Title: Infrared Issues in de Sitter Space: Secular Growth of Fluctuations and the Breakdown of the Semiclassical Approximation.

Date: Jun 16, 2010 10:30 AM

URL: http://pirsa.org/10060015

Abstract: I'll discuss some work done in collaboration with Cliff Burgess, Louis Leblond and Sarah Shandera on the significance of the IR divergences in de Sitter space. First, I'll talk about how large fluctuations at long distances can induce the failure of the loop expansion for interacting field theories with massless degrees of freedom in de Sitter space, much in the same manner as happens in thermal field theories. Then I'll shift gears slightly and describe work involving the use of the dynamical renormalization group in resumming the secularly growing perturbative corrections to correlation functions for massless, minimally coupled scalar fields in de Sitter.

IR Issues in Inflationary Cosmology

C.P.Burgess, R.H, L. Leblond, S. Shandera

arXiv: 0912.1608, 1005.3551

Outline

- What IR issues?
- Breakdown of the Semiclassical Approximation in de Sitter
- Dynamical RG resummation of secular terms
- A Potpourri of further directions

What IR Issues?

- Light fields in DS have long been known to have IR problems.
    - Secular growth in time of the two point function (time dependent logs)
    - Box-size dependent logs
- These long-distance issues impair our ability to trust perturbative corrections to the power spectrum, bi-spectrum etc.

The Breakdown of the Semiclassical Approximation

IR divergences at Finite T

Low-k Bose-Einstein behavior makes IR divergences stronger than at T=0

$$n_B(k) \propto \frac{T}{k}$$

Now let's compute the most IR divergent amplitude with E external lines I internal ones and V vertices.

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

Schematically

$$A(\mathbf{k},T) \simeq \left[\lambda(2\pi)^3\delta^3(\mathbf{p}+\mathbf{k})\,\frac{1}{T}\,\delta_{nn'}\right]^V \left[T\sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\frac{1}{p_n^2+\mathbf{p}^2+\mathbf{k}^2+m^2}\right]^I$$

Set n=0, use m as the IR cutoff for the internal integrals

$$A_{\mathrm{IR}}(\mathbf{k},T) \simeq \delta^3(\mathbf{k})\left(\frac{m^4}{\lambda T}\right)\left(\frac{\lambda}{m^2}\right)^{\frac{E}{2}}\left[\frac{\lambda T}{(2\pi)^2 m}\right]^L$$

At a critical point m(T*)=0 and loop expansion/mean field approx breaks down

IR Divergences in De Sitter Space

Claim: IR fluctuations for light fields in deS also generate a change in the loop expansion parameter

In fact:

$$\lambda^L \to \left(\frac{\sqrt{\lambda}H}{2\pi m}\right)^{2L}$$

Loop expansion breaks down if

$$m \lesssim \frac{\sqrt{\lambda}H}{2\pi}$$

Some Consequences for inflation models

$$V = V_0 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

Case 1: Large Field

Slow roll ends: $\frac{\phi_{SR}}{M_{\rm Pl}} \sim \mathcal{O}(1)$

Eternal inflation: $\phi \gg \phi_{EI} \simeq \frac{M_{\rm Pl}}{\lambda^{\frac{1}{6}}}$

Low energy EFT approximation: $\phi \ll \phi_{HE} \simeq \frac{M_{\rm Pl}}{\lambda^{\frac{1}{4}}}$

Semiclassical approximation: $\phi \ll \phi_{BD} \simeq \frac{M_{\rm Pl}}{\lambda^{\frac{1}{2}}}$

Case 2: Hybrid Inflation

$$U(\phi,\chi) = \frac{1}{4}\zeta\,(\chi^2 - v^2)^2 + \frac{g^2}{2}\chi^2\phi^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

Along trough, have same potential as before, but now we look at small field regime.

