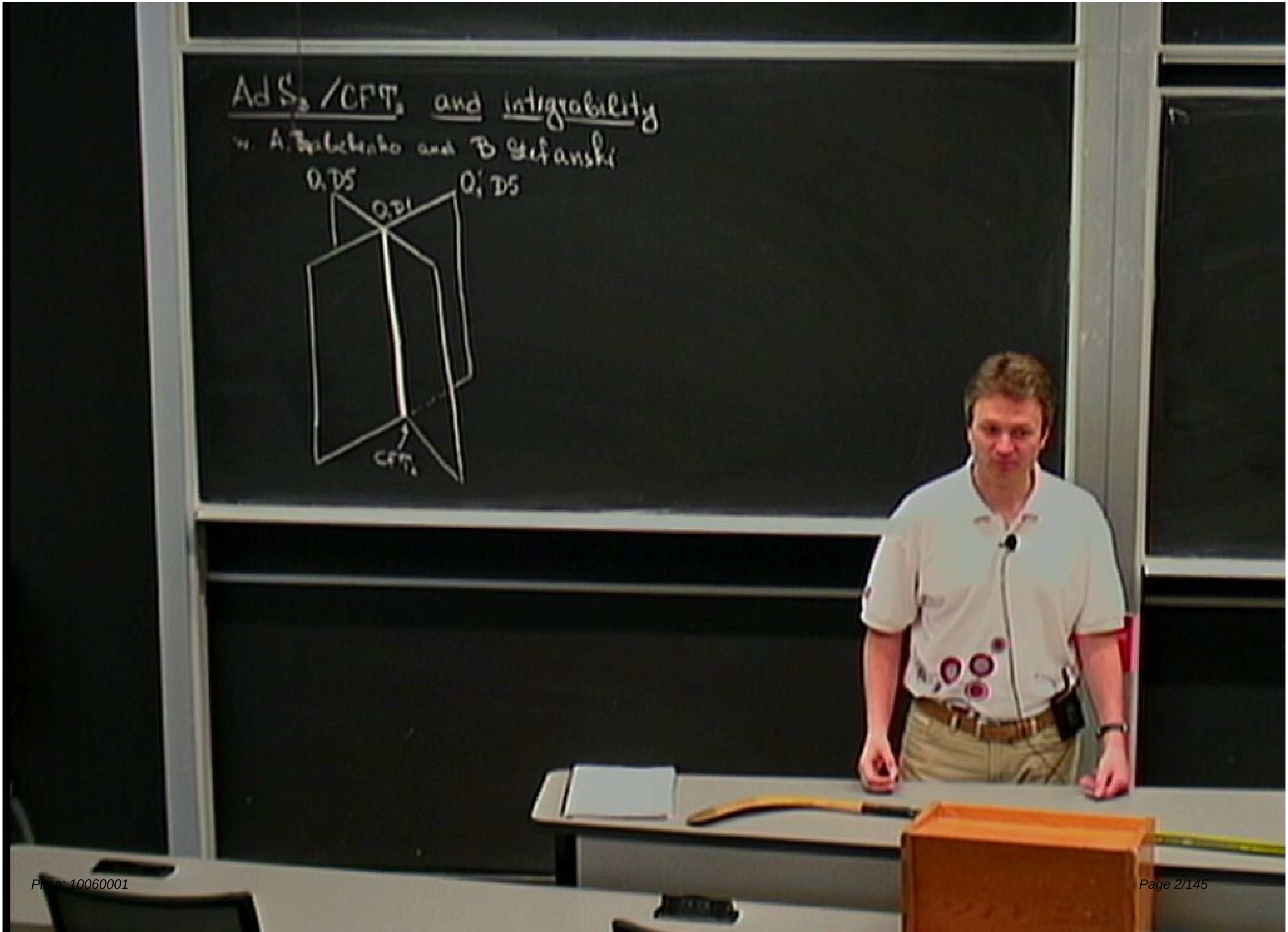
Title: AdS(3)/CFT(2) and integrability

Date: Jun 01, 2010 11:00 AM

URL: http://pirsa.org/10060001

Abstract: Quantization of string theory on the AdS(3) backgrounds with the RR flux, such as AdS(3)xS(3)xT(4) or AdS(3)xS(3)xS(3)xS(1), is an unsolved problem. Since the sigma model on these backgrounds is classically integrable, one can try to implement powerful methods of integrability similar to those used to solve AdS(5)/CFT(4) and AdS(4)/CFT(3). I will describe the integrability approach to the AdS(3) backgrounds, emphasizing the differences to the better understood cases of AdS(5) and AdS(4).


AdS3 /CFT2 and intigrability
w. A. Babichenko and B Stefanski
Q1D5
Q1' D5
Q1D1
CFT2

AdS$_3$/CFT$_2$ and integrability

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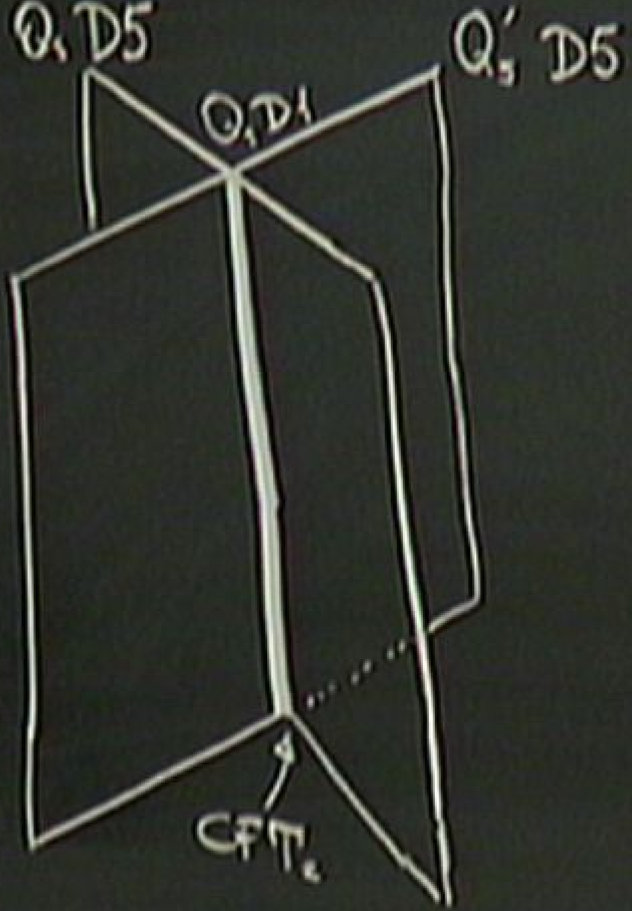

AdS_3/CFT_2 and integrability

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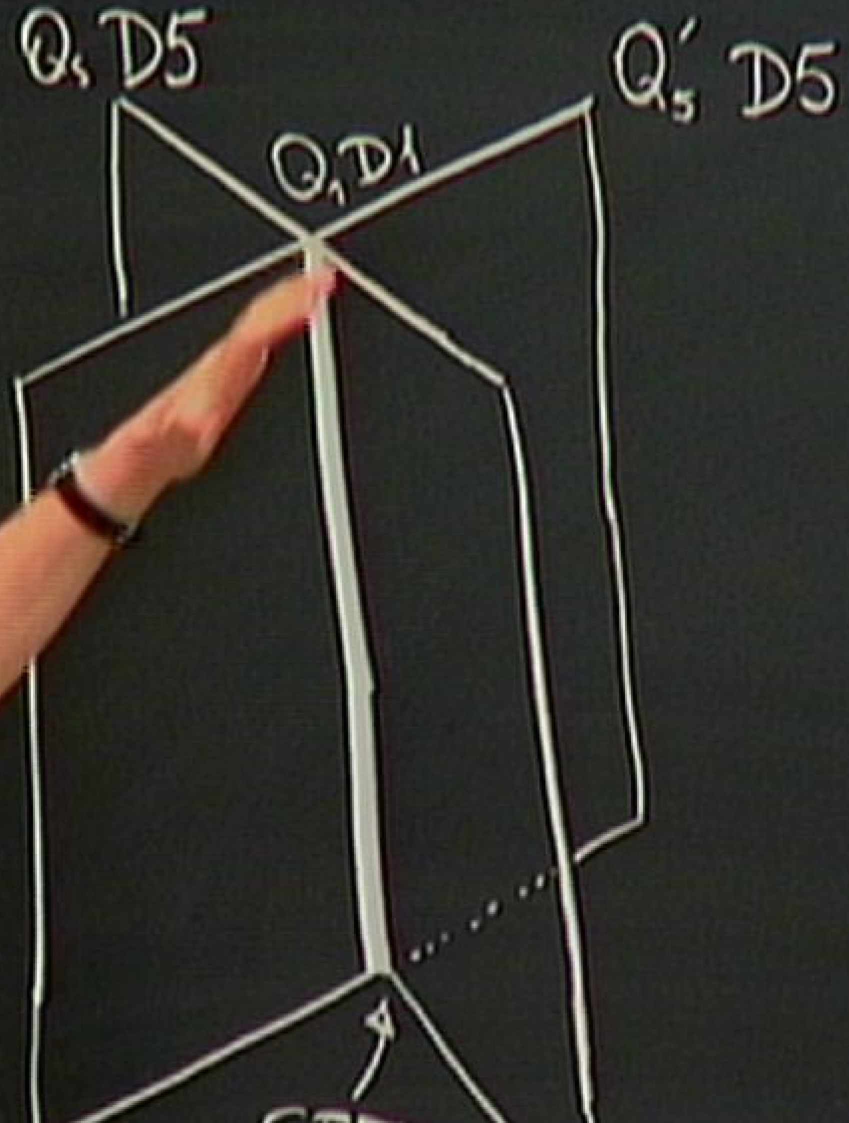

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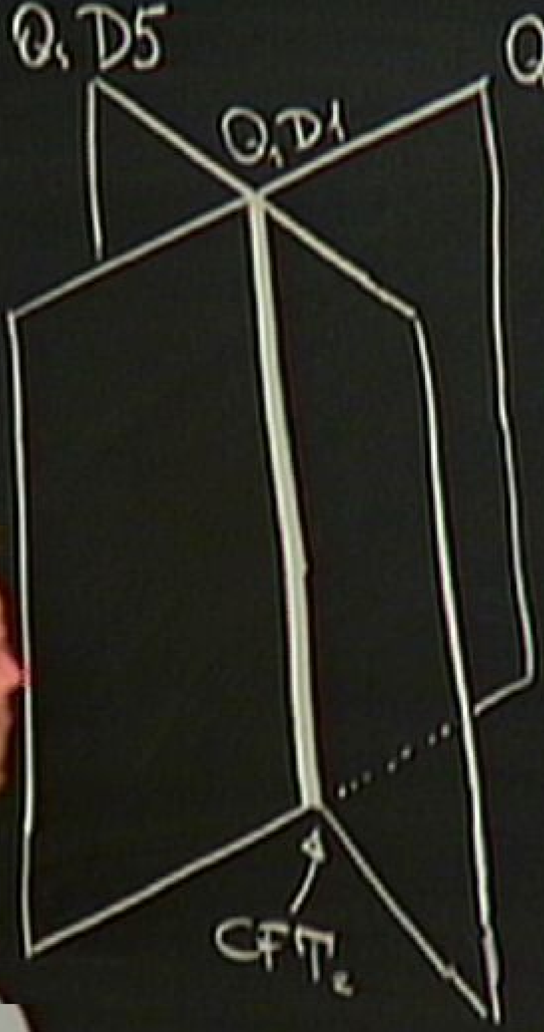
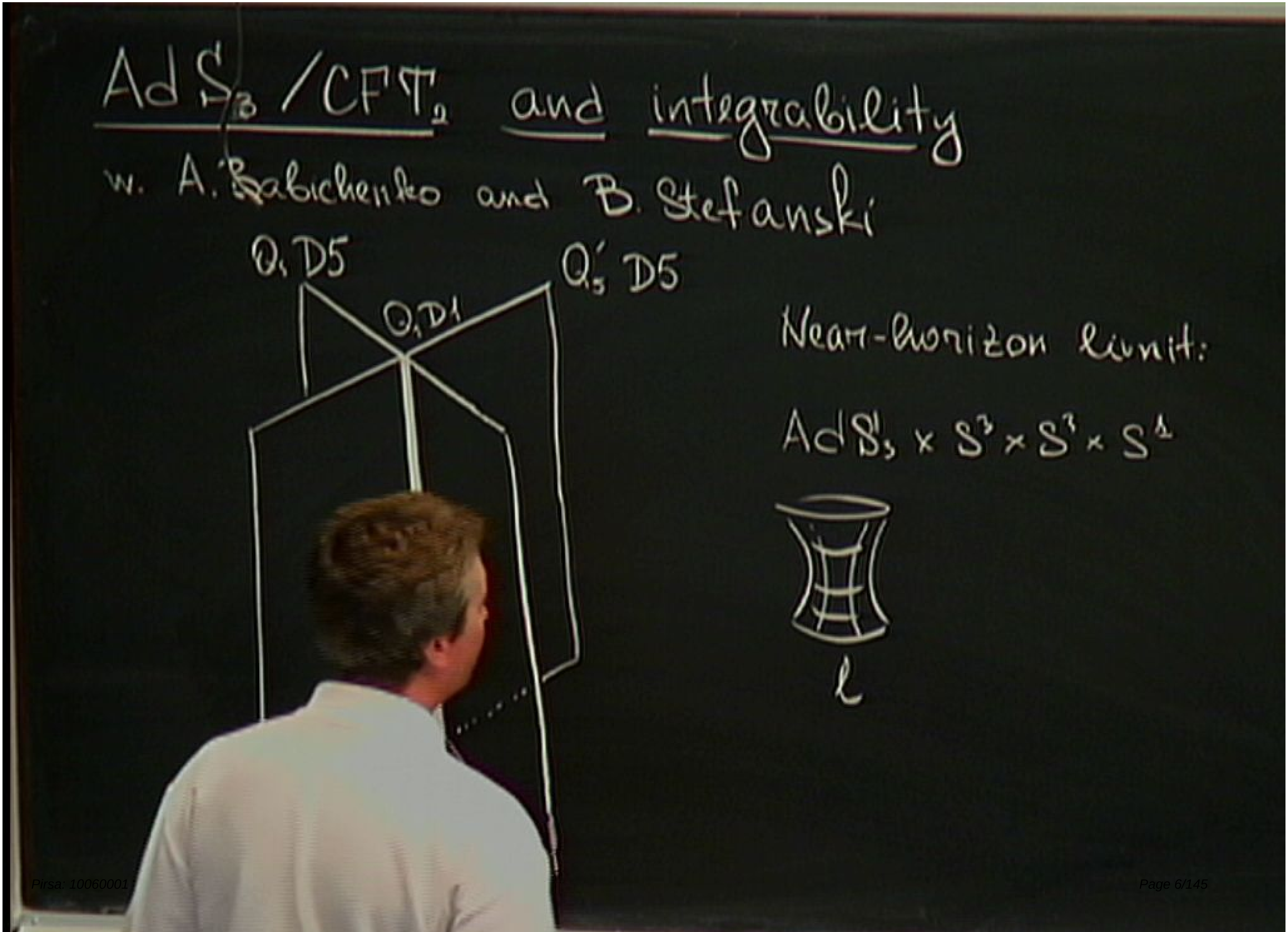


Near-horizon limit:

$AdS_3 \times S^3 \times S^3 \times S^1$



AdS_3 / CFT_2 and integrability

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Near-horizon limit:

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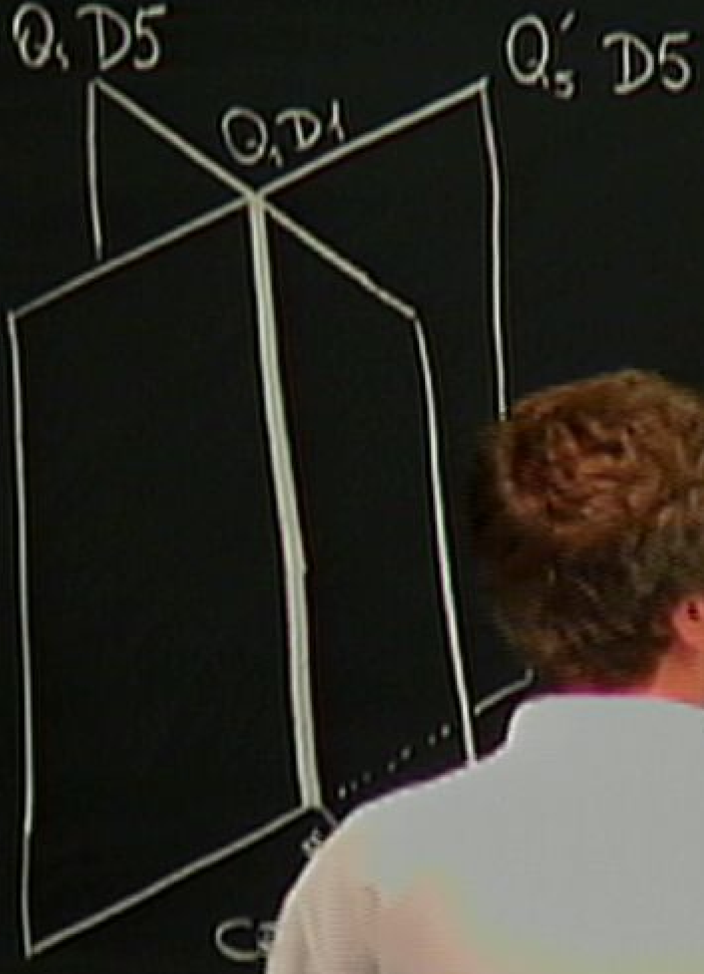

Near-horizon limit:

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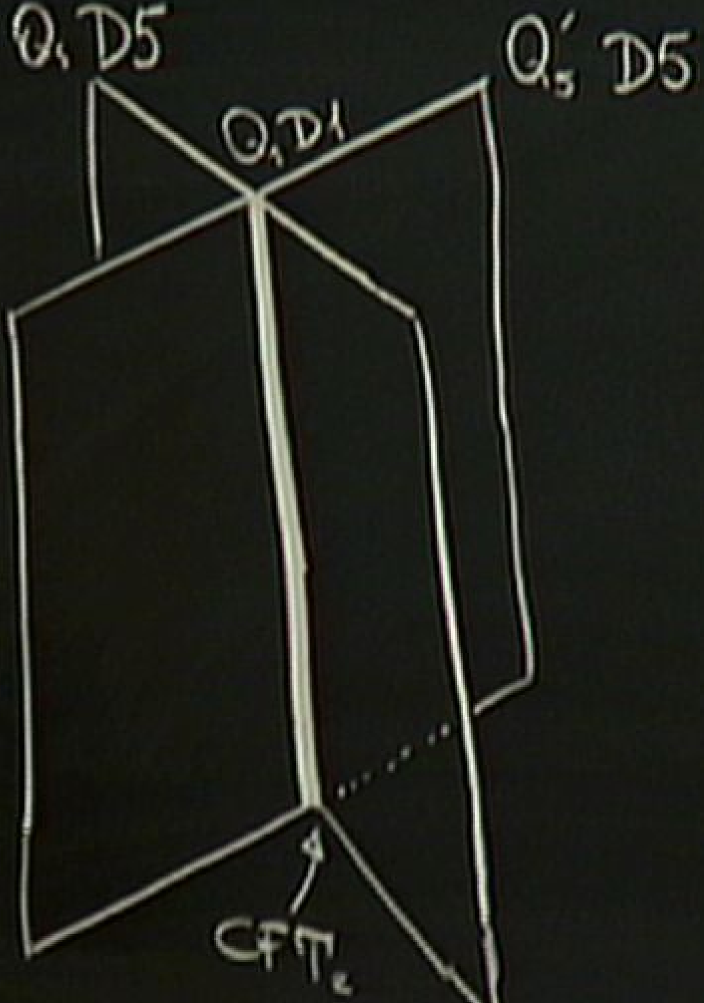

Near-horizon limit:

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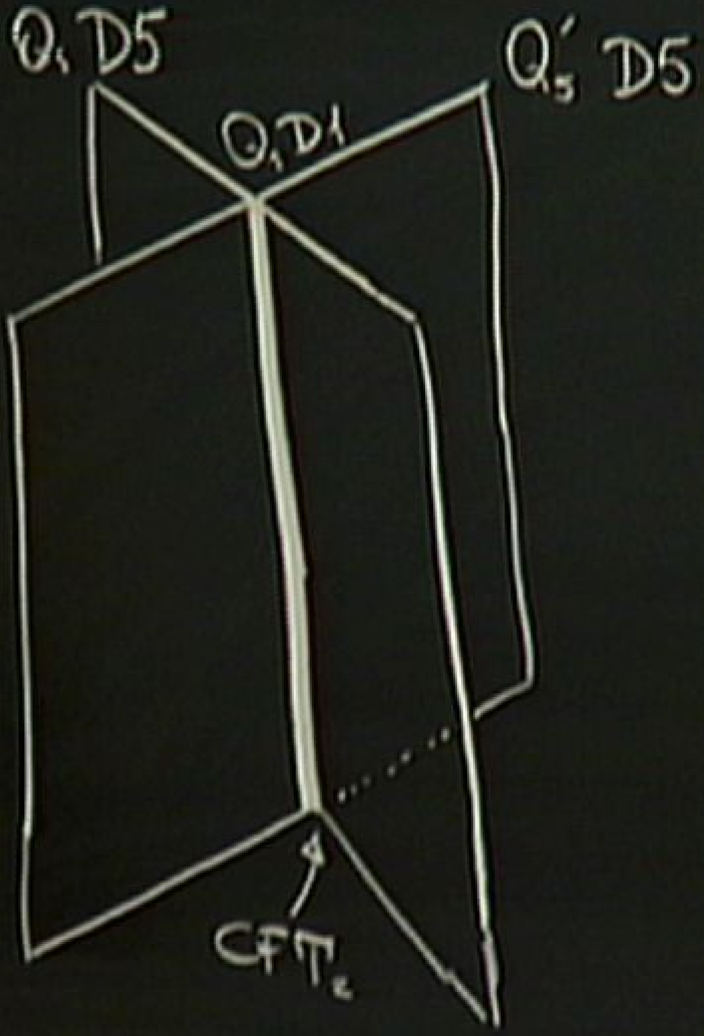
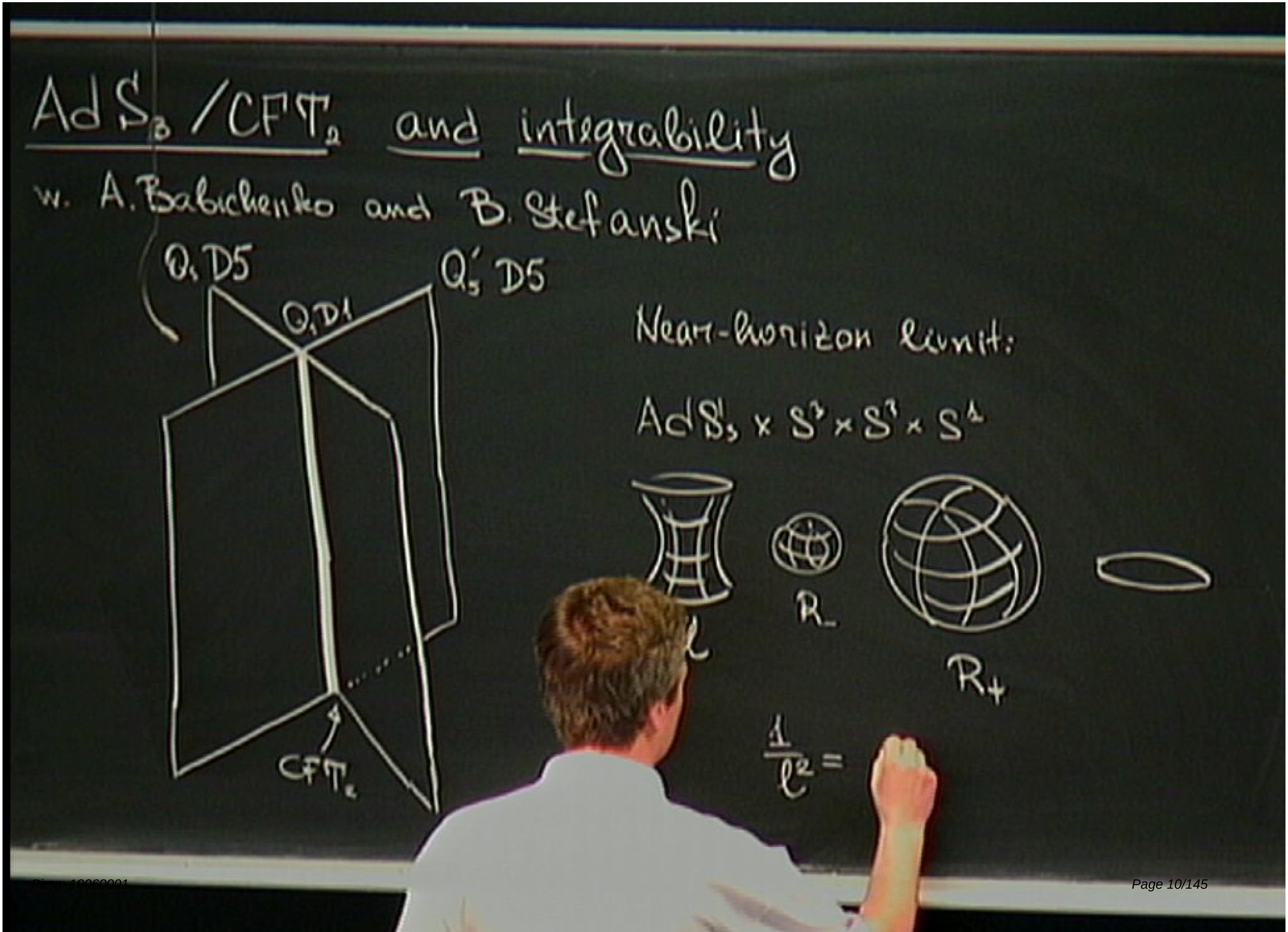


Near-horizon limit:

$AdS_3 \times S^3 \times S^3 \times S^1$



AdS_3/CFT_2 and integrability

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Near-horizon limit:

$AdS_3 \times S^3 \times S^3 \times S^1$

$\frac{1}{\ell^2} =$

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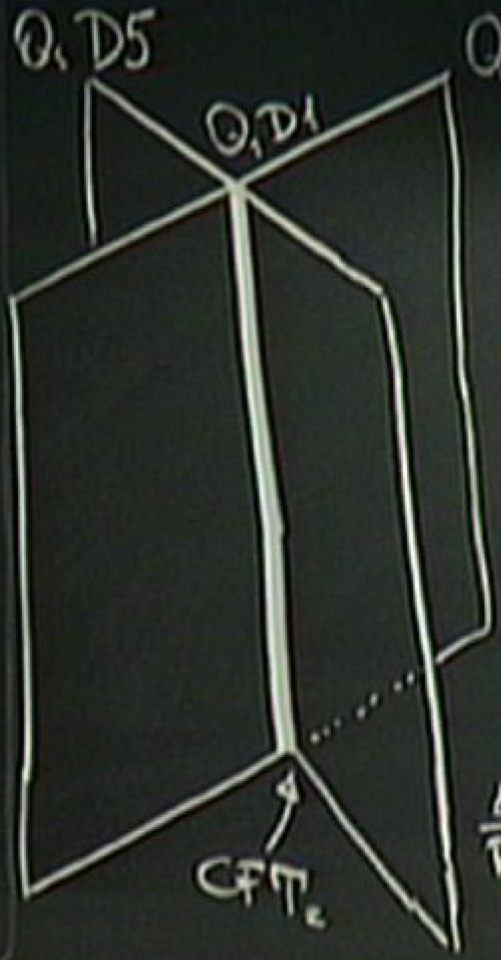
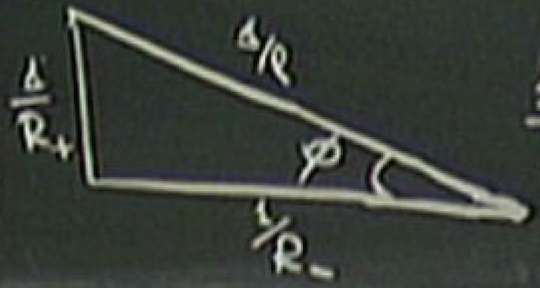


Near-horizon limit:

$AdS_3 \times S^3 \times S^3 \times S^1$



$$\frac{1}{\ell^2} = \frac{1}{R_-^2} + \frac{1}{R_+^2}$$

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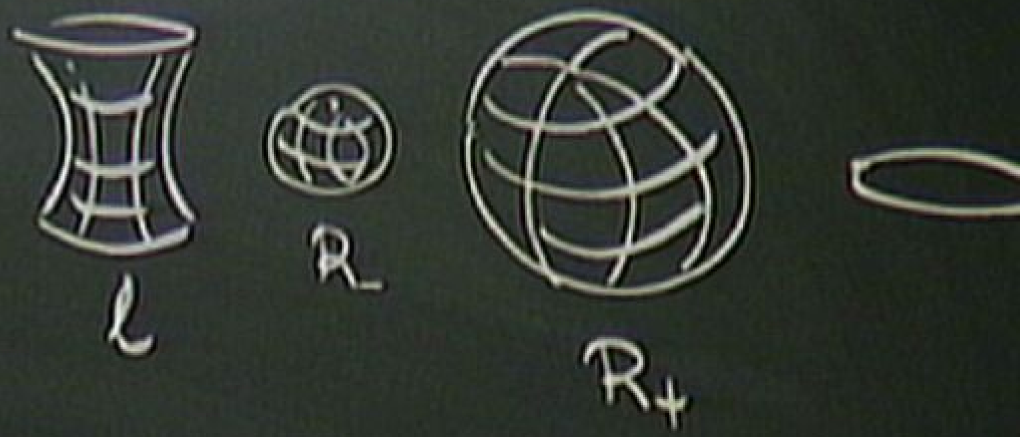
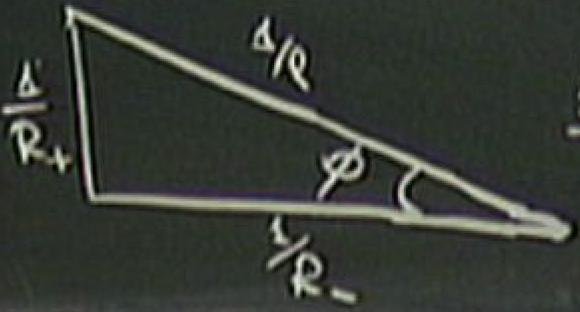
Near-horizon limit:

$AdS_3 \times S^3 \times S^3 \times S^1$





$$\frac{1}{\ell^2} = \frac{1}{R_-^2} + \frac{1}{R_+^2}$$

$R_+ \to \infty$

R_-

$R_+ \to \infty \quad (\phi \to 0)$

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$$AdS_3 \times S^3 \times T^4$$

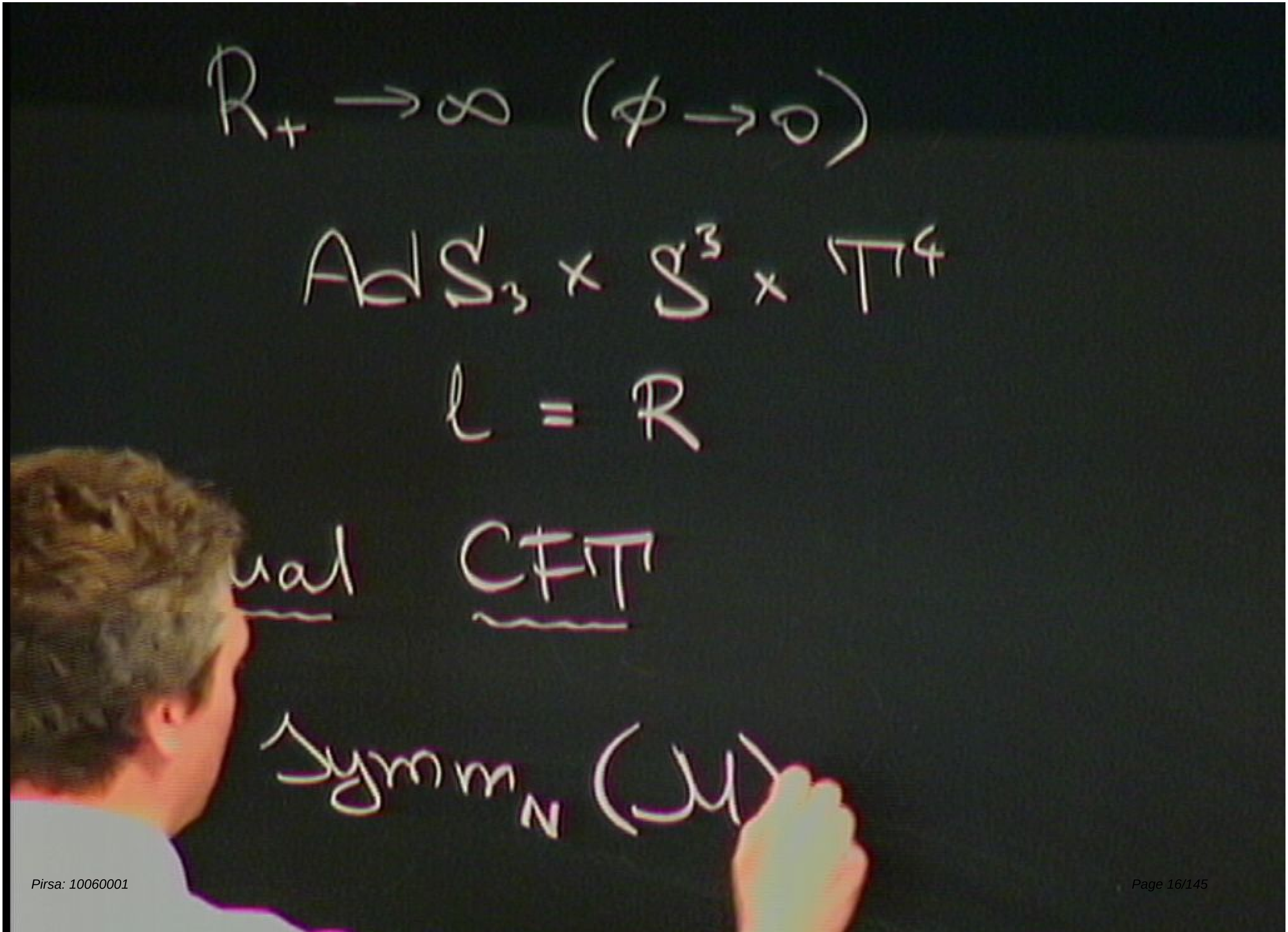
$$l = R$$

$$R_+ \to \infty \quad (\phi \to 0)$$

$$AdS_3 \times S^3 \times T^4$$

$$\ell = R$$

$\underset{\sim\sim\sim}{\text{Dual}}$ $\underset{\sim\sim\sim}{\text{CFT}}$



$R_+ \to \infty \quad (\phi \to 0)$

$AdS_3 \times S^3 \times T^4$

$\ell = R$

ual CFT

$\mathrm{Symm}_N(\mathcal{M})$

$$R_+ \to \infty \quad (\phi \to 0)$$

$$AdS_3 \times S^3 \times T^4$$

$$l = R$$

<u>Dual</u> <u>CFT</u>

$$\text{Symm}_N(\mathcal{M}) + \lambda \int d^2z \, O_{1,1}$$

$$R_+ \to \infty \quad (\phi \to 0)$$

$$AdS_3 \times S^3 \times T^4$$

$$l = R$$

Dual CFT

$$\mathrm{Symm}_N(\mathcal{M}) + \lambda \int d^2z \, O_{1,1}$$

$$O_{1,1} = G_{-1/2} \, \bar{G}_{-1/2} \, \underset{\uparrow \text{ twist operator}}{\sigma_{\text{transp}}}$$

Dual CFT

$$\mathrm{Symm}_N(\mathcal{M}) + \lambda \int d^2z \, \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2} \, \sigma_{\mathrm{transp}}$$

