Title: Extracting the three- and four-graviton vertices from binary pulsars and coalescing binaries

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Abstract: Using a formulation of the post-Newtonian expansion in terms of Feynman graphs, we discuss how various tests of General Relativity (GR) can be translated into measurement of the three- and four-graviton vertices. The timing of the Hulse-Taylor binary pulsar provides a bound on the deviation of the three-graviton vertex from the GR prediction at the 0.1% level. For coalescing binaries at interferometers, because of degeneracies with other parameters in the template such as mass and spin, the effects of modified three- and four-graviton vertices at the level of the restricted PN approximation, is to induce an error in the determination of these parameters and it is not possible to use coalescing binaries for constraining deviations of the vertices from the GR prediction.

Gravitational waves from coalescing binaries & fundamental physics

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PI @ Waterloo, June 29th, 2010

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Outline

1. Gravitational wave theory
   - Linearized Einstein equations
   - GW interaction with light/matter

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   - Natural detectors
   - Man-made detectors
     - Working principle
     - Status
     - Prospects

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4. Coalescing binaries
   - Rates
   - Source modelling
     - Non-spinning waveforms
     - Spinning waveforms
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Einstein equations and their linearization

Weak field approximation, approximatly Cartesian coor.:

$$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}, \qquad ||h_{\mu\nu}|| \ll 1$$

A gravity wave is the radiative, high frequency part of $h_{\mu\nu}$
Einstein equations + Lorentz gauge condition: $\partial^\mu \bar{h}_{\mu\nu} = 0$:

$$\Box \bar{h}_{\mu\nu} = 16\pi G_N T_{\mu\nu}\,, \qquad \bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^\alpha_\alpha$$

Diffeomorphism invariance:

$$h_{\mu\nu} \to h_{\mu\nu} + \xi_{\mu,\nu} + \xi_{\nu,\mu}$$

reduced (not completely fixed) to $\Box \xi^\mu = 0$:
4 f's of 4 variables $\to$ 8 f's of 3 variables

The Transverse-Traceless gauge

$h(\bar{h})_{\mu\nu}$ includes

1. 4 gauge degrees of freedom
2. 2 physical, radiative degrees of freedom
3. 4 physical, non-radiative degrees of freedom

1&3 propagate with "the speed of thought" (Eddington '22)

After fixing the diffeomorphism invariance:

$$h_{\mu\nu} = \begin{pmatrix} -2\Phi & \Xi_i \\ \Xi_i & h^{TT}_{ij} + \theta\delta_{ij} \end{pmatrix}$$

$\partial_i \Xi^i = h^{TT}_{ij}\delta^{ij} = \partial^i h_{ij} = 0$: 6 degrees of freedom left, 4 eaten by gauge fixing

Einstein eq's:

$$\begin{aligned} \nabla^2\Phi = \nabla^2\Xi_i = \nabla^2\Theta &= 0 \\ \Box h^{TT}_{ij} &= 0 \end{aligned}$$

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GW interaction with point-particles

Physical distances are affected by GW's:

$$L = \int_0^{\bar{L}} dx \sqrt{1 + h_{xx}} \simeq \bar{L}\left(1 + \frac{1}{2} h_{xx}\right)$$

or by geodesic equation deviation

$$\delta \ddot{L}^i = R^i{}_{tjt} L^j = -\frac{1}{2} \ddot{h}_{ij}^{TT} L^j$$

$$\begin{array}{rrcl} \text{Light path :} & \delta\phi & = & 4\pi\delta L/\lambda \\ \text{EoM for test particle :} & \ddot{x}^i + \omega^2 x^i & = & -\frac{1}{2}\ddot{h}^i_j x^j \end{array}$$

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Localized source:

$$h_{ij}^{TT}(t, x) \simeq \frac{4 G_N}{|x|} \Lambda^{TT}_{ij;kl} \int d^3x' \, T_{ij}(t - |x - x'|) \sim \frac{G_N}{r} \ddot{Q}_{ij}$$

They carry energy

$$\frac{dE}{\ldots} = \frac{1}{\ldots} \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \qquad Flux = \frac{G_N}{\ldots} \langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle$$

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The Hulse-Taylor binary pulsar

GW's have been observed in the NS-NS binary system:

PSR B1913+16

Observation of orbital parameters ($a_p \sin\iota$, e, P, $\dot{\theta}$, γ, $\dot{P}$)

$\downarrow$

determination of m_p, m_c (1PN physics, GR)

Energy dissipation in GW's $\rightarrow \dot{P}^{(GR)}(m_p, m_c, P, e)$, compared with $\dot{P}^{(obs)}$

$$\frac{1}{2\pi}\phi = \int_0^T \frac{1}{P(t)}\,dt \simeq \frac{T}{P_0} - \frac{\dot{P}_0}{P_0^2}\frac{T^2}{2}$$

Weisberg and Taylor (2004)

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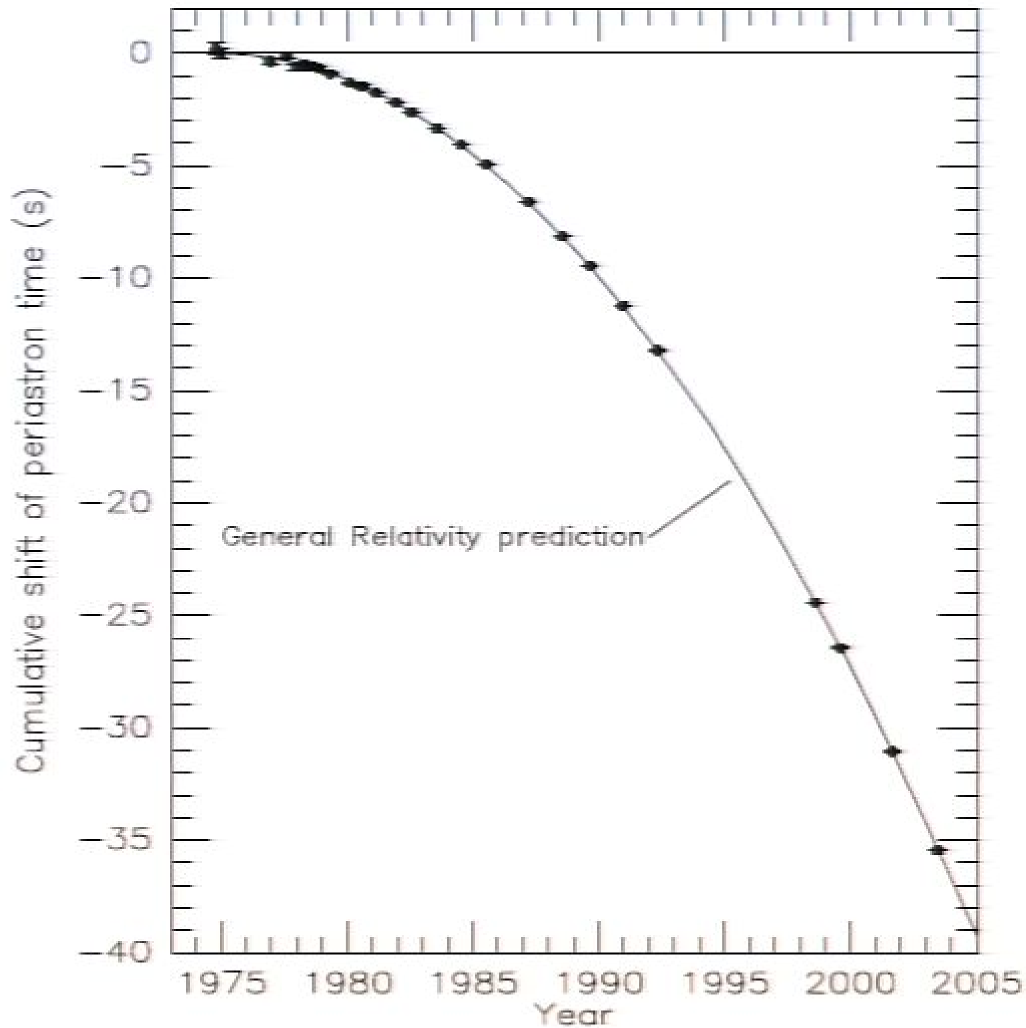
$$\frac{\dot{P}_{GR}-\dot{P}_{exp}}{\dot{P}} \sim 10^{-3}$$



