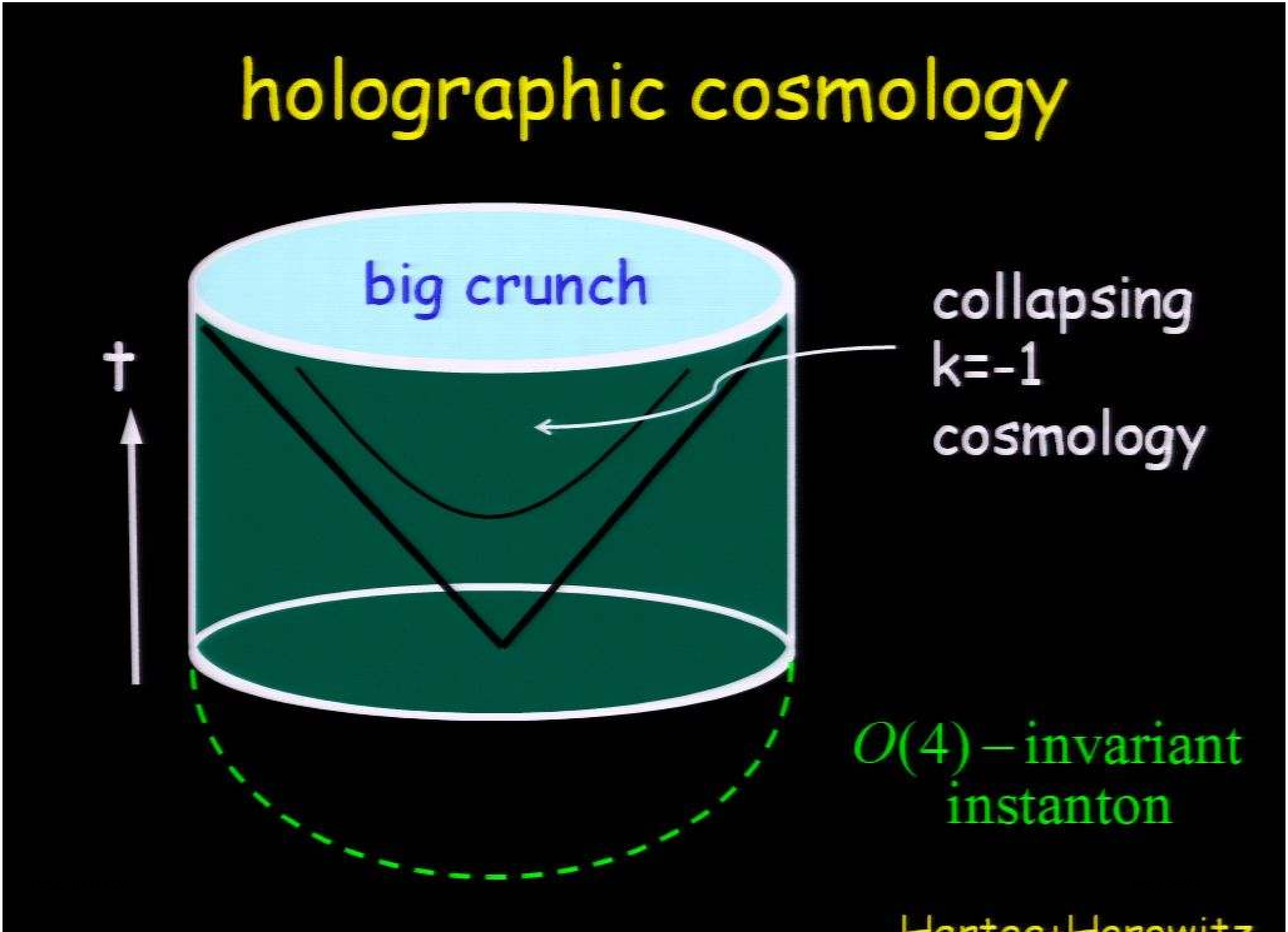
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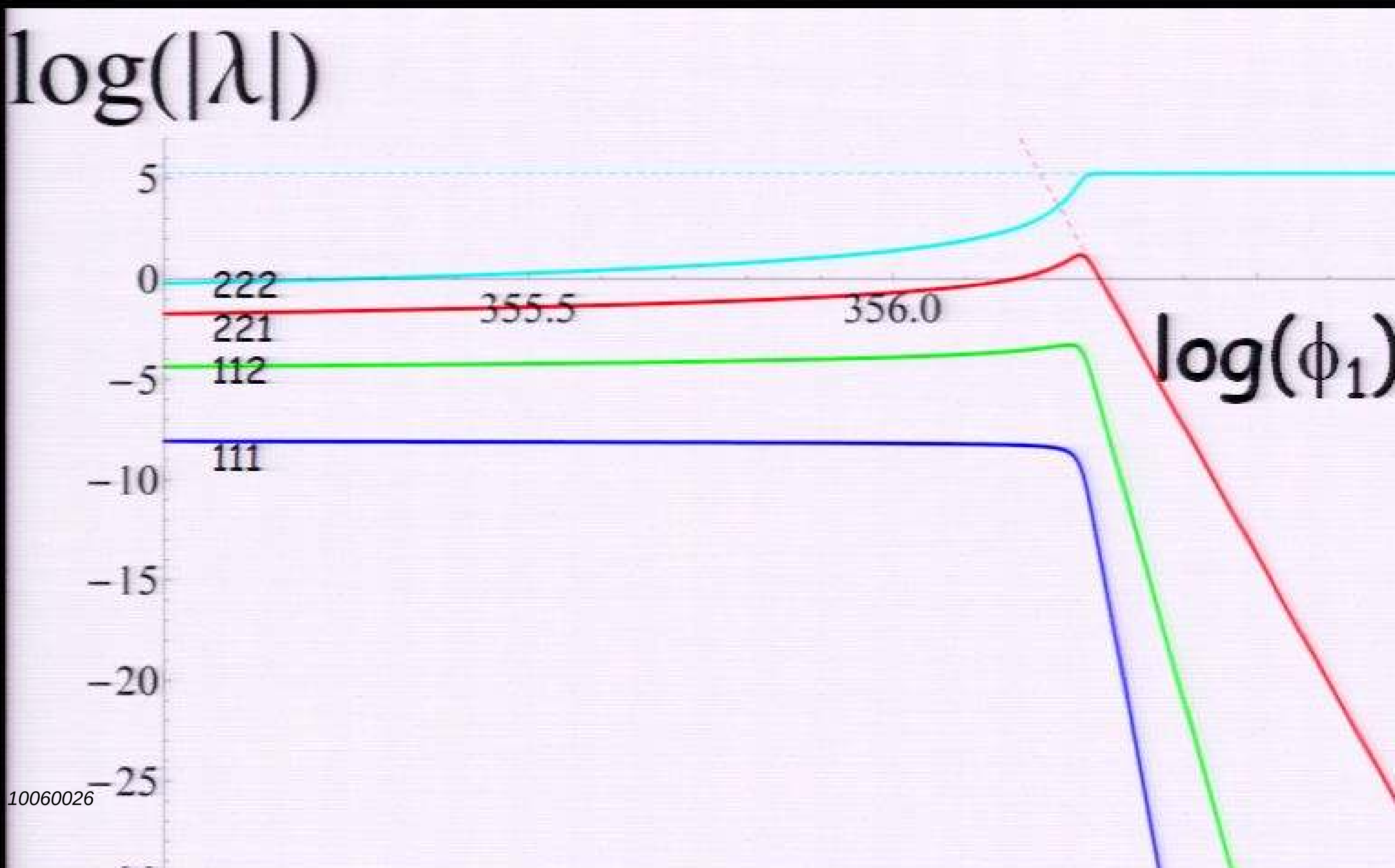