Note that end of slow roll is now decoupled from quartic coupling

$$M_{\rm eff}^2 = m^2 + \frac{\lambda}{2}\phi^2 \Rightarrow$$

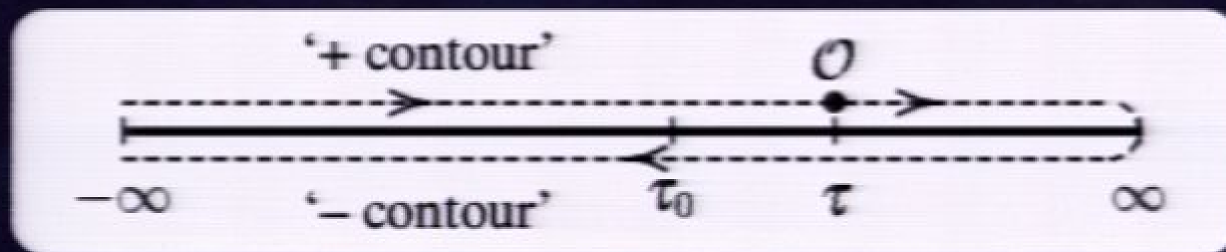
$$m^2 \gg \frac{\lambda H^2}{4\pi^2} = \frac{\lambda V_0}{12\pi^2 M_{\rm Pl}^2}$$

$$\Rightarrow \quad \eta \simeq \frac{m^2 M_{\rm Pl}^2}{V_0} \gg \frac{\lambda}{12\pi^2}$$

Loop size	$\mathcal{O}(1)$	$\mathcal{O}(\sqrt{\lambda})$	$\mathcal{O}(\lambda)$
	m_C	m_{dyn}	m_{max}
Thermal	λT	$\sqrt{\lambda} T$	T
de Sitter	$\sqrt{\lambda} H$	$\lambda^{1/4} H$	H

Power Counting

In-In Formalism



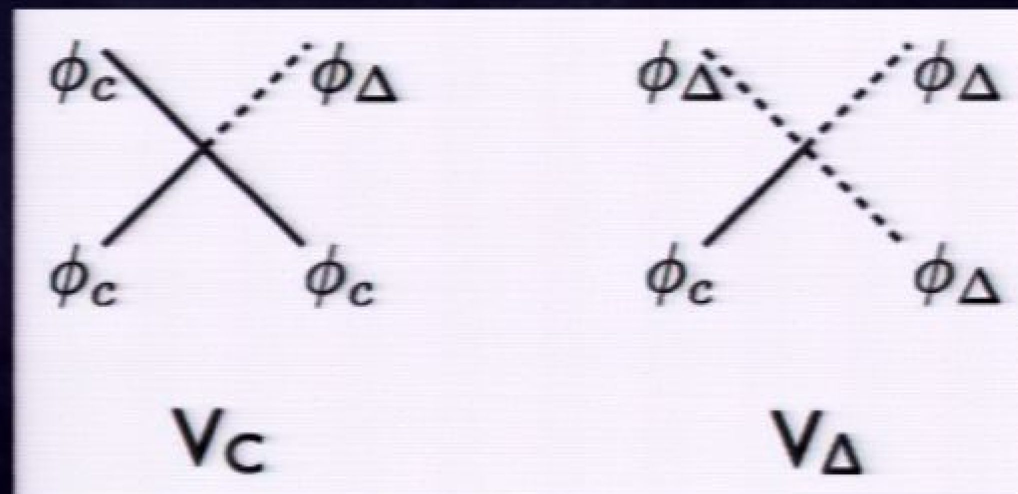
$$S = S[\phi_+] - S[\phi_-] \ni V(\phi_+) - V(\phi_-)$$
$$= m^2\phi_C\phi_\Delta + \frac{\lambda}{4!}\left(4\phi_C^3\phi_\Delta + \phi_C\phi_\Delta^3\right)$$

$$\phi_C = \frac{1}{2}(\phi_+ + \phi_-)$$
$$\phi_\Delta = \phi_+ - \phi_-$$

IR propagators

$$\langle \phi_C(\tau_1)\phi_C(\tau_2)\rangle \simeq \frac{H^2}{2k^3}\left\{1+\mathcal{O}(k^2\tau_i^2)\right\}$$

