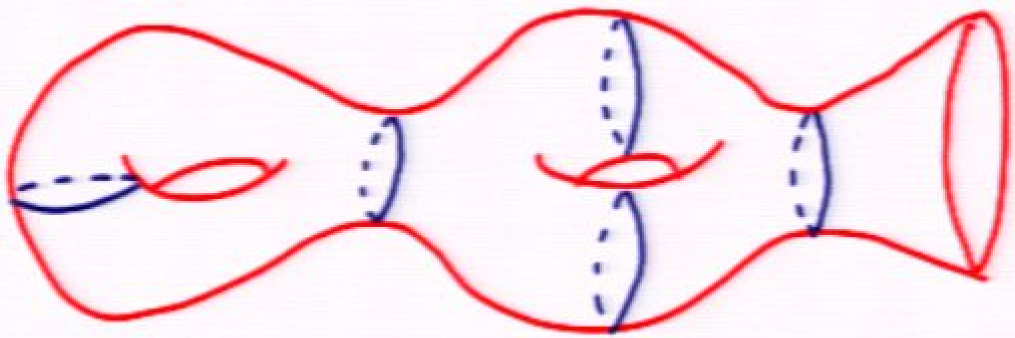
Title: On the Microscopic Description of the Kerr Black Hole

Date: Jun 17, 2010 10:00 AM

URL: http://pirsa.org/10060020

Abstract: We describe recent progress on the quantum description of the Kerr black hole. Previous descriptions of black hole microstates have relied on the existence of near-horizon regions with conformal symmetry, and hence have only worked for extremal or supersymmetric black holes. We argue that the states of non-extremal black holes can also be understood in terms of a conformal symmetry, the difference being that this symmetry is not geometrically realized. Thus a Kerr black hole is an excited state of a conformal field theory. By making certain (natural) assumptions about the nature of this dual CFT we can compute its density of states. This gives a microscopic computation of the Bekenstein-Hawking entropy of a Kerr black hole with arbitrary mass and angular momentum.

Microstates of the Kerr Black Hole



Cosmological Frontiers, 6-17-2010
Alex Maloney, McGill University

A. Castro, A. M. & A. Strominger
A. M. et al, to appear

Overview

The Problem:

Can we understand the quantum states of a realistic black hole?

String theory provides an accounting of the microstates of certain extremal black holes. These differ qualitatively from astrophysical black holes. They have $M = Q$ or $M = J^2$ and zero Hawking temperature.

Today I will discuss realistic black holes, with arbitrary mass M and angular momentum J.

I will argue that the states of $3+1$ dimensional Kerr are described by a dual two dimensional conformal field theory.

This allows us to derive the Bekenstein-Hawking entropy microscopically, provided the CFT satisfies certain (reasonable) criteria.

The Idea:

Every derivation of Black Hole entropy in string theory can be understood in terms of AdS/CFT.

The near-horizon region of a non-extremal black hole is Rindler space, not AdS.

Nevertheless, the states of quantum gravity still organize themselves into representations of the conformal group. The difference is that the conformal symmetry is not geometrically realized.

Aside from this, the computation proceeds in the exact same way as for extremal black holes. This works far from extremality.

But many features of this CFT remain mysterious.

Plan for Today:

- The "Near" Region
- Conformal Structure
- Counting States

Extremal Black Holes (A Caricature)

For an extremally charged ($M = Q$) or rotating ($M = J^2$) black hole we define the near horizon region by

$$r - r_{hor} << M$$

The geometry of the near horizon region includes an AdS factor.

The isometry group of AdS is the same as the conformal group in one less dimension; this symmetry group acts as conformal transformations on the asymptotic boundary of the near horizon geometry.

So the Hilbert space of states is that of a conformal theory.

Brown & Henneaux, Maldacena, ...

Non-Extremal Black Holes

A non-extremal black hole is unstable, but this does not preclude a CFT description. It just means that the CFT must be coupled to external degrees of freedom.

What is the analog of the "near-horizon" region?

For an extremal black hole the near-horizon region

$$r - r_{hor} << M$$

is the part of the geometry probed by low energy modes

$$\omega << M^{-1}$$

For non-extremal black holes these two definitions do not coincide. The first definition gives Rindler space.

What about the second?

The Near Region

Probe a non-extremal black hole by low energy modes

$$\omega << M^{-1}$$

These modes do not live near the horizon. But we can define the "near" region by

$$r << \omega^{-1}$$

This definition is probe-dependent, so is not a limit of the geometry.

When ω is small it includes

- The inner and outer horizons at $r = r_\pm$
- The ergosphere
- Regions outside the black hole

Matching Surface

Since $\omega << M^{-1}$ the two regions

- Near: $r << \omega^{-1}$
- Far: $r >> M$

overlap. We divide the geometry into near and far regions, and match together along a *matching surface* at

$$M << r_{match} << \omega^{-1}$$

This surface plays the same role as the boundary of AdS (or NHEK) in the extremal case.

Claim: Physics in the near region has a conformal symmetry which is realized as conformal transformations of the matching surface.

Kerr Metric

A Kerr black hole with mass M and angular momentum J has inner and outer horizons at $r = r_\pm$ given by

$$M = \frac{r_+ + r_-}{2}, \qquad \frac{J}{M} = \sqrt{r_+ r_-} \equiv a$$

In Boyer-Lindquist coordinates the Kerr metric is

$$ds^2 = \frac{\rho^2}{\Delta} dr^2 - \frac{\Delta}{\rho^2}(dt - a \sin^2\theta d\phi)^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}((r^2 + a^2) d\phi - a dt)^2$$

$$\Delta = (r - r_+)(r - r_-), \qquad \rho^2 = r^2 + a^2 \cos^2\theta$$

The Ergosphere is at $\rho = 0$.

The Wave Equation

Consider a field ϕ in the Kerr background.

$$\phi = e^{i\omega t}\phi(r,\theta,\phi)$$

In the limit $r << \omega^{-1}$ the Kerr Laplacian is

$$\nabla^2 = H^2 + \ell(\ell+1) = \bar{H}^2 + \ell(\ell+1)$$

where

$$H^2 = -H_0^2 + \{H_1, H_{-1}\}$$

is the Casimir of $SL(2,\mathbb{R})$ and $\ell(\ell+1)$ is the S^2 Laplacian.

Thus states organize into representations of

$$SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R = SO(2,2)$$

the rigid conformal group in $1+1$ dimensions.

Conformal Action

The generators act as conformal transformations on the matching surface. If we let

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R \phi}, \qquad w^- = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L \phi - t/2M}$$

then the $SL(2,\mathbb{R})$ generators are

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i(w^+\partial_+ + \frac{1}{2}y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

where y is a "radial" coordinate

$$y = \sqrt{\frac{r_+ - r_-}{r-r_-}}\, e^{\pi(T_L+T_R)\phi - \frac{t}{4M}}$$

The Wave Equation

Consider a field ϕ in the Kerr background.

$$\phi = e^{i\omega t}\phi(r,\theta,\phi)$$

In the limit $r << \omega^{-1}$ the Kerr Laplacian is

$$\nabla^2 = H^2 + \ell(\ell+1) = \bar{H}^2 + \ell(\ell+1)$$

where

$$H^2 = -H_0^2 + \{H_1, H_{-1}\}$$

is the Casimir of $SL(2,\mathbb{R})$ and $\ell(\ell+1)$ is the S^2 Laplacian.

Thus states organize into representations of

$$SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R = SO(2,2)$$

the rigid conformal group in $1+1$ dimensions.

Conformal Action

The generators act as conformal transformations on the matching surface. If we let

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}} e^{2\pi T_R \phi}, \qquad w^- = \sqrt{\frac{r-r_+}{r-r_-}} e^{2\pi T_L \phi - t/2M}$$

then the $SL(2,\mathbb{R})$ generators are

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i(w^+\partial_+ + \frac{1}{2}y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

where y is a "radial" coordinate

$$y = \sqrt{\frac{r_+ - r_-}{r-r_-}} e^{\pi(T_L+T_R)\phi - \frac{t}{4M}}$$

A CFT

This argument works for any field, including the graviton. It also extends to local conformal symmetries $Vir_L \times Vir_R$.

These conformal transformations are not geometrically realized, but they still act on the phase space of the theory.

More precisely, they act on the sector of the phase space defined by the "near" limit.

So states of the near region organize into representations of the conformal group, just like in AdS/CFT.

To describe the black hole microstates we need to know

- which state describes the black hole
- the central charge of the CFT

Conformal Action

The generators act as conformal transformations on the matching surface. If we let

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R \phi}, \qquad w^- = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L \phi - t/2M}$$

then the $SL(2,\mathbb{R})$ generators are

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i(w^+\partial_+ + \frac{1}{2}y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

where y is a "radial" coordinate

$$y = \sqrt{\frac{r_+ - r_-}{r-r_-}}\, e^{\pi(T_L+T_R)\phi - \frac{t}{4M}}$$

A CFT

This argument works for any field, including the graviton. It also extends to local conformal symmetries $Vir_L \times Vir_R$.

These conformal transformations are not geometrically realized, but they still act on the phase space of the theory.

More precisely, they act on the sector of the phase space defined by the "near" limit.

So states of the near region organize into representations of the conformal group, just like in AdS/CFT.

To describe the black hole microstates we need to know

- which state describes the black hole
- the central charge of the CFT

Finite Temperature

To find the state, note that because

$$\phi \sim \phi + 2\pi$$

the conformal generators are not globally defined. We must identify

$$w^+ \sim e^{4\pi^2 T_R} w^+, \qquad w^- \sim e^{4\pi^2 T_L} w^-$$

These identifications define a state a finite temperature

$$T_R = \frac{r_+ - r_-}{4\pi\sqrt{r_+ r_-}}, \qquad T_L = \frac{r_+ + r_-}{4\pi\sqrt{r_+ r_-}}$$

The state breaks $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ down to $U(1) \times U(1)$.

The theory is conformally invariant but the state is not.

Conformal Action

The generators act as conformal transformations on the matching surface. If we let

$$w^+ = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_R \phi}, \qquad w^- = \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_L \phi - t/2M}$$

then the $SL(2, \mathbb{R})$ generators are

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i(w^+\partial_+ + \frac{1}{2} y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

where y is a "radial" coordinate

$$y = \sqrt{\frac{r_+ - r_-}{r - r_-}} \, e^{\pi(T_L + T_R)\phi - \frac{t}{4M}}$$

Finite Temperature

To find the state, note that because

$$\phi \sim \phi + 2\pi$$

the conformal generators are not globally defined. We must identify

$$w^+ \sim e^{4\pi^2 T_R} w^+, \qquad w^- \sim e^{4\pi^2 T_L} w^-$$

These identifications define a state a finite temperature

$$T_R = \frac{r_+ - r_-}{4\pi\sqrt{r_+ r_-}}, \qquad T_L = \frac{r_+ + r_-}{4\pi\sqrt{r_+ r_-}}$$

The state breaks $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ down to $U(1) \times U(1)$.

The theory is conformally invariant but the state is not.

Conformal Action

The generators act as conformal transformations on the matching surface. If we let

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R \phi}, \qquad w^- = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L \phi - t/2M}$$

then the $SL(2,\mathbb{R})$ generators are

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i(w^+\partial_+ + \frac{1}{2}y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

where y is a “radial” coordinate

$$y = \sqrt{\frac{r_+ - r_-}{r-r_-}}\, e^{\pi(T_L+T_R)\phi - \frac{t}{4M}}$$

A CFT

This argument works for any field, including the graviton. It also extends to local conformal symmetries $Vir_L \times Vir_R$.

These conformal transformations are not geometrically realized, but they still act on the phase space of the theory.

More precisely, they act on the sector of the phase space defined by the "near" limit.

So states of the near region organize into representations of the conformal group, just like in AdS/CFT.

To describe the black hole microstates we need to know

- which state describes the black hole
- the central charge of the CFT

Conformal Action

The generators act as conformal transformations on the matching surface. If we let

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R \phi}, \qquad w^- = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L \phi - t/2M}$$

then the $SL(2,\mathbb{R})$ generators are

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i(w^+\partial_+ + \frac{1}{2}y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

where y is a “radial” coordinate

$$y = \sqrt{\frac{r_+ - r_-}{r-r_-}}\, e^{\pi(T_L+T_R)\phi - \frac{t}{4M}}$$

Finite Temperature

To find the state, note that because

$$\phi \sim \phi + 2\pi$$

the conformal generators are not globally defined. We must identify

$$w^+ \sim e^{4\pi^2 T_R} w^+, \qquad w^- \sim e^{4\pi^2 T_L} w^-$$

These identifications define a state a finite temperature

$$T_R = \frac{r_+ - r_-}{4\pi\sqrt{r_+ r_-}}, \qquad T_L = \frac{r_+ + r_-}{4\pi\sqrt{r_+ r_-}}$$

The state breaks $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ down to $U(1) \times U(1)$.

The theory is conformally invariant but the state is not.

Central Charge

We need to find a vacuum state invariant under the symmetries.

If we set $T_R = 0$ the black hole is extremally rotating $M = J^2$:

- The "near" region is the "near-horizon extremal Kerr" geometry of Bardeen & Horowitz
- the $SL(2,\mathbb{R})_R$ is unbroken and geometrically realized.

