

Title: AdS(3)/CFT(2) and integrability

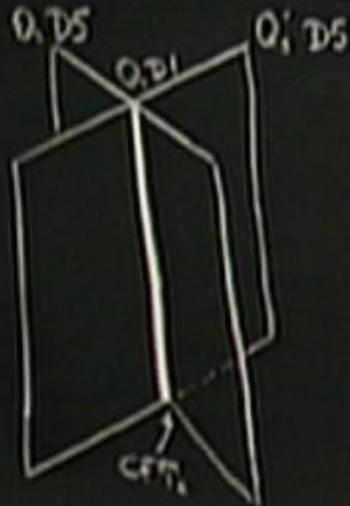
Date: Jun 01, 2010 11:00 AM

URL: <http://pirsa.org/10060001>

Abstract: Quantization of string theory on the AdS(3) backgrounds with the RR flux, such as AdS(3) $\times$ S(3) $\times$ T(4) or AdS(3) $\times$ S(3) $\times$ S(3) $\times$ S(1), is an unsolved problem. Since the sigma model on these backgrounds is classically integrable, one can try to implement powerful methods of integrability similar to those used to solve AdS(5)/CFT(4) and AdS(4)/CFT(3). I will describe the integrability approach to the AdS(3) backgrounds, emphasizing the differences to the better understood cases of AdS(5) and AdS(4).

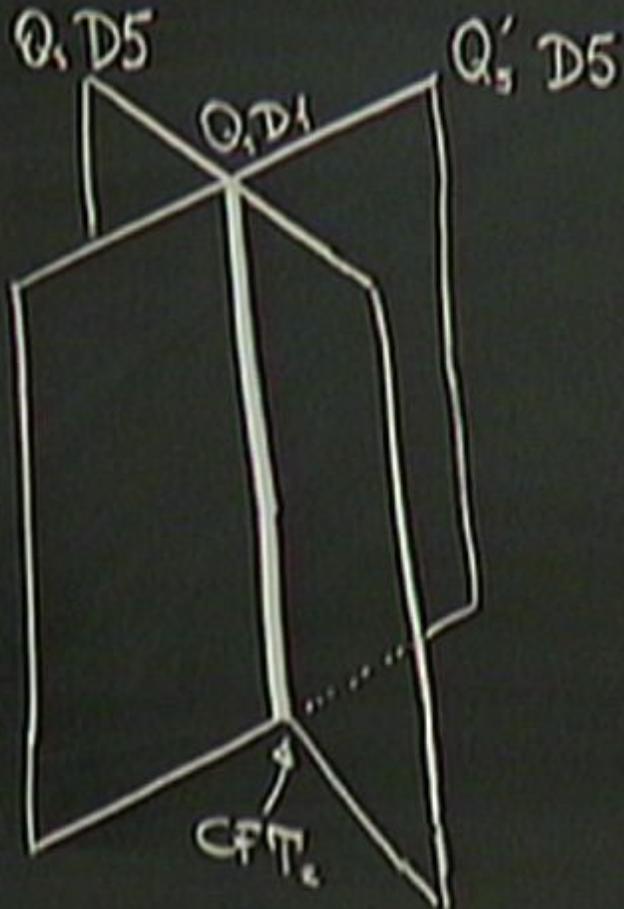
# AdS<sub>3</sub>/CFT<sub>2</sub> and integrability

w. A. Babelenko and B. Stefanski



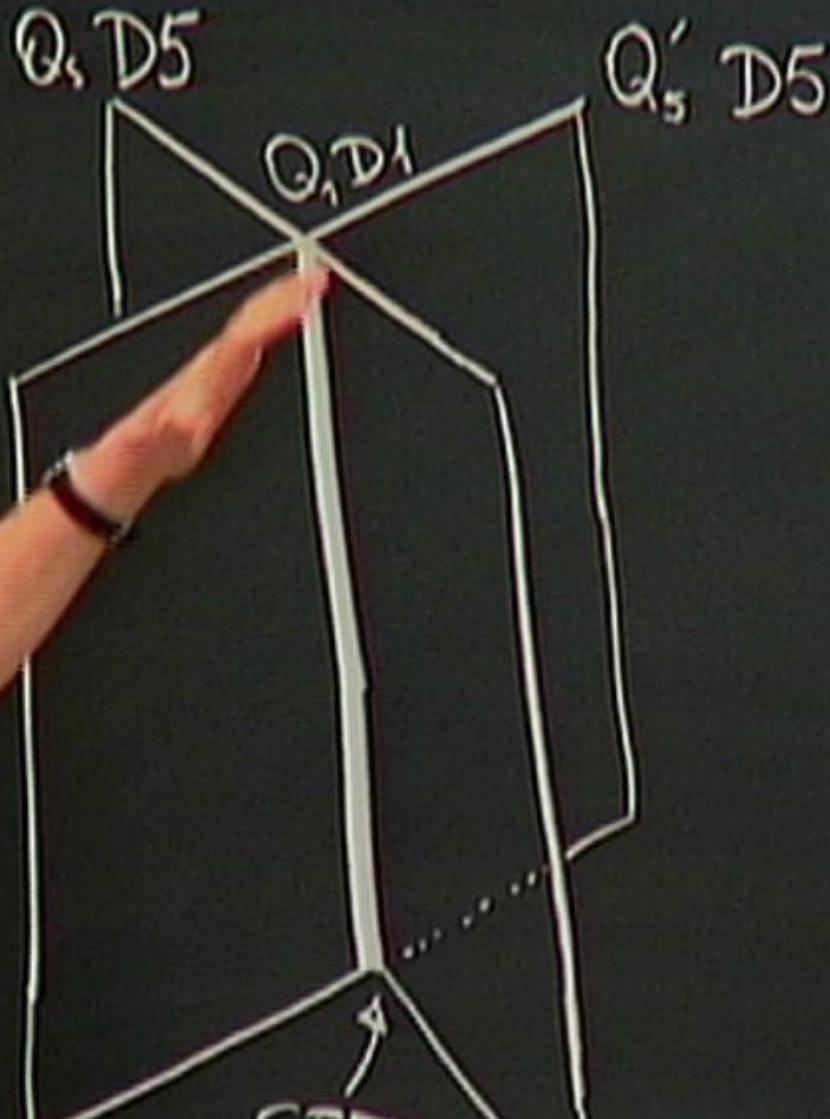
AdS<sub>3</sub> / CFT<sub>2</sub> and integrability

w. A. Babichenko and B. Stefanski



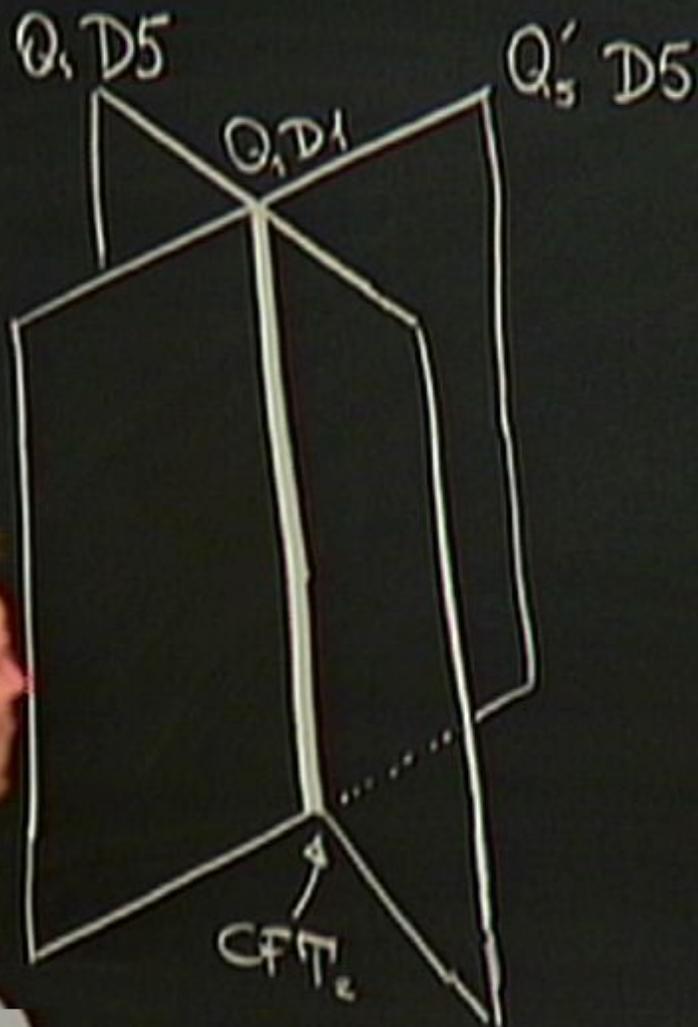
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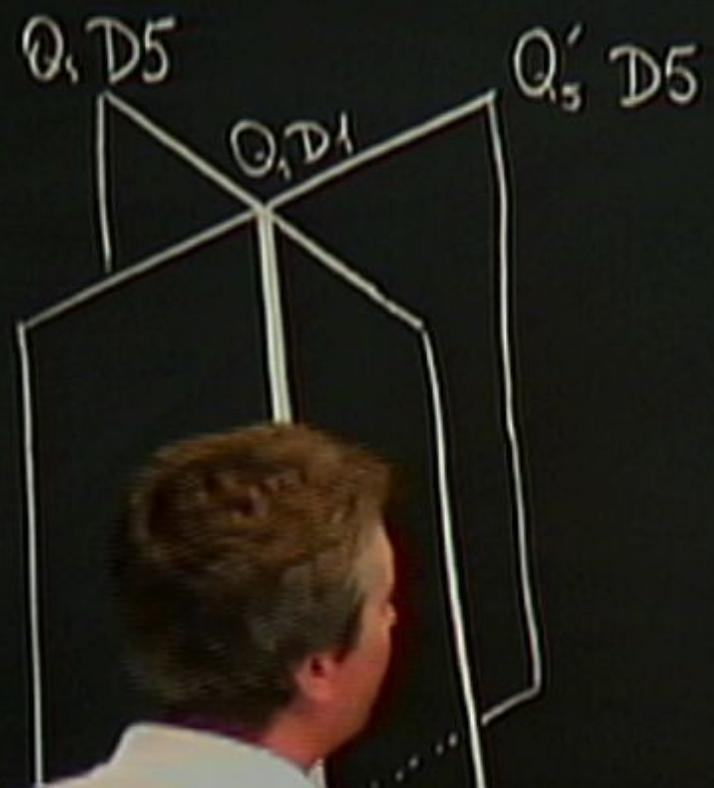


Near-horizon limit:

$$AdS_3 \times S^3 \times S^3 \times S^1$$

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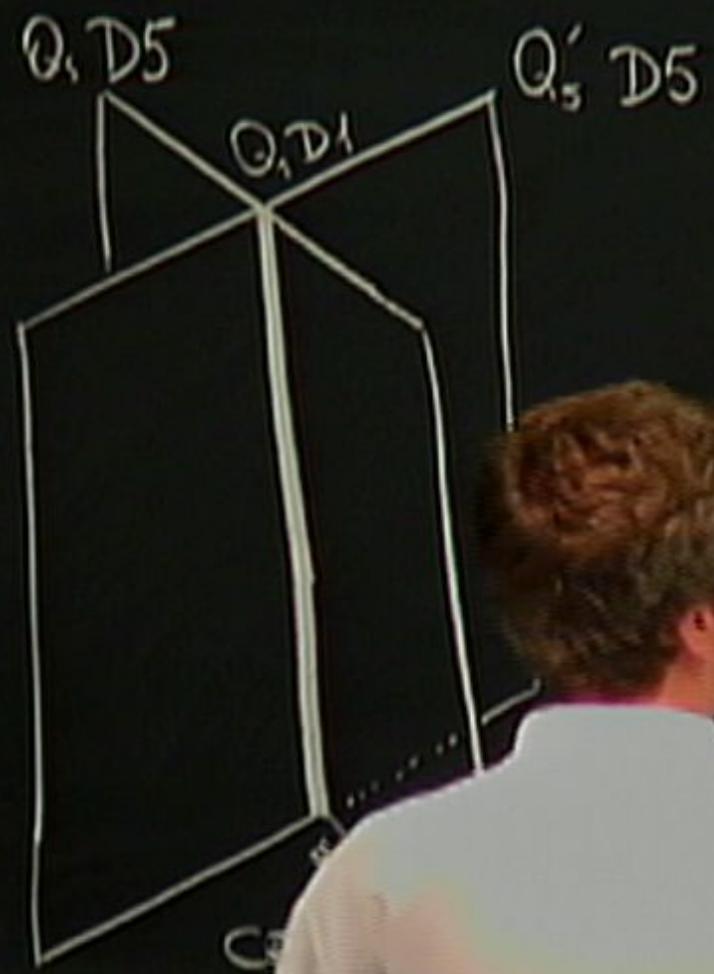
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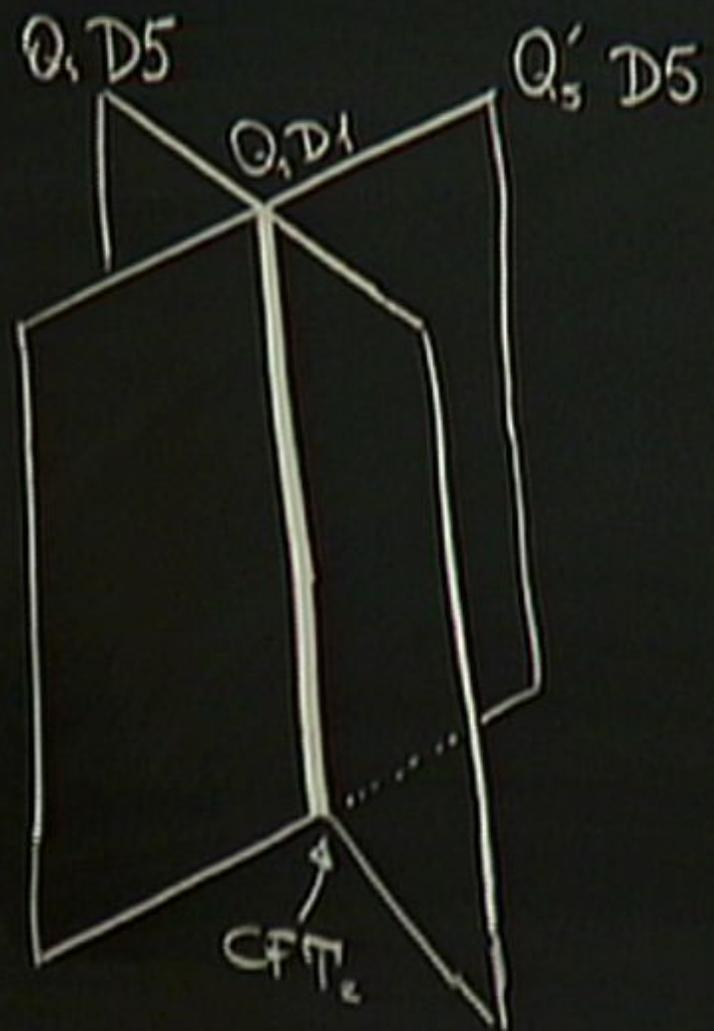
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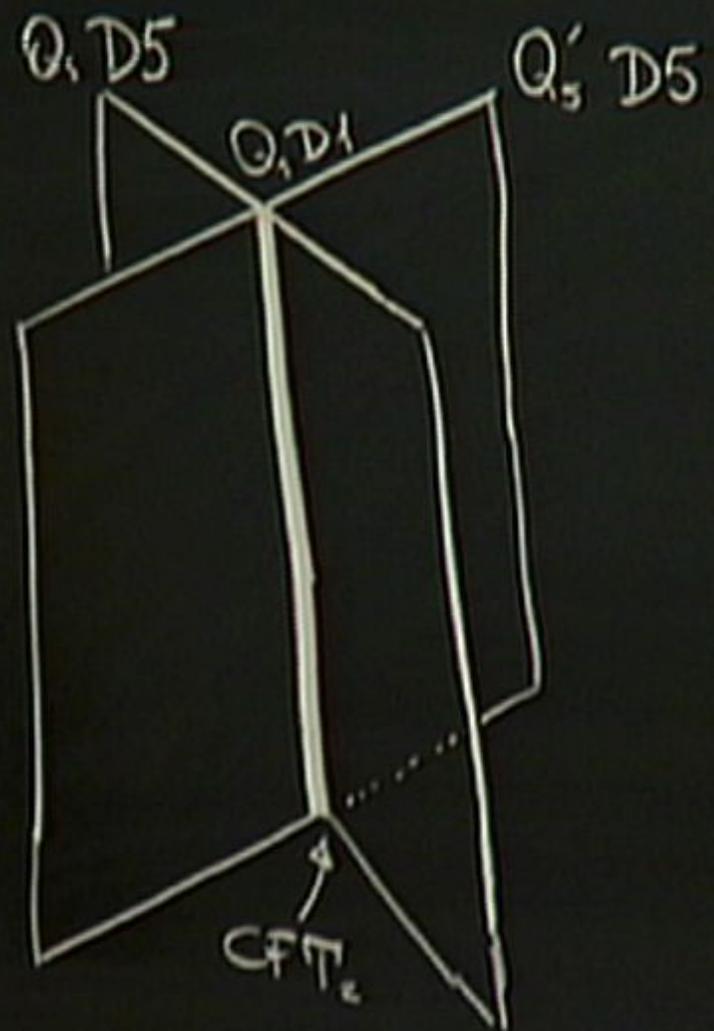
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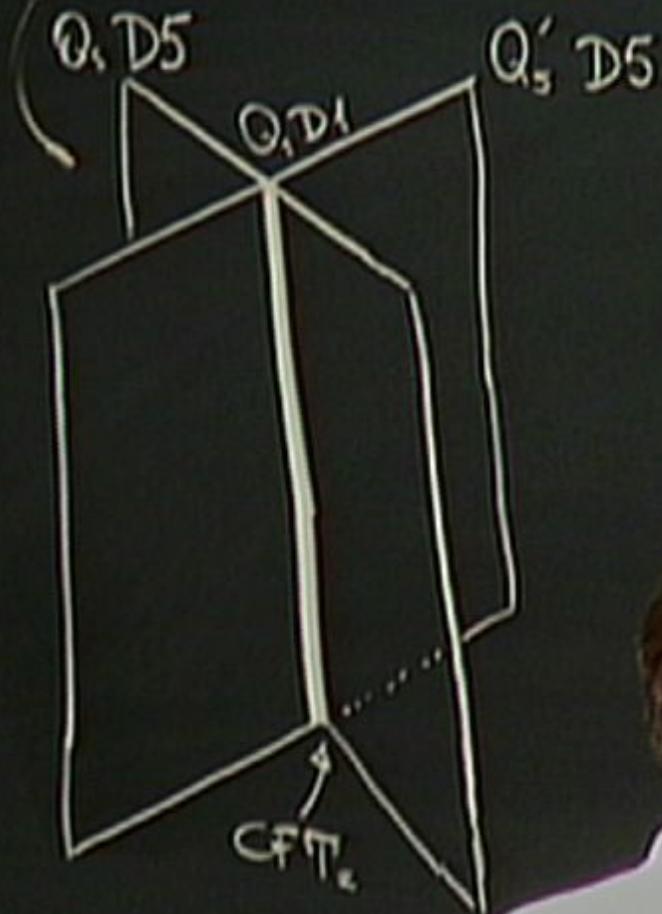
Near-horizon limit:

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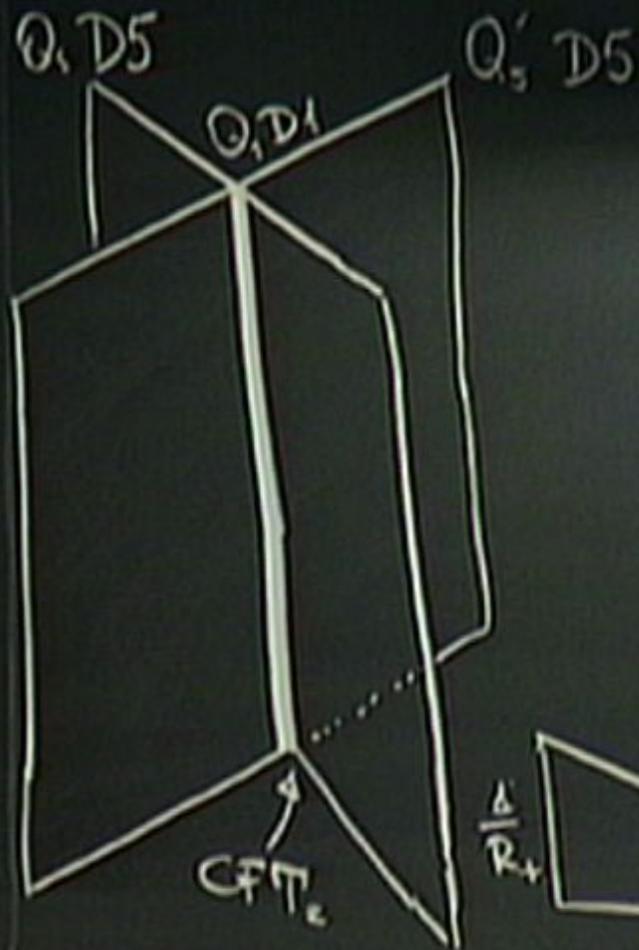
Near-horizon limit:

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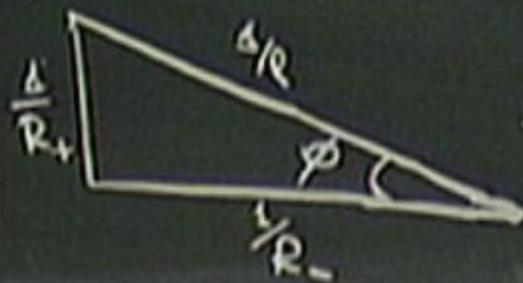
$$\frac{1}{\ell^2} =$$

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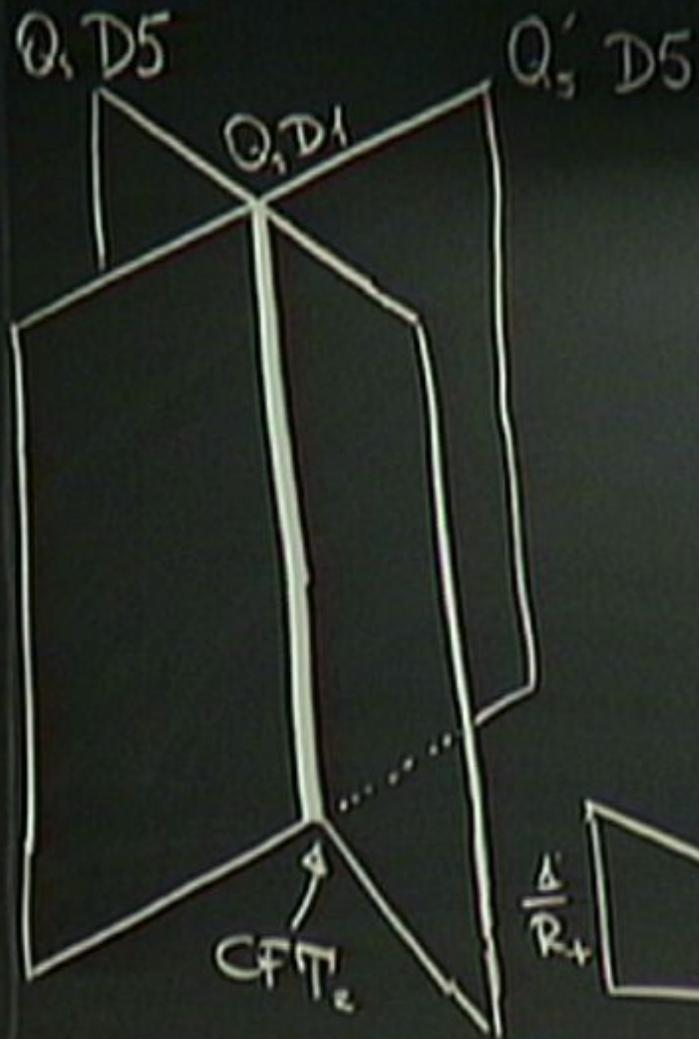
Near-horizon limit:

$$AdS_3 \times S^2 \times S^3 \times S^1$$



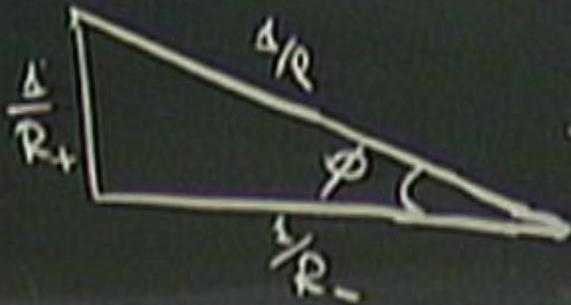
$$\frac{1}{\mathcal{L}^2} = \frac{1}{R_-^2} + \frac{1}{R_+^2}$$

A. Fabiachenko and B. Stefanski



Near-horizon limit:

$$AdS_3 \times S^2 \times S^2 \times S^1$$



$$\frac{1}{l^2} = \frac{1}{R_-^2} + \frac{1}{R_+^2}$$

$R_+ \rightarrow \infty$

$$R_+ \rightarrow \infty \quad (\phi \rightarrow 0)$$

$$\mathbb{R}_+ \rightarrow \infty \quad (\phi \rightarrow 0)$$

$$\text{AdS}_3 \times \mathcal{S}^3 \times \mathbb{T}^4$$

$$\mathcal{L} = \mathcal{R}$$

$$\mathbb{R}_+ \rightarrow \infty \quad (\phi \rightarrow 0)$$

$$\text{AdS}_3 \times \mathcal{S}^3 \times \mathbb{T}^4$$

$$\mathcal{L} = \mathcal{R}$$

Dual      CFT

$$R_+ \rightarrow \infty \quad (\phi \rightarrow 0)$$

$$\text{AdS}_3 \times S^3 \times T^4$$

$$L = R$$

ual CFIT

$\text{Symm}_N(\mathcal{M})$

$$R_+ \rightarrow \infty \quad (\phi \rightarrow 0)$$

$$AdS_3 \times S^3 \times T^4$$

$$l = R$$

Dual CFT

$$\text{Symm}_N(\mathcal{M}) + 2 \int d^2x \mathcal{O}_{1,1}$$

$$R_+ \rightarrow \infty \quad (\phi \rightarrow 0)$$

$$AdS_3 \times S^3 \times T^4$$

$$L = \mathbb{R}$$

Dual CFT

$$\text{Sym}_N(\mathcal{M}) + a \int d^2z \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2}$$

$\sigma_{\text{transp}}$

twist operator

# Dual CFT

$$\text{Sym}_N(\mathcal{M}) + 2 \int d^2z \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2} \sigma_{\text{transp}}$$

twist operator

$$\mathcal{M} = \left\{ \begin{array}{l} \mathbb{P}^4 \end{array} \right.$$

# Dual CFIT

$$\text{Symm}_N(\mathcal{M}) + \alpha \int d^2z \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2} \sigma_{\text{transp}}$$

↖ twist operator

$$\mathcal{M} = \left\{ \begin{array}{l} \mathbb{P}^4 \end{array} \right.$$

# Dual CFT

$$\text{Symm}_N(\mathcal{M}) + \alpha \int d^2z \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2} \sigma_{\text{transp}}$$

↖ twist operator

$$\mathcal{M} = \left\{ \mathbb{P}^4 \right.$$

AdS<sub>3</sub> × S<sup>3</sup> × T<sup>4</sup>

$$L = \mathbb{R}$$

Dual CFT

$$\text{Sym}_N(\mathcal{M}) + 2 \int d^2z \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2} \sigma_{\text{twist}}$$

twist operator

$$\mathcal{M} = \begin{cases} \mathbb{T}^4, & \text{for AdS}_3 \times S^3 \times \mathbb{T}^4 \\ \text{WZW}_{\text{SU}(2), k=2}, & \text{for AdS}_3 \times S^3 \times S^2 \times S^1 \end{cases}$$

$$R_+ \rightarrow \infty \quad (\phi \rightarrow 0)$$

$$\text{AdS}_3 \times S^3 \times \mathbb{T}^4$$

$$L = \mathbb{R}$$

Dual CFT

$$\text{Sym}_N(\mathcal{M}) + 2 \int d^2 z \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{-1/2} \tilde{G}_{-1/2} \sigma_{\text{twist}}$$

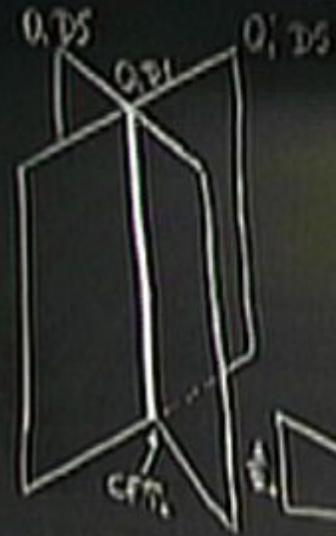
$\uparrow$   
 twist operator

$$\mathcal{M} = \begin{cases} \mathbb{T}^4, & \text{for AdS}_3 \times S^3 \times \mathbb{T}^4 \end{cases}$$

$$\text{WZW}_{\text{su}(2), k=2}, \text{ for AdS}_3 \times S^3 \times \mathbb{T}^4$$

AdS<sub>2</sub> / CFT<sub>1</sub>

by A. Bilchenko and B. Grefanski



Near-horizon limit

$$AdS_2 \times S^1 \times S^1 \times S^1$$



$$\frac{1}{L} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_1 \rightarrow \infty \quad (r \rightarrow 0)$$

$$AdS_2 \times S^1 \times T^1$$

$$L = R$$

Dual CFT

$$Sym_{\mathbb{Z}_2}(J) + 2 \int d^2x \mathcal{O}_{1,1}$$

$$\mathcal{O}_{1,1} = G_{\mu\nu} \tilde{G}_{\mu\nu} \text{ Gauss constraint operator}$$

$$J = \begin{cases} T^{\mu\nu} & \text{for } AdS_2 \times S^1 \times T^1 \\ \text{WZV current} & \text{for } AdS_2 \times S^1 \times S^1 \end{cases}$$

# AdS<sub>3</sub> / CFT<sub>2</sub>

w. A. Bilchenko and B. Stefanski



Non-horizon limit:

$$AdS_3 \times S^1 \times S^1 \times S^1$$



$$\frac{1}{L} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_+ \rightarrow \infty \quad (\gamma \rightarrow 0)$$

$$AdS_3 \times S^1 \times \mathbb{T}^2$$

$$L = R$$

Dual CFT

$$Sym_{\mathbb{N}}(\mathcal{H}) + \alpha \int d^2x \mathcal{O}_{i,1}$$

$$\mathcal{O}_{i,1} = G_{i,1} \tilde{G}_{i,1} \sigma_{i,1}$$

↖ least operator

$$\mathcal{H} = \begin{cases} \mathbb{T}^2 & \text{for } AdS_3 \times S^1 \times \mathbb{T}^2 \\ \text{M2V} & \text{for } AdS_3 \times S^1 \times S^1 \end{cases}$$

AdS<sub>3</sub> / CFT<sub>2</sub>

w. A. Bilchenko and B. Stefanski



Non-horizon limit:

$AdS_3 \times S^1 \times S^1 \times S^1$



$\frac{1}{L} = \frac{1}{R_+} + \frac{1}{R_-}$

$R_+ \rightarrow \infty \quad (\nu \rightarrow 0)$

$AdS_3 \times S^1 \times \mathbb{T}^2$

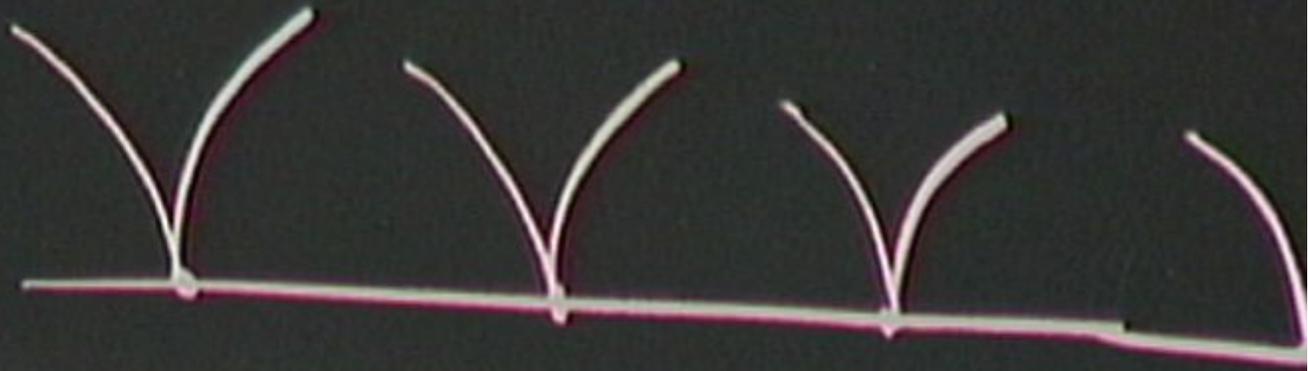
$L = R$

Dual CFT

$Sym_{U(1)}(J) + \alpha \int d^2x \mathcal{O}_{1,1}$

$\mathcal{O}_{1,1} = G_{\mu\nu} \tilde{G}_{\mu\nu} \sigma_{\text{trace}}$   
 (with an arrow pointing to  $\sigma_{\text{trace}}$  labeled "trace operator")

$J = \begin{cases} \mathbb{T}^2, & \text{for } AdS_3 \times S^1 \times \mathbb{T}^2 \\ \text{M2V vacuum, for } AdS_3 \end{cases}$

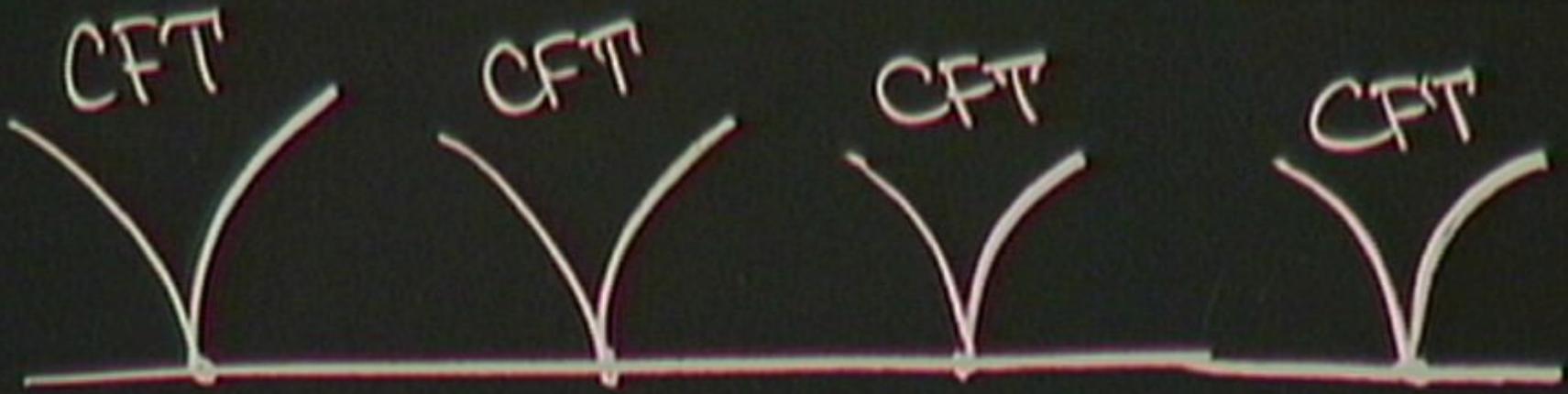


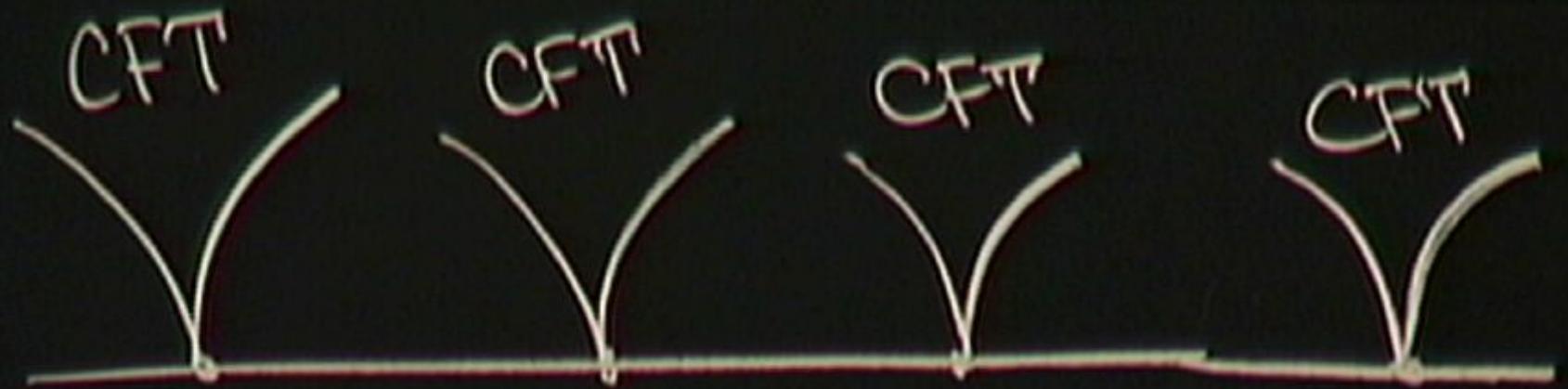
$$\int d^2z \mathcal{O}_{1,1}$$

$\sigma_{\text{transp}}$   
↑  
twist operator

$AdS_3 \times S^3 \times T^4$



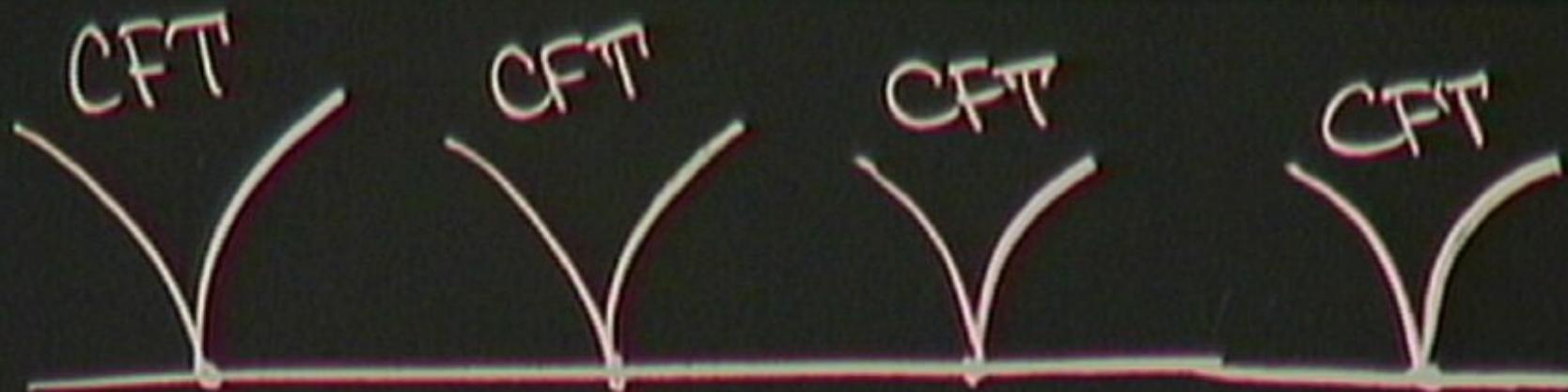




$$\mathcal{H} = F \otimes \mathcal{L}$$

$$H: \mathcal{H} \rightarrow \mathcal{H}$$

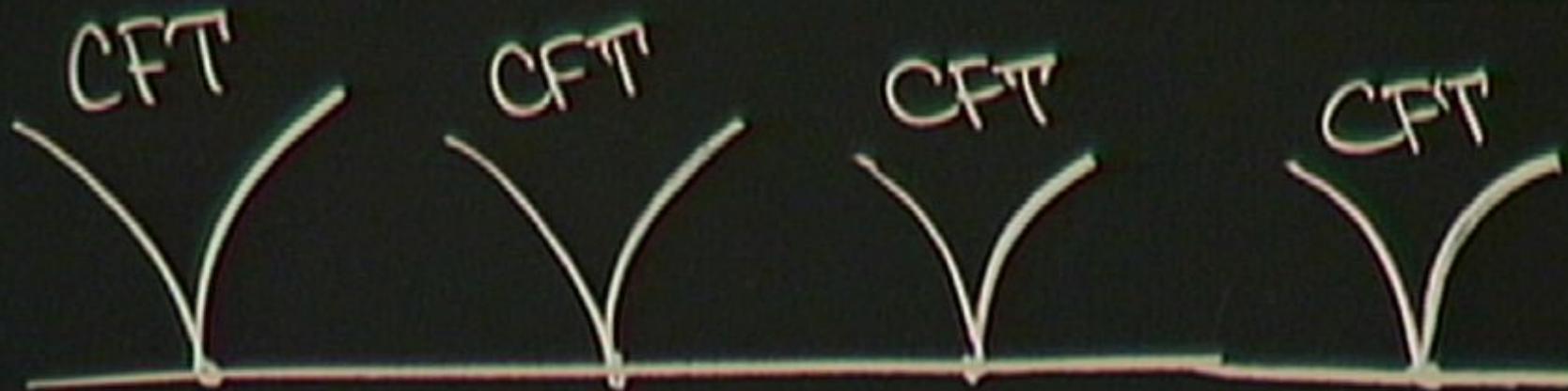
$$H = \sum_{\ell=1}^{\Delta} H_{\ell, \ell+1}$$



$$\mathcal{H} = F \otimes \mathcal{L}$$

$$H: \mathcal{H} \rightarrow \mathcal{H}$$

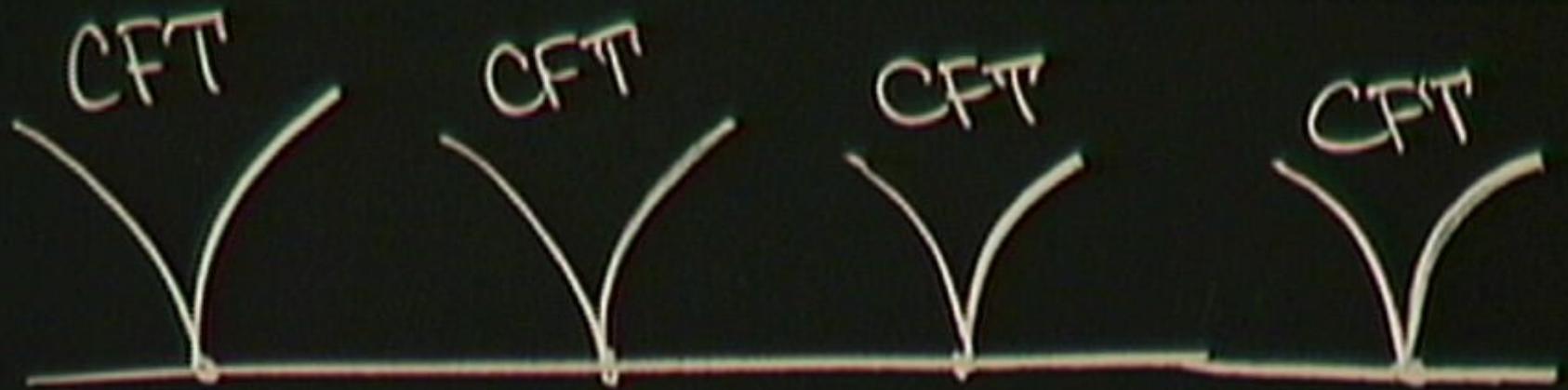
$$H = \sum_{\ell=1}^{\Delta} H_{\ell, \ell+1}$$



$$H = F \otimes \mathcal{L}$$

$$H: \mathcal{K} \rightarrow \mathcal{K}$$

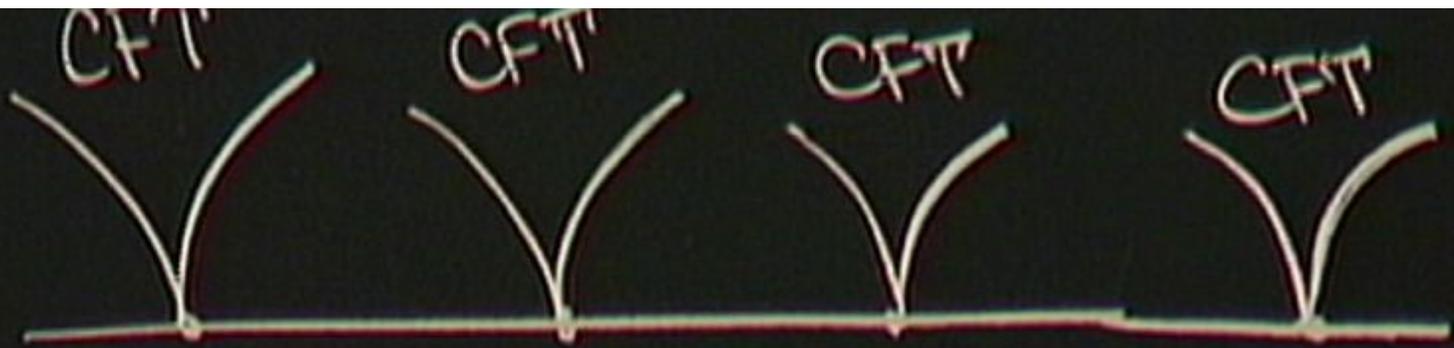
$$H = \sum_{\ell=1}^L H_{\ell, \ell+1}$$



$$\mathcal{H} = F \otimes \mathcal{L}$$

$$H: \mathcal{H} \rightarrow \mathcal{H}$$

$$H = \sum_{\ell=1}^{\infty} H_{\ell, \ell+1}$$



$$\mathcal{H} = F \otimes \mathcal{L}$$

$$H: \mathcal{H} \rightarrow \mathcal{H}$$

$$H = \sum_{\ell=1}^4 H_{\ell, \ell+1}$$

$$H_{12}: F \otimes F \rightarrow F \otimes F$$

Strings on  $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \text{Vol}(A$$

Strings on AdS<sub>3</sub> × S<sup>3</sup> × S<sup>3</sup> × S<sup>1</sup>

RR flux:

$$F = \text{Vol}(\text{AdS}_3) + \frac{1}{\cos^2 \phi} \text{Vol}(S_+^3) + \frac{1}{\sin^2 \phi} \text{Vol}(S_-^3)$$

Strings on AdS<sub>3</sub> × S<sup>3</sup> × S<sup>3</sup> × S<sup>1</sup>

RR flux:

$$F = \text{Vol}(\text{AdS}_3) + \frac{1}{\cos^2 \phi} \text{Vol}(S_+^3) + \frac{1}{\sin^2 \phi} V$$

$$\mathcal{L} = (\dot{X})^2$$

145  
Strings on  $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \text{Vol}(AdS_3) + \frac{1}{\cos^2 \phi} \text{Vol}(S_+^3) + \frac{1}{\sin^2 \phi} \text{Vol}(S_-^3)$$

$$\mathcal{L} = (\partial X)^2 + \bar{\Theta} \partial X \cdot \Gamma \partial \Theta + \bar{\Theta} F \cdot \Gamma \Theta$$

Strings on AdS<sub>3</sub> × S<sup>3</sup> × S<sup>3</sup> × S<sup>1</sup>

RR flux:

$$F = \text{Vol}(\text{AdS}_3) + \frac{1}{\cos^2 \phi} \text{Vol}(S_+^3) + \frac{1}{\sin^2 \phi} \text{Vol}(S_-^3)$$

$$\mathcal{L} = (\partial X)^2 + \bar{\Theta} \partial X \cdot \Gamma \partial \Theta + \bar{\Theta} F \cdot \Gamma \Theta$$

# Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \text{Vol}(AdS_3) + \frac{1}{\cos^2 \phi} \text{Vol}(S_+^3) + \frac{1}{\sin^2 \phi} \text{Vol}(S_-^3)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta} \partial X \cdot \Gamma \partial \Theta + \bar{\Theta} F \cdot \Gamma \Theta$$

$$F \cdot \Gamma = 12 \Gamma^{012} \mathcal{K}^+$$

↑  
projection operator

↑  
RR flux

# Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \text{Vol}(AdS_3) + \frac{1}{\cos^2 \phi} \text{Vol}(S^3_+) + \frac{1}{\sin^2 \phi} \text{Vol}(S^3_-)$$

$$\mathcal{L}_{as} = (\partial X)^2 + \bar{\Theta} \partial X \cdot \Gamma \partial \Theta + \bar{\Theta} F \cdot \Gamma \Theta$$

$$F \cdot \Gamma = \omega \Gamma^{012} K^+$$

↑ projection operator

x-symmetry gauge:

↑ RR flux

Strings on  $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \text{Vol}(AdS_3) + \frac{1}{\cos^2 \phi} \text{Vol}(S^3_+) + \frac{1}{\sin^2 \phi} \text{Vol}(S^3_-)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta} \partial X \cdot \Gamma \partial \Theta + \bar{\Theta} F \cdot \Gamma \Theta$$

$$F \cdot \Gamma = 12 \Gamma^{012} K^+$$

↑ projection operator

↑ RR flux

$x$ -symmetry gauge:

# Strings on $AdS_3 \times S^3 \times S^3 \times S^1$

RR flux:

$$F = \text{Vol}(AdS_3) + \frac{1}{\cos^2 \phi} \text{Vol}(S^3_+) + \frac{1}{\sin^2 \phi} \text{Vol}(S^3_-)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\Theta} \partial X \cdot \Gamma \partial \Theta + \bar{\Theta} F \cdot \Gamma \Theta$$

$$F \cdot \Gamma = 12 \Gamma^{012} K^+$$

↑ projection operator

↑ RR flux

$x$ -symmetry gauge:

$$K^+ \Theta = 0$$

Strings on  $AdS_5 \times S^1 \times S^1 \times S^1$

RR flux:

$$F = \text{Vol}(AdS_5) + \frac{1}{\cos\theta} \text{Vol}(S^1_+) + \frac{1}{\sin\theta} \text{Vol}(S^1_-)$$

$$\mathcal{L}_{\text{as}} = (\partial X)^2 + \bar{\theta} \partial X \cdot \Gamma \partial \theta + \bar{\theta} F \Gamma \theta$$

$$F \cdot \Gamma = 10 \Gamma^{012} K^+$$

$K^+$   
projection operator

$x$ -symmetry gauge:

$$K^+ \theta = 0$$

$$\Gamma^3 K^- = 0 \Rightarrow X^3 \text{ direction}$$

RR flux:

$$F = \text{Vol}(AdS_5) + \frac{L}{\cos^2\phi} \text{Vol}(S_+^3) + \frac{L}{\sin^2\phi} \text{Vol}(S_-^3)$$

$$\mathcal{L}_{GS} = (\partial X)^2 + \bar{\theta} \partial X \cdot \Gamma \partial \theta + \bar{\theta} F \cdot \Gamma \theta$$

↑  
RR flux

$$F \cdot \Gamma = \omega \Gamma^{012} K^+$$

↑  
projection operator

$x$ -symmetry gauge:

$$K^+ \theta = 0$$

$$K^+ \Gamma^3 K^- = 0 \Rightarrow X^3 \text{ decouples.}$$

What is left?

What is left?

What is left?

- 1 compact free ~~base~~  $(X^g)$

What is left?

- 1 compact free boson  $(X^g)$
- GS-type sigma-model

target:  $\Delta$

What is left?

- 1 compact free boson ( $X^9$ )

- GS-type sigma-model

target:  $AdS_3 \times S^3 \times S^3$

What is left?

- 1 compact free boson ( $X^9$ )

- GS-type sigma-model

target:  $AdS_3 \times S^3 \times S^3$

16 supersymmetries

What is left?

- 1 compact free boson ( $X^9$ )

- GS-type sigma-model

target:  $AdS_3 \times S^3 \times S^3$

16 supersymmetries

$D(2,1;\alpha) \times D(2,1;\alpha)$

What is left?

- 1 compact free boson ( $X^9$ )

- GS-type sigma-model

target:  $AdS_3 \times S^3 \times S^3$

16 supersymmetries

$D(2,1;\alpha) \times D(2,1;\alpha)$

What is left?

- 1 compact free boson ( $X^9$ )

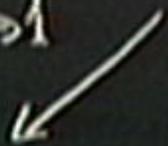
- GS-type sigma-model

target:  $AdS_3 \times S^3 \times S^3$

16 supersymmetries

$D(2,1;\alpha) \times D(2,1;\alpha)$

$R_+ \rightarrow \infty \quad \alpha \rightarrow 1$



$\alpha = \cos^2 \phi$

What is left?

- 1 compact free boson ( $X^9$ )

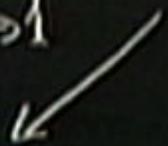
- GS-type sigma-model

target:  $AdS_3 \times S^3$

16 supersymmetries

$D(2,1;\alpha) \times D(2,1;\alpha)$

$R_+ \rightarrow \infty \quad \alpha \rightarrow 1$



$\alpha = \cos^2 \phi$

$PSU(1,1|2) \times PSU(1,1|2)$

What is left?

- 1 compact free boson ( $X^9$ )

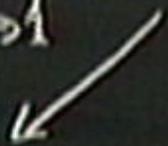
- GS-type sigma-model

target:  $AdS_3 \times S^3 \times S^3$

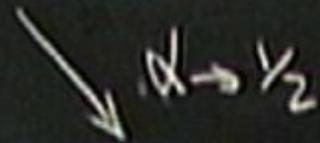
16 supersymmetries

$D(2,1;\alpha) \times D(2,1;\alpha)$

$R_+ \rightarrow \infty \quad \alpha \rightarrow 1$



$PSU(1,1|2) \times PSU(1,1|2)$



$\alpha = \cos^2 \phi$

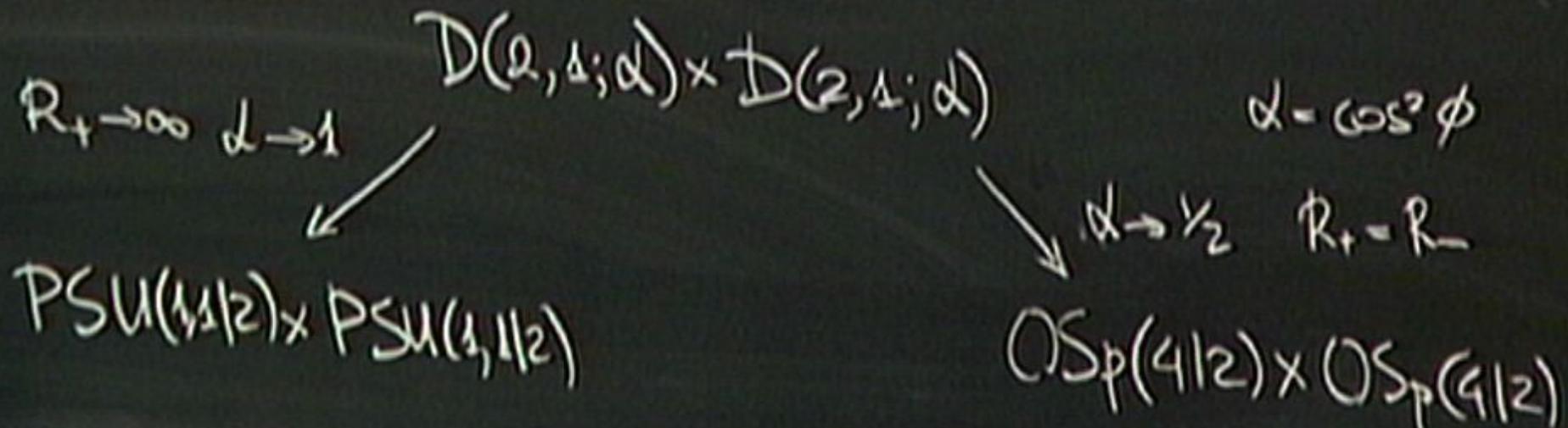
What is left?

• 1 compact free boson ( $X^9$ )

• GS-type sigma-model

target:  $AdS_3 \times S^3 \times S^3$

16 supersymmetries



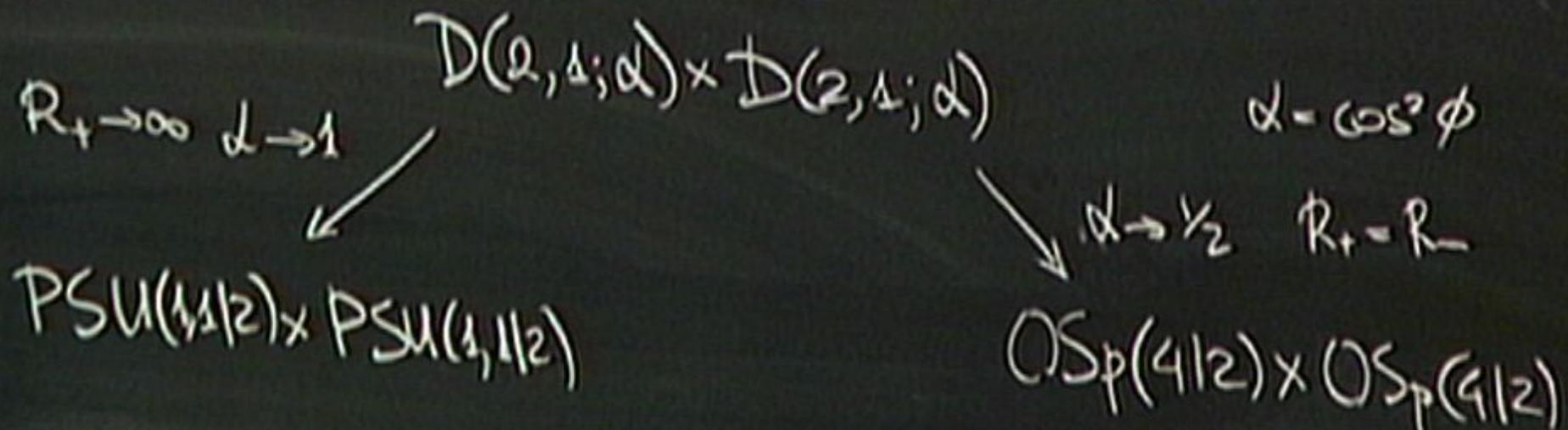
What is left?

- 1 compact free boson ( $X^9$ )

- GS-type sigma-model

target:  $AdS_3 \times S^3 \times S^3$

16 supersymmetries



$\sigma$ -model is a coset:

$$D(A, \alpha; \alpha) \times DC$$

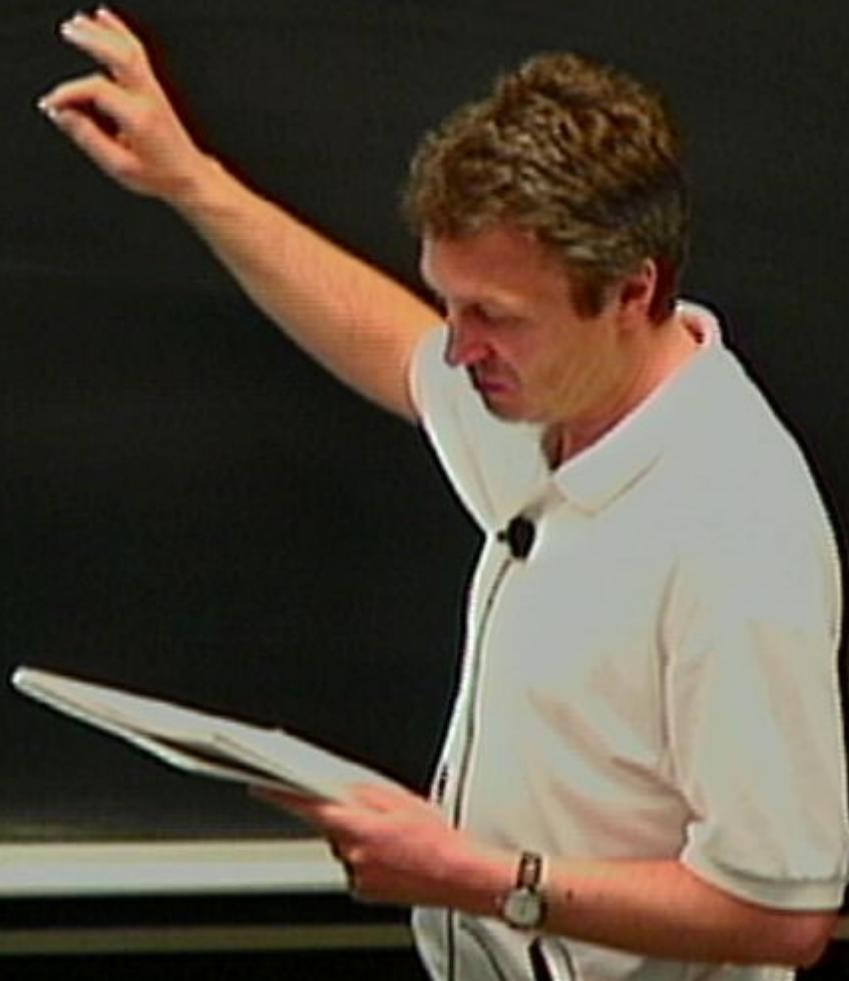
$\sigma$ -model is a coset:

$$D(\mathbb{R}, \mathbb{1}; \alpha) \times D(\mathbb{R}, \mathbb{1}; \alpha) / SL(2, \mathbb{R}) \times SU(4) \text{ diag}$$

$\sigma$ -model is a coset:

$$D(a, \alpha; d) \times D(a, \alpha; d) / SL(2, \mathbb{R}) \times SU(1)$$

$\mathbb{Z}_2$  symmetry



$\sigma$ -model is a coset:

$$D(A, \Lambda; d) \times D(R, \Lambda; d) / SL(2, \mathbb{R}) \times SO(n)$$

$\mathbb{Z}_n$  symmetry

$$M = G/H_0$$



$\sigma$ -model is a coset:

$$D(A, \Lambda; d) \times D(R, \Lambda; d) / SL(2, \mathbb{R}) \times SU(4)_{diag}$$

$\mathbb{Z}_2$  symmetry

$$M = G/H_0$$

$\Omega$  - automorphism of  $\Omega^{\mathbb{R}} = i$

$\sigma$ -model is a coset:

$$D(A, \Delta; d) \times D(A, \Delta; d) / SL(2, \mathbb{R}) \times SO(4)_{diag}$$

$\mathbb{Z}_2$  symmetry

$$M = G/H_0$$

$\Omega$  - automorphism of  $\hat{\mathfrak{g}}$  :  $\Omega^2 = id$

$\sigma$ -model is a coset:

$$D(A, \Lambda; d) \times D(A, \Lambda; d) / SL(2, \mathbb{R}) \times SO(4)_{diag}$$

$\mathbb{Z}_2$  symmetry

$$M = G/H_0$$

$\Omega$  - automorphism of  $\hat{\mathfrak{g}}$  :  $\Omega^n = id$

$\sigma$ -model is a coset:

$$D(\mathbb{R}, \mathbb{S}^1; \alpha) \times D(\mathbb{R}, \mathbb{S}^1; \alpha) / SL(2, \mathbb{R}) \times SO(4) \text{ diag}$$

$\mathbb{Z}_2$  symmetry

$$M = G / H_0$$

$\Omega$  - automorphism of  $\hat{\mathfrak{g}}$  .  $\Omega$

$$\Omega(h_n) = i^n h_n$$

$$\hat{\mathfrak{g}} = \underbrace{\mathfrak{h}_0}_{\mathfrak{B}} \oplus \underbrace{\mathfrak{h}_1}_{\mathfrak{F}} \oplus \underbrace{\mathfrak{h}_2}_{\mathfrak{B}} \oplus \underbrace{\mathfrak{h}_3}_{\mathfrak{F}}$$

$\sigma$ -model is a coset:

$$D(A, \Lambda; \alpha) \times D(R, \Lambda; \alpha) / SL(2, \mathbb{R}) \times SO(4) \text{ diag}$$

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$\sigma$ -model is a coset:

$$D(\alpha, \lambda; \alpha) \times D(\alpha, \lambda; \alpha) / SL(2, \mathbb{R}) \times SO(4) \text{ diag}$$

$\mathbb{Z}_2$  symmetry

$$M = G / H_0$$

$\Omega$  - automorphism of  $\hat{\mathfrak{g}}$  :  $\Omega^F = \text{id}$

$$\Omega(\mathfrak{h}_n) = i^n \mathfrak{h}_n$$

$$\Omega^2 = (-1)^F$$

$$\hat{\mathfrak{g}} = \mathfrak{h}_0 \oplus \mathfrak{h}_1 \oplus \mathfrak{h}_2 \oplus \mathfrak{h}_3$$

$\mathfrak{B} \qquad \mathfrak{F} \qquad \mathfrak{B} \qquad \mathfrak{F}$

$$g(x) \sim g(x) \cdot h(x)$$

$\Downarrow$   
 $G$                        $\Downarrow$   
                                  $H_1$

$$g^{-1} \partial_x g = J_{\mu_0} + J_{\mu_1} + J_{\mu_2} + J_{\mu_3}$$

$$g(x) \sim g(x) \cdot h(x)$$

$\Downarrow$   
 $G$

$\Downarrow$   
 $H$

$$g^{-1} \partial_\mu g = J_{\mu_0} + J_{\mu_1} + J_{\mu_2} + J_{\mu_3}$$

$\uparrow$   
gauge field

$$g(\sigma) \sim g(\omega) \quad h(\sigma) \in \mathbb{H}$$

$$g^{-1} \partial_\mu g = J_{\mu_0} + J_{\mu_1} + J_{\mu_2} + J_{\mu_3}$$

↑  
gauge field

$$\mathcal{S} = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \operatorname{Str} \left( \sqrt{-h} h \right)$$

$$g(\sigma) \sim g(\sigma) h(\sigma)$$

$$\mathbb{S}^1 \ni \sigma \quad \mathbb{H}^1$$

$$g^{-1} \partial_\mu g = J_{\mu 0} + J_{\mu 1} + J_{\mu 2} + J_{\mu 3}$$

↑  
gauge field

$$\mathcal{S} = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \operatorname{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu 2} J_{\nu 2} \right)$$

$$g(\omega) \sim g(\omega) h(\omega)$$

$$\ni \ni$$

$$G \ni H,$$

$$g^{-1} \partial_\mu g = J_{\mu 0} + J_{\mu 1} + J_{\mu 2} + J_{\mu 3}$$

↑  
gauge field

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \operatorname{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu 2} J_{\nu 2} + \epsilon^{\mu\nu} J_{\mu 1} \right)$$

$$g(\sigma) \sim g(\sigma) \quad h(\sigma)$$

$$\Downarrow \quad \Downarrow$$

$$G \quad H$$

$$g^{-1} \partial_\mu g = J_{\mu 0} + J_{\mu 1} + J_{\mu 2} + J_{\mu 3}$$

↑  
gauge field

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \text{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu 2} J_{\nu 1} J_{\mu 3} J_{\nu 0} \right)$$

Integrable!

$$g(\sigma) \sim g(\sigma) h(\sigma)$$

$$\Downarrow \quad \Downarrow$$

$$G \quad H$$

$$g^{-1} \partial_r g = J_{\mu_0} + J_{\mu_1} + J_{\mu_2} + J_{\mu_3}$$

↑  
gauge field

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \text{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu_2} J_{\nu_2} + \epsilon^{\mu\nu} J_{\mu_1} J_{\nu_3} \right)$$

Integrable!

$$g(\sigma) \sim g(\sigma) h(\sigma)$$

$$\mathbb{S}^1 \ni \mathbb{H}_1$$

$$g^{-1} \partial_x g = J_{\mu_0} + J_{\mu_1} + J_{\mu_2} + J_{\mu_3}$$

↑  
gauge field

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \text{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu_2} J_{\nu_2} + \epsilon^{\mu\nu} J_{\mu_1} J_{\nu_3} \right)$$

Integrable!

$$g(\sigma) \sim g(\sigma) h(\sigma)$$

$$\sigma \ni H,$$

$$g^{-1} \partial_\mu g = J_{\mu 0} + J_{\mu 1} + J_{\mu 2} + J_{\mu 3}$$

↑  
gauge field

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \text{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu 2} J_{\nu 2} + \epsilon^{\mu\nu} J_{\mu 1} J_{\nu 3} \right)$$

Integrable!

$$g(\sigma) \sim g(\sigma) h(\sigma)$$

$$\Downarrow$$

$$H.$$

$$g^{-1} \partial_r g = J_{\mu_0} + J_{\mu_1} + J_{\mu_2} + J_{\mu_3}$$

↑  
gauge field

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \text{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu_2} J_{\nu_2} + \epsilon^{\mu\nu} J_{\mu_1} J_{\nu_1} \right)$$

Integrable!