$$\langle \phi_C(\tau_1)\phi_\Delta(\tau_2) \simeq \theta(\tau_1-\tau_2)\frac{H^2}{3}(\tau_1^3-\tau_2^3)\left\{1+\mathcal{O}(k^2\tau_i^2)\right\}$$



$$\mathcal{A}(k,\tau_a) \propto H^{2(I_C+I_R)}\left(\frac{H^2}{2k^3}\right)^{E_C}\left[\lambda\int\frac{d\tau_i}{H^4\tau_i^4}\right]^{V_C+V_\Delta}\left[H^2\theta(\tau_a-\tau_i)(\tau_a^3-\tau_i^3)\right]^{E_R^{(1)}+E_R^{(2)}}$$
$$\times\left[\int\frac{d^3p}{(2\pi)^3}\frac{1}{2p^3}\right]^{I_C}\left[\int\frac{d^3p}{(2\pi)^3}\frac{\theta(\tau_j-\tau_l)}{3}(\tau_j^3-\tau_l^3)\right]^{I_R}\left[(2\pi)^3\delta^3(p)\right]^{V_C+V_\Delta-1}$$

Problems:

1. External times complicate dimensional analysis

2. Time integrals have k-dependence and are nested

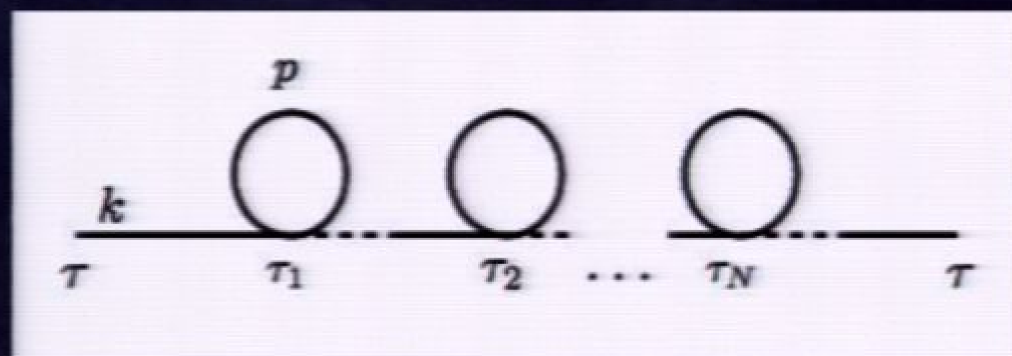
Better to compute in position space

$$\langle \phi_C^N(x)\,\phi_\Delta^{N_\Delta}(x)\rangle \propto \left[\frac{\sqrt{\lambda}\,H}{(2\pi)^2}\right]^{N_C+N_\Delta}\left(\frac{\lambda}{4\pi^2}\right)^{L-1}\Lambda_{IR}^{3P}$$

$$P=(N_C+N_\Delta)-E_C-1+(I_C+I_R)-I_C-(V_C+V_\Delta-1)+(V_C+V_\Delta)-E_R-I_R=0$$

Only Logs show up!

Logs get worse at higher order



$$G_C^{(L)}(k,\tau)\propto G_C^{(0)}(k,\tau)\left[\frac{\lambda}{(2\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon}\ln(-k\tau)\right]^L,\ \epsilon=\frac{m^2}{3H^2}$$

Conclusions Part I

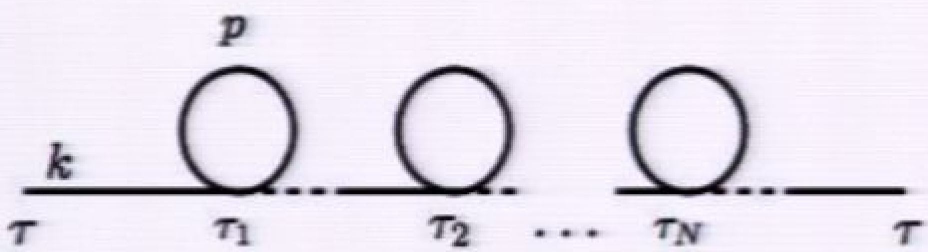
- De Sitter looks like a finite T system to the extent that IR fluctuations induce a breakdown of the loop expansion.
- Different in that the coupling constant dependence of the critical mass at which the breakdown occurs is parametrically different.
- In some models this gives constraints on slow roll parameters.
- Haven't shown things effects don't resum, but no reason for them to.

