Title: Holographic Non-Gaussianity

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Abstract: We discuss holographic models for the inflationary epoch. We show how cosmological observables such as the primordial spectrum and non-Gaussianities can be computed via computations of correlation functions of a dual three dimensional QFT (without gravity!) We present a general class of models that have the following universal features: (i) they have a nearly scale invariant spectrum of small amplitude primordial fluctuations, (ii) the scalar spectral index runs as alpha_s = - (n_s-1), (iii) the three point function of density perturbations is exactly equal to the sum of the local and equilateral form with f_{NL}^{local} = 6 f_{NL}^{equil} = 20/3.

Introduction

- Over the last two decades, striking new observations have transformed cosmology from a *qualitative* to a *quantitative* science: a minimal set of just six parameters characterizes the observed universe, all of which are now known to within a few percent.
- With future observations promising an unprecedented era of precision cosmology, the constraints on cosmological parameters are expected to tighten further still, particularly as regards the inflationary sector.
- Despite its successes, the theory of inflation is still unsatisfactory in a number of ways (*e.g.*, fine-tuning, initial conditions, trans-Planckian issues).

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- Despite its successes, the theory of inflation is still unsatisfactory in a number of ways (*e.g.*, fine-tuning, initial conditions, trans-Planckian issues).

Introduction

It is thus imperative that

- inflation is embedded in a UV complete theory. Indeed there is increasing amount of effort devoted to embedding inflation in string theory.
- alternative scenarios are developed.

The holographic approach that we undertake provides both.

Holography for cosmology

- Specifically, in this talk I will address the question:

Can a four-dimensional inflationary cosmology be described in terms of a three-dimensional QFT? (*without* gravity!)

Main results

Our two main results are:

Standard inflation is holographic.

There are holographic models that have different phenomenology than slow-roll inflation but they are nevertheless consistent with current observations.

References

The talk is based on

- Paul McFadden, KS,
  Holography for Cosmology,
  arXiv:0907.5542
- Paul McFadden, KS,
  The Holographic Universe,
  arXiv:1007.2007
- Paul McFadden, KS,
  Holographic Non-Gaussianity,
  to appear
- Adam Bzowski, Paul McFadden, KS
  on-going

Holography for cosmology

Any holographic proposal for cosmology should specify

1. what the dual QFT is
2. how it can be used to compute cosmological observables *(the holographic dictionary)*

Having defined the duality,

- the new description should recover established results in the regime where the weakly coupled gravitational description is valid
- new results should follow by using the duality in the regime where gravity is strongly coupled.

Plan

In the first part I will explain the sense in which inflation is holographic.

- Review standard inflationary computations.
- Review how to compute strong coupling QFT results using standard gauge/gravity duality.
- Show that the inflationary results can be fully expressed in terms of correlators of strongly coupled QFTs.

New holographic models

In the second part I will discuss new holographic models. While standard inflation is linked to strongly coupled QFTs, the new models are based on weakly coupled three dimensional QFT.

- In these models gravity is strongly coupled at early times.
- They provide a new mechanism for a scale invariant spectrum.
- They are compatible with current observations, yet they have different phenomenology than standard slow-roll inflation.

→ Alternative scenarios to standard inflation.

Outline

Cosmological Perturbations

We start by reviewing standard inflationary cosmology.

- We will discuss (for simplicity) single field four dimensional inflationary models,

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi))$$

- We assume a spatially flat background (for simplicity)

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)dx^i dx^i \\ \Phi &= \varphi(t) \end{aligned}$$

Cosmological perturbations

Perturbing the solution one finds that the physical degrees of freedom are a scalar field ζ and a transverse traceless metric γ_{ij}.

- The equations the perturbations satisfy to linear order are:

$$
\begin{aligned}
0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} + a^{-2}q^2\zeta \\
0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + a^{-2}q^2\gamma_{ij}
\end{aligned}
$$

where H is the Hubble function and $\epsilon = 2(H'/H)^2$ is the slow-roll parameter. We are not assuming that ϵ is small.

Power spectrum

In the inflationary paradigm, cosmological perturbations are assumed to originate at sub-horizon scales as quantum fluctuations.

- Quantising the perturbations in the usual manner,

$$\begin{aligned} \langle \zeta(t,\vec{q})\zeta(t,-\vec{q})\rangle &= |\zeta_q(t)|^2 \\ \langle \gamma_{ij}(t,\vec{q})\gamma_{kl}(t,-\vec{q})\rangle &= 2|\gamma_q(t)|^2\Pi_{ijkl}, \end{aligned}$$

  where Π_{ijkl} is the transverse traceless projection operator and $\zeta_q(t)$ and $\gamma_q(t)$ are the mode functions.

- The superhorizon power spectra are obtained by

$$\Delta_S^2(q) = \frac{q^3}{2\pi^2}|\zeta_q(0)|^2, \quad \Delta_T^2(q) = \frac{2q^3}{\pi^2}|\gamma_q(0)|^2,$$

  where $\gamma_q(0)$ and $\zeta_q(0)$ are the constant late-time values of the cosmological mode functions.