Lax current:  $L_\mu(\sigma; x) = J_{\mu_0} + \dots$

$$g(\sigma) \sim g(\sigma) h(\sigma)$$

$$\cong \mathbb{H}$$

$$g^{-1} \partial_r g = J_{\mu_0} + J_{\mu_1} + J_{\mu_2} + J_{\mu_3}$$

↑  
gauge field

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \text{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu_2} J_{\nu_2} + \epsilon^{\mu\nu} J_{\mu_1} J_{\nu_3} \right)$$

Integrable!

Lax current:  $L_{\mu}(\sigma; x) = J_{\mu_0} + \dots$

$$g(\sigma) \sim g(\sigma) h(\sigma)$$

$$\Downarrow \quad \Downarrow$$

$$G \quad H$$

$$g^{-1} \partial_r g = J_{\mu_0} + J_{\mu_1} + J_{\mu_2} + J_{\mu_3}$$

↑  
gauge field

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \operatorname{Str} \left( \sqrt{-h} h^{\mu\nu} J_{\mu_2} J_{\nu_2} + \epsilon^{\mu\nu} J_{\mu_1} J_{\nu_3} \right)$$

Integrable!

Lax current:  $L_\mu(\sigma; x) = J_{\mu_0} + \dots$

$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$

$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$

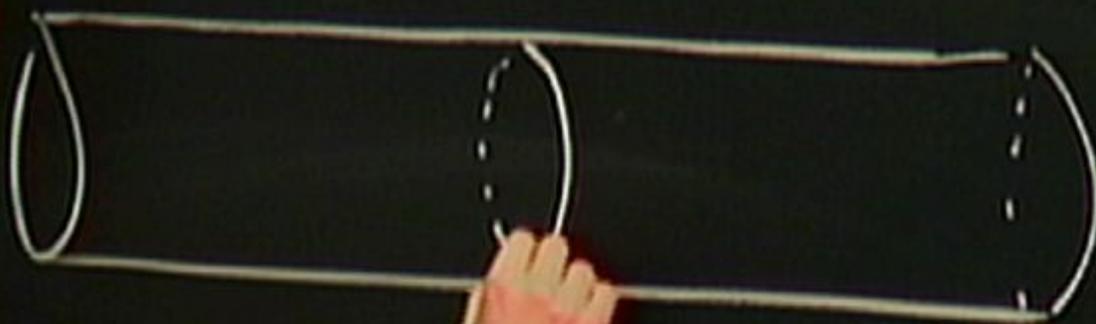


Eqs. of motion

$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$



Eqs. of motion



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Eqs. of motion



$$\partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$



Eqs. of motion



$$te \text{ Pexp} \oint ds^\mu L_\mu(\sigma; x)$$

$$\partial_\mu L_1 - \partial_\nu L_2 + [L_2, L_1] = 0 \quad \forall x$$



Eqs. of motion



to  $P_{exp} \oint ds^n L_\mu(\sigma; x) \Rightarrow \infty$  set of integrals of motion

(NZ)  $\{s_1, s_2, \dots\}$  for  $A_1 S_1 \times S_1 \times S_1 \times S_1$

$$\partial_t L_1 - \partial_x L_1 + [L_1, L_1] = 0 \quad \forall x$$



Eqs. of motion



to Perp  $\int ds^1 L_1(\sigma; x) \Rightarrow \infty$  set of integrals of motion

$$i\hbar \frac{d}{dt} L_1 - \partial_t L_1 + [L_1, L_1] = 0 \quad \forall x$$



Eq. of motion



to Pexp  $\int ds L_1(\sigma; x) \Rightarrow \infty$  set of integrals of motion

$$0 = \mathbb{H} \left[ \frac{d}{ds} + L_1(\sigma; x; x) \right] \psi = 0$$

$$\partial_t L_1 - \partial_i L_i + [L_H, L_1] = 0 \quad \forall x$$



Eq<sub>s</sub> of motion



to  $P_{exp} \int d\sigma^\mu L_\mu(\sigma; x) \Rightarrow \infty$  set of integrals of motion

$$\left[ \frac{d}{d\sigma} + L_1(\sigma; \tau; x) \right] \psi = 0$$

$$\partial_\mu L_1 - \partial_\nu L_1 + [L_\mu, L_\nu] = 0 \quad \forall x$$



Eqs of motion



to  $P_{exp} \int d\sigma^\mu L_\mu(\sigma; x) \Rightarrow \infty$  set of integrals of motion

$$\left[ \frac{d}{d\sigma} + L_1(\sigma; \tau; x) \right] \psi = 0$$

$$i\hbar \dot{L}_\mu - \partial_\mu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$



Eqs of motion



to Pexp  $\oint ds^\mu L_\mu(\sigma; x) \Rightarrow \infty$  set of integrals of motion

$$\left[ \frac{d}{ds} - \partial_\mu \dot{x}^\mu \right] \psi = 0$$

Band spectrum:

$$i\hbar \partial_t L_1 - \partial_1 L_1 + [L_1, L_1] = 0 \quad \forall x$$



Eqs. of motion



to Perp  $\oint d\sigma^k L_k(\sigma; x) \Rightarrow \infty$  set of integrals of motion

$$\left[ \frac{d}{d\sigma} + L_1(\sigma; x; x) \right] \psi = 0$$

Band spectrum:

$$L_1 - \tau_1 L_2 + [L_1, L_2] = 0 \quad \forall x$$



Eqs of motion



to  $P \exp \oint d\sigma^\mu L_\mu(\sigma; x) \Rightarrow \infty$  set of integrals of motion

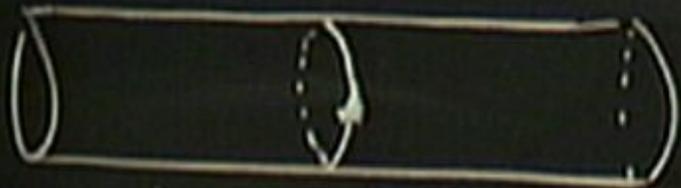
$$\left[ \frac{d}{d\sigma} + L_1(\sigma; z; x) \right] \psi = 0$$

Band spectrum:

$$i\hbar \dot{L}_\mu - \dot{\tau}_\mu L_\mu + [L_\mu, L_\nu] = 0 \quad \forall x$$



Eqs of motion



$\int_0^{2\pi} d\sigma^\mu L_\mu(\sigma; x) \Rightarrow \infty$  set of integrals of motion

$$\left[ \frac{d}{dt} + \mathcal{L}(\sigma; z; x) \right] \Psi = 0$$

Band spectrum:

$$\Psi(\sigma + 2\pi; x) = \mathcal{U}(x) \Psi(\sigma; x)$$

$$L_1 - \tau_1 L_2 + [L_1, L_2] = 0 \quad \forall x$$



Eqs. of motion



$$L_1 \exp \int d\sigma^{\mu\nu} L_{\mu\nu}(\sigma; x)$$

$\Rightarrow \infty$  set of integrals of motion

$$\left[ \frac{d}{d\sigma} + L_1(\sigma; z; x) \right] \Psi = 0$$

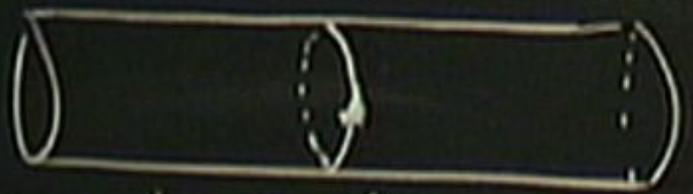
Band spectrum:

$$\Psi(\sigma + 2\pi; x) = \mathcal{M}(x) \Psi(\sigma; x)$$

$$L_1 - \tau_1 L_2 + [L_1, L_2] = 0 \quad \forall x$$



Eqs of motion



$$L_1 \exp \int d\sigma L_2(\sigma; x)$$

$\Rightarrow \infty$  set of integrals of motion

$$\left[ \frac{d}{d\sigma} + L_1(\sigma; x) \right] \Psi = 0$$

Band spectrum:

$$\Psi(\sigma + 2\pi; x) = \mathcal{M}(x) \Psi(\sigma; x)$$

$$U(x) = U^{-1} e^{i H_0 P_0(x)} U$$

Cartan

$$\mathcal{U}(x) = U^{-1} e^{i H_2 P_e(x)} U$$

Cartan generators

$P_e(x)$

$$\mathcal{U}(x) = U^{-1} e^{i H_x P_e(x)} U$$

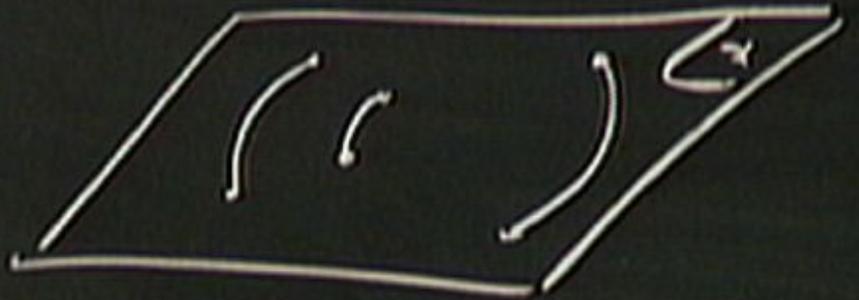
Cartan generators

$P_e(x)$  - quasi-momenta

$$U(x) = U^{-1} e^{\int^x H_1 P_c(z)} U$$

Cartan generators

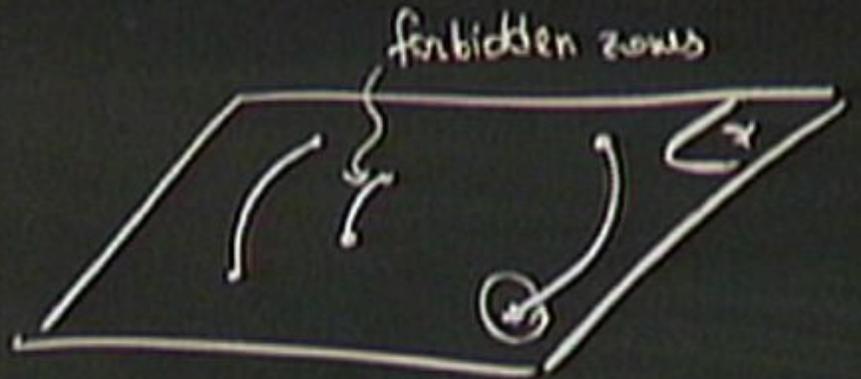
$P_c(x)$  - quasi-momenta



$$U(x) = U^{-1} e^{iH_1 P_0(x)} U$$

Cartan generators

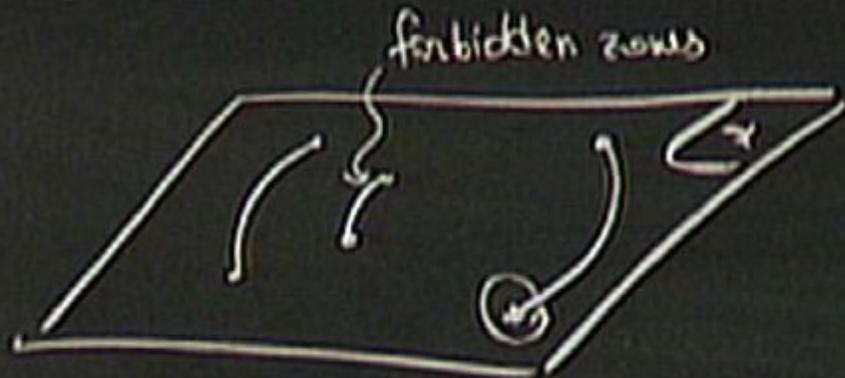
$P_0(x)$  - quasi-momenta



$$U(x) = U^{-1} e^{iH_1 P_1(x)} U$$

Cartan generators

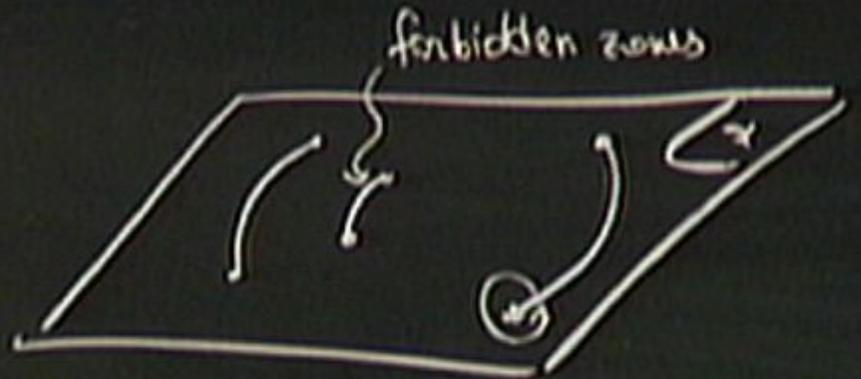
$P_1(x)$  - quanta momenta



$$U(x) = U^{-1} e^{iH_1 P_c(x)} U$$

Cartan generators

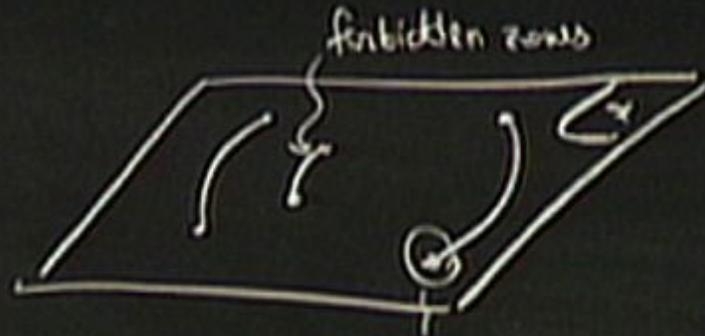
$P_c(x)$  - quasi-momenta



$$\mathcal{U}(x) = \mathcal{U}^{-1} e^{iH_x P_c(x)} \mathcal{U}$$

Cartan generators

$P_c(x)$  - quasi-momenta

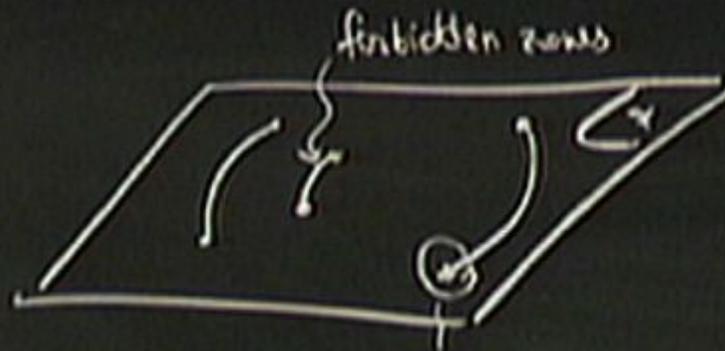


Monodromy & Weyl group

$$U(x) = U^{-1} e^{iH_1 P_1(x)} U$$

Cartan generators

$P_1(x)$  - quasi-momenta

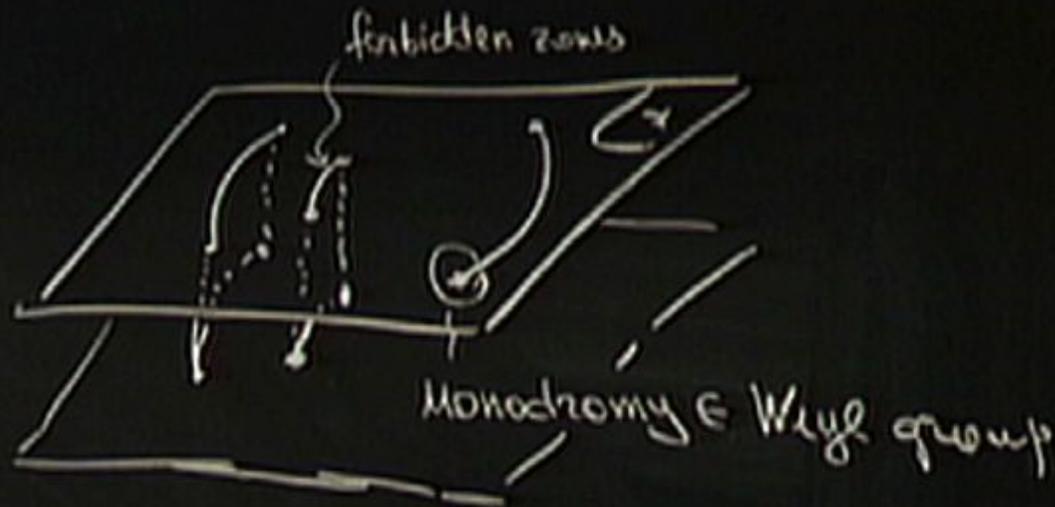


Monochromy & Weyl group

$$U(x) = U^{-1} e^{iH_2 P_c(x)} U$$

Cartan generators

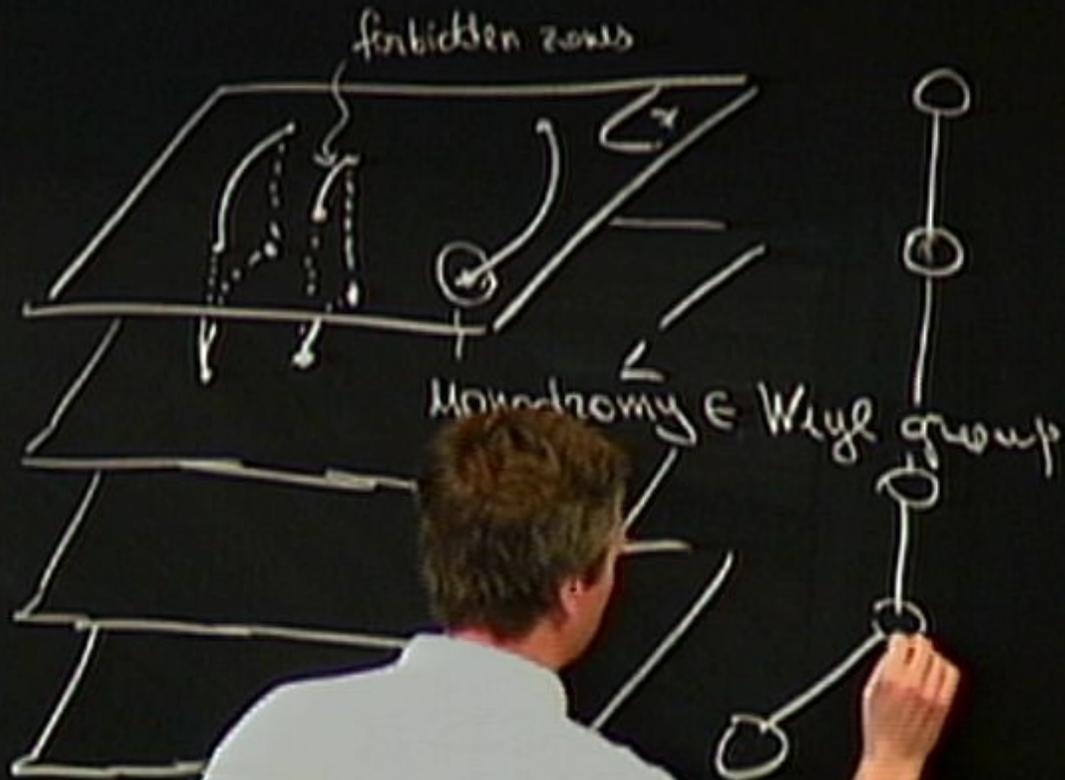
$P_c(x)$  - quasi-momenta



$$U(x) = U^{-1} e^{\int H_1 P_c(x)} U$$

Cartan generators

$P_c(x)$  - quasi-momenta



$$U(x) = U^{-1} e^{iH_2 P_c(x)} U$$

Cartan generators

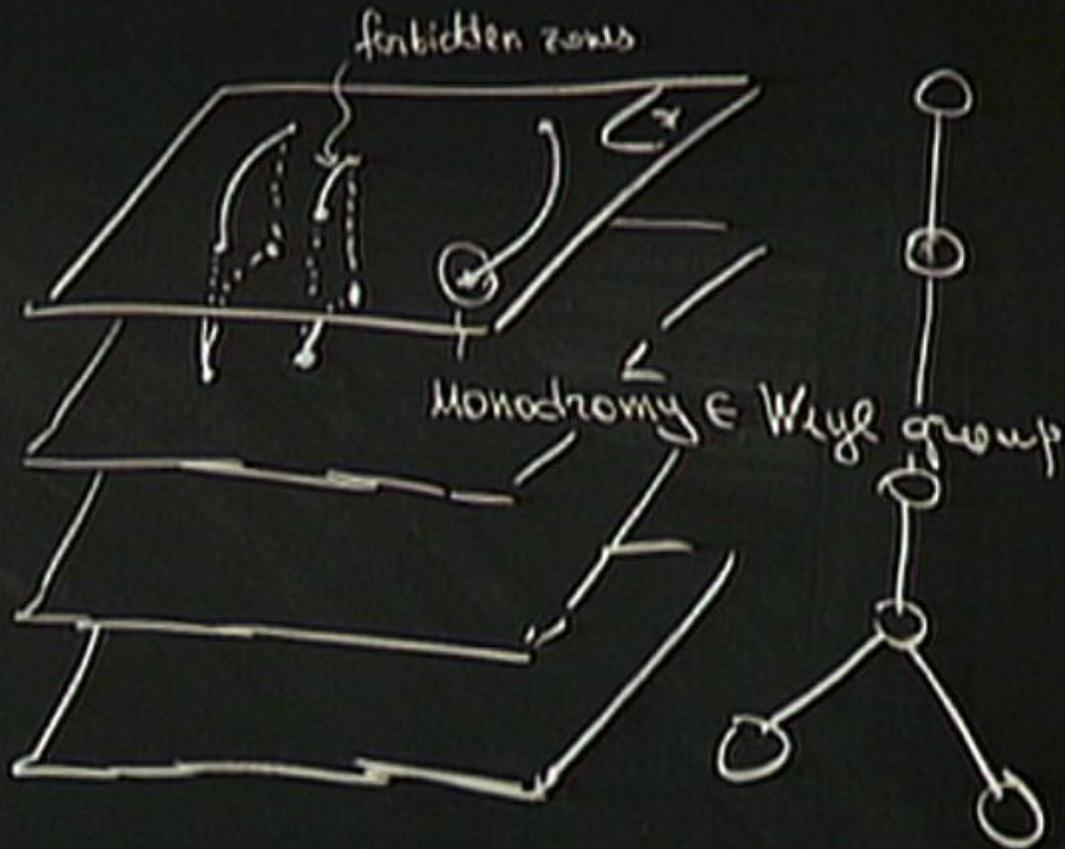
$P_c(x)$  - quasi-momenta



$$U(x) = U^{-1} e^{iH_0 P_c(x)} U$$

Cartan generators

$P_c(x)$  - quasi-momenta

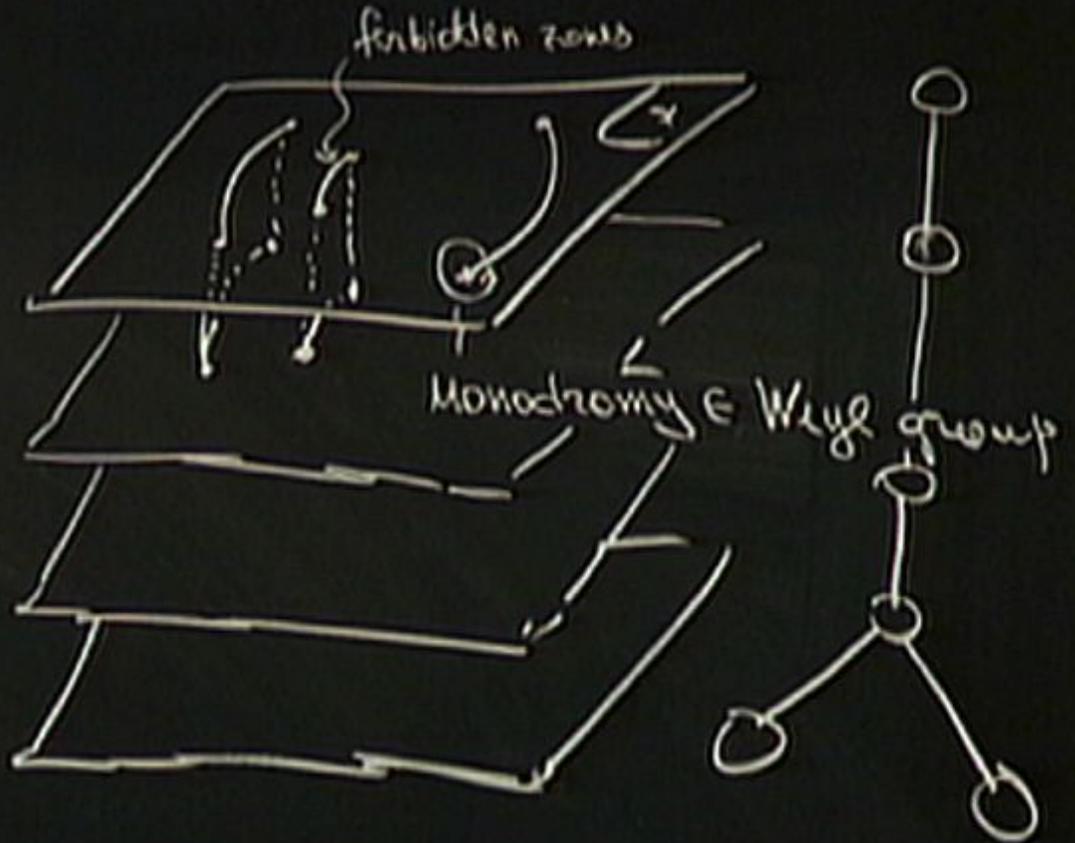


$$U(x) = U^{-1} e^{\int^x P_2(z) dz} U$$

Cartan generators

$P_2(x)$  - quasi-momenta

$$P_2(x) = \frac{\alpha - \alpha + 2\alpha m_1}{\alpha - 1} + \int dy \frac{P_2(y)}{\alpha - 1}$$

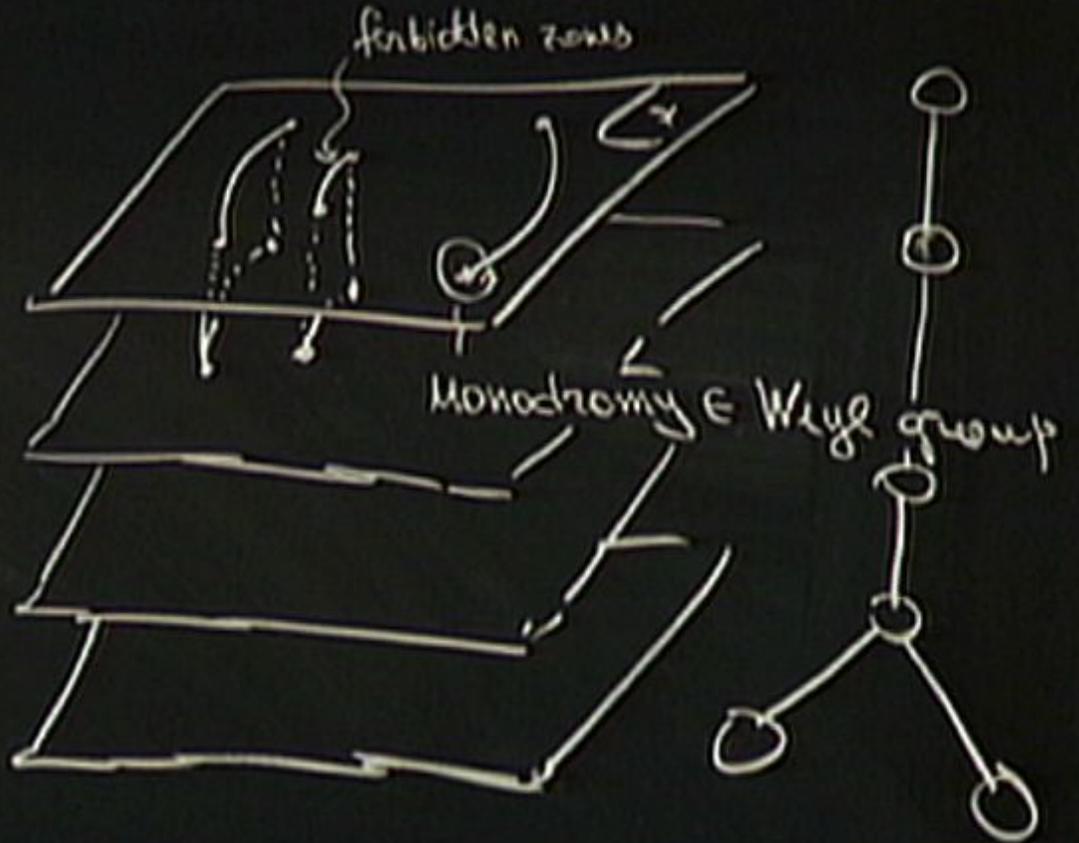


$$U(x) = U^{-1} e^{\int^x P_2(z) dz} U$$

Cartan generators

$P_2(x)$  - quasi-momenta

$$P_2(x) = -\frac{x_1 x + 2m_1}{x^2 - 1} + \int dy \frac{P_2(y)}{x - y}$$



$$\mathcal{M}(x) = \mathcal{U} \quad e^{\mathcal{H}} \quad \mathcal{U}^{-1}$$

Cartan generators

$P_\ell(x)$  - quasi-momenta

$$P_\ell(x) = -\frac{x_\ell x + 2\pi i m_\ell}{x^2 - 1} + \int dy \frac{P_\ell(y)}{x - y}$$

$$\Rightarrow S_{\ell m} \int dy \frac{P_m(y)}{x - y}$$

$$U(x) = U \quad e^{\eta} \quad U$$

Cartan generators

$P_\ell(x)$  - quasi-momenta

$$P_\ell(x) = -\frac{x_\ell x + 2\pi i m_\ell}{x^2 - 1} + \int dy \frac{P_\ell(y)}{x - y}$$

$$\Rightarrow \sum_{\ell m} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

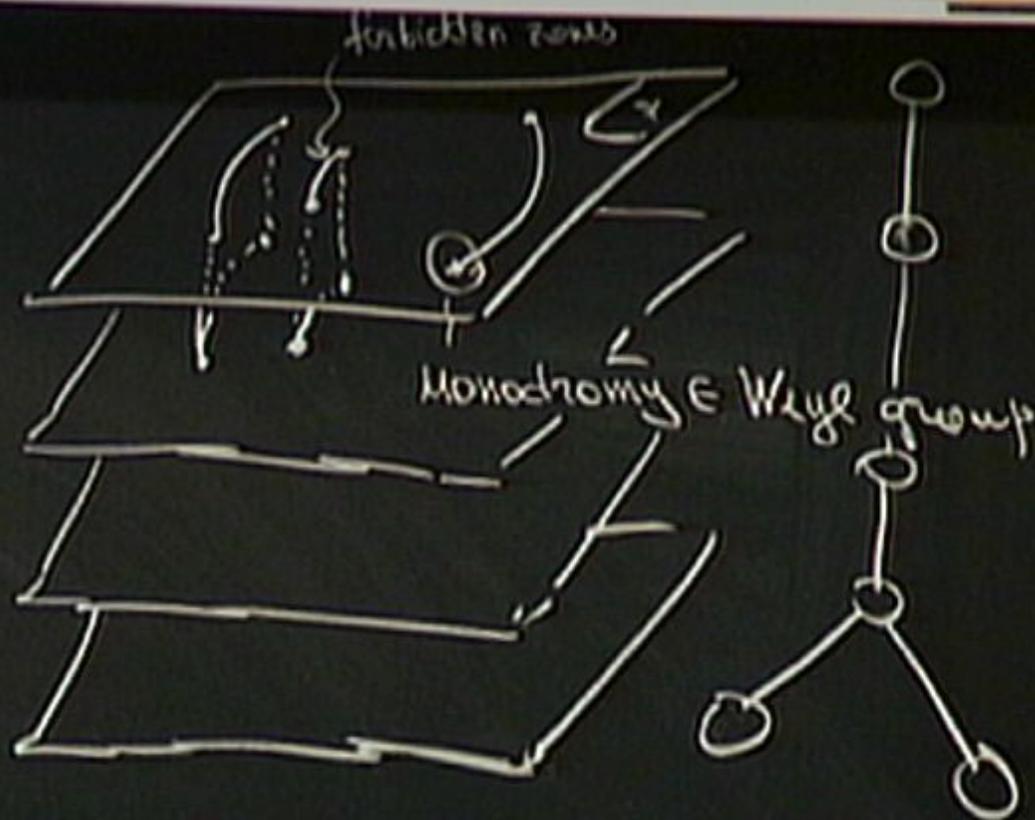
$$U(x) = U \quad e^{i\pi} \quad U$$

Cartan generators

$P_\ell(x)$  - quasi-momenta

$$P(x) = -\frac{x(x+2m_1)}{x^2-1} + \int dy \frac{P_1(y)}{x-y}$$

$$\sim \int \frac{dy}{dy} \frac{P_1(y)}{x-\frac{1}{y}}$$



$$PSU(4|2) \times PSU(1|2)$$

$$OSp(4|2) \times OSp(4|2)$$

$P_\ell(x)$  - quasi-momenta

$$P_\ell(x) = -\frac{x_\ell x + 2\pi m_\ell}{x^2 - 1} + \int dy \frac{P_\ell(y)}{x - y}$$

$$S_{\ell m} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$$m \int dy \frac{P_m(y)}{x - y} - A_{\ell k} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$P_e(x)$  - quasi-momenta

$$P_e(x) = -\frac{x_c x + 2\pi M_c}{x^2 - 1} + \int dy \frac{P_e(y)}{x - y}$$

$$S_{em} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$$A_{em} \int dy \frac{P_m(y)}{x - y} - A_{ek} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$P_\ell(x)$  - quasi-momenta

$$P_\ell(x) = -\frac{x_\ell x + 2\pi M_\ell}{x^2 - 1} + \int dy \frac{P_\ell(y)}{x - y}$$

$$S_{\ell m} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$$A_{\ell m} \int dy \frac{P_m(y)}{x - y} - A_{\ell k} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$x$ -symmetry gauge:

$$K^+ \theta = 0$$

$$K^+ P^3 K^- = 0 \Rightarrow x^3 \text{ decouples.}$$

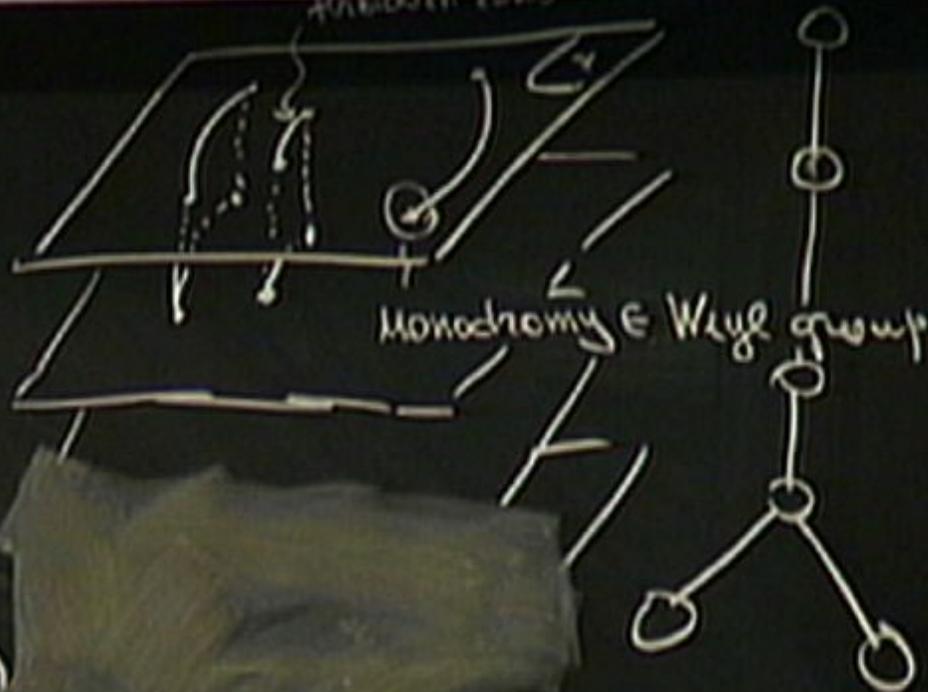
Cartan generators

$P_i(x)$  - quasi-momenta

$$P_i(x) = -\frac{x_i x + 2m_i}{x^2 - 1} + \int dy \frac{P_i(y)}{x - y}$$

$$\rightarrow S_{im} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

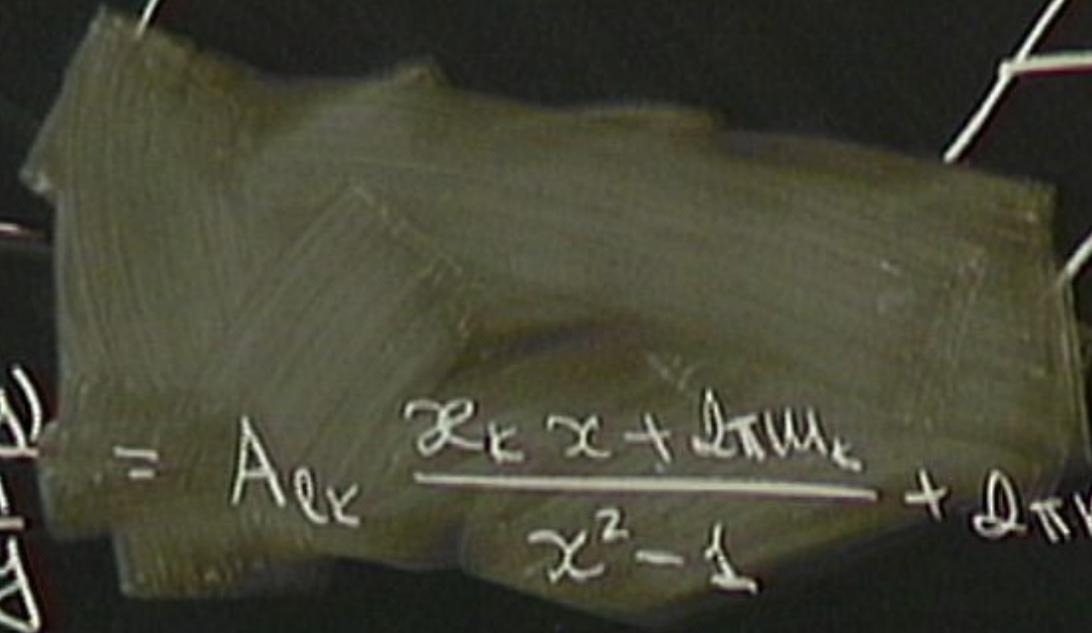
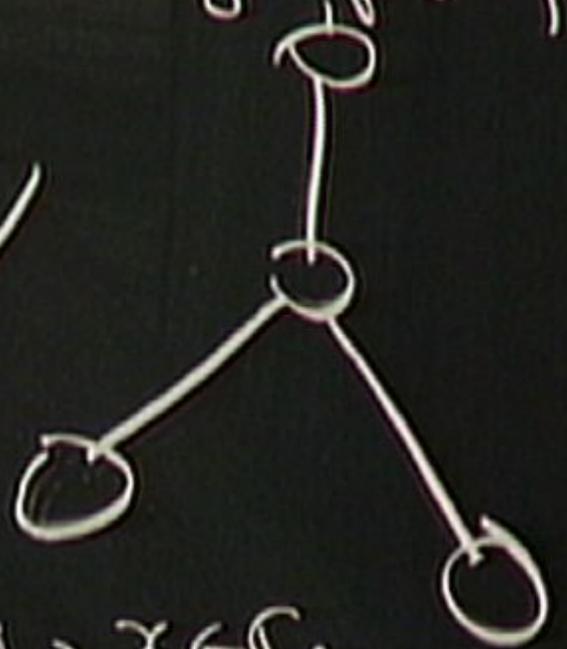
$$A_{em} \int dy \frac{P_m(y)}{x - y} - A_{ek} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$



$$PSU(1|2) \times PSU(1|2)$$

$$OSp(4|2) \times OSp(4|2)$$

Monodromy & Weyl group



$$p_m(y) = A_{\ell x} \frac{x_{\ell} x + \sum_{i=1}^{\ell} m_i x_i}{x^2 - 1} + \sum_{i=1}^{\ell} m_i x_i, \quad x \in \mathbb{C}_{\neq 0}$$

$$\rightarrow \text{OSp}(4|2) \times \text{OSp}(4|2)$$

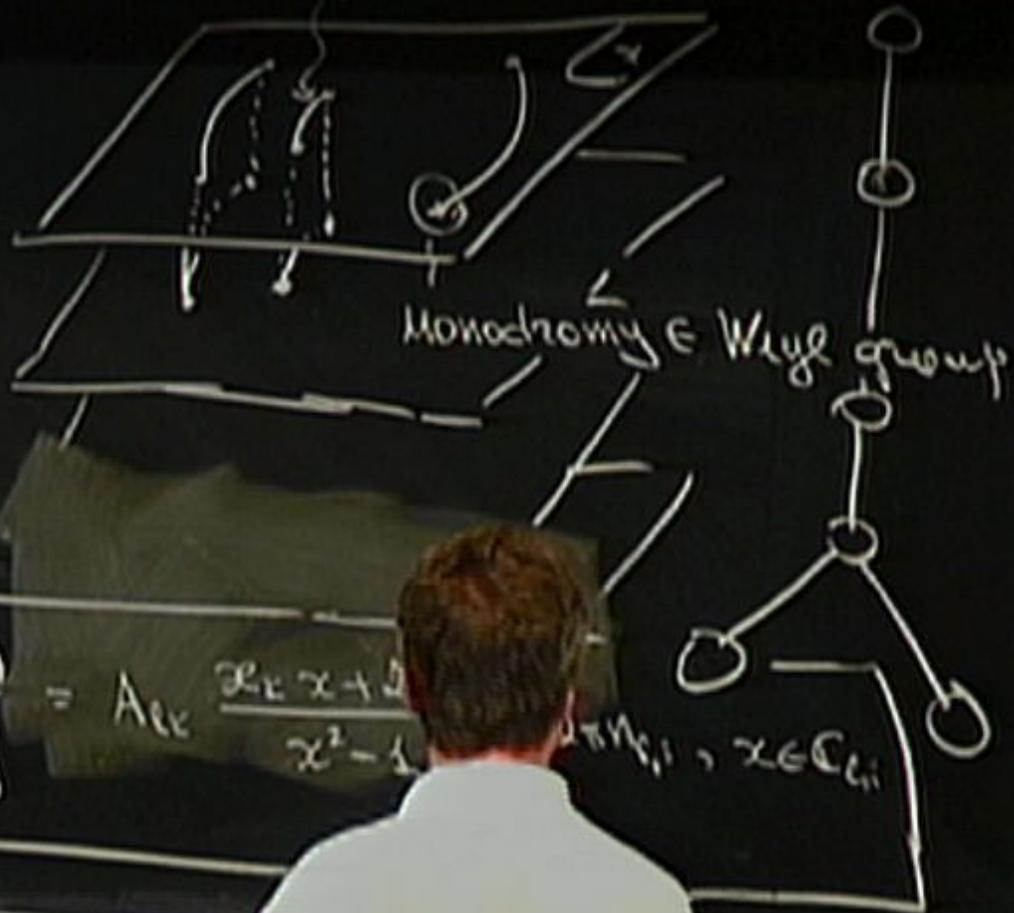
Cartan generators

$P_i(x)$  - quasi-momenta

$$P_i(x) = -\frac{x_i x + 2m_i}{x^2 - 1} + \int dy \frac{P_i(y)}{x - y}$$

$$\Rightarrow S_{im} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$$A_{em} \int dy \frac{P_m(y)}{x - y} - A_{em} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}} = A_{ek} \frac{x_k x + 2m_k}{x^2 - 1}$$



$$PSU(1,1|2) \times PSU(1,1|2)$$

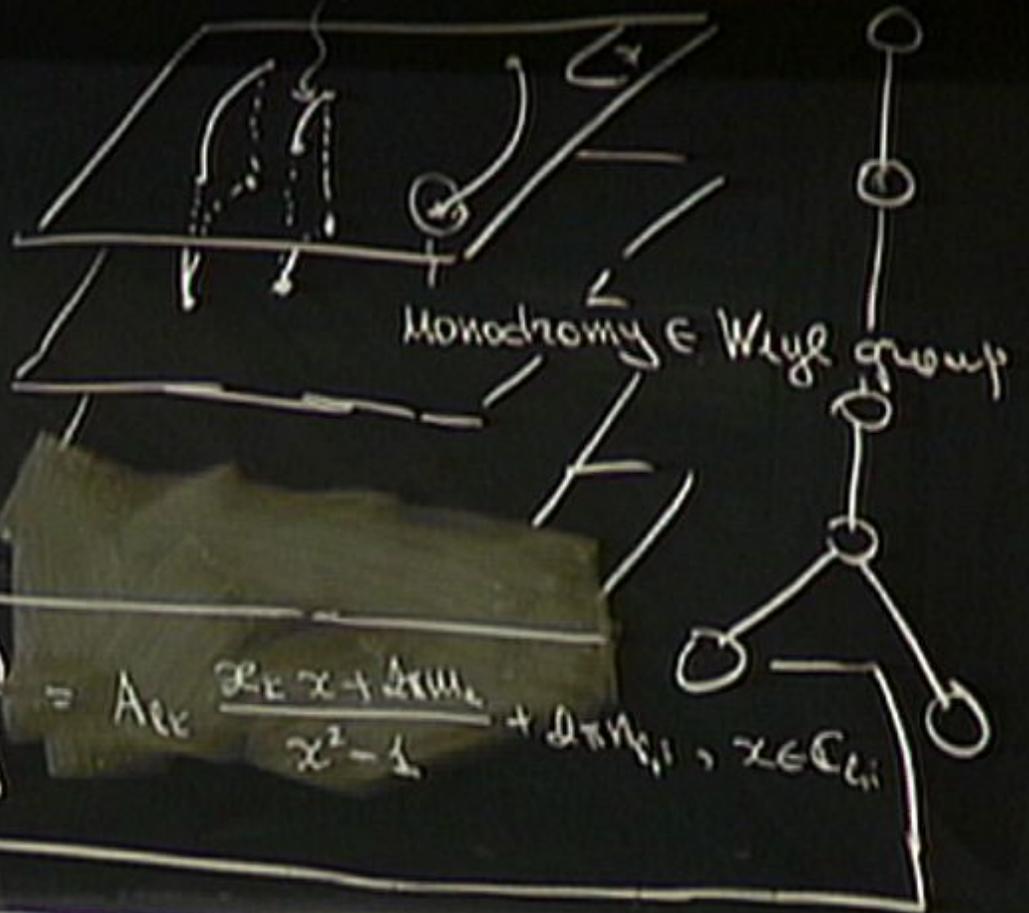
Cartan generators

$P_i(x)$  - quasi-momenta

$$P_i(x) = -\frac{x_i x + 2\pi m_i}{x^2 - 1} + \int dy \frac{P_i(y)}{x - y}$$

$$\rightarrow S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$$A_{cm} \int dy \frac{P_m(y)}{x - y} - A_{cl} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}} = A_{cl} \frac{x_i x + 2\pi m_i}{x^2 - 1} + 2\pi n_{i,1}, x \in \mathbb{C}_{cl}$$



$$PSU(4|2) \times PSU(4|2)$$

$$OSp(4|2) \times OSp(4|2)$$

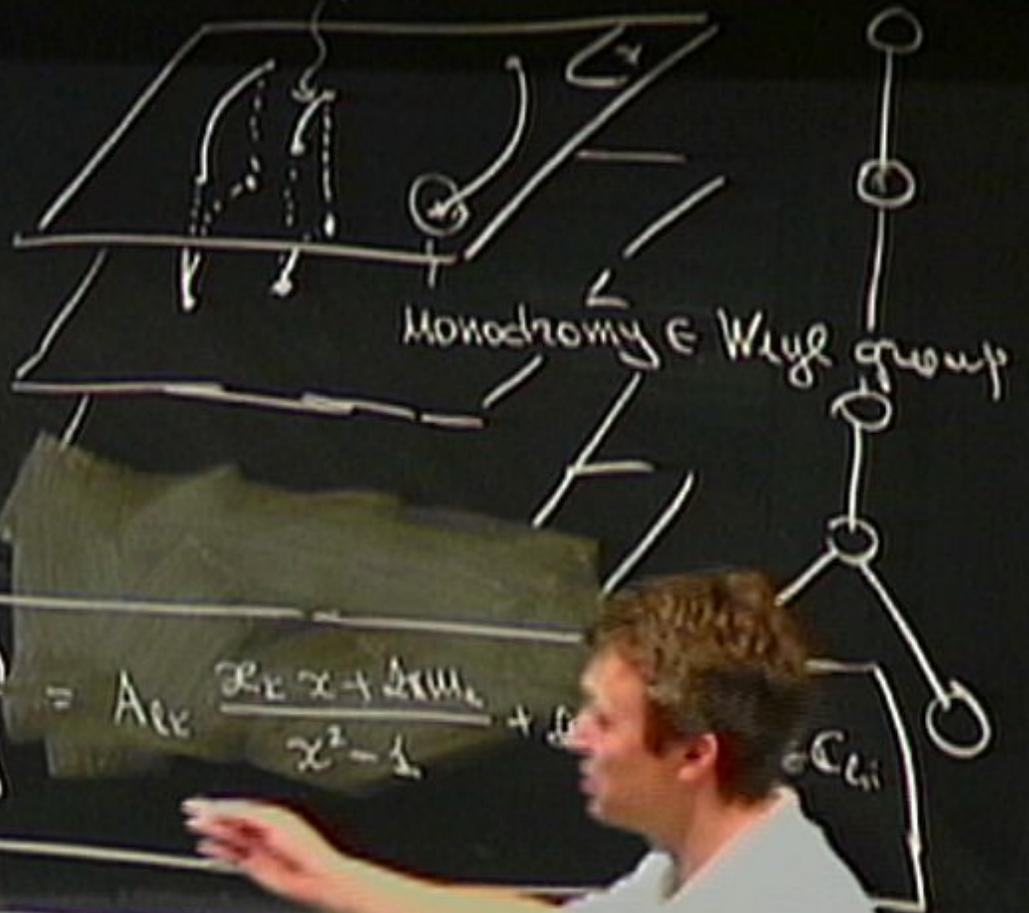
# Cartan generators

$P_i(x)$  - quasi-momenta

$$P_i(x) = -\frac{x_L x + 2\mu_{Li}}{x^2 - 1} + \int dy \frac{P_i(y)}{x - y}$$

$$\rightarrow S_{im} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$$A_{em} \int dy \frac{P_m(y)}{x - y} - A_{ek} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}} = A_{ek} \frac{x_L x + 2\mu_{Li}}{x^2 - 1} + \dots$$



