Title: The generation of primordial magnetic fields

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Abstract: Magnetic fields are ubiquitous in our Universe. The are observed in galaxies and clusters in our vicinity and at high redshifts.

In my talk I outline the possibilities to generate magnetic fields in the early Universe during inflation or during a first order phase transition. I explain the form of the magnetic field spectrum obtained in the different cases. I also discuss the subsequent evolution of helical and non-helical magnetic fields in the cosmological plasma and argue that fields generated at the electroweak phase transition do not have enough large scale power to represent the magnetic fields observed in galaxies and clusters, even if they are maximally helical.

Cosmic Magnetic fields

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Perimeter Institute, June 2010

Outline

Introduction: Cosmology is a search for relics

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- or the electroweak transition at $t \simeq 10^{-10}$ sec which may have led to the observed baryon asymmetry in the Universe.
- It has been proposed that confinement and, especially the electroweak phase transition but also inflation might generate primordial magnetic fields which represent seeds for the magnetic fields observed in galaxies and clusters.

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- Estimates by equi-partition (e.g. of magnetic field and thermal or turbulent energy).

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- Intergalactic space, voids: The fact that certain blazars do emit TeV γ-radiation but not GeV, means that lower energy electrons which are produced by scattering with intergalactic background light and which then generate a cascade of GeV photons by inverse Compton scattering must be deflected out of the beam. This requires intergalactic fields of $B \gtrsim 3 \times 10^{-16}$Gauss with coherence scales of 1Mpc (Neronov & Vovk, 2010).

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Because of the difficulty to permeate voids with magnetic fields at late times, we look only at primordial processes in this talk.

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It is not surprising that all these limits are comparable, since

$$\Omega_B = 10^{-5} \Omega_\gamma \left(\frac{B}{10^{-8}\mathrm{Gauss}} \right)^2$$

Magnetic fields of the order 3×10^{-9}Gauss (on CMB scales) will leave 10% effects on the CMB anisotropies while 10^{-9}Gauss will leave 1% effects. It is thus clear that we can never detect magnetic fields of the order of 10^{-16} or even 10^{-20}Gauss (on galactic scales) in the CMB.

Other effects on the CMB

Magnetic fields effect the CMB via

- their energy-momentum tensor which leads to metric perturbations $\Rightarrow$ perturbed photon geodesics
- magnetosonic waves affect the acoustic peaks in the CMB spectrum
- Alvèn waves (vector perturbations)
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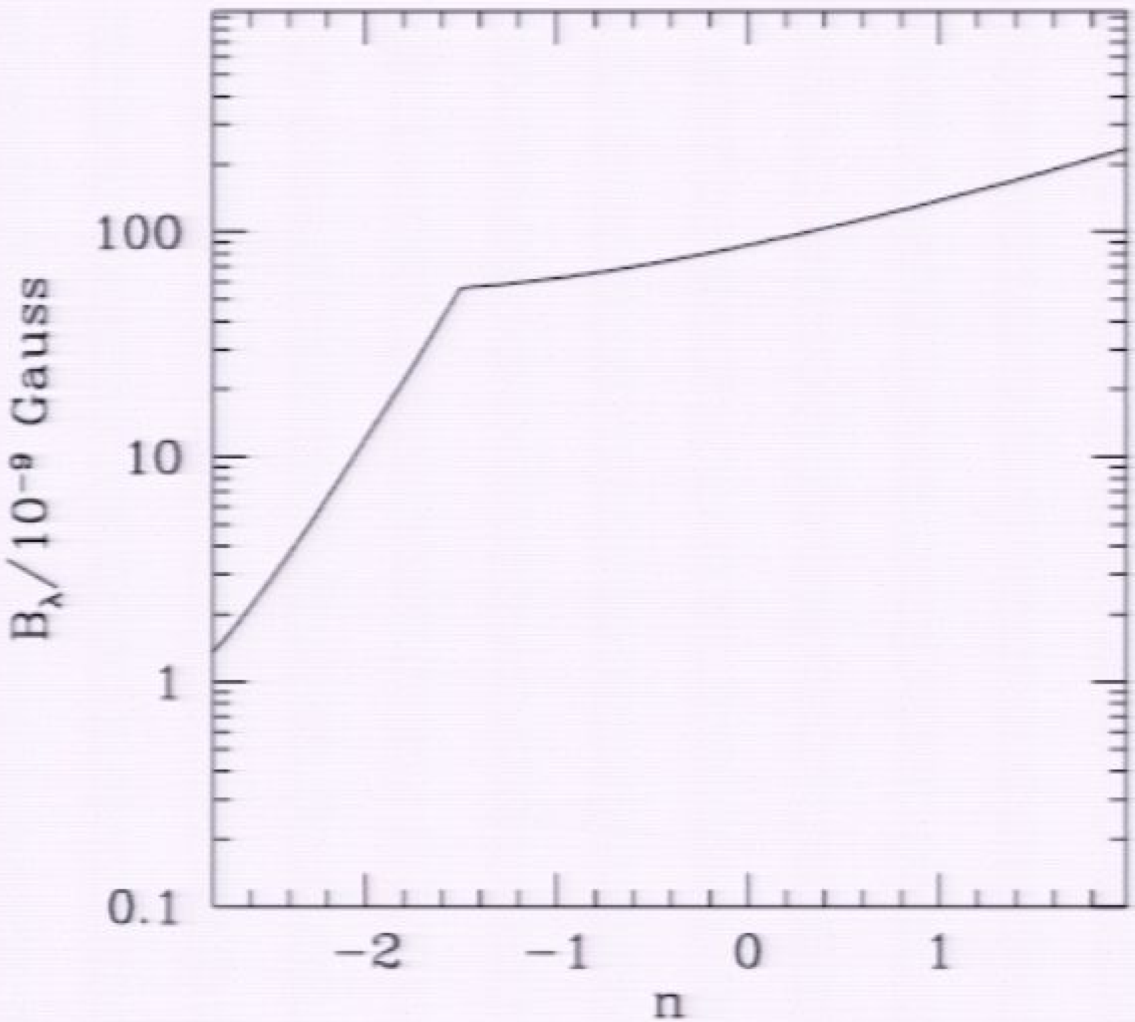