↖ twist operator

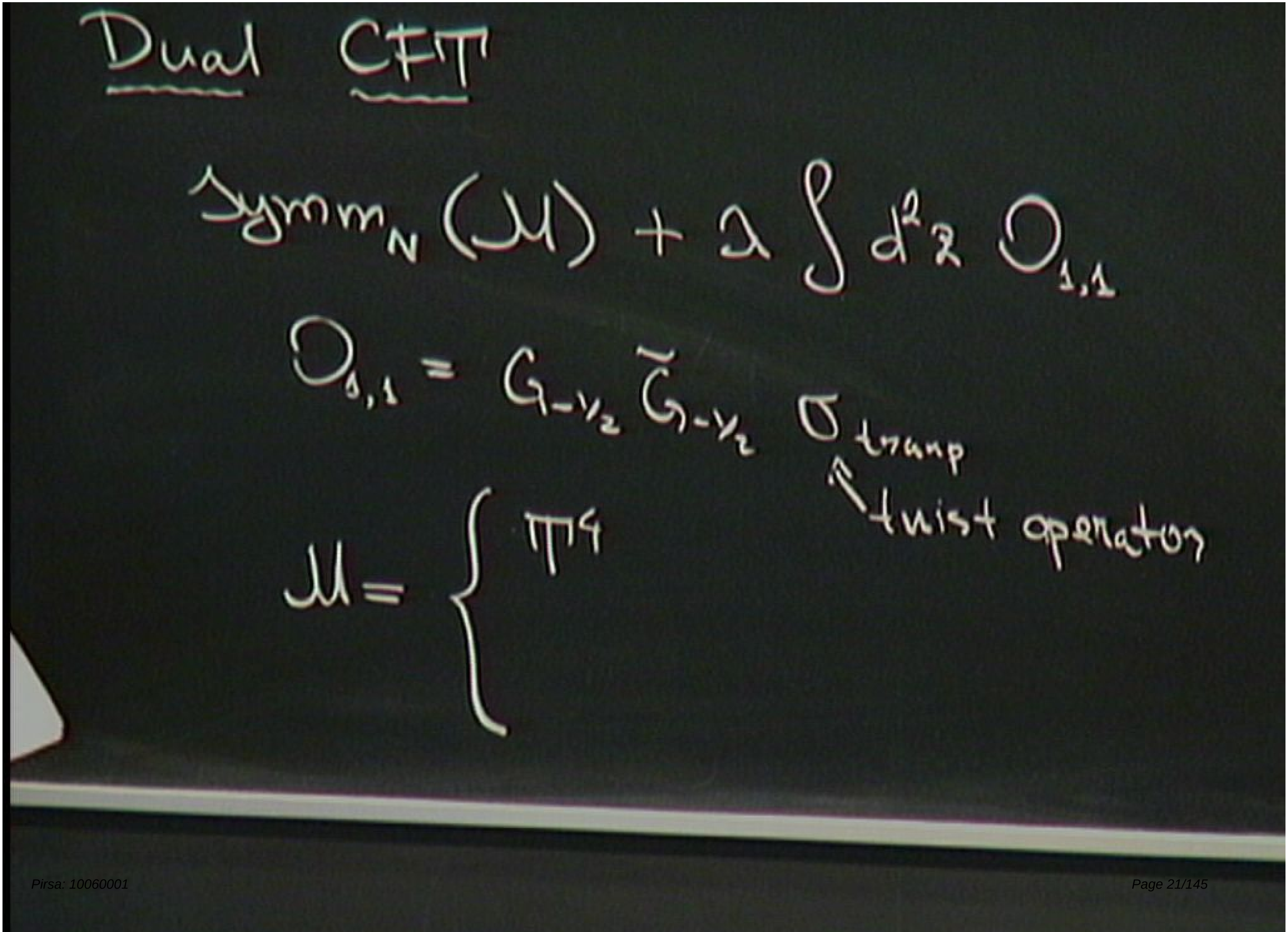
$$\mathcal{M} = \begin{cases} T^4 \\ \end{cases}$$

Dual CFT

$$\mathrm{Symm}_N(\mathcal{M}) + \lambda \int d^2z \, \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \, \tilde{G}_{-1/2} \, \underset{\nwarrow \text{ twist operator}}{\sigma_{\text{transp}}}$$

$$\mathcal{M} = \left\{ \begin{array}{l} T^4 \\ \\ \\ \end{array} \right.$$



Dual CFT

$$\mathrm{Symm}_N(\mathcal{M}) + \lambda \int d^2z \, \mathcal{O}_{1,1}$$

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↖ twist operator

$$\mathcal{M} = \begin{cases} T^4 \\ \\ \end{cases}$$

$AdS_3 \times S^3 \times T^4$

$l = R$

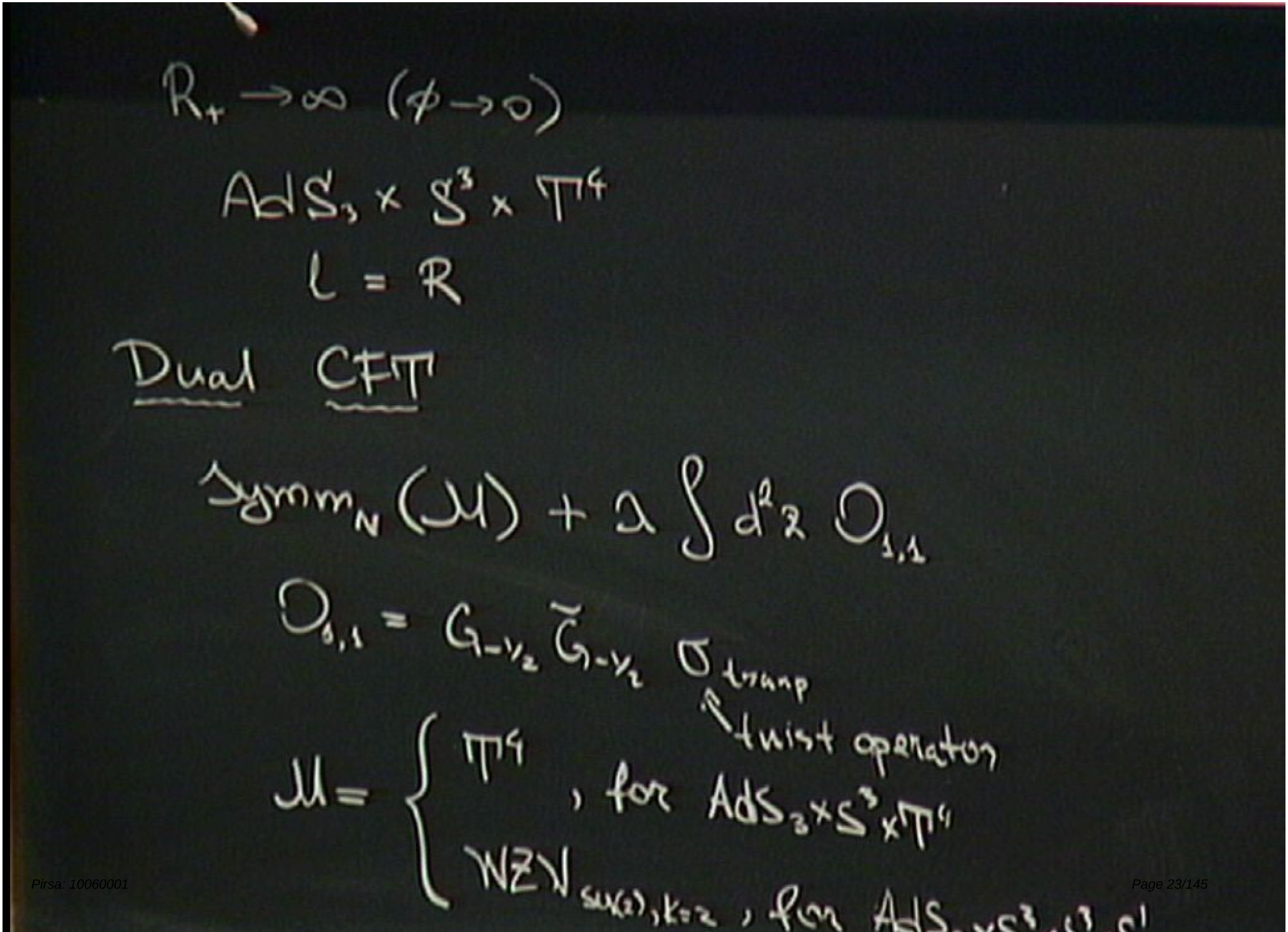
Dual CFT

$$\mathrm{Symm}_N(\mathcal{M}) + \lambda \int d^2z \, \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2} \, \sigma_{\text{transp}}$$

twist operator

$$\mathcal{M} = \begin{cases} T^4 & , \text{ for } AdS_3 \times S^3 \times T^4 \\ WZW_{SU(2), k=2} & , \text{ for } AdS_3 \times S^3 \times S^3 \times S^1 \end{cases}$$



$R_+ \to \infty \quad (\phi \to 0)$

$AdS_3 \times S^3 \times T^4$

$l = R$

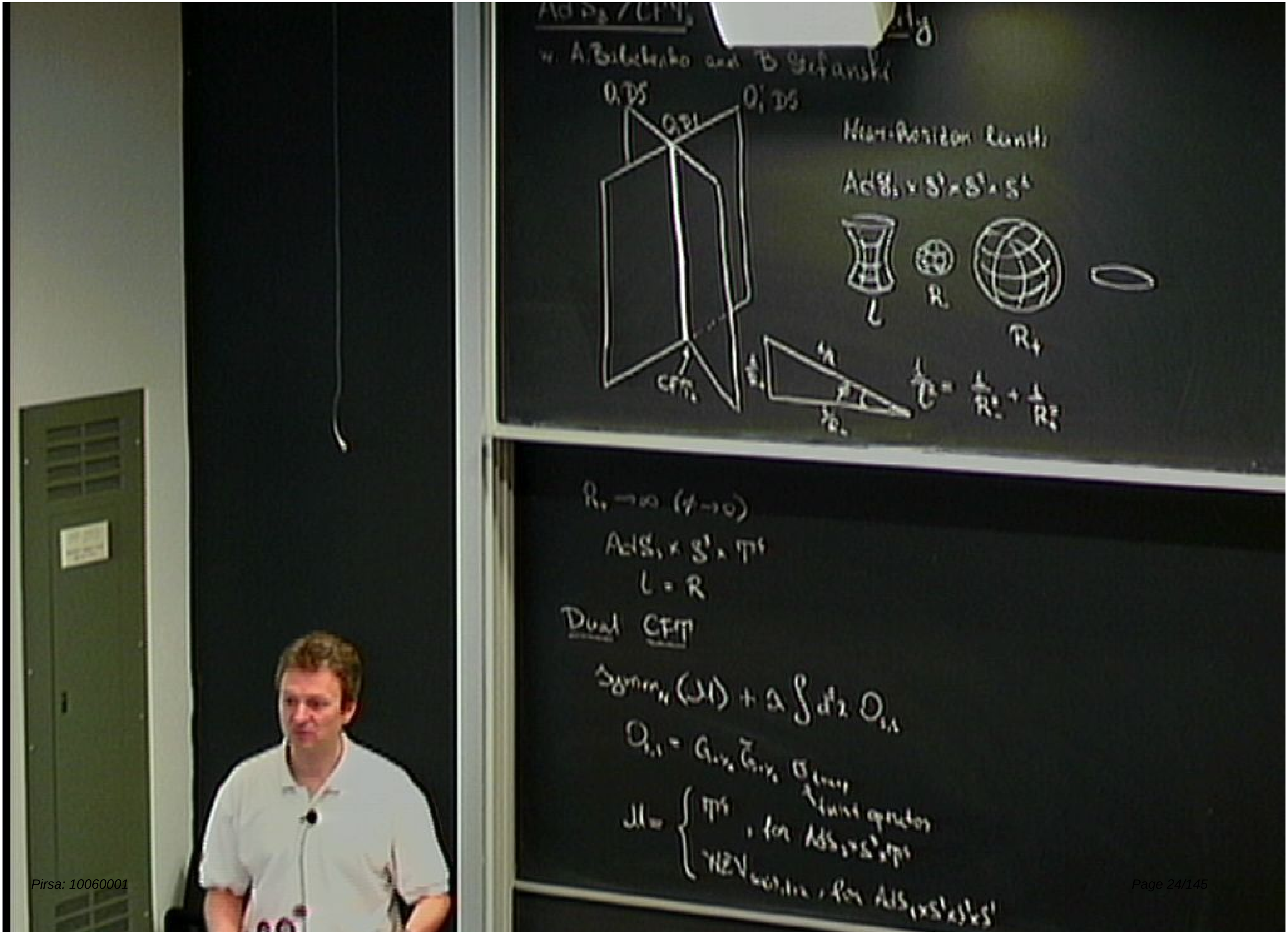
<u>Dual</u> <u>CFT</u>

$$\mathrm{Symm}_N(\mathcal{M}) + \lambda \int d^2 z \, \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2} \, \sigma_{\text{transp}}$$

twist operator

$$\mathcal{M} = \begin{cases} T^4 & , \text{for } AdS_3 \times S^3 \times T^4 \\ WZW_{SU(2),k=2} & , \text{for } AdS_3 \times S^3 \times S^3 \times S^1 \end{cases}$$



Near-horizon limit

$AdS_3 \times S^3 \times S^3 \times S^1$

$$\frac{1}{l^2} = \frac{1}{R_-^2} + \frac{1}{R_+^2}$$

$R_+ \to \infty \ (\phi \to 0)$

$AdS_3 \times S^3 \times T^4$

$l = R$

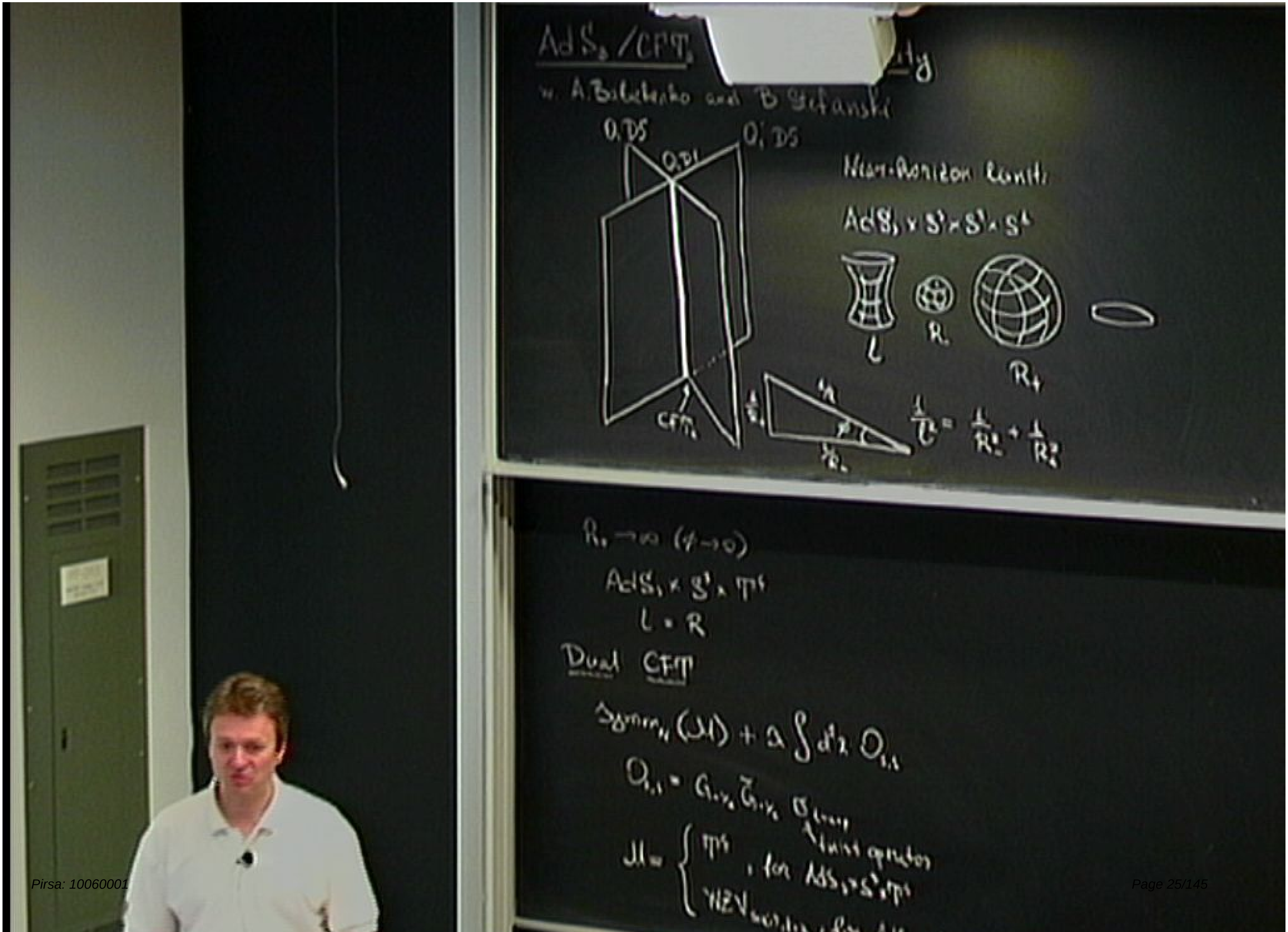
Dual CFT

$$\mathrm{Sym}^N(\mathcal{M}) + \lambda \int d^2z \, O_{1,1}$$

$$O_{1,1} = G_{-1/2} \bar{G}_{-1/2} \, \sigma_{twist}$$

twist operator

$$\mathcal{M} = \begin{cases} T^4 & , \text{ for } AdS_3 \times S^3 \times T^4 \\ WZW_{\dots} & , \text{ for } AdS_3 \times S^3 \times S^3 \times S^1 \end{cases}$$



AdS_3/CFT_2 ity

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Near-horizon limit

$AdS_3 \times S^3 \times S^3 \times S^1$

l R_- R_+

$$\frac{1}{l^2} = \frac{1}{R_-^2} + \frac{1}{R_+^2}$$

$R_+ \to \infty$ $(\alpha \to 0)$

$AdS_3 \times S^3 \times T^4$

$l = R$

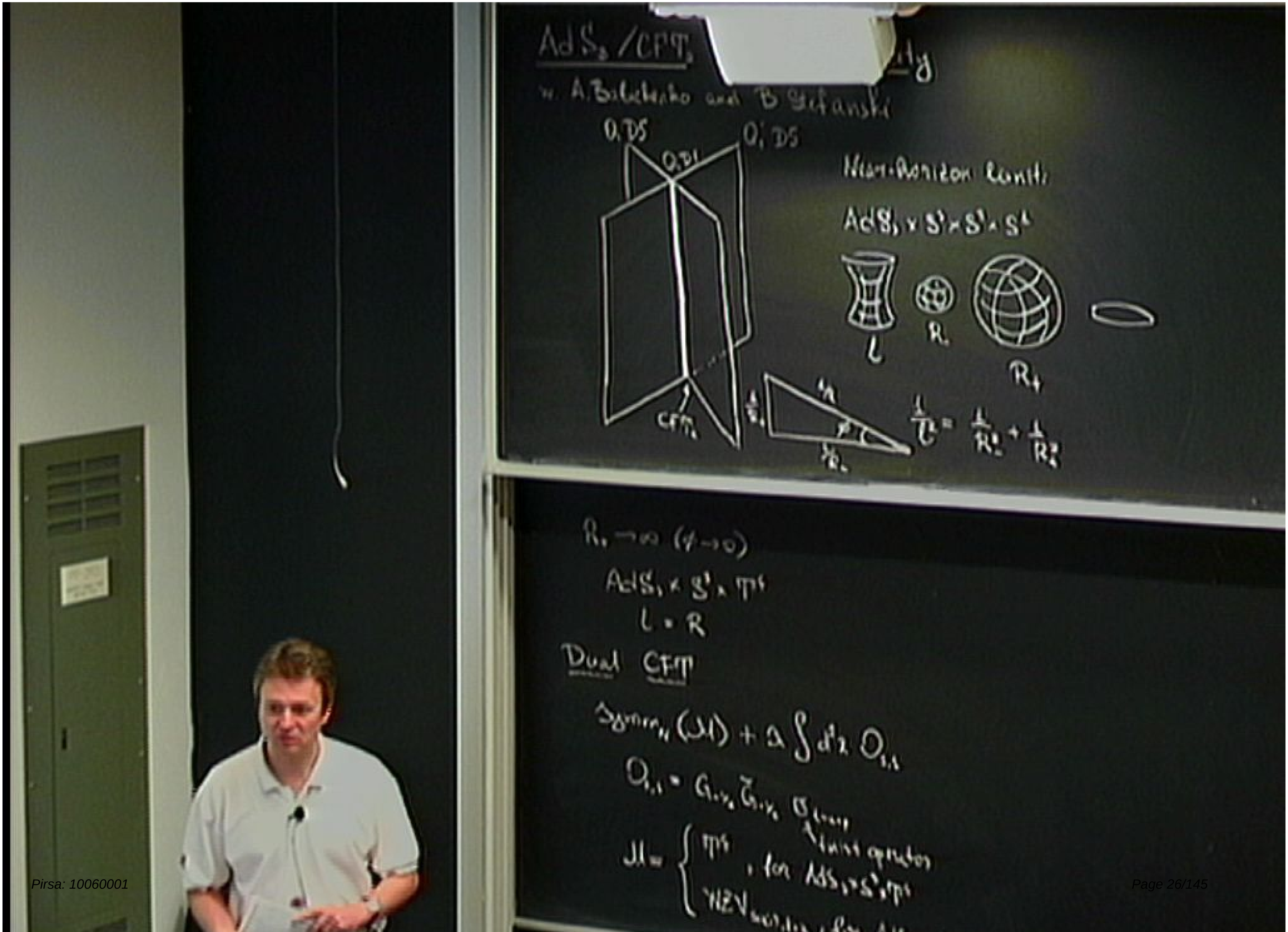
Dual CFT

$$\mathrm{Sym}^N(\mathcal{M}) + \lambda \int d^2z \, \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-\frac{1}{2}} \bar{G}_{-\frac{1}{2}} \sigma_{2,\text{twist}}$$

twist operator

$$\mathcal{M} = \begin{cases} T^4 & \text{, for } AdS_3 \times S^3 \times T^4 \\ \text{WZW models, for } AdS \end{cases}$$



AdS_3/CFT_2 [illegible] ty

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Near-horizon limit

$AdS_3 \times S^3 \times S^3 \times S^1$

$$\frac{1}{l^2} = \frac{1}{R_-^2} + \frac{1}{R_+^2}$$

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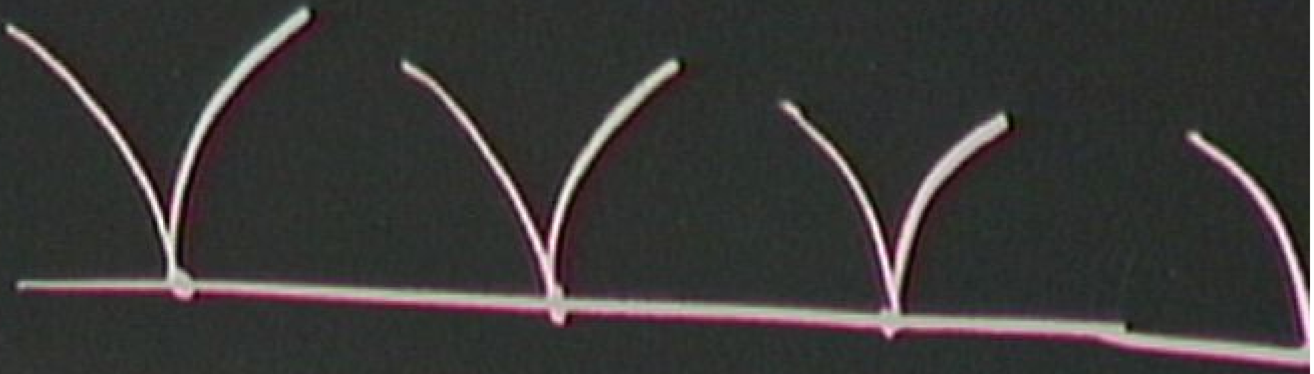
$$\mathrm{Sym}^N(\mathcal{M}) + \lambda \int d^2z \, \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \bar{G}_{-1/2} \sigma_{twist}$$

twist operator

$\mathcal{M} = \begin{cases} T^4 & \text{for } AdS_3 \times S^3 \times T^4 \\ \text{WZW models} & \text{for } AdS \end{cases}$

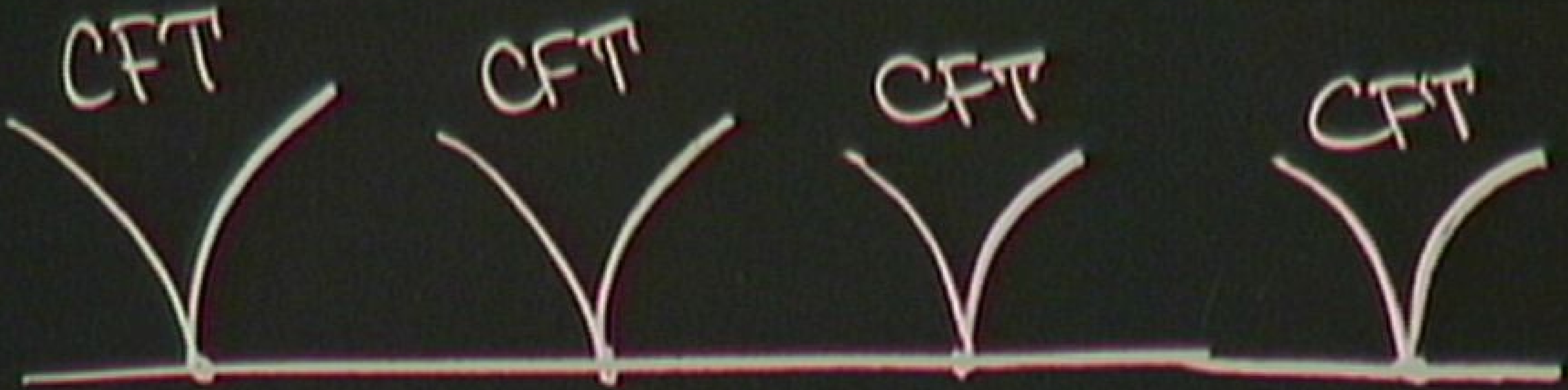


$\int d^2z\, \mathcal{O}_{1,1}$

σ_{tranp}

↑ twist operator

$AdS_3 \times S^3 \times T^4$


CFT
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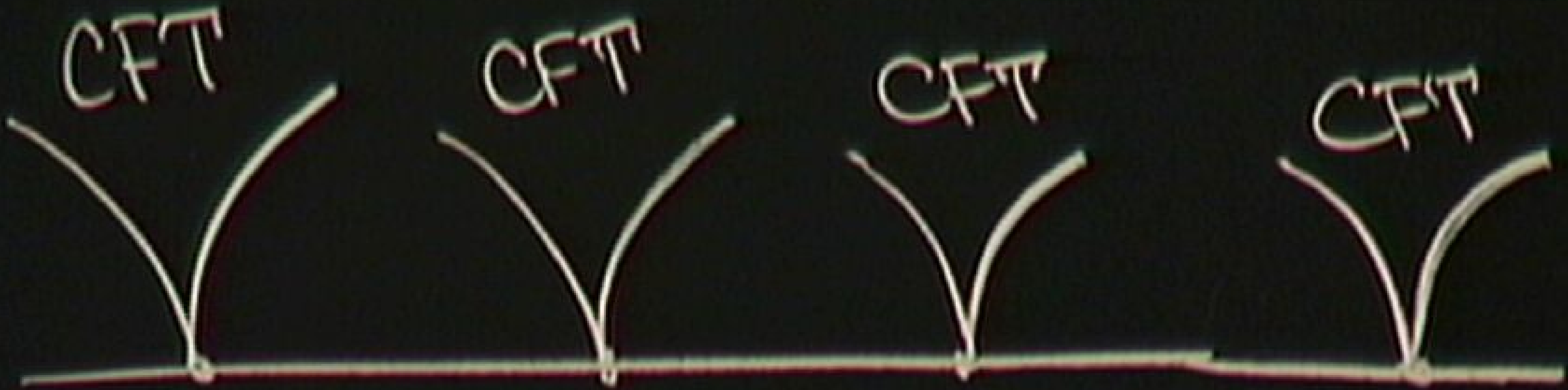
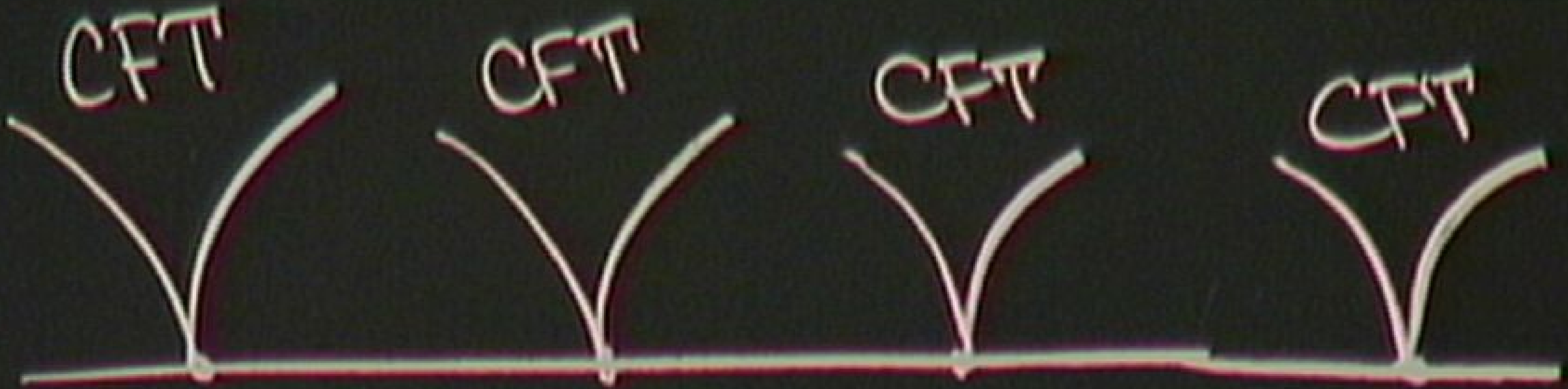


$$\mathcal{H} = F^{\otimes L}$$

$$H : \mathcal{H} \to \mathcal{H}$$

$$H = \sum_{\ell=1}^{L} H_{\ell,\ell+1}$$



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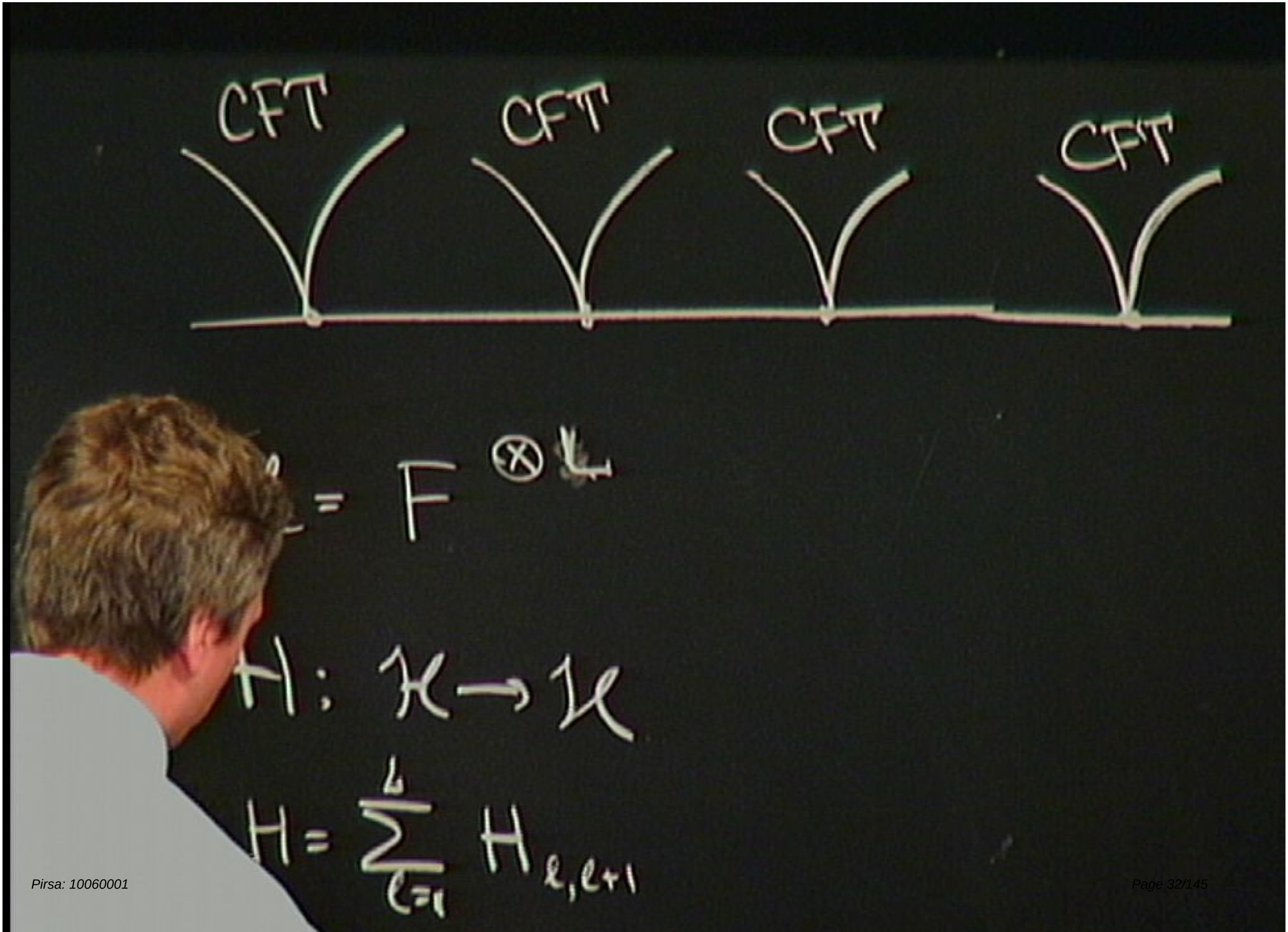

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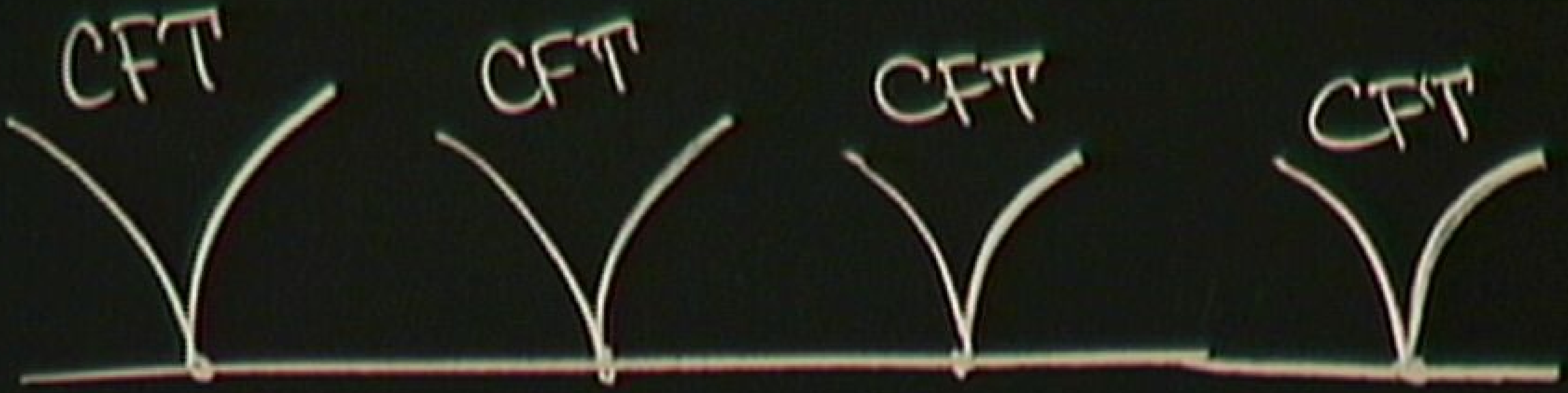

