Title: Probing the physical and astrophysical nature of black holes with gravitational waves

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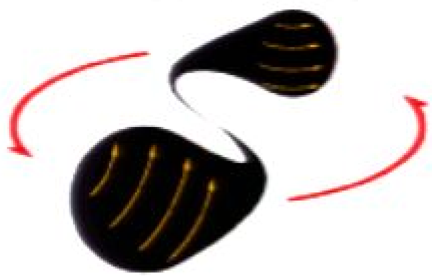
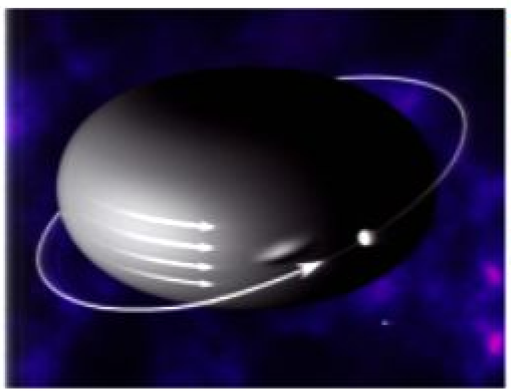
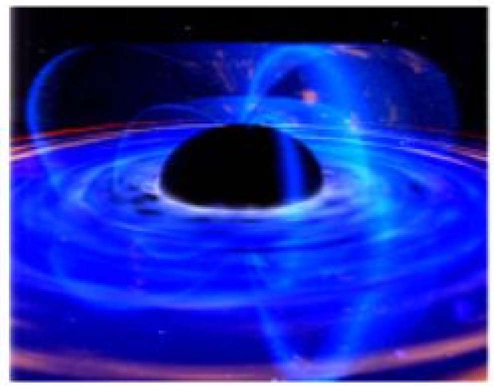
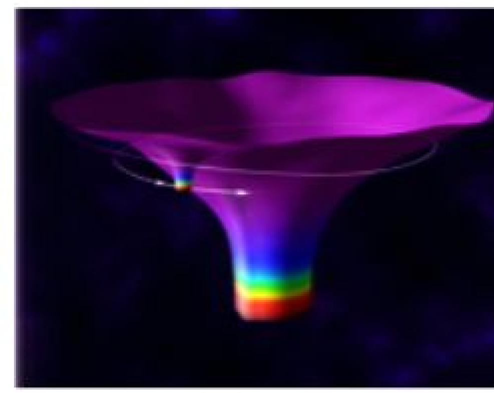
Abstract: Black holes play a central role in astrophysics and in physics more generally. Candidate black holes are nearly ubiquitous in nature. They are found in the cores of nearly all galaxies, and appear to have resided there since the earliest cosmic times. They are also found throughout the galactic disk as companions to massive stars. Though these objects are almost certainly black holes, their properties are not very well constrained. We know their masses (often with errors that are factors of a few), and we know that they are dense. In only a handful of cases do we have information about their spins. Gravitational-wave measurements will enable us to rectify this situation. Focusing largely on measurements with the planned space-based detector LISA, I will describe how gravitational-wave measurements will allow us to measure both the masses and spins of black holes with percent-level accuracy even to high redshift, allowing us to track their growth and evolution over cosmic time. I will also describe how a special class of sources will allow us to measure the multipolar structure of candidate black hole spacetimes. This will make it possible to test the no-hair theorem, and thereby to test the hypothesis that black hole candidates are in fact black holes are described by general relativity.

Probing the physical and astrophysical nature of black holes with gravitational waves









Using gravitational waves to learn about black holes in astrophysics and to test strong-field gravity

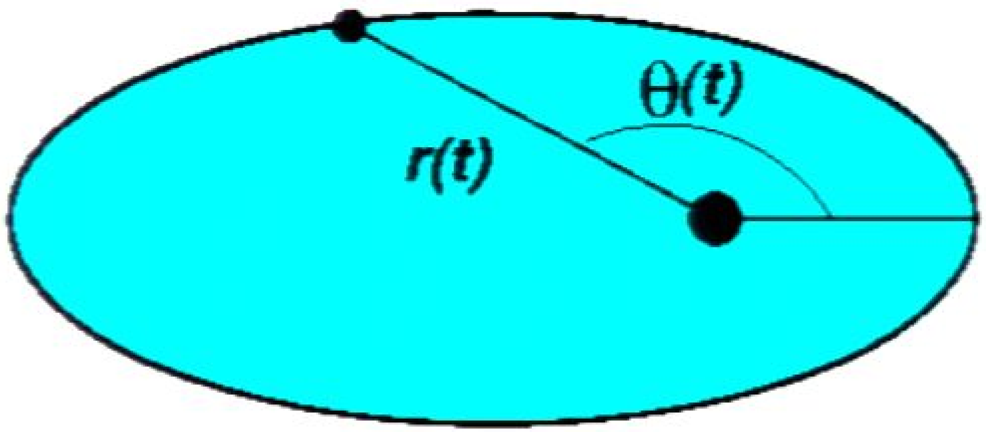
Current workshops

Capra: Workshop on the problem of motion around black holes (including self interaction) with applications to GW sources.

NRDA: Workshop on connecting numerical relativity binary models to analysis (ongoing & future) of data from GW detectors.

Unifying theme: Connection of two-body problem in GR to gravitational-wave generation, measurement ... and exploiting it to learn about GW sources.

2 bodies a la Newton



$$M = m_1 + m_2$$
$$\mu = m_1 m_2 / M$$
Choose energy E

Choose ang. mom. L

Define $$p = \frac{L^2}{G\mu^2 M}\,, \quad \epsilon = \sqrt{1 + \frac{2EL^2}{G^2\mu^2 M^3}}$$

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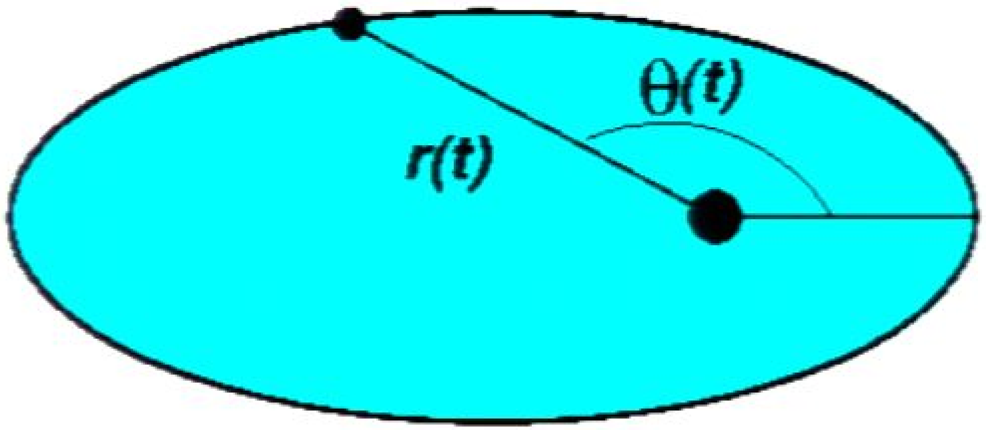

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$$r(\theta) = \frac{p}{1 + \epsilon\cos\theta}$$
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Complete solution fits on a single slide!

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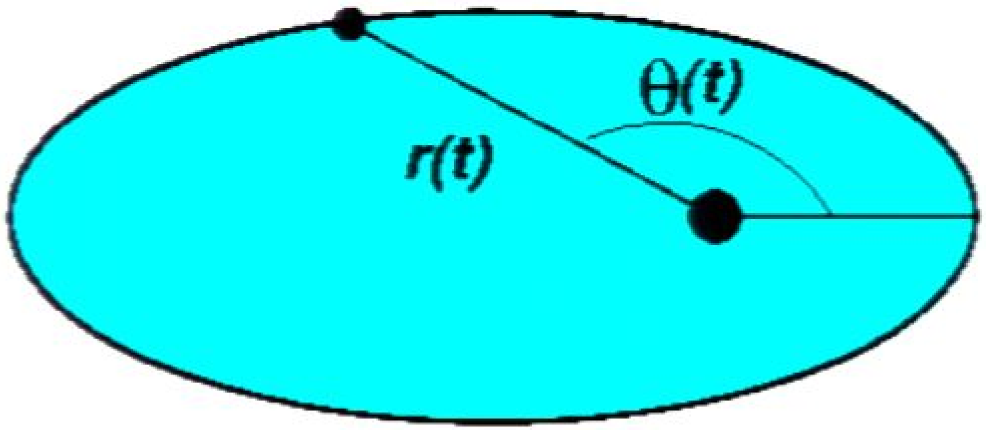

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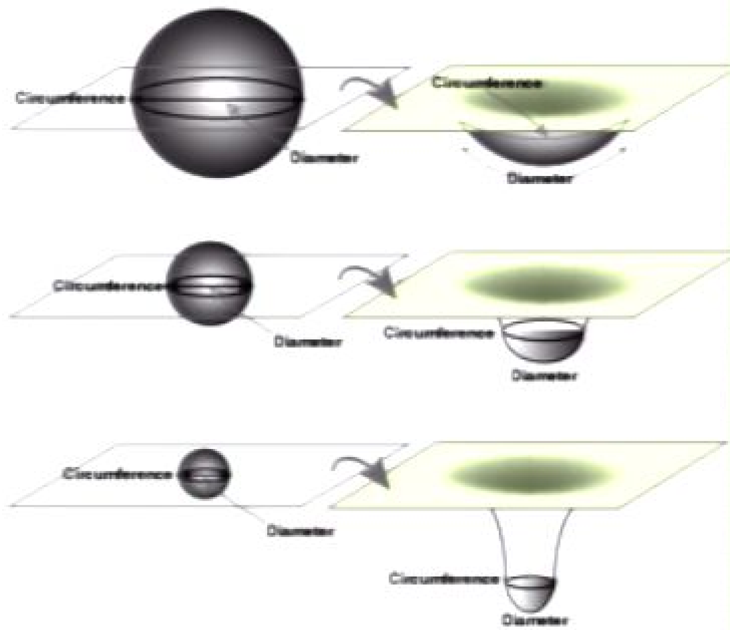
1. Build the spacetime of a large, gravitating object

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



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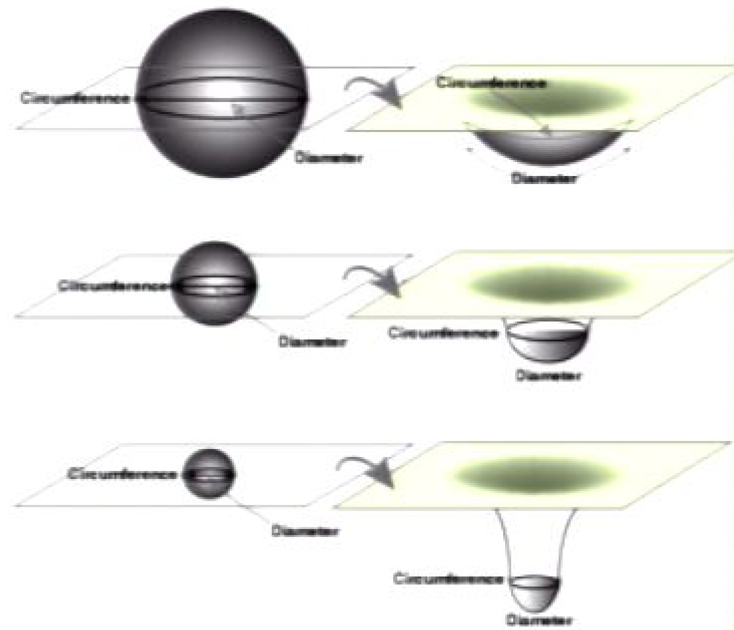


1. Build the spacetime of a large, gravitating object

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

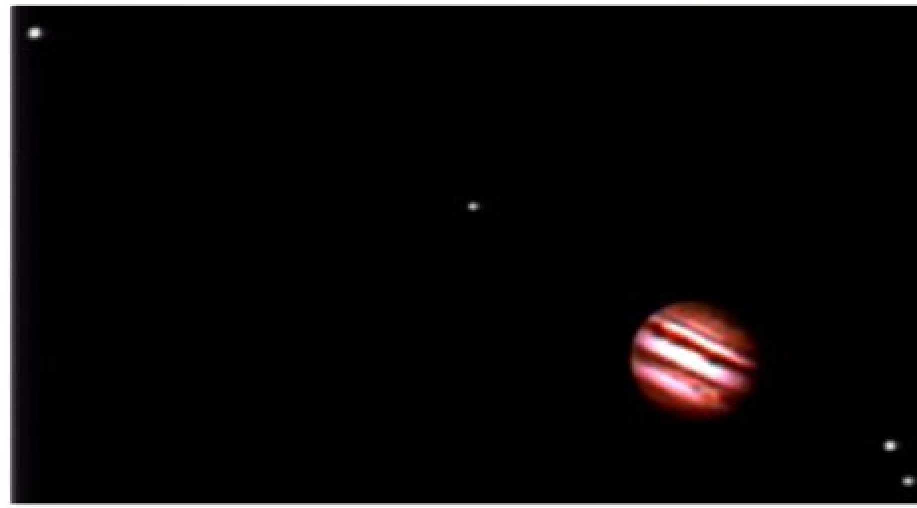
2. Freely-falling objects respond to the spacetime by following *geodesics*: Trajectories of extremal time as measured by the object.

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

Step 2: Simple

Computing geodesics:
One of the first exercises
a student learns in a
general relativity class.

Reproduces well-tested
aspects of Newton's gravity



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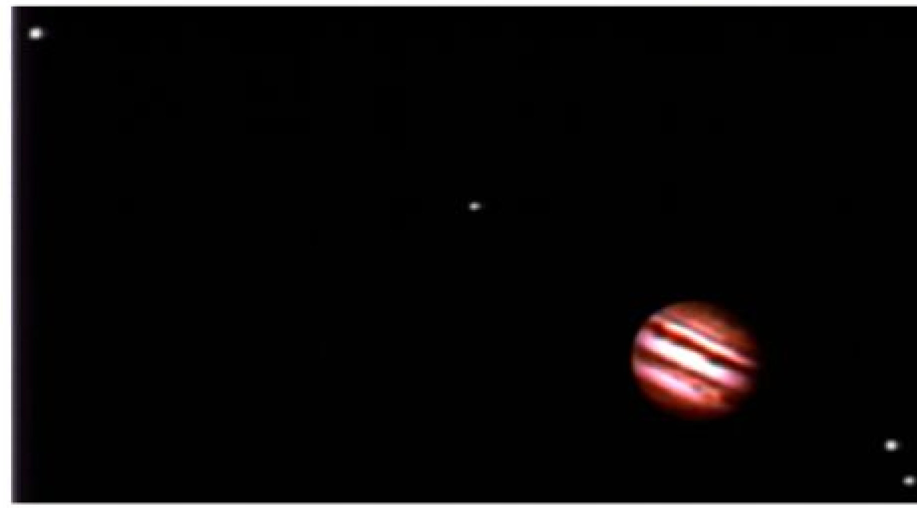

Introduces new features
which have passed all
tests to date.

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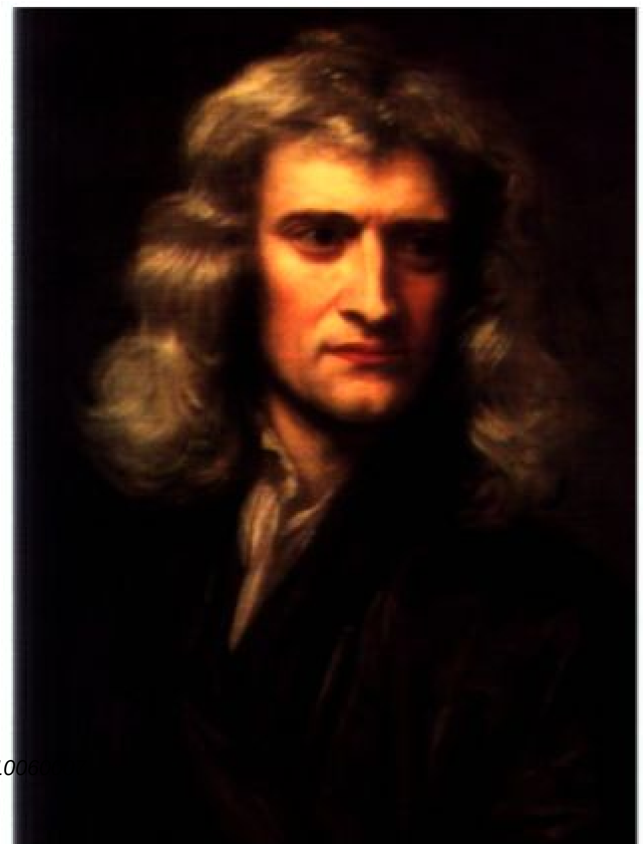

Linear relationship of potential and matter density in Newton's gravity: Simple to set boundary conditions, see how field varies as source varies.

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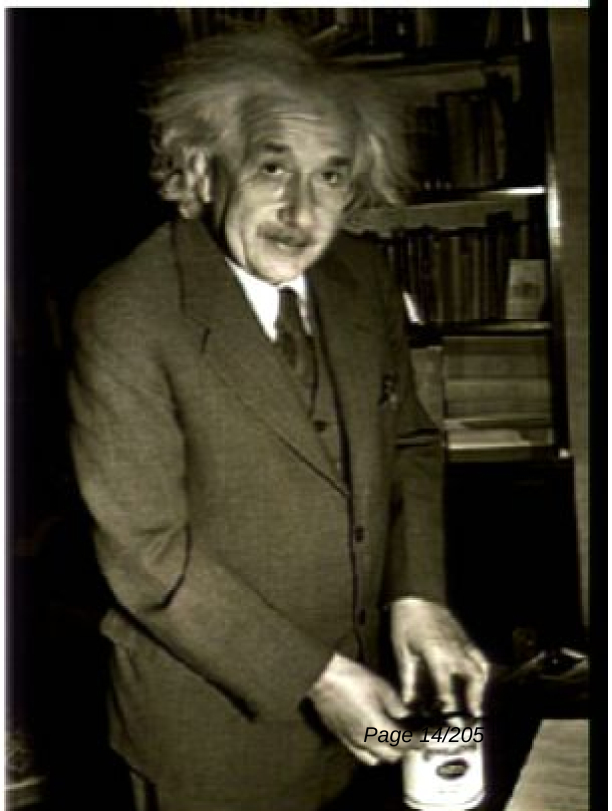
$$\nabla^2\phi = -4\pi G\rho_M \longrightarrow G_{\alpha\beta} = \frac{8\pi G}{c^4}T_{\alpha\beta}$$

$G_{\alpha\beta}$: *non*linear, coupled differential operator acting on the metric.

$T_{\alpha\beta}$: Stress-energy of source. Includes matter density, but also *flow* of energy and momentum in the spacetime.



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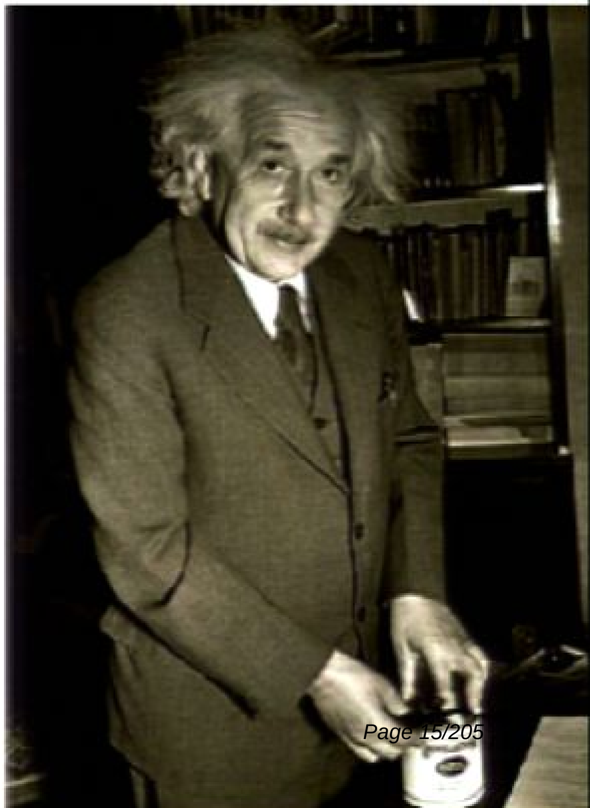

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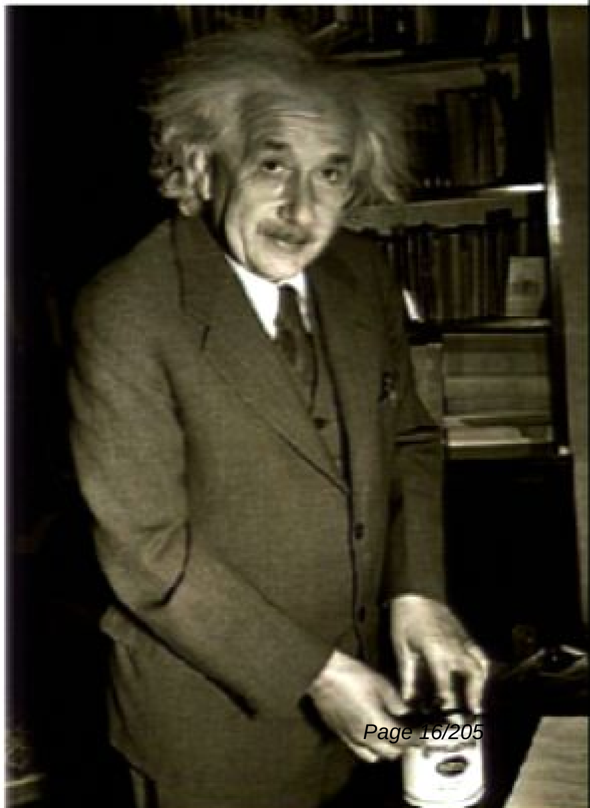
Nothing can travel faster than light ... including information about gravity.

*Radiation is **required***



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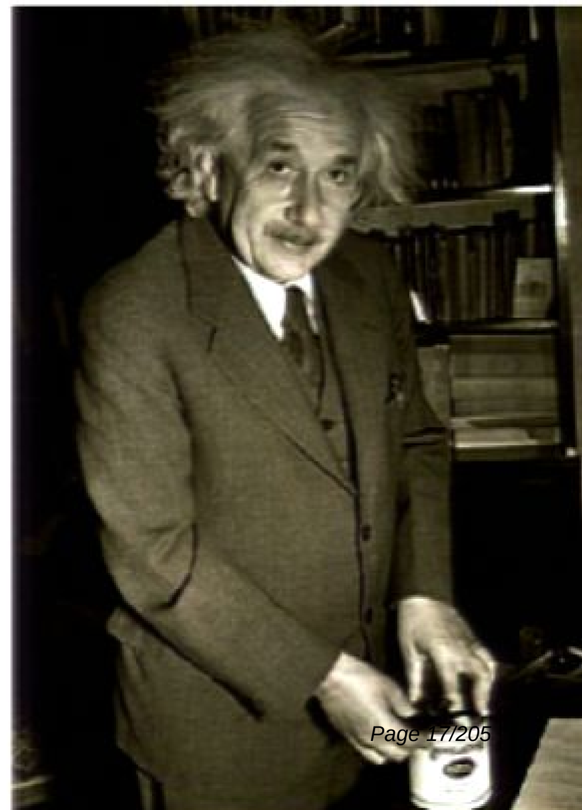

Newton vs Einstein: Different notions of gravity

Newton: Gravity is a force. All objects respond to a field and are accelerated. "Charge" setting the response is object's mass.

Apollo 15: Hammer and feather hit ground at the same time on moon.

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John Wheeler conducted a vote to determine the answer (1957): Split 50-50!

Even harder when both bodies "generate" spacetime

In examples so far, small body is a test object: It responds to spacetime, but does not bend it.

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Directly solve the Einstein field equations and infer two-body dynamics from the solution.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad \nabla^\mu G_{\mu\nu} = 0$$

Gravitational waves

Well known that general relativity has *radiative* spacetime solutions.

Typically start with linearized wave equation:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{becomes} \quad \Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

with $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad ||h_{\mu\nu}|| \ll 1$

and

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\left(\eta^{\alpha\gamma}\eta^{\beta\delta}h_{\gamma\delta}\right)$$

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Note: Fully ***non***linear wave equation well known (Penrose 1960).

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Character of waves

Linearized limit useful for characterizing the most important aspects of radiation.

Leading solution shows that radiation depends on variations of a source's *mass quadrupole*:

$$h_{ij} = \frac{2G}{c^4}\frac{1}{r}\frac{d^2Q_{ij}}{dt^2} \qquad Q_{ij} \simeq \int \rho\left(x_i x_j - \frac{1}{3}\delta_{ij}x^2\right) d^3x$$

Rough scale of radiation:

(where v is typical velocity of quadrupole variations.)

$$h_{ij} \sim \frac{G}{c^4}\frac{mv^2}{r}$$

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Very weak

Need large *m* and *v* to overcome gravity's intrinsic weakness.

$$h_{ij} \sim \frac{G}{c^4}\frac{m v^2}{r}$$

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Blessing:

Waves propagate from source to us with practically no absorption or scatter.

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Curse:

Waves barely interact with our detectors!

$$h_{ij} \sim \frac{G}{c^4}\frac{mv^2}{r}$$

Imprint of waves

Radiation is an oscillation in spacetime geometry.

In principle, measure it by bouncing light between freely falling mirrors [Bondi 1957]:

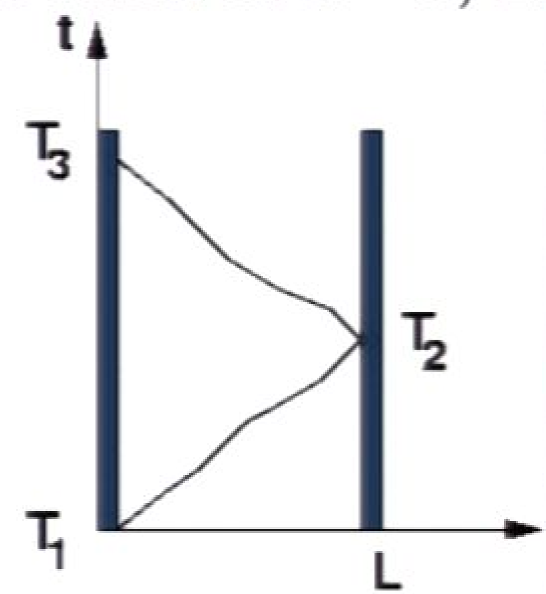
Light follows null geodesic in spacetime with wave:

$$ds^2 = 0 = -c^2 dt^2 + [1 + h(t,x)]\, dx^2$$

Coordinate velocity of light:

$$\frac{dx}{dt} = \frac{c}{\sqrt{1 + h(x,t)}}$$

Lagrangian coordinates: mirrors fixed at x = 0, L.



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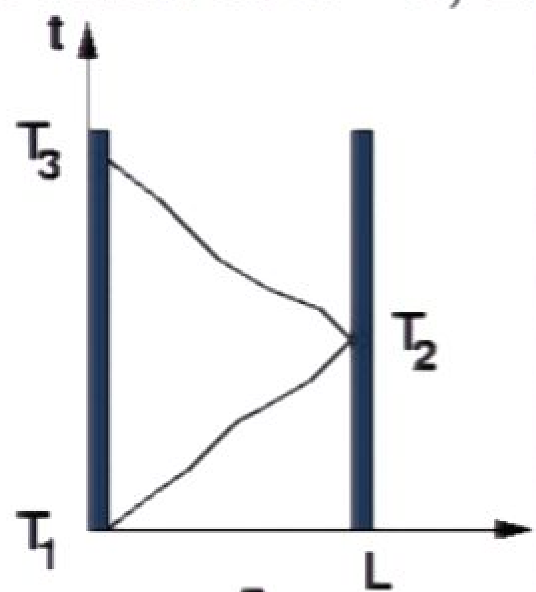

$$T_3 - T_1 = \int \frac{dx}{dx/dt} \simeq \frac{1}{c} \int \left[1 - \frac{1}{2} h(t,x)\right] dx$$

How much effect do we typically expect?

Estimate of the timing impact:

$$h \simeq \frac{G}{c^4}\frac{mv^2}{r} \simeq \frac{G}{c^4}\frac{2KE^{\rm ns}}{r}$$

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Results in the following *h* estimate:

$$h \simeq 10^{-21} - 10^{-22}$$

Indirect detection

Gravitational waves also carry energy and angular momentum away from a radiator, just like electromagnetic radiation:

$$\left(\frac{dE}{dt}\right)_{\mathrm{E\&M}} = \frac{1}{3}\frac{d^2 d_a}{dt^2}\frac{d^2 d^a}{dt^2}$$

$$\left(\frac{dE}{dt}\right)_{\mathrm{GW}} = \frac{1}{5}\frac{d^3 Q_{ab}}{dt^3}\frac{d^3 Q^{ab}}{dt^3}$$

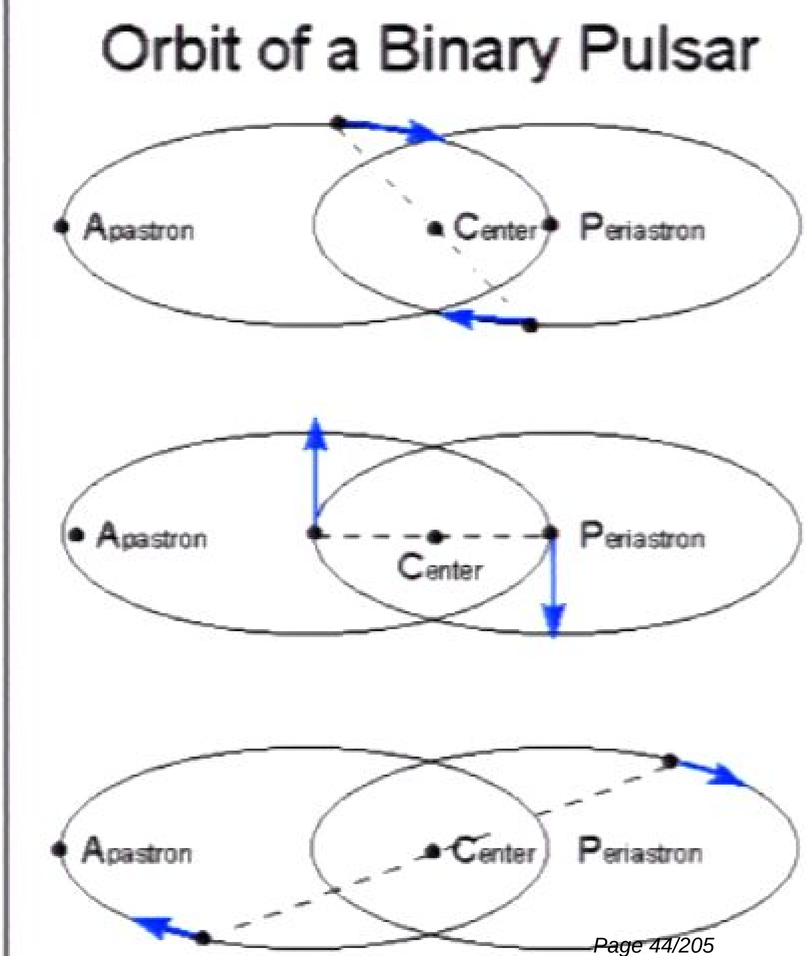
Perhaps we can find a system in which the effects of backreaction are apparent.

Binary pulsars: Laboratories for strong-field gravity

Binary systems in which both members are neutron stars — high mass, high density, *very* strong gravity.

One member is a pulsar: extremely precise clock.

Allows high precision timing measurements of orbital characteristics.

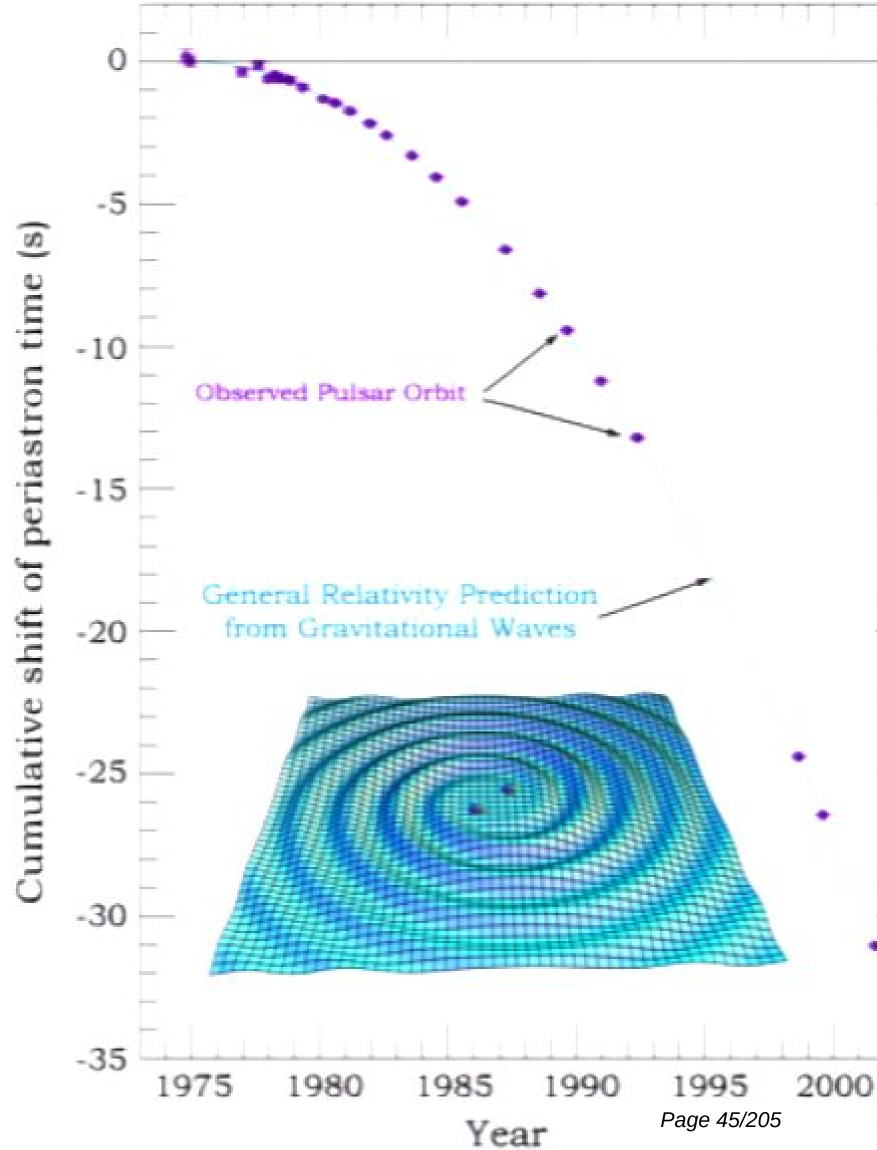


Hulse-Taylor binary

Discovery by Russell Hulse and Joe Taylor of exactly such a binary in 1975.

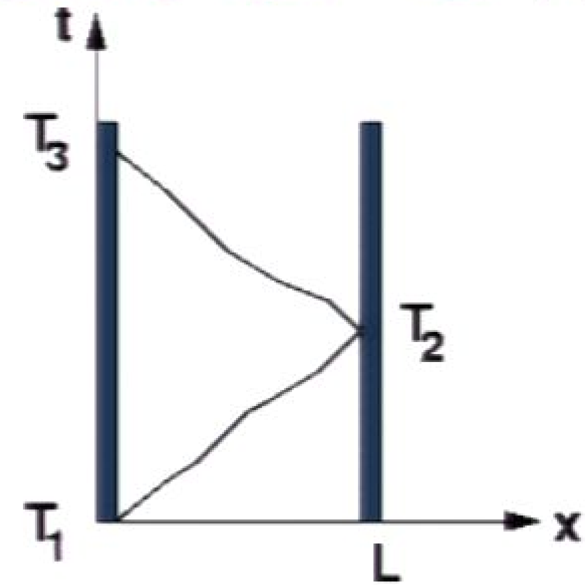
Detailed study over next decades showed that the orbit was losing energy …

… at ***exactly*** the rate predicted for gravitational wave emission!

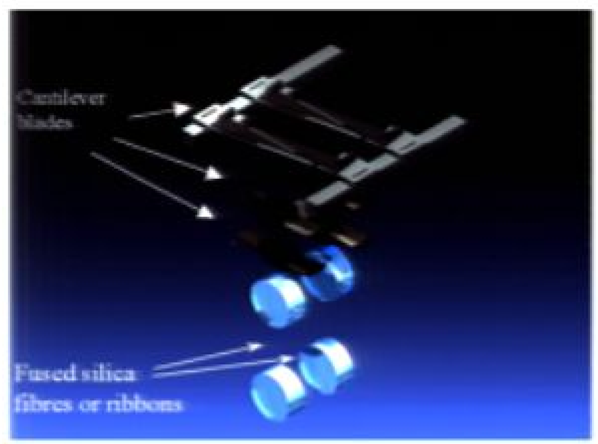


Direct measurement of GWs

Direct detection is based on implementing the principle of Bondi's freely falling mirrors:



On ground, free fall replaced by pendulum suspension: Roughly free fall for $f >> (g/l)^{1/2}$



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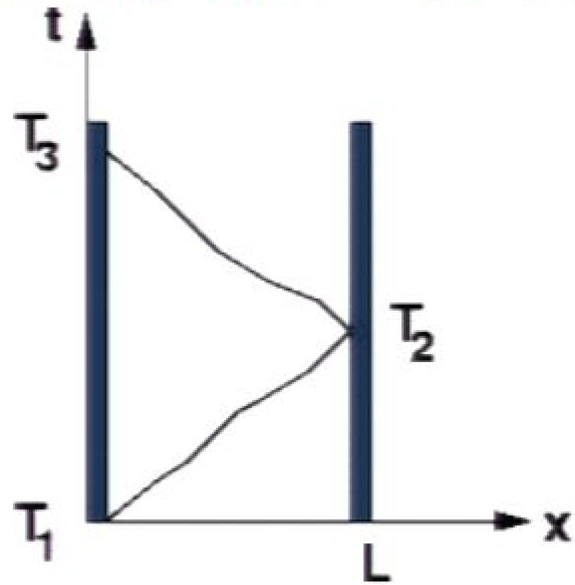
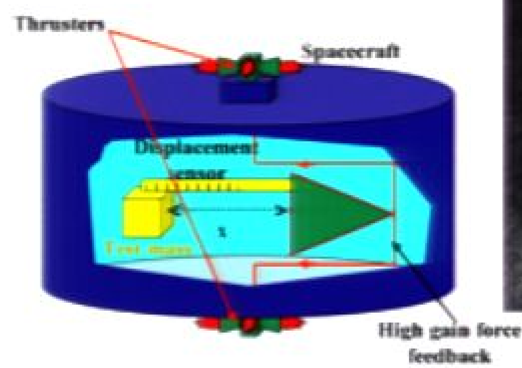
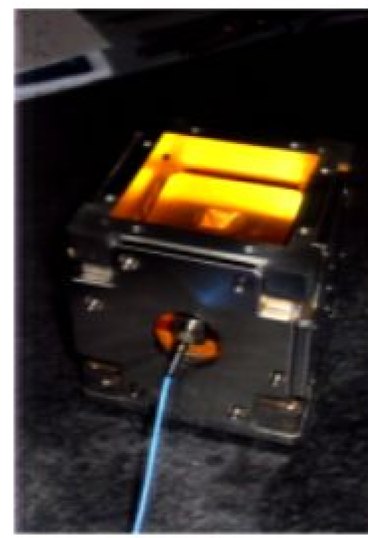


In space, free fall achieved in "drag free" spacecraft. Measure changes in rate of arrival of laser phase fronts to measure timing fluctuations.



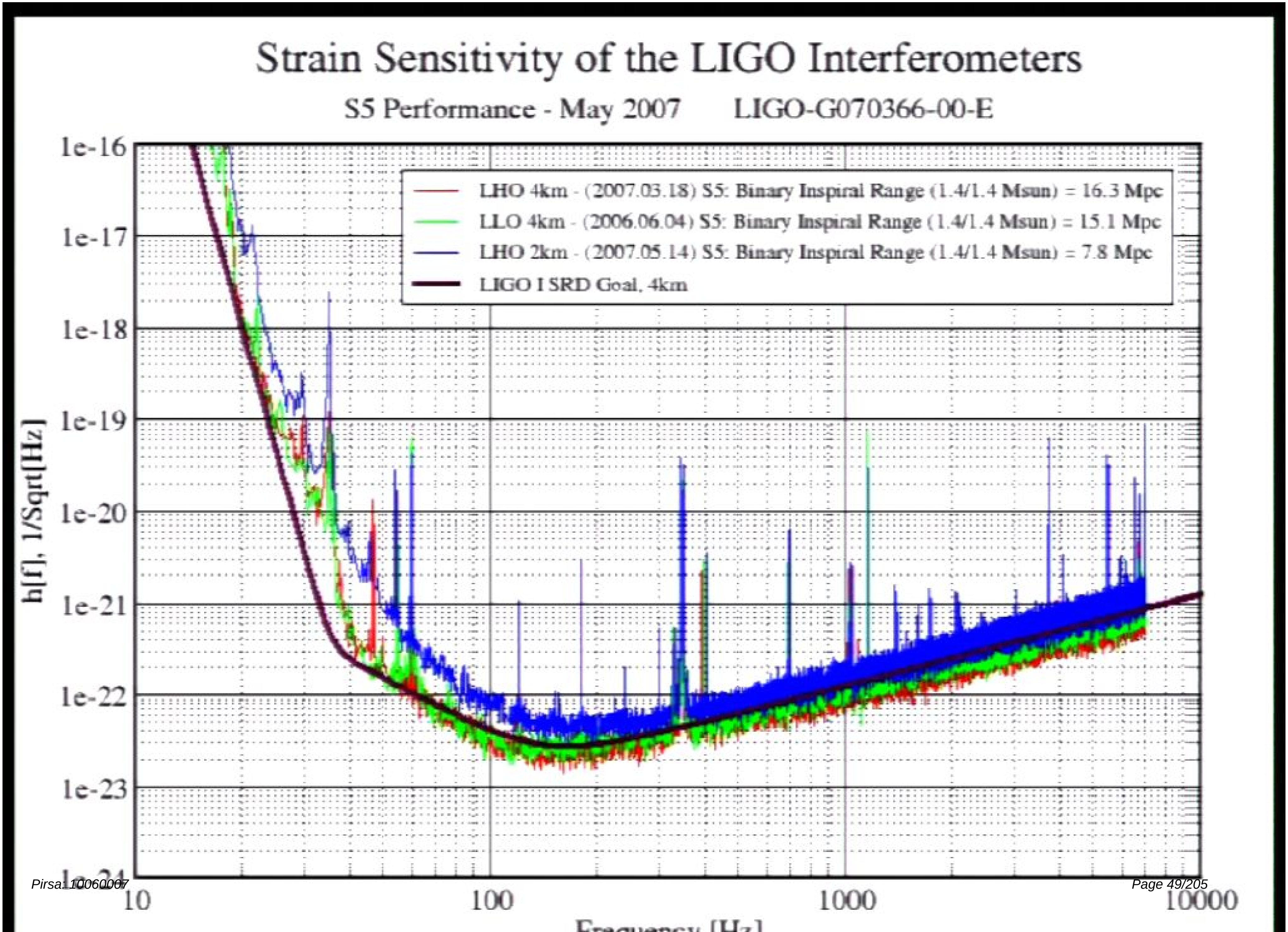


LIGO

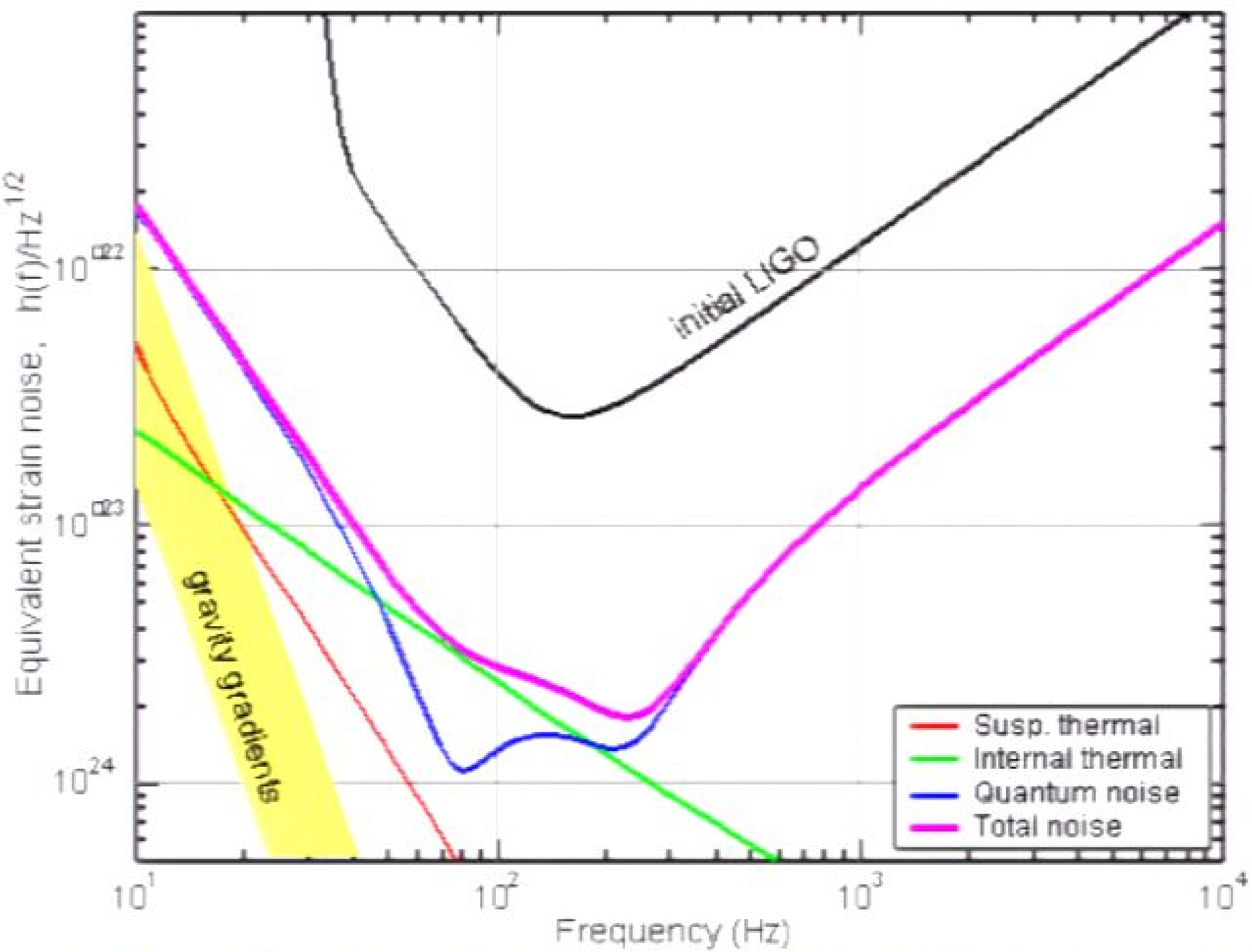
Two separated sites:
Hanford, WA (top) &
Livingston, LA (bottom).
4 kilometer long arms.
Sensitive in band
~10 Hz < f < (a few) kHz.

