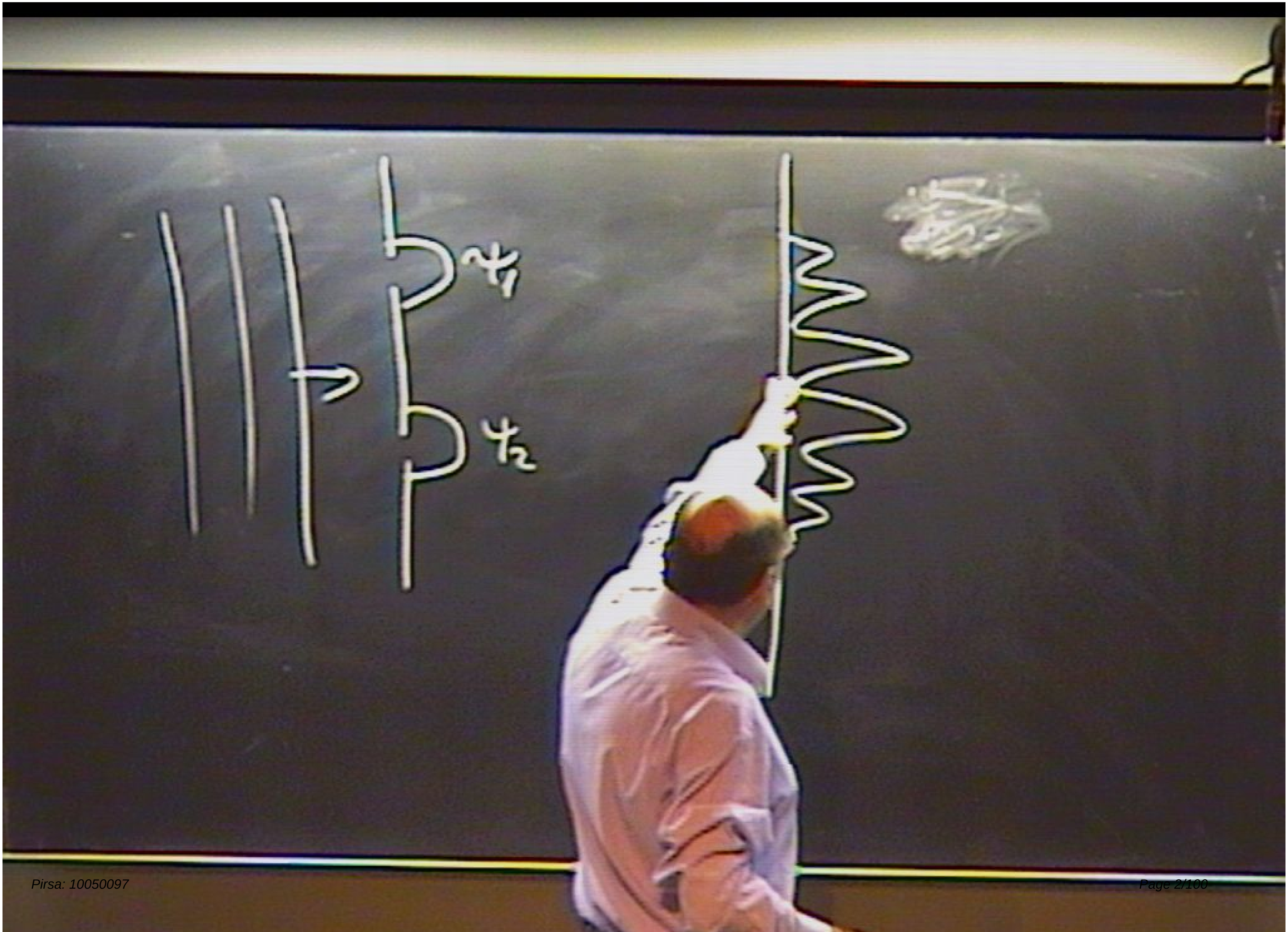
Title: Dynamical quantum nonlocality

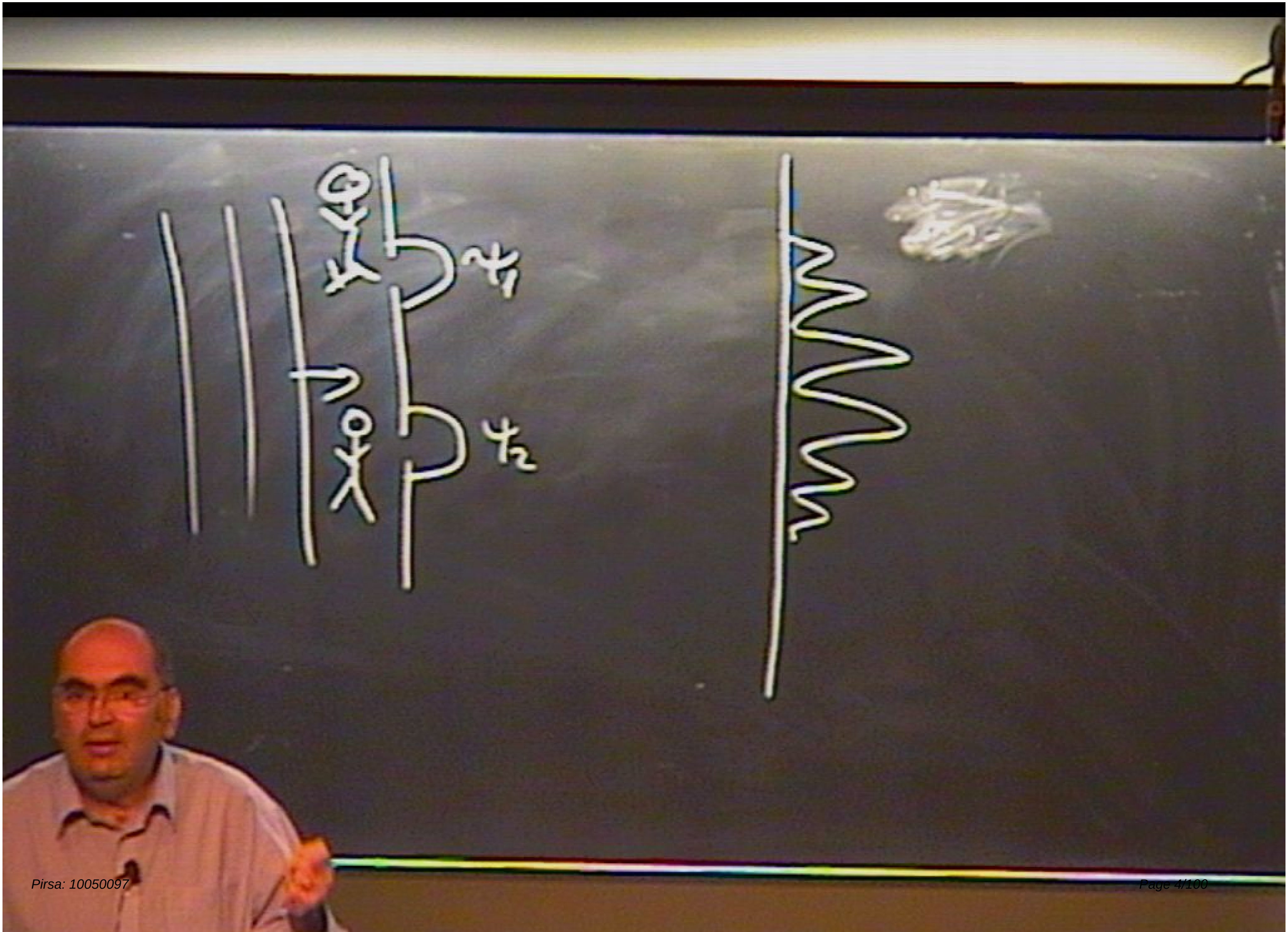
Date: May 11, 2011 09:30 AM

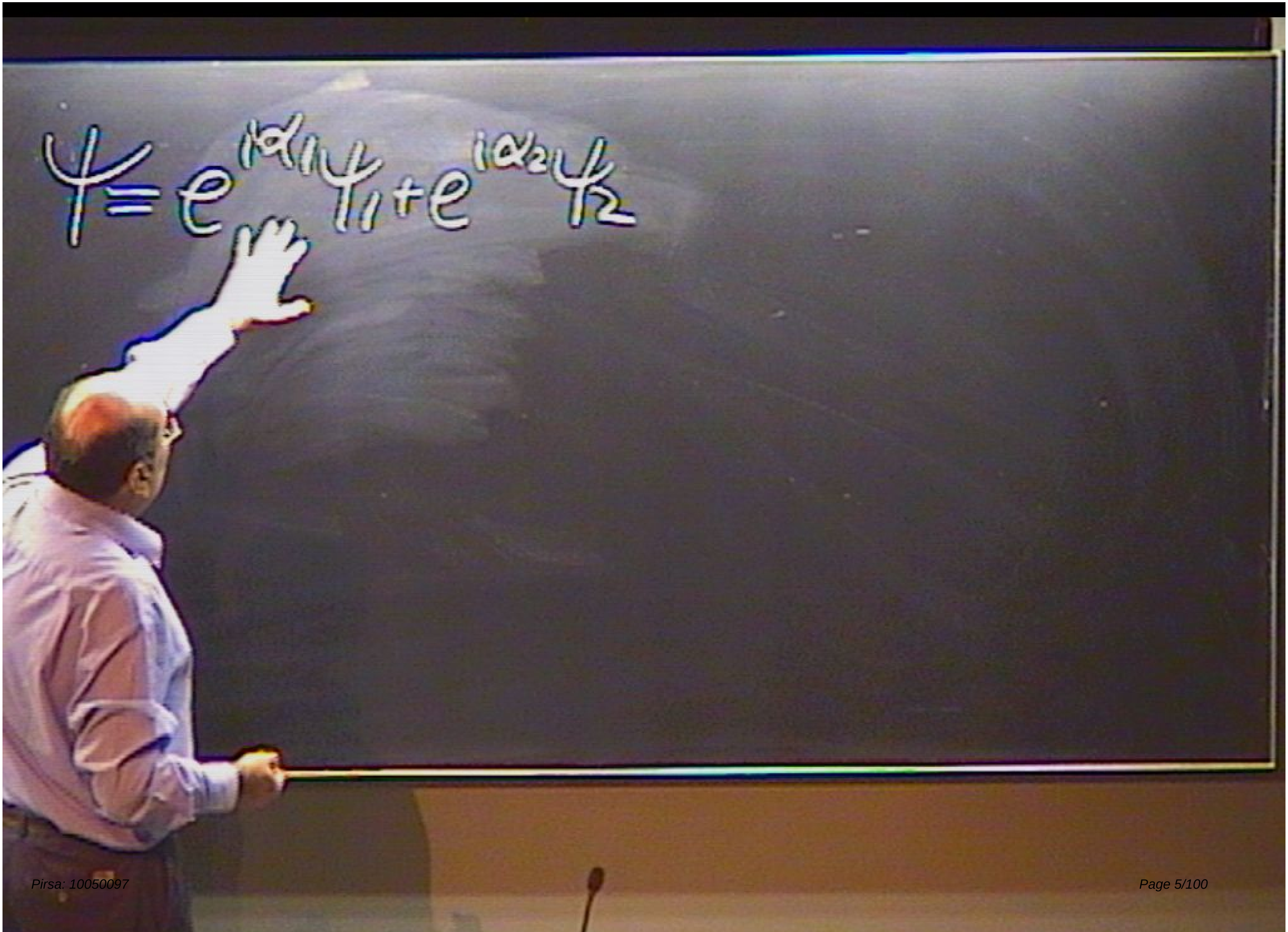
URL: http://pirsa.org/10050097

Abstract: In my talk I raise the question of the fundamental limits to the size of thermal machines - refrigerators, heat pumps and work producing engines - and I will present the smallest possible ones. I will also discuss the issue of a possible complementarity between size and efficiency and show that even the smallest machines could be maximally efficient. Finally I will present a new point of view over what is work and what do thermal machines actually do.


ψ_1
ψ_2

$$\Psi = e^{i\alpha_1}\Psi_1 + e^{i\alpha_2}\Psi_2$$


ψ_1
ψ_2

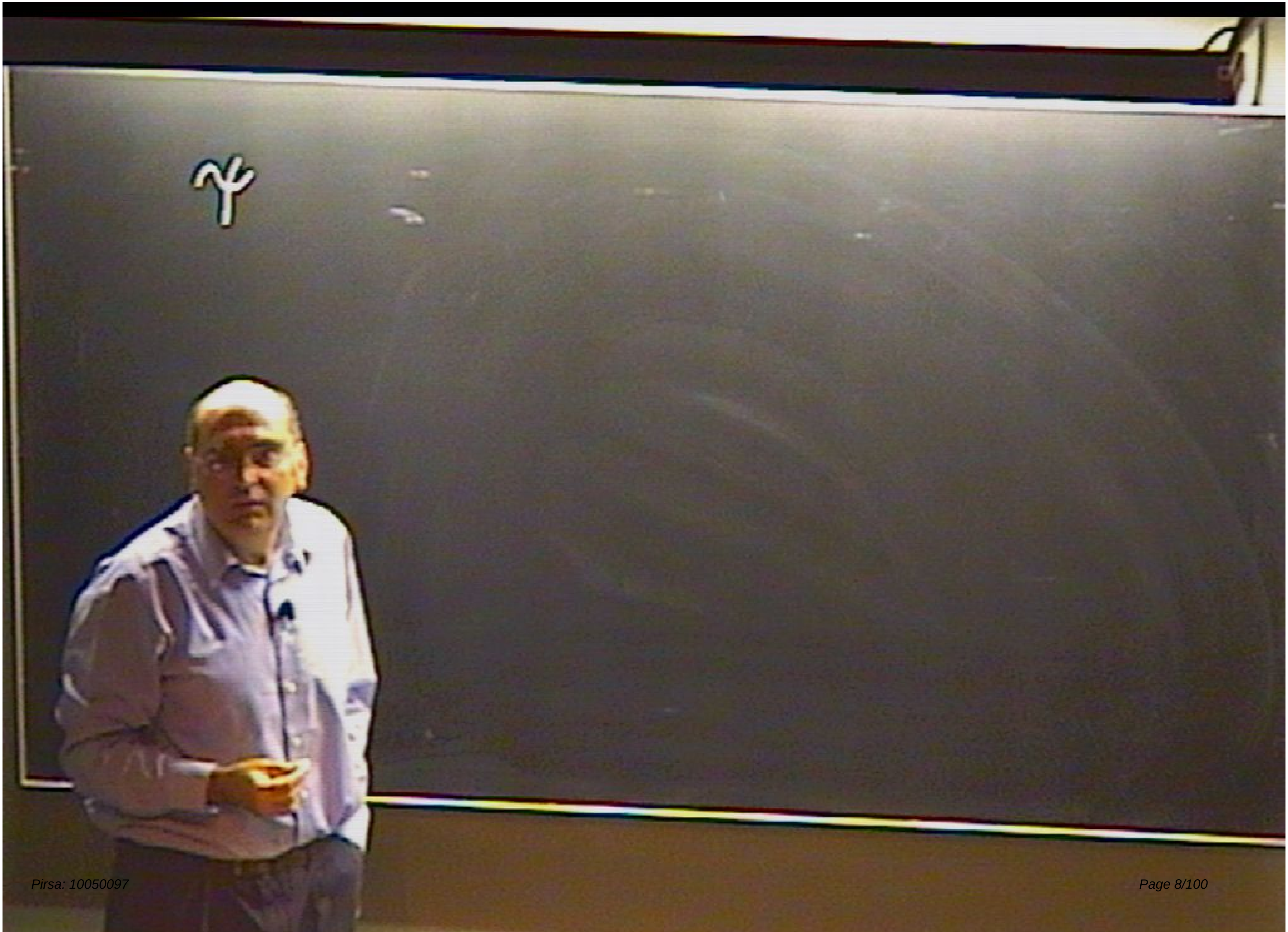

$\Psi = e^{i\alpha_1}\psi_1 + e^{i\alpha_2}\psi_2$

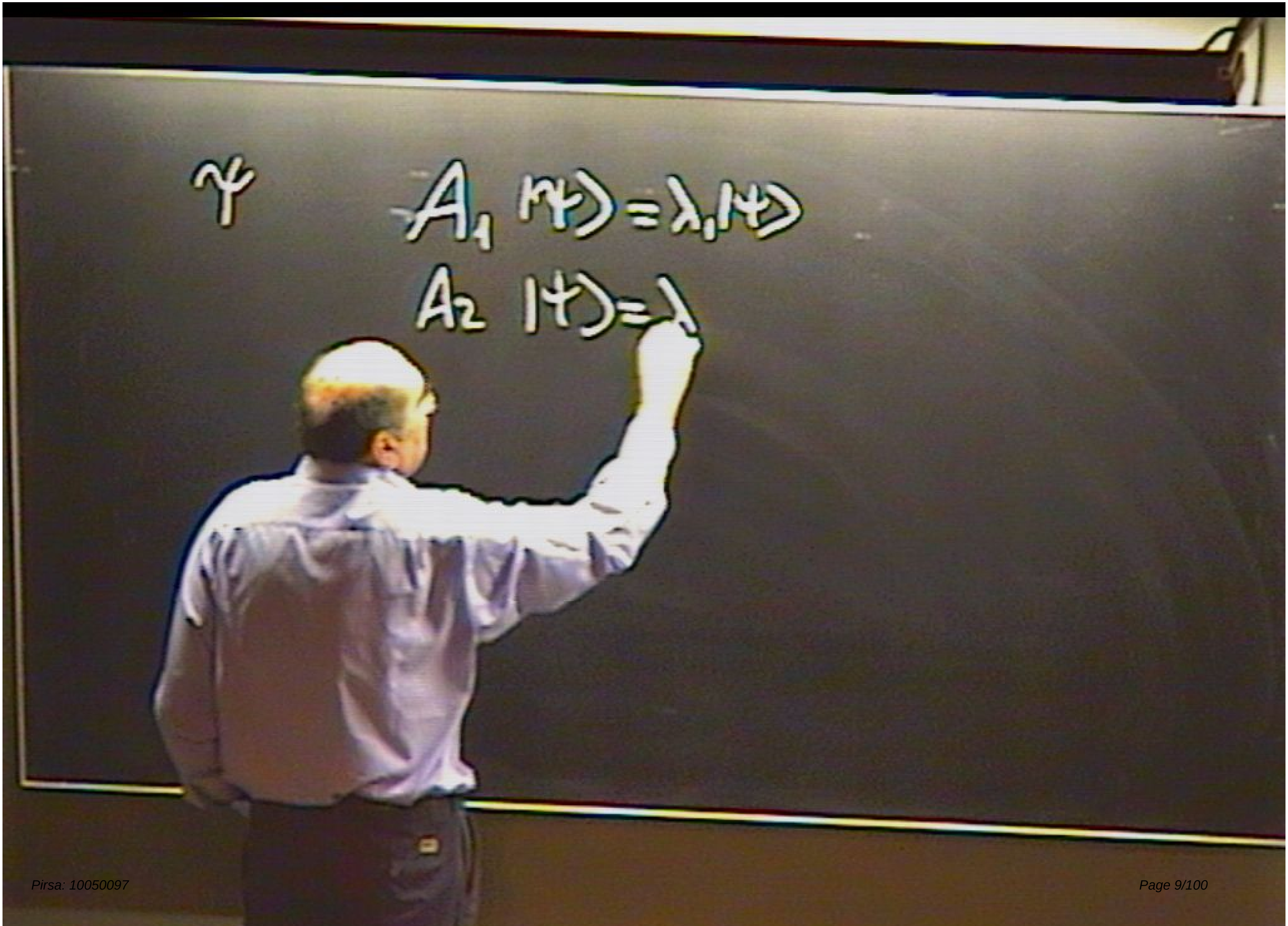
$$\Psi = e^{i\alpha_1}\psi_1 + e^{i\alpha_2}\psi_2$$

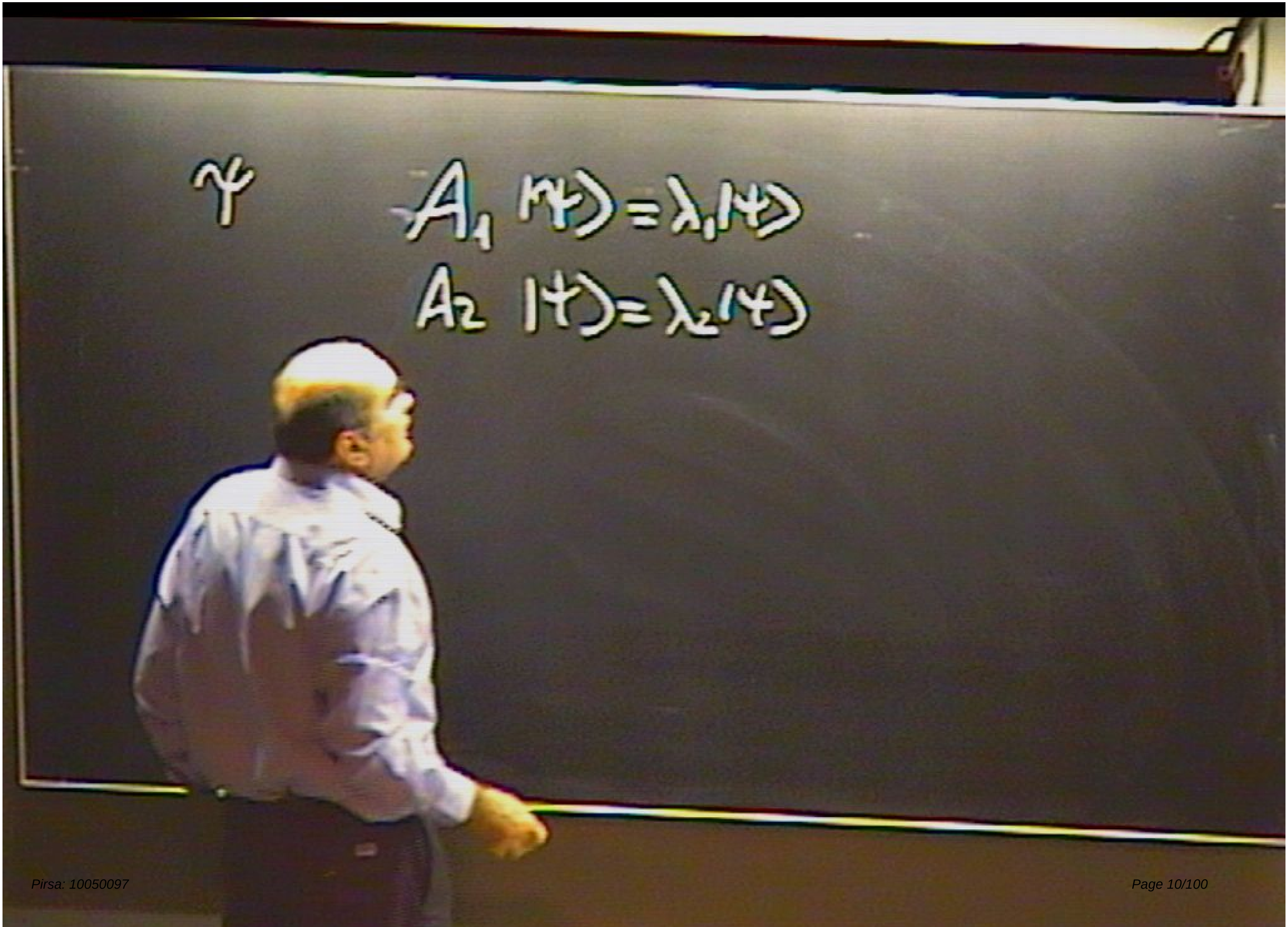
$$= \cancel{e^{i\alpha_1}}\left(\psi_1 + e^{i(\alpha_2-\alpha_1)}\psi_2\right)$$

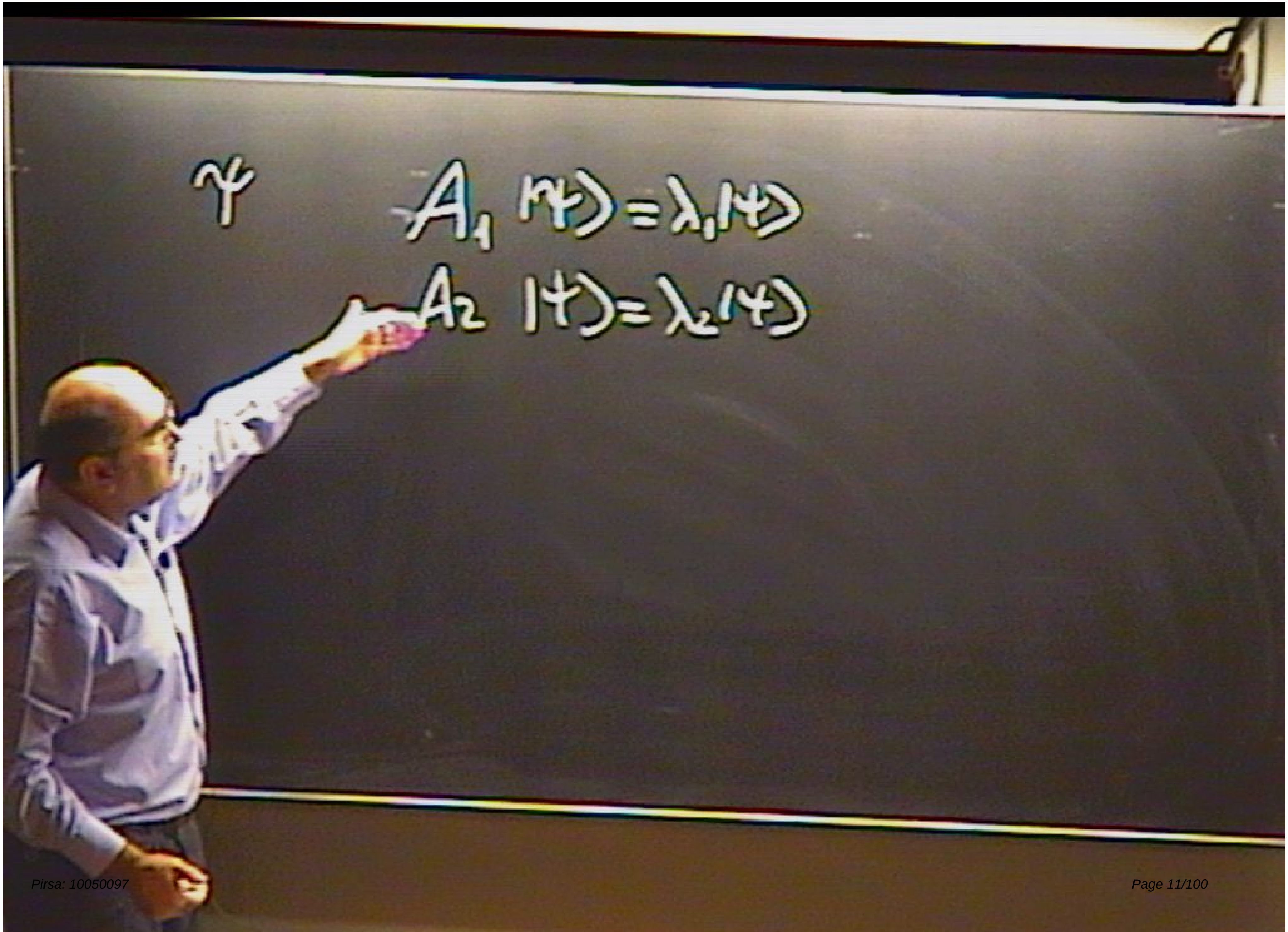
$$\Psi = e^{i\alpha_1}\psi_1 + e^{i\alpha_2}\psi_2$$

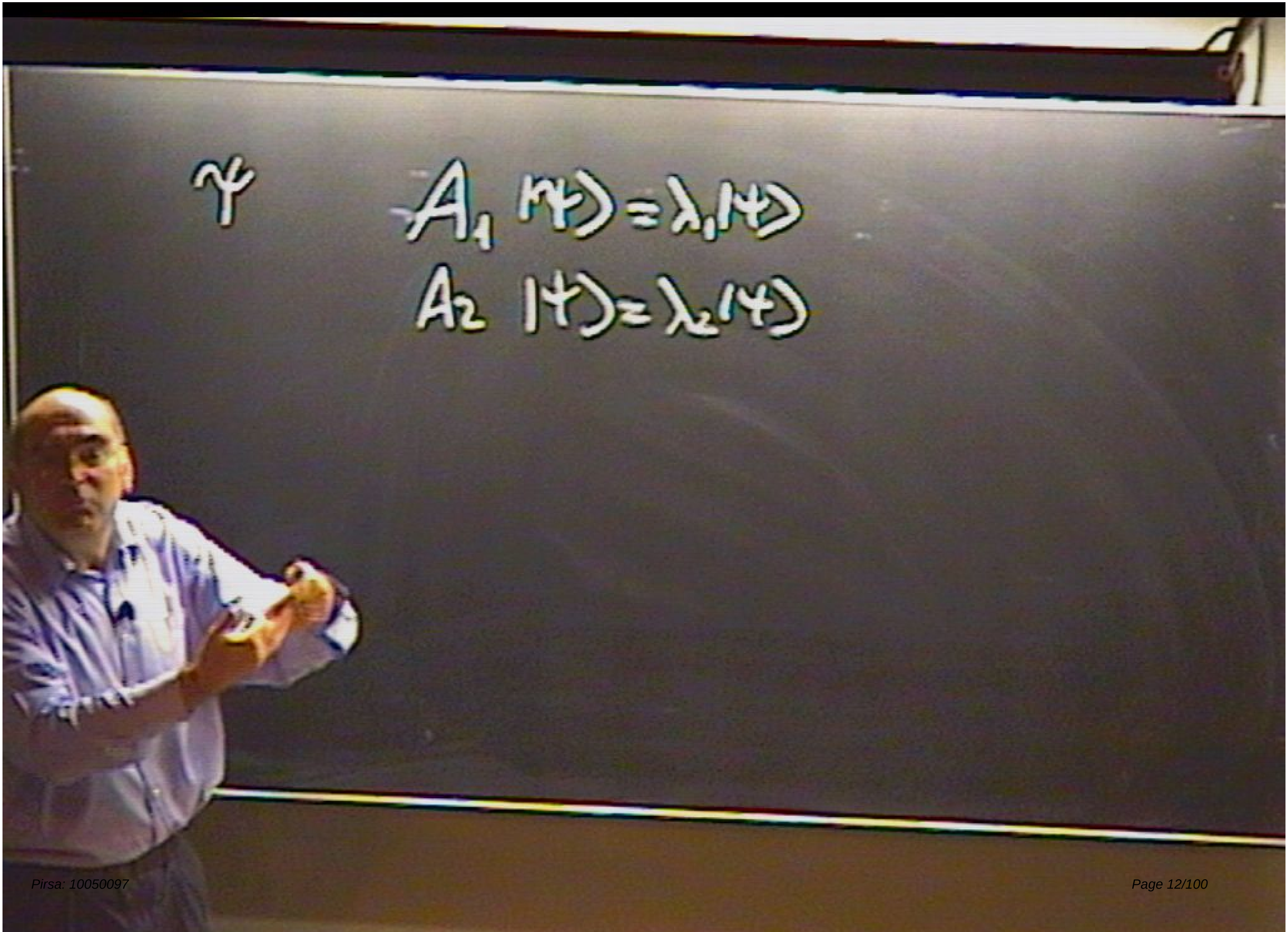
$$= \cancel{e^{i\alpha_1}}\left(\psi_1 + e^{i(\alpha_2-\alpha_1)}\psi_2\right)$$