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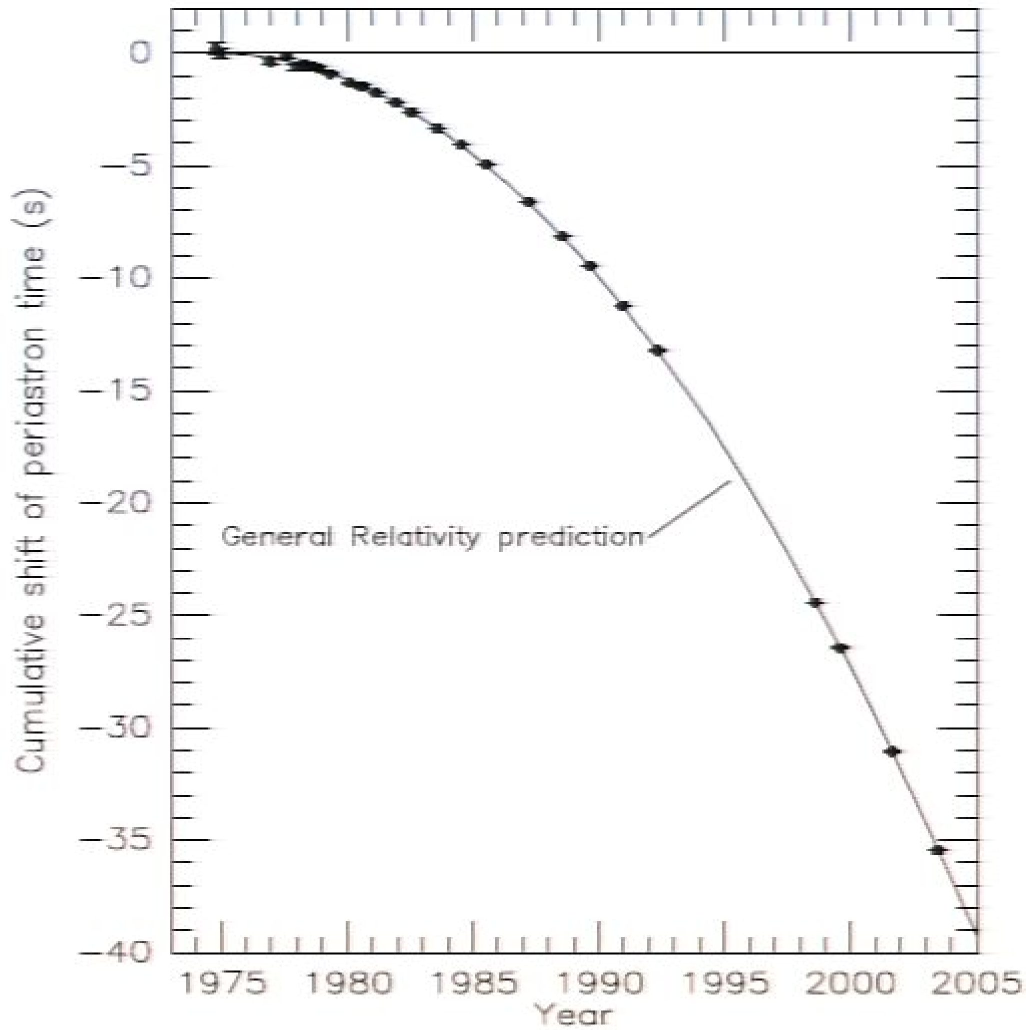
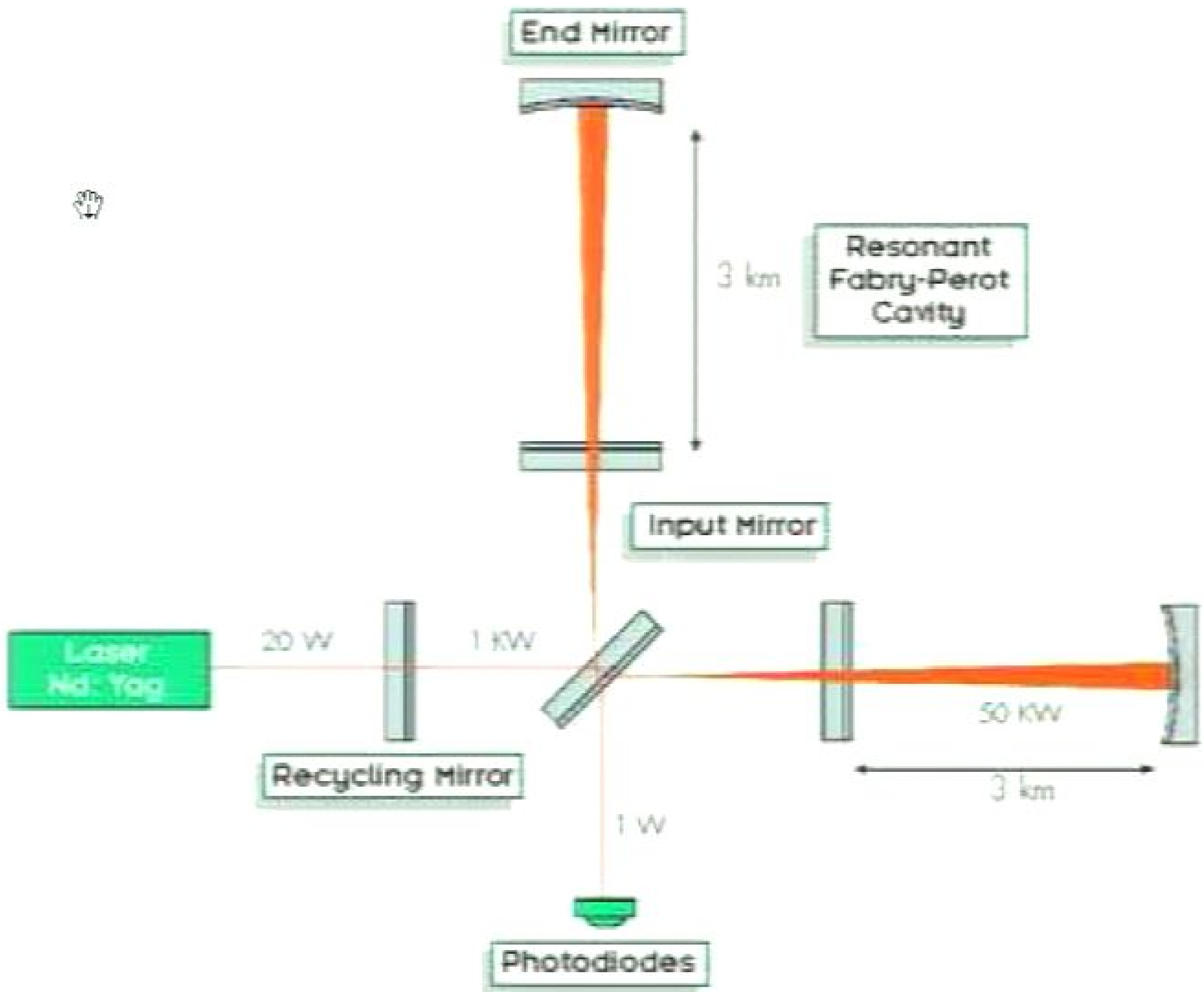


