Title: Probe Branes, time dependent couplings and thermallization in AdS/CFT

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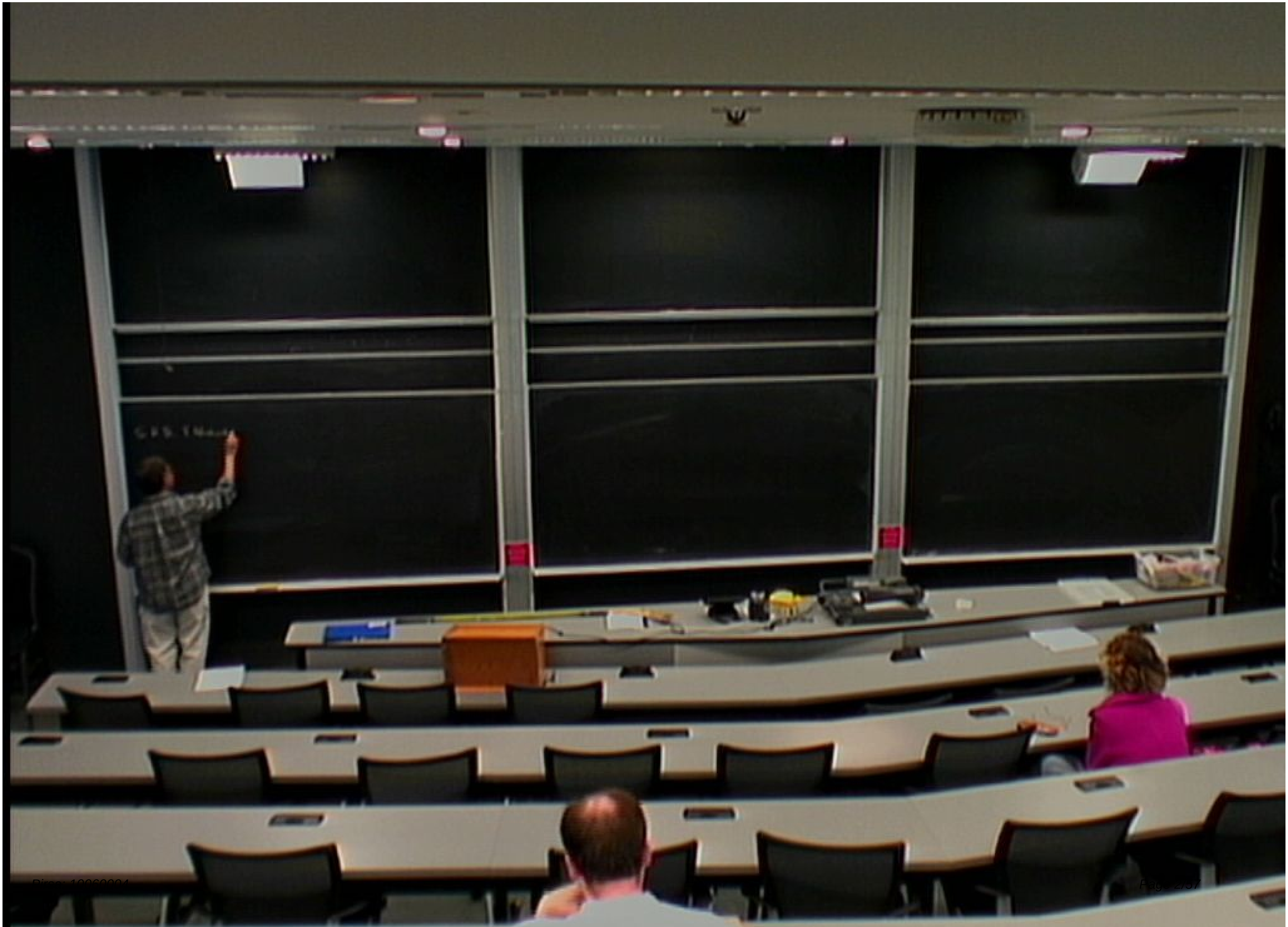
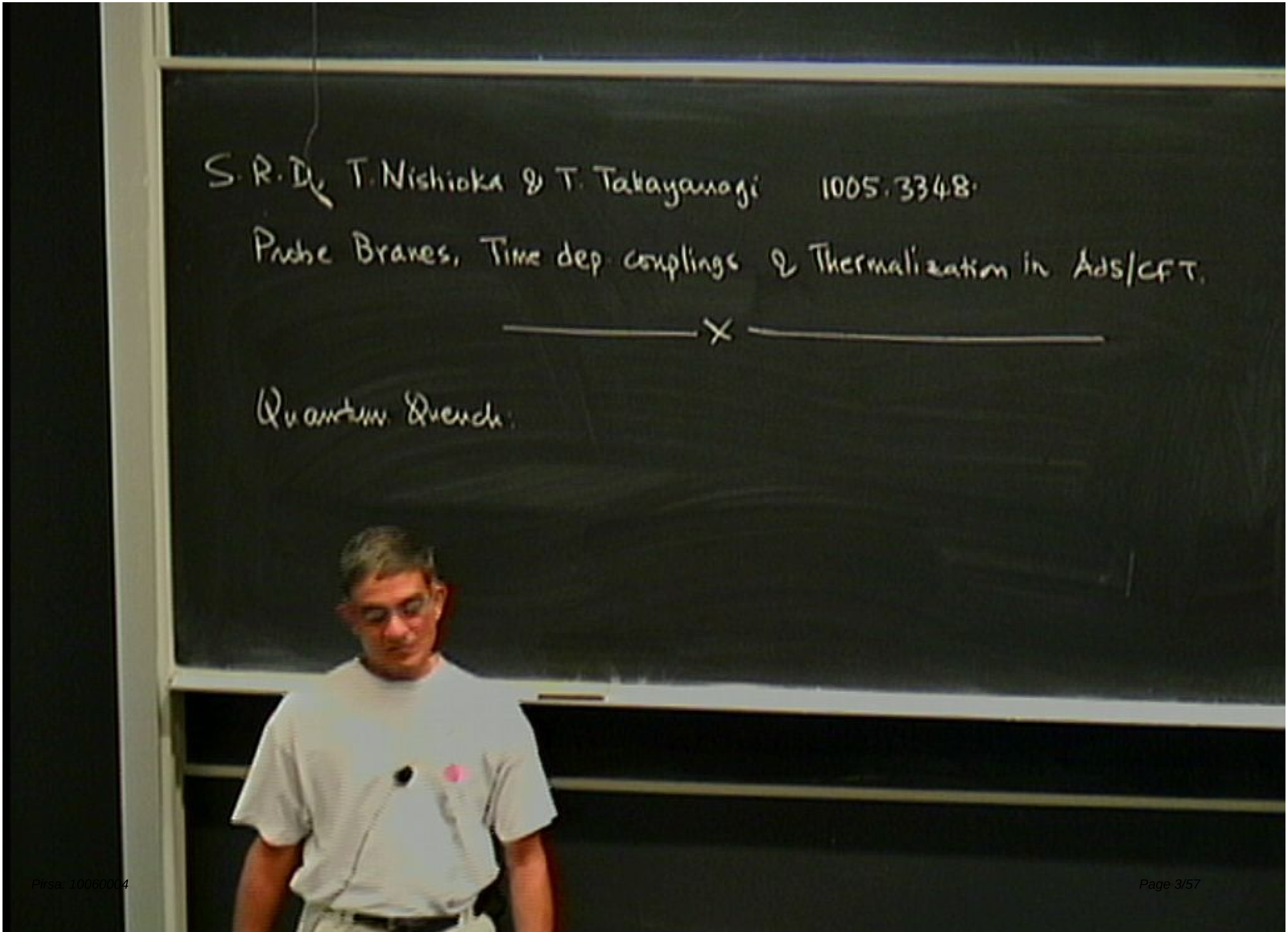
URL: http://pirsa.org/10060004

Abstract: We study time dependent couplings in conformal field theories using rotating probe branes in AdS X S spacetimes. We find that induced metrics on the brane worldvolumes develop horizons with characteristic Hawking temperatures even when there is no black hole in the bulk. This framework is used to obtain toy models for quantum quench.



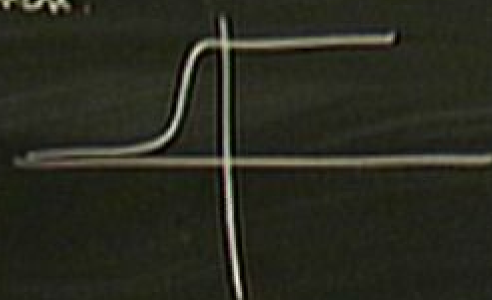

S.R.Das, T. Nishioka & T. Takayanagi 1005.3348
Probe Branes, Time dep couplings & Thermalization in AdS/CFT.
Quantum Quench.

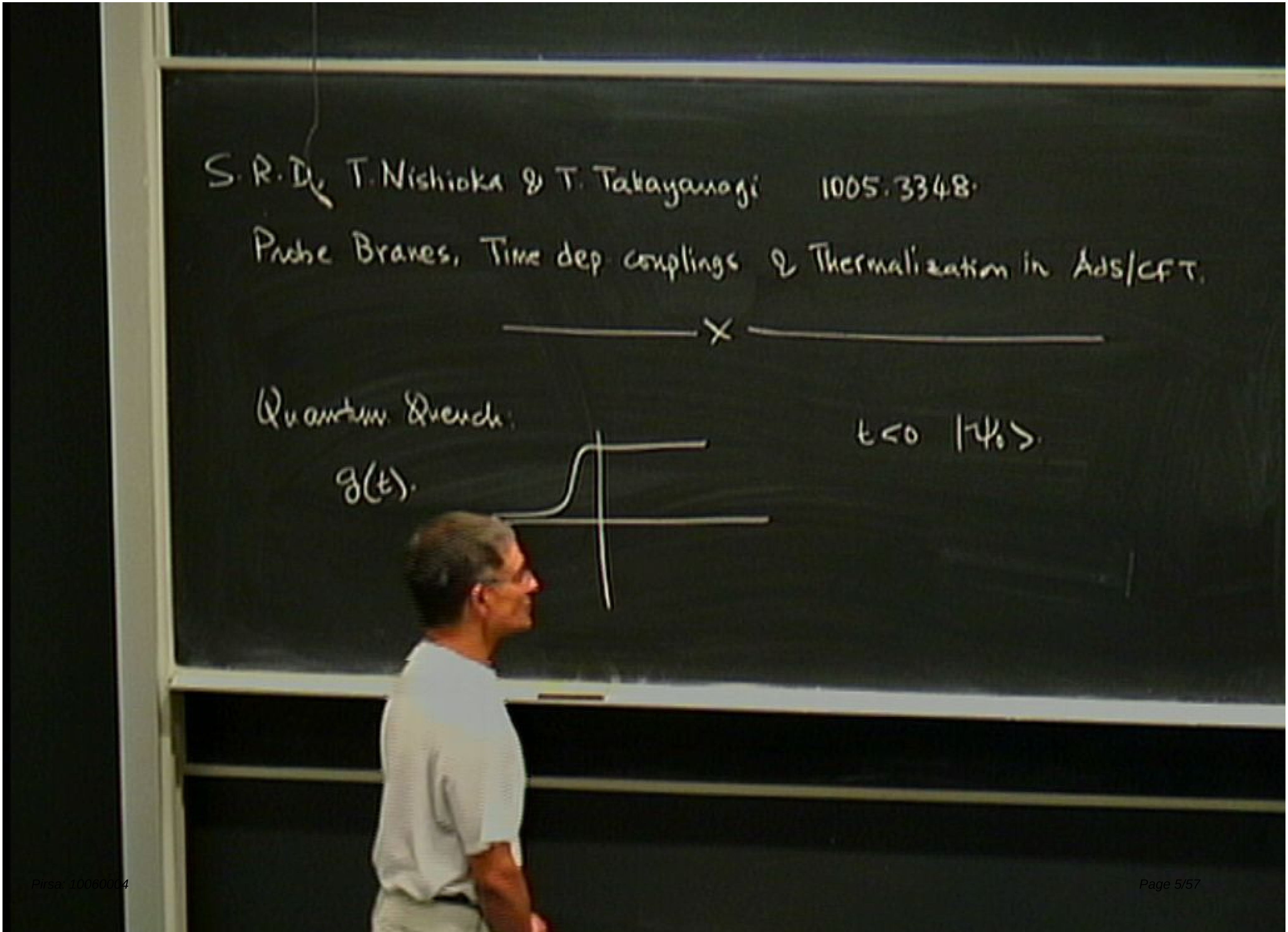
S. R. Das, T. Nishioka & T. Takayanagi   1005.3348.

Probe Branes, Time dep. couplings & Thermalization in AdS/CFT.

Quantum Quench:

$g(t)$.

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Probe Branes, Time dep couplings & Thermalization in AdS/CFT.
Quantum Quench:
g(t).
t<0 |Ψ0>.

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Probe Branes, Time dep couplings & Thermalization in AdS/CFT.

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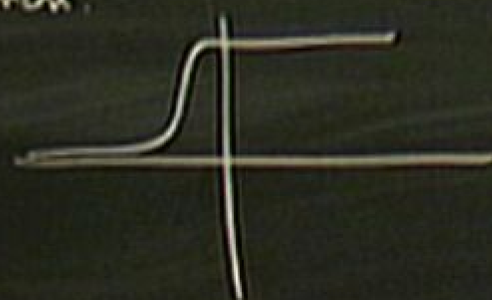
Quantum Quench:

$g(t)$.

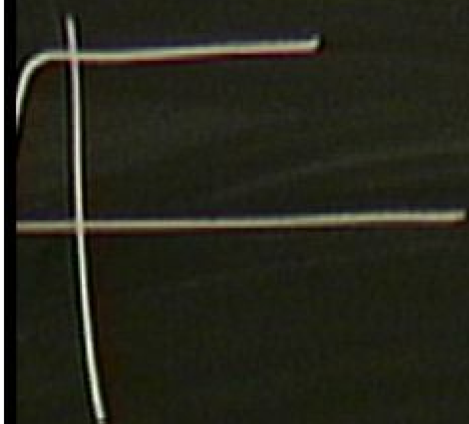


Sudden:

$t<0 \quad |\Psi_0\rangle. \quad H_0|\Psi_0\rangle = 0.$

$t>0 \quad H_0 \longrightarrow H.$

...p Couplings & Thermalization in AdS/CFT.



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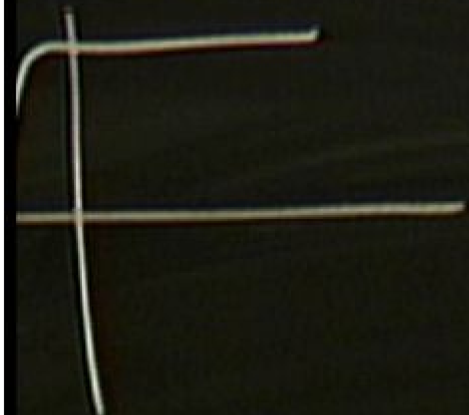
$t>0 \quad H_0 \longrightarrow H.$

$\delta t \ll \frac{1}{(\Delta M)}$

ep. Couplings & Thermalization in AdS/CFT.