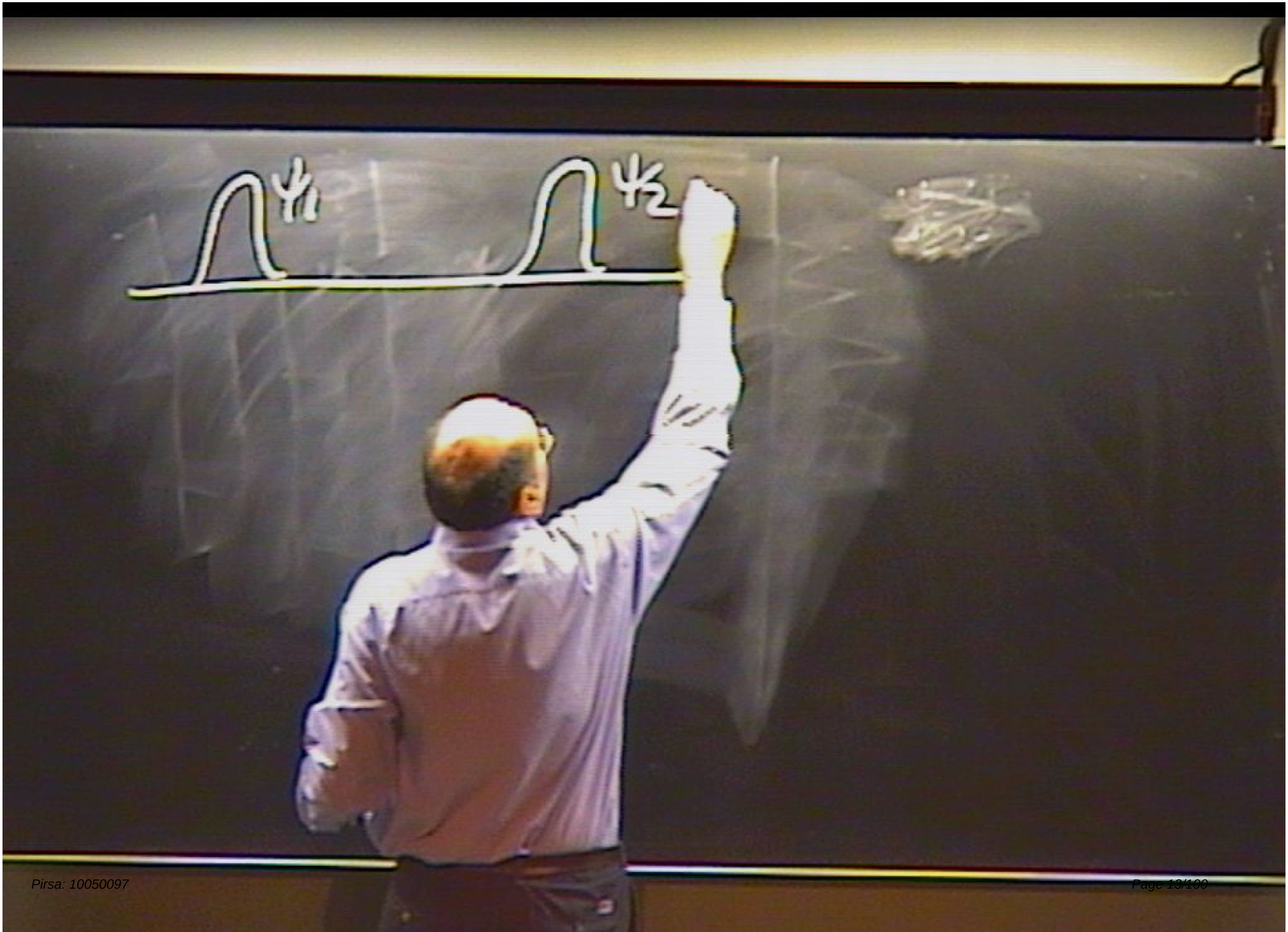

ψ

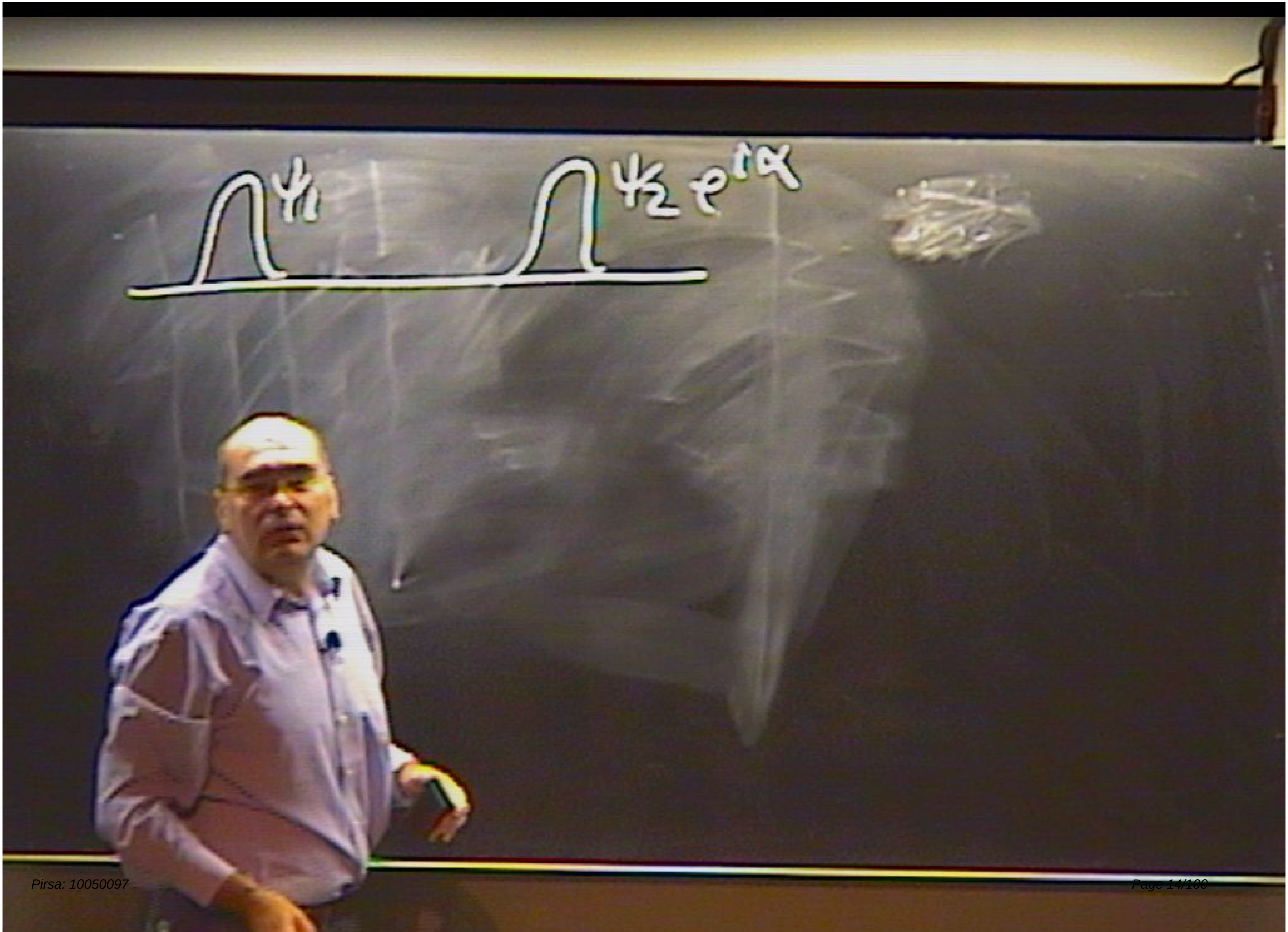

ψ
$A_1 |\psi\rangle = \lambda_1 |\psi\rangle$
$A_2 |\psi\rangle = \lambda$

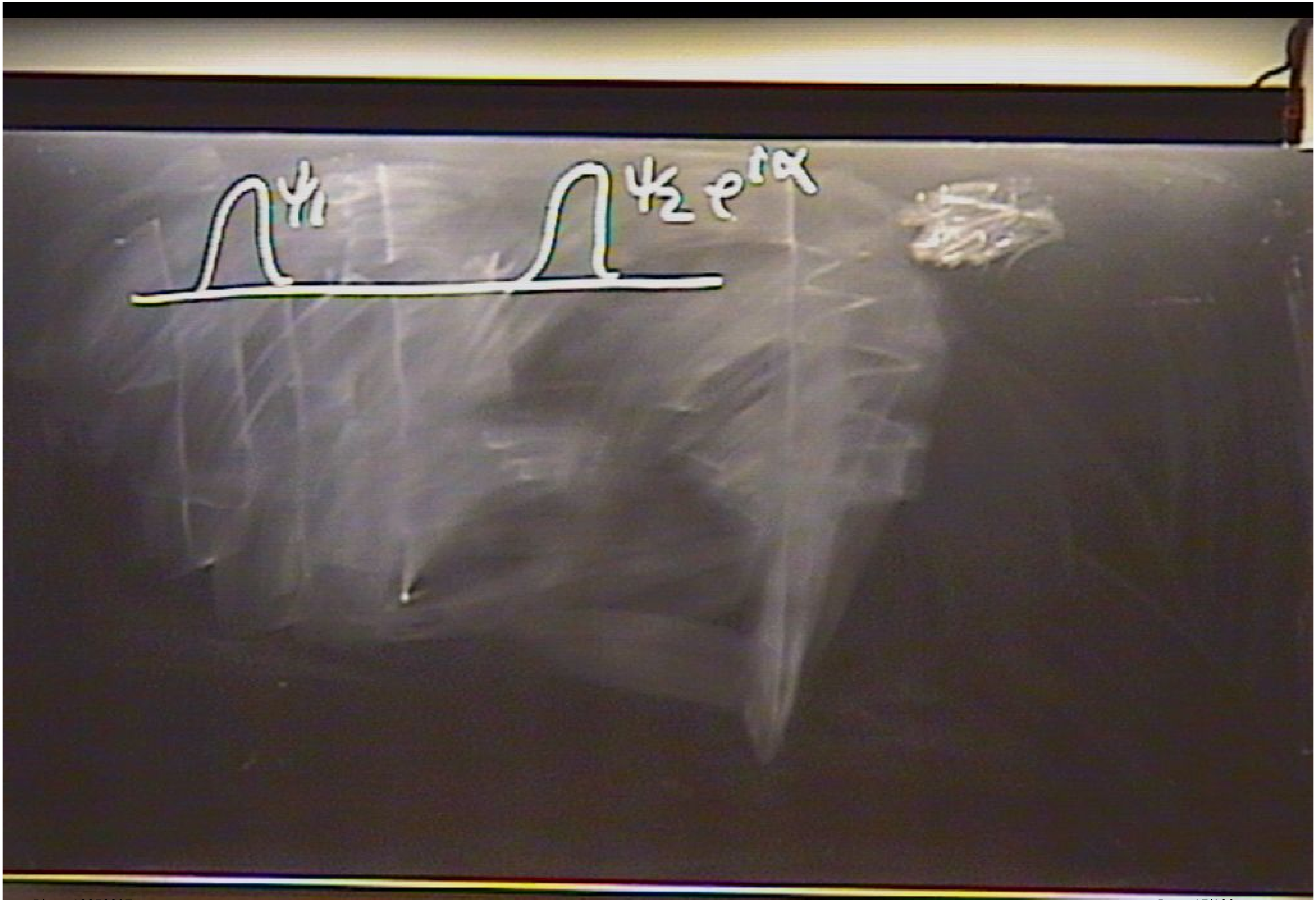

ψ
$A_1 |\psi\rangle = \lambda_1 |\psi\rangle$
$A_2 |\psi\rangle = \lambda_2 |\psi\rangle$

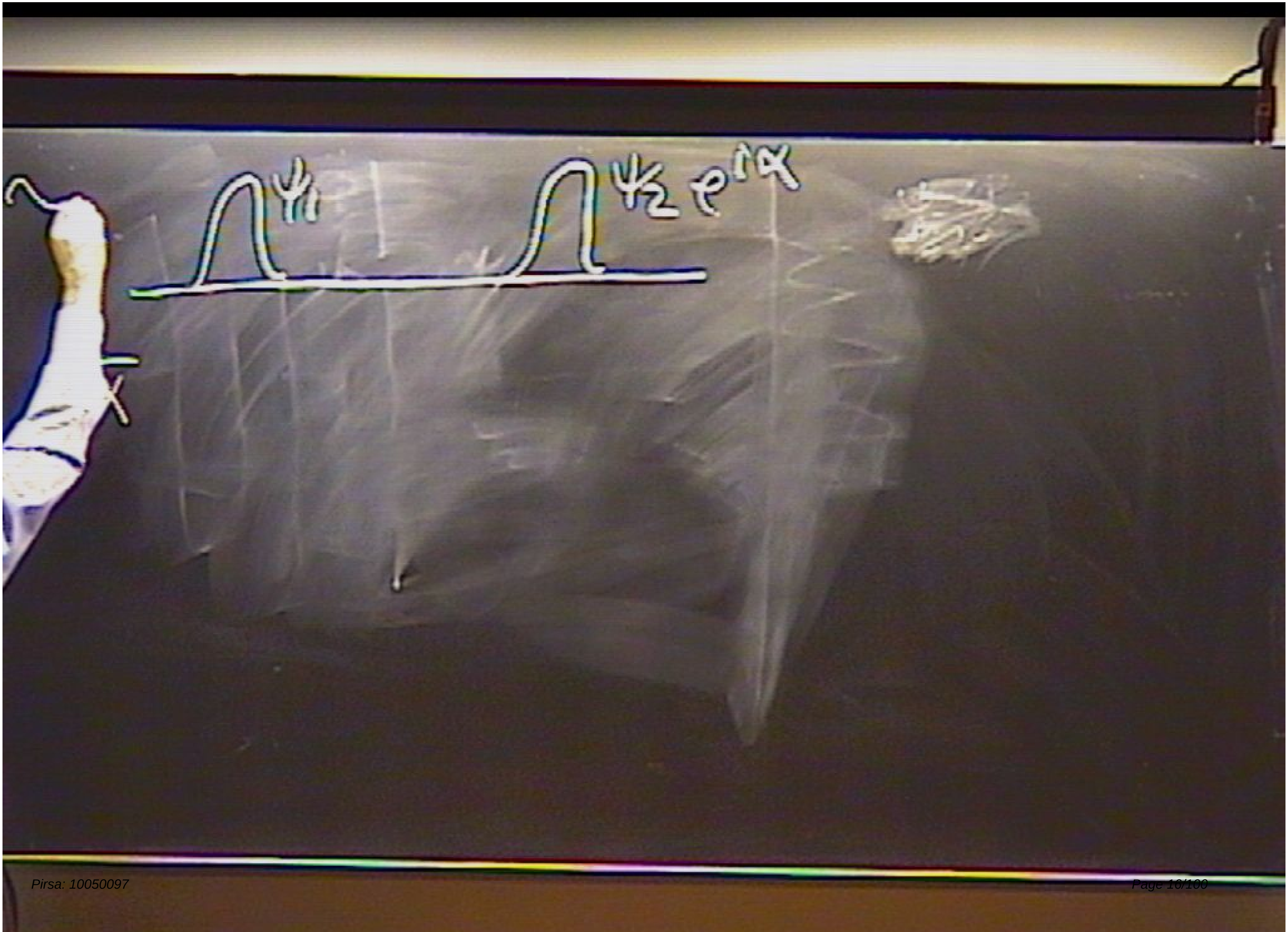

ψ
$A_1 |\psi\rangle = \lambda_1 |\psi\rangle$
$A_2 |\psi\rangle = \lambda_2 |\psi\rangle$

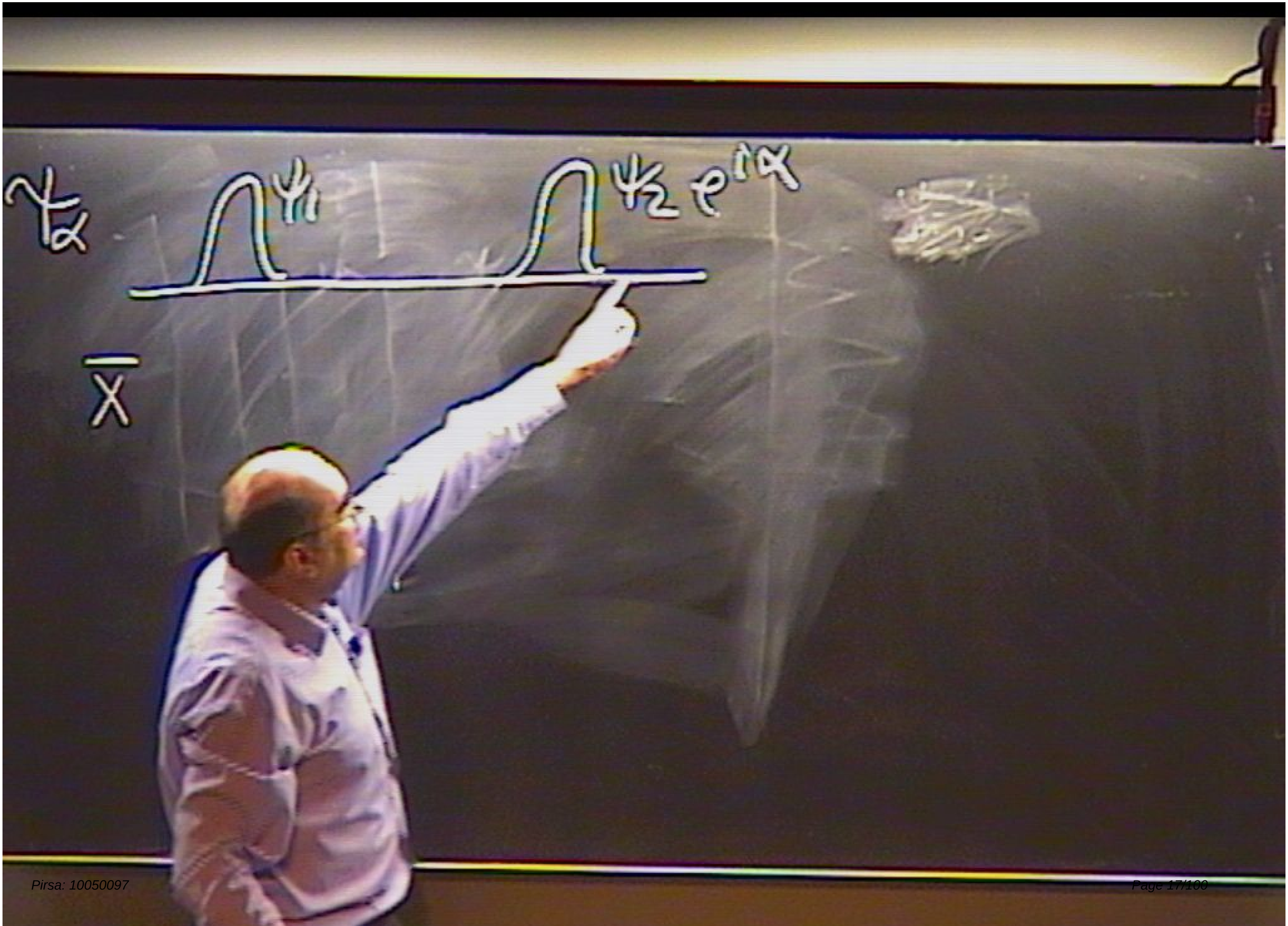

ψ
$A_1 |\psi\rangle = \lambda_1 |\psi\rangle$
$A_2 |\psi\rangle = \lambda_2 |\psi\rangle$

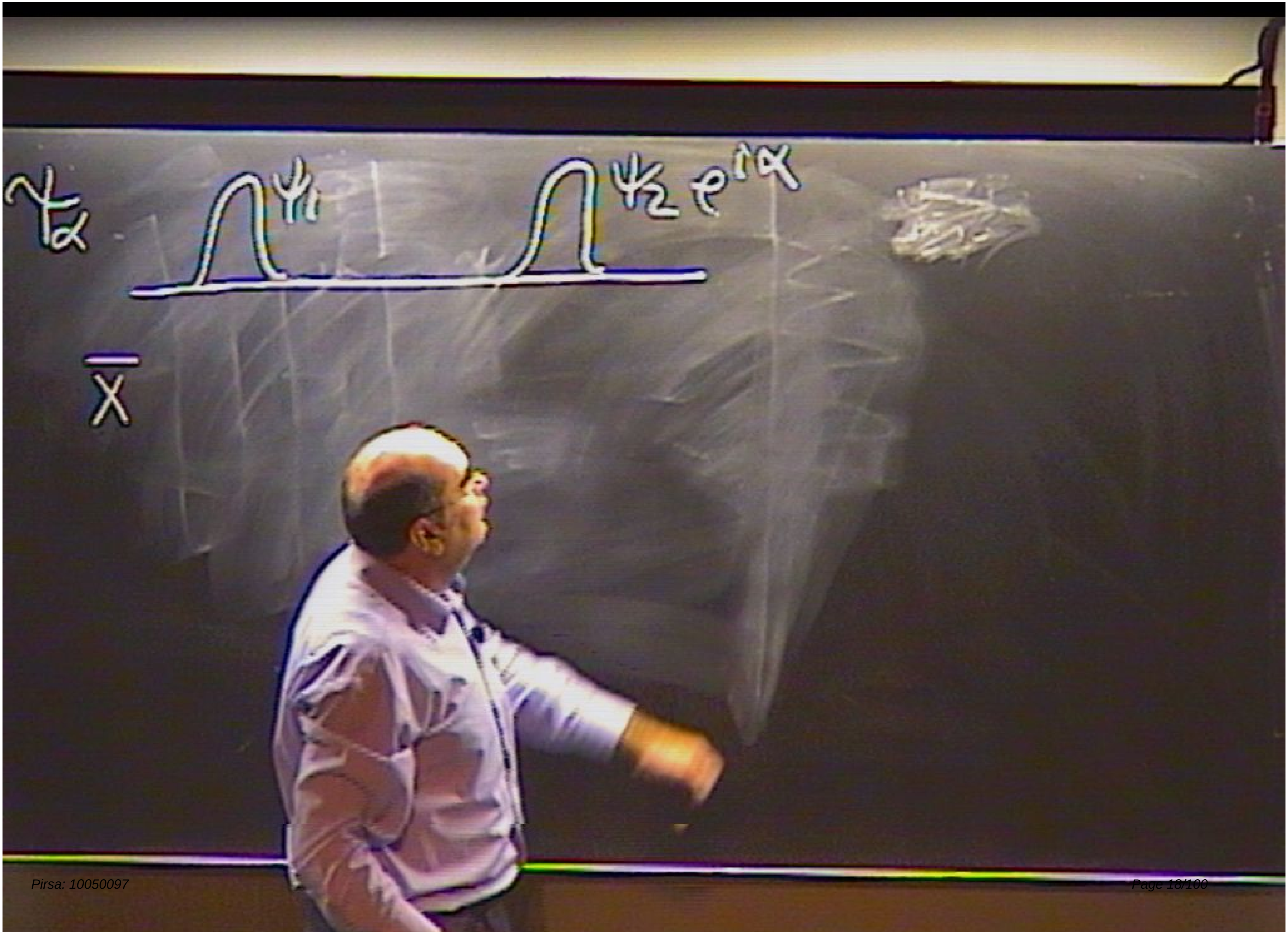

ψ_1
ψ_2

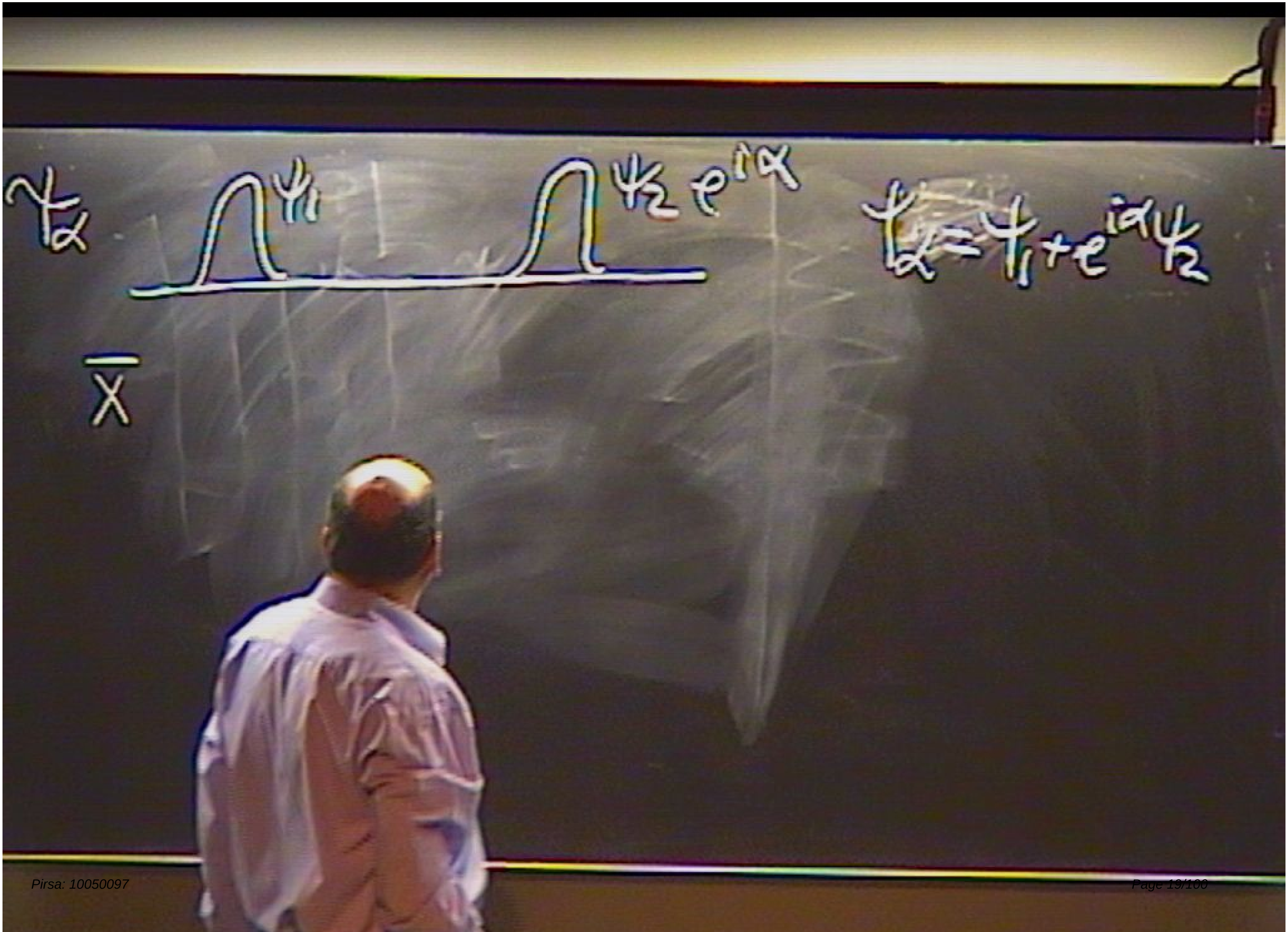

ψ_1
$\psi_2 e^{i\alpha}$

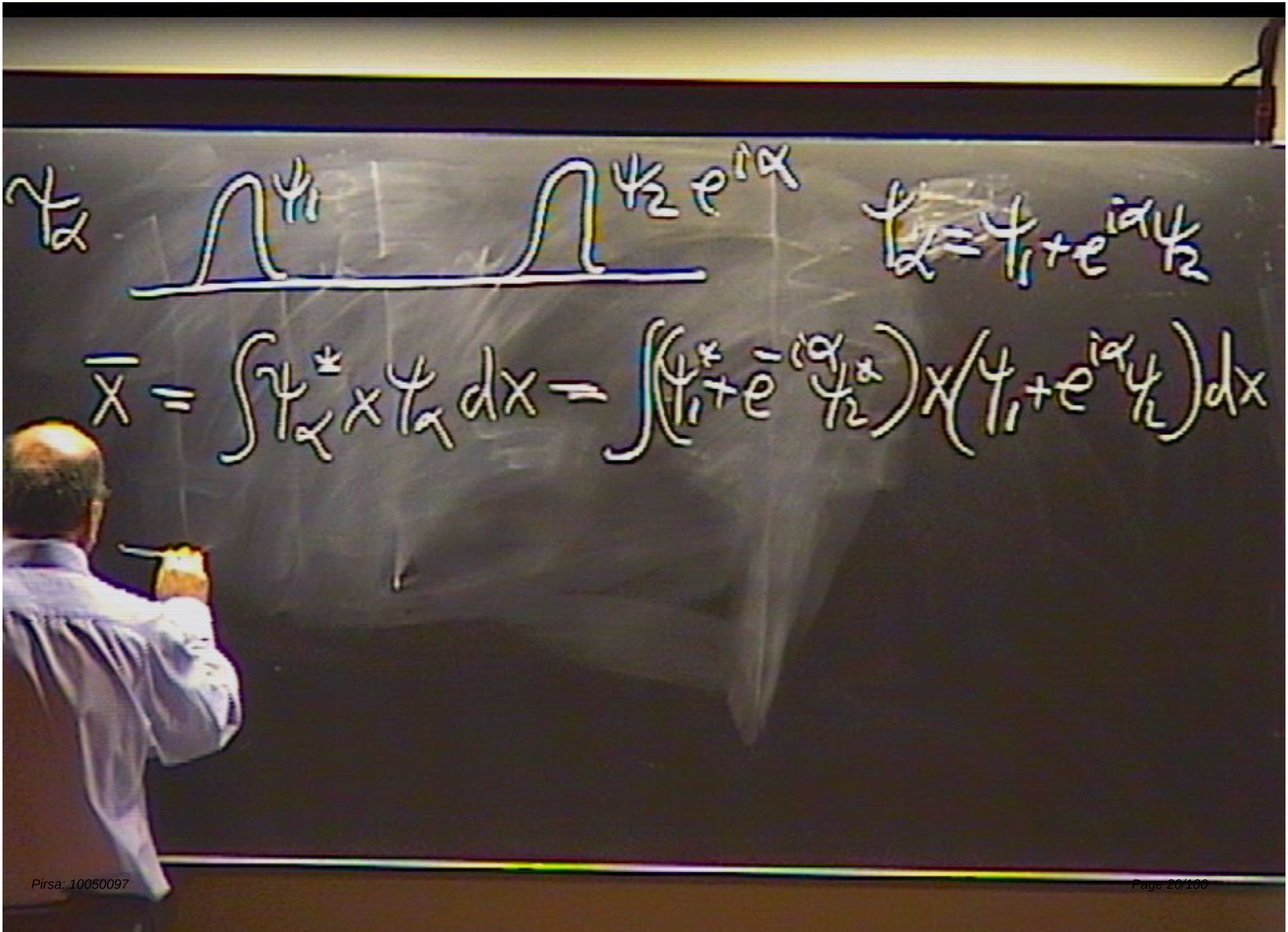

ψ_1
$\psi_2 e^{i\alpha}$


ψ_1
$\psi_2 e^{i\alpha}$


ψ_1
$\psi_2 e^{i\alpha}$
$\bar{X}$


ψ_α
ψ_1
$\psi_2 e^{i\alpha}$
$\bar{X}$


ψ_α
ψ_1
$\psi_2 e^{i\alpha}$
$\psi_\alpha = \psi_1 + e^{i\alpha}\psi_2$
$\bar{X}$

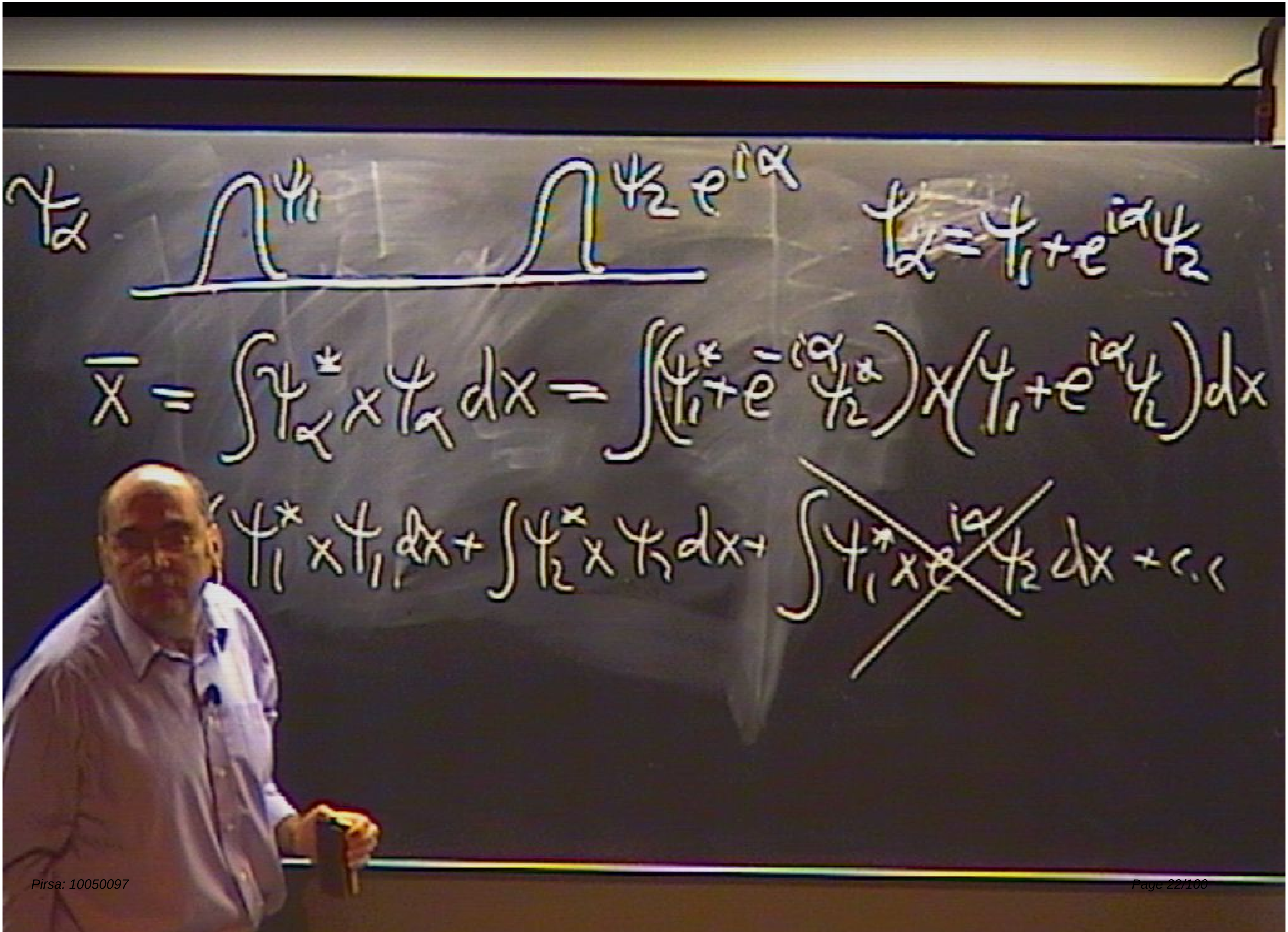

ψ_α
ψ_1
$\psi_2 e^{i\alpha}$
$\psi_\alpha = \psi_1 + e^{i\alpha}\psi_2$
$\bar{x} = \int \psi_\alpha^* x \psi_\alpha dx = \int (\psi_1^* + e^{-i\alpha}\psi_2^*) x (\psi_1 + e^{i\alpha}\psi_2) dx$

ψ_α

$\psi_\alpha = \psi_1 + e^{i\alpha}\psi_2$

$$\bar{x} = \int \psi_\alpha^* \, x \, \psi_\alpha \, dx = \int \left(\psi_1^* + e^{-i\alpha}\psi_2^*\right) x \left(\psi_1 + e^{i\alpha}\psi_2\right) dx$$

$$= \int \psi_1^* \, x \, \psi_1 \, dx + \int \psi_2^* \, x \, \psi_2 \, dx + \int \psi_1^* \, x \, e^{i\alpha}\psi_2 \, dx + c.c.$$



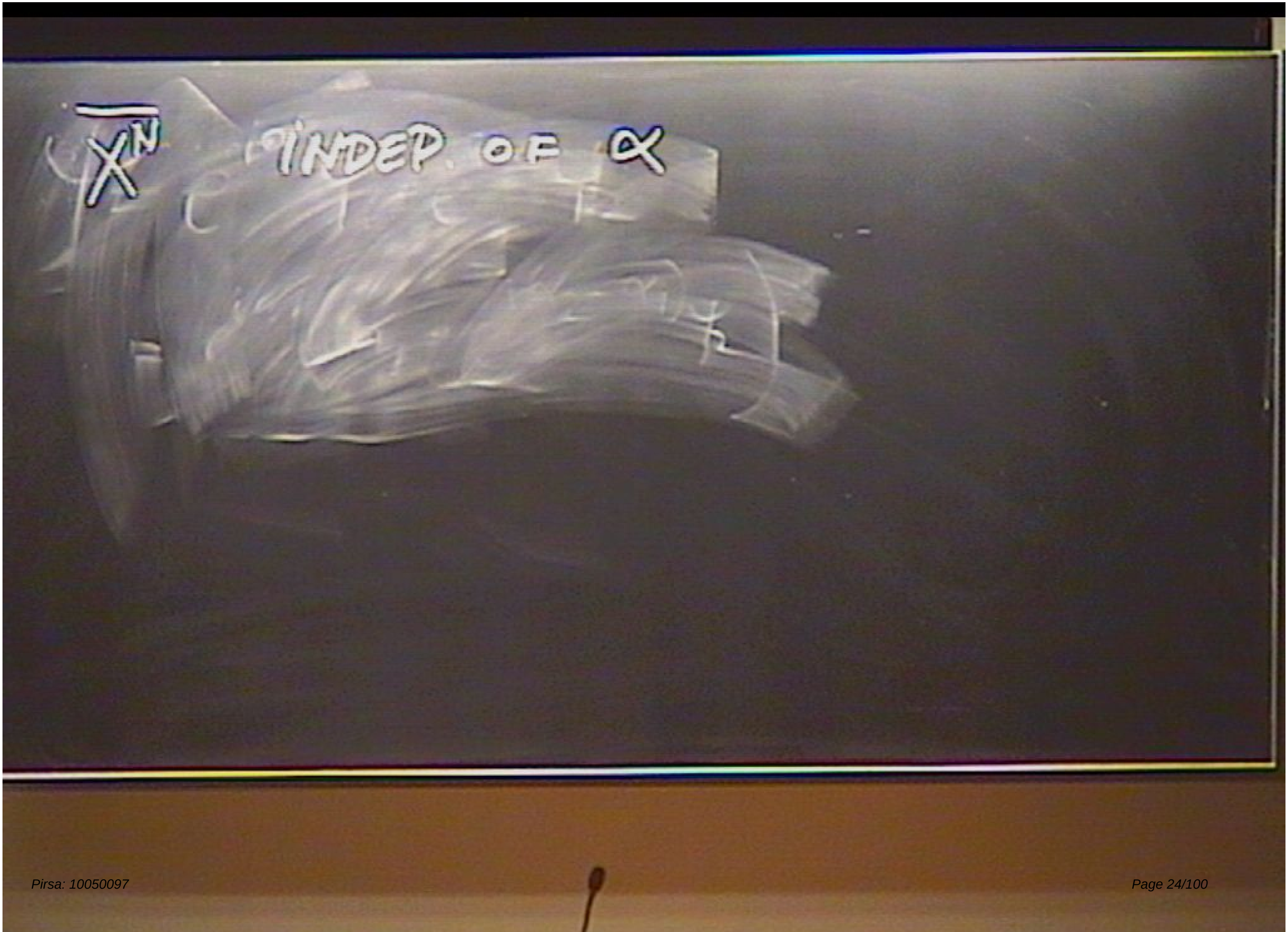
ψ_α
ψ_1
$\psi_2 e^{i\alpha}$
$\psi_\alpha = \psi_1 + e^{i\alpha}\psi_2$
$\bar{x} = \int \psi_\alpha^* x \psi_\alpha dx = \int (\psi_1^* + e^{-i\alpha}\psi_2^*) x (\psi_1 + e^{i\alpha}\psi_2) dx$
$\int \psi_1^* x \psi_1 dx + \int \psi_2^* x \psi_2 dx + \cancel{\int \psi_1^* x e^{i\alpha} \psi_2 dx} + c.c$