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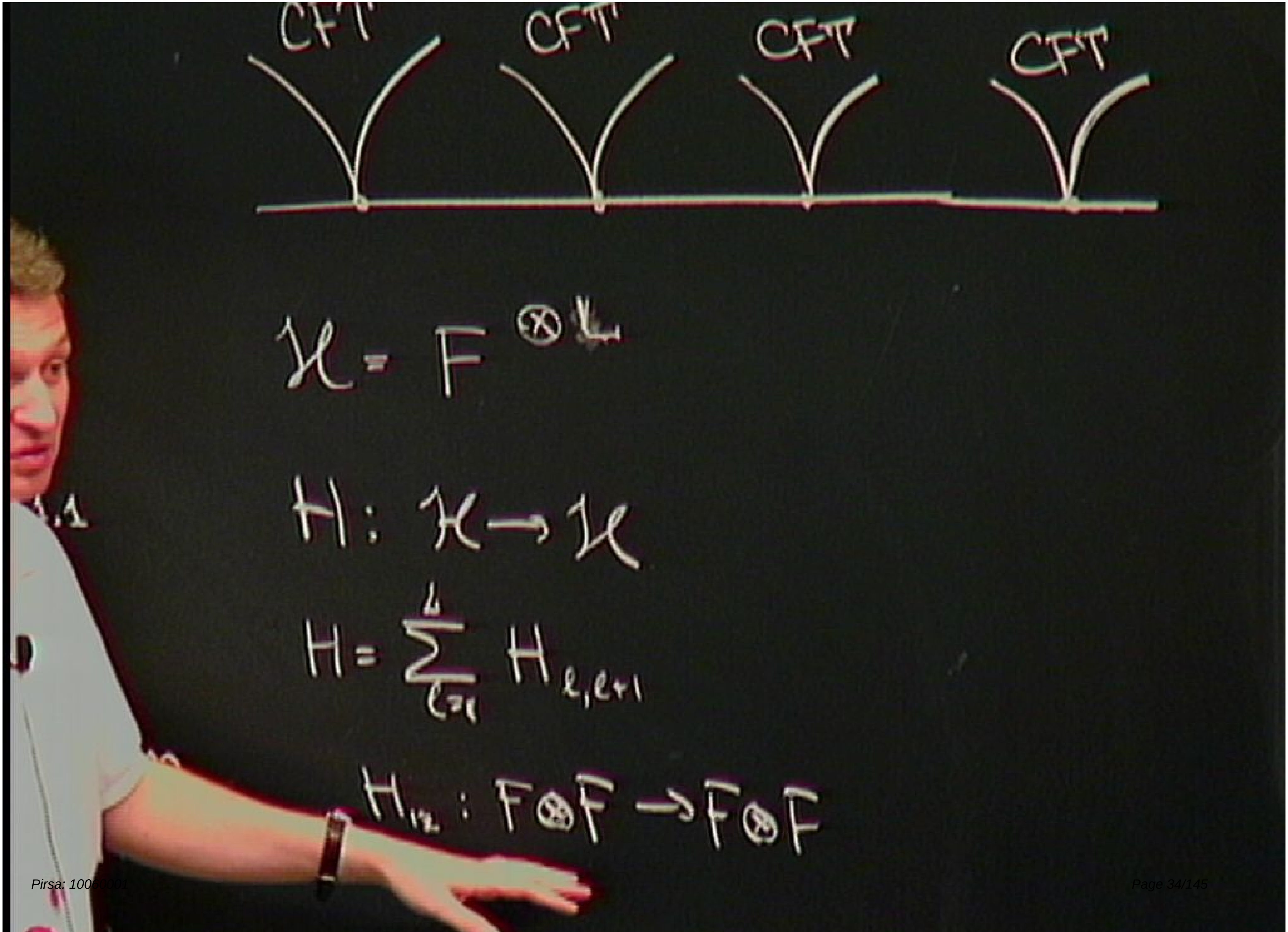
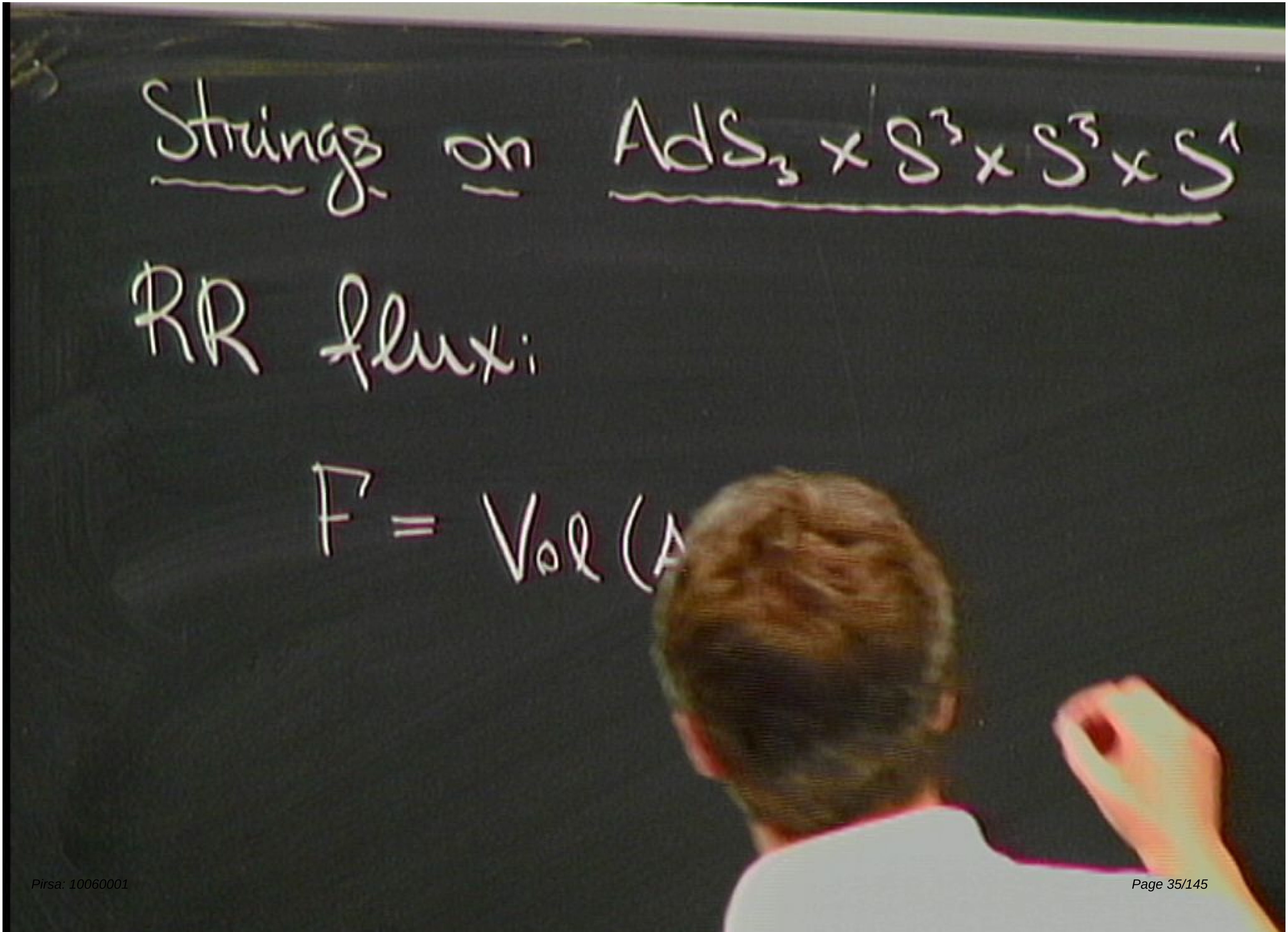


$$\mathcal{H} = F^{\otimes L}$$

$$H : \mathcal{H} \to \mathcal{H}$$

$$H = \sum_{\ell=1}^{L} H_{\ell,\ell+1}$$

$$H_{12} : F \otimes F \to F \otimes F$$



Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

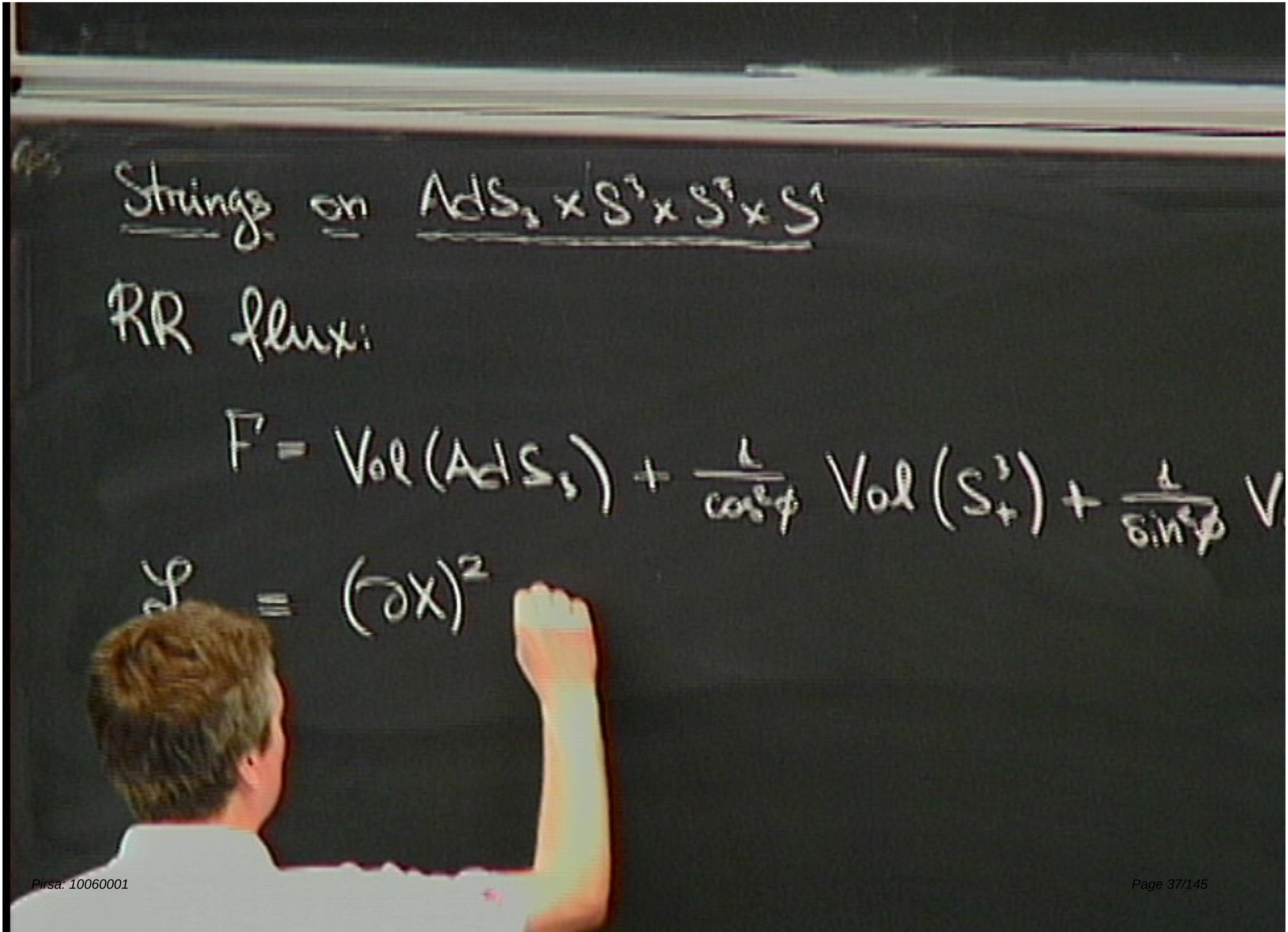
RR flux:

$$F = Vol(A$$

Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\mathrm{Vol}(S^3_-)$$

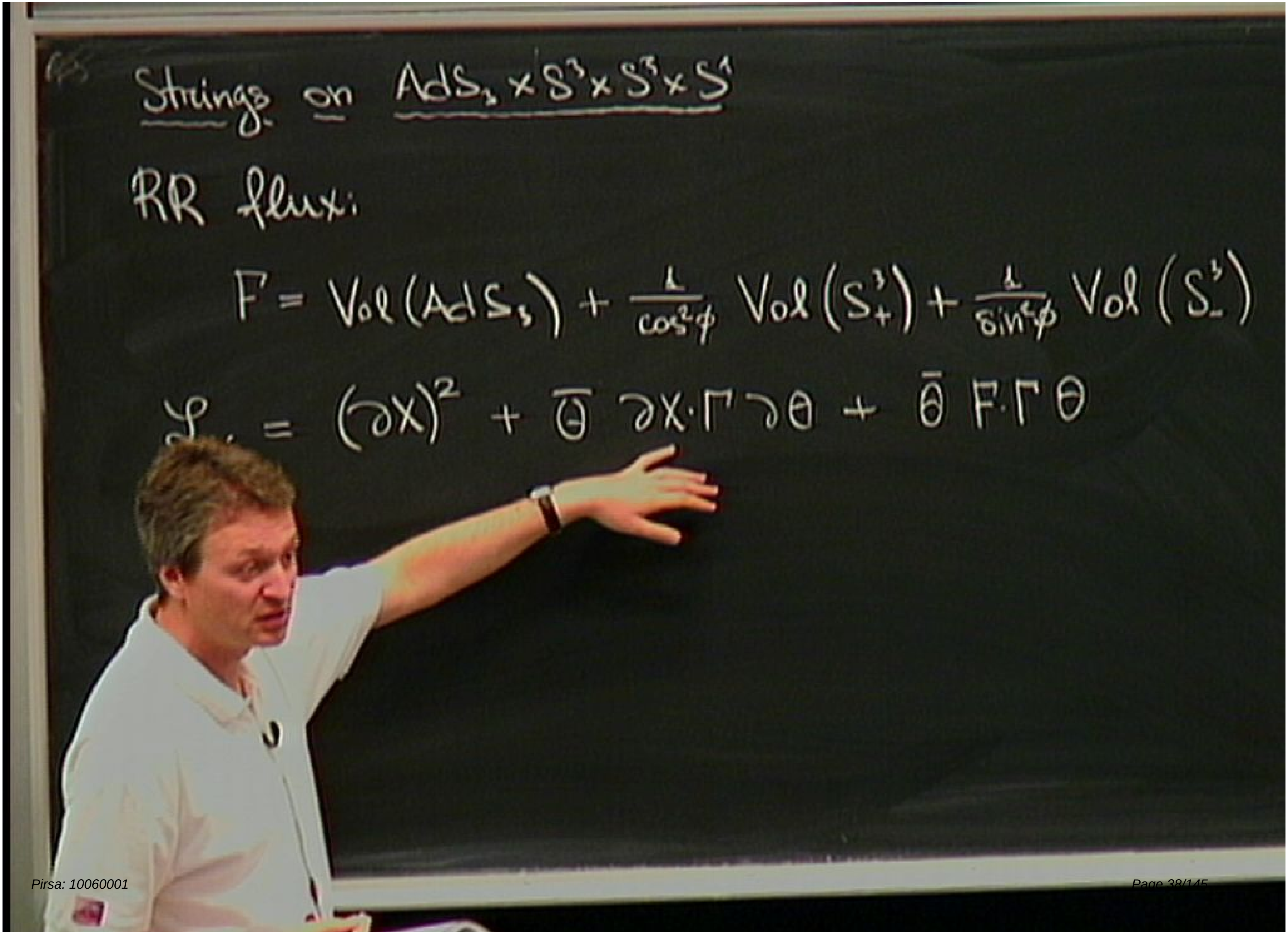


Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,V$$

$$\mathcal{L} = (\partial X)^2$$

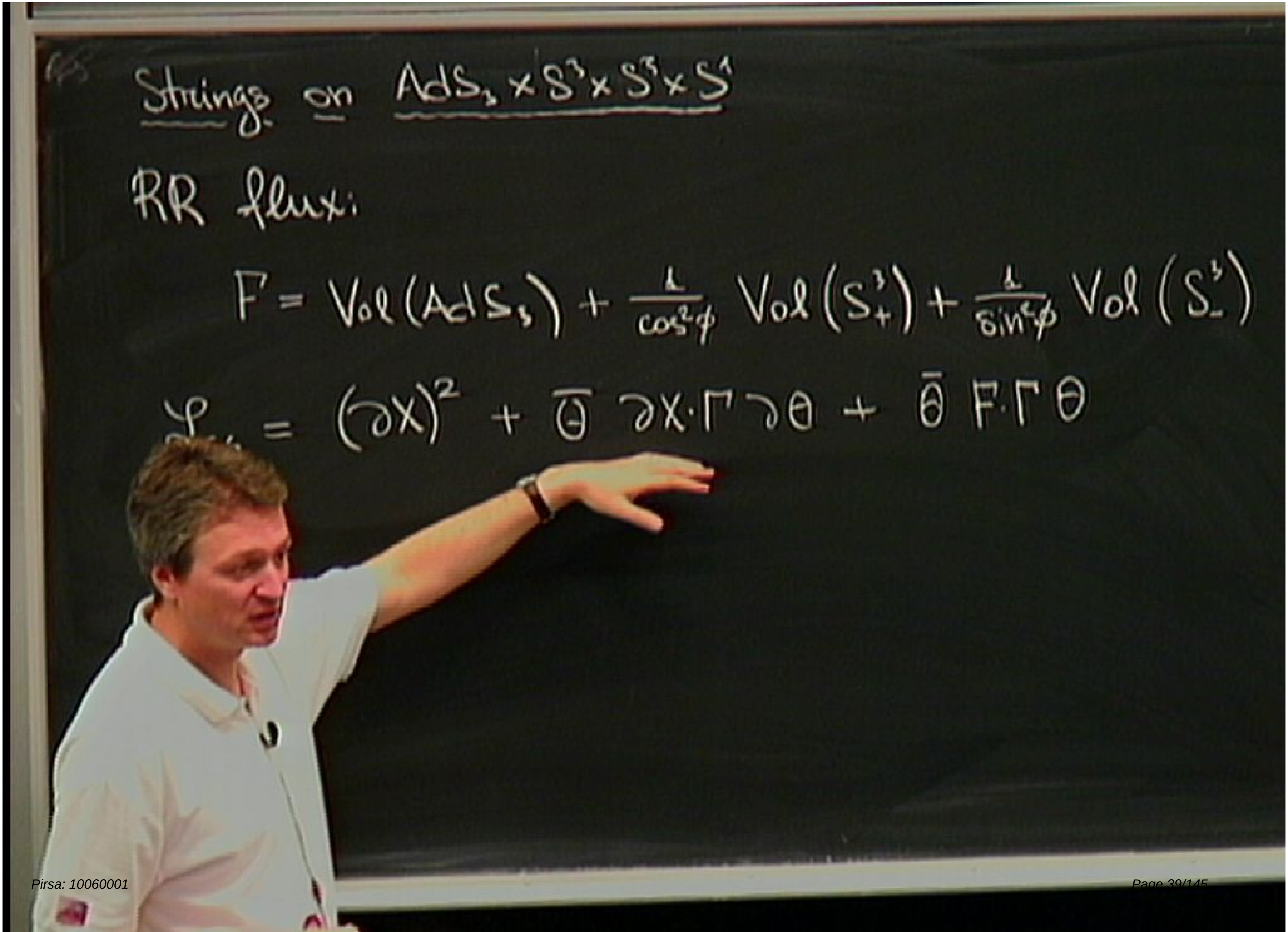


Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\mathrm{Vol}(S^3_-)$$

$$\mathcal{L} = (\partial X)^2 + \bar{\Theta}\,\partial X\cdot\Gamma\,\partial\Theta + \bar{\Theta}\,F\cdot\Gamma\,\Theta$$



Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\mathrm{Vol}(S^3_-)$$

$$\mathcal{L} = (\partial X)^2 + \bar{\Theta}\,\partial X\cdot\Gamma\,\partial\Theta + \bar{\Theta}\,F\cdot\Gamma\,\Theta$$

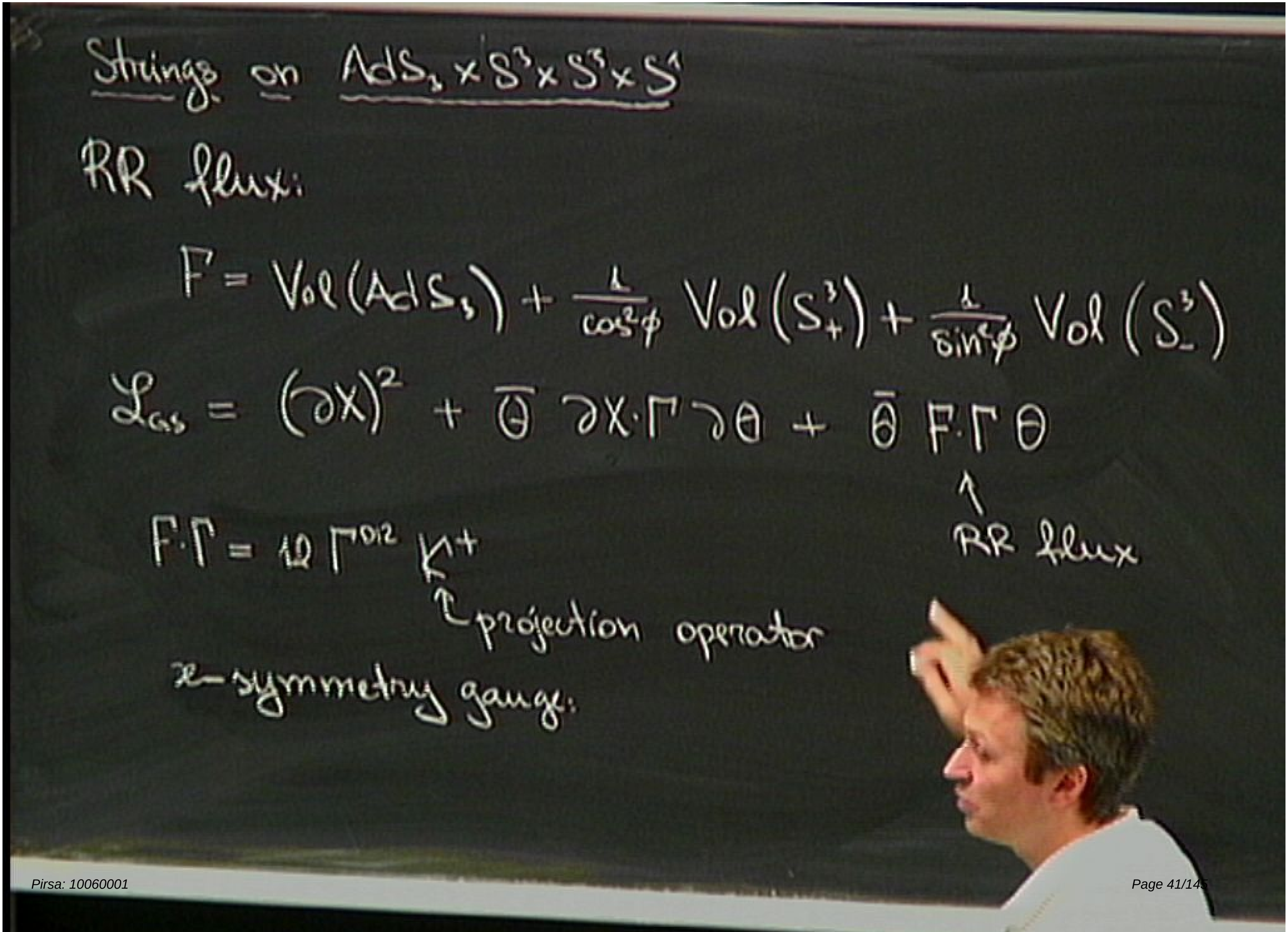
Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\mathrm{Vol}(S^3_-)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta}\,\partial X\cdot\Gamma\,\partial\Theta + \bar{\Theta}\,\underset{\substack{\uparrow\\ \text{RR flux}}}{F\cdot\Gamma}\,\Theta$$

$$F\cdot\Gamma = 12\,\Gamma^{012}\,\underset{\substack{\uparrow\\ \text{projection operator}}}{K^+}$$



Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

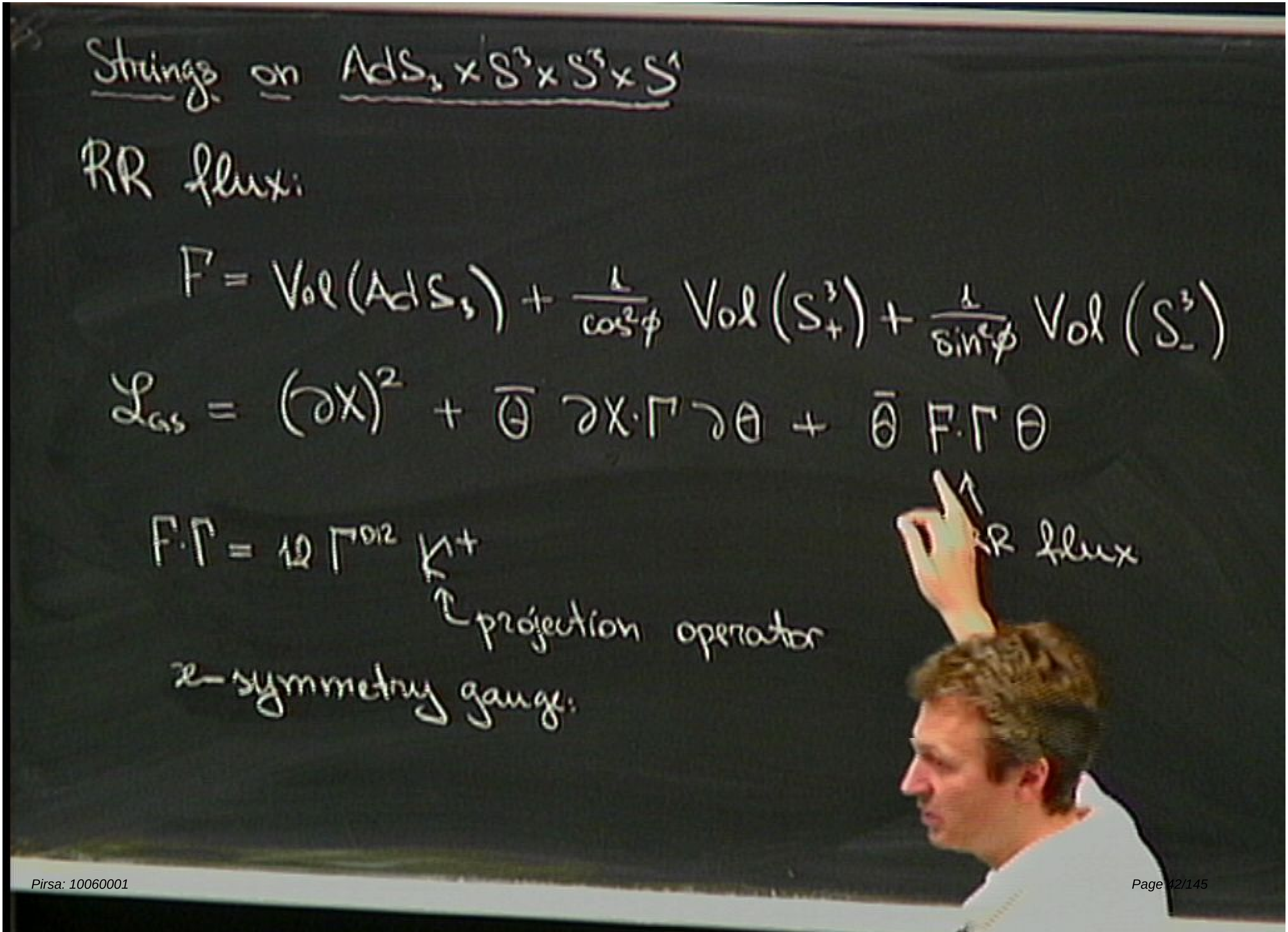
RR flux:

$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\mathrm{Vol}(S^3_-)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta}\,\partial X\cdot\Gamma\,\partial\Theta + \bar{\Theta}\,\underset{\substack{\uparrow\\ \text{RR flux}}}{F\cdot\Gamma}\,\Theta$$

$$F\cdot\Gamma = 12\,\Gamma^{012}\,\underset{\substack{\uparrow\\ \text{projection operator}}}{K^+}$$

κ-symmetry gauge:



Strings on $AdS_3\times S^3\times S^3\times S^1$

RR flux:

$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\mathrm{Vol}(S^3_-)$$

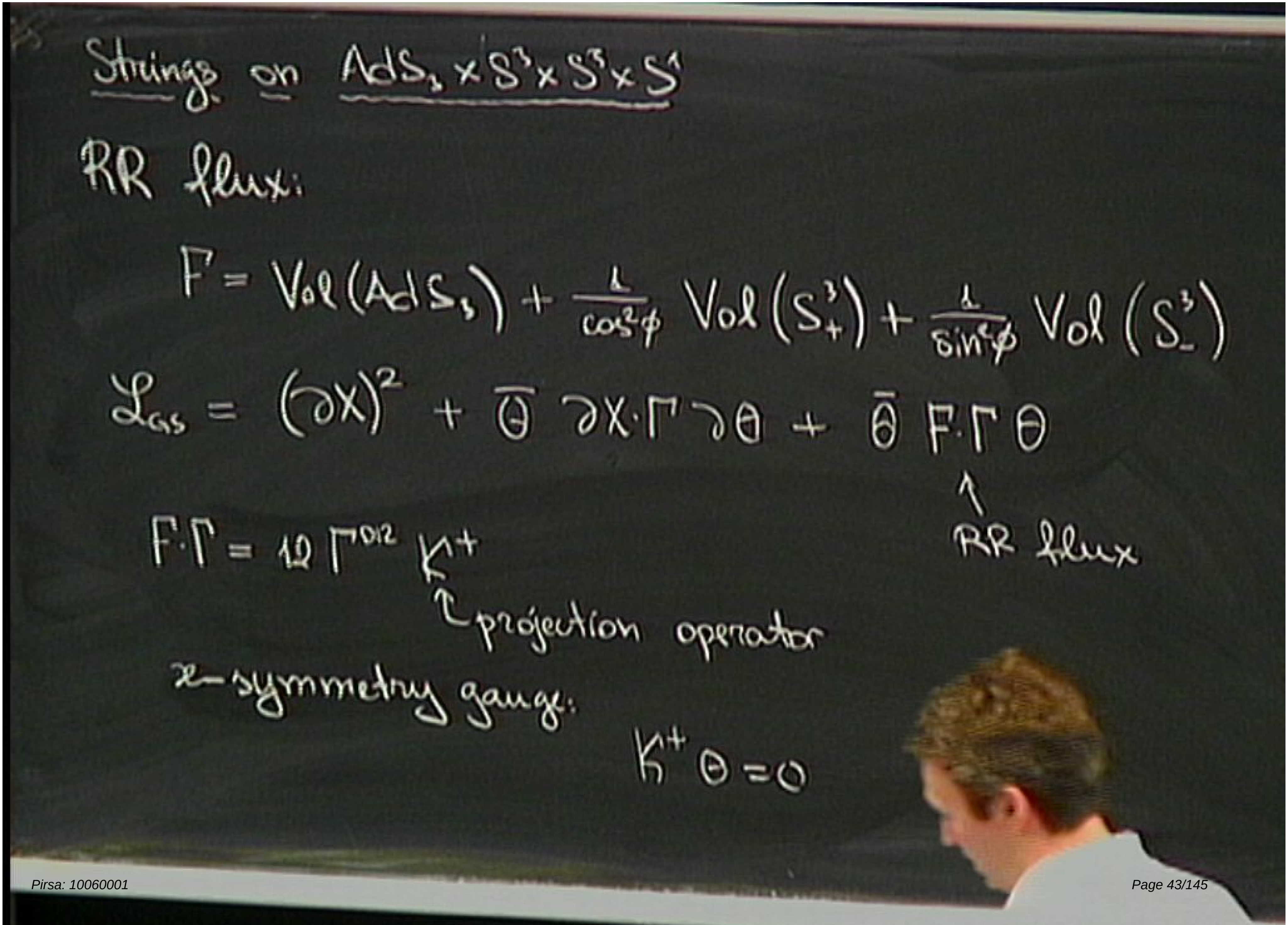
$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta}\,\partial X\cdot\Gamma\,\partial\Theta + \bar{\Theta}\,F\cdot\Gamma\,\Theta$$

↑ R flux

$$F\cdot\Gamma = 12\,\Gamma^{012}\,K^+$$

↑ projection operator

κ-symmetry gauge:



Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

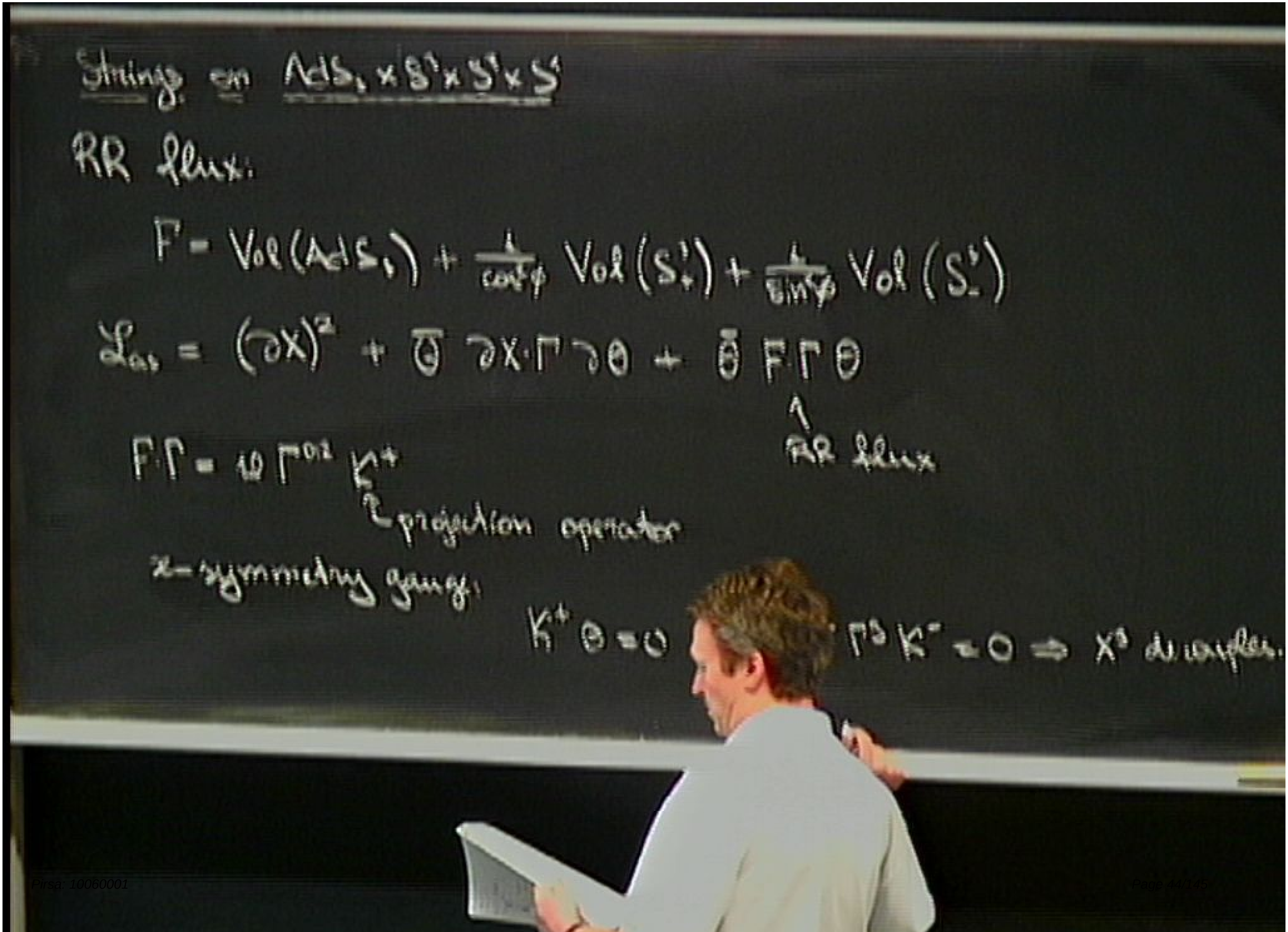
$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\mathrm{Vol}(S^3_-)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta}\,\partial X \cdot \Gamma\,\partial\Theta + \bar{\Theta}\,\underset{\substack{\uparrow \\ \text{RR flux}}}{F\cdot\Gamma}\,\Theta$$

$$F\cdot\Gamma = 12\,\Gamma^{012}\,\underset{\substack{\uparrow \\ \text{projection operator}}}{K^+}$$

κ-symmetry gauge:

$$K^+\Theta = 0$$



Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

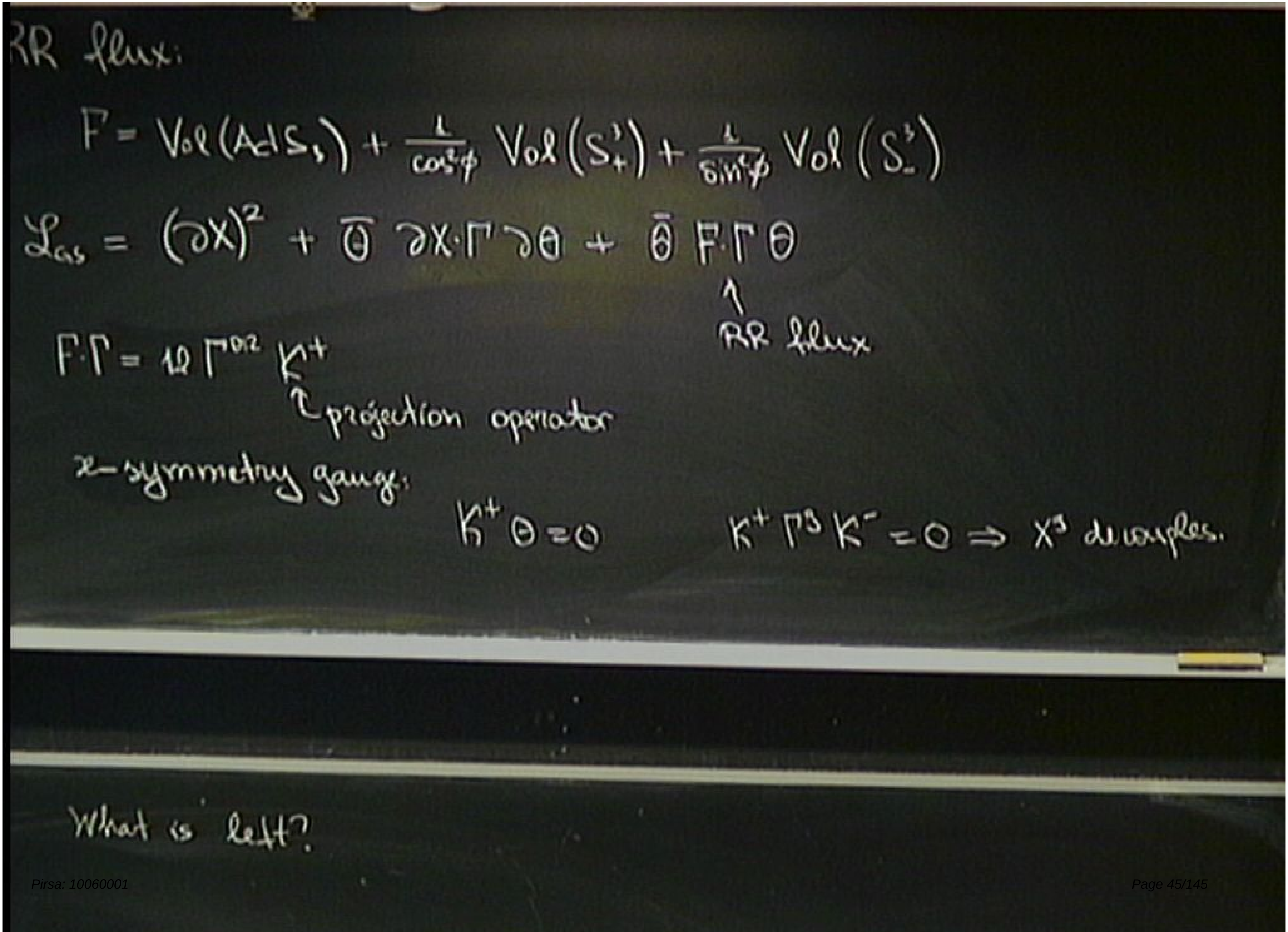
$$F = \mathrm{Vol}(AdS_3) + \frac{1}{\cos^2\phi}\,\mathrm{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\mathrm{Vol}(S^3_-)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta}\,\partial X\cdot\Gamma\,\partial\Theta + \underset{\substack{\uparrow\\ \text{RR flux}}}{\bar{\Theta}\,F\,\Gamma\,\Theta}$$

$$F\cdot\Gamma = 12\,\Gamma^{012}\,\underset{\text{projection operator}}{K^+}$$

κ-symmetry gauge:

$$K^+\Theta = 0 \quad \ldots \quad \Gamma^3 K^- = 0 \Rightarrow X^3 \text{ decouples.}$$