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Non-gaussianity

- Non-Gaussianity is related to higher-point functions. In this talk we focus on the three-point function of ζ. This is computed using the in-in formalism as

$$\langle \zeta^3(t) \rangle = -i \int_{t_0}^{t} dt' \langle [\zeta^3(t), H_{int}(t')] \rangle$$

where H_{int} is obtained by expanding the action to cubic order.

- This leads to

$$\langle \zeta_{q_1} \zeta_{q_2} \zeta_{q_3} \rangle = (2\pi)^3 \delta(q_1 + q_2 + q_3) F(q_1, q_2, q_3)$$

Different models are characterized by different $F(q_1, q_2, q_3)$.

Response functions

In preparation to the holographic discussion, we rewrite these results as follows.

- We define the response functions:

$$\Pi^{\zeta} = \Omega_2 \zeta + \Omega_3 \zeta^2 + \cdots, \quad \Pi^{\gamma}_{ij} = E_2 \gamma_{ij} + \cdots,$$

  where Π^{ζ} and Π^{γ}_{ij} are the canonical momenta and the dots indicate other terms that are quadratic and higher order in fluctuations.

- One can show that

$$|\zeta_q|^{-2} = -2\mathrm{Im}[\Omega_2(q)], \quad |\gamma_q|^{-2} = -4\mathrm{Im}[E_2(q)].$$

  so the power spectra can be expressed in terms of the late time behavior of the response functions.

Response functions and 3-point functions

- One can also show that

$$F(q_1, q_2, q_3) \sim \frac{\mathrm{Im}[\Omega_3(q_1, q_2, q_3)]}{\prod_{i=1}^{3} \mathrm{Im}[\Omega_2(q_i)]}$$

evaluated at late times.

We will next show that $\Omega_2(q)$, $E_2(q)$ and $\Omega_3(q_1, q_2, q_3)$ are related to two- and three-point functions of a **strongly coupled** $3d$ **QFT**.

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Domain-wall/cosmology correspondence

The springboard for our discussion is a correspondence between cosmologies and domain-wall spacetimes.

- Domain-wall spacetime:

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\begin{aligned}
ds^2 &= dr^2 + e^{2A(r)} dx^i dx^i \\
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- This solves the field equations that follow from

$$S_{DW} = \frac{1}{2\bar{\kappa}^2} \int d^4x \sqrt{g}\, [-R + (\partial\Phi)^2 + 2\bar{\kappa}^2 \bar{V}(\Phi)],$$

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- One can prove the following:

Domain-wall/Cosmology correspondence

For **every** domain-wall solution of a model with potential $\bar{V}$ there is a FRW solution for a model with potential $(V = -\bar{V})$. [Cvetic, Soleng (1994)], [KS, Townsend (2006)]

- The correspondence can be understood as analytic continuation for the metric. The flip in the sign of V guarantees that the scalar field remains real.
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Domain-walls and holography

Domain-wall spacetimes enter prominently in holography. They describe holographic RG flows.

- The AdS_{d+1} metric is the unique metric whose isometry group is the same as the conformal group in d dimensions. This is the main reason why the bulk dual of a CFT is AdS.
- The domain-wall spacetimes are the most general solutions whose isometry group is the Poincaré group in d dimensions. Thus, if a QFT has a holographic dual the bulk solution must be of the domain-wall type.

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Holographic RG flows

There are two different types of domain-wall spacetimes whose holographic interpretation is fully understood.

1. The domain-wall is asymptotically AdS_{d+1},

$$A(r) \to r, \qquad \Phi(r) \to 0, \qquad \text{as} \quad r \to \infty$$

   This corresponds to a QFT that in the UV approaches a fixed point. The fixed point is the CFT which is dual to the AdS spacetime approached as $r \to \infty$.

Holographic RG flows

2. The domain-wall has the following asymptotics

$$A(r) \to n \log r, \qquad \Phi(r) \to \sqrt{2n} \log r, \qquad \text{as} \quad r \to \infty$$

This case has only been understood recently [Kanitscheider, KS, Taylor (2008)] [Kanitscheider, KS (2009)].

→ Specific cases of such spacetimes are ones obtained by taking the near-horizon limit of the non-conformal branes (D0, D1, F1, D2, D4).

→ These solutions describe QFTs with a "generalized conformal structure": all terms in the action have the same scaling and there is a dimensionful coupling constant.

Domain-wall/cosmology correspondence

Let us see how the correspondence acts on the domain-walls describing holographic RG flows.

1. Asymptotically AdS domain-walls are mapped to inflationary cosmologies that approach de Sitter spacetime at late times,

$$ds^2 \to ds^2 = -dt^2 + e^{2t} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

2. The second type of domain-walls is mapped to solutions that approach power-law scaling solutions at late times,

$$ds^2 \to ds^2 = -dt^2 + t^{2n} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

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Holography: a primer

The holographic dictionary for cosmology will be based on the standard holographic dictionary, so we now briefly review standard holography:

1. There is 1-1 correspondence between local gauge invariant operators $\mathcal{O}$ of the boundary QFT and bulk supergravity modes Φ.
   - $\rightarrow$ The bulk metric corresponds to the energy momentum tensor of the boundary theory.
2. Correlation functions of gauge invariant operators can be extracted from the asymptotics of bulk solutions.