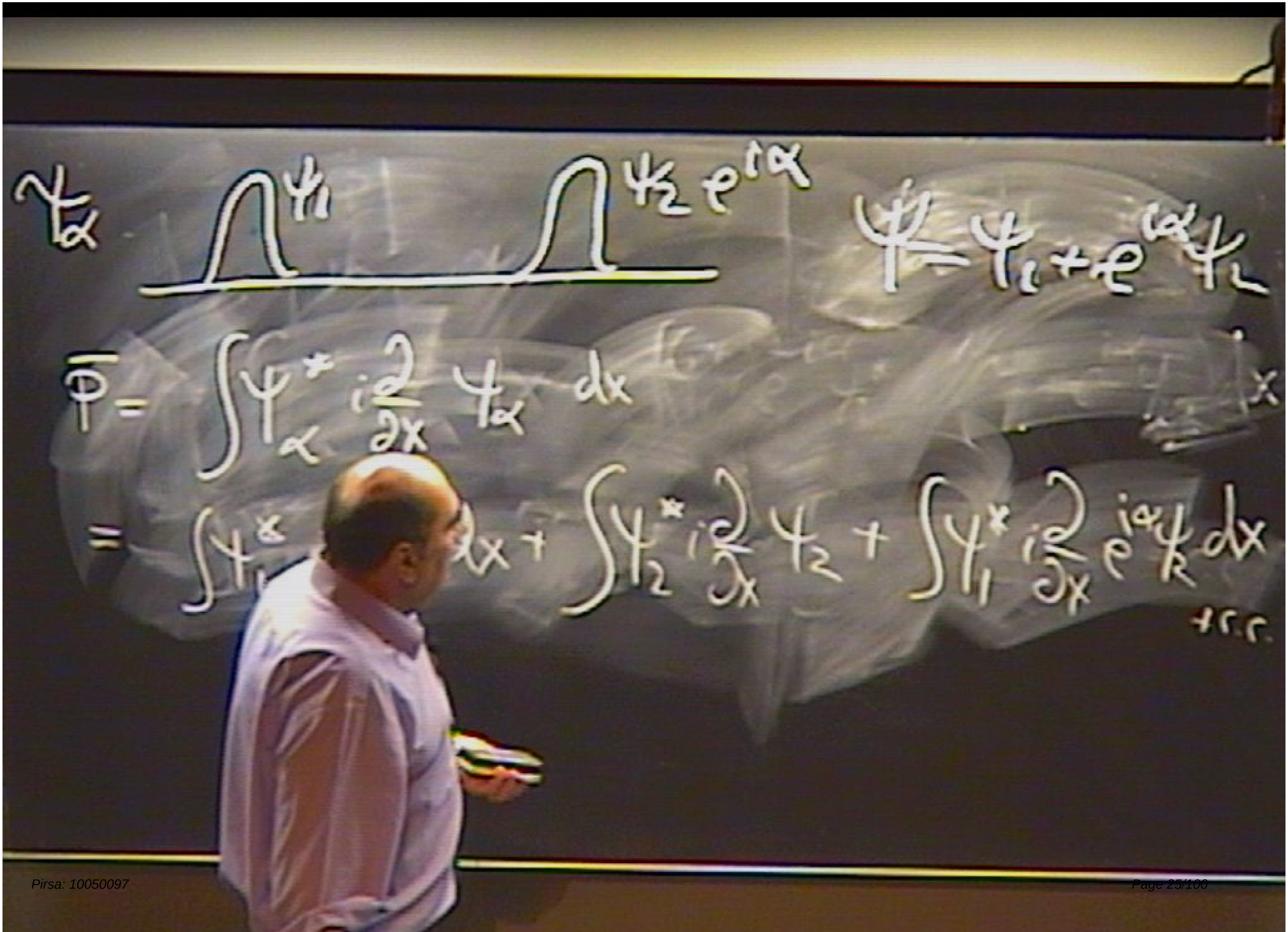
ψ_α

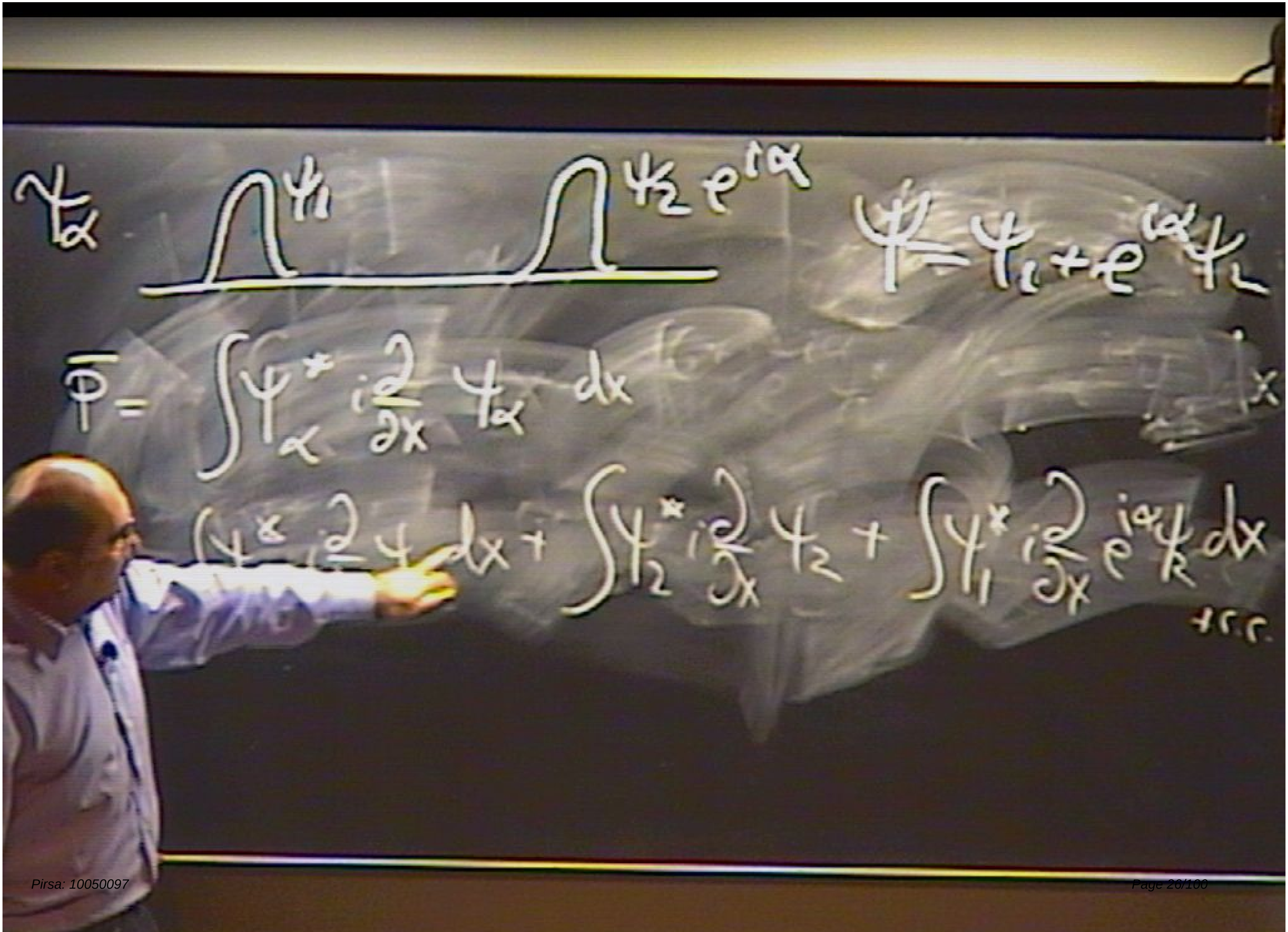
$$\psi_\alpha = \psi_1 + e^{i\alpha}\psi_2$$

$$\overline{x^N} = \int \psi_\alpha^* x^N \psi_\alpha \, dx = \int (\psi_1^* + e^{-i\alpha}\psi_2^*) x^N (\psi_1 + e^{i\alpha}\psi_2) \, dx$$

$$= \int \psi_1^* x^N \psi_1 \, dx + \int \psi_2^* x^N \psi_2 \, dx + \int \psi_1^* \cancel{x^N e^{i\alpha}} \psi_2 \, dx + c.c.$$


$\overline{X^N}$ INDEP. OF α

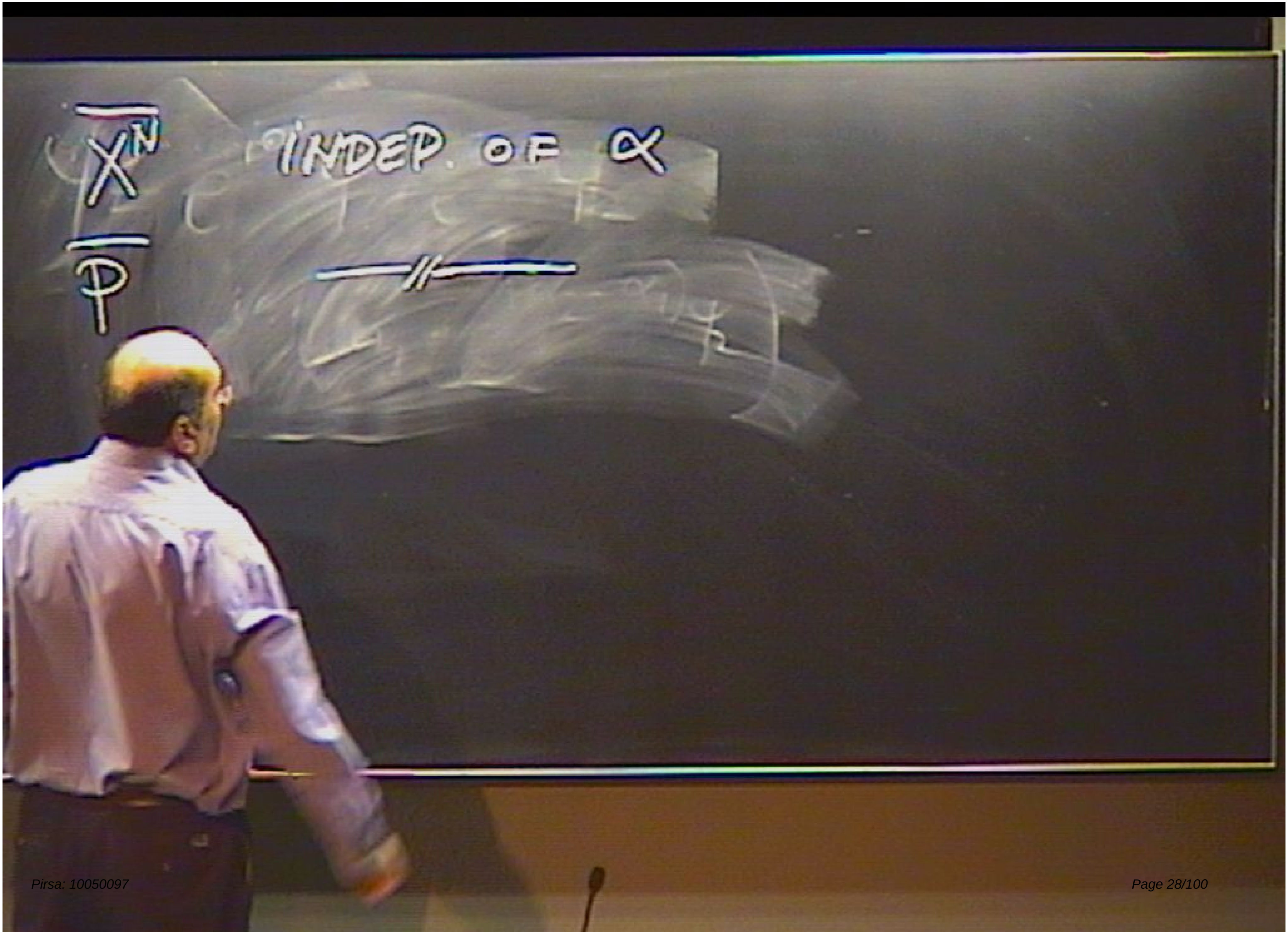

ψ_α
ψ_1
$\psi_2 e^{i\alpha}$
$\psi = \psi_1 + e^{i\alpha}\psi_2$
$\bar{P} = \int \psi_\alpha^* i\frac{\partial}{\partial x}\psi_\alpha \, dx$
$= \int \psi_1^*$
$dx + \int \psi_2^* i\frac{\partial}{\partial x}\psi_2 + \int \psi_1^* i\frac{\partial}{\partial x} e^{i\alpha}\psi_2 \, dx$
$+ c.c.$


ψ_α
ψ_1
$\psi_2 e^{i\alpha}$
$\psi = \psi_1 + e^{i\alpha}\psi_2$
$\bar{P} = \int \psi_\alpha^* \, i\frac{\partial}{\partial x} \psi_\alpha \, dx$
$\int \psi_1^* \, i\frac{\partial}{\partial x} \psi_1 \, dx + \int \psi_2^* \, i\frac{\partial}{\partial x} \psi_2 + \int \psi_1^* \, i\frac{\partial}{\partial x} e^{i\alpha}\psi_2 \, dx$
$+ c.c.$

$$\psi = \psi_1 + e^{i\alpha}\psi_2$$

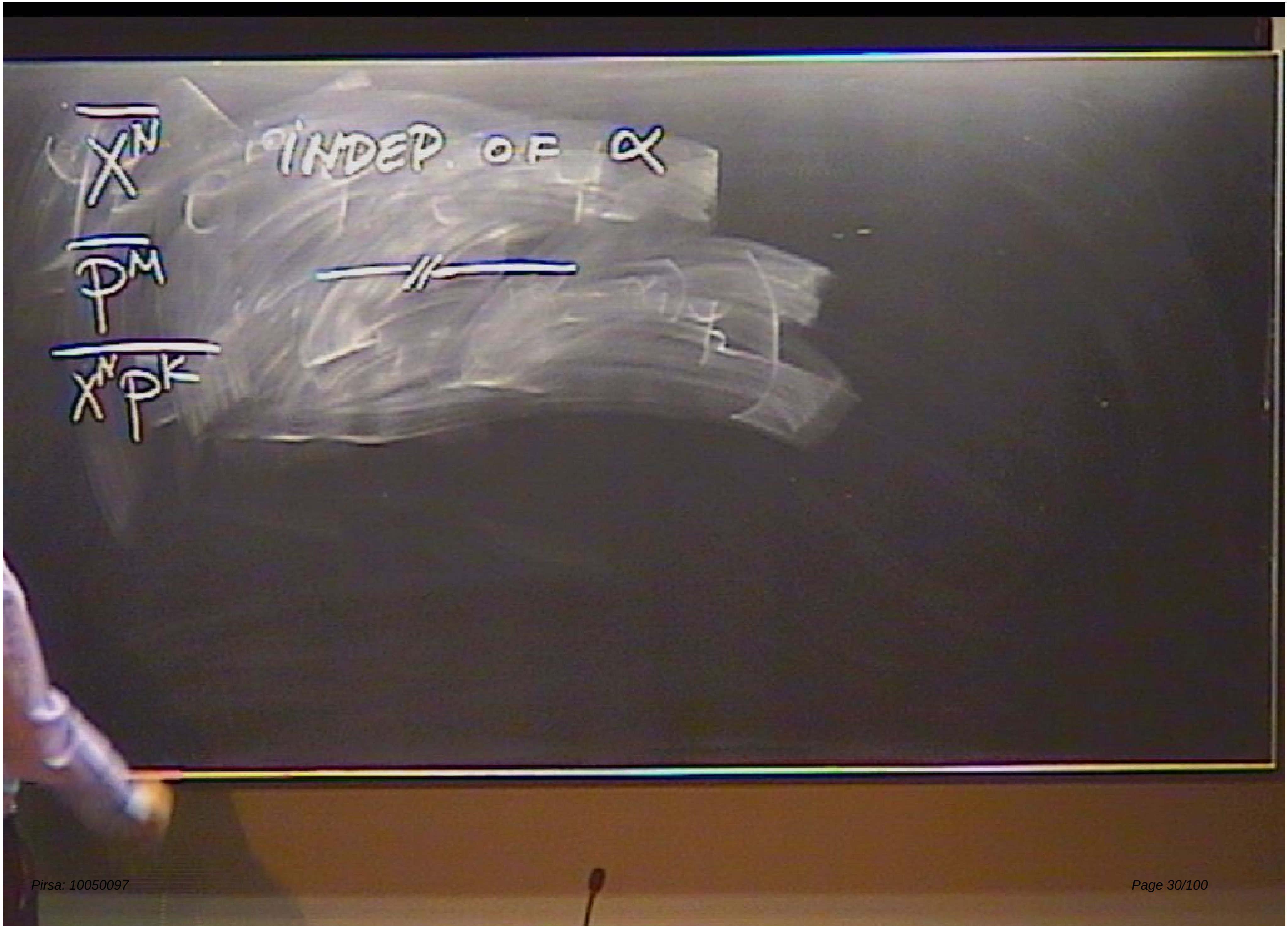
$$\bar{\Phi} = \int \psi_\alpha^* \, i\frac{\partial}{\partial x} \psi_\alpha \, dx$$

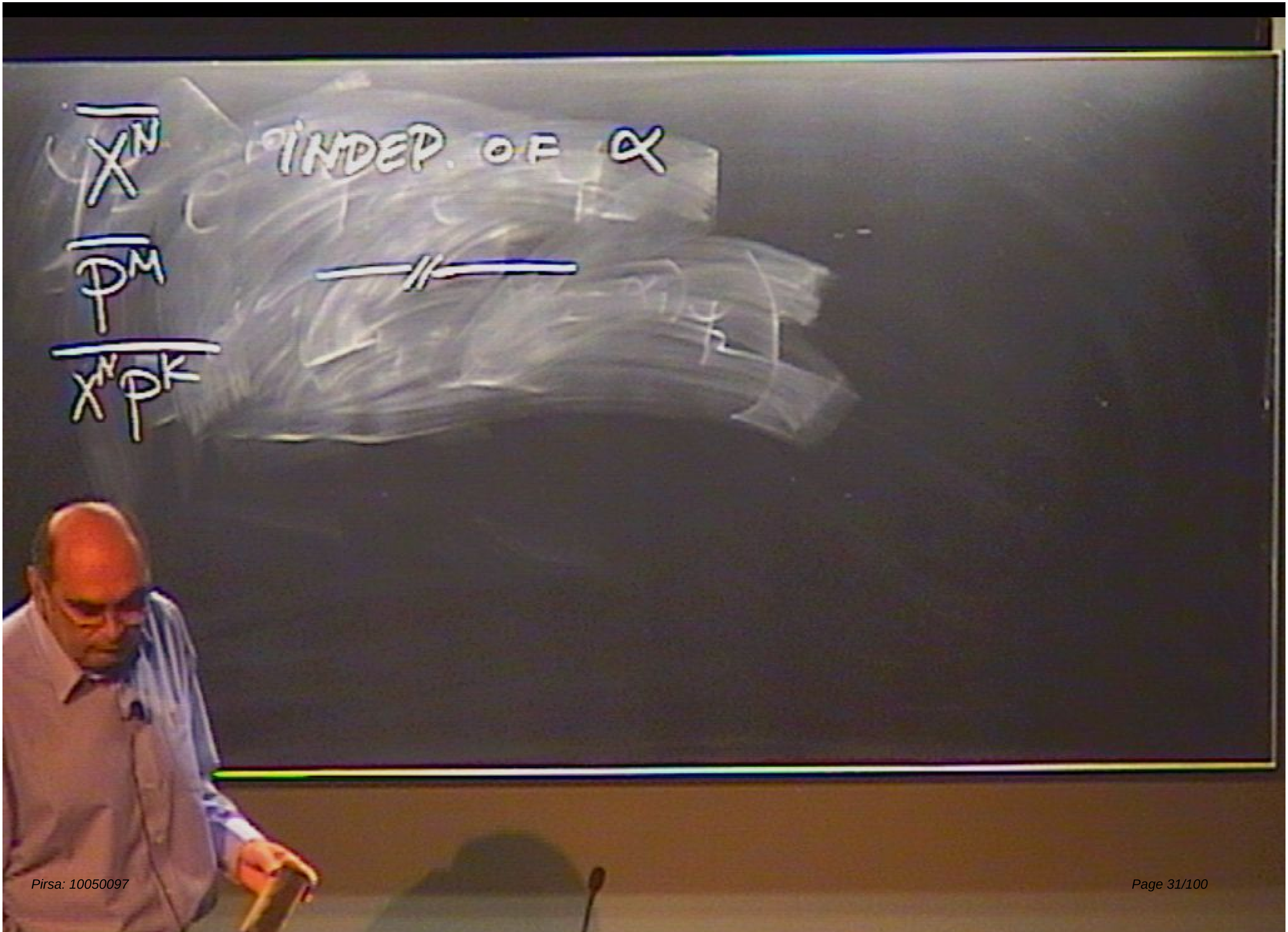
$$= \int \psi_1^* \, i\frac{\partial}{\partial x}\psi_1 \, dx + \int \psi_2^* \, i\frac{\partial}{\partial x}\psi_2 + \int \psi_1^* \, i\frac{\partial}{\partial x} e^{i\alpha}\psi_2 \, dx + \text{c.c.}$$


INDEP. OF α

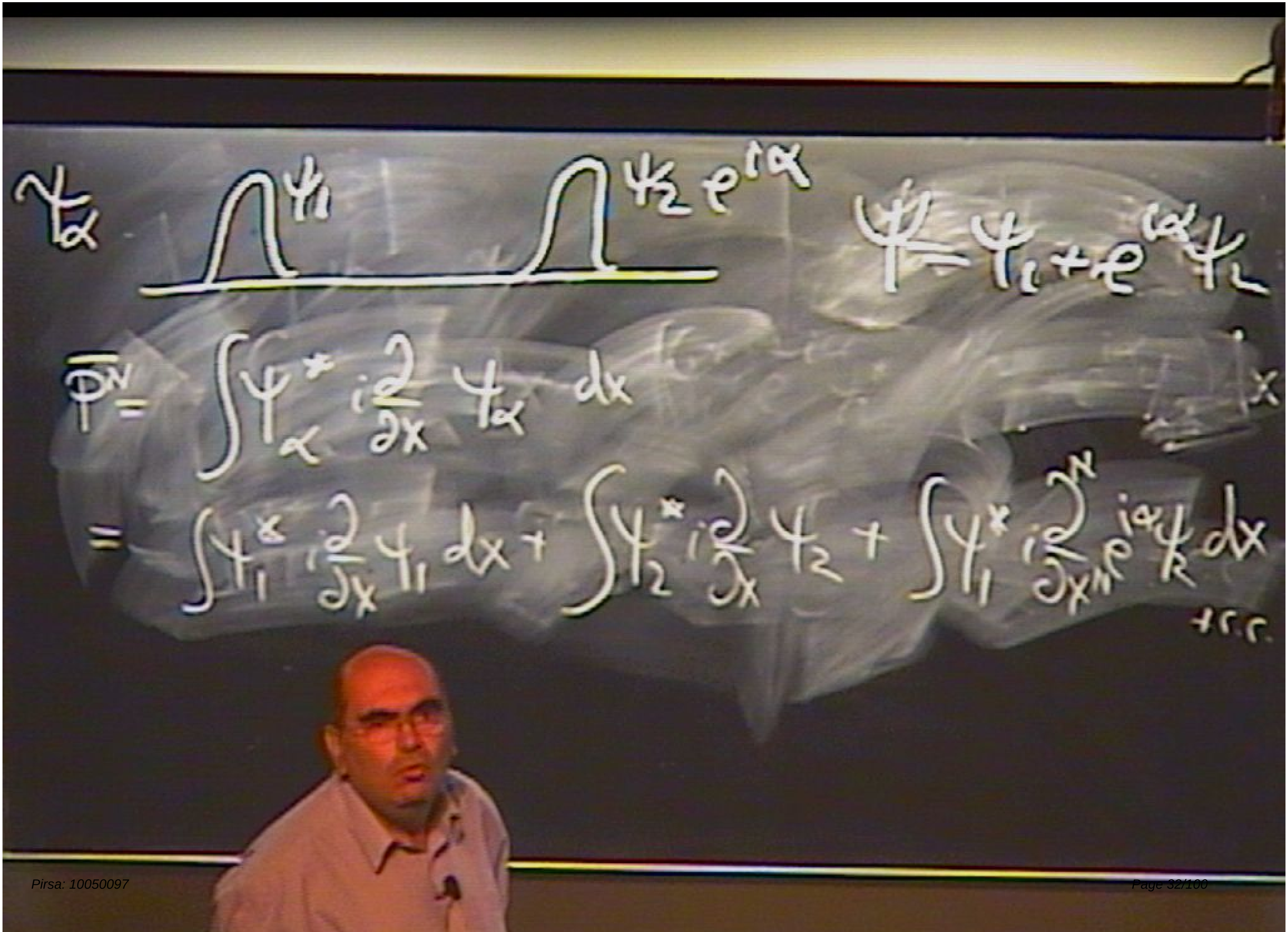
$\overline{X^N}$ INDEP. OF α

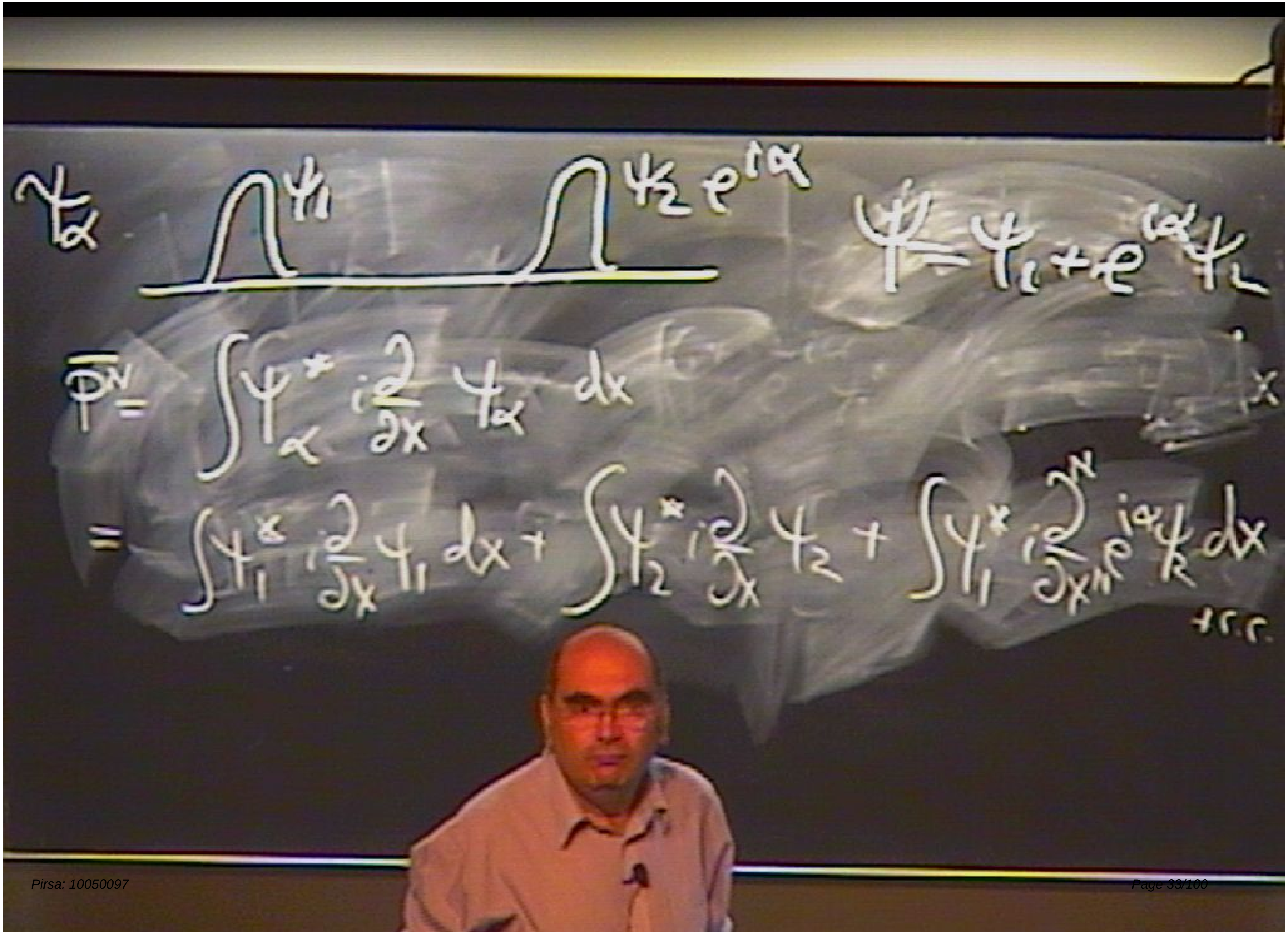
$\overline{P^M}$ ——//——


$\overline{X^N}$ INDEP. OF α
$\overline{P^M}$ —//—
$\overline{X^N P^K}$



$\overline{X^N}$ INDEP. OF α
$\overline{P^M}$
$\overline{X^N P^K}$

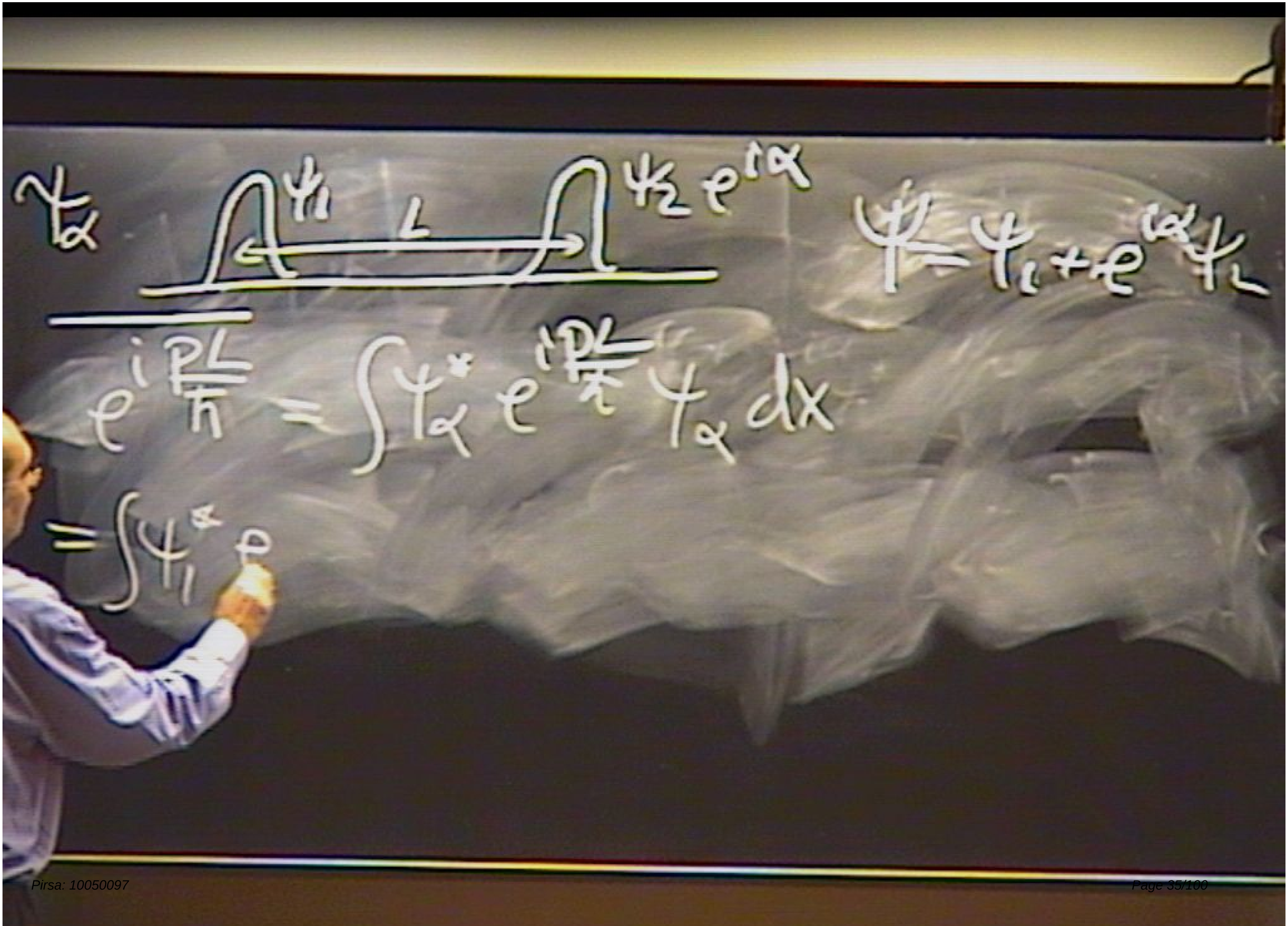

ψ_α
ψ_1
$\psi_2 e^{i\alpha}$
$\psi = \psi_1 + e^{i\alpha}\psi_2$
$\bar{P} = \int \psi_\alpha^* \, i\frac{\partial}{\partial x} \psi_\alpha \, dx$
$= \int \psi_1^* \, i\frac{\partial}{\partial x} \psi_1 \, dx + \int \psi_2^* \, i\frac{\partial}{\partial x} \psi_2 + \int \psi_1^* \, i\frac{\partial}{\partial x} e^{i\alpha} \psi_2 \, dx$
$+ c.c.$
x


ψ_α
ψ_1
$\psi_2 e^{i\alpha}$
$\Psi = \psi_1 + e^{i\alpha}\psi_2$
$\bar{P} = \int \psi_\alpha^* \, i\frac{\partial}{\partial x} \psi_\alpha \, dx$
$= \int \psi_1^* \, i\frac{\partial}{\partial x} \psi_1 \, dx + \int \psi_2^* \, i\frac{\partial}{\partial x} \psi_2 + \int \psi_1^* \, i\frac{\partial}{\partial x} e^{i\alpha} \psi_2 \, dx$
$+ c.c.$

Ψ_α

$$\Psi = \psi_1 + e^{i\alpha}\psi_2$$

$$e^{i\frac{pL}{\hbar}}$$



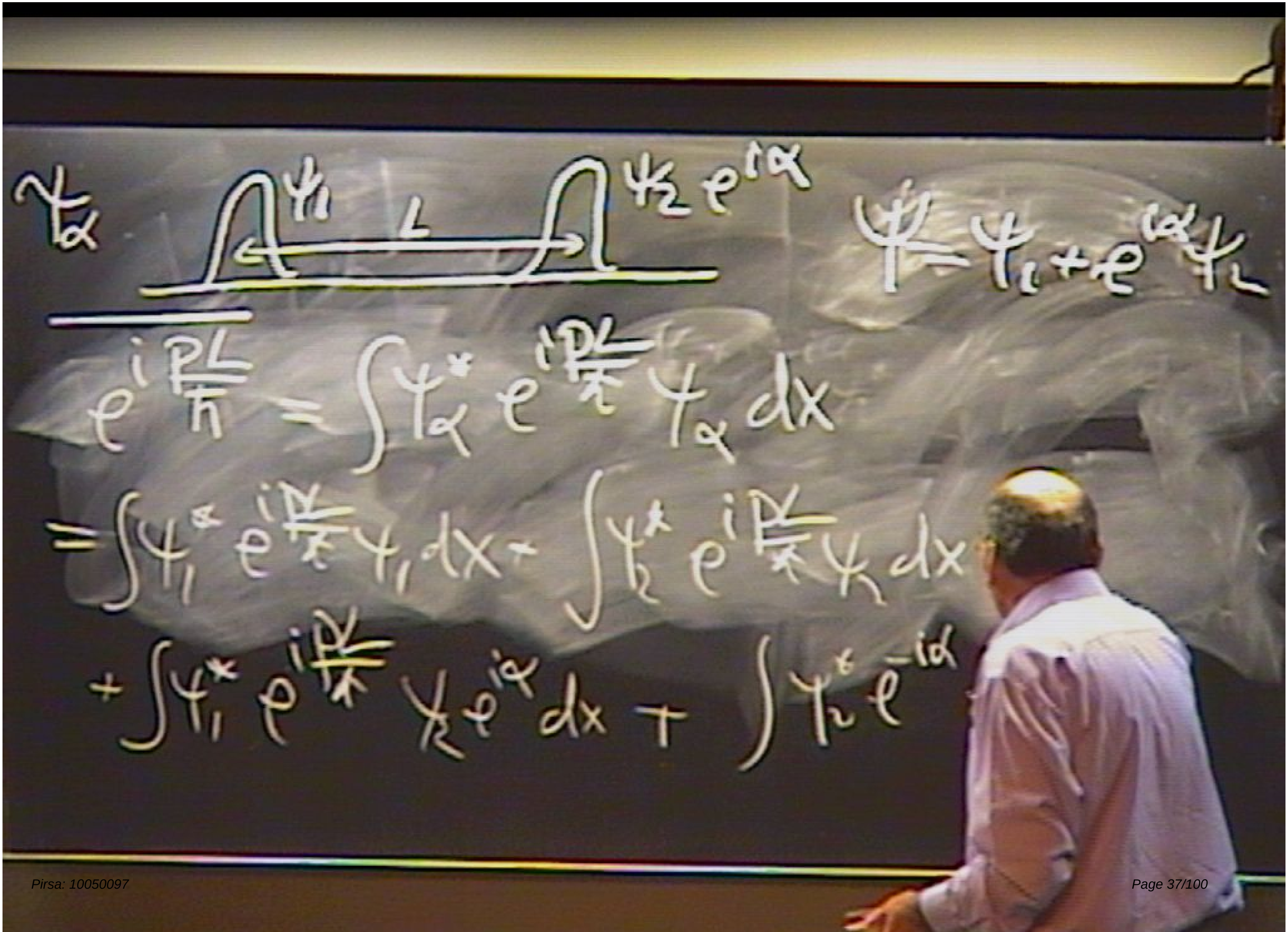
ψ_α
ψ_1
L
$\psi_2 e^{i\alpha}$
$\Psi = \psi_1 + e^{i\alpha}\psi_2$
$e^{i\frac{PL}{\hbar}} = \int \psi_\alpha^* e^{i\frac{PL}{\hbar}} \psi_\alpha \, dx$
$= \int \psi_1^* e$