RR flux:

$$F = \text{Vol}(AdS_5) + \frac{1}{\cos^2\phi}\,\text{Vol}(S^3_+) + \frac{1}{\sin^2\phi}\,\text{Vol}(S^3_-)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta}\,\partial X\cdot\Gamma\,\partial\Theta + \bar{\Theta}\,\underset{\substack{\uparrow \\ \text{RR flux}}}{F\cdot\Gamma}\,\Theta$$

$$F\cdot\Gamma = 12\,\Gamma^{012}\,\underset{\substack{\uparrow \\ \text{projection operator}}}{K^+}$$

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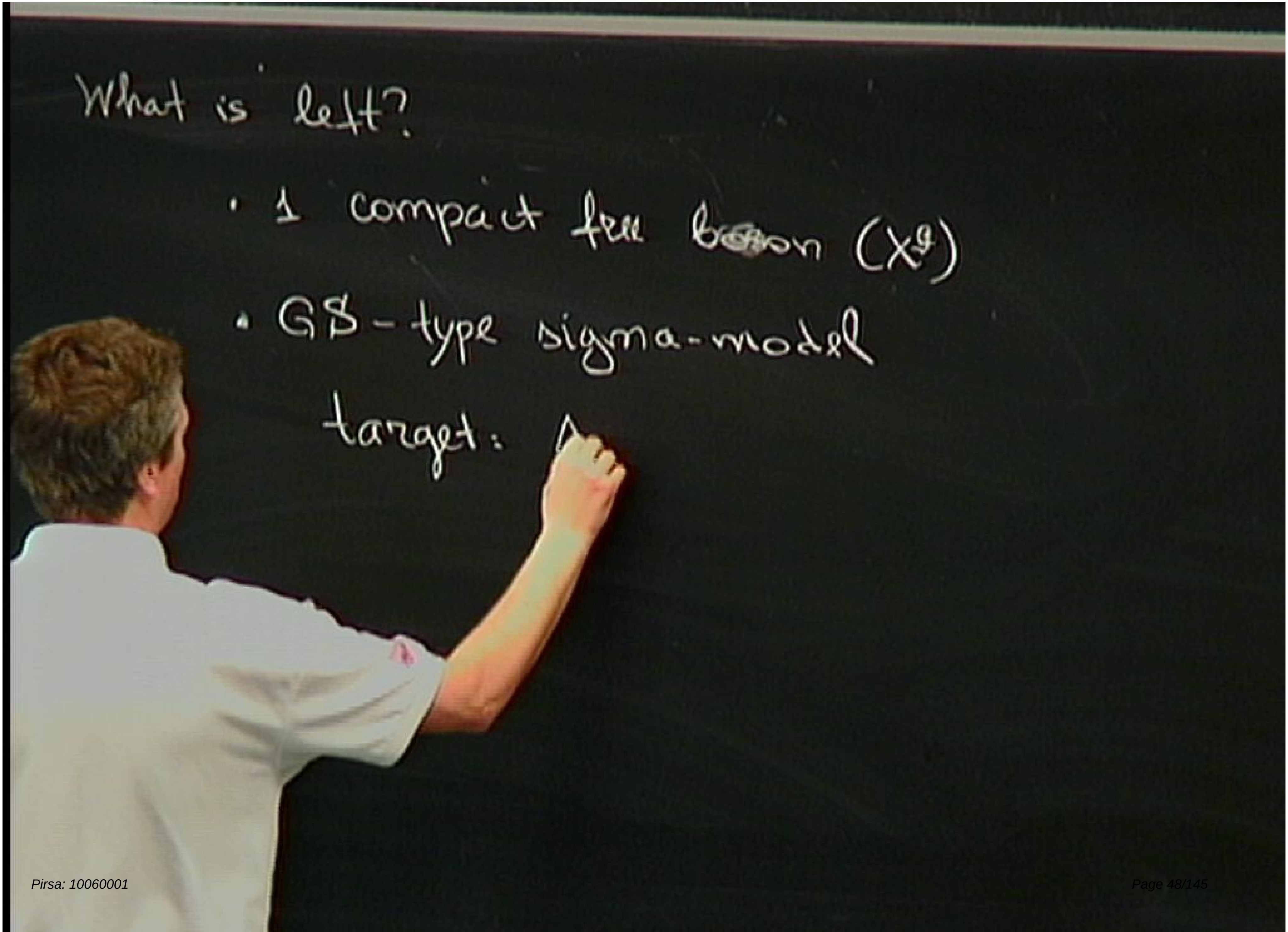
- 1 compact free boson (X^9)



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target: Δ

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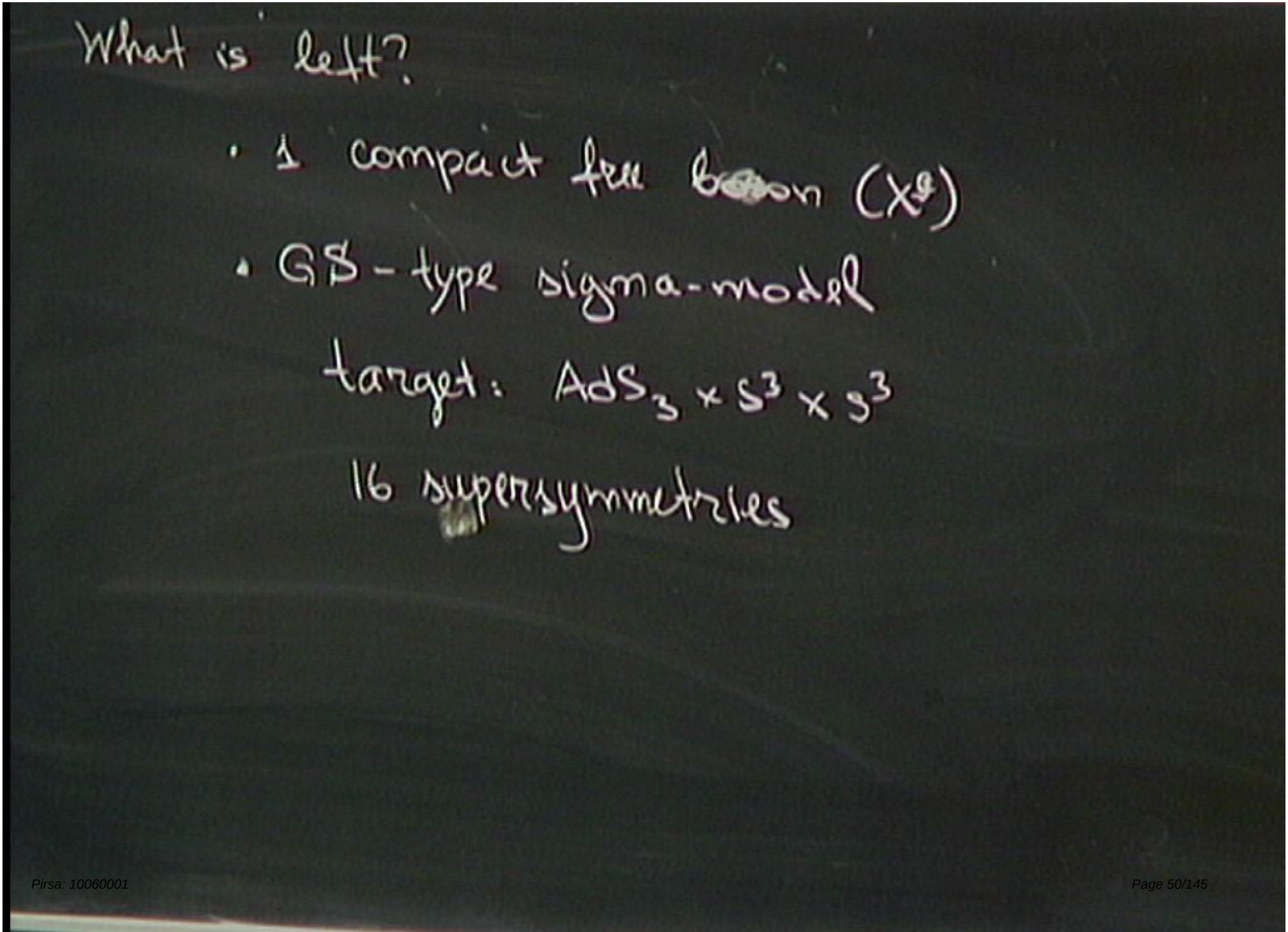

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16 supersymmetries

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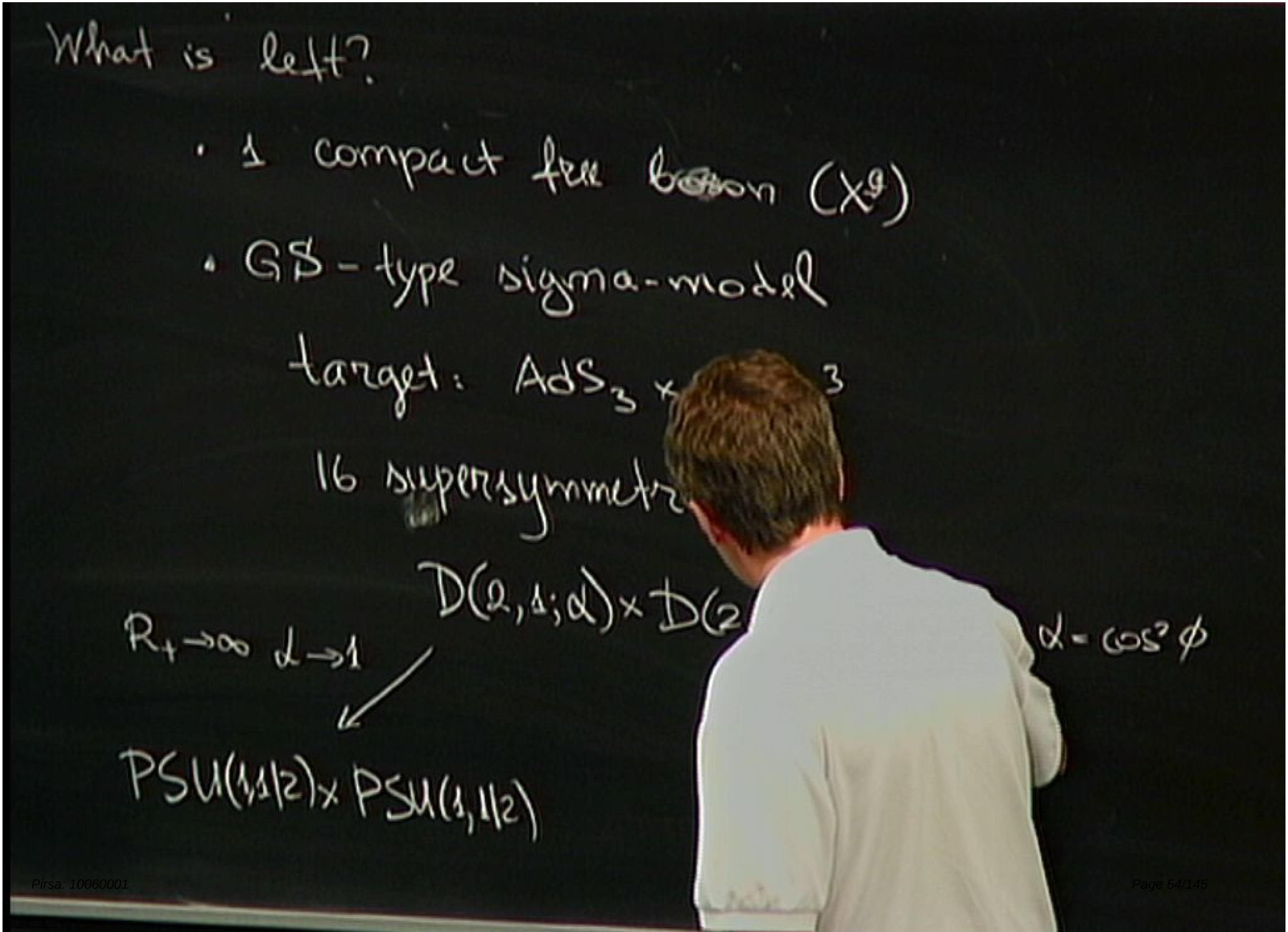
16 supersymmetries

$$D(2,1;\alpha) \times D(2,1;\alpha)$$

$R_+ \to \infty$ $\alpha \to 1$ ↙

$\alpha = \cos^2\phi$



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target: $AdS_3 \times$ [illegible] 3

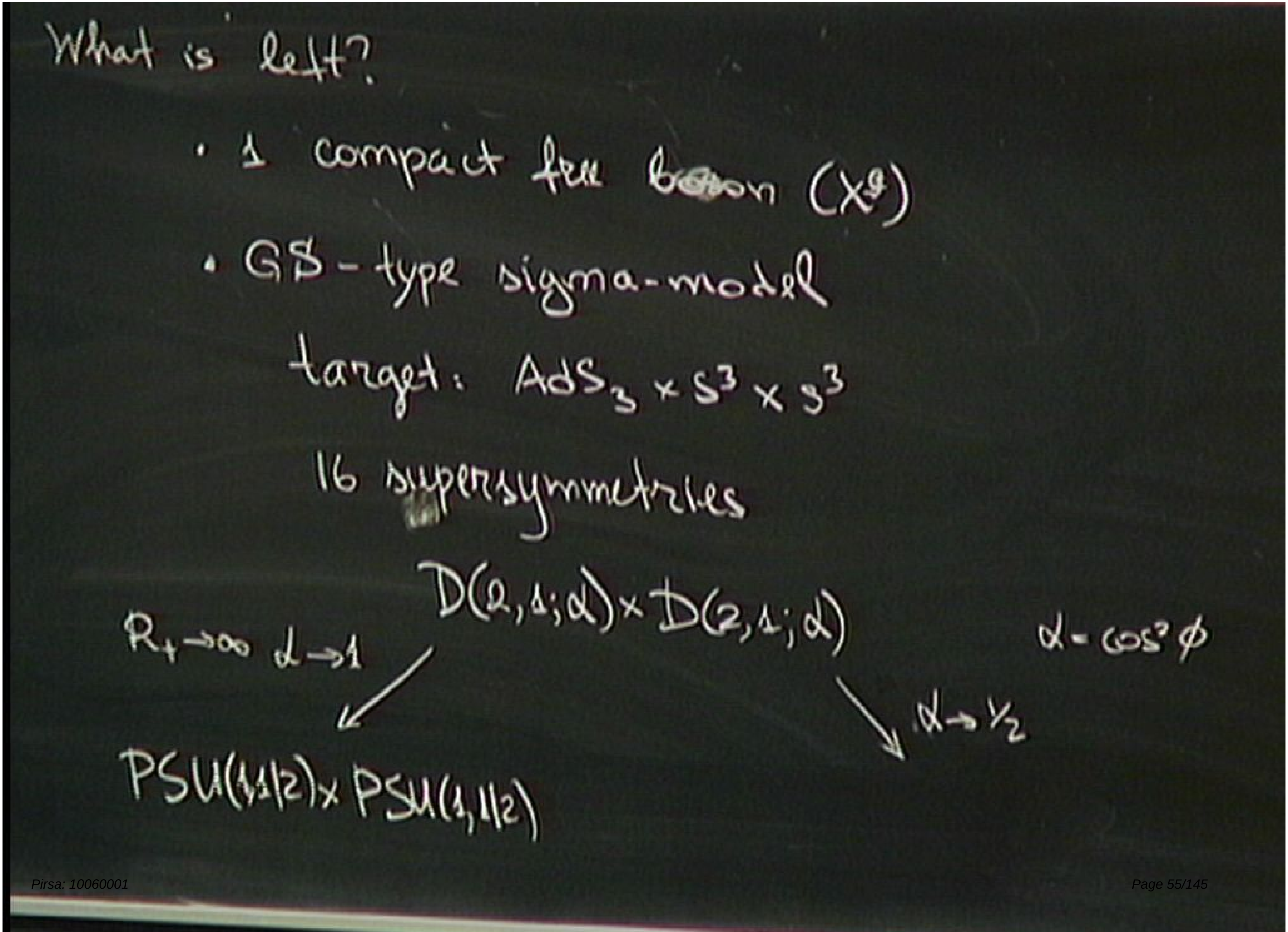
16 supersymmetr[illegible]

$D(2,1;\alpha) \times D(2$ [illegible] $\alpha = \cos^2\phi$

$R_+ \to \infty \quad \alpha \to 1$

$\swarrow$

$PSU(1,1|2) \times PSU(1,1|2)$



What is left?

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$$\begin{tikzcd}
 & D(2,1;\alpha)\times D(2,1;\alpha) \arrow[dl, "R_+\to\infty \quad \alpha\to 1"'] \arrow[dr, "\alpha\to 1/2"] & \\
PSU(1,1|2)\times PSU(1,1|2) & & {}
\end{tikzcd}$$

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σ-Model is a coset:

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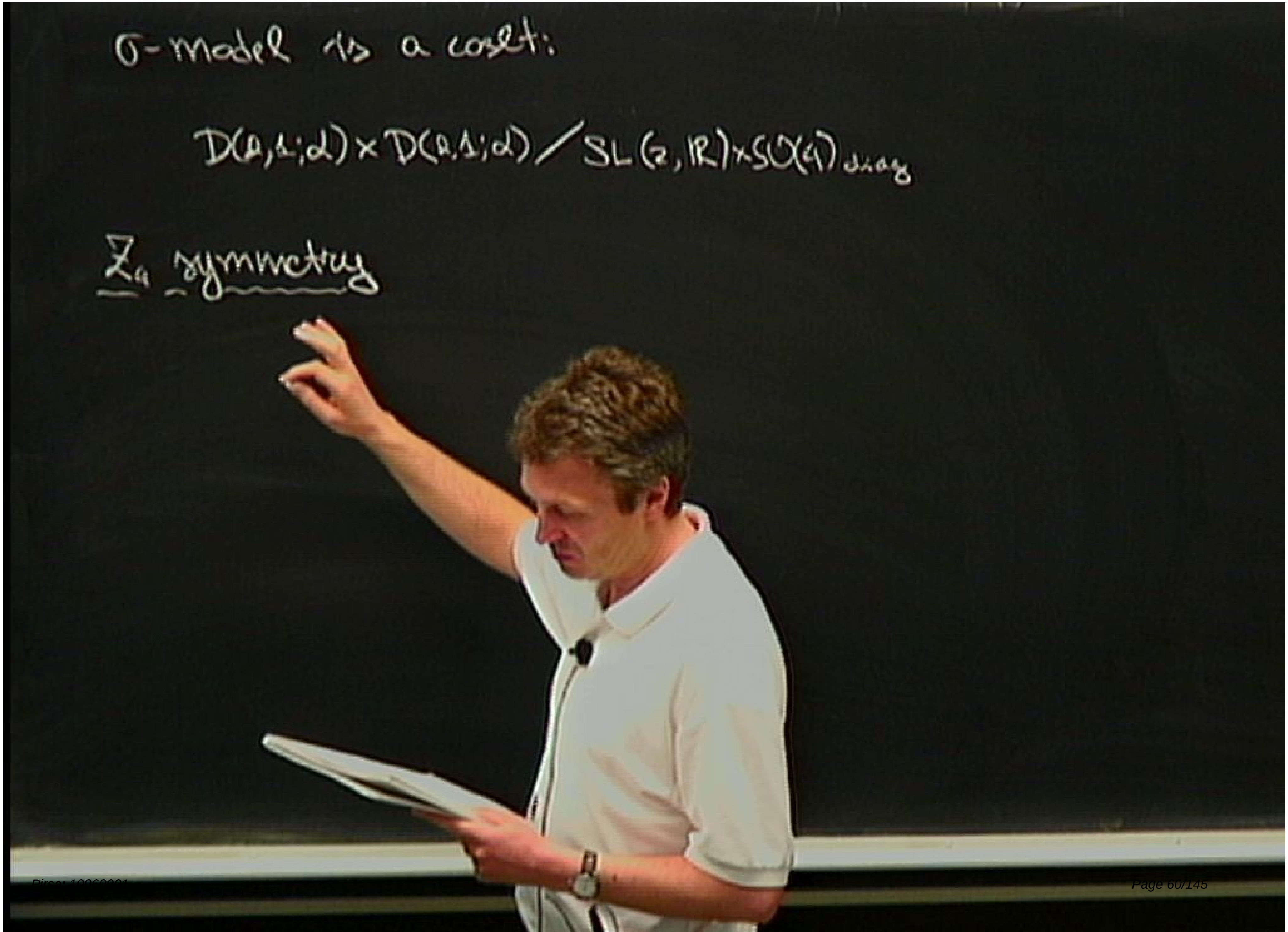

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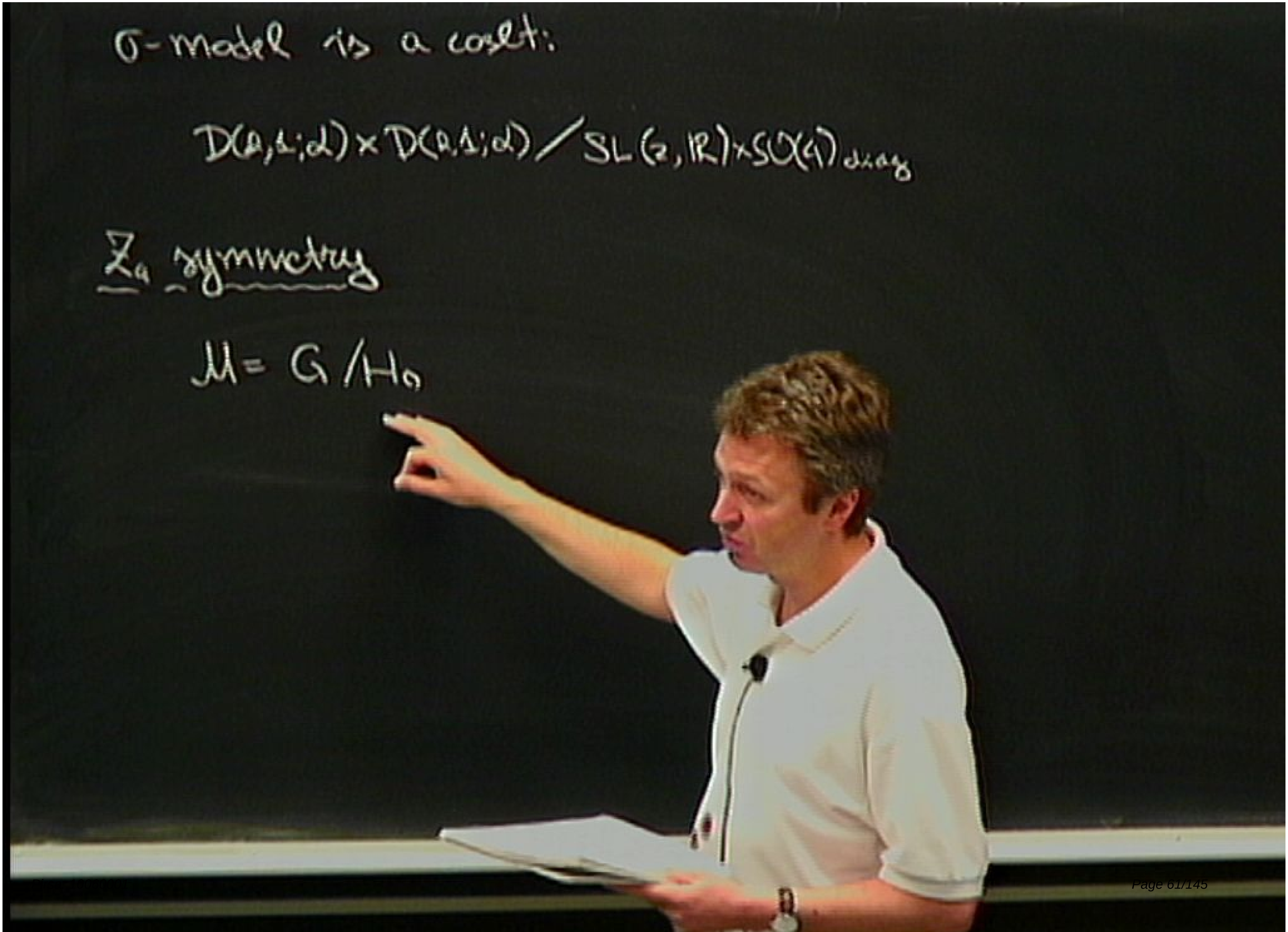
$\underline{\mathbb{Z}_4 \text{ symmetry}}$



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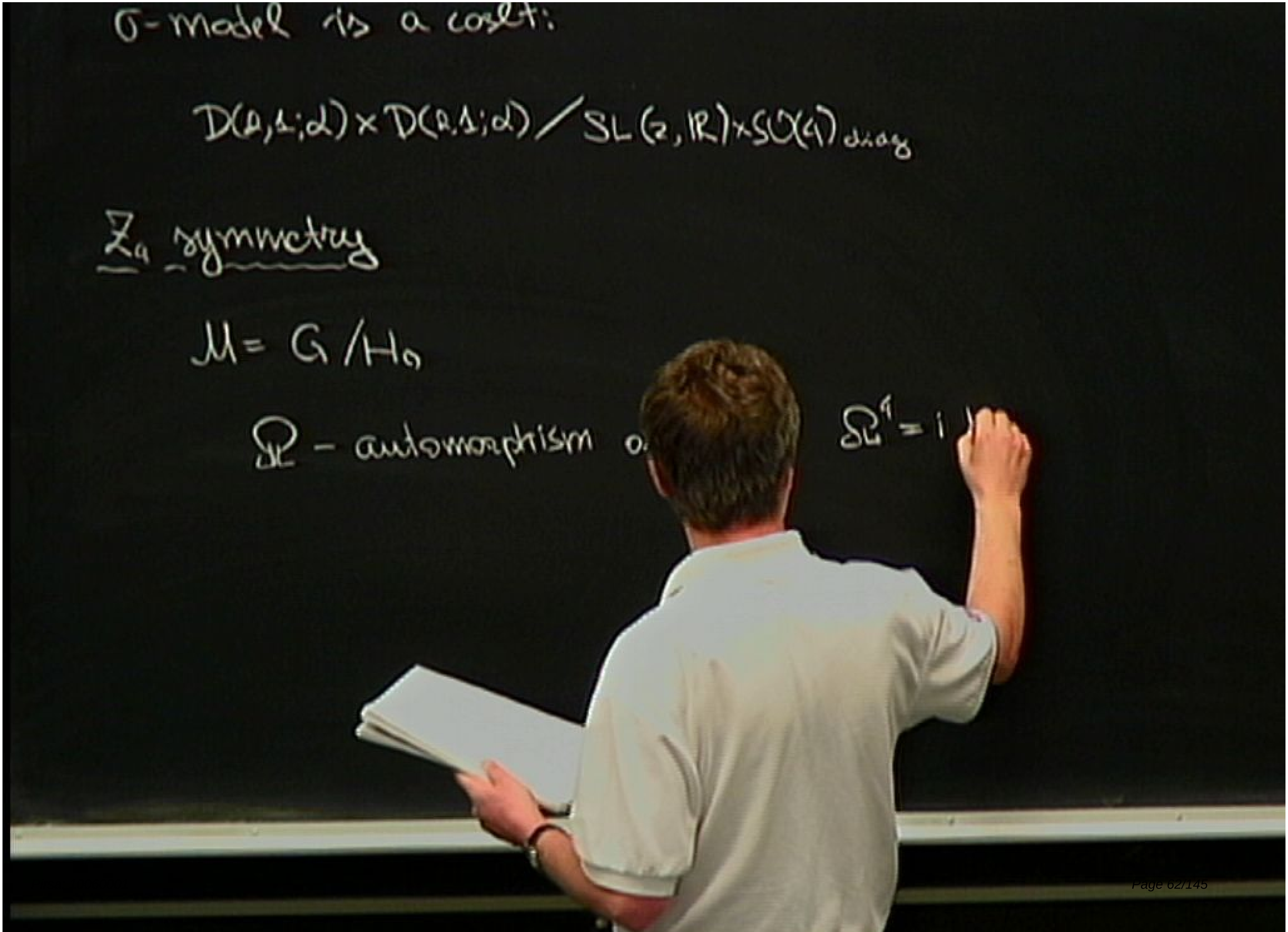
$$\mathcal{M} = G\,/\,H_0$$



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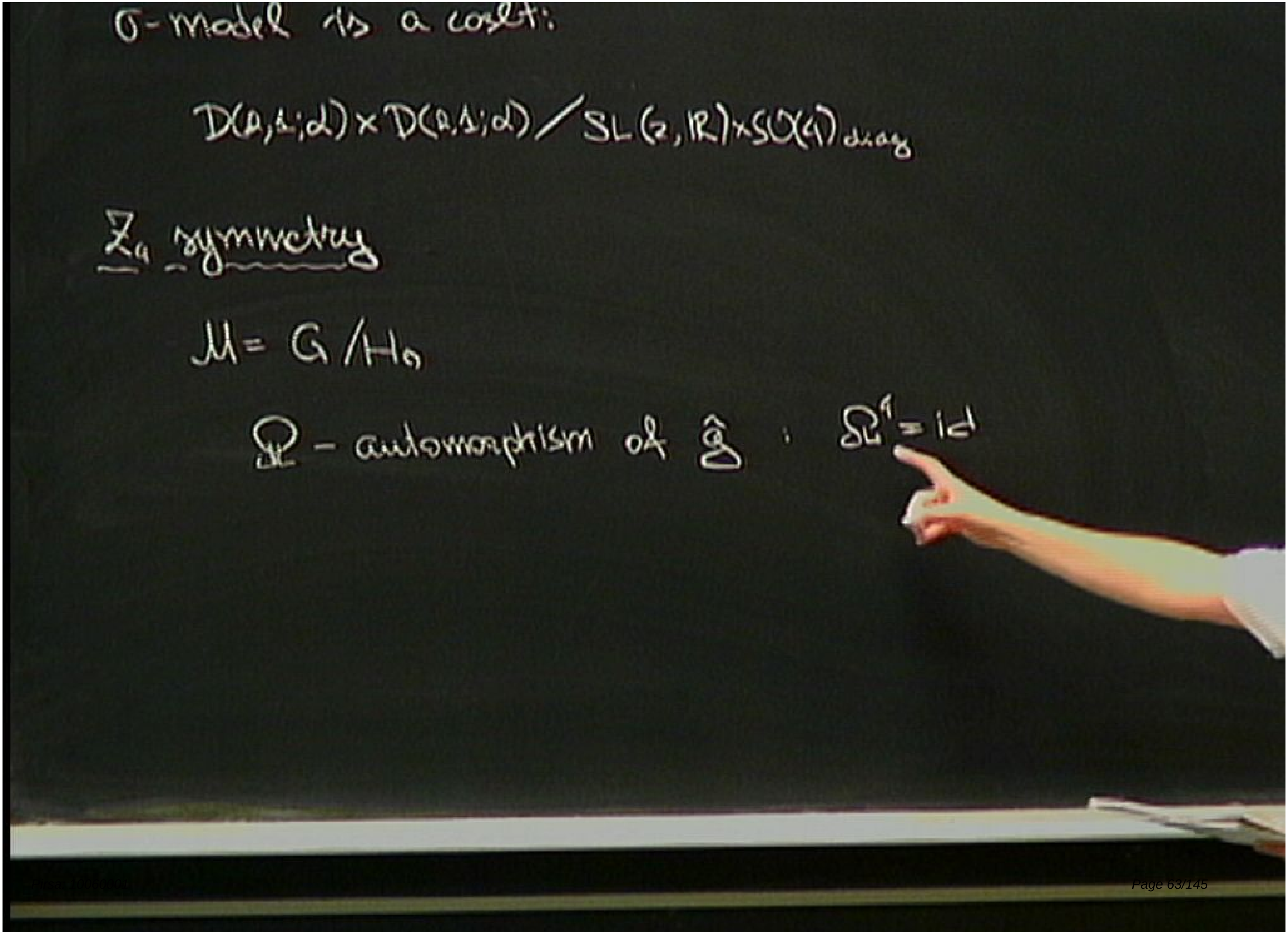
Ω - automorphism o $\Omega^4 = 1$



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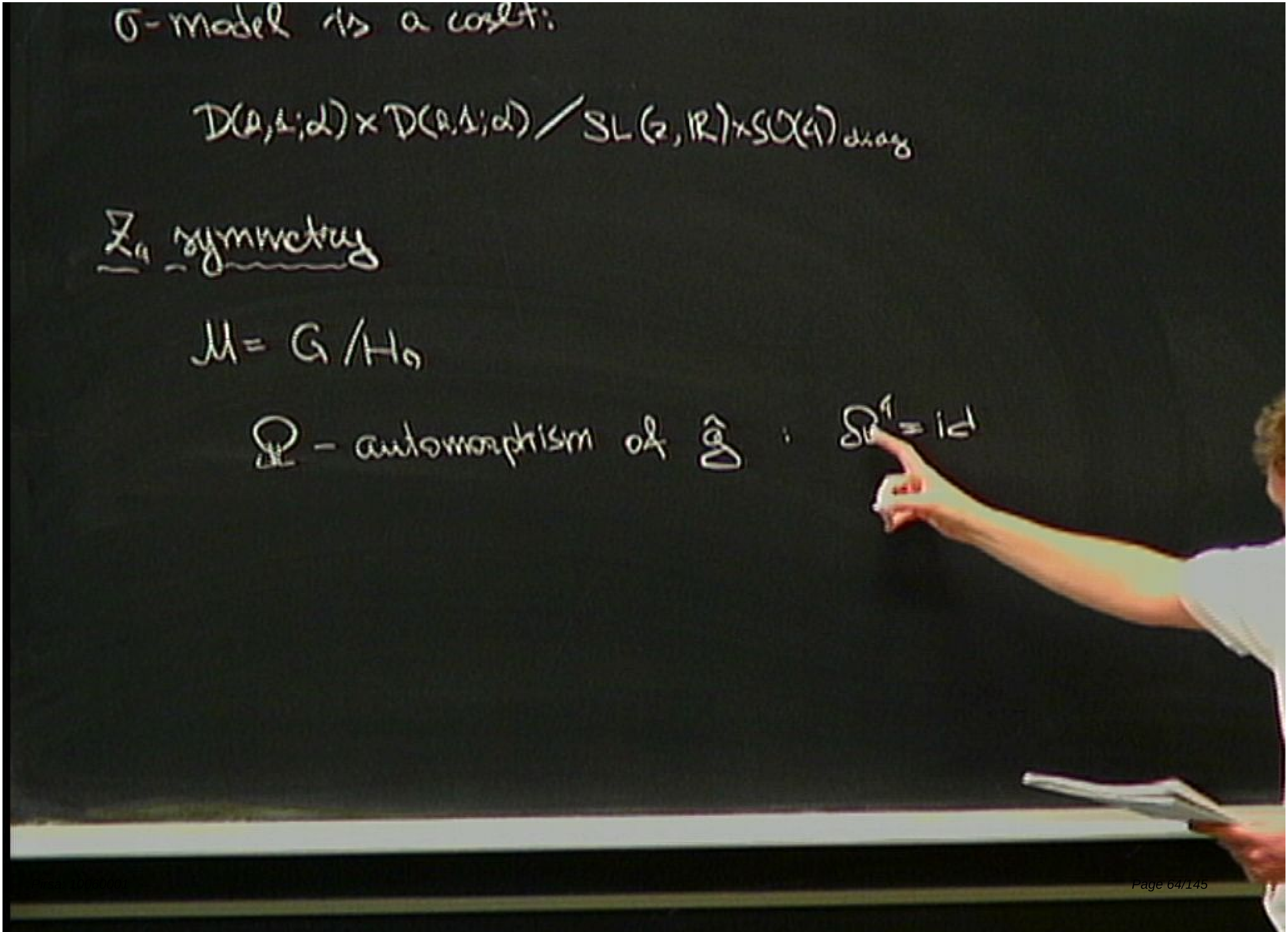
Ω - automorphism of $\hat{g}$: $\Omega^4 = \mathrm{Id}$