(from: RD, Ferreira & Kahniashvili '98)

Generation of primordial magnetic fields

There are three ideas how magnetic fields may have formed in the early Universe:

- Second order perturbations: To generate magnetic fields in the cosmic fluid one needs vorticity and a charge and current density. The first can be obtained only in second order perturbation theroy (first order vector perturbations decay) and the second only in second order in the tight coupling limit. Estimates have shown that typical fields do not exceed 10^{-23}Gauss. This is far too small to be consistent with the Neronov-Vovk bound or with the minimal amplitude needed for dynamo amplification.

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In the remainder of this talk I restrict to the 2nd and 3rd possibility. I shall mainly concentrate in generic arguments which help to determine the spectrum and limit the amplitude. You will see that these are already very strong.

The spectrum

We assume that the process leading to a magnetic field is statistically homogeneous and isotropic. A magnetic field spectrum generated by such a process is of the form

$$\langle B_i(\mathbf{k})B_j^*(\eta,\mathbf{q})\rangle = \frac{(2\pi)^3}{2}\delta(\mathbf{k}-\mathbf{q})\left\{(\delta_{ij}-\hat{k}_i\hat{k}_j)P_S(k) - i\epsilon_{ijn}\hat{k}_n P_A(k)\right\}$$

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$P_A \propto |B_+|^2 - |B_-|^2$ determines the helicity of the magnetic field. Its integral is the helicity density while the integral of $P_S \propto |B_+|^2 + |B_-|^2$ determines the energy density in magnetic fields.
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On small scales the magnetic field is damped by the viscosity of the cosmic plasma, $P_S = P_A = 0$ for $k > k_d(t)$. Here $k_d(t)$ is a time-dependent damping scale.

