Title: Grasping quantum many-body systems in terms of tensor networks

Date: May 25, 2010 10:30 AM

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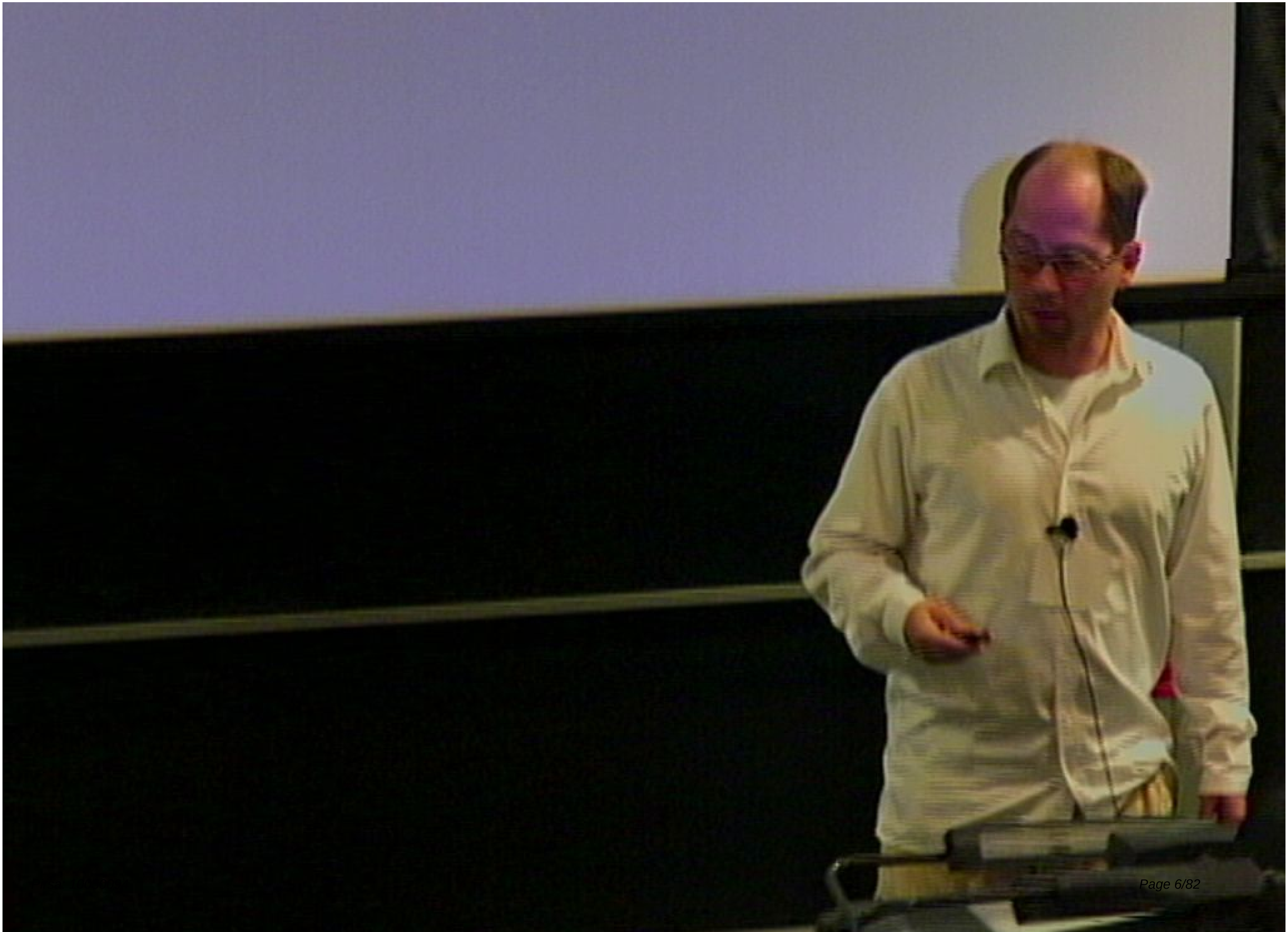
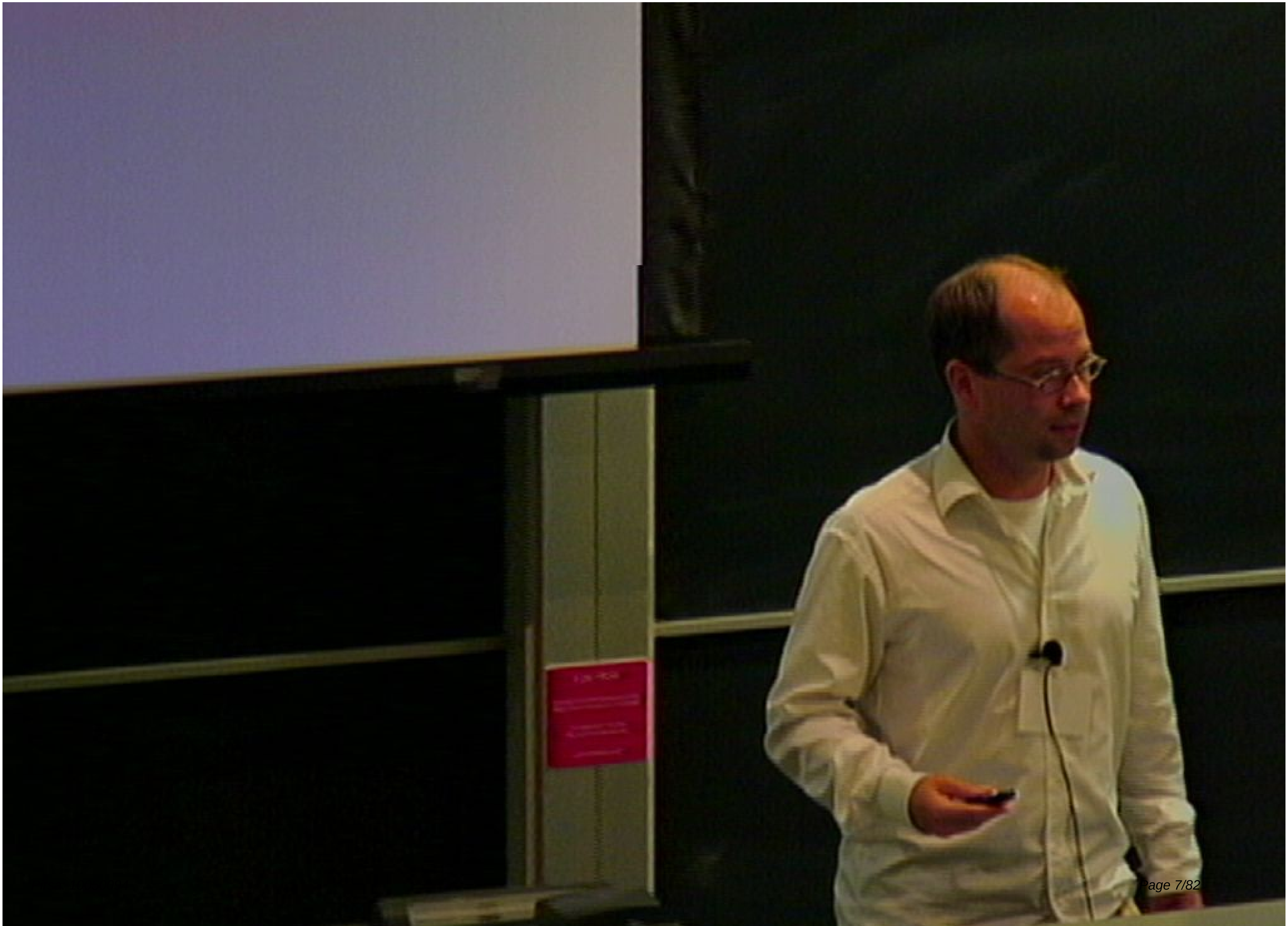
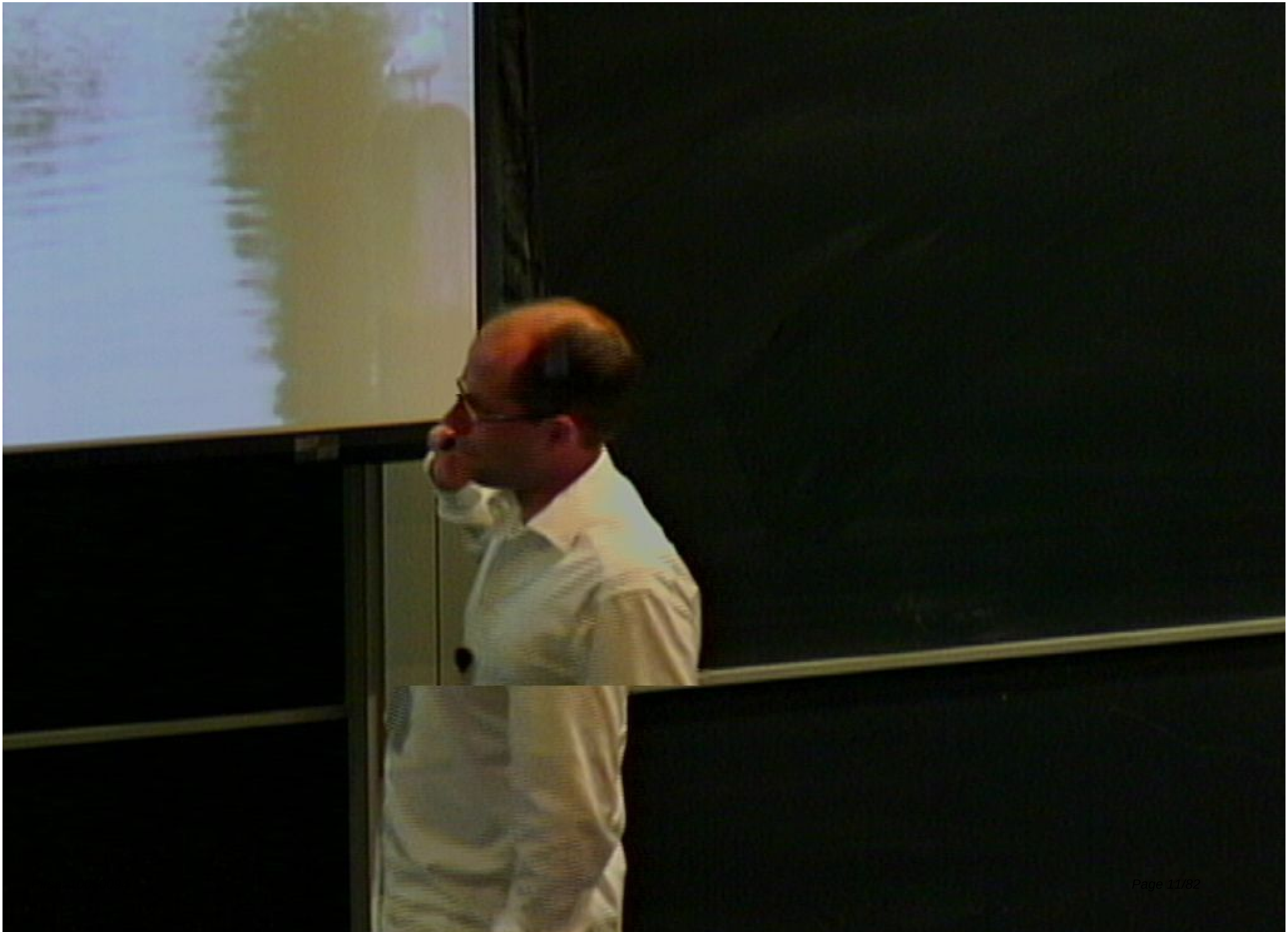
Abstract: This talk will be concerned with three new results (or a subset thereof) on the idea of grasping quantum many-body systems in terms of suitable tensor networks, such as finitely correlated states (FCS), tree tensor networks (TTN), projected entangled pair states (PEPS) or entanglement renormalization (MERA). We will first briefly introduce some basic ideas and relate the feasibility of such approaches to entanglement properties and area laws.

We will then see that (a) surprisingly, any MERA can be efficiently encoded in a PEPS, hence in a sense unifying these approaches. (b) We will also find that the ground state-manifold of any frustration-free spin-1/2 nearest neighbor Hamiltonian can be completely characterized in terms of tensor networks, how all such ground states satisfy an area law, and in which way such states serve as ansatz states for simulating almost frustration-free systems. (c) The last part will be concerned with using flow techniques to simulate interacting quantum fields with finitely correlated state approaches, and with simulating interacting fermions using efficiently contractible tensor networks.







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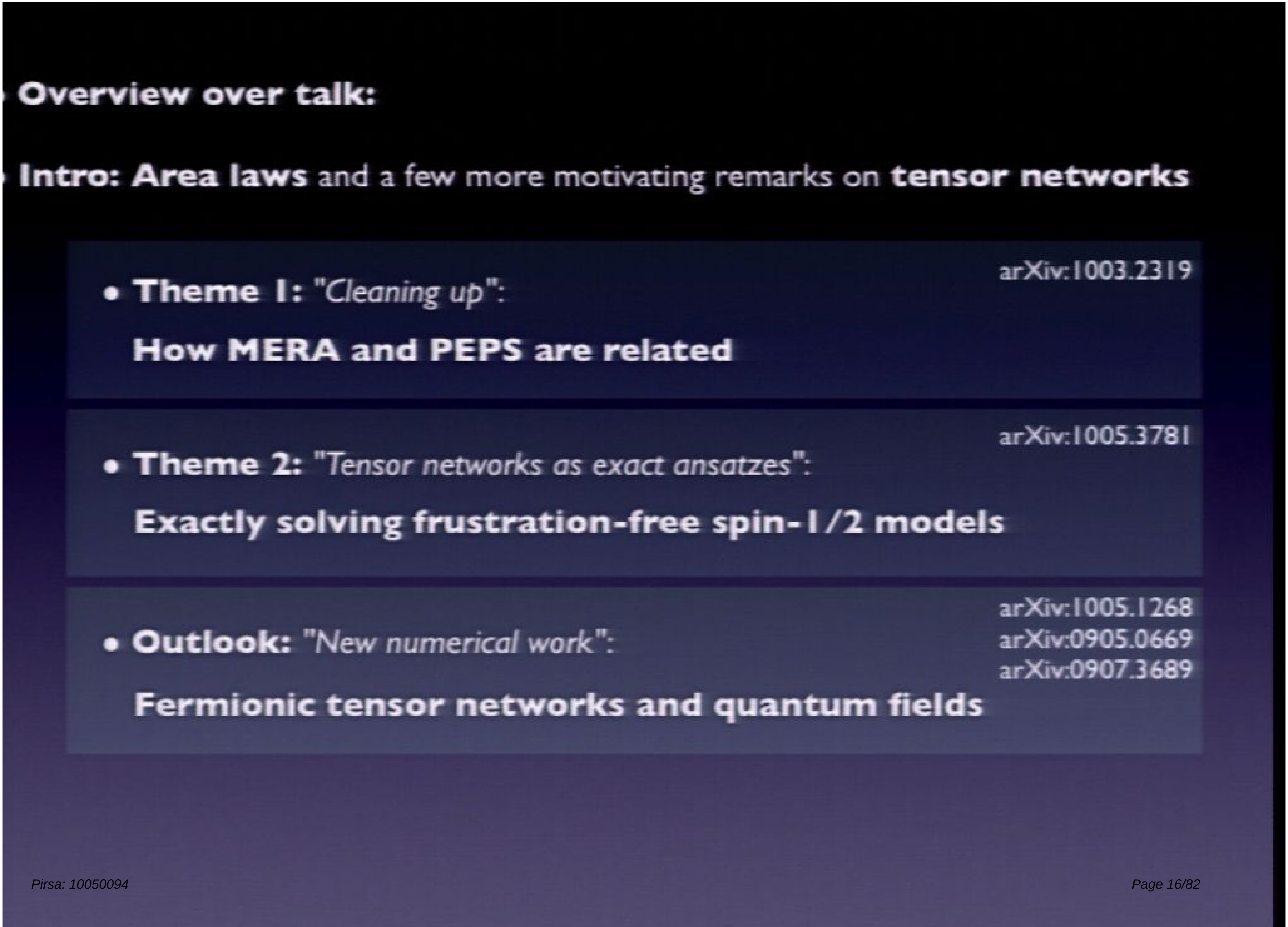
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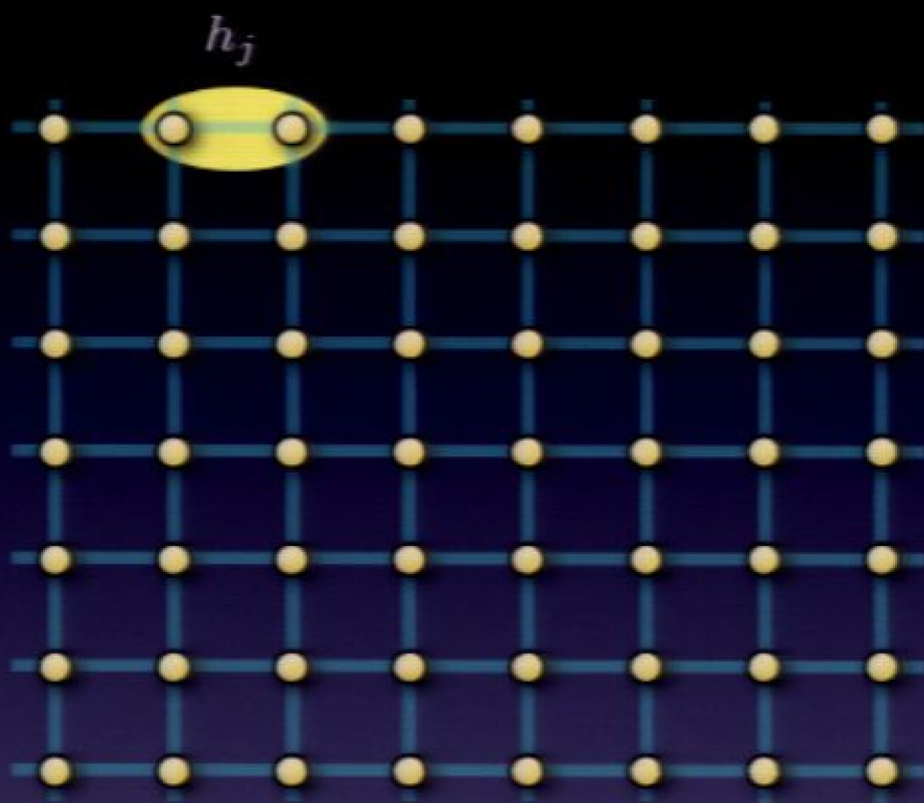
$$H = \sum_j h_j$$



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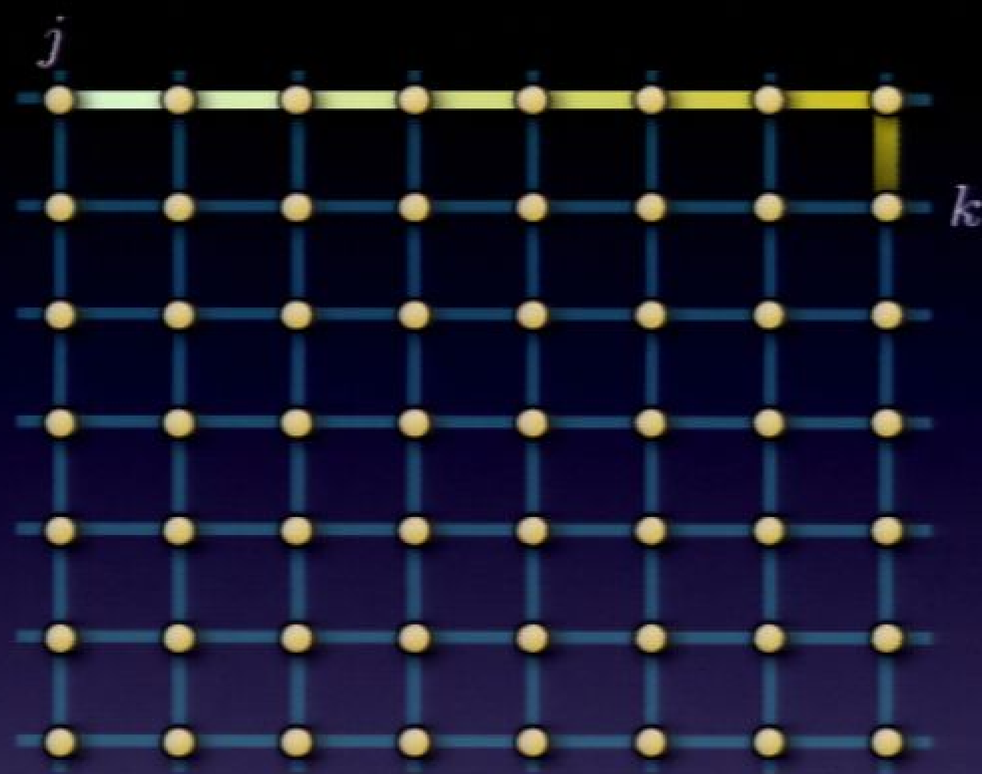
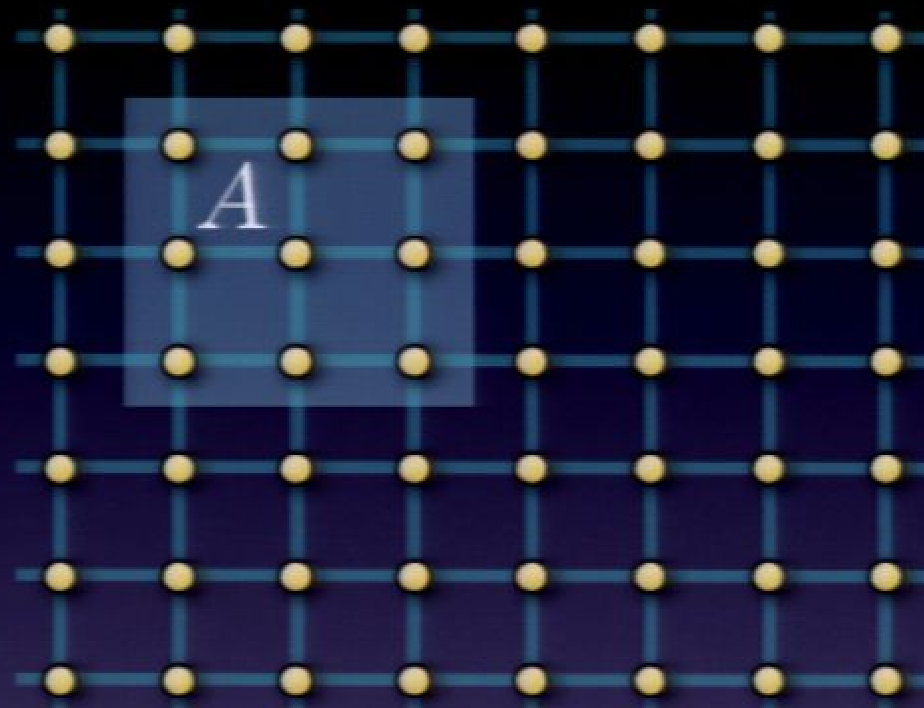


- **Correlation functions** decay with distance, for gapped (non-critical) models even exponentially

$$|\langle o_j o_k \rangle - \langle o_j \rangle \langle o_k \rangle| \leq c e^{-\xi d(j,k)}$$

- Imagine **ground state** of **local Hamiltonian**, and think of **entropy** $S(\rho_A) = -\mathrm{tr}(\rho_A \log \rho_A)$ of subsystem of some sites A

$$\rho_A = \mathrm{tr}_{\setminus A}(\rho)$$



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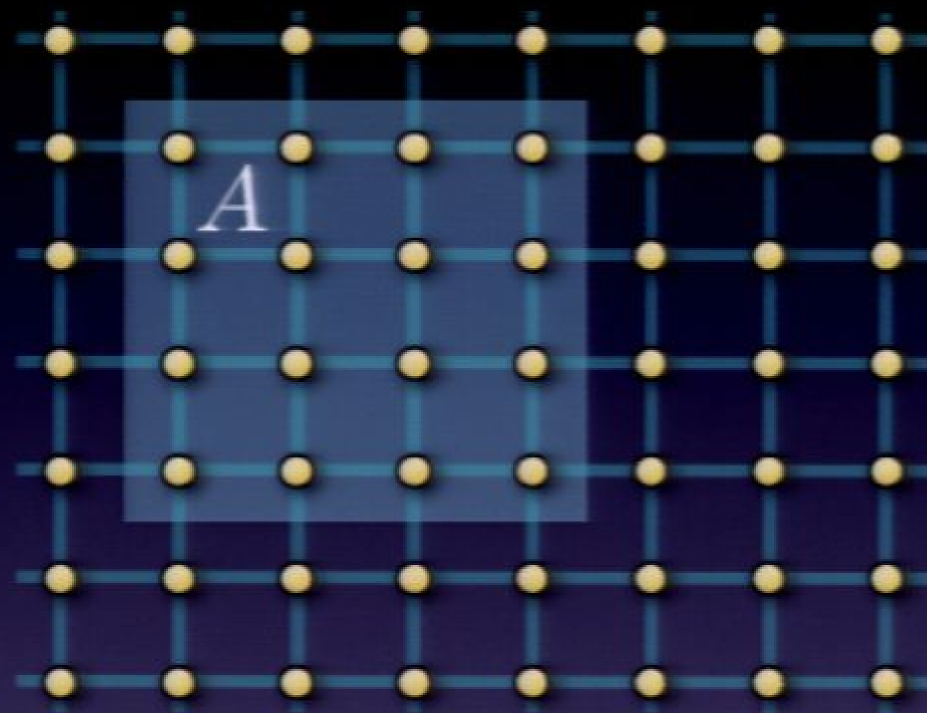

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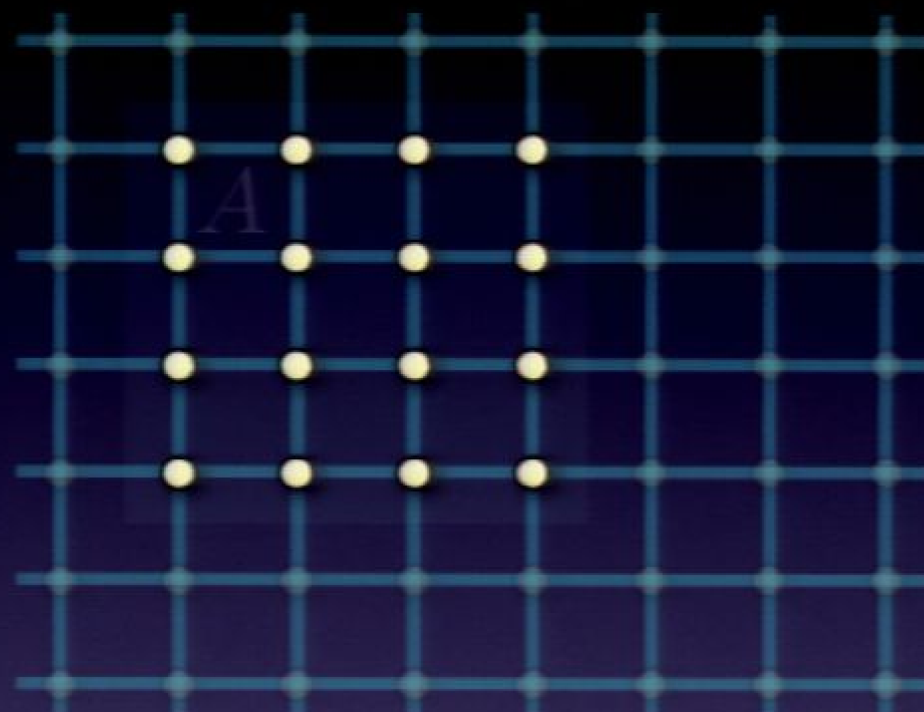
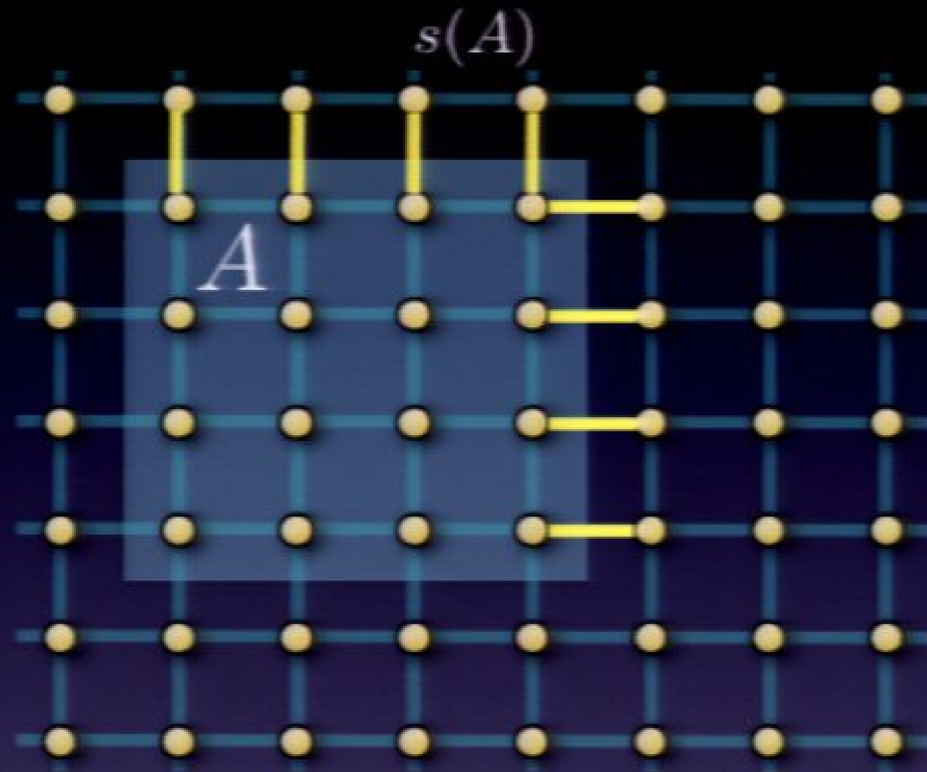
- "Naive answer": Should be extensive, i.e., **volume law**



Correct answer: **Entanglement scales like boundary area ("area law")**



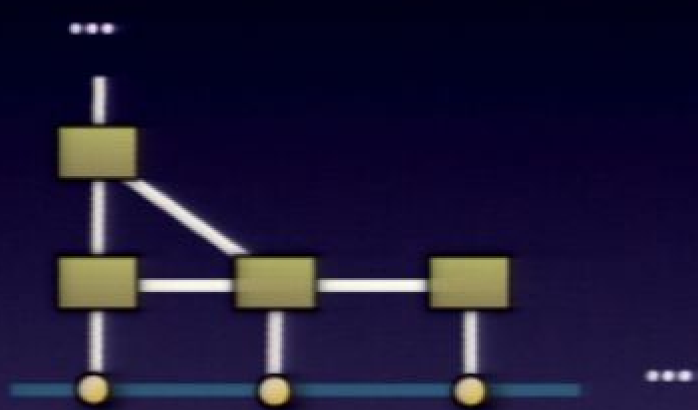
Observation 1: For ground states of either **free bosonic** on an arbitrary lattice (graph) or an **arbitrary** gapped strongly correlated systems in 1D, or **time-evolved** systems, for some constant $c > 0, S(\rho_A) \leq cs(A)$

Area laws for the entanglement entropy (with Cramer, Plenio), *Rev Mod Phys* **82**, 277 (2010)

Entropy, entanglement and area: Analytical results for harmonic lattice models (with Cramer, Dreissig, Plenio), *Phys Rev Lett* **94**, 060503 (2005)
An area law for one-dimensional quantum systems, Hastings, *J Stat Mech* P08024 (2007)
General entanglement-scaling laws from time evolution (with Osborne), *Phys Rev Lett* **97**, 150404 (2006)

- **Tensor network states** (MPS, PEPS, TTN, MERA, IPEPS, cMPS) usually parametrize such low entropy states

(see Ignacio's, Frank's, Guifre's talks today)



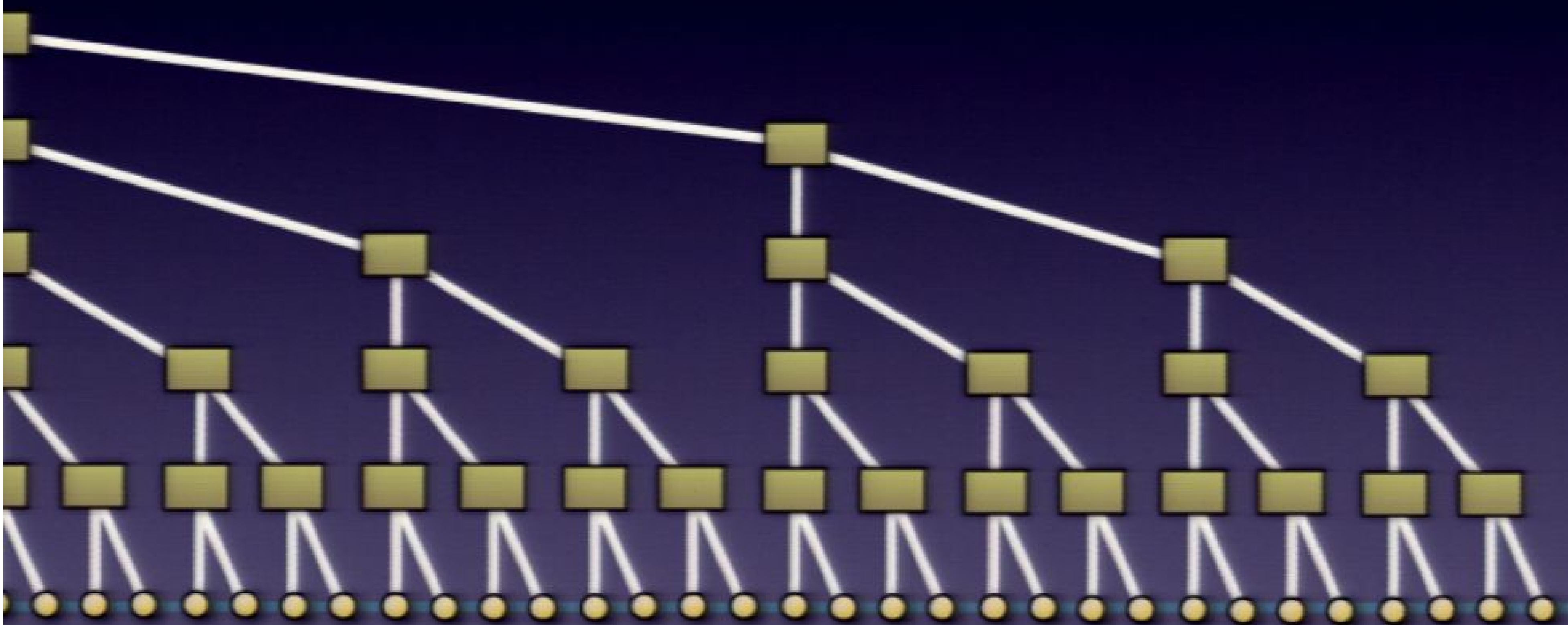
- **Lesson:** "Since nature explores a small subspace anyway, one can often efficiently parameterize this subspace using tensor networks"

'heme 1:

leaning up: Relating MERA and PEPS

al-space renormalization yields finite correlations (with Barthel, Kliesch), arXiv:1003.2319

- **MERA**, "Multiscale entanglement renormalization" (compare Guifre's talk)
- Sequence of **isometries** and **disentanglers**
- Instance of **real-space renormalization,** "scale invariant", like critical systems
- Makes perfect sense in 2D, 3D, ...



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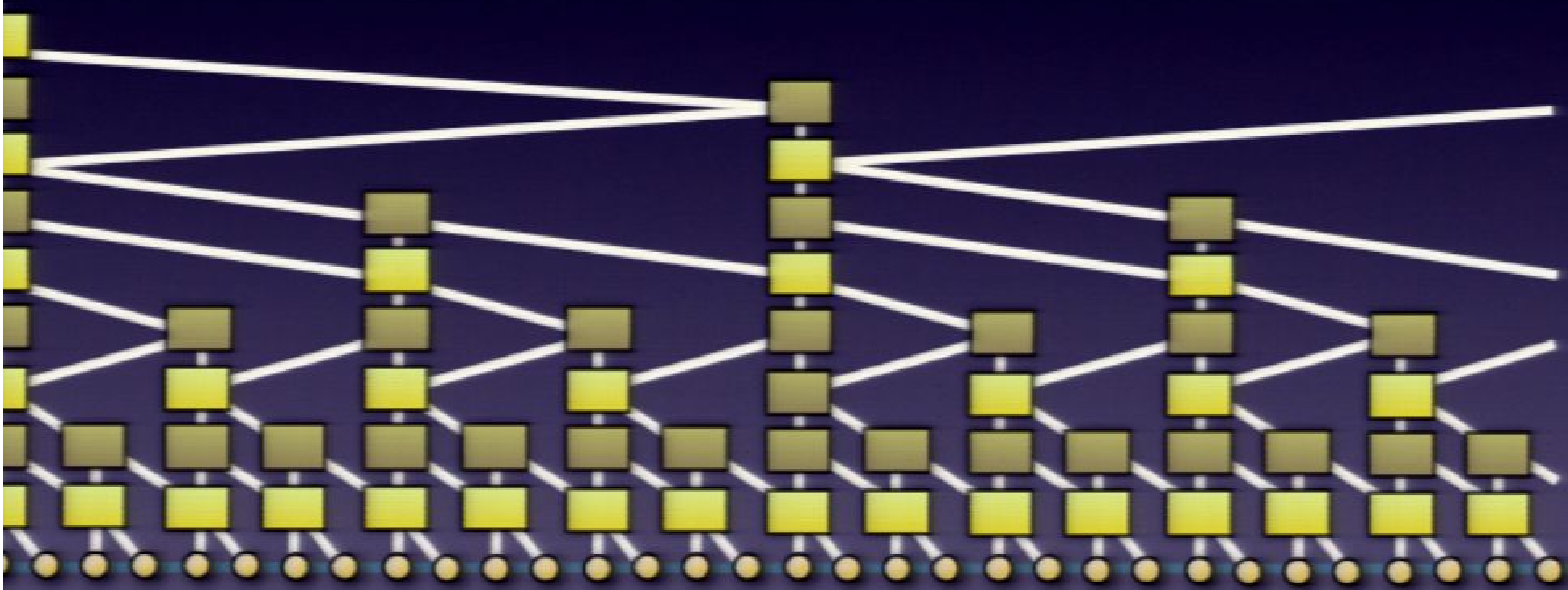

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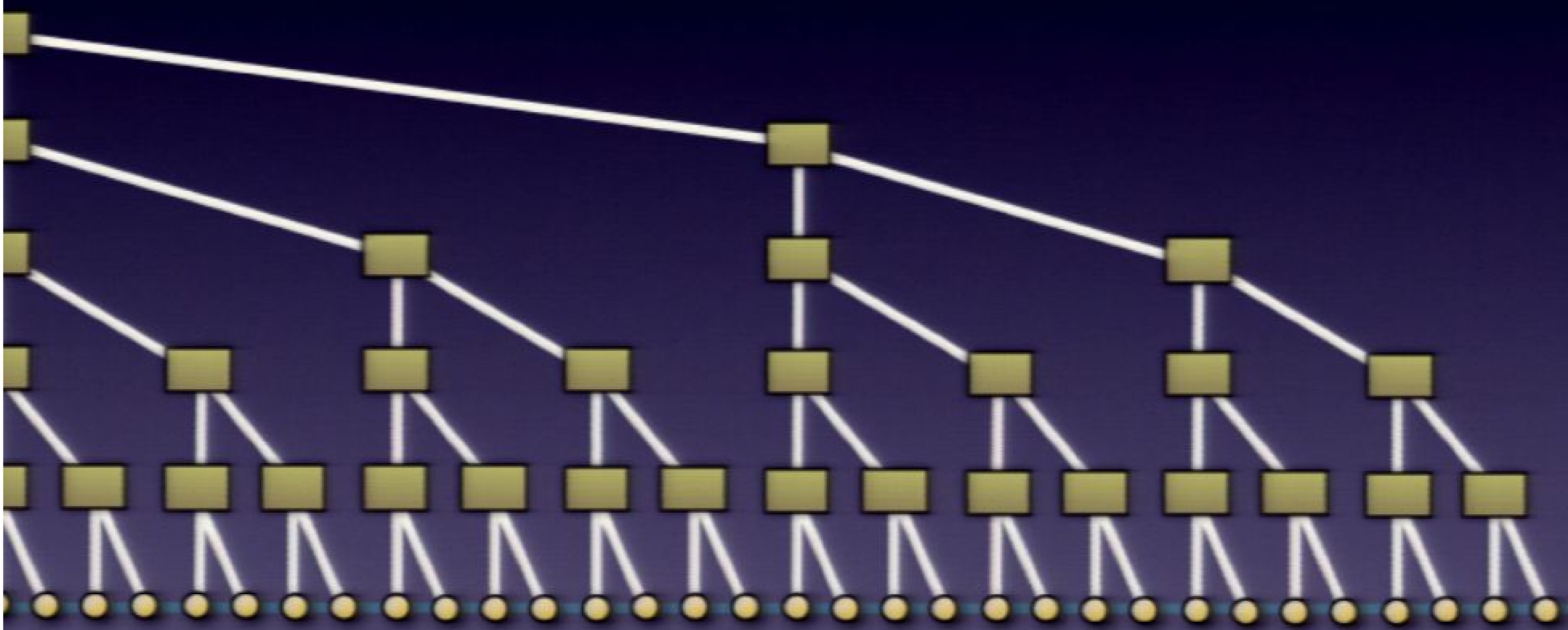

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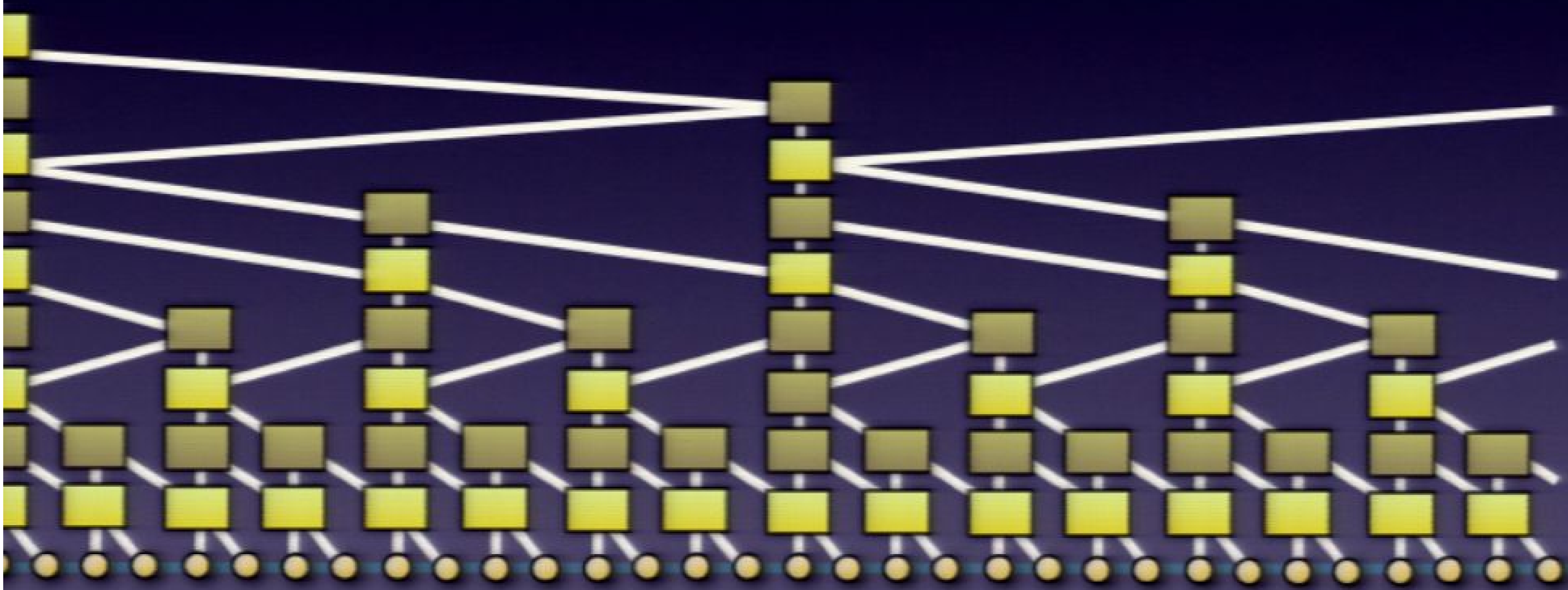

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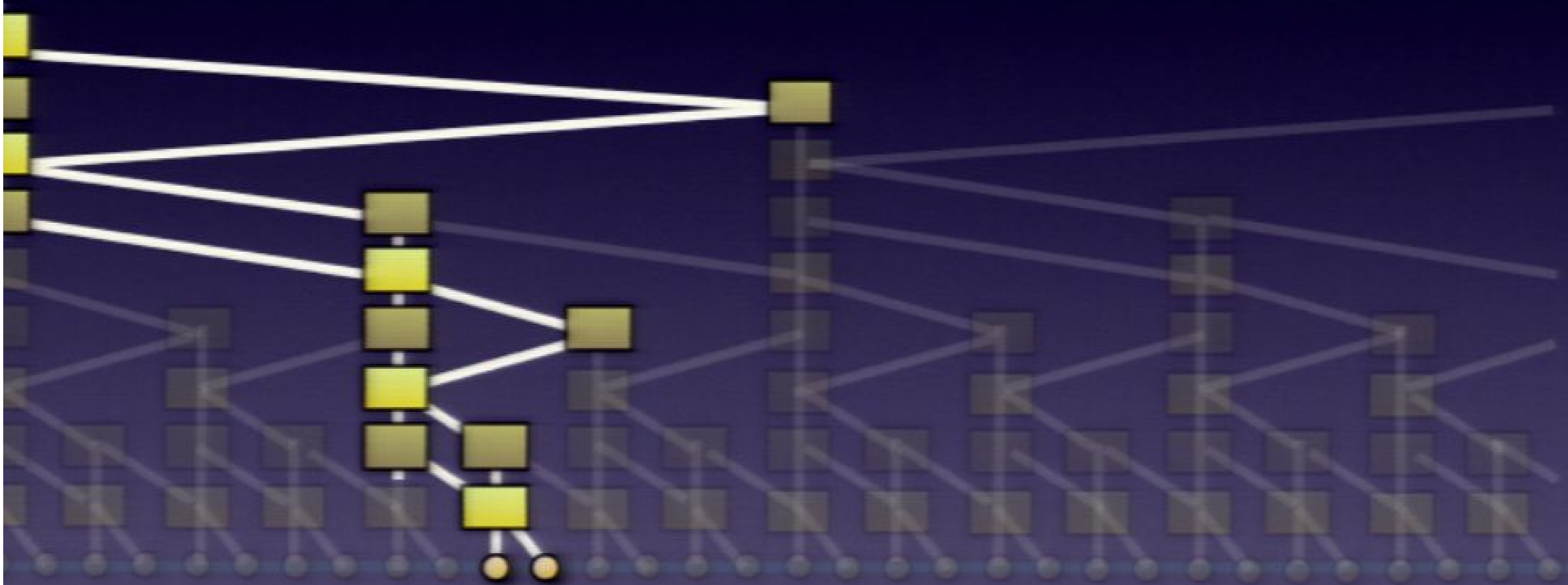
MERA features a **causal cone**, can be efficiently contracted



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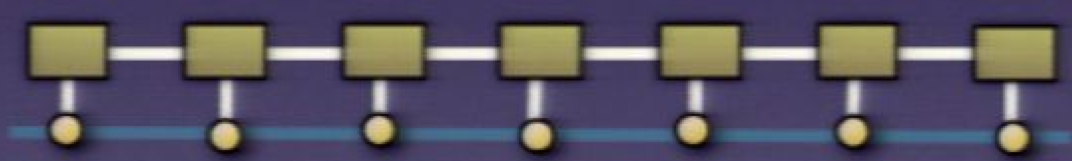

ensity matrix formulation for quantum renormalization groups, White, *Phys Rev Lett* **69**, 2863 (1992)

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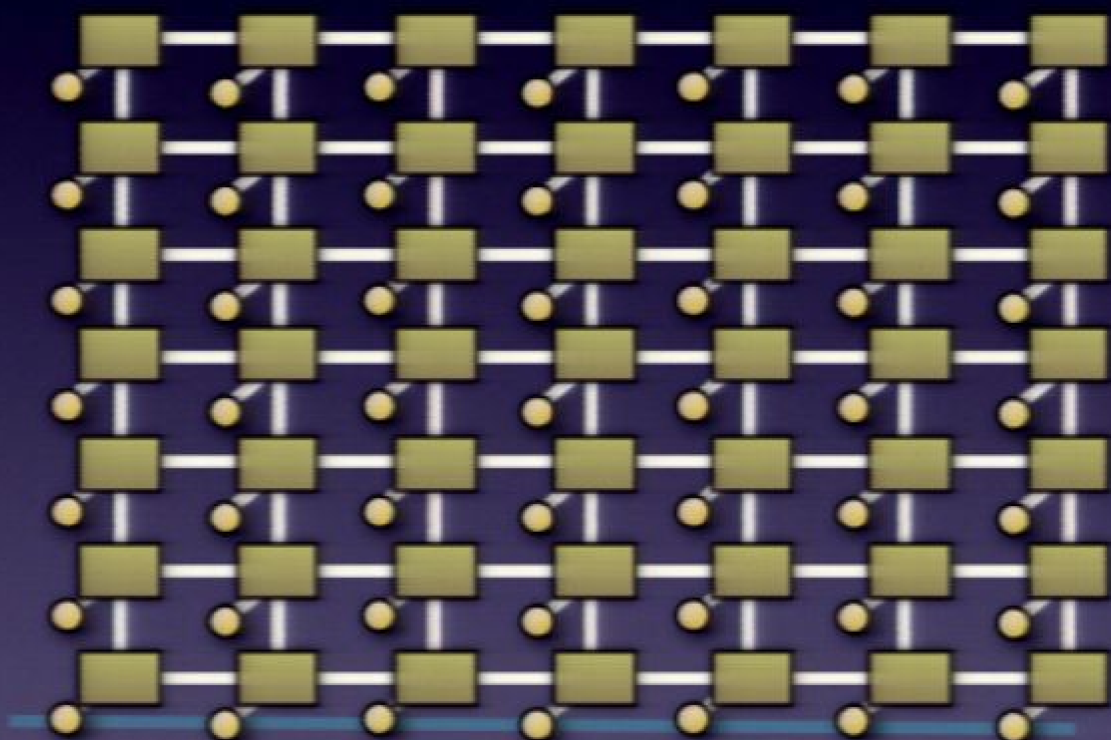

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Two major classes of tensor networks to simulate strongly correlated models

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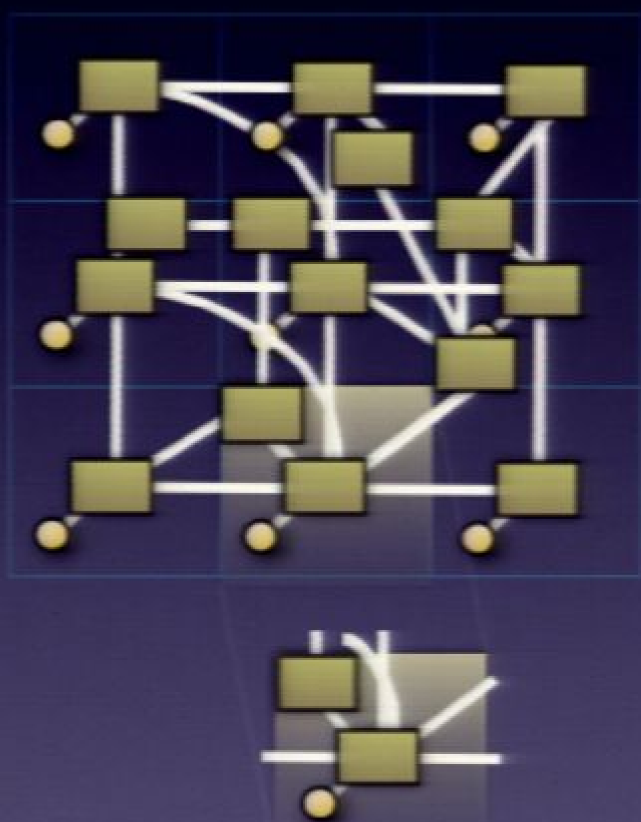

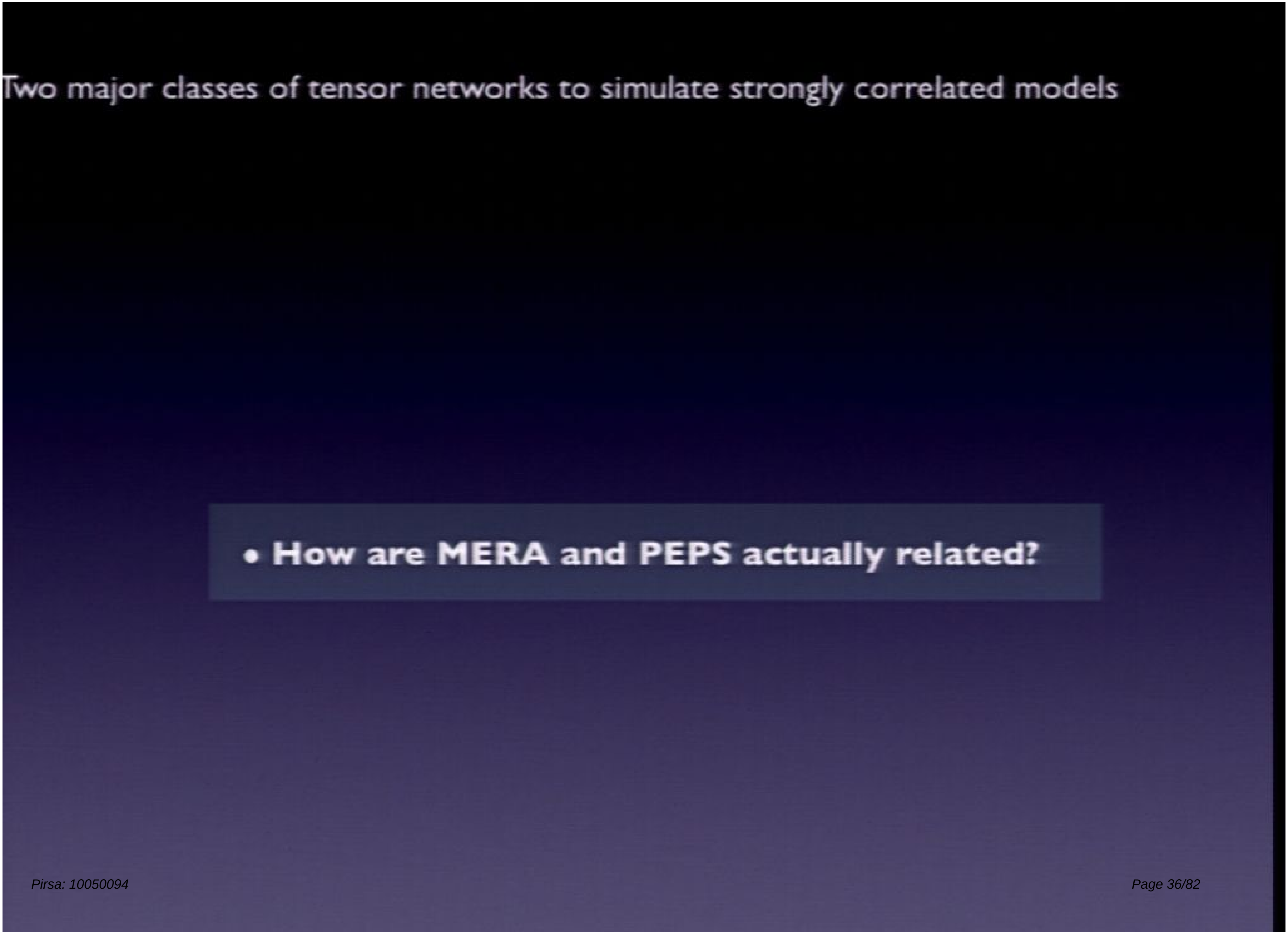
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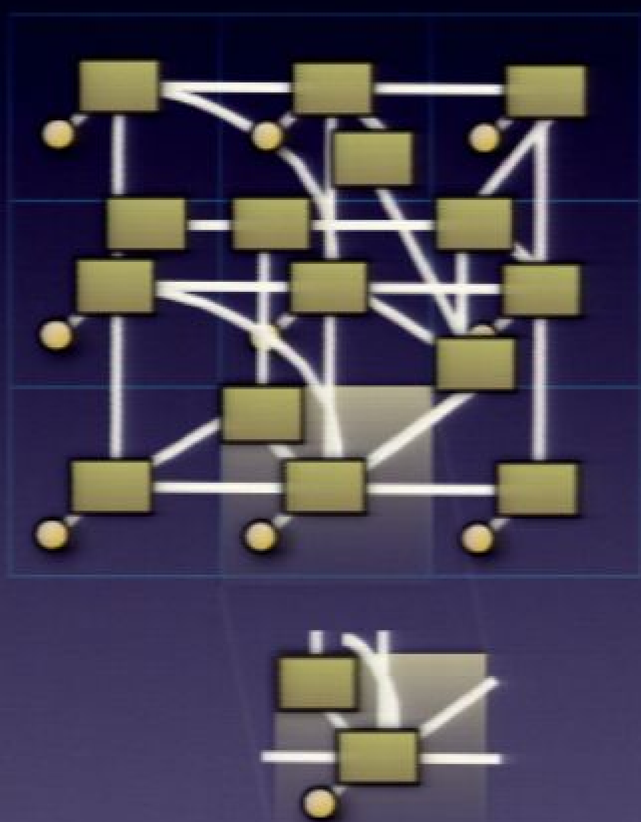

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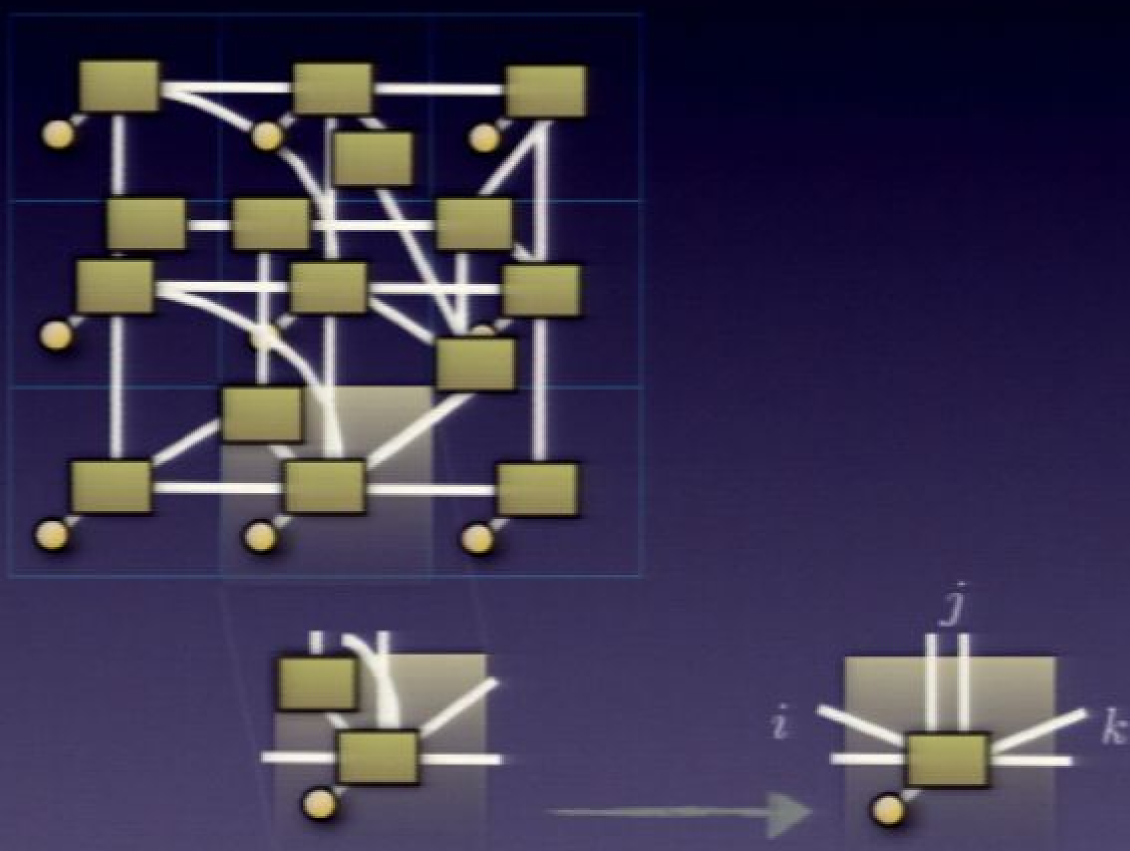

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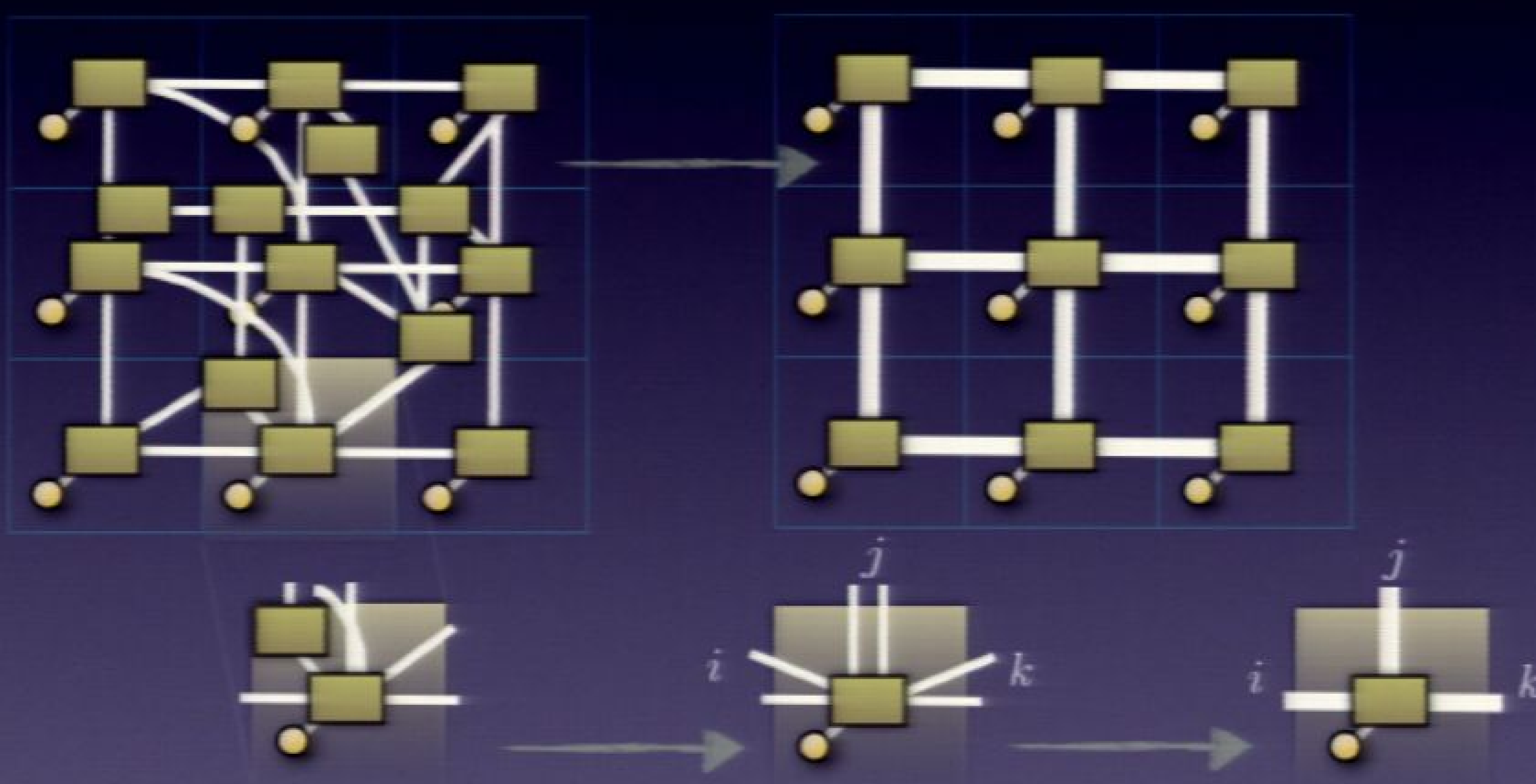

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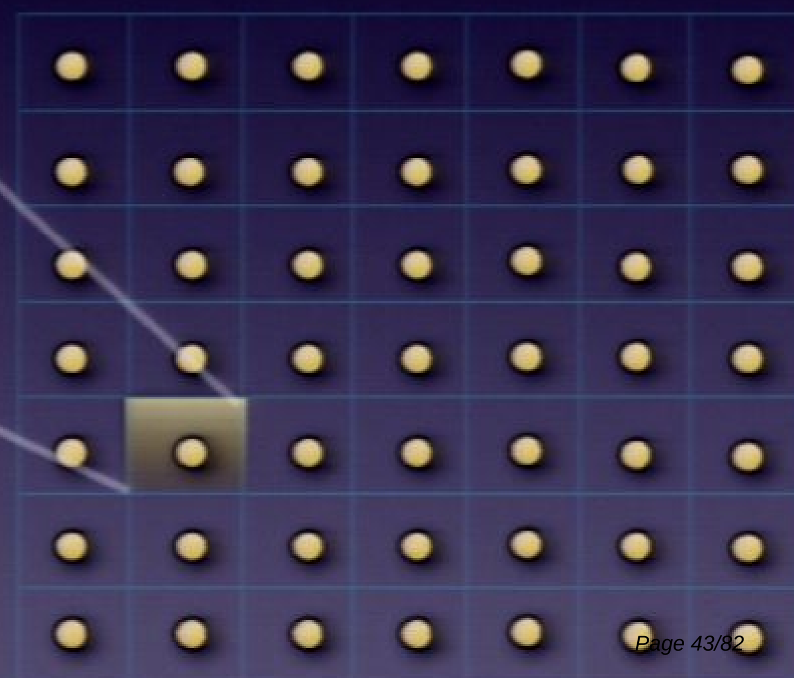
- Suitable **placement** of tensors:



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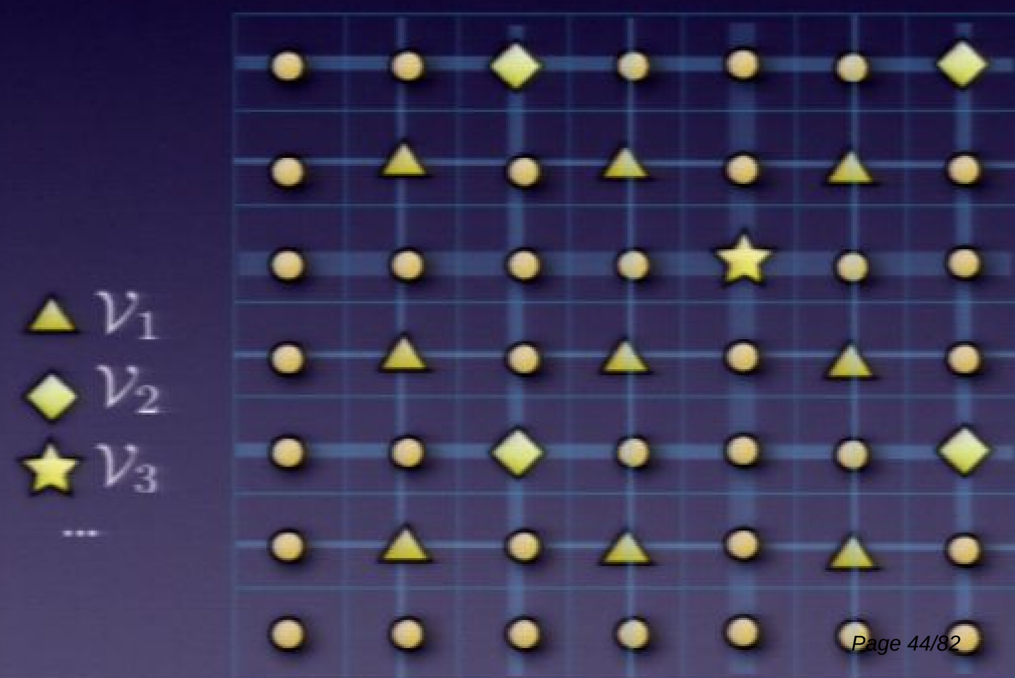

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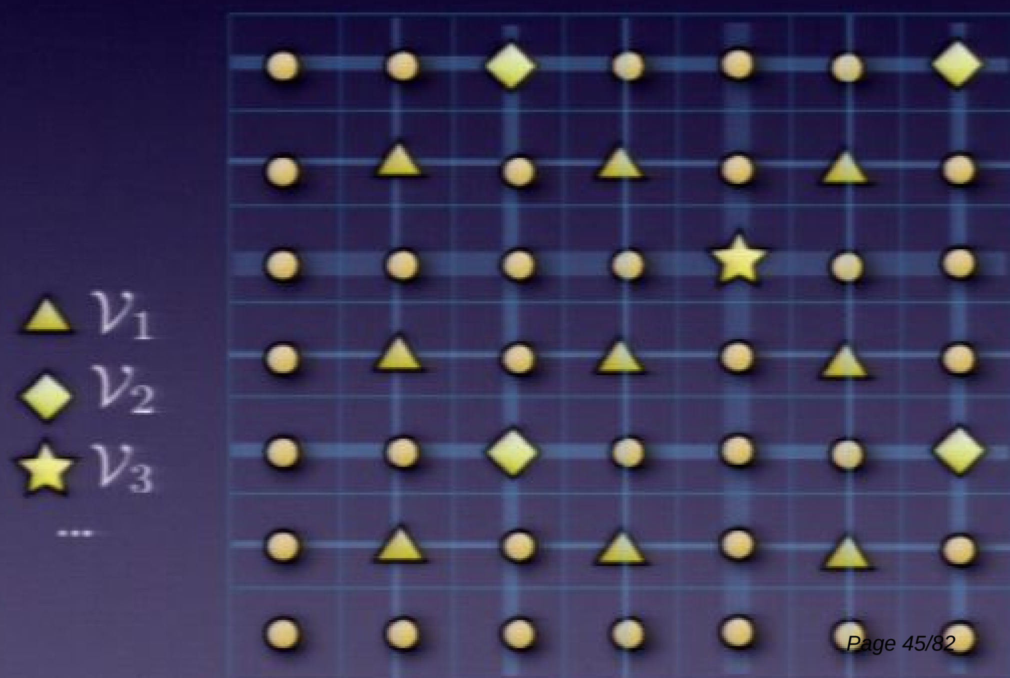
- **Can show:** Gives rise to PEPS with bond dimension bounded from above by

$$\log_\chi(\chi_{\mathrm{PEPS}}) \leq (eC_r)^D (C_T + b^{D(C_T+1)}) C_t C_o = O(1)$$



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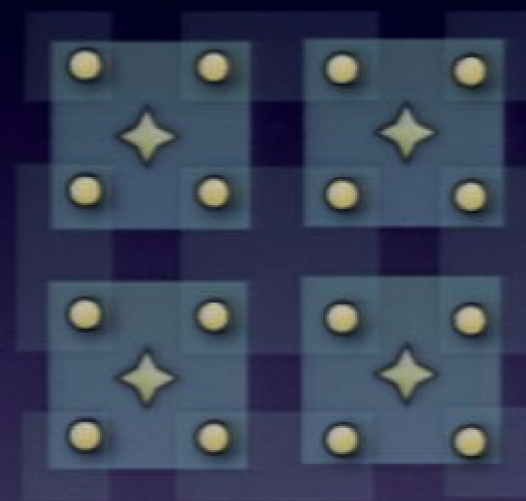

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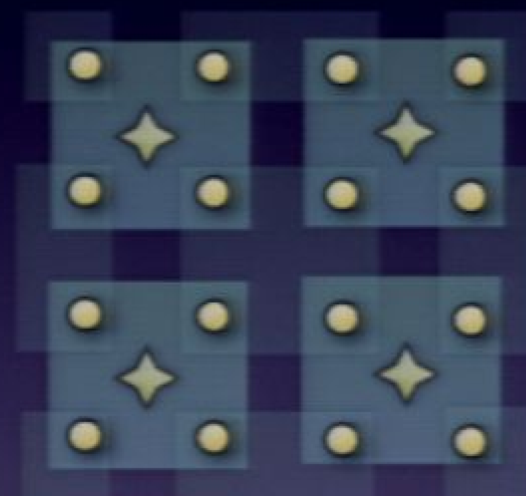

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  "Real-space renormalization captured in PEPS"

  (but note that MERA are efficiently contractible)...

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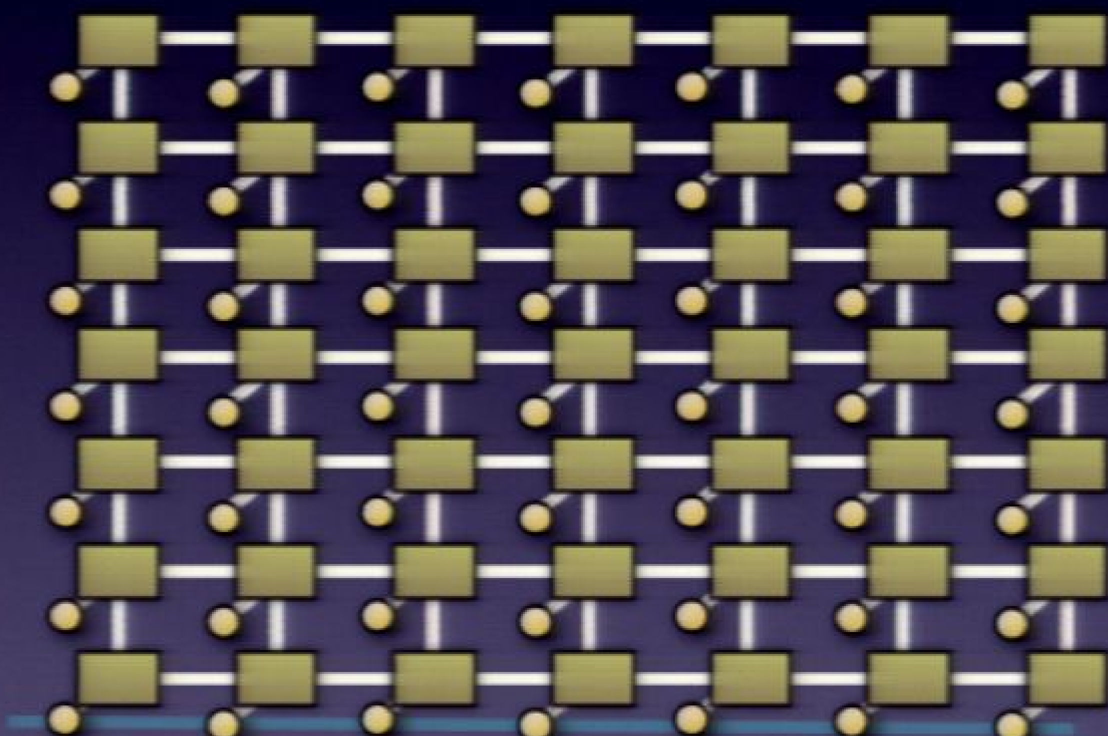

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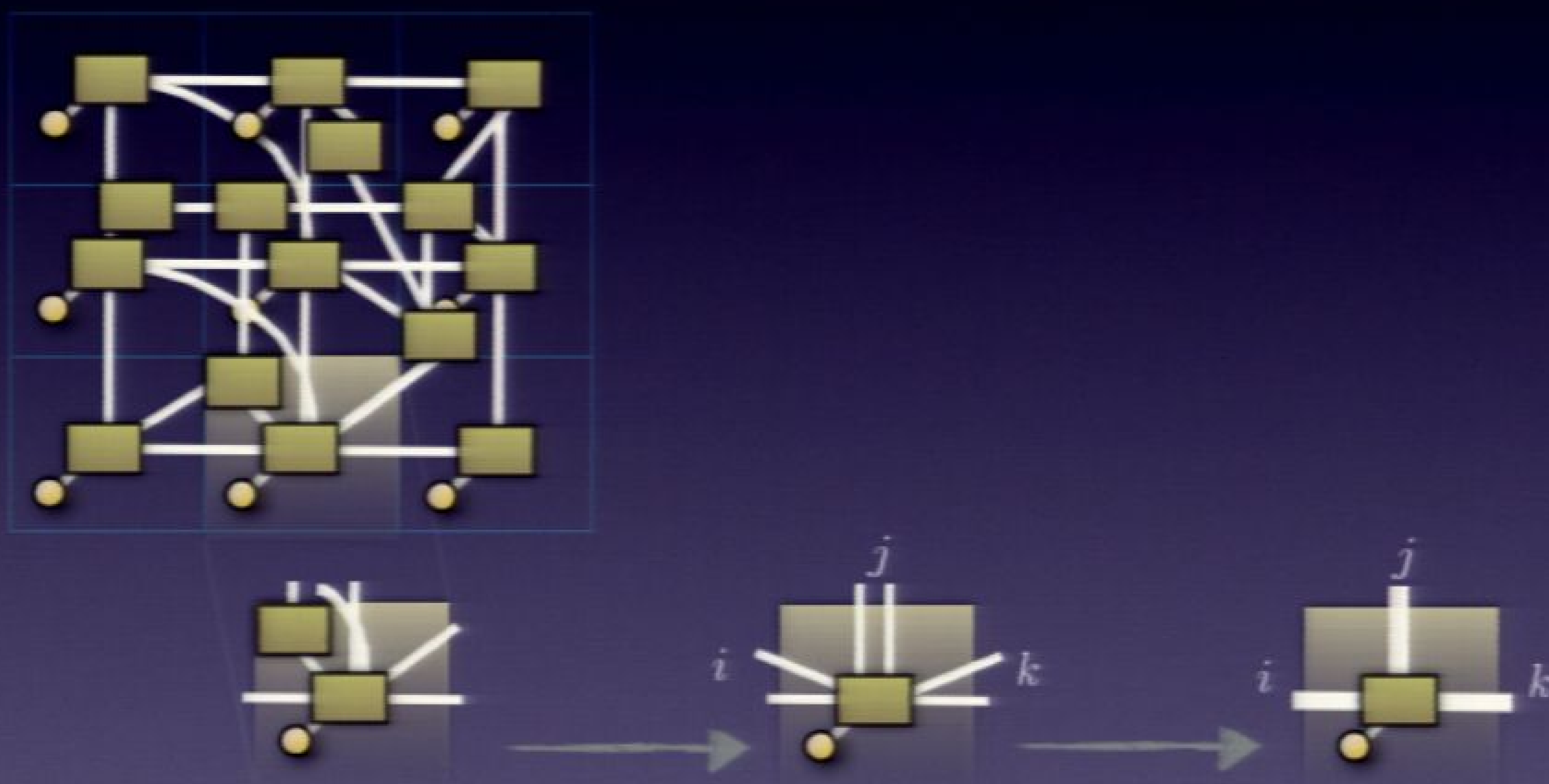

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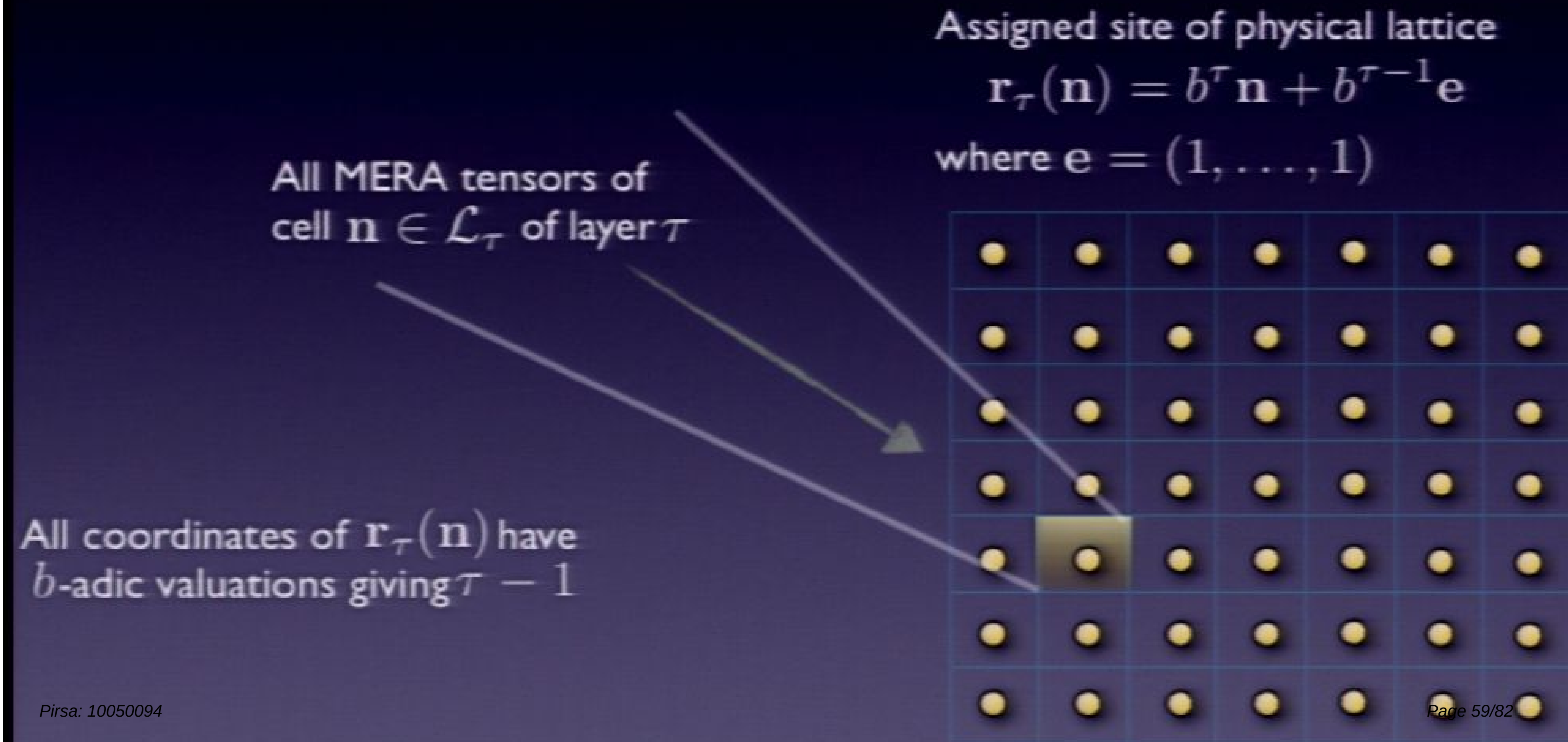

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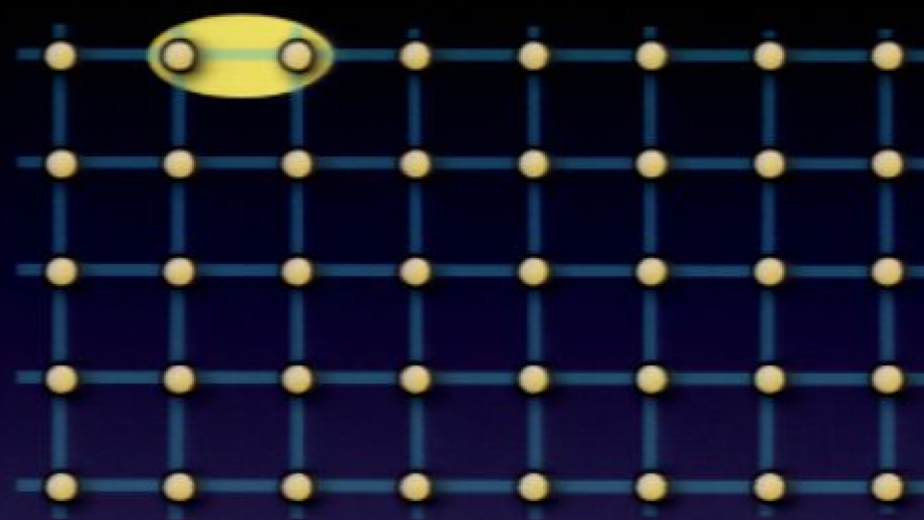
:inding ground states of local quantum many-body systems
;, in general, computationally **difficult**

$$E = \min \langle \psi | H | \psi \rangle$$



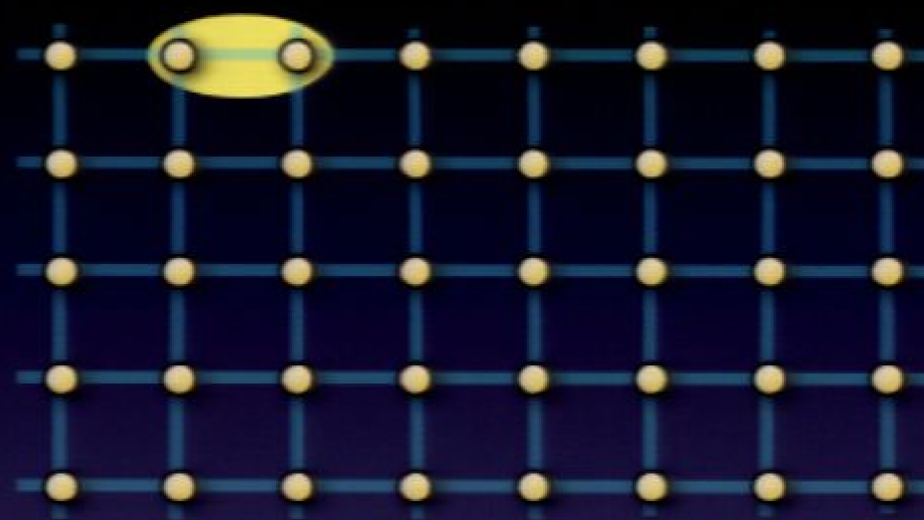
'lodels for which approximation of **ground state energy** is **QMA-complete**

'hysically: "Glassy" models

'he *power of quantum computing on a line*, Aharonov, Gottesman, Irani, Kempe, *Common Math Phys* **287**, 51 (2009)
omplexity of the local Hamiltonian problem, Kempe, Kitaev, Regev, *SIAM J Comp* **35**, 1070 (2006)

Here: Identify models for which finding **entire ground state manifold** is **easy:**

Frustration-free nearest-neighbor natural spin-1/2 systems on **arbitrary lattices**



- GS local **expectation values** $\langle A \rangle$ can be computed exactly and efficiently

("natural" means, excited states of Hamiltonian terms contain entangled one)

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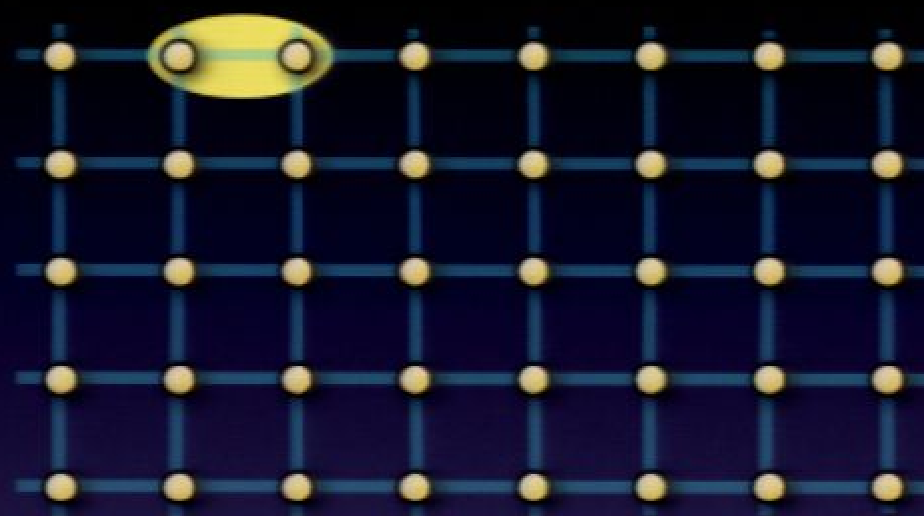
Here: Identify models for which finding **entire ground state manifold** is **easy:**

Frustration-free nearest-neighbor natural spin-1/2 systems on **arbitrary lattices**



- **Reminder:** Hamiltonian $H = \sum_{\{a,b\}} h_{a,b}$ is **unfrustrated** if not only

  $H|\psi\rangle = 0$ for every $|\psi\rangle \in M$ (GS manifold), but

  $h_{a,b}|\psi\rangle = 0$ for all $|\psi\rangle \in M$ and all $h_{a,b}$

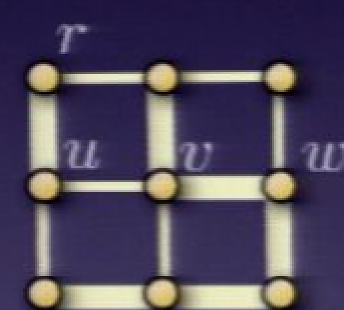
Solving frustration-free systems (with de Beaudrap, Ohliger, Osborne), arXiv:1005.3781

dea: Start from ingredients from Bravyi's algorithm for quantum 2-SAT, generalize
o capture entire GS manifold

f Hamiltonian term $h_{u,v}$ of **rank 2,3**: $\ker(h_{u,v}) \subset \mathrm{span}(|\psi_0\rangle, |\psi_1\rangle\}$, apply
isometry $U_{u:uv} : \mathcal{H}_2^{\otimes u} \to \mathcal{H}_2^{\otimes \{u,v\}}$ such that

$$\sum_{x=0,1} \alpha_x |x\rangle_u \to \sum_{x=0,1} \alpha_x |\psi_x\rangle_{u,v}$$

onsider new Hamiltonian $H' = U^{\dagger}_{u:u,v} H U_{u:u,v}$



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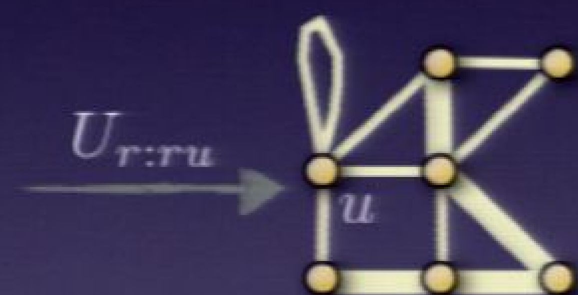

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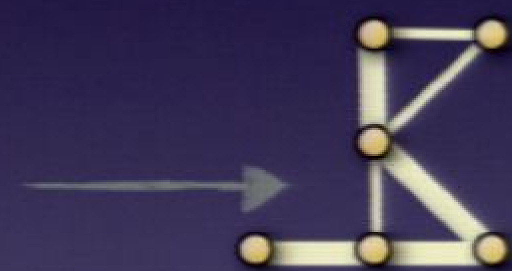
If encounter **single-spin** operator h_u then delete isometrically
to new Hamiltonian $H' = (\langle\psi|_u \otimes \mathbf{1}) H (|\psi\rangle_u \otimes \mathbf{1})$



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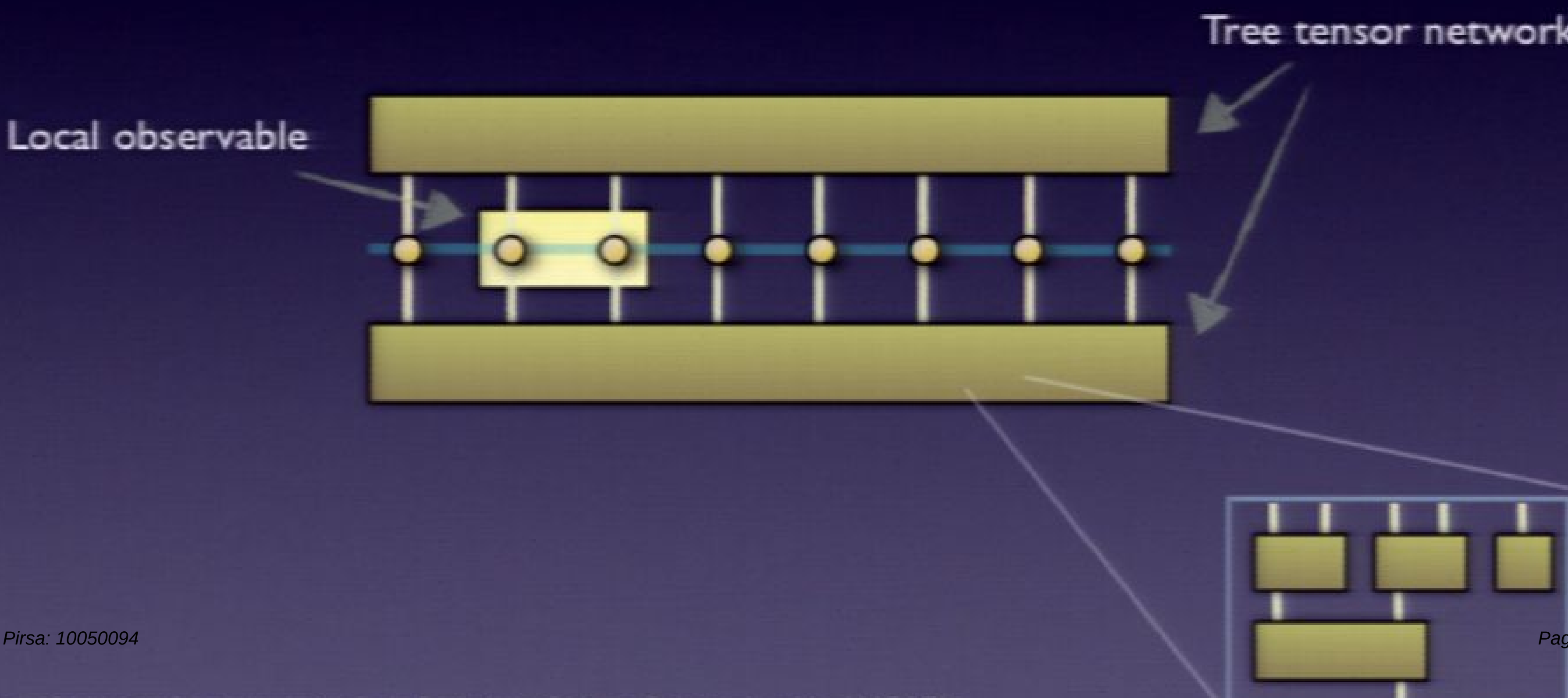
- rove that for our purposes, **contraction-order** does not matter
- rove that property of being **"natural"** is conserved (somewhat technical)
- ind normal form of Hamiltonian of rank-1, $h_{a,b} = |\beta_{a,b}\rangle\langle\beta_{a,b}|$ closed under

$$\langle \beta'_{a,c}| = (\langle \beta_{a,b}| \otimes \langle \beta_{b,c}|(\mathbf{1} \otimes |\psi^-\rangle \otimes \mathbf{1})$$

- **Ground space** of this remaining core of $|V_c|$ spins is image of a symmetric subspace $\mathrm{Symm}(\mathcal{H}_2^{\otimes V_c})$ under known local invertible transformations

olving *frustration-free systems* (with de Beaudrap, Ohliger, Osborne), arXiv:1005.3781

Observation 4: Expectation values of **local observables** can be **efficiently exactly computed**



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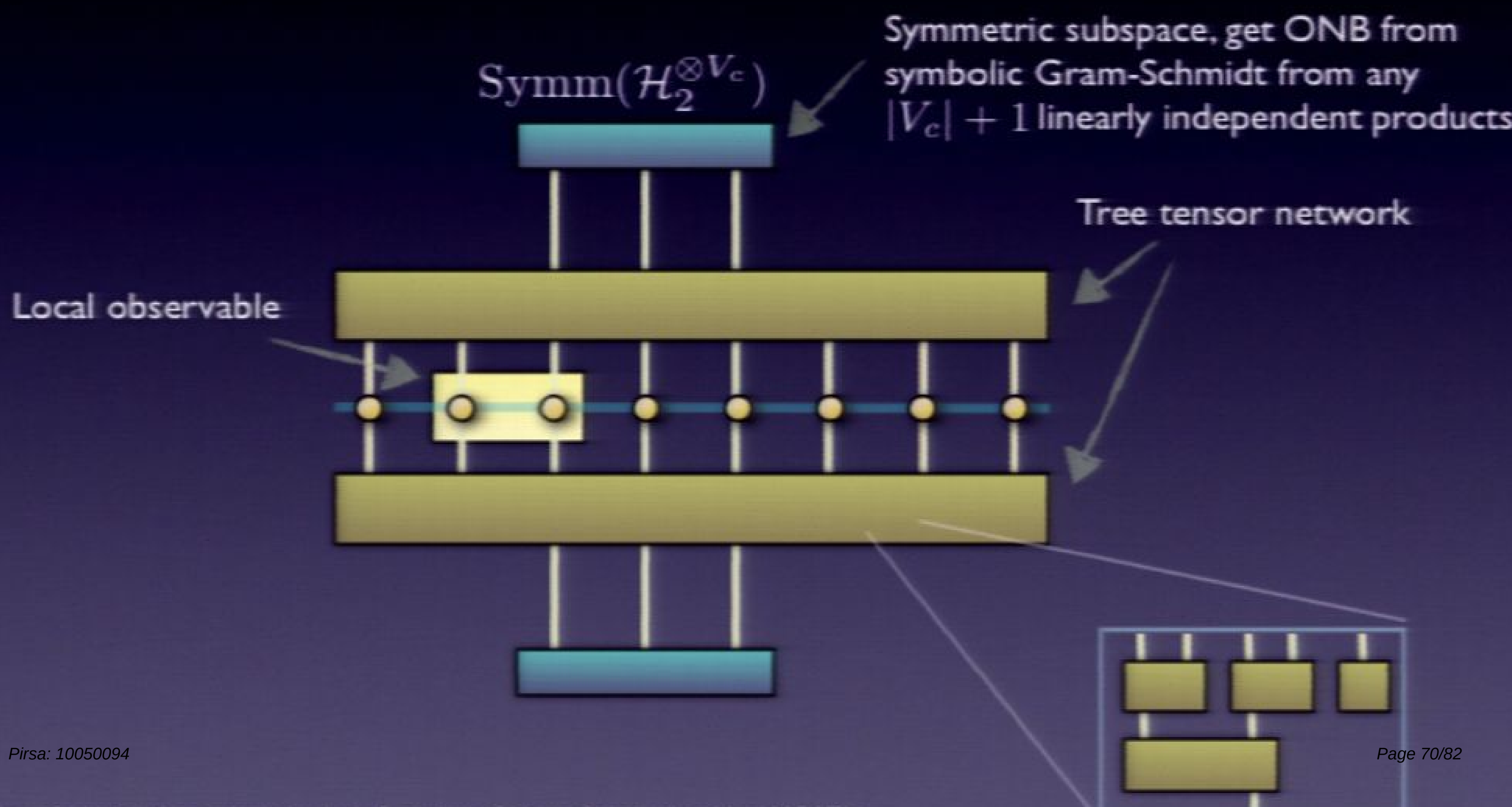
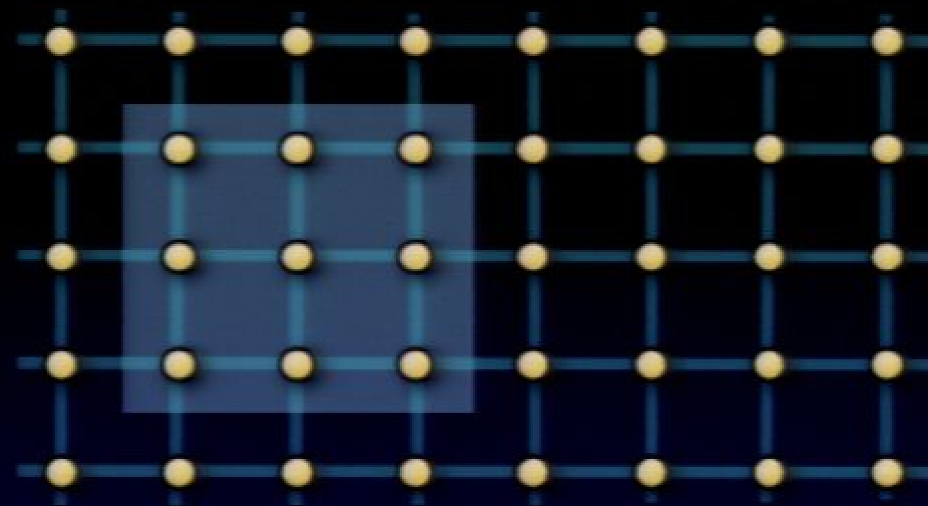




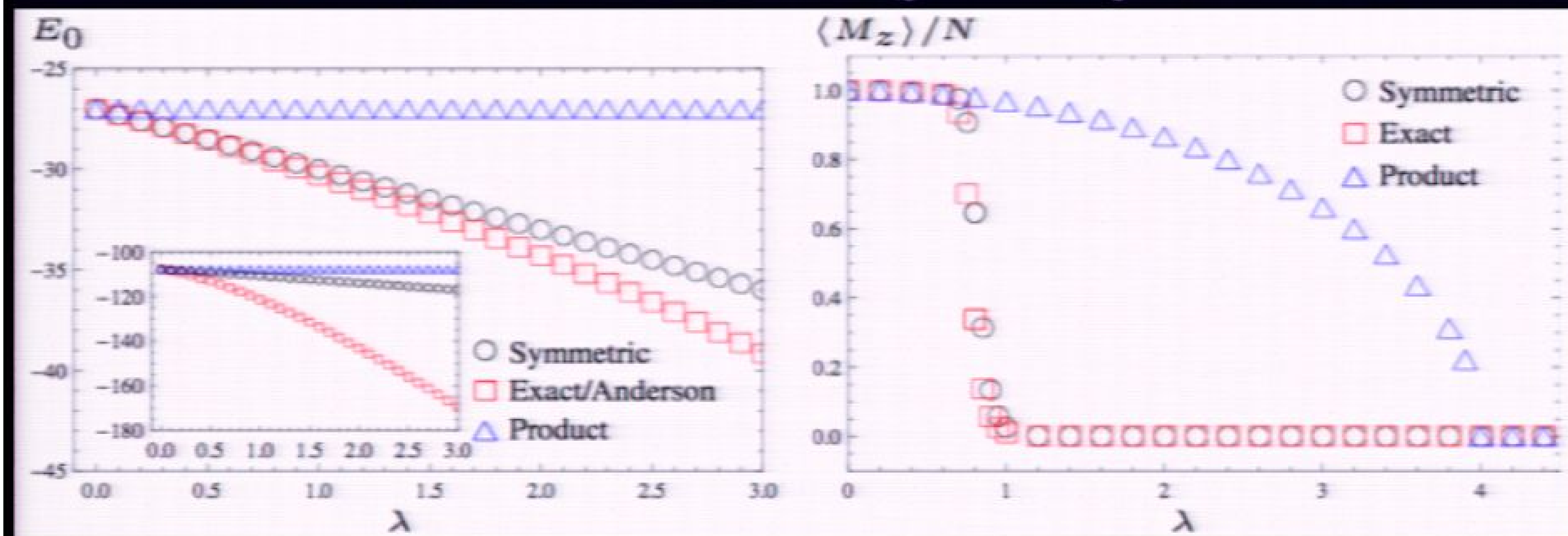
- **Observation 5:** All such ground states satisfy **area laws**, now for mixed-state entanglement
- Novel class of models for which area law is known
- **Proof:** For each connected component of interaction graph, explicitly bound maximum Schmidt rank

Observation 6: Provides ansatz class of **almost frustration-free models**

Consider $H = H_0 + \lambda V$, H_0 unfrustrated, small examples:

GS energy of XXZ model on 3x3 torus $h_{i,j} = -X_iX_j - Y_iY_j - (1-\lambda)Z_iZ_j$

Magnetization in 4x4 Ising model $h_{i,j} = -Z_iZ_j \;\; h_i = -\lambda X_i$



- Sample from GS manifold of exactly frustration-free system
- Very **simple**, but significantly outperforms Gutzwiller mean field

- **Lesson:** Such frustration-free models can be solved exactly
- Instance of **real-space renormalization** which is exact
- "Tensor networks with an input"

utlook: What else ...

ermionic tensor networks and quantum fields

lographic quantum states (with Osborne, Verstraete), arXiv:1005.1268

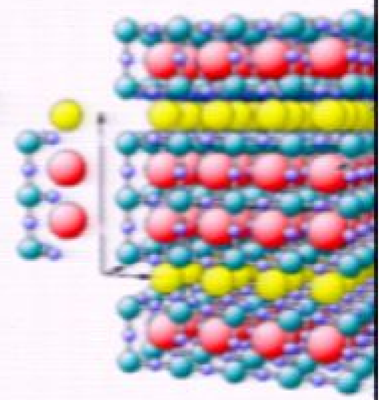
itary circuits for simulating strongly correlated fermions (with Pineda, Barthel), arXiv:0905.0669

ntraction of fermionic operator circuits and the simulation of strongly correlated fermions

ith Barthel, Pineda), *Phys Rev A* **80**, 042333 (2009)

ontinuum limits of MPS for quantum fