Title: Non-Gaussianities from Inflation

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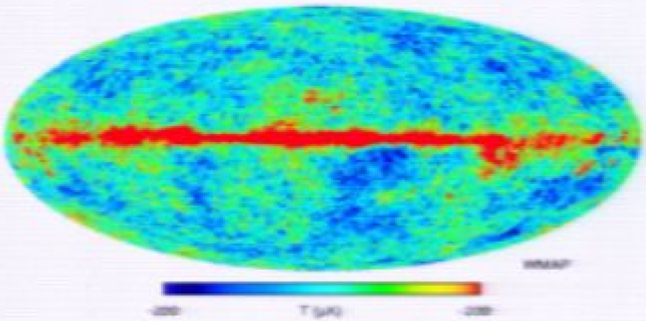
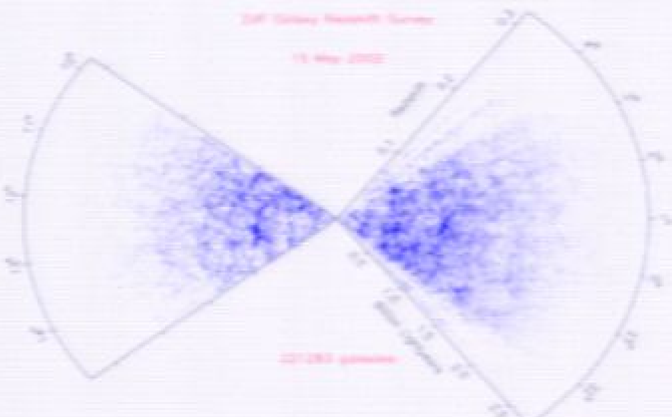
Abstract: TBA

Non-Gaussianities from Inflation

June 2010

The cosmological model





WMAP Cosmological Parameters

Model: lcdm+sz+lens

Data: wmap7+bao+h0

$10^2\Omega_b h^2$	2.260 ± 0.053	$1-n_s$	0.037 ± 0.012
$1-n_s$	$0.013 < 1-n_s < 0.061$ (95% CL)	$A_{BAO}(z=0.35)$	0.468 ± 0.011
C_{220}	5762^{+38}_{-37}	$d_A(z_{eq})$	14238^{+128}_{-129} Mpc
$d_A(z_*)$	14073^{+129}_{-130} Mpc	$\Delta^2_{\mathcal{R}}$	$(2.441^{+0.088}_{-0.092}) \times 10^{-9}$
h	$0.704^{+0.013}_{-0.014}$	H_0	$70.4^{+1.3}_{-1.4}$ km/s/Mpc
k_{eq}	0.00985 ± 0.00026	ℓ_{eq}	$138.6^{+2.6}_{-2.5}$
ℓ_*	302.40 ± 0.73	n_s	0.963 ± 0.012
Ω_b	0.0456 ± 0.0016	$\Omega_b h^2$	0.02260 ± 0.00053
Ω_c	0.227 ± 0.014	$\Omega_c h^2$	0.1123 ± 0.0035
Ω_Λ	$0.728^{+0.015}_{-0.016}$	Ω_m	$0.272^{+0.016}_{-0.015}$
$\Omega_m h^2$	0.1349 ± 0.0036	$r_{hor}(z_{dec})$	284.6 ± 1.9 Mpc
$r_s(z_d)$	152.7 ± 1.3 Mpc	$r_s(z_d)/D_v(z=0.2)$	$0.1904^{+0.0037}_{-0.0038}$
$r_s(z_d)/D_v(z=0.35)$	0.1143 ± 0.0029	$r_s(z_*)$	146.2 ± 1.1 Mpc
R	$1.7239^{+0.0100}_{-0.0099}$	σ_8	0.809 ± 0.024
A_{SZ}	$0.96^{+0.49}_{-0.96}$	t_0	13.75 ± 0.11 Gyr
τ	0.087 ± 0.014	θ_*	0.010389 ± 0.000025
θ_*	0.5953 ± 0.0014 °	t_*	377730^{+3205}_{-3200} yr
z_{dec}	1088.2 ± 1.1	z_d	1020.5 ± 1.3
z_{eq}	3232 ± 87	z_{reion}	10.4 ± 1.2
z_*	$1090.89^{+0.68}_{-0.69}$		

We have a standard cosmological model which fits the data remarkably well.
A universe filled with radiation, cold dark matter and a cosmological constant with perturbations already present "initially". These perturbations are scalar, scale invariant, adiabatic and Gaussian.

TABLE 1

SUMMARY OF THE COSMOLOGICAL PARAMETERS OF ΛCDM MODEL

Class	Parameter	WMAP 7-year ML[a]	WMAP+BAO+H_0 ML	WMAP 7-year Mean[b]	WMAP+BAO+H_0 Mean
Primary	$100\Omega_b h^2$	2.270	2.246	$2.258^{+0.057}_{-0.056}$	2.260 ± 0.053
	$\Omega_c h^2$	0.1107	0.1120	0.1109 ± 0.0056	0.1123 ± 0.0035
	Ω_Λ	0.738	0.728	0.734 ± 0.029	$0.728^{+0.015}_{-0.016}$
	n_s	0.969	0.961	0.963 ± 0.014	0.963 ± 0.012
	τ	0.086	0.087	0.088 ± 0.015	0.087 ± 0.014
	$\Delta^2_{\mathcal{R}}(k_0)$[c]	2.38×10^{-9}	2.45×10^{-9}	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.441^{+0.088}_{-0.092}) \times 10^{-9}$
Derived	σ_8	0.803	0.807	0.801 ± 0.030	0.809 ± 0.024
	H_0	71.4 km/s/Mpc	70.2 km/s/Mpc	71.0 ± 2.5 km/s/Mpc	$70.4^{+1.3}_{-1.4}$ km/s/Mpc
	Ω_b	0.0445	0.0455	0.0449 ± 0.0028	0.0456 ± 0.0016
	Ω_c	0.217	0.227	0.222 ± 0.026	0.227 ± 0.014
	$\Omega_m h^2$	0.1334	0.1344	$0.1334^{+0.0056}_{-0.0055}$	0.1349 ± 0.0036
	$z_{\rm reion}$[d]	10.3	10.5	10.5 ± 1.2	10.4 ± 1.2
	t_0[e]	13.71 Gyr	13.78 Gyr	13.75 ± 0.13 Gyr	13.75 ± 0.11 Gyr

[a] Larson et al. (2010). "ML" refers to the Maximum Likelihood parameters.

[b] Larson et al. (2010). "Mean" refers to the mean of the posterior distribution of each parameter. The quoted errors show the 68% confidence levels (CL).

[c] $\Delta^2_{\mathcal{R}}(k) = k^3 P_{\mathcal{R}}(k)/(2\pi^2)$ and $k_0 = 0.002$ Mpc^{-1}.

[d] "Redshift of reionization," if the universe was reionized instantaneously from the neutral state to the fully ionized state at $z_{\rm reion}$. Note that these values are somewhat different from those in Table 1 of Komatsu et al. (2009b), largely because of the changes in the treatment of reionization history in the Boltzmann code CAMB (Lewis 2008).

[e] The present-day age of the universe.

WMAP 7yr

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3\langle|\mathcal{R}_k|^2\rangle}{2\pi^2} = \Delta^2_{\mathcal{R}}(k_0)\left(\frac{k}{k_0}\right)^{n_s-1}.$$

where $k_0 = 0.002$ Mpc^{-1}, we find

$$n_s = 0.963 \pm 0.012 \text{ (68\% CL)}.$$

ns is 3 sigma different from 1 in the basic LCDM fit.

TABLE 2

SUMMARY OF THE 95% CONFIDENCE LIMITS ON DEVIATIONS FROM THE SIMPLE (FLAT, GAUSSIAN, ADIABATIC, POWER-LAW) ΛCDM MODEL EXCEPT FOR DARK ENERGY PARAMETERS

Sec.	Name	Case	WMAP 7-year	WMAP+BAO+SN[a]	WMAP+BAO+H_0
§ 4.1	Grav. Wave[b]	No Running Ind.	$r < 0.36$[c]	$r < 0.20$	$r < 0.24$
§ 4.2	Running Index	No Grav. Wave	$-0.084 < dn_s/d\ln k < 0.020$[c]	$-0.065 < dn_s/d\ln k < 0.010$	$-0.061 < dn_s/d\ln k < 0.017$
§ 4.3	Curvature	$w = -1$	N/A	$-0.0178 < \Omega_k < 0.0063$	$-0.0133 < \Omega_k < 0.0084$
§ 4.4	Adiabaticity	Axion	$\alpha_0 < 0.13$[c]	$\alpha_0 < 0.064$	$\alpha_0 < 0.077$
		Curvaton	$\alpha_{-1} < 0.011$[c]	$\alpha_{-1} < 0.0037$	$\alpha_{-1} < 0.0047$
§ 4.5	Parity Violation	Chern-Simons[d]	$-5.0° < \Delta\alpha < 2.8°$[e]	N/A	N/A
§ 4.6	Neutrino Mass[f]	$w = -1$	$\sum m_\nu < 1.3$ eV[c]	$\sum m_\nu < 0.71$ eV	$\sum m_\nu < 0.58$ eV[g]
		$w \neq -1$	$\sum m_\nu < 1.4$ eV[c]	$\sum m_\nu < 0.91$ eV	$\sum m_\nu < 1.3$ eV[h]
§ 4.7	Relativistic Species	$w = -1$	$N_{\rm eff} > 2.7$[c]	N/A	$4.34^{+0.86}_{-0.88}$ (68% CL)[i]
§ 6	Gaussianity[j]	Local	$-10 < f_{NL}^{\rm local} < 74$[k]	N/A	N/A
		Equilateral	$-214 < f_{NL}^{\rm equil} < 266$	N/A	N/A
		Orthogonal	$-410 < f_{NL}^{\rm orthog} < 6$	N/A	N/A

[a] "SN" denotes the "Constitution" sample of Type Ia supernovae compiled by Hicken et al. (2009b), which is an extension of the "Union" sample (Kowalski et al. 2008) that we used for the 5-year "WMAP+BAO+SN" parameters presented in Komatsu et al. (2009b). Systematic errors in the supernova data are not included. While the parameters in this column can be compared directly to the 5-year WMAP+BAO+SN parameters, they may not be as robust as the "WMAP+BAO+H_0" parameters, as the other compilations of the supernova data do not give the same answers (Hicken et al. 2009b; Kessler et al. 2009). See Section 3.2.4 for more discussion. The SN data will be used to put limits on dark energy properties. See Section 5 and Table 4.