$$\langle \phi_C^N(x)\, \phi_\Delta^{N_\Delta}(x)\rangle \propto \left[\frac{\sqrt{\lambda}\, H}{(2\pi)^2}\right]^{N_C+N_\Delta} \left(\frac{\lambda}{4\pi^2}\right)^{L-1} \Lambda_{IR}^{3P}$$

$$P = (N_C + N_\Delta) - E_C - 1 + (I_C + I_R) - I_C - (V_C + V_\Delta - 1) + (V_C + V_\Delta) - E_R - I_R = 0$$

Only Logs show up!

Logs get worse
at higher order



$$G_C^{(L)}(k,\tau) \propto G_C^{(0)}(k,\tau) \left[\frac{\lambda}{(2\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon} \ln(-k\tau)\right]^L, \ \epsilon = \frac{m^2}{3H^2}$$

Conclusions Part I

- De Sitter looks like a finite T system to the extent that IR fluctuations induce a breakdown of the loop expansion.
- Different in that the coupling constant dependence of the critical mass at which the breakdown occurs is parametrically different.
- In some models this gives constraints on slow roll parameters.
- Haven't shown things effects don't resum, but no reason for them to.

DRG Resummation of super-Hubble Fluctuations

DRG Resummation of Secular Growth

Two types of secular logs

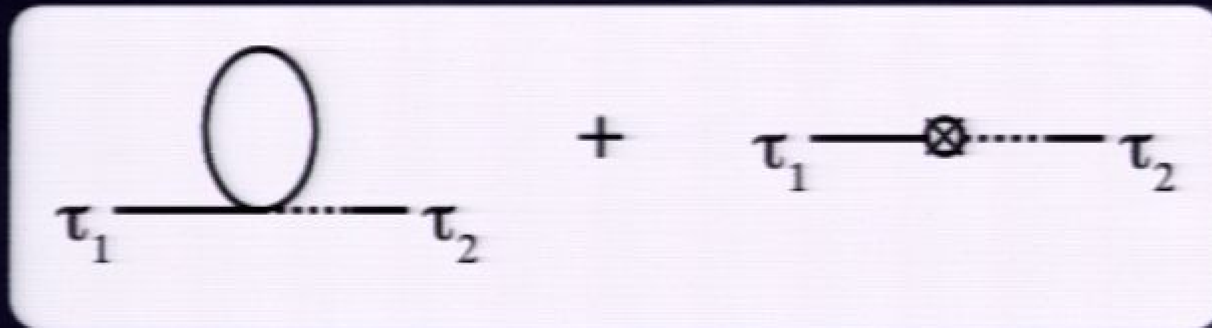
$$\ln\frac{\tau_0}{\tau} = \ln\frac{a(\tau)}{a(\tau_0)}$$

DeS inv. broken by a beginning of inflation

$$\ln(-k\tau) = \ln\frac{a(\tau_k)}{a(\tau)}, \; a(\tau_k) = \frac{k}{H}$$

These show up in higher order corrections even in De S

Let's go back to tadpole graph



$$
\begin{aligned}
\Lambda(\tau) \equiv \langle \phi^2(x) \rangle = -iG^{-+}(x,x) = G^0_C(x,x) &= \int \frac{d^3k}{(2\pi)^3} G^0_C(k,\tau,\tau) + \text{c.t.} \\
&= \frac{1}{(2\pi)^2}\left[\int_{a\Lambda_{IR}}^{a\mu} \frac{dk}{k}\left\{H^2\left[1+\left(\frac{k}{aH}\right)^2\right]\right\}\right] \\
&\simeq \frac{1}{(2\pi)^2}\left[H^2 \ln\left(\frac{\mu}{\Lambda_{IR}}\right) + \frac{1}{2}(\mu^2 - \Lambda_{IR}^2)\right]
\end{aligned}
$$