ψ_α

$$\psi = \psi_1 + e^{i\alpha}\psi_2$$

$$\overline{e^{i\frac{PL}{\hbar}}} = \int \psi_\alpha^* e^{i\frac{PL}{\hbar}} \psi_\alpha \, dx$$

$$= \int \psi_1^* e^{i\frac{PL}{\hbar}} \psi_1 \, dx + \int \psi_2^* e^{i\frac{PL}{\hbar}} \psi_2 \, dx$$


ψα
ψ1 L ψ2 e^{iα}
Ψ = ψ1 + e^{iα} ψ2
e^{iPL/ħ} = ∫ψα* e^{iPL/ħ} ψα dx
= ∫ψ1* e^{iPL/ħ} ψ1 dx + ∫ψ2* e^{iPL/ħ} ψ2 dx
+ ∫ψ1* e^{iPL/ħ} ψ2 e^{iα} dx + ∫ψ2* e^{-iα}

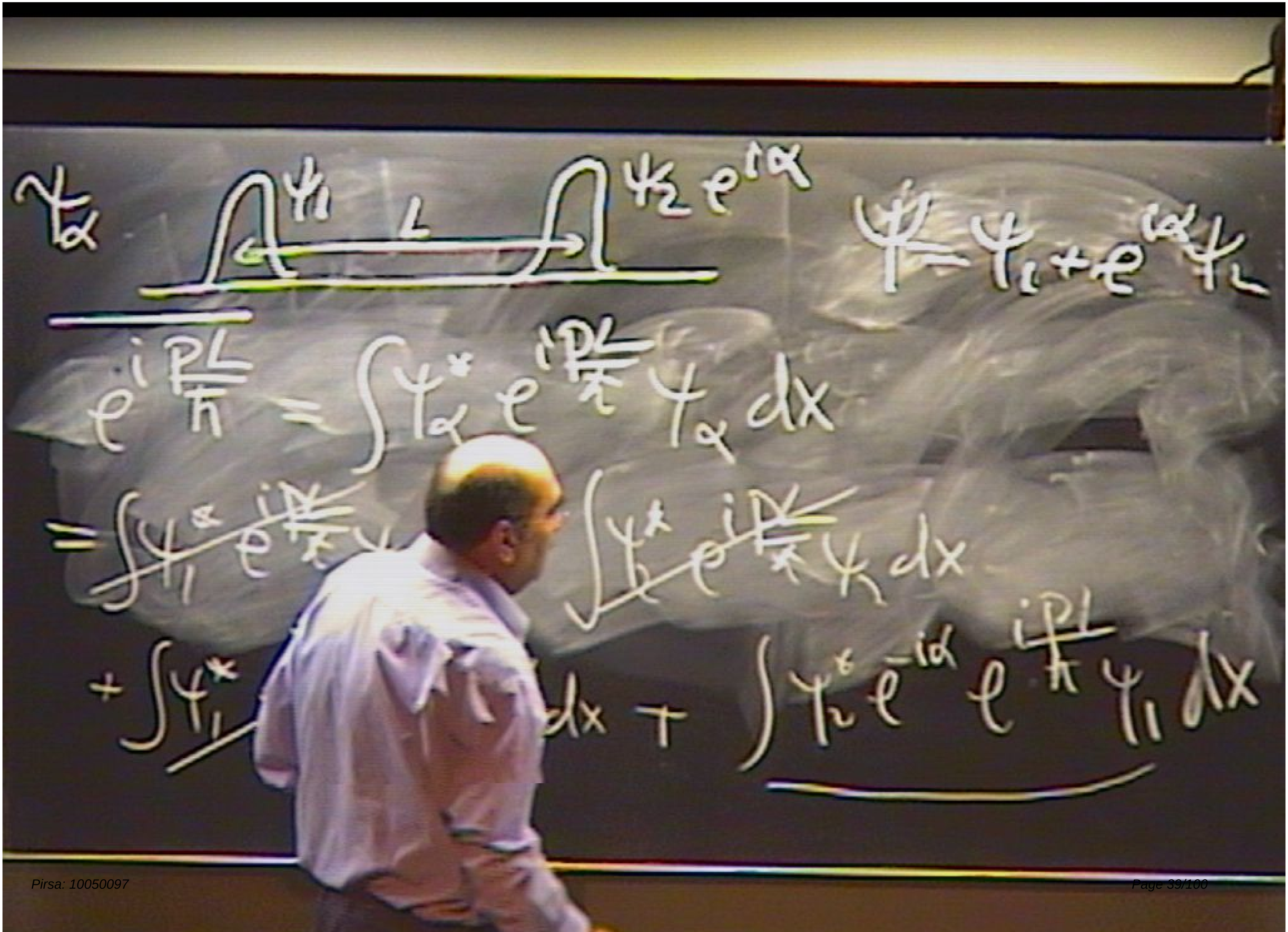
ψ_α

$$\psi = \psi_1 + e^{i\alpha}\psi_2$$

$$\overline{\langle e^{i\frac{pL}{\hbar}} \rangle} = \int \psi_\alpha^* e^{i\frac{pL}{\hbar}} \psi_\alpha \, dx$$

$$= \int \psi_1^* e^{i\frac{pL}{\hbar}} \psi_1 \, dx + \int \psi_2^* e^{i\frac{pL}{\hbar}} \psi_2 \, dx$$

$$+ \int \psi_1^* e^{i\frac{pL}{\hbar}} \psi_2 e^{i\alpha} \, dx + \int \psi_2^* e^{-i\alpha} e^{i\frac{pL}{\hbar}} \psi_1 \, dx$$

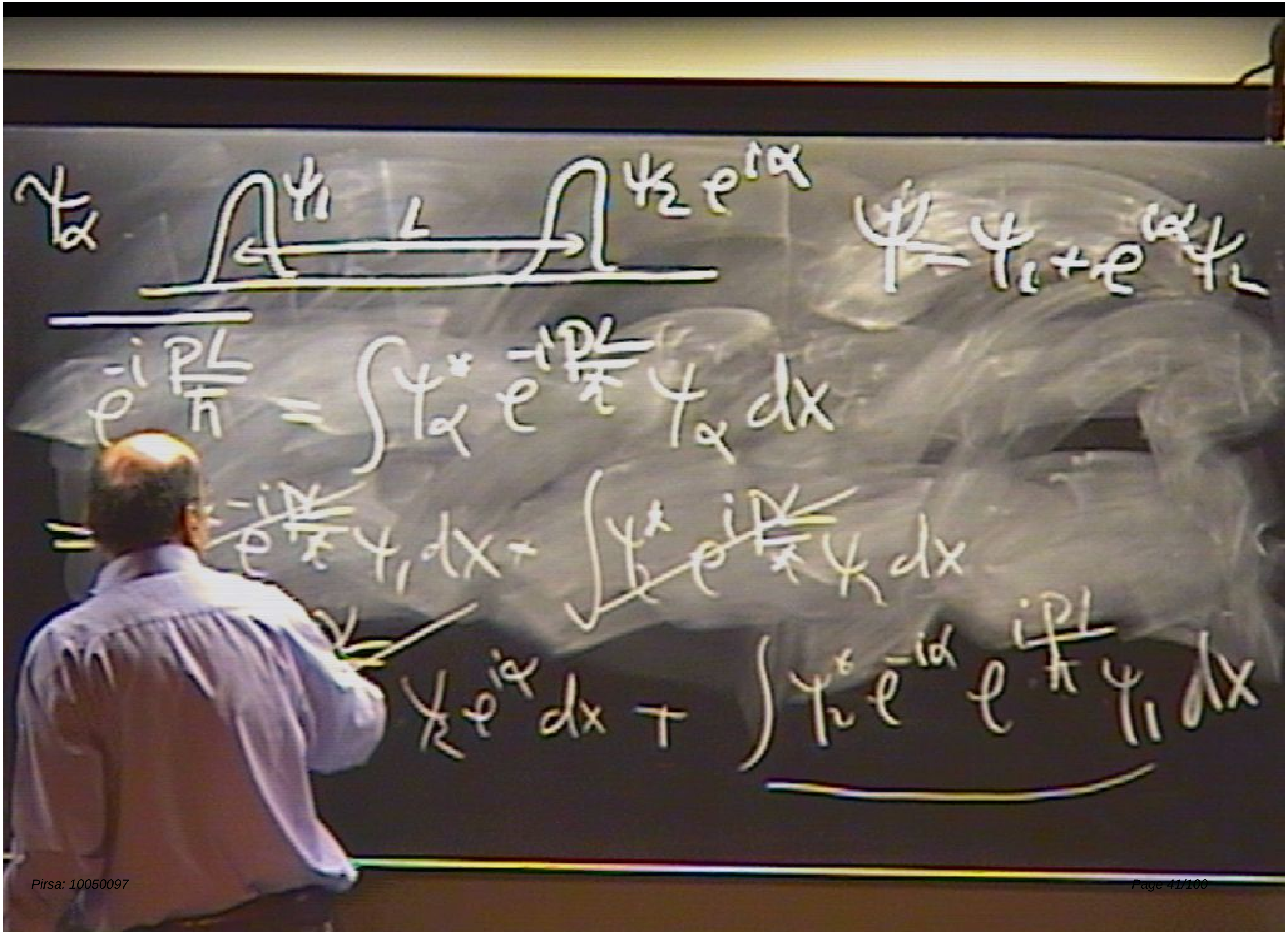

ψ_α
ψ_1
L
$\psi_2 e^{i\alpha}$
$\psi = \psi_1 + e^{i\alpha}\psi_2$
$e^{i\frac{PL}{\hbar}} = \int \psi_\alpha^* e^{i\frac{PL}{\hbar}} \psi_\alpha \, dx$
$= \int \psi_1^* e^{i\frac{PL}{\hbar}} \psi$
$\int \psi_2^* e^{i\frac{PL}{\hbar}} \psi_2 \, dx$
$+ \int \psi_1^*$
$dx +$
$\int \psi_2^* e^{-i\alpha} e^{i\frac{PL}{\hbar}} \psi_1 \, dx$

$\overline{X^N}$ INDEP. OF α

$\overline{P^M}$ ——//——

$\overline{X^N P^K}$

$$e^{iP\frac{L}{\hbar}} = e^{i\alpha}$$



ψα
ψ1
L
ψ2 e^{iα}
ψ = ψ1 + e^{iα} ψ2
e^{-iPL/ħ} = ∫ ψα* e^{-iPL/ħ} ψα dx
= ∫ ψ1* e^{-iPL/ħ} ψ1 dx + ∫ ψ2* e^{-iPL/ħ} ψ2 dx
ψ2 e^{iα} dx + ∫ ψ2* e^{-iα} e^{iPL/ħ} ψ1 dx

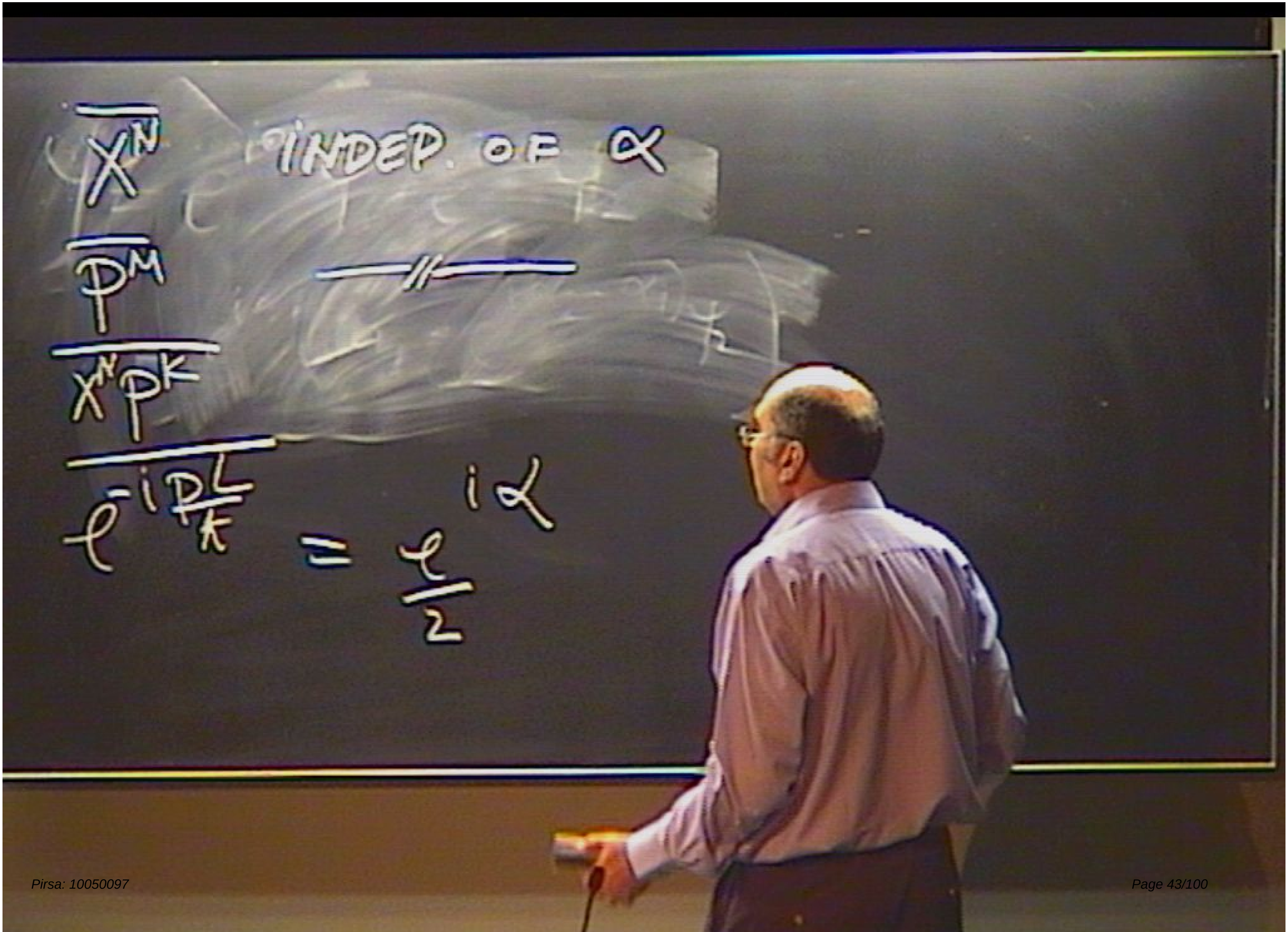
Ψ_α

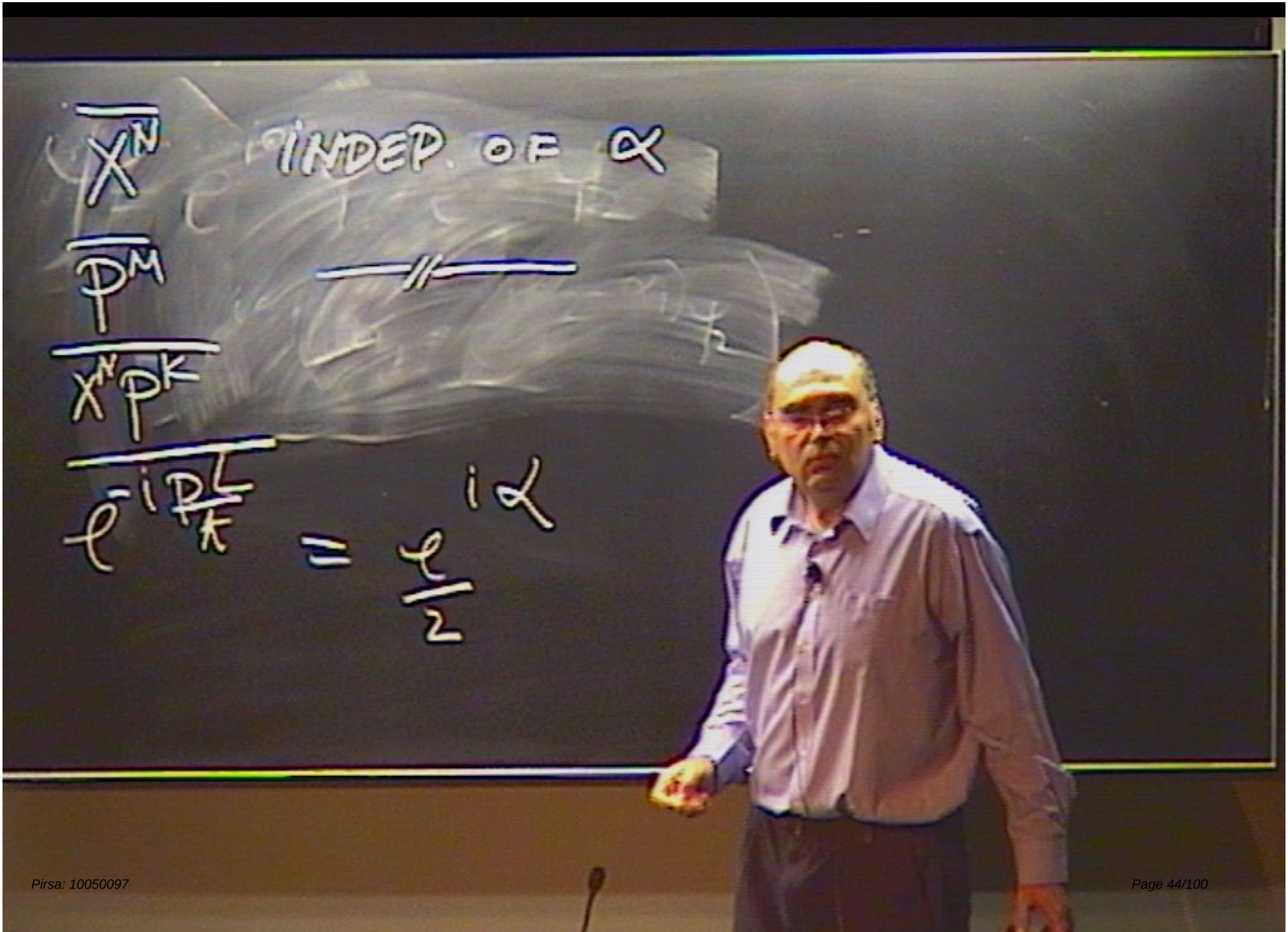
$$\Psi = \psi_1 + e^{i\alpha}\psi_2$$

$$\overline{e^{-i\frac{PL}{\hbar}}} = \int \Psi_\alpha^* e^{-i\frac{PL}{\hbar}} \Psi_\alpha \, dx$$

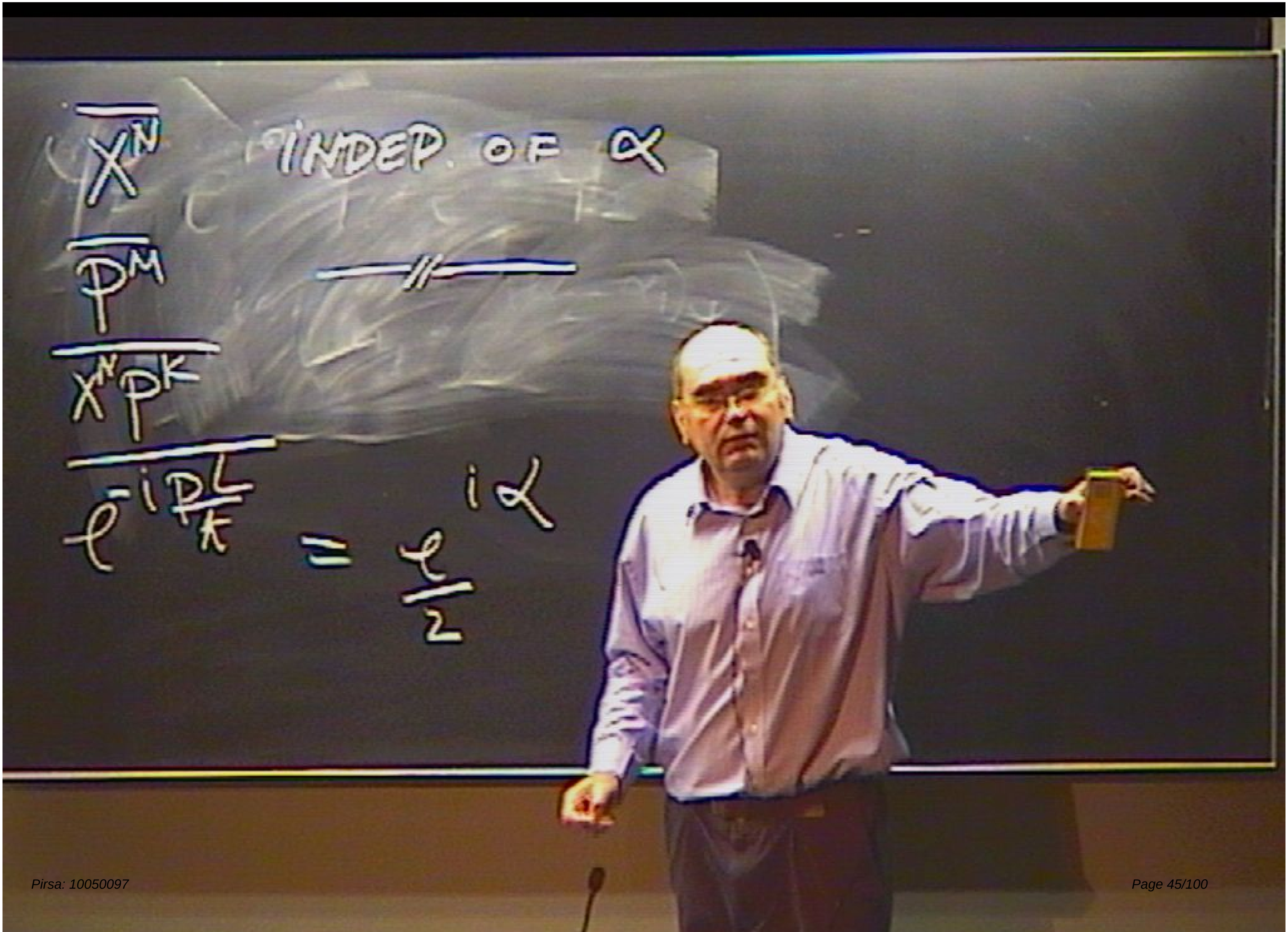
$$= \cancel{\int \psi_1^* e^{-i\frac{PL}{\hbar}} \psi_1 \, dx} + \cancel{\int \psi_2^* e^{-i\frac{PL}{\hbar}} \psi_2 \, dx}$$

$$+ \cancel{\int \psi_1^* e^{-i\frac{PL}{\hbar}}} \psi_2 e^{i\alpha} dx + \underline{\int \psi_2^* e^{-i\alpha} e^{-i\frac{PL}{\hbar}} \psi_1 \, dx}$$


$\overline{X^N}$ INDEP. OF α
$\overline{P^M}$ —//—
$\overline{X^N P^K}$
$e^{-iP\frac{L}{\hbar}} = e^{i\alpha}$
2



$\overline{X^N}$ INDEP. OF α
$\overline{P^M}$ — // —
$\overline{X^N P^K}$
$\overline{e^{-iP\frac{L}{\hbar}}} = \frac{e^{i\alpha}}{2}$



$\overline{X^N}$ INDEP. OF α
$\overline{P^M}$ —— // ——
$\overline{X^N P^K}$
$\overline{e^{-i p \frac{L}{\hbar}}} = e^{\frac{i\alpha}{2}}$

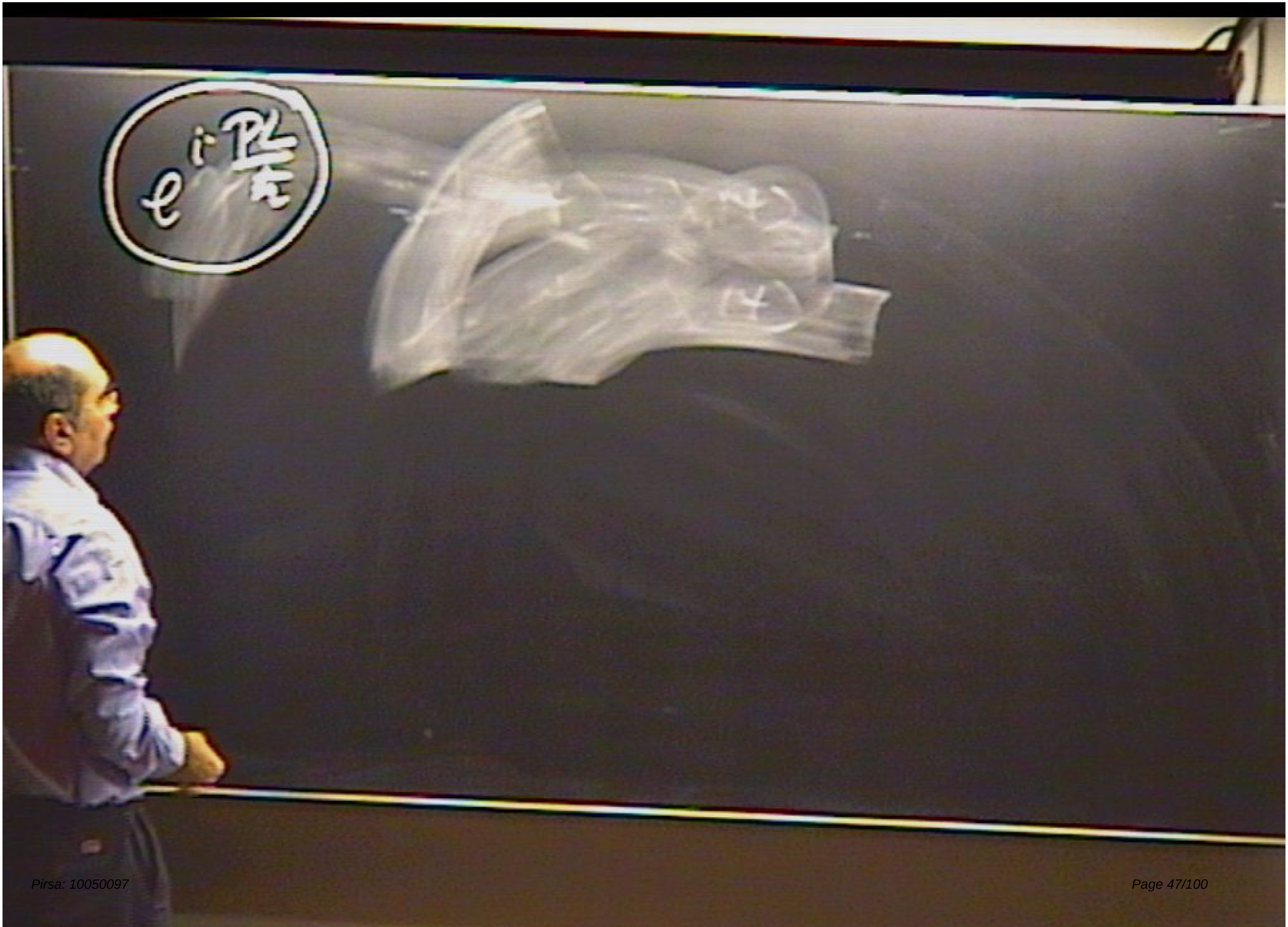
Ψ_α

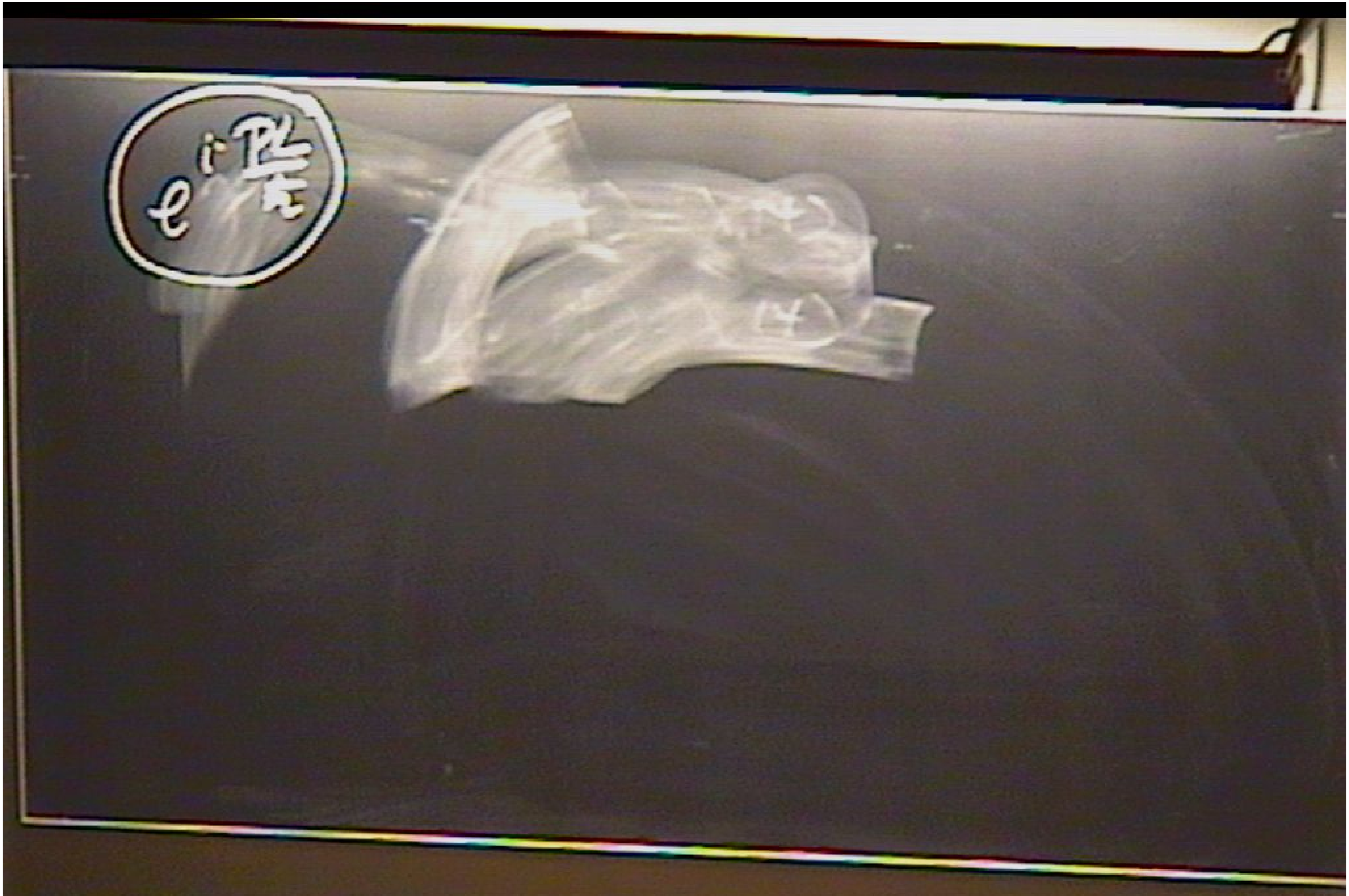
$\Psi = \psi_1 + e^{i\alpha}\psi_2$

$$e^{-i\frac{PL}{\hbar}} = \int \Psi_\alpha^* e^{-i\frac{PL}{\hbar}} \Psi_\alpha \, dx$$

$$= \cancel{\int \psi_1^* e^{-i\frac{PL}{\hbar}} \psi_1 \, dx} + \cancel{\int \psi_2^* e^{-i\frac{PL}{\hbar}} \psi_2 \, dx}$$

$$+ \cancel{\int \psi_1^* e^{-i\frac{PL}{\hbar}}} \psi_2 e^{i\alpha} \, dx + \underline{\int \psi_2^* e^{-i\alpha} e^{-i\frac{PL}{\hbar}} \psi_1 \, dx}$$


$e^{iPL/\hbar}$


$e^{iPL/\hbar}$

ψ_α

$\Psi = \psi_1 + e^{i\alpha}\psi_2$

$$e^{-i\frac{PL}{\hbar}} = \int \psi_\alpha^* e^{-i\frac{PL}{\hbar}} \psi_\alpha \, dx$$

$$= \cancel{\int \psi_1^* e^{-i\frac{PL}{\hbar}} \psi_1 \, dx} + \cancel{\int \psi_2^* e^{-i\frac{PL}{\hbar}} \psi_2 \, dx}$$

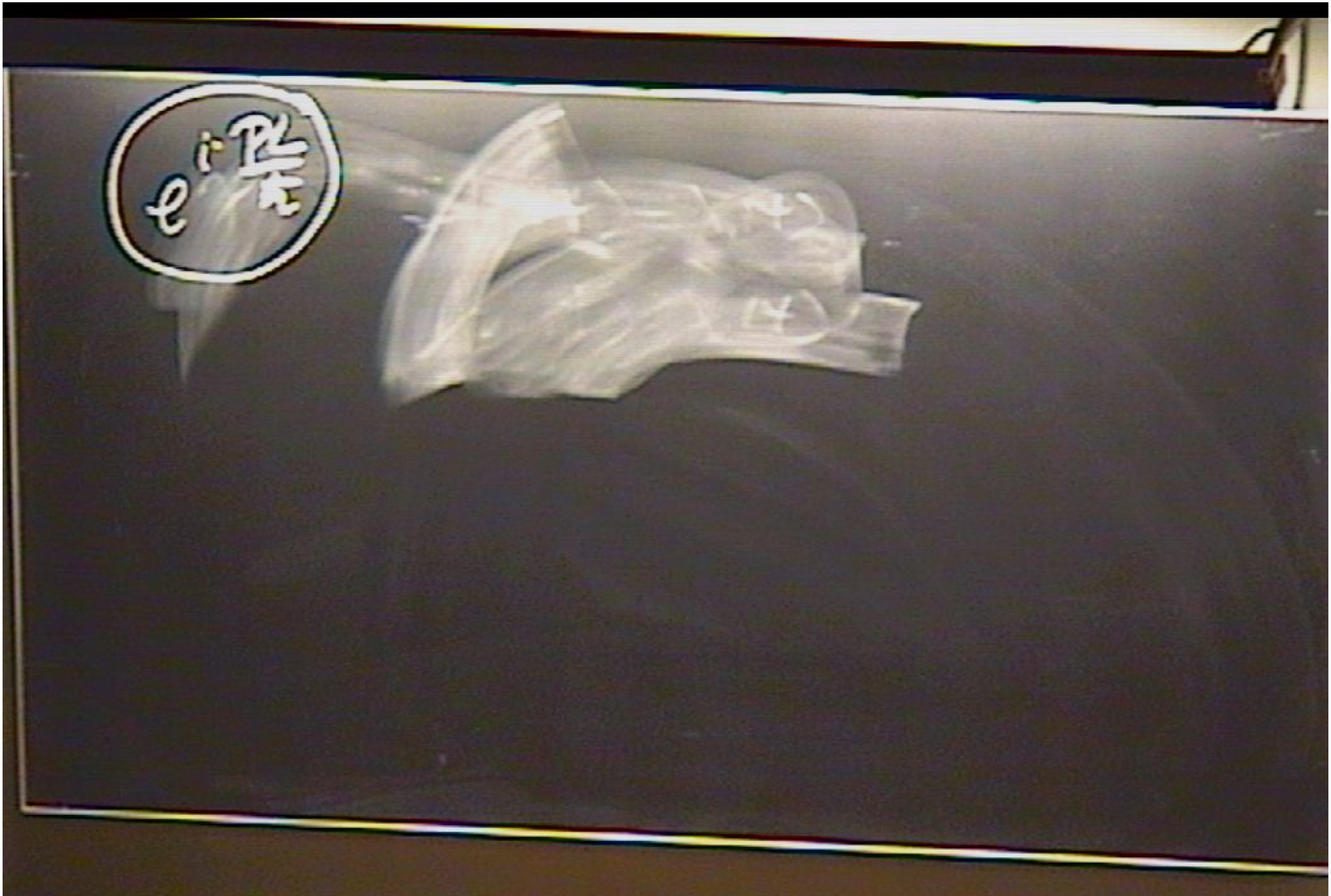
$$+ \cancel{\int \psi_1^* e^{-i\frac{PL}{\hbar}}} \psi_2 e^{i\alpha} \, dx + \underline{\int \psi_2^* e^{-i\alpha} e^{-i\frac{PL}{\hbar}} \psi_1 \, dx}$$

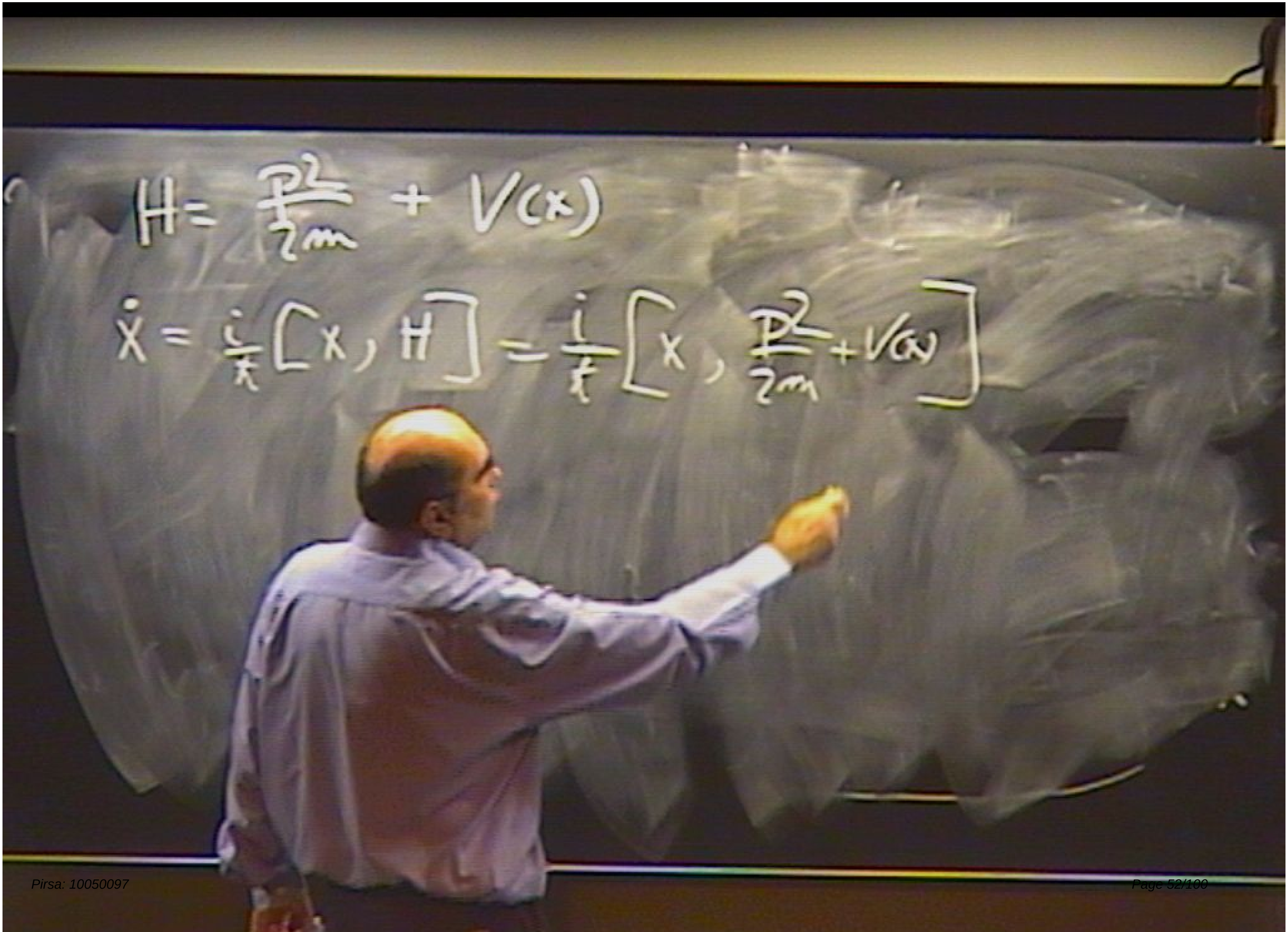
$\overline{X^N}$ INDEP. OF α

$\overline{P^M}$ —— // ——

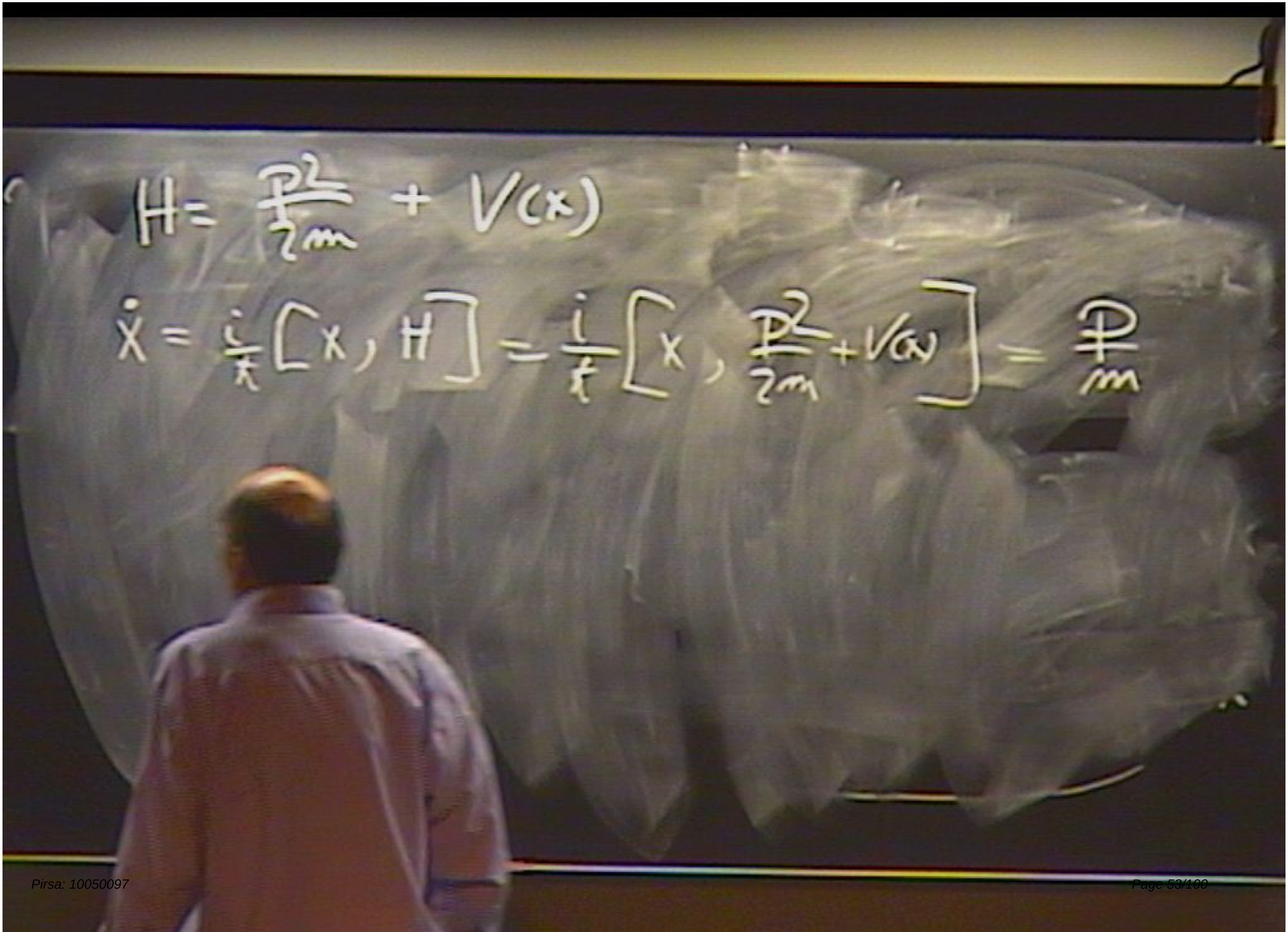
$\overline{X^N P^K}$

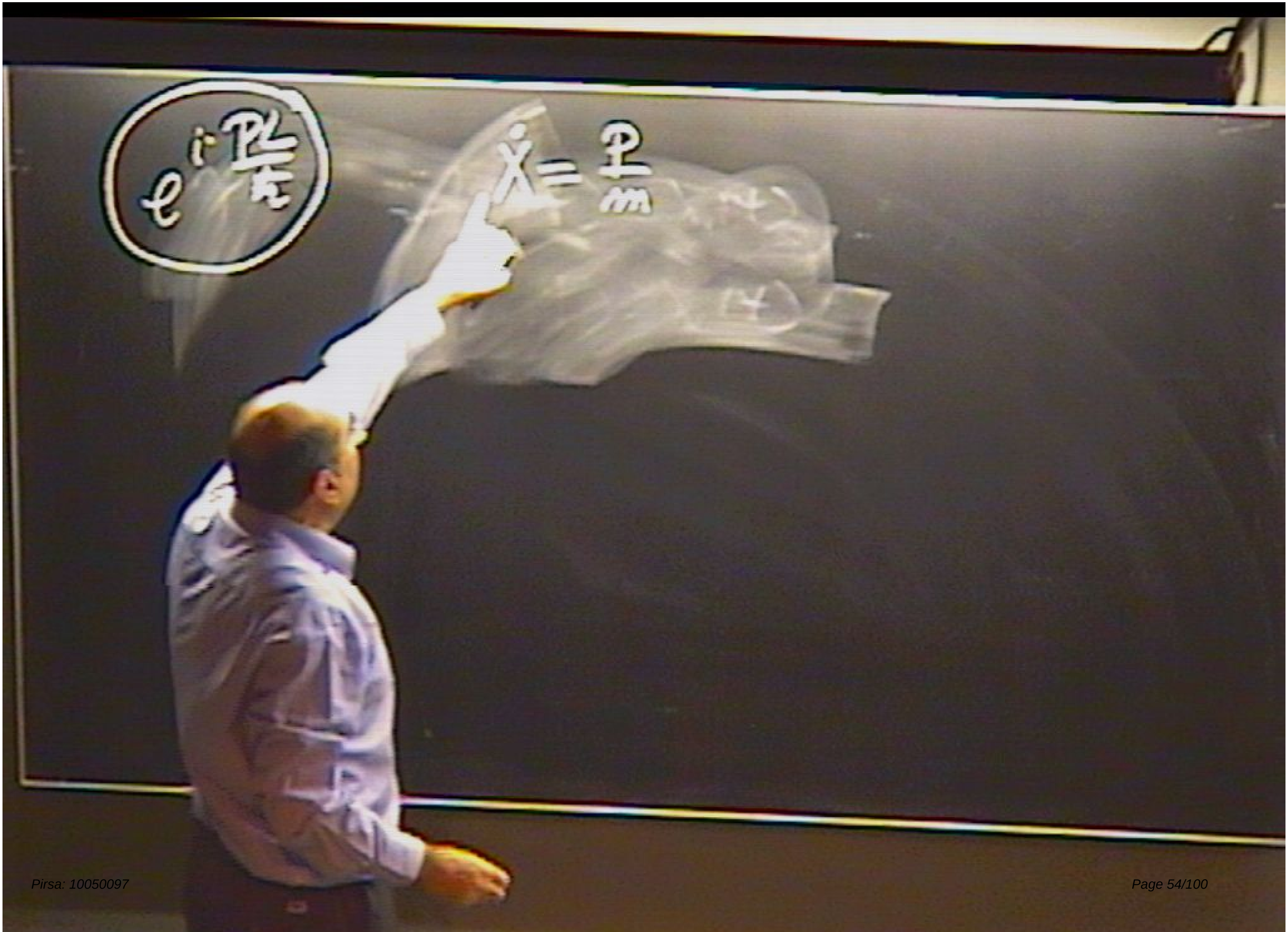
$\overline{e^{-i\frac{PL}{\hbar}}} = e^{\frac{i\alpha}{2}}$

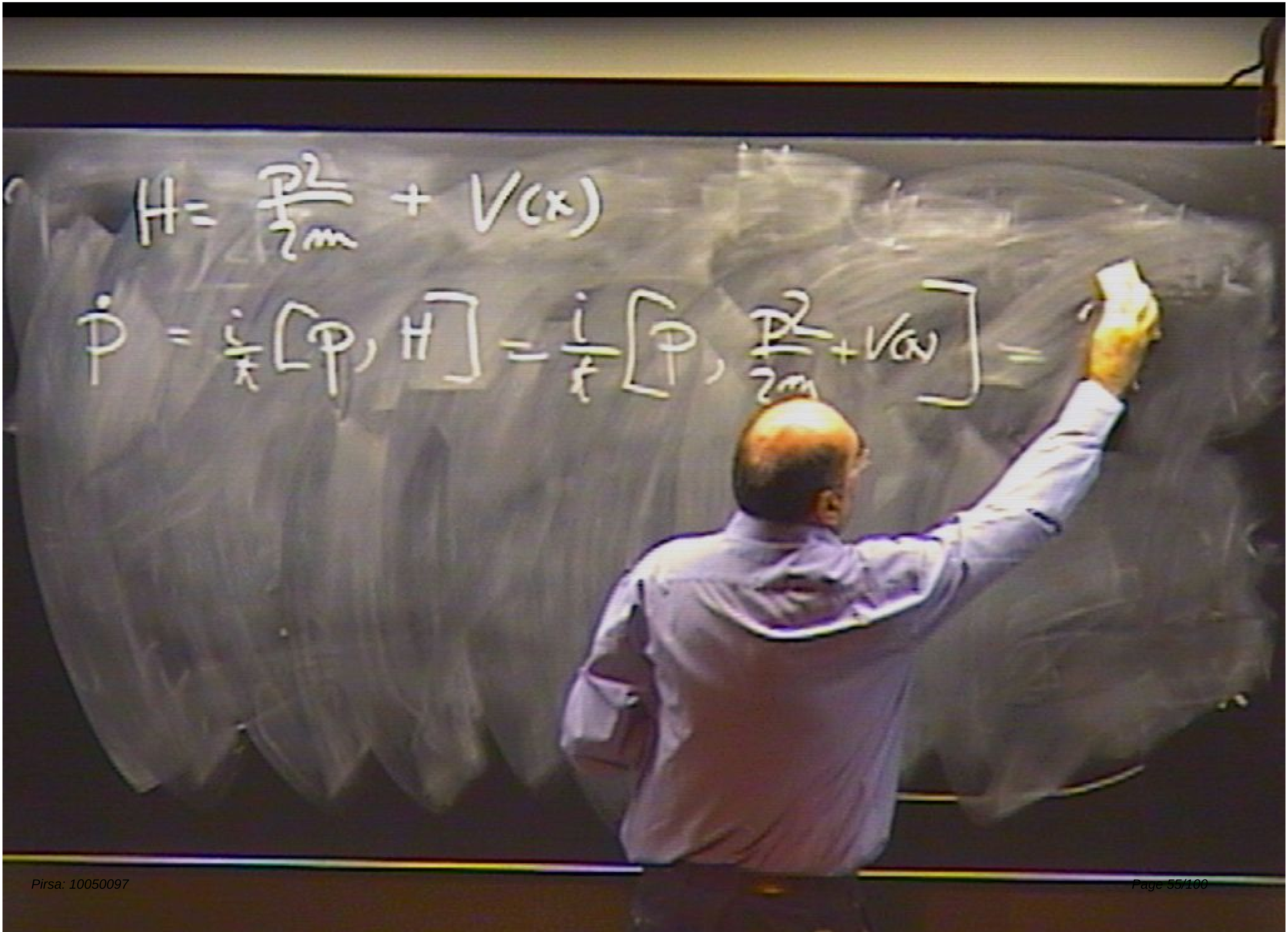


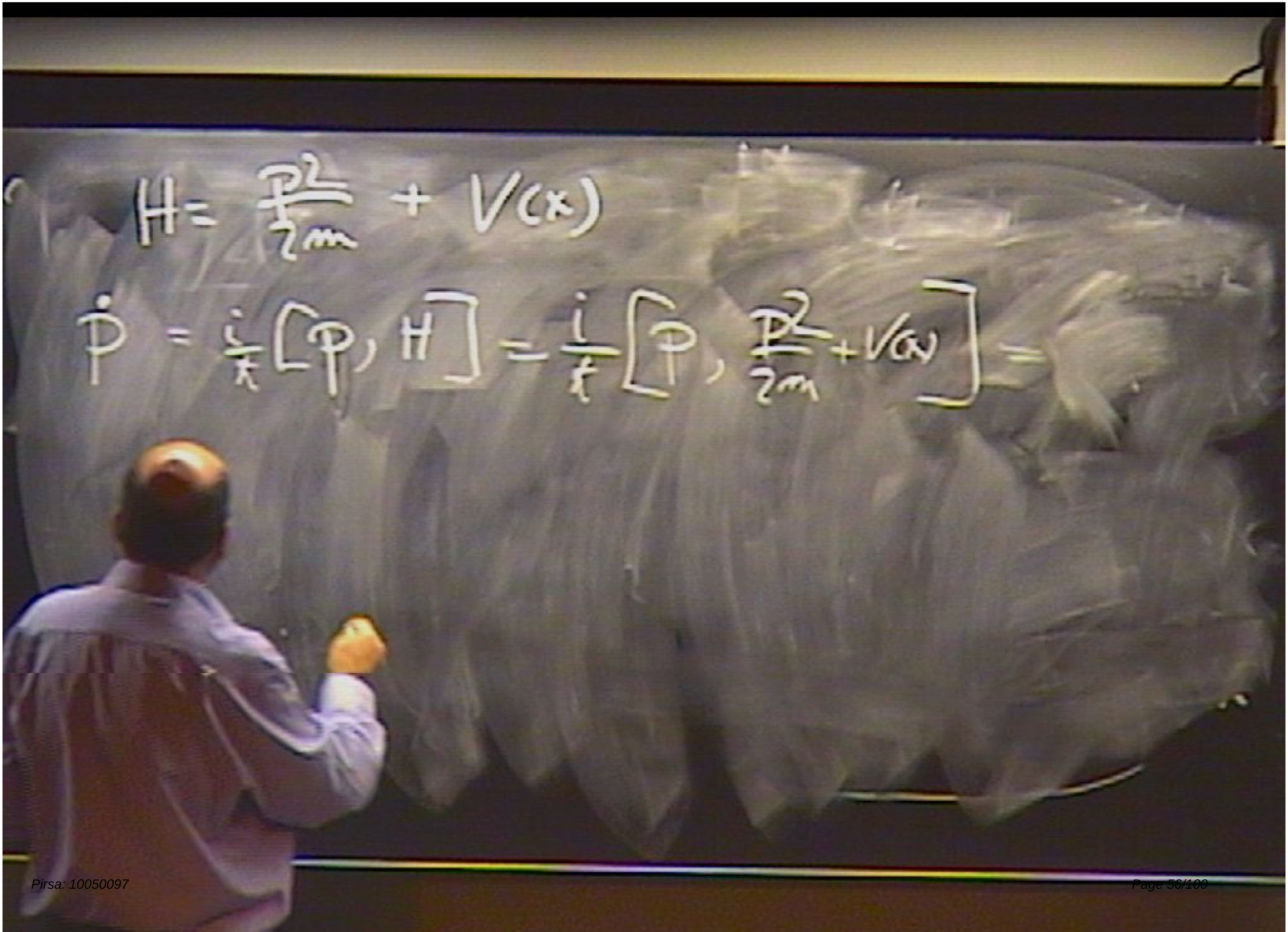


$H = \frac{P^2}{2m} + V(x)$
$\dot{x} = \frac{i}{\hbar}[x, H] = \frac{i}{\hbar}\left[x, \frac{P^2}{2m} + V(x)\right]$


$H = \frac{p^2}{2m} + V(x)$
$\dot{x} = \frac{i}{\hbar}[x, H] = \frac{i}{\hbar}\left[x, \frac{p^2}{2m} + V(x)\right] = \frac{p}{m}$


$e^{i\frac{PL}{\hbar}}$
$\dot{x} = \frac{p}{m}$

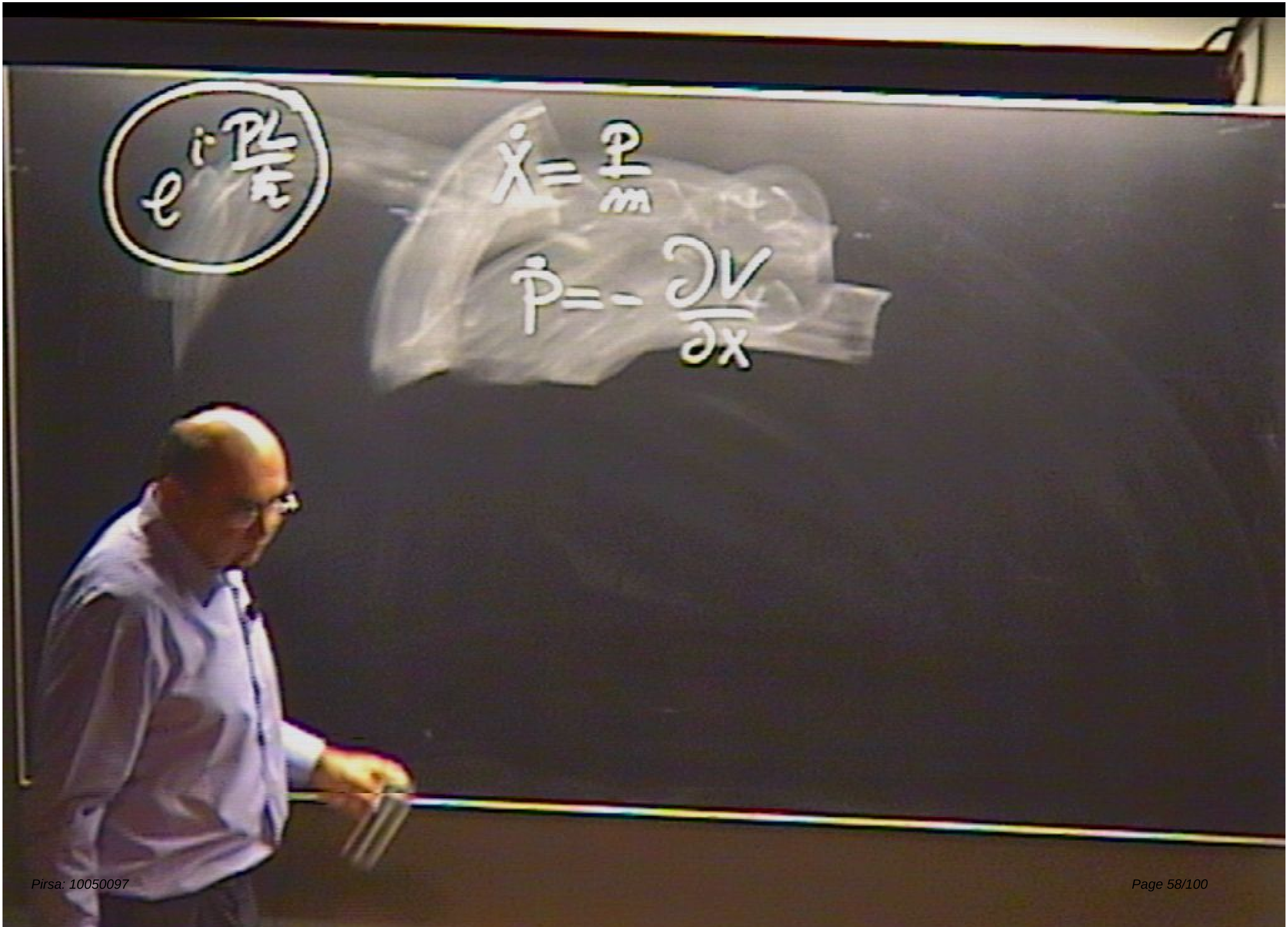

$H = \frac{p^2}{2m} + V(x)$
$\dot{p} = \frac{i}{\hbar}[p, H] = \frac{i}{\hbar}\left[p, \frac{p^2}{2m} + V(x)\right] =$


$H = \frac{p^2}{2m} + V(x)$
$\dot{p} = \frac{i}{\hbar}[p, H] = \frac{i}{\hbar}\left[p, \frac{p^2}{2m} + V(x)\right] =$

$$H = \frac{p^2}{2m} + V(x)$$

$$\dot{p} = \frac{i}{\hbar}[p, H] = \frac{i}{\hbar}\left[p, \frac{p^2}{2m} + V(x)\right] =$$

$$= \frac{i}{\hbar}[p, V(x)] = \frac{-i\,i\hbar}{\hbar}\left(-\frac{\partial V}{\partial x}\right) = -\frac{\partial V}{\partial x}$$


$e^{iPL/\hbar}$
$\dot{x} = \frac{p}{m}$
$\dot{p} = -\frac{\partial V}{\partial x}$

$$e^{i\frac{PL}{\hbar}}$$

$$\dot{X} = \frac{P}{m}$$

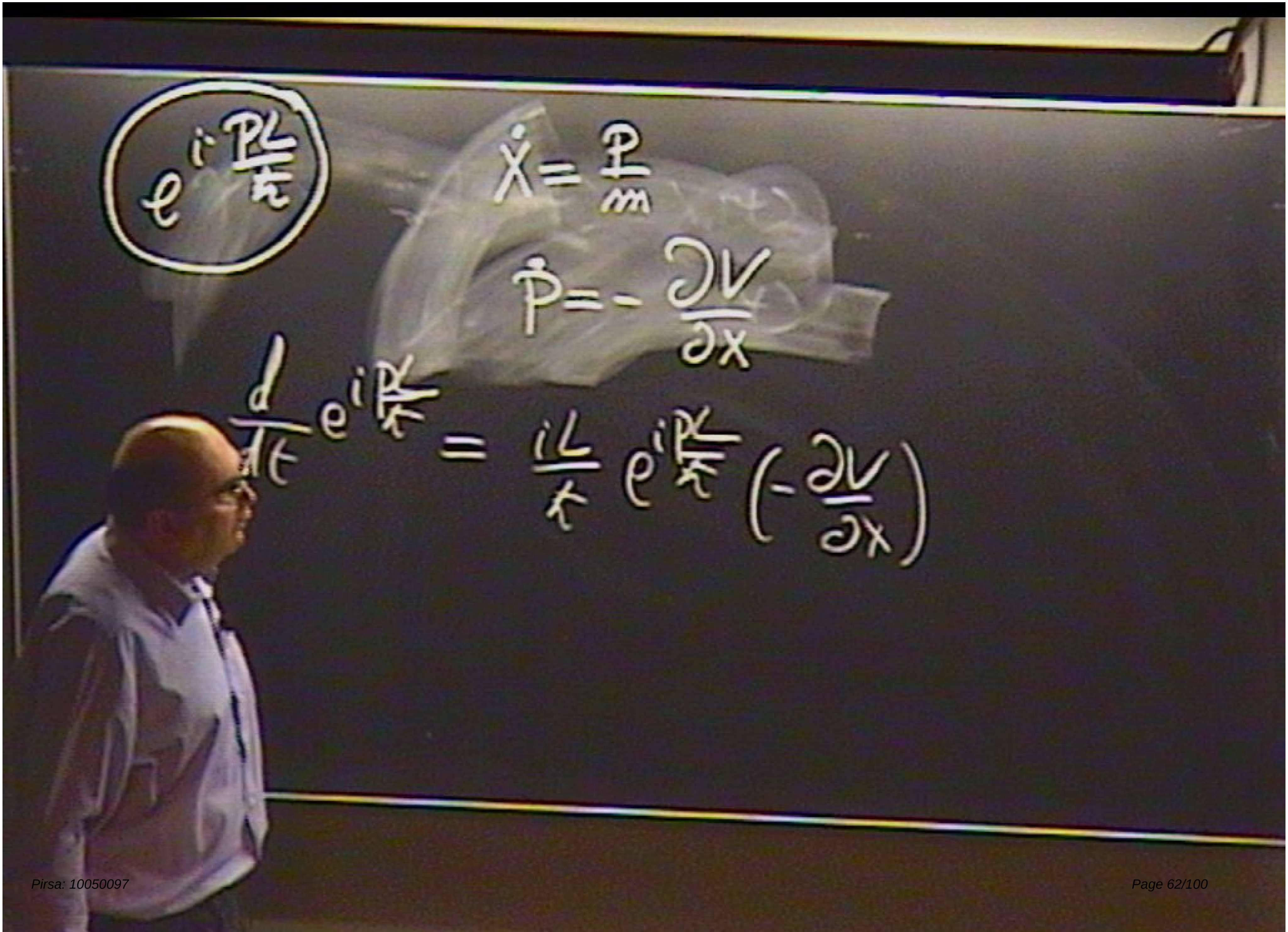
$$\dot{P} = -\frac{\partial V}{\partial x}$$

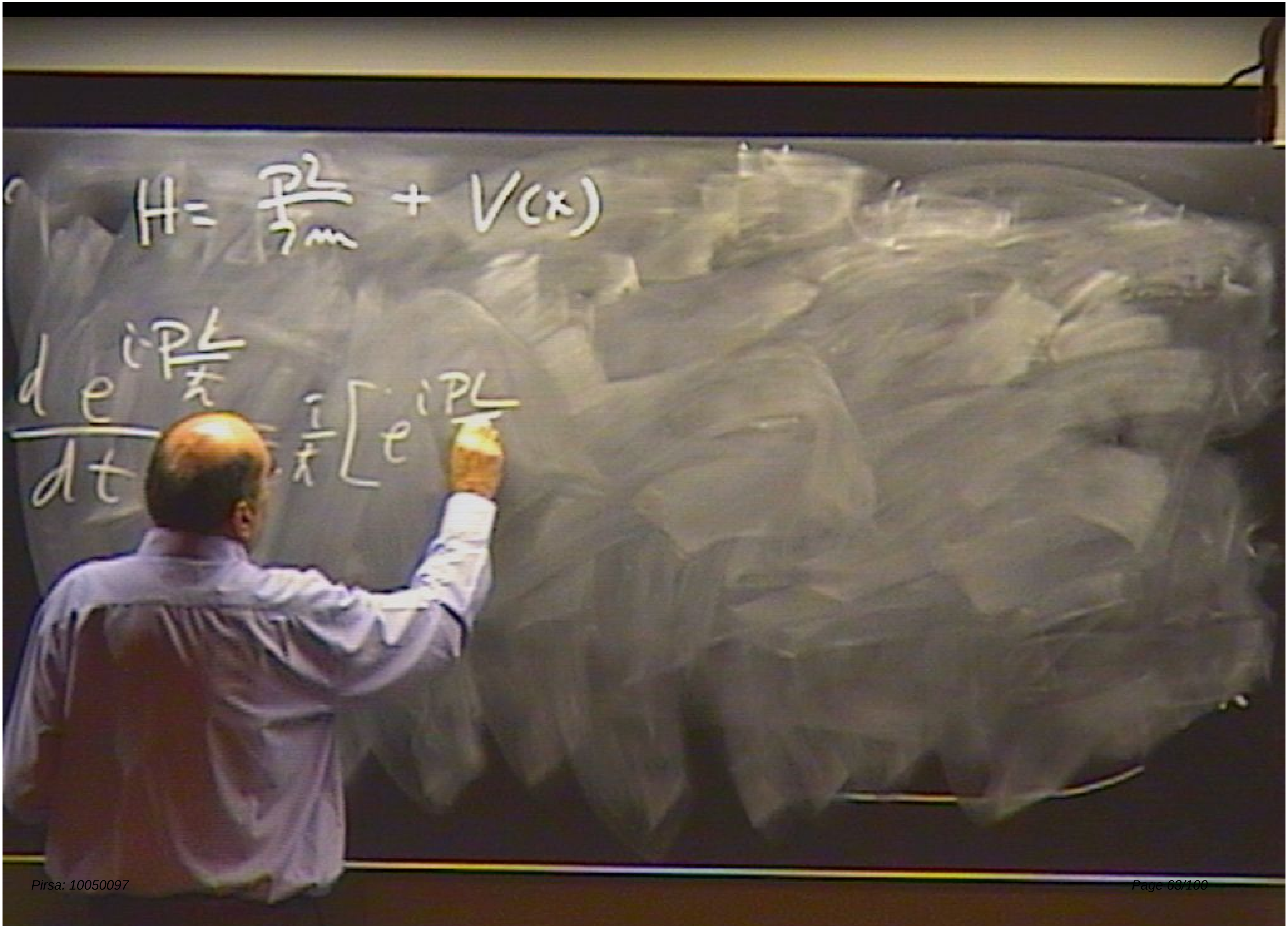
$$H = \frac{p^2}{2m} + V(x)$$

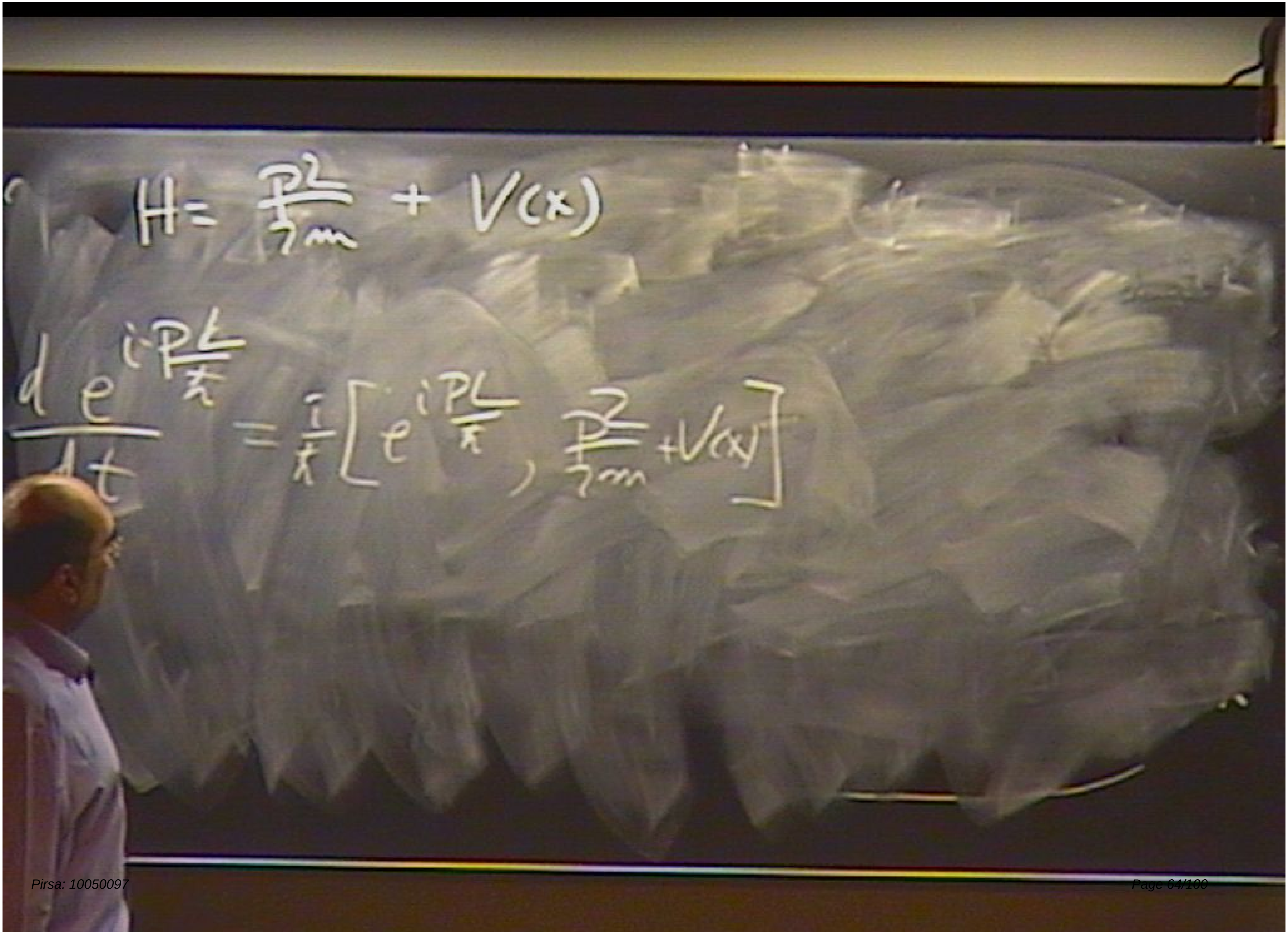
$$\frac{d\, e^{iPt/\hbar}}{dt}$$

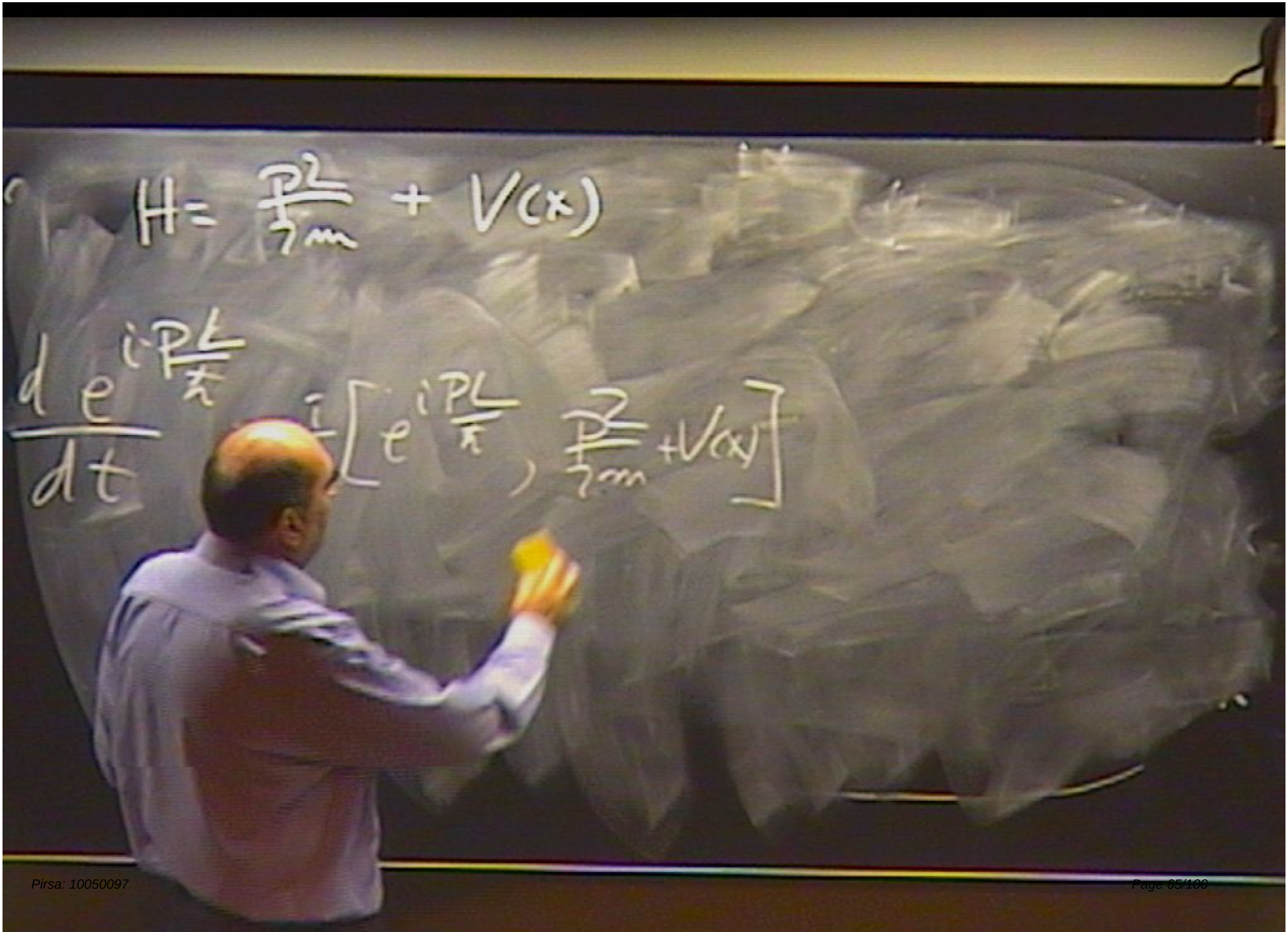
$$H = \frac{P^2}{2m} + V(x)$$

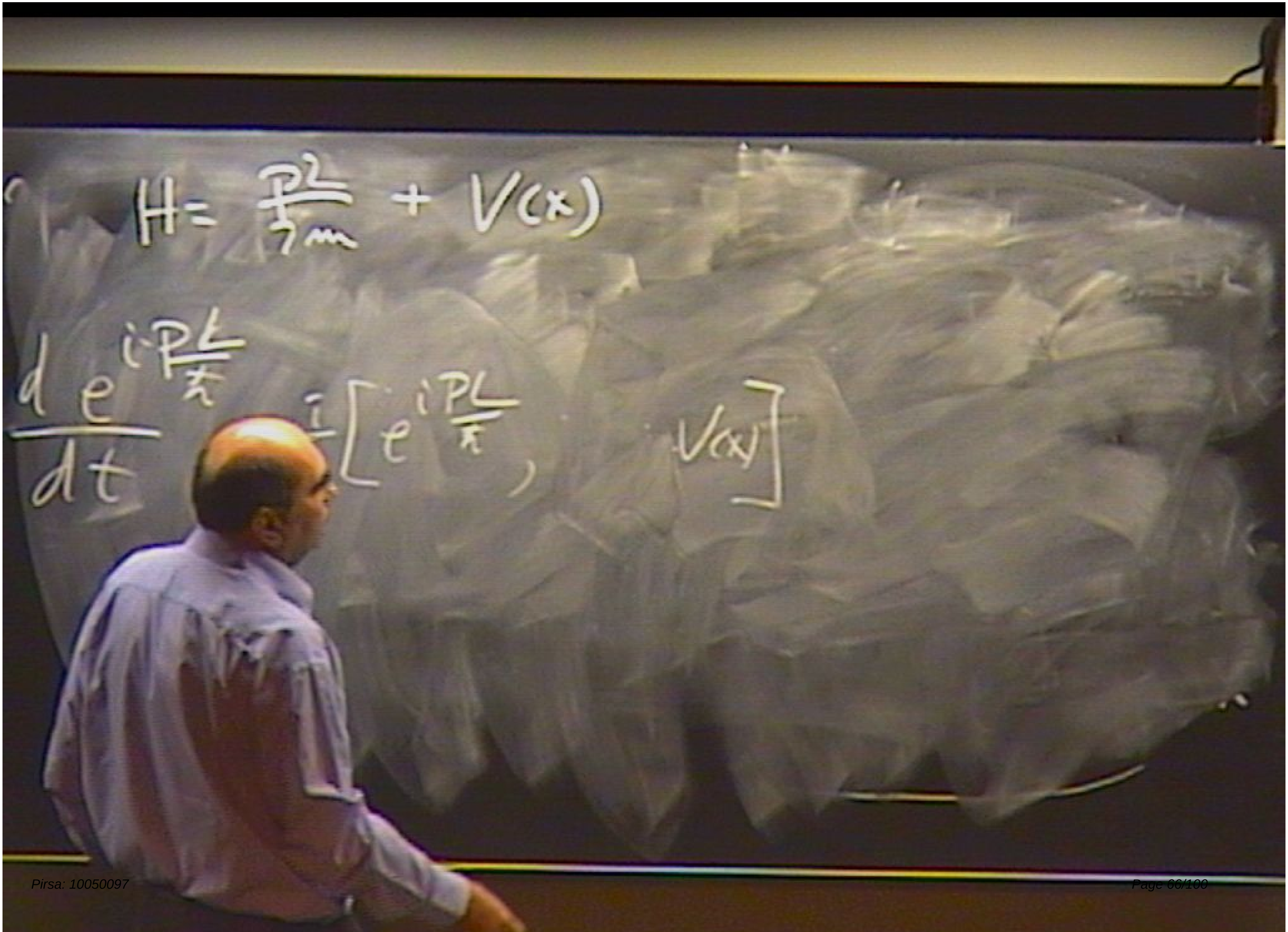
$$\frac{d\, e^{iP\frac{L}{\hbar}}}{dt} = \frac{d\, e^{iP\frac{L}{\hbar}}}{dP} \quad \frac{dP}{dt} = \frac{iL}{\hbar} e^{iP\frac{L}{\hbar}} \left(-\frac{\partial V}{\partial x}\right)$$


$e^{i\frac{PL}{\hbar}}$
$\dot{X} = \frac{P}{m}$
$\dot{P} = -\frac{\partial V}{\partial x}$
$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \left(-\frac{\partial V}{\partial x}\right)$


$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iP\frac{L}{\hbar}}}{dt} = \frac{i}{\hbar}\left[e^{iP\frac{L}{\hbar}}\right.$


$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iPL/\hbar}}{dt} = \frac{i}{\hbar}\left[e^{iPL/\hbar}, \frac{P^2}{2m} + V(x)\right]$

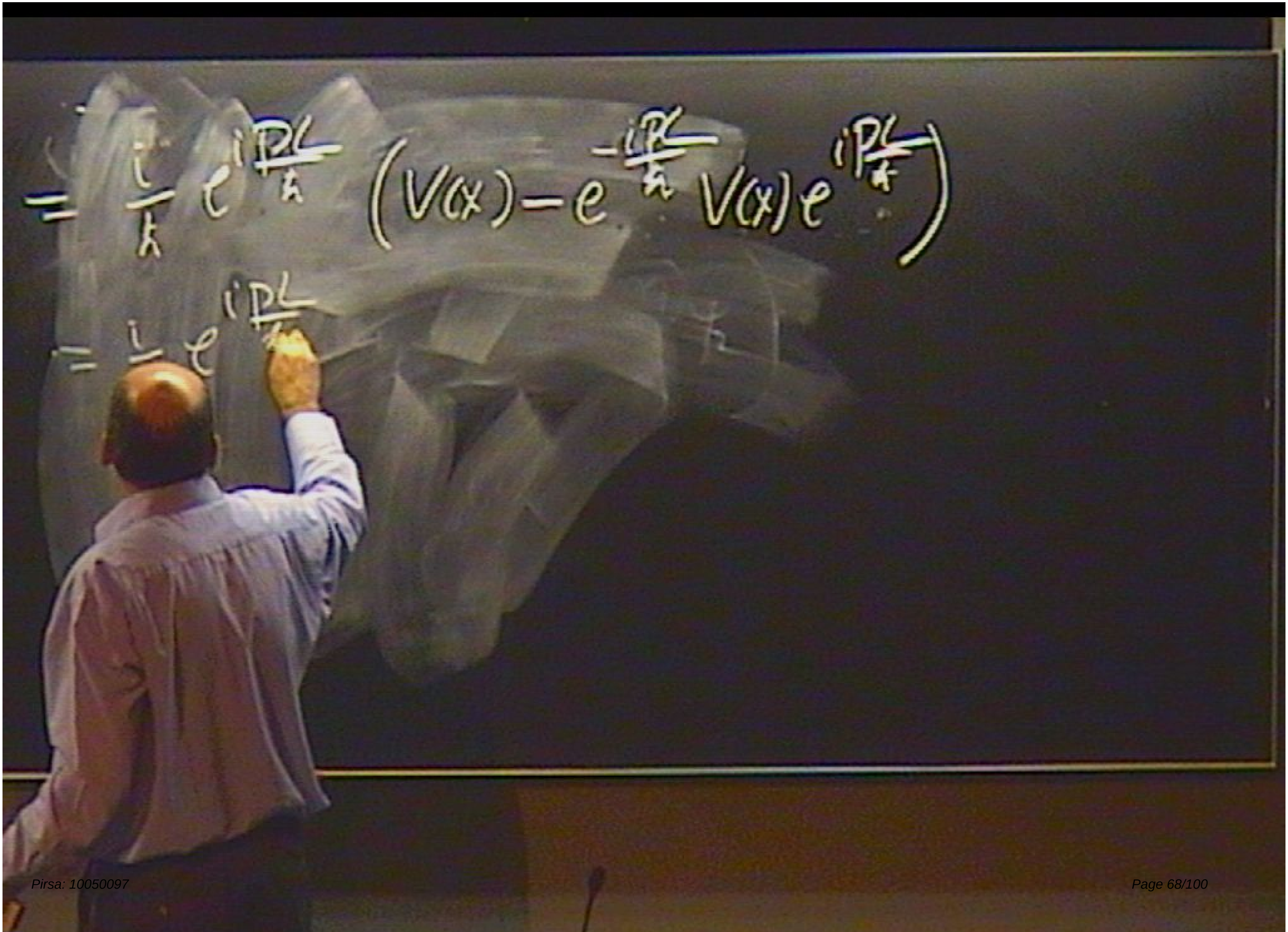

$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iPL/\hbar}}{dt}$
$[e^{iPL/\hbar}, \frac{P^2}{2m} + V(x)]$

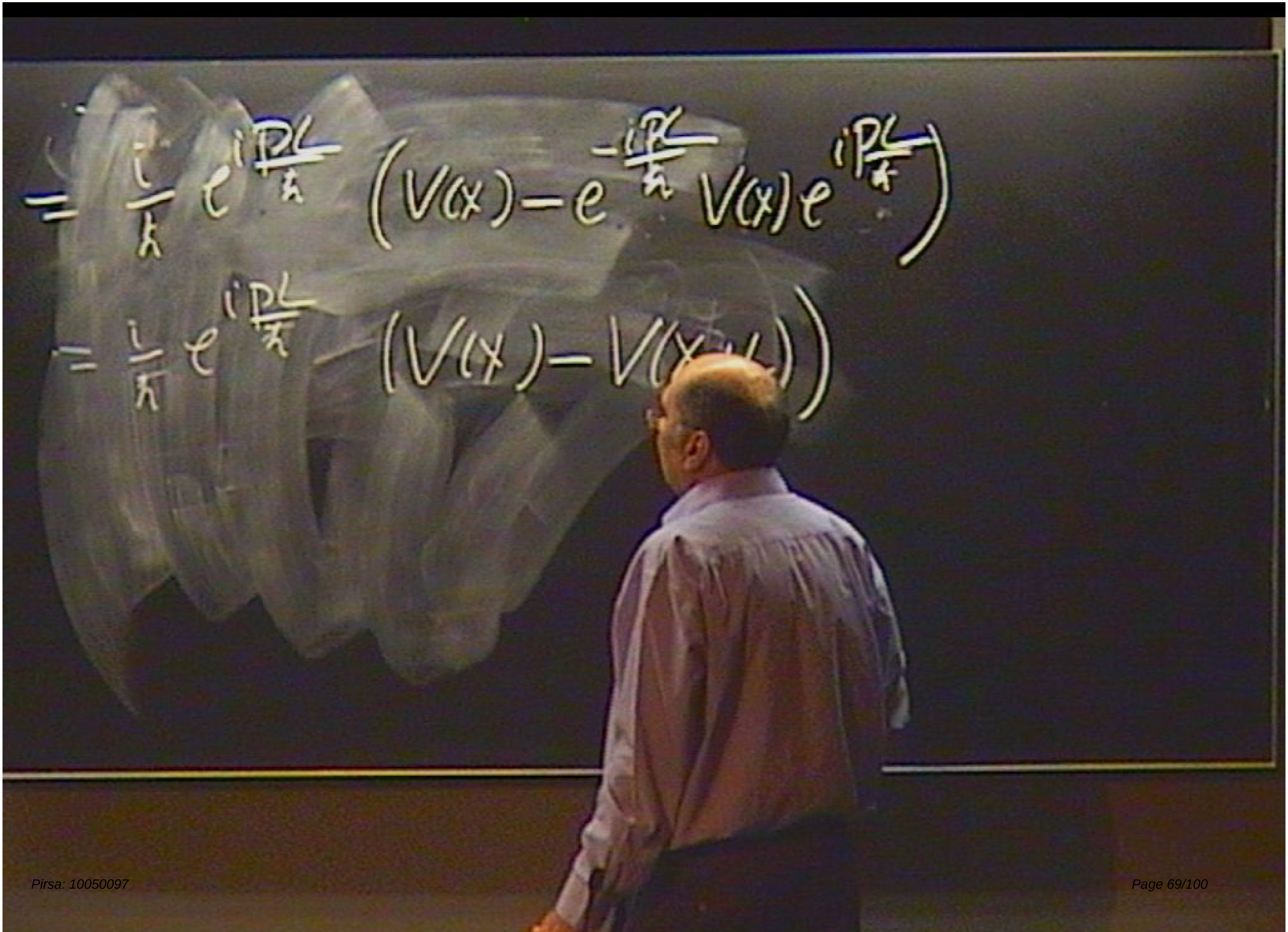

$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iPL/\hbar}}{dt} = \frac{i}{\hbar}\left[e^{iPL/\hbar}, V(x)\right]$

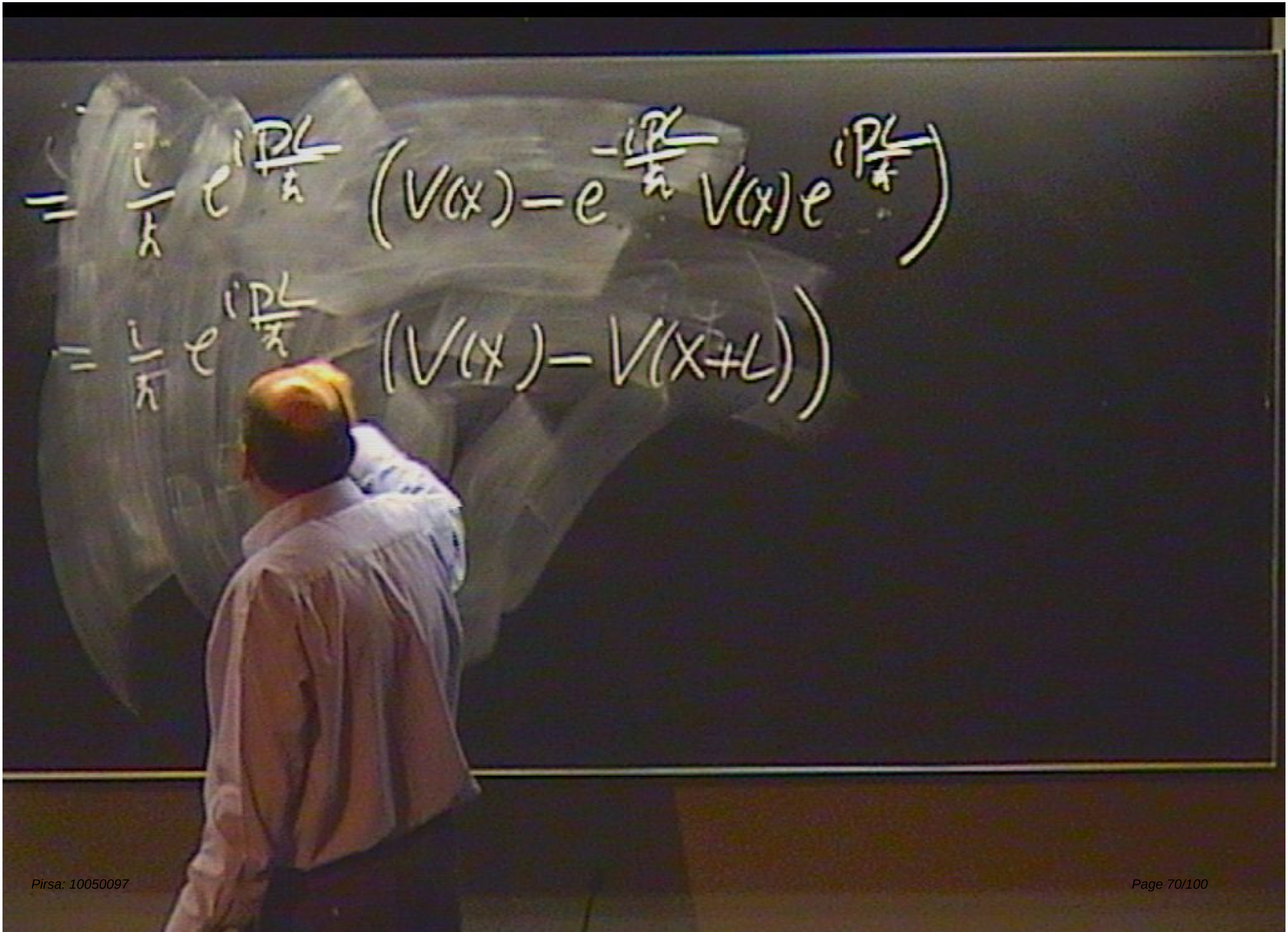
$$H = \frac{P^2}{2m} + V(x)$$

$$\frac{d\, e^{\frac{iPL}{\hbar}}}{dt} = \frac{i}{\hbar}\left[e^{\frac{iPL}{\hbar}}, V(x)\right] =$$

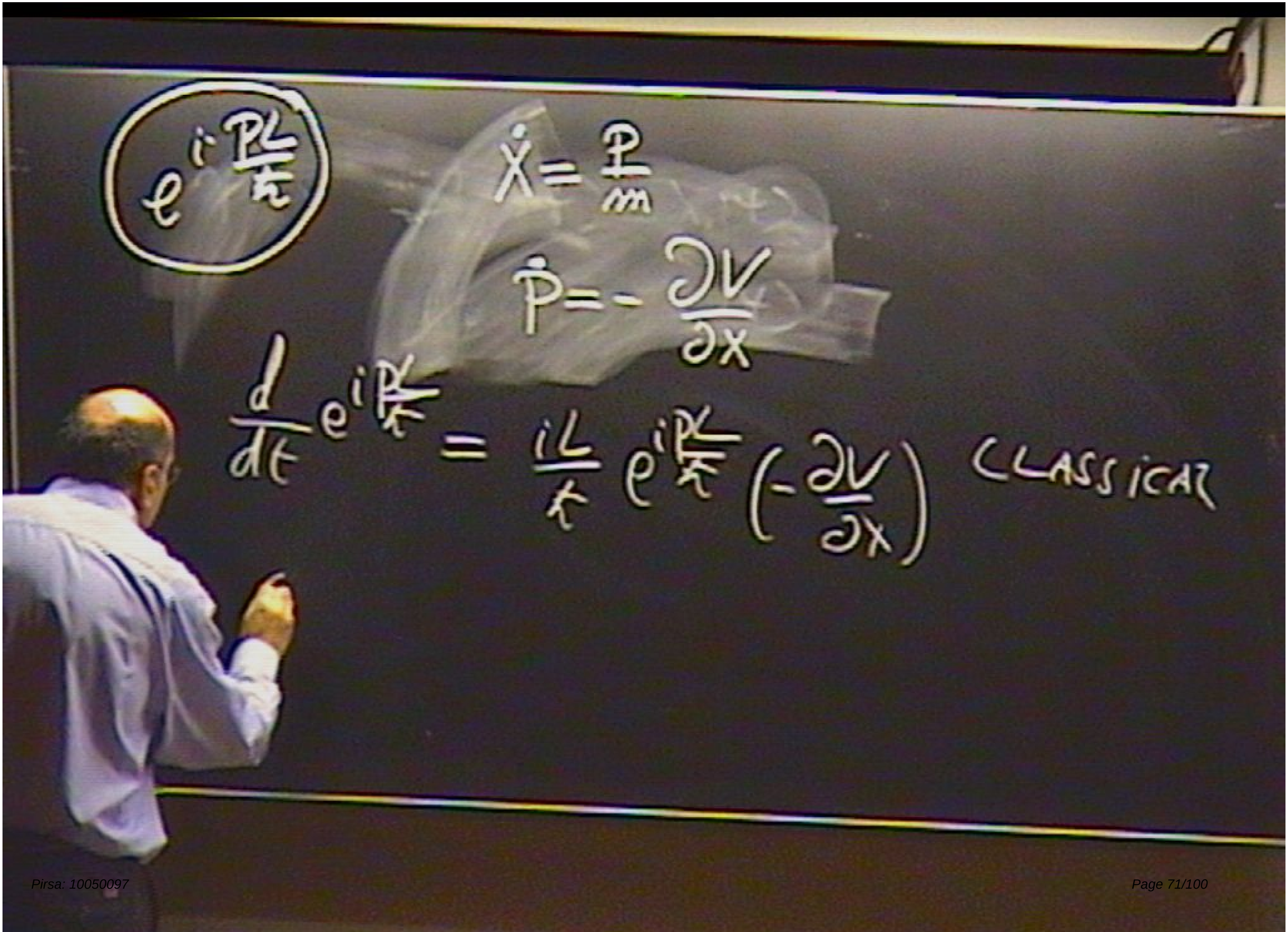
$$= \frac{i}{\hbar}\left(e^{\frac{iPL}{\hbar}} V(x) - V(x) e^{\frac{iPL}{\hbar}}\right) =$$


$= \frac{i}{\hbar} e^{iPL/\hbar}\left(V(x) - e^{-iPL/\hbar} V(x) e^{iPL/\hbar}\right)$
$= \frac{i}{\hbar} e^{iPL/\hbar}$

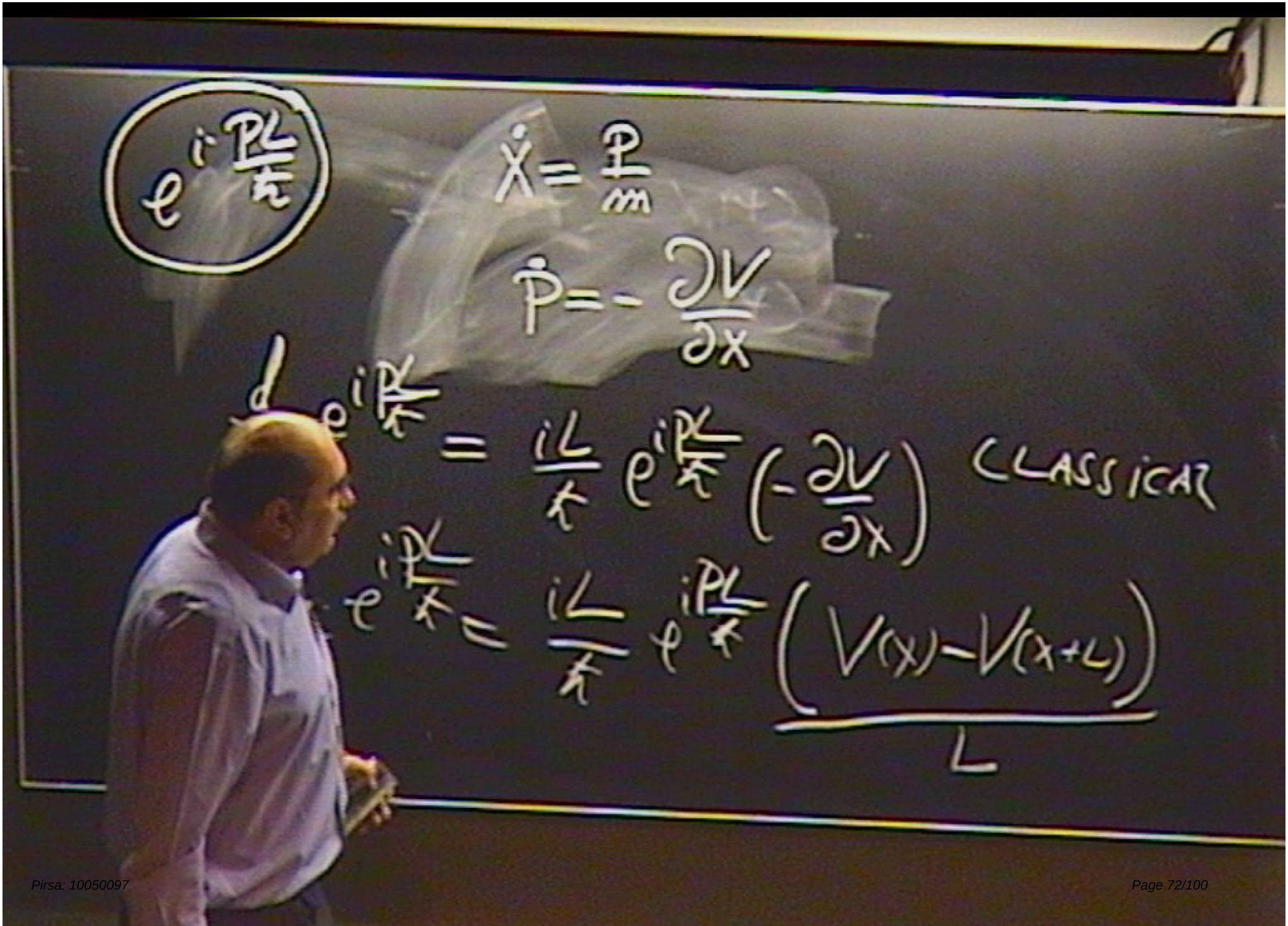

$= \frac{i}{\hbar} e^{\frac{ipL}{\hbar}} \left(V(x) - e^{-\frac{ipL}{\hbar}} V(x) e^{\frac{ipL}{\hbar}} \right)$
$= \frac{i}{\hbar} e^{\frac{ipL}{\hbar}} \left(V(x) - V(x+L) \right)$

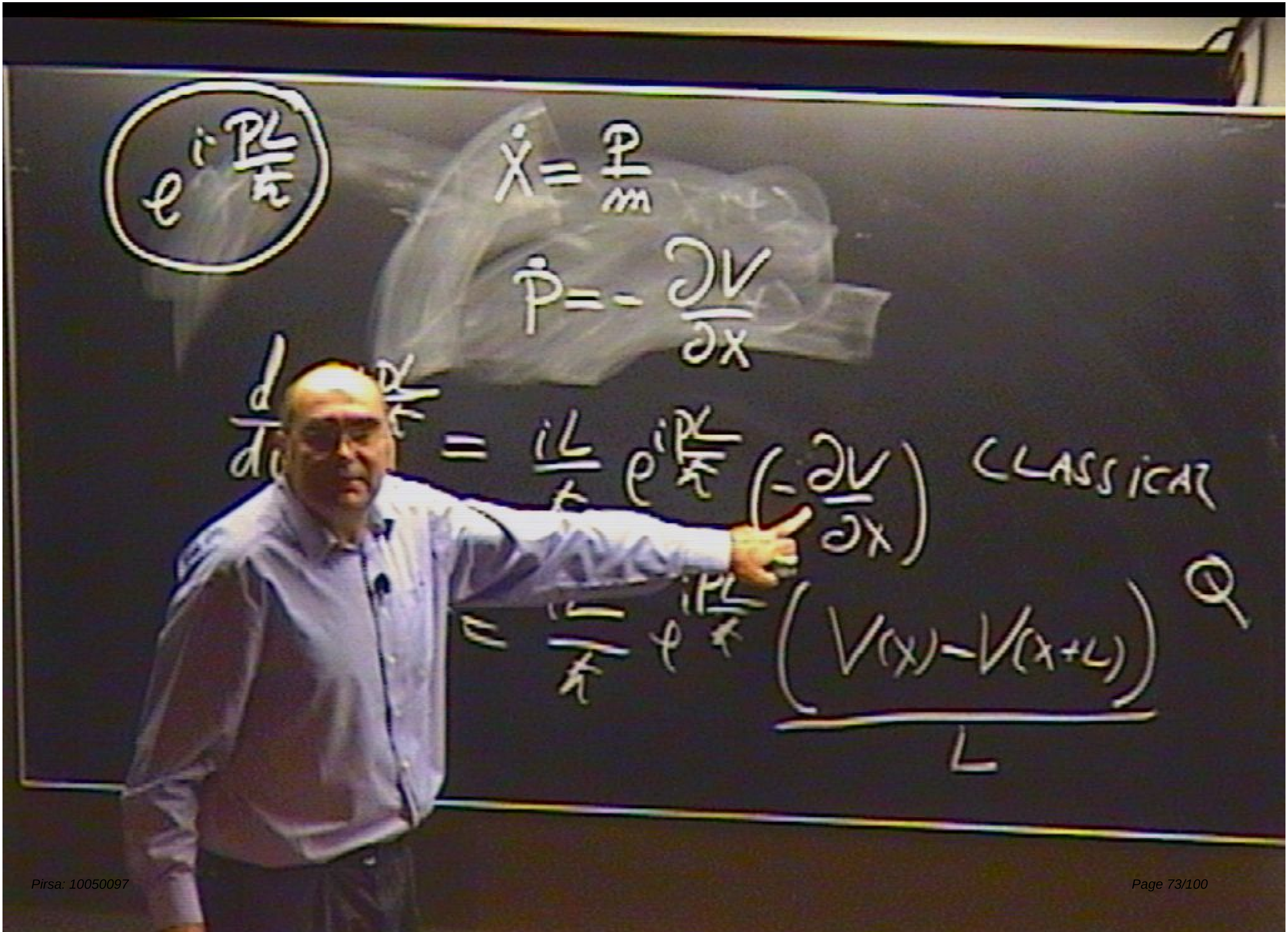


$= \frac{i}{\hbar} e^{\frac{iPL}{\hbar}} \left(V(x) - e^{-\frac{iPL}{\hbar}} V(x) e^{\frac{iPL}{\hbar}} \right)$
$= \frac{i}{\hbar} e^{\frac{iPL}{\hbar}} \left(V(x) - V(x+L) \right)$



$e^{i\frac{PL}{\hbar}}$
$\dot{x} = \frac{P}{m}$
$\dot{P} = -\frac{\partial V}{\partial x}$
$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \left(-\frac{\partial V}{\partial x}\right)$
CLASSICAL


$e^{i\frac{PL}{\hbar}}$
$\dot{X} = \frac{P}{m}$
$\dot{P} = -\frac{\partial V}{\partial x}$
$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \left(-\frac{\partial V}{\partial x}\right)$
CLASSICAL
$e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \frac{\left(V(x) - V(x+L)\right)}{L}$


$e^{i\frac{PL}{\hbar}}$
$\dot{x} = \frac{P}{m}$
$\dot{P} = -\frac{\partial V}{\partial x}$
$= \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \left(-\frac{\partial V}{\partial x}\right)$
CLASSICAL
$= \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \frac{\left(V(x) - V(x+L)\right)}{L}$
Q

