Title: Eternal Inflation without Metaphysics

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Abstract: TBA

Eternal Inflation without Metaphysics

Perimeter Institute

June 2010

Thomas Hertog
(APC – Paris)

w/ Jim Hartle (UCSB), Stephen Hawking (DAMTP)

arXiv:0803:1663
arXiv:0905.3877
arXiv:1001.0262
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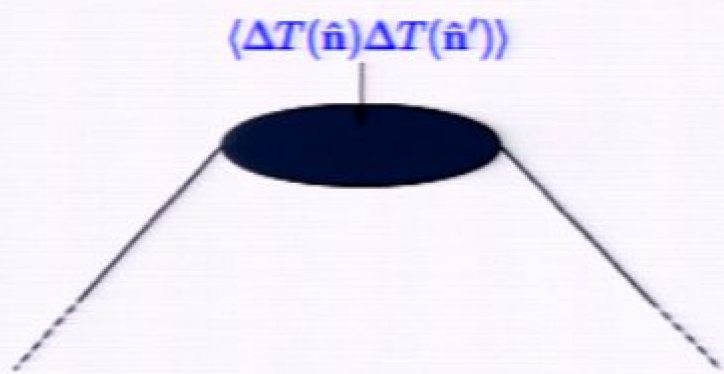
Introduction

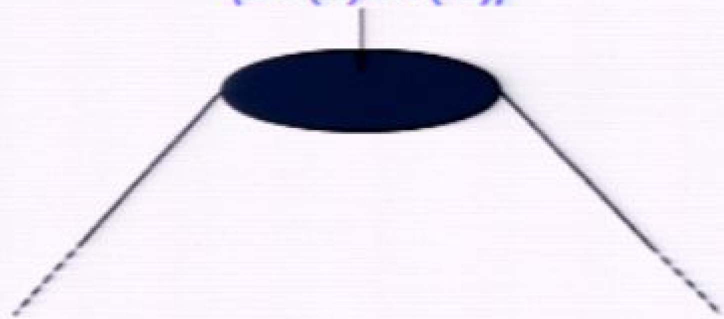


$$\Psi[h, \chi] = \int_C \delta g \delta \phi \exp(-I[g, \phi])$$

"The integral is over all regular metrics g and matter fields ϕ that match (h, χ) on their only boundary."
[Hartle & Hawking '83]

Introduction

$$\langle \Delta T(\hat{\mathbf{n}}) \Delta T(\hat{\mathbf{n}}') \rangle$$



$$C_l = \frac{16\pi^2 T_0^2}{9} \int_0^\infty k^2 dk \langle \delta\zeta_k \delta\zeta_{k'} \rangle j_l^2(kr_l)$$

In a given homogeneous isotropic background, standard inflationary theory predicts

$$\langle \delta\zeta_k \delta\zeta_{k'} \rangle = \frac{A^2}{k^{4-n}}$$

where e.g. for $V(\phi) = \mu\phi^n$

$$A^2 \approx \frac{V_{exit}}{\epsilon_{exit}}, \qquad n = 1 - \frac{2+n}{N_0}$$

Quantum Cosmology

A quantum state does not predict a unique FRW background. At best it provides a probability distribution of an ensemble of possible universes.

To evaluate $\langle \delta\zeta_k \delta\zeta_{k'} \rangle$ from NBWF one first needs:

- probabilities of ensemble of universes, and of Hubble volumes of different kinds.
- probability of inflation?
- state of perturbations $\delta\zeta_k$ for each type
- probability distributions of perturbations

$$\rightarrow \quad \langle \delta\zeta_k \delta\zeta_{k'} \rangle \approx \sum_i p(i) \; {}_i\langle \delta\zeta_k \delta\phi_{k'} \rangle_i$$

where i labels the Hubble volumes with different distributions for the observable of interest.

Model

$$\Psi[h,\chi_i] = \int_C \delta g \delta\phi \exp(-I[g,\phi_i])$$

"The integral is over all regular metrics g and matter fields ϕ that match (h,χ) on their only boundary." [Hartle & Hawking '83]

Matter model:

$$I[g,\phi_i] = -\frac{1}{2}\int_M R - 2\Lambda - \sum_i \left[(\nabla\phi_i)^2 + \mu_i \phi_i^{n_i}\right]$$

Minisuperspace:

$$ds^2 = (3/\Lambda)[d\tau^2 + a^2(\tau) d\Omega_3^2 + h_{\mu\nu}dx^\mu dx^\nu]$$

What is the ensemble of cosmologies, with perturbations, predicted by the NBWF?

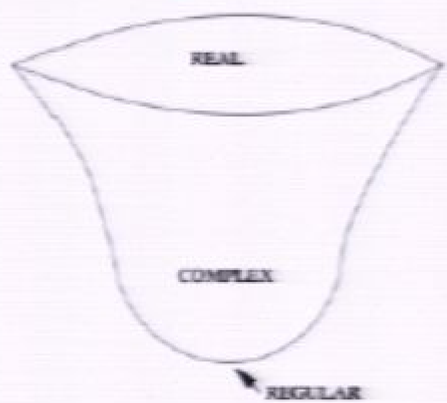
Semiclassical Approximation

In some regions of (mini)superspace the wave function can be evaluated in the steepest descents approximation.