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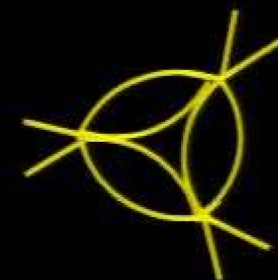

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Including O(1/N) running, renormalised potential continues to take form

$$V_{ren} = \frac{1}{6N^2}\lambda_{222}(\vec{\phi}_2^{\,2} - \gamma\vec{\phi}_1^{\,2})^3$$

where γ runs to zero in UV

Dynamics: ϕ_1 is classically unstable
-> drives system to UV

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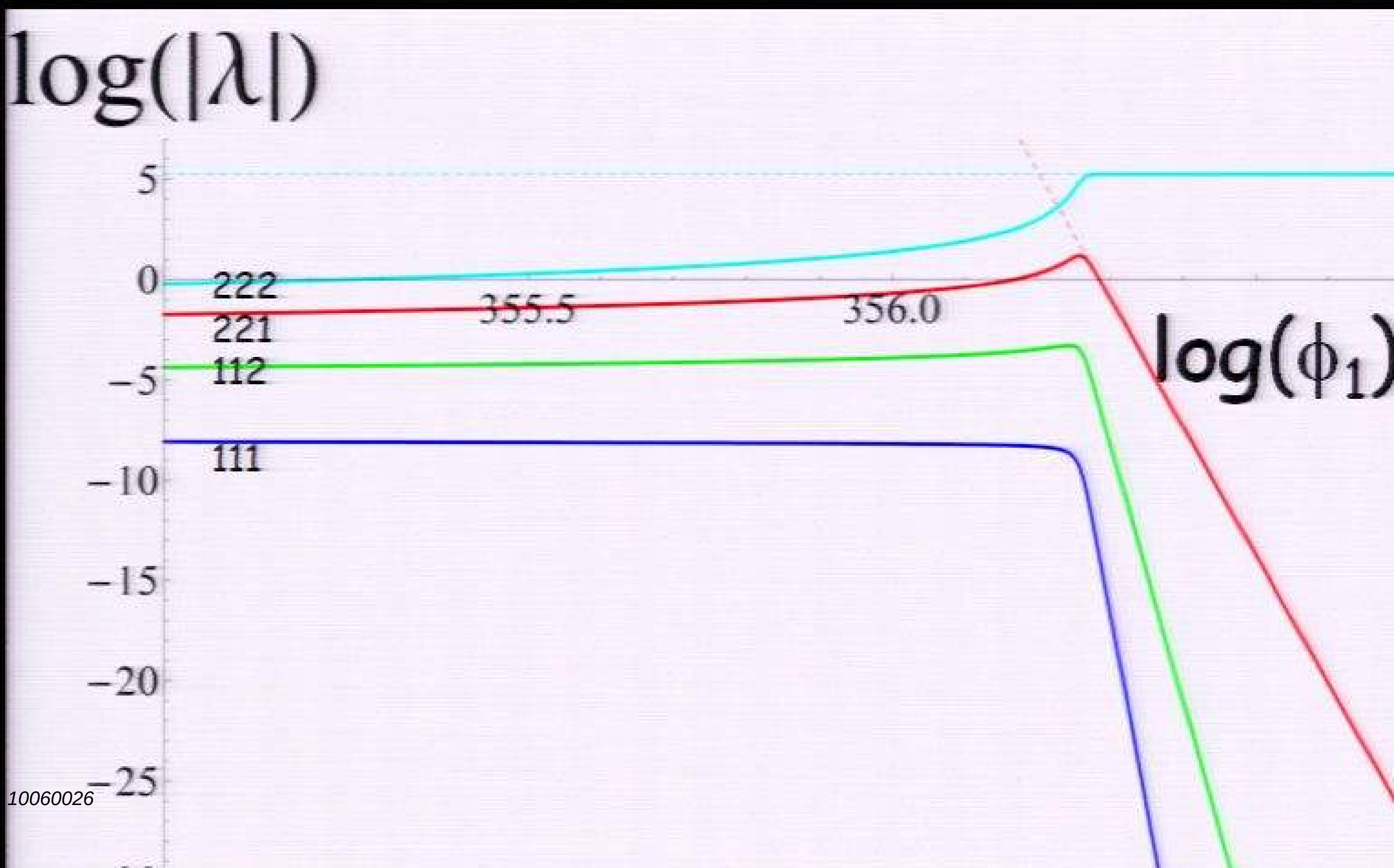

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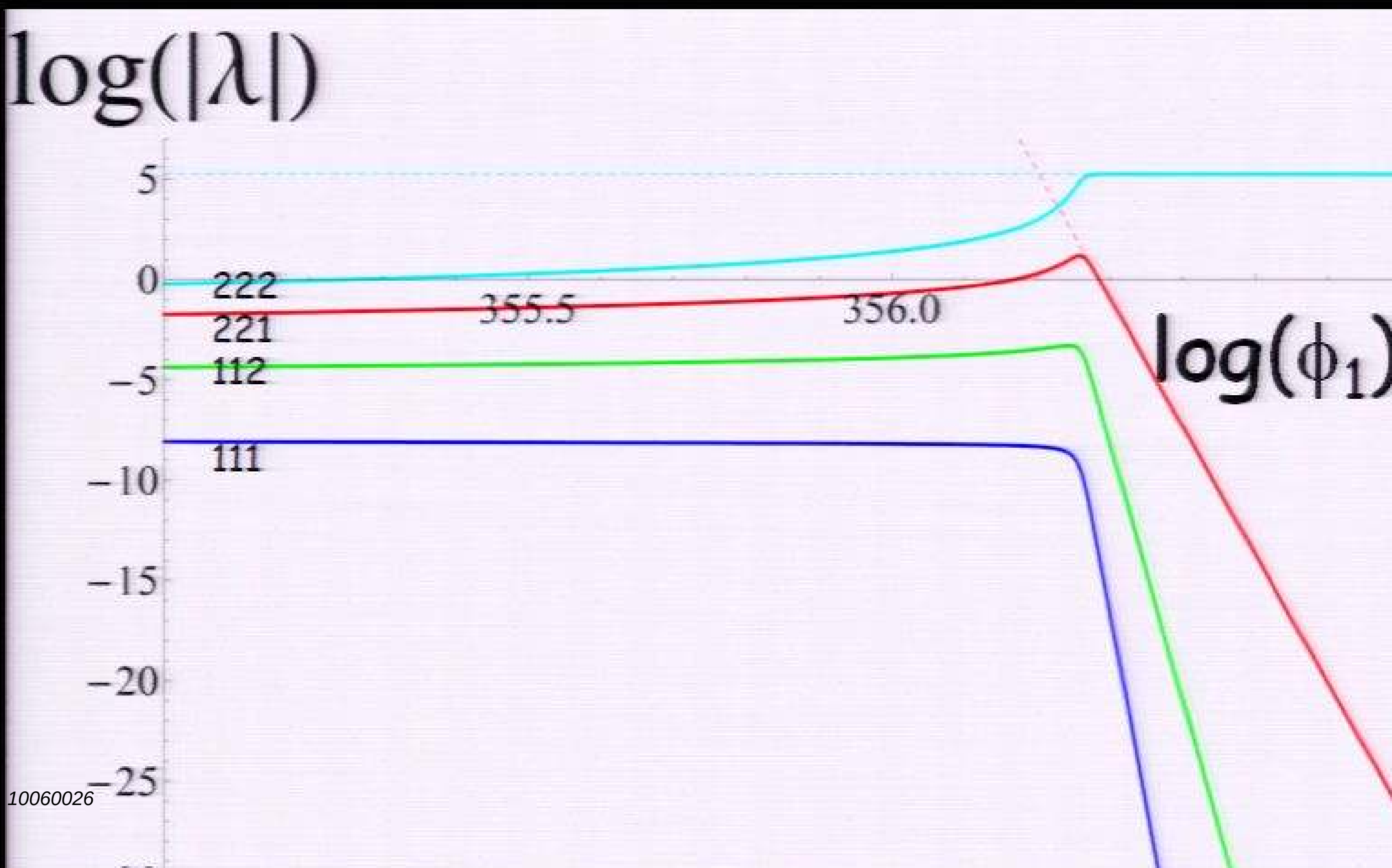