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$$\Omega(h_n) = i^n h_n$$

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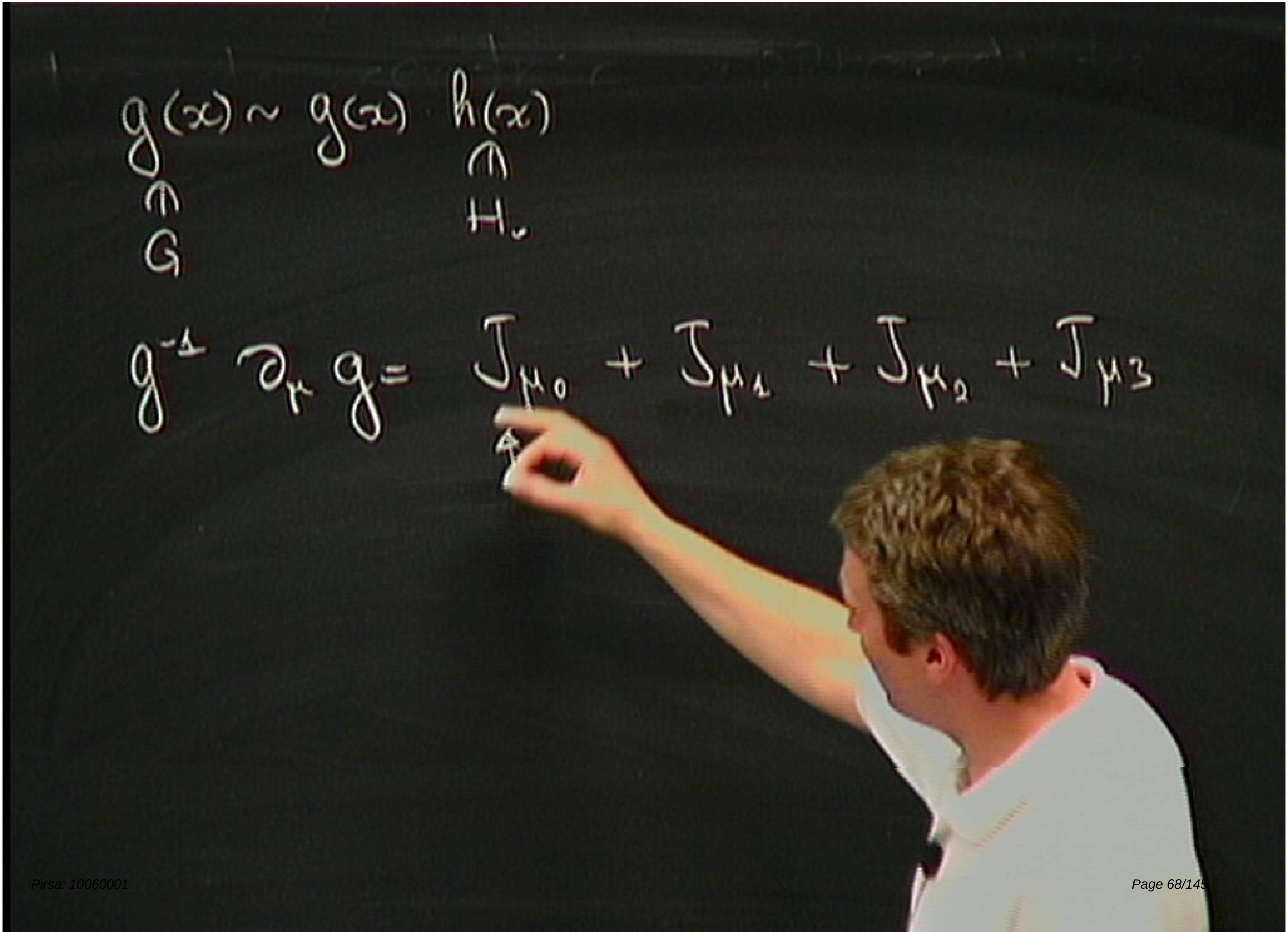

$$\underset{\underset{G}{\pitchfork}}{g}(x) \sim g(x)\;\underset{\underset{H_0}{\pitchfork}}{h}(x)$$

$$g^{-1}\partial_\mu g = \underset{\uparrow}{J_{\mu_0}} + J_{\mu_1} + J_{\mu_2} + J_{\mu 3}$$

$$\underset{\substack{\rotatebox{90}{$\in$}\\ G}}{g}(x) \sim g(x)\ \underset{\substack{\rotatebox{90}{$\in$}\\ H_0}}{h}(x)$$

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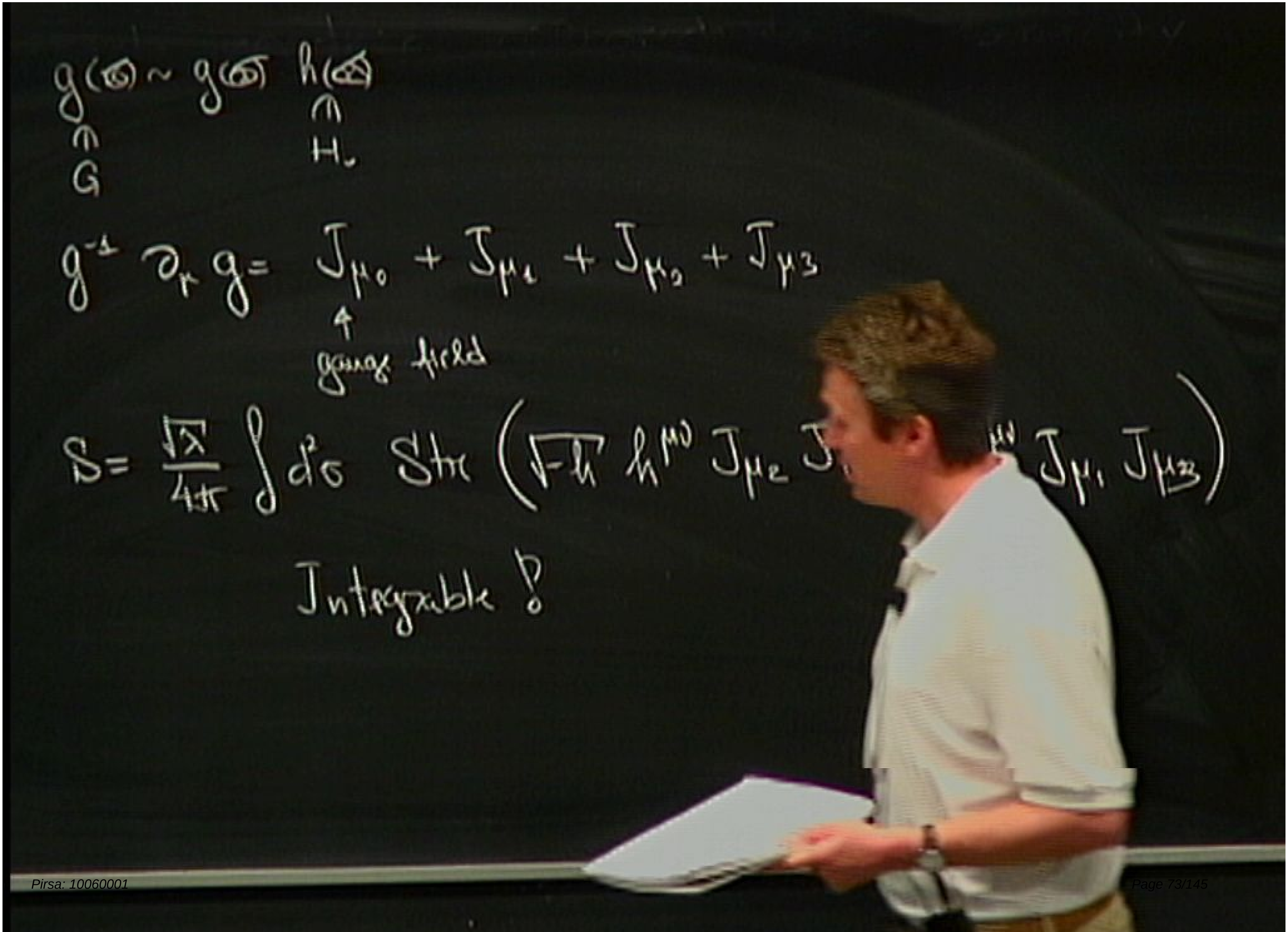

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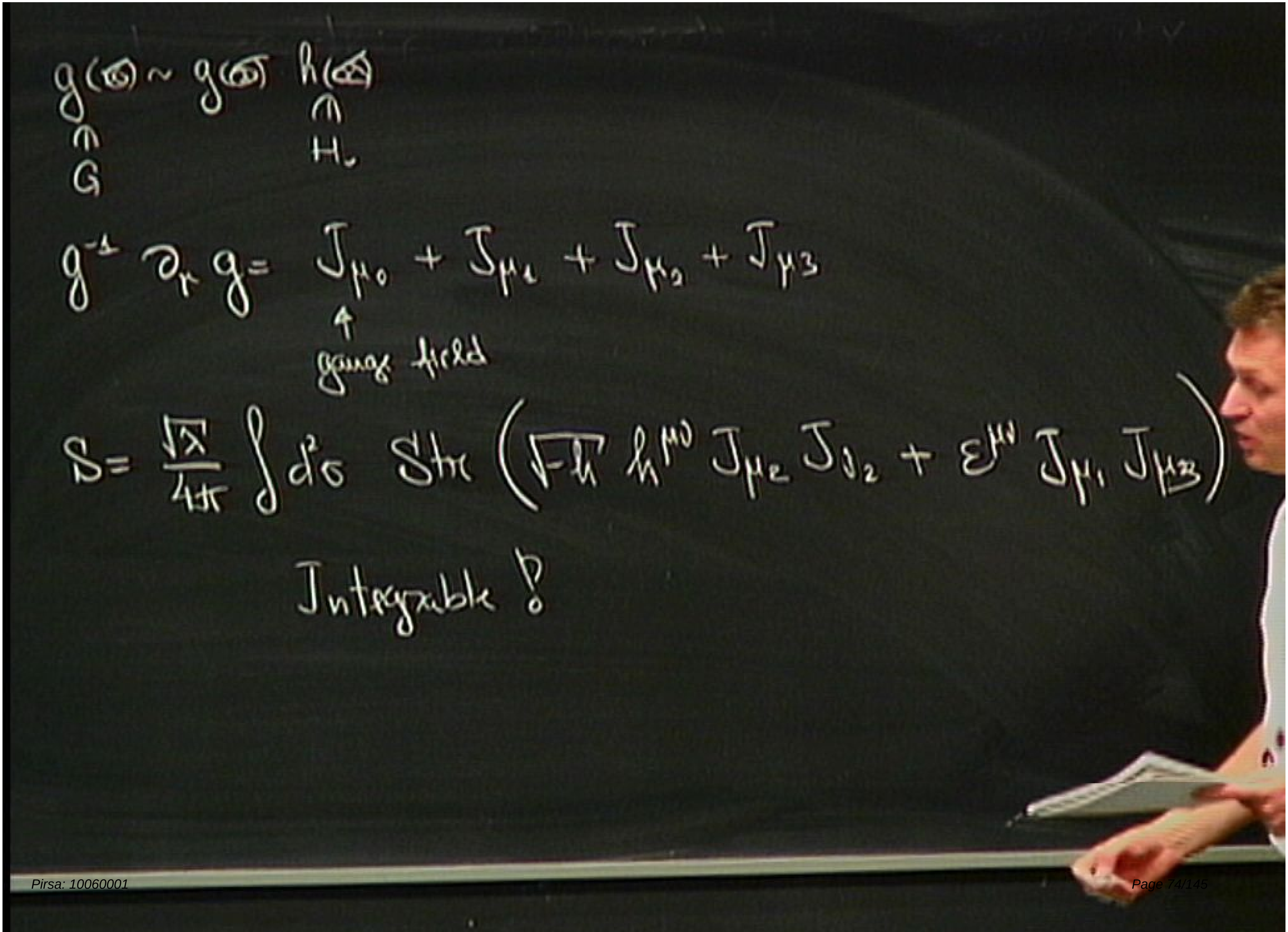
Integrable !



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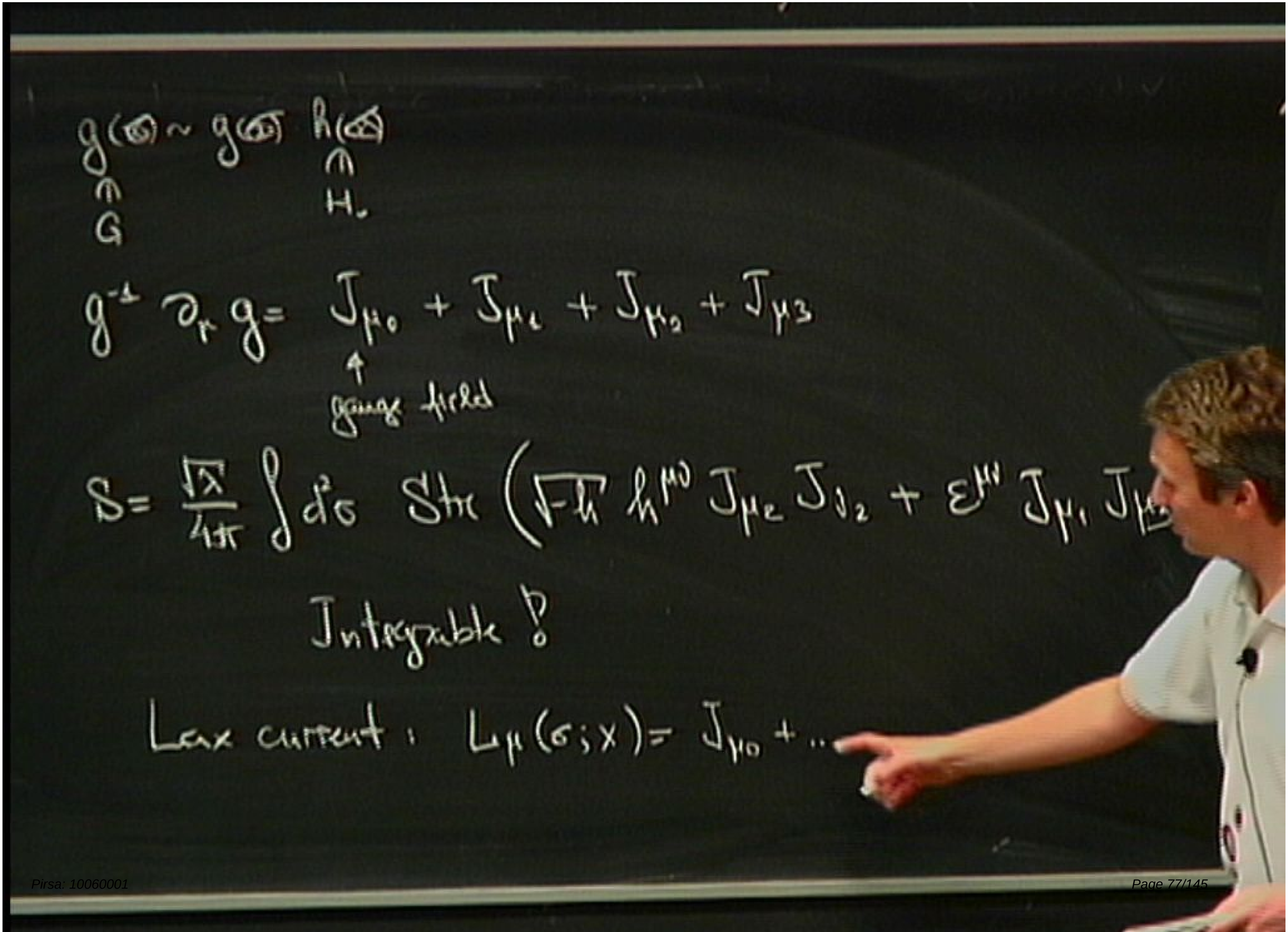

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Integrable ?

Lax current : $L_\mu(\sigma; x) = J_{\mu 0} + \ldots$

$$\begin{array}{lll} g(\sigma) \sim & g(\sigma) & h(\sigma) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ G & & H_0 \end{array}$$

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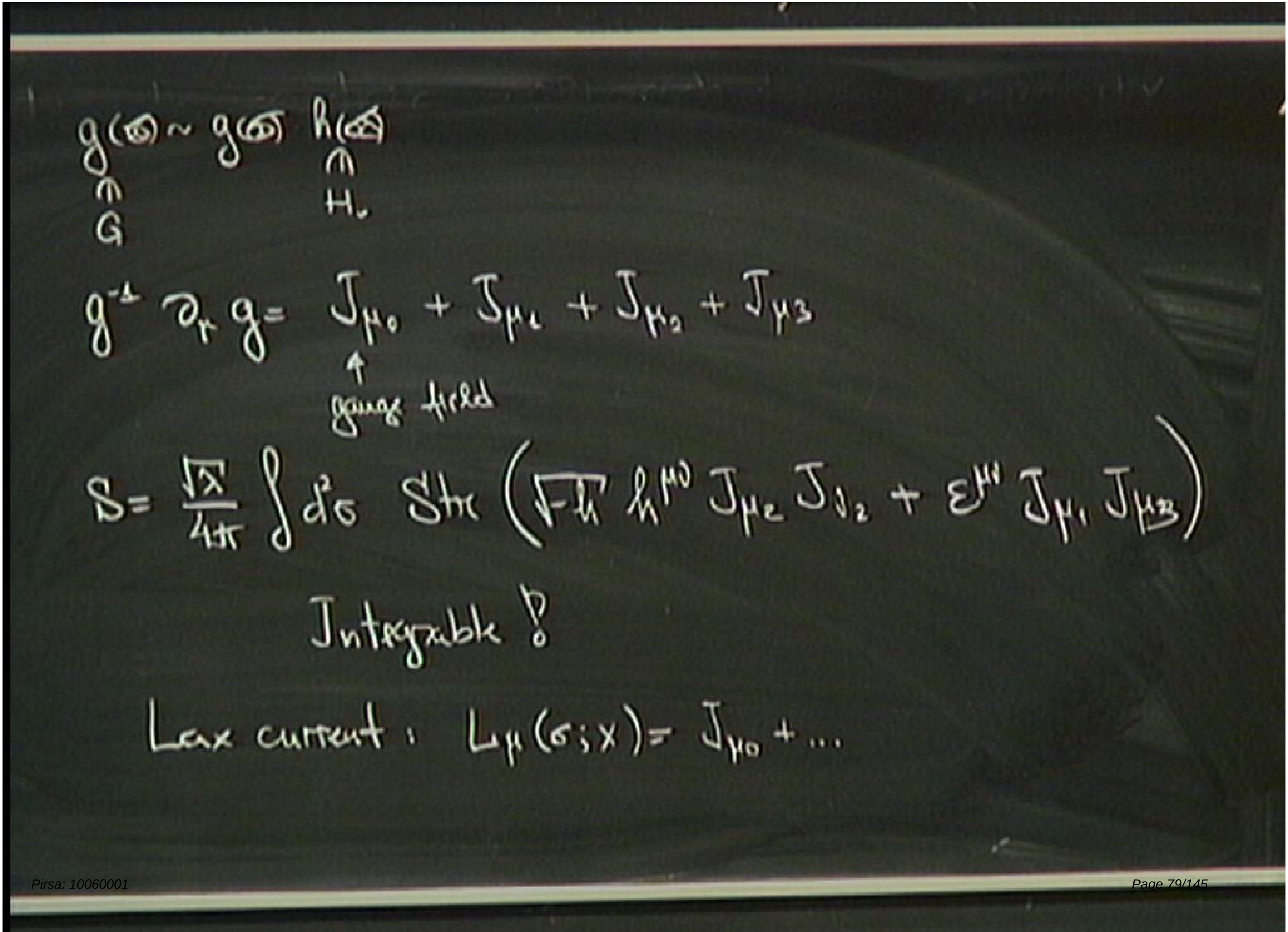

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$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$

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$$\Updownarrow$$

Eqs. of motion

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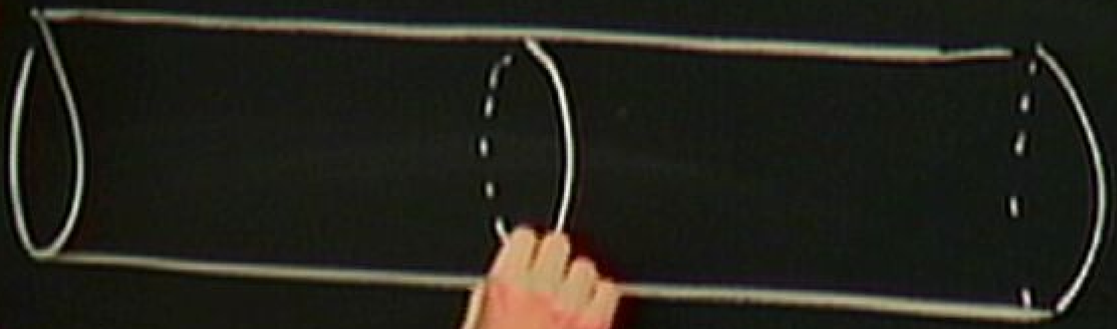
Eqs. of motion



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Eqs. of motion

$$\mathrm{tr}\, P\exp \oint d\sigma^\mu \, L_\mu(\sigma; x)$$

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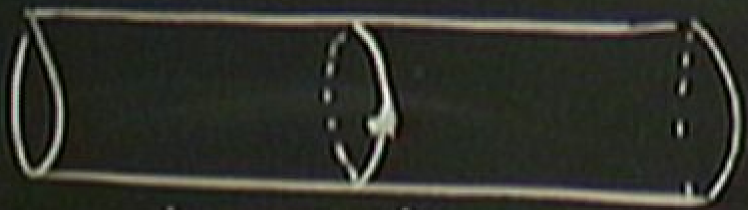
Eqs. of motion



$\mathrm{tr}\, P\exp \oint d\sigma^\mu\, L_\mu(\sigma; x) \Rightarrow \infty$ set of integrals of motion

($WZW_{SU(2), k=2}$, for $AdS_3 \times S^3 \times S^3 \times S^1$

$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$

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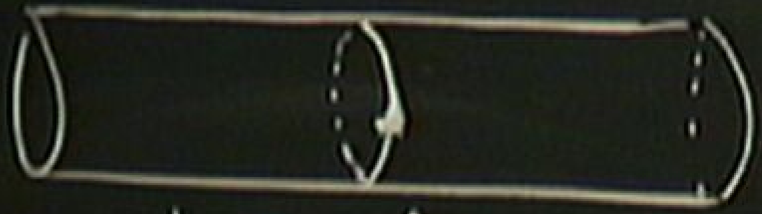
Eqs of motion



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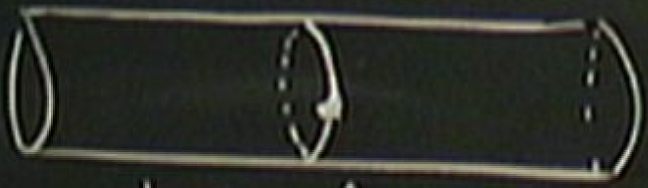
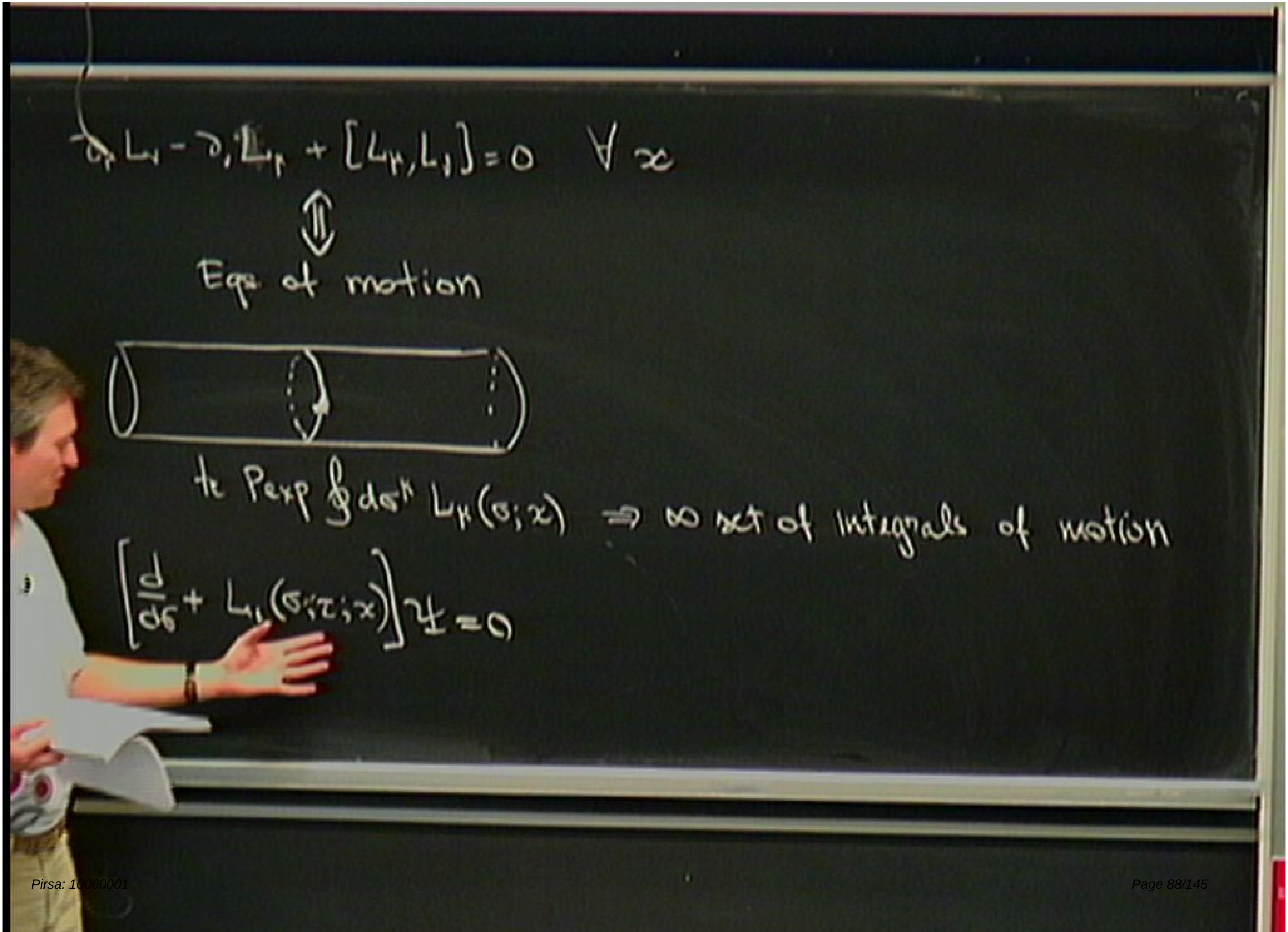
Eqs of motion



$$\mathrm{tr}\, P\exp \oint d\sigma^\mu L_\mu(\sigma; x) \Rightarrow \infty \text{ set of integrals of motion}$$

$$\left[\frac{d}{d\sigma} + L_1(\sigma; \tau; x)\right]\Psi = 0$$



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Eqs of motion

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$$\left[\frac{d}{d\sigma} + L_1(\sigma;\tau;x)\right]\Psi = 0$$

$$\partial_\mu L_1 - \partial_1 L_\mu + [L_\mu, L_1] = 0 \quad \forall x$$

$$\Updownarrow$$

Eqs of motion

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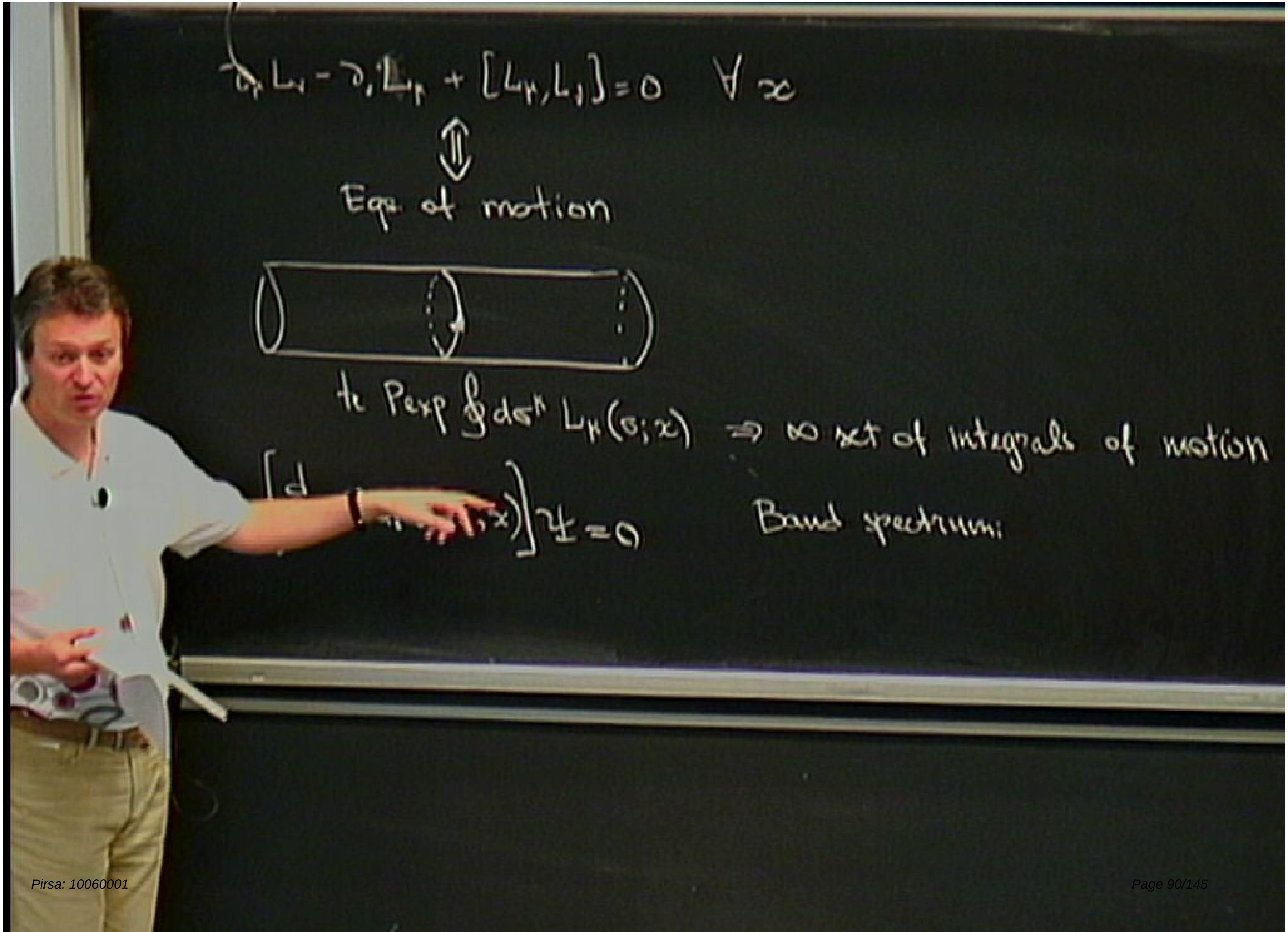

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Eqs. of motion

$\mathrm{tr}\, P\exp \oint d\sigma^\mu L_\mu(\sigma; x)$ ⇒ ∞ set of integrals of motion

$\Big[d$ [illegible] $, x) \Big] \Psi = 0$

Band spectrum:

$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$

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Eqs of motion

$\mathrm{tr}\, P\exp \oint d\sigma^\mu L_\mu(\sigma;x) \Rightarrow \infty$ set of integrals of motion

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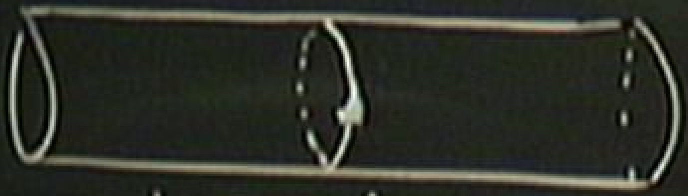
Eqs of motion



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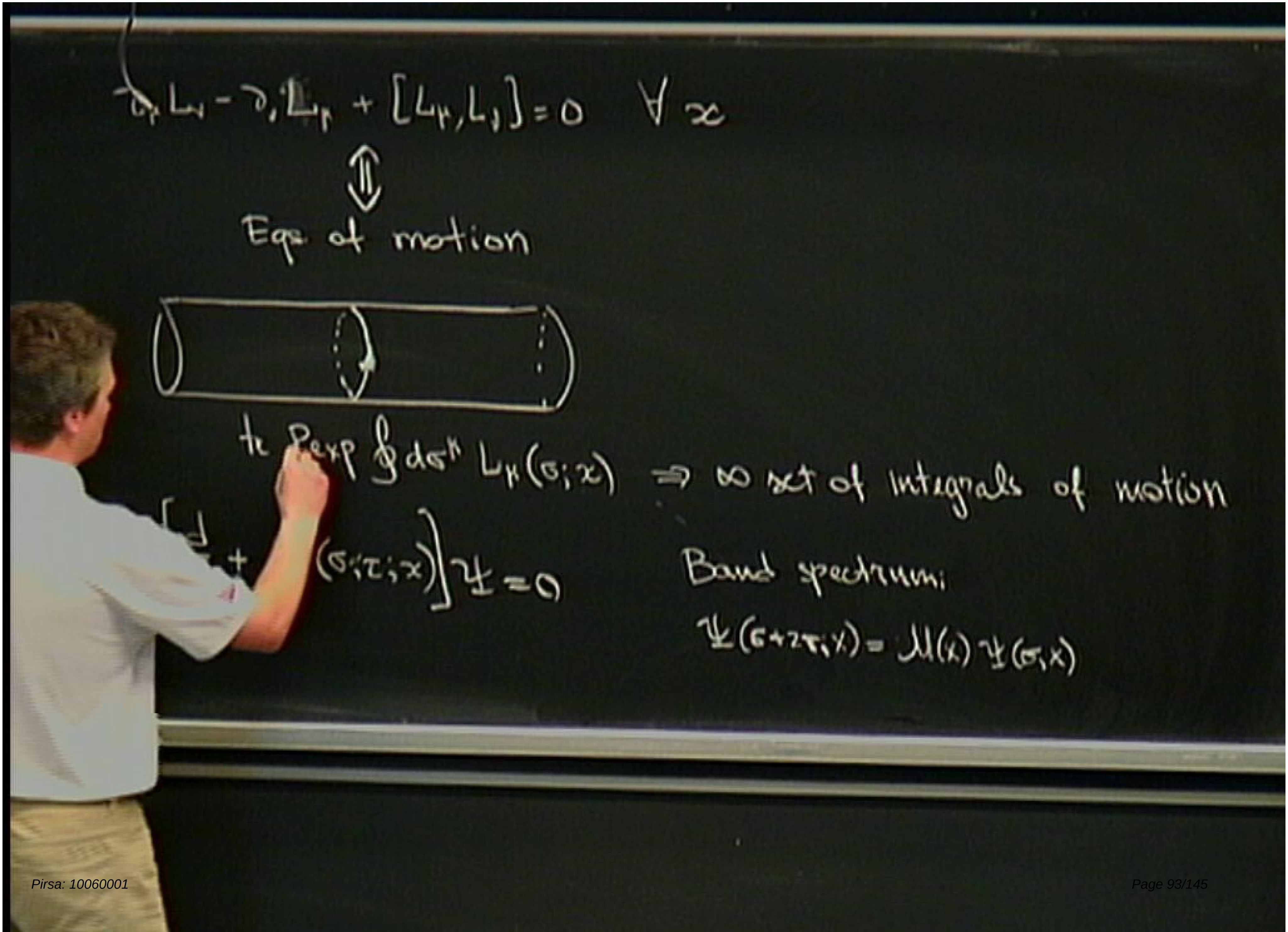
Band spectrum.