IR cutoff is unphysical; should be replaced by physical scale L due to missing physics

$$
\begin{aligned}
G_C(k,L) &= G_C^{UV}(\mu/\Lambda_{IR}) + G_C^{IR}(\Lambda_{IR}L) \\
&= \left[A + B\ln\left(\frac{\mu}{\Lambda_{IR}}\right) + \cdots\right] + \left[C + B\ln(\Lambda_{IR}L) + \cdots\right] \\
&= (A+C) + B\ln(\mu L) + \cdots
\end{aligned}
$$

For logs, IR cutoff from UV calculation can give us full dependence on L

In our case, the choice is whether L depends on time or not.

mass term: L is time indep

Pre-inflationary physics: L time dep

Now use this to correct propagator

$$G_C(k,\tau) = \frac{H^2}{2k^3}\left[1 + \frac{\lambda}{3(2\pi)^2}\ln\left(\frac{\mu}{\Lambda_{IR}}\right)\ln(-k\tau) + \cdots\right]$$
$$\Rightarrow \frac{H^2}{2k^3}\left[1 + \frac{\lambda}{3(2\pi)^2}\ln(\mu L)\,\ln(-k\tau) + \cdots\right]$$

How to resum the secular terms?

How can we fix L?

Let's recall how the RG works:

1. Compute 1-loop corrected coupling

$$\alpha(\mu) = \alpha(\mu_0) + b\,\alpha^2(\mu_0)\ln\left(\frac{\mu}{\mu_0}\right)$$

valid for $\alpha(\mu_0) \ll 1,\ \alpha(\mu_0)\ln(\mu/\mu_0) \ll 1$

2. Differentiate then integrate wrt subtraction point

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\mu_0)} - b\ln\left(\frac{\mu}{\mu_0}\right), \text{ valid for } \alpha \ll 1$$

Domain of validity has been extended

What is the time dependent analog?

Suppose approximation scheme generates secular growth

$$y(t) = y_0(t) + \varepsilon\, y_1(t) + c_0 + \mathcal{O}\left(\varepsilon^2\right)$$
$$= y_0(c,t) + \varepsilon\, y_1(c,t) + c_0 + \mathcal{O}\left(\varepsilon^2\right),$$
with c the integration constant for $y_0(t)$.

1. Introduce an arbitrary time scale

$$y(t) = y_0(t) + \varepsilon\left[y_1(t) - y_1(\vartheta) + y_1(\vartheta)\right] + \mathcal{O}\left(\varepsilon^2\right) \Rightarrow$$
$$y(t) = y_0[c(\vartheta), t] + \varepsilon\left[y_1(t) - y_1(\vartheta)\right] + \mathcal{O}\left(\varepsilon^2\right)$$
with $y_0[c(\vartheta), t] \equiv y_0(c,t) + \varepsilon\, y_1(\vartheta)$

2. Use independence from new time scale to get DRG eqn

$$\left(\frac{\partial y_0}{\partial c}\right)\frac{dc}{d\vartheta} - \varepsilon\,\frac{\partial y_1(c,\vartheta)}{\partial \vartheta} \Rightarrow c = \tilde{c}(\vartheta).$$

3. Set new scale equal to t. Solution has greater domain of validity

$$y(t) = y_0[\tilde{c}(\vartheta), t] + \varepsilon\left[y_1(t) - y_1(\vartheta)\right] + \mathcal{O}\left(\varepsilon^2\right)$$
$$= y_0[\tilde{c}(t), t] + \mathcal{O}\left(\varepsilon^2\right)$$