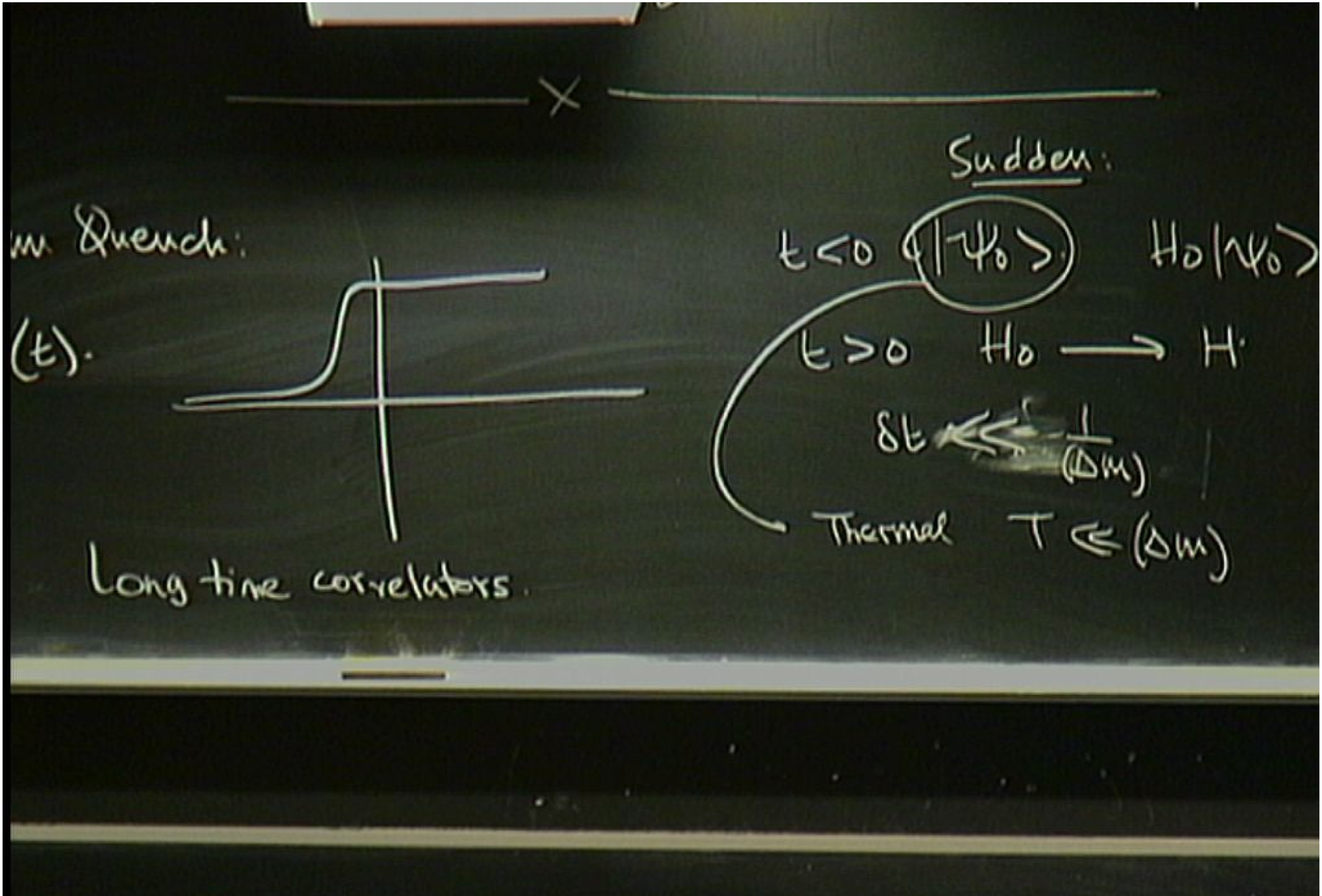
Sudden:

$t<0 \quad |\Psi_0\rangle \qquad H_0|\Psi_0\rangle = 0$

$t>0 \quad H_0 \longrightarrow H$

$\delta t \lll \frac{1}{(\Delta m)}$

Thermal $\quad T \ll (\Delta m)$


Quench:
(t).
Long time correlators.
Sudden:
t<0 |ψ0> H0|ψ0>
t>0 H0 → H
δt << 1/(Δm)
Thermal T << (Δm)

Calabrese & Cardy

Calabrese, Cardy & Sotiriadis.

Calabrese & Cardy

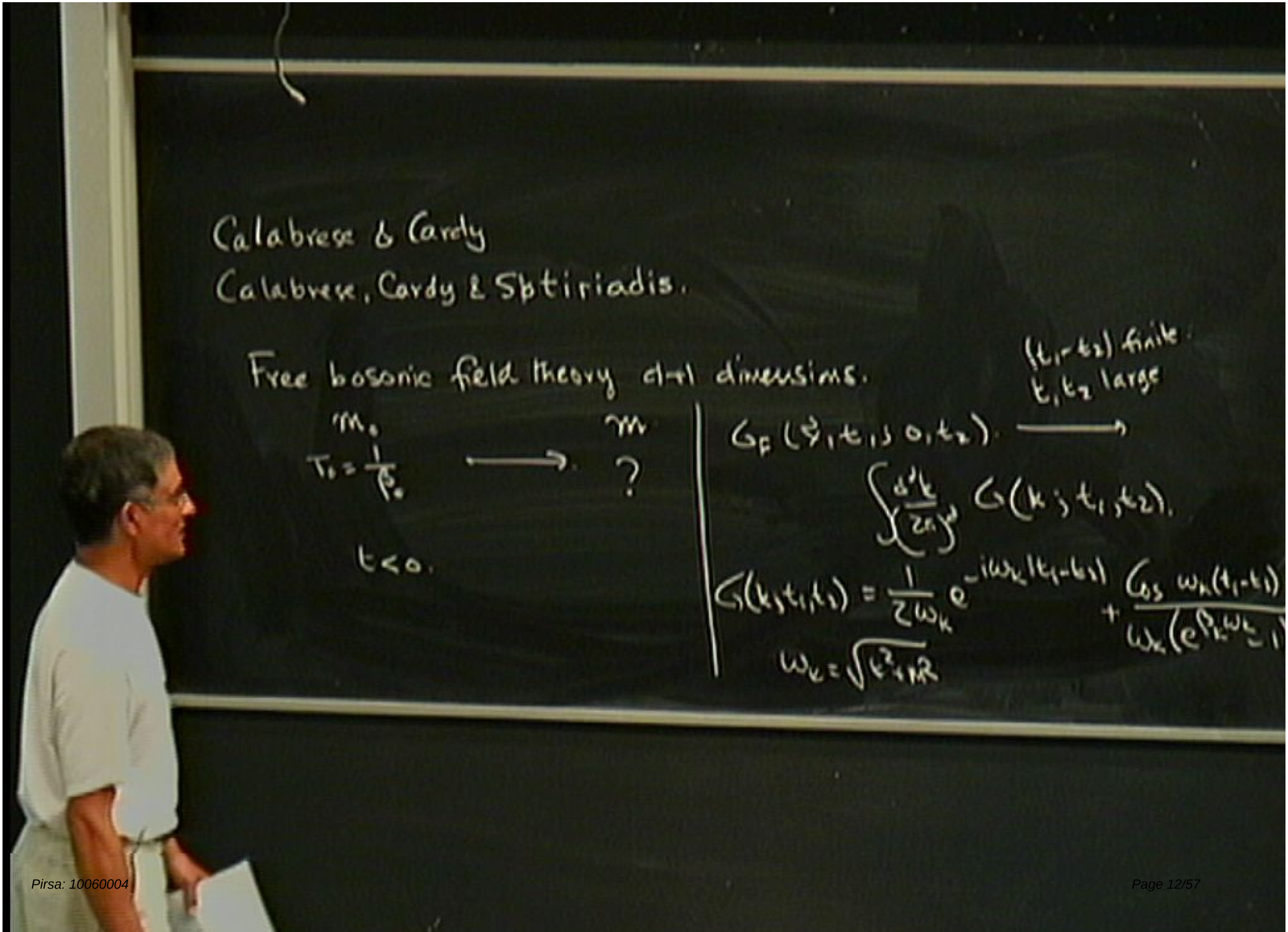
Calabrese, Cardy & Sotiriadis.

Free bosonic field theory $d+1$ dimensions.

m_0

$T_0 = \frac{1}{\beta_0}$

$t<0$.


Calabrese & Cardy
Calabrese, Cardy & Sotiriadis.
Free bosonic field theory d+1 dimensions.
m_0 → m
$T_0 = \frac{1}{\beta_0}$ → ?
$t < 0$.
$G_F(\vec{x}_1, t_1; 0, t_2)$ $\xrightarrow{\substack{(t_1 - t_2)\text{ finite} \\ t_1, t_2 \text{ large}}}$
$\int \frac{d^d k}{(2\pi)^d} G(k; t_1, t_2)$.
$G(k; t_1, t_2) = \frac{1}{2\omega_k} e^{-i\omega_k |t_1 - t_2|} + \frac{\cos \omega_k(t_1 - t_2)}{\omega_k(e^{\beta_k \omega_k} - 1)}$
$\omega_k = \sqrt{k^2 + m^2}$

riadis.

ory d+1 dimensions.

m

?

$(t_1 - t_2)$ finite;
t_1, t_2 large

$$G_F(\vec{x}_1, t_1; 0, t_2) \longrightarrow \int \frac{d^d k}{(2\pi)^d} G(k; t_1, t_2).$$

$$G(k; t_1, t_2) = \frac{1}{2\omega_k} e^{-i\omega_k |t_1 - t_2|} + \frac{\cos \omega_k (t_1 - t_2)}{\omega_k \left(e^{\beta_k \omega_k} - 1\right)}.$$

$$\omega_k = \sqrt{k^2 + m^2}$$

$$\beta_k = \frac{1}{\omega_k} \log \left[\frac{(\omega_k - \omega_{0k})^2 + e^{\beta_0 \omega_{0k}} (\omega_k + \omega_{0k})^2}{(\omega_k + \omega_{0k})^2 + e^{\beta_0 \omega_{0k}} (\omega_k - \omega_{0k})^2} \right]$$

eory d+1 dimensions.

m

$\Rightarrow$. ?

$|t_1 - t_2|$ finite.
t_1, t_2 large

$$G_F(\vec{x}_1, t_1; 0, t_2). \longrightarrow \int \frac{d^d k}{(2\pi)^d}\, G(k; t_1, t_2).$$

$$G(k; t_1, t_2) = \frac{1}{2\omega_k} e^{-i\omega_k |t_1 - t_2|} + \frac{\cos \omega_k (t_1 - t_2)}{\omega_k (e^{\beta_k \omega_k} - 1)}$$

$\omega_k = \sqrt{k^2 + m^2}$

$\omega_{0k} = \sqrt{k^2 + m_0^2}$

$$\beta_k = \frac{1}{\omega_k} \log \left[\frac{(\omega_k - \omega_{0k})^2 + e^{\beta_0 \omega_k} (\omega_k + \omega_{0k})^2}{(\omega_k + \omega_{0k})^2 + e^{\beta_0 \omega_k} (\omega_k - \omega_{0k})^2} \right]$$

If $T_0 = 0$ $\quad \beta_k = \frac{2}{\omega_k} \log \frac{\omega_k + \omega_{0k}}{|\omega_k - \omega_{0k}|}$

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If $T_0=0$ $\qquad \beta_k = \frac{2}{\omega_k}\log\frac{\omega_k+\omega_{0k}}{|\omega_k-\omega_{0k}|}.$

$m_0\neq 0,\ m=0$
$T_0\neq 0$ $\quad \rightarrow \beta(k=0) = \frac{4}{m_0}\tanh\left(\frac{\beta_0 m_0}{2}\right). \longrightarrow \frac{4}{m_0} \quad T_0=0.$

$$\beta_k = \frac{1}{\omega_k} \log \left[\frac{(\omega_k - \omega_{0k})^2 + e^{\beta_0 \omega_k} (\omega_k + \omega_{0k})^2}{(\omega_k + \omega_{0k})^2 + e^{\beta_0 \omega_k} (\omega_k - \omega_{0k})^2} \right]$$

If $T_0 = 0$ $\quad \beta_k = \frac{2}{\omega_k} \log \frac{\omega_k + \omega_{0k}}{|\omega_k - \omega_{0k}|}$

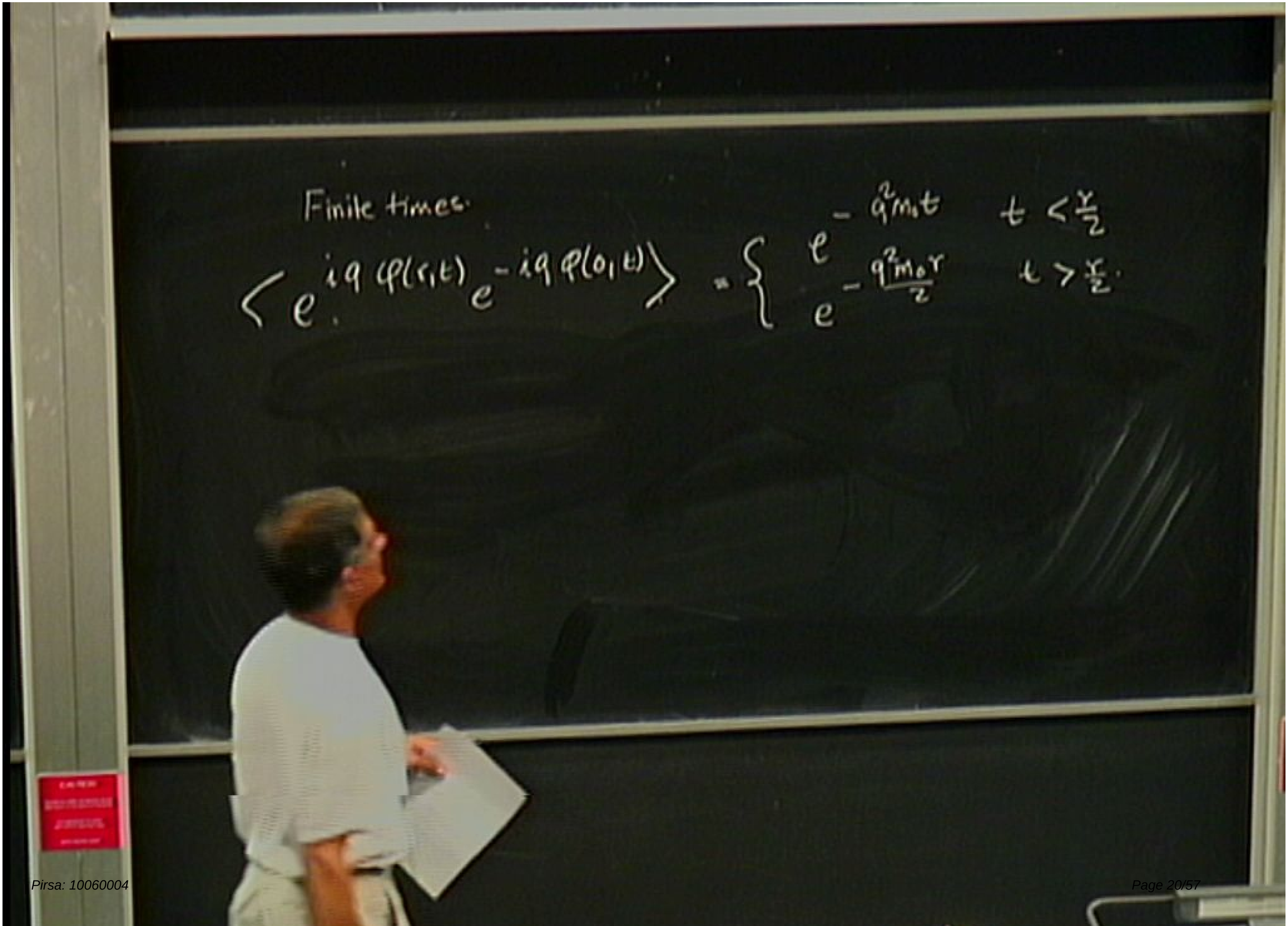
$m_0 \neq 0, \; m = 0, \; T_0 \neq 0 \quad \rightarrow \beta(k=0) = \frac{4}{m_0} \tanh\left(\frac{\beta_0 m_0}{2}\right) \longrightarrow \frac{4}{m_0} \quad T_0 = 0$