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BUT: as λ_{222} grows, ϕ_2 goes quantum unstabl

Trial wavefunction: free field mass m

$$\left\langle \phi^2 \right\rangle = \int_0^\Lambda \frac{d^2k}{(2\pi)^2}\frac{1}{2\omega_k} = \tfrac{1}{4\pi}(\Lambda - m);$$

Bardeen,Moshe,Bander

$$\left\langle \phi^2 \right\rangle_{ren} = \tfrac{1}{4\pi}(-m) \quad \Rightarrow \left\langle H \right\rangle = N\frac{m^3}{24\pi}\left(1 - \frac{\lambda}{16\pi^2}\right)$$

For $\lambda > 16\pi^2$, theory has no ground state

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Renormalization:

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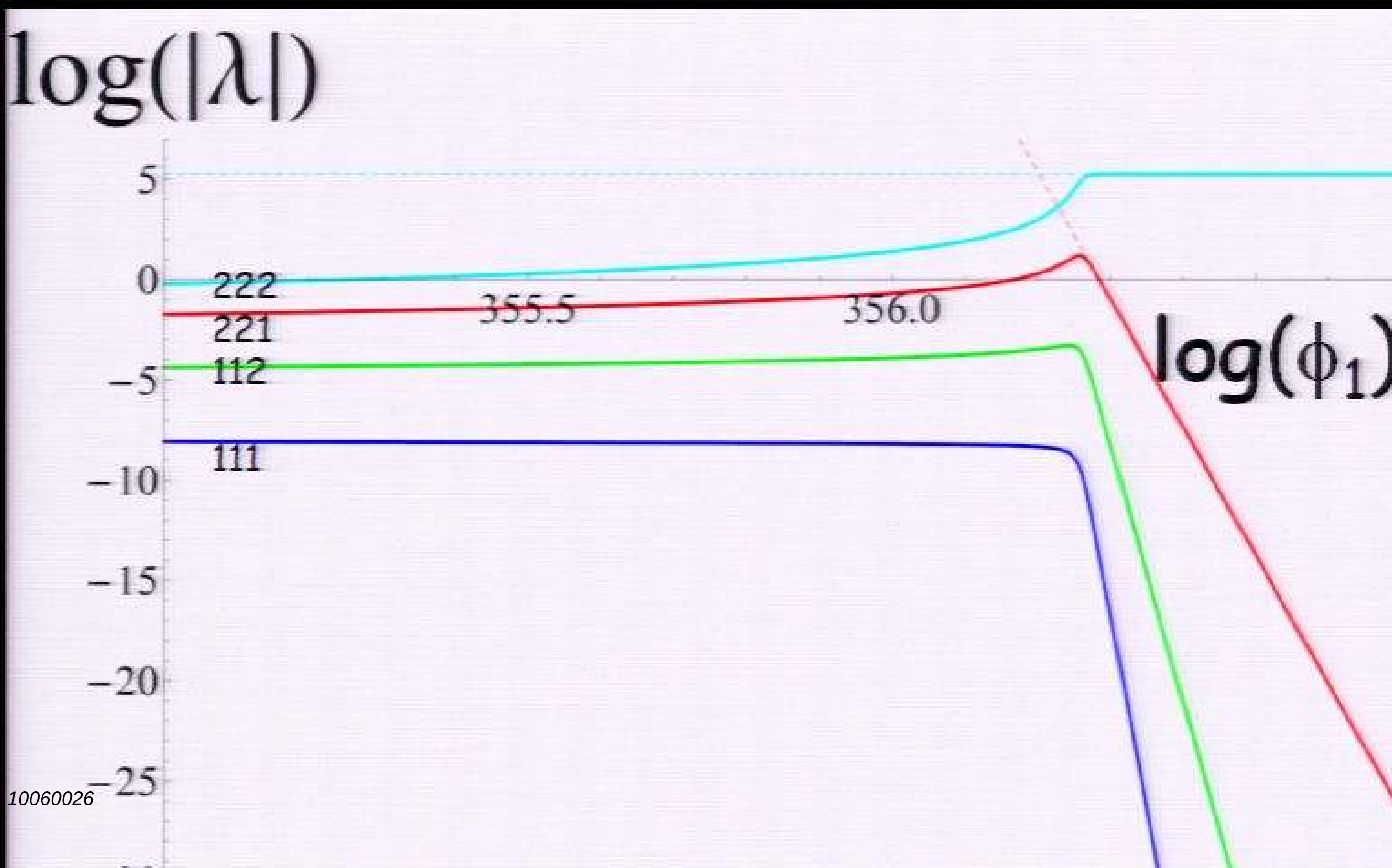

UV fixed pt

$\lambda_{222} = 192$

10060026

Why?

Pisarski 80's - for O(N) model

$$\beta_\lambda = \frac{3}{\pi^2 N}\left(\lambda^2 - \frac{1}{192}\lambda^3\right)$$

* suppressed by 1/N
* UV fixed pt exists as long as N sufficiently large, k>>N
* In opposite regime, of strong 'tHooft coupling , k<<N, AdS/CFT calculation (a la Witten) also indicates UV fixed point

Including O(1/N) running, renormalised potential continues to take form

$$V_{ren} = \frac{1}{6N^2}\lambda_{222}(\vec{\phi}_2^{\,2} - \gamma\vec{\phi}_1^{\,2})^3$$

where γ runs to zero in UV

Dynamics: ϕ_1 is classically unstable
-> drives system to UV

$$\ddot{\phi}_1 \sim |\lambda_{111}|\phi_1^5 \Rightarrow \phi_1 \sim \frac{1}{|\lambda_{111}|^{\frac{1}{4}}|t|^{\frac{1}{2}}},\ t \to 0$$