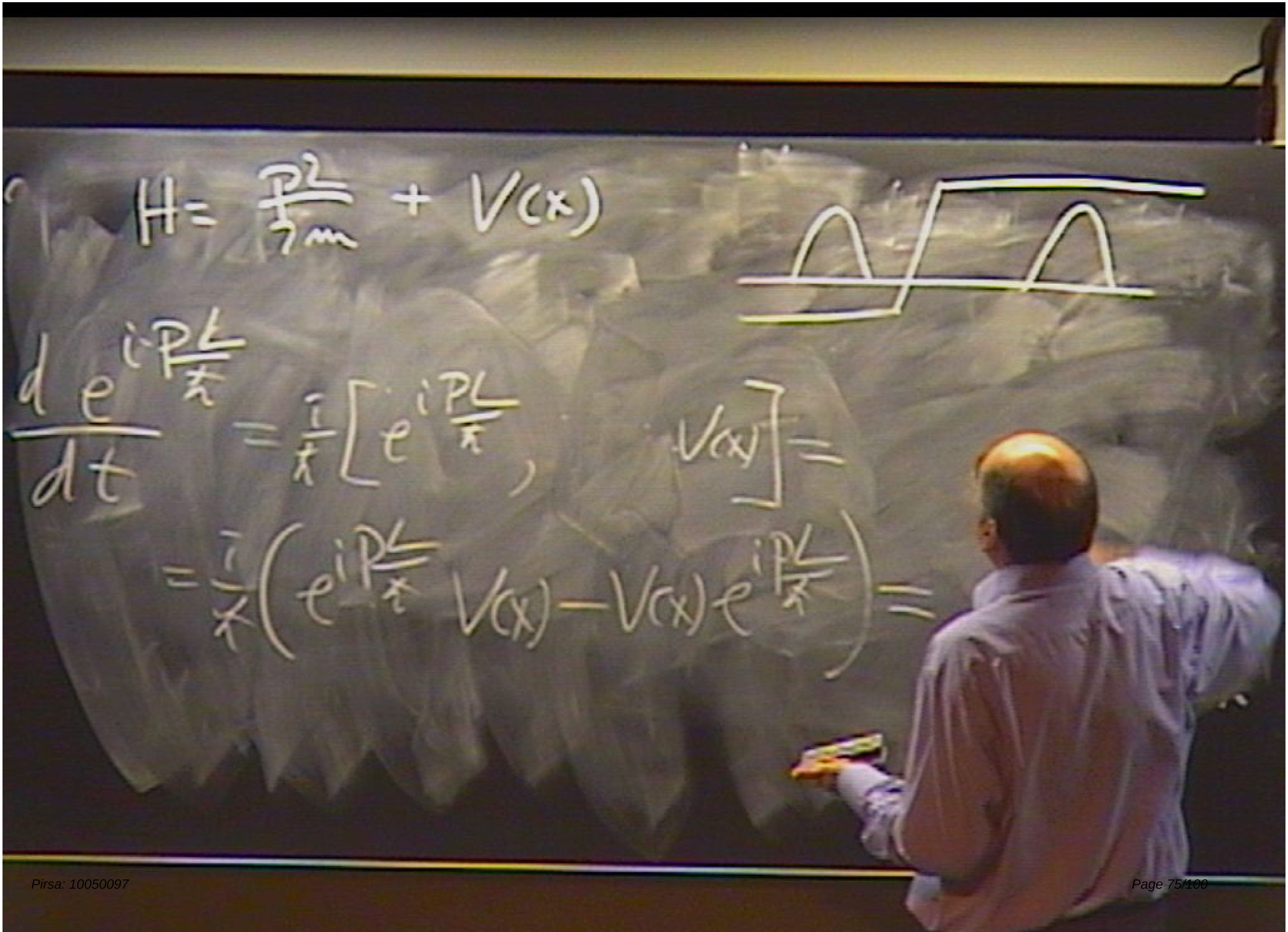
$$e^{i\frac{pL}{\hbar}}$$

$$\dot{x} = \frac{p}{m}$$

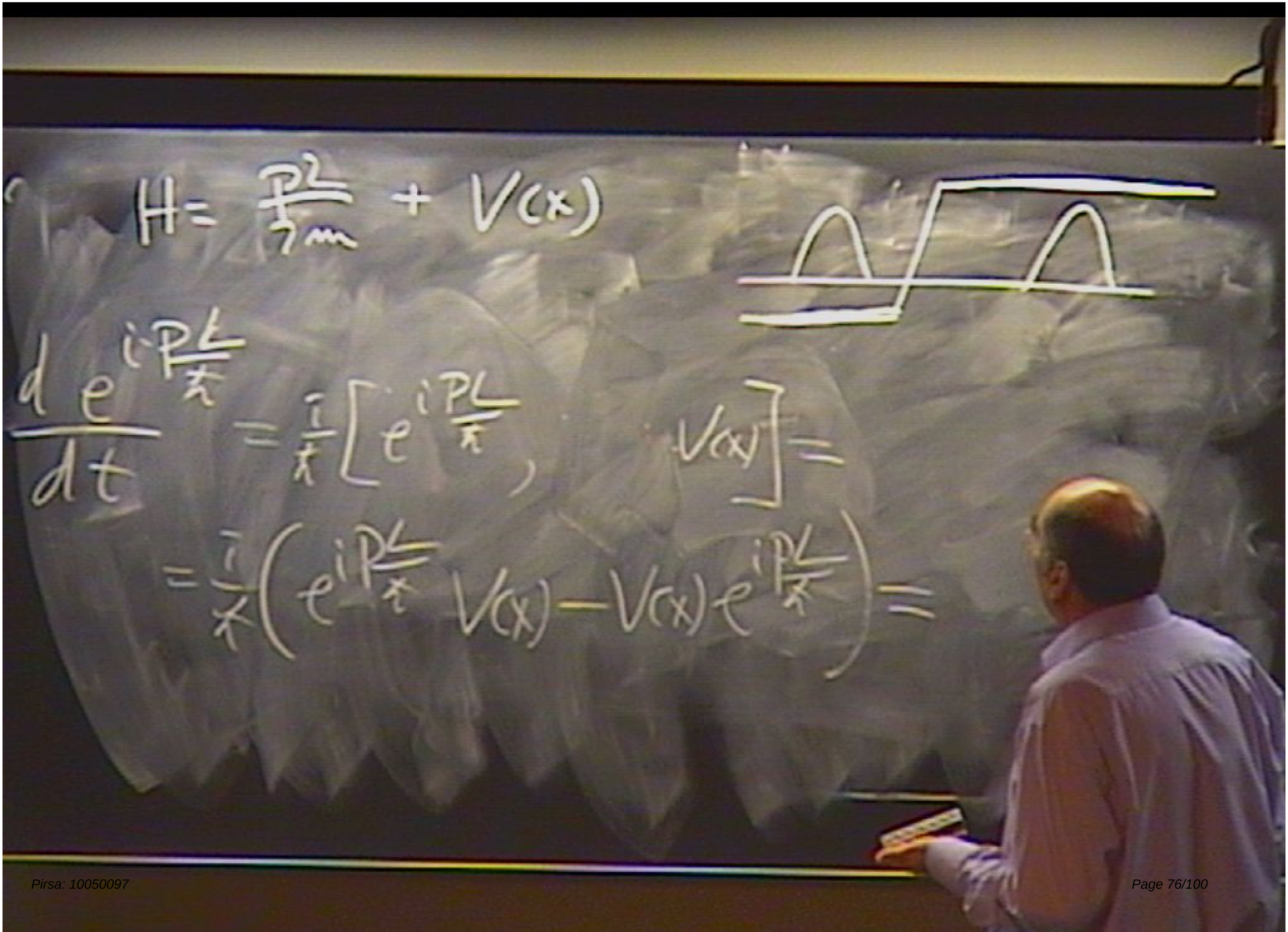
$$\dot{p} = -\frac{\partial V}{\partial x}$$

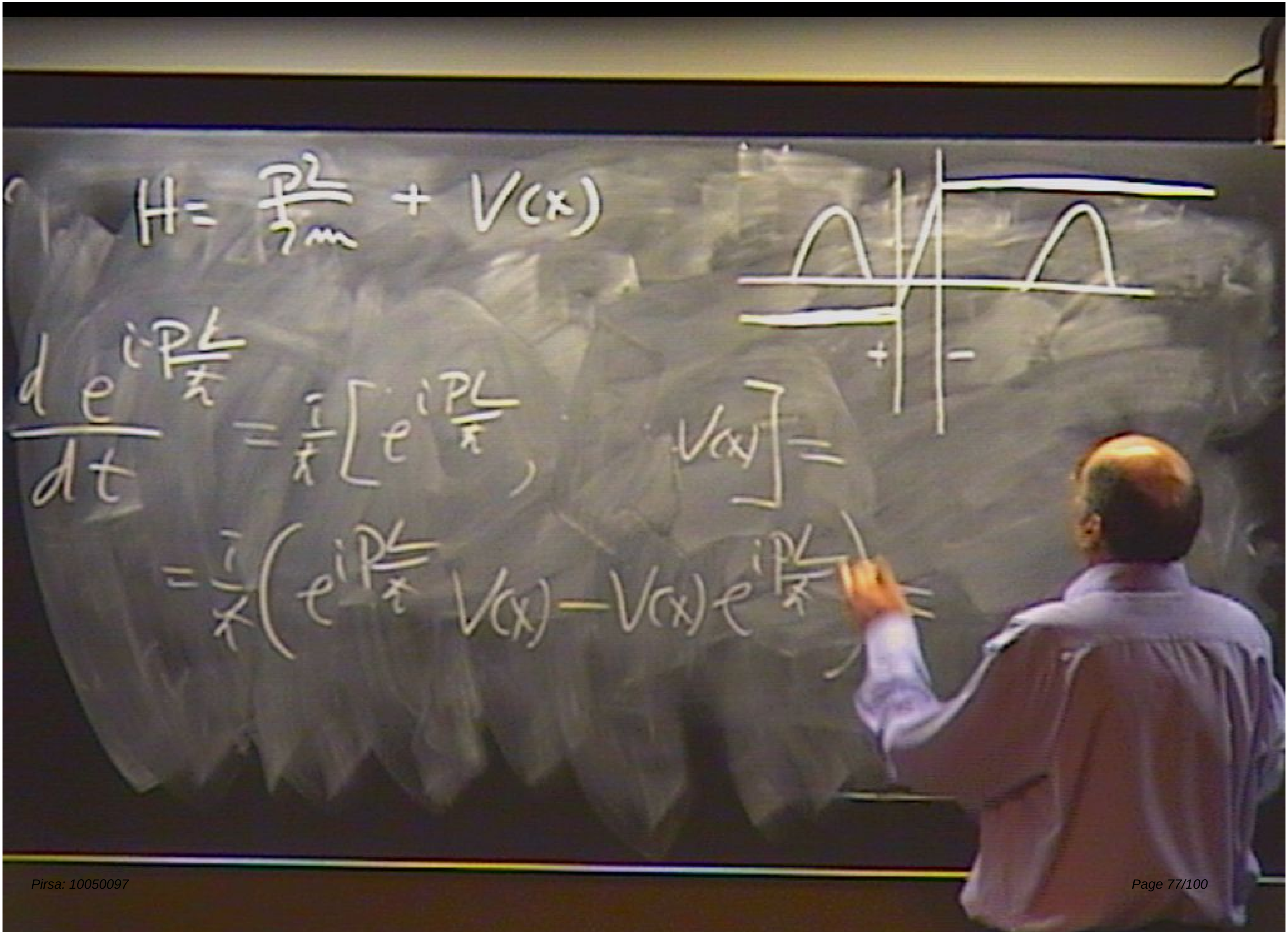
$$\frac{d}{dt} e^{i\frac{pL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{pL}{\hbar}} \left(-\frac{\partial V}{\partial x}\right) \quad \text{CLASSICAL}$$

$$\frac{d}{dt} e^{i\frac{pL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{pL}{\hbar}} \frac{\left(V(x) - V(x+L)\right)}{L} \quad Q$$

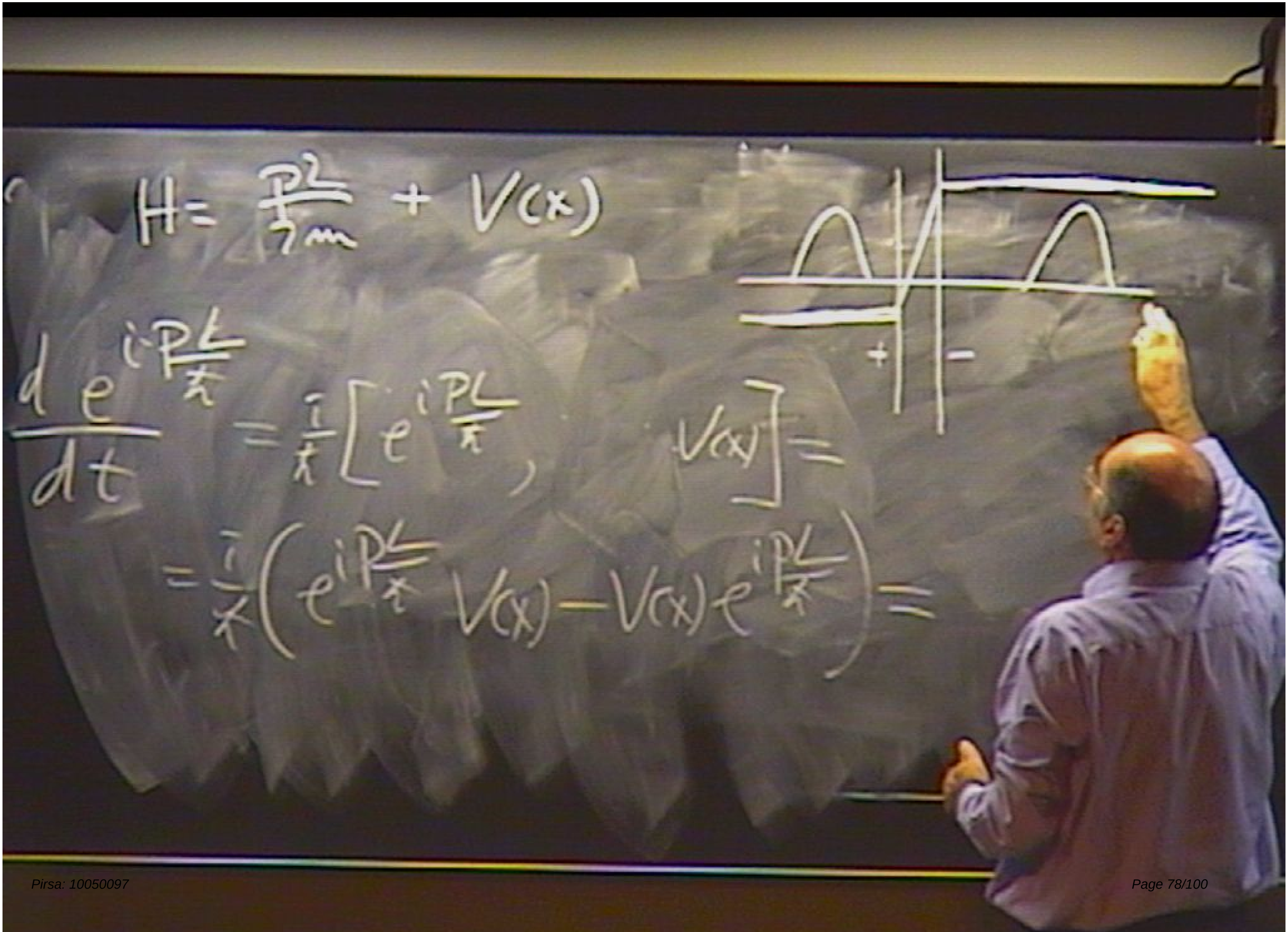


$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iP\frac{L}{\hbar}}}{dt} = \frac{i}{\hbar}\left[e^{iP\frac{L}{\hbar}}, V(x)\right] =$
$= \frac{i}{\hbar}\left(e^{iP\frac{L}{\hbar}} V(x) - V(x) e^{iP\frac{L}{\hbar}}\right) =$


$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iP\frac{L}{\hbar}}}{dt} = \frac{i}{\hbar}\left[e^{iP\frac{L}{\hbar}}, V(x)\right] =$
$= \frac{i}{\hbar}\left(e^{iP\frac{L}{\hbar}} V(x) - V(x) e^{iP\frac{L}{\hbar}}\right) =$



$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iP\frac{L}{\hbar}}}{dt} = \frac{i}{\hbar}\left[e^{iP\frac{L}{\hbar}}, V(x)\right] =$
$= \frac{i}{\hbar}\left(e^{iP\frac{L}{\hbar}} V(x) - V(x) e^{iP\frac{L}{\hbar}}\right) =$
$+$
$-$

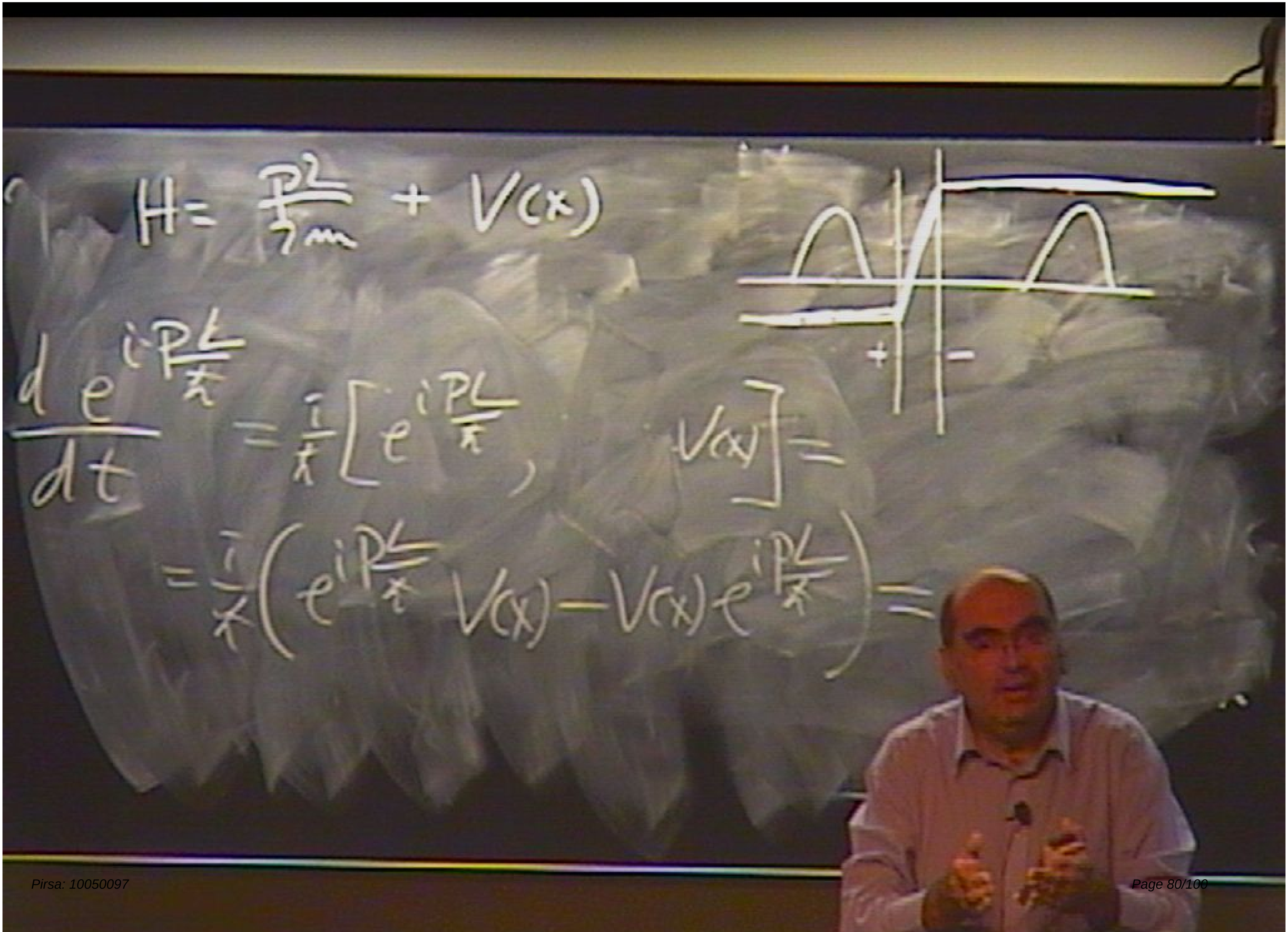


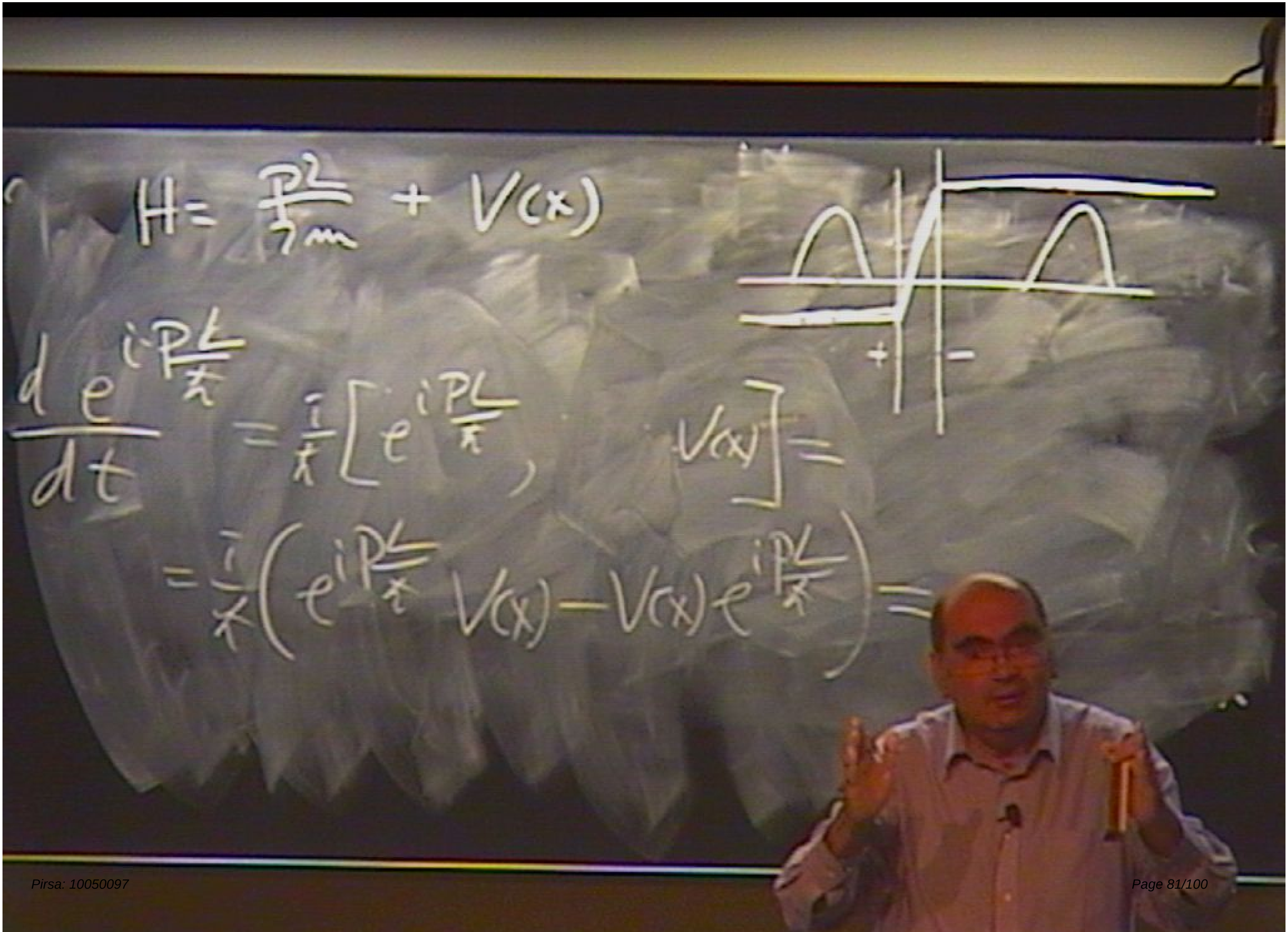
$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iPL/\hbar}}{dt} = \frac{i}{\hbar}\left[e^{iPL/\hbar}, V(x)\right] =$
$= \frac{i}{\hbar}\left(e^{iPL/\hbar} V(x) - V(x) e^{iPL/\hbar}\right) =$
$+$
$-$

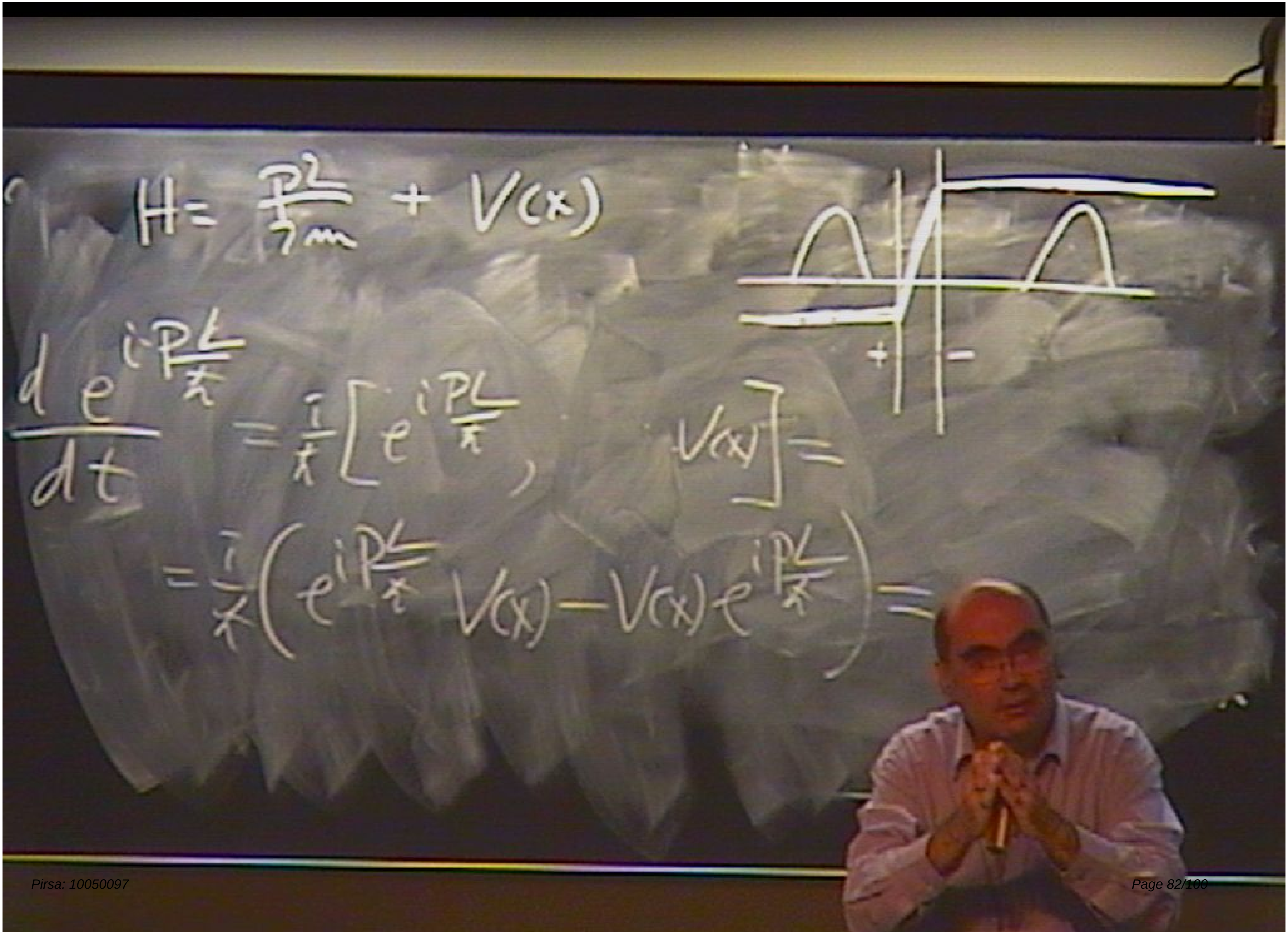
$$H = \frac{P^2}{2m} + V(x)$$

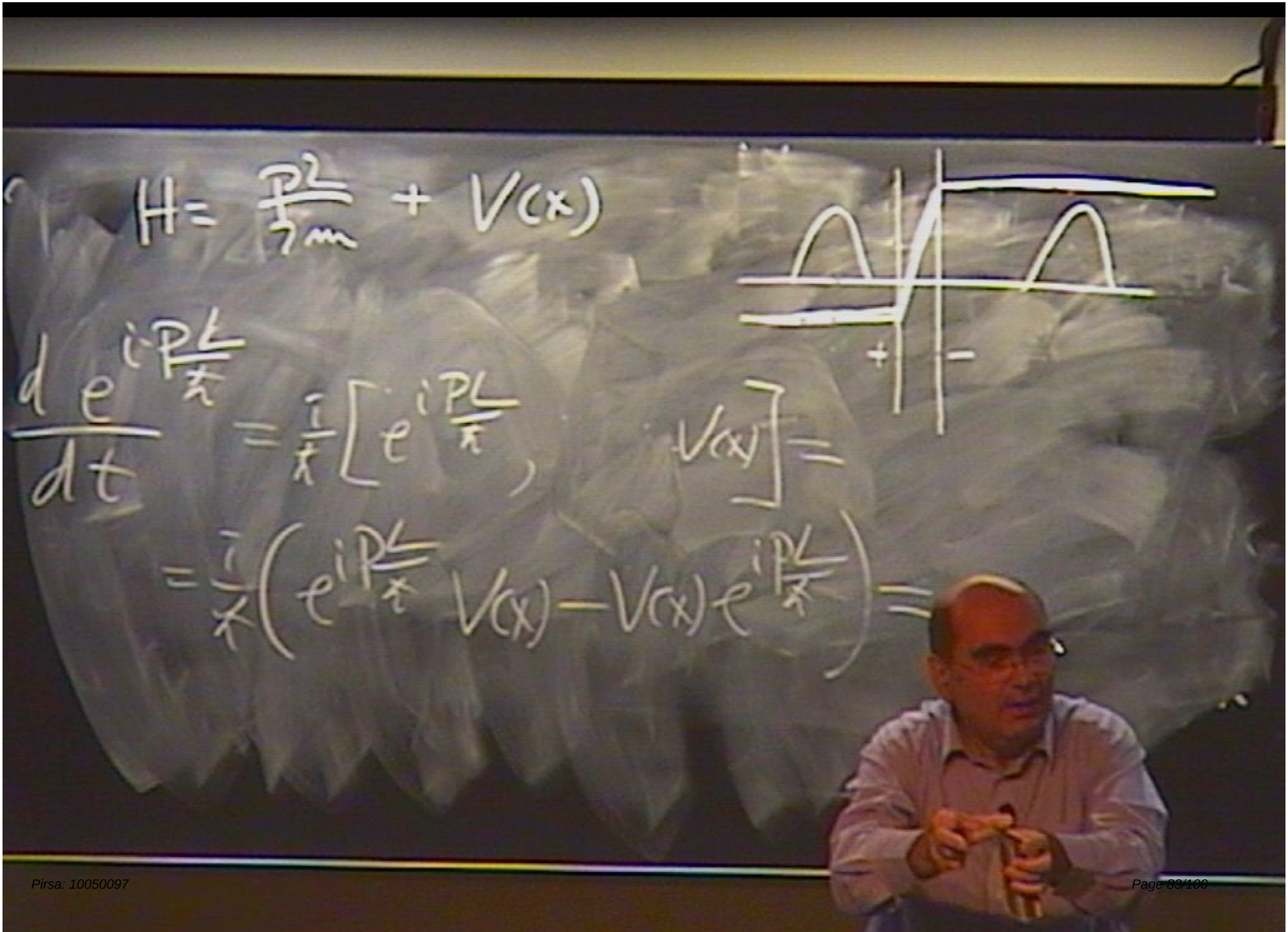
$$\frac{d\, e^{iP\frac{L}{\hbar}}}{dt} = \frac{i}{\hbar}\left[e^{iP\frac{L}{\hbar}}, V(x)\right] =$$

$$= \frac{i}{\hbar}\left(e^{iP\frac{L}{\hbar}} V(x) - V(x)\, e^{iP\frac{L}{\hbar}}\right) =$$

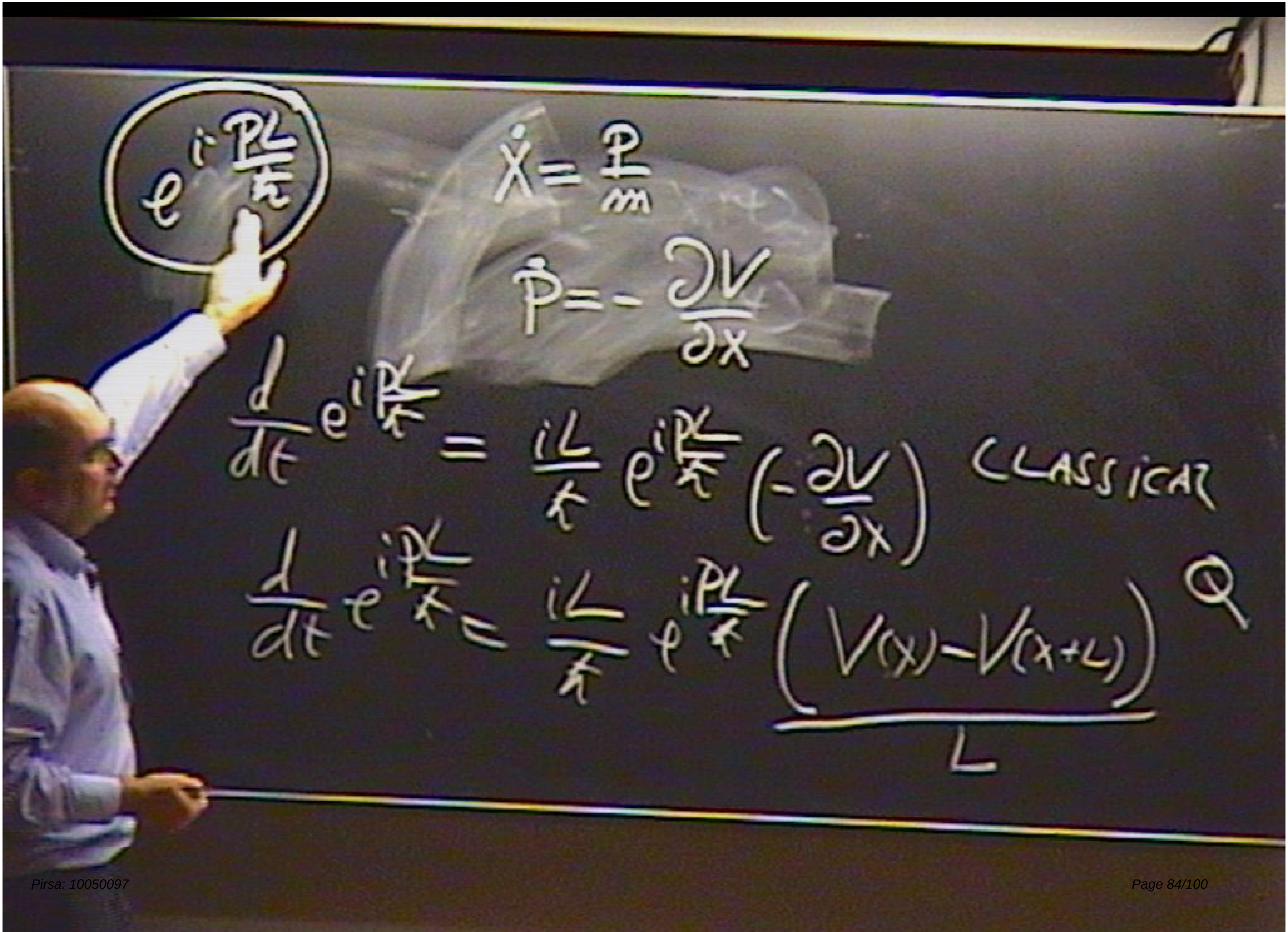

$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iP\frac{L}{\hbar}}}{dt} = \frac{i}{\hbar}\left[e^{iP\frac{L}{\hbar}}, V(x)\right] =$
$= \frac{i}{\hbar}\left(e^{iP\frac{L}{\hbar}} V(x) - V(x) e^{iP\frac{L}{\hbar}}\right) =$
+
-


$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iPL/\hbar}}{dt} = \frac{i}{\hbar}\left[e^{iPL/\hbar}, V(x)\right] =$
$= \frac{i}{\hbar}\left(e^{iPL/\hbar} V(x) - V(x) e^{iPL/\hbar}\right) =$
+
-


$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iPL/\hbar}}{dt} = \frac{i}{\hbar}\left[e^{iPL/\hbar}, V(x)\right] =$
$= \frac{i}{\hbar}\left(e^{iPL/\hbar} V(x) - V(x) e^{iPL/\hbar}\right) =$
+
-



$H = \frac{P^2}{2m} + V(x)$
$\frac{d\, e^{iP\frac{L}{\hbar}}}{dt} = \frac{i}{\hbar}\left[e^{iP\frac{L}{\hbar}}, V(x)\right] =$
$= \frac{i}{\hbar}\left(e^{iP\frac{L}{\hbar}} V(x) - V(x) e^{iP\frac{L}{\hbar}}\right) =$
+
−


$e^{i\frac{PL}{\hbar}}$
$\dot{x} = \frac{P}{m}$
$\dot{P} = -\frac{\partial V}{\partial x}$
$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \left(-\frac{\partial V}{\partial x}\right)$ CLASSICAL
$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \frac{\left(V(x) - V(x+L)\right)}{L}$ Q