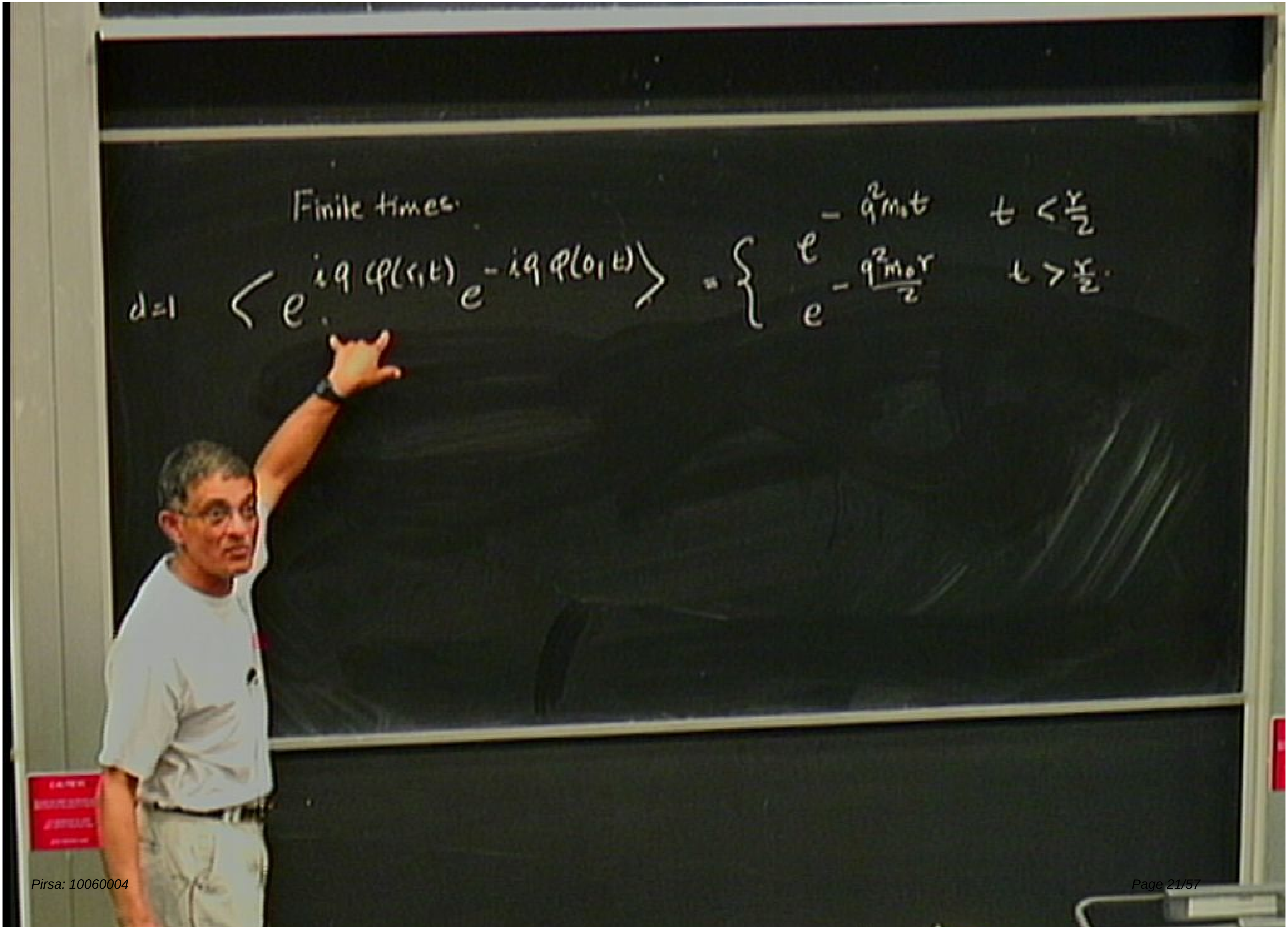
$d = 1 \quad T_0 = 0 \quad$ Large m_0

$$\frac{G(r,t)}{m_0} \longrightarrow \begin{cases} 0 & t < \end{cases}$$

Finite times

$$\langle e^{iq\varphi(r,t)} e^{-iq\varphi(0,t)} \rangle$$


Finite times.
$\langle e^{iq\varphi(r,t)} e^{-iq\varphi(0,t)} \rangle = \begin{cases} e^{-q^2 m_0 t} & t < \frac{r}{2} \\ e^{-\frac{q^2 m_0 r}{2}} & t > \frac{r}{2}. \end{cases}$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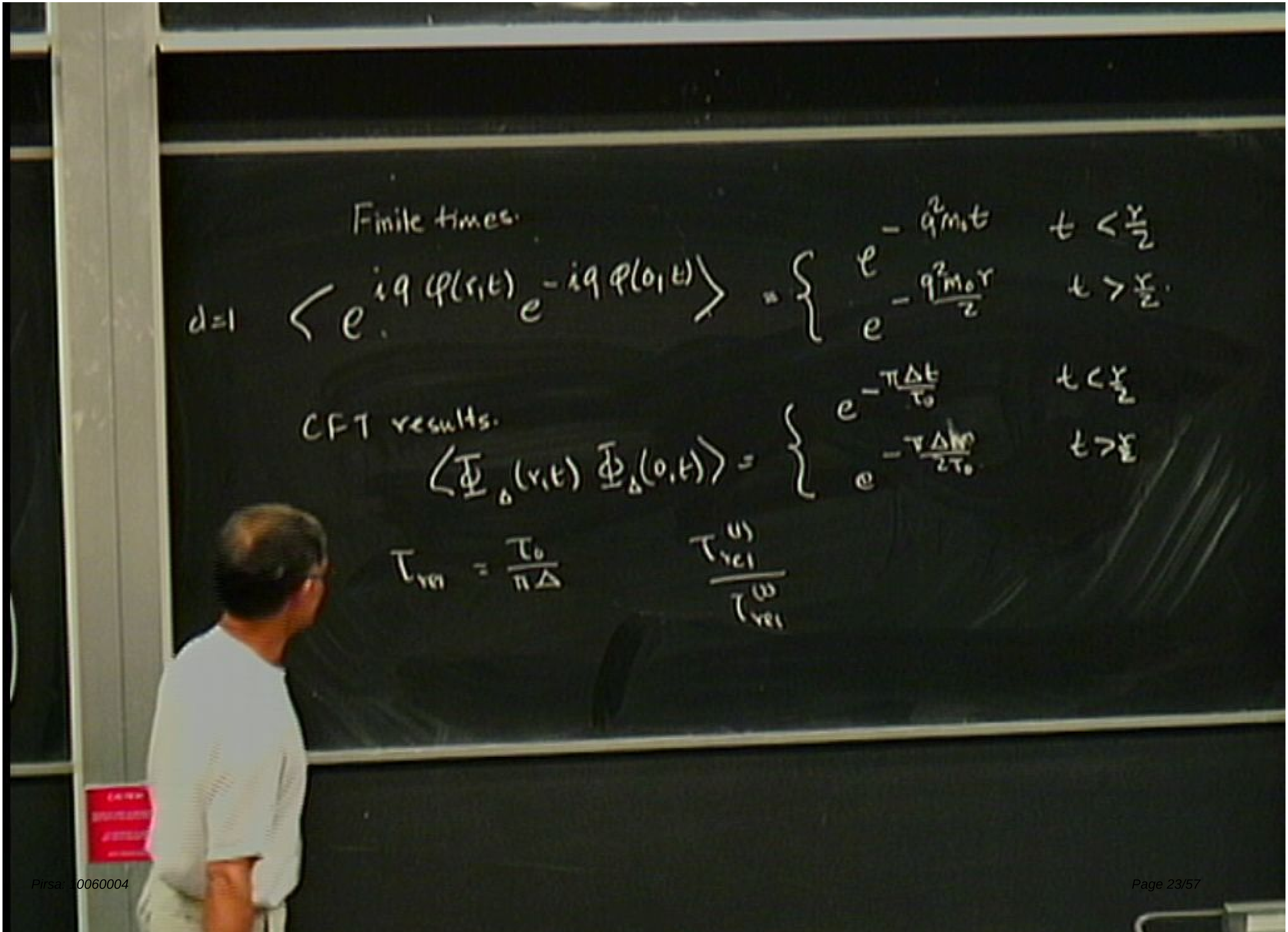
$d=1 \quad \langle e^{iq\varphi(r,t)} e^{-iq\varphi(0,t)} \rangle = \begin{cases} e^{-q^2 m_0 t} & t < \frac{r}{2} \\ e^{-\frac{q^2 m_0 r}{2}} & t > \frac{r}{2} \end{cases}$

Finite times.

$$d=1 \quad \left\langle e^{iq\varphi(r,t)} e^{-iq\varphi(0,t)} \right\rangle = \begin{cases} e^{-q^2 m_0 t} & t < \frac{r}{2} \\ e^{-\frac{q^2 m_0 r}{2}} & t > \frac{r}{2} \end{cases}$$

CFT results.

$$\langle \Phi_\Delta(r,t)\, \Phi_\Delta(0,t) \rangle = \begin{cases} e^{-\frac{\pi \Delta t}{\tau_0}} & t < \frac{r}{2} \\ e^{-\frac{\pi \Delta r}{2\tau_0}} & t > \frac{r}{2} \end{cases}$$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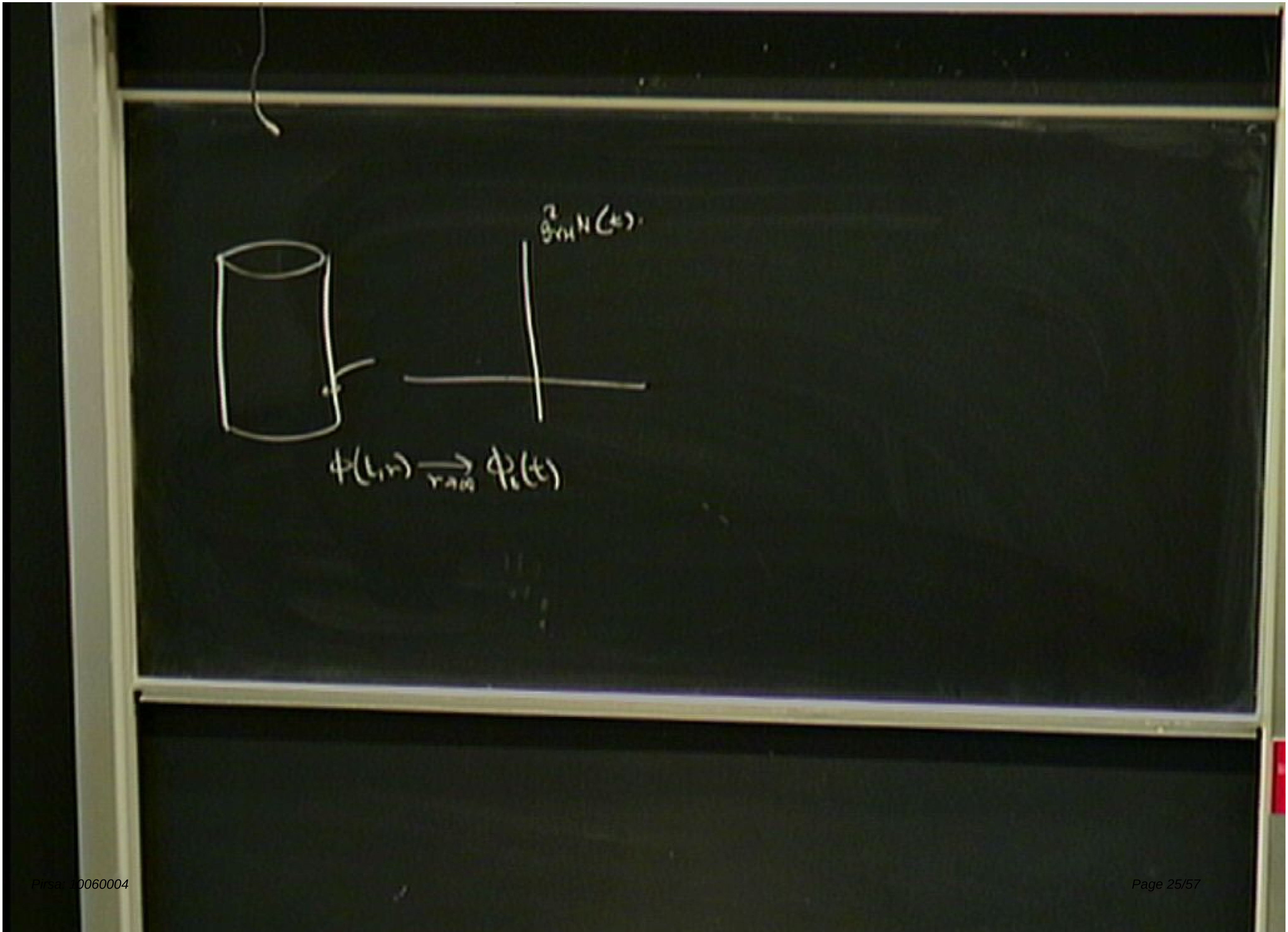
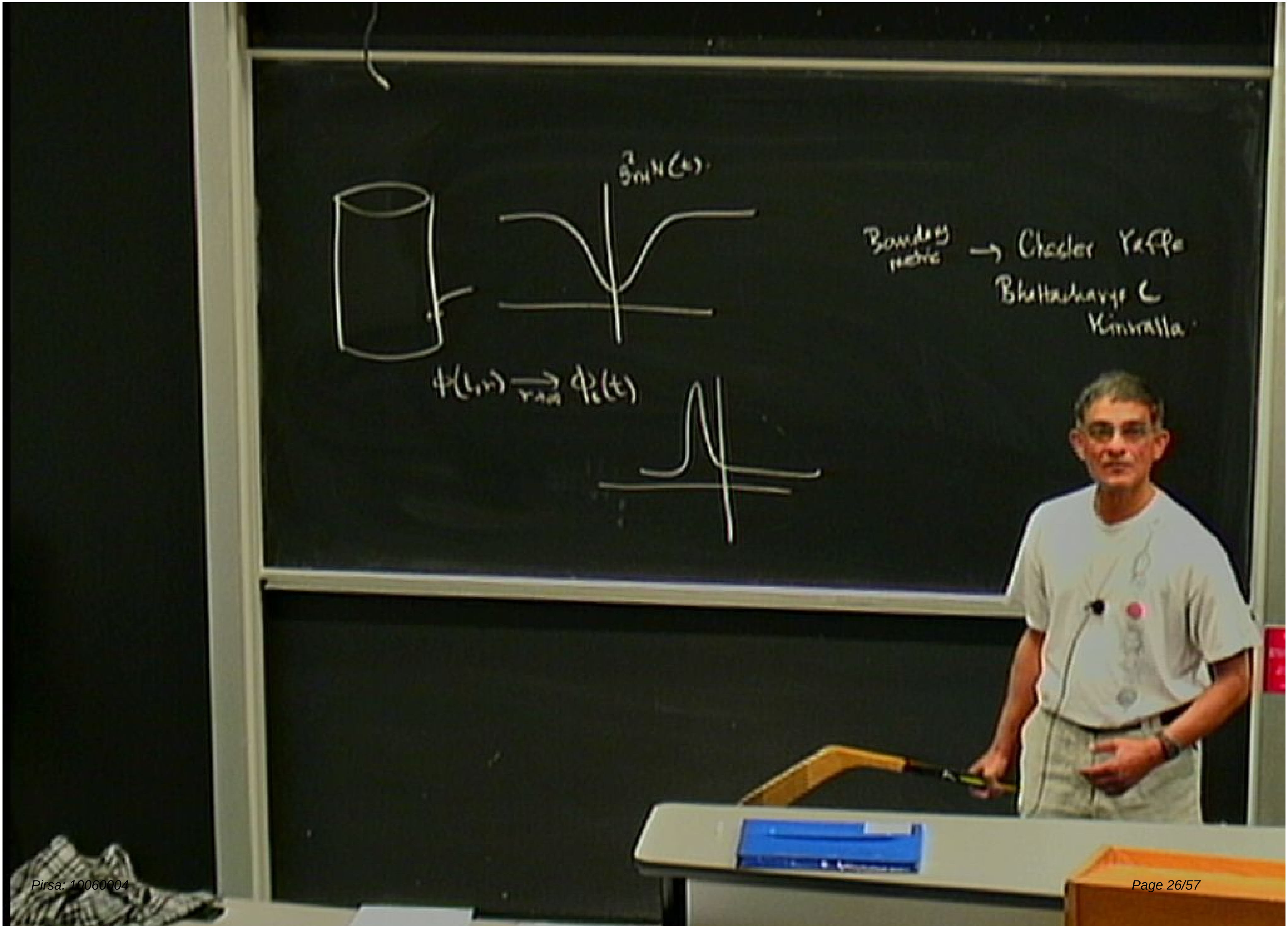
$\langle \Phi_\Delta(r,t)\, \Phi_\Delta(0,t) \rangle = \begin{cases} e^{-\frac{\pi\Delta t}{\tau_0}} & t < \frac{r}{2} \\ e^{-\frac{\pi\Delta r}{2\tau_0}} & t > \frac{r}{2} \end{cases}$
$\tau_{rel} = \frac{\tau_0}{\pi\Delta}$
$\frac{\tau_{rel}^{(1)}}{\tau_{rel}^{(2)}}$