BUT: as λ_{222} grows, ϕ_2 goes quantum unstabl

Trial wavefunction: free field mass m

$$\left\langle \phi^2 \right\rangle = \int_0^\Lambda \frac{d^2k}{(2\pi)^2}\frac{1}{2\omega_k} = \tfrac{1}{4\pi}(\Lambda - m);$$

Bardeen,Moshe,Bander

$$\left\langle \phi^2 \right\rangle_{ren} = \tfrac{1}{4\pi}(-m) \quad \Rightarrow \left\langle H \right\rangle = N\frac{m^3}{24\pi}\left(1 - \frac{\lambda}{16\pi^2}\right)$$

For $\lambda > 16\pi^2$, theory has no ground state

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For $\lambda > 16\pi^2$, theory has no ground state

However... there is a quantum scaling solution, which is stable

Ansatz: $\left\langle \vec{\phi}^2 \right\rangle_{ren} = -\frac{C}{|t|} N$ (at $N=\infty$; at finite N, exponent gets N^{-1} corrns)

$$\phi = \sum_{\vec{k}} \chi_{\vec{k}}(t)\, e^{i\vec{k}\cdot\vec{x}},\ \vec{\phi}^2 \to \left\langle \vec{\phi}^2 \right\rangle \text{ in } N \to \infty \text{ limit}$$

Field eq $\Rightarrow \ddot{\chi}_{\vec{k}} = -k^2 \chi_{\vec{k}} - \lambda \frac{C^2}{t^2} \chi_{\vec{k}}$, Bessel $\nu^2 = \frac{1}{4} - \lambda C^2$

Gap equation $\frac{1}{4} - \nu^2 = \frac{\lambda \nu^2}{16\pi^2} \left(\cot \nu\pi\right)^2$

$$\lambda \to \lambda_* = 192 \Rightarrow \nu \to \nu_* = N_* i,\ N_* = 1.061..$$

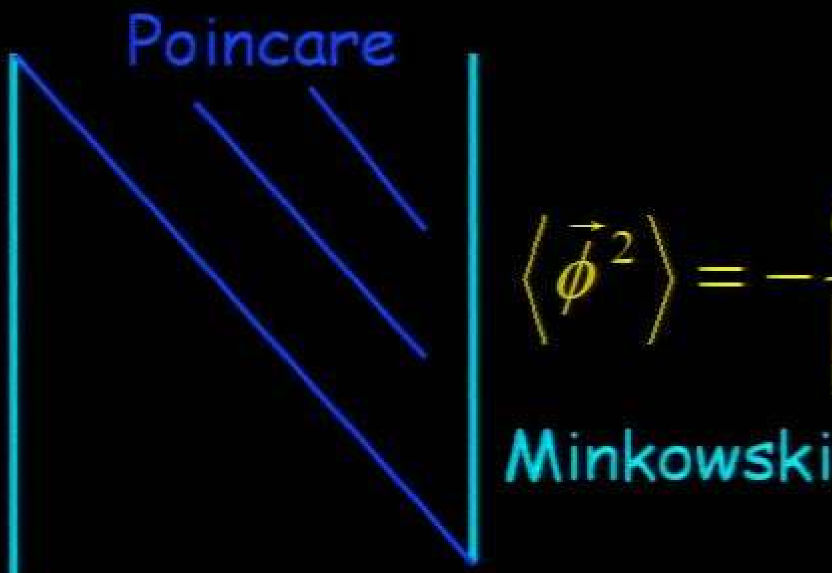
Instability dynamically breaks conformal symm

$$SO(3,2) \to SO(3,1)$$

As ϕ_1 rolls, it induces a transition to the quantum scaling phase

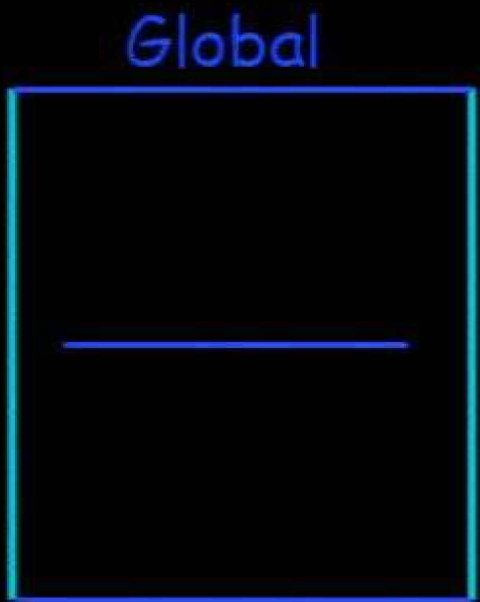
QM instability of ϕ_2 dominates nr singularity

1) UV fixed pt
2) D=3-> no trace anomaly
-> free to change conformal frame



$$\left\langle \vec{\phi}^{2} \right\rangle = -\frac{C}{|t|}N,\ \ \vec{\phi} \to |t|^{\frac{1}{2}}\vec{\phi},\ \ \eta \to \frac{1}{|t|^{2}}\eta$$

de Sitter!



$S^2 \times \mathbb{R}$

Just Weyl-transform from global dS_3:

$$ds^2 = \frac{1}{(\cos\tau)^2}(-d\tau^2 + d\Omega_2^2), \quad \langle \vec{\phi}^2 \rangle = -CN$$

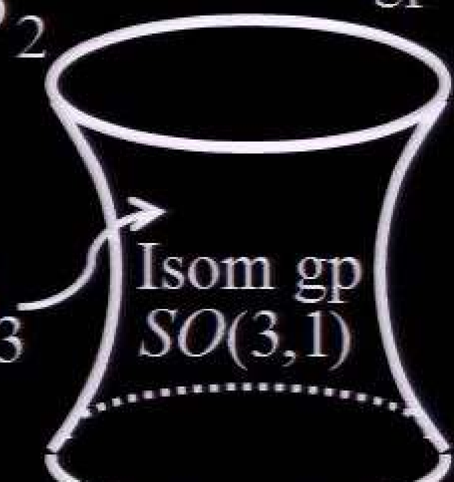
$$\rightarrow \quad ds^2 = (-d\tau^2 + d\Omega_2^2), \qquad \langle \vec{\phi}^2 \rangle = -\frac{CN}{\cos\tau}$$

bulk- boundary symmetry correspondence

boundary $= S_2$

Conf gp $SO(3,1)$

bulk $= \mathrm{dS}_3$



- no gravity
- no divergences

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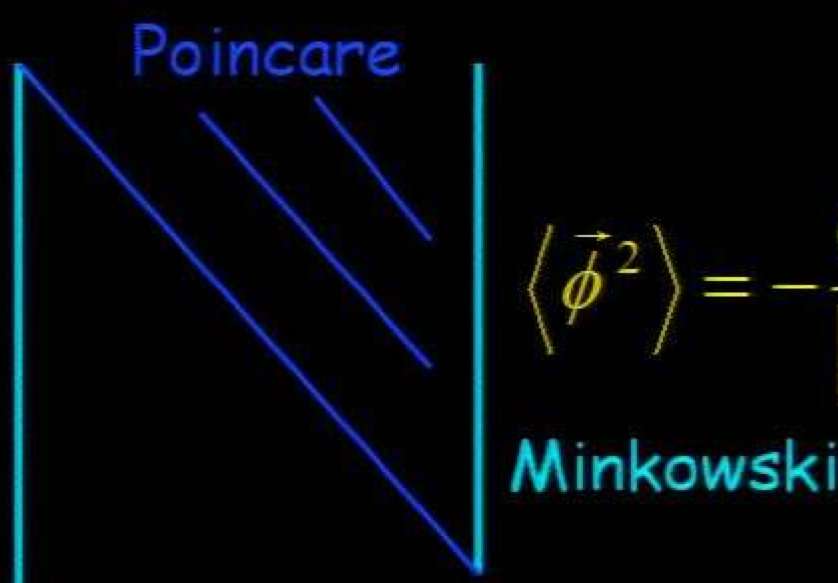