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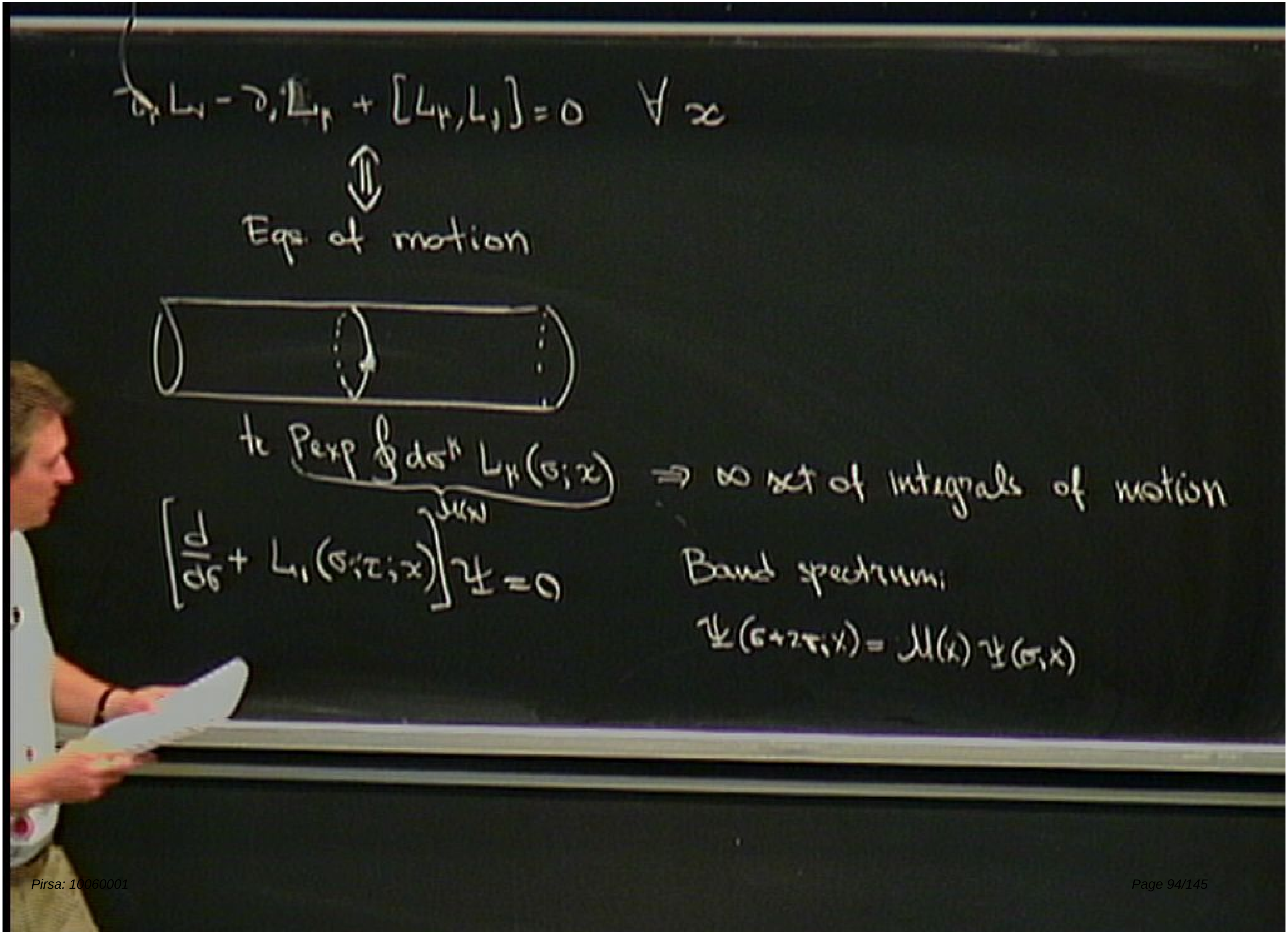
Eqs of motion

$$\mathrm{tr}\, P\exp \oint d\sigma^\mu\, L_\mu(\sigma; x) \Rightarrow \infty \text{ set of integrals of motion}$$

$$\left[\frac{d}{\ } + \quad (\sigma;\tau;x)\right]\Psi = 0$$

Band spectrum:

$$\Psi(\sigma+2\pi; x) = \lambda(x)\,\Psi(\sigma; x)$$



$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$

$$\Updownarrow$$

Eqs of motion

$$\mathrm{tr}\, \underbrace{P\exp \oint d\sigma^\mu L_\mu(\sigma;x)}_{M(x)} \Rightarrow$$ ∞ set of integrals of motion

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$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$

$$\Updownarrow$$

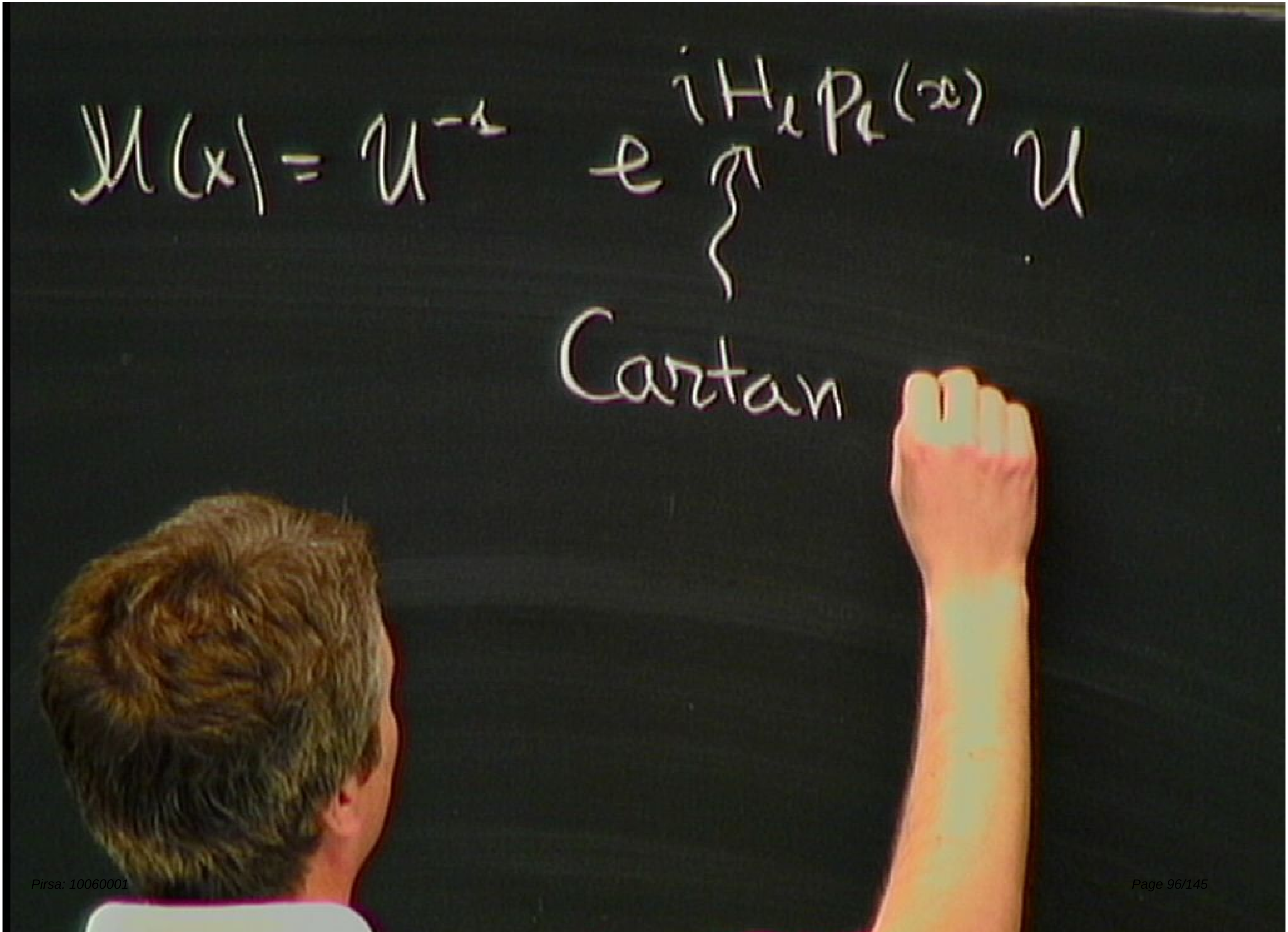
Eqs of motion

$$\mathrm{tr}\, \underbrace{P\exp \oint d\sigma^\mu L_\mu(\sigma;x)}_{\mathcal{M}(x)} \Rightarrow \infty \text{ set of integrals of motion}$$

$$\left[\frac{d}{d\sigma} + L_1(\sigma;\tau;x)\right]\Psi = 0$$

Band spectrum:

$$\Psi(\sigma+2\pi;x) = \mathcal{M}(x)\,\Psi(\sigma,x)$$



$$\mathcal{U}(x) = \mathcal{U}^{-1}\, e^{\underset{\underset{\text{Cartan}}{\uparrow}}{iH_\ell}\, P_\ell(x)}\, \mathcal{U}$$

$$\mathcal{U}(x) = \mathcal{U}^{-1} \, e^{\overset{\substack{\text{Cartan generators}\\ \uparrow}}{i H_\ell} P_\ell(x)} \, \mathcal{U}$$

Cartan generators

$P_\ell(x)$

$$\mathcal{U}(x) = U^{-1}\, e^{i \underset{\uparrow}{H_\ell} p_\ell(x)}\, U$$

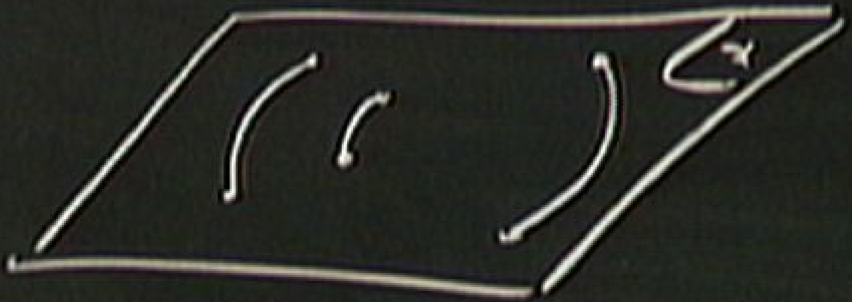
Cartan generators

$p_\ell(x)$ - quasi-momenta

$$\mathcal{M}(x) = \mathcal{U}^{-1}\, e^{i\underset{\uparrow}{H_l} p_l(x)}\, \mathcal{U}$$

Cartan generators

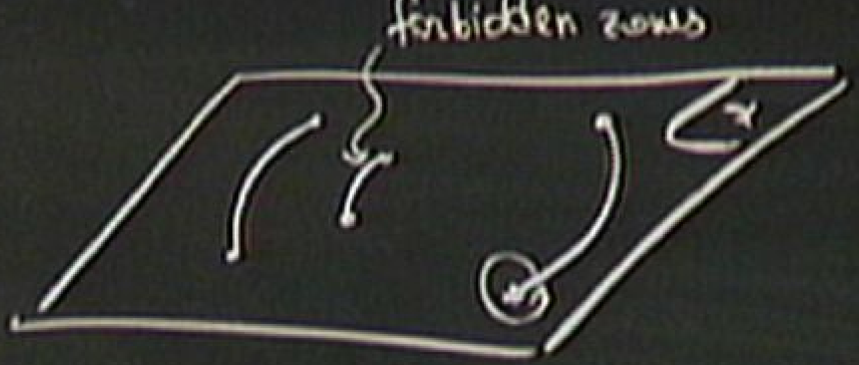
$p_l(x)$ - quasi-momenta

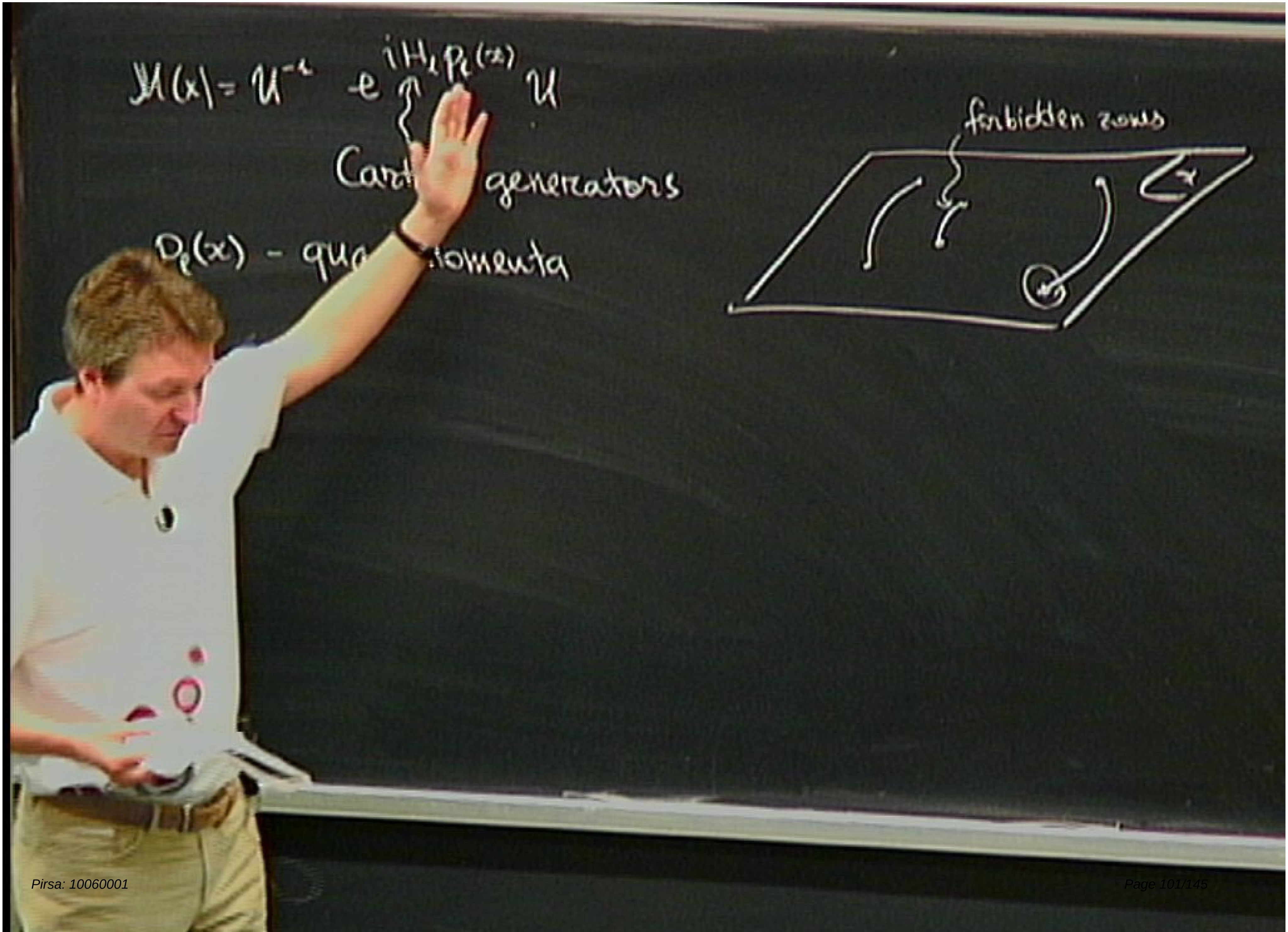


$$\mathcal{U}(x) = \mathcal{U}^{-1}\, e^{i \underset{\substack{\uparrow \\ \wr}}{H_l} p_l(x)}\, \mathcal{U}$$

Cartan generators

$p_l(x)$ - quasi-momenta





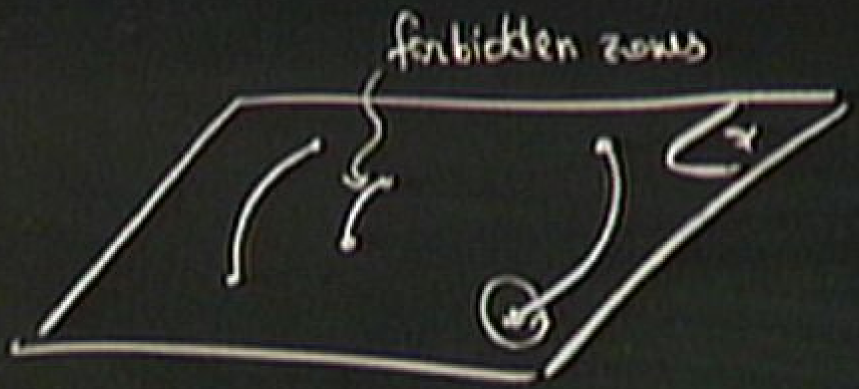
$$\mathcal{M}(x) = U^{-1}\, e^{iH_l p_l(x)}\, U$$

Cart generators

$p_l(x)$ - qua omenta

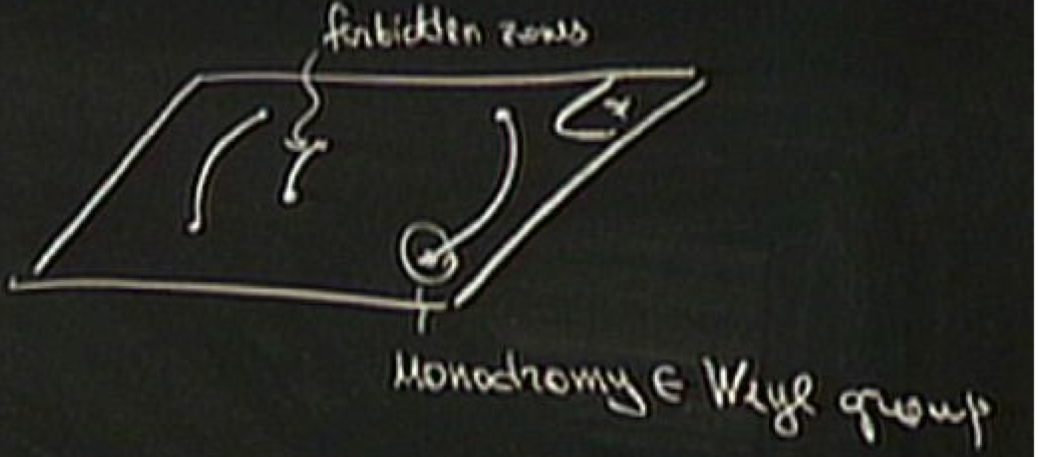
$$\mathcal{M}(x) = U^{-1}\, e^{i \underset{\substack{\uparrow \\ \text{Cartan generators}}}{H_l} p_l(x)}\, U$$

$p_l(x)$ - quasi-momenta



$$\mathcal{U}(x) = \mathcal{U}^{-1}\, e^{i \underset{\substack{\uparrow \\ \text{Cartan generators}}}{H_l}\, p_l(x)}\, \mathcal{U}$$

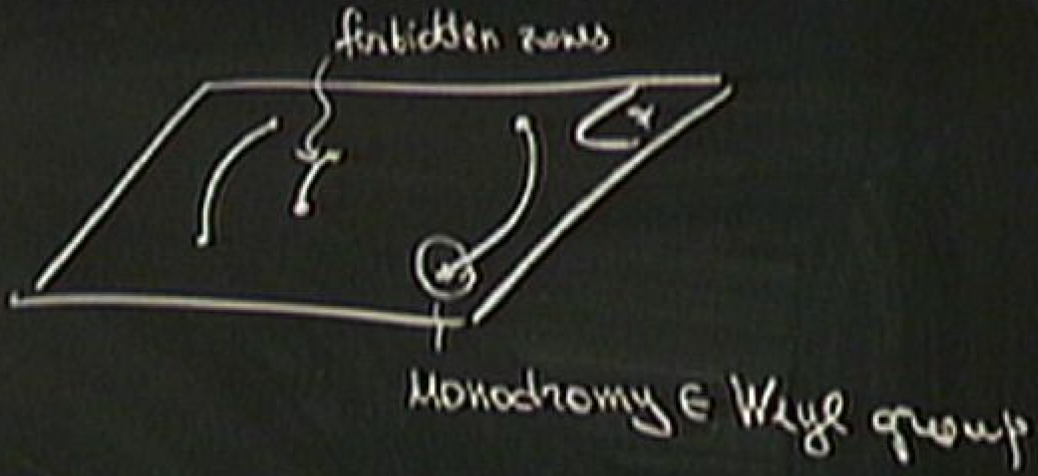
$p_l(x)$ - quasi-momenta

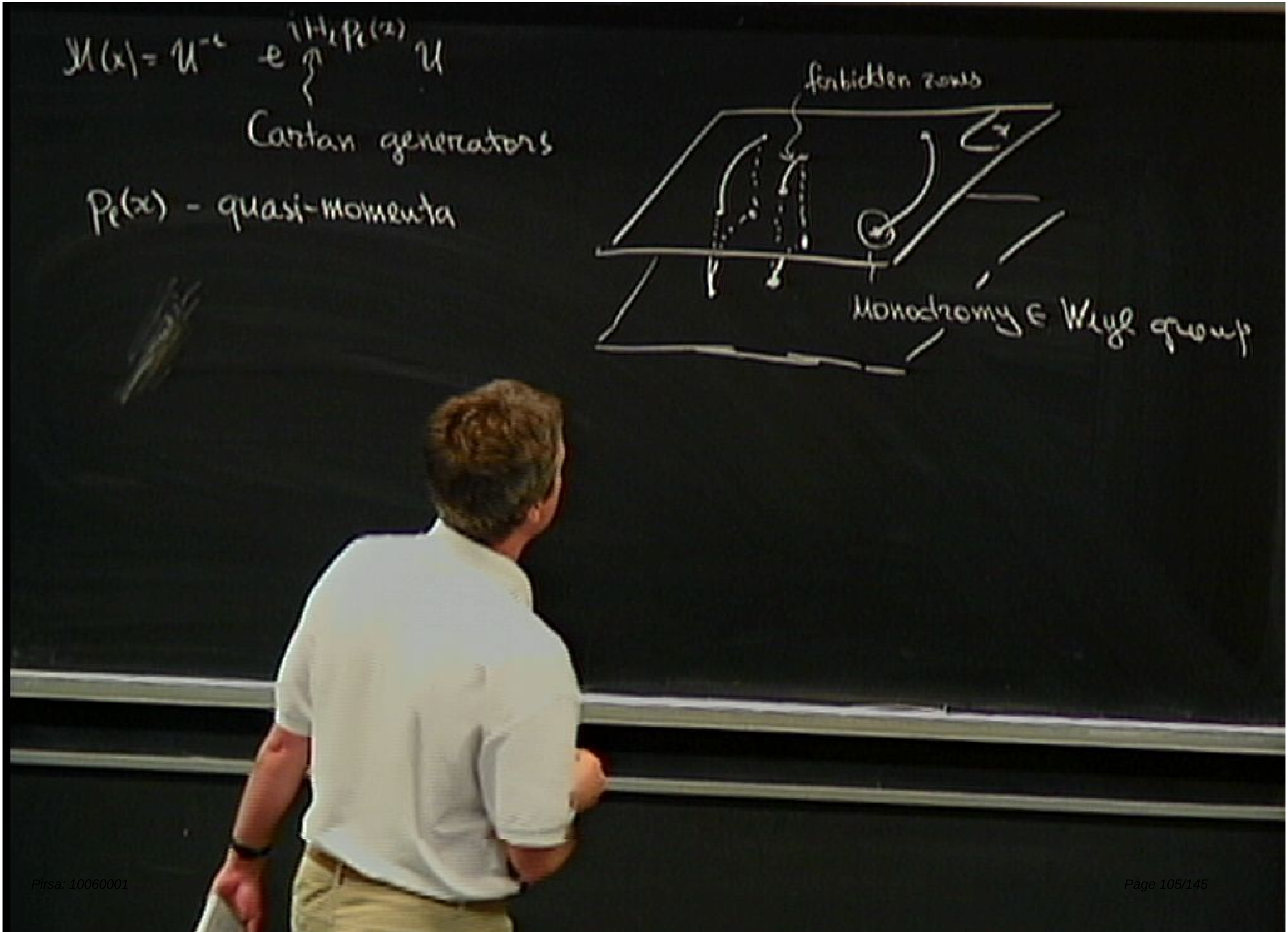


$$\mathcal{U}(x) = \mathcal{U}^{-1}\, e^{i\underset{\uparrow}{H_l} p_l(x)}\, \mathcal{U}$$

Cartan generators

$p_l(x)$ - quasi-momenta



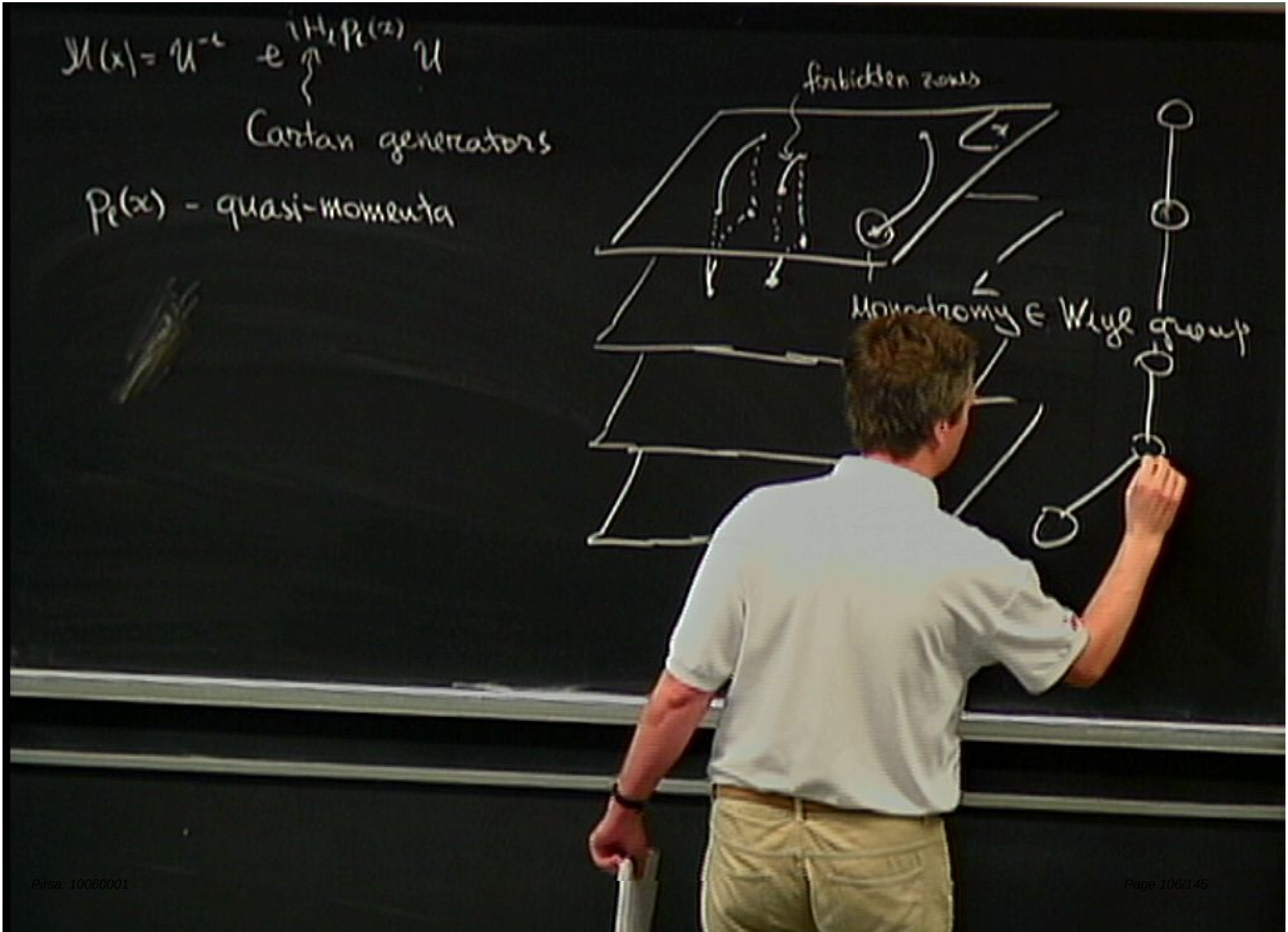


$$\mathcal{M}(x) = U^{-1}\, e^{i H_l p_l(x)}\, U$$

Cartan generators

$p_l(x)$ - quasi-momenta

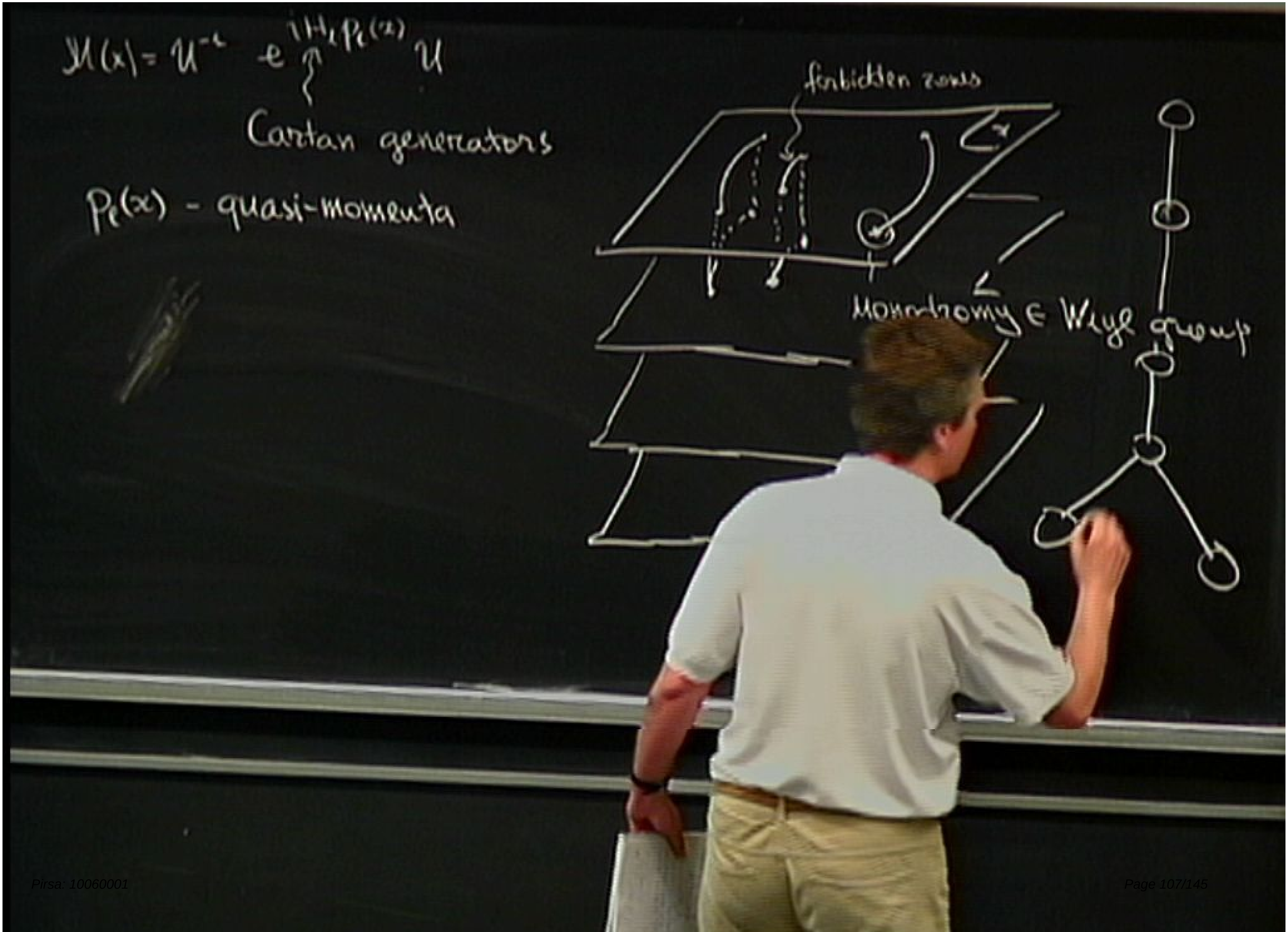
Monodromy ∈ Weyl group



$$\mathcal{U}(x) = U^{-1}\, e^{\overset{\uparrow}{iH_l}\, p_l(x)}\, U$$

Cartan generators

$p_l(x)$ - quasi-momenta

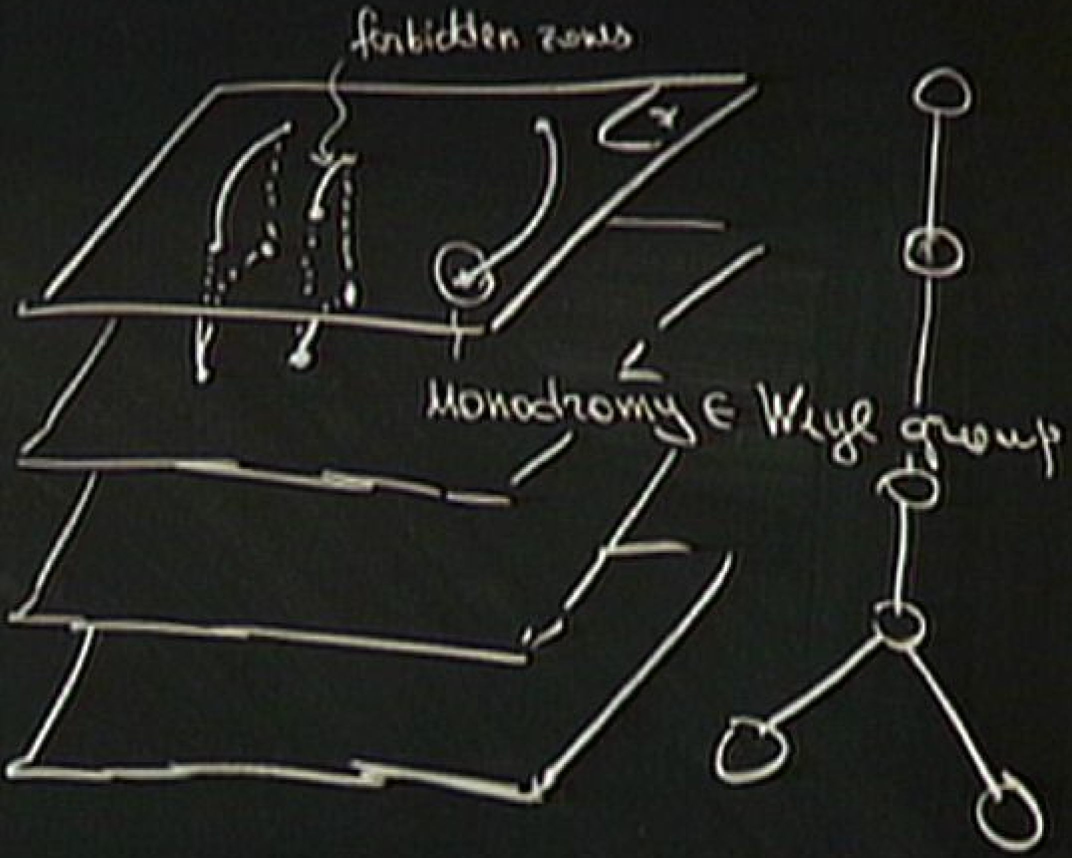


$$\mathcal{U}(x) = U^{-1}\, e^{i\underbrace{H_\ell}_{\text{Cartan generators}} p_\ell(x)}\, U$$

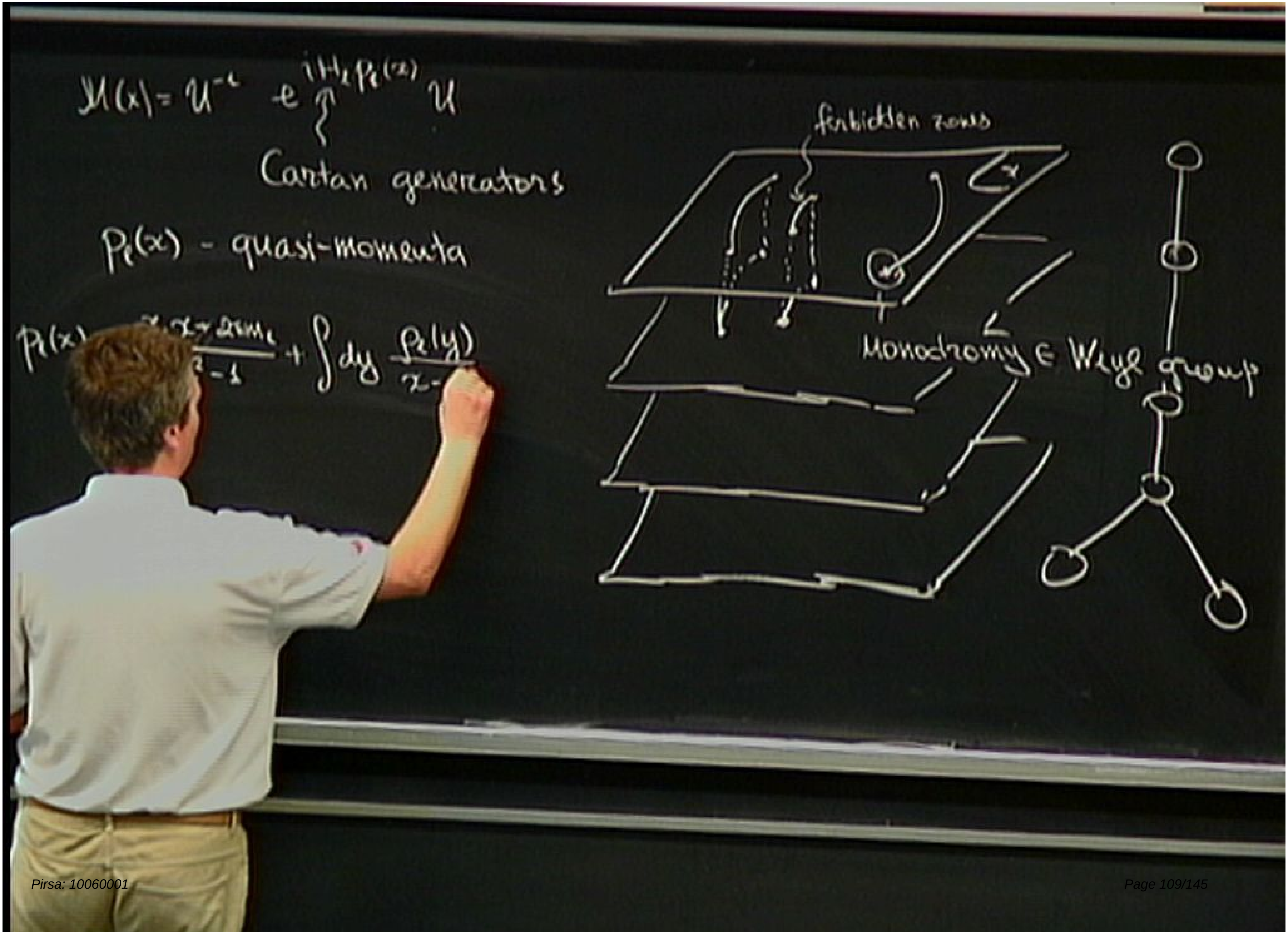
$p_\ell(x)$ - quasi-momenta

$$\mathcal{U}(x) = \mathcal{U}^{-1} \, e^{i \underbrace{H_l}_{\text{Cartan generators}} p_l(x)} \, \mathcal{U}$$