Magnetic field evolution

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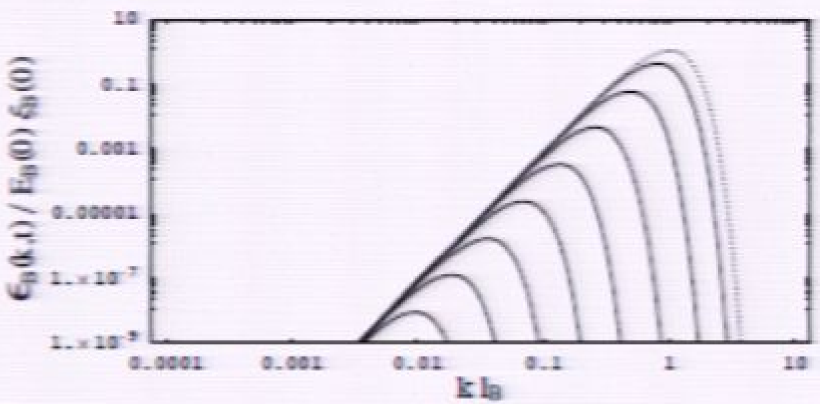
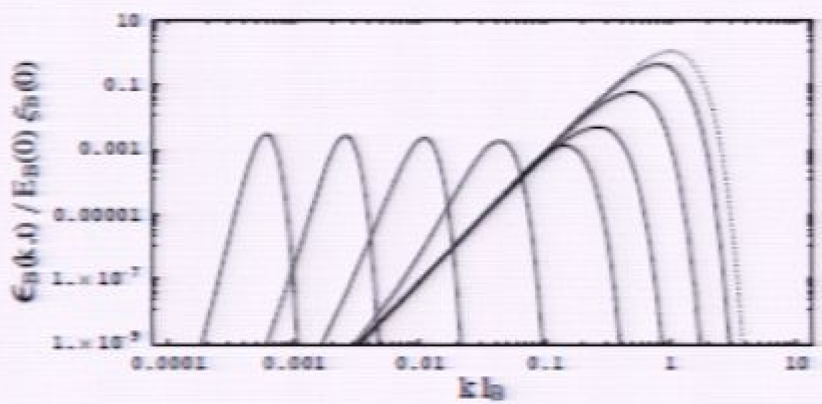
However, iof the field is helical, helcity conservation leads to an inverse cascade which moves the correlation scale to larger and larger scales (numerical simulations by Jedamzik et al. 2000-2005, Campanelli, 2007)





Campanelli, 2007

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Hence the correlation function is a function of compact support; and therefore its Fourier transform is analytic. Usually this signifies white noise (flat) on large scales but since we have to additional condition $\nabla \cdot \mathbf{B} = 0$, for magnetic fields we must require $P_S \propto k^2$ on large scales, $n_S = 2$. Correspondingly n_A must be odd and the physical requirement $|P_A| \leq P_S$ then implies $P_A \propto k^3$, $n_A = 3$.

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For the energy density per log-k-interval this implies

$$\frac{d\rho_B}{d\log(k)} \propto k^5$$

Limits on magnetic fields from phase transitions

As we now show this already implies very stringent limits on magnetic fields from phase transitions. Be $\epsilon = \Omega_B^*/\Omega_r^*$ the ratio of the magnetic field to the radiation energy density at the moment of formation and k_* the cutoff scale. Since radiation and magnetic fields scale the same way, at later times and scales larger than the cutoff, the magnetic field to radiation density is given by

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For the electroweak phase transition with $k_* > \mathcal{H}_* \simeq 10^{-3}$Hz and $k_1 = 1/0.1$Mpc$\simeq 10^{-13}Hz$ this yields the following limit for the field at scale k_1:

$$\left(\frac{B(k_1)}{10^{-6}\mathrm{Gauss}}\right)^2 \simeq \Omega_r^{-1}\frac{d\Omega_B}{d\log(k_1)} < \epsilon \times 10^{-50}$$

Since also $\epsilon < 1$ this implies $B(k_1) < 10^{-31}$Gauss. Using slightly more model dependent but also more realistic numbers (e.g. $k_* \simeq 100\mathcal{H}_*$ one arrives at $B(k_1) < 10^{-36}$Gauss.

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$$\left(\frac{B(k_1)}{10^{-6}\text{Gauss}}\right)^2 \simeq \Omega_r^{-1}\frac{d\Omega_B}{d\log(k_1)} < \epsilon \times 10^{-50}$$

Since also $\epsilon < 1$ this implies $B(k_1) < 10^{-31}$Gauss. Using slightly more model dependent but also more realistic numbers (e.g. $k_* \simeq 100\mathcal{H}_*$ one arrives at $B(k_1) < 10^{-36}$Gauss.

The limits from the QCD phase transition are somewhat less stringent but still discouraging, $B(k_1) < 10^{-30}$Gauss.