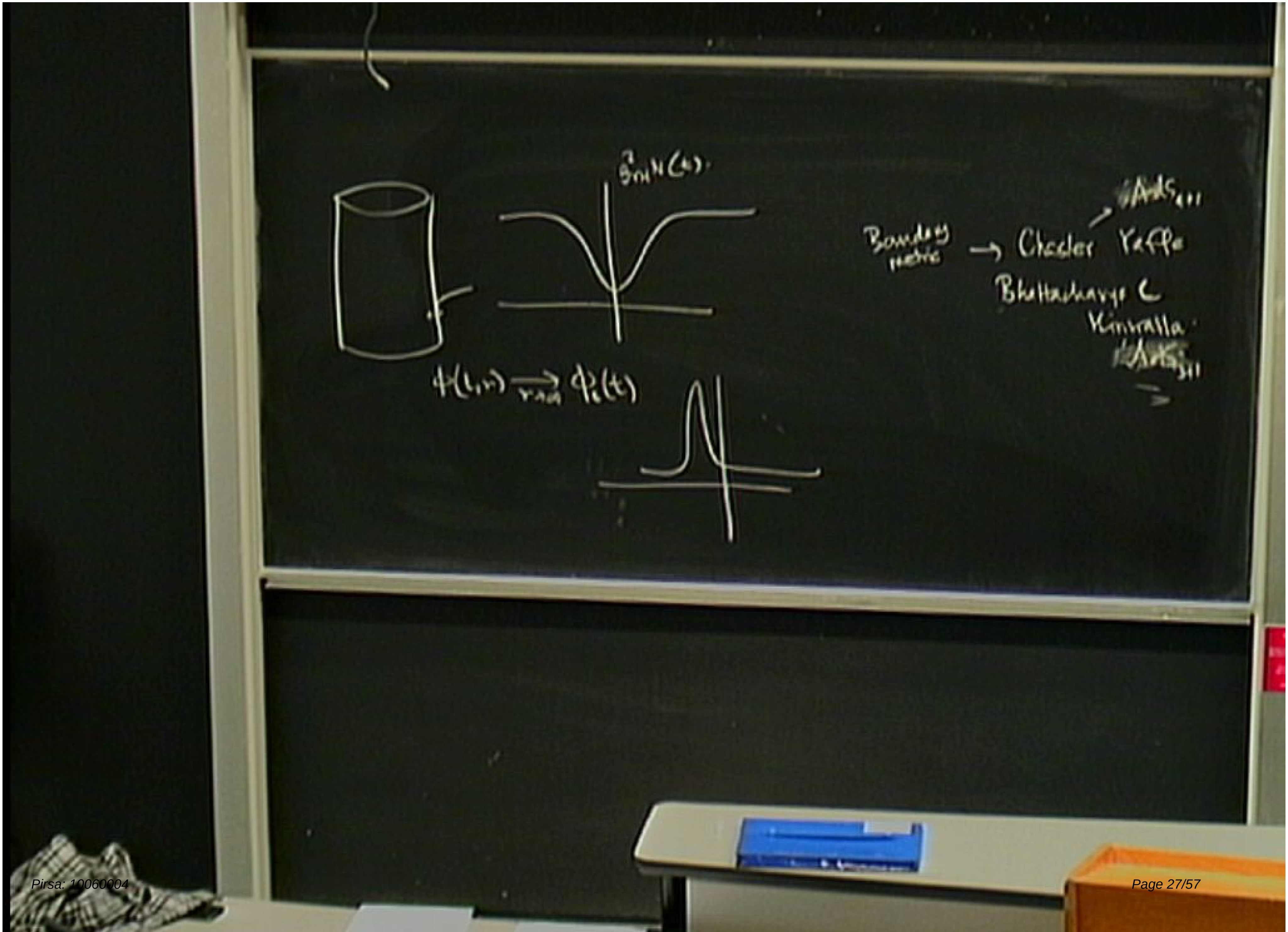
CFT results.

$$\langle \Phi_\Delta(r,t)\, \Phi_\Delta(0,t) \rangle = \begin{cases} e^{-\frac{\pi \Delta t}{\tau_0}} \\ e^{-\frac{\pi \Delta r}{2\tau_0}} \end{cases}$$

$$\tau_{rel} = \frac{\tau_0}{\pi \Delta} \qquad \frac{\tau_{rel}^{(1)}}{\tau_{rel}^{(2)}} = \frac{\Delta_2}{\Delta_1}$$


$\phi(t,r) \xrightarrow[r\to\infty]{} \phi_0(t)$


$g_{\mu\nu}(t)$
$\phi(t,r) \xrightarrow{r\to\infty} \phi_0(t)$
Boundary metric → Chesler Yaffe
Bhattacharya &
Minwalla


Boundary metric
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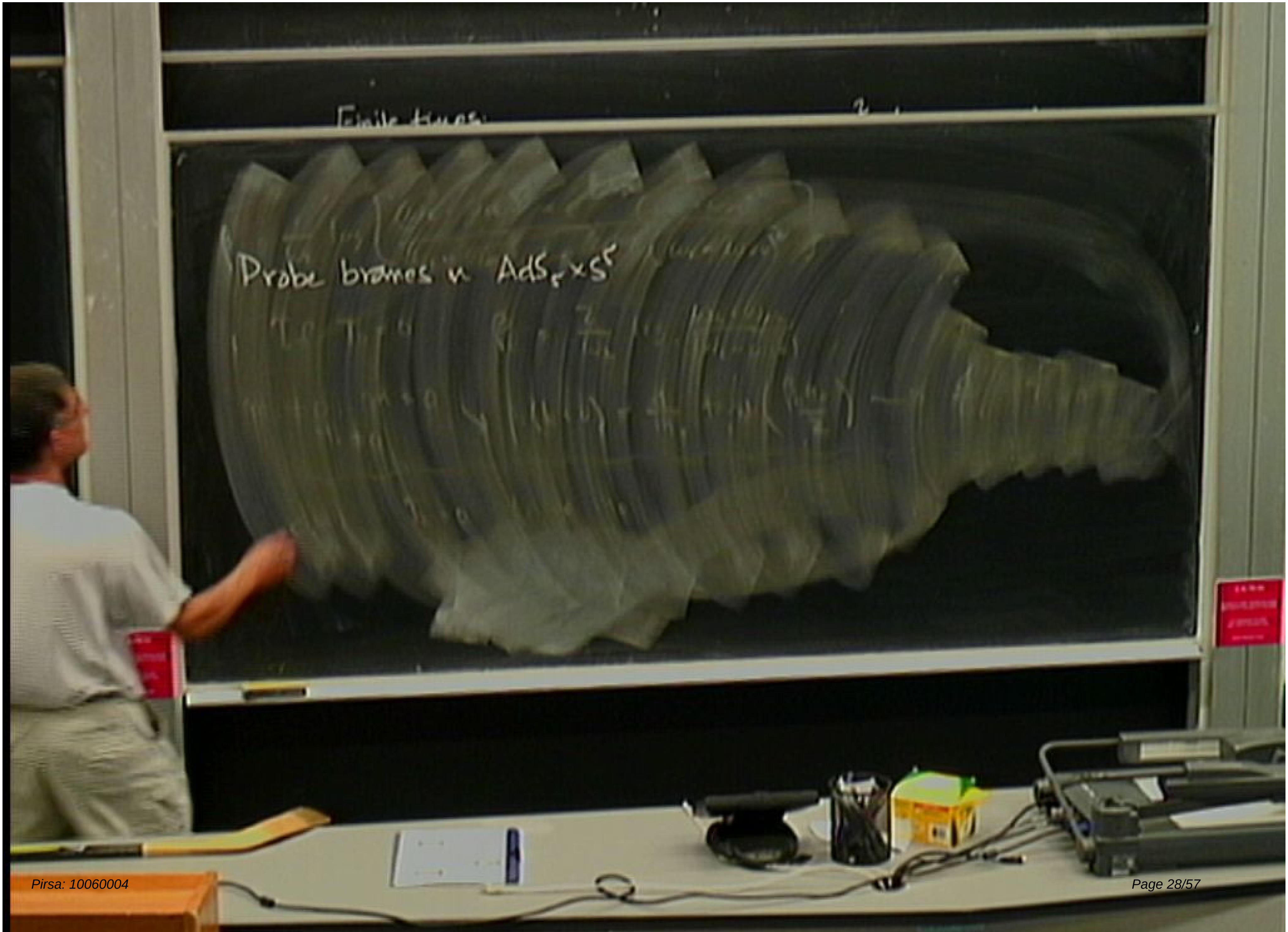
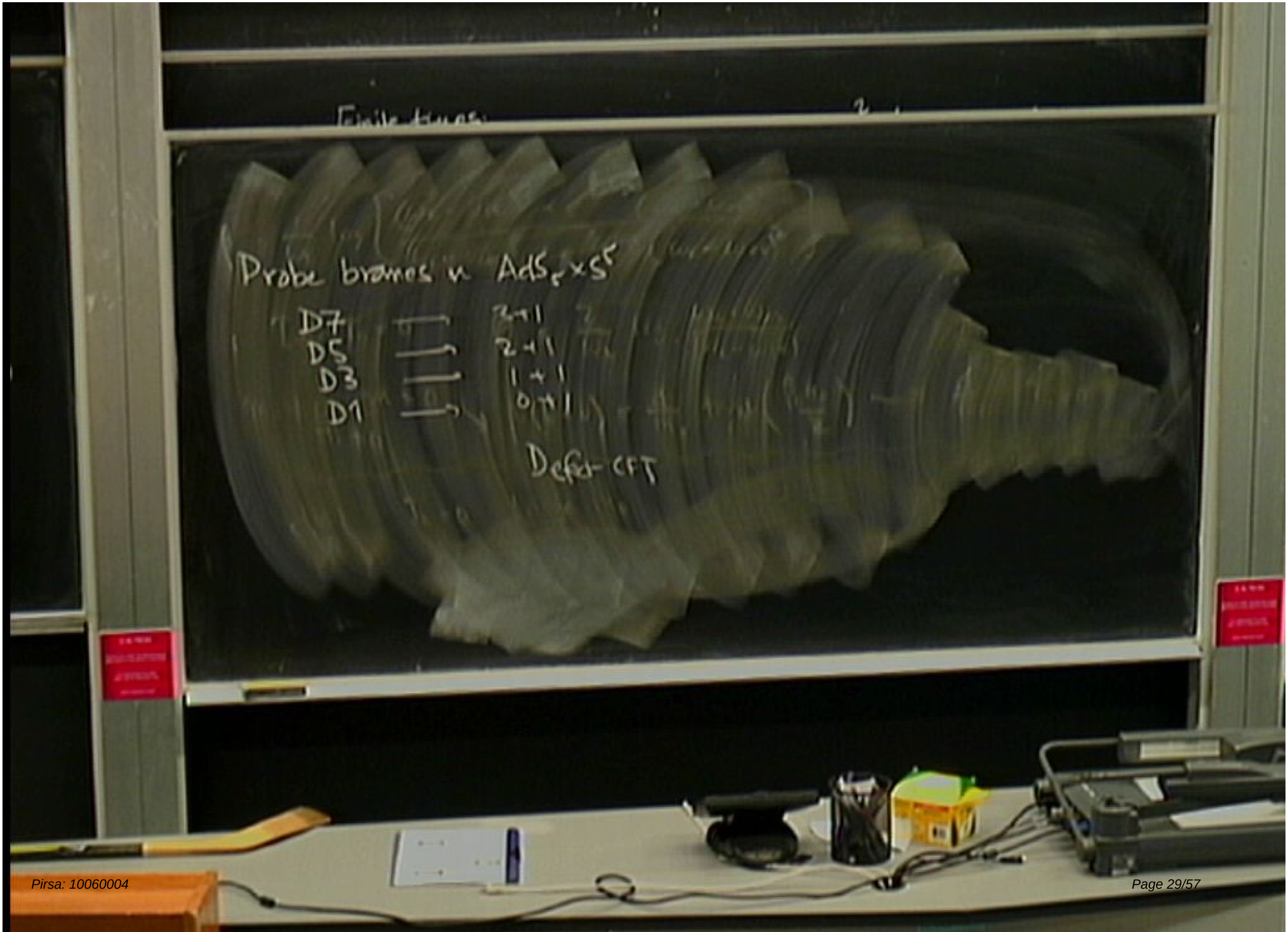
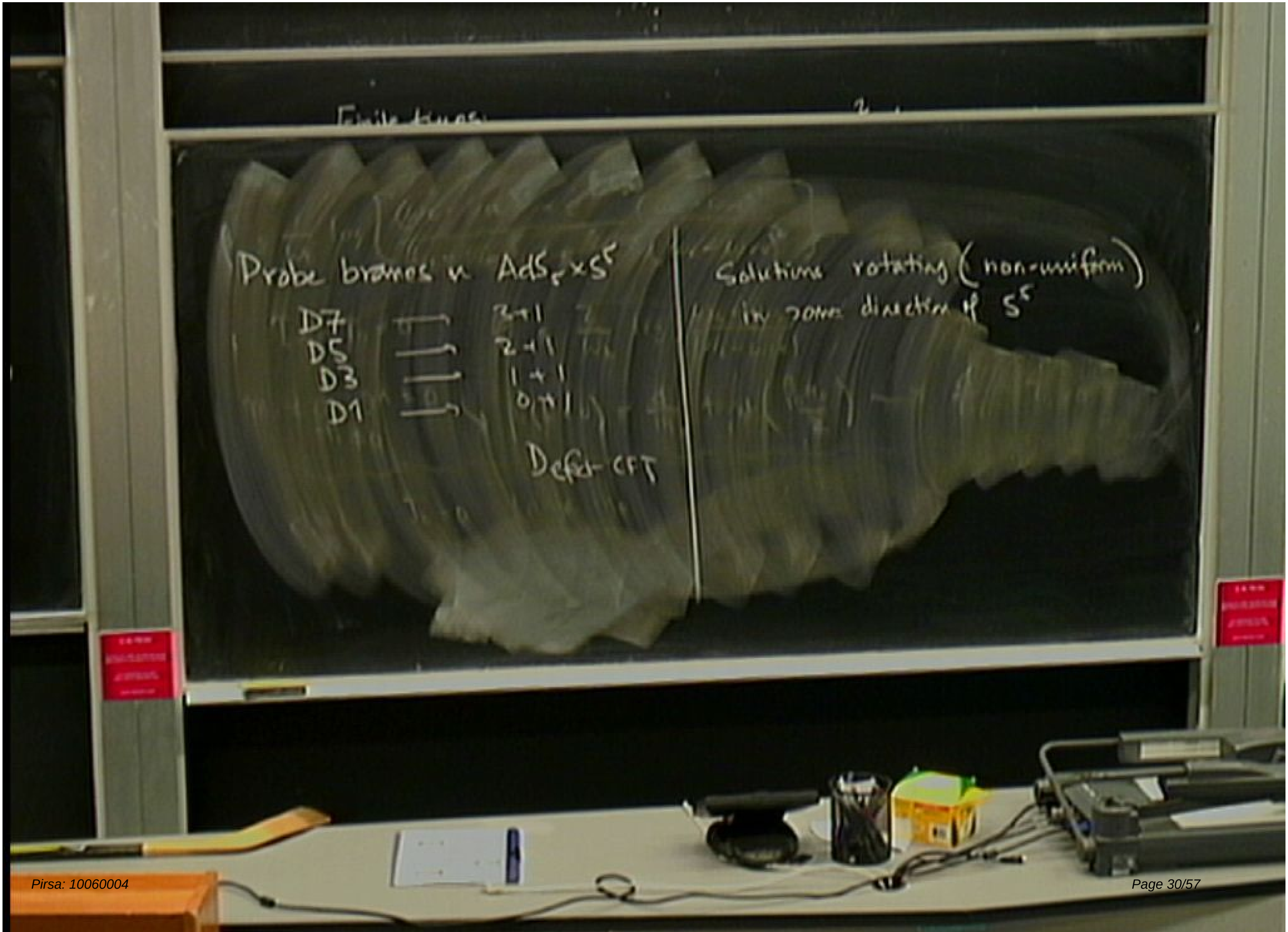