$p_l(x)$ - quasi-momenta



Monodromy ∈ Weyl group



$$\mathcal{M}(x) = \mathcal{U}^{-1}\, e^{iH_\ell p_\ell(x)}\, \mathcal{U}$$

Cartan generators

$p_\ell(x)$ - quasi-momenta

$$p_\ell(x) \quad \frac{x + 2\pi m_\ell}{-1} + \int dy\, \frac{\rho_\ell(y)}{x-}$$

forbidden zones

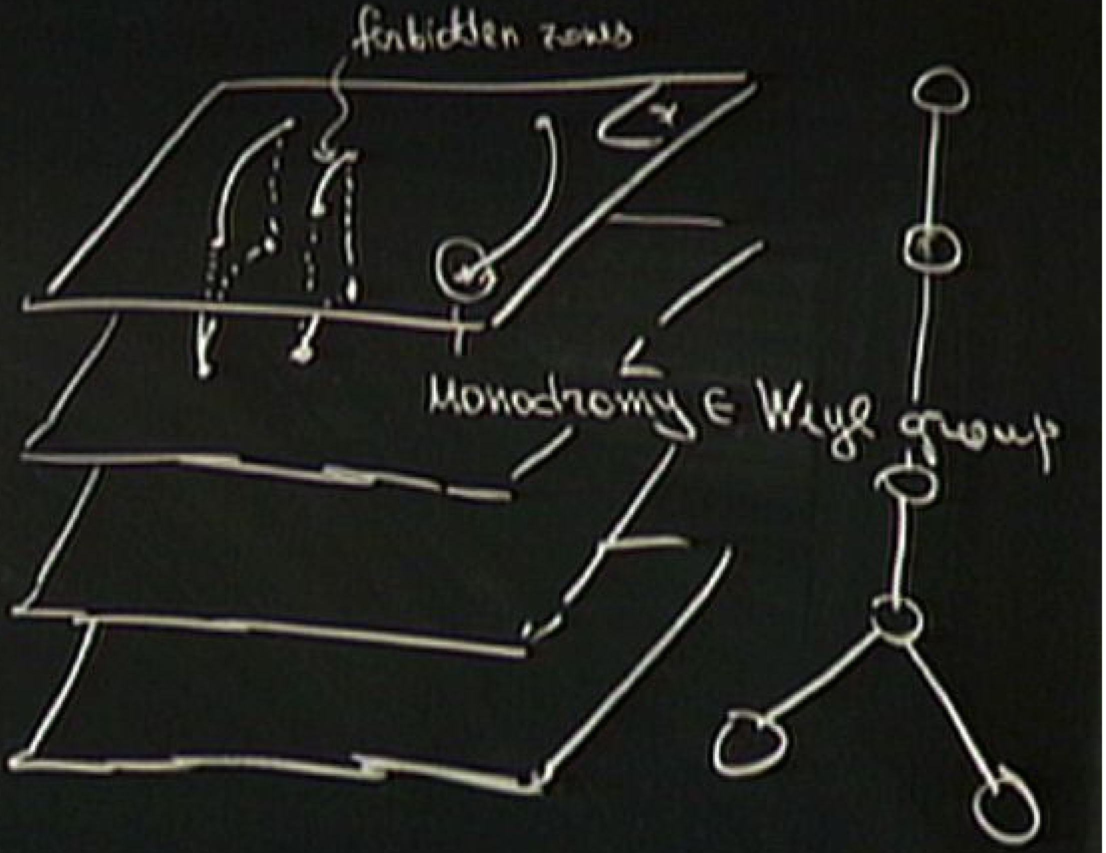
Monodromy ∈ Weyl group

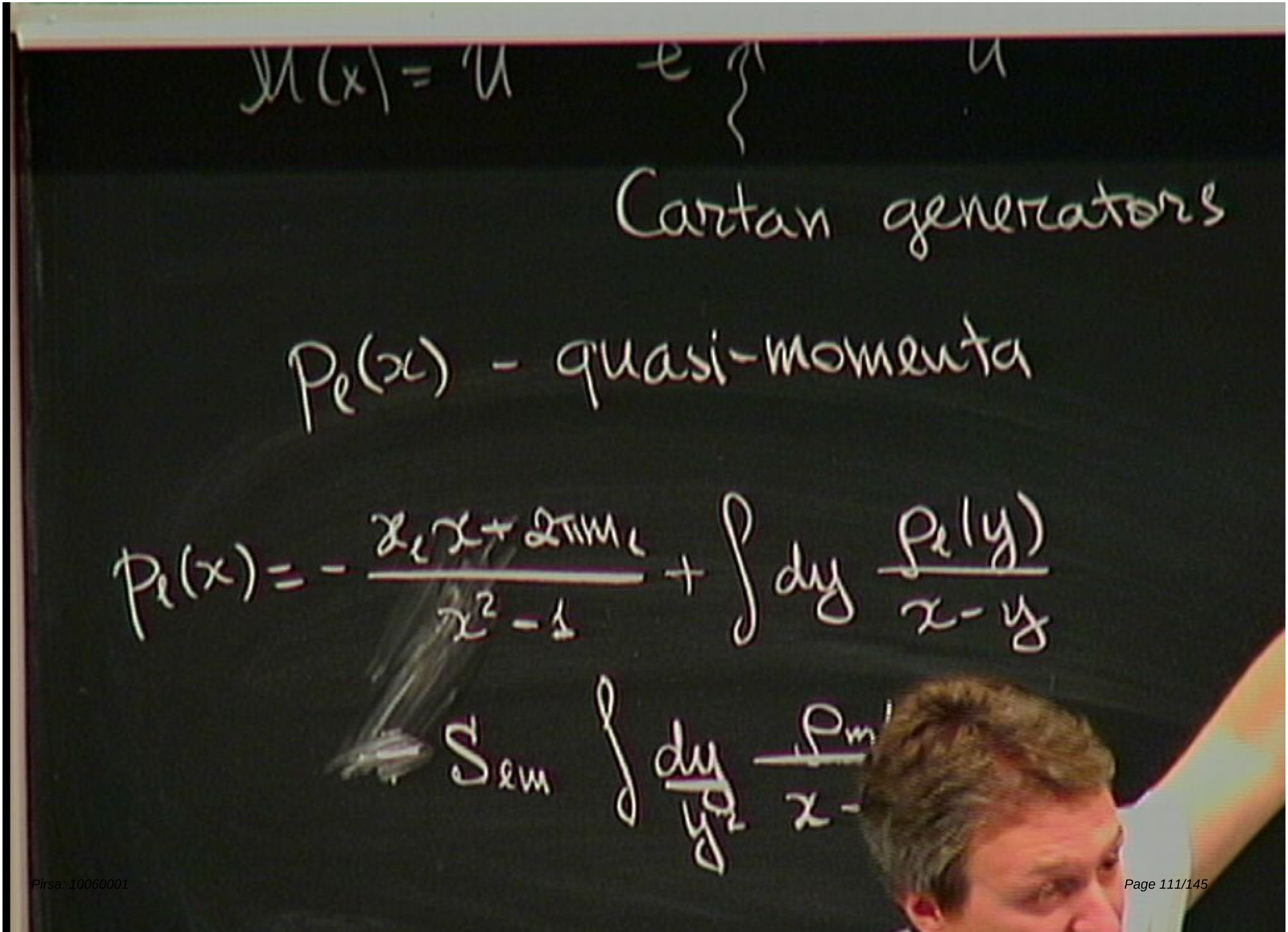
$$\mathcal{M}(x) = \mathcal{U}^{-1}\, e^{\overset{\uparrow}{i H_l} p_l(x)}\, \mathcal{U}$$

Cartan generators

$p_l(x)$ - quasi-momenta

$$p_l(x) = -\frac{\kappa_l x + 2\pi m_l}{x^2 - 1} + \int dy \frac{\rho_l(y)}{x - y}$$





$$\mathcal{U}(x) = u \quad e \quad u$$

Cartan generators

$p_\ell(x)$ - quasi-momenta

$$p_\ell(x) = -\frac{\varkappa_\ell x + 2\pi m_\ell}{x^2-1} + \int dy \, \frac{\rho_\ell(y)}{x-y}$$

$$S_{\ell m} \int \frac{dy}{y^2} \frac{\rho_m}{x-}$$

$$\mathfrak{M}(x) = U \quad e \quad U$$

Cartan generators

$p_l(x)$ - quasi-momenta

$$p_l(x) = -\frac{\varkappa_l x + 2\pi m_l}{x^2 - 1} + \int dy \, \frac{\rho_l(y)}{x - y}$$

$$+ S_{lm} \int \frac{dy}{y^2} \, \frac{\rho_m(y)}{x - \frac{1}{y}}$$

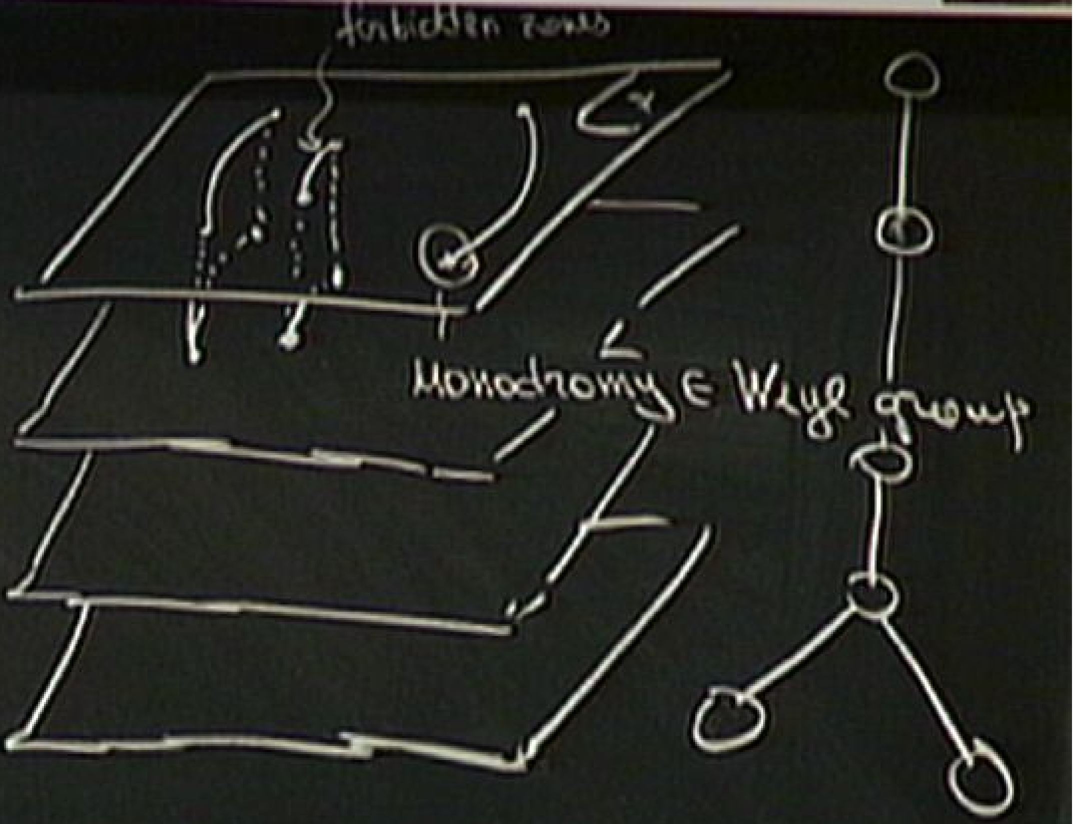
$\mathcal{U}(x) = u \quad e \quad u$

Cartan generators

$p_l(x)$ - quasi-momenta

$$p_l(x) = -\frac{x_l x + 2\pi m_l}{x^2-1} + \int dy \frac{\rho_l(y)}{x-y}$$

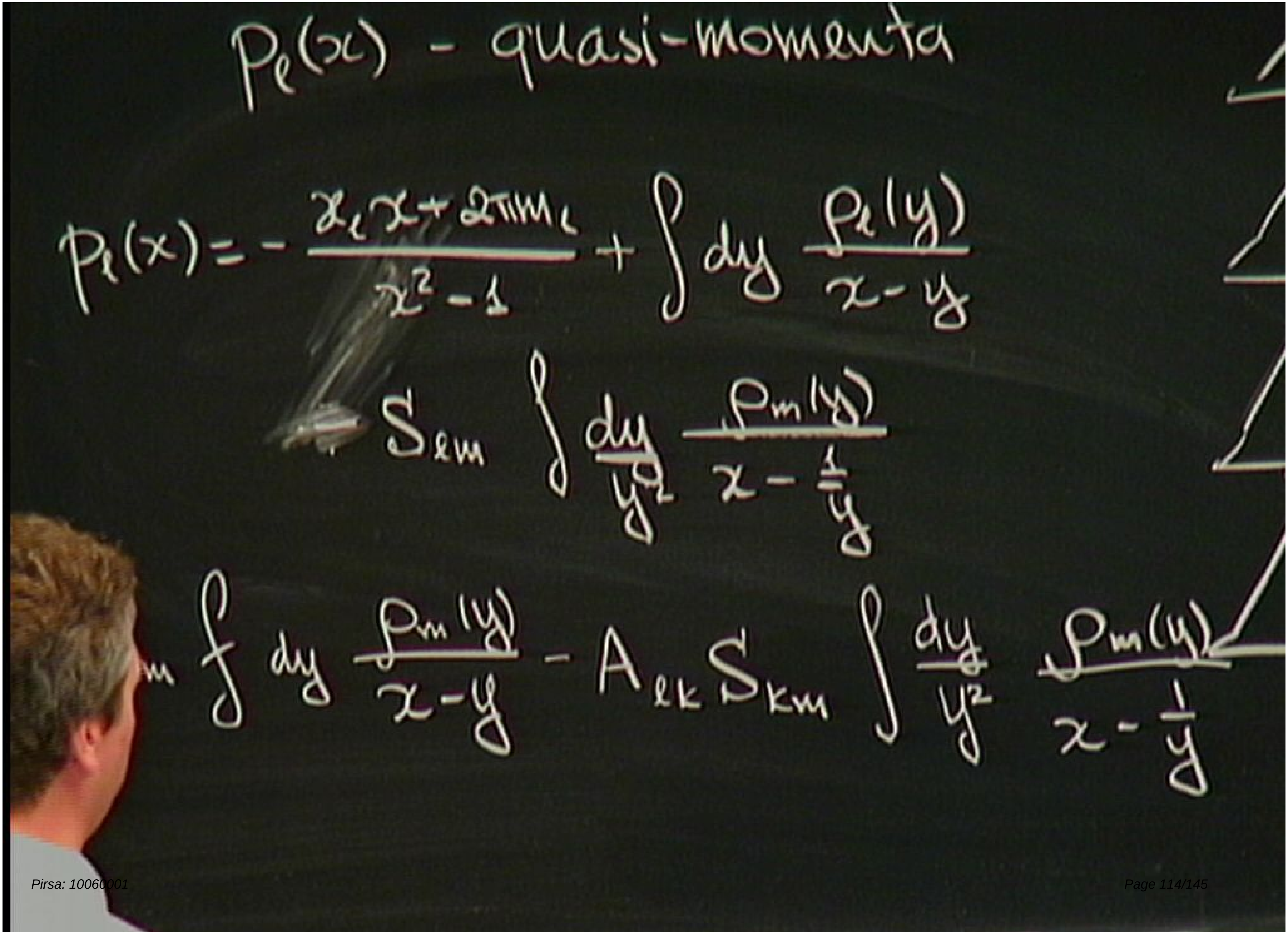
$$S_{lm} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$



Monodromy ∈ Weyl group

PSU(1,1|2) × PSU(1,1|2)

OSp(4|2) × OSp(4|2)



$p_\ell(x)$ - quasi-momenta

$$p_\ell(x) = -\frac{\varkappa_\ell x + 2\pi m_\ell}{x^2-1} + \int dy \, \frac{\rho_\ell(y)}{x-y}$$

$$S_{\ell m} \int \frac{dy}{y^2} \, \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$$_{m} \int dy \, \frac{\rho_m(y)}{x-y} - A_{\ell k} S_{km} \int \frac{dy}{y^2} \, \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$p_l(x)$ - quasi-momenta

$$p_l(x) = -\frac{\varkappa_l x + 2\pi i m_l}{x^2 - 1} + \int dy \, \frac{\rho_l(y)}{x - y}$$

$$- S_{lm} \int \frac{dy}{y^2} \, \frac{\rho_m(y)}{x - \frac{1}{y}}$$

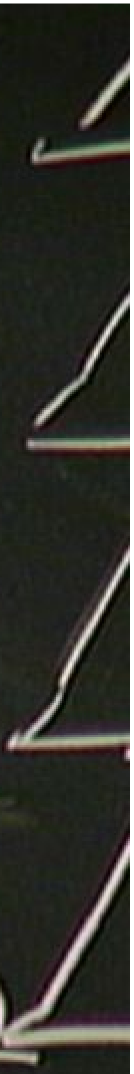
$$A_{lm} \fint dy \, \frac{\rho_m(y)}{x - y} - A_{lk} S_{km} \int \frac{dy}{y^2} \, \frac{\rho_m(y)}{x - \frac{1}{y}}$$

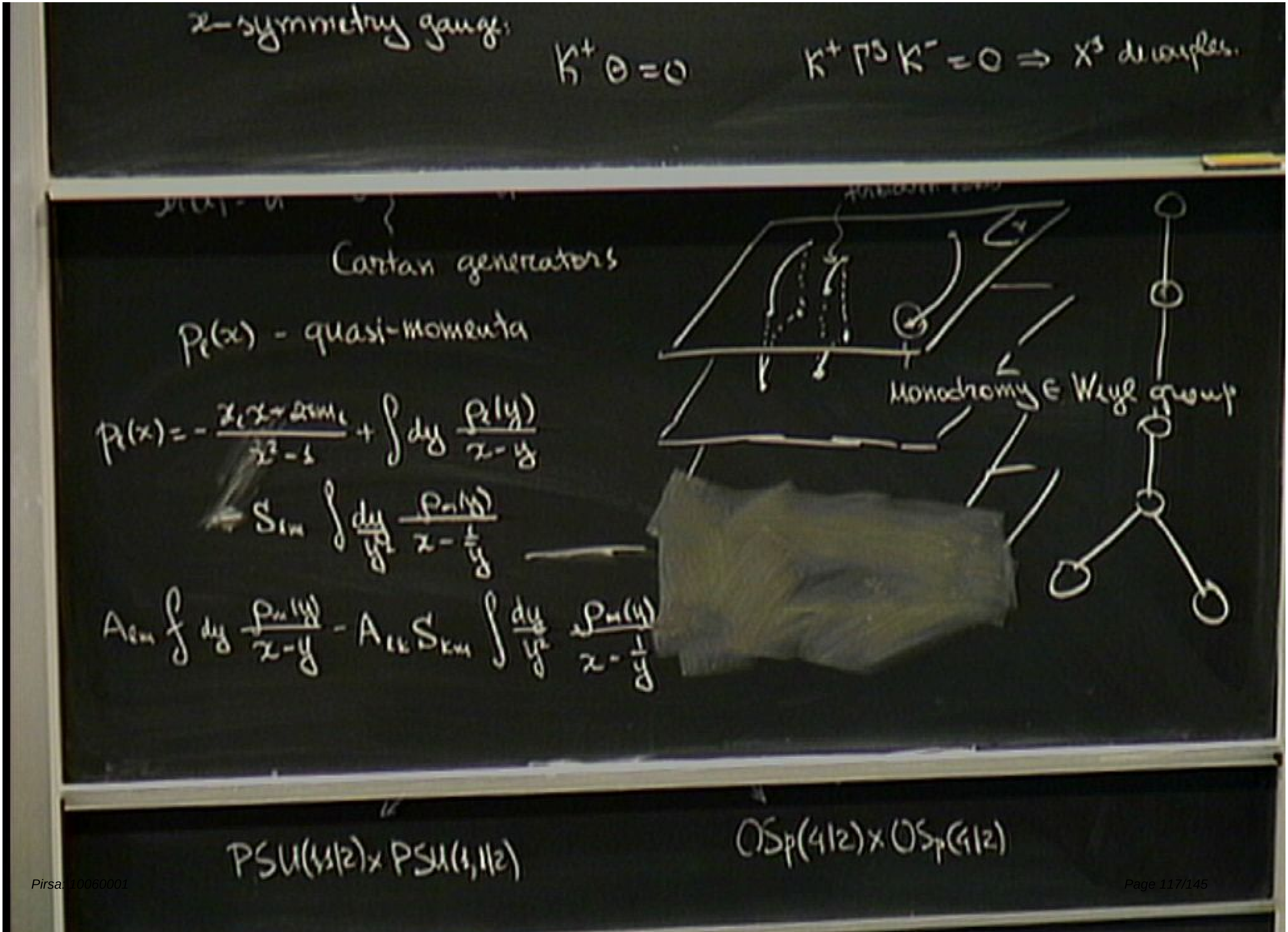
$P_\ell(x)$ - quasi-momenta

$$P_\ell(x) = -\frac{\varkappa_\ell x + 2\pi m_\ell}{x^2-1} + \int dy \, \frac{\rho_\ell(y)}{x-y}$$

$$S_{\ell m} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$$A_{\ell m} \fint dy \, \frac{\rho_m(y)}{x-y} - A_{\ell k} S_{km} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$





x-symmetry gauge: $K^+\Theta=0$ $K^+\Gamma^3K^-=0 \Rightarrow X^3$ decouples.

Cartan generators

$p_l(x)$ - quasi-momenta

$$p_l(x) = -\frac{\varkappa_l x + 2\pi m_l}{x^2-1} + \int dy \frac{\rho_l(y)}{x-y}$$

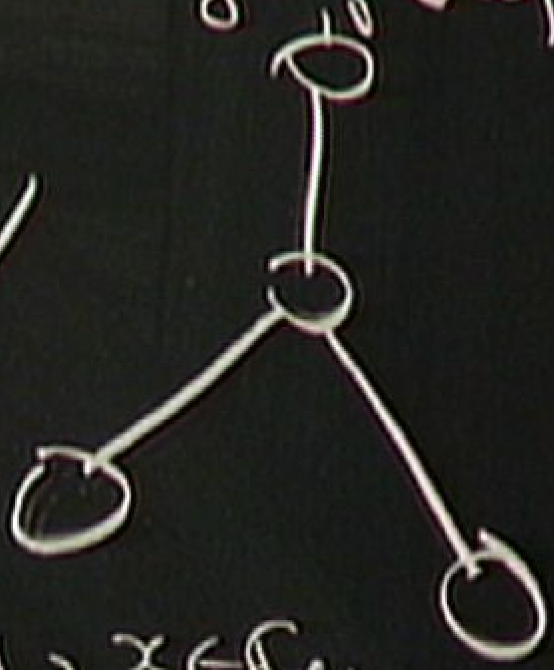
$$+ S_{lm} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$$A_{lm} \fint dy \frac{\rho_m(y)}{x-y} - A_{lk} S_{km} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$

Monodromy ∈ Weyl group

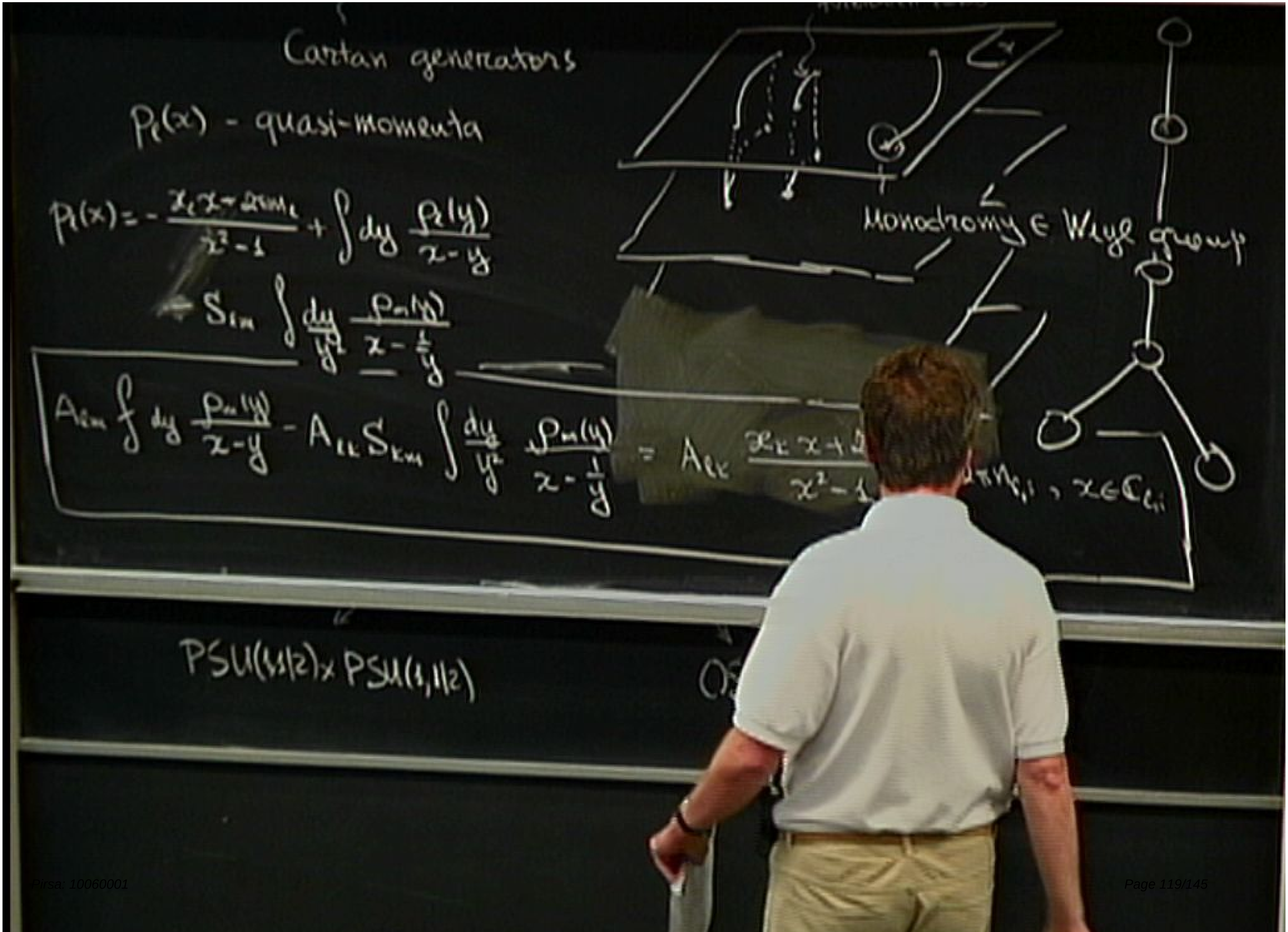
$PSU(2,2|4)$ $PSU(1,1|2)$ $OSp(4|2) \times OSp(4|2)$

Monodromy ∈ Weyl group



$$\frac{\rho_m(y)}{x-\frac{1}{y}} = A_{\ell k}\frac{\varkappa_k x + 2\pi m_k}{x^2-1} + 2\pi n_{\ell,i}\ , \quad x \in \mathcal{C}_{\ell,i}$$

$OSp(4|2) \times OSp(4|2)$



Cartan generators

$p_\ell(x)$ - quasi-momenta

$$p_\ell(x) = -\frac{\mathcal{X}_\ell x - 2\pi m_\ell}{x^2-1} + \int dy \frac{\rho_\ell(y)}{x-y}$$

$$+ S_{\ell m} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$$\boxed{A_{\ell m} \fint dy \frac{\rho_m(y)}{x-y} - A_{\ell k} S_{km} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}} = A_{\ell k} \frac{\mathcal{X}_k x + 2}{x^2-1} \quad \pi n_{\ell,i} \,,\; x \in \mathcal{C}_{\ell,i}}$$

Monodromy ∈ Weyl group

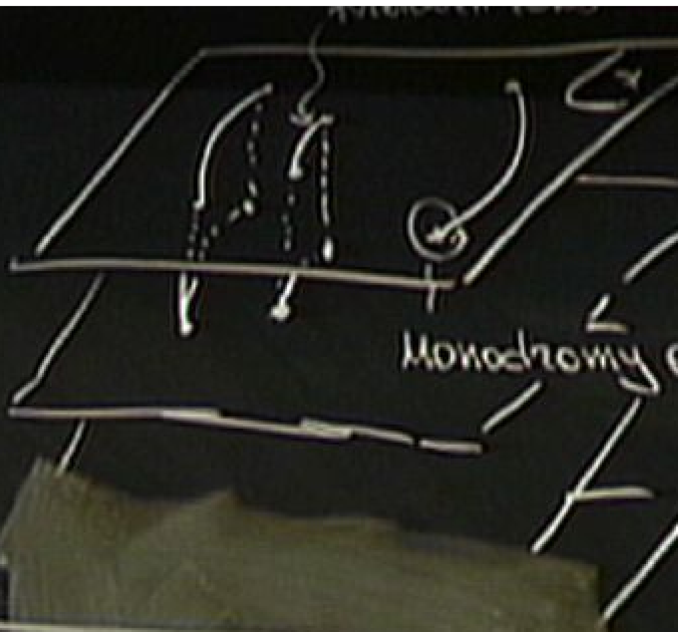
$PSU(1,1|2) \times PSU(1,1|2)$

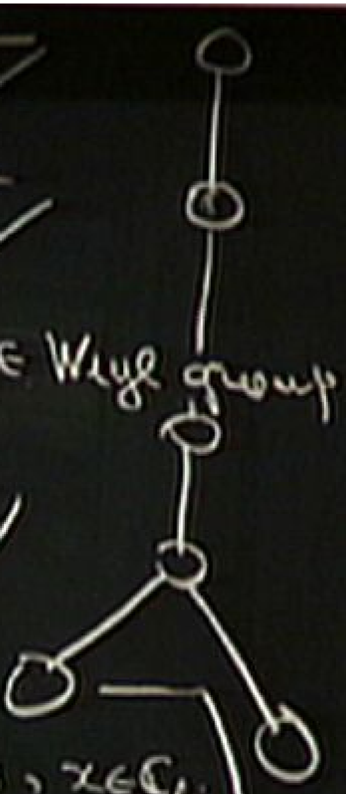
OS

Cartan generators

$p_l(x)$ - quasi-momenta

$$p_l(x) = -\frac{x_l x + 2\pi m_l}{x^2-1} + \int dy \, \frac{\rho_l(y)}{x-y} + S_{lm} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$

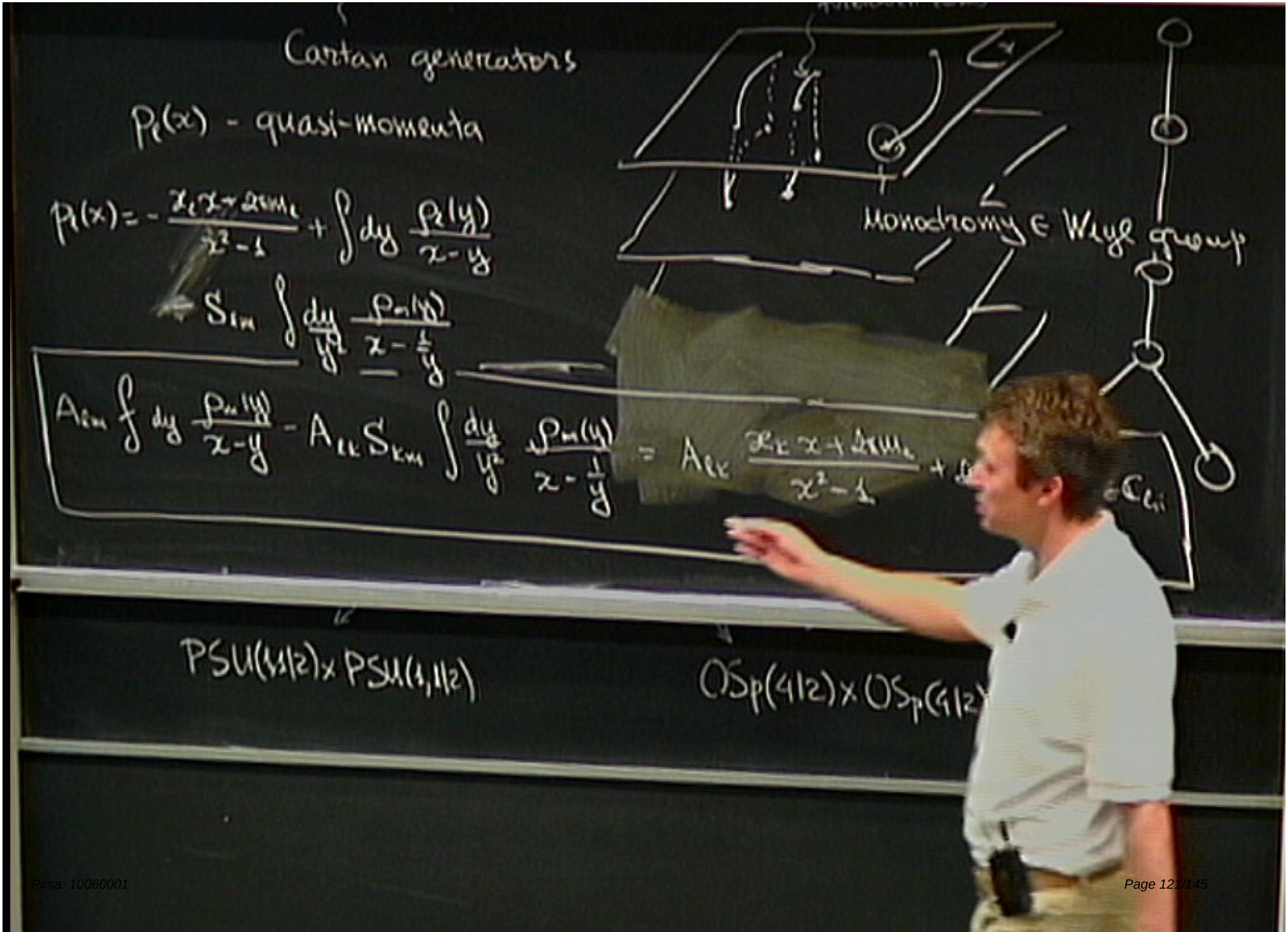

Monodromy ∈ Weyl group



$$\boxed{A_{lm} \fint dy \, \frac{\rho_m(y)}{x-y} - A_{ll} S_{lm} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}} = A_{ll} \frac{x_l x + 2\pi m_l}{x^2-1} + 2\pi n_{l,i} \, , \quad x \in \mathcal{C}_{l,i}}$$

PSU(1,1|2)×PSU(1,1|2)

OSp(4|2)×OSp(4|2)



Cartan generators

$p_\ell(x)$ - quasi-momenta

$$p_\ell(x) = -\frac{\varkappa_\ell x + 2\pi m_\ell}{x^2-1} + \int dy \frac{\rho_\ell(y)}{x-y}$$

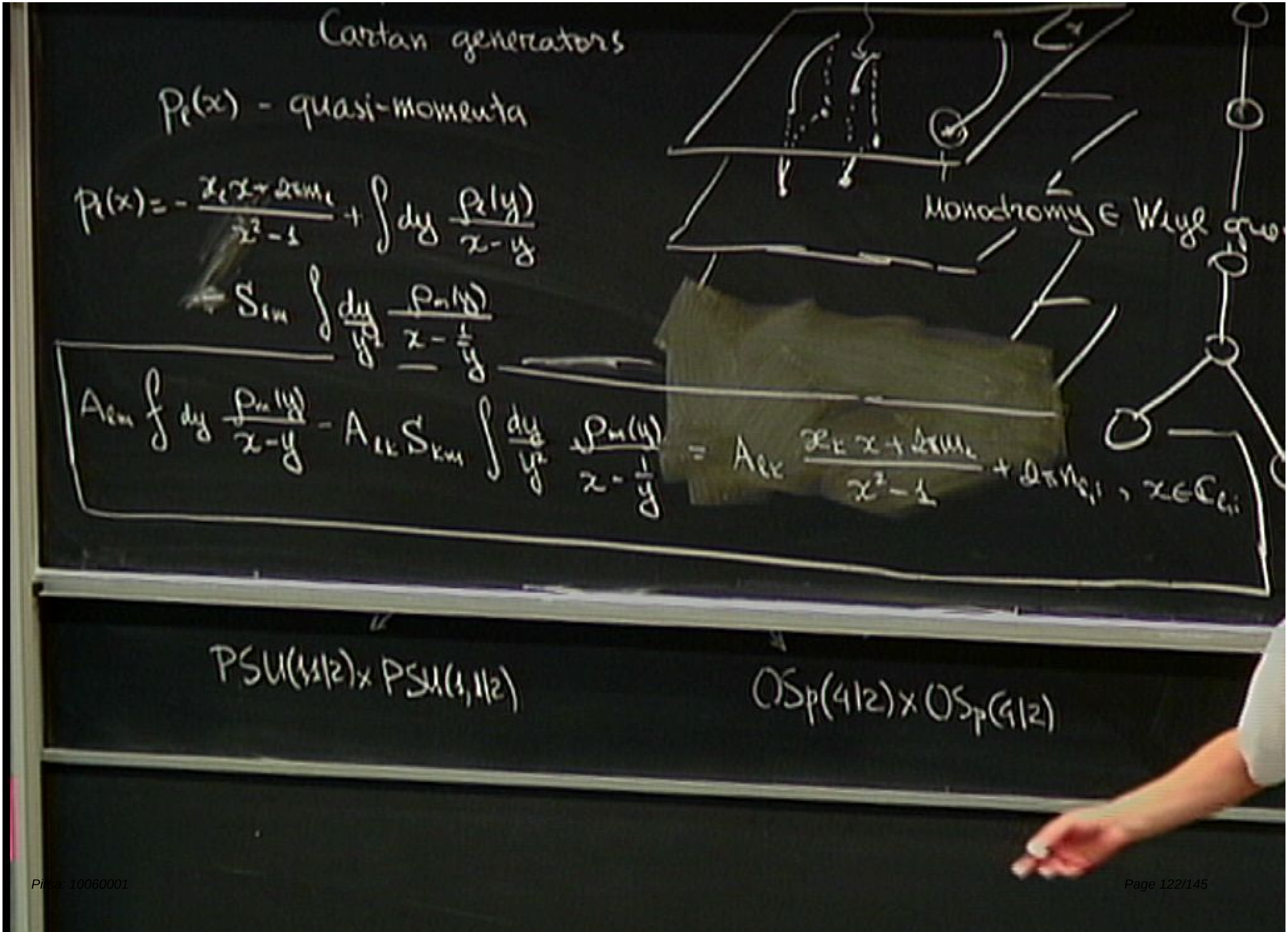
$$S_{\ell m} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$$\boxed{A_{\ell m} \int dy \frac{\rho_m(y)}{x-y} - A_{\ell\ell} S_{\ell m} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}} = A_{\ell\ell} \frac{\varkappa_\ell x + 2\pi m_\ell}{x^2-1} + \quad \mathcal{C}_{\ell i}}$$