[b] In the form of the tensor-to-scalar ratio, r, at $k = 0.002$ Mpc^{-1}.

[c] Larson et al. (2010).

[d] For an interaction of the form given by $[\phi(t)/M]F_{\alpha\beta}\tilde{F}^{\alpha\beta}$, the polarization rotation angle is $\Delta\alpha = M^{-1}\int \frac{dt}{a}\dot{\phi}$.

[e] The 68% CL limit is $\Delta\alpha = -1.1° \pm 1.3°$ (stat.) $\pm 1.5°$ (syst.), where the first error is statistical and the second error is systematic.

[f] [illegible]) eV.

[g] For WMAP+LRG+H_0, $\sum m_\nu < 0.44$ eV.

[h] For WMAP+LRG+H_0, $\sum m_\nu < 0.71$ eV.

[i] The 95% limit is $2.7 < N_{\rm eff} < 6.2$. For WMAP+LRG+H_0, $N_{\rm eff} = 4.25 \pm 0.80$ (68%) and $2.8 < N_{\rm eff} < 5.9$ (95%).

WMAP 7yr

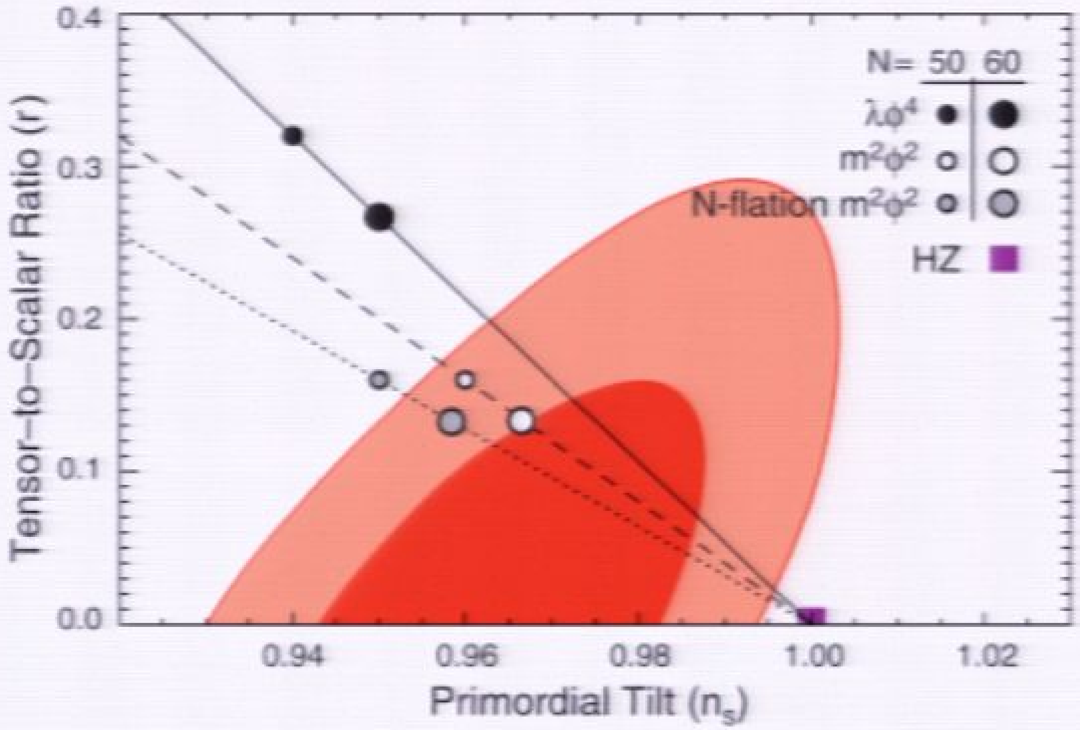
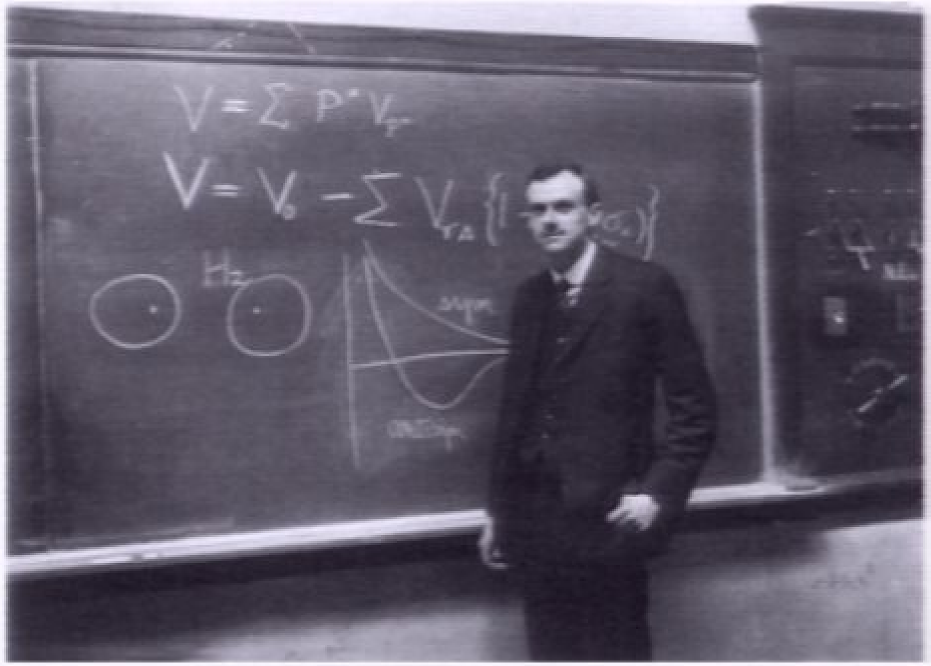


FIG. 19.— Two-dimensional joint marginalized constraint (68% and 95% CL) on the primordial tilt, n_s, and the tensor-to-scalar ratio, r, derived from the data combination of *WMAP*+BAO+H_0. The symbols show the predictions from "chaotic" inflation models whose potential is given by $V(\phi) \propto \phi^\alpha$ (Linde 1983), with $\alpha = 4$ (solid) and $\alpha = 2$ (dashed) for single-field models, and $\alpha = 2$ for multi-axion field models with $\beta = 1/2$ (dotted; Easther & McAllister 2006).

The initial conditions

How did the initial seeds for structure come about?
Quantum Mechanics



Paul A. M. Dirac
1939 Lecture

"Let us return to dynamical questions. With the new cosmology the universe must have started off in some very simple way. What, then, becomes of the initial conditions required by dynamical theory? Plainly there cannot be any, or they must be trivial. We are left in a situation which would be untenable with the old mechanics. *If the universe were simply the motion which follow from a given scheme of equations of motion with trivial initial conditions, it could not contain the complexity we observe. Quantum mechanics provides an escape from the difficulty. It enables us to ascribe the complexity to the quantum jumps, lying outside the scheme of equations of motion.*"

The initial conditions

How did the initial seeds for structure come about?

We have a standard paradigm: Inflation.

Very well motivated theoretically but basically none of the basic properties of these period have been determined observationally while theoretically there appear to be many options.

Can we improve our constraints of the Inflationary period. Could it some day be a part of our standard history of the Universe with the same level of confidence as Big Bang Nucleosynthesis?

The most constrained set up

A period of accelerated expansion with:

$$\dot{H} << H^2$$

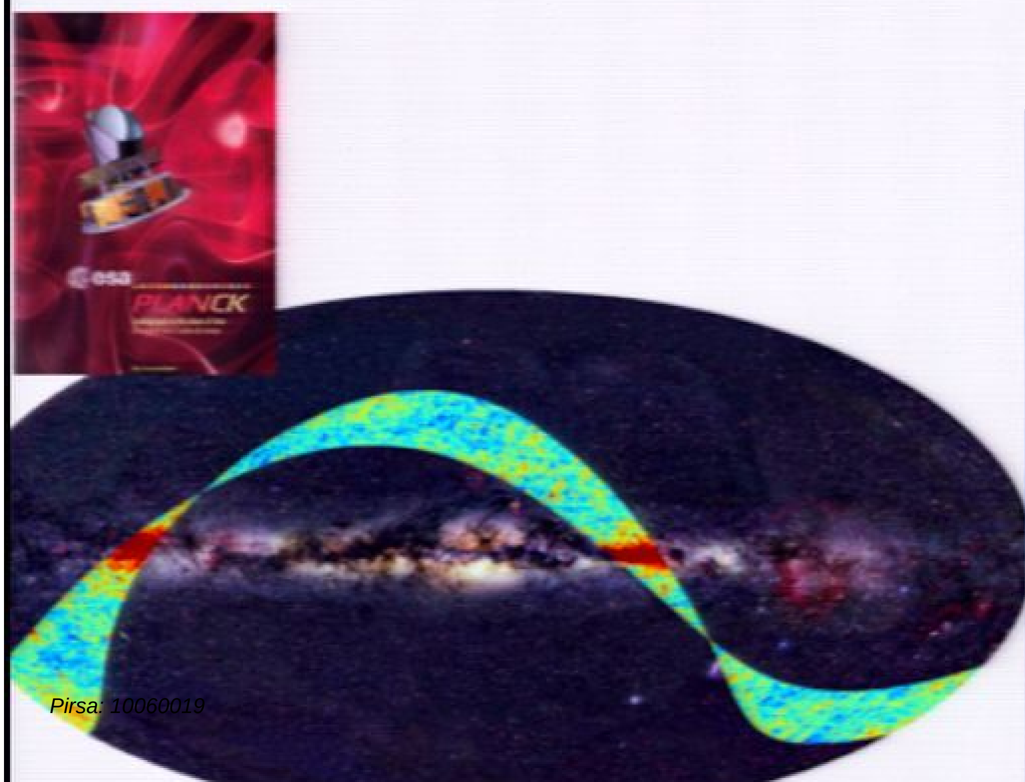
Quantum fluctuations in the clock that determines when inflation ends (the inflaton) are the source of the initial seeds.