Currently operational!

Vigorous R&D for upgrade to "advanced" sensitivity in the near future.

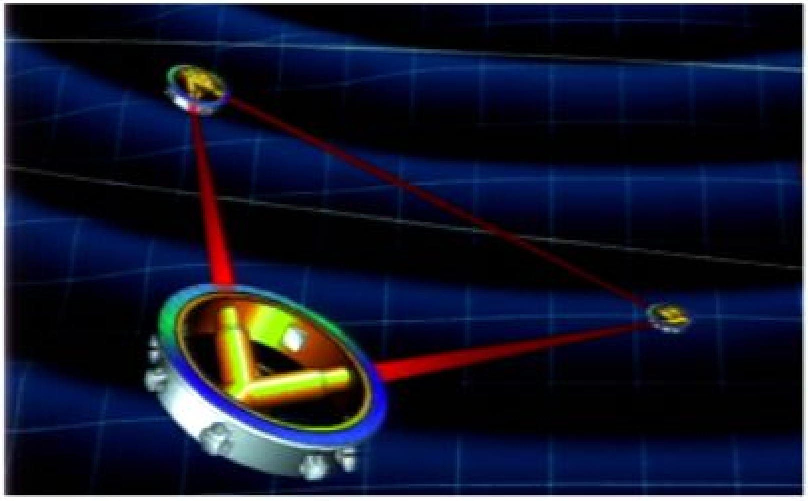

Strain Sensitivity of the LIGO Interferometers
S5 Performance - May 2007 LIGO-G070366-00-E
LHO 4km - (2007.03.18) S5: Binary Inspiral Range (1.4/1.4 Msun) = 16.3 Mpc
LLO 4km - (2006.06.04) S5: Binary Inspiral Range (1.4/1.4 Msun) = 15.1 Mpc
LHO 2km - (2007.05.14) S5: Binary Inspiral Range (1.4/1.4 Msun) = 7.8 Mpc
LIGO I SRD Goal, 4km
h[f], 1/Sqrt[Hz]
1e-16
1e-17
1e-18
1e-19
1e-20
1e-21
1e-22
1e-23
10
100
1000
10000

Advanced detectors



By increasing laser power and mirror masses, can reach a detector with sensitivity essentially limited by the uncertainty principle.

LISA

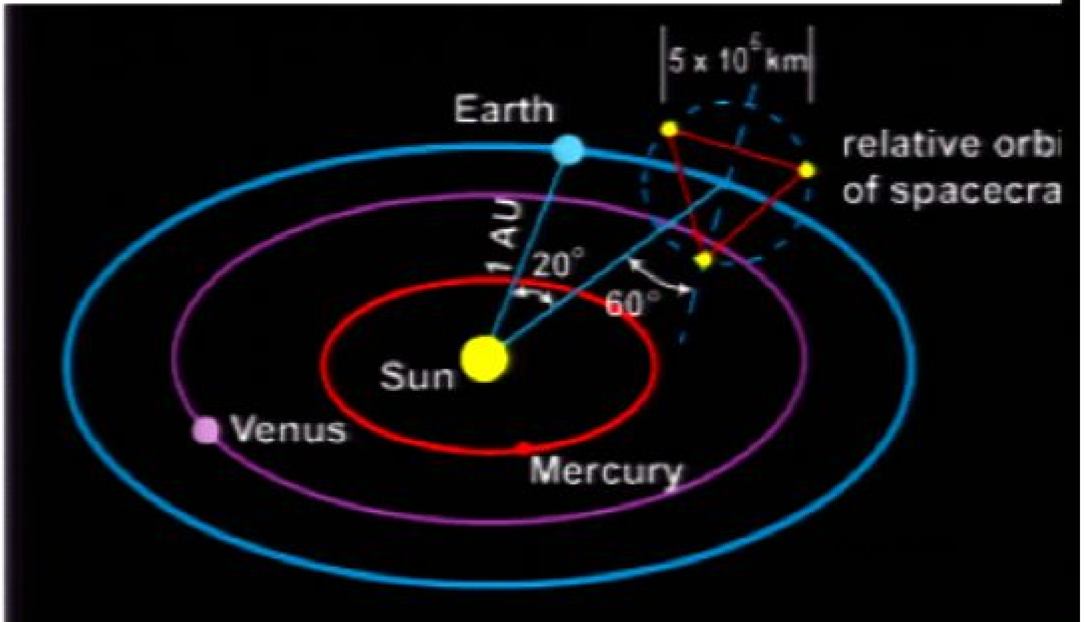


5 *million* kilometer space-based interferometer.

Under development as a joint NASA-ESA mission for launch c. 202*n* (*n* < 5?)

Sensitive in band
$\sim 10^{-4}$ Hz < f < 0.1 Hz

Very different astrophysics!



Largely targets processes involving massive black holes.

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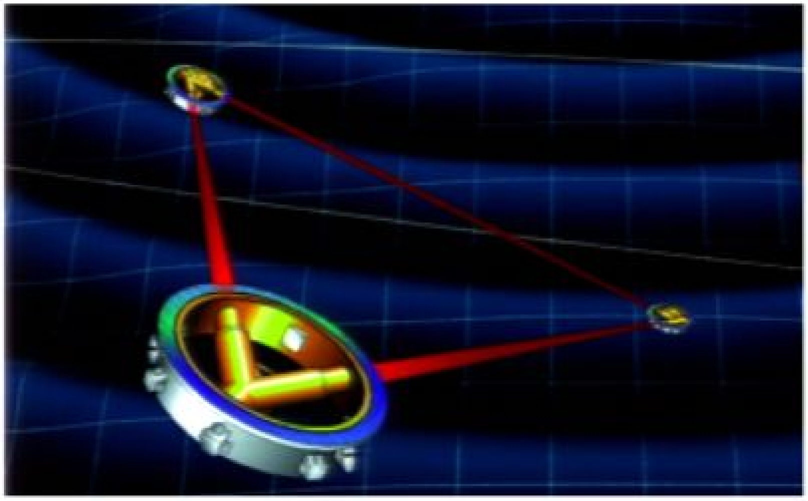

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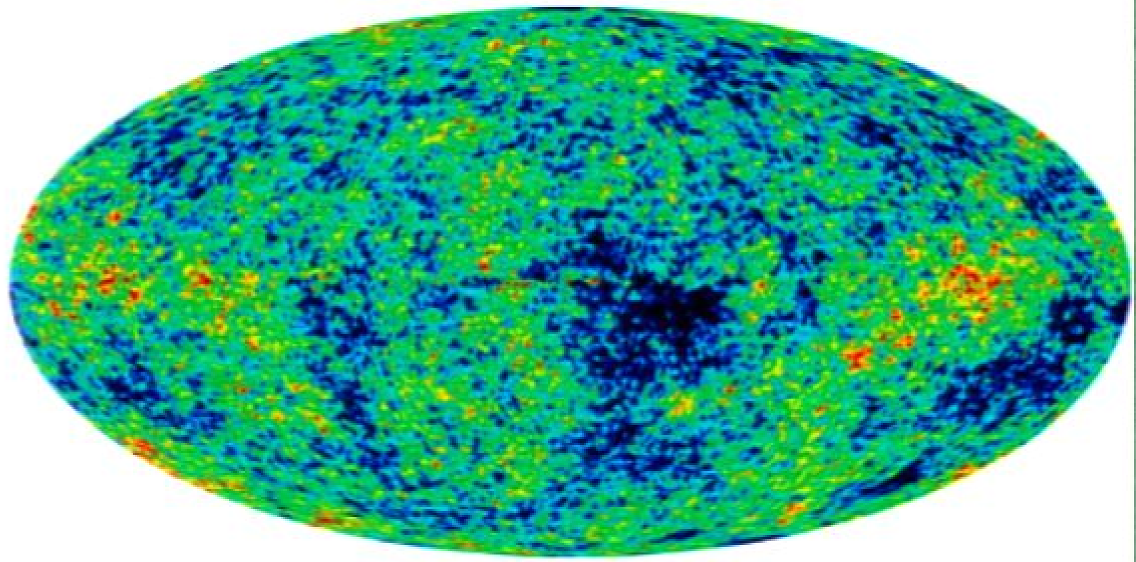
Optical requirements similar to Michelson-Morley

Slide courtesy Karsten Danzmann, AEI-Hannover

A brief history of galaxies

Cosmic microwave background: First glimpse of the universe's largest structures



Gravity grows overdensities: Slight overdensity at $z = 1100$ becomes more dense (compared to mean) as that region attracts more matter.

Initial overdensity tiny ($\delta\rho/\rho \sim 10^{-6}$), can treat evolution with simple linear theory

(Zeldovich 1970, Astronomy & Astrophysics, 5, 84)

A brief history of galaxies

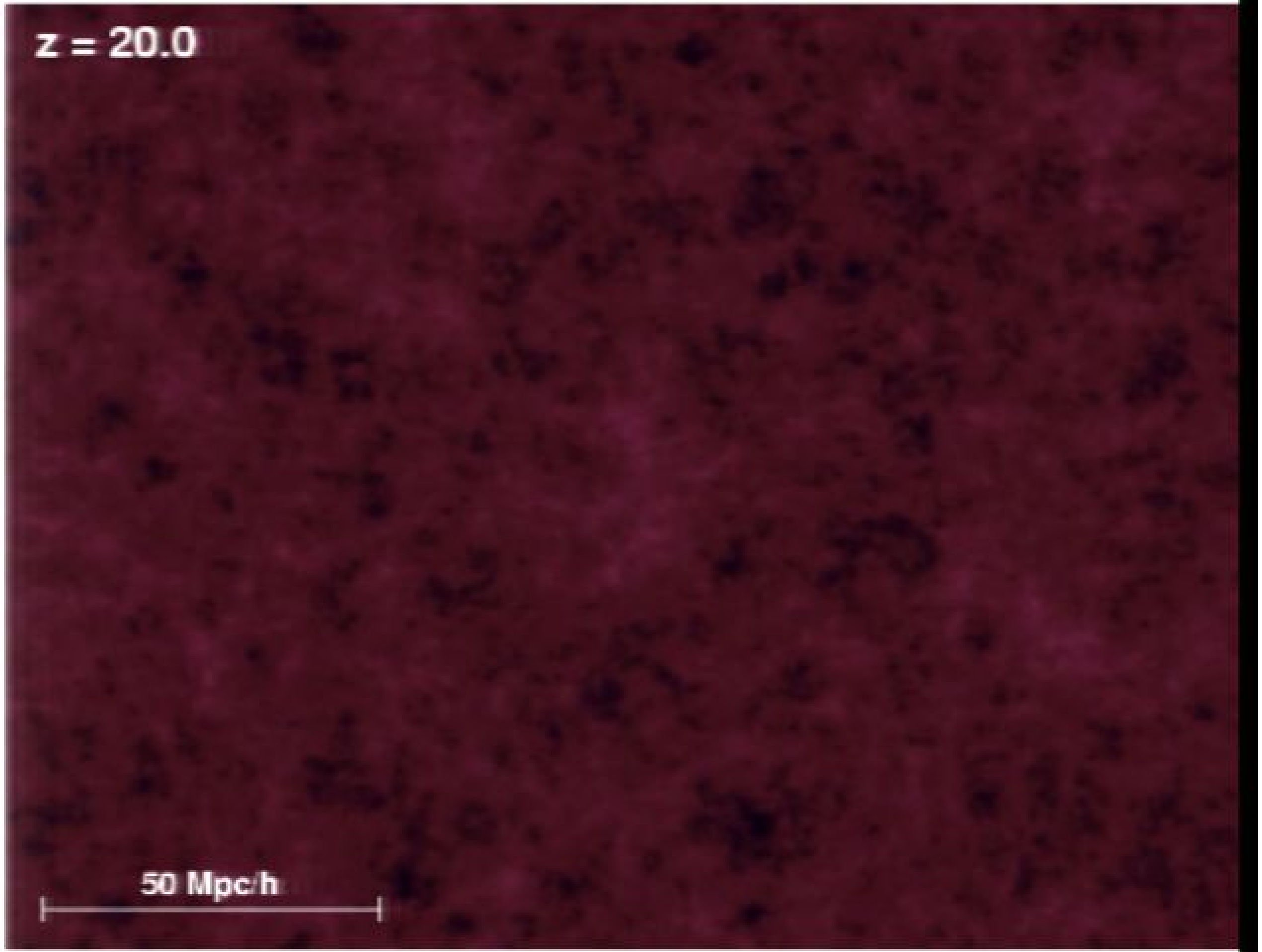
Evolution of density inhomogeneities

At z ~ 20, find

$\delta\rho/\rho \sim 1$:

Linear evolution no longer accurate. Now model using massive N-body simulations.

Credit:
The VIRGO Cosmological N-body Project



Density evolution, "comoving" coordinates.
Dark matter distribution followed in simulation.

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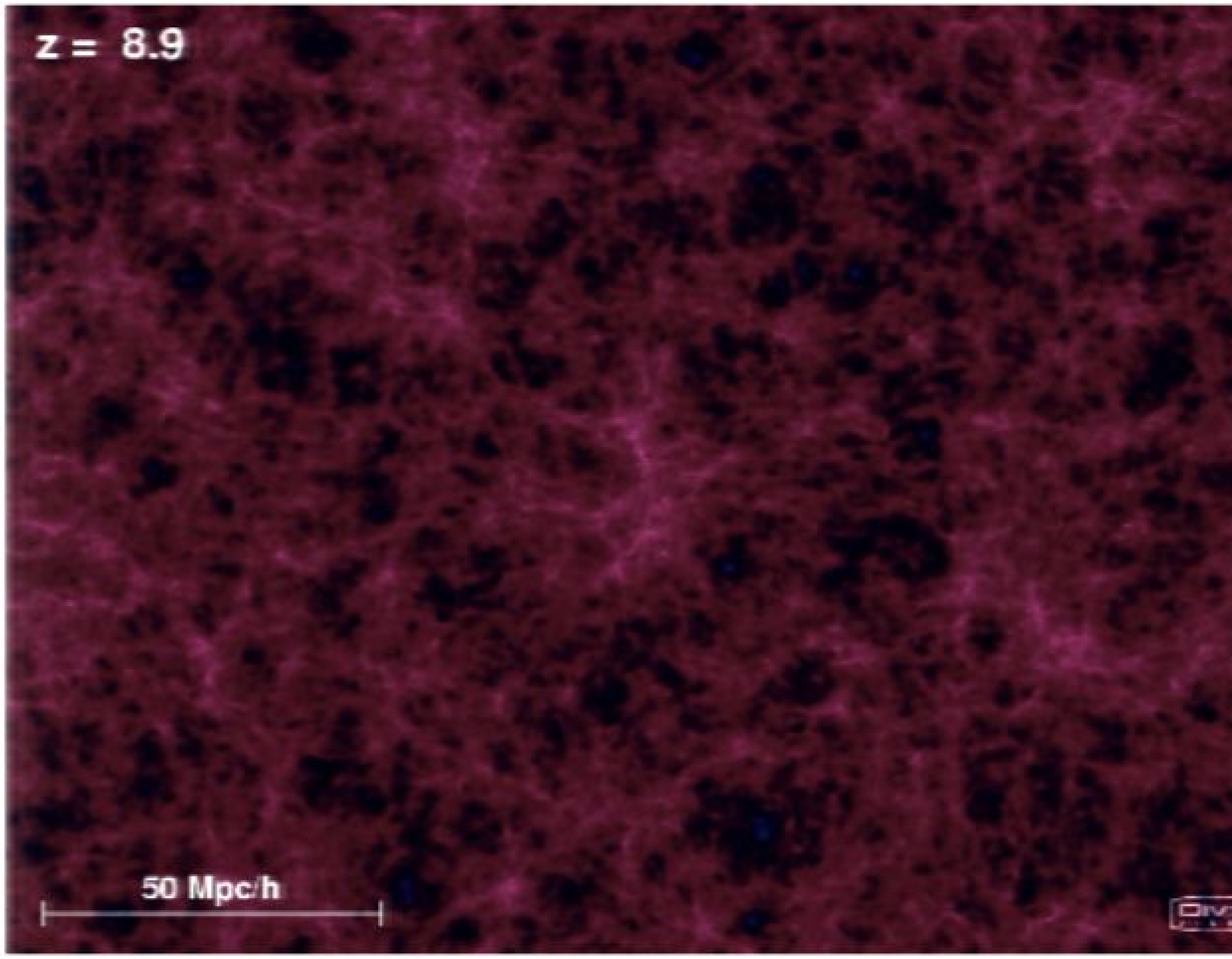

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A brief history of galaxies

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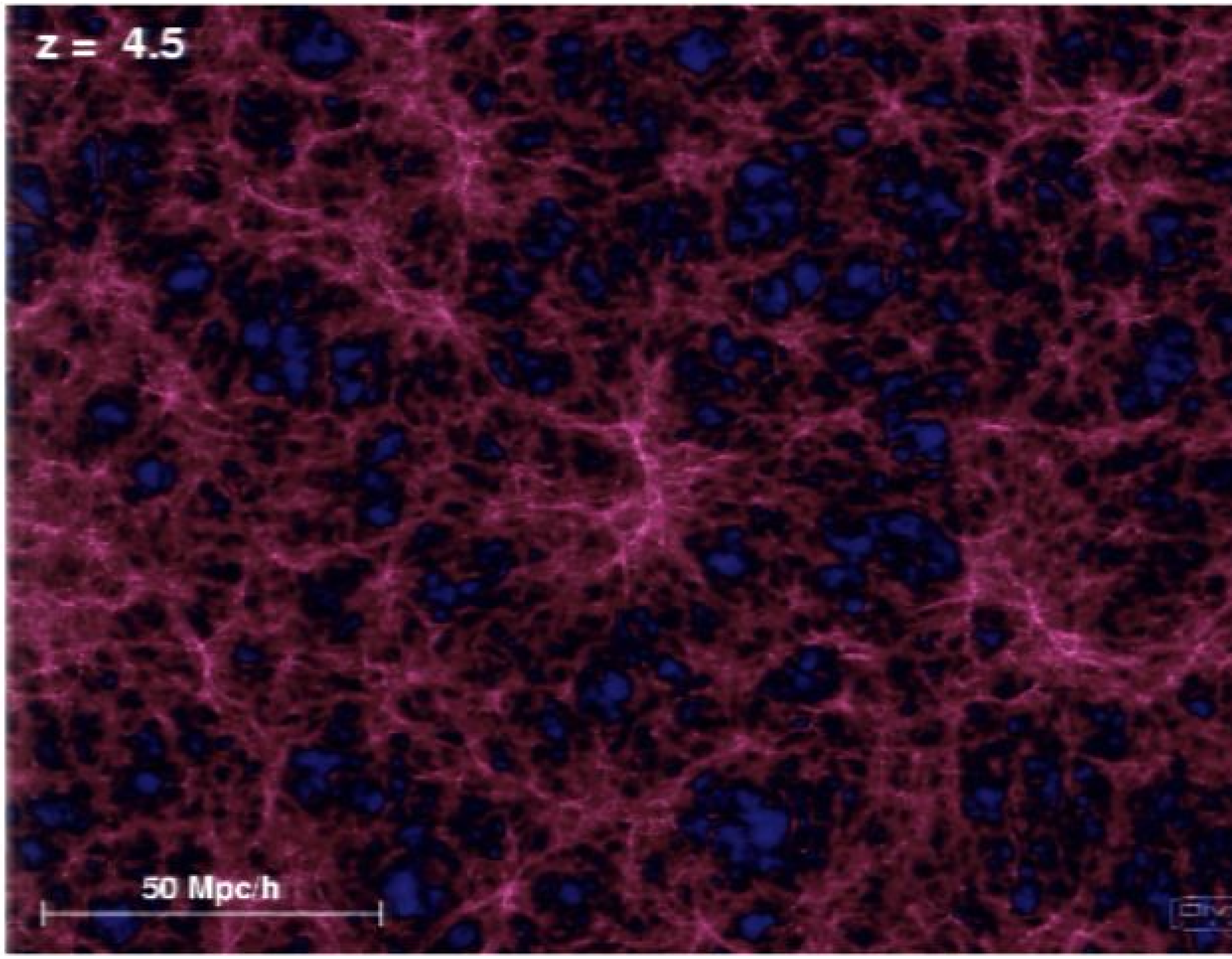
Structure is built *hierarchically*. Big things made of many mergers of small things.

Dark matter halos & galaxies merge a lot!



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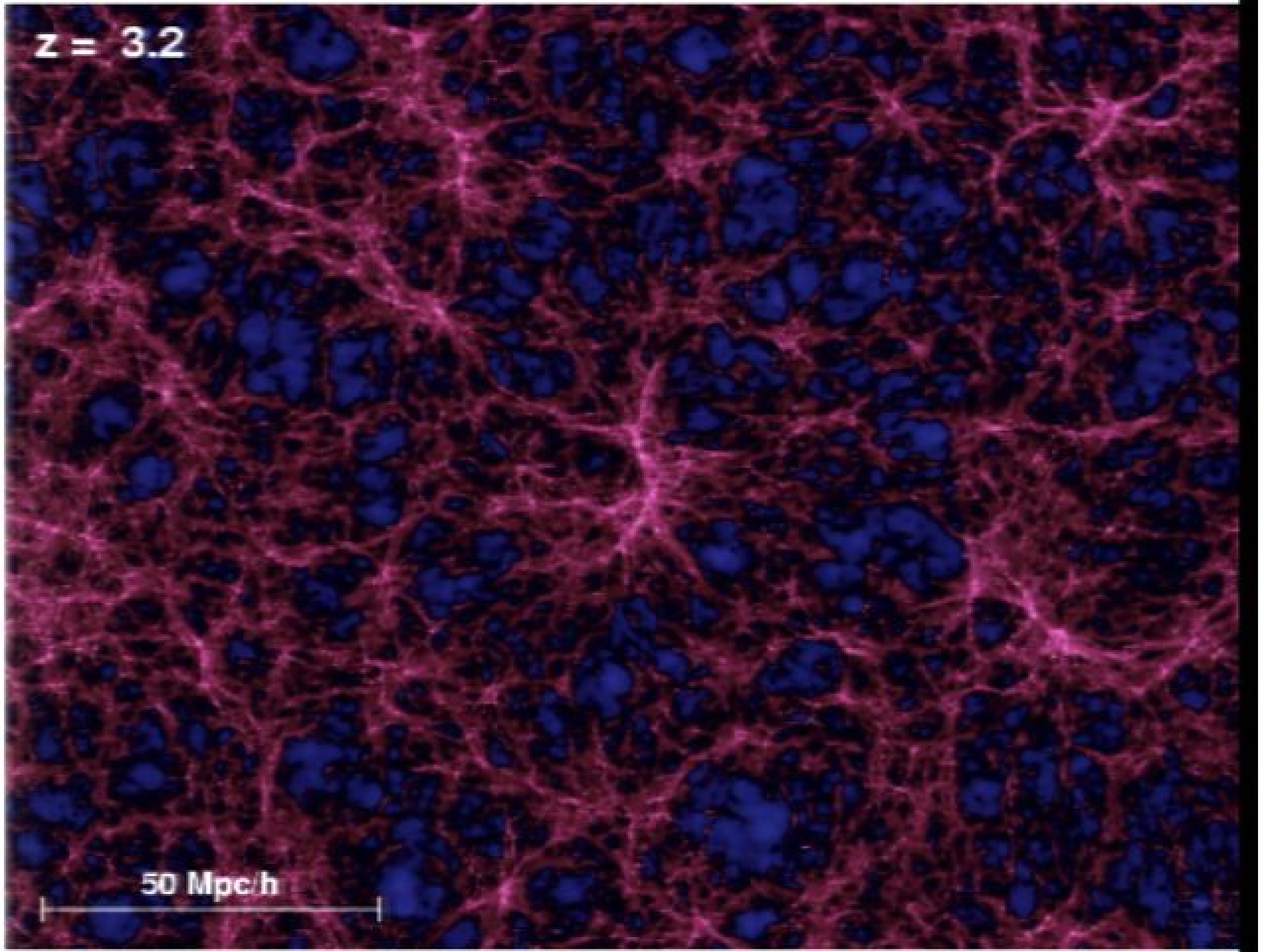

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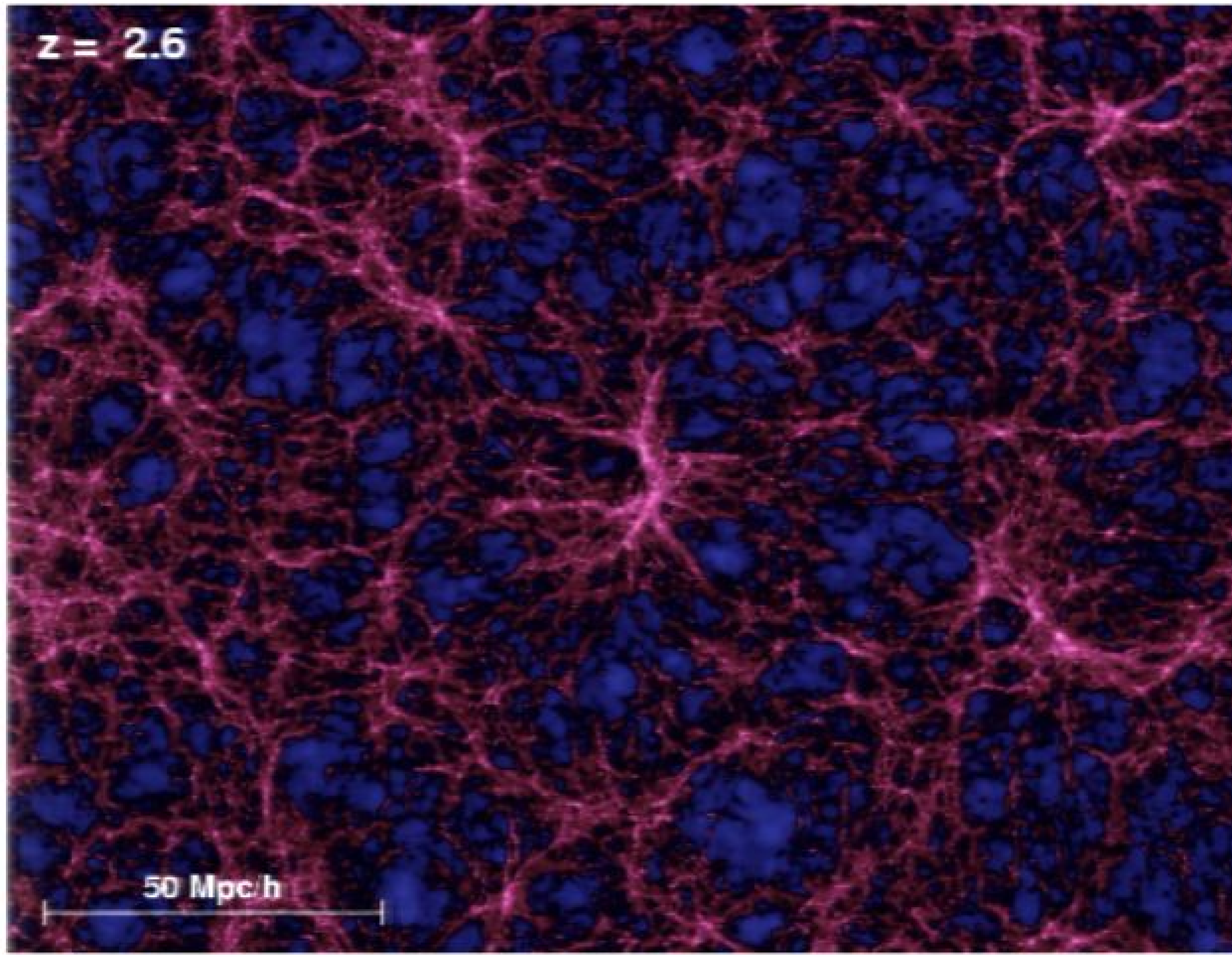

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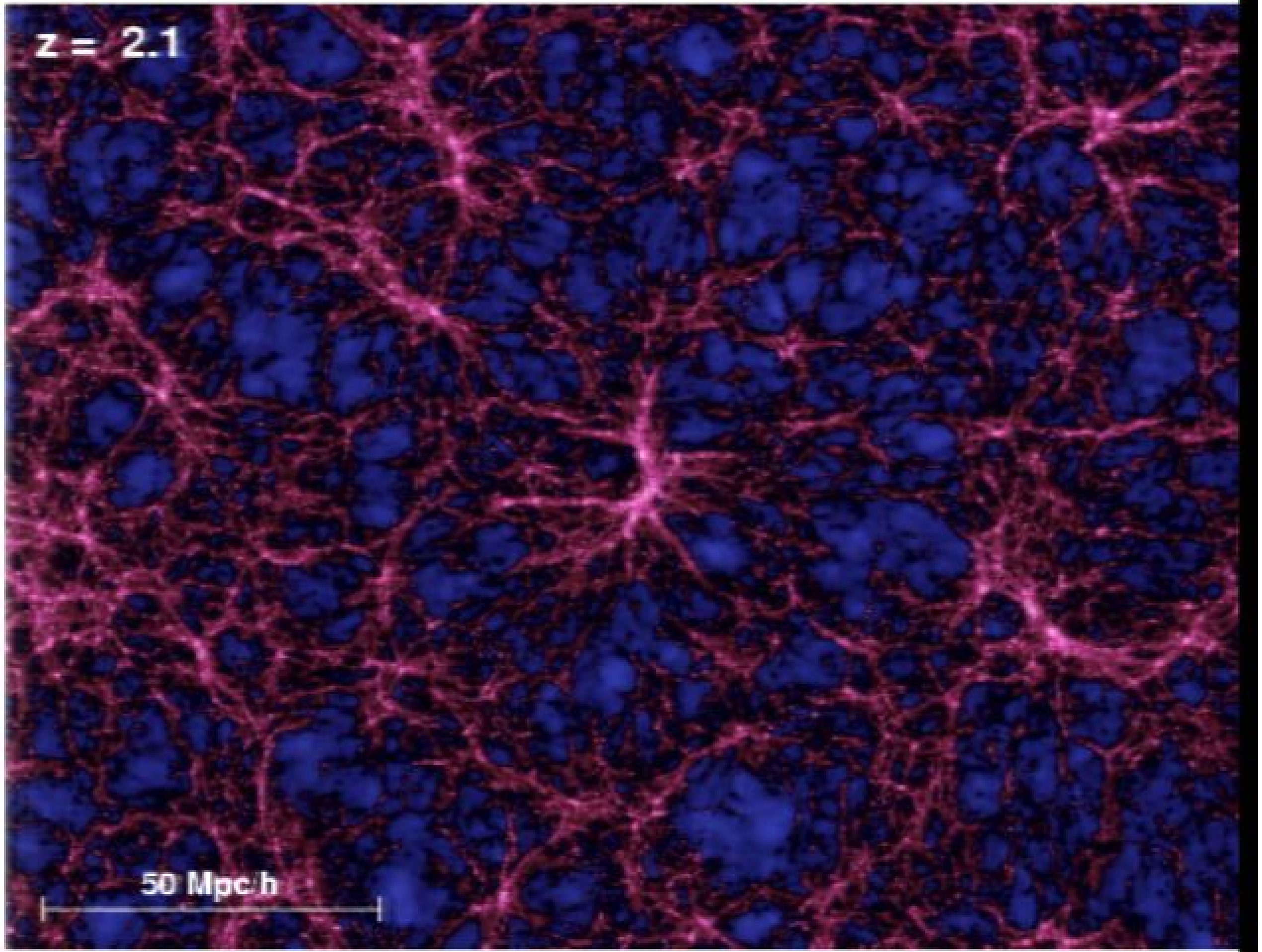

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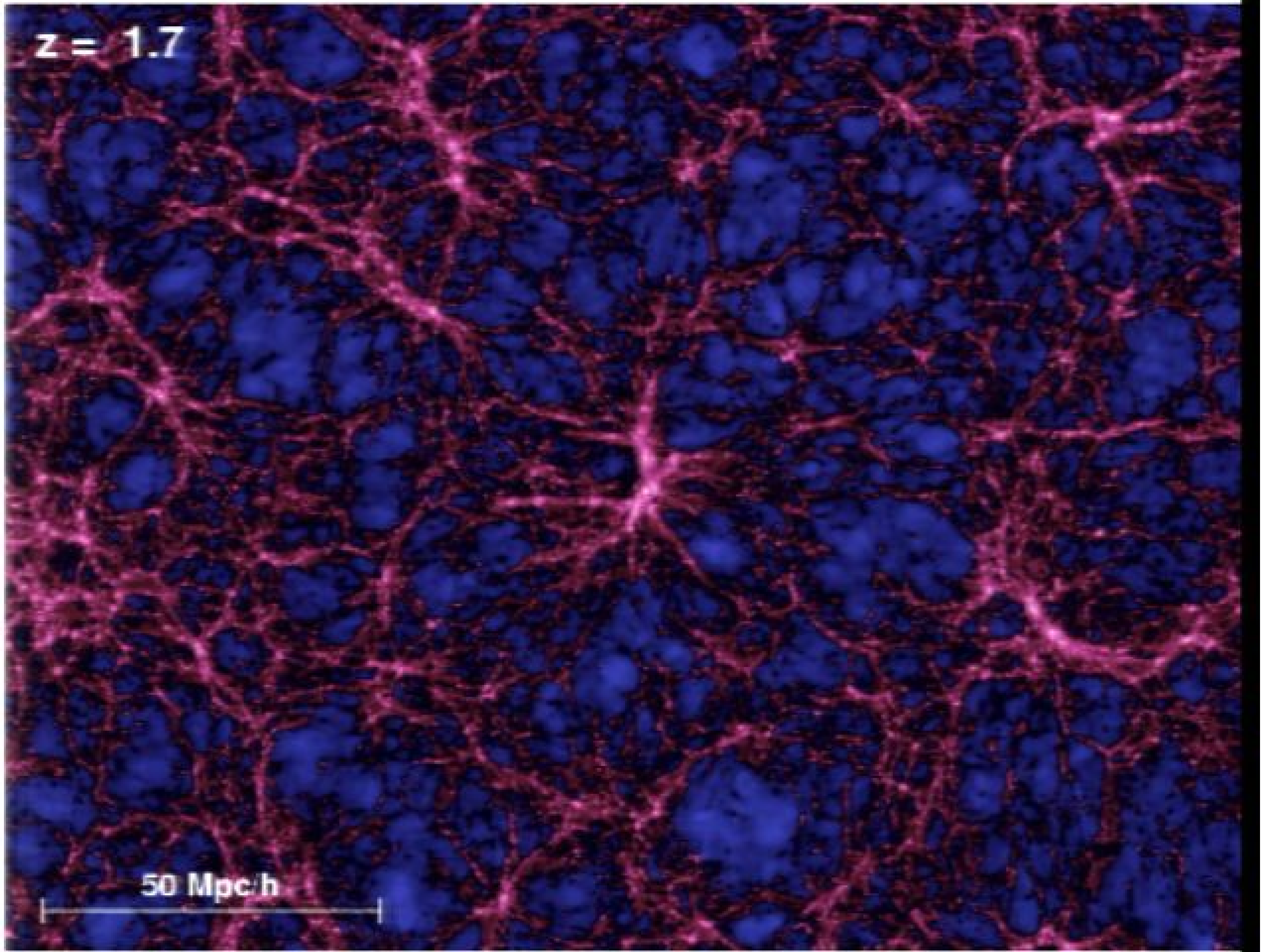

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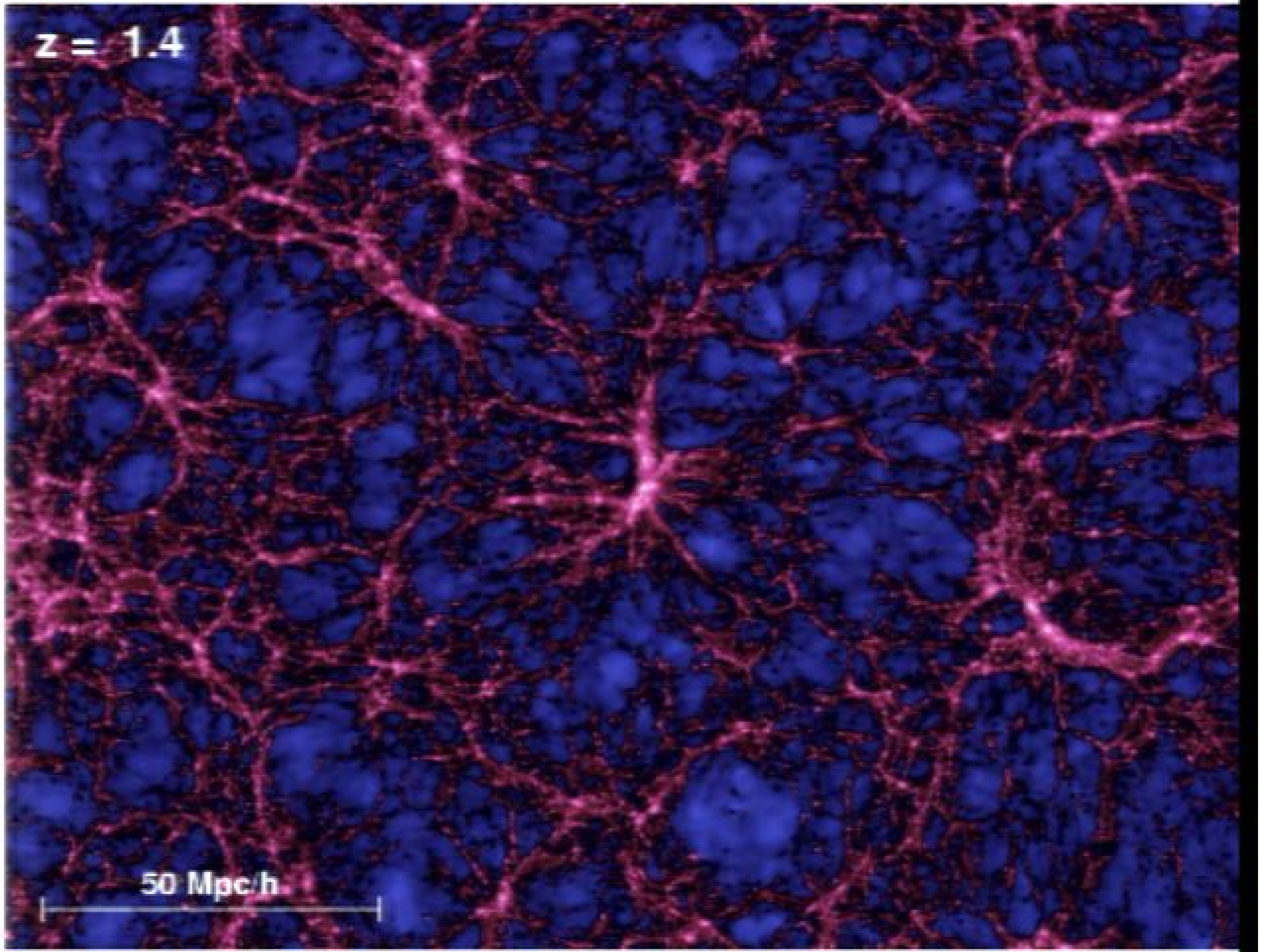

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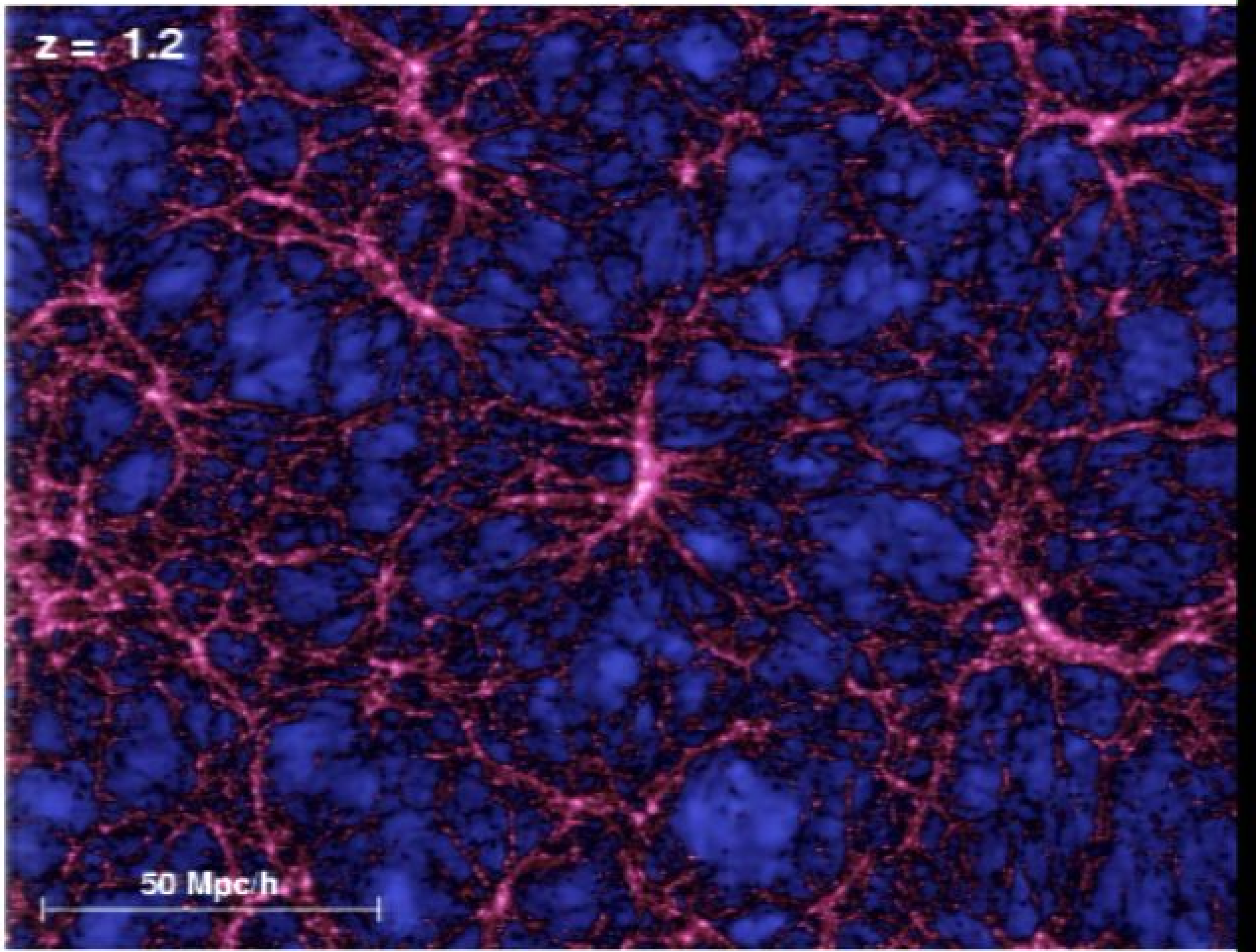