Probe branes in $AdS_5 \times S^5$


Probe branes in AdS5×S5
D7 → 3+1
D5 → 2+1
D3 → 1+1
D1 → 0+1
Defect CFT


Probe branes in AdS5 × S5
D7 → 3+1
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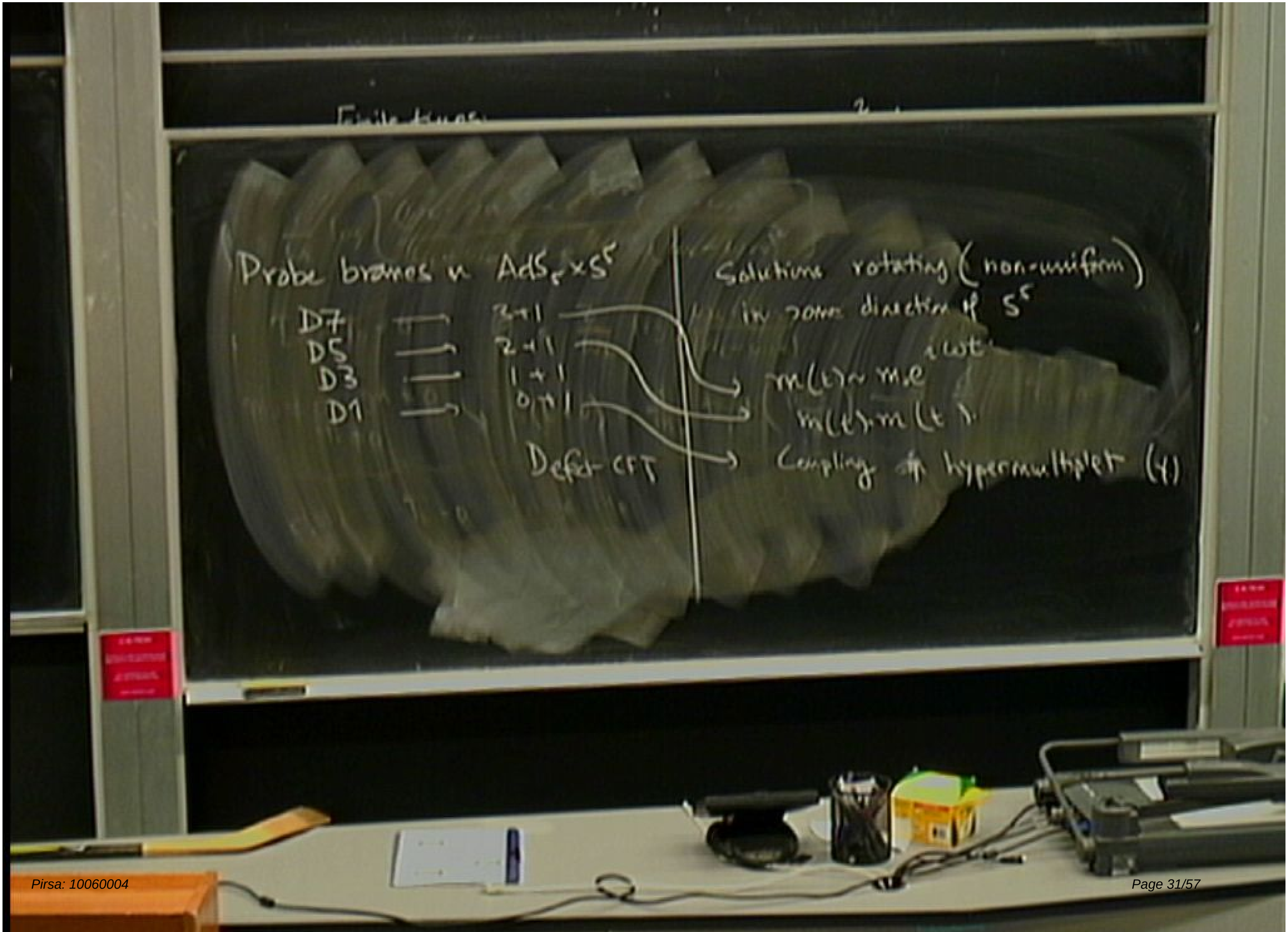
D1 → 0+1
Defect CFT
Solutions rotating (non-uniform)
in some direction of S5


Probe branes in AdS5 x S5
D7 → 3+1
D5 → 2+1
D3 → 1+1
D1 → 0+1
Defect CFT
Solutions rotating (non-uniform)
in some direction of S5
m(t) ~ m e^iωt
m(t) ~ m(t)
Coupling of hypermultiplet (ψ)

$1+1$ $\rightarrow$ $m(t) \sim m_0 e$

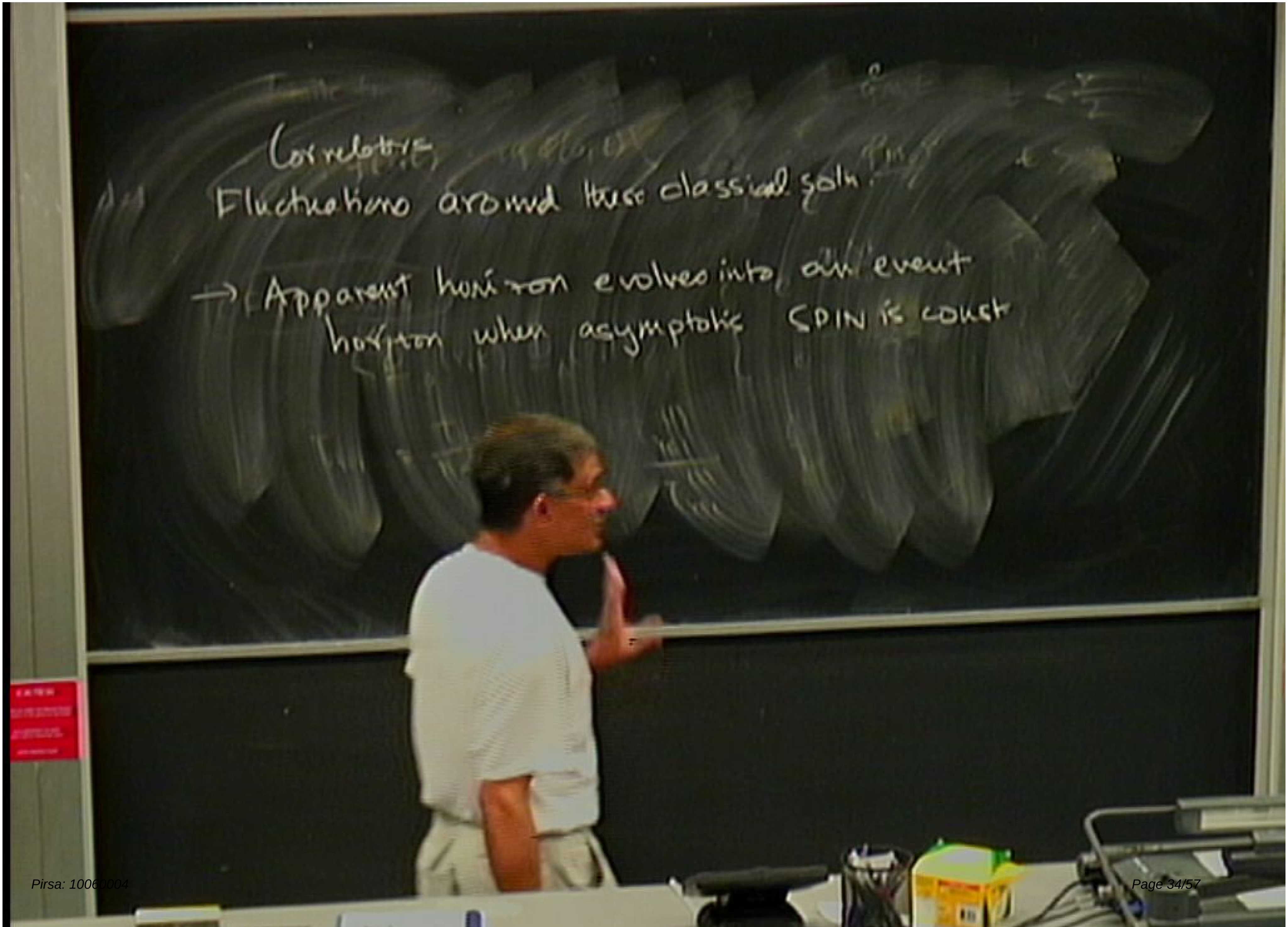
$0+1$ $\rightarrow$ $m(t) \cdot m(t)$.

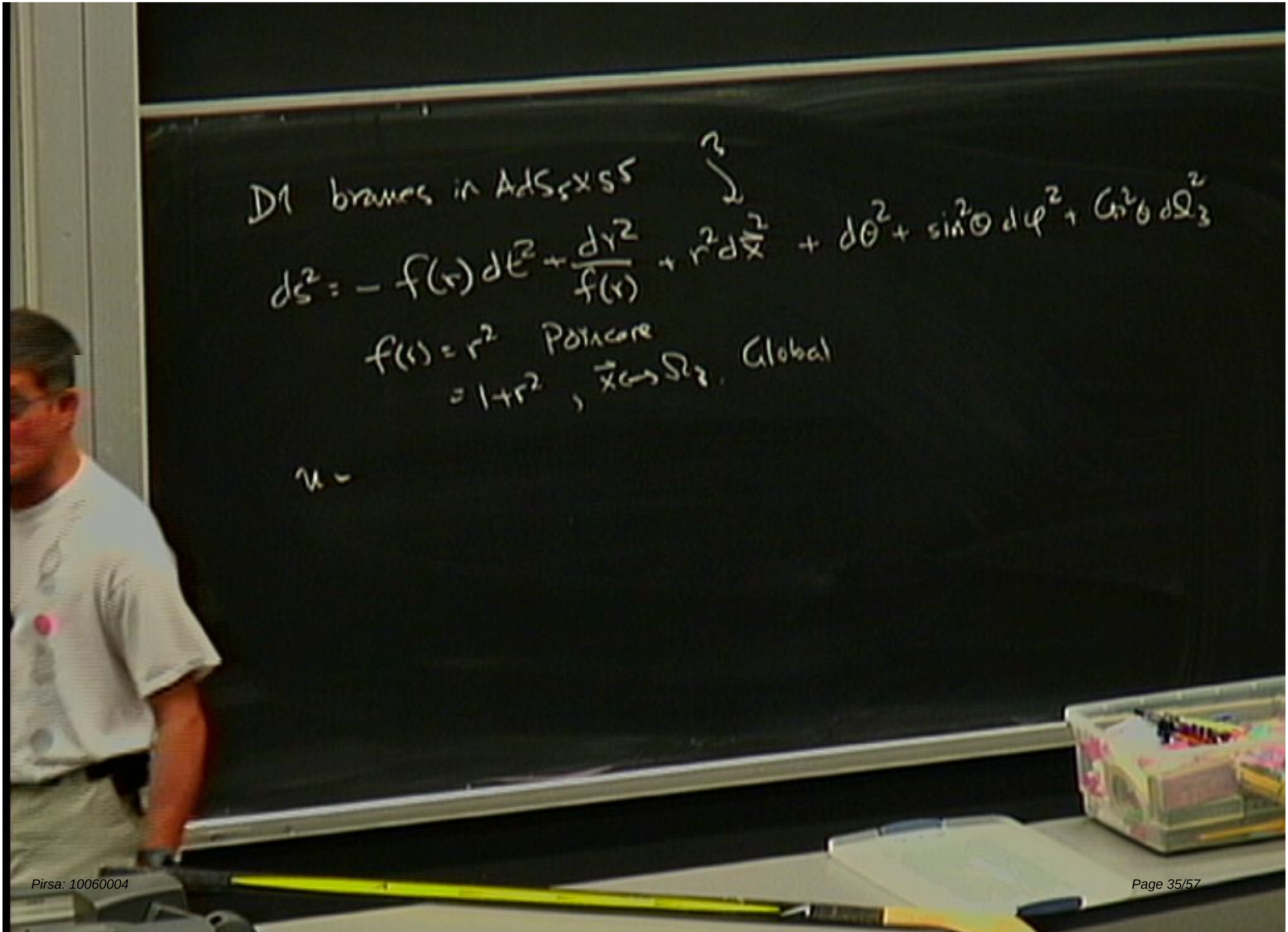
Defect CFT $\rightarrow$ Coupling hypermultipl

× INDUCED METRIC ON BRANE WORLDVOLUME DEVELOPS APPARENT HORIZONS.

Correlators

Fluctuations around these classical soln.


Correlators
Fluctuations around these classical soln.
→ Apparent horizon evolves into an event
horizon when asymptotic SDIN is const


D1 branes in $AdS_5 \times S^5$
$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\vec{x}^2 + d\theta^2 + \sin^2\theta\, d\varphi^2 + \cos^2\theta\, d\Omega_3^2$
$f(r) = r^2$ Poincare
$= 1 + r^2$, $\vec{x} \to \Omega_3$. Global
$u =$

D1 branes in $AdS_5 \times S^5$?

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\vec{x}^2 + d\theta^2 + \sin^2\theta\, d\varphi^2 + \cos^2\theta\, d\Omega_3^2$$

$\varphi(t, r)$

$f(r) = r^2$ Poincare

$= 1 + r^2$, $\vec{x} \to \Omega_3$ Global

$$\left.\begin{matrix} u \\ v \end{matrix}\right\} = t \pm \int \frac{dr}{f(r)}$$ Ingoing EF (v, r)

D1 branes in $AdS_5 \times S^5$

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\vec{x}^2 + d\theta^2 + \sin^2\theta\, d\varphi^2 + \cos^2\theta\, d\Omega_3^2$$

$\varphi(t,r)$

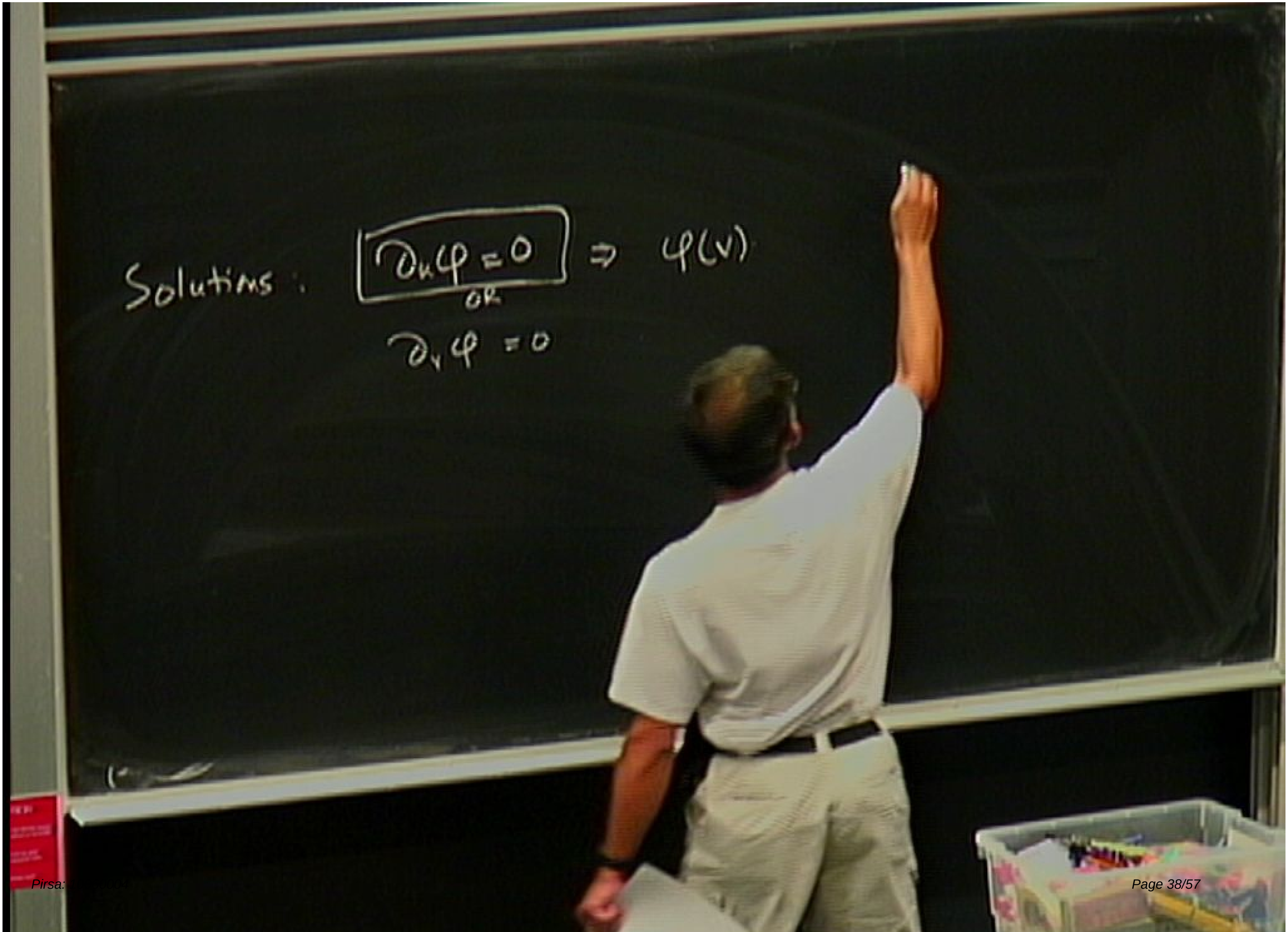
$f(r) = r^2$ Poincare

$= 1 + r^2$, $\vec{x} \leftrightarrow \Omega_3$ Global

$$\left.\begin{matrix} u \\ v \end{matrix}\right\} \; t \pm \int \frac{dr}{f(r)}$$ Ingoing EF (v, r)