Asymptotic solutions

- The standard gauge/gravity duality is based on spacetimes that are asymptotically locally Anti-de Sitter.
- These spacetimes have a conformal boundary and *near the conformal boundary* Einstein equations (with negative cosmological constant) hold.
- This implies that the metric has the following asymptotic form (in 4 bulk dimensions) [Fefferman, Graham (1985)]

$$ds^2 = dr^2 + e^{2r} g_{ij}(x,r) dx^i dx^j$$

$$g_{ij}(x,r) = \mathbf{g_{(0)ij}(x)} + e^{-2r} g_{(2)ij}(x) + e^{-3r} g_{(3)ij}(x) + \dots$$

- $\mathbf{g_{(0)}(x)}$ is the metric of the spacetime where the boundary theory lives and (as such) it is also the source of the boundary energy momentum tensor.

Domain-wall/cosmology correspondence

Let us see how the correspondence acts on the domain-walls describing holographic RG flows.

1. Asymptotically AdS domain-walls are mapped to inflationary cosmologies that approach de Sitter spacetime at late times,

$$ds^2 \to ds^2 = -dt^2 + e^{2t} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

2. The second type of domain-walls is mapped to solutions that approach power-law scaling solutions at late times,

$$ds^2 \to ds^2 = -dt^2 + t^{2n} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

Holographic RG flows

2. The domain-wall has the following asymptotics

$$A(r) \to n \log r, \qquad \Phi(r) \to \sqrt{2n} \log r, \qquad \text{as} \quad r \to \infty$$

This case has only been understood recently [Kanitscheider, KS, Taylor (2008)] [Kanitscheider, KS (2009)].

→ Specific cases of such spacetimes are ones obtained by taking the near-horizon limit of the non-conformal branes (D0, D1, F1, D2, D4).

→ These solutions describe QFTs with a "generalized conformal structure": all terms in the action have the same scaling and there is a dimensionful coupling constant.

Outline

Asymptotic solutions

- The standard gauge/gravity duality is based on spacetimes that are asymptotically locally Anti-de Sitter.
- These spacetimes have a conformal boundary and *near the conformal boundary* Einstein equations (with negative cosmological constant) hold.
- This implies that the metric has the following asymptotic form (in 4 bulk dimensions) [Fefferman, Graham (1985)]

$$ds^2 = dr^2 + e^{2r} g_{ij}(x,r) dx^i dx^j$$

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Correlation functions

- Using the formalism of holographic renormalization, we then find a precise relation between correlation functions and asymptotics [de Haro, Solodukhin, KS (2000)]

$$\langle T_{ij} \rangle = \frac{3}{2\kappa^2} g_{(3)ij}.$$

- This formula only requires that Einstein equations hold near the conformal boundary. *In particular, it is also valid when curvatures are large in the interior.*
- Higher-point functions are obtained by differentiating the 1-point functions w.r.t. sources and then setting the sources to their background value

$$\langle T_{i_1j_1}(x_1) T_{i_2j_2}(x_2) \cdots T_{i_nj_n}(x_n) \rangle \sim \frac{\delta^{(n-1)} g_{(3)i_1j_1}(x_1)}{\delta g_{(0)i_2j_2}(x_2) \cdots \delta g_{(0)i_nj_n}(x_n)} \Big|_{g_{(0)}=\eta}$$

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Correlation functions

Thus to solve the theory we need to know $g_{(3)}$ as a function of $g_{(0)}$. This can be obtained perturbatively.

→ From gravity to QFT
2-point functions are obtained by solving linearized fluctuations, 3-point functions by solving quadratic fluctuations etc. Here it is crucial that the gravitational approximation is valid and this results in correlators of strongly coupled QFT.

→ From QFT to gravity
Given QFT correlators one obtains an asymptotic solution. If the QFT correlators are that of weakly coupled QFT then the bulk description has the prescribed asymptotic behavior and is strongly coupled in the interior.

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Correlation functions for holographic RG flows

- To compute correlation functions we perturb around the domain-wall. The linearized equations are given by [Bianchi, Freedman, KS (2001)], [Papadimitriou, KS (2004)],

$$\begin{aligned} 0 &= \ddot{\zeta} + (3H + \dot{\epsilon}/\epsilon)\dot{\zeta} - \bar{q}^2 e^{-2A}\zeta \\ 0 &= \ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} - \bar{q}^2 e^{-2A}\gamma_{ij}, \end{aligned}$$

- Comparing with the cosmological perturbations, we find that the equations are mapped to each other provided

$$\bar{q} = -iq$$

- The same holds to all order: the fluctuation equations are mapped to each other provided the momenta are continued as above.

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We now want to extract 2- and 3-point functions.