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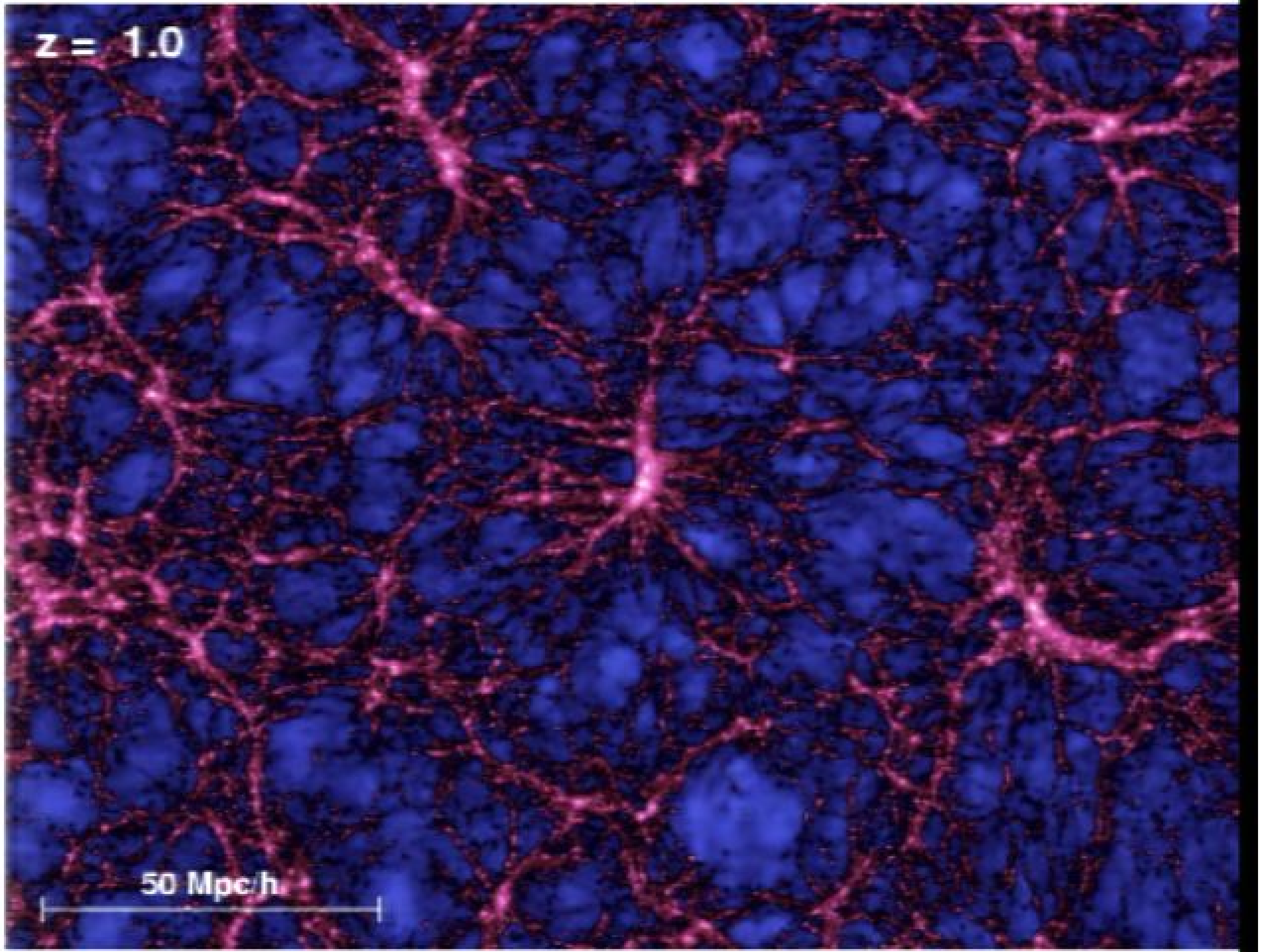

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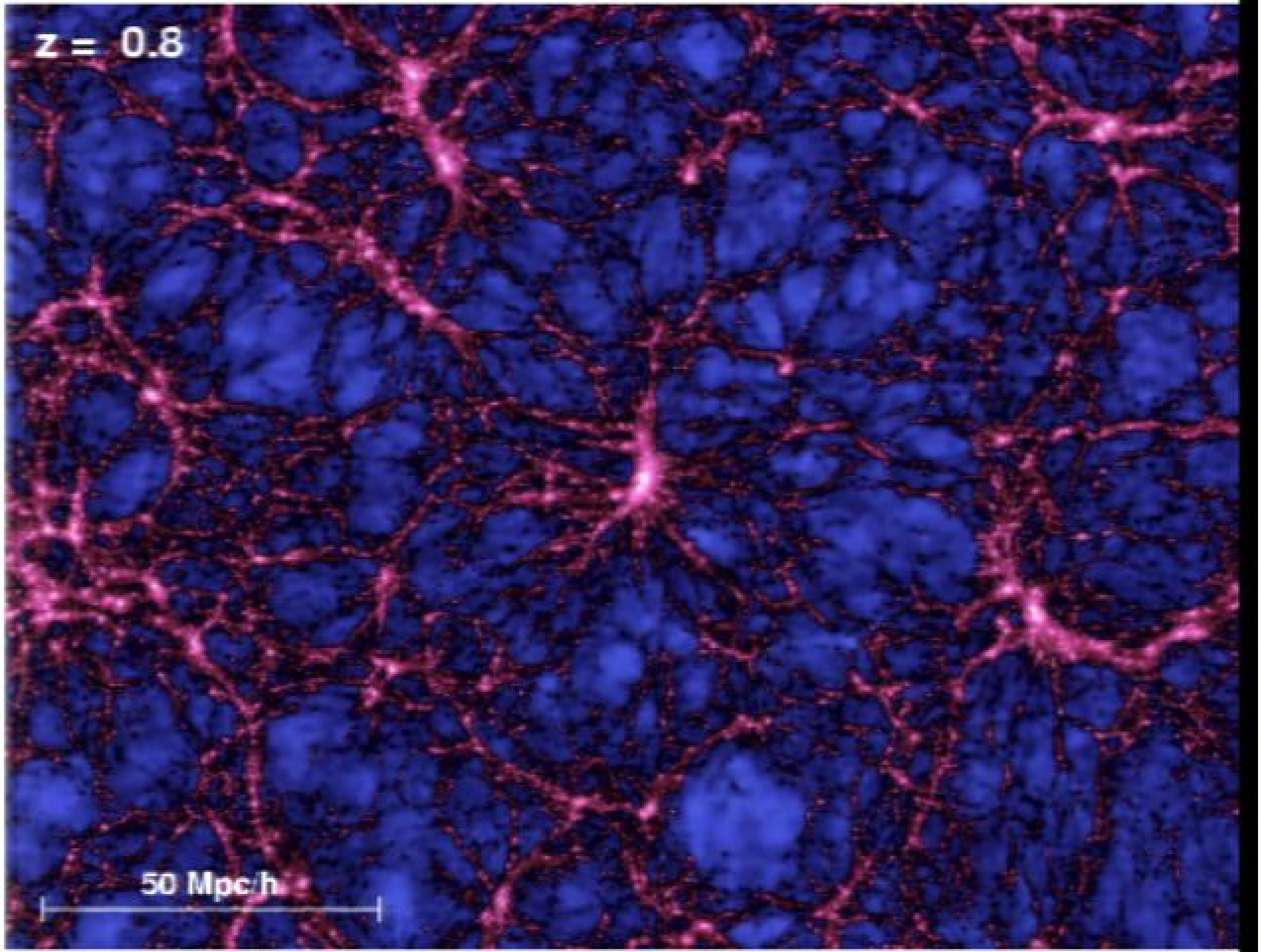

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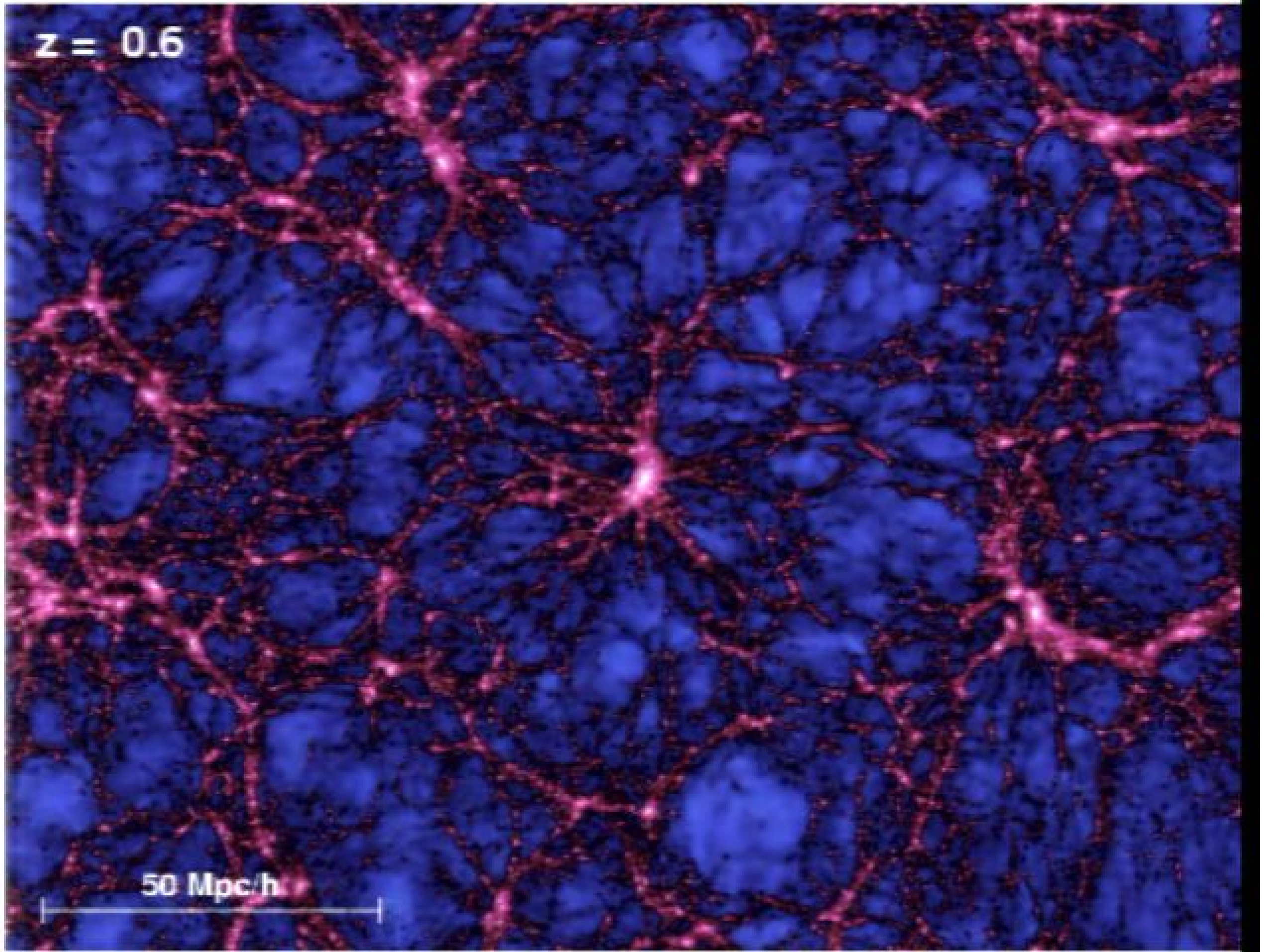

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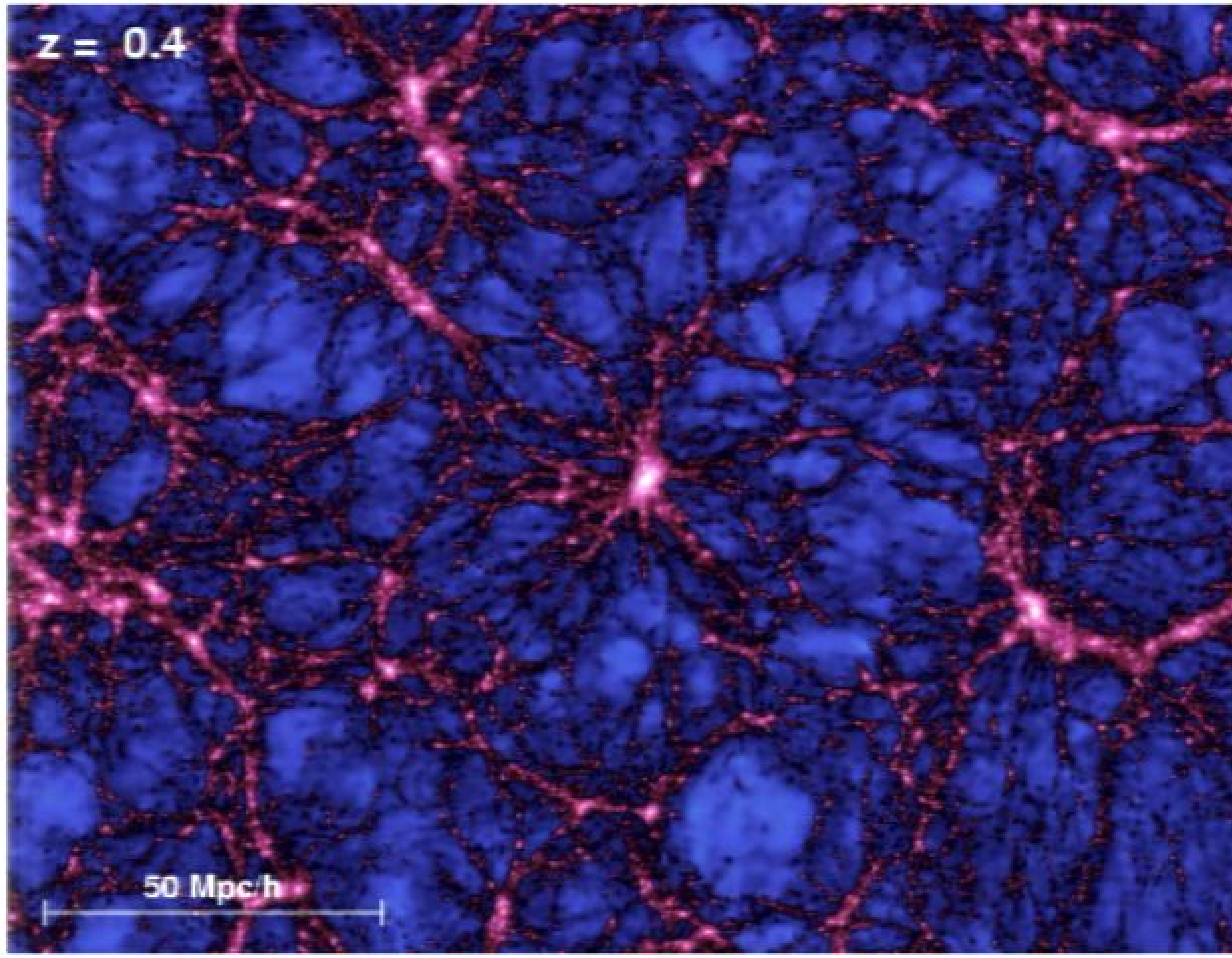

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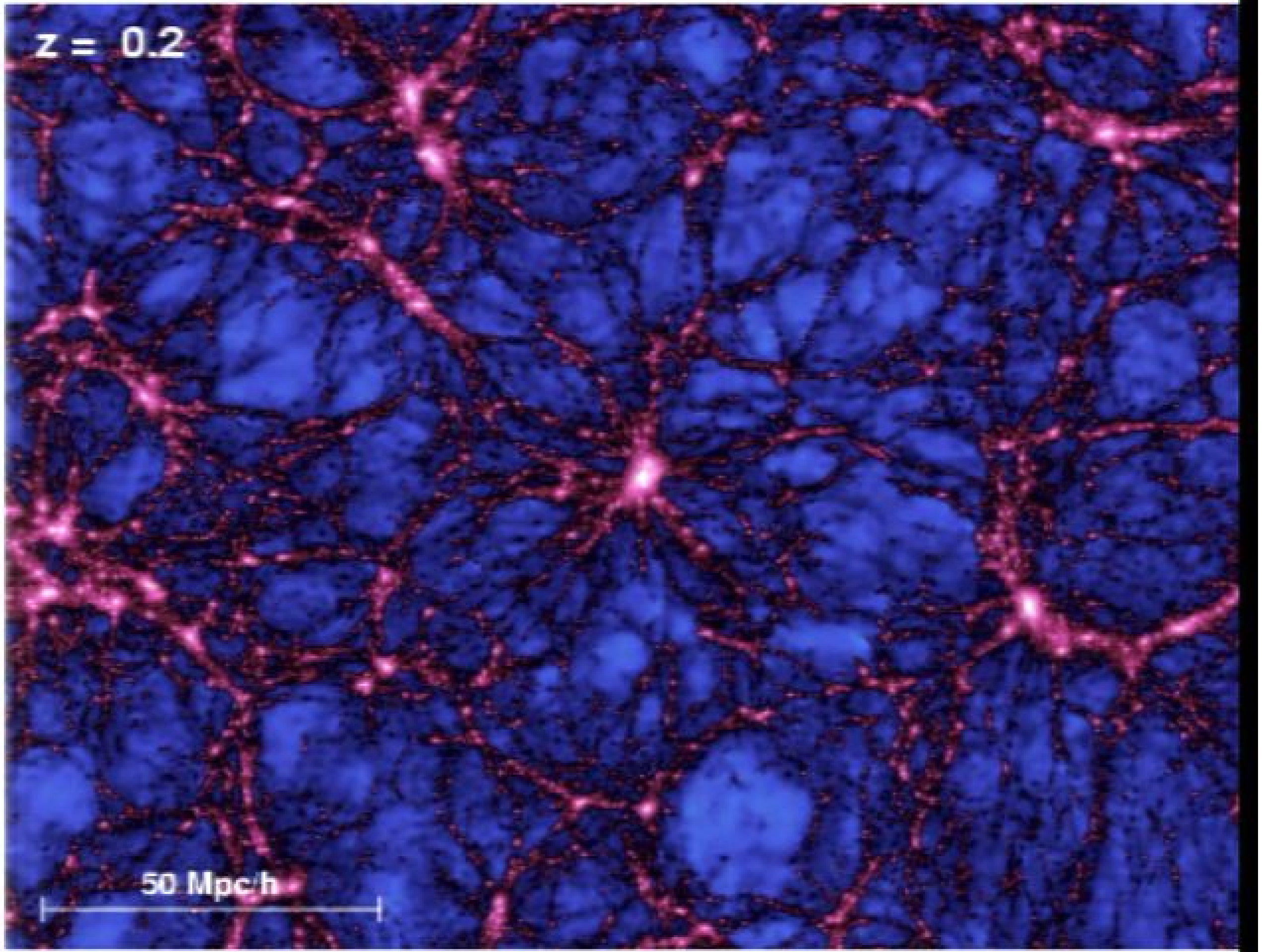

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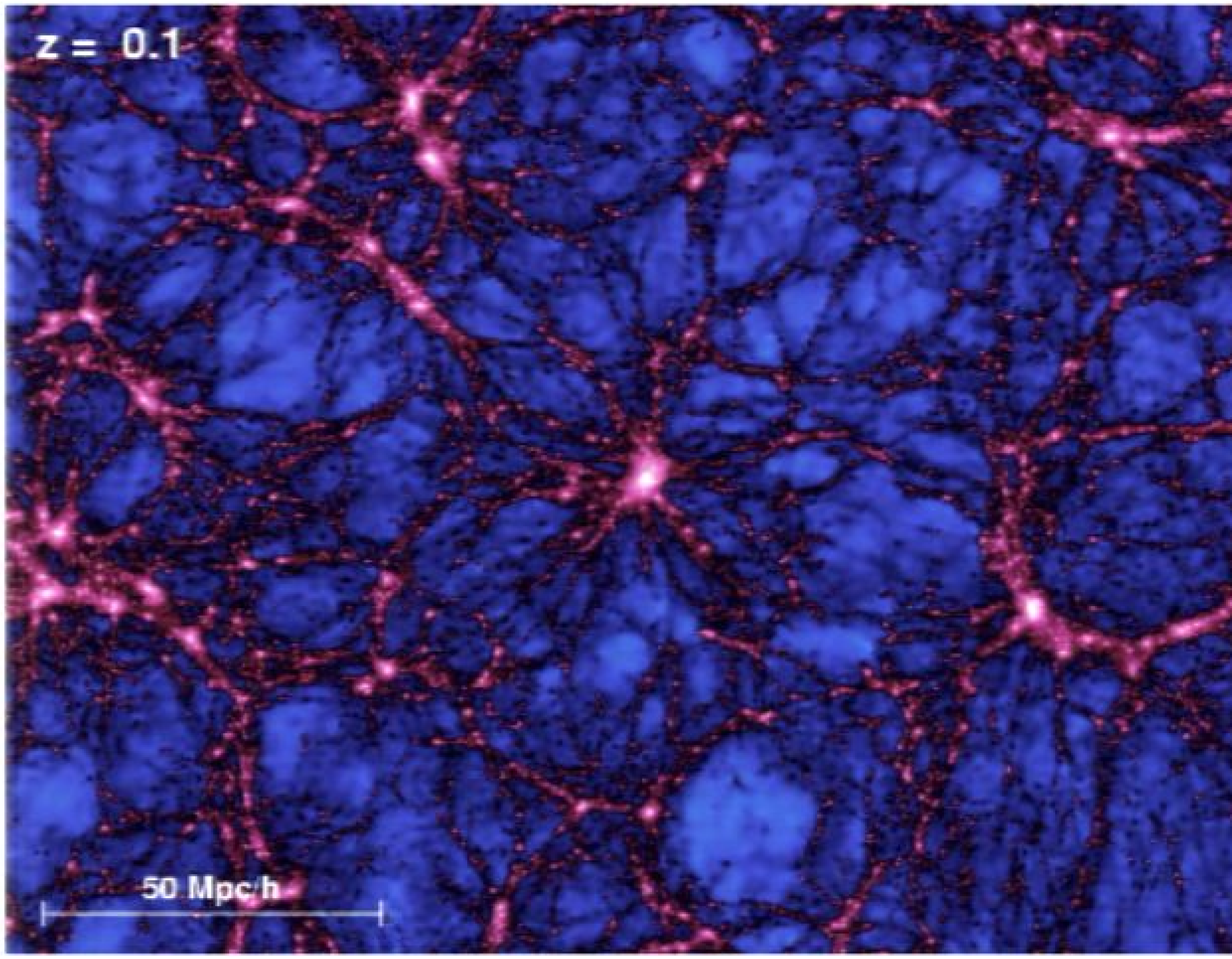

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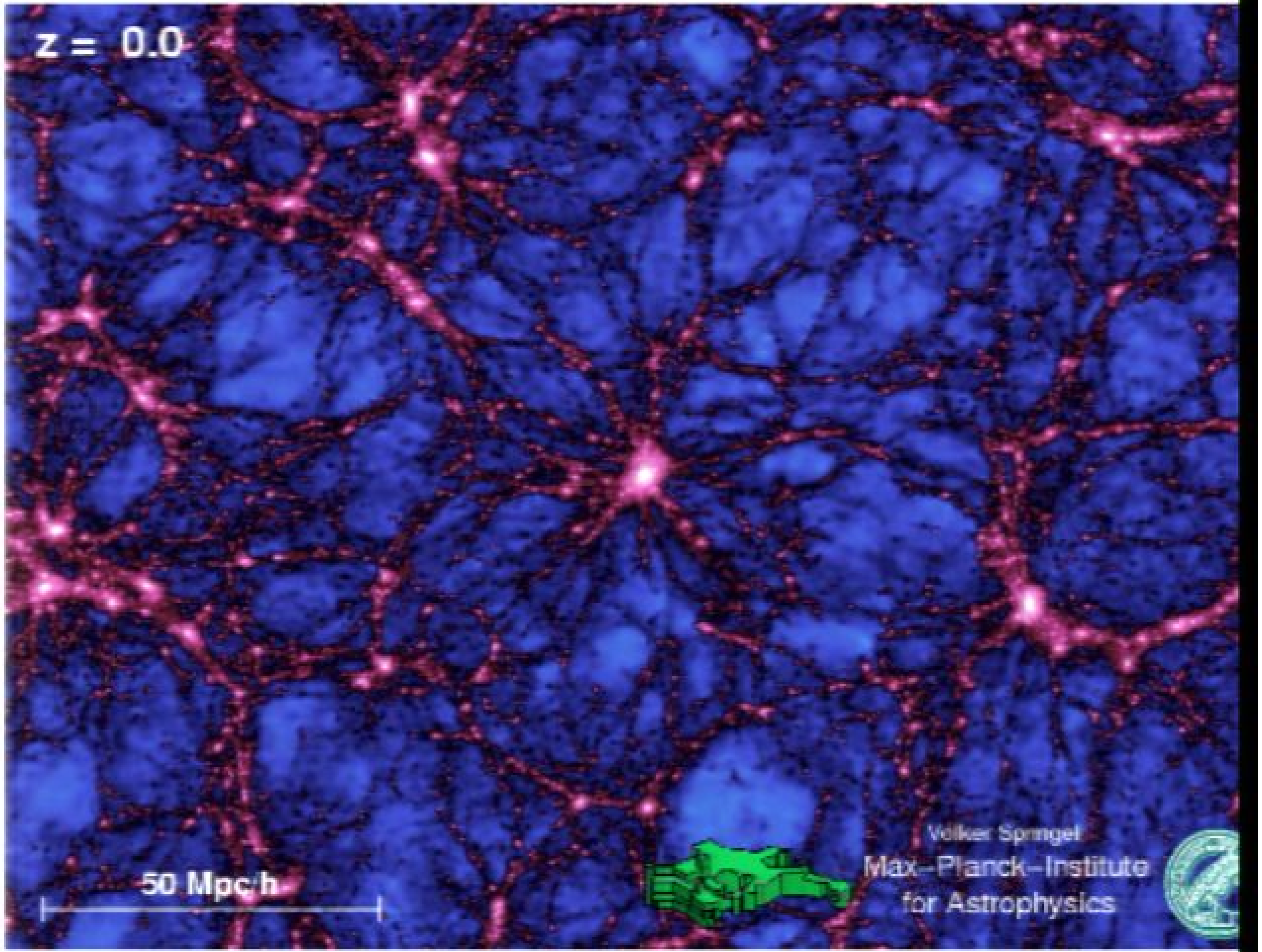


Many mergers at moderate to high redshift.

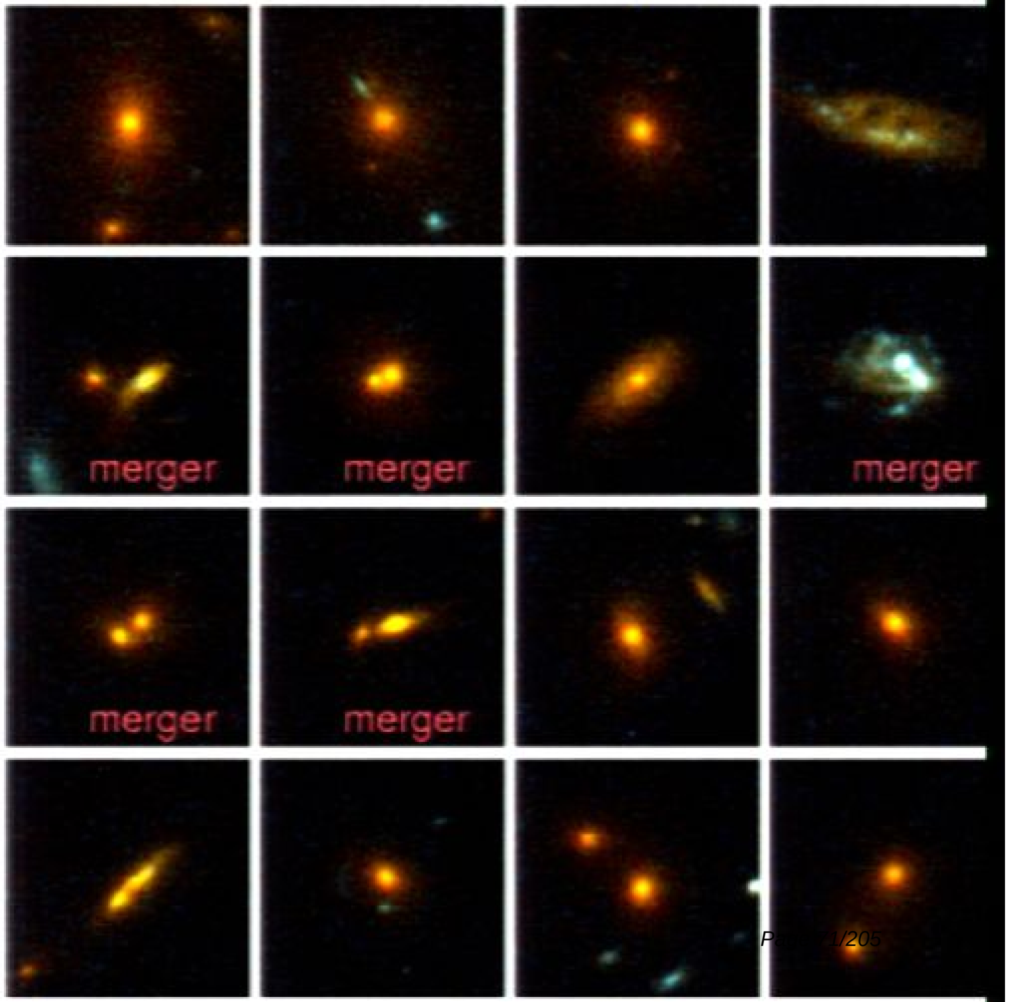
Observational evidence

Simulations backed up by observations: Galaxy mergers much more common in the past.

Action shot: galaxies in rich cluster MS 1054-03 ($z = 0.83$).

About 20% of the galaxies in this cluster are merging

van Dokkum et al 1999, Astrophys J 520, L95



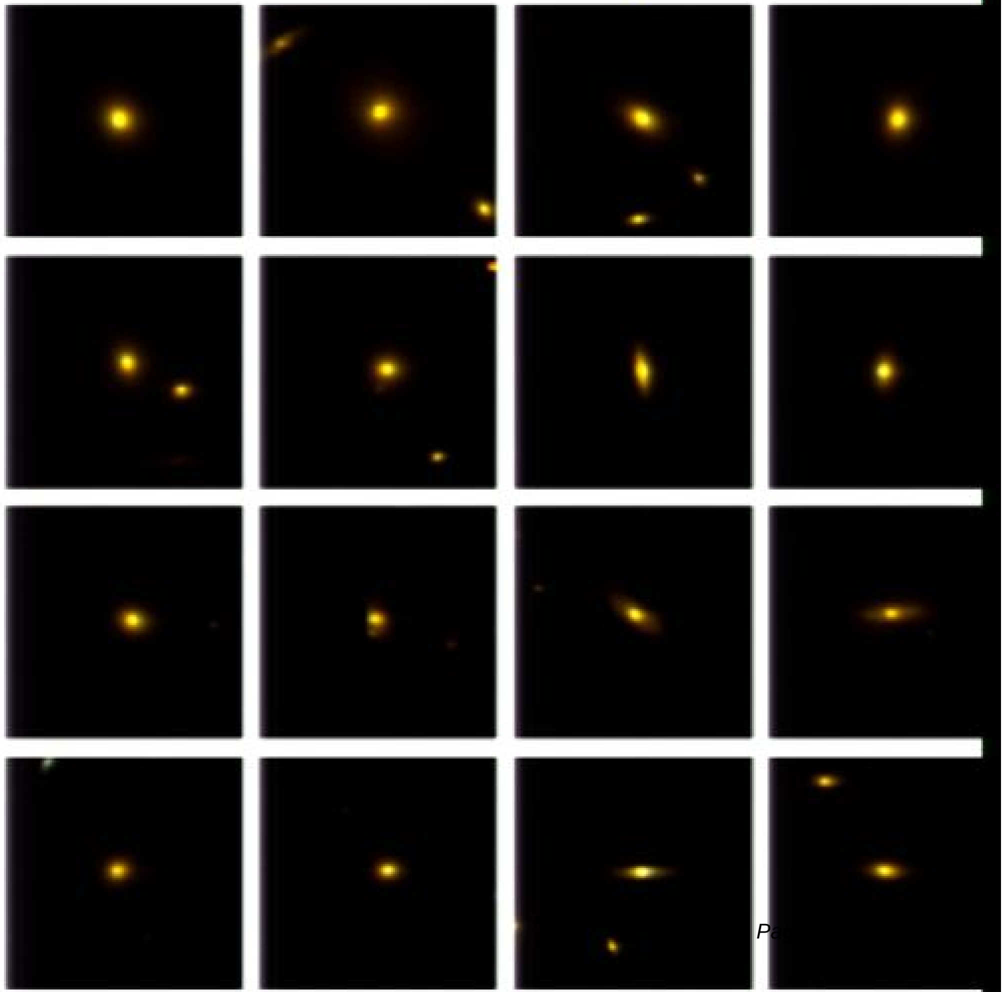
Observational evidence

Simulations backed up by observations: Galaxy mergers much more common in the past.

Inaction shot: galaxies in rich cluster MS 1358-62 ($z = 0.32$).

No mergers apparent in this cluster

van Dokkum et al 1999, Astrophys J 520, L95



Observational evidence

Simulations backed up by observations: Galaxy mergers much more common in the past.

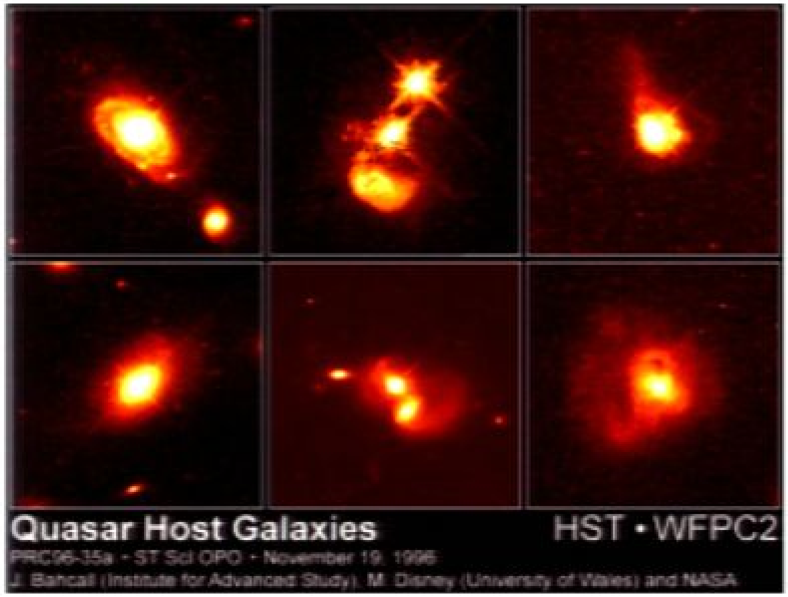
Trend continues to high redshift: Merger rate grows out to z ~ 5 or greater.

Observations indicate that the young universe was busy building the structures we observe today hierarchically.

Black holes in galaxies

Other key fact: Most galaxies have black holes at their cores.

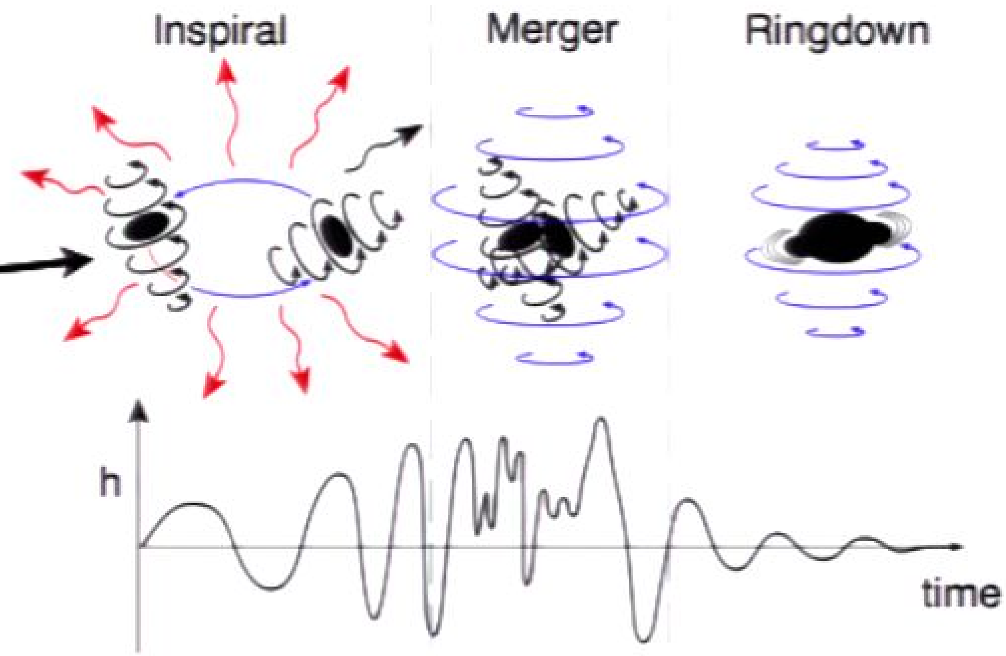
Precise measurements in nearby universe; quasars indicate presence at highest observed redshifts.



As host galaxies and structures merge, black holes form binaries: *Extremely* strong GW sources.

Coalescence waves

Inspiral: Slow evolution driven by GW loss of orbital energy and angular momentum.



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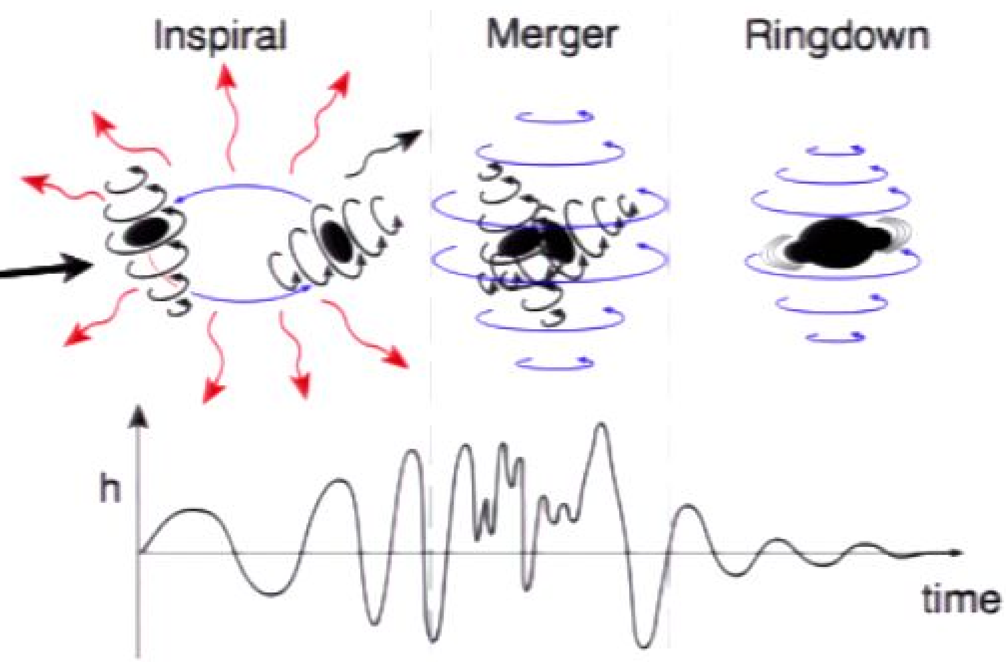
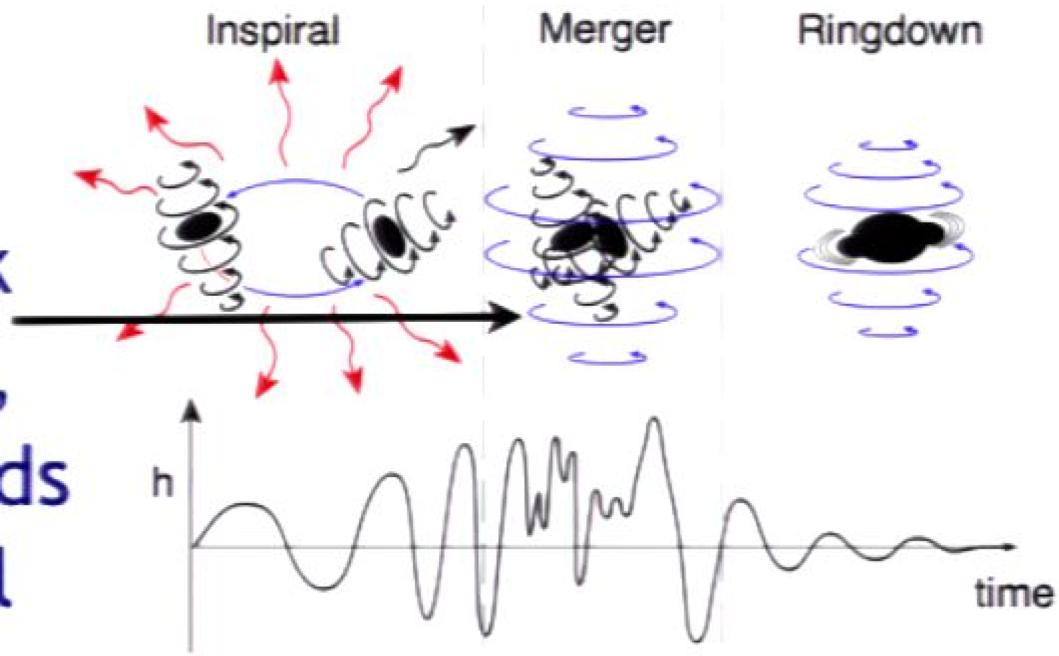


Rather well understood. Waveform described by 17 parameters in general.

2 masses
6 spin components
2 position angles
2 orientation angles
1 distance
1 initial semi-major axis
1 initial orbit anomaly
1 initial eccentricit
1 initial periapsis longitude

Coalescence waves

Merger: Extremely violent dynamics of spacetime: Two black holes smash together, leaving one behind; ends in "ringdown" of final quasi-normal modes.



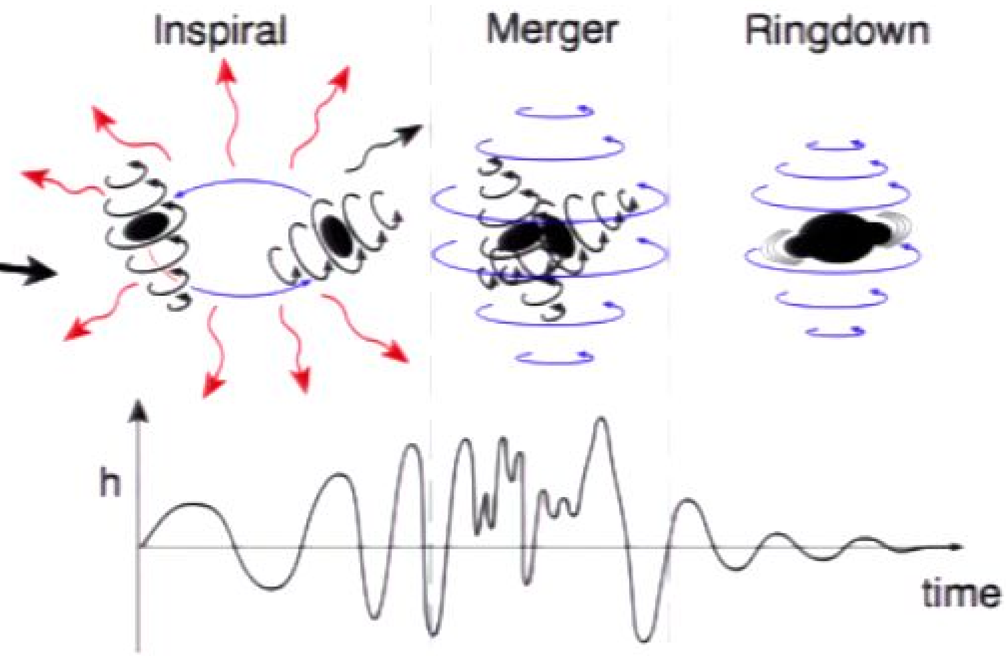
Ultimate confrontation of classical gravity theory with data!

Modeling requires rather large numerical simulations. Recent breakthroughs have opened this up — race is on to explore parameter space, develop merger phenomenology: **NRDA!**

Focus on inspiral for this talk

Inspiral: Slow, strong dependence on system parameters ...

Measuring waves allows us to determine those parameters.



Post-Newtonian description good for most of the inspiral: Expansion in orbital speed and field strength that gives an analytic description of waves as parameterized by masses, spins etc of the binary.

Post-Newtonian description

Example: Equations describing center of mass motion of each member of the binary.

$$a_1^i \quad = \quad -\frac{Gm_2 n_{12}^i}{r_{12}^2}$$

Leading term: Newtonian gravity.