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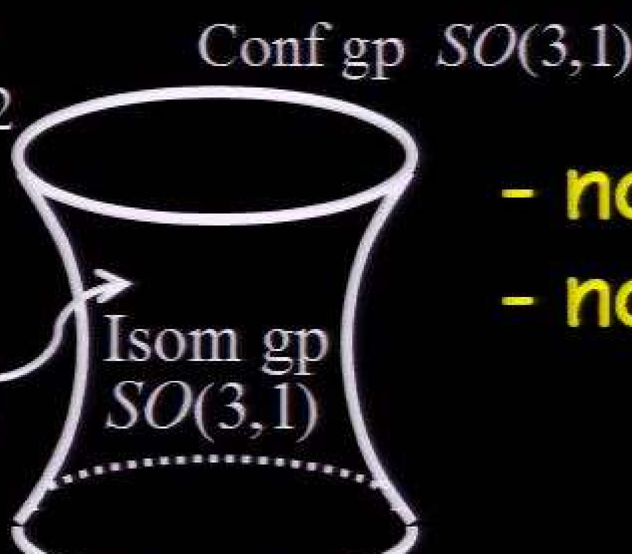

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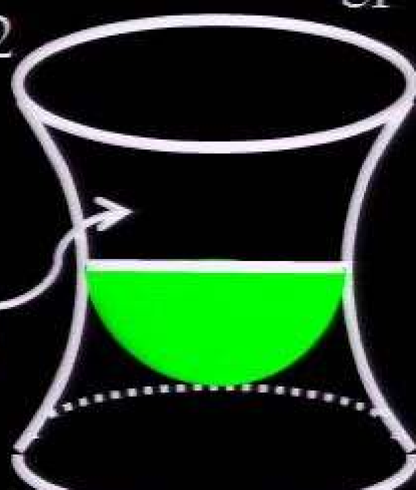
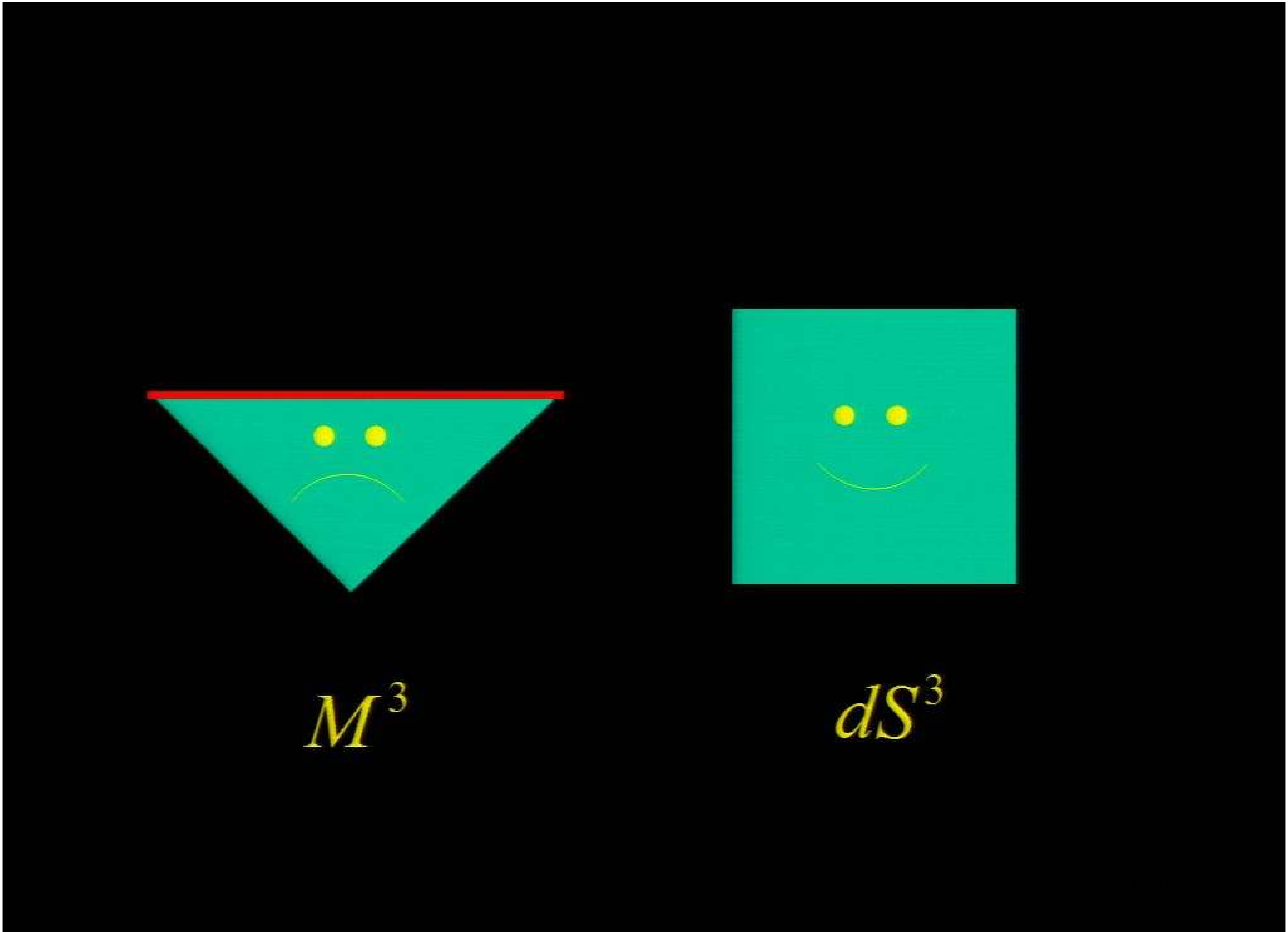
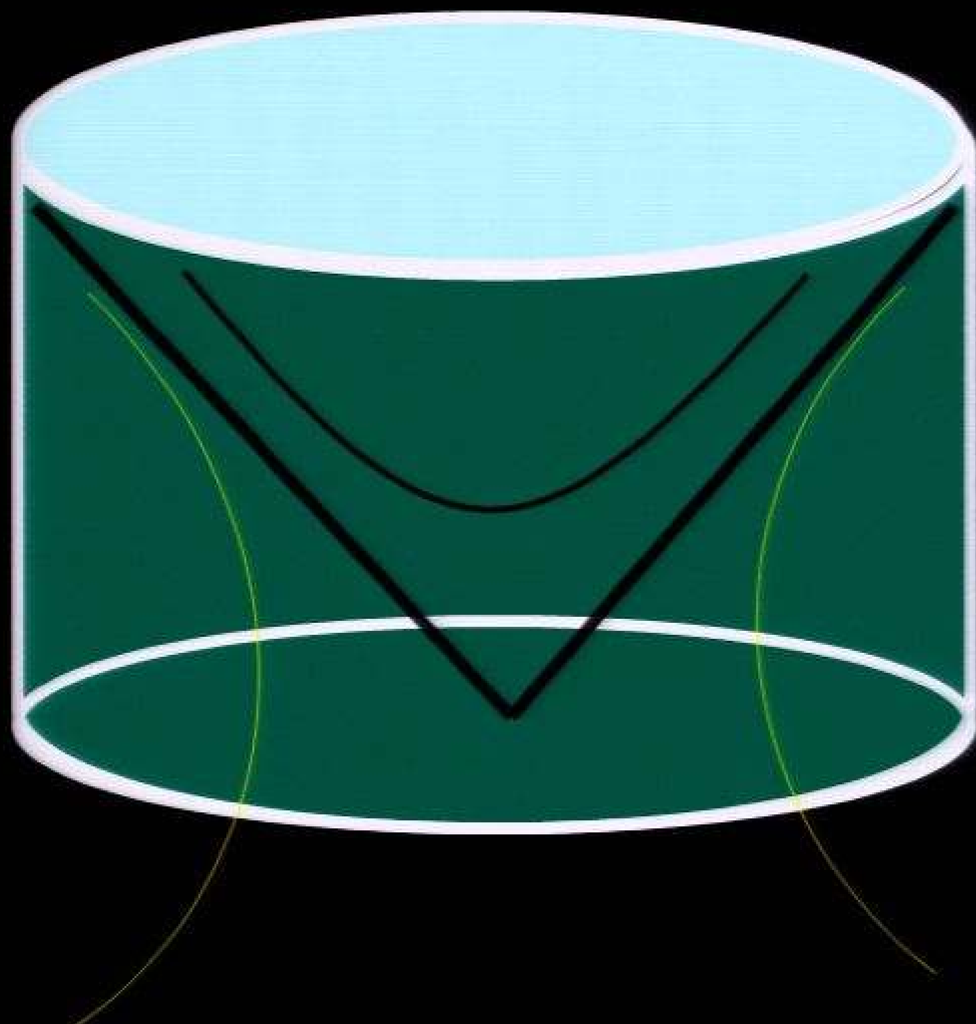