To leading order in $\hbar$ the NBWF will then have the semiclassical form,

$$\Psi(b, \chi_i) \approx \exp\{[-I_R(b, \chi_i) + iS(b, \chi_i)]/\hbar\}$$

In general the extremal geometries will be complex:



Classical Universes in Quantum Cosmology

$$\Psi(b,\chi_i) \approx \exp\{[-I_R(b,\chi_i) + iS(b,\chi_i)]/\hbar\}$$

The semiclassical wave function predicts Lorentzian, classical evolution in regions of superspace where [Hawking '84, Grischuk & Rozhansky '90]

$$|\nabla_A I_R| \ll |\nabla_A S_L|$$

The allowed classical histories of the universe are the integral curves of S_L:

$$p_A = \nabla_A S_L$$

which have probability

$$P_{history} \propto \exp[-2I_R/\hbar]$$

Saddle points

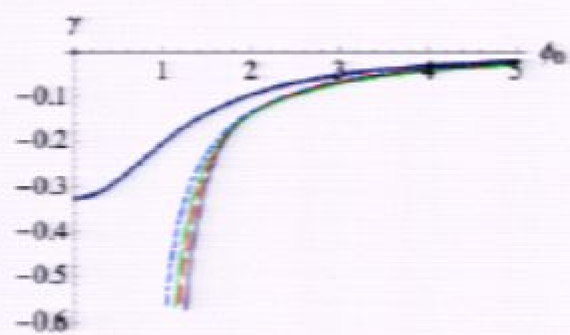
Regularity at SP: $a(0) = 0, \quad \dot{a}(0) = 1, \quad \dot{\phi}_i(0) = 0$

Free parameter at SP: $\quad \phi_i(0) = \phi_{i0} e^{i\gamma}$

At boundary τ_f: $\quad a(\tau_f) = b, \quad \phi_i(\tau_f) = \chi_i$

• For large scale factor b, saddle points for which $|\nabla_A I_R| \ll |\nabla_A S_L|$ at boundary provide dominant contribution → classical spacetime predicted.

• Classicality requires

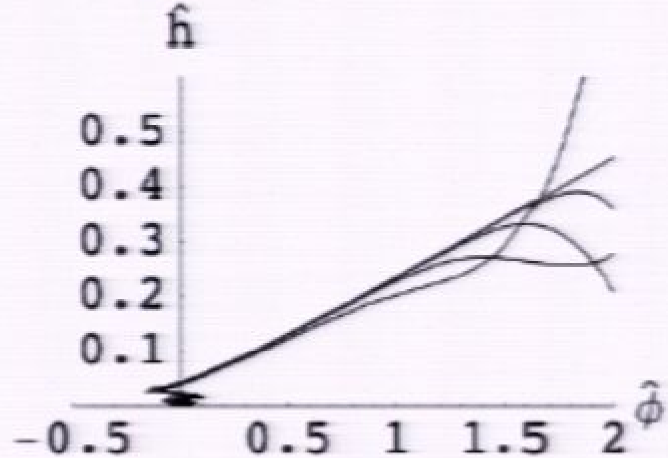


→ NBWF specifies slice through phase space

Inflation

Complex extrema specify Cauchy data for Lorentzian histories at the boundary $a = b, \phi = \chi$.

Lorentzian evolution backwards from boundary yields



All classical universes predicted by NBWF inflate at early times.

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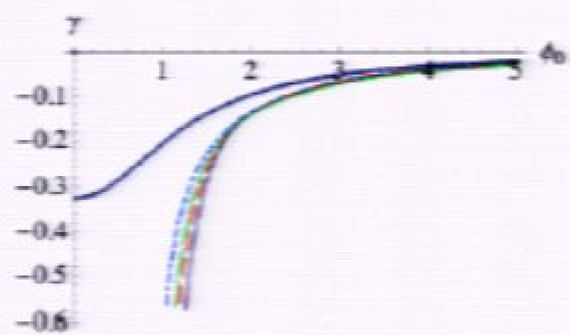

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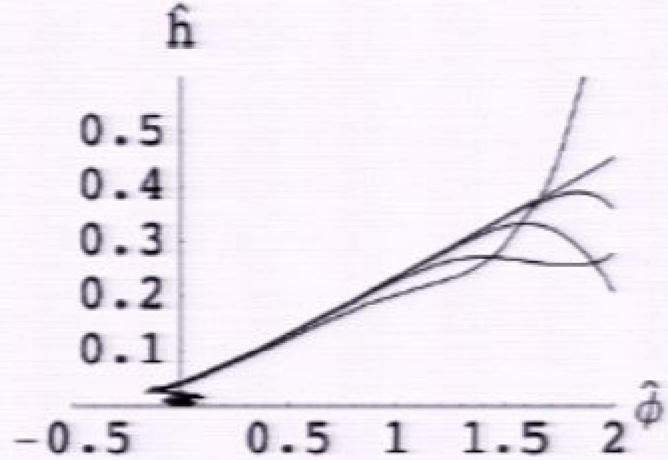