Post-Newtonian description

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$$+\frac{1}{c^2}\left\{ \left[\frac{5G^2 m_1 m_2}{r_{12}^3} + \frac{4G^2 m_2^2}{r_{12}^3} + \frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2 - v_1^2 + 4(v_1 v_2) - 2v_2^2\right)\right] n_1^i \right.$$

$$\left. + \frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1) - 3(n_{12}v_2)\right) v_{12}^i \right\}$$

Relativity corrections

Post-Newtonian description

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Relativity corrections

More corrections ...

$$
\begin{aligned}
&+\frac{1}{c^4}\Bigg\{\Bigg[-\frac{57G^3m_1^2m_2}{4r_{12}^4}-\frac{69G^3m_1m_2^2}{2r_{12}^4}-\frac{9G^3m_2^3}{r_{12}^4}\\
&\quad+\frac{Gm_2}{r_{12}^2}\Bigg(-\frac{15}{8}(n_{12}v_2)^4+\frac{3}{2}(n_{12}v_2)^2v_1^2-6(n_{12}v_2)^2(v_1v_2)-2(v_1v_2)^2+\frac{9}{2}(n_{12}v_2)^2v_2^2\\
&\qquad\qquad+4(v_1v_2)v_2^2-2v_2^4\Bigg)\\
&\quad+\frac{G^2m_1m_2}{r_{12}^3}\left(\frac{39}{2}(n_{12}v_1)^2-39(n_{12}v_1)(n_{12}v_2)+\frac{17}{2}(n_{12}v_2)^2-\frac{15}{4}v_1^2-\frac{5}{2}(v_1v_2)+\frac{5}{4}v_2^2\right)\\
&\quad+\frac{G^2m_2^2}{r_{12}^3}\left(2(n_{12}v_1)^2-4(n_{12}v_1)(n_{12}v_2)-6(n_{12}v_2)^2-8(v_1v_2)+4v_2^2\right)\Bigg]n_{12}^i\\
&\quad+\Bigg[\frac{G^2m_2^2}{r_{12}^3}\left(-2(n_{12}v_1)-2(n_{12}v_2)\right)+\frac{G^2m_1m_2}{r_{12}^3}\left(-\frac{63}{4}(n_{12}v_1)+\frac{55}{4}(n_{12}v_2)\right)\\
&\qquad+\frac{Gm_2}{r_{12}^2}\Bigg(-6(n_{12}v_1)(n_{12}v_2)^2+\frac{9}{2}(n_{12}v_2)^3+(n_{12}v_2)v_1^2-4(n_{12}v_1)(v_1v_2)\\
&\qquad\qquad+4(n_{12}v_2)(v_1v_2)+4(n_{12}v_1)v_2^2-5(n_{12}v_2)v_2^2\Bigg)\Bigg]v_{12}^i\Bigg\}\\
&+\frac{1}{c^5}\Bigg\{\left[\frac{208G^3m_1m_2^2}{15r_{12}^4}(n_{12}v_{12})-\frac{24G^3m_1^2m_2}{5r_{12}^4}(n_{12}v_{12})+\frac{12G^2m_1m_2}{5r_{12}^3}(n_{12}v_{12})v_{12}^2\right]n_{12}^i\\
&\quad+\left[\frac{8G^3m_1^2m_2}{5r_{12}^4}-\frac{32G^3m_1m_2^2}{5r_{12}^4}-\frac{4G^2m_1m_2}{5r_{12}^3}v_{12}^2\right]v_{12}^i\Bigg\}
\end{aligned}
$$

And some more.

[Blanchet 2006, Liv Rev Rel 9, 4, Eq. (168)]

Gravitomagnetism

Magnetic-like contribution to the spacetime drives magnetic-like precession of binary members' spins.

$$\frac{d\mathbf{S}_1}{dt} = \frac{1}{r^3}\left[\left(2+\frac{3}{2}\frac{m_2}{m_1}\right)\mu\sqrt{Mr}\hat{\mathbf{L}}\right]\times\mathbf{S}_1 + \frac{1}{r^3}\left[\frac{1}{2}\mathbf{S}_2 - \frac{3}{2}(\mathbf{S}_2\cdot\hat{\mathbf{L}})\hat{\mathbf{L}}\right]\times\mathbf{S}$$

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Leads to new forces, modifying the orbital acceleration felt by each body.

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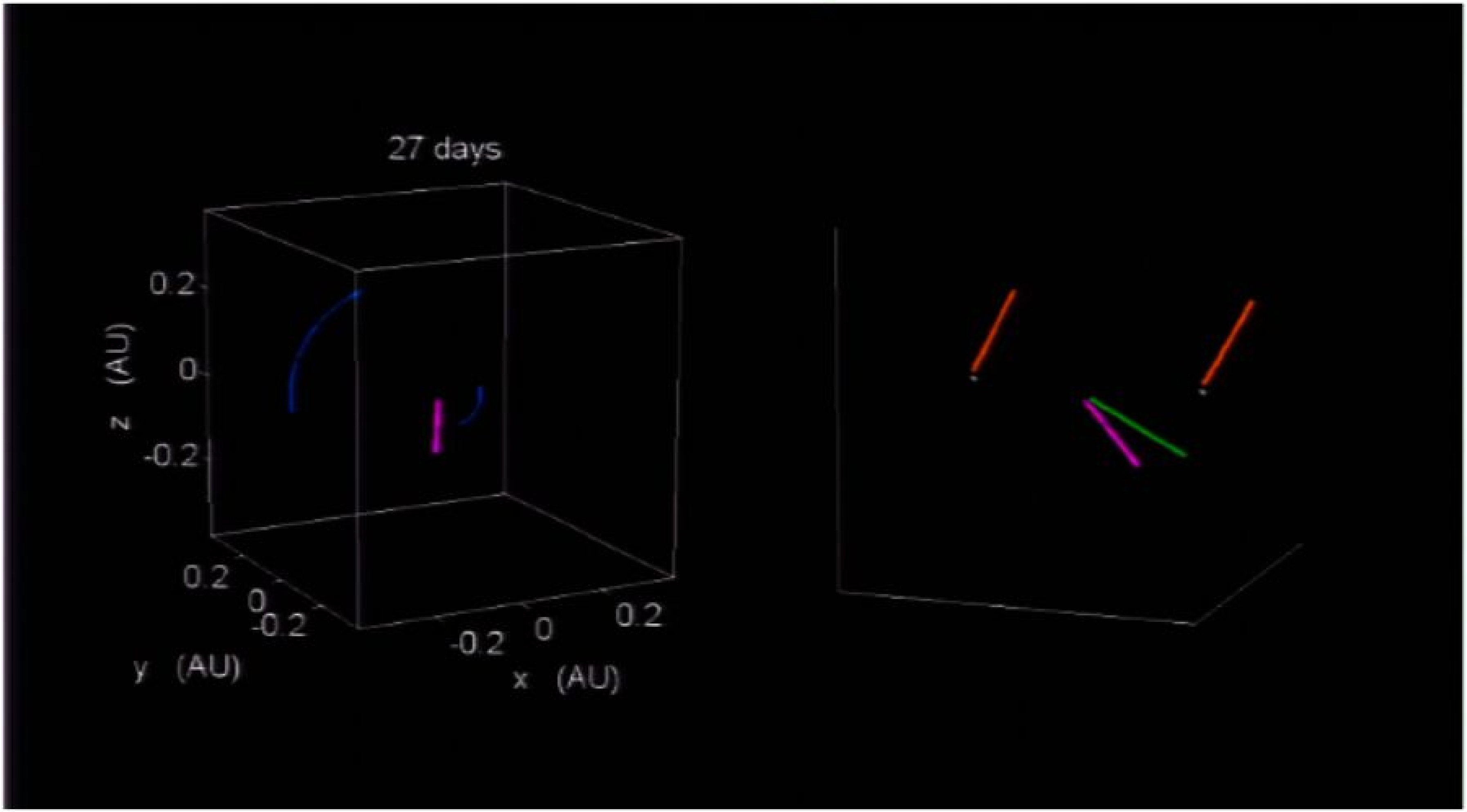
Angular momentum is *globally* conserved:

$$\mathbf{J} = \mathbf{L} + \mathbf{S}_1 + \mathbf{S}_2 = \text{constant}$$

Orbital plane precesses to compensate for precession of the individual spins.

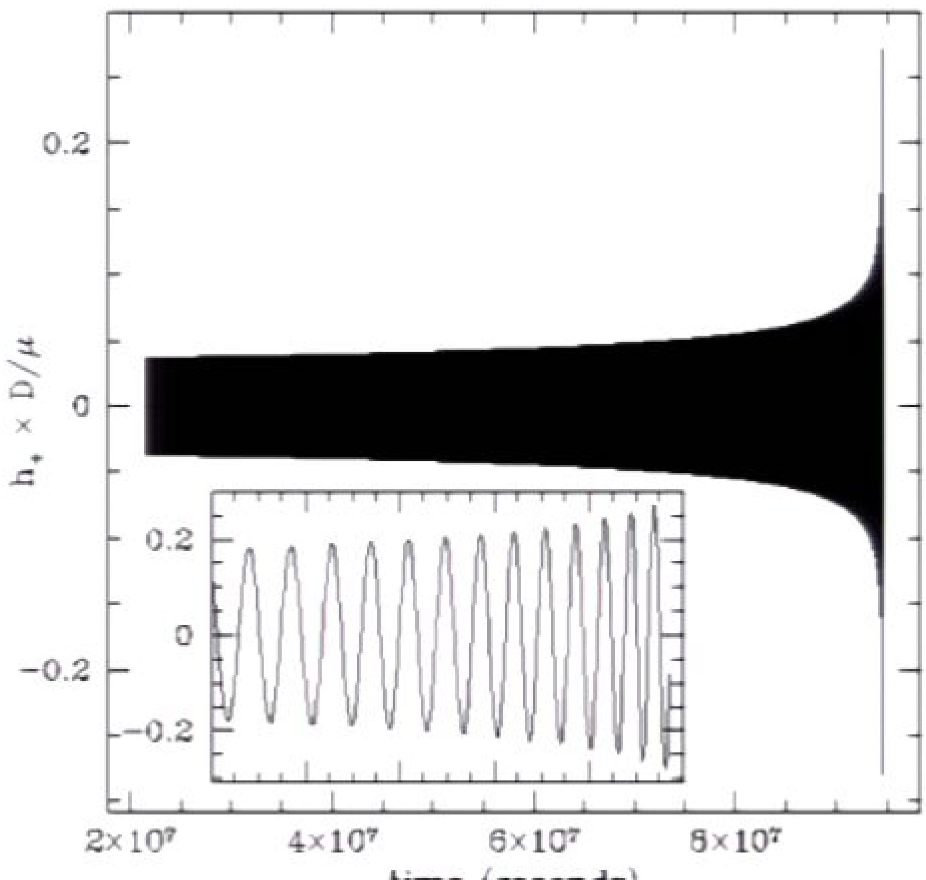
Gravitomagnetism



Video by Peter Reinhardt, MIT

Waveforms

Using the equations of motion and precession, not too difficult to build waveform describing two massive black holes spiralling together.



Example: Two non-spinning black holes.

Composer: Ryan Lang

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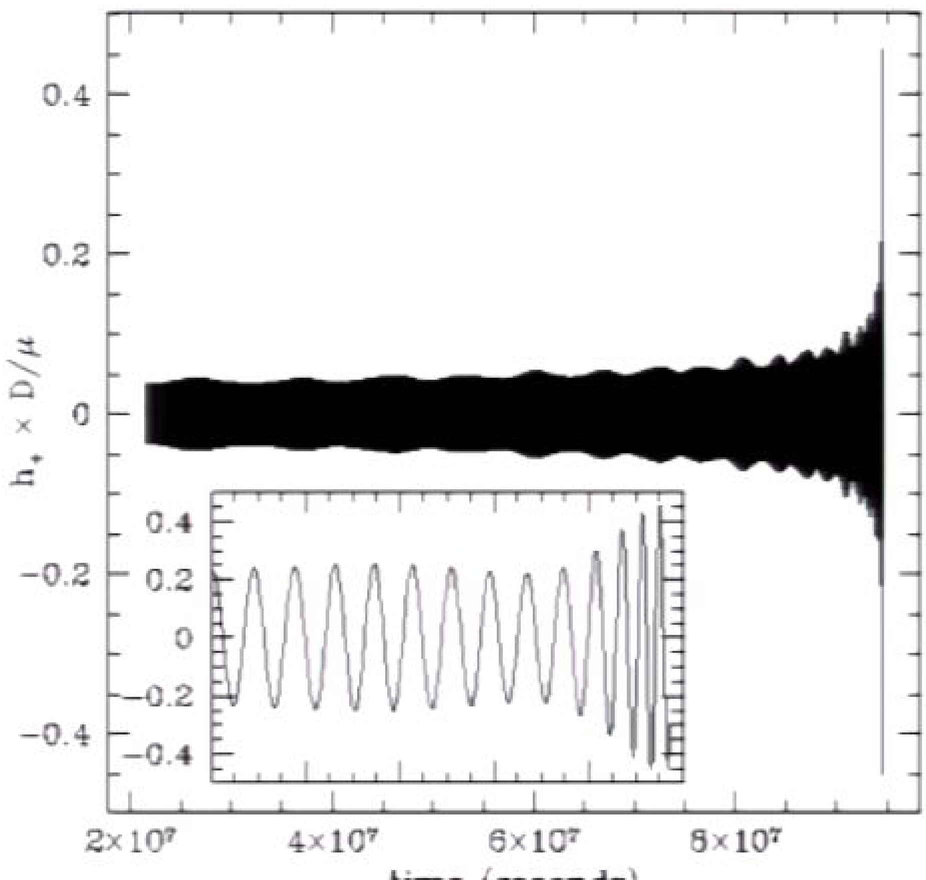
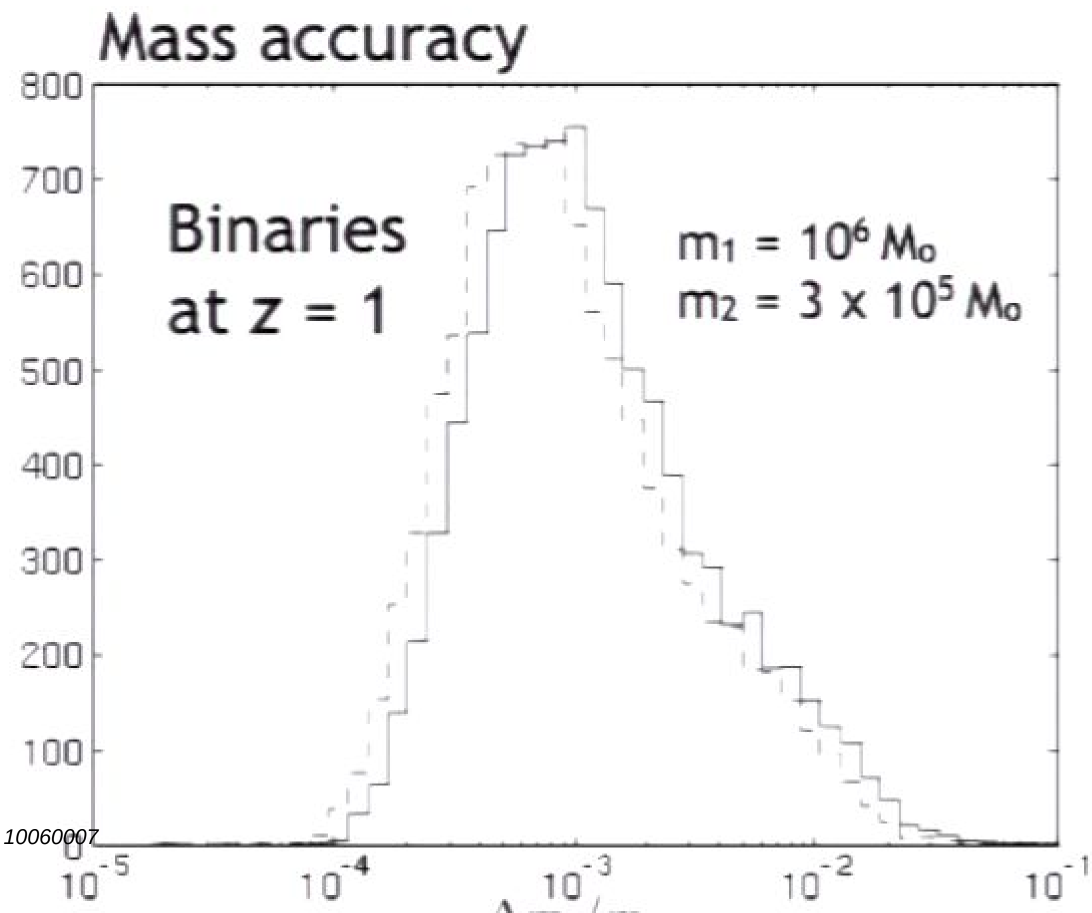


Example:
Two *rapidly* spinning black holes.

Composer:
Ryan Lang

Inspiral measurements

Analysis of parameter dependence, plus Monte Carlo exploration, lets us assess the kinds of measurement accuracies we should achieve:



Find masses are typically measured with 0.03—1% fractional precision

(Similar results at higher redshift, degrading as 1 over distance.)

Inspiral measurements

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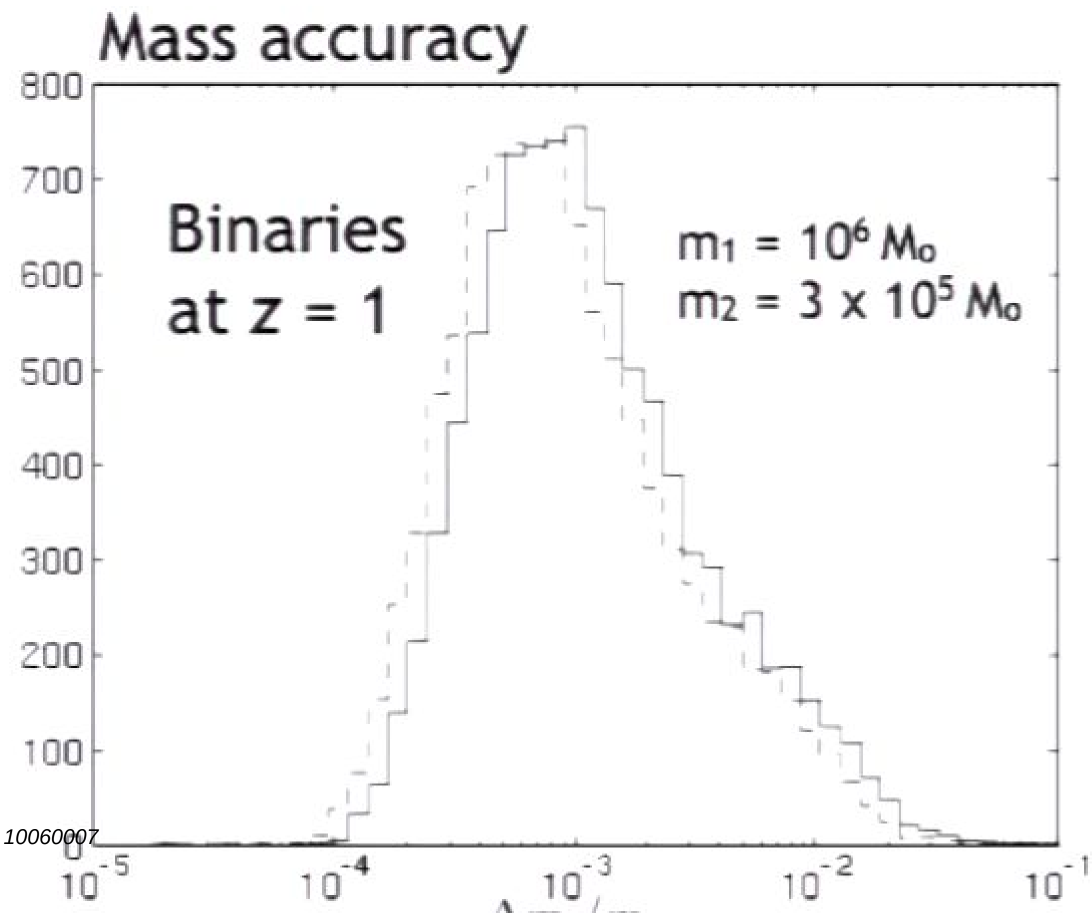
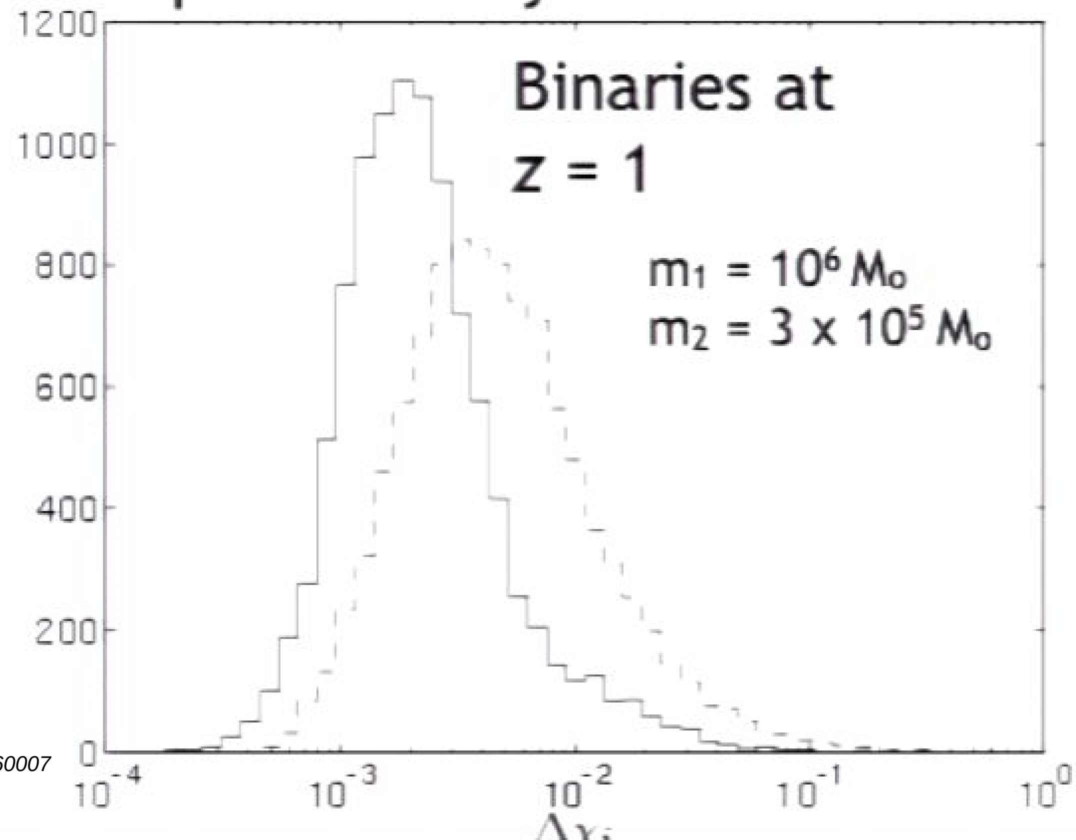


Current black hol
mass knowledge:
Best case, mass
known to ~10%
accuracy (Sgr-A*)
others, generally
known to a facto
of 2 — 3.

Inspiral measurements

More careful analysis, plus broad Monte Carlo exploration, lets us assess the kinds of measurement accuracies we should achieve:

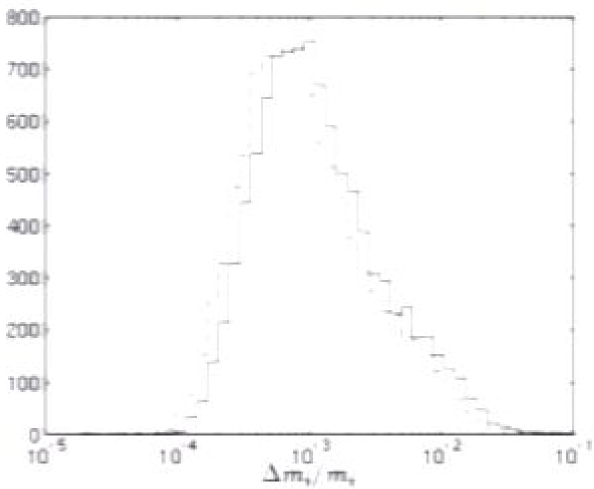
Spin accuracy

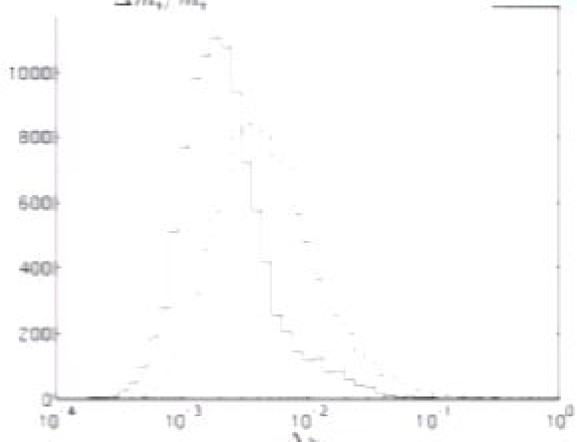


Find spins are measured with precision of roughly 0.1 – 10%.

Precision black hole physics

Measurement precision on mass and spin typically below percent level, even for cosmologically distant sources.





Since mass and spin totally characterize black holes, this allows us to trace their growth ... and thus to learn about the mergers of structures early in the universe.

Window onto early growth of structure in universe.

Precision distance measure

Waveform also gives us a direct measure of the distance to the wave's source:

$$h_+ = \frac{[G(1+z)\mathcal{M}/c^2]^{5/3}[\pi f(t)/c]^{2/3}}{D_L}\mathcal{F}(\text{angles})\cos[\Phi(t)]$$

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Waveform phase: Directly encodes mass and spins of binary's members.

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Portion of the amplitude dependent on angles which define binary's sky position and orientation ... pinned down by detector orbital motion and spin-induced precession of binary's members.

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Once angles and phase are known, distance to source is determined by measurement of the wave amplitude.

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Inspiralling binaries are a standard candle ("siren") ... standardized by GR.

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Detailed analysis: Distances typically measured with accuracy (a few)/(signal-to-noise)

Nearby (z ~ 1): δD/D ≈ 0.2 - 1% is typical

Distant (z ~ 5): δD/D ≈ 3 - 10% is typical

Problem: Redshift degeneracy

What we would really like to do: Simultaneously determine redshift and distance.

Problem: Masses & spins enter wave as *timescales*:

$$m \to \tau_m = Gm/c^3$$

$$a \equiv S/m \to \tau_s = S/mc^2$$

Timescales undergo cosmological redshift; inferred masses/spins likewise redshift.

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Redshift is degenerate with intrinsic binary parameters that we measure.

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Solution: Associate event with an electromagnetic counterpart.

Locating the merger

Big challenge: Identifying the host of the merger in a relatively large field.

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Good GW localization:
10 - 30 arcminutes by
3 - 10 arcminutes.

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Hubble
Deep Field!

Locating the merger

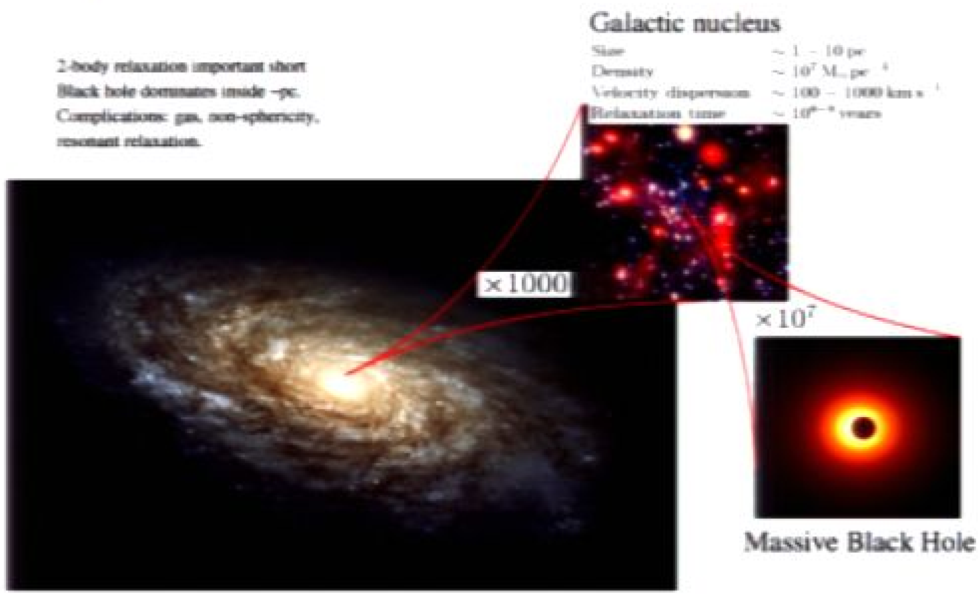
Very difficult to locate ***the*** host in a field that is 10 - 20 times larger than this!

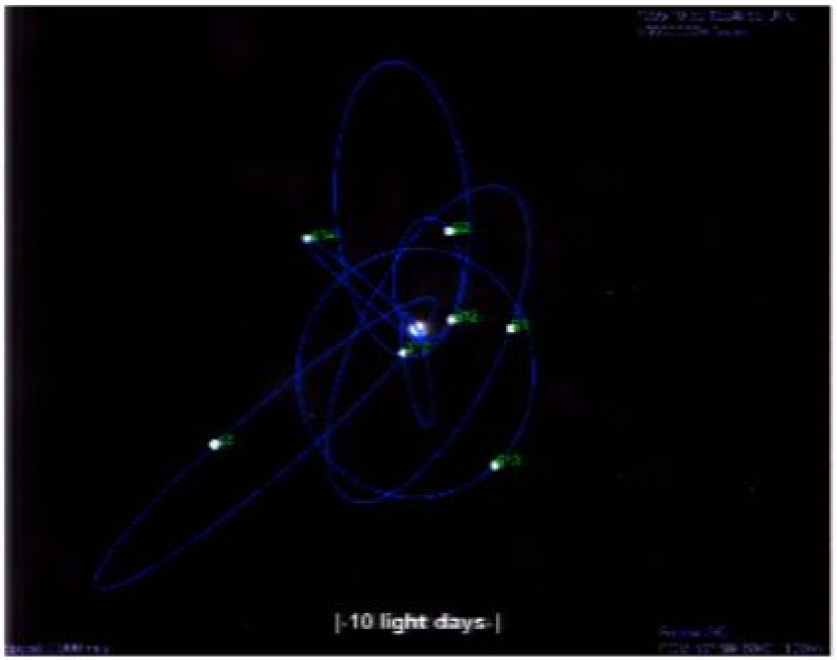
Hopefully something goes “boom”: Transient activity accompanies the GW merger (e.g.,

Palenzuela, Lehner, Liebling, Science, in press

Capture binaries

Another LISA source: The capture of stellar mass compact bodies by ~10^6 Msun black holes. Given black hole demographics & properties of galaxy centers, we expect dozens to hundreds of events per year.





Get "extreme mass ratio inspiral" ... or "EMRI"

Courtesy Max-Planck-Institut & Reinhard Genzel

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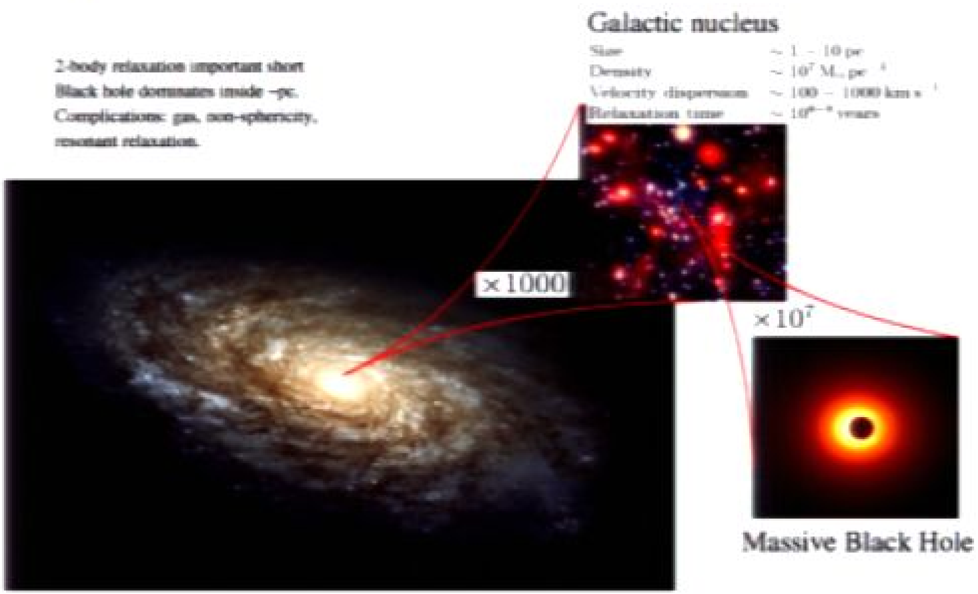

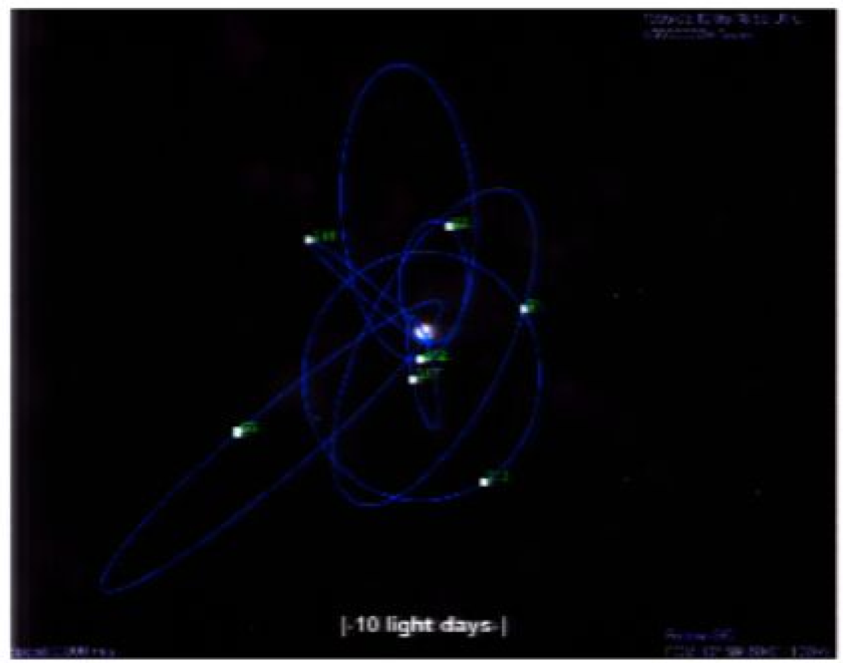

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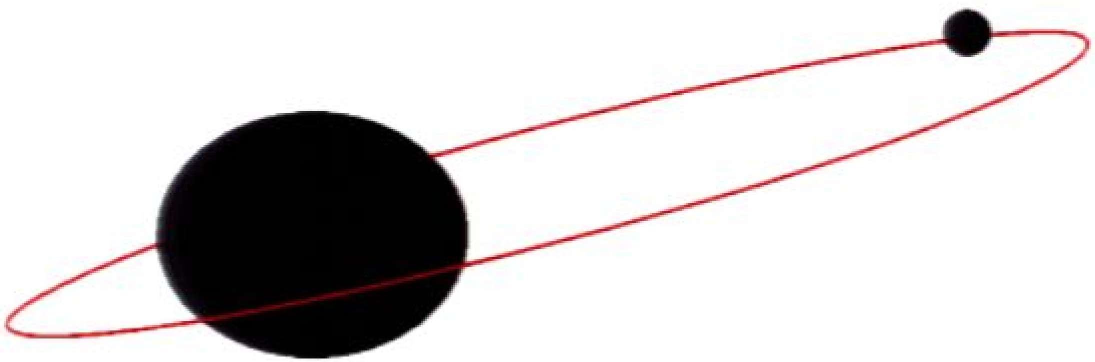
Perturbation theory

In the extreme mass ratio limit, spacetime dominated by the binary's larger member.



$$g_{\alpha\beta} = g^{\rm BH}_{\alpha\beta} + h_{\alpha\beta}$$

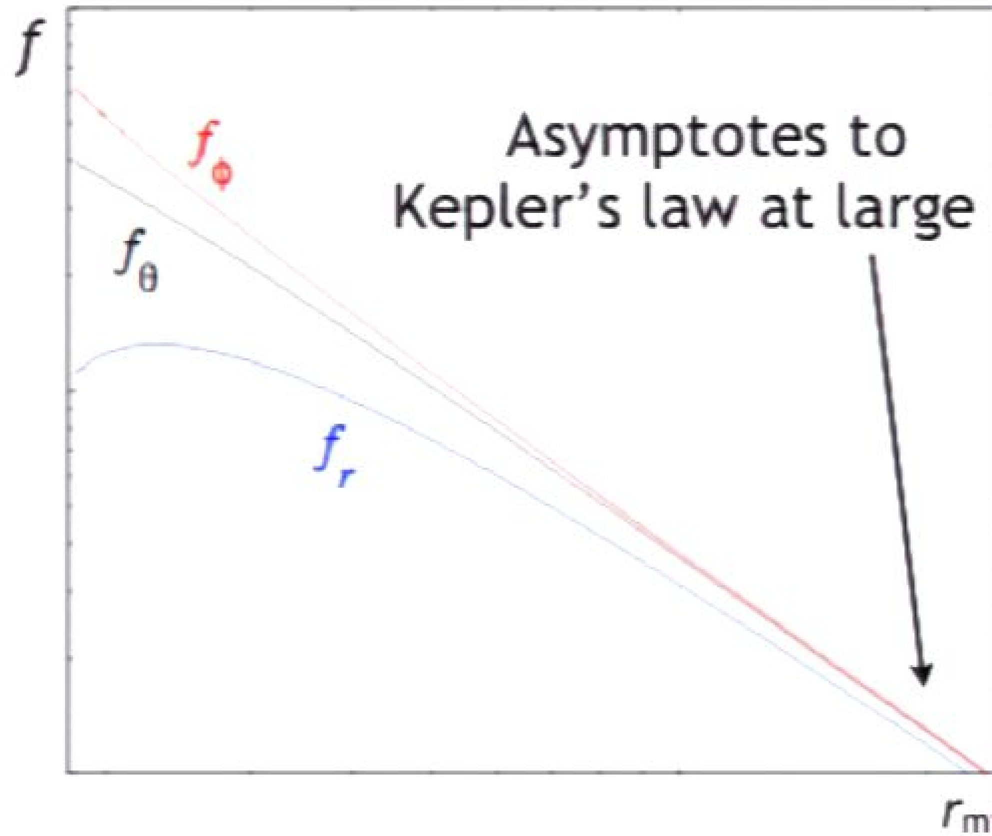
$$\frac{||h_{\alpha\beta}||}{||g^{\rm BH}_{\alpha\beta}||} \approx \frac{m}{M}$$

Expand Einstein field equations in mass ratio; develop system to describe how black hole spacetime is modified by small body.

Strong-field dynamics

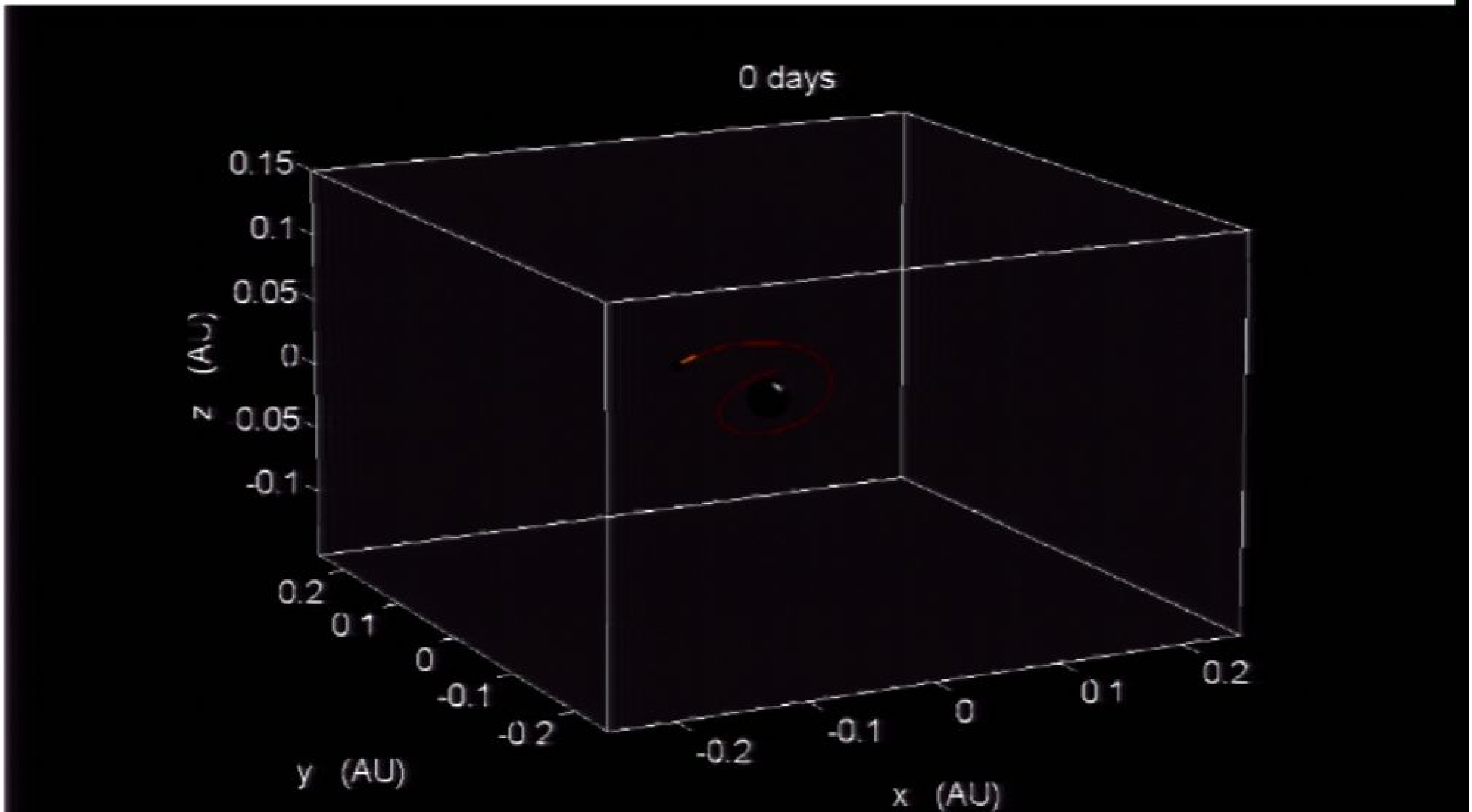
Strong-field character of black hole spacetime leaves a distinctive imprint on orbit and GW frequencies.

Large *r*: Frequencies lie on Kepler track.



As we move to small radius, frequencies split and become distinct: Strong gravity splits the Kepler "line."