It would be good get additional observational clues.

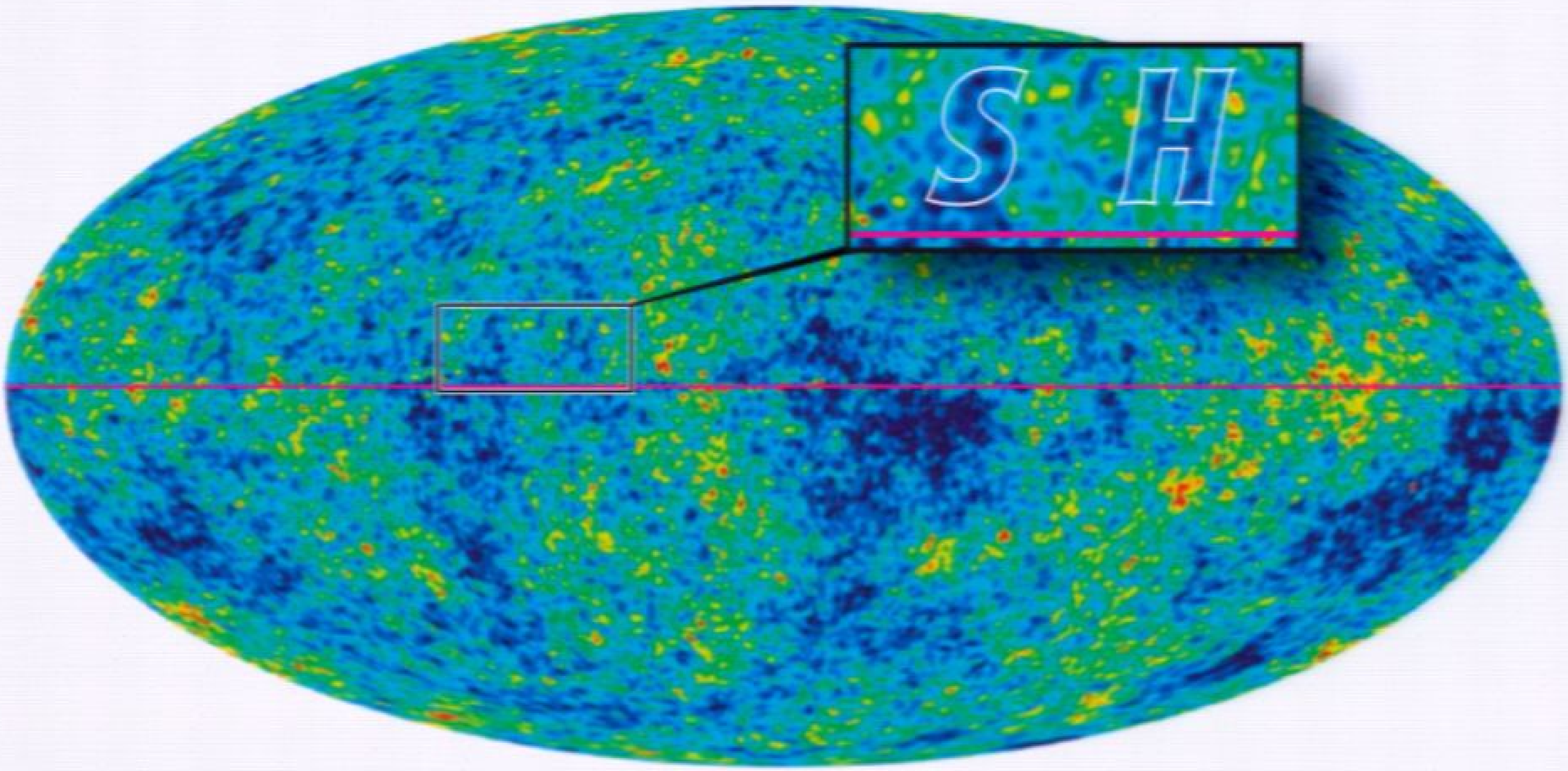
There are several possible clues we might get.

The initial conditions for the scalar fluctuations are consistent with being a Gaussian random field. Could there be departures?

What kind of departures are could we expect? What should we be looking for?



Did we expect to have the map initialized?



The information in the 2-pt function

When interactions are small the fluctuations in the inflaton field are just described by a collection of harmonic oscillators. Wave function of the vacuum state is a harmonic oscillator is a Gaussian resulting in Gaussian initial fluctuations.

In this limit all that is needed to describe it is the two point correlation function. Translational invariance demands only a function of distance.
Simple arguments demand it to be close to scale invariant, ie power law with specific slope.

Not that much to measure. No robust predictions.

Scale invariance a consequence of time translation invariance during inflation:

$$k^3|\zeta_k|^2 = A(\frac{k}{k_*})^{(n-1)}$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} << 1$$

$n - 1 = 0.037 \pm 0.012$ (WMAP)

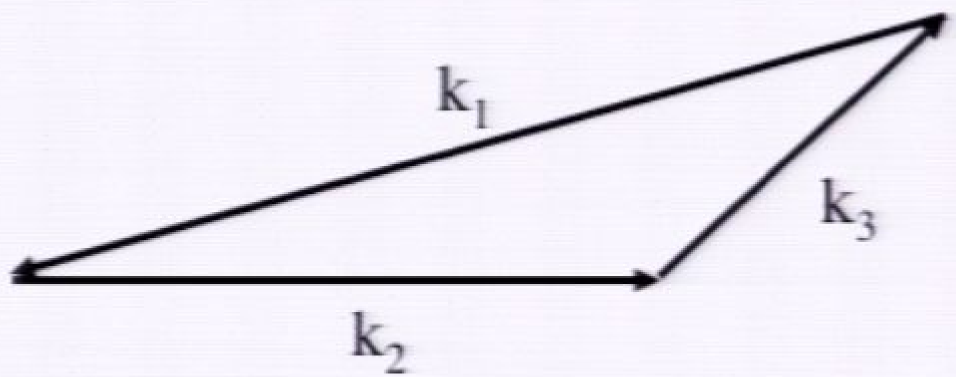
The information in the 2-pt function of gravitational waves

Inflation predicts the presence of a stochastic Background of Gravitational Waves

$$h_{ij} \sim \frac{H}{M_{PL}}$$

The amplitude of a GW background is probably more constraining than a measurement of the tilt because under standard assumptions it directly measures the expansion rate during Inflation.

The 3-pt function

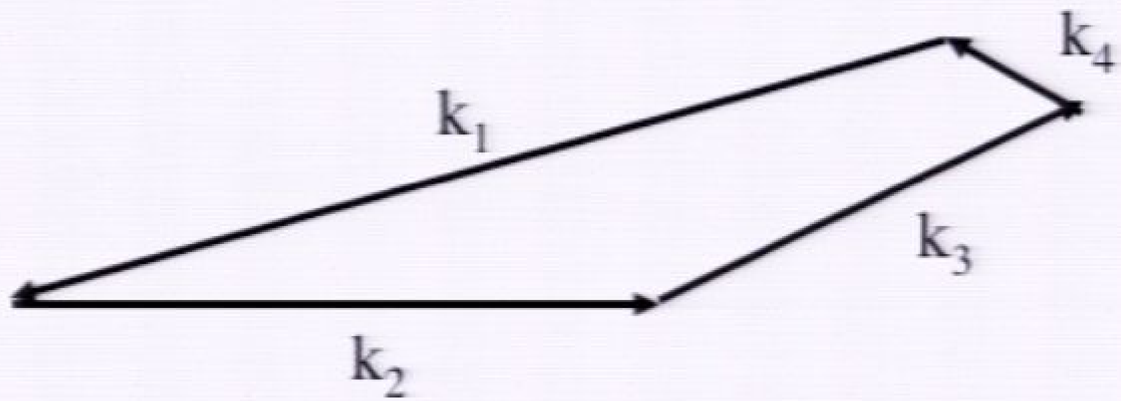


$$k_1^6 \langle \zeta_{k1} \zeta_{k2} \zeta_{k3} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k1})$$

Departure from Gaussianity are sensitive to the interactions of the field.

Even after requiring scale invariance and translation invariance the three point function is still an arbitrary function of two continuous variables.

The 4-pt function



$$k_1^9 \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k_1}, \frac{k_4}{k_1}, \theta, \phi)$$

Higher order moments have even more freedom.
Even after requiring scale invariance and translation invariance the four point function is still an arbitrary function of five continuous variable.

Higher order moments can have a rich structure even after homogeneity and scale invariance are taken into account.

What is the best way of characterizing the departures from Gaussianity one might expect?

Are there rules about the possible interactions between the clock fluctuations? or does anything go?

Effective theory of inflation: Chung, Creminelli, Fitzpatrick, Kaplan & Senatore. 0709.0293

Use the measured time in the clock as the time coordinate (unitary gauge).
The clock disappears from the action, everything is in the metric.
Can still make time dependent transformations of the spatial coordinates but time has been fixed. Terms must respect the residual symmetry.

$$S_{\rm E.H.+S.F.} = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2(3H^2+\dot{H}) + $$
$$+\frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + \;....$$
$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + ...\Big]$$

$\delta K_{\mu\nu}$ the variation of the extrinsic curvature of constant time surfaces
Has one more derivative.

Expansion in fluctuations and in derivatives. Coefficients in the first line are such that the action starts quadratic.