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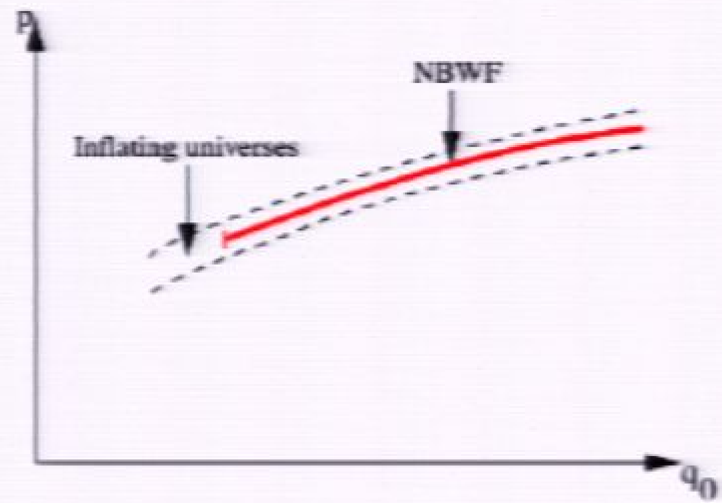
No-Boundary Measure

→ NBWF selects inflating histories/directions ϕ_i in field space



These are exponentially improbable with a flat measure on phase space. [Gibbons & Turok '06]

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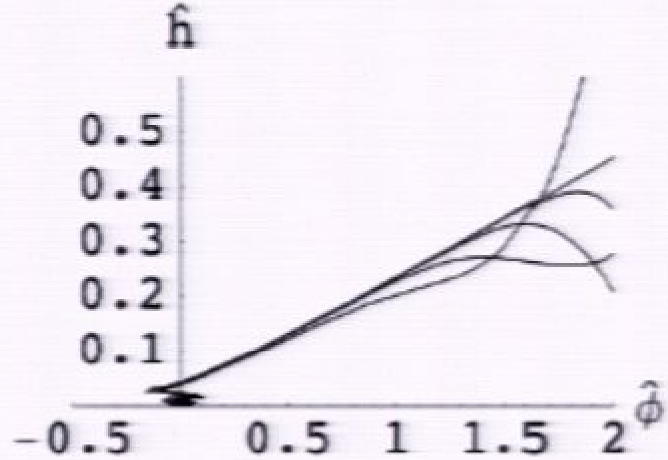

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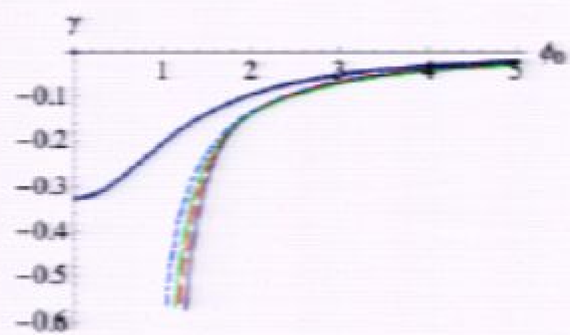

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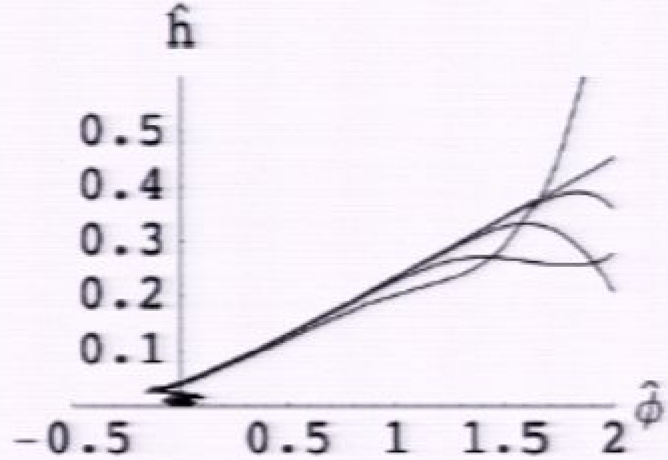

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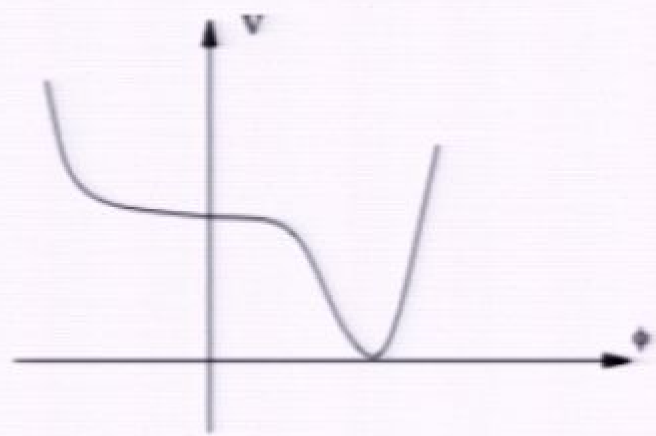
Extension to Landscape Models

Suppose low energy string theory has a multidimensional landscape potential with many vacua...