Monodromy ∈ Weyl group

PSU(1,1|2)×PSU(1,1|2)

OSp(4|2)×OSp(4|2



Cartan generators

$p_l(x)$ - quasi-momenta

$$p_l(x) = -\frac{\varkappa_l x + 2\pi m_l}{x^2-1} + \int dy \, \frac{\rho_l(y)}{x-y} + S_{lm} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$$\boxed{A_{lm} \fint dy \, \frac{\rho_m(y)}{x-y} - A_{lk} S_{km} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}} = A_{lk} \frac{\varkappa_k x + 2\pi m_k}{x^2-1} + 2\pi n_{l,i} \, , \; x \in \mathcal{C}_{l,i}}$$

Monodromy ∈ Weyl gro

$PSU(1,1|2) \times PSU(1,1|2)$

$OSp(4|2) \times OSp(4|2)$

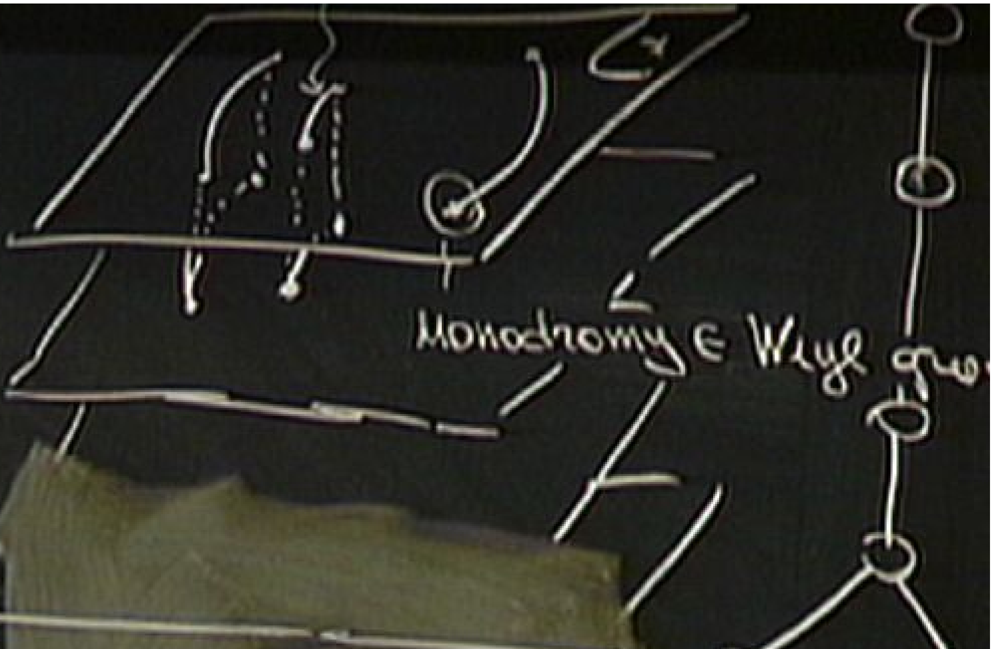
Cartan generators

$p_l(x)$ - quasi-momenta

$$p_l(x) = -\frac{x_l x + 2\pi m_l}{x^2-1} + \int dy \frac{\rho_l(y)}{x-y}$$

$$S_{lm} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$$\boxed{A_{lm} -\!\!\!\!\!\!\int dy \frac{\rho_m(y)}{x-y} - A_{lk} S_{km} \int \frac{dy}{y^2} \frac{\rho_m(y)}{x-\frac{1}{y}} = A_{lk} \frac{x_k x + 2\pi m_k}{x^2-1} + 2\pi n_{l,i} \,, \quad x \in \mathcal{C}_{l,i}}$$



Monodromy ∈ Weyl gro

$PSU(1,1|2) \times PSU(1,1|2)$

$OSp(4|2) \times OSp(4|2)$

x-symmetry gauge: $K^+\Theta = 0$ $K^+\Gamma^3 K^- = 0 \Rightarrow X^3$ decouples.

$$\mathcal{U}(x) = U^{-1}\, e^{\overset{\uparrow}{iH_l}\, p_l(x)}\, U$$

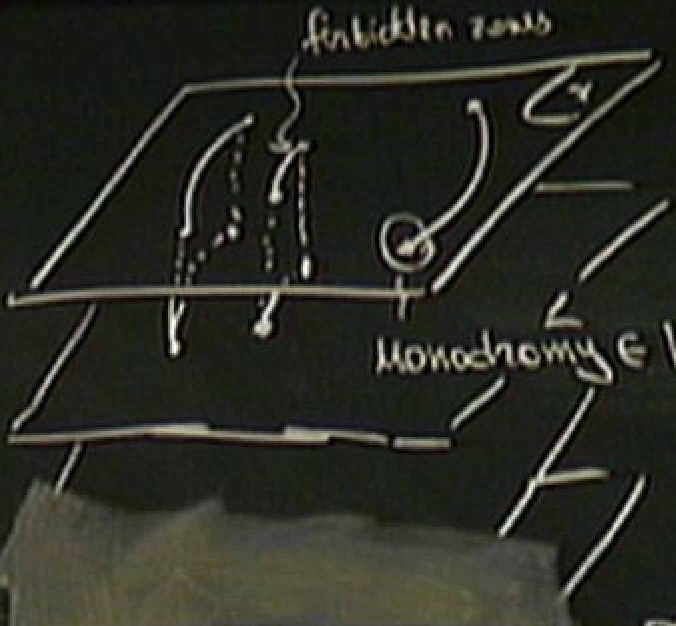
Cartan generators

$p_l(x)$ - quasi-momenta

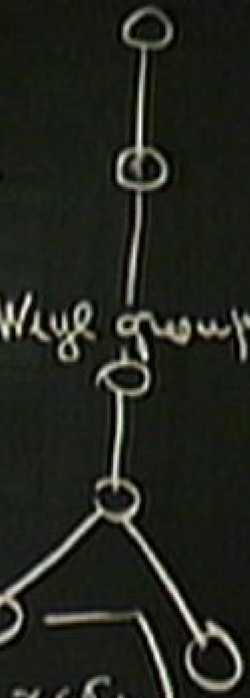
$$p_l(x) = -\frac{\mathcal{X}_l x + 2\pi m_l}{x^2-1} + \int dy\, \frac{\rho_l(y)}{x-y}$$

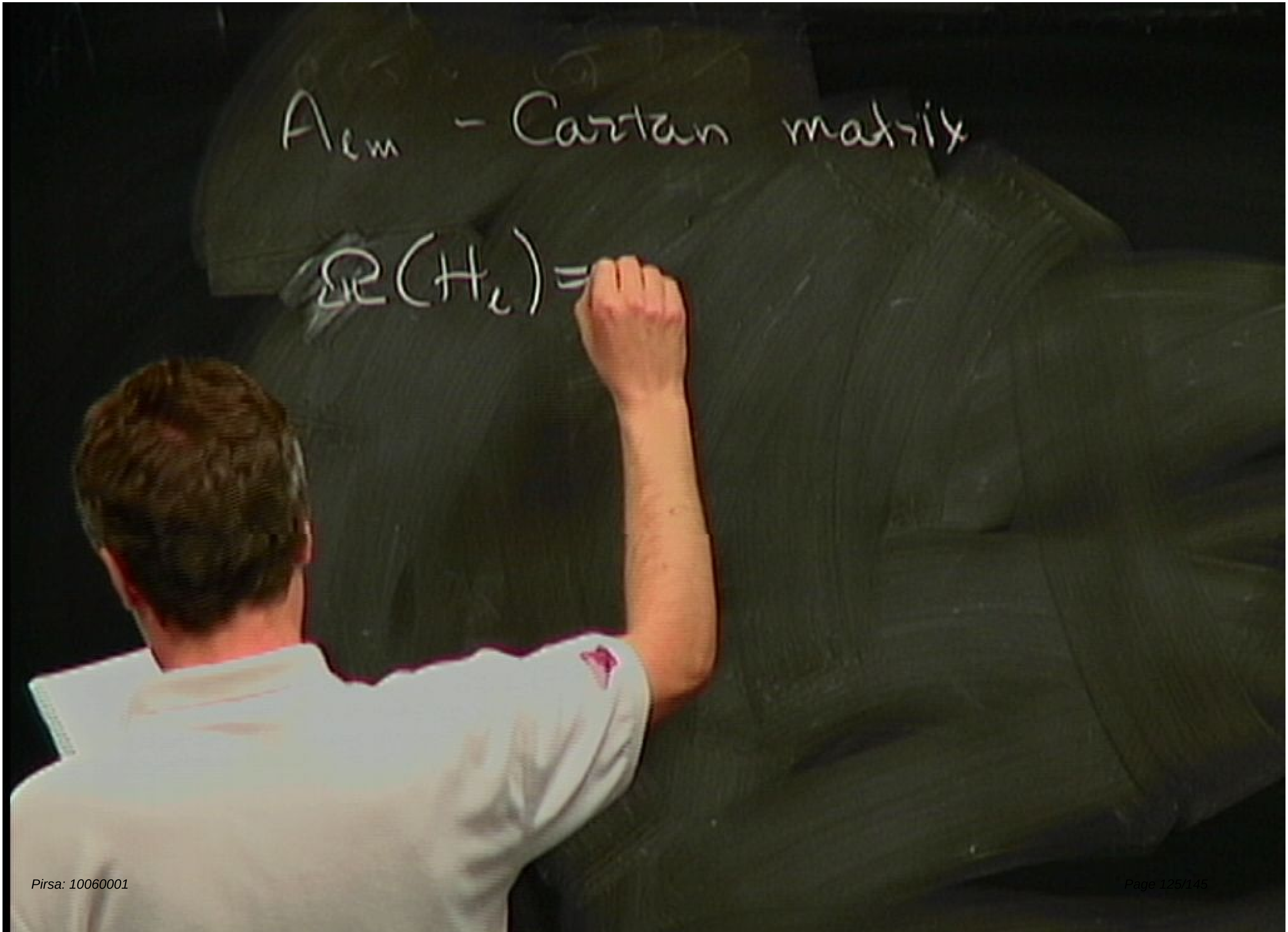
$$+ S_{lm} \int \frac{dy}{y^2}\, \frac{\rho_m(y)}{x-\frac{1}{y}}$$

$$\boxed{A_{lm} ⨍ dy\, \frac{\rho_m(y)}{x-y} - A_{lk} S_{km} \int \frac{dy}{y^2}\, \frac{\rho_m(y)}{x-\frac{1}{y}} = A_{lk}\, \frac{\mathcal{X}_k x + 2\pi m_k}{x^2-1} + 2\pi n_{l,i}\,,\quad x \in \mathcal{C}_{l,i}}$$



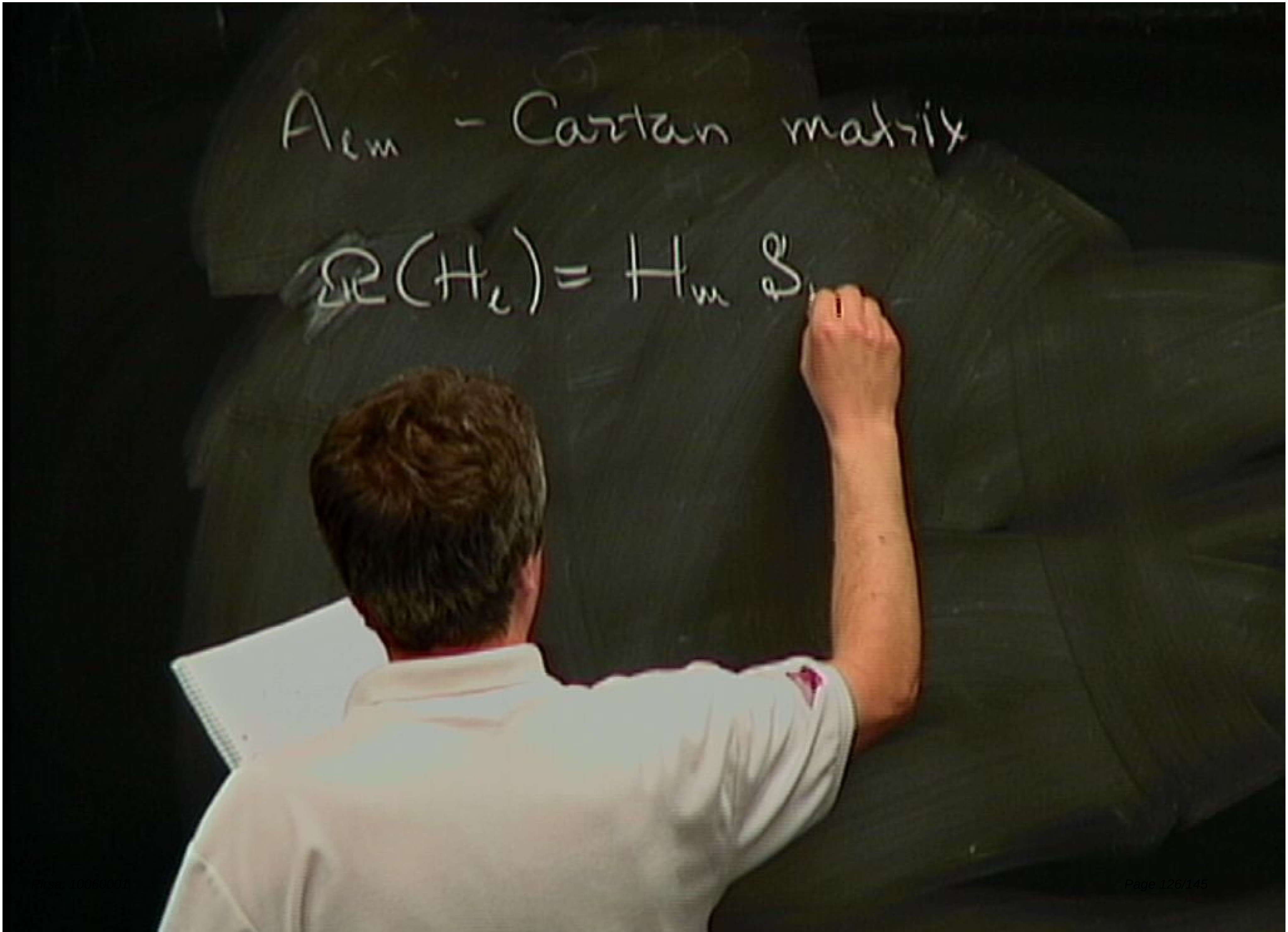
Monodromy ∈ Weyl group





$A_{\ell m}$ - Cartan matrix

$\Omega(H_\ell) =$

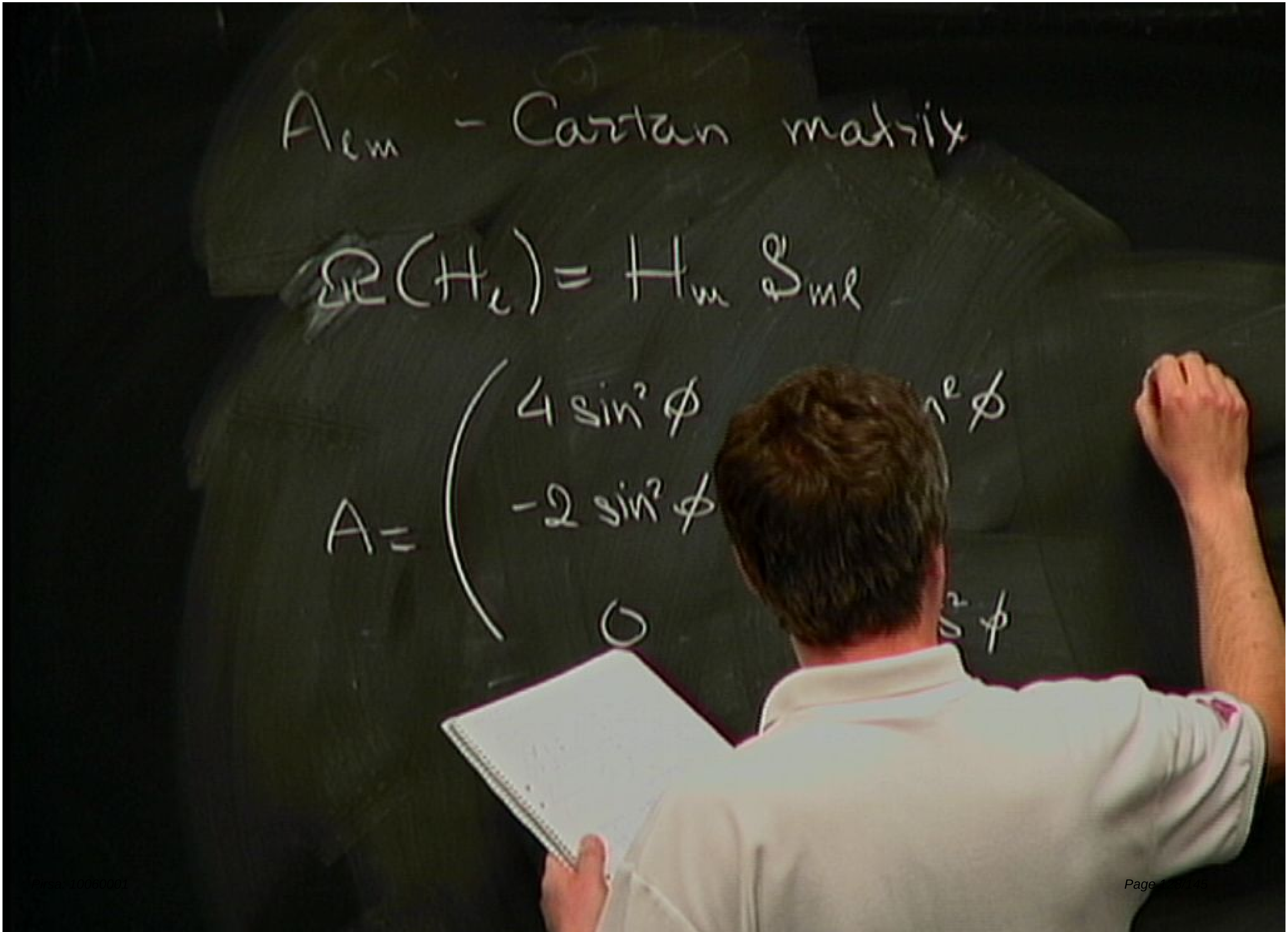


A_{lm} - Cartan matrix

$\Omega(H_l) = H_m \delta$

$A_{\ell m}$ – Cartan matrix

$$\mathcal{R}(H_\ell) = H_m \, \delta_{m\ell}$$



A_{lm} – Cartan matrix

$$\Omega(H_l) = H_m \delta_{ml}$$

$$A = \left(\begin{matrix} 4\sin^2\phi & n^2\phi \\ -2\sin^2\phi & \\ 0 & s^2\phi \end{matrix} \right.$$

A_{lm} - Cartan matrix

$$\Omega(H_l) = H_m \mathcal{S}_{ml}$$

$$A = \begin{pmatrix} 4\sin^2\phi & -2\sin^2\phi & 0 \\ -2\sin^2\phi & 0 & \\ 0 & -2\cos^2\phi & \end{pmatrix}$$

$A_{\ell m}$ - Cartan matrix

$$\Omega(H_\ell) = H_m \, \delta_{m\ell}$$

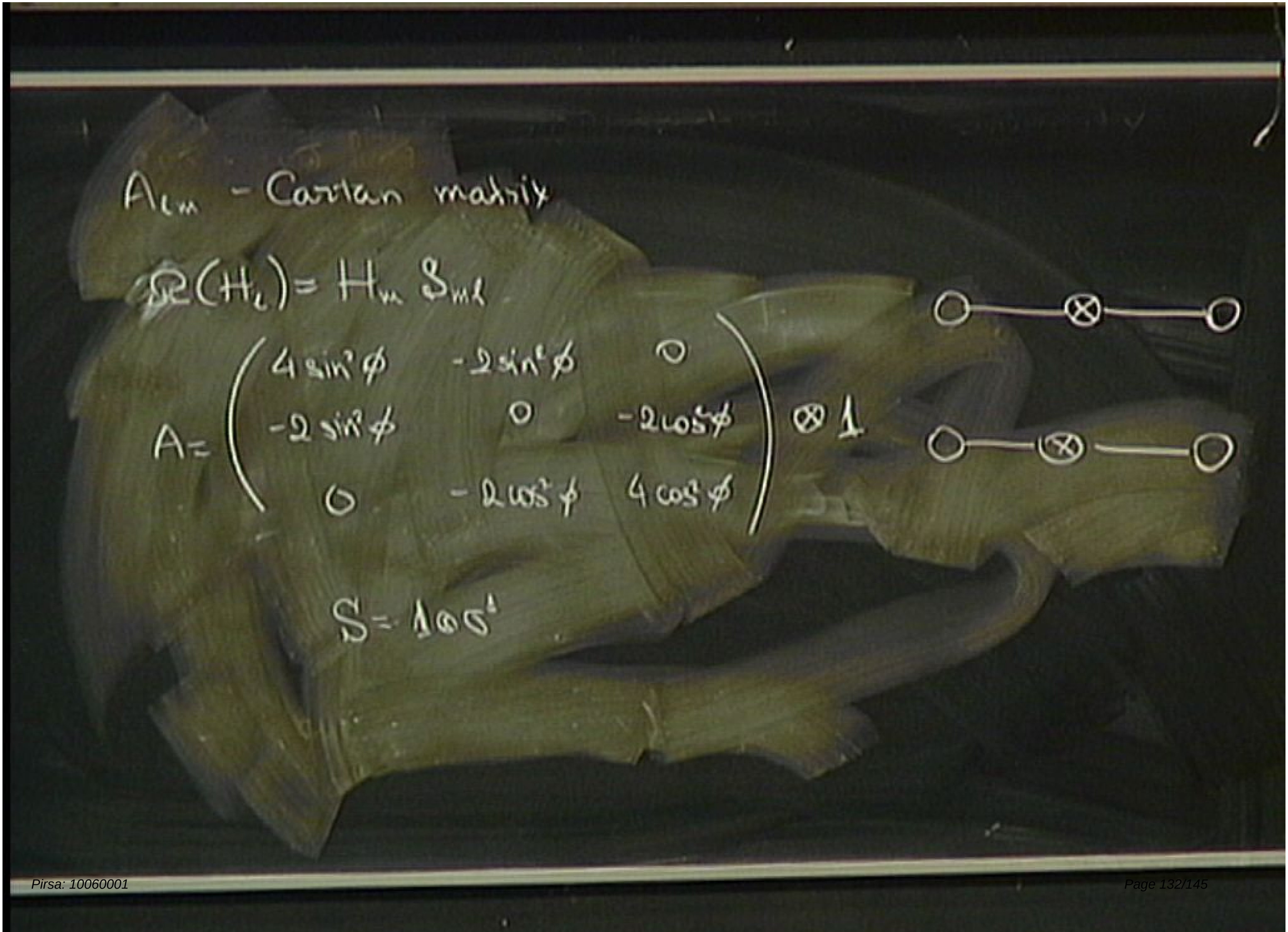
$$A = \begin{pmatrix} 4\sin^2\phi & -2\sin^2\phi & 0 \\ -2\sin^2\phi & 0 & -2\cos^2\phi \\ 0 & -2\cos^2\phi & 4\cos^2\phi \end{pmatrix}$$

A_{lm} - Cartan matrix

$$\Omega(H_l) = H_m S_{ml}$$

$$A = \begin{pmatrix} 4\sin^2\phi & -2\sin^2\phi & 0 \\ -2\sin^2\phi & 0 & -2\cos^2\phi \\ 0 & -2\cos^2\phi & 4\cos^2\phi \end{pmatrix} \otimes 1$$

$$S = 1 \otimes \sigma^1$$



A_{lm} - Cartan matrix

$$\Omega(H_l) = H_m S_{ml}$$

$$A = \begin{pmatrix} 4\sin^2\phi & -2\sin^2\phi & 0 \\ -2\sin^2\phi & 0 & -2\cos^2\phi \\ 0 & -2\cos^2\phi & 4\cos^2\phi \end{pmatrix} \otimes 1$$

$$S = 1 \otimes \sigma^1$$

A_{lm} - Cartan matrix

$$\Omega(H_l) = H_m S_{ml}$$

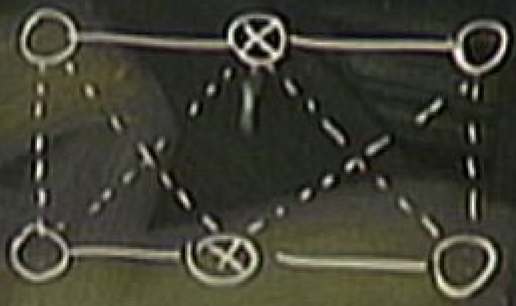
$$A = \begin{pmatrix} 4\sin^2\phi & -2\sin^2\phi & 0 \\ -2\sin^2\phi & 0 & -2\cos^2\phi \\ 0 & -2\cos^2\phi & 4\cos^2\phi \end{pmatrix} \otimes 1$$

$$S = 1 \otimes \sigma^1$$

A_{lm} - Cartan matrix

$$\Omega(H_l) = H_m S_{ml}$$

$$A = \begin{pmatrix} 4\sin^2\phi & -2\sin^2\phi & 0 \\ -2\sin^2\phi & 0 & -2\cos^2\phi \\ 0 & -2\cos^2\phi & 4\cos\phi \end{pmatrix} \otimes 1$$



$$S = 1 \otimes \sigma^1$$

Bethe eqs:

$$e^{ip_j L} = \prod_{k\neq j} S(p_j, p_k)$$

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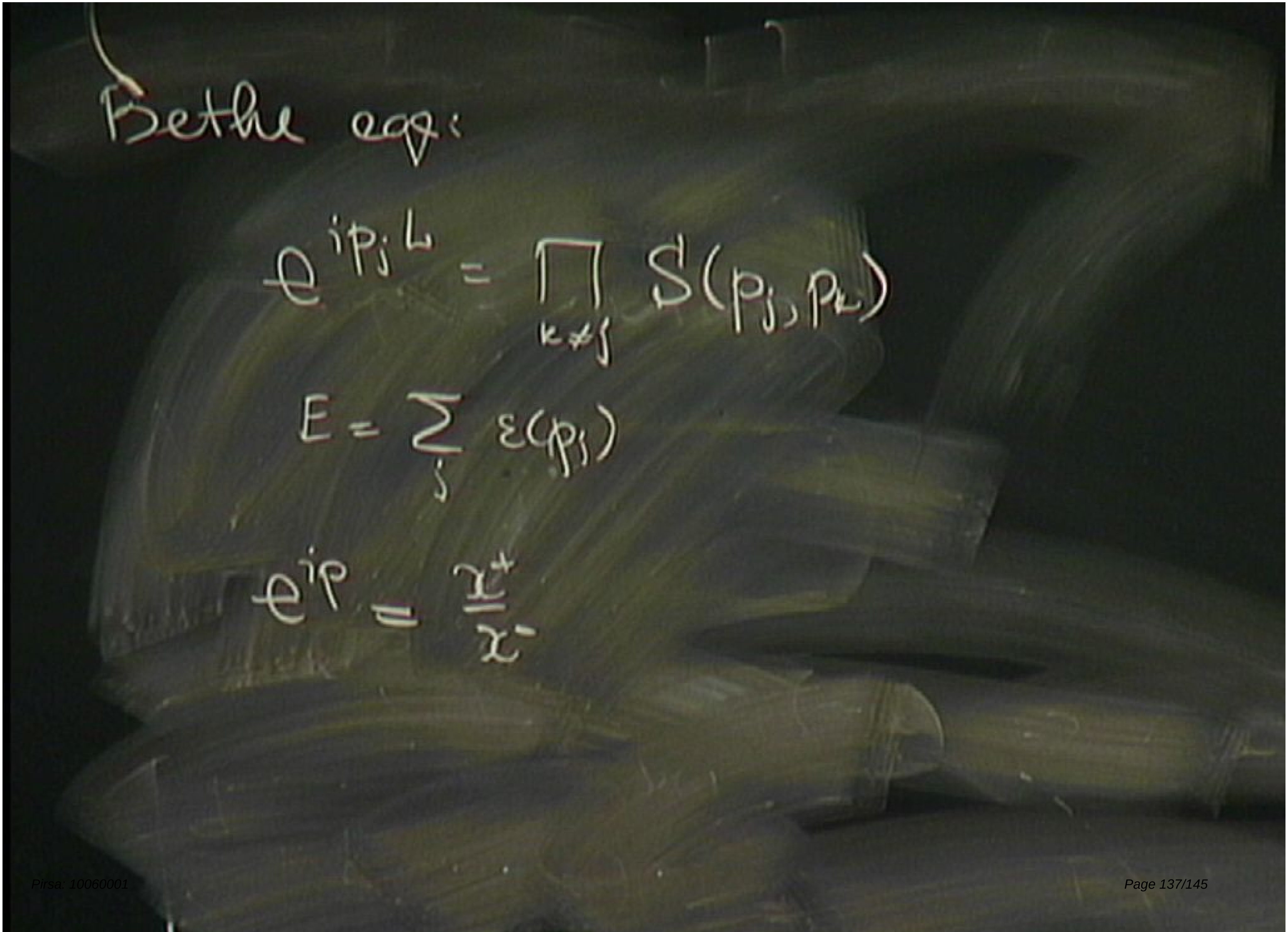

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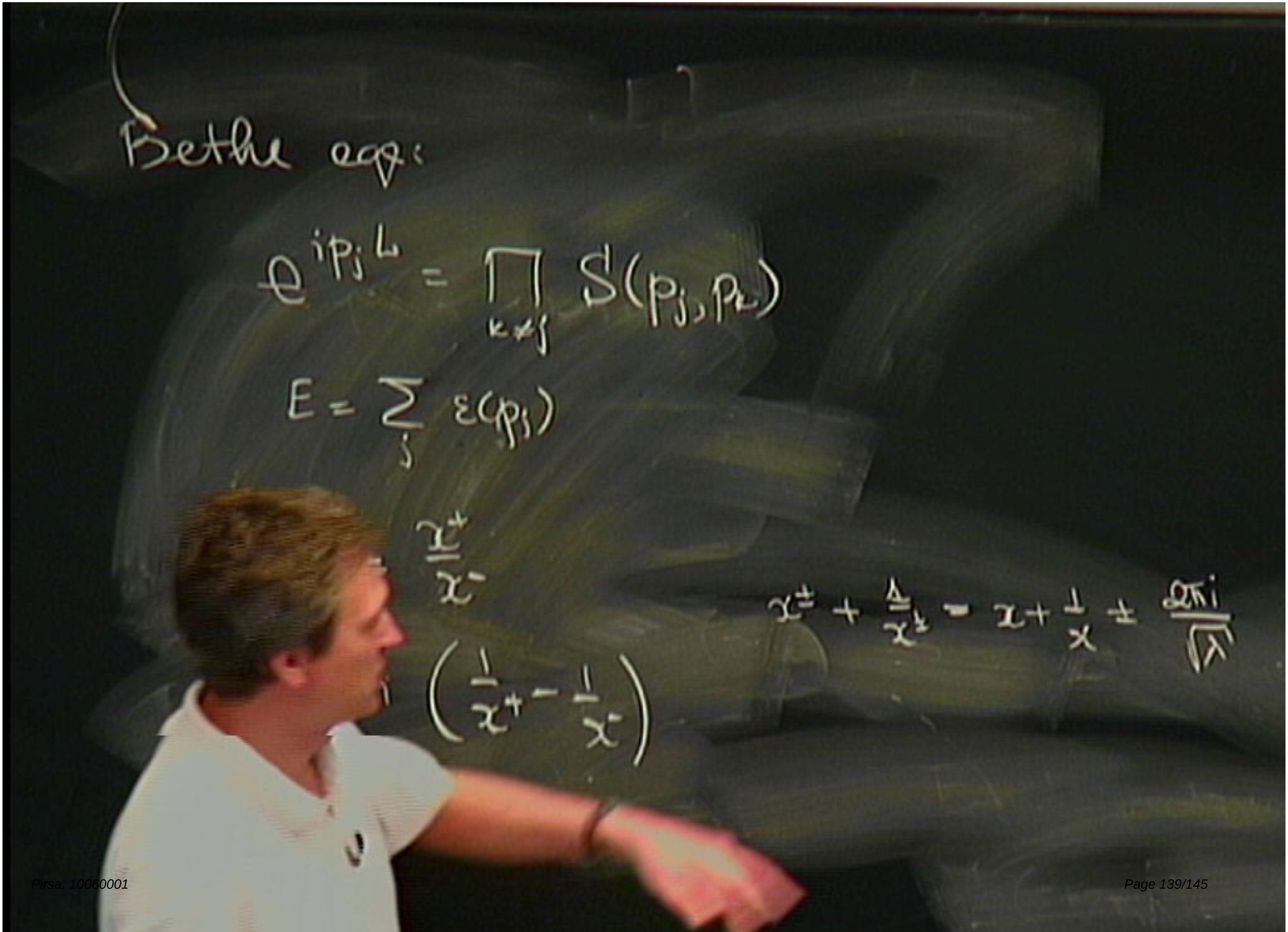

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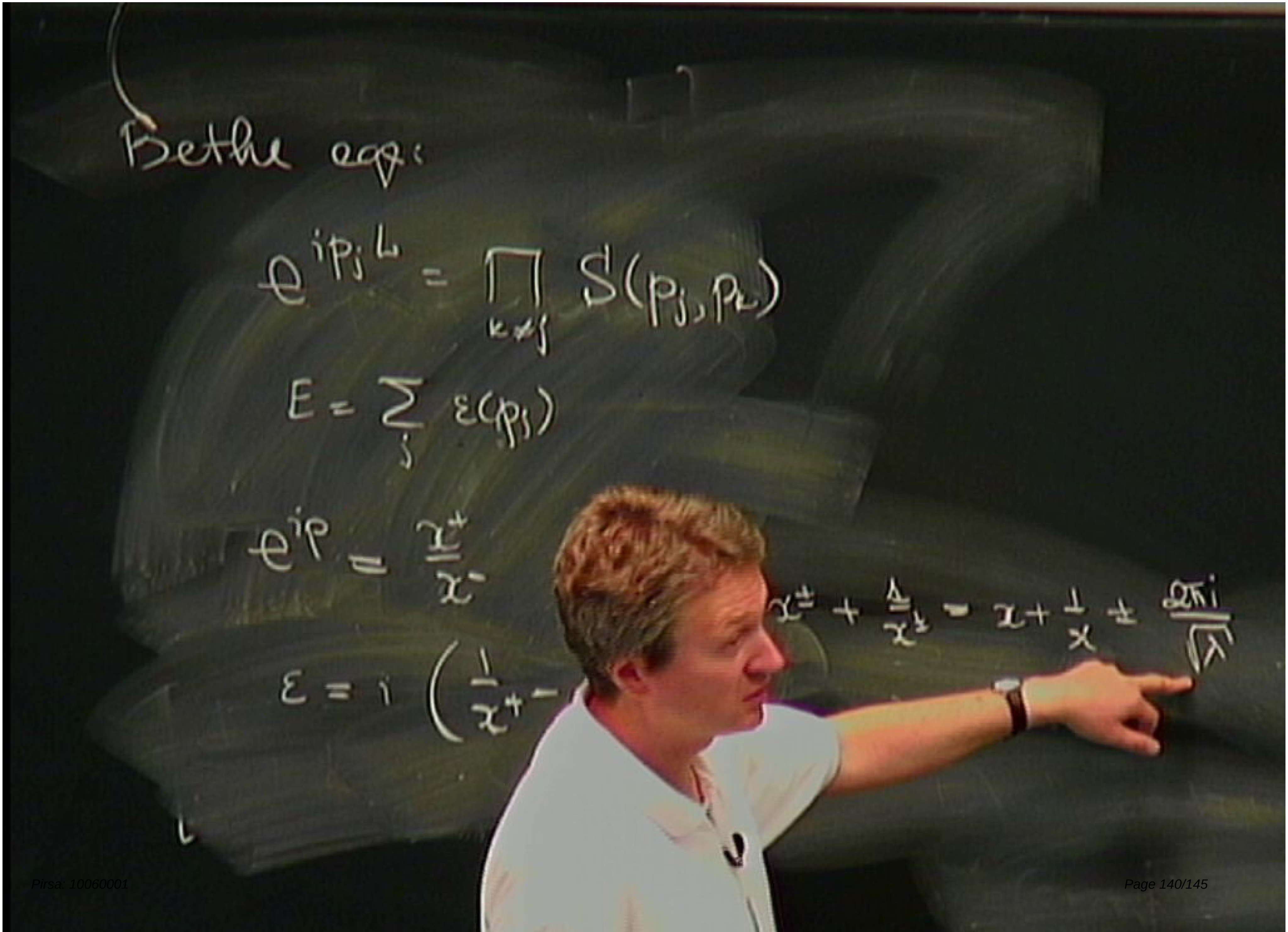

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σ-model is a coset:

$$D(2,1;\alpha)\times D(2,1;\alpha)\,/\,SL(2,\mathbb{R})\times SO(4)_{diag}$$

$\underline{\mathbb{Z}_4}$ symmetry

$$\mathcal{M} = G/H_0$$

Ω - automorphism of $\hat{g}$: $\Omega^4 = id$

$\Omega^2 = (-1)^F$

$$\Omega(h_n) = i^n h_n$$

$$\hat{g} = \underset{B}{h_0} \oplus \overset{F}{h_1} \oplus \underset{B}{h_2} \oplus \overset{F}{h_3}$$