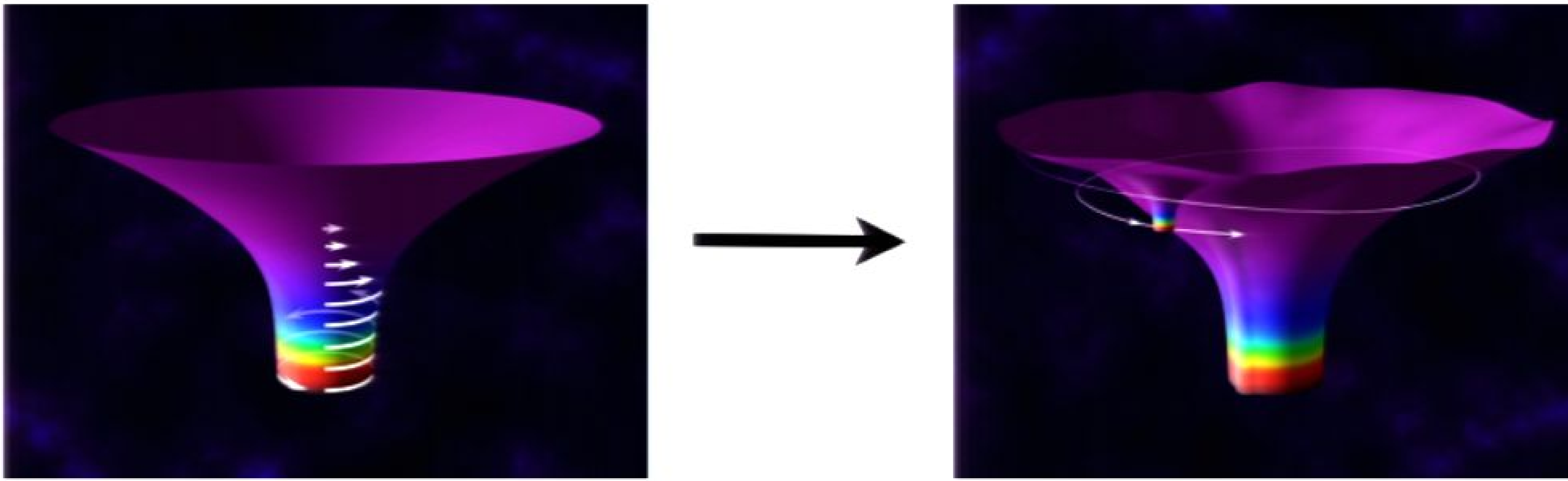
Illustration



Video by Peter Reinhardt, MIT

Self interaction

Small body deforms black hole's spacetime:



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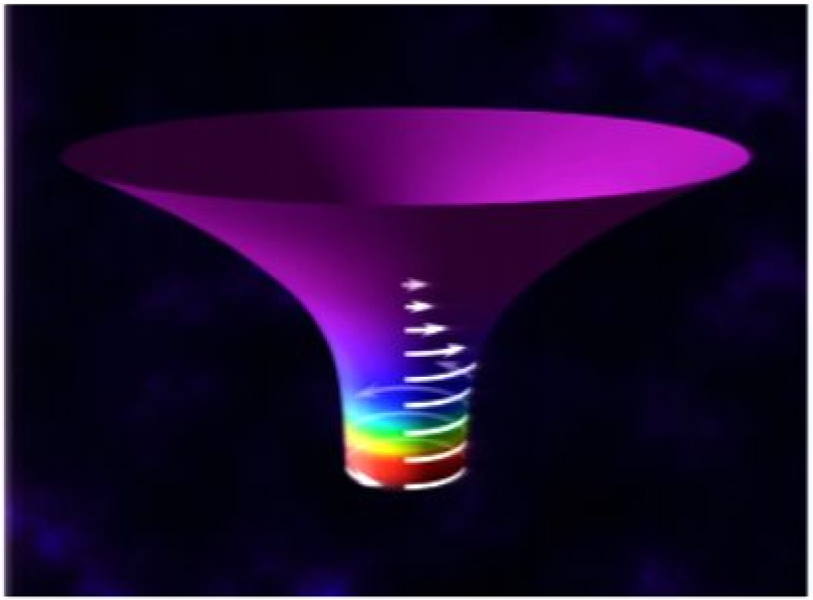

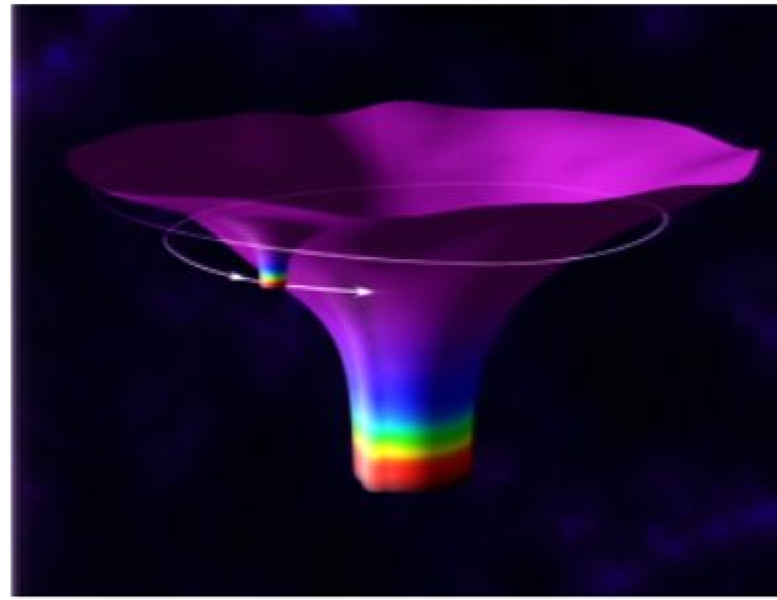

Calculate Einstein tensor from this deformed spacetime, require that it satisfy the identity

$$\nabla^\alpha G_{\alpha\beta} = 0$$

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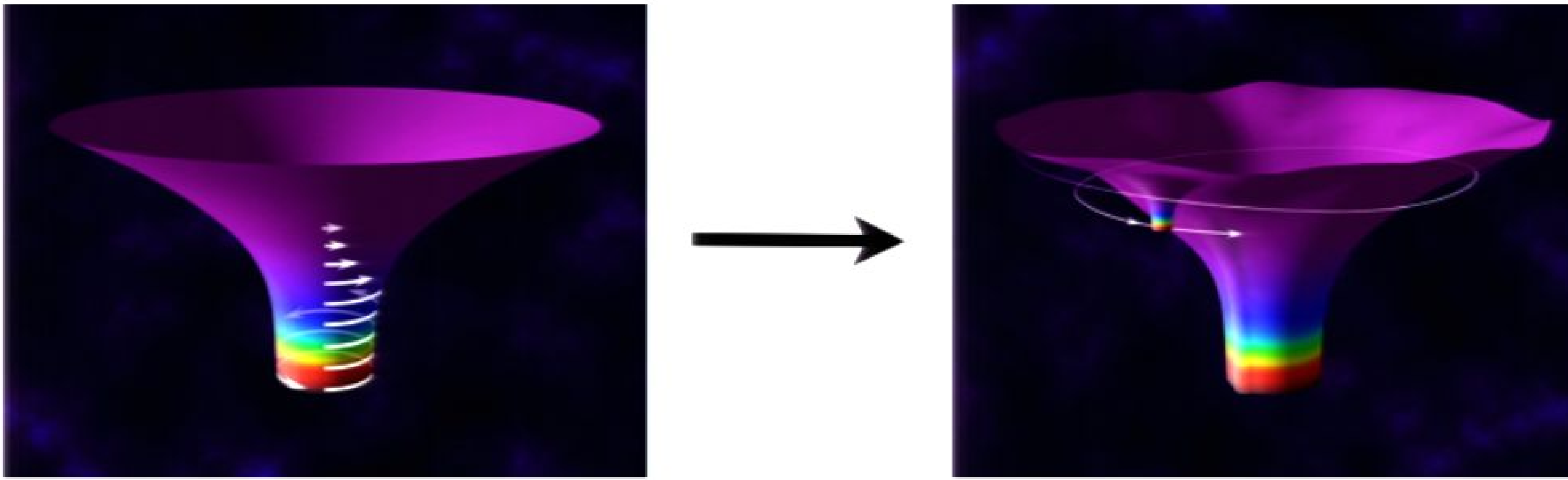

Result: Find that the small body interacts with its own spacetime deformation via a *self force* f^a:

$$\frac{d^2x^\alpha}{d\tau^2} + (\Gamma^\alpha{}_{\mu\nu})^{\text{Kerr}}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = f^\alpha$$

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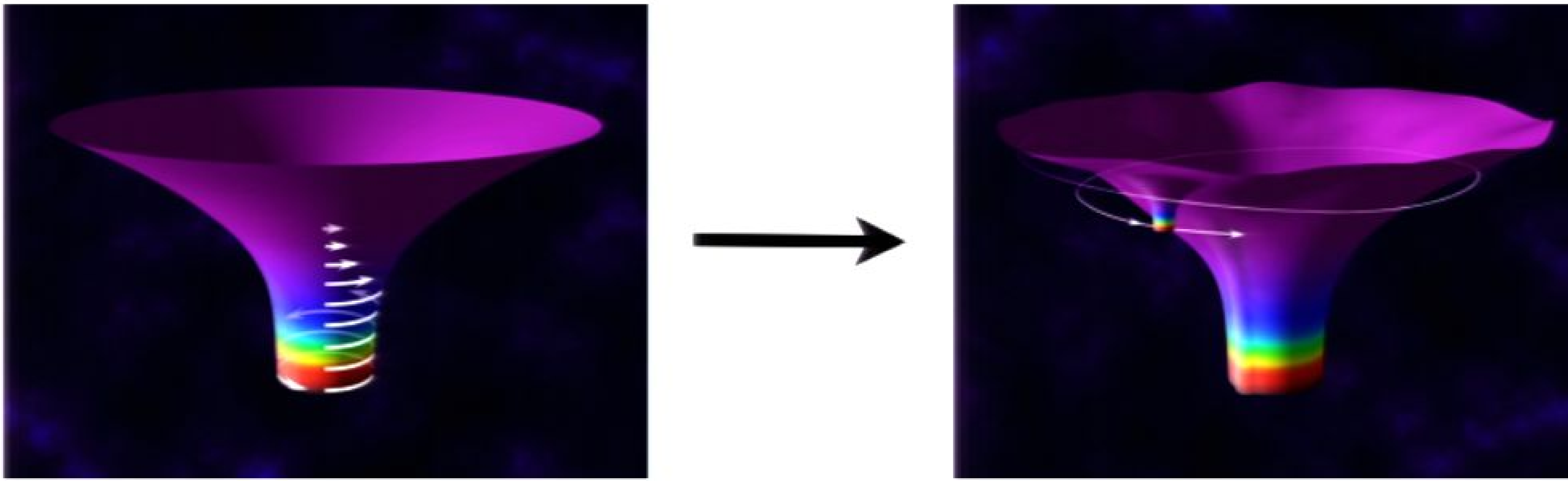


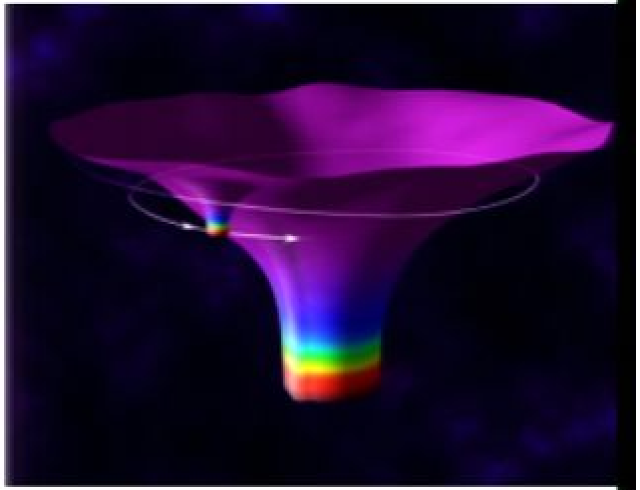
"MiSaTaQuWa" formalism for computing force f^a

(Mino, Sasaki, & Tanaka 1997; Quinn and Wald 1997)

$$\frac{d^2x^\alpha}{d\tau^2} + (\Gamma^\alpha{}_{\mu\nu})^{\mathrm{Kerr}}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = f^\alpha$$

Impact of self interaction

Self interaction has two major effects:



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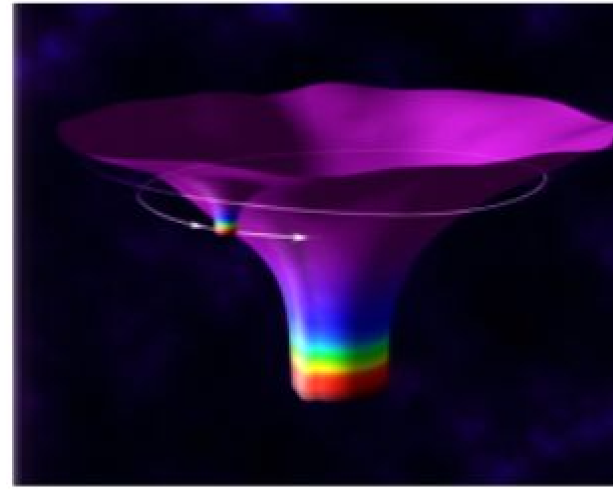

1. A *conservative* interaction, which modifies orbital frequencies:

$$\Omega_x \to \Omega_x + \delta\Omega_x \qquad \delta\Omega_x \propto \mu/M$$

Shift drives "anomalous" precessions, which leave an observable imprint.

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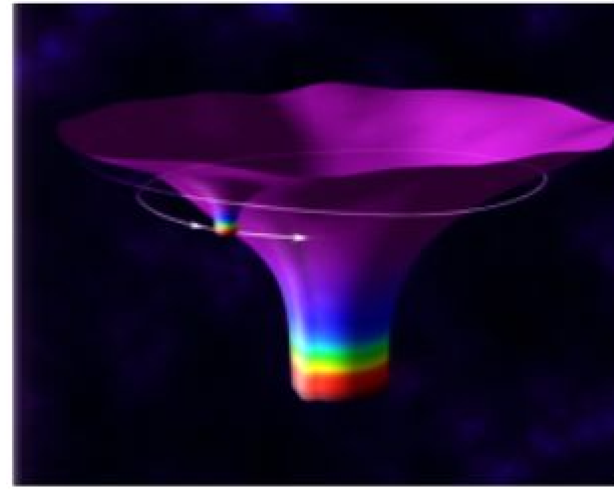

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2. A *dissipative* interaction, which causes frequencies to evolve ... equivalent to loss of energy and angular momentum from gravitational-wave emission.

Including gravitational waves

Perturbative nature makes this relatively easy:

We expand around a quiescent background, so

Einstein field equations simplify:

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta} \qquad \longrightarrow \qquad \mathcal{D}^2 h = \mathcal{T}$$

$$\nabla^\alpha G_{\alpha\beta} = 0$$

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Linear operator

Perturbation to spacetime

Source describing small body

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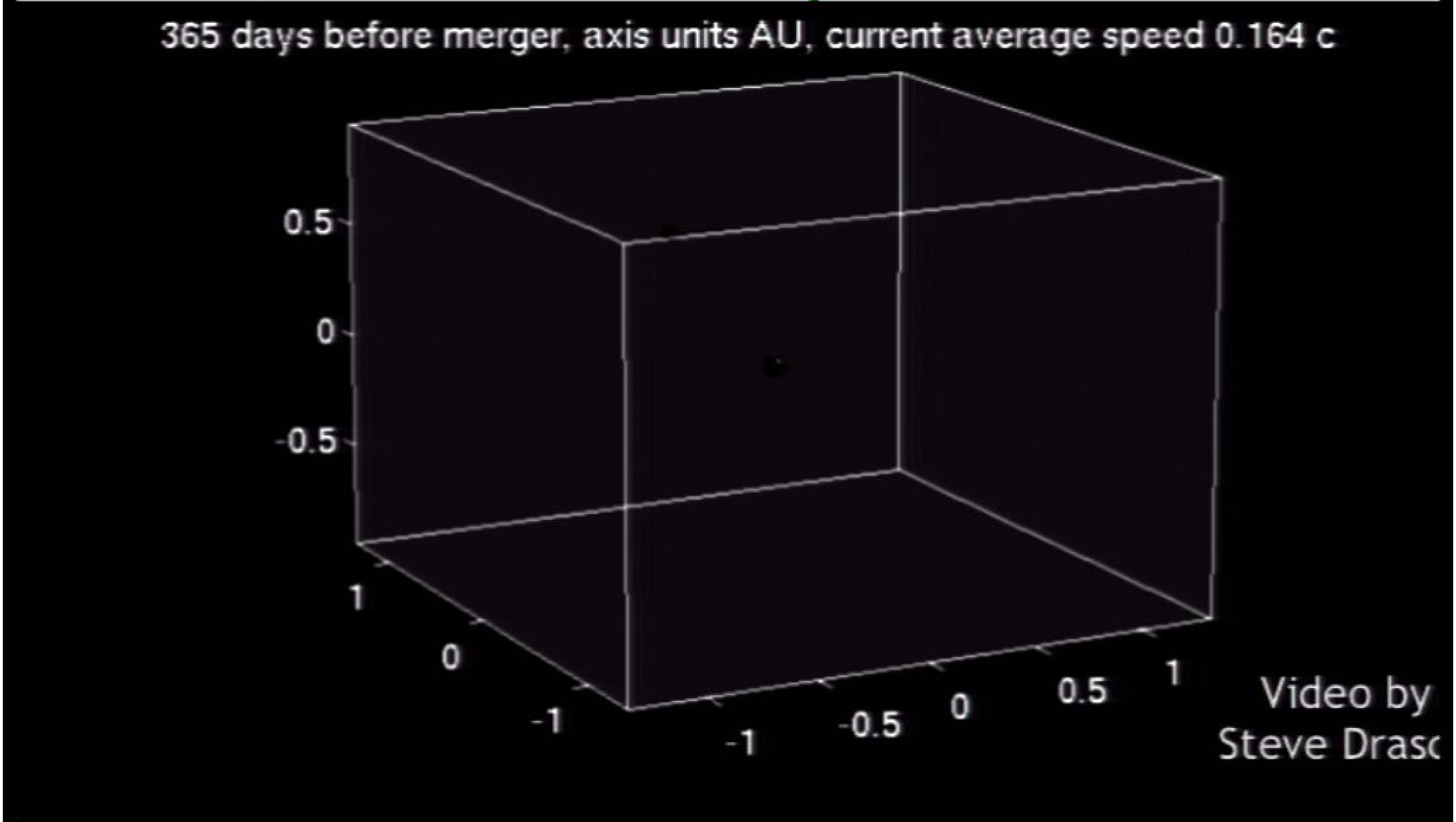
Linear operator

Perturbation to spacetime

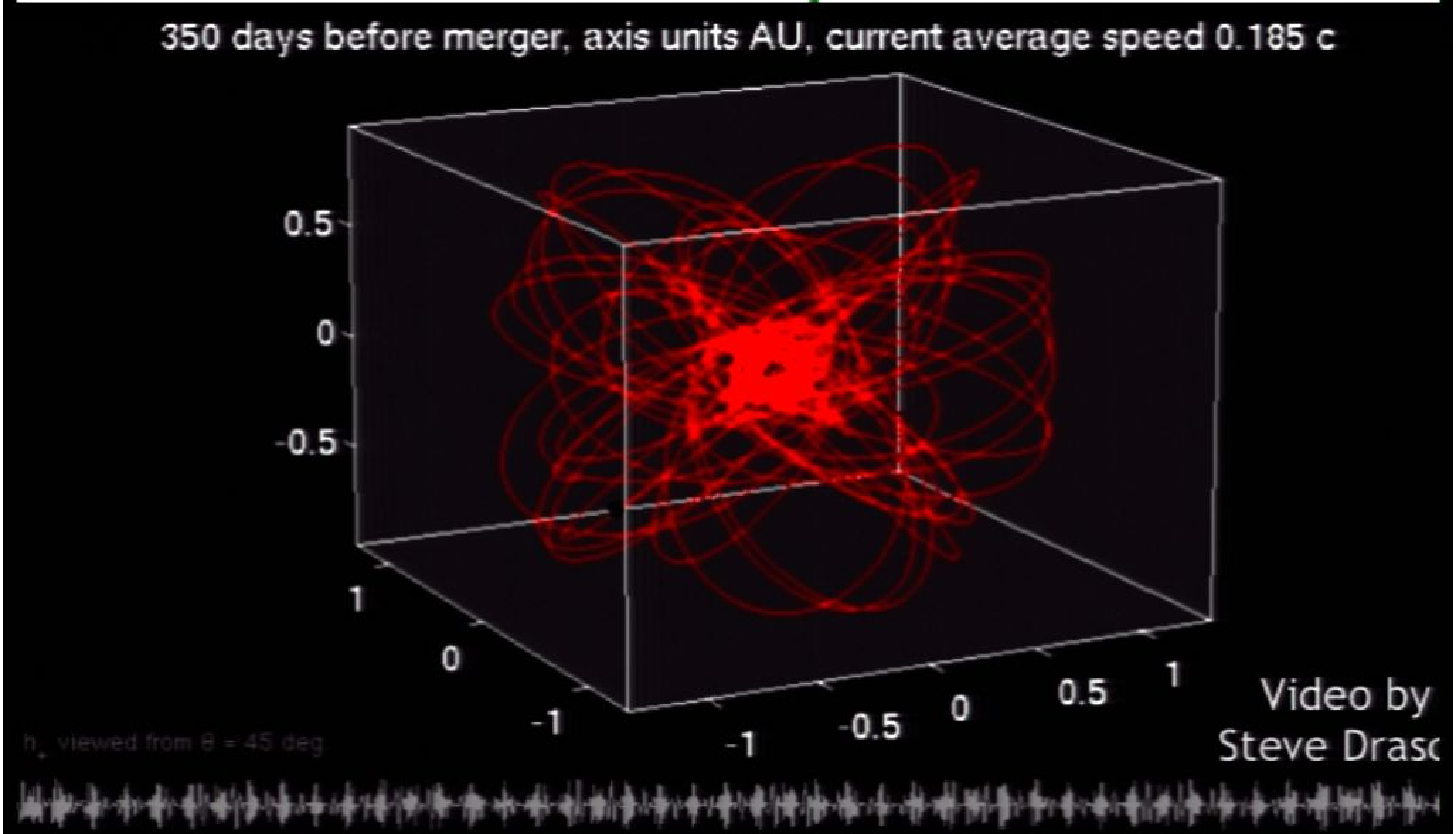
Source describing small body

Hope is that perturbative nature means we can solve this very precisely, build phase-coherent models of inspiral over ~10^5 orbits.

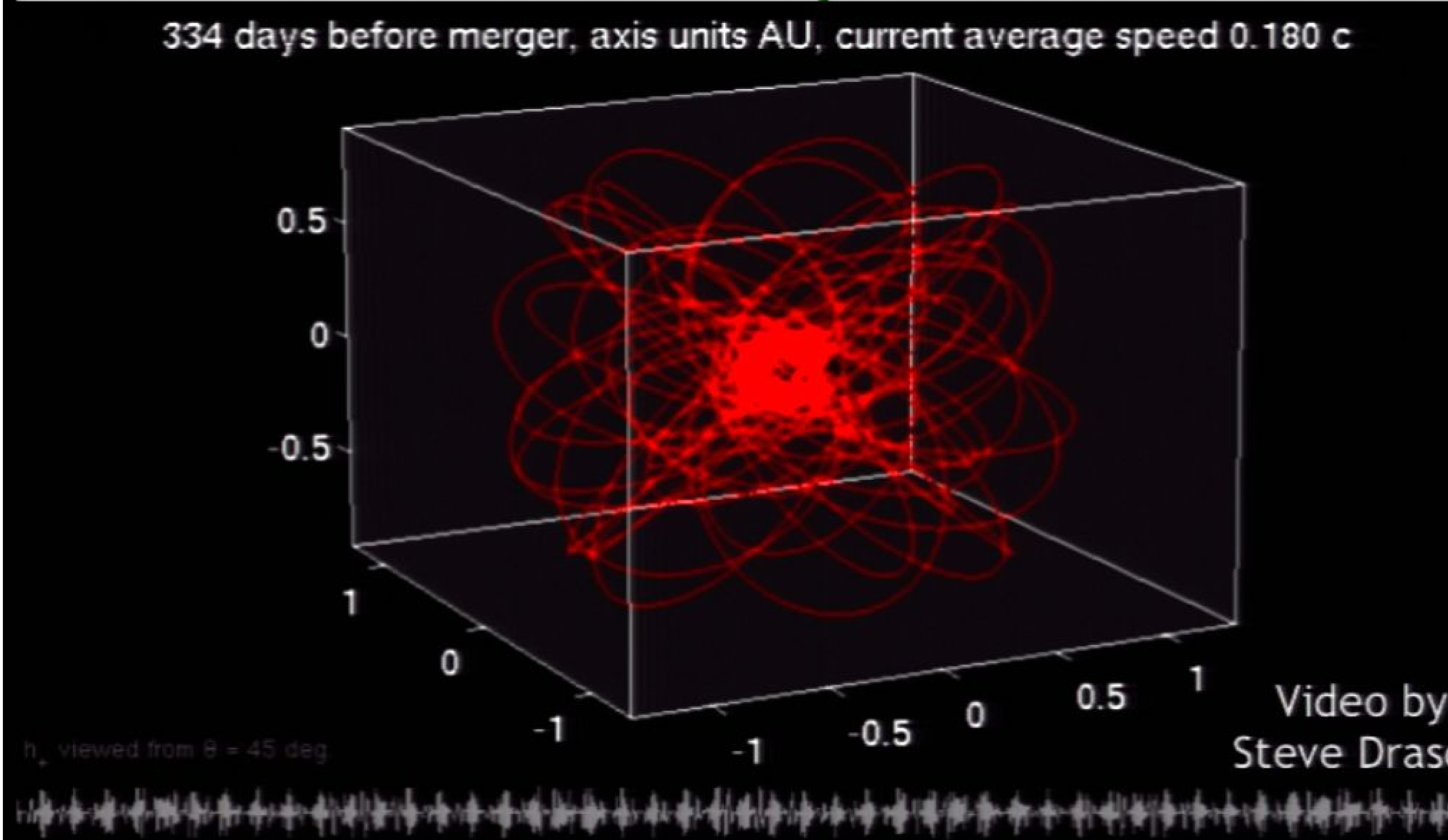
Example


365 days before merger, axis units AU, current average speed 0.164 c
0.5
0
-0.5
1
0
-1
-1
-0.5
0
0.5
1
Video by
Steve Drasc

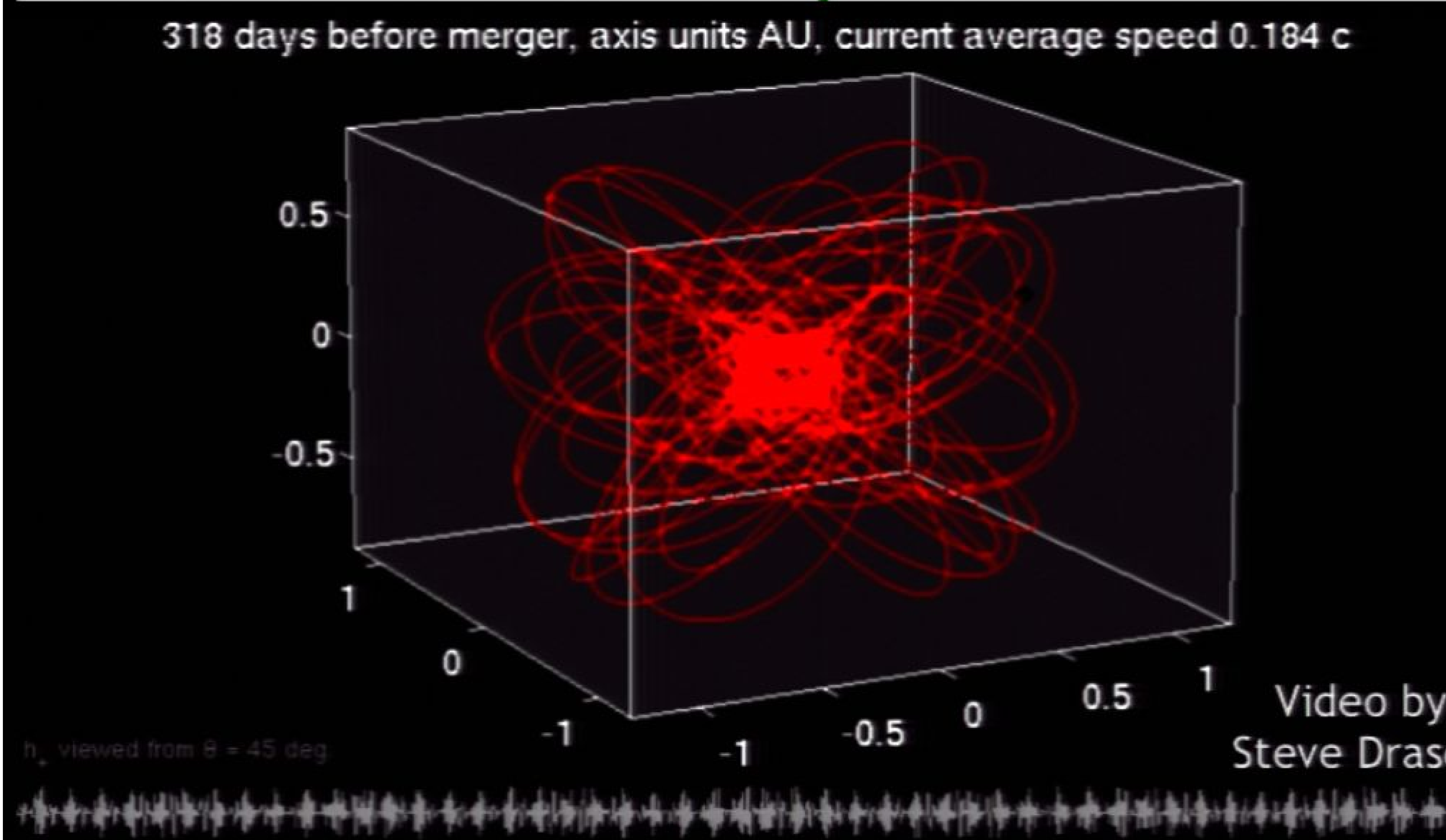
Example


350 days before merger, axis units AU, current average speed 0.185 c
0.5
0
-0.5
1
0
-1
-1
-0.5
0
0.5
1
Video by
Steve Drasc

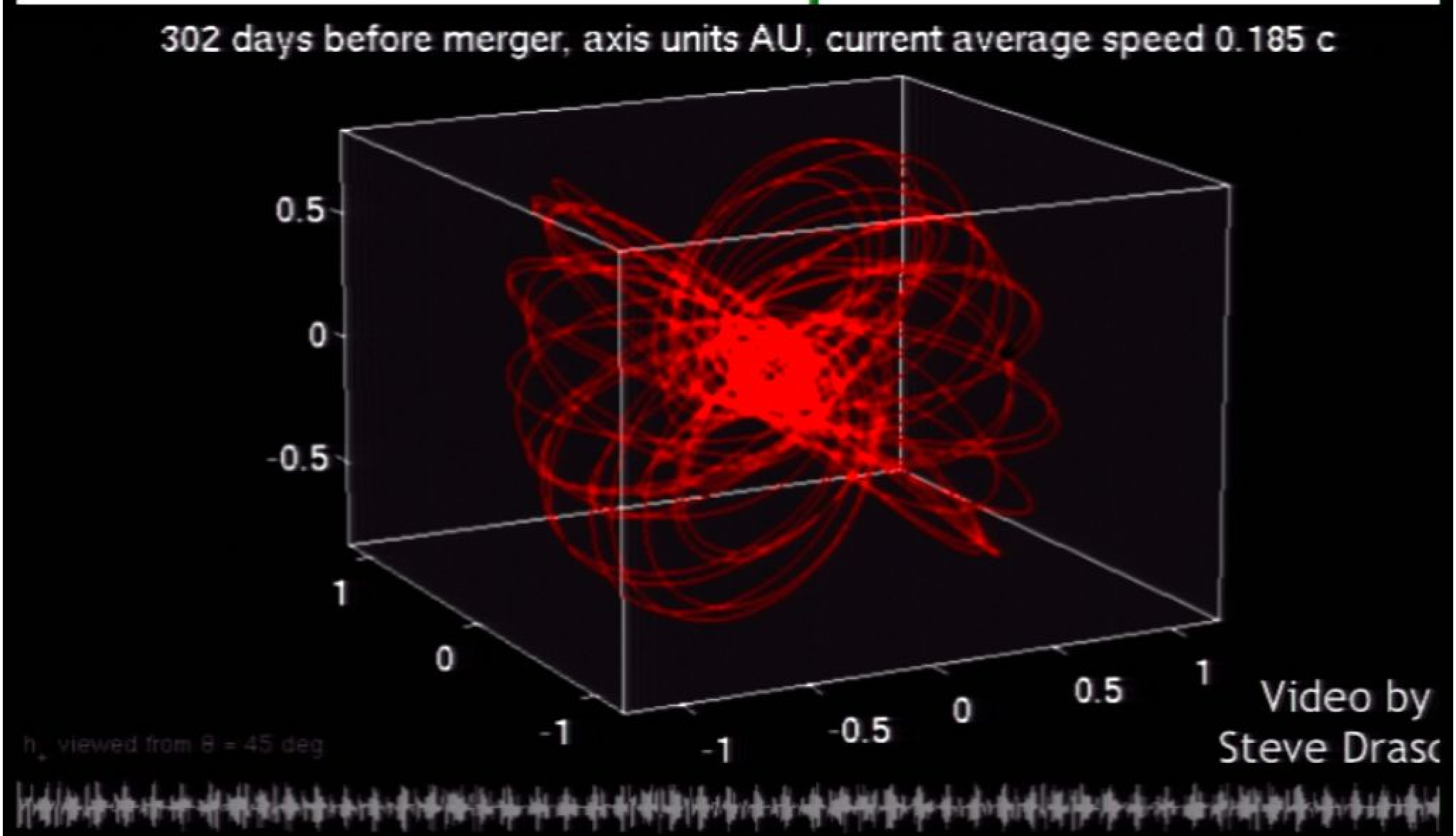
Example


334 days before merger, axis units AU, current average speed 0.180 c
0.5
0
-0.5
1
0
-1
-1
-0.5
0
0.5
1
h+ viewed from θ = 45 deg
Video by
Steve Drasc

Example


318 days before merger, axis units AU, current average speed 0.184 c
0.5
0
-0.5
1
0
-1
-1
-0.5
0
0.5
1
Video by
Steve Drasc

Example


302 days before merger, axis units AU, current average speed 0.185 c
0.5
0
-0.5
1
0
-1
-1
-0.5
0
0.5
1
Video by
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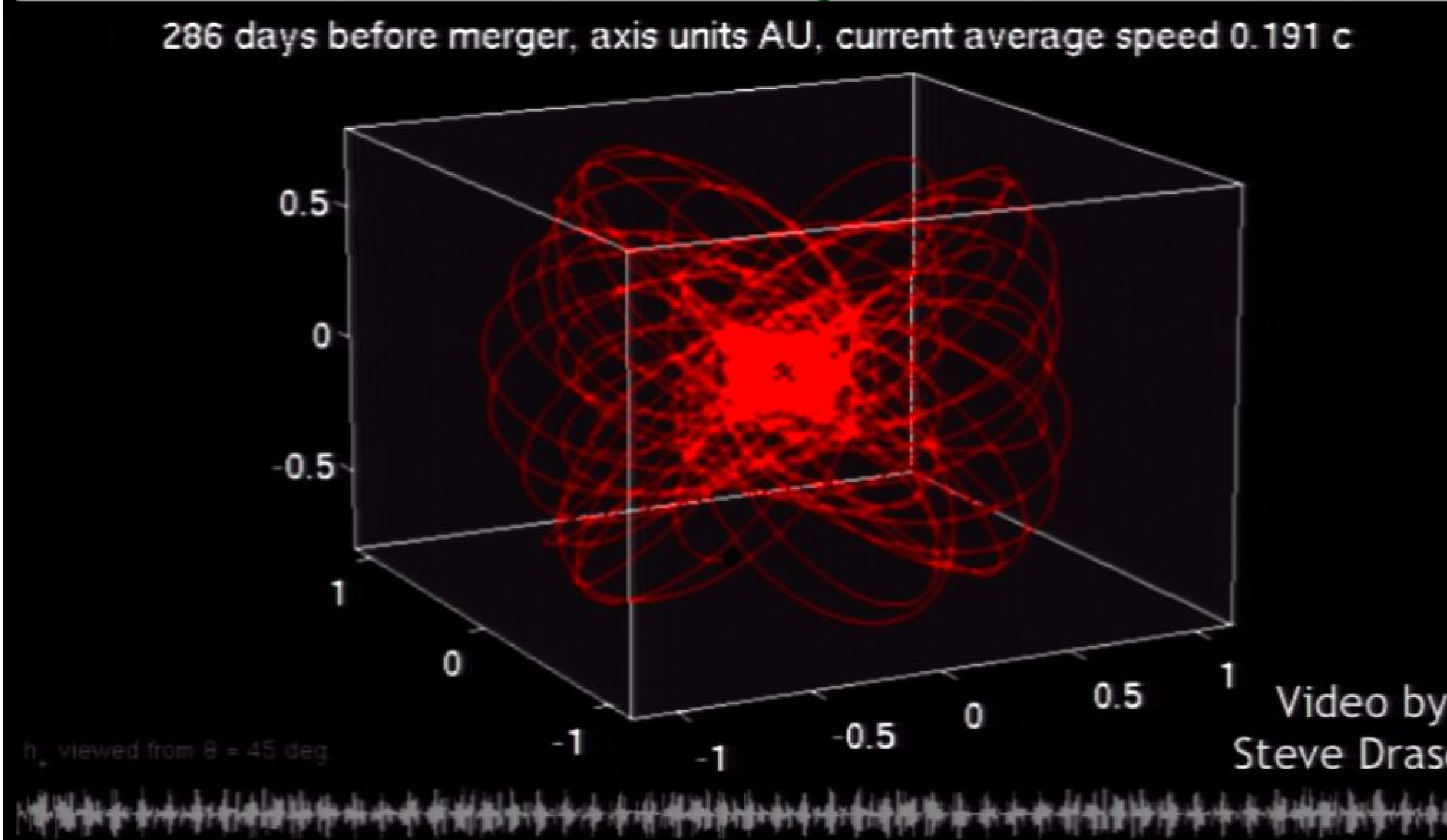
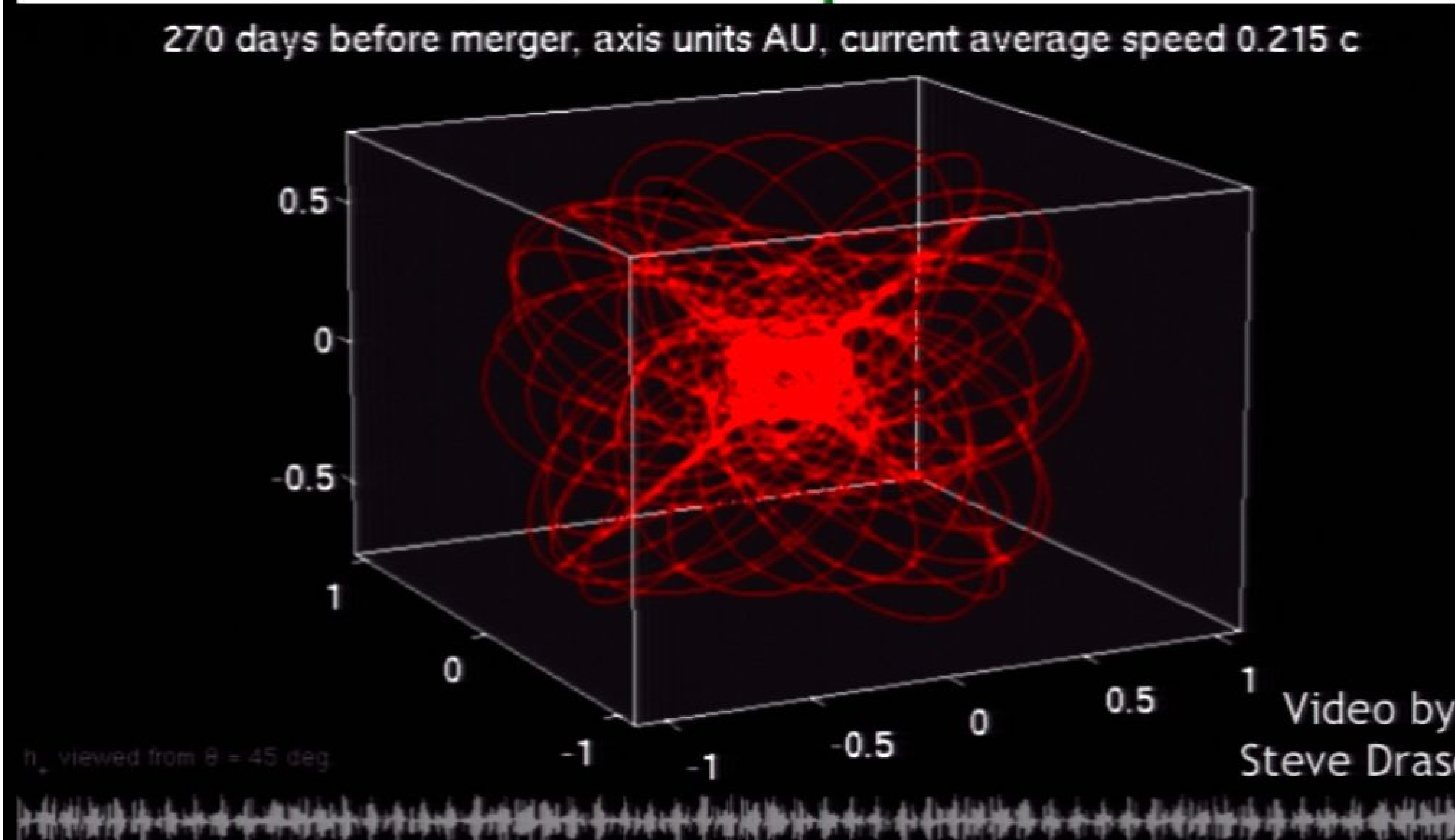
Example



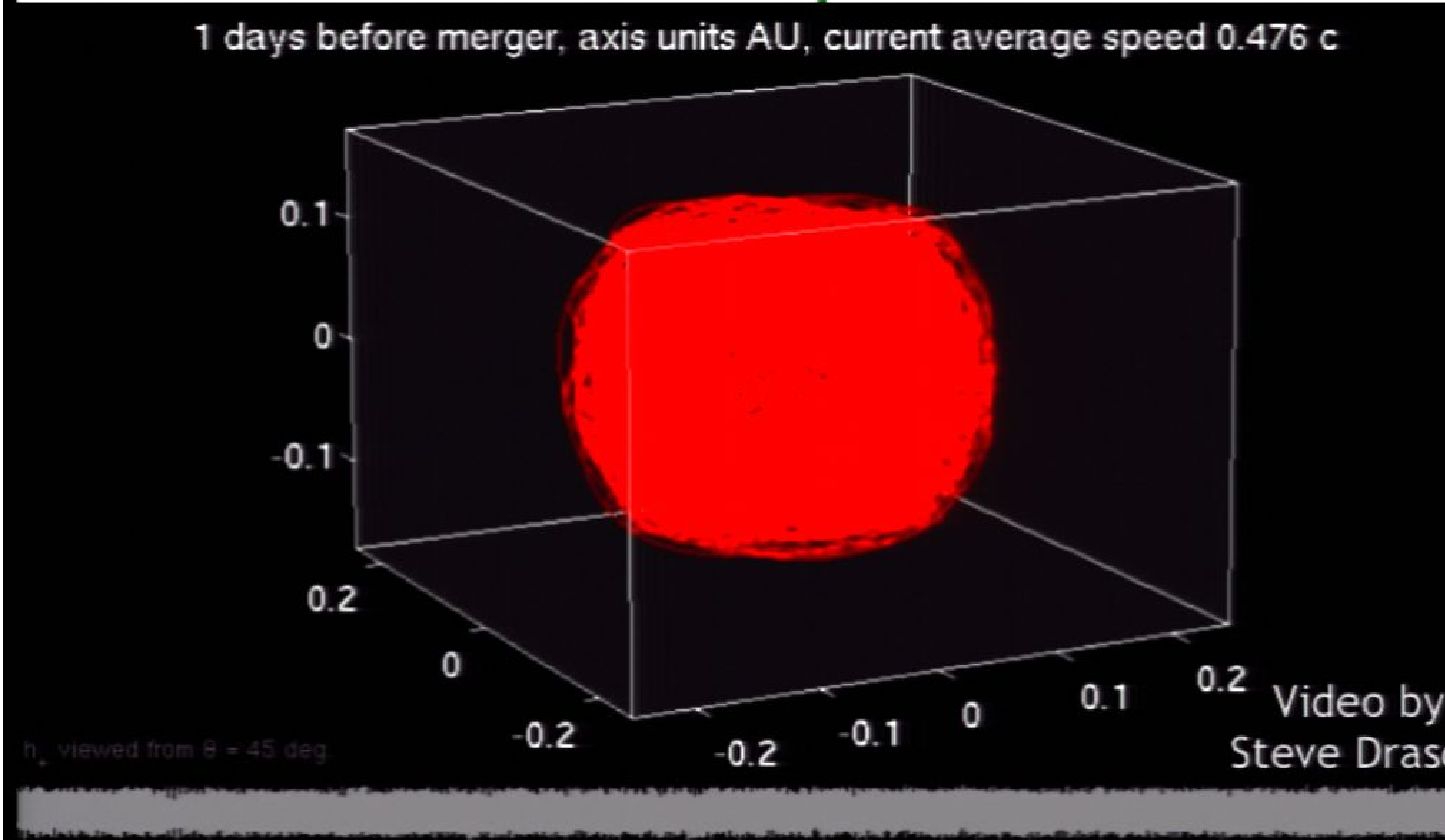
286 days before merger, axis units AU, current average speed 0.191 c
0.5
0
-0.5
1
0
-1
-1
-0.5
0
0.5
1
Video by
Steve Drasc

Example



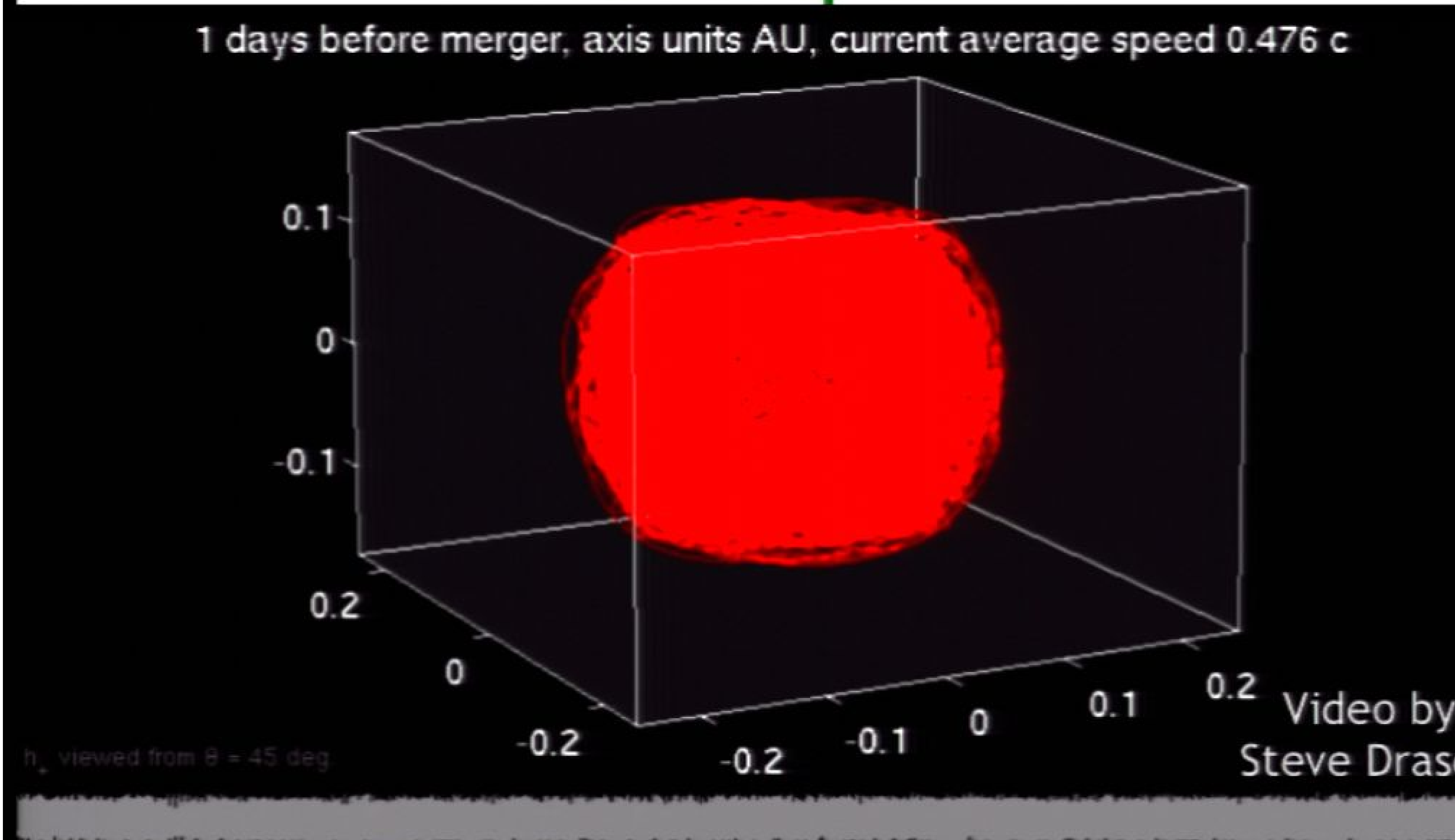
270 days before merger, axis units AU, current average speed 0.215 c
0.5
0
-0.5
1
0
-1
-1
-0.5
0
0.5
1
Video by
Steve Drasc

Example



1 days before merger, axis units AU, current average speed 0.476 c
0.1
0
-0.1
0.2
0
-0.2
-0.2
-0.1
0
0.1
0.2
Video by
Steve Drasc

Example


1 days before merger, axis units AU, current average speed 0.476 c
0.1
0
-0.1
0.2
0
-0.2
-0.2
-0.1
0
0.1
0.2
Video by
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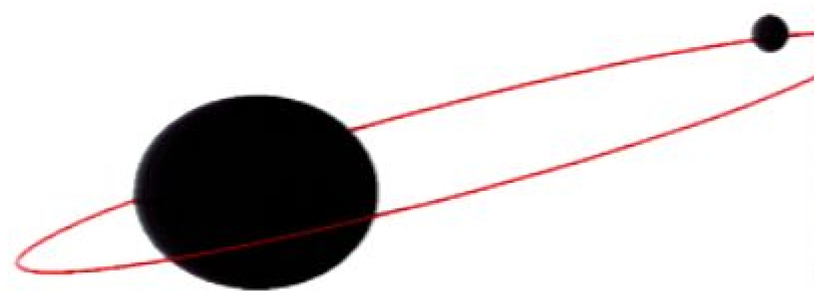
Application

Testing the black hole hypothesis by observing strong field orbits of candidate black holes.