Example: If $$y(t) = c\left[1 + \varepsilon f(t) + \mathcal{O}\left(\varepsilon^2\right)\right]$$

the DRG improvement is $$y(t) = c\, e^{\varepsilon f(t)}\left[1 + \mathcal{O}\left(\varepsilon^2\right)\right]$$

Now let's work this on the two point function:

$$G_C(k,\tau) = \frac{H^2}{2k^3}\left[1 + \frac{\lambda}{3(2\pi)^2}\ln(\mu L)\ln(-k\tau) + \cdots\right] \Rightarrow$$

$$G_C(k,\tau) = \frac{H^2}{2k^3}\exp\left[+\frac{\lambda}{3(2\pi)^2}\ln(\mu L)\ln(-k\tau)\right](1+\cdots)$$

$$= \frac{H^2}{2k^3}\left(\frac{k}{aH}\right)^{\delta}(1+\mathcal{O}(\delta^2))$$

$$\delta = \frac{\lambda}{3(2\pi)^2}\ln(\mu L)$$

We can do this for other situations

Massive (but light) field

$$G^0_C(k,\tau_1,\tau_2) \simeq \frac{H^2}{2k^3}(k^2\tau_1\tau_2)^\epsilon$$

$$G^0_R(k,\tau_1,\tau_2) \simeq \theta(\tau_1-\tau_2)\frac{H^2}{3}(\tau_1^{3-\epsilon}\tau_2^{\epsilon} - \tau_1^{\epsilon}\tau_2^{3-\epsilon})$$

$$\epsilon = \frac{m^2}{3H^2}$$

$$G_C(k,\tau) \simeq \frac{H^2}{2k^3}(-k\tau)^{2\epsilon}\left[1+\frac{\lambda}{6(2\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon}\ln(-k\tau)+\cdots\right]$$

Identify $$\ln(\mu L) \to \frac{1}{2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon} = \frac{3H^2}{2M^2}\left(\frac{\mu}{H}\right)^{2M^2/3H^2}$$

What does the DRG say?

$$G_C(k,\tau) \simeq \frac{H^2}{2k^3}(-k\tau)^{2\epsilon+\delta_m}$$
$$\delta_m = \frac{\lambda}{6(2\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon}$$

Coupling dominates mass if

$$\frac{\lambda}{(4\pi)^2} > 3\epsilon^2\left(\frac{H}{\mu}\right)^{2\epsilon} = \frac{M^4}{3H^4}\left(\frac{H}{\mu}\right)^{2M^2/3H^2}$$

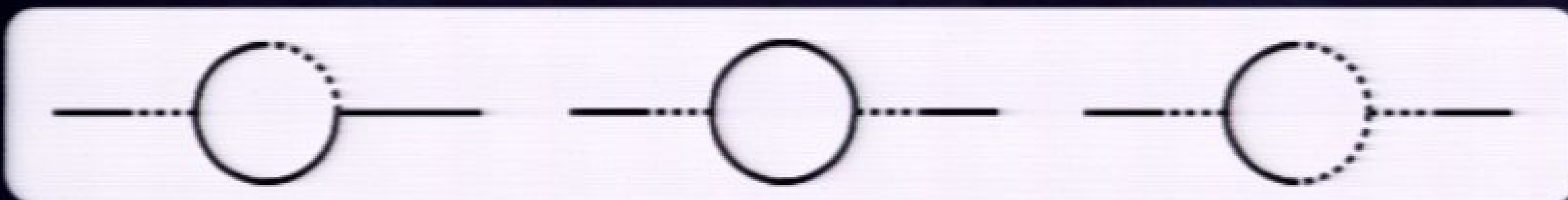
Equivalent mass

$$M^2_{\text{eff}} = \frac{3H^2}{2}\,\delta_m = \frac{\lambda H^2}{(4\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon} \simeq \frac{3\lambda H^4}{(4\pi)^2 M^2}$$

Same result as for mean field! Also

$$M^2_{\text{mf}} = \frac{1}{2}\lambda\langle\phi^2\rangle \Rightarrow$$
$$\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 M^2}$$

Finally, try a cubic theory. Does the IR regulating physics look like a mass?



$$G_C(k,\tau,\tau) = G_C^0(k,\tau,\tau)\exp\left\{\frac{h^2}{9H^2}\left[\frac{1}{(2\pi)^2}\ln^3(-k\tau) + \frac{4\Lambda}{H^2}\ln^2(-k\tau) + \dots\right]\right\}$$

Not a mass; no surprise since potential is ill behaved.