Can reintroduce the clock fluctuations: π
(Goldstone bosons of broken time diffs)

$$t \rightarrow t - \pi$$

$$g^{00} \quad \rightarrow \quad \frac{\partial(t+\pi)}{\partial x^{\mu}} \frac{\partial(t+\pi)}{\partial x^{\nu}} g^{\mu\nu}$$

For the models discussed here the terms involving π are the dominant. At sufficiently high energies the mixing between the Goldstone and the metric is negligible, it is decoupled. In this context the Hubble scale is at sufficiently high energy.

$$g^{00} \quad \rightarrow \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i \pi)^2$$

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$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big]$$

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$$S_\pi = \int d^4x \sqrt{-g}\left[-M_{\rm Pl}^2 \dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3\right]$$

Parameters: $H, \quad \dot{H}, \quad \ddot{H}, \quad M_2, \quad M_3$

Structure is set by the symmetries, the requirement that everything can be incorporated into the metric by a suitable choice of coordinates. Specific signs and coefficients in front of various terms, requirement of certain interactions, difference between time and

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Cubic Lagrangian

$$S_\pi = \int d^4x \sqrt{-g} \left[-M_{\rm Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 \right]$$

M$_2$ changes the dispersion relation of modes, introducing a "sound speed". Same term that changes the propagation speed generates interactions.

$$M_2^4 = -\frac{1-c_s^2}{c_s^2} \frac{M_{\rm Pl}^2 \dot{H}}{2}$$

$$\omega^2 = c_s^2 k^2$$

$$S_\pi = \int d^4x \sqrt{-g} \left[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} - \frac{2}{3} \frac{\tilde{c}_3}{c_s^4} M_{\rm Pl}^2 \dot{H} \dot{\pi}^3 \right]$$

$$S_{\text{E.H.}+\text{S.F.}} = \int d^4x \, \sqrt{-g} \Big[\frac{1}{2} M_{\text{Pl}}^2 R + M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2 (3H^2 + \dot{H}) +$$

$$+ \frac{1}{2!} M_2(t)^4 (g^{00}+1)^2 + \frac{1}{3!} M_3(t)^4 (g^{00}+1)^3 +$$

$$- \frac{\bar{M}_1(t)^3}{2} (g^{00}+1) \delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2} \delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2} \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots \Big]$$

$$g^{00} \quad \rightarrow \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2} (\partial_i \pi)^2$$

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Parameters: $H, \quad \dot{H}, \quad \ddot{H}, \quad M_2, \quad M_3$

Structure is set by the symmetries, the requirement that everything can be incorporated into the metric by a suitable choice of coordinates. Specific signs and coefficients in front of various terms, requirement of certain interactions, difference between time and

Cubic Lagrangian

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Just two specific shapes for this two shapes are predicted from each of the two possible interactions:

$$\dot{\pi}^3 \quad \dot{\pi}(\partial_i \pi)^2$$

Caveats:

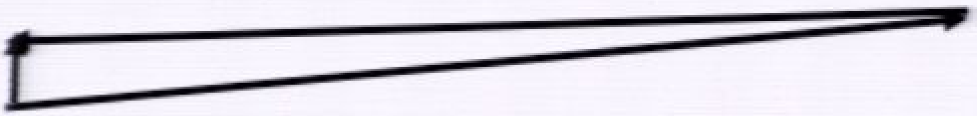
- Dispersion relation may be dominated by higher derivative terms when history is very close to de-Sitter.
- Higher derivative term may be the leading interaction.

$$\ddot{\pi}(\partial_i \pi)^2$$

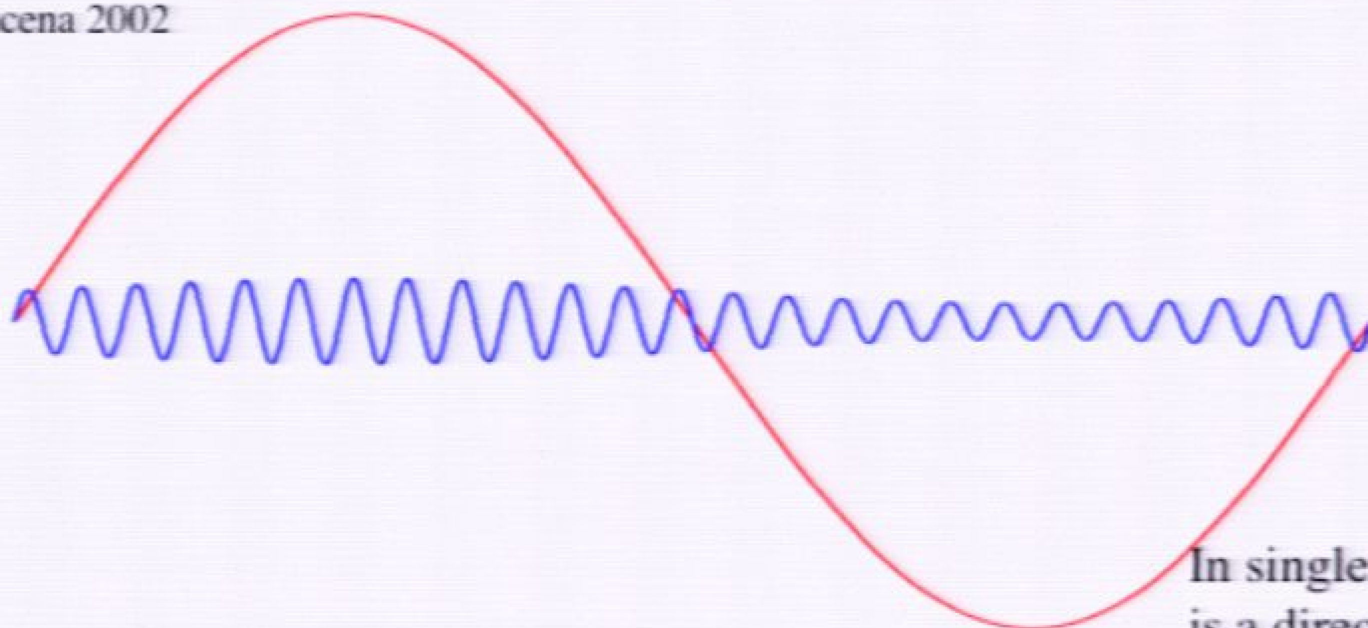
The Squeezed limit

$k_3 << k_1 \sim k_2$

These are derivative interactions, they shut down when one mode is outside the horizon.



Maldacena 2002

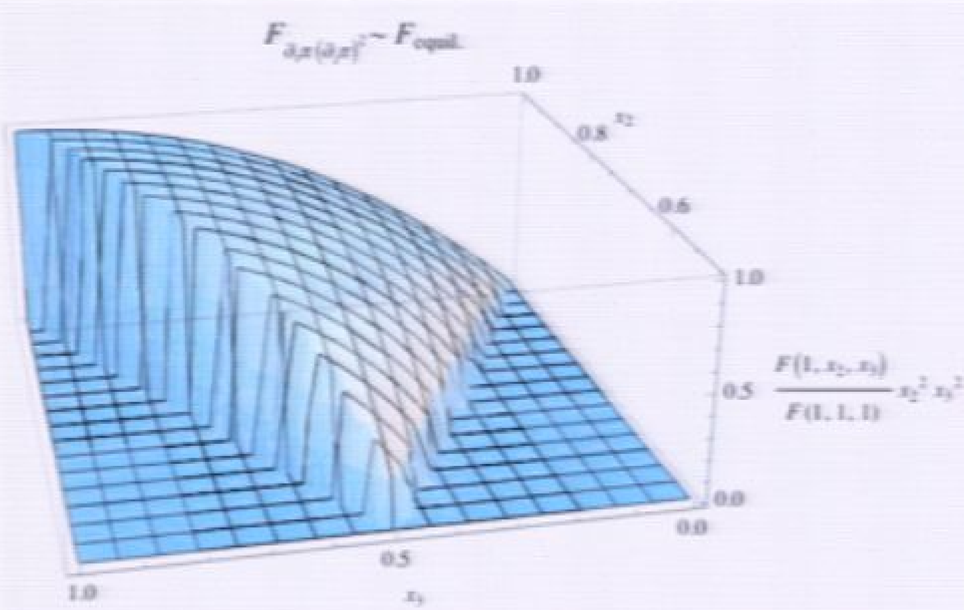


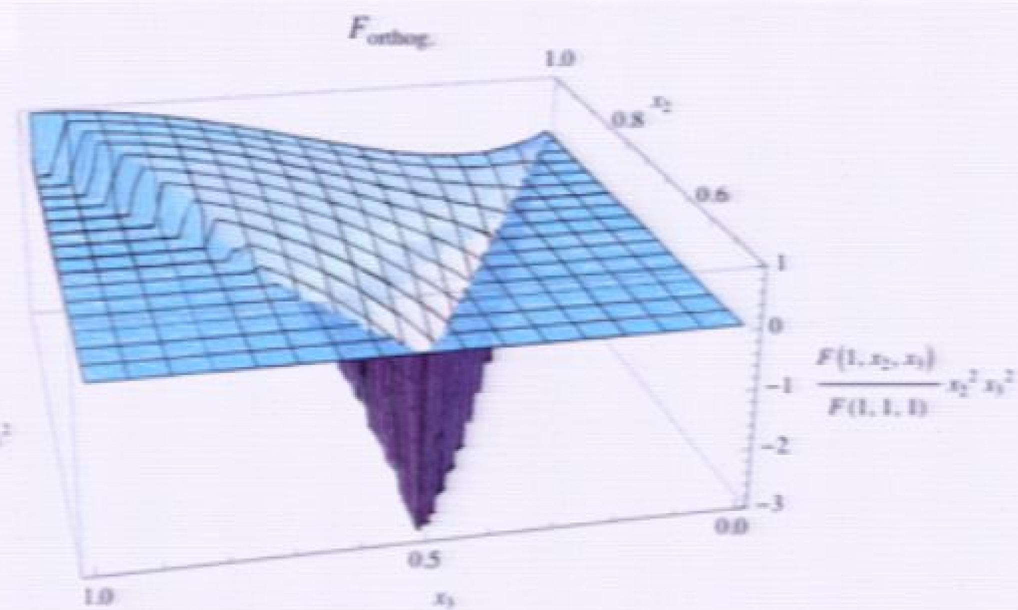
In single clock models there is a direct connection between the departures from scale invariance and the three point function.