Waves from an extreme mass ratio binary depend most strongly on the properties of the larger member of the binary ... which is presumably a Kerr black hole.



Goal of these observations: Use precision measurements of their waves to *test* the hypothesis that they are Kerr black holes.

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity provides the ***absolutely exact representation*** of untold numbers of black holes that populate the universe."

Subramanyan Chandrasekhar (Nobelist 1983), The Nora and Edward Ryerson Lecture (U. Chicago), 22 April 1975.

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In practice,
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(Wisdom of the Internet)

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How?

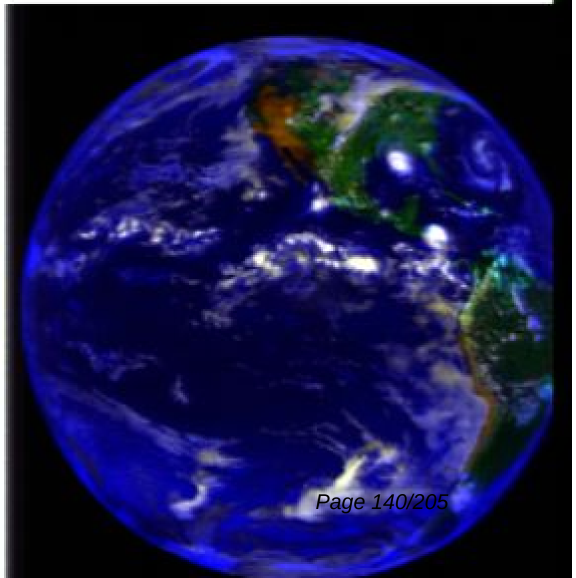
How do we actually go about formulating a tes
of spacetime nature with such measurements?

Conceptual framework similar to *geodesy*.

Expand Earth's potential in spherical harmonics:

$$\Phi = -\frac{GM}{r} + \frac{GM}{R}\sum_{lm}\left(\frac{R}{r}\right)^{l+1} B_{lm}Y_{lm}(\theta,\phi)$$

B coefficients determine the potential's "shape." Mapped by satellite orbits.



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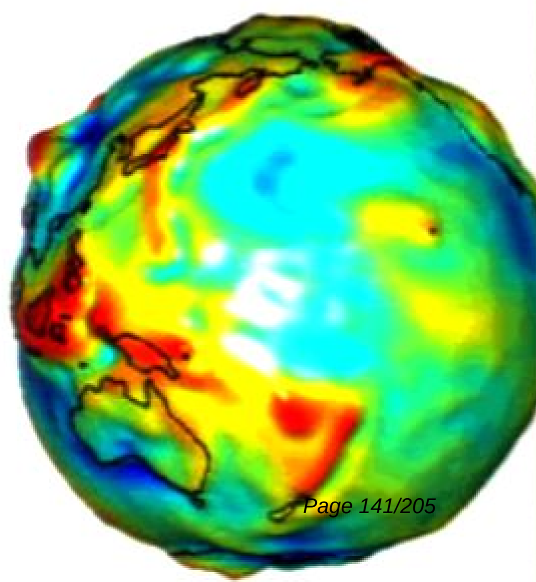

Distribution of matter as inferred by measurements with GRACE, a recent geodesy mission.

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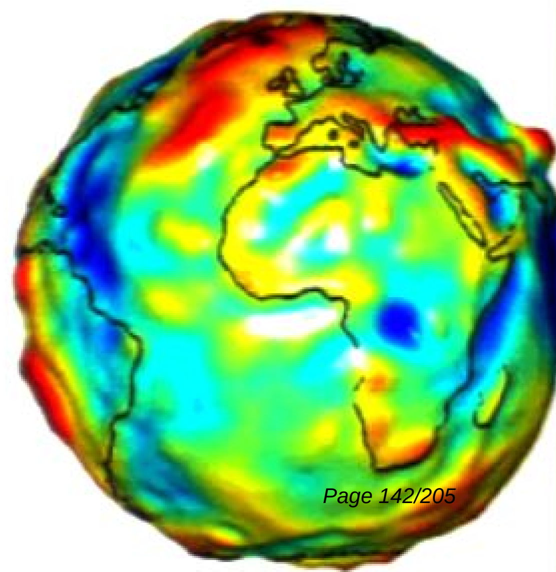

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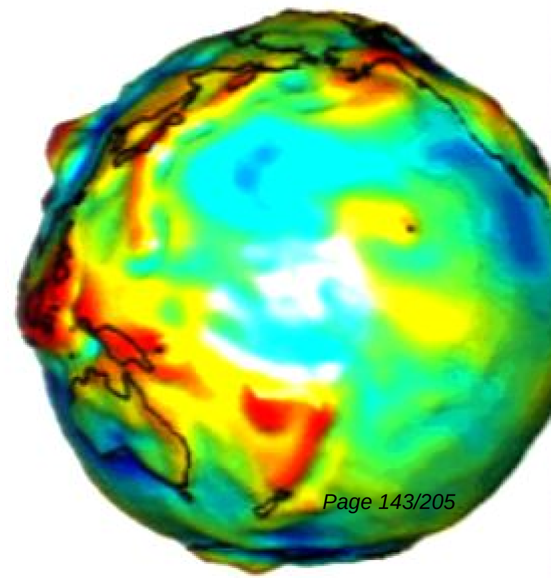

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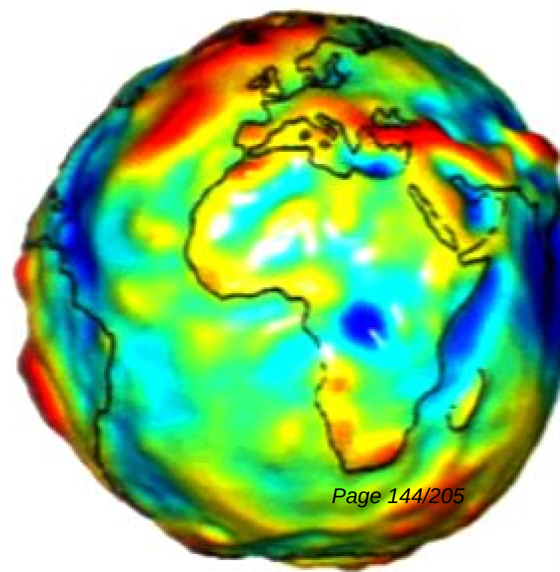

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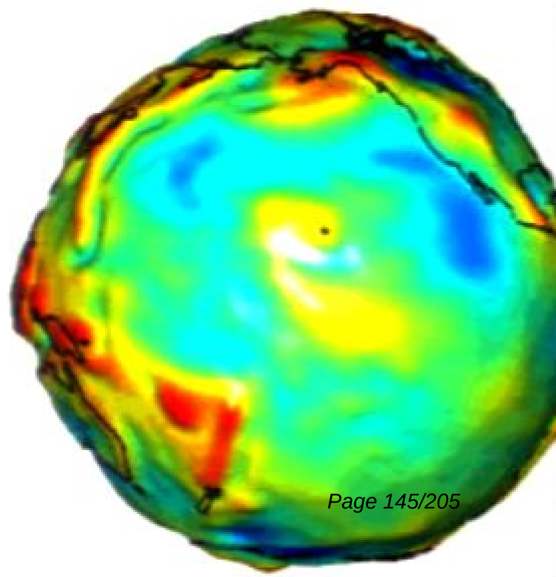

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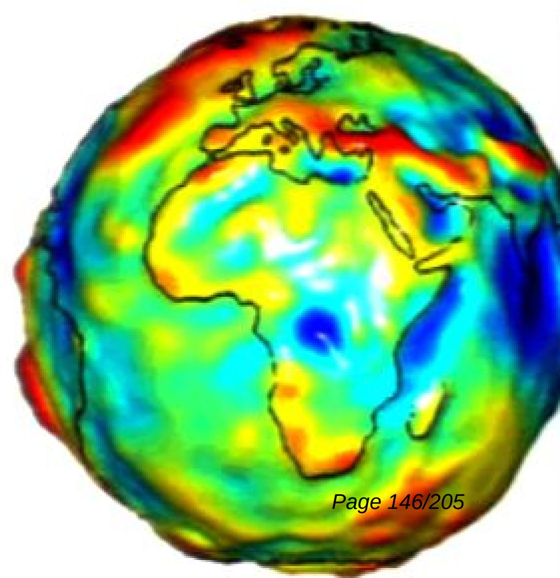

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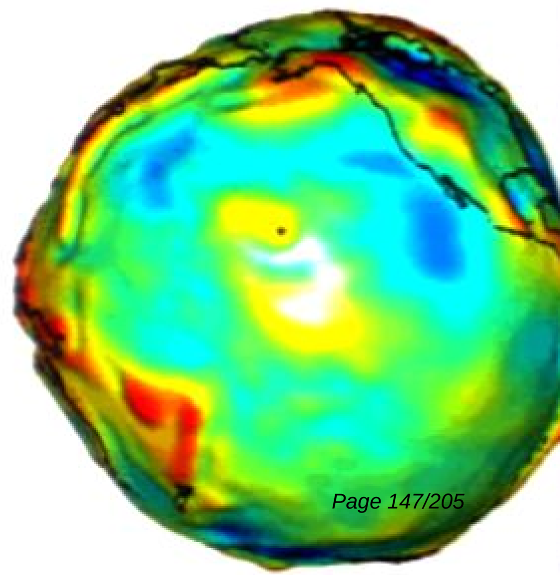

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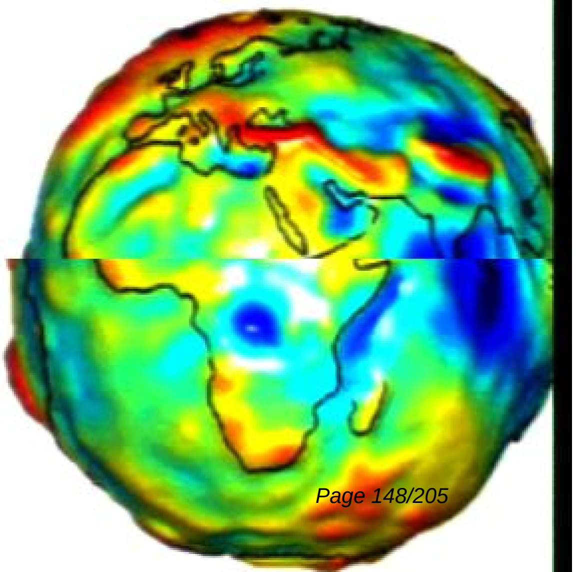

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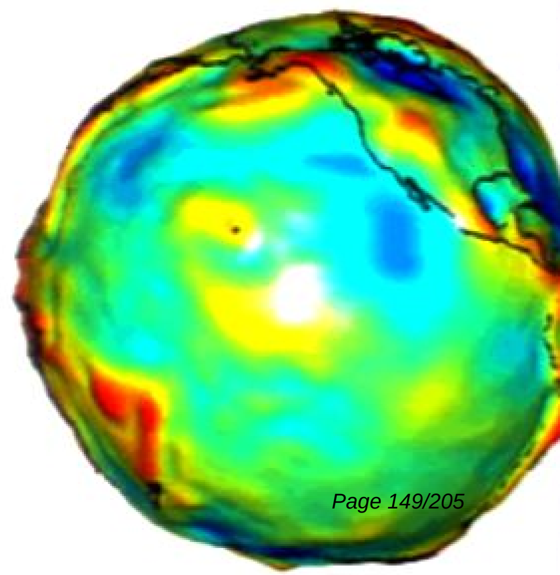

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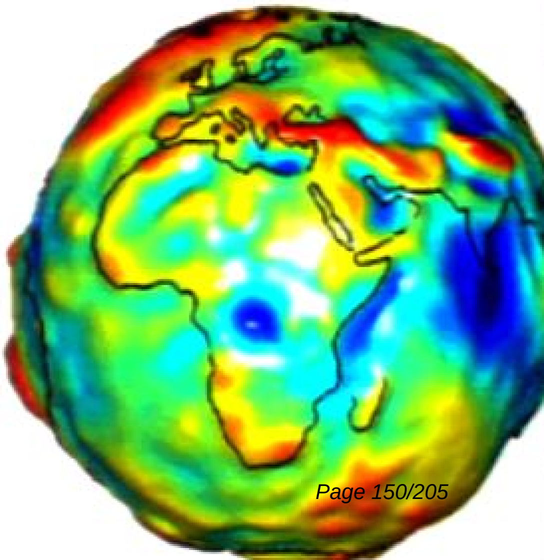

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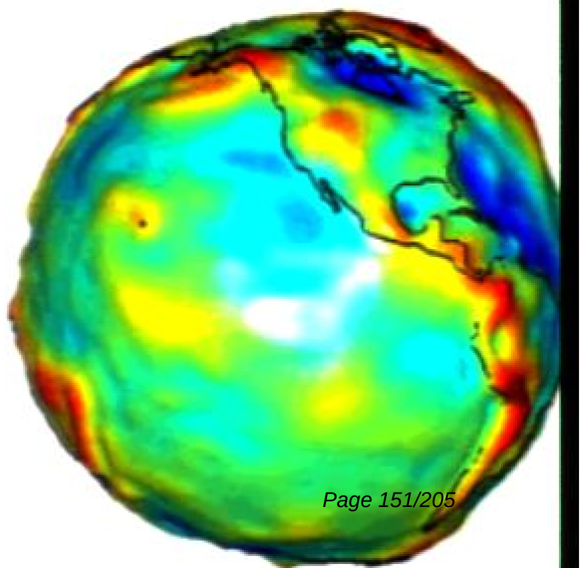

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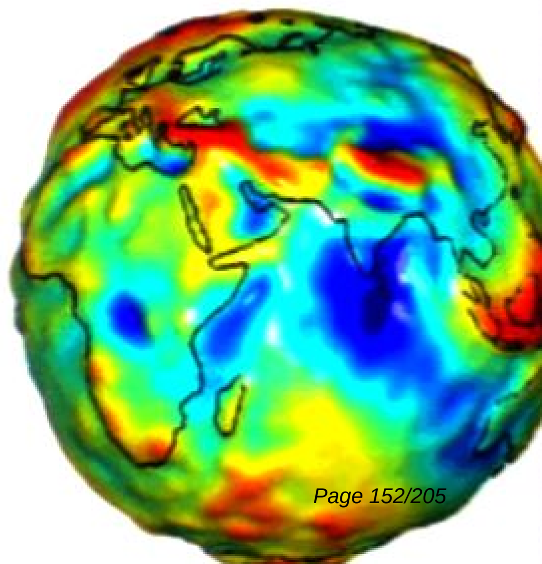

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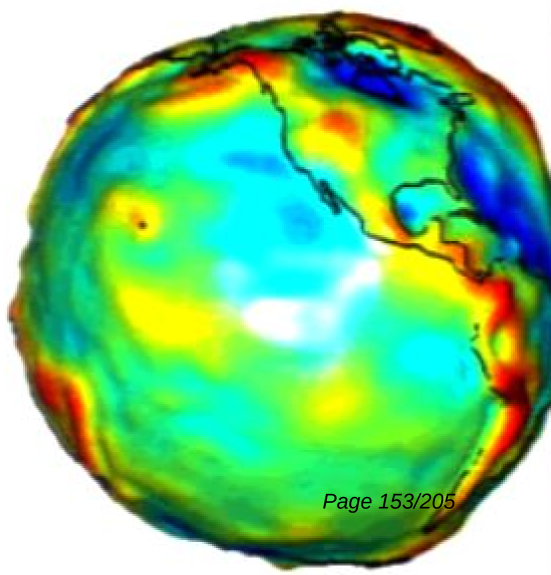

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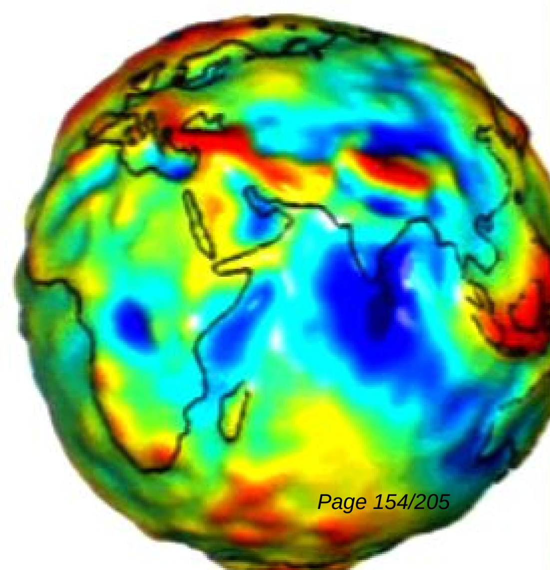

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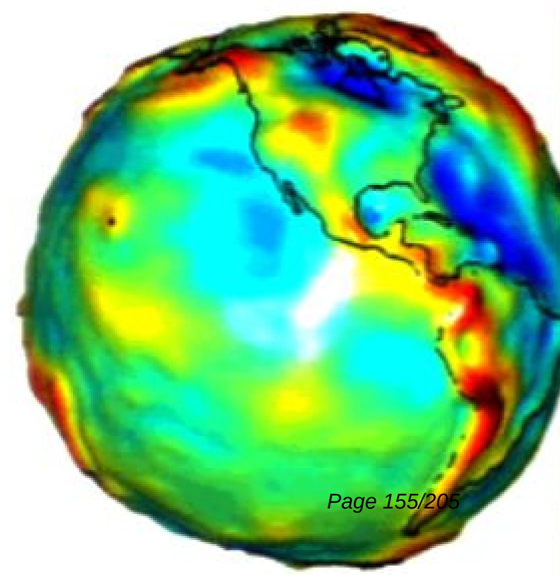

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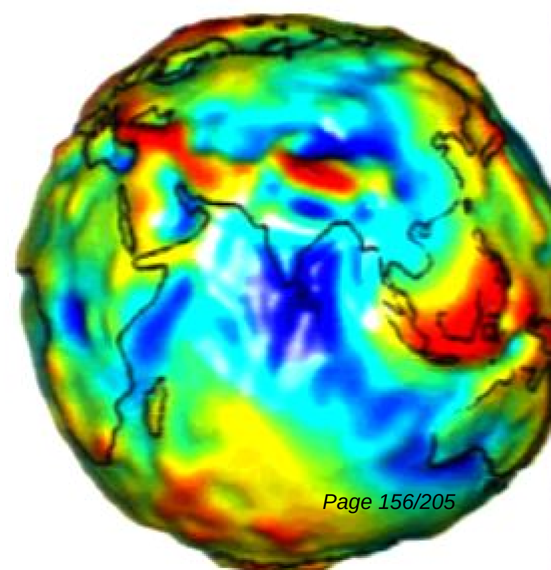

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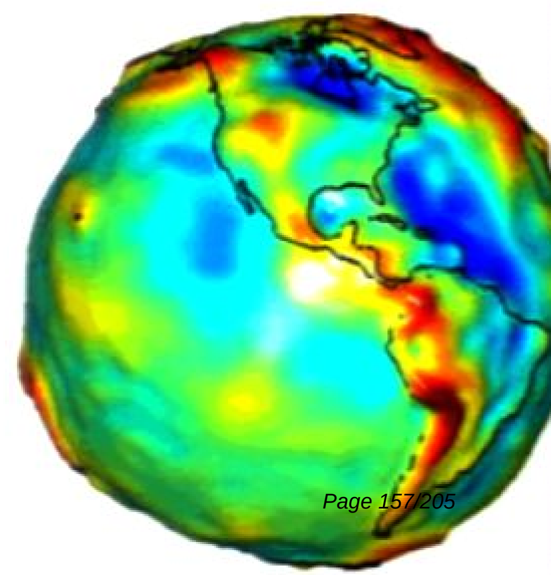

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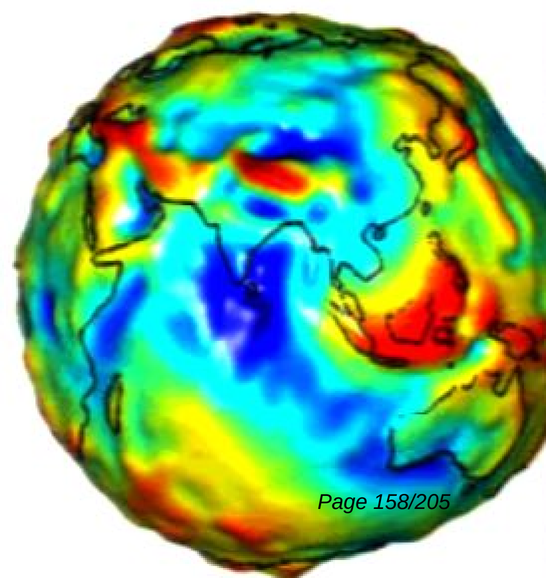

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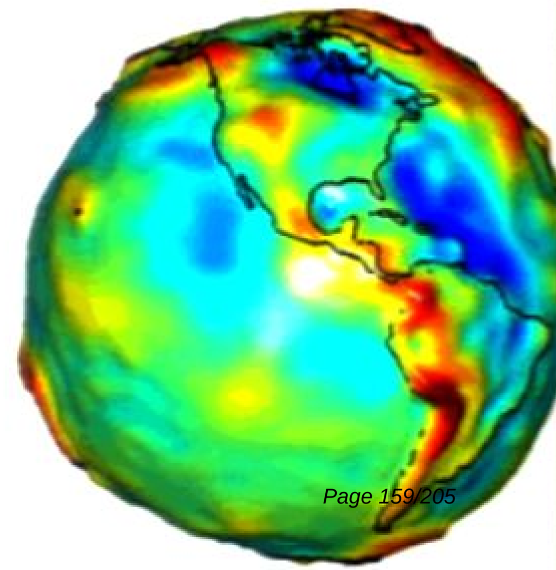

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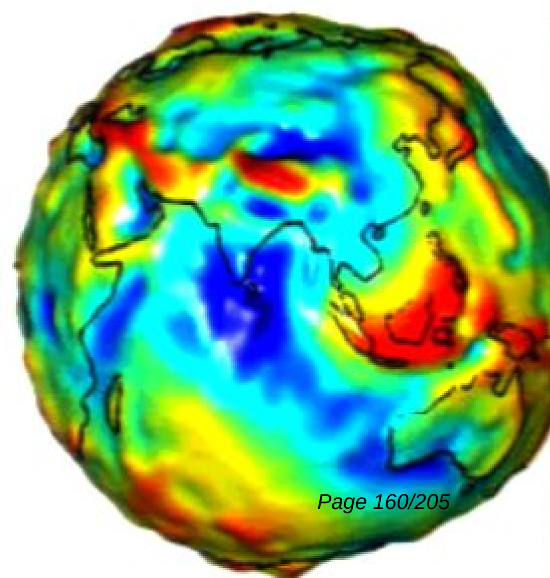

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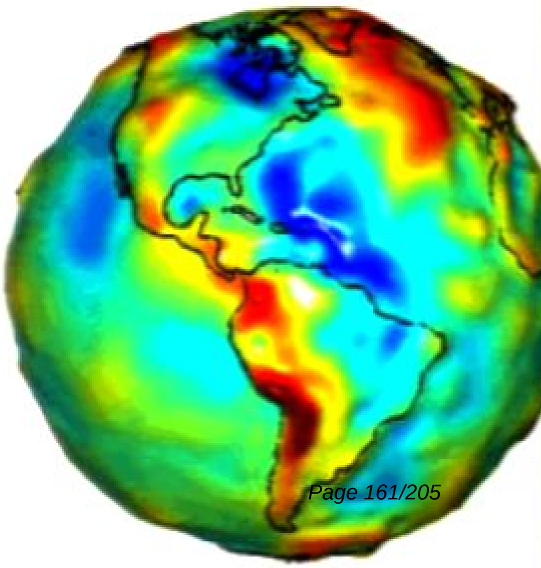

Geodesy for black holes

Basic idea: "map" spacetimes of candidate BHs as we map multipole distribution of earth's mass.

Particularly powerful for black holes: Their moments can only depend on hole's mass and spin (no hair theorem).

$$M_l + iS_l = M(ia)^l$$

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Mass of BH

Spin parameter

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Only TWO moments are independent

Measure more than two: Have enough information to falsify the black hole hypothesis.

Mapping black holes

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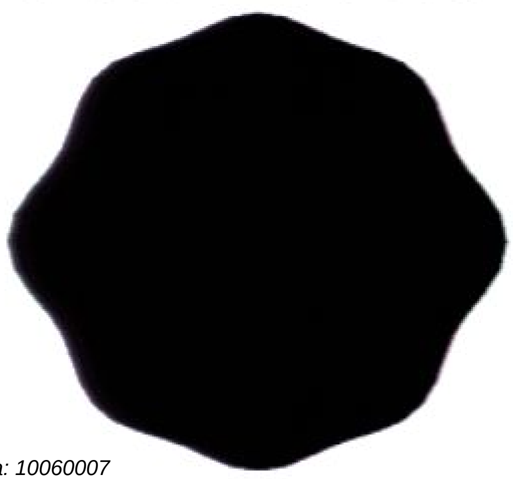
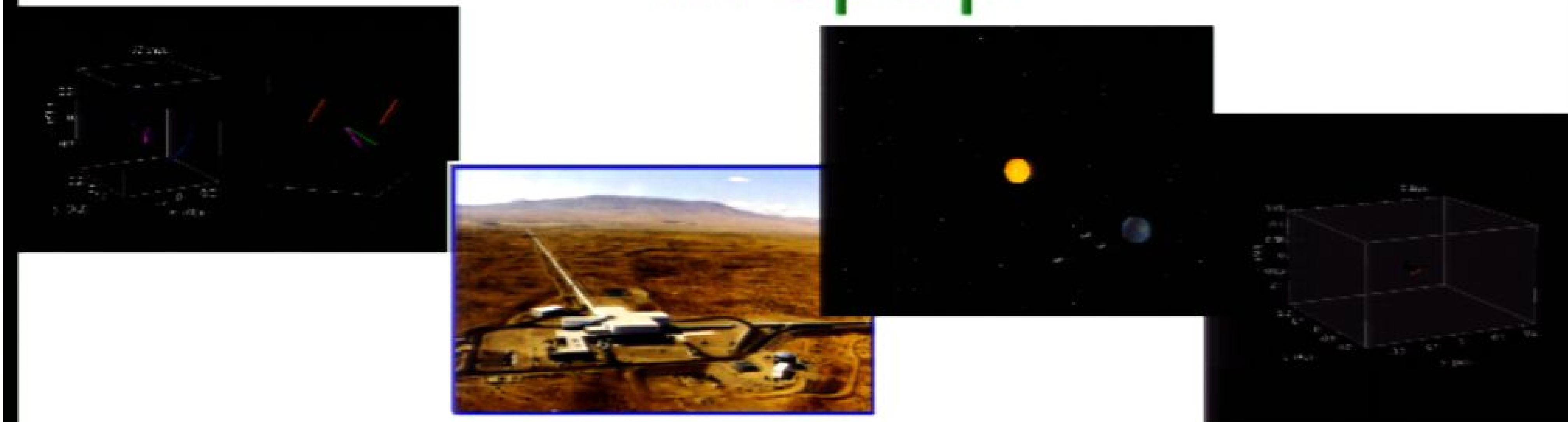


One idea:

1. Develop spacetime for black hole with "wrong" multipoles.
2. Compute how "bumpiness" is encoded in orbital frequencies.
3. Use as foundation for a ***null experiment***: If BH candidates are GR's BHs, their bumpiness is ***zero***.

Wrapup



Black hole theory and experiment are coming together as never before!

On threshold of *strong-field gravity phenomenology* ... Capra + NRDA + GW Data will make especially fun meeting!

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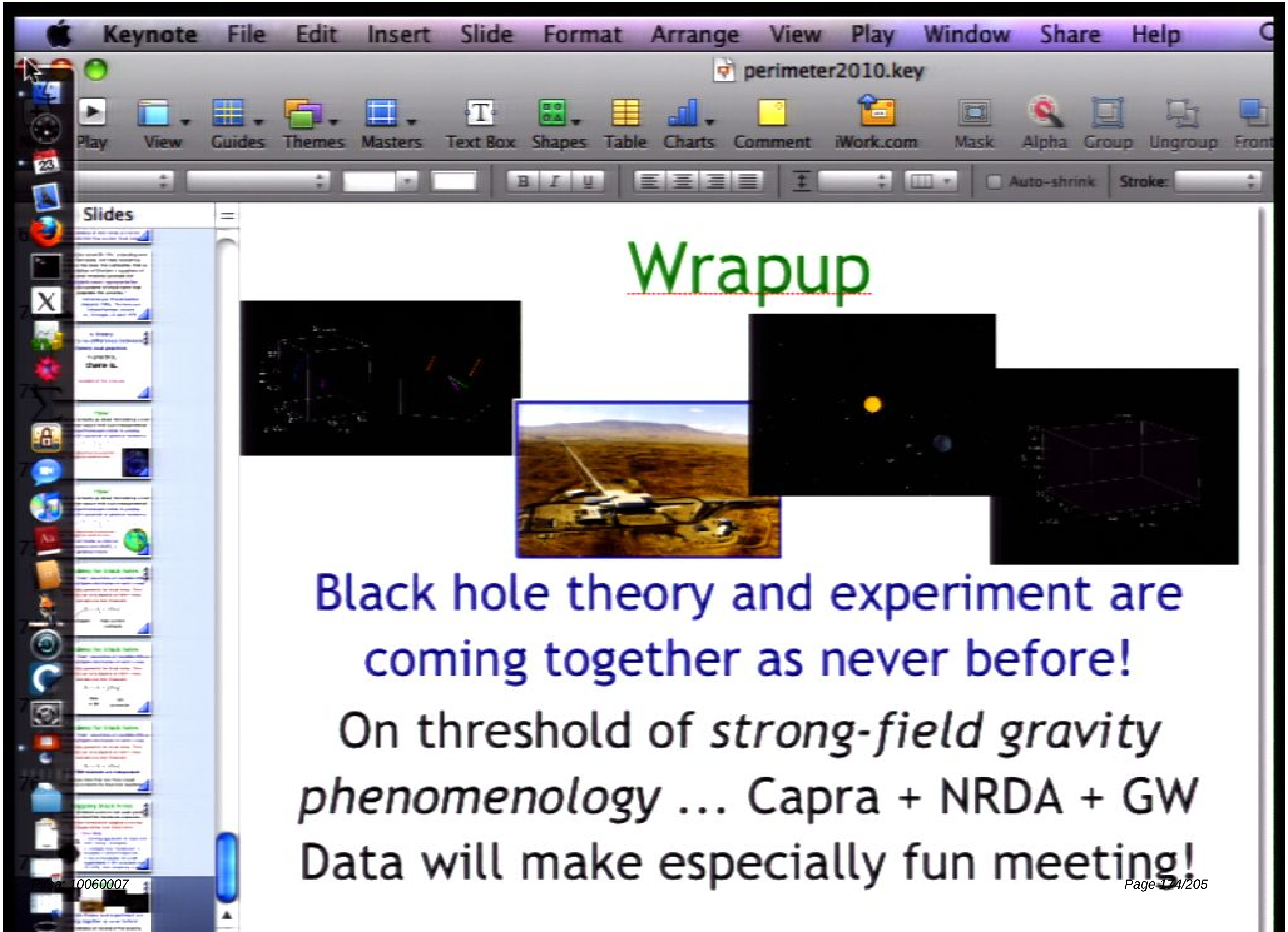
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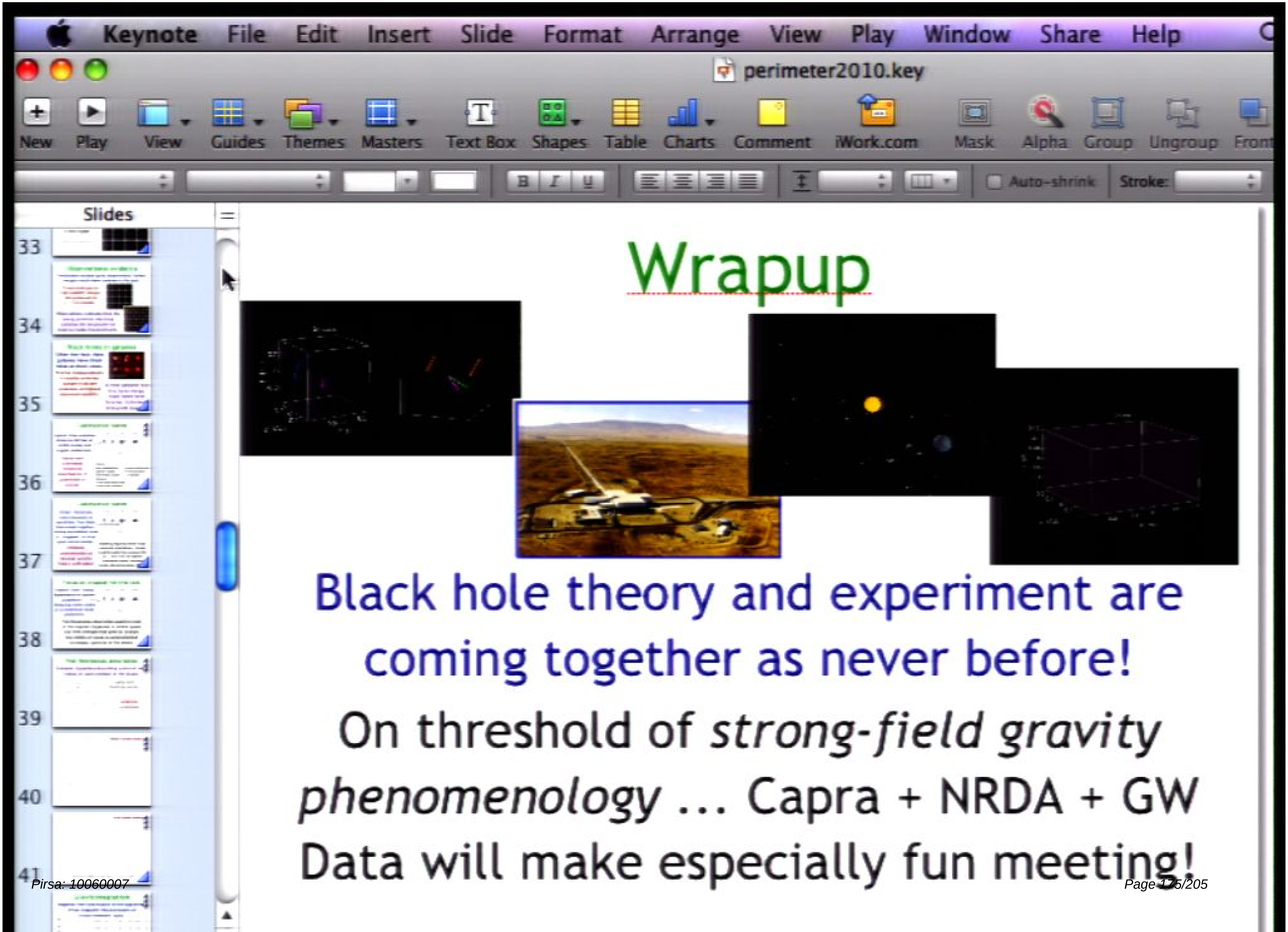
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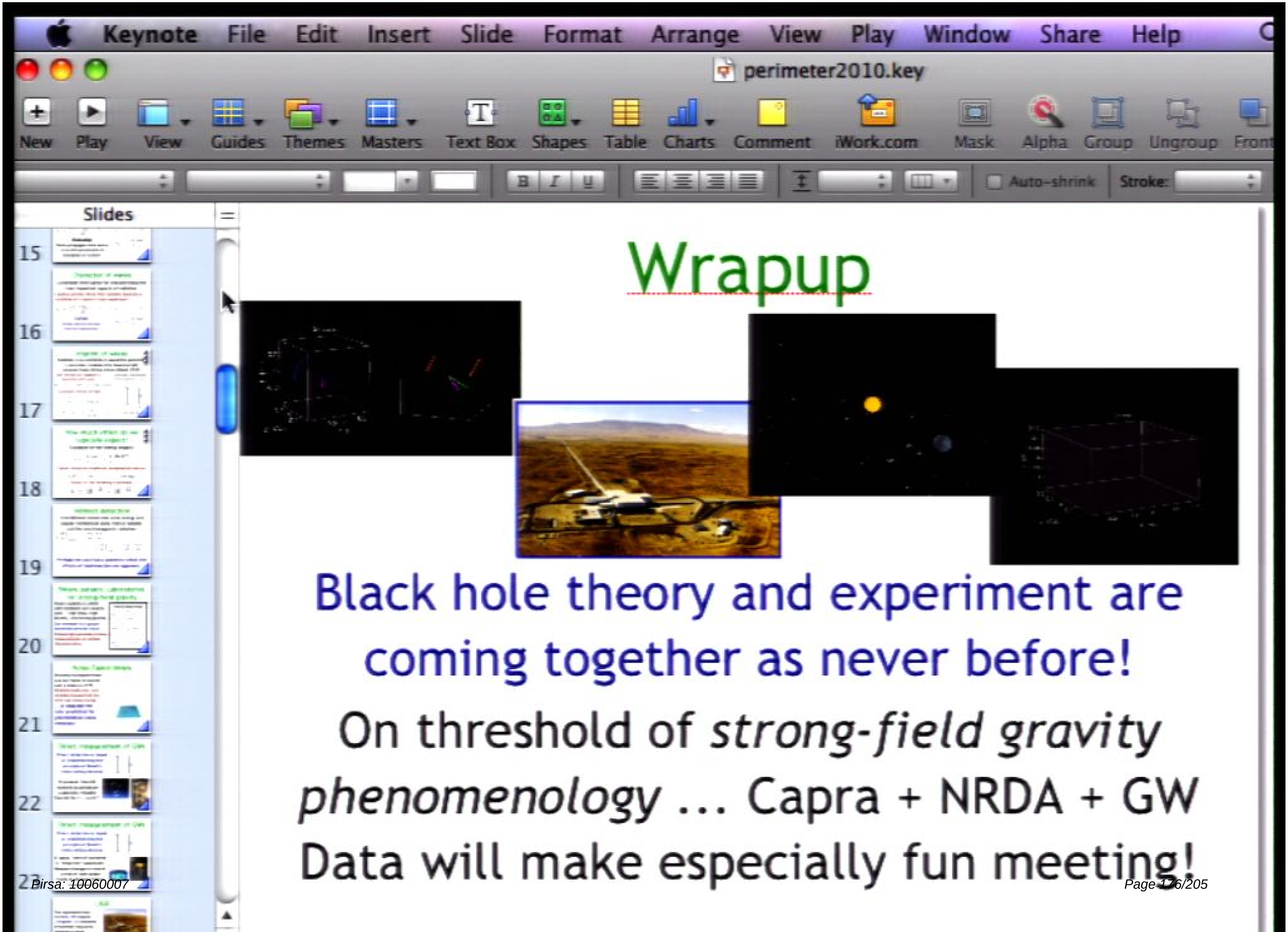
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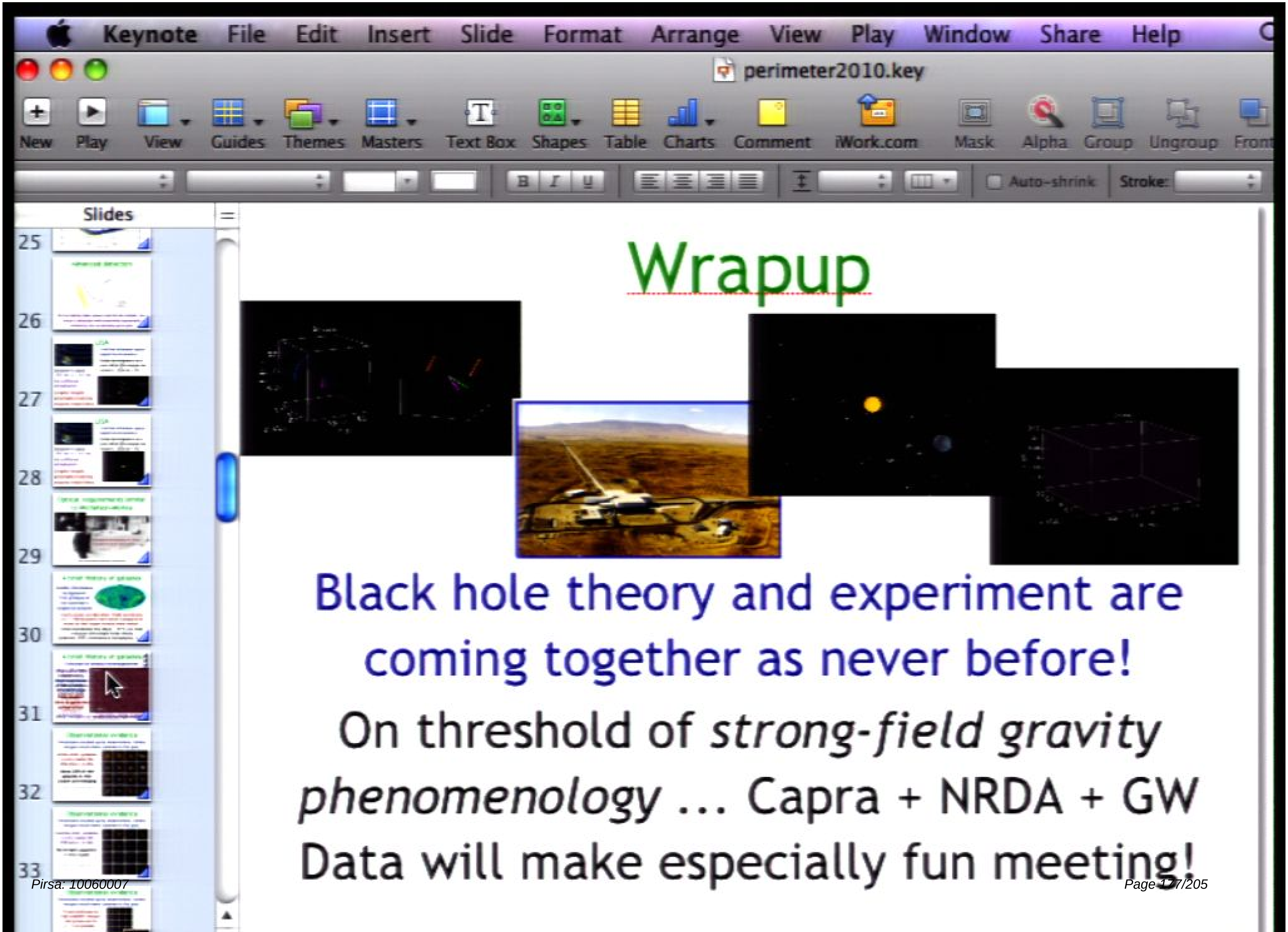
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A brief history of galaxies

Evolution of density inhomogeneities

At $z \sim 20$, find $\delta\rho/\rho \sim 1$:

Linear evolution no longer accurate.