10 pulsars in NS-NS, still $\sim$ 100Myr for coalescence

Outline

Large interferometers



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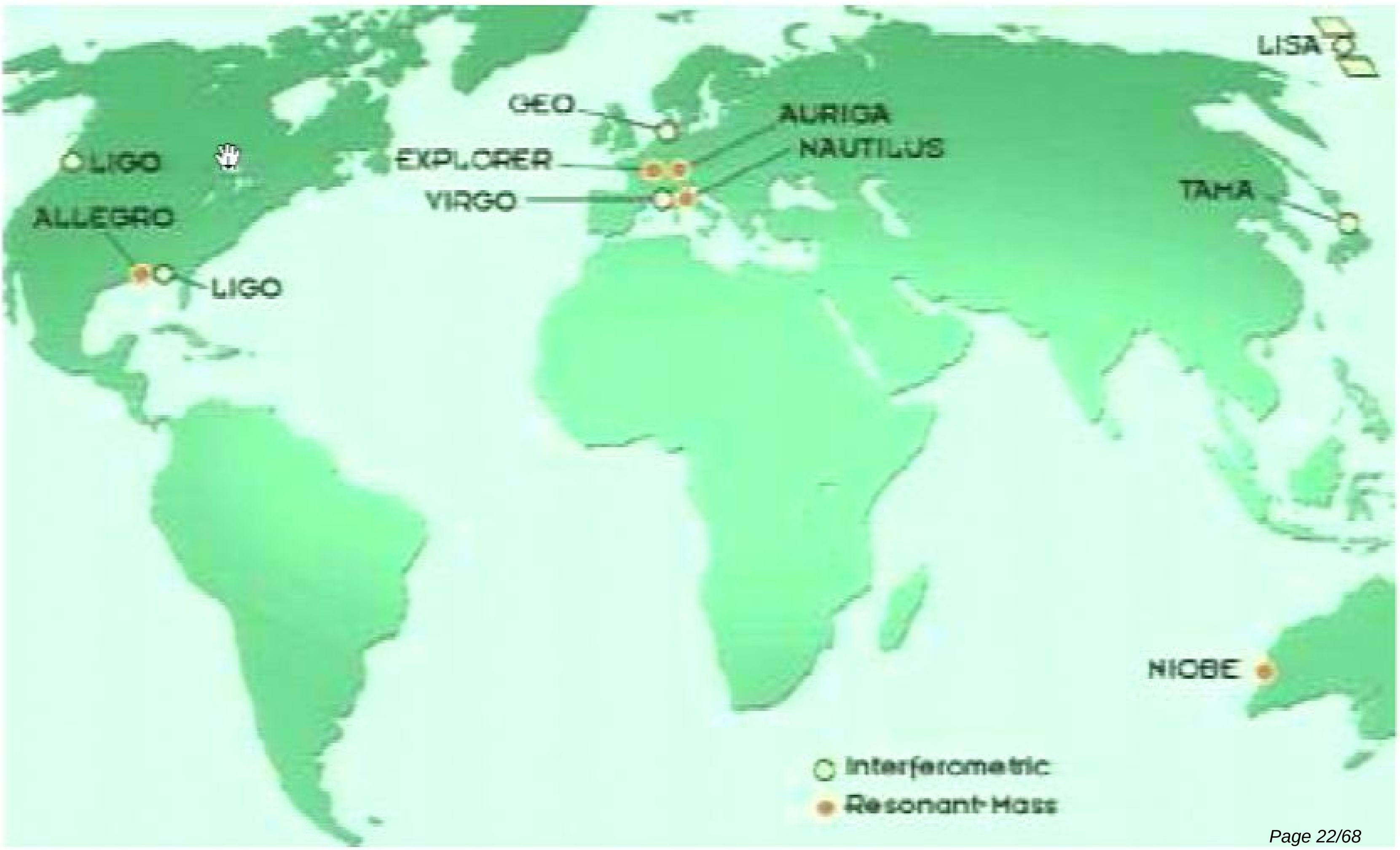
Detector Network



Sensitivity

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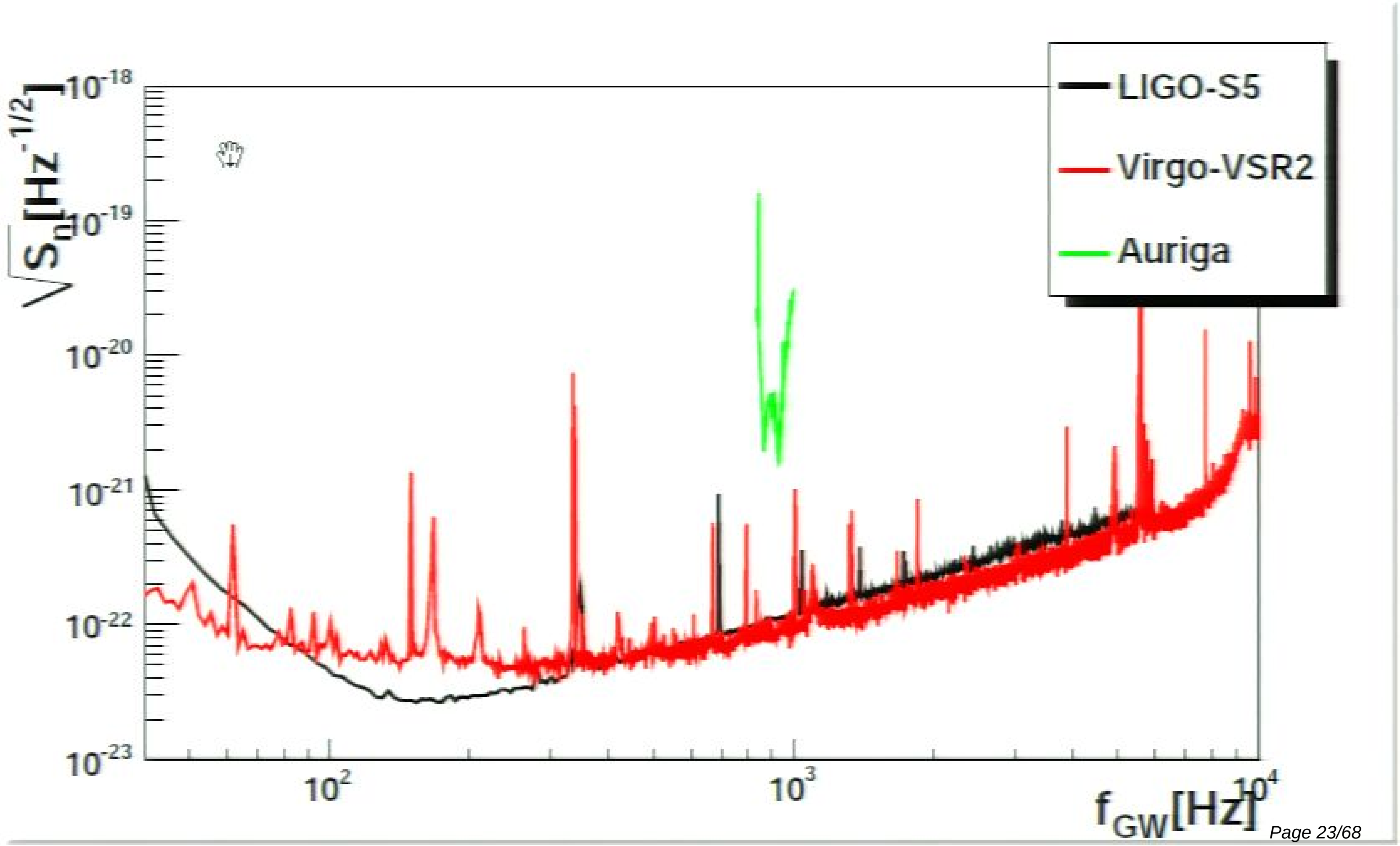

Interferometers & resonant antennas

Interferometers:

- 2 LIGO's (Hanford, Livingston 4Km)
- Virgo (Cascina, 3Km)

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- CLIO (Kamioka mine, 100m)

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Cylindrical antennas

- AURIGA (Legnaro, 3m)
- NAUTILUS (Frascati, 3m) EXPLORER (CERN, 3m)

Interferometers & resonant antennas

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Spheres

- miniGRAIL (Leiden, sphere ⌀ 68 cm)
- Mario Schenberg (S. Paulo)

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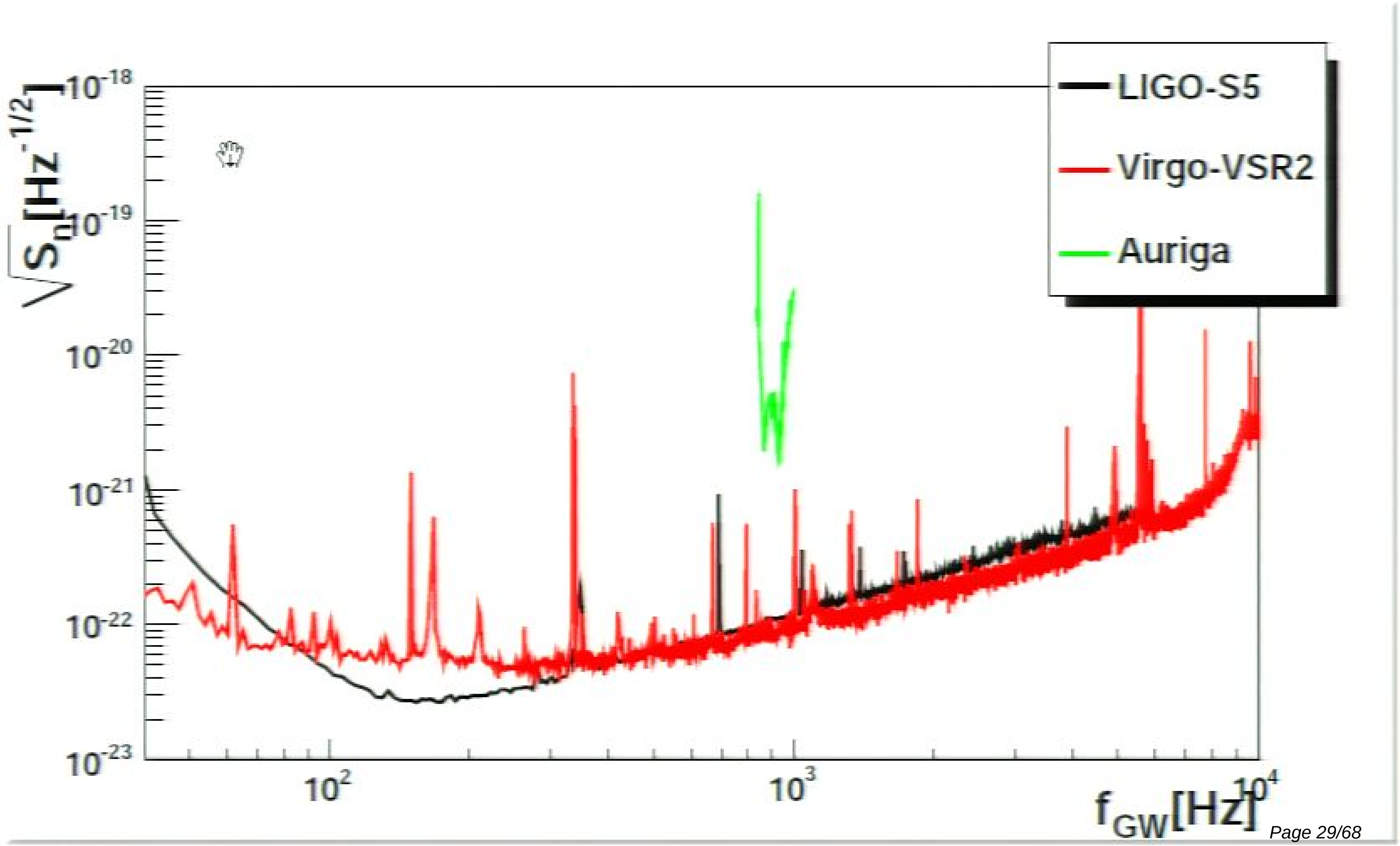

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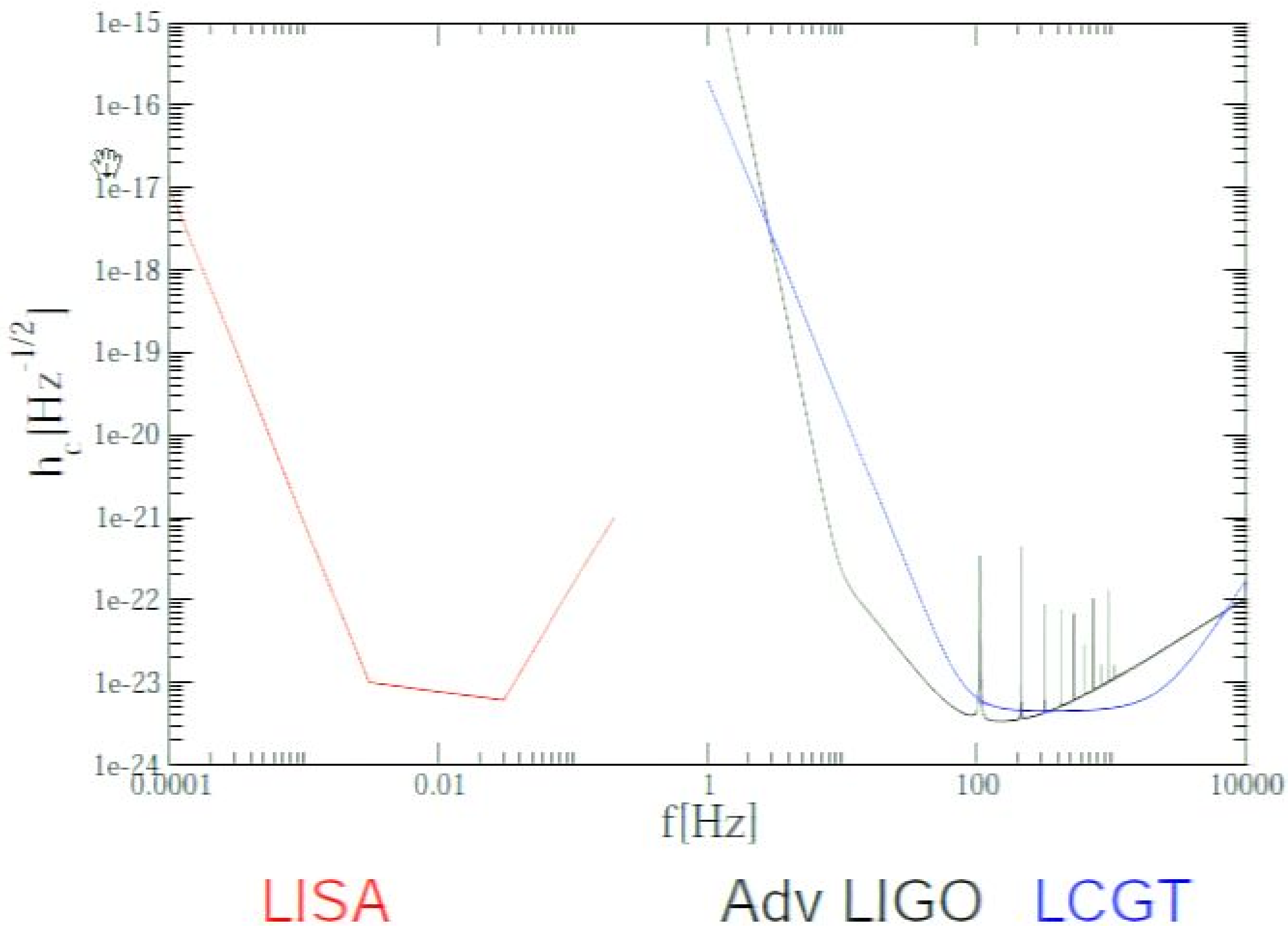
Prospects

LIGO+/Virgo present run started on July 9th 2009
Virgo is now off, since January the 7th for hardware upgrade
Joint LIGO/Virgo run starting in late July 2010

end of joint run due in fall 2010

GEO600 and (possibly) bar detectors will be operative in 2011-2014
LIGO/Virgo Advanced: from 2014-2015
LISA (>2018, pathfinder due in 2011/2012)
LCGT in $\sim$ 10 years, first 3 years funded last week!
AIGO project for a large interferometer in Australia $\sim$ 10 yrs

Sensitivity of future detectors



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Data analysis techniques in GW interferometry

An experimental apparatus output: time series

$$s(t) = h(t) + n(t) \qquad h(t) = D^{ij} h_{ij}(t)$$

Noise is conveniently characterized by its spectral function

$$\langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \delta(f - f') S_n(f) \qquad [Hz^{-1}]$$

Matched filter enhances the sensitivity

$$\frac{1}{T}\int_0^T s(t)h(t)\,dt = \frac{1}{T}\int_0^T h^2(t)\,dt + \frac{1}{T}\int_0^T n(t)h(t)\,dt \sim$$

$$h_0^2 + \sqrt{\frac{\tau_0}{T}} n_0 h_0$$

4 kind of searches

Unmatched:

- burst: search for excess noise (SN explosions, NS oscillations, compact object merging . . .)

Matched:

- stochastic: excess in the cross-correlation (cosmological and astrophysical background)
- periodic: matched filtering with (modulated) sine/cosine (rotating NS)
- coalescing binaries: emit all throughout coalescence, if a filter is matched, source parameters as well as details of gravitational dynamic are uncovered
  Signal-to-noise-ratio measured by

$$SNR^2 = \int \frac{f|\tilde{h}(f)|^2}{S_n} d\ln f$$

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How many coalescences can LIGO/Virgo see?

LIGO S5 (ended in Sep 2007) could have seen a pair of $1.4 M_\odot$ NS @ $r \sim 30$ Mpc
LIGO/Virgo Advanced

	NS-NS	$10\ M_\odot$ BH-BH
Distance (Mpc)	300Mpc	1GPc
Rates $\mathrm{MWEG}^{-1}\mathrm{Myear}^{-1}$	$1 \div 10^3$	$4 \cdot 10^{-2} \div 100$

$$N = 0.011 \times \frac{4}{3}\pi \left(\frac{D_H}{2.26 Mpc}\right)^3 \mathrm{MWEG}$$

Best case:

$$r_{NS-NS} \sim 400 \mathrm{yr}^{-1} \qquad r_{BH-BH} \sim 10^3 \mathrm{yr}^{-1}$$

I. Mandell et al. PRD 2010

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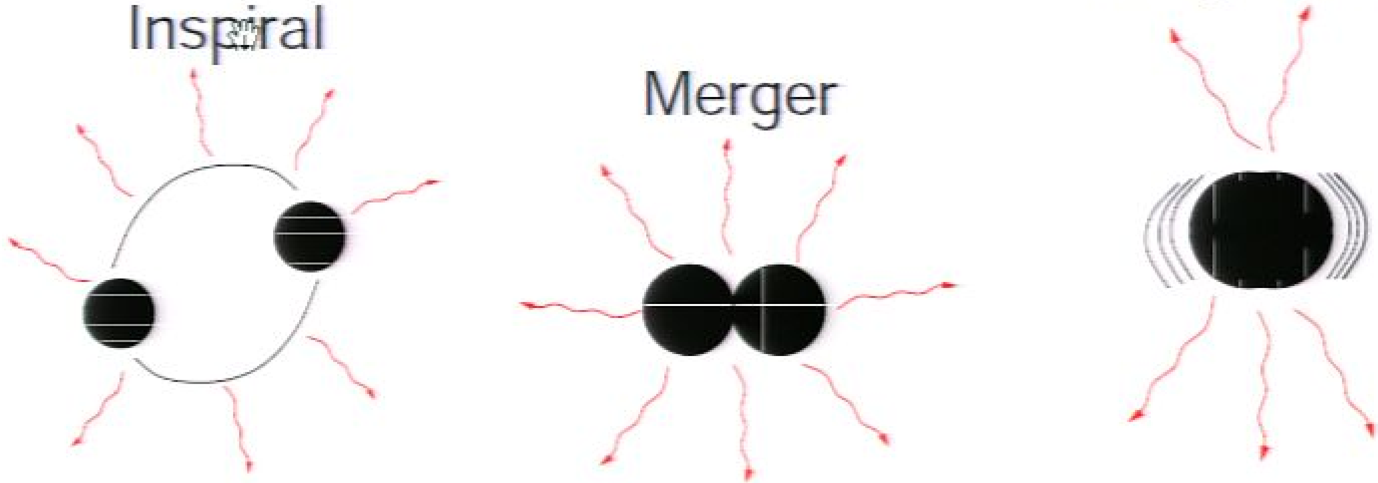
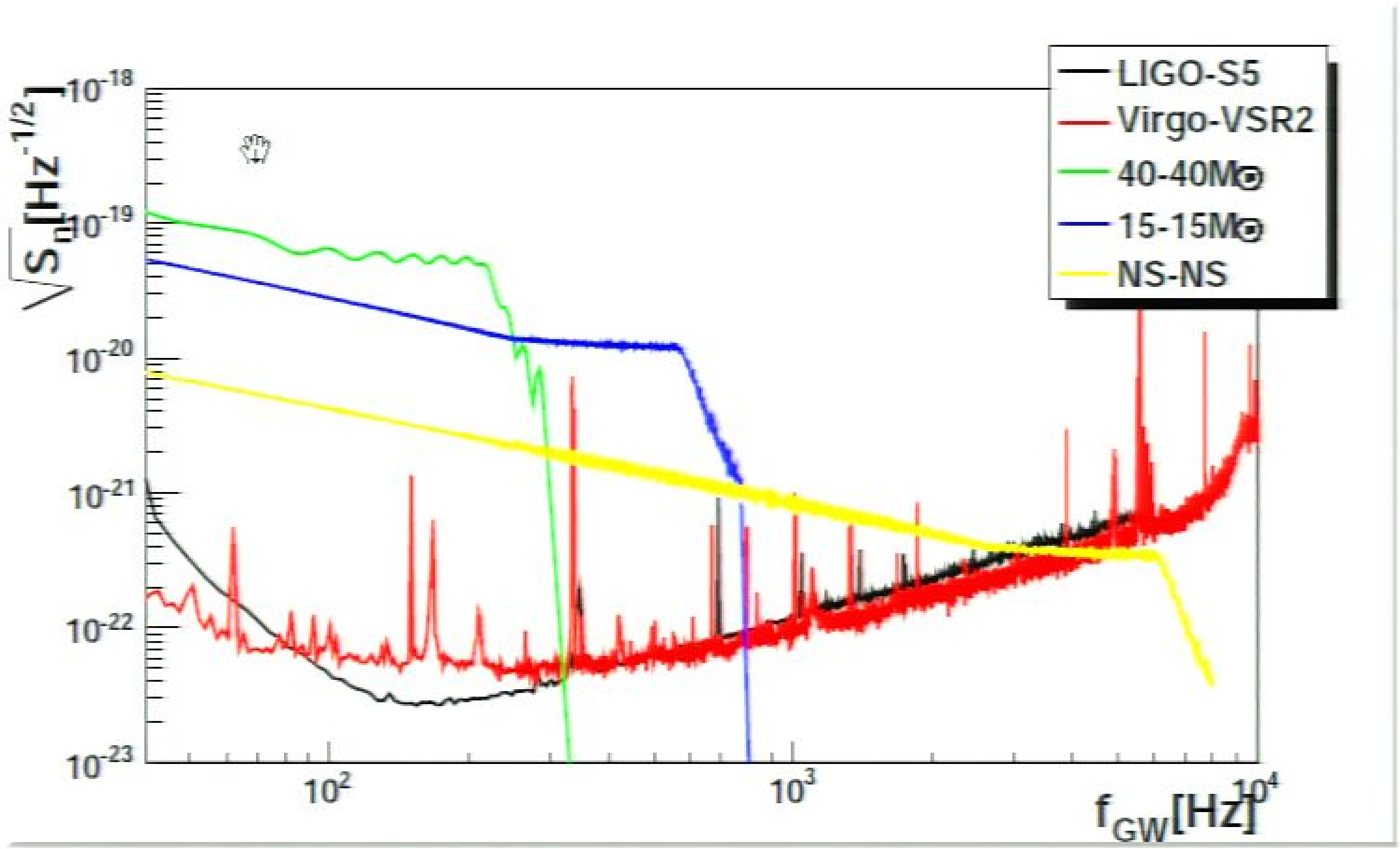
Signal templates

Inspiral	Merger	Ring-down
Perturbative PN-series	Non Perturbative	Expansion in pseudo-normal modes



Sensitivity to binary inspiral



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Binary system & PN corrections: spinless

Inspiral

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$E(v) = -\tfrac{1}{2}\nu M v^2 \left(1 + \#(\nu)v^2 + \#(\nu)v^4 + \dots\right)$$

$$P(v) \equiv -\frac{dE}{dt} = \tfrac{32}{5G_N} v^{10} \left(1 + \#(\nu)v^2 + \#(\nu)v^3 + \dots\right)$$

E(v)(P(v)) known up to 3(3.5)PN, see Damour, Blanchet ...

$$\frac{1}{2\pi}\phi(T) = \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)} dv$$

$$\sim \int \left(1 + \#(\nu)v^2 + \dots + \#(\nu)v^6 + \dots\right)\frac{dv}{v^6}$$

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Perturbatively described phases

- Inspiral $N_{cycles} \simeq 1.6 \cdot 10^4 \left(\frac{10\text{Hz}}{f_{min}}\right)^{5/3} \left(\frac{1.2 M_\odot}{M_c}\right)^{5/3}$
  Sensitivity $\propto M_c^{5/3}\sqrt{N_{cycles}} \propto M_c^{5/6}$, $f_{Max} \propto M^{-1}$

$$\text{ChirpMass } M_c \equiv M\left[\frac{m_1 m_2}{(m_1+m_2)^2}\right]^{3/5} = \nu^{3/5} M$$

- Ring-down

$$h(t) = \sum_{lmn} e^{-\tau_{lmn}(M,S)} \times \left[A\cos(\omega_{lmn}(M,S)t) + B\sin(\omega_{lmn}(M,S)t)\right]$$

Binary system & PN corrections: spinless

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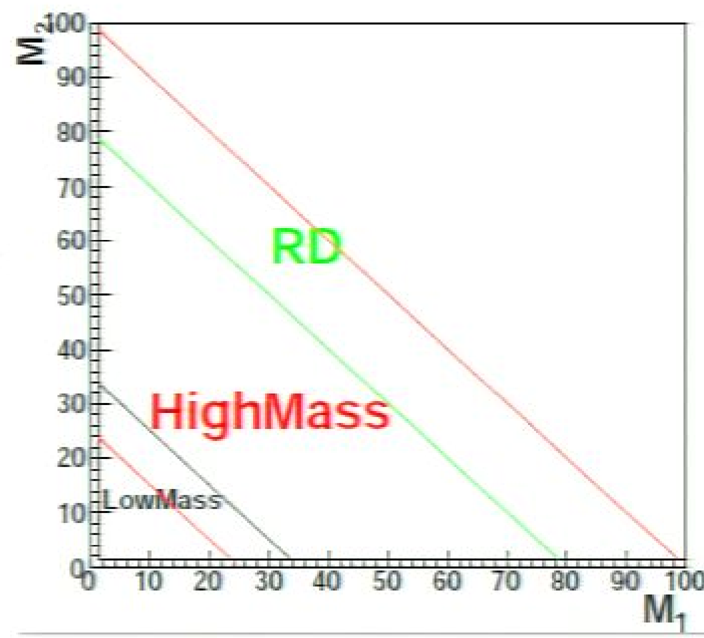
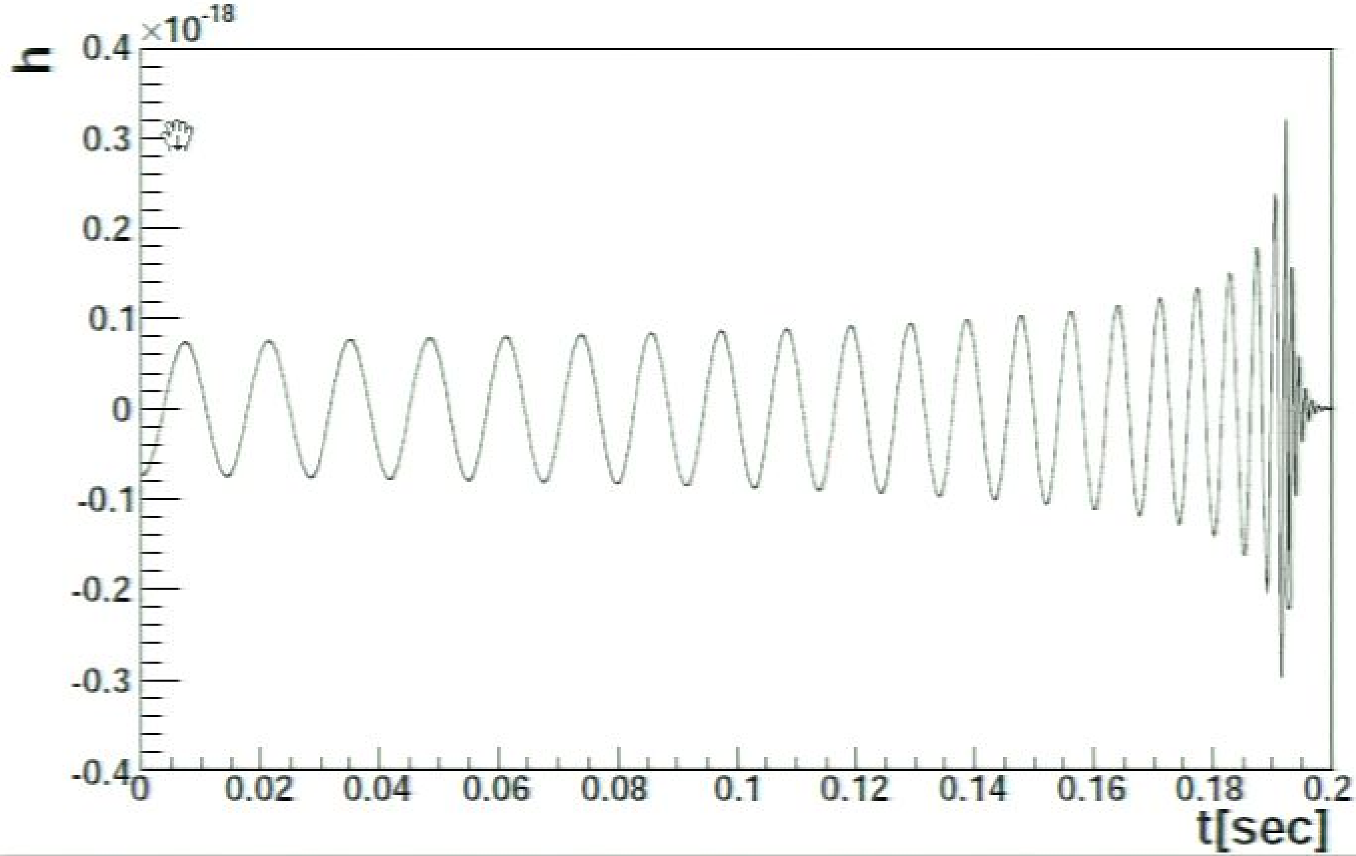
$$[A \cos(\omega_{lmn}(M,S)t) + B \sin(\omega_{lmn}(M,S)t)]$$

Matched filtering and templates

- Inspiral only
  $2.8 < M/M_{\odot} < 35$
- Inspiral+Merger+ RingDown
  $25 < M/M_{\odot} < 100$,
  EOBNR non-perturbative template banks, calibrated on PN inspiral and numerically generated wf's
- Ring down only
  $80 < M/M_{\odot} < 500$



Example of an EOBNR waveform



Complete waveform (spin-less)

Spinning binary system & PN corrections

Inspiral

$$\frac{d\phi}{dv} \propto \frac{1}{v^6}\left[1 + \#(\nu)v^2 + \#(\nu, \mathbf{L}\cdot\mathbf{S}_{1,2})v^3 + \#(\nu, \mathbf{L}, \mathbf{S}_{1,2})v^4 + \ldots\right.$$

$$\frac{d\mathbf{L}}{dt} \propto \Omega(\nu, v, \mathbf{S}_{1,2}) \times \mathbf{L} \qquad \frac{d\mathbf{S}_{1,2}}{dt} \propto \Omega(\nu, v, \mathbf{L}, \mathbf{S}_{2,1}) \times \mathbf{S}_{1,2}$$

How to estimate binary's parameters?

template bank impractical → Bayesian inference methods:
15-dimensional parameter space sampling →
posterior probabilities and posterior density functions

$$\delta m/m \sim 10^{-4} \div 10^{-5}$$
$$\delta|\mathbf{S}|/|\mathbf{S}| \sim 10^{-1} \div 10^{-3} \qquad @\ SNR \sim 10^2 \div 10^3$$

Mock LISA data challenge, CQG 2010

Binary system & PN corrections: spinless

Inspiral

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$E(v) = -\tfrac{1}{2}\nu M v^2 \left(1 + \#(\nu)v^2 + \#(\nu)v^4 + \ldots\right)$$

$$P(v) \equiv -\frac{dE}{dt} = \frac{32}{5G_N} v^{10} \left(1 + \#(\nu)v^2 + \#(\nu)v^3 + \ldots\right)$$

E(v)(P(v)) known up to 3(3.5)PN, see Damour, Blanchet ...

$$\frac{1}{2\pi}\phi(T) = \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)}dv$$

$$\sim \int \left(1 + \#(\nu)v^2 + \ldots + \#(\nu)v^6 + \ldots\right)\frac{dv}{v^6}$$

Spinning binary system & PN corrections

Inspiral

$$\frac{d\phi}{dv} \propto \frac{1}{v^6}\left[1 + \#(\nu)v^2 + \#(\nu, \mathbf{L}\cdot\mathbf{S}_{1,2})v^3 + \#(\nu, \mathbf{L}, \mathbf{S}_{1,2})v^4 + \ldots\right.$$

$$\frac{d\mathbf{L}}{dt} \propto \Omega(\nu, v, \mathbf{S}_{1,2}) \times \mathbf{L} \qquad \frac{d\mathbf{S}_{1,2}}{dt} \propto \Omega(\nu, v, \mathbf{L}, \mathbf{S}_{2,1}) \times \mathbf{S}_{1,2}$$

How to estimate binary's parameters?

template bank impractical $\rightarrow$ Bayesian inference methods:
15-dimensional parameter space sampling $\rightarrow$
posterior probabilities and posterior density functions

$$\delta m/m \sim 10^{-4} \div 10^{-5}$$
$$\delta|\mathbf{S}|/|\mathbf{S}| \sim 10^{-1} \div 10^{-3} \qquad @\ SNR \sim 10^2 \div 10^3$$

Mock LISA data challenge, CQG 2010

Complete waveforms from spinnig binaries

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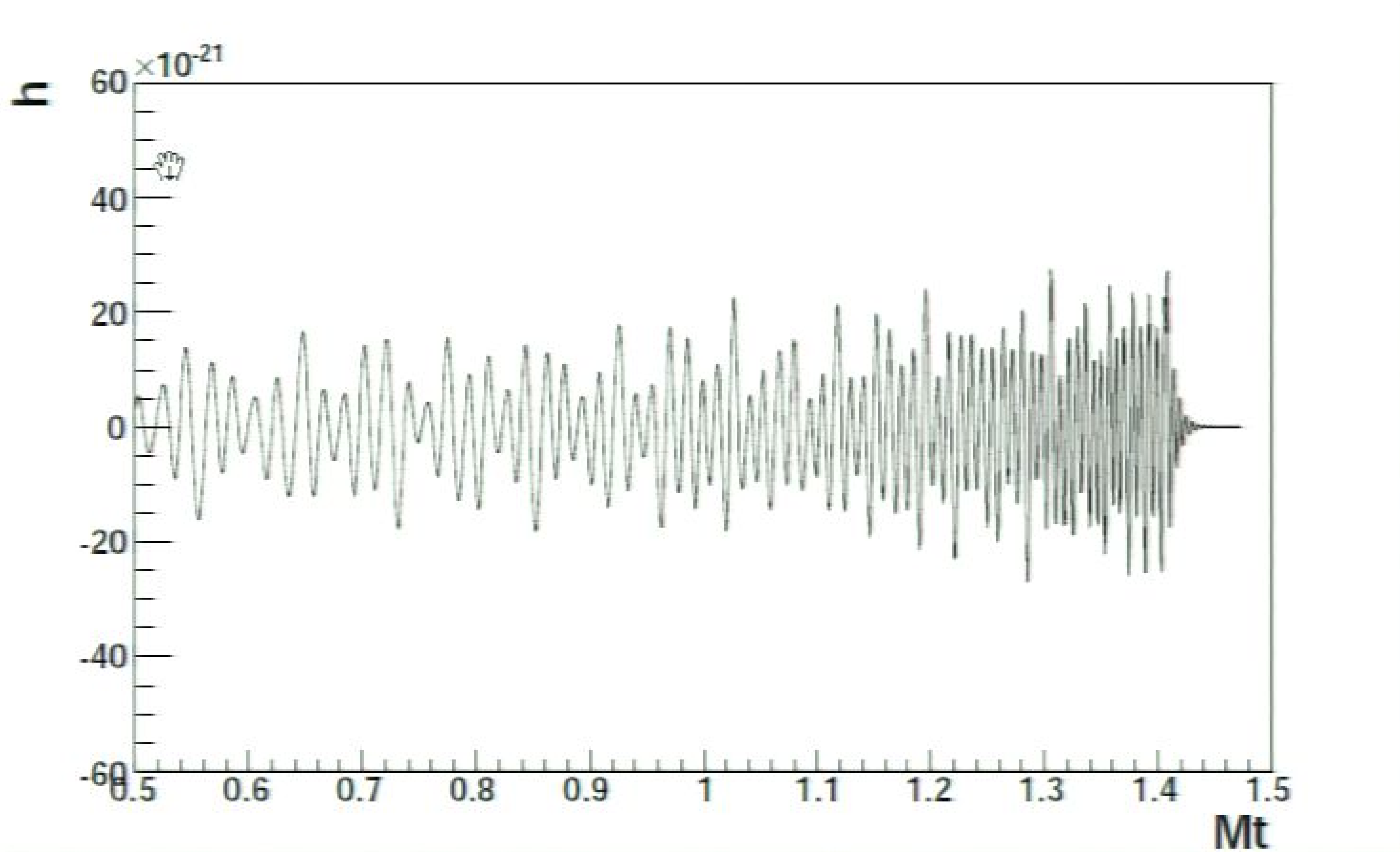


Amplitude modulation due to orbital plane precession

Outline

1. Gravitational wave theory
   - Linearized Einstein equations
   - GW interaction with light/matter
2. Gravitational wave detectors
   - Natural detectors
   - Man-made detectors
     - Working principle
     - Status
     - Prospects
3. Data Analysis
4. Coalescing binaries
   - Rates
   - Source modelling
     - Non-spinning waveforms
     - Spinning waveforms
   - Fundamental physics

Effective field theory of Gravity

PN expansion of the fundamental GR Lagrangean can be computed via EFT methods

Goldberger and Rothstein 2004, Chu 2008

Integrate out short-distance d.o.f. $\rightarrow$ coefficients of operators consistent with long-wavelength physics

- scaling arguments (dimensional analysis)
- Feynman diagrams
- clear separation of scales and regularization schemes
  - Very short, $r < r_S$, internal structure: 5PN-effect
  - Short distance $\rightarrow$ potential gravitons $k_\mu \sim (v/r, 1/r)$
  - Long wavelength $\rightarrow$ gravity waves $k_\mu \sim (v/r, v/r)$, background field

Internal structure

Fundamental coupling: $m \int d\tau$

Very short distance physics : eff. operators 2PN-correction to the potential:

$$c_R r_s^2 \int d\tau R + c_V r_s^2 \int d\tau R_{\mu\nu} \dot{x}^\nu \dot{x}^\nu$$

unphysical source bare-parameter redefinition (wiped away by coordinate re-definition)

First correction, 5PN (effacement principle, Damour '82):

$$c_e m r_s^4 \int d\tau R_{\mu\alpha\nu\beta} R^{\mu\alpha\nu\beta}$$

$$c_m m r_s^4 \int d\tau \epsilon^{\mu\nu\rho\sigma} R_{\mu\alpha\nu\beta} R_{\rho\ \sigma}^{\ \alpha\ \beta}$$

Integrating out potential gravitons

- Fundamental

$$
\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} + H_{\mu\nu} \\
S_{EH} &= -\frac{1}{32\pi G_N}\int d^4x\sqrt{g}\,R(h+H) \\
S_{pp} &\simeq -m\int dt\left(1 + \frac{h_{00}+H_{00}}{2} + (h_{0i}+H_{0i})\,v_i + \frac{(h_{ij}+H_{ij})v^i v^j}{2}\right)
\end{aligned}
$$

- Effective

$$
\begin{aligned}
S_{EH} &= -\frac{1}{32\pi G_N}\int d^4x\sqrt{g}\,R(h) \\
S_{pp} &= \int dt\left(\frac{1}{2}\sum_a m_a v_a^2 + \frac{G_N m_1 m_2}{r} + \dots\right) + \\
&\quad h_{00}\left(\frac{1}{2}\sum_a m_a v^2 - \frac{G_N m_1 m_2}{r}\right) + h_{0i}L_i + h_{ij}\ddot{Q}_{ij} + \dots
\end{aligned}
$$

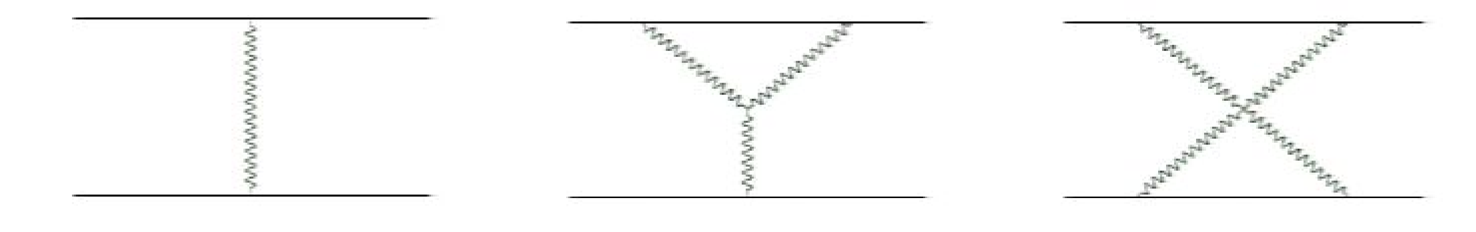
Conservative dynamics

Classical massive particles (neutron stars, black holes ...)

Scaling arguments: h-M Vertex: $\sim dt\, d^3k \frac{M_c}{M_{Pl}}$

Propagator: $\delta(t)\frac{1}{k^2}\delta^{(3)}(k)$

h^3 Vertex: $\frac{k^2}{M_{Pl}} dt (d^3k)^3 \delta^{(3)}(k) \ldots$

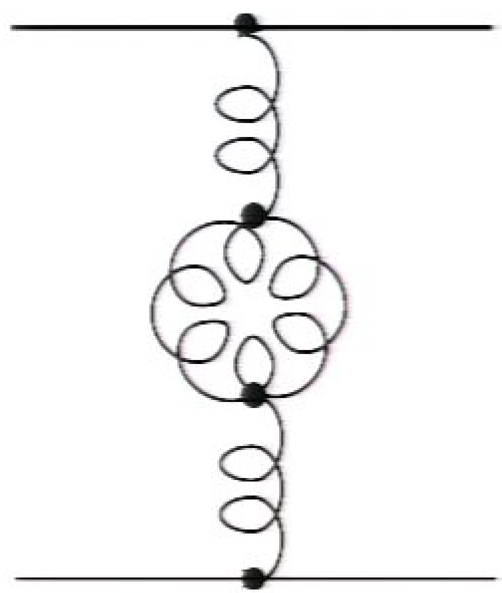


L $\quad$ Lv^2 $\quad$ Lv^4

$$V = -\frac{Gm_1 m_2}{r}\left[1 - \frac{r_s}{2r} + \frac{1}{4}\left(\frac{r_s}{r}\right)^2\left(1 - 2\nu + 5\nu^2\right)\right]$$