$\theta = \frac{\pi}{2}$

$$S = -T_1 \int du\,dv\, f(r) \left[1 - \frac{4}{f(r)} \partial_u\varphi\, \partial_v\varphi\right]^{1/2}$$


Solutions :
$\boxed{\partial_u \varphi = 0} \Rightarrow \varphi(v)$
OR
$\partial_v \varphi = 0$

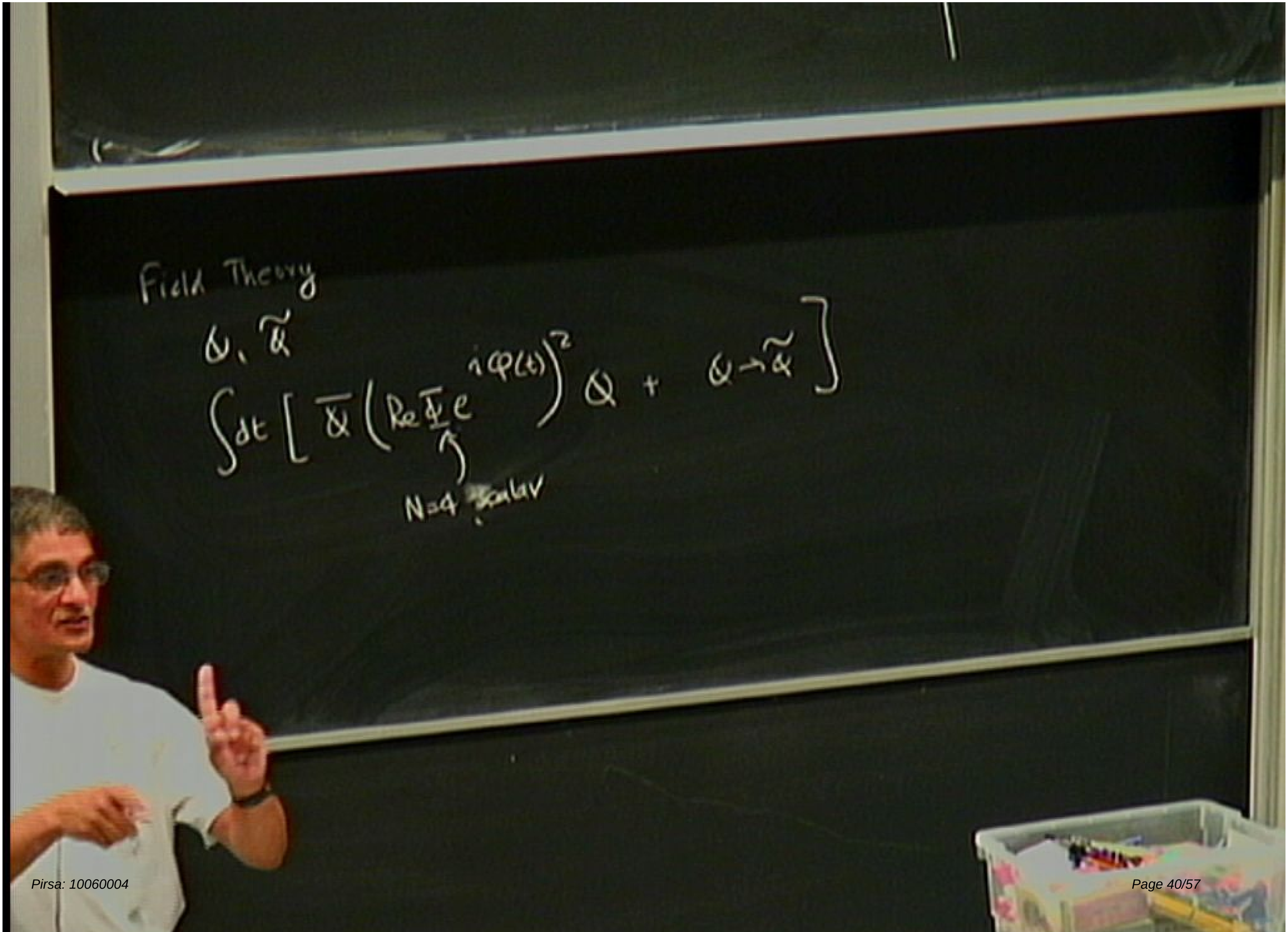
Poincar

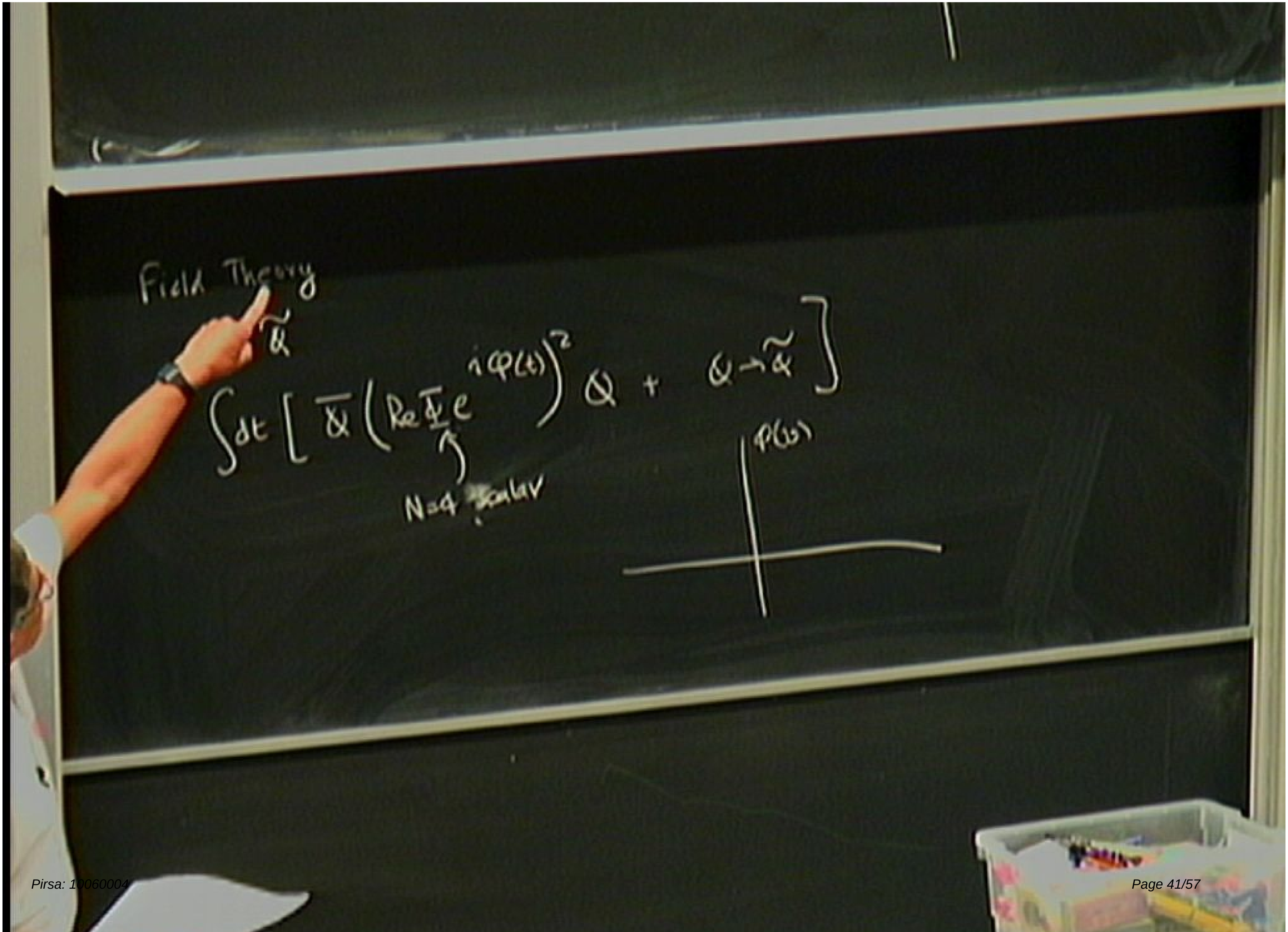
$v = t - \frac{1}{r}$

Solutions : $\boxed{\partial_u \varphi = 0} \Rightarrow \varphi(v)$

OR

$\partial_v \varphi = 0$


Field Theory
$Q, \tilde{Q}$
$\int dt \left[\bar{Q} \left(\mathrm{Re}\, \Phi\, e^{i\varphi(t)} \right)^2 Q + Q \to \tilde{Q} \right]$
N=4 scalar

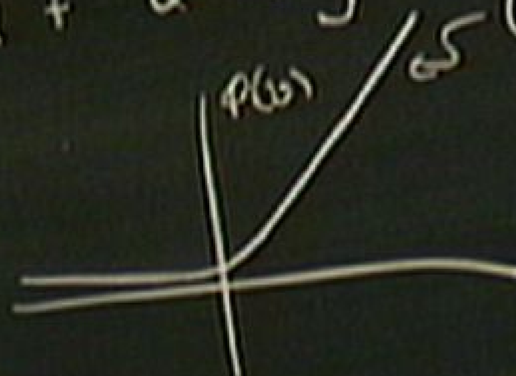

Field Theory
$\tilde{Q}$
$\int dt \left[\bar{Q} \left(\mathrm{Re}\,\Phi\, e^{i\varphi(t)} \right)^2 Q + Q \to \tilde{Q} \right]$
N=4 scalar
$\varphi(\omega)$

Field Theory

$Q, \tilde{Q}$

$$\int dt \left[\bar{Q} \left(\mathrm{Re}\,\Phi\, e^{i\varphi(t)} \right)^2 Q + Q \to \tilde{Q} \right]$$

N=4 scalar



Event horizon

$r > \omega$

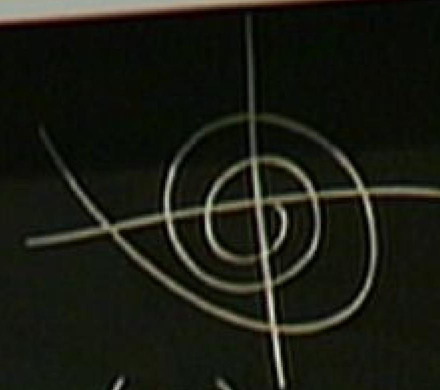
$T_H = \frac{\omega}{2\pi}$

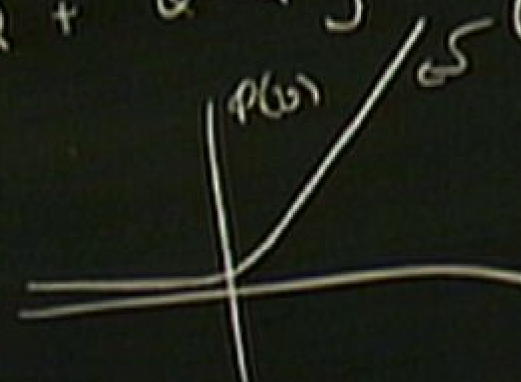
Field Theory

$Q, \tilde{Q}$

$$\int dt \left[\bar{Q} \left(\mathrm{Re}\,\Phi\, e^{i\varphi(t)} \right)^2 Q + Q \to \tilde{Q} \right]$$

N=4 scalar





Event horizon

$r_> \omega$

$T_H = \frac{\omega}{2\pi}$

$Q, \tilde{Q}$

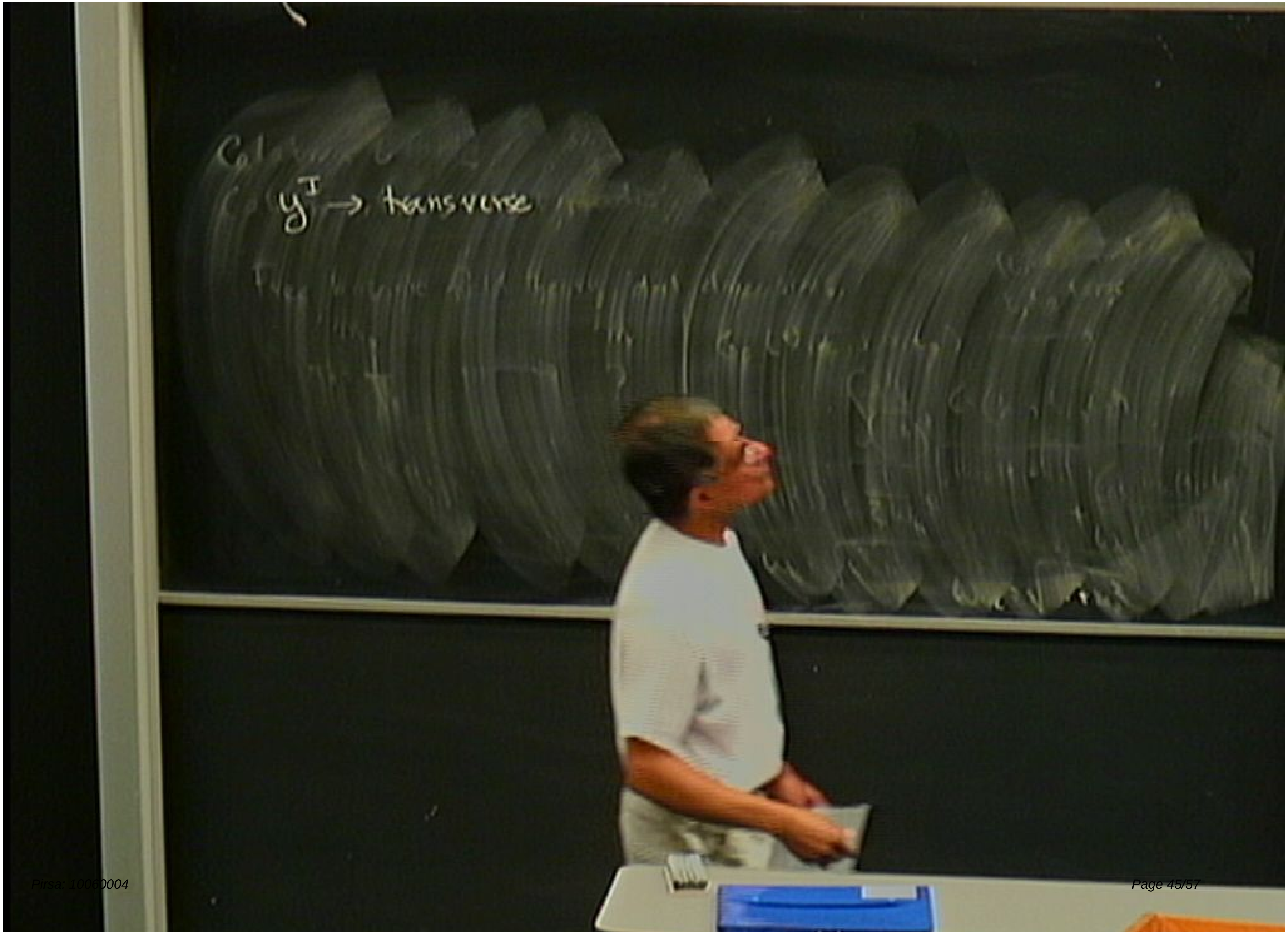
$$\int dt \left[\bar{Q} \left(\mathrm{Re}\,\Phi\, e^{i\varphi(t)} \right)^2 Q \right. + \quad Q \to \tilde{Q}$$