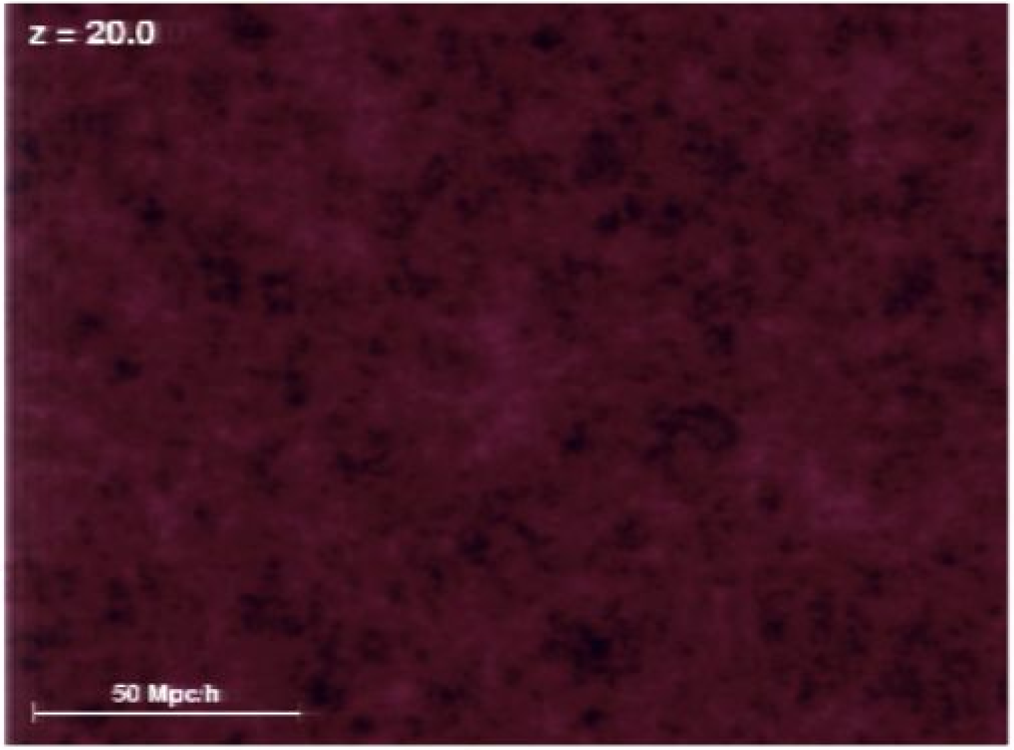
New model using massive N-body simulations.

Credit: The VIRGO Cosmological N-body Project

Structure is built *hierarchically*.

Big things made of many mergers of small things.

Dark matter halos & galaxies merge a lot!



Density evolution, "comoving" coordinates.

Dark matter distribution followed in simulation.

Many mergers at moderate to high redshift.

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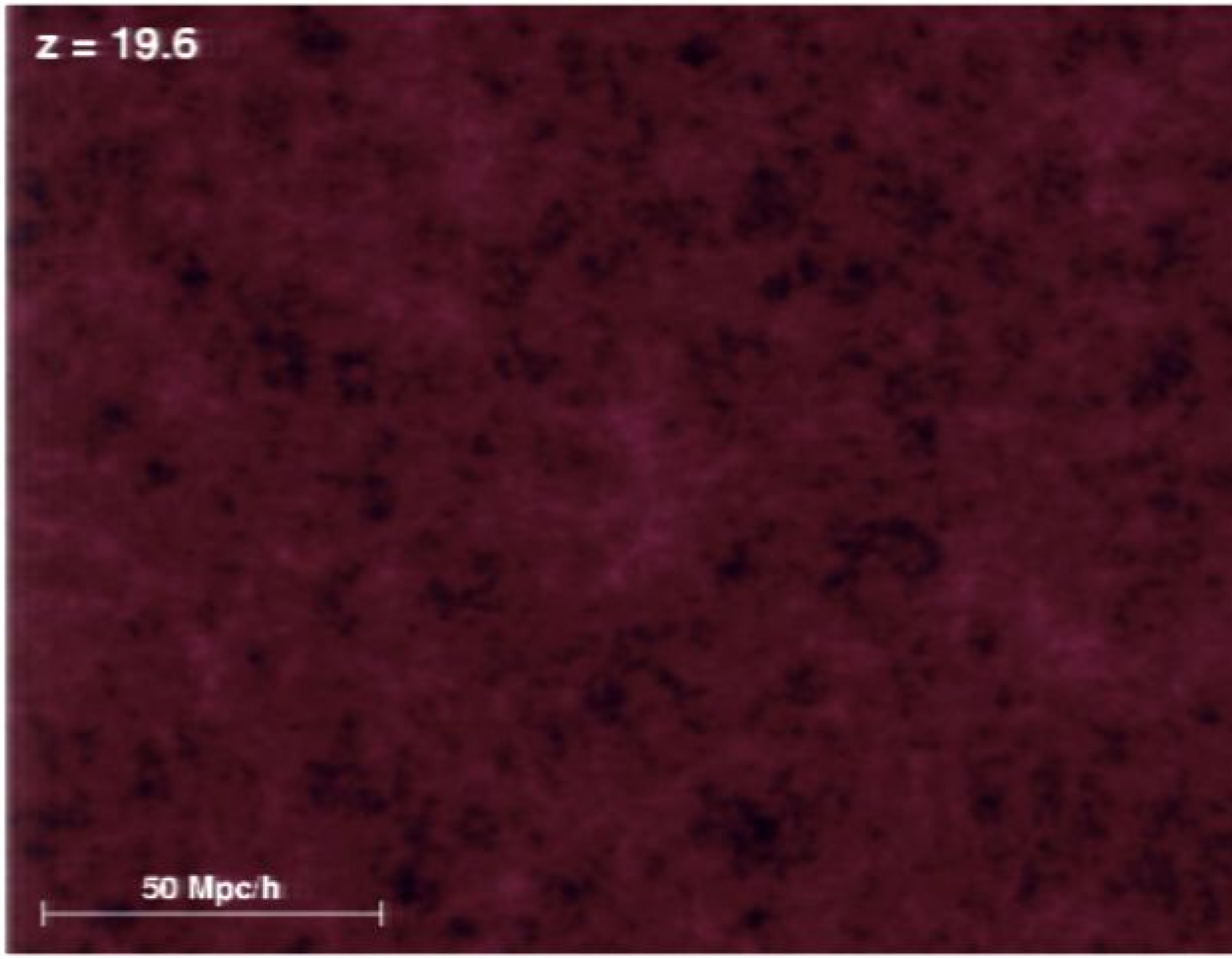

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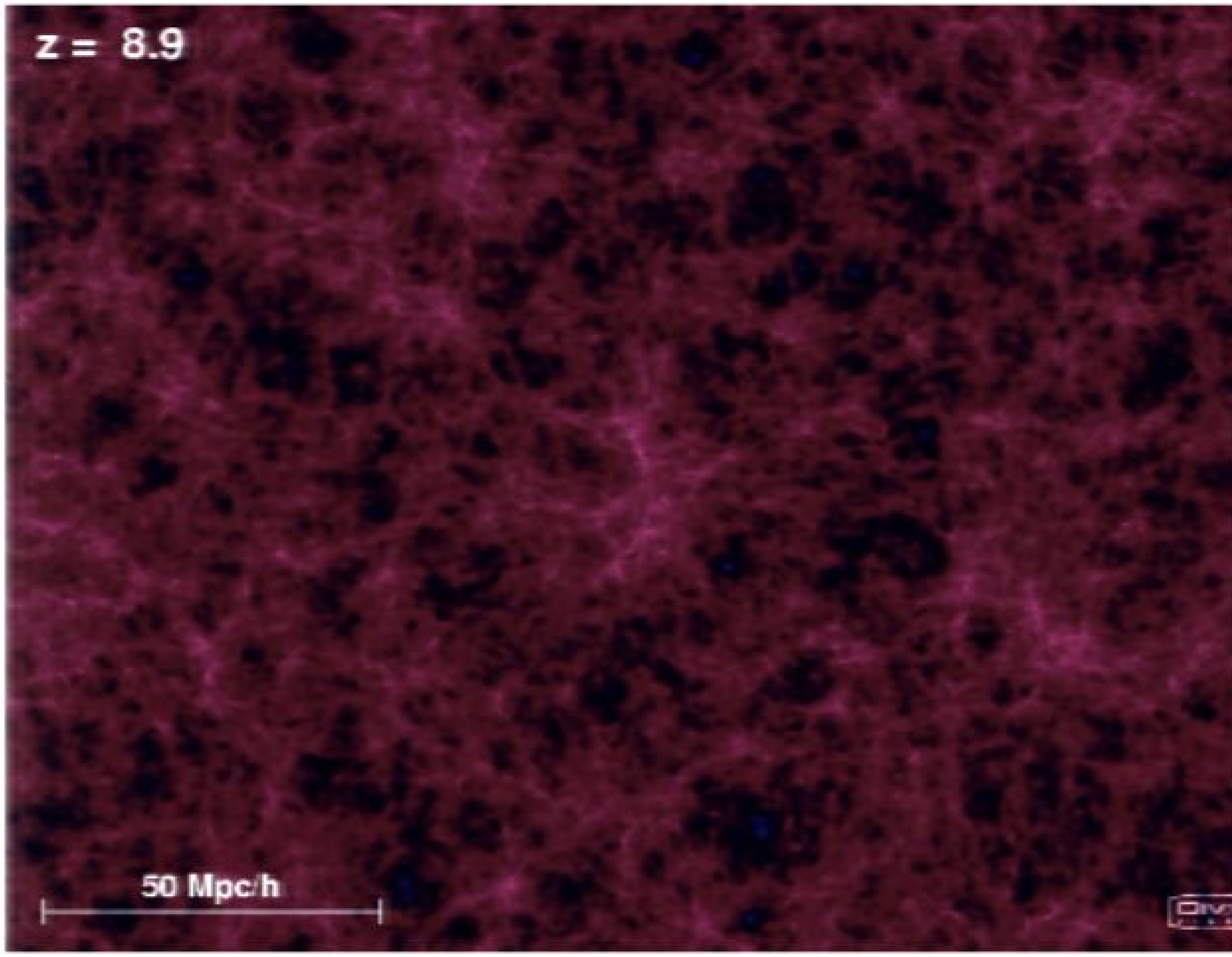

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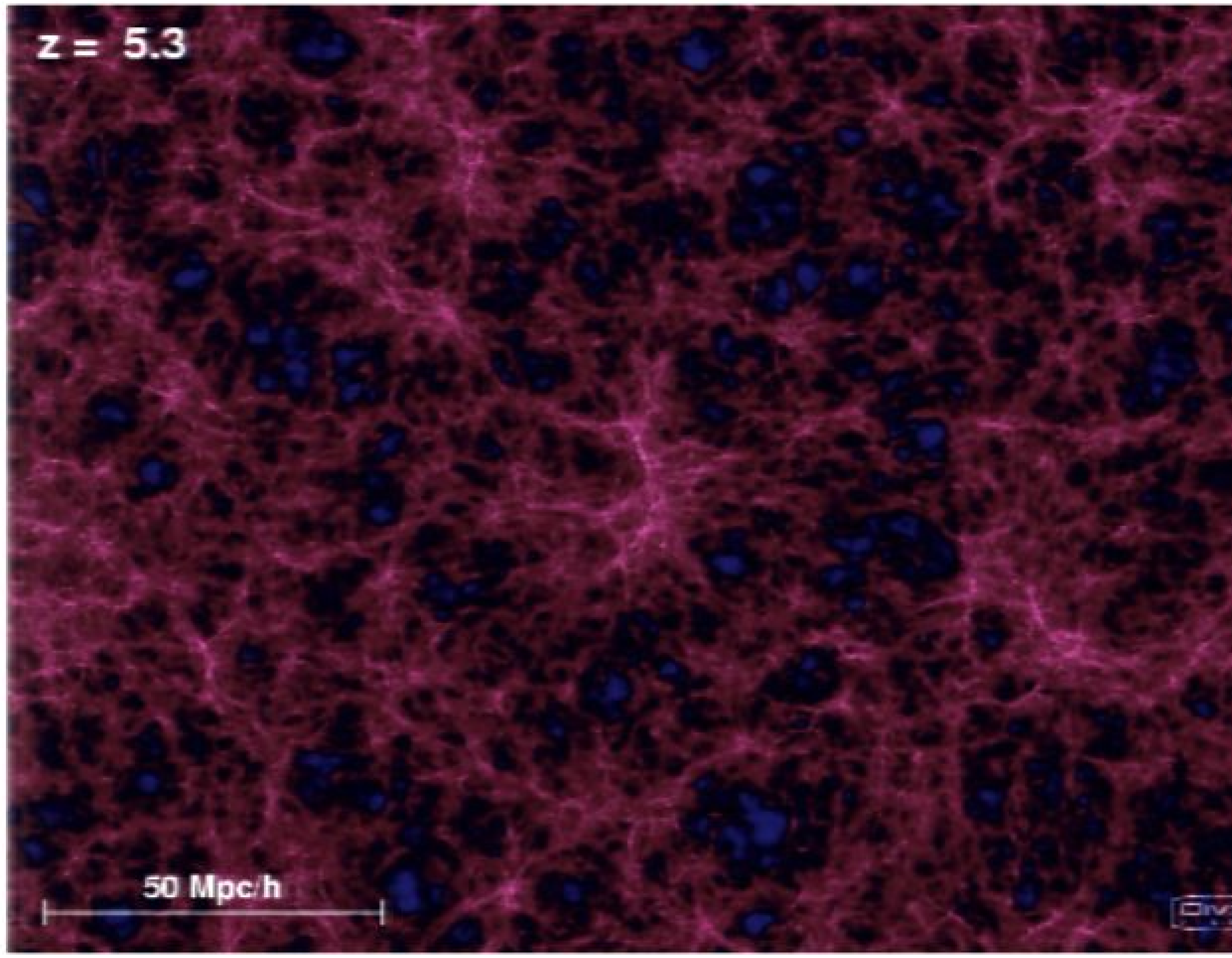

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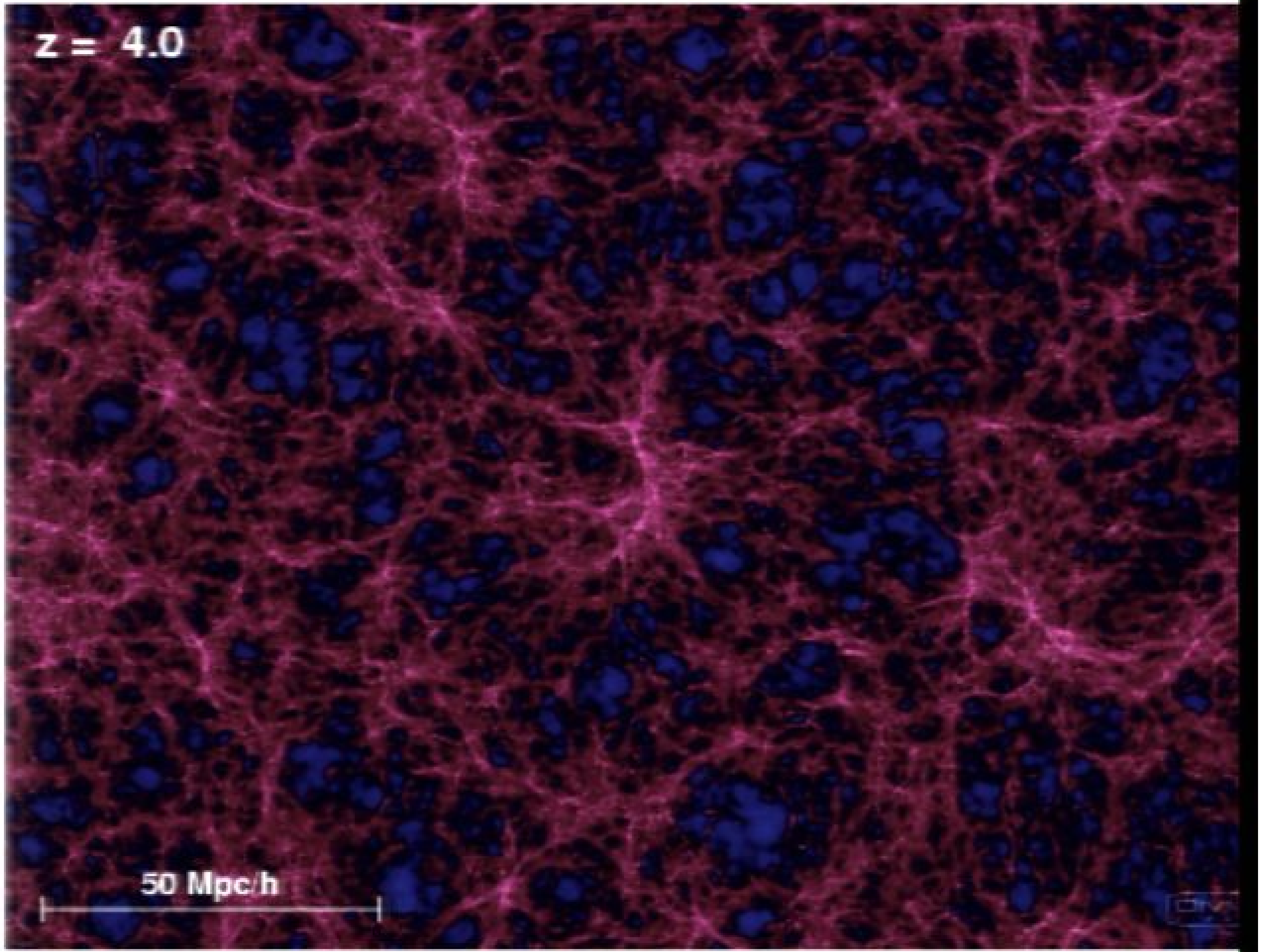

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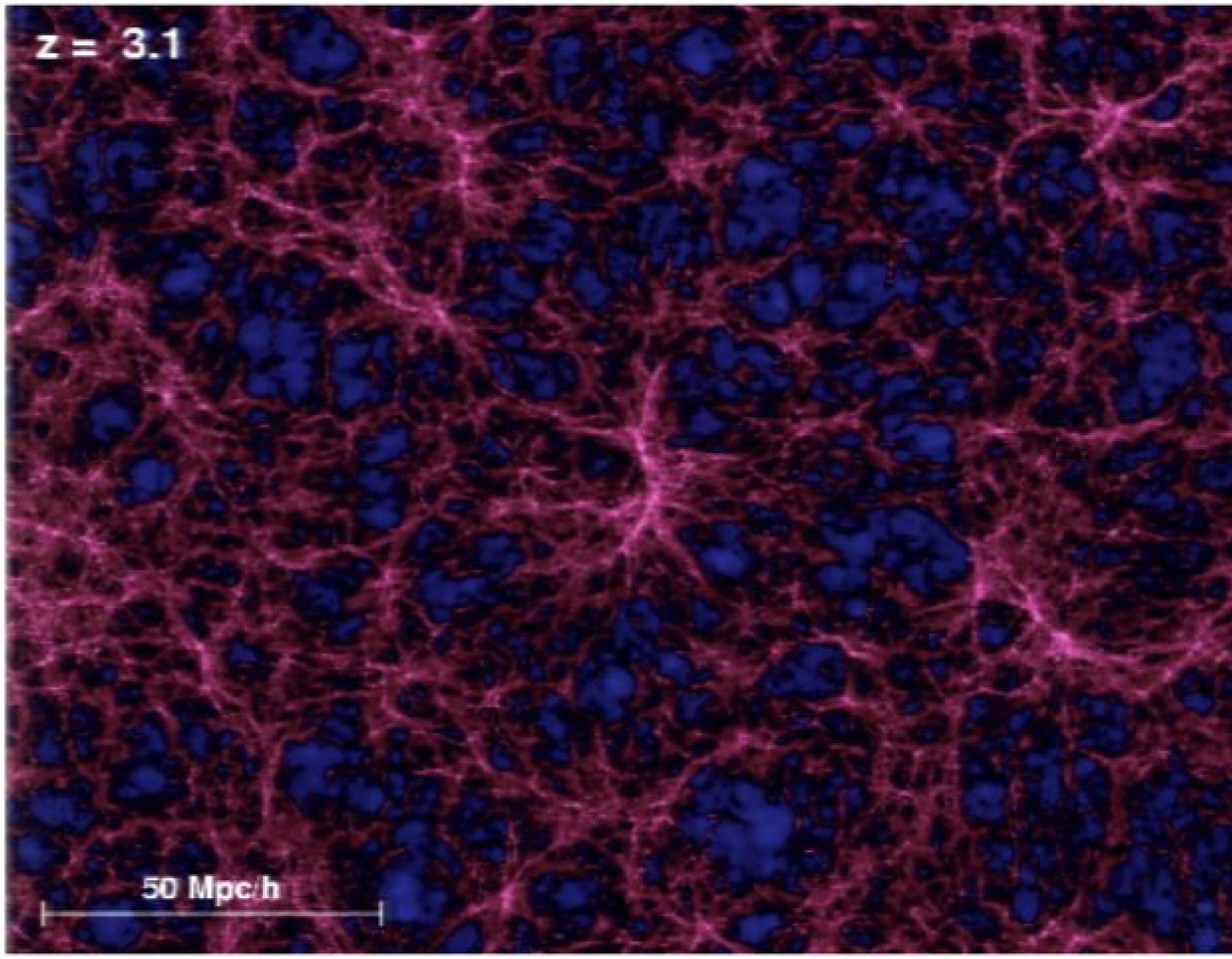

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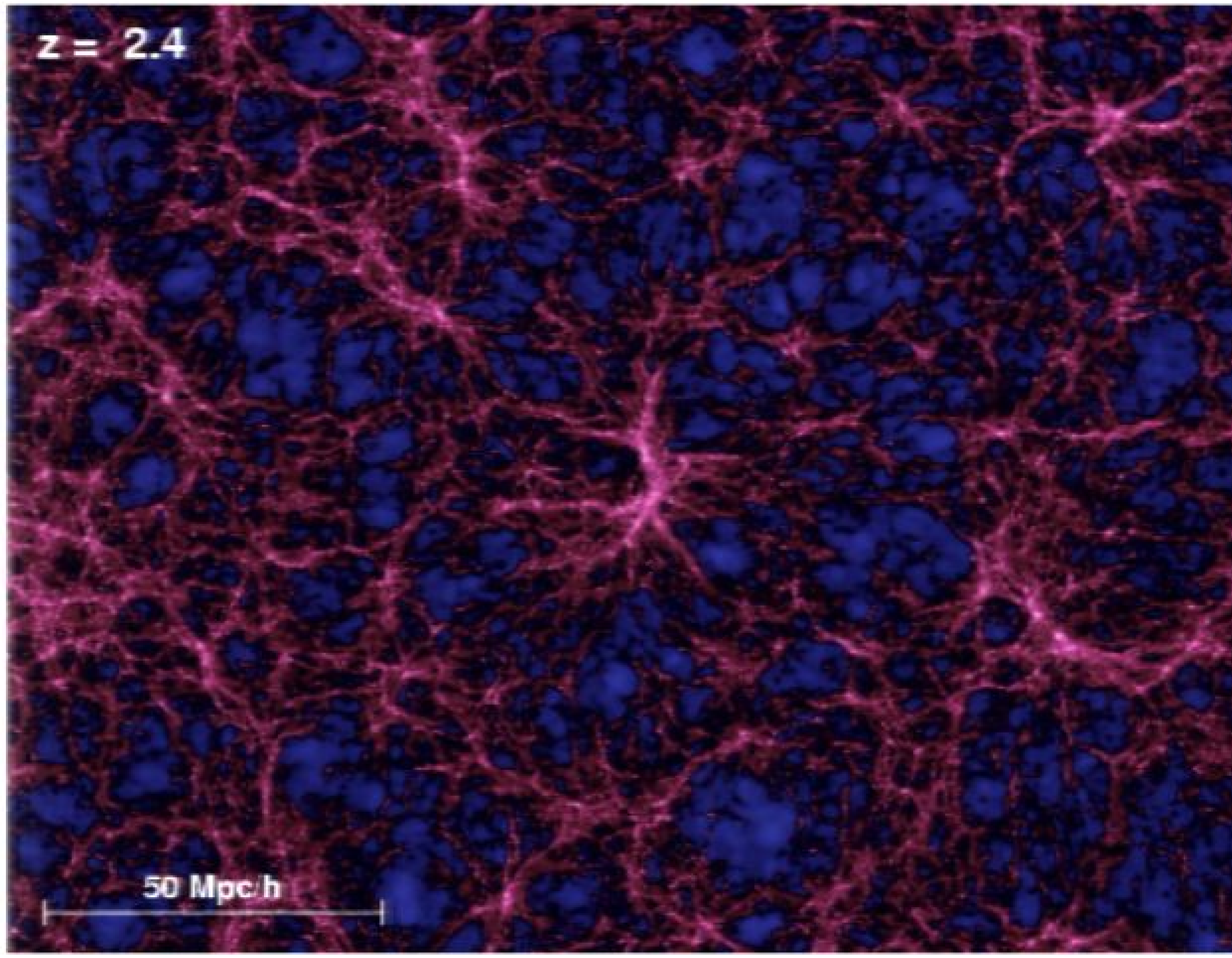

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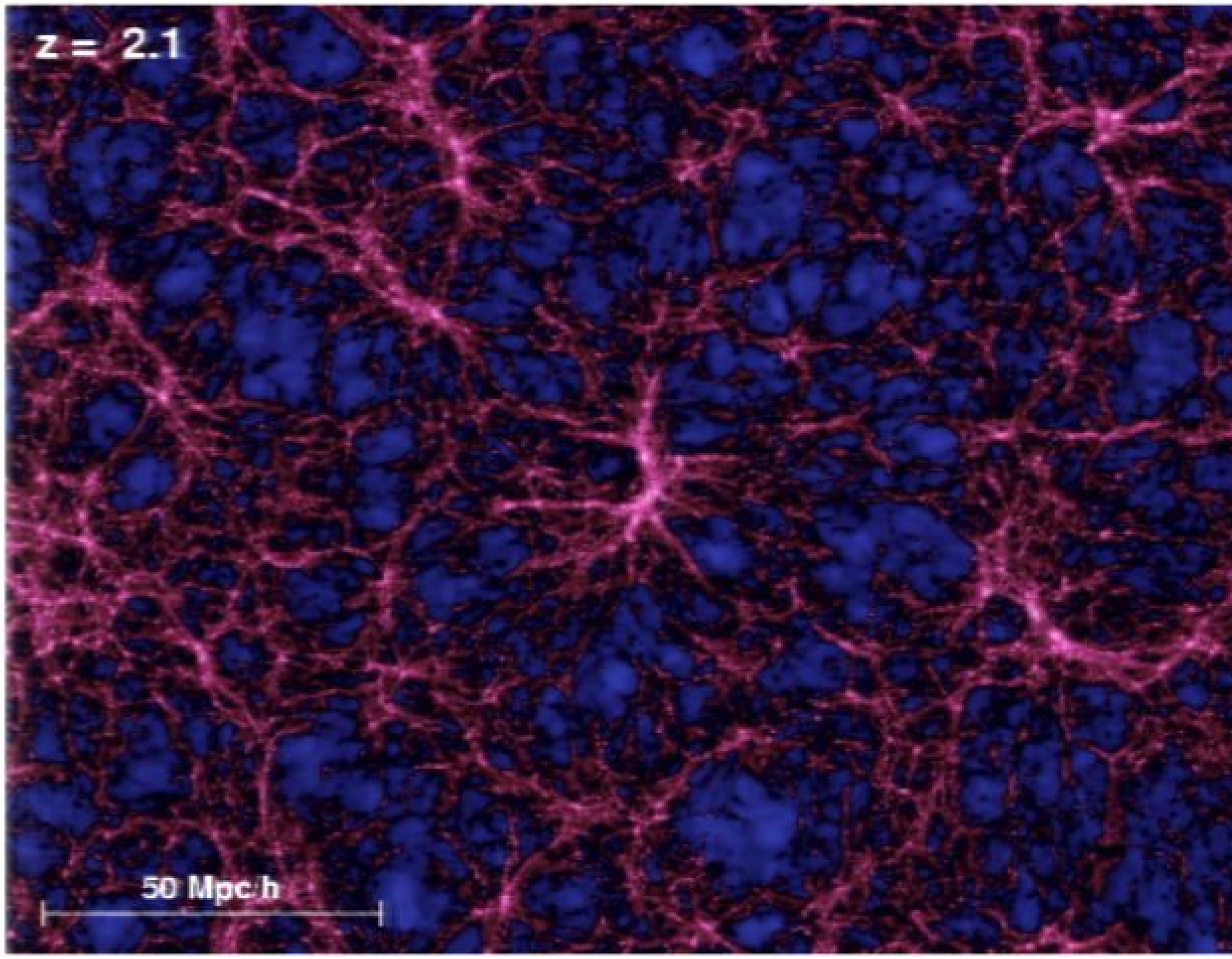

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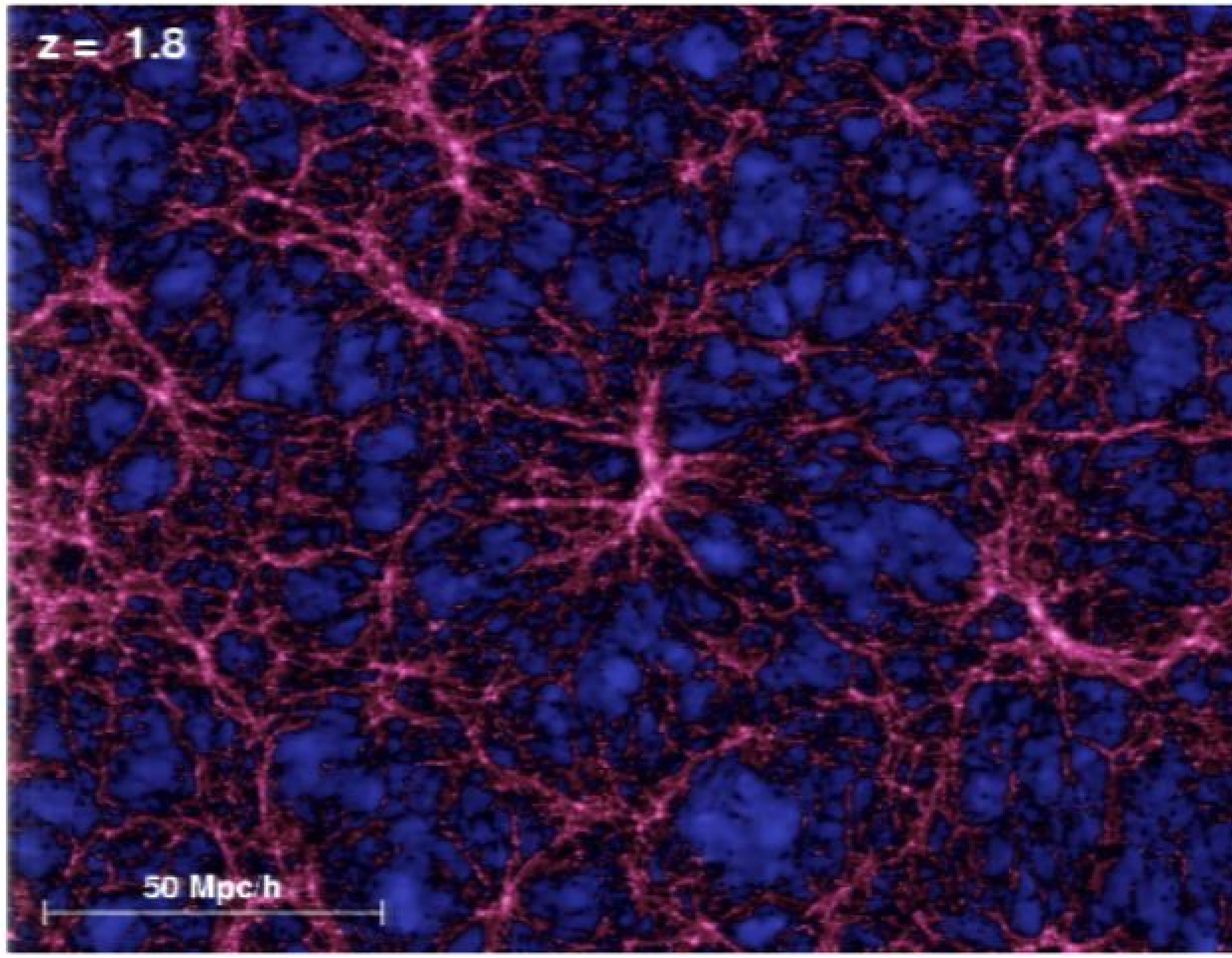

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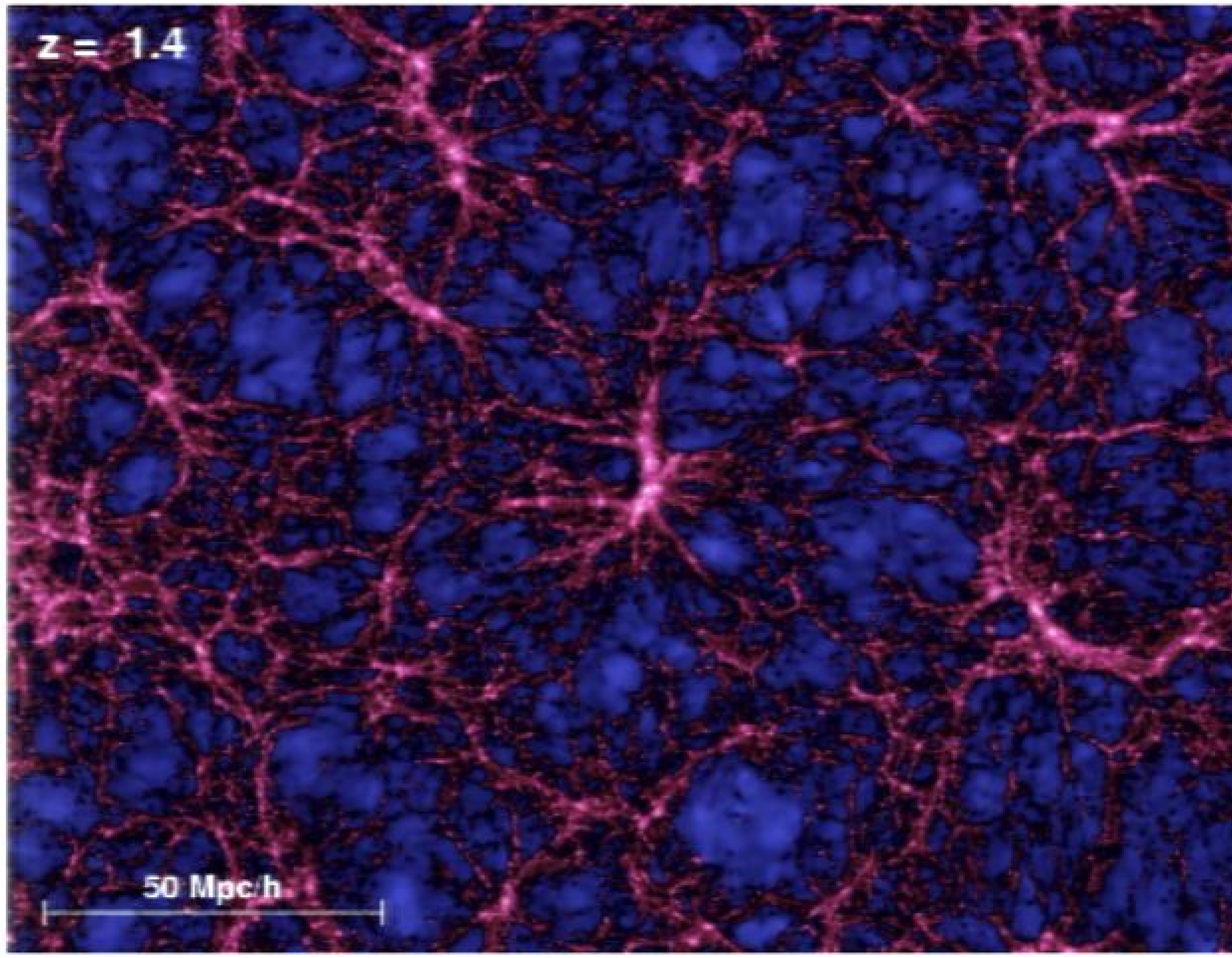

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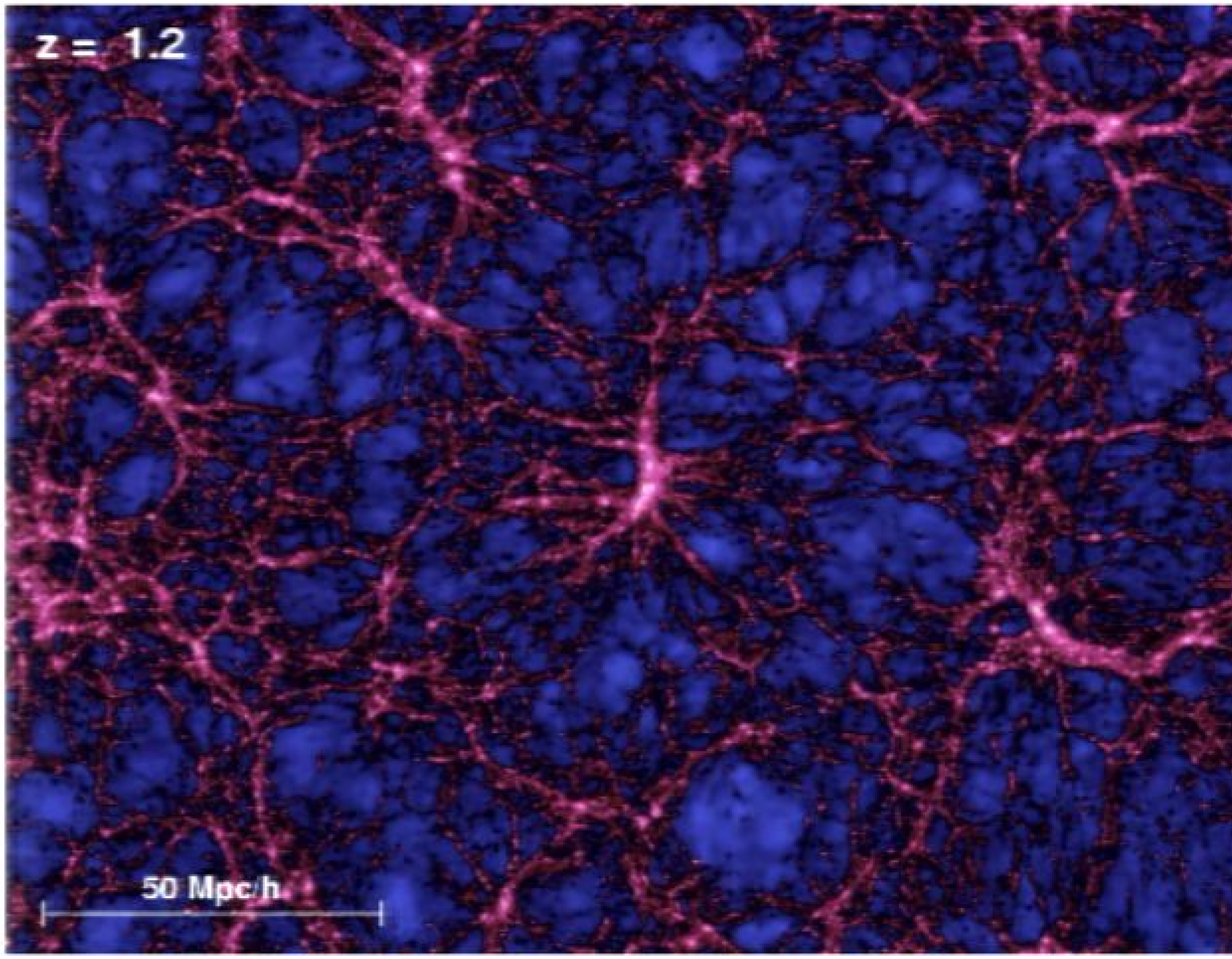

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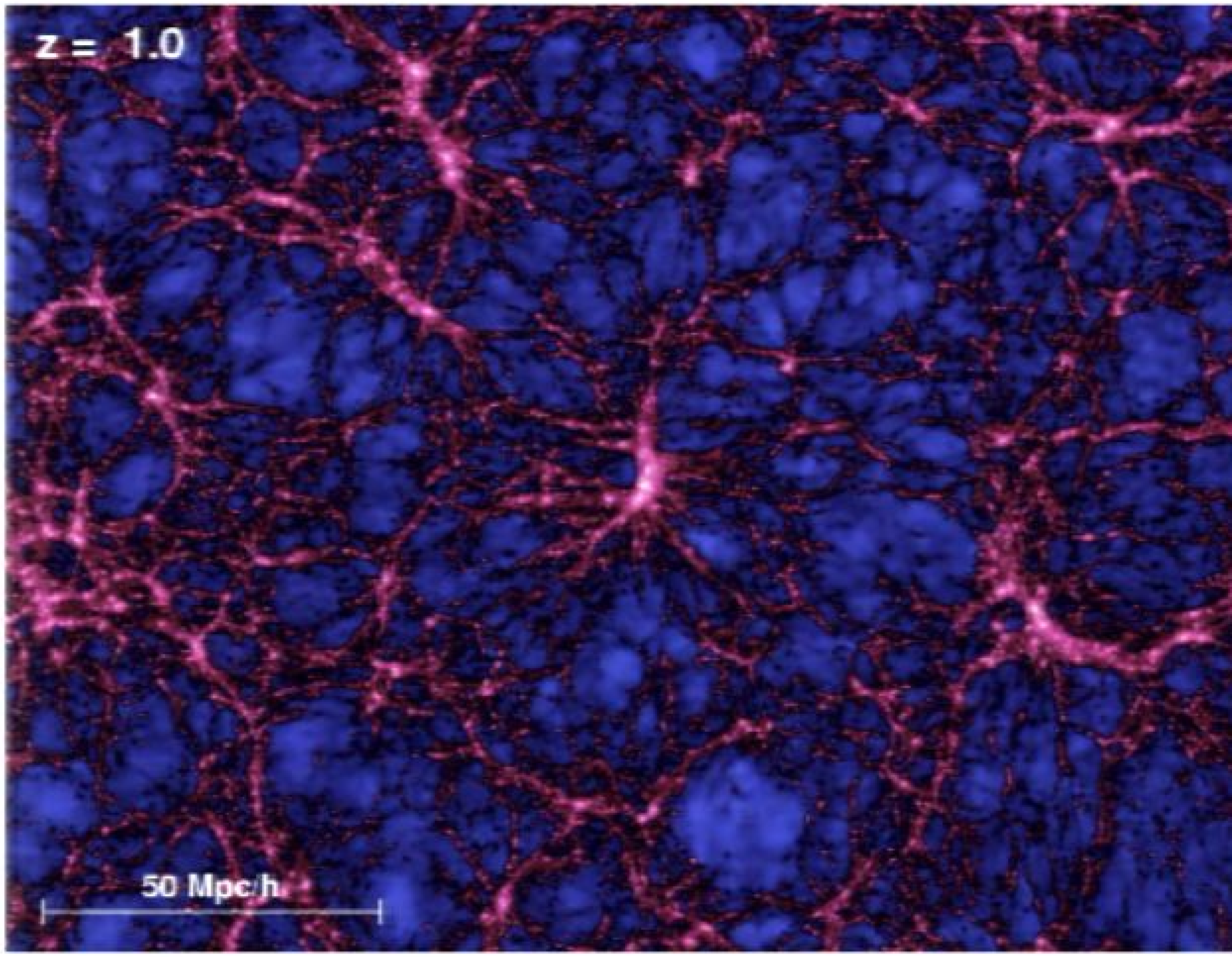

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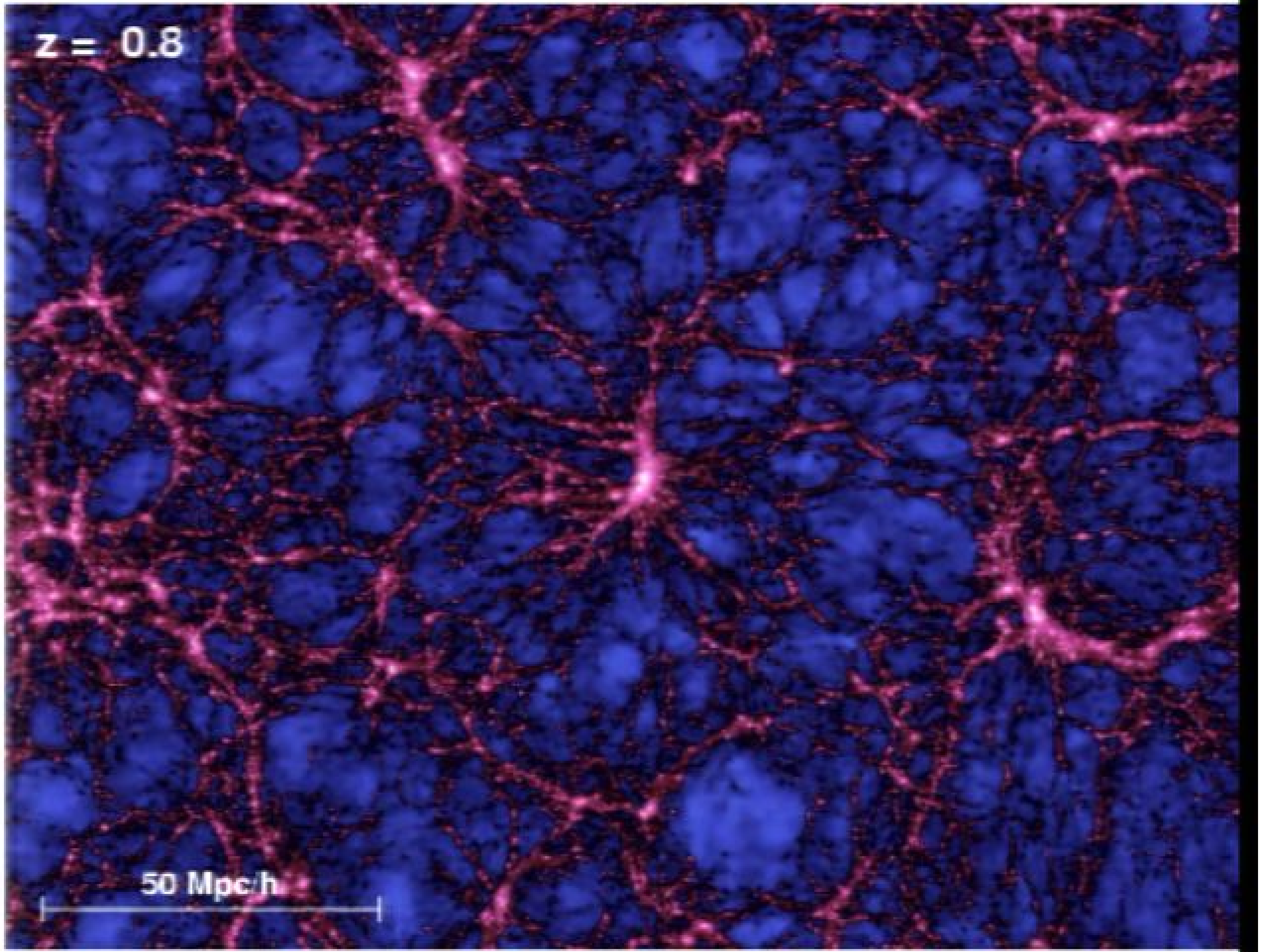

Density evolution, "comoving" coordinates.
Dark matter distribution followed in simulation.