In NBWF, classical universes emerge only where potential admits inflation.

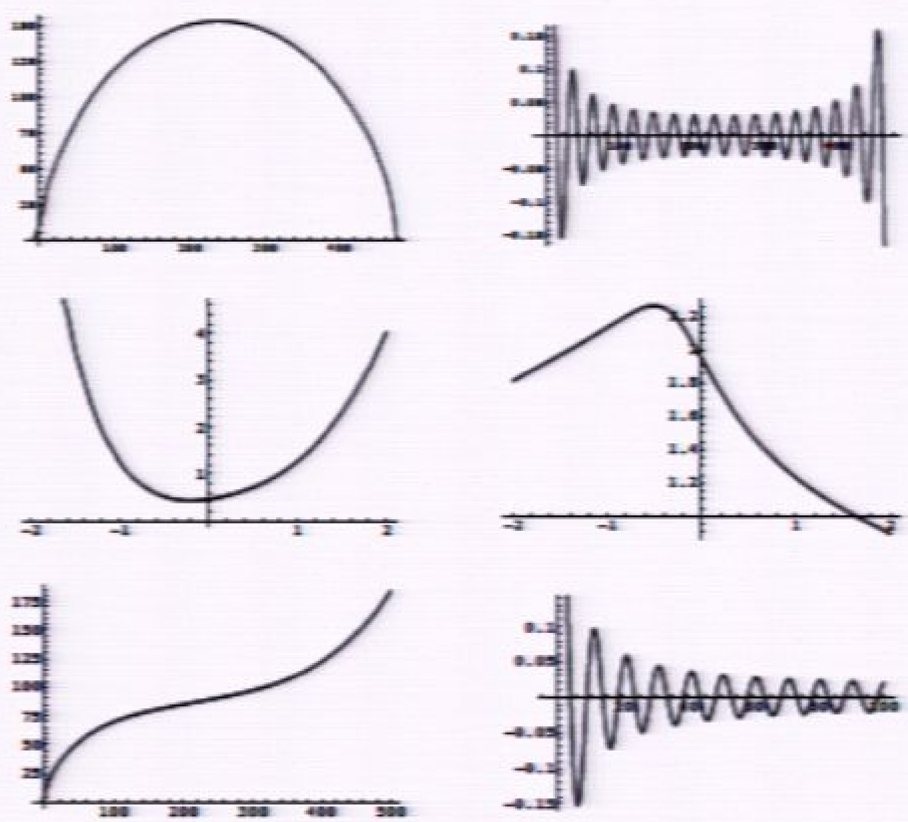


Other vacua have exceedingly small probability.

→ the observation of a quasiclassical realm acts as vacuum selection principle.

Sample of histories

scale factor | scalar field



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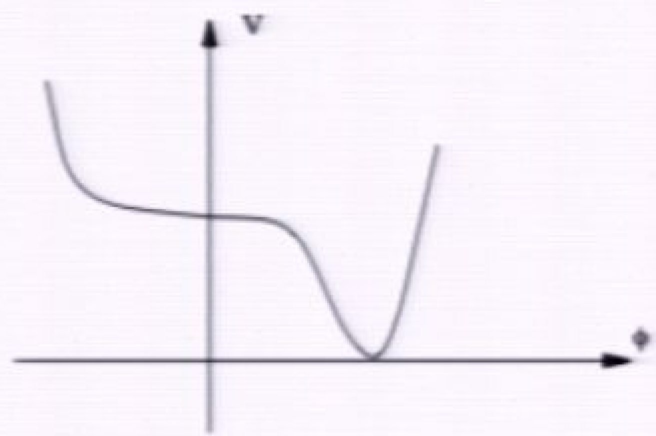