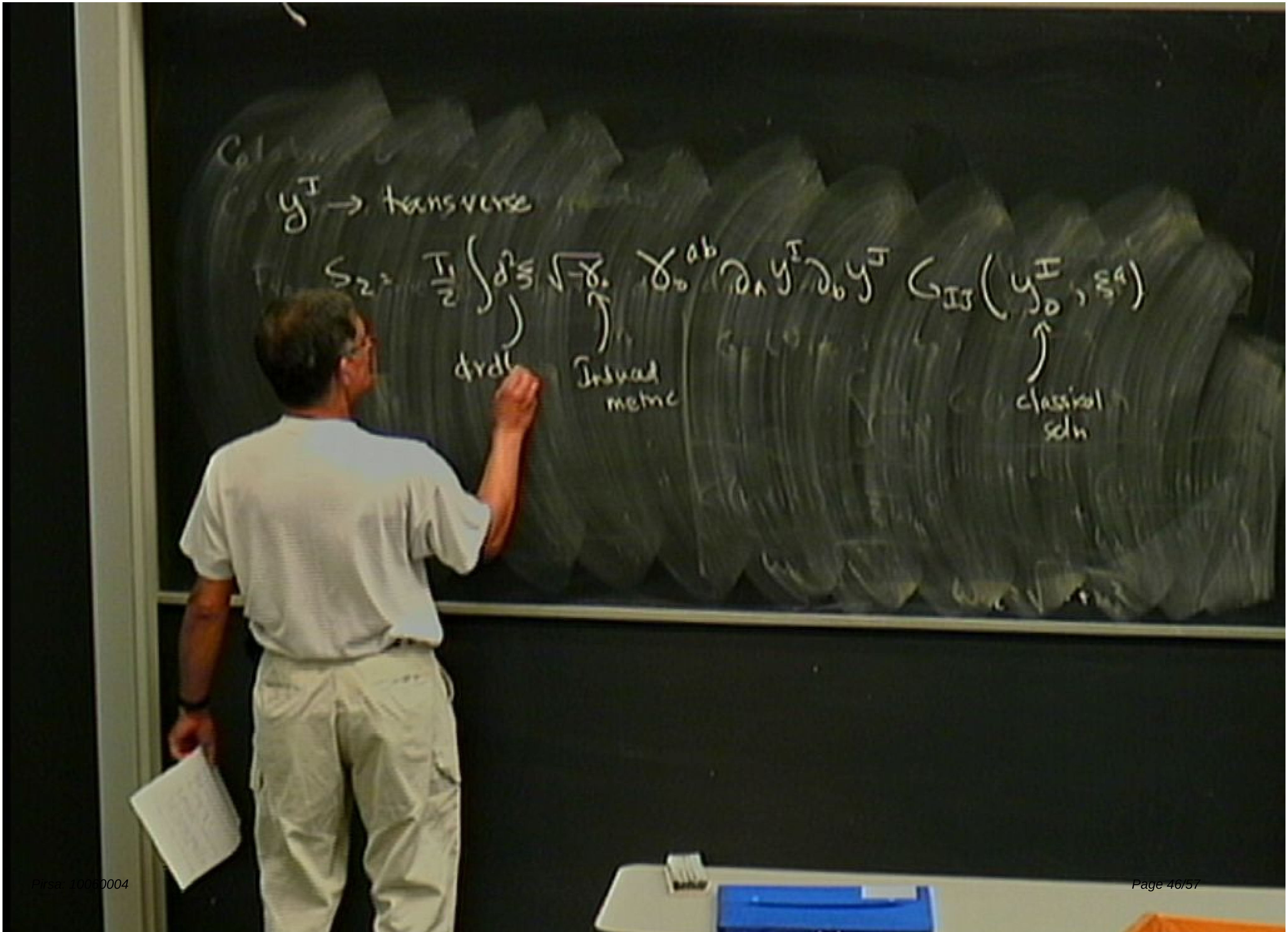
N=4 scalar

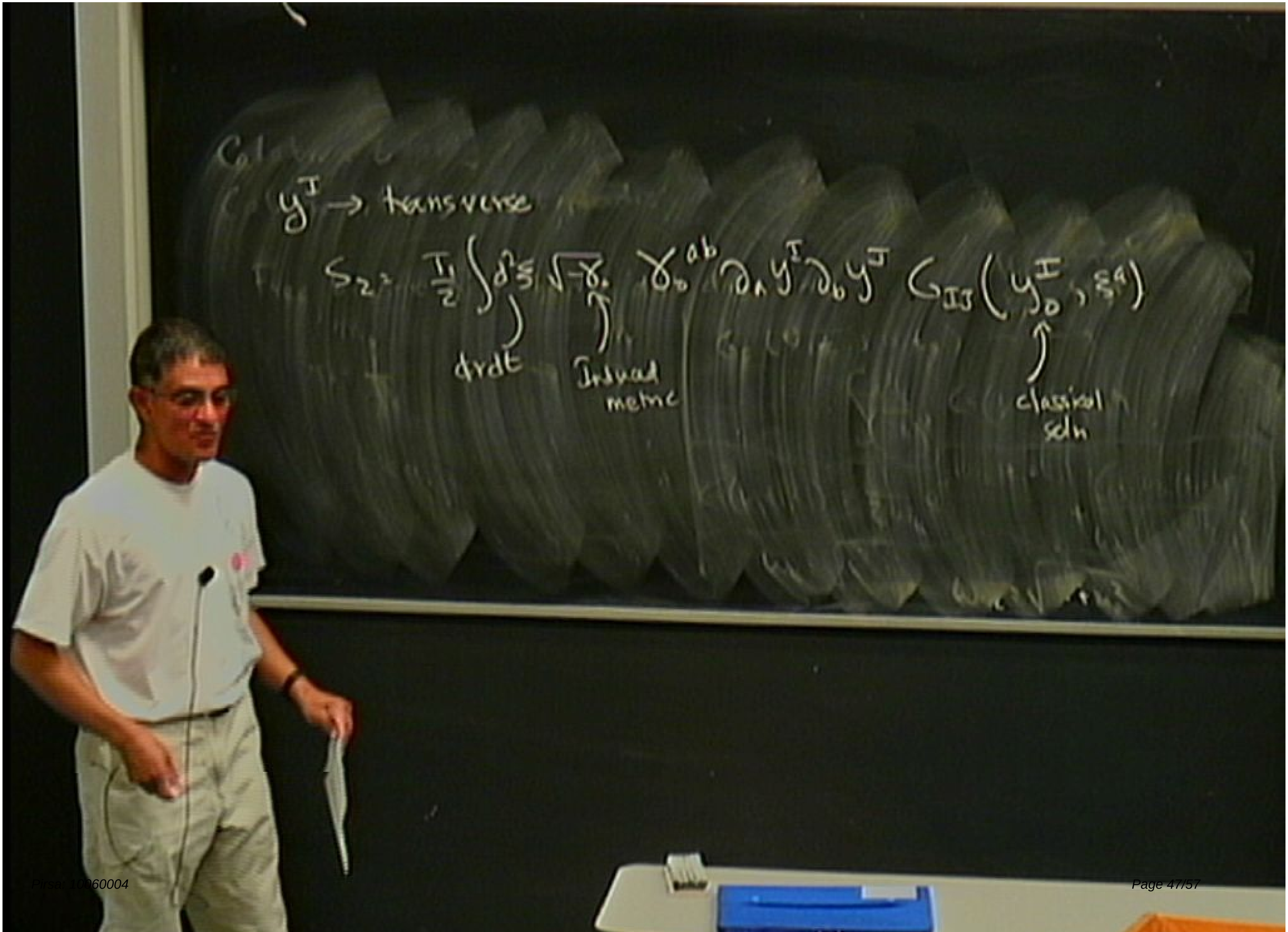
Global horizon ($\varphi = \omega \tau$)

$\omega > 1$

$\varphi($


y^I → transverse


y^I → transverse
S_2 = T_1/2 ∫ d^2ξ √-γ_0 γ_0^ab ∂_a y^I ∂_b y^J G_IJ(y_0^I, ξ^a)
dτdl
Induced metric
classical soln


$y^I \rightarrow$ transverse
$S_2 = \frac{T_1}{2} \int d^2\xi \sqrt{-\gamma_0}\, \gamma_0^{ab} \partial_a y^I \partial_b y^J G_{IJ}(y_0^I, \xi^a)$
$d\tau dt$
Induced metric
classical soln

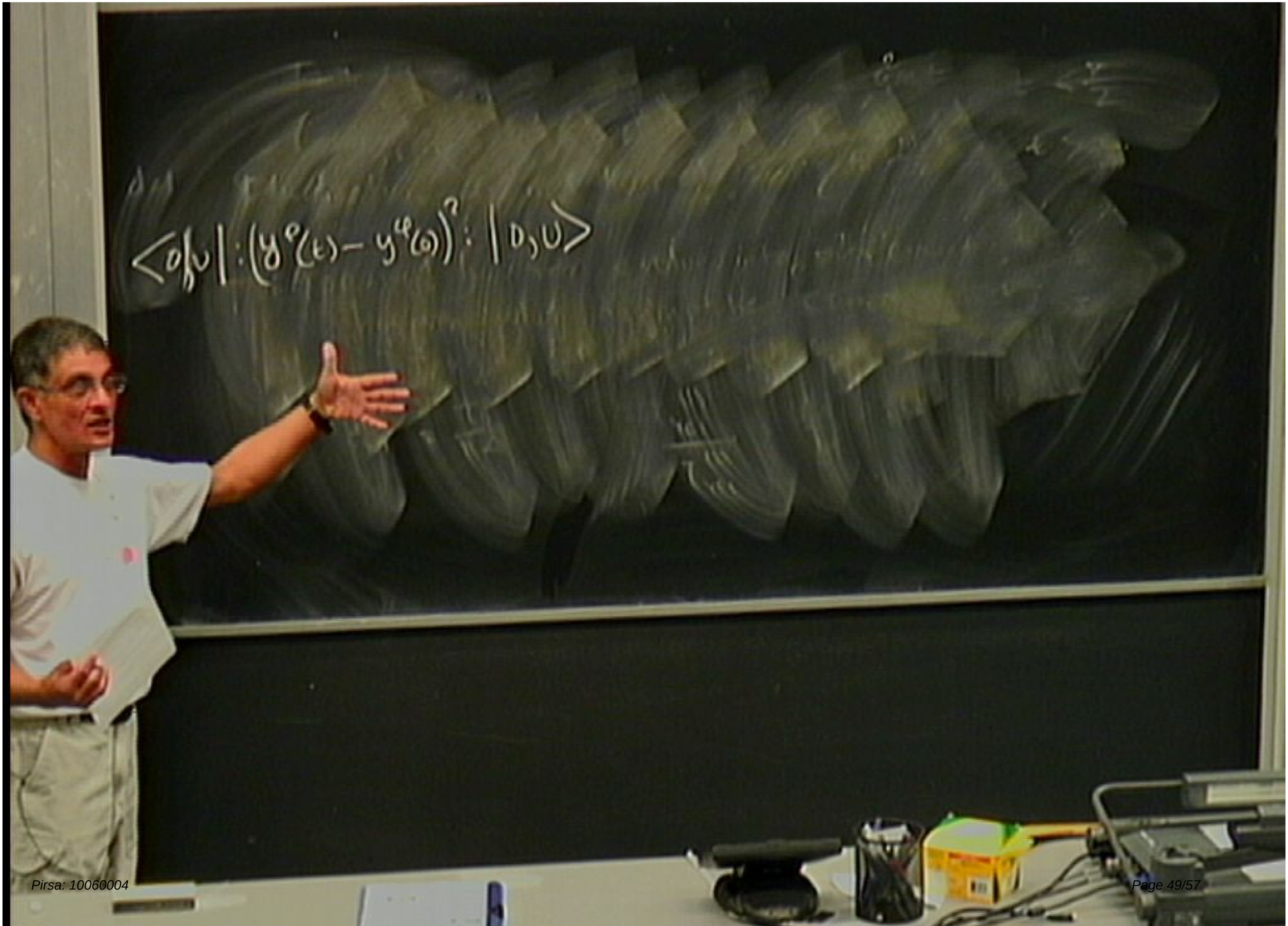
$y^I \to$ transverse fluctuation

$$S_2 = \frac{T_1}{2}\int d^2\xi \,\sqrt{-\gamma_0}\;\gamma_0^{ab}\,\partial_a y^I \partial_b y^J \, G_{IJ}(y_0^I, \xi^a)$$