- Schematically, we must expand the perturbed solution near $r \to \infty$ and extract the piece that scales like e^{-3r}.
  - The part linear in fluctuation gives the 2-point function.
  - The part quadratic in fluctuation gives the 3-point function.
- It is convenient to work in terms of response functions [Papadimitriou, KS (2004)]

$$\bar{\Pi}^{\zeta} = -\bar{\Omega}_2\zeta - \bar{\Omega}_3\zeta^2 + \cdots , \quad \bar{\Pi}^{\gamma}_{ij} = -\bar{E}_2\gamma_{ij} + \cdots ,$$

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The 2-point function of the energy momentum tensor is then given by

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl},$$

where $\Pi_{ijkl} = \frac{1}{2}(\pi_{ik}\pi_{lj} + \pi_{il}\pi_{kj} - \pi_{ij}\pi_{kl}), \quad \pi_{ij} = \delta_{ij} - \bar{q}_i \bar{q}_j / \bar{q}^2.$

$$A(\bar{q}) = 4 \left[\bar{E}_2(\bar{q})\right]_{(0)}, \qquad B(\bar{q}) = \frac{1}{4} \left[\bar{\Omega}_2(\bar{q})\right]_{(0)}.$$

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- Similarly, one can derive a holographic formula for the 3-point function

$$\langle T_{i_1j_1}(\bar{q}_1)T_{i_2j_2}(\bar{q}_2)T_{i_3j_3}(\bar{q}_3)\rangle = ...$$

  in terms of response functions.
- For the 3-point function for the trace of stress energy tensor, $T = T^i_i$, it is given by

$$\langle T(\bar{q}_1)T(\bar{q}_2)T(\bar{q}_3)\rangle \sim \left[\bar{\Omega}_3(\bar{q}_1,\bar{q}_2,\bar{q}_3)\right]_{(0)}$$

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Holography for cosmology

We are now ready to present the holographic dictionary for cosmology.

- The DW/cosmology correspondence maps the near boundary region to the late time region.
- Under the analytic continuation

$$\bar{\kappa}^2 = -\kappa^2, \qquad \bar{q} = -iq$$

the response functions continue as follows

$$\bar{\Omega}_2(\bar{q}) = \Omega_2(-iq), \quad \bar{E}_2(\bar{q}) = E_2(-iq),$$
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- The analytic continuations translate in QFT language to

$$\bar{N} \to -iN, \qquad \bar{q} \to -iq$$

Holographic dictionary: Power spectrum

- We have shown earlier that

$$\Delta_S^2(q) = \frac{-q^3}{4\pi^2 \mathrm{Im}\Omega_{(0)}(q)}, \quad \Delta_T^2(q) = \frac{-q^3}{2\pi^2 \mathrm{Im}E_{(0)}(q)}.$$

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Holographic dictionary: Non-Gaussianity

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$$\langle \zeta_{q_1}\zeta_{q_2}\zeta_{q_3}\rangle = (2\pi)^3\delta(q_1+q_2+q_3)F(q_1,q_2,q_3)$$

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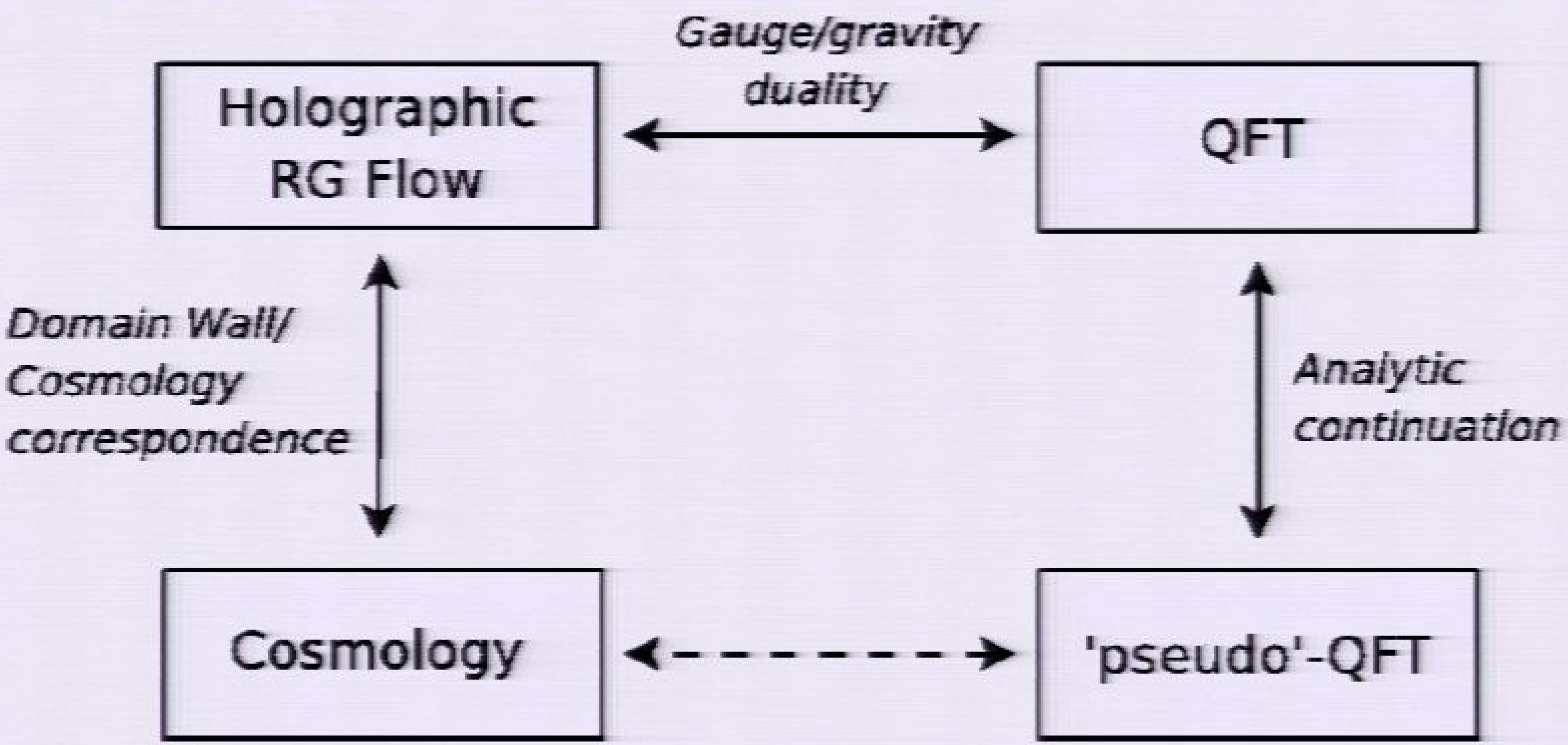
Summary