Limits on magnetic fields from phase transitions

If the magnetic field is helical, the inverse cascade which moves power from small to larger scales can help. But a detailed calculation (Caprini, RD, Fenu, 2009) shows

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$B(k_1) < 10^{-21}$Gauss for the QCD transition. This result could be marginally sufficient dynamo amplification, but is still several orders of magnitude below the Neronov-Vovk-bound.

Magnetic fields from inflation

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Let us discuss a simple case where we couple the inflation to the electromagnetic field.

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This is a wave equation with a time-dependent mass term. We know how to calculate the generation of its modes out of the vacuum. This case has been discussed for the first time in (Ratra '92).

Magnetic fields from inflation: example

For example if $f \propto a^\gamma$ is a simple power law, we can compute the resulting magnetic fields spectrum to

$$P_S \propto k^n \quad \text{with} \quad n = \begin{cases} 1-2\gamma & \text{if } \gamma \geq -1/2 \\ 3+2\gamma & \text{if } \gamma \leq -1/2 \end{cases}$$

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During inflation we cannot assume that the Universe is highly conducting and the electric field is damped. We therefore also have to compute the electric field spectrum. One finds (Martin & Yokoyama '08, Subramanian '10)

$$P_E \propto k^m \quad \text{with} \quad m = \begin{cases} 3 - 2\gamma & \text{if } \gamma \geq 1/2 \\ 1 + 2\gamma & \text{if } \gamma \leq 1/2 \end{cases}$$

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Since there is infrared cutoff, the spectral index should not be less than -3 otherwise $\frac{d\rho_B}{d\log(k)} \propto k^3 P_S \propto k^{3+n}$ or $\frac{d\rho_E}{d\log(k)} \propto k^3 P_E \propto k^{3+m}$ diverges. This limits

$$-2 \lesssim \gamma \lesssim 2.$$

Problems

On the other hand, if the spectrum is too blue, the fact that magnetic fields should not dominate the energy density of the Universe leads to very stringent constraints on small scales. Since the Hubble scale at the end of inflation is so small, the spectrum needs not be very blue for this to happen.

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Hence during inflation the electron charge was much larger than 1. In this regime we cannot trust perturbation theory and our calculation does actually not apply... (Demozzi et al. '09).
Note that choosing $f_i = 1$ does not help since then $f_0 \gg 1$ and the presently measured electron charge is $e_N = e/\sqrt{f_0}$, again we need $e/\sqrt{f_i} \gg 1$.

Generation of helical magnetic fields during inflation

These problems motivated us to see what can be gained from a helical coupling of the inflaton to the magnetic field. The idea was that this does not mix with the electron charge and because of the inverse cascade invoked by helicity conservation in the evolution of the field after inflation, a somewhat bluer spectral index might be admissible.

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where $\tilde{F}^{\mu\nu} = \frac{1}{2}\eta^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual of the electromagnetic field tensor.

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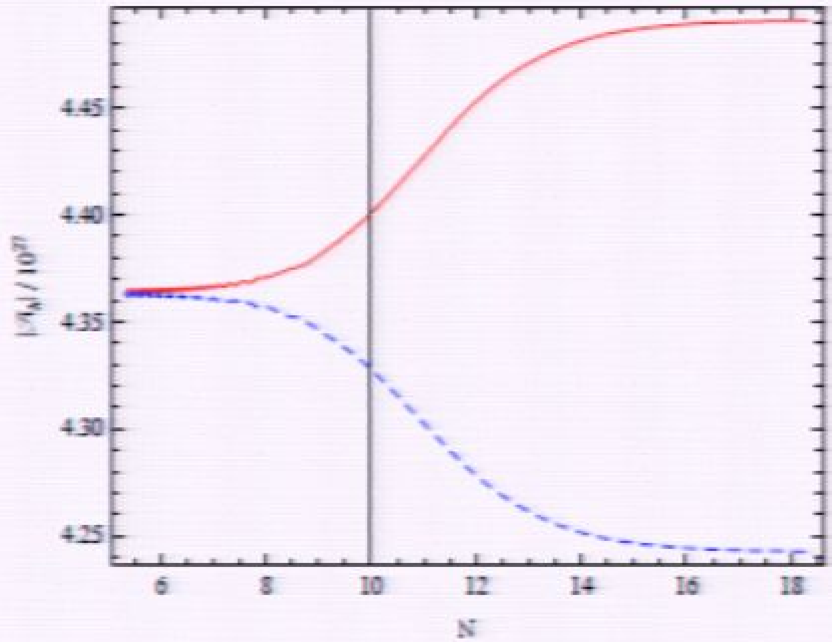
Again, a wave equation with time-dependent mass term. There are two main differences to the non-helical case: Now one of the helicity modes is amplified while the other is reduced depending on the sign of f', and the mass-term is proportional to k.