The algebra of charges which generate Vir_R diffeomorphisms is the Virasoro algebra, with central charge

$$c_R = 12J$$

This is the "Kerr/CFT" used to describe extremal black holes of Guica, Hartman, Song & Strominger.

Conformal Action

The generators act as conformal transformations on the matching surface. If we let

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_R \phi}, \qquad w^- = \sqrt{\frac{r-r_+}{r-r_-}}e^{2\pi T_L \phi - t/2M}$$

then the $SL(2,\mathbb{R})$ generators are

$$\begin{aligned} H_1 &= i\partial_+ \\ H_0 &= i(w^+\partial_+ + \frac{1}{2}y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

where y is a "radial" coordinate

$$y = \sqrt{\frac{r_+ - r_-}{r-r_-}}\, e^{\pi(T_L+T_R)\phi - \frac{t}{4M}}$$

Finite Temperature

To find the state, note that because

$$\phi \sim \phi + 2\pi$$

the conformal generators are not globally defined. We must identify

$$w^+ \sim e^{4\pi^2 T_R} w^+, \qquad w^- \sim e^{4\pi^2 T_L} w^-$$

These identifications define a state a finite temperature

$$T_R = \frac{r_+ - r_-}{4\pi\sqrt{r_+ r_-}}, \qquad T_L = \frac{r_+ + r_-}{4\pi\sqrt{r_+ r_-}}$$

The state breaks $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ down to $U(1) \times U(1)$.

The theory is conformally invariant but the state is not.

Central Charge

We need to find a vacuum state invariant under the symmetries.

If we set $T_R = 0$ the black hole is extremally rotating $M = J^2$:

- The "near" region is the "near-horizon extremal Kerr" geometry of Bardeen & Horowitz
- the $SL(2,\mathbb{R})_R$ is unbroken and geometrically realized.

The algebra of charges which generate Vir_R diffeomorphisms is the Virasoro algebra, with central charge

$$c_R = 12J$$

This is the "Kerr/CFT" used to describe extremal black holes of Guica, Hartman, Song & Strominger.

Finite Temperature

To find the state, note that because

$$\phi \sim \phi + 2\pi$$

the conformal generators are not globally defined. We must identify

$$w^+ \sim e^{4\pi^2 T_R} w^+, \qquad w^- \sim e^{4\pi^2 T_L} w^-$$

These identifications define a state a finite temperature

$$T_R = \frac{r_+ - r_-}{4\pi\sqrt{r_+ r_-}}, \qquad T_L = \frac{r_+ + r_-}{4\pi\sqrt{r_+ r_-}}$$

The state breaks $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ down to $U(1) \times U(1)$.

The theory is conformally invariant but the state is not.

Central Charge

We need to find a vacuum state invariant under the symmetries.

If we set $T_R = 0$ the black hole is extremally rotating $M = J^2$:

- The "near" region is the "near-horizon extremal Kerr" geometry of Bardeen & Horowitz
- the $SL(2,\mathbb{R})_R$ is unbroken and geometrically realized.

The algebra of charges which generate Vir_R diffeomorphisms is the Virasoro algebra, with central charge

$$c_R = 12J$$

This is the "Kerr/CFT" used to describe extremal black holes of Guica, Hartman, Song & Strominger.

Density of States

Assume that the CFT

- is parity invariant,
- is modular invariant, and
- possesses a normailzable ground state.

Then we can compute the density of states using Cardy's formula

$$N \sim \exp\left\{\frac{\pi^2}{3}(c_L T_L + c_R T_R)\right\}$$

This is a good approximation when T_L and T_R are large, which happens when the mass M is large.

Note that T_L and T_R are *not* the usual Hawking temperature and angular potential.

Entropy

This reproduces the Bekenstein-Hawking entropy

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = \frac{\text{Area}}{4}$$

including the numerical coefficient.

We have reproduced the entropy of a *realistic* black hole – not just an extremal one – using a dual CFT.

Moreover,

- Scattering amplitudes reproduce CFT correlation functions.
- This works for a variety of other non-extremal black holes.

Conclusions

We are closing in on a microscopic description of realistic, astrophysical black holes.

Using holographic techniques,

- States live in a dual CFT; conformal symmetries act on the phase space but not on the geometry.
- This reproduces the Bekenstein-Hawking entropy, including the precise numerical coefficient.

This was not a string theory construction so we don't know the dual CFT. We don't even know the vacuum state of the theory.

Are the microstates geometric in nature? Probably not...