Gauge/gravity duality
Holographic RG Flow
QFT
Domain Wall/ Cosmology correspondence
Analytic continuation
Cosmology
'pseudo'-QFT

Outline

New holographic models

- We are now going to obtain new models by using weakly coupled QFT. This correspond to the gravitational theory being strongly coupled at early times.
- The boundary theory will be a combination of gauge fields, fermions and scalars and it should admit a large N expansion.
- To extract predictions we need to compute n-point functions of the stress energy tensor analytically continue the result and insert them in the holographic formulae.

Holographic phenomenology for cosmology

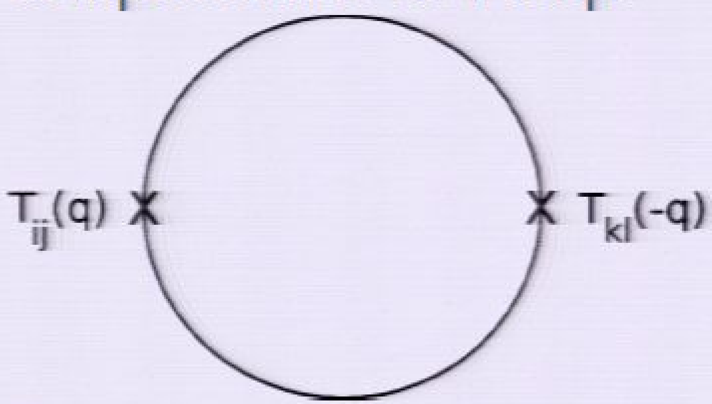
- As a starting point one can consider the strong coupling version of asymptotically dS cosmologies and power-law cosmology.
- In this talk we focus on QFTs dual to the latter. These are super-renormalizable QFTs that depend on a single dimensionful coupling:

$$S = \frac{1}{g_{\rm YM}^2}\int d^3x \mathrm{tr}\Big[\frac{1}{2}F^I_{ij}F^{Iij} + \frac{1}{2}(D\phi^J)^2 + \frac{1}{2}(D\chi^K)^2 + \bar{\psi}^L \not{D}\psi^L + \lambda_{M_1M_2M_3M_4}\Phi^{M_1}\Phi^{M_2}\Phi^{M_3}\Phi^{M_4} + \mu^{\alpha\beta}_{ML_1L_2}\Phi^M\psi^{L_1}_\alpha\psi^{L_2}_\beta\Big].$$

- All terms in this Lagrangian have dimension 4.

A new mechanism for scale invariant spectrum

We need to compute the 2-point function of T_{ij}. The leading order computation is at 1-loop:



The answer follows from general considerations:

- The stress energy tensor has dimension 3 in three dimensions.
- 1-loop amplitudes are independent of g_{YM}^2
- There is a factor of $\bar{N}^2$ because of the trace over the gauge indices.

$$\langle T_{ij} T_{kl} \rangle \sim \bar{N}^2 \bar{q}^3$$

A new mechanism for scale invariant spectrum

Recalling the holographic map:

$$\Delta_S^2 \sim \frac{\bar{q}^3}{\langle TT \rangle} \sim \frac{1}{N^2}$$

- Spectrum is scale invariant to leading order, independent of the details of the holographic theory.

Furthermore,

- Amplitude of power spectrum $\mathcal{A} \sim 1/N^2$.
- Small $\mathcal{A} \sim 10^{-9} \Rightarrow$ large $N \sim 10^4$, justifying the large N limit.