$d\tau dt$

Induced metric

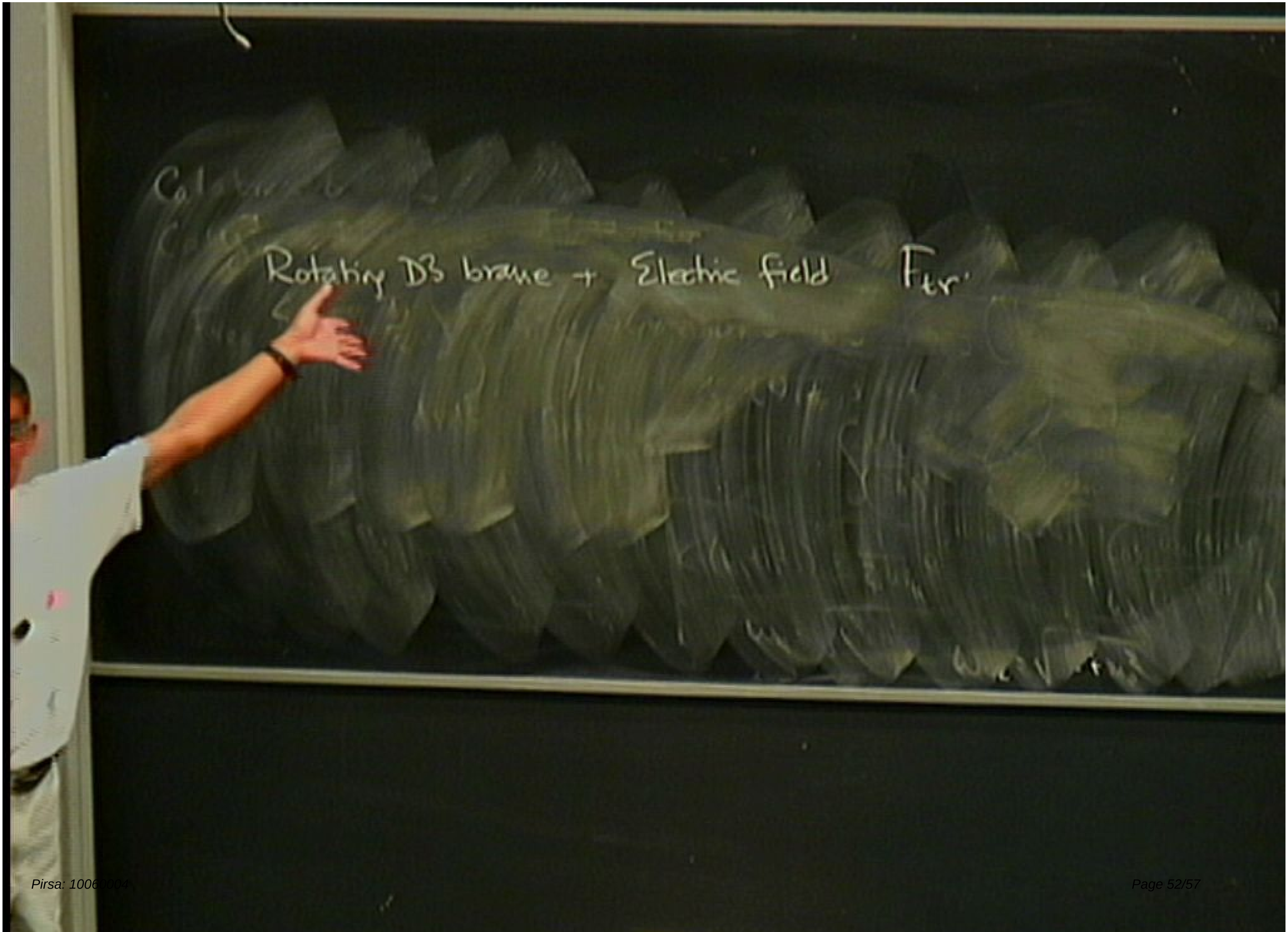
classical soln

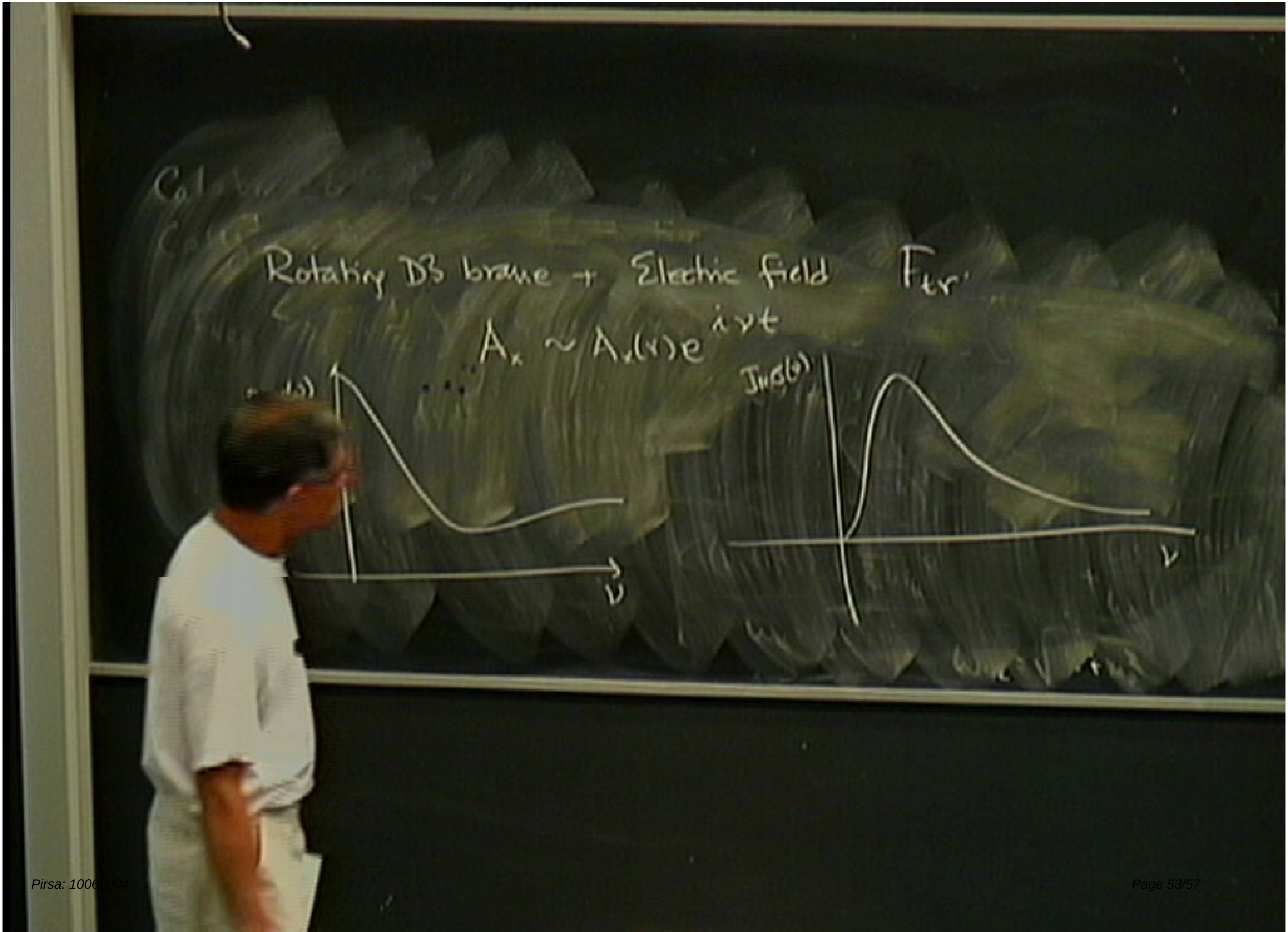

$\langle 0,u | :(y^{\varphi}(t) - y^{\varphi}(0))^2: | 0,u \rangle$

$$\langle 0_g, U | :\left(y^{\varphi}(t) - y^{\varphi}(0)\right)^2: | 0, U \rangle = \begin{cases} \dfrac{\pi t^2}{12 \beta_H^2} & \pi t \ll \beta_H \\ \dfrac{\pi |t|}{2 \beta_H} - \dfrac{1}{2\pi} \log \dfrac{2\pi |t|}{\beta_H} & \pi t \gg \beta_H \end{cases}$$

$$\langle 0_f, v | :\left(y^{\varphi}(t) - y^{\varphi}(0)\right)^2: | 0, v \rangle = \begin{cases} \dfrac{\pi t^2}{12 \beta_H^2} & \pi t \ll \beta_H \\ \dfrac{\pi |t|}{2 \beta_H} - \dfrac{1}{2\pi} \log \dfrac{2\pi |t|}{\beta_H} & \pi t \gg \beta_H \end{cases}$$

$$y^x \longrightarrow \begin{cases} \dfrac{\pi t^2}{1 m \beta_H} & t \ll \beta_H^2 m \\ \beta_H |t| & t \gg m \beta_H^2 \end{cases}$$

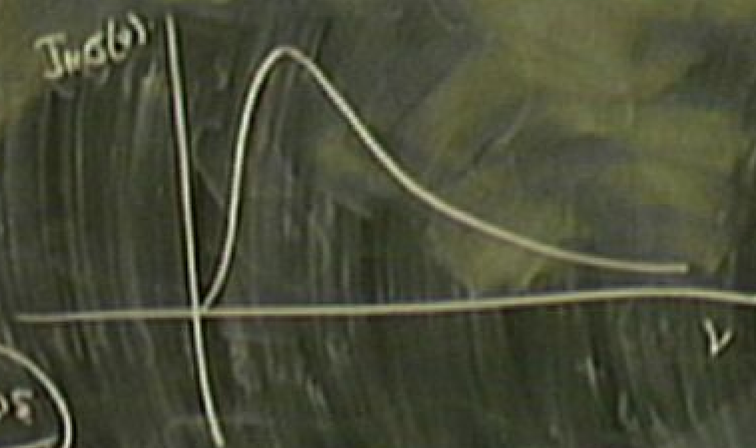

Rotating D3 brane + Electric field F_{tr}

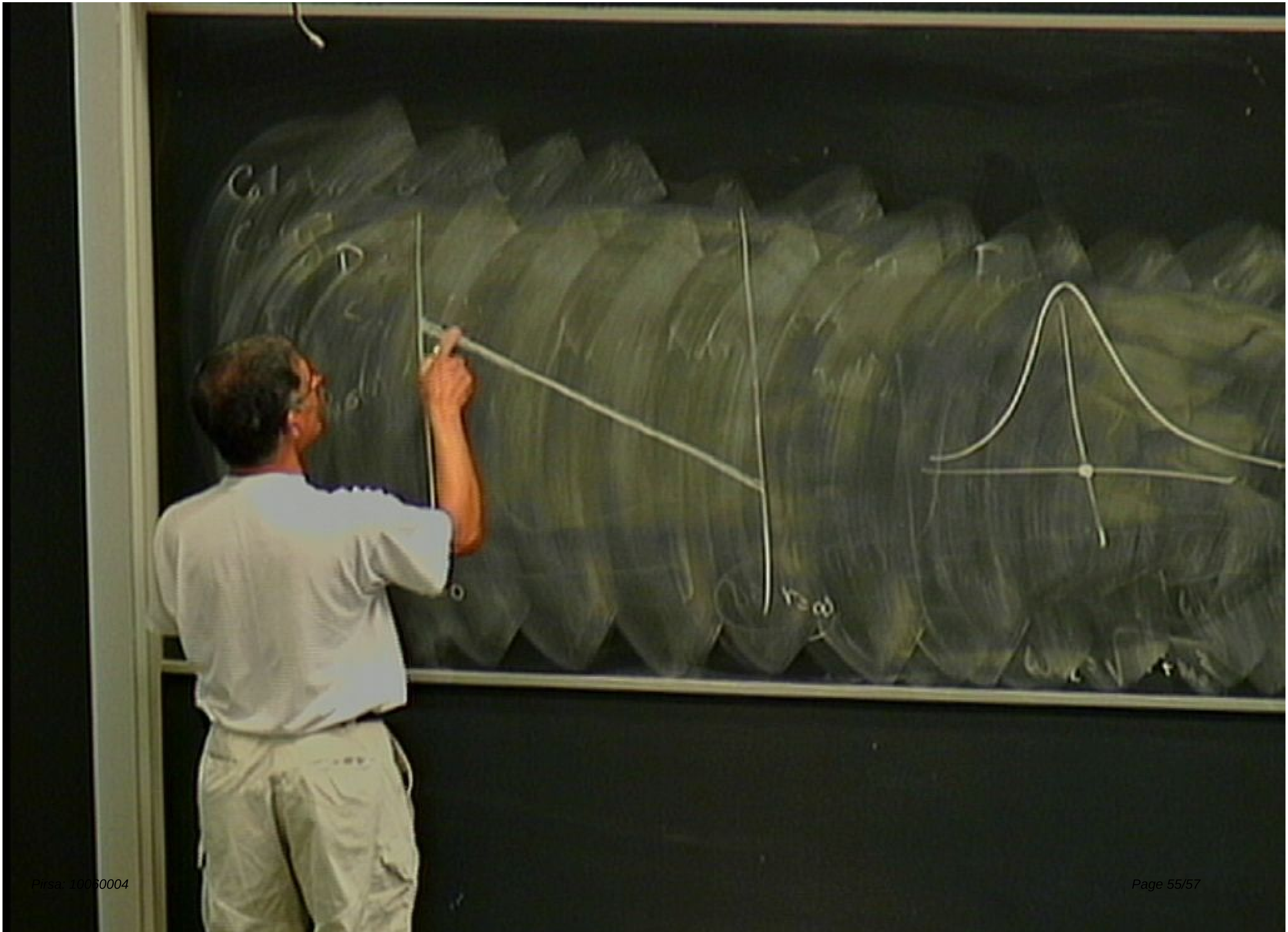

Rotating D3 brane + Electric field Ftr
Ax ~ Ax(r)e^iνt
ImG(ν)
ν
ν

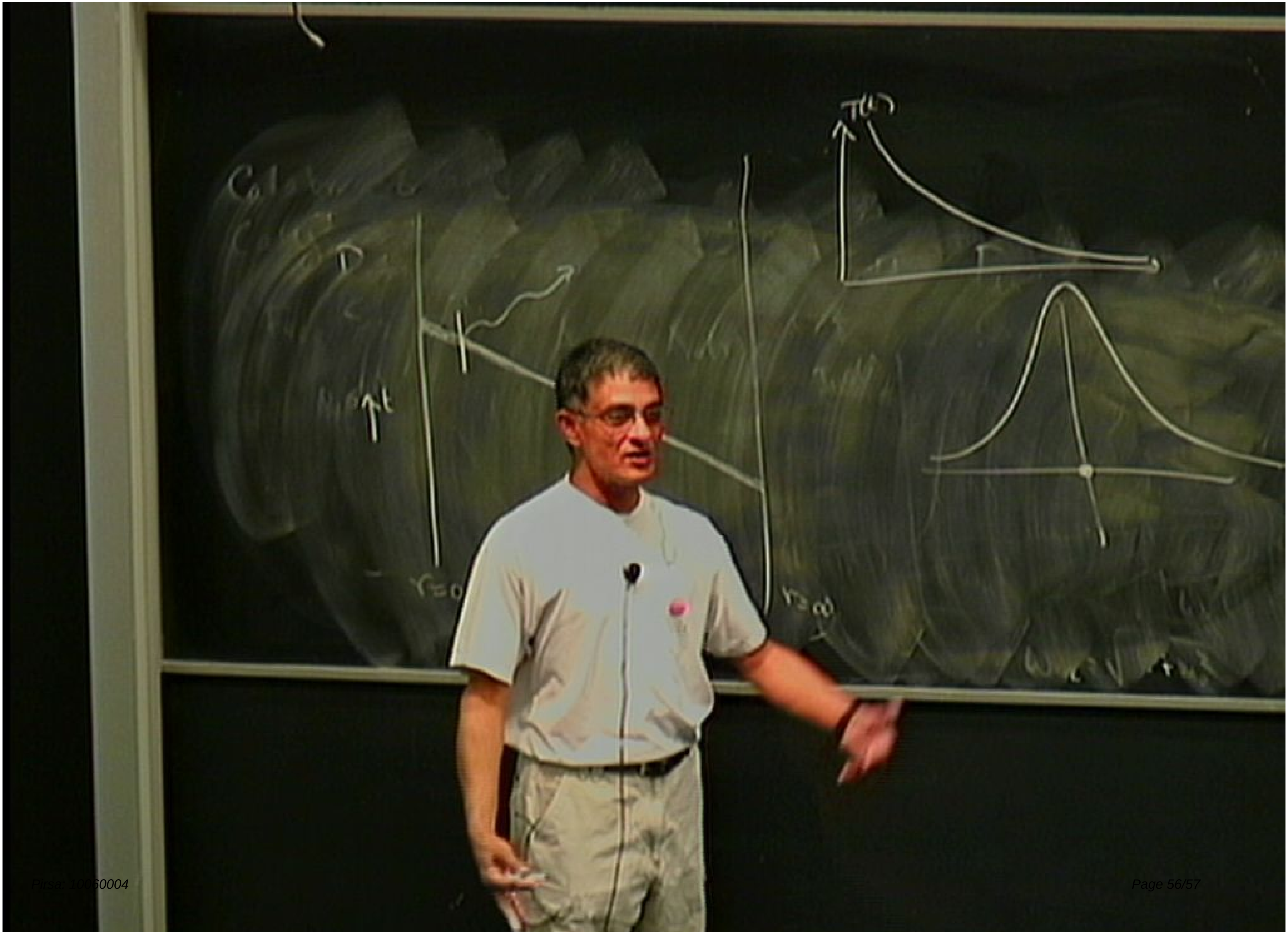
Rotating D3 brane + Electric field F_{tr}

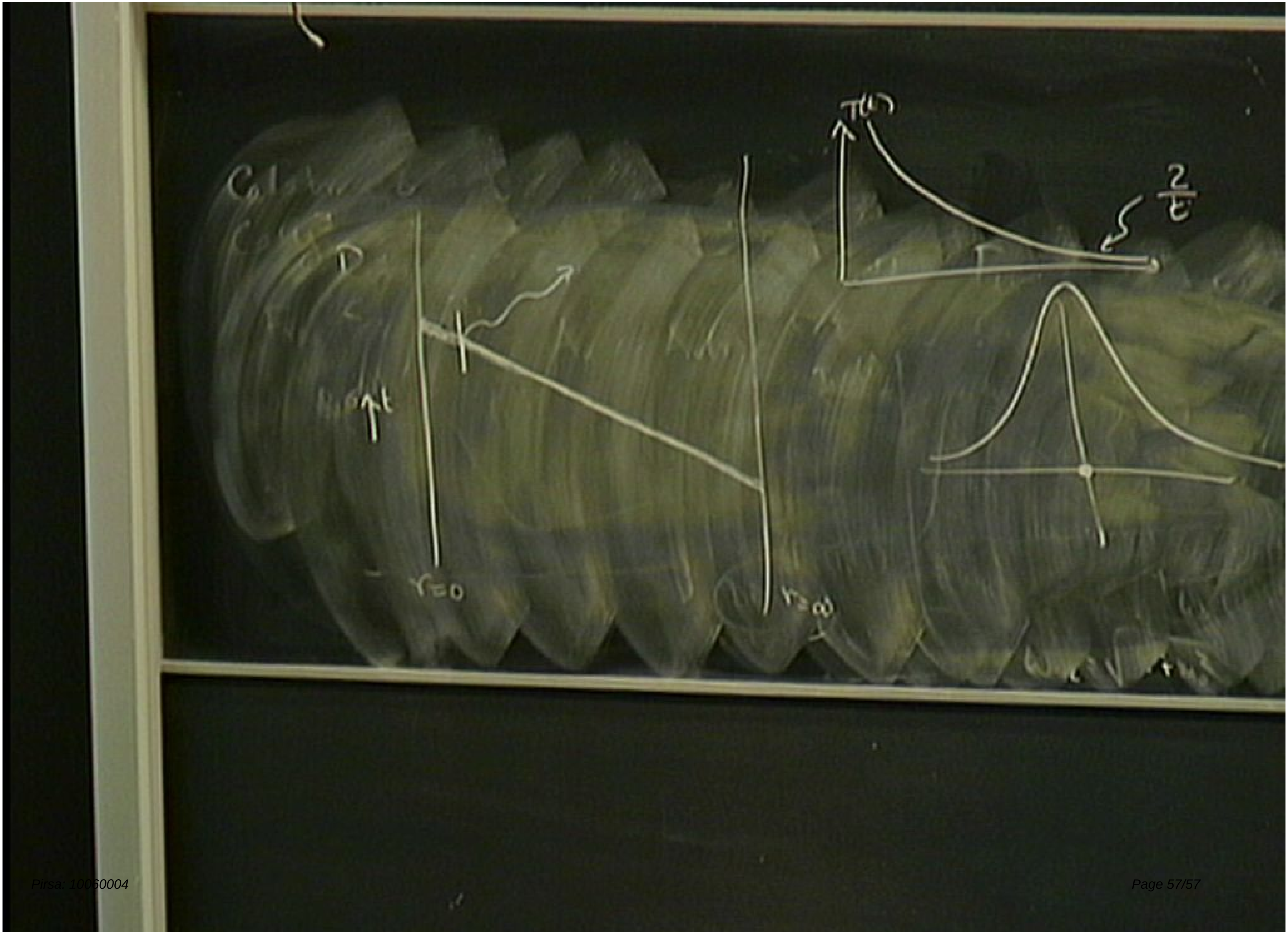
$$A_x \sim A_x(r)\, e^{i\nu t}$$

DRUDE








T(ω)
r=0
r=∞