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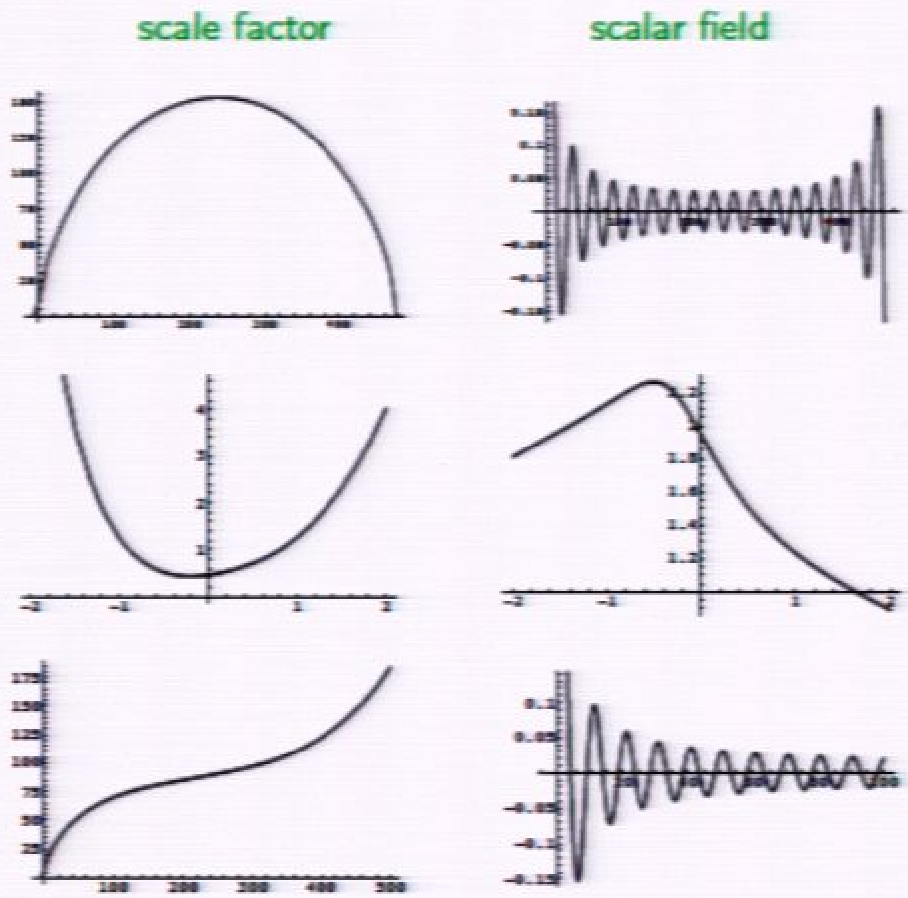
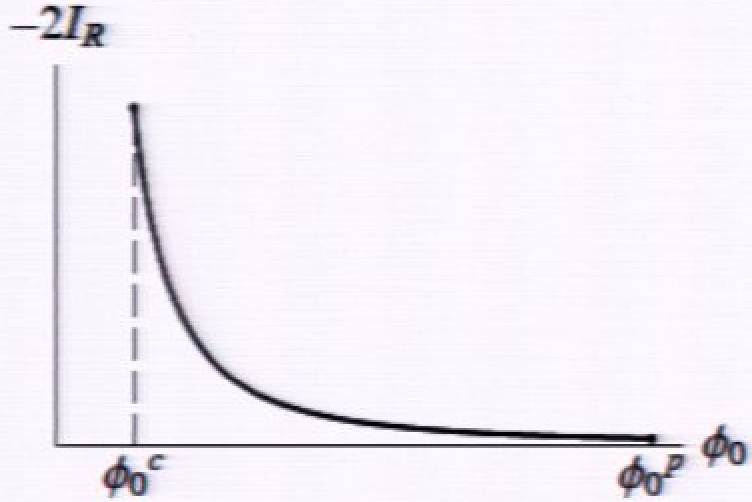


Probabilities of histories

The value of the real part of the Euclidean action of the saddle points is conserved along the Lorentzian histories and determines their bottom-up probability.

$$p(\phi_0) \propto \exp[-2I_R/\hbar]$$



$$I_R \approx -\frac{\pi}{2}\frac{1}{\mu_i \phi_{i0}^{n_i}} \approx -\frac{1}{V(\phi_{i0})}$$

The NBWF favors histories with a small amount of inflation.

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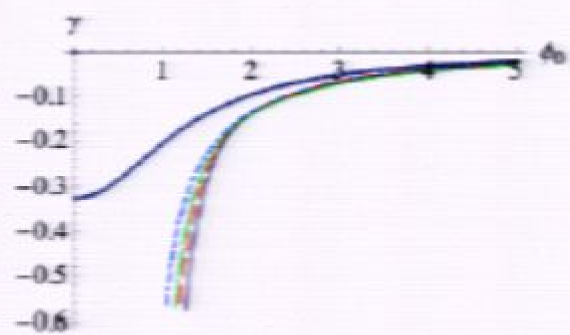


→ NBWF specifies slice through phase space

Probabilities for Observation

Our observations are restricted to part of a light cone that extends over a Hubble volume, located somewhere in space.

$$\langle \delta\zeta_k \delta\zeta_{k'} \rangle \approx \sum_i p(i) \;_i\langle \delta\zeta_k \delta\zeta_{k'} \rangle_i$$

A 'phase space' factor connects $p(i)$ to $p(\phi_0)$, because the number of possible locations of our light cone differs from history to history in the ensemble.

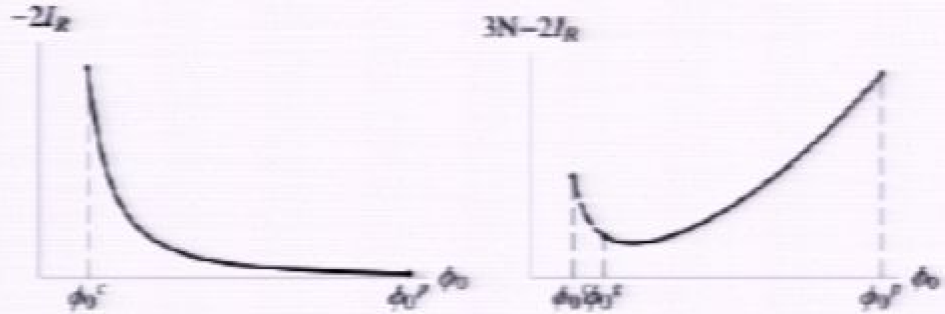
In histories where we are rare, this amounts to a volume weighting of the probabilities of histories, [Hawking, 08; Hartle & TH, 09]

$$\langle \delta\zeta_k \delta\zeta_{k'} \rangle \approx \sum_i \int d\phi_{i0} \left[p_E N_h(\phi_{i0}) \right] \; p(\phi_{i0}) \; \langle \delta\zeta_k \delta\phi_{k'} \rangle_{\phi_{i0}}$$