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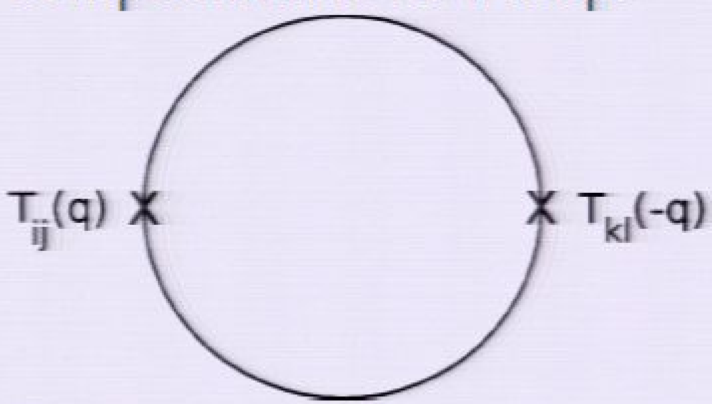

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$$\langle T_{ij} T_{kl} \rangle \sim \bar{N}^2 \bar{q}^3$$

A new mechanism for scale invariant spectrum

Recalling the holographic map:

$$\Delta_S^2 \sim \frac{\bar{q}^3}{\langle TT \rangle} \sim \frac{1}{N^2}$$

- Spectrum is scale invariant to leading order, independent of the details of the holographic theory.

Furthermore,

- Amplitude of power spectrum $\mathcal{A} \sim 1/N^2$.
- Small $\mathcal{A} \sim 10^{-9} \Rightarrow$ large $N \sim 10^4$, justifying the large N limit.

Power spectra

The complete answer is

$$A(\bar{q}) = C_A \bar{N}^2 \bar{q}^3 + O(g_{\rm YM}^2), \qquad B(\bar{q}) = C_B \bar{N}^2 \bar{q}^3 + O(g_{\rm YM}^2),$$

where

$$C_A = (\mathcal{N}_A + \mathcal{N}_\phi + \mathcal{N}_\chi + 2\mathcal{N}_\psi)/256, \qquad C_B = (\mathcal{N}_A + \mathcal{N}_\phi)/256.$$

It follows

$$\Delta_S^2(q) = \frac{1}{16\pi^2 N^2 C_B} + O(g_{\rm YM}^2), \qquad \Delta_T^2(q) = \frac{2}{\pi^2 N^2 C_A} + O(g_{\rm YM}^2).$$

$\mathcal{N}_A$: # of gauge fields, $\mathcal{N}_\phi$: # of minimally coupled scalars,
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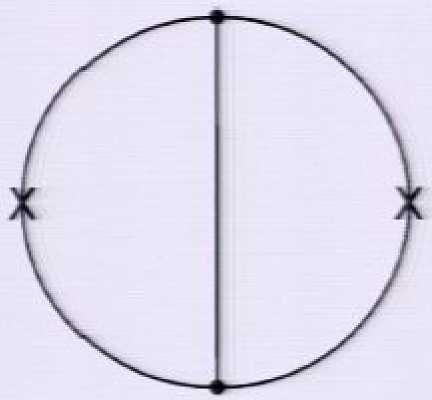
Subleading corrections



Subleading corrections give small deviations from scale invariance:

$$n_s - 1 \sim g_{\text{eff}}^2 = g_{\text{YM}}^2 N/q.$$

The observational value $(n_s - 1) \sim 10^{-2}$ is then consistent with the QFT being weakly interacting.

- To determine the sign of $(n_s - 1)$ (positive: red-tilted spectrum, negative: blue-tilted spectrum) requires summing all 2-loop graphs, and will in general depend on the field content of the dual QFT.

[Work in progress]

2-loop details

Super-renormalizable theories often have infrared problems. The specific type of theories we consider however are well-defined: g^2_{YM} acts as an infrared cut-off. [Jackiw, Templeton (1981)] [Appelquist, Pisarski (1981)].
The 2-loop integrals are indeed finite and one obtains:

$$A(\bar{q}) = C_A \bar{N}^2 \bar{q}^3 [1 + D_A g^2_{\text{eff}} \ln g^2_{\text{eff}} + O(g^4_{\text{eff}})],$$
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where $g^2_{\text{eff}} = g^2_{\text{YM}} \bar{N}/\bar{q}$ and D_A and D_B are numerical constants.
This leads to

$$n_S(q) - 1 = -D_B g^2_{\text{eff}} + O(g^4_{\text{eff}}), \qquad n_T(q) = -D_A g^2_{\text{eff}} + O(g^4_{\text{eff}}).$$

Running

- Independent of the details of the theory, the scalar spectral index runs as
$$\alpha_s = \frac{dn_s}{d\ln q} = -(n_s - 1) + O(g_{\text{eff}}^4).$$
- This prediction is qualitatively different from slow-roll inflation, for which $\alpha_s/(n_s - 1)$ is of first-order in slow-roll.