+v-dependent terms

Quantum corrections are irrelevant

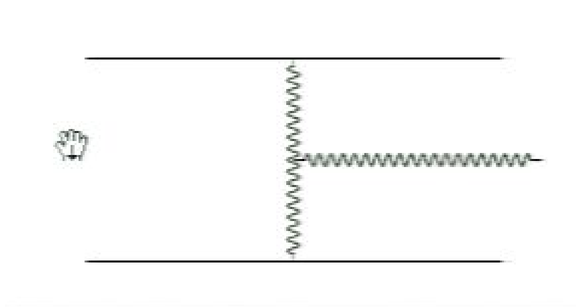


V^3

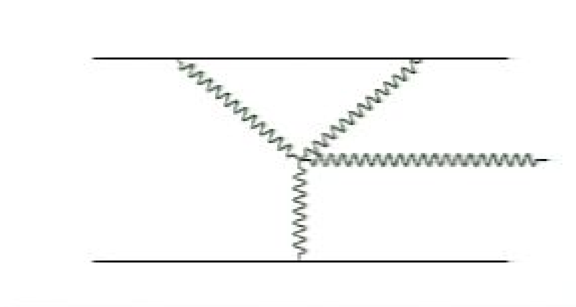
"Usual" rule for quantum weight $\hbar^{I-V} = \hbar^{L-1}$

Radiation diagrams

Long wavelength emitted gravitons



$0PN$

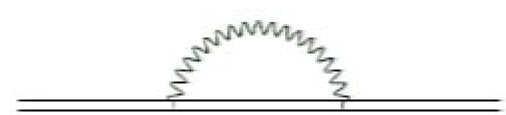


$1PN$

give rise to radiation coupling:

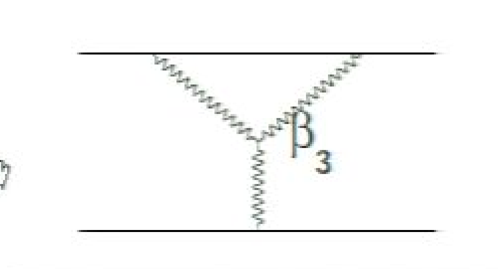
$$h_{ij}\left[\ddot{Q}_{ij}+\ldots\right]$$

Emitted power: Im

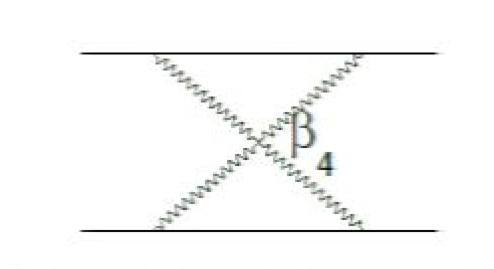


Graviton self-interaction vertices

- Conservative dynamics



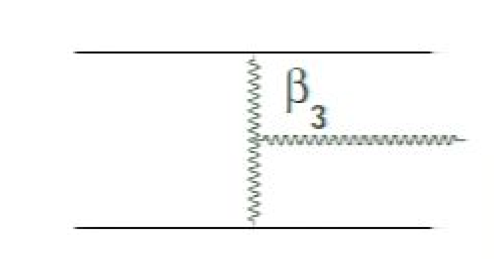
1*PN*



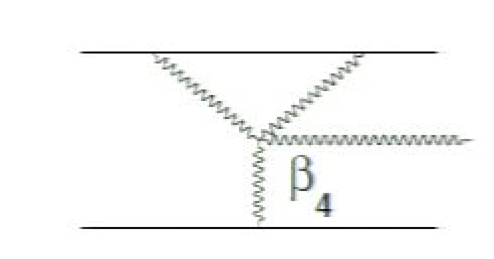
2*PN*

$$V \supset \beta_3 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2} + \beta_4 \frac{G_N^3 m_1^2 m_2^2}{r^3}$$

- Emission



0*PN*



1*PN*

Example of tagging of fundamental physics effects

$\beta_{3,4}$ is a tag, not a viable modification of General Relativity
Effect on the phase:

$$\phi \propto \left(\frac{|t-t_c|}{M_c}\right)^{5/8} \times \Big[1 - \frac{5}{2}\beta_3 + (a_1(\nu) + b_1(\beta_3,\beta_4,\nu))\, v^2$$
$$+ (a_2(\nu) + b_2(\beta_3,\beta_4,\beta_{LS}))\, v^3 \ldots\Big]$$

$\beta_{3,4}$ effect $\rightarrow$ can be reabsorbed by shifting M_c, ν (m_1, m_2)
at PN order ≥ 1.5 degeneracy with spin-dependent terms
Need for use of other harmonics than the fundamental one to constrain $\beta_{3,4}$

Constraints

At present: Hulse-Taylor Pulsar gives best constraint on non-conservative effect from β_3

$$\dot{P}_{\beta_3} = \dot{P}_{GR}(1 + c\beta_3) \qquad c \simeq 3.21$$

Given that $\frac{\dot{P}_{obs}}{\dot{P}_{GR}} - 1 \simeq 0.1\% \implies \beta_3 = (4.0 \pm 6.4) \cdot 10^{-4}$

β_3 already constrained by Lunar Laser Ranging, as @ 1PN

$$\beta_3 = \beta_{PPN} < 2 \cdot 10^{-4}$$

No interesting existing bound on β_4

Cannella et al. '09

Corrections to GR phase

Pick your effects:

- scalar-tensor theories (-1PN, 0PN, . . . effects, see Damour '94)
- modified dispersion relations (1PN effects, see Will '98)
- Chern-Simon terms for gravity (see Yunes '09)
- modified potential by extra dimensions
- . . .

Longer inspiral signals are favoured, inspiral detectable by LISA are best candidates

Coalescing binaries as standard sirens

LISA and/or ground network can localize the sources (triangolation)
Complementarity with astrophysics: distance vs. red-shift

$$h_c \simeq \frac{1}{D}(G_N M_c)^{5/3}(f_e)^{2/3}\cos(\phi(t_e/M_c,\nu)) \xrightarrow{t_r=t_e(1+z)}$$

$$\frac{1}{D_L}(G_N M_c(z))^{5/3}(f_r)^{2/3}\cos(\phi(t_r/M_c(z),\nu))$$

$$D_L \equiv D(1+z),\ M_c(z) \equiv M_c(1+z)$$

can measure the luminosity distance, complementarity with astrophysics: distance vs.red-shift

Standard sirens

Schutz '86, Holz & Hughes '08

Conclusions

GW detection will open a new window on the Universe:

- New way to observe old and new objects in the Universe
- Test of General Relativity, even though high SNR required

Coalescing binaries as standard sirens

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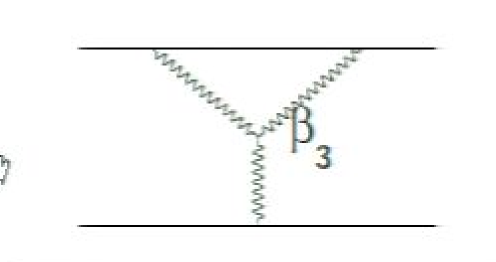
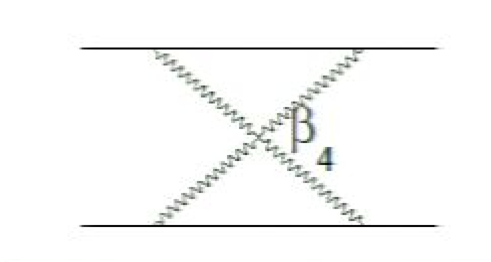
Graviton self-interaction vertices

- Conservative dynamics



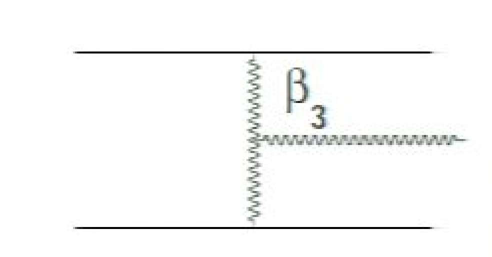
$1PN$



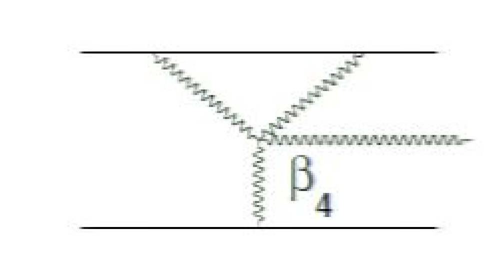
$2PN$

$$V \supset \beta_3 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2} + \beta_4 \frac{G_N^3 m_1^2 m_2^2}{r^3}$$

- Emission



$0PN$



$1PN$