A brief history of galaxies

Evolution of density inhomogeneities

At z ~ 20, find $\delta\rho/\rho \sim 1$:

Linear evolution no longer accurate. Now model using massive N-body simulations.

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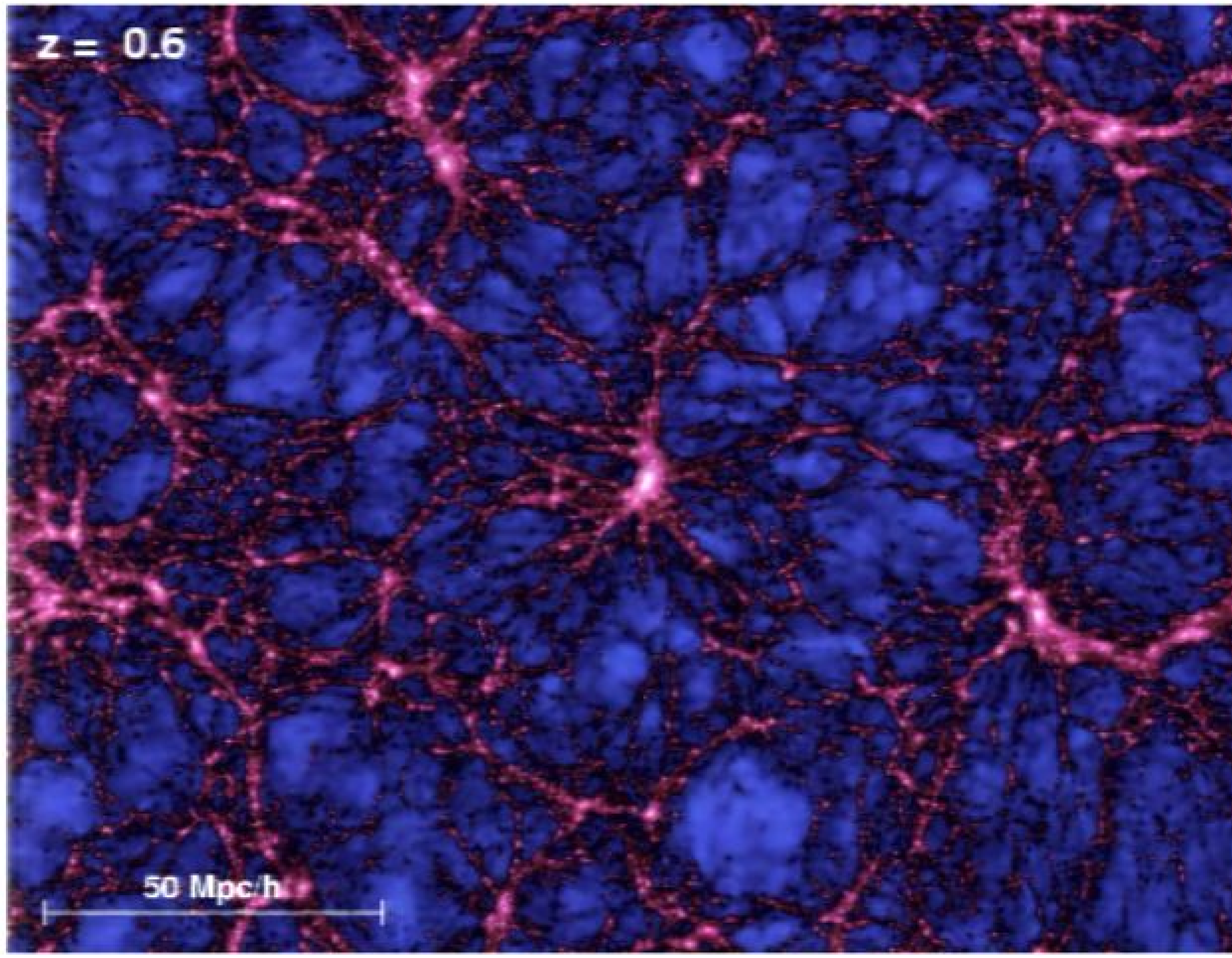

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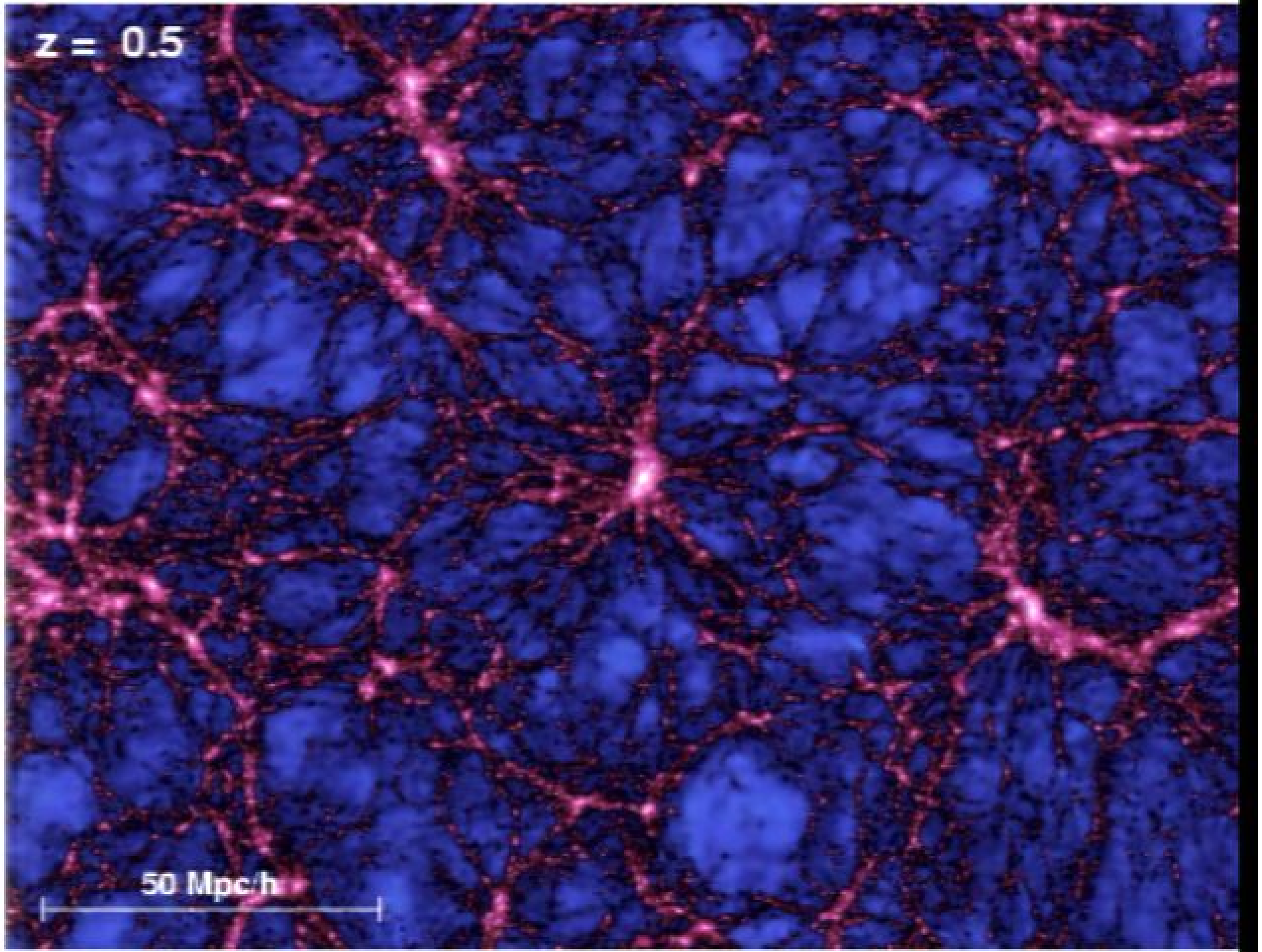

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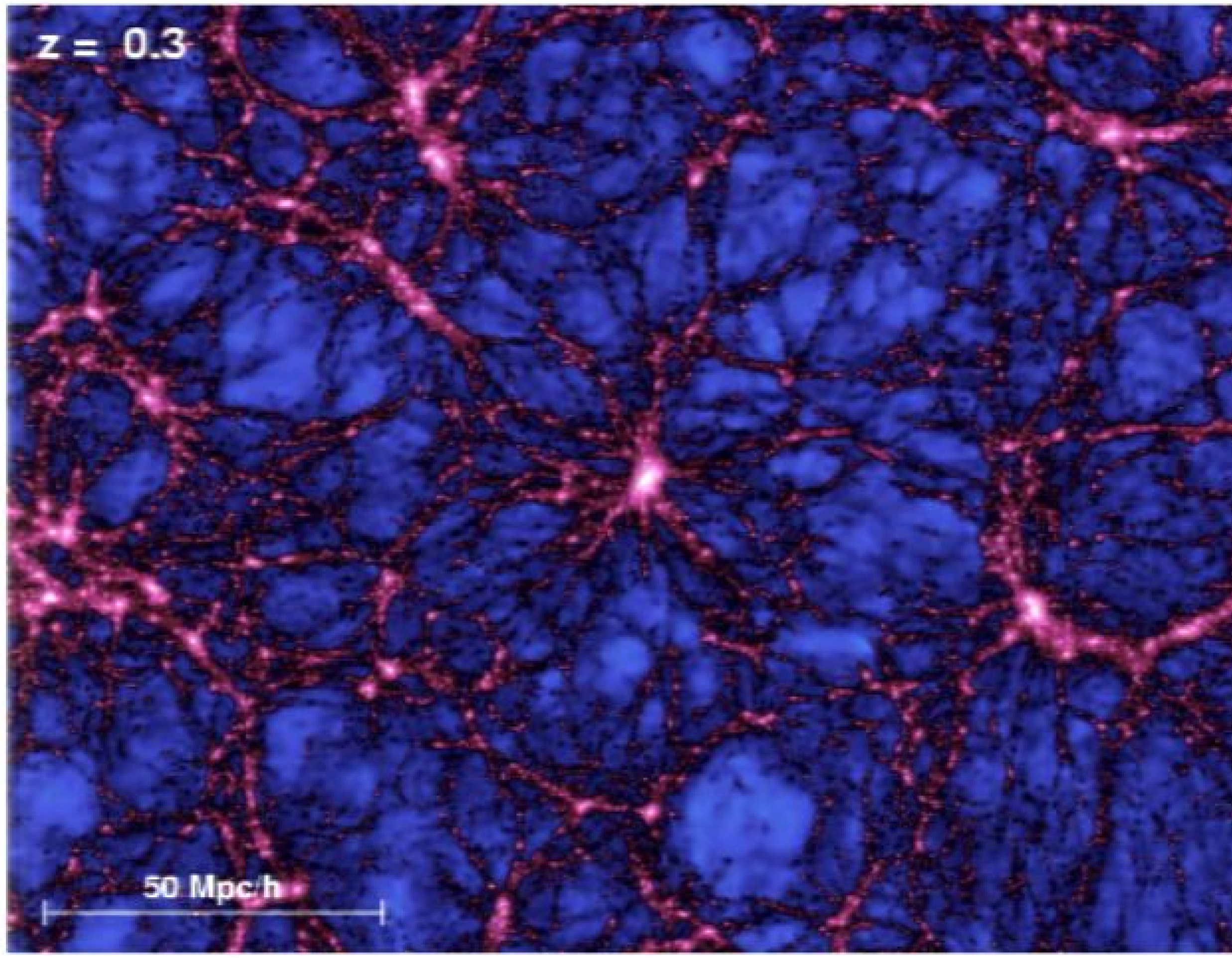

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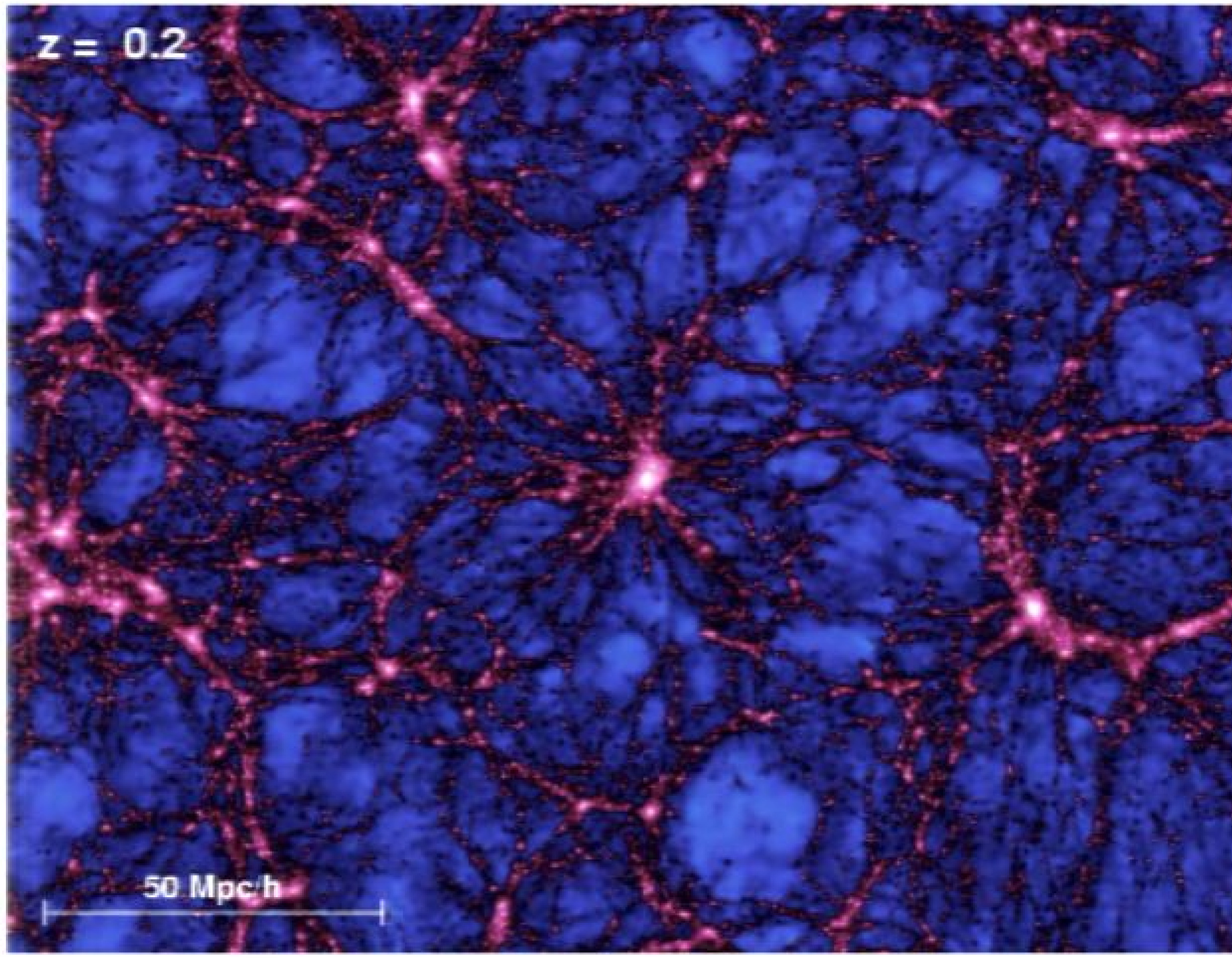

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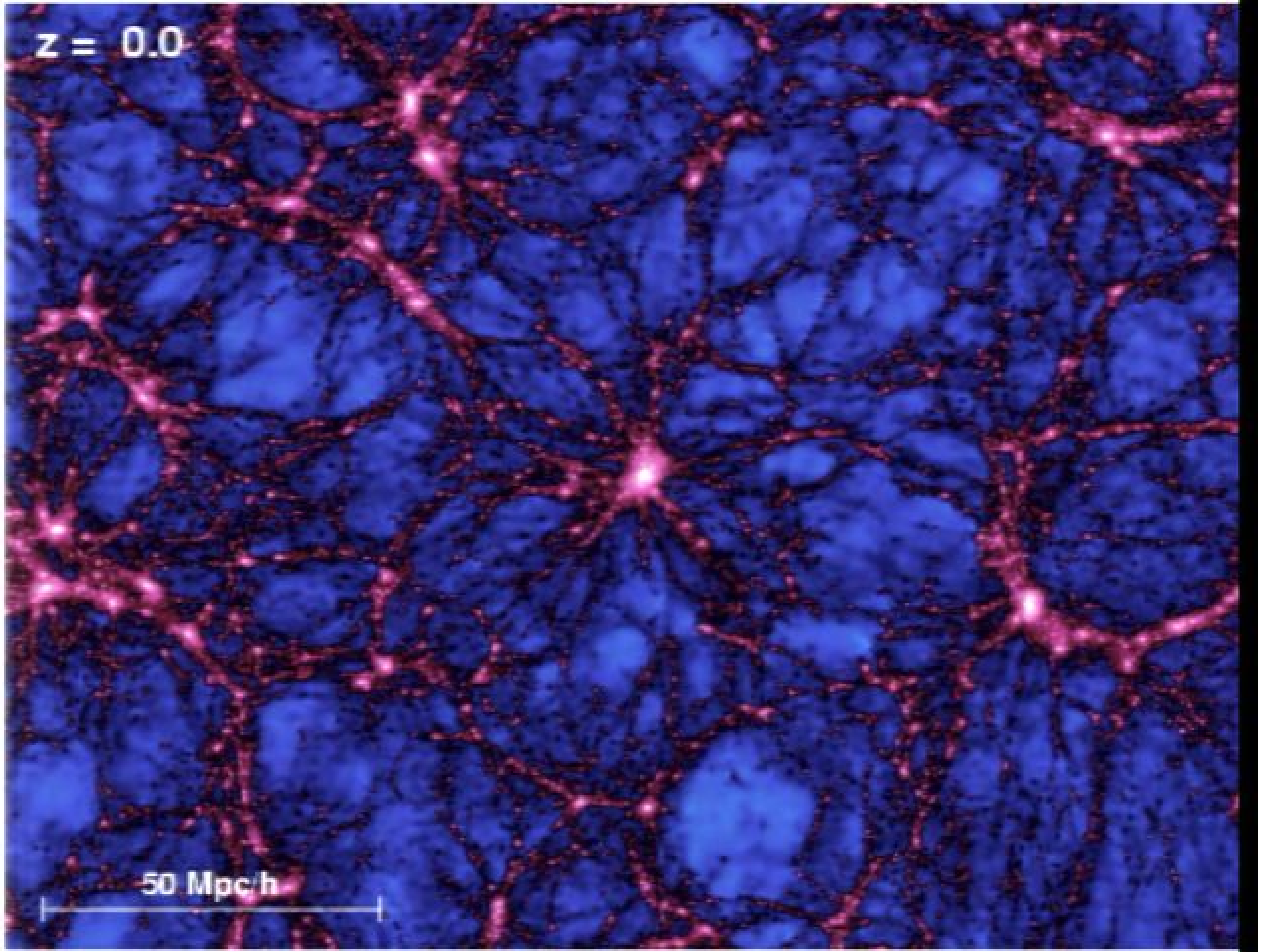

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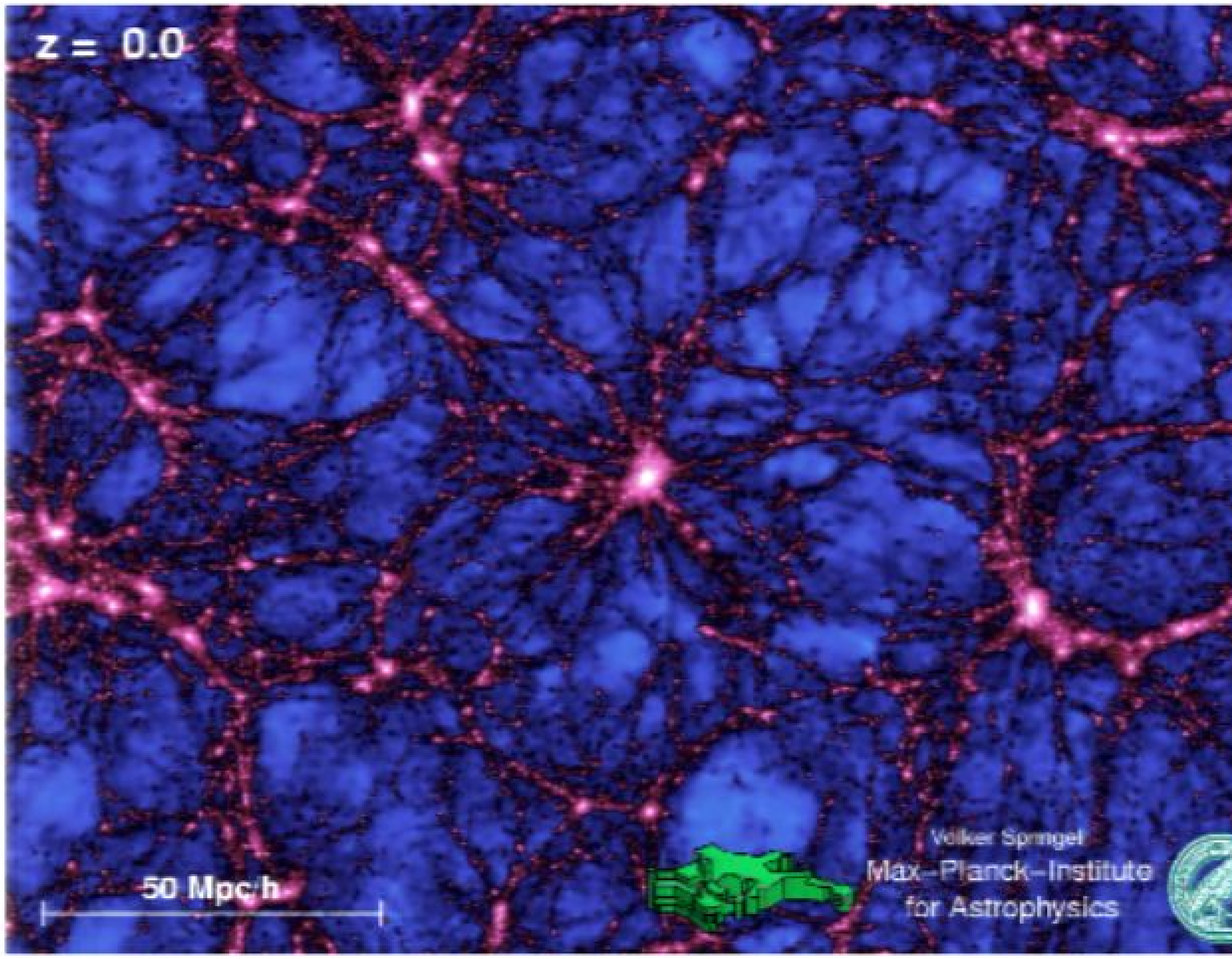

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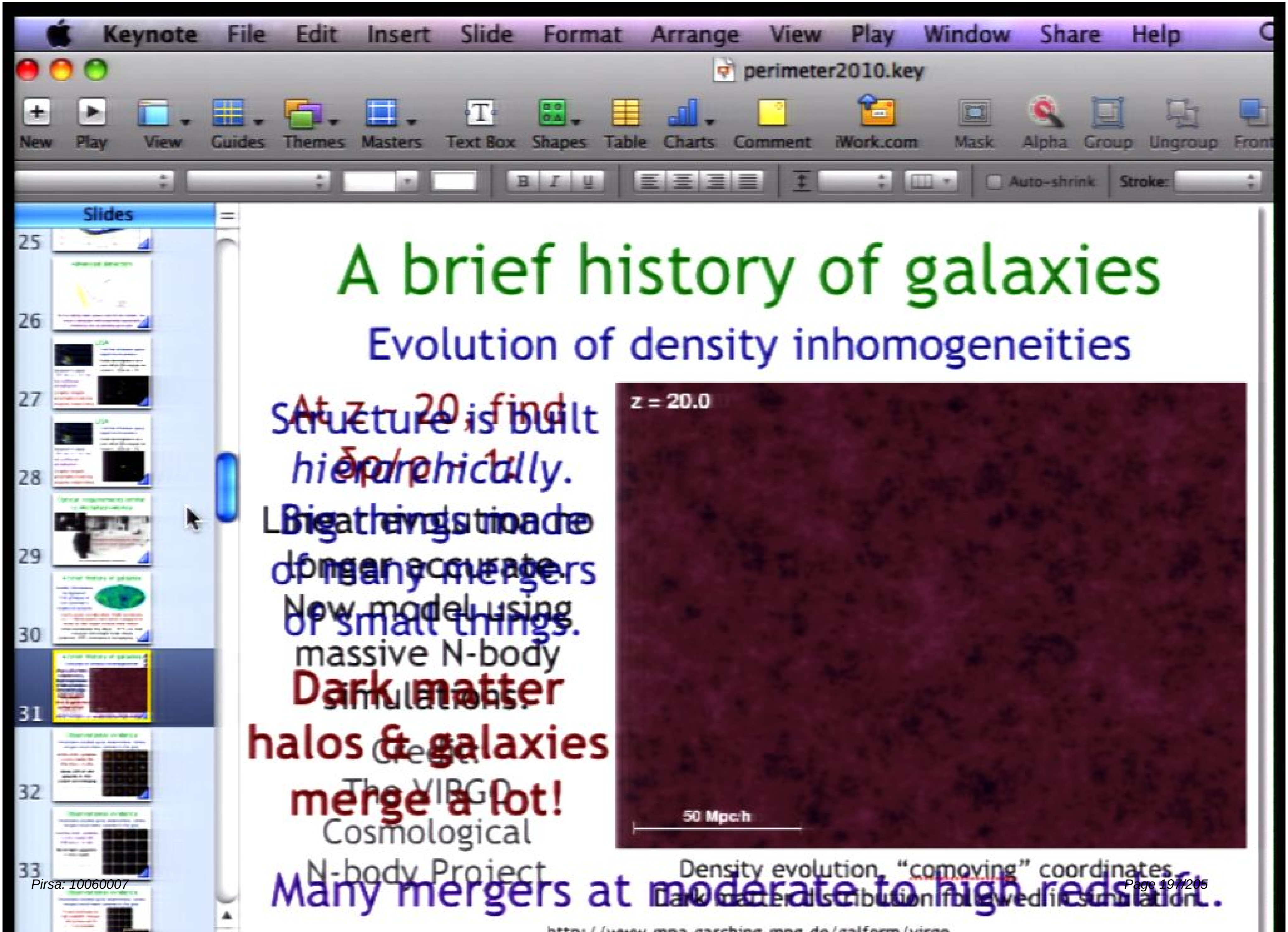
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Evolution of density inhomogeneities
Structure is built hierarchically.
of many mergers
of small things.
Linear evolution no
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Now model using
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Dark matter
halos & galaxies
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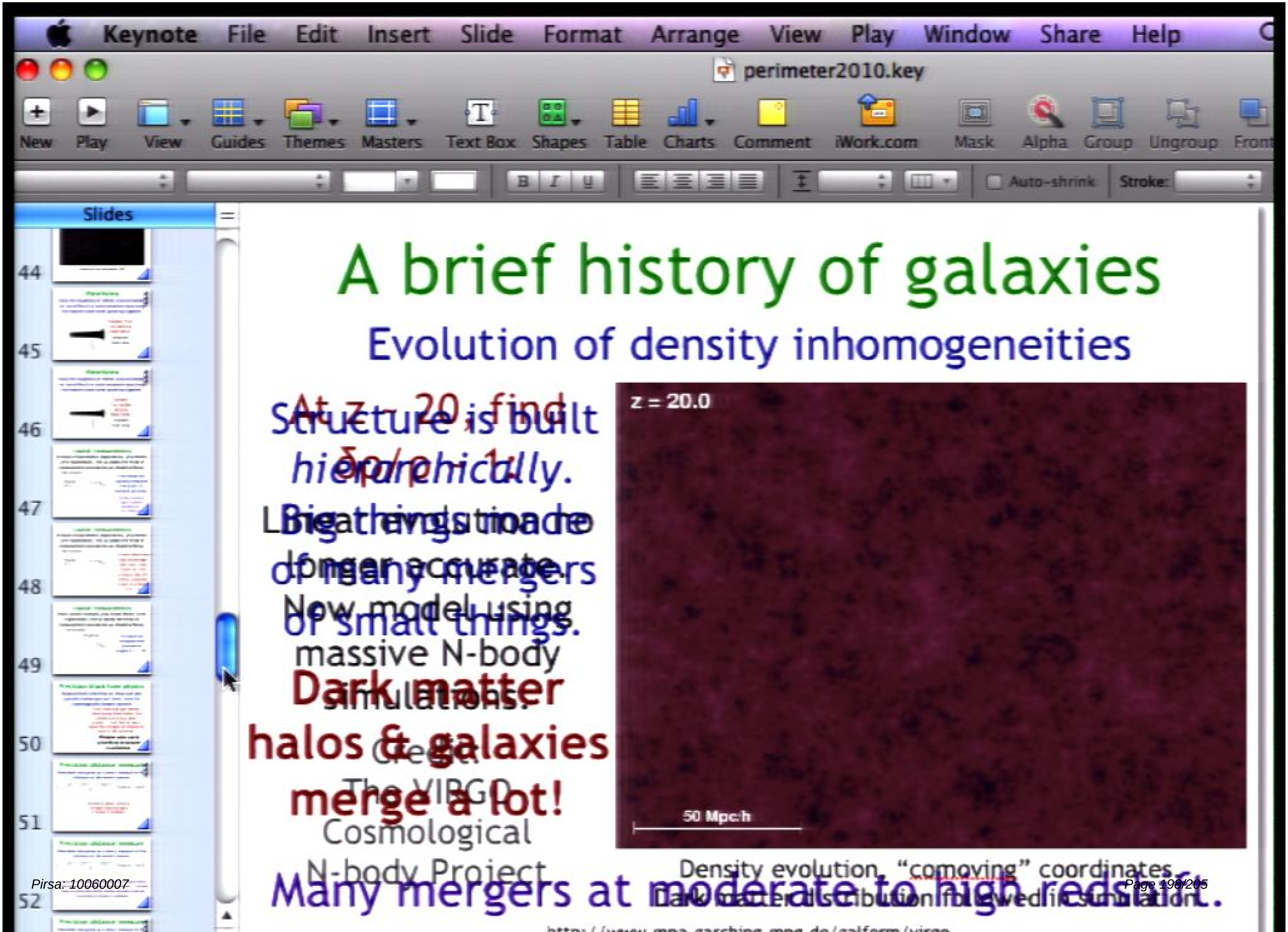
Cosmological
N-body Project
z = 20.0
50 Mpc/h
Density evolution, "comoving" coordinates.
Dark matter distribution followed in simulation.
Many mergers at moderate to high redshift.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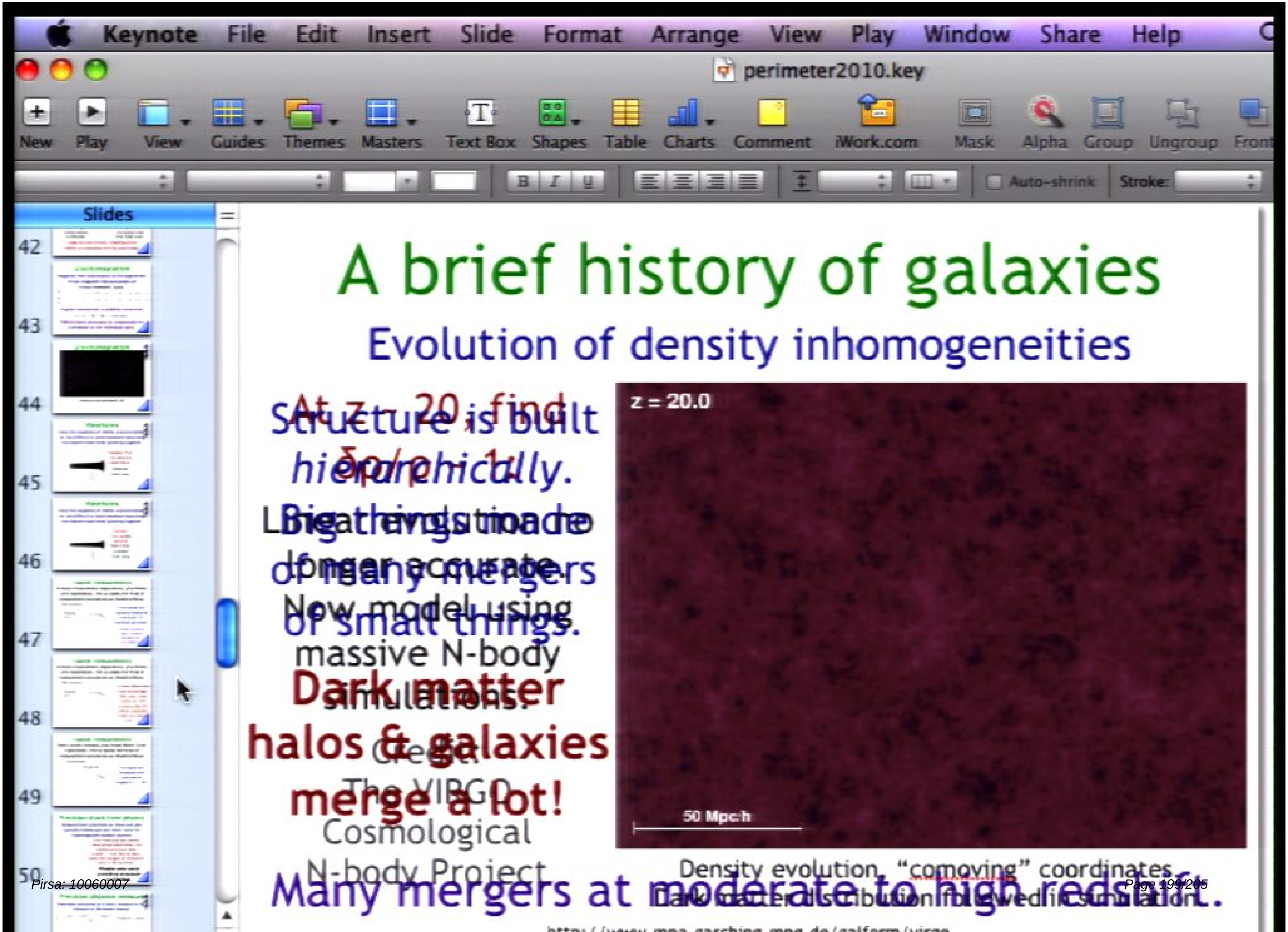
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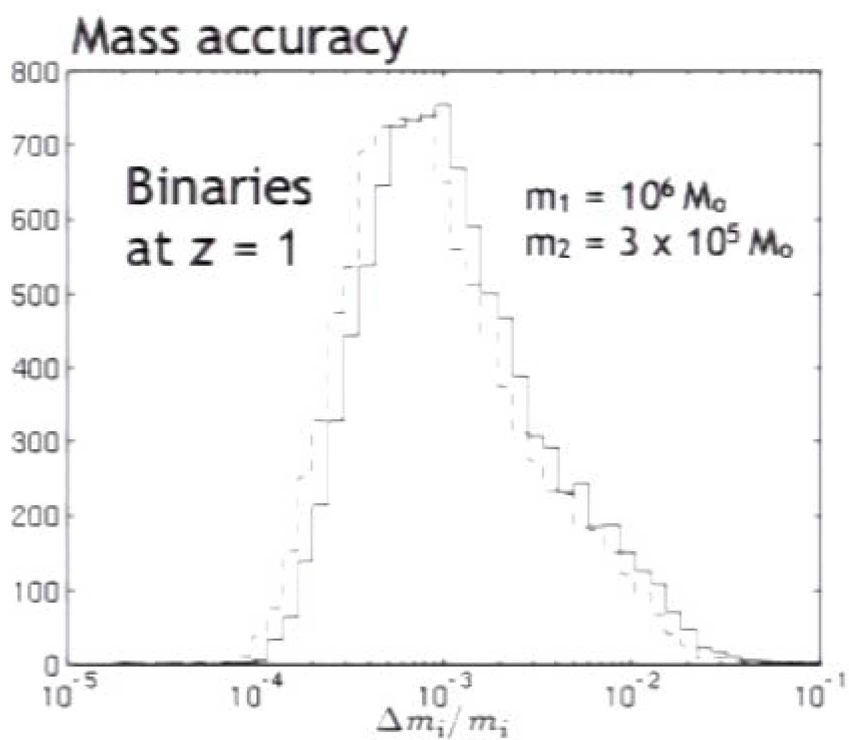
Inspiral measurements

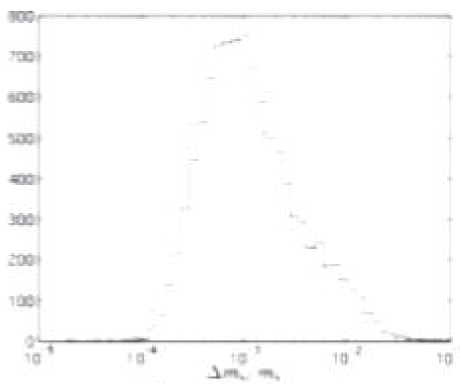
Analysis of parameter dependence, plus Monte Carlo exploration, lets us assess the kinds of measurement accuracies we should achieve:

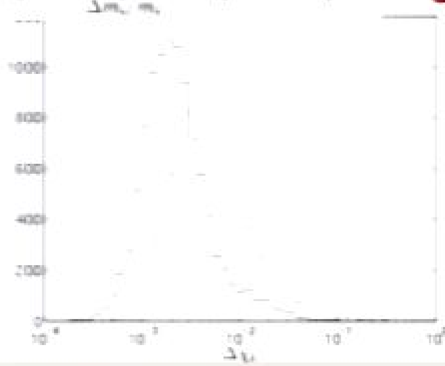


Current black hole mass knowledge: Best case, mass known to ~10% accuracy (Sgr-A*); others, generally known to a factor of 2 – 3.

Precision black hole physics

Measurement precision on mass and spin typically below percent level, even for cosmologically distant sources.





Since mass and spin totally characterize black holes, this allows us to trace their growth ... and thus to learn about the mergers of structures early in the universe.

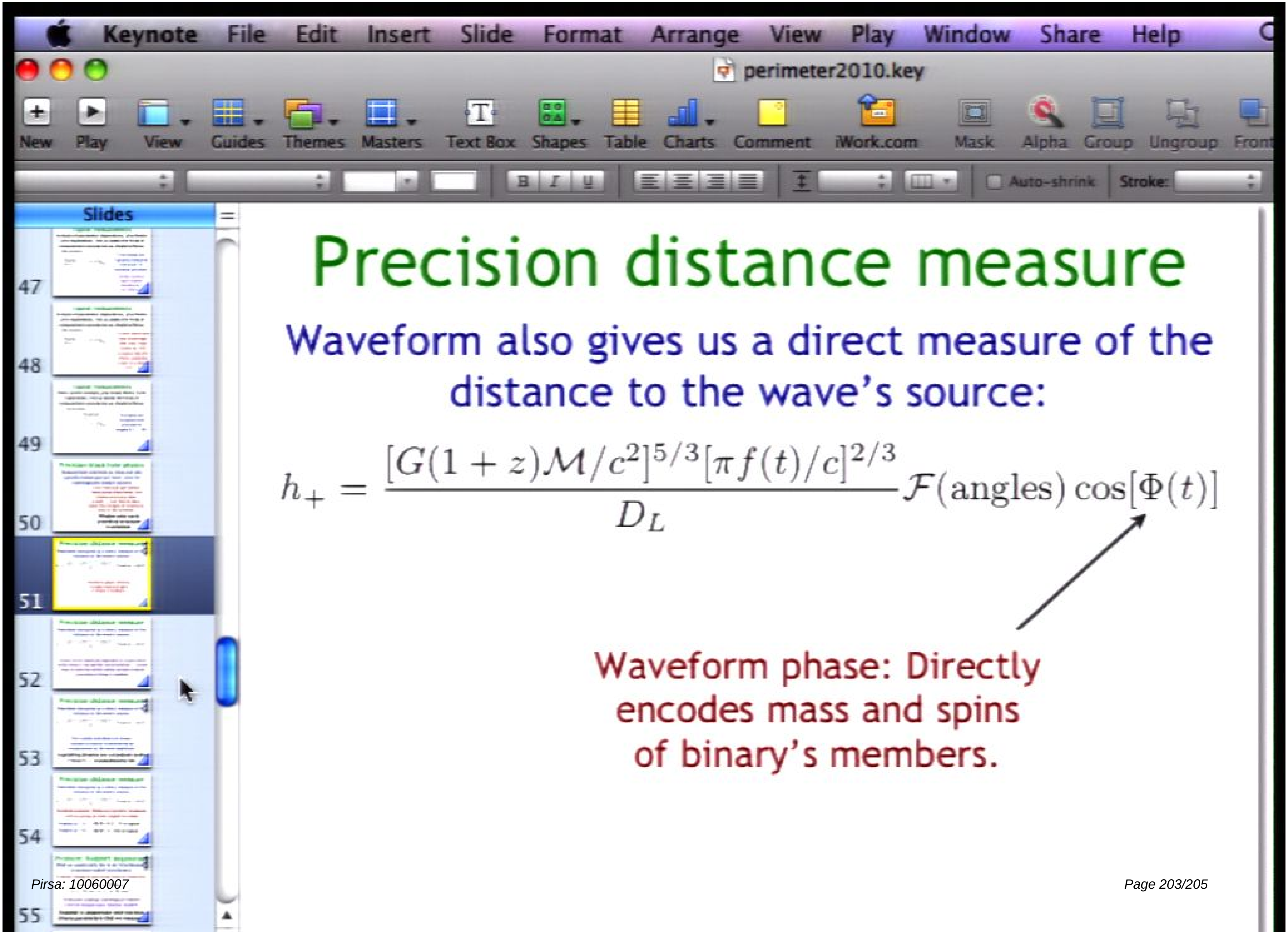
Window onto early growth of structure in universe.

Precision distance measure

Waveform also gives us a direct measure of the distance to the wave's source:

$$h_+ = \frac{[G(1+z)\mathcal{M}/c^2]^{5/3}[\pi f(t)/c]^{2/3}}{D_L}\mathcal{F}(\text{angles})\cos[\Phi(t)]$$

Waveform phase: Directly encodes mass and spins of binary's members.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Detailed analysis: Distances typically measured with accuracy (a few)/(signal-to-noise)

Nearby (z ~ 1): $\delta D/D \simeq 0.2$ - 1% is typical

Distant (z ~ 5): $\delta D/D \simeq 3$ - 10% is typical

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Detailed analysis: Distances typically measured with accuracy (a few)/(signal-to-noise)

Nearby ($z \sim 1$): $\delta D/D \approx 0.2$ - 1% is typical

Distant ($z \sim 5$): $\delta D/D \approx 3$ - 10% is typical