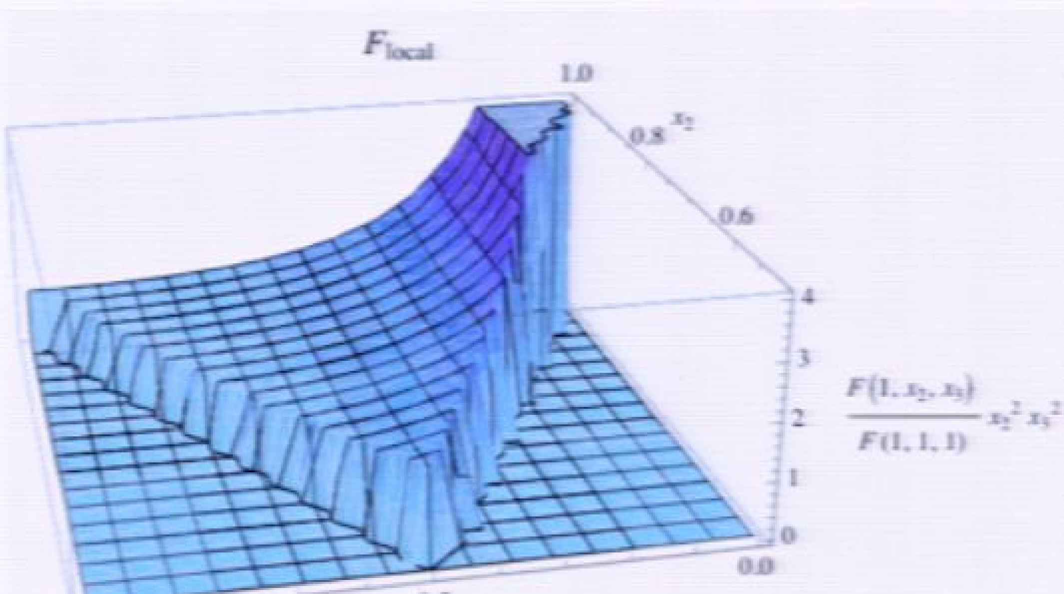
Non-Gaussianities $\sim (n-1)10^{-5}$

There is an easy way to get

The shapes in pictures







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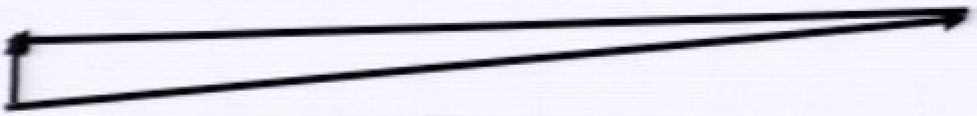

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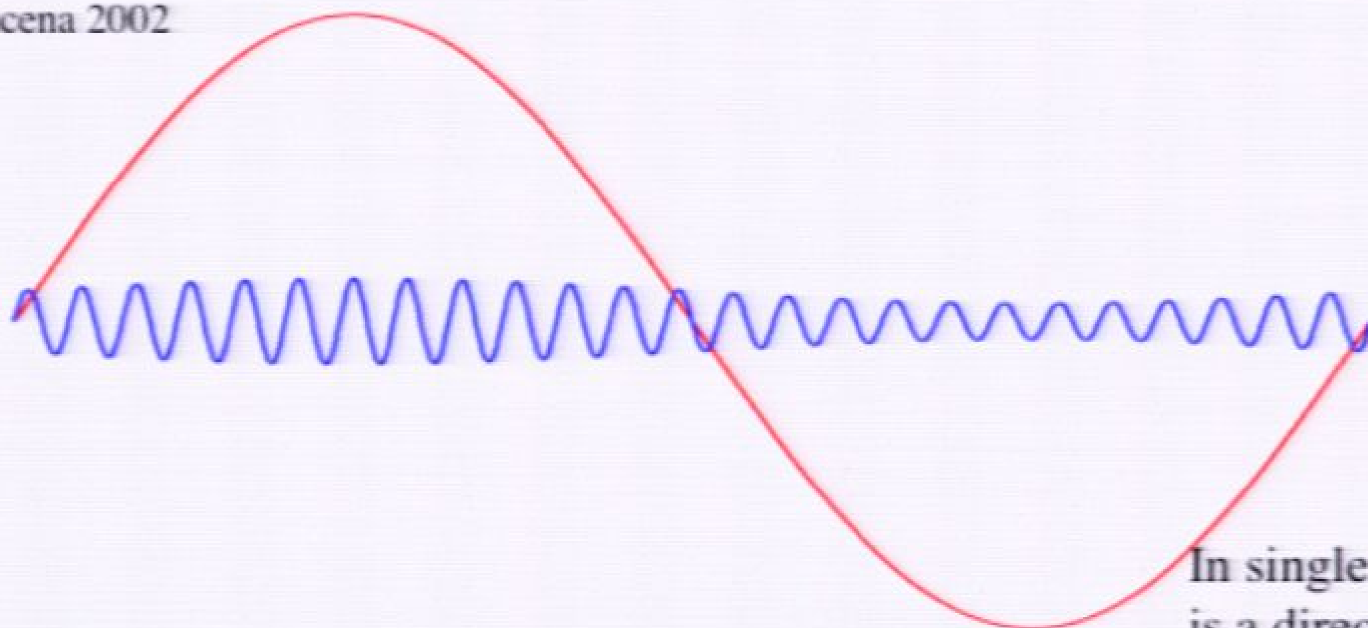

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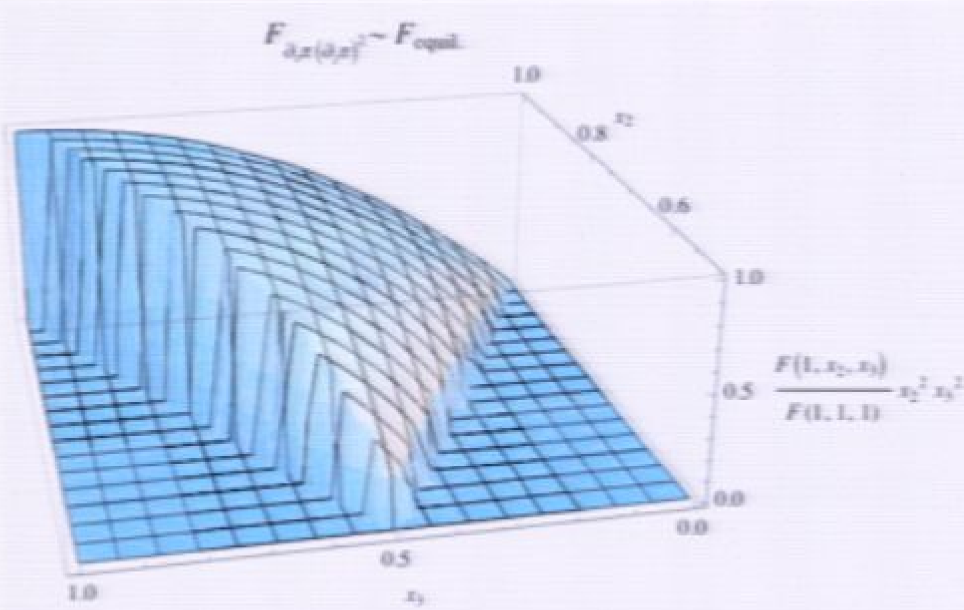
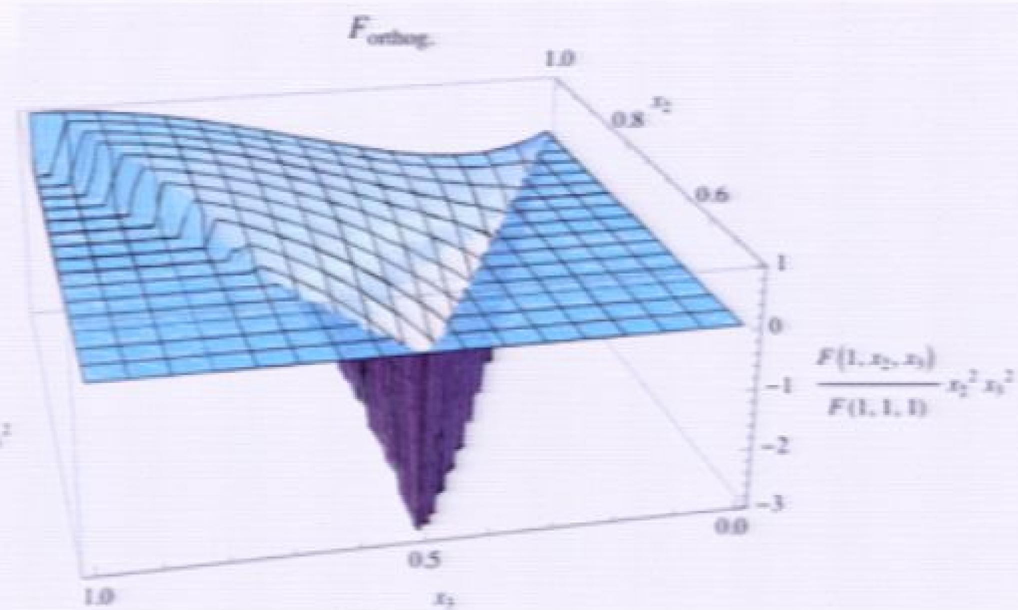
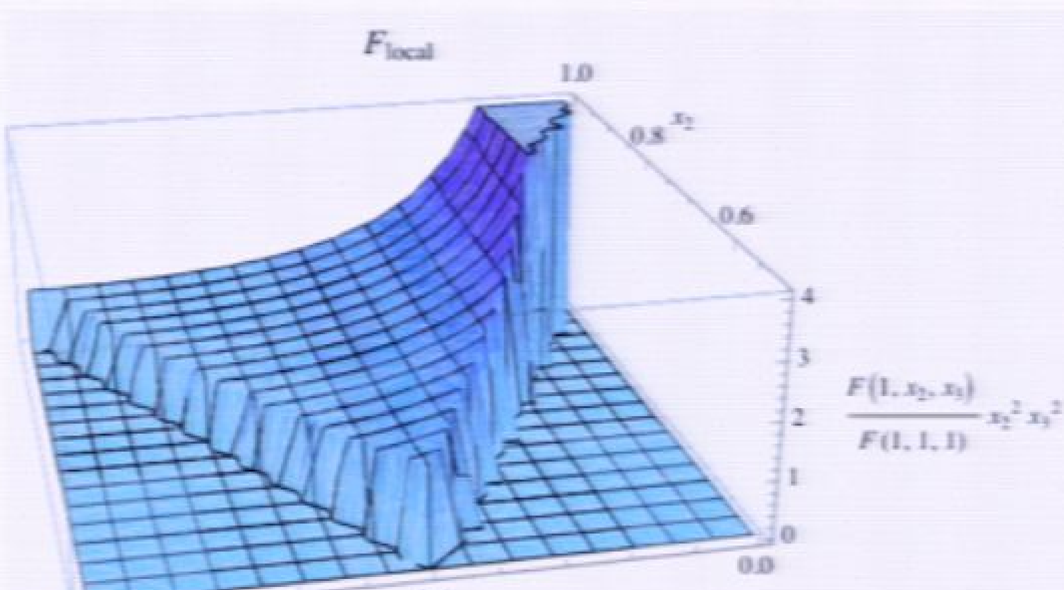






The structure of the interactions is very constrained, leading to much fewer possibilities than allowed by translational invariance and scale invariance.