Conclusions Part II

- DRG resums secular terms in two point function
- DRG automatically resums leading logs; actual diagrams need not be singled out
- For quartic potential, missing IR physics is the generation of a dynamical mass.
- This is the same mass found in gap equations in stochastic program.
- DRG can distinguish different types of IR physics: quartic vs cubic potential.

A Potpourri of Further Directions

- Can the DRG work when the IR physics breaks De S invariance?
- Can the DRG shed light on the systematics of stochastic inflation?
- Where does the dynamical mass come from? It smells of a Coleman-Weinberg type of mechanism, but now time-dependent.
- DRG resumming eases IR properties of the two-point function. How does that feed into the loop expansion?

What does the DRG say?

$$G_C(k,\tau) \simeq \frac{H^2}{2k^3}(-k\tau)^{2\epsilon+\delta_m}$$
$$\delta_m = \frac{\lambda}{6(2\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon}$$

Coupling dominates mass if

$$\frac{\lambda}{(4\pi)^2} > 3\epsilon^2\left(\frac{H}{\mu}\right)^{2\epsilon} = \frac{M^4}{3H^4}\left(\frac{H}{\mu}\right)^{2M^2/3H^2}$$

Equivalent mass

$$M_{\rm eff}^2 = \frac{3H^2}{2}\,\delta_m = \frac{\lambda H^2}{(4\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon} \simeq \frac{3\lambda H^4}{(4\pi)^2 M^2}$$

Same result as for mean field! Also

$$M_{\rm mf}^2 = \frac{1}{2}\lambda\langle\phi^2\rangle \Rightarrow$$
$$\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 M^2}$$

Example: If $$y(t) = c\left[1+\varepsilon f(t) + \mathcal{O}\left(\varepsilon^2\right)\right]$$

the DRG improvement is $$y(t) = c\, e^{\varepsilon f(t)}\left[1 + \mathcal{O}\left(\varepsilon^2\right)\right]$$

Now let's work this on the two point function:

$$G_C(k,\tau) = \frac{H^2}{2k^3}\left[1 + \frac{\lambda}{3(2\pi)^2}\ln(\mu L)\ln(-k\tau) + \cdots\right] \Rightarrow$$

$$G_C(k,\tau) = \frac{H^2}{2k^3}\exp\left[+\frac{\lambda}{3(2\pi)^2}\ln(\mu L)\ln(-k\tau)\right](1+\cdots)$$

$$= \frac{H^2}{2k^3}\left(\frac{k}{aH}\right)^{\delta}(1+\mathcal{O}(\delta^2))$$

$$\delta = \frac{\lambda}{3(2\pi)^2}\ln(\mu L)$$

We can do this for other situations

Massive (but light) field

$$G_C^0(k,\tau_1,\tau_2) \simeq \frac{H^2}{2k^3}(k^2\tau_1\tau_2)^\epsilon$$

$$G_R^0(k,\tau_1,\tau_2) \simeq \theta(\tau_1-\tau_2)\frac{H^2}{3}(\tau_1^{3-\epsilon}\tau_2^\epsilon - \tau_1^\epsilon\tau_2^{3-\epsilon})$$

$$\epsilon = \frac{m^2}{3H^2}$$

$$G_C(k,\tau) \simeq \frac{H^2}{2k^3}(-k\tau)^{2\epsilon}\left[1+\frac{\lambda}{6(2\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon}\ln(-k\tau)+\cdots\right]$$

Identify

$$\ln(\mu L) \to \frac{1}{2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon} = \frac{3H^2}{2M^2}\left(\frac{\mu}{H}\right)^{2M^2/3H^2}$$