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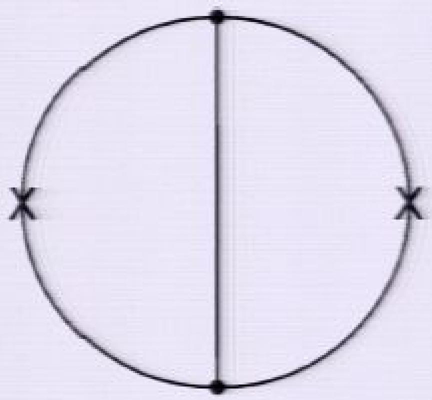

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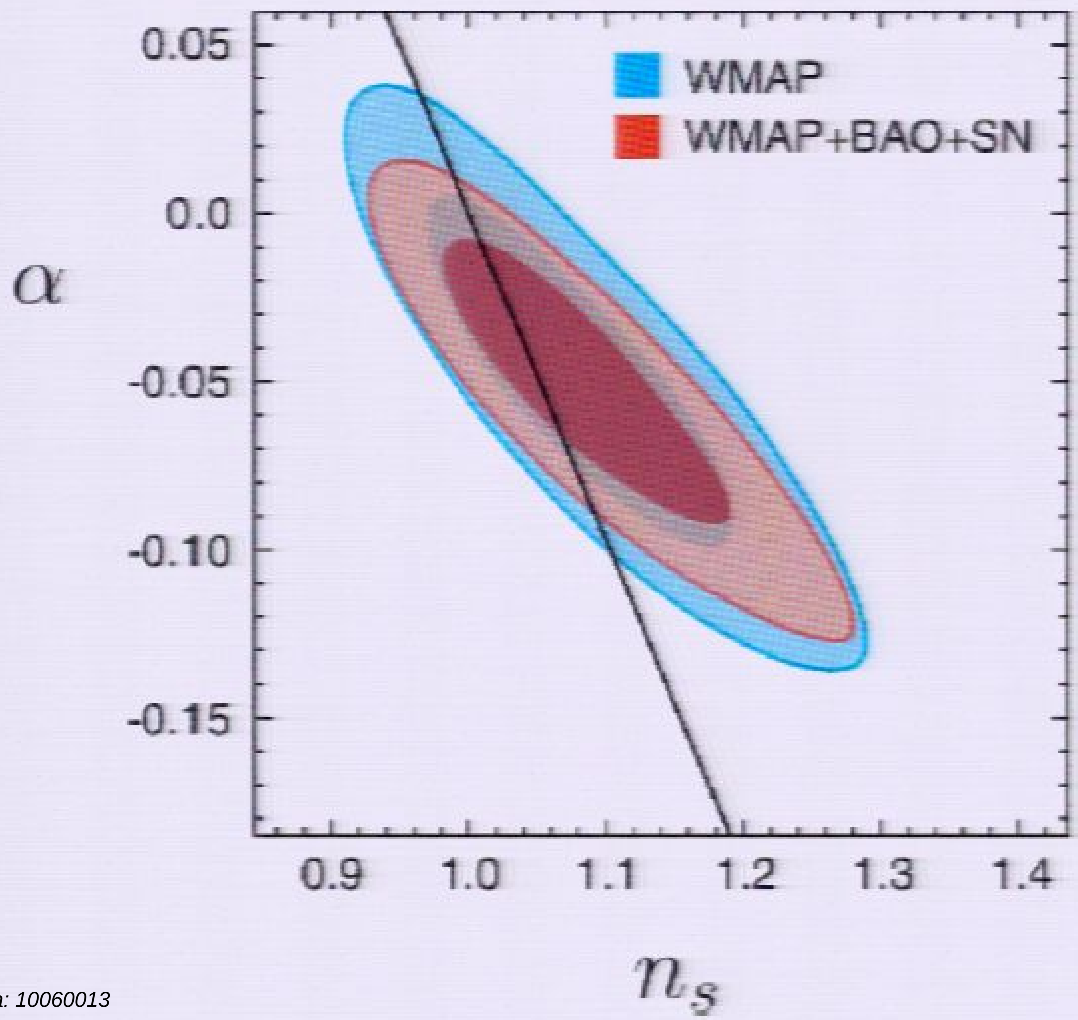
WMAP data

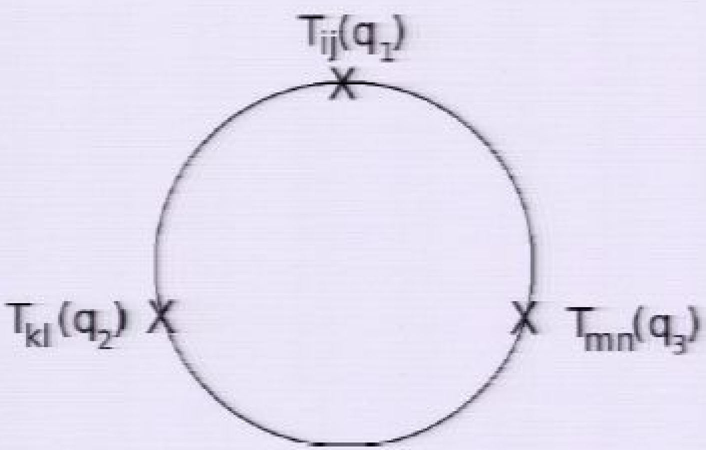
Komatsu et al. arXiv:0803.0547.