- no gravity
- no divergences
- Euclidean contin^n


M^3
dS^3

Bulk picture



By choosing boundary to be dS^3, we avoid the singularity!

Dual theory spontaneously generates a mass $\propto H_{dS} = R_{AdS}^{-1}$

No infinities enter at singularity

Euclidean continuation exists

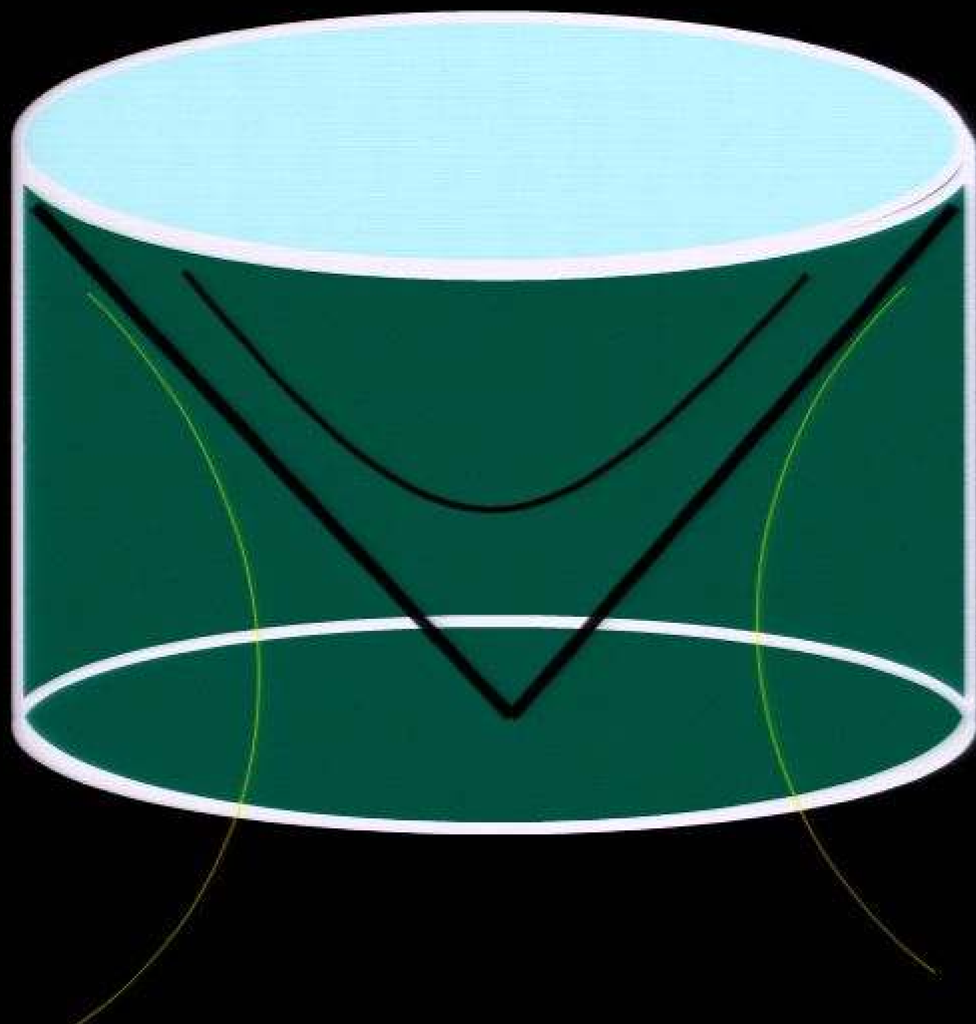
No tachyonic modes

(with M. Smolkin)

New, stable quantum phase on dS^3

(no bubble nucleation)

Bulk picture



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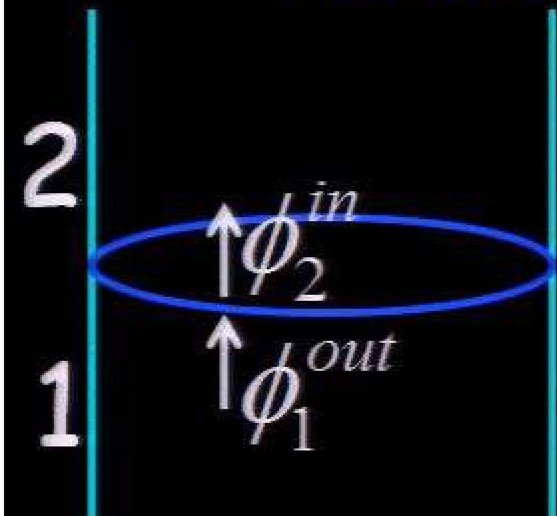
Crossing the singularity

dS_3 conformal to Einstein cylinder



need "S matrix": ϕ_1^{out} to ϕ_2^{in}

demand $SO(3,1)$ invariance**

conformal weight

$$\phi_1 \sim (\tfrac{\pi}{2} - \tau)^{1+iN_*} f_1^{out}(\Omega) + \text{h.c.},\ \tau \to \tfrac{\pi}{2}$$