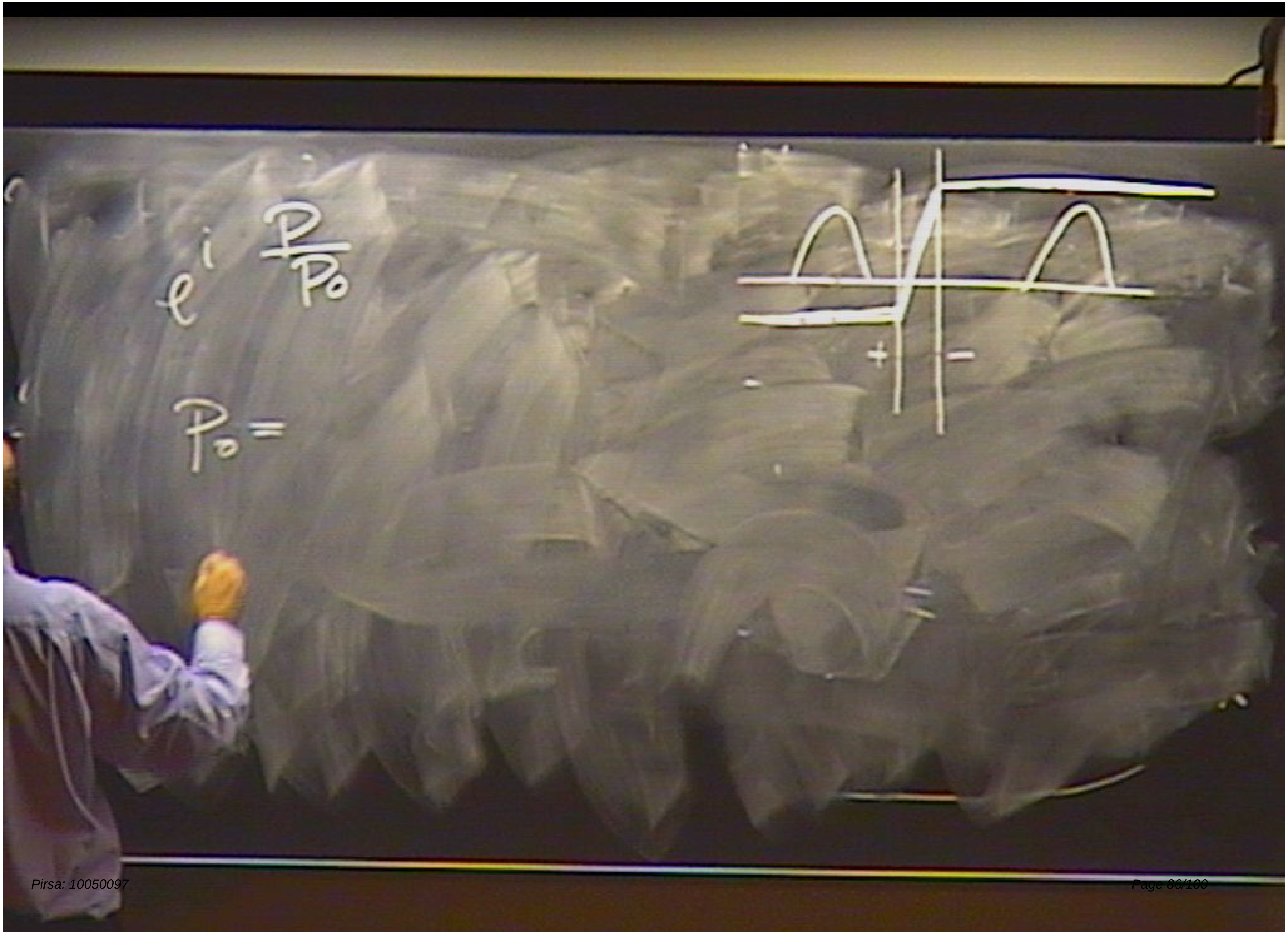
$$e^{i\frac{PL}{\hbar}}$$

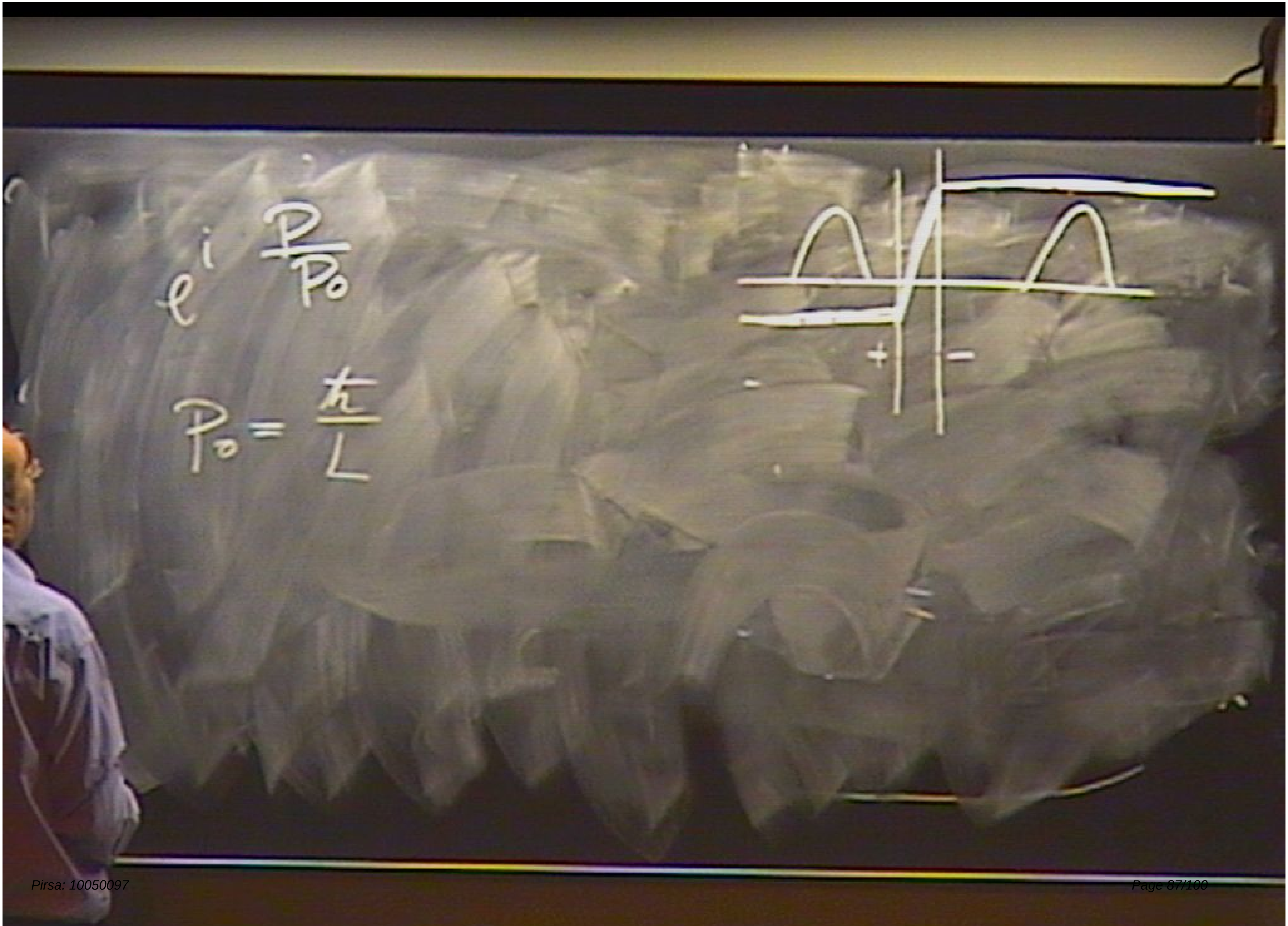
$$\dot{X} = \frac{P}{m}$$

$$\dot{P} = -\frac{\partial V}{\partial x}$$

$$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \left(-\frac{\partial V}{\partial x}\right)$$ CLASSICAL

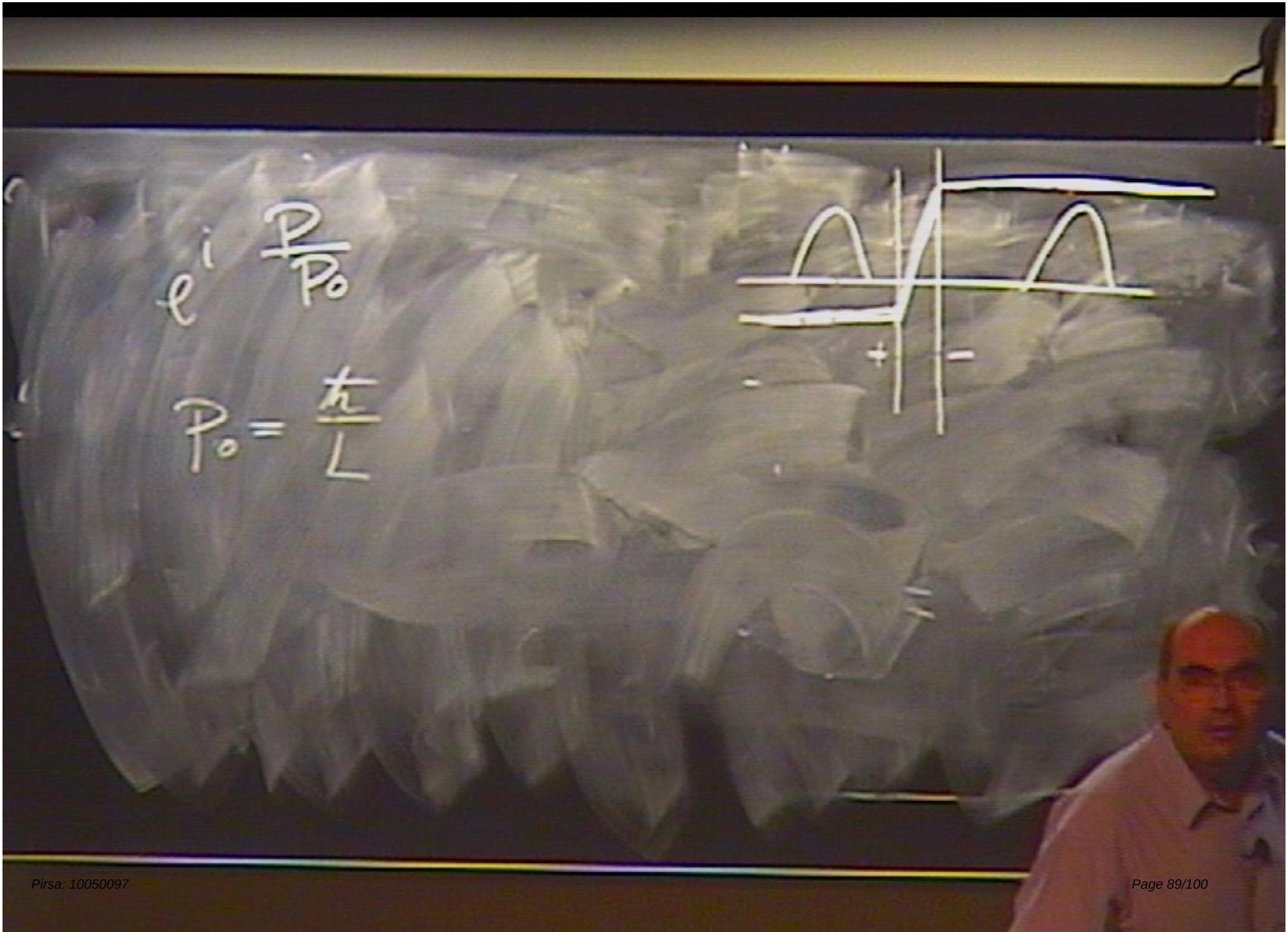
$$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \frac{\left(V(x)-V(x+L)\right)}{L}$$ Q


$e^{i \frac{p}{p_0}}$
$p_0 =$
+ –


$e^{i \frac{p}{p_0}}$
$p_0 = \frac{\hbar}{L}$
+
-

$$e^{i\frac{p}{p_0}}$$

$$p_0 = \frac{\hbar}{L}$$


$e^{i \frac{p}{p_0}}$
$p_0 = \frac{\hbar}{L}$

$$e^{i\frac{p}{p_0}}$$

$$p_0 = \frac{\hbar}{L}$$

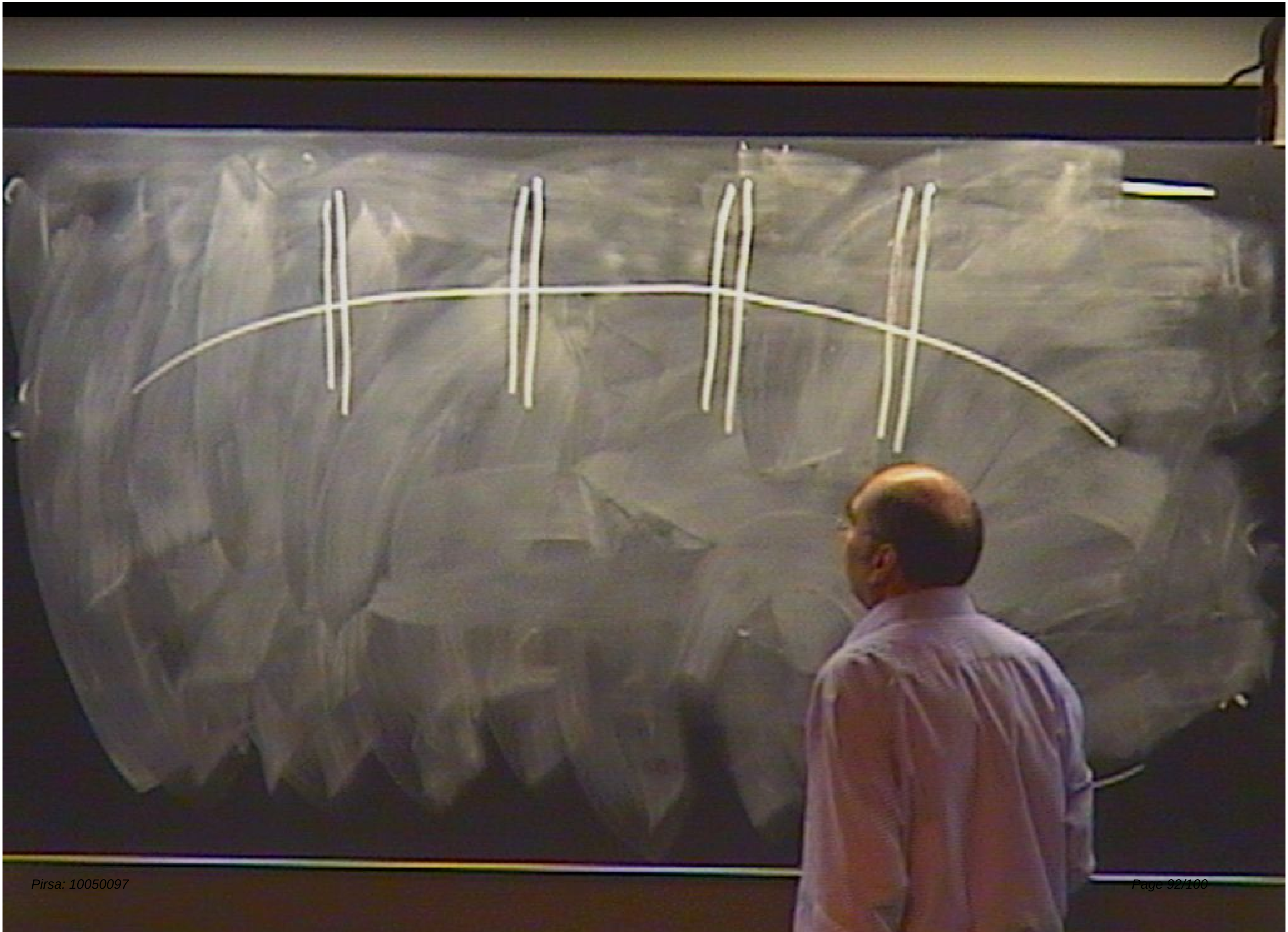
$$e^{i\frac{PL}{\hbar}}$$

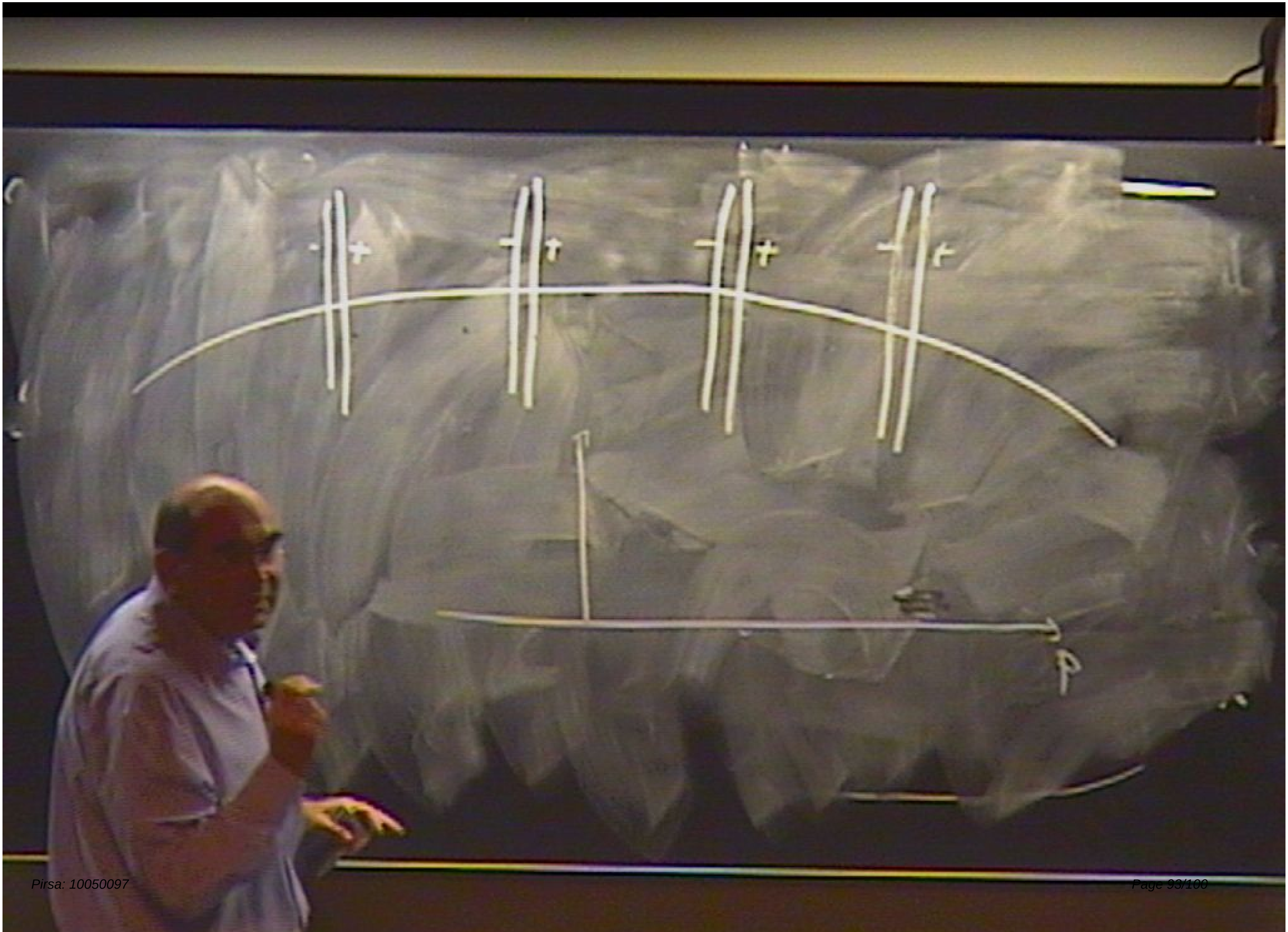
$$\dot{X} = \frac{P}{m}$$

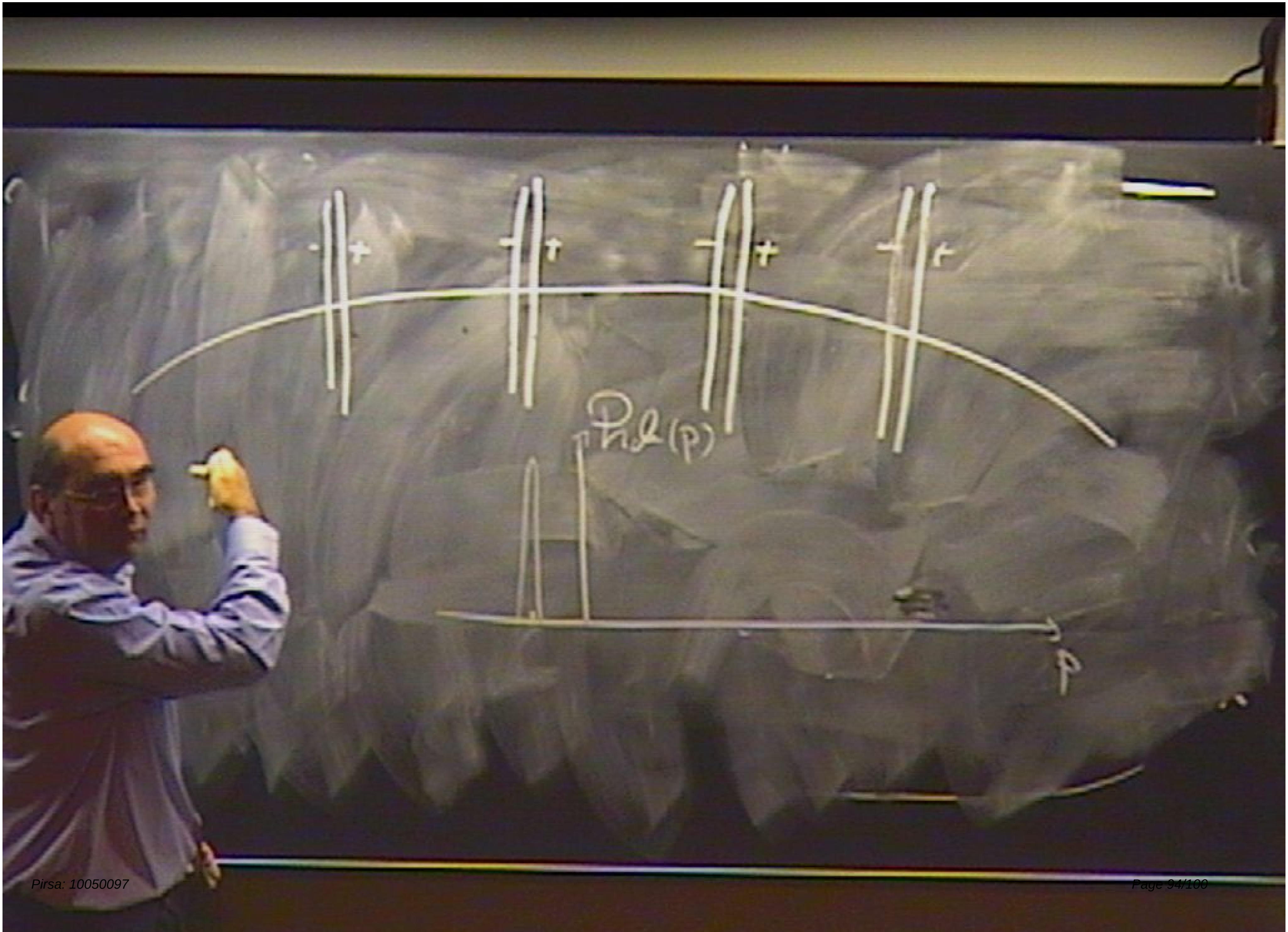
$$\dot{P} = -\frac{\partial V}{\partial x}$$

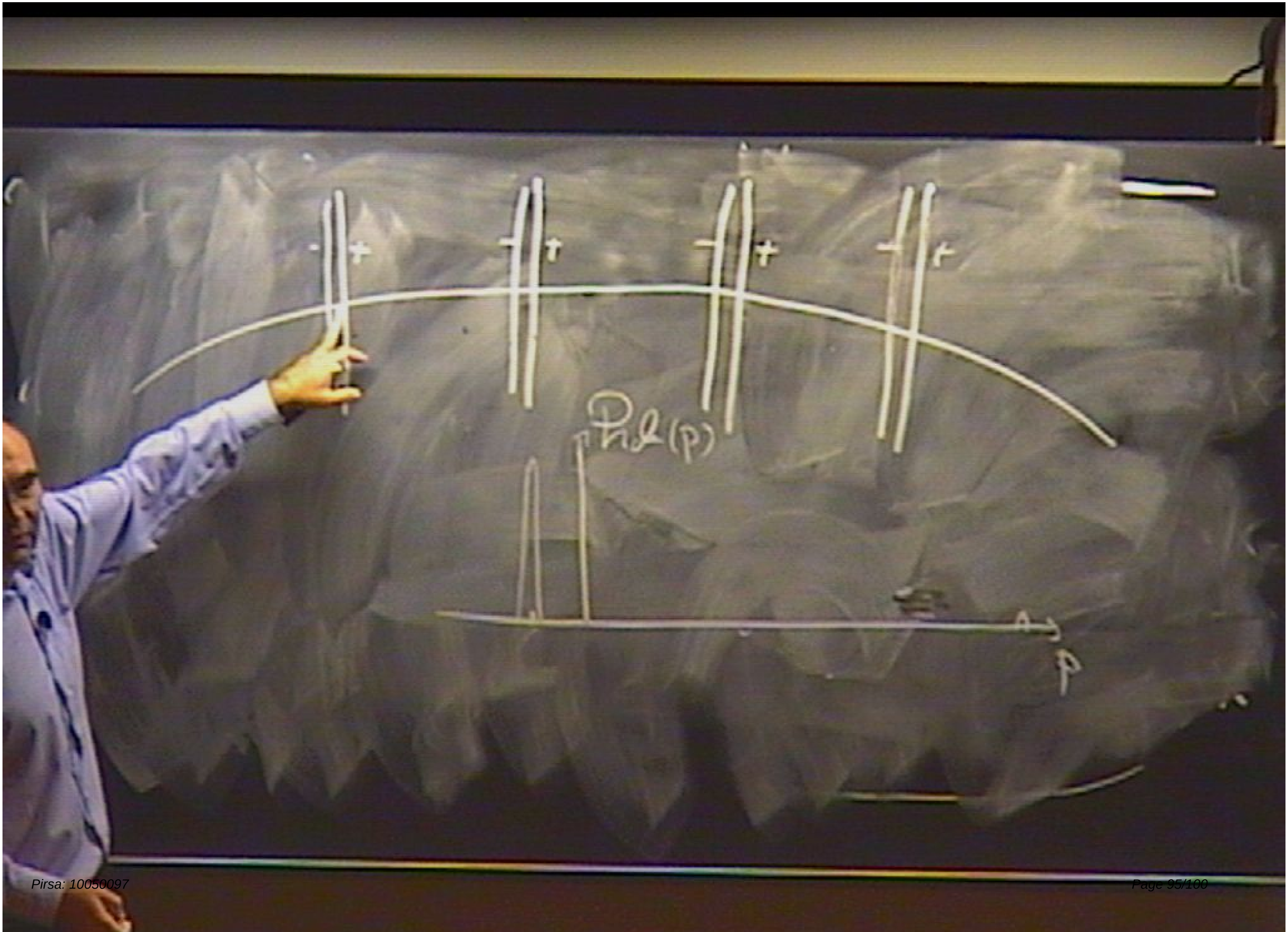
$$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \left(-\frac{\partial V}{\partial x}\right) \quad \text{CLASSICAL}$$

$$\frac{d}{dt} e^{i\frac{PL}{\hbar}} = \frac{iL}{\hbar} e^{i\frac{PL}{\hbar}} \frac{\left(V(x)-V(x+L)\right)}{L} \quad Q$$

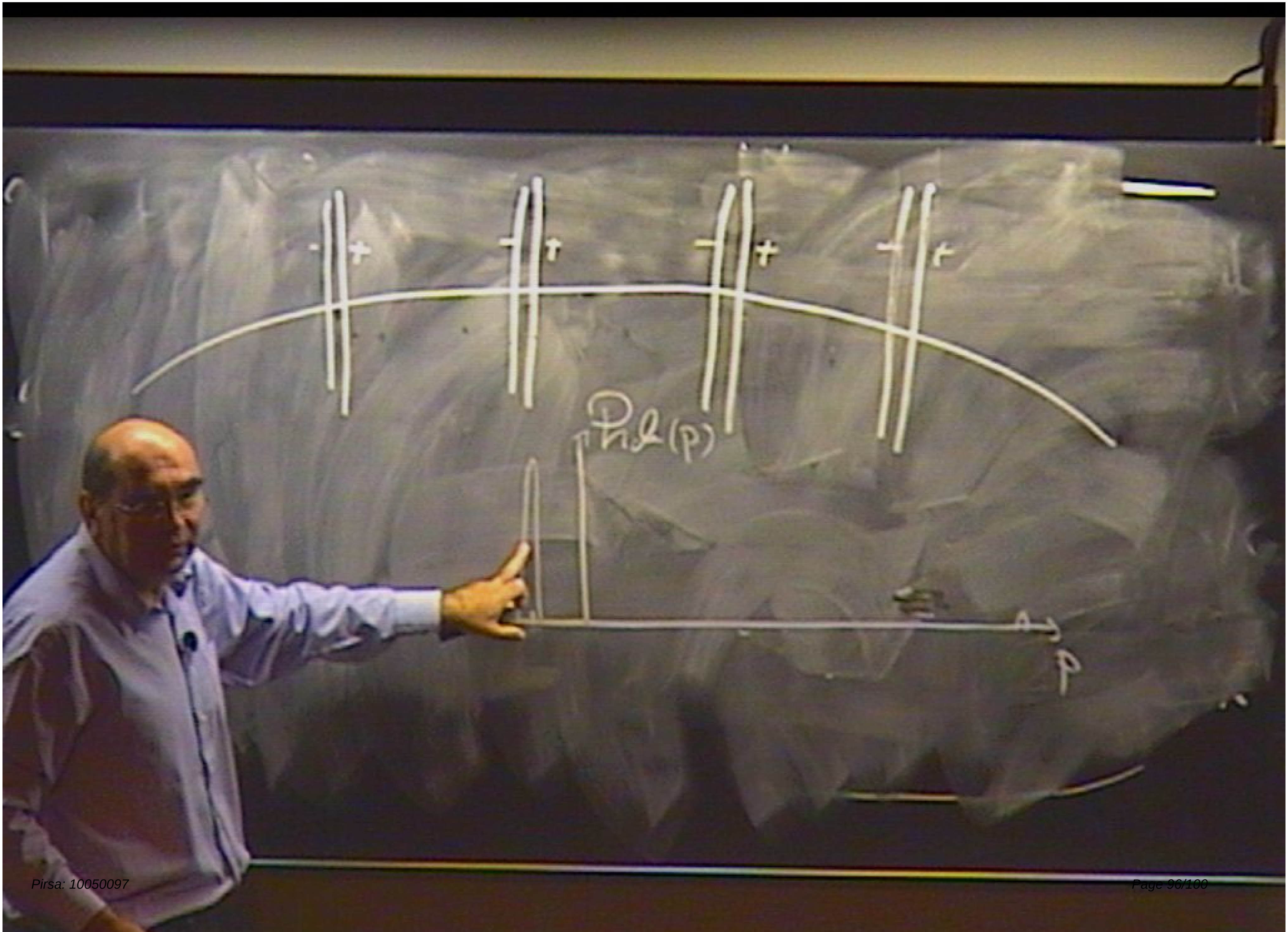




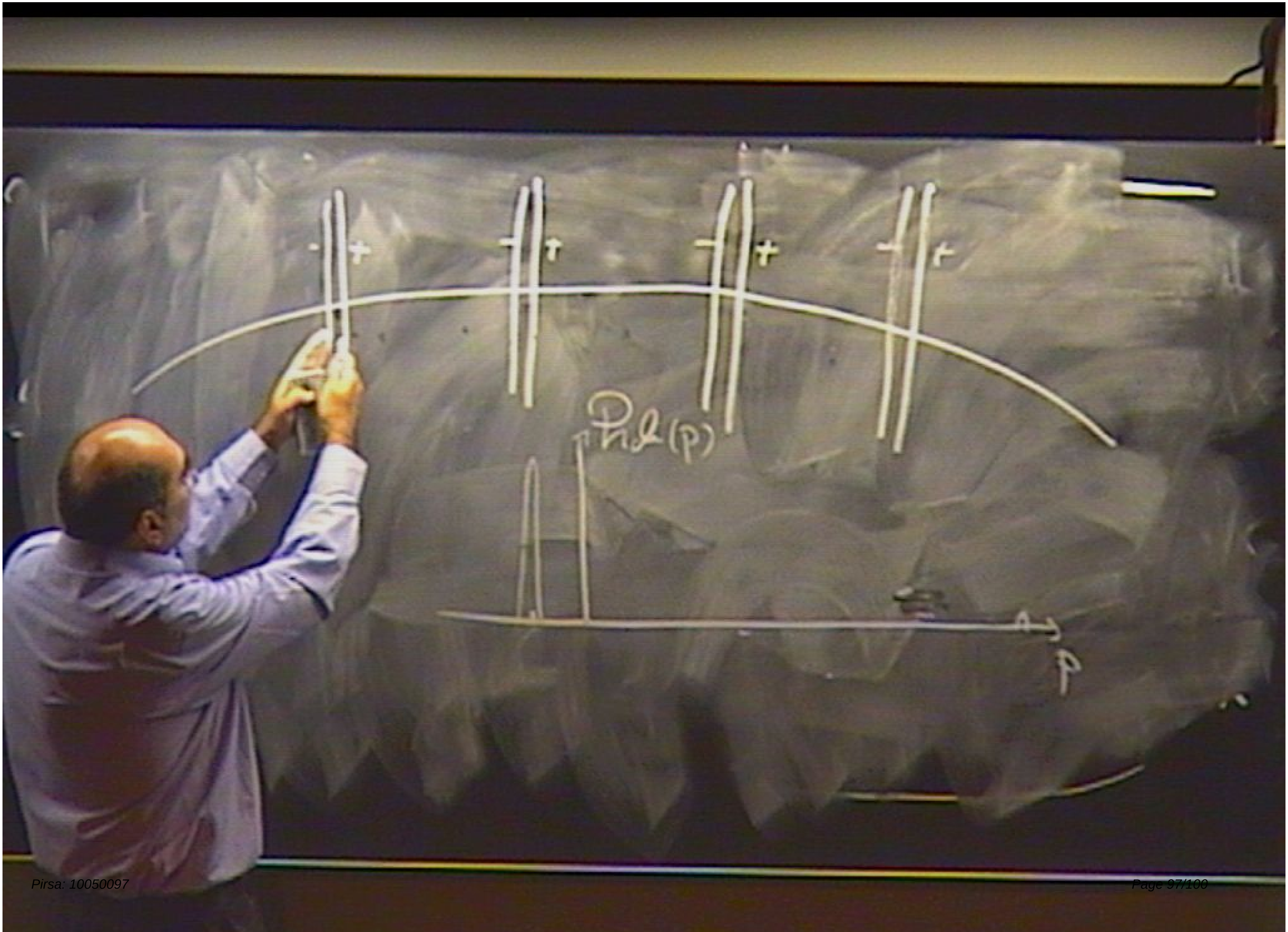

Prob(p)
p

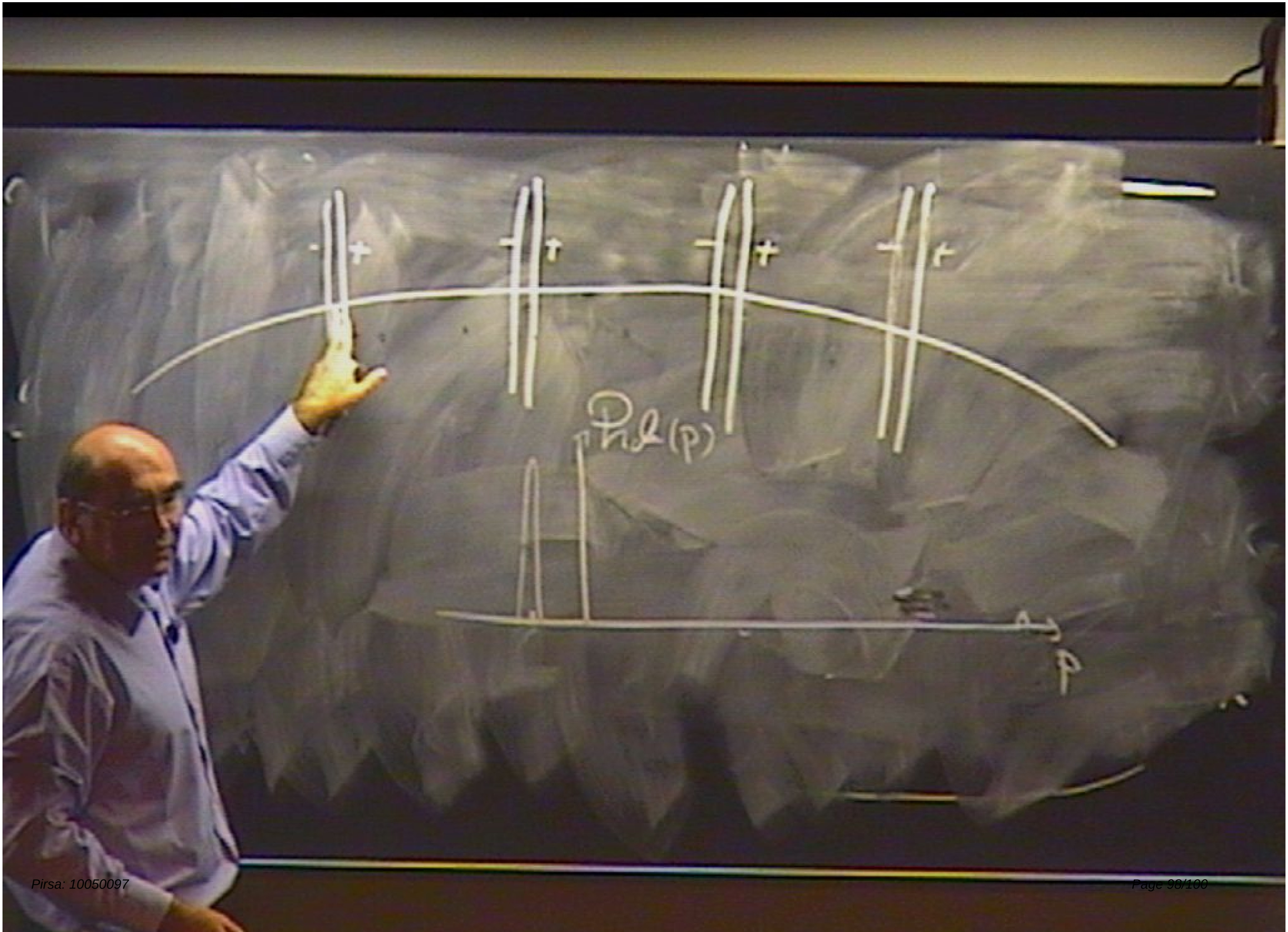


$P_{rel}(p)$
p



- +
- +
- +
- +
Prob(p)
p


p


Prob(p)
p