$$PSU(4|2) \times PSU(4|2)$$

$$OSp(4|2) \times OSp(4|2)$$

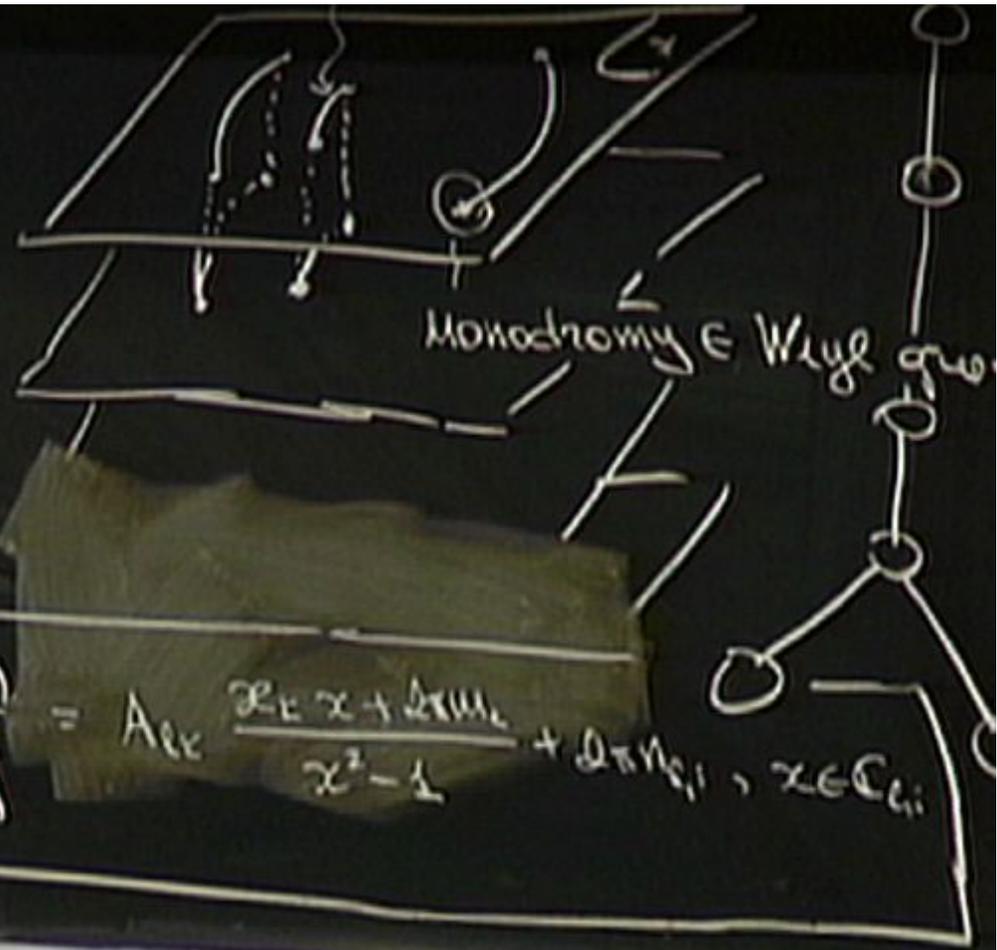
# Cartan generators

$P_c(x)$  - quasi-momenta

$$P(x) = -\frac{x_c x + 2\pi m_c}{x^2 - 1} + \int dy \frac{P(y)}{x - y}$$

$$\leftrightarrow S_{em} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$$A_{em} \int dy \frac{P_m(y)}{x - y} - A_{ek} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}} = A_{ek} \frac{x_c x + 2\pi m_c}{x^2 - 1} + 2\pi n_{c1}, \quad x \in \mathbb{C}_{c1}$$



$$PSU(1|1|2) \times PSU(1|1|2)$$

$$OSp(4|2) \times OSp(4|2)$$

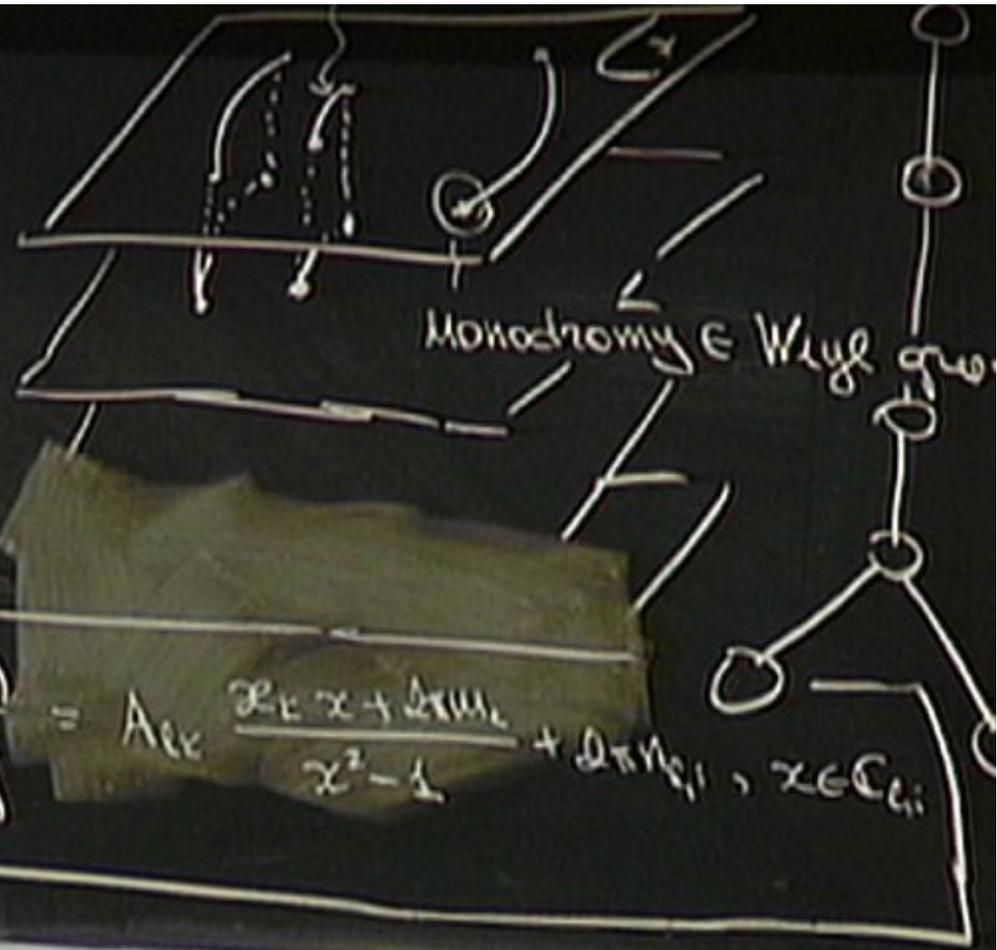
# Cartan generators

$P_c(x)$  - quasi-momenta

$$P_c(x) = -\frac{x_c x + 2\pi m_c}{x^2 - 1} + \int dy \frac{P_c(y)}{x - y}$$

$$\rightarrow S_{em} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}}$$

$$A_{em} \int dy \frac{P_m(y)}{x - y} - A_{ek} S_{km} \int \frac{dy}{y^2} \frac{P_m(y)}{x - \frac{1}{y}} = A_{ek} \frac{x_c x + 2\pi m_c}{x^2 - 1} + 2\pi n_{c1}, \quad x \in \mathbb{C}_{cl}$$



$$PSU(1|2) \times PSU(1|2)$$

$$OSp(4|2) \times OSp(4|2)$$

$x$ -symmetry gauge:

$$K^+ \theta = 0$$

$$K^+ P^3 K^- = 0 \Rightarrow x^3 \text{ decouples.}$$

$$U(x) = U^{-1} e^{iH_L P_L(x)} U$$

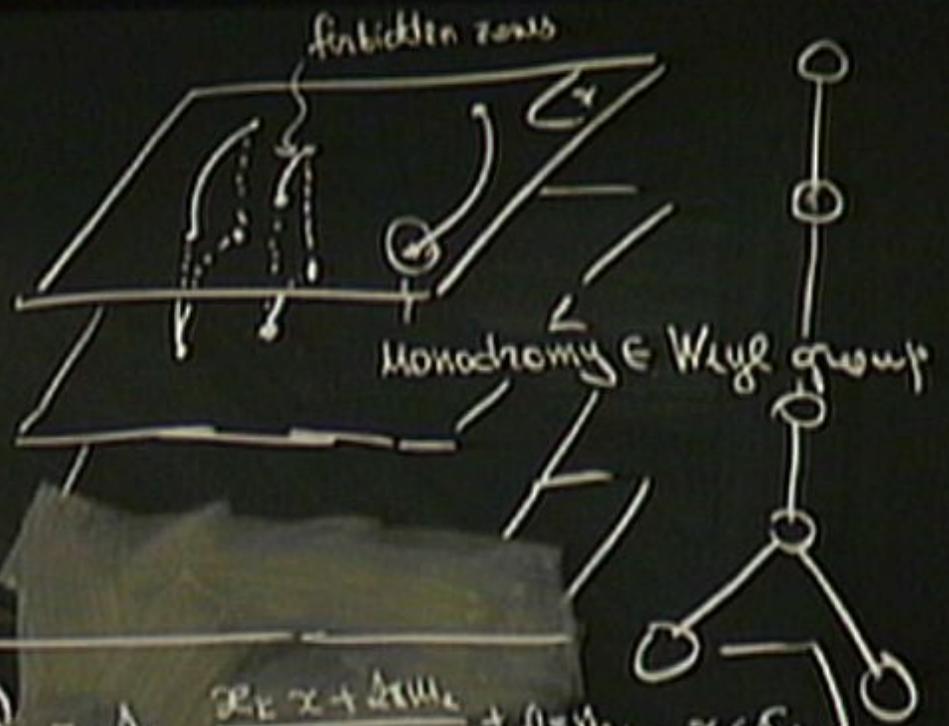
Cartan generators

$P_L(x)$  - quasi-momenta

$$P_L(x) = -\frac{x_L x + 2\eta_{11} x}{x^2 - 1} + \int dy \frac{P_L(y)}{x-y}$$

$$\rightarrow S_{11} \int \frac{dy}{y^2} \frac{P_L(y)}{x - \frac{1}{y}}$$

$$A_{11} \int dy \frac{P_L(y)}{x-y} - A_{11} S_{11} \int \frac{dy}{y^2} \frac{P_L(y)}{x - \frac{1}{y}} = A_{11} \frac{x_L x + 2\eta_{11} x}{x^2 - 1} + 2\eta_{11} \eta_{11}, x \in \mathbb{C}_1$$



$A_{em}$  - Cartan matrix

$$\alpha_{\mathbb{R}}(H_i) =$$

$A_{lm}$  - Cartan matrix

$$\alpha_{\mathbb{R}}(H_l) = H_m \varrho_l$$

$A_{lm}$  - Cartan matrix

$$\sum_l \alpha_l(H_k) = H_m S_{ml}$$

$A_{em}$  - Cartan matrix

$$\mathcal{R}(H_k) = H_m S_{ml}$$

$$A = \begin{pmatrix} 4 \sin^2 \phi & 1 \sin \phi \\ -2 \sin^2 \phi & 0 \end{pmatrix}$$

$A_{em}$  - Cartan matrix

$$\mathcal{R}(H_k) = H_m S_{ml}$$

$$A = \begin{pmatrix} 4 \sin^2 \phi & -2 \sin^2 \phi & 0 \\ -2 \sin^2 \phi & 0 & 0 \\ 0 & -2 \cos^2 \phi & 0 \end{pmatrix}$$

$A_{em}$  - Cartan matrix

$$\Sigma_{\mu\nu}(H_\nu) = H_\mu S_{\mu\nu}$$

$$A = \begin{pmatrix} 4 \sin^2 \phi & -2 \sin^2 \phi & 0 \\ -2 \sin^2 \phi & 0 & -2 \cos^2 \phi \\ 0 & -2 \cos^2 \phi & 4 \cos^2 \phi \end{pmatrix}$$

$A_{em}$  - Cartan matrix

$$\Delta_{\mathbb{R}}(H_L) = H_m S_{ml}$$

$$A = \begin{pmatrix} 4 \sin^2 \phi & -2 \sin^2 \phi & 0 \\ -2 \sin^2 \phi & 0 & -2 \cos^2 \phi \\ 0 & -2 \cos^2 \phi & 4 \cos^2 \phi \end{pmatrix} \otimes 1$$

$$S = 1 \otimes \sigma^z$$

$A_{cm}$  - Cartan matrix

$$\Omega(H_i) = H_m S_{mi}$$

$$A = \begin{pmatrix} 4 \sin^2 \phi & -2 \sin^2 \phi & 0 \\ -2 \sin^2 \phi & 0 & -2 \cos^2 \phi \\ 0 & -2 \cos^2 \phi & 4 \cos^2 \phi \end{pmatrix} \otimes 1$$

$$S = 1 \otimes \sigma^z$$

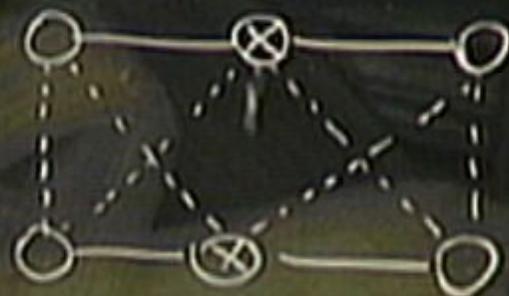


$A_{lm}$  - Cartan matrix

$$\Omega(H_i) = H_m S_{mi}$$

$$A = \begin{pmatrix} 4 \sin^2 \phi & -2 \sin^2 \phi & 0 \\ -2 \sin^2 \phi & 0 & -2 \cos^2 \phi \\ 0 & -2 \cos^2 \phi & 4 \cos^2 \phi \end{pmatrix} \otimes 1$$

$$S = 1 \otimes S^1$$

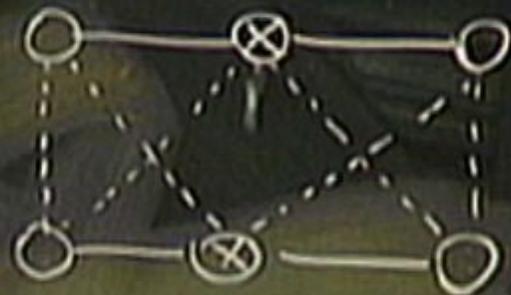


$A_{cm}$  - Cartan matrix

$$\Omega(H_i) = H_m S_{mi}$$

$$A = \begin{pmatrix} 4 \sin^2 \phi & -2 \sin^2 \phi & 0 \\ -2 \sin^2 \phi & 0 & -2 \cos^2 \phi \\ 0 & -2 \cos^2 \phi & 4 \cos^2 \phi \end{pmatrix} \otimes 1$$

$$S = 105^4$$



Bethe eq:

$$e^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

F

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Bethe eq:

$$e^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = \sum_j \varepsilon(p_j)$$

$$e^{ip} = \frac{\chi^+}{\chi^-}$$

$$\varepsilon = i \left( \frac{1}{\chi^+} - \frac{1}{\chi^-} \right)$$

Bethe eq:

$$e^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = \sum_j \varepsilon(p_j)$$

$$\frac{\lambda^+}{\lambda^-}$$

$$\left( \frac{1}{\lambda^+} - \frac{1}{\lambda^-} \right)$$

$$\lambda^+ + \frac{\Delta}{\lambda^+} = \lambda + \frac{1}{\lambda} \pm \frac{\Delta}{\lambda}$$

Bethe eq:

$$e^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = \sum_j \varepsilon(p_j)$$

$$e^{ip} = \frac{x^+}{x^-}$$

$$\varepsilon = i \left( \frac{-1}{x^+} - \frac{1}{x^-} \right)$$

$$x^+ + \frac{\Delta}{x^+} = x + \frac{1}{x} \pm \frac{ikL}{2}$$



Bethe eqs:

$$e^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = \sum_j \varepsilon(p_j)$$

$$e^{ip} = \frac{x^+}{x^-}$$

$$\varepsilon = i \left( \frac{1}{x^+} - \frac{1}{x^-} \right)$$

$$\frac{ik}{L} = \frac{1}{x} + x + \frac{1}{x^2} + x^2 + \frac{1}{x^3} + x^3 + \dots$$

Bethe eqs:

$$\theta^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = \sum_j \varepsilon(p_j)$$

$$\theta^{ip} = \frac{x^+}{x^-}$$

$$\varepsilon = i \left( \frac{1}{x^+} - \frac{1}{x^-} \right)$$

$$\frac{i\sqrt{K}}{x} \neq x + \frac{1}{x} + \frac{\Delta}{x^2} + \frac{1}{x^2}$$

Bethe eqs:

$$\psi^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = \sum_j \varepsilon(p_j)$$

$$\psi^{ip} = \frac{\lambda^+}{\lambda^-}$$

$$\varepsilon = i \left( \frac{1}{\lambda^+} - \frac{1}{\lambda^-} \right)$$

$$\frac{ik}{L} = \frac{1}{\lambda} + \lambda + \frac{\Delta}{\lambda^2} - \lambda + \frac{1}{\lambda} + \lambda$$

Bethe eq:

$$\varphi^{ip_j L} = \prod_{k \neq j} S(p_j, p_k)$$

$$E = \sum_j \varepsilon(p_j)$$

$$\varphi^{ip} = \frac{\lambda^+}{\lambda^-}$$

$$\varepsilon = i \left( \frac{1}{\lambda^+} - \frac{1}{\lambda^-} \right)$$

$$\frac{E}{L} = \frac{1}{L} \sum_j \varepsilon(p_j) = \frac{1}{L} \sum_j \left( \frac{1}{\lambda_j^+} - \frac{1}{\lambda_j^-} \right)$$

$\sigma$ -model is a coset:

$$D(\alpha, \lambda; \alpha) \times D(\alpha, \lambda; \alpha) / SL(2, \mathbb{R}) \times SO(4) \text{ diag}$$

$\mathbb{Z}_4$  symmetry

$$\mathcal{M} = G/H_0$$

$\Omega$  - automorphism of  $\hat{\mathfrak{g}}$  :  $\Omega^4 = \text{id}$

$$\Omega(\mathfrak{h}_n) = i^n \mathfrak{h}_n$$

$$\Omega^2 = (-1)^F$$

$$\hat{\mathfrak{g}} = \mathfrak{h}_0 \oplus \mathfrak{h}_1 \oplus \mathfrak{h}_2 \oplus \mathfrak{h}_3$$