Factor out τ dependence -> correlators take CFT form, weight $h=1+iN_*$

** may be shown to be anomaly free to all orders

Theorem: if $\quad f_2^{in}(\underline{\Omega}) = \int d\underline{\Omega}' \; G(\underline{\Omega}, \underline{\Omega}') \, f_1^{out}(\underline{\Omega}')$

and

$$\hat{O}_{\underline{\Omega}} f_2^{in}(\underline{\Omega}) = \int d\underline{\Omega}' \; G(\underline{\Omega}, \underline{\Omega}') \; \hat{O}_{\underline{\Omega}'} f_1^{out}(\underline{\Omega}') \quad \forall f_1^{out}(\underline{\Omega})$$

↑ conf gp gen

then $\; G(\underline{\Omega}, \underline{\Omega}') = \delta(\underline{\Omega}, \underline{\Omega}')$

Also, for dS_3, adiabatic in -> adiabatic out -> zero particle production in limit $N \to \infty$

$\Longrightarrow$ a perfectly cyclic universe, with calculable 1/N corrections

Proof: dS_3 : embed in Mink$_4$ $\;-X_0^2 + X_1^2 + X_2^2 + X_3^2 = 1$

$$ds^2 = -dt_C^2 + (\cosh t_C)^2 \, d\Omega_2$$

apply $\hat{O} = X_3 \partial_0 - X_0 \partial_3 = \cos\theta \; \partial_{t_C} - \tanh t_C \sin\theta \; \partial_\theta$

to $\mathrm{G}(\Omega, \Omega') = \mathrm{G}(\vec{\mathrm{n}} \cdot \vec{\mathrm{n}}\,') \equiv G(\Delta)$ (by $SO(3)$ invariance)

$\Rightarrow$

$$(h(\cos\theta' - \cos\theta) - 2\cos\theta')G(\Delta) = (\cos\theta + \cos\theta')(\Delta - 1)G'(\Delta)$$

Choose $\cos\theta = -\cos\theta' \Rightarrow rhs = 0$

$\Rightarrow G(\Delta) = 0 \;\; \forall \; -1 \leq \Delta < 1$

$\Rightarrow \; \mathrm{G}(\Delta) = \delta(\Omega, \Omega')$

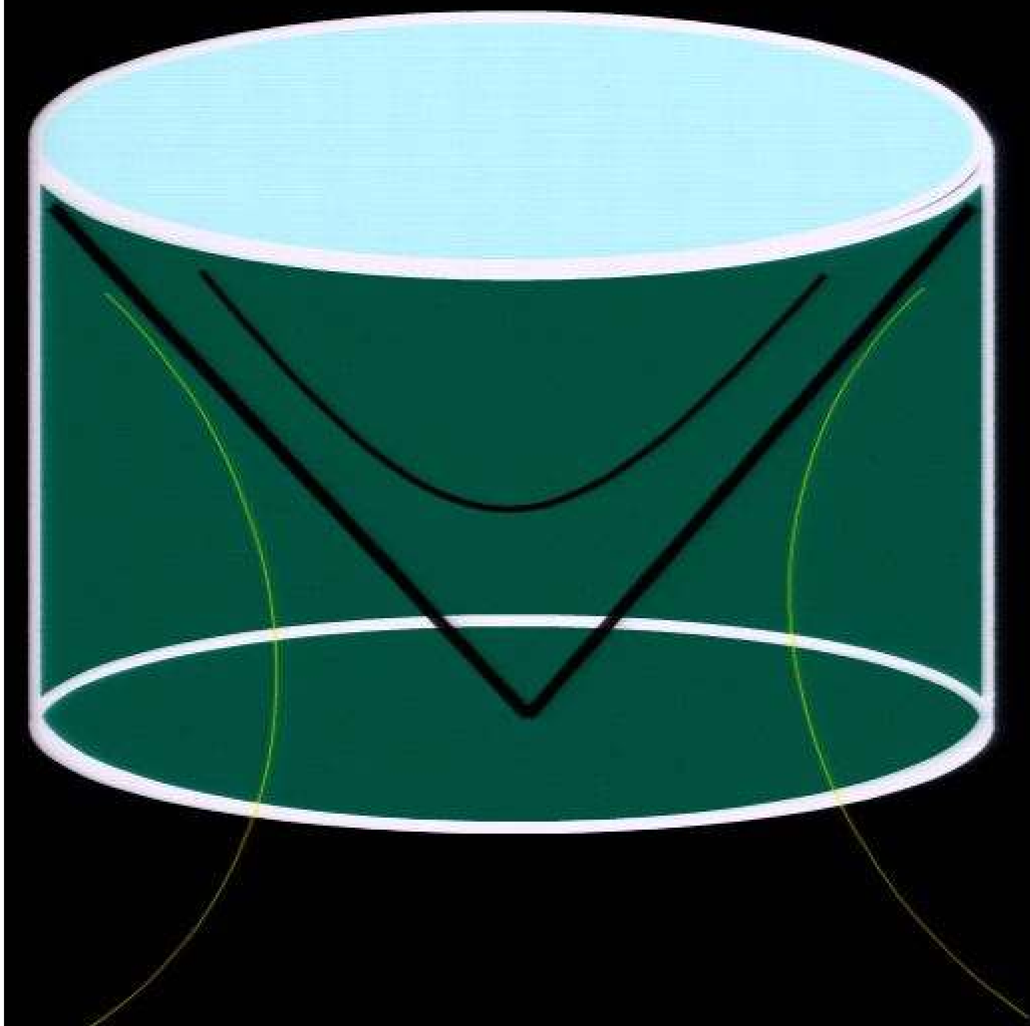
Particle creation across bounce
(in all modes) only occurs due to running
of couplings i.e. violation of scale invariance

Bogoliubov $\beta \sim \beta$ function $\sim f/M^2$

Parametrically small in large M limit:

4d cosmology bounces whereas 5d
cosmology (dual to deformed N=4 SYM)
does not!

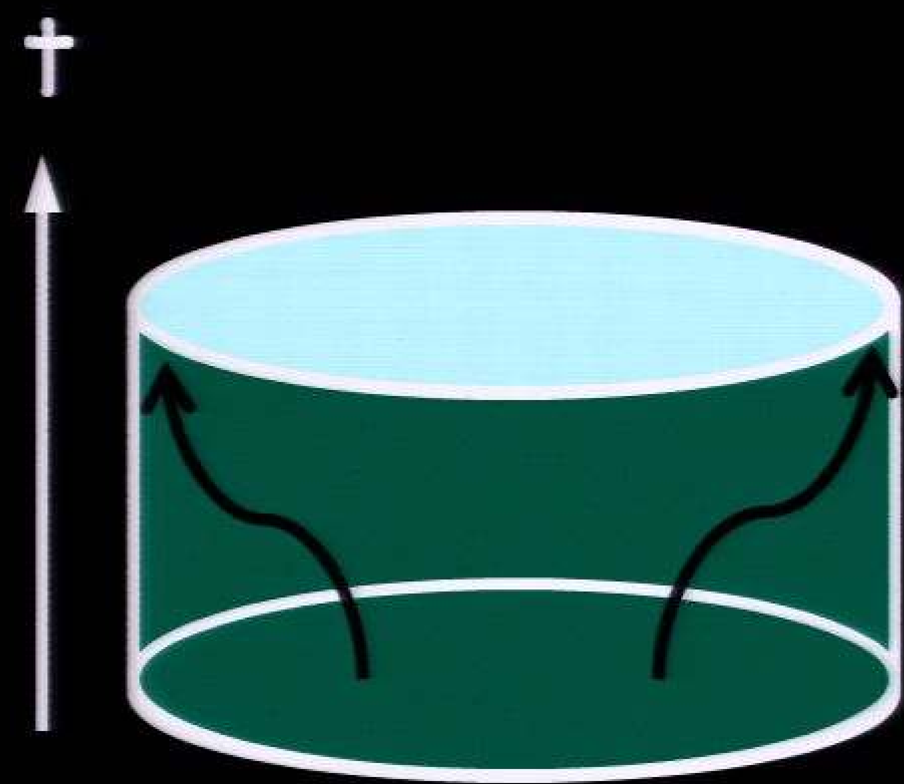
Strong 't Hooft coupling



bulk equations
with α' corrections
-> strong coupling
solution of CFT

Bulk solution for
O(6)xO(2) case - lifted
to 11d - is asymptotically
Milne near the singularity
i.e. the same bulk as for
a brane collision in 11d

a holographic bounce



an alternative to inflation?

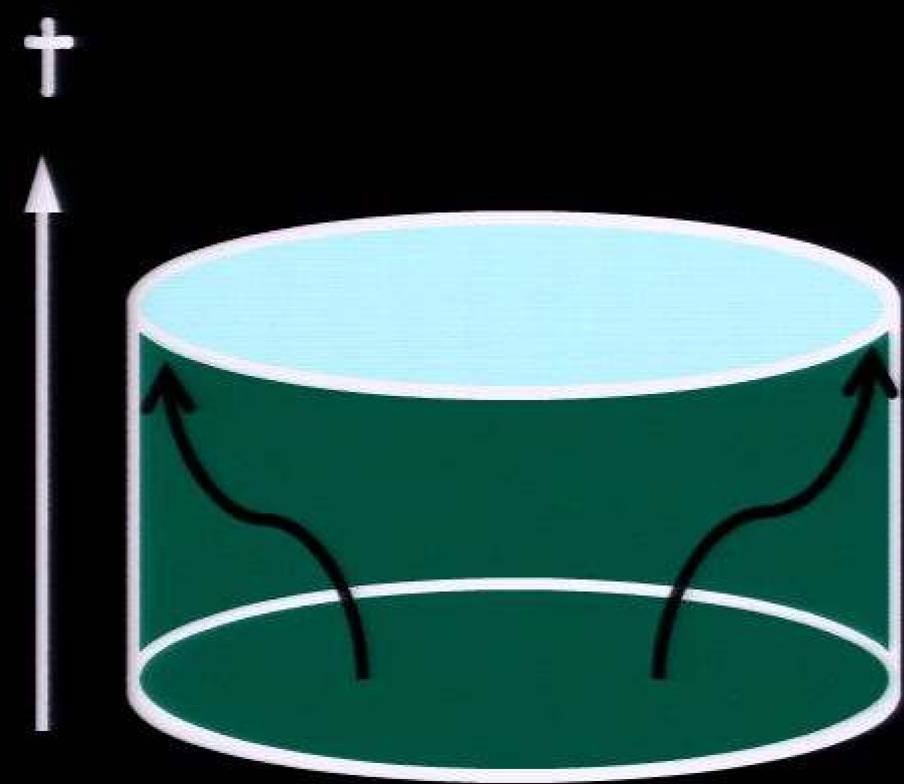
"scale invariance from scale invariance"

nearly scale-invariant perturbations are automatically generated (in the dual theory)
nearly Gaussian
small-amplitude ($1/N^6$ suppressed)
adiabatic
scalar
slightly red (perts go to zero at UV fixed pt)

need to translate these into the bulk, but expect these properties to survive

Thank you

a holographic bounce



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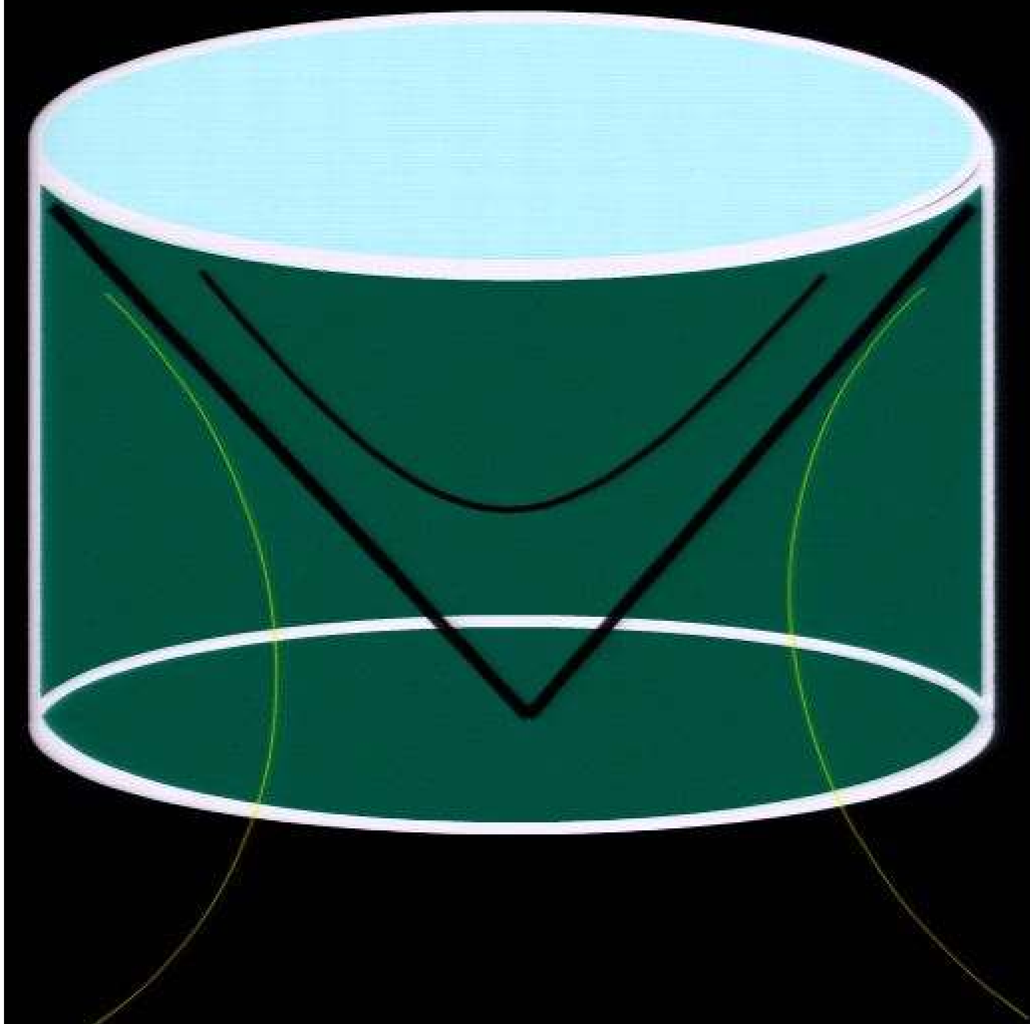

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