Volume weighting connects probabilities for histories to probabilities relevant for observation

Eternal Inflation

$$N_h(\phi_{i0})\; p(\phi_{i0}) \;\propto \exp\left[\frac{3\phi_{i0}^{n_i}}{2n_i} + \frac{2\pi}{\mu_i \phi_{i0}^{n_i}}\right]$$



$dp/d\phi_i > 0$ when $\phi_{i0} > 1/\mu^{1/2+n_i}$

The NBWF predicts a large number of efoldings N in our past in models with a regime of eternal inflation.

$$\langle \delta\zeta_k \delta\phi_{k'} \rangle \approx \sum_i \int d\phi_{i0} \left[p_E N_h(\phi_{i0})\right]\; p(\phi_{i0})\; \langle \delta\zeta_k \delta\phi_{k'} \rangle_{\phi_{i0}}$$

Perturbed Saddle Points

[Hawking, LaFlamme, Lyons '93]

Perturbations $\delta\phi$ and metric perturbations δg,

$$ds^2 = (1+2\varphi)d\tau^2 + 2a(\tau)B_{|i}dx^i d\tau$$
$$+a(\tau)^2[(1-2\psi)\gamma_{ij} + 2E_{|ij}]dx^i dx^j$$

Constraints: $\Psi(b, \chi, \psi, E, \delta\phi) \to \Psi(b, \chi, \zeta)$

Semiclassical appr:

$$\Psi(b, \chi, \zeta) = \exp[-I(b, \chi, \zeta)/\hbar]$$

where ζ is the real boundary value of

$$\zeta = -\psi - \frac{H}{\dot{\phi}}\delta\phi$$

Mode expansion on S^3:

$$I(b, \chi, \zeta) = I^{(0)}(b, \chi) + \sum_n I^{(n)}(b, \chi, \zeta_n)$$

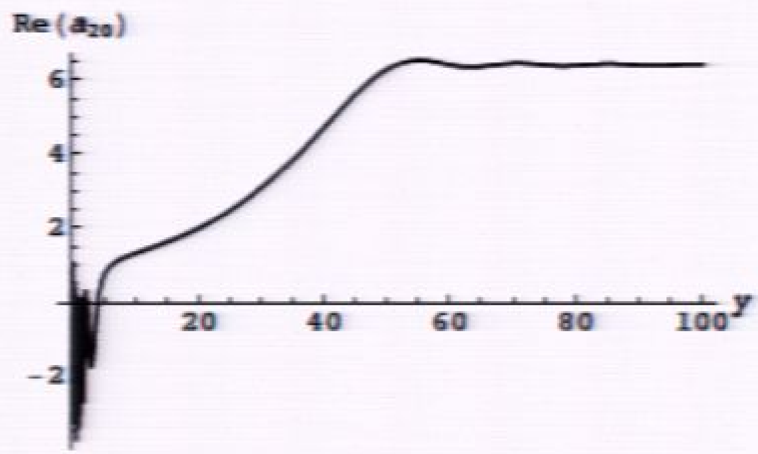
with $\quad \zeta_n = -[\psi_n(\tau) + \frac{H}{\dot{\phi}}\delta\phi_n(\tau)]Q^n(\Omega)$

Complex perturbations

Regularity at South Pole: $\zeta_n \to 0$

Phase $\zeta_n(0)$ taken so that $\zeta \to$ real z at boundary.

$\to$ ensemble of perturbed histories labeled by ζ_{n0}.



At horizon crossing $n/a \sim H$, solutions change from oscillating to slowly growing matter/metric perturbations.

Gauge-invariant variable ζ_n tends to a constant

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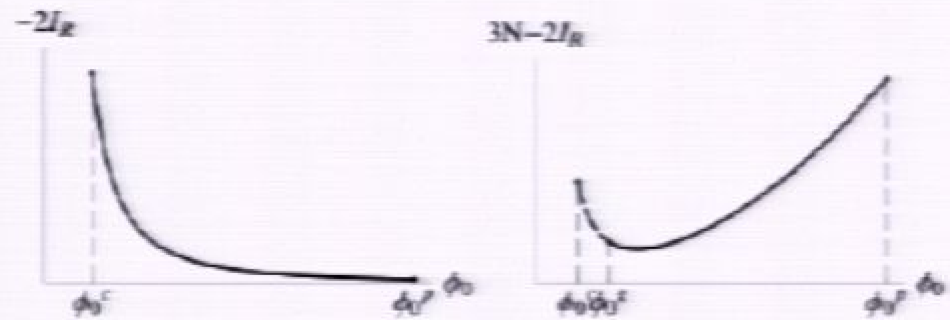