Non-Gaussianities may provide an avenue to get convinced that perturbations were created during an inflationary period.

The interactions cannot be made arbitrarily small. There is also a minimum level of non-Gaussianity (first calculated by Maldacena in 2002). This level is smaller than we can measure using CMB observations but may be reachable with other techniques.
The shape and amplitude of this non-Gaussianities is completely fixed once the expansion history is known.

Observational Status:

Data was first used to search for the local type of non-Gaussianity, the shape that cannot be created by single field models. This shape however is the "simplest" one to write down. $\zeta(x) = \zeta_g(x) + f_{NL}\zeta_g^2(x) + \cdots$

After it was realized that single field models could not produce the local type of non-Gaussianity it was soon realized that inflationary models could be constructed that had larger non-Gaussianities with another shape.

The development of the effective theory of inflation showed that for single field at this stage there are really two shapes of the three point that need to be searched for. We now have upper limits for this two shapes that can be directly translated into constraints on the two parameters in the Lagrangian for the fluctuations.

Current limits are roughly:

$$\frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^{3/2}} < 10^{-2} \qquad c_s \geq 0.011 \quad 95\% \;\; \text{CL}$$

Observational Status:

The local type of non-Gaussianity is particularly interesting observationally because it shows up in the two point function of objects (such as galaxies) as a scale dependent bias (Dalal et al). Constraints coming from this method are competitive with the CMB. The relevant point is that the 3 or 4-point that show up in this way must have a squeezed limit.

Current limits:

Senatore,Smith and MZ
JCAP 0909:006 (2009)
JCAP 1001:028 (2010)

Optimal analysis of WMAP data (foreground template corrections) are ~ compatible with Gaussianity

Optimal limits on NG

$$-10 < f_{NL}^{local} < 74 \quad \text{at } 95\% \text{ C.L.}$$

Komatsu *et al*. **WMAP 7yr**

$$(-5 < f_{NL}^{local} < 59 \quad \text{at } 95\% \text{ C.L.})$$

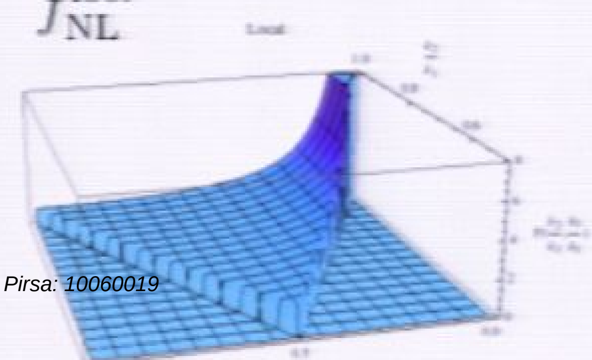
after combining with LSS
Slosar *et al*. **JCAP 0808:031, 2008**

$$-214 < f_{NL}^{equil.} < 266 \quad \text{at } 95\% \text{ C.L.}$$

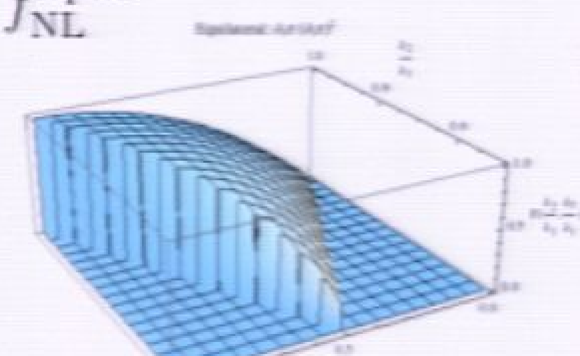
$$-410 < f_{NL}^{orthog.} < 6 \quad \text{at } 95\% \text{ C.L.}$$

Komatsu *et al*. **WMAP 7yr**

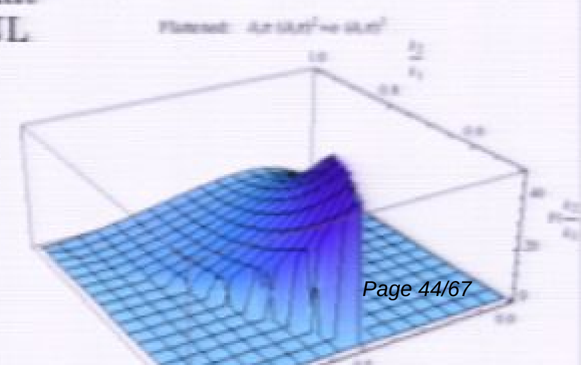
$f_{NL}^{loc.}$



$f_{NL}^{equil.}$



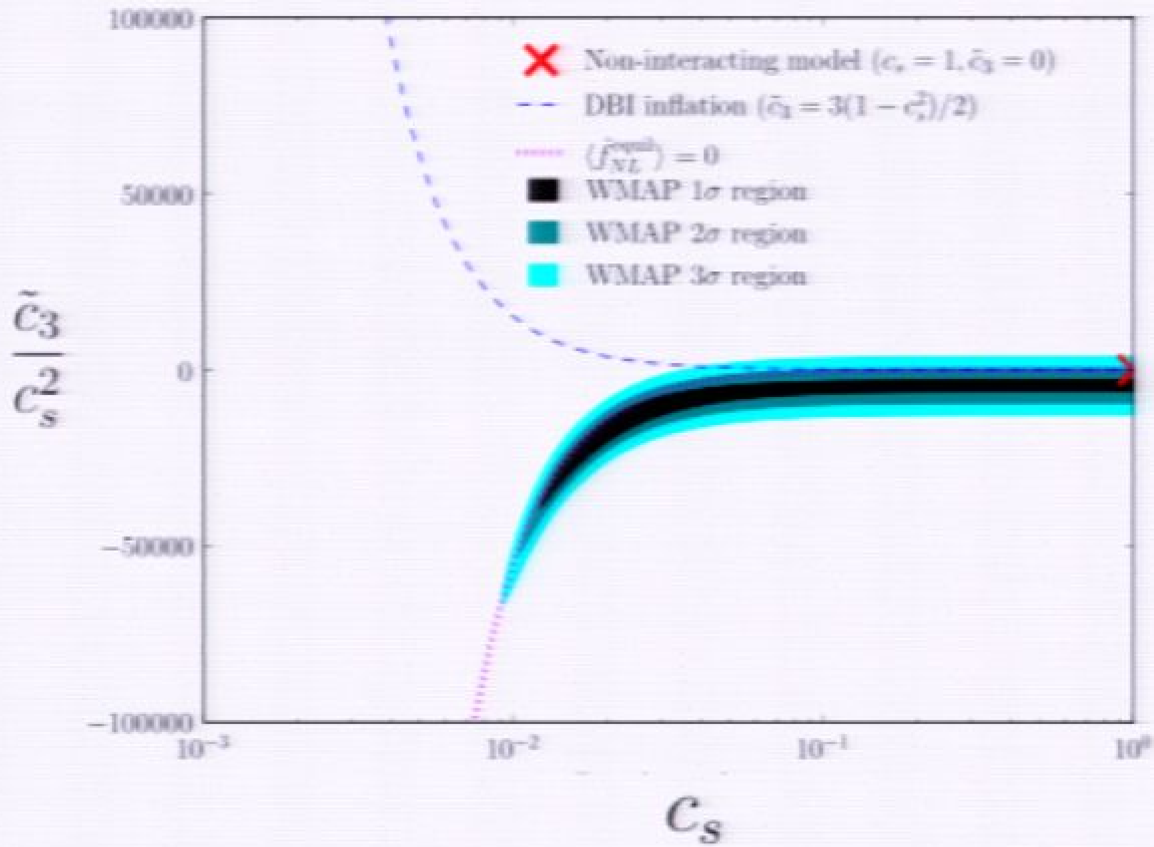
f_{NL}^{flat}



(Optimal) Limits on the parameters of the Lagrangian

$$S_\pi = \int d^4x\,\sqrt{-g}\left[M_{\rm Pl}^2\dot{H}(\dot{\pi}^2-(\partial_i\pi)^2)+M_2^4\left(\dot{\pi}^2+\dot{\pi}^3-\dot{\pi}(\partial_i\pi)^2\right)-M_3^4\dot{\pi}^3+\dots\right]$$

- Limits on f_{NL}'s get translated into limits on the parameters
- For models not-very-close to de Sitter (like DBI): c_s , $\tilde{c}_3$

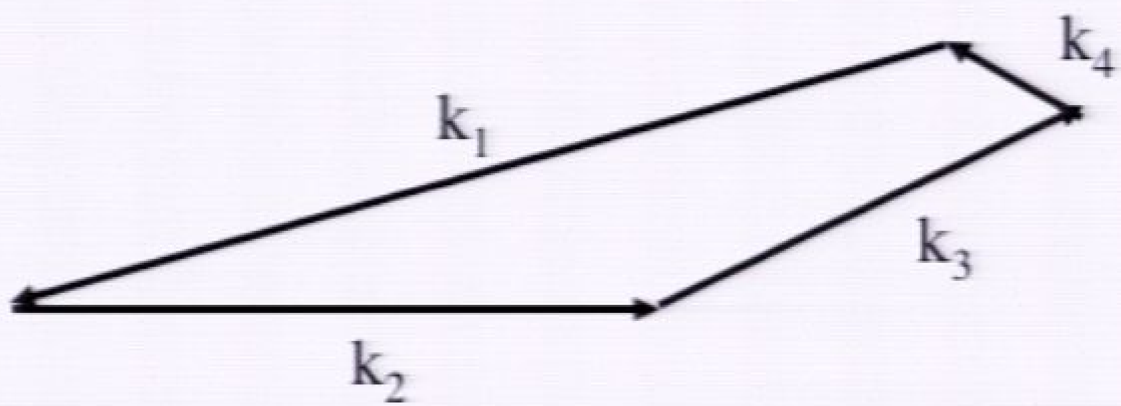


Very similar in spirit to Peskin and Takeuchi **PRD46:381,1992**

$$\frac{1}{c_s^2}\dot{\pi}(\partial_i\pi)^2+\frac{\tilde{c}_3}{c_s^2}\dot{\pi}^3$$

- Limit on the speed of sound: $c_s \gtrsim 0.011$!