Generation of helical magnetic fields during inflation

Both difference are very important: the first leads to helicity and the second is the cause of the short duration of the amplification phase



RD, Hollenstein & Jain, 2010

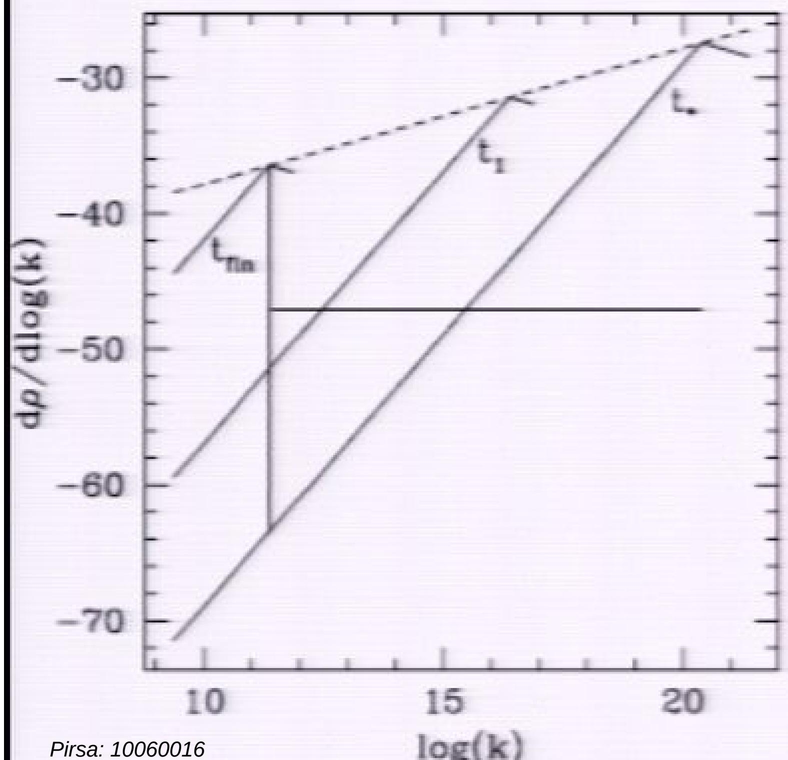
Because the duration during which the mass term is relevant is always just the Hubble exit time $k \sim \mathcal{H}$, before the $\ddot{A}$ term is much larger and later the $k^2 A$ term dominates, the vacuum fluctuation of one mode are always amplified while those of the other mode are suppressed. And this by the same, k-independent factor.

Generation of helical magnetic fields during inflation

Since the vacuum fluctuations of the vector potential behave like k^{-1}, this yields $P_S \propto P_A \propto k$,

$$\frac{d\rho_B}{d\log(k)} \propto k^4.$$

Despite the inverse cascade, this spectrum is too steep to satisfy both, $\frac{d\rho_B}{d\log(k_*)} < \rho_r$ and $B(k_1, t_0) > 10^{-20}$Gauss.



(ρ_B in Gauss^2 and k in 1/Mpc)

RD, Hollenstein & Jain, 2010

Conclusions

- Magnetic fields are observed on all cosmological scales (galaxies, clusters, filaments and probably even voids) with significant amplitudes. Intergalactic fields with coherence length of about 1Mpc and amplitudes of 10^{-20}Gauss (for dynamo amplification) or even 3×10^{-16}Gauss (Neronov-Vovk-bound) are required.

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- Fields from phase transition are too blue, they do not have enough power on large scales.
- Inverse cascade of helical magnetic fields can mitigate this problem but seems not quite sufficient to solve it.

Conclusions

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No Signal

VGA-1