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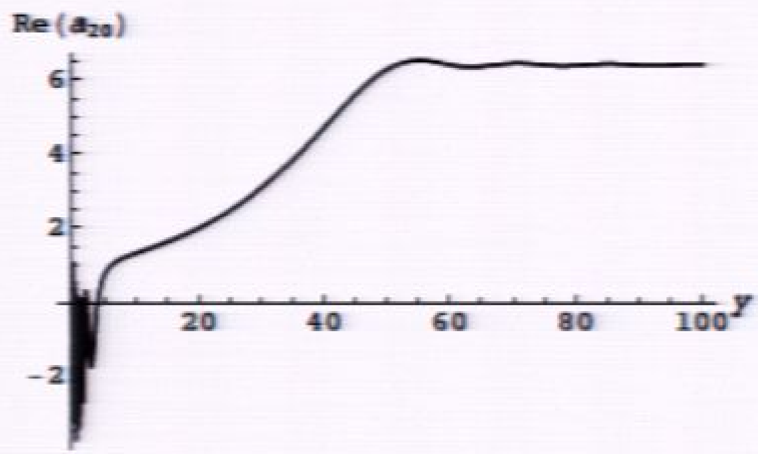

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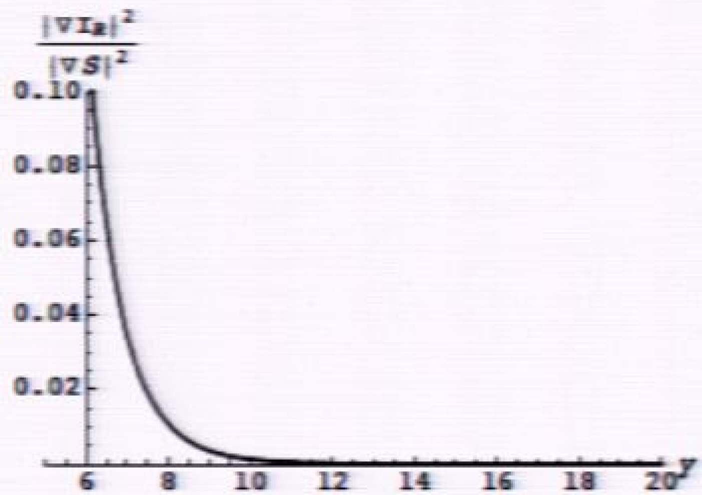
Classical Perturbations

Perturbations behave classically outside horizon:



→ ensemble of perturbed histories with probabilities

$$I_R^{(n)} \to \frac{n^3 \epsilon_*}{2V_*} \zeta_n^{-2}, \qquad n = a_* H_*$$

→ Gaussian spectrum with $< (\Delta T/T)^2 > = V_*/\epsilon_*$

Probabilities of Perturbations

Distributions in different directions ϕ_i:

$$I_R^{(n)} \to \frac{n^3}{2\sigma_n^2(i)}\zeta_n^2, \qquad n = a_* H_*$$

The variance $\sigma_n^2(i) > 1$ of modes leaving horizon in the regime of eternal inflation.

$\to$ NBWF predicts significant large-scale inhomogeneities in histories with $\phi_{i0} > 1/\mu^{1/2+n_i}$

This increases the possible locations of our light cone [e.g. Creminelli et al. 08]:

$$p_E N_h(\phi_{i0}) \to 1 \quad \text{when} \quad \phi_{i0} \geq 1/\mu^{1/2+n_i}$$

Hence for directions in field space with a regime of eternal inflation,

$$\langle \delta\zeta_k \delta\phi_{k'} \rangle \approx \sum_i \int_{\phi_{i0}^c} d\phi_{i0}\, p(\phi_{i0})\, \langle \delta\zeta_k \delta\phi_{k'} \rangle_{\phi_{i0}}$$

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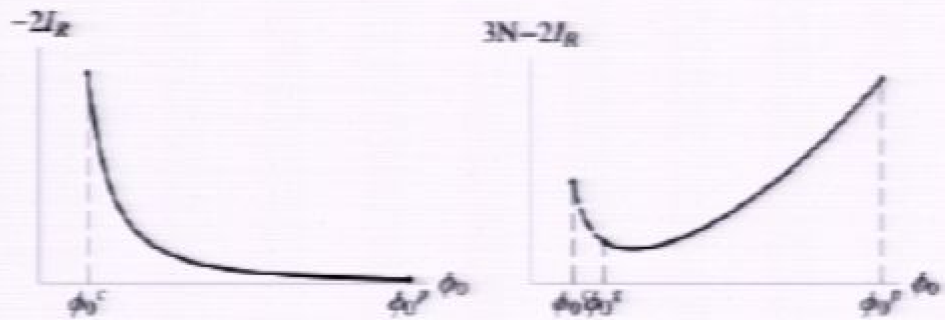