4-pt function



$$k_1^9 \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = F(\frac{k_2}{k_1}, \frac{k_3}{k_1}, \frac{k_4}{k_1}, \theta, \phi)$$

Could one have an observable 4-pt function without a much larger 3-pt?

Could one have an observable 4-pt function without a much larger 3-pt?

$$S_{\rm E.H. + S.F.} = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2(3H^2 + \dot{H}) +$$
$$+\frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 +$$
$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big]$$

$$g^{00} \quad\to\quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i\pi)^2$$

Although there are quartic interactions inside $(g_{00}+1)^2$ and $(g_{00}+1)^3$ quartic terms, those are very small given current constraints on the three point function.

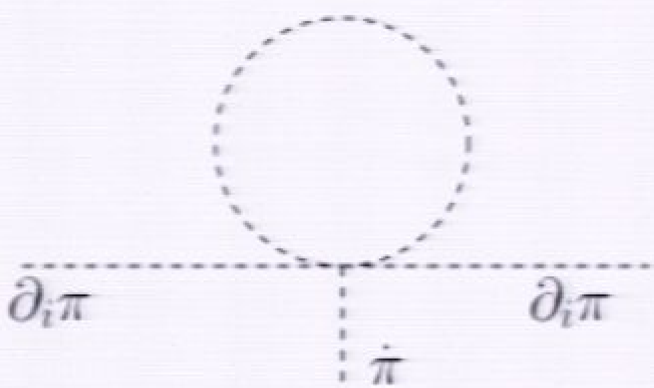
$$\frac{\langle\zeta^n\rangle}{\langle\zeta^2\rangle^{n/2}}$$

$$\tau_{NL}\zeta^2 \sim \frac{L_4}{L_2}\Big|_{E\sim H} \sim \frac{1}{c_s^4}\zeta^2 \Rightarrow \tau_{NL} \sim \frac{1}{c_s^4}$$

$$f_{NL}\zeta \sim \frac{L_3}{L_2}\Big|_{E\sim H} \sim \frac{1}{c_s^2}\zeta \Rightarrow f_{NL} \sim \frac{1}{c_s^2}$$

Could one have an observable 4-pt function without a much larger 3-pt?

Could one have a $(g_{00}+1)^4$ terms without $(g_{00}+1)^2$ and $(g_{00}+1)^3$ ones?



$$M_4^4(\delta g^{00})^4 \rightarrow M_4^4\left(16\dot{\pi}^4 - 32\dot{\pi}^3(\partial_\mu\pi)^2 + 24\dot{\pi}^2(\partial_\mu\pi)^4 - 8\dot{\pi}(\partial_\mu\pi)^6 + (\partial_\mu\pi)^8\right)$$

Although loop corrections generate 3-pt interactions they can be consistently small.

Could one have an observable 4-pt function without a much larger 3-pt?

$$S_{\rm E.H. + S.F.} = \int d^4x \, \sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2 (3H^2 + \dot{H}) +$$

$$+\frac{1}{2!}M_2(t)^4 (g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4 (g^{00}+1)^3 +$$

$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big]$$

$$g^{00} \quad \to \quad -1 - 2\dot{\pi} - \dot{\pi}^2 + \frac{1}{a^2}(\partial_i \pi)^2$$

Although there are quartic interactions inside $(g_{00}+1)^2$ and $(g_{00}+1)^3$ quartic terms, those are very small given current constraints on the three point function.

$$\frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n/2}}$$

$$\tau_{NL}\zeta^2 \sim \frac{L_4}{L_2}\Big|_{E\sim H} \sim \frac{1}{c_s^4}\zeta^2 \Rightarrow \tau_{NL} \sim \frac{1}{c_s^4}$$

$$f_{NL}\zeta \sim \frac{L_3}{L_2}\Big|_{E\sim H} \sim \frac{1}{c_s^2}\zeta \Rightarrow f_{NL} \sim \frac{1}{c_s^2}$$

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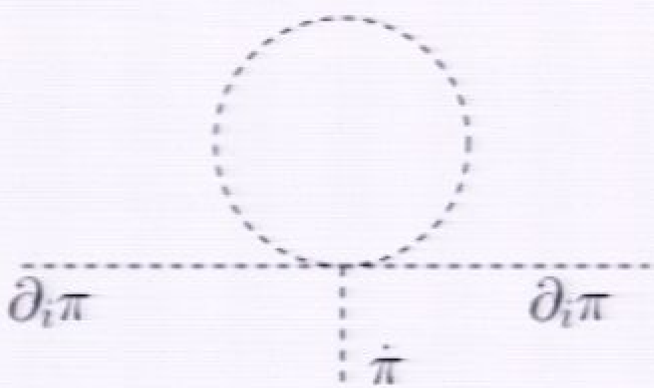

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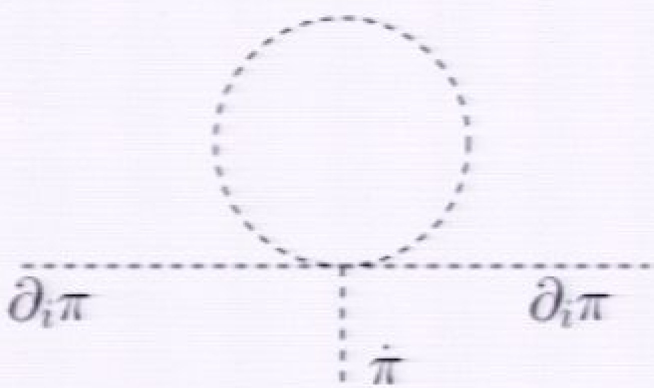

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Although an arbitrary function of five continuous variable only one possibly large shape from single clock models, created by the interaction

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Caveats:

• Dispersion relation may be dominated by higher derivative terms when history is very close to de-Sitter. In that case shape is different

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Observational Status:

The vast majority of the observational effort so far has been concentrated on the three point function. There is a strong case to look at the 4-point function as well, although this is significantly more complicated.

Inflation

A period of accelerated expansion with

$$\dot{H} << H^2$$

~~Quantum fluctuation in the clock that determines when inflation ends leads to the structure we see.~~

What if fluctuations had another source that dominated over the fluctuations of the clock? Can an analogous effective theory be constructed? What are the consequences?

Effective theory of multi-field inflation:

(Senatore & MZ, in preparation)

Consider the case with other fields in their vacuum at horizon crossing. Fields need to be lighter than H, so for example can be the goldstone bosons of some spontaneously broken group. There can be a soft breaking at the time of horizon crossing.
Need to couple this field to the inflaton (go to the unitary gauge).
For the U(1) case:

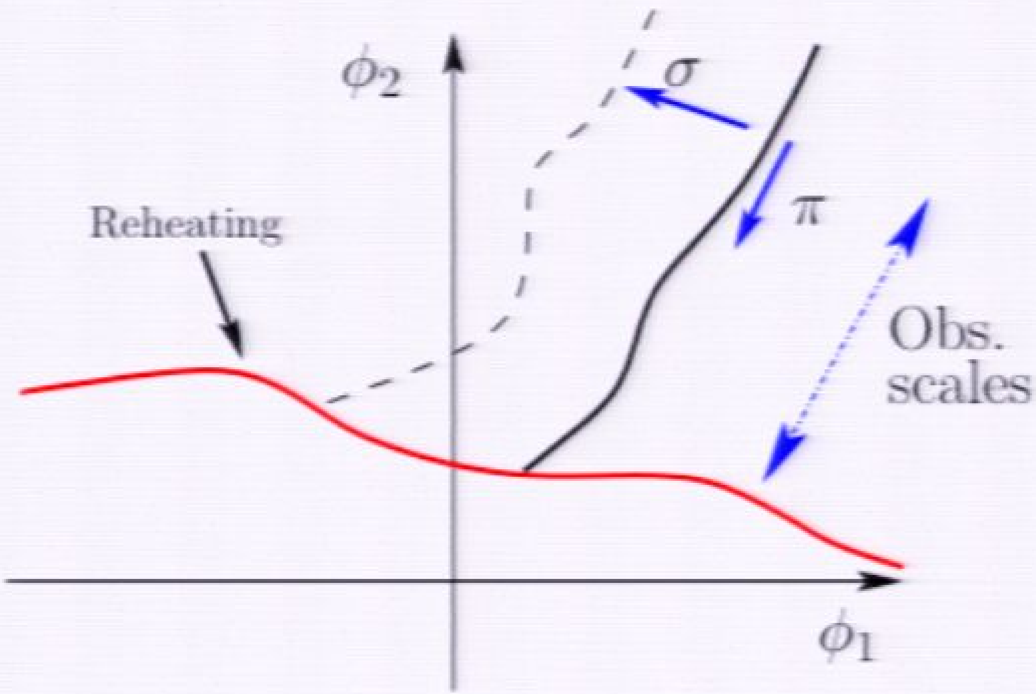
$$S_{\rm M.F.} = \int d^4x \sqrt{-g}\Big[\bar{M}_1(t)^{2\,I}(g^{00}+1)(g^{0\mu}\partial_\mu\sigma_I)$$
$$-e_1(t)^{IJ}(g^{\mu\nu}\partial_\mu\sigma_I\partial_\nu\sigma_J) + e_2(t)^{IJ}(g^{0\mu}\partial_\mu\sigma_I)(g^{0\mu}\partial_\mu\sigma_J) +$$
$$+e_3(t)^{IJ}(g^{00}+1)(g^{0\mu}\partial_\mu\sigma_I)(g^{0\mu}\partial_\mu\sigma_J) + e_4(t)^{IJ}(g^{00}+1)(g^{\mu\nu}\partial_\mu\sigma_I\partial_\nu\sigma_J) +$$
$$+\bar{M}_2(t)^{2\,I}(g^{00}+1)^2(g^{0\mu}\partial_\mu\sigma_I) +$$
$$+\bar{M}_3(t)^{-2,\,IJK}(g^{0\mu}\partial_\mu\sigma_I)(g^{0\mu}\partial_\mu\sigma_J)(g^{0\mu}\partial_\mu\sigma_K) +$$
$$+\bar{M}_4(t)^{-2,\,IJK}(g^{0\mu}\partial_\mu\sigma_I)(g^{\mu\nu}\partial_\mu\sigma_J\partial_\nu\sigma_K) + \ldots\Big]\,.$$

Classify the various cases to see if there are new signatures. In other words are there new shapes for the 3 and 4-point functions? Are there other situations in which the 4-point might be there without and already detected 3-point?

Effective theory of multi-field inflation:

(Senatore & MZ, in preparation)

How are the additional fields related to what we observe?
Everything is happening outside the horizon.



$$\zeta(x) = \left.\frac{\partial\zeta}{\partial\sigma_I}\right|_0 \sigma_I(x) + \frac{1}{2!}\left.\frac{\partial^2\zeta}{\partial\sigma_I\sigma_J}\right|_0 \sigma_I\sigma_J + \frac{1}{3!}\left.\frac{\partial^3\zeta}{\partial\sigma_I\partial\sigma_J\partial\sigma_K}\right|_0 \sigma_I(x)\sigma_J(x)\sigma_K(x) + \ldots .$$

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There are new signatures but there are still several rules. For example in the U(1) case:

- Local 3- and 4-point from reheating (conversion). The 4-point should be very small. Could create isocurvature perturbations.
- Additional way to get large 4-point relative to 3-point in addition to $\sigma \to -\sigma$. Lorentz invariance of the perturbation Lagrangian implies that $(\partial\sigma)^4$ is the leading interaction, a new 4-point function shape which could be there without a larger 3-point.
- The soft breaking Lagrangian results in a new 3 point function shape, from the operator $\sigma(\partial\sigma)^2$. It can also result in a local 4-point function without the associated local 3-point function (contrary to what happens at reheating).

Observational signatures: new 3 and 4 point function shapes

Multi-field

Operator	Dispersion		Type	Origin	Squeezed L.
	$w = c_s k$	$w \propto k^2$			
$\dot\sigma^4$, $\dot\sigma^2(\partial_i\sigma)^2$, $(\partial_i\sigma)^4$	X		Ad., Iso.	Ab., non-Ab.	
$(\partial_\mu\sigma)^4$	X		Ad., Iso.	Ab., non-Ab.	
σ^4	X	X	Ad., Iso.	Ab.$_s$, non-Ab.$_s$, S.*	X
$\dot\sigma\sigma^3$	X	X	Ad., Iso.	Ab.$_s^\dagger$, non-Ab.$_s^\dagger$.	X
$\sigma^2\dot\sigma^2$, $\sigma^2(\partial_i\sigma)^2$	X	X†*	Ad.†*, Iso.	non-Ab, Ab.$_s^{\dagger\star}$, non-Ab.$_s^{\dagger\star}$,	X
$\sigma^2(\partial_\mu\sigma)^2$	X		Ad.†*, Iso.	non-Ab, Ab.$_s^{\dagger\star}$, non-Ab.$_s^{\dagger\star}$, S.*	X
$\sigma(\partial\sigma)^3$	X		Iso.	non-Ab.$_s^\star$.	X
$\dot\sigma^3$, $\dot\sigma(\partial_i\sigma)^2$	X		Ad., Iso.	Ab., non-Ab.	
$\dot\sigma(\partial_i\sigma)^2$, $\partial_j^2\sigma(\partial_i\sigma)^2$		X	Ad., Iso.	Ab.	
σ^3	X	X	Ad., Iso.	Ab.$_s$, non-Ab.$_s$, S, R	X
$\dot\sigma\sigma^2$	X	X	Ad., Iso.	Ab.$_s$, non-Ab.$_s$	X
$\sigma\dot\sigma^2$, $\sigma(\partial_i\sigma)^2$	X	X	Ad., Iso.	Ab.$_s^{\dagger\star}$, non-Ab.$_s^{\dagger\star}$	X
$\sigma(\partial_\mu\sigma)^2$	X		Ad., Iso.	Ab.$_s^{\dagger\star}$, non-Ab.$_s^{\dagger\star}$.	X

Single field

Operator	Dispersion		Squeezed L.
	$w = c_s k$	$w \propto k^2$	
$\dot\pi^4$	X		
$(\partial_j^2\pi)^4$, $\dot\pi(\partial_j^2\pi)^3$, ...		X	
$\dot\pi^3$, $\dot\pi(\partial_i\pi)^2$	X		
$\dot\pi(\partial_i\pi)^2$, $\partial_j^2\pi(\partial_i\pi)^2$		X	

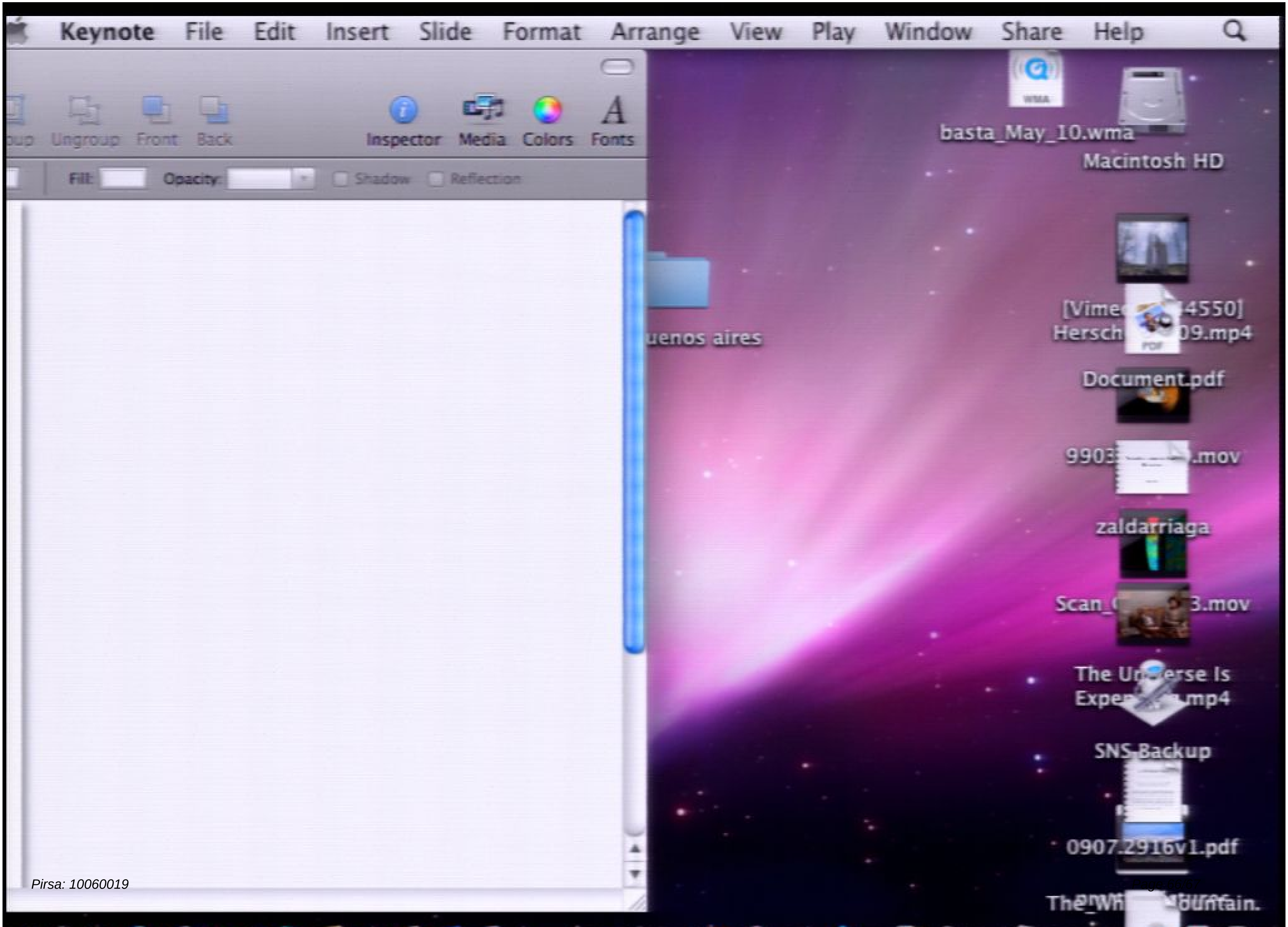
Summary:

In the last few years our understanding of non-Gaussianities from single clock models has improved substantially. They tell us about the interactions between fluctuations in the clock and symmetries dictate the structure of these interactions.

In single field:
There are only two important interactions so the shape of the possible three point functions is fully determined by two parameters. Only a single 4 point function shape could be large. Not yet been searched for in the data.

Currently the best upper limits on equilateral shapes come from WMAP. Planck should improve things by at least an order of magnitude. Local shapes can be searched for in large scale structure surveys.

Relaxing the single field assumption results in a relatively small number of new shapes. It strengthens the case for looking at the 4-point function.


Keynote
File
Edit
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Slide
Format
Arrange
View
Play
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Share
Help
Ungroup
Front
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Colors
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No Signal

VGA-1