Solid line:

$$\alpha = -(n_s - 1)$$

Non-Gaussianity



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$$\langle T(\bar{q}_1)T(\bar{q}_2)T(\bar{q}_3)\rangle =$$
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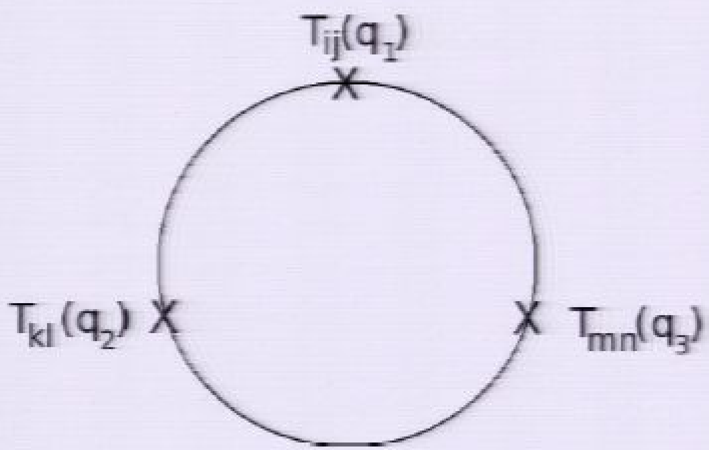

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The shape of non-Gaussianity

- Local form [Gangui etal (1994)]; [Verde etal(2000)] ; Komatsu & Spergel (2001)]

$$F_{\rm local}(q_1,q_2,q_3) = f_{NL}^{\rm local}\frac{2A^2}{q_1^3q_2^3q_3^3}\sum_{i=1}^{3}q_i^3$$

→ Observationally: $f_{NL}^{\rm local} = 32 \pm 21 (68\% CL)$
→ Single scalar slow-roll inflation: $f_{NL}^{\rm local} = 0.015$

- Equilateral form [Creminelli etal, astro-ph/0509029]]

$$F_{\rm equil}(q_1,q_2,q_3) = f_{NL}^{\rm equil}\frac{6A^2}{q_1^3q_2^3q_3^3}\left(-2q_1q_2q_3 - \sum_{i=1}^{3}q_i^3 + (q_1q_2^2 + 5\ perm)\right)$$

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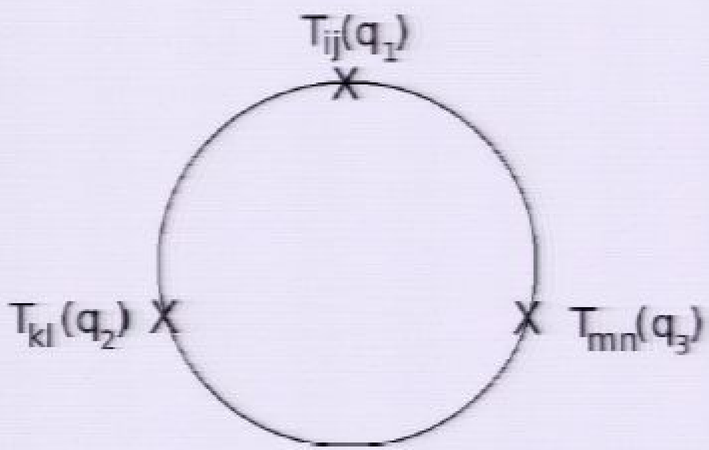

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Outline

Conclusions

- I have presented a holographic description of inflationary cosmology in terms of a 3-dimensional QFT (without gravity!)
- When gravity is weakly coupled, holography correctly reproduces standard inflationary predictions for cosmological observables.
- When gravity is strongly coupled, one finds new models that have a QFT description.
- We initiated a holographic phenomenological approach to cosmology.

Holographic phenomenology

I presented models with the following universal features:

- they have a nearly scale invariant spectrum of small amplitude primordial fluctuations.
- the scalar spectral index runs as $\alpha_s = -(n_s - 1)$.
- the three point function of curvature perturbations is exactly equal to the sum of the local and equilateral form with $f_{NL}^{\text{local}} = 6 f_{NL}^{\text{equil}} \sim 5$.

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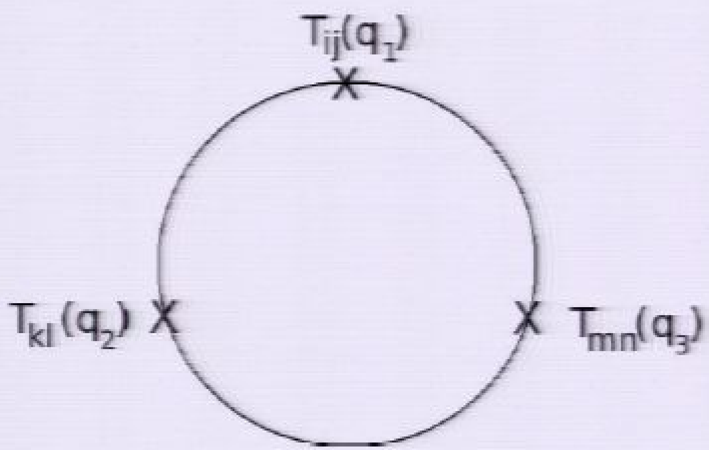

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