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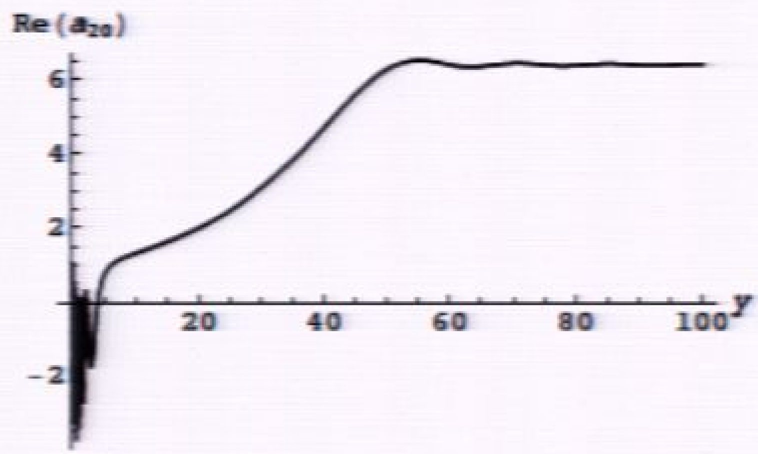

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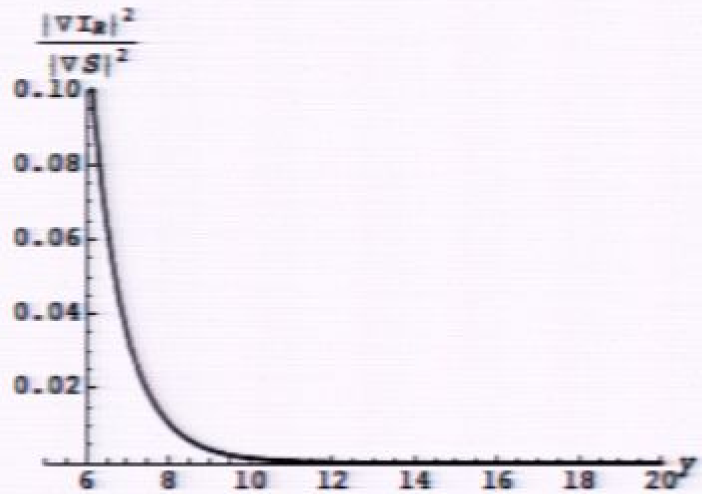

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where $\phi_{i0}^c = 1/\mu^{1/2+n_i}$

CMB Predictions

$$\langle \delta\zeta_k \delta\phi_{k'} \rangle \approx \sum_i \langle \delta\zeta_k \delta\phi_{k'} \rangle_{\phi_{i0}} \int_{\phi^c_{i0}} d\phi_{i0}\, p(\phi_{i0})$$

But

$$p(\phi_{i0})) \approx \exp[\frac{1}{V(\phi_{i0})}]$$

Hence

$$\langle \delta\zeta_k \delta\phi_{k'} \rangle \approx \sum_i p(\phi^c_{i0})\ \langle \delta\zeta_k \delta\phi_{k'} \rangle_{\phi^c_{i0}}$$

Summary, for a multidimensional potential:

• NBWF selects inflationary directions

• Volume weighting selects directions with regime of eternal inflation

• NBWF gives the relative probabilities p(i) of eternal inflation in different directions ϕ_i,

$$p(i) \approx \exp[\frac{1}{V(\phi^c_{i0})}] = \exp[\frac{1}{\mu_i^{2/2+n_i}}]$$

where ϕ^c_{i0} is the scalar field value at the threshold of eternal inflation in direction ϕ_i.

CMB Predictions

$$\langle \delta\zeta_k \delta\phi_{k'} \rangle \approx \sum_i p(\phi_{i0}^c)\ \langle \delta\zeta_k \delta\phi_{k'} \rangle_{\phi_{i0}^c}$$

CMB multipole coefficients:

$$C_l \approx \frac{16\pi^2 T_0^2}{9} \int_0^\infty k^2 dk \langle \delta\zeta_k \delta\zeta_{k'} \rangle j_l^2(k r_l)$$

$$\rightarrow C_l \approx \sum_i p(i) C_l(i)$$

• Direction with lowest $V(\phi_i^c)$ provides the dominant contribution to the predictions for the $C_l's$.

e.g. $V(\phi_1, \phi_2) = m^2\phi_1^2 + \lambda\phi_2^4$ with COBE normalization: dominant contribution from the ϕ_1^2 direction.

• Other directions lead to non-Gaussian corrections, which are significant in models with several comparable p(i).

Conclusion

- The NBWF provides a measure on classical phase space that selects inflating histories.
- In histories where we are rare, volume weighting connects probabilities for 4D histories to probabilities relevant for observation.
- In single field models of eternal inflation, this implies the NBWF predicts a long period of inflation in our past, with Gaussian perturbations
- In landscape models, the dominant contribution to correlators of local observables such as the C_l comes from the direction(s) in field space where the condition for eternal inflation holds at the lowest value of the potential.
- Other directions give non-Gaussian corrections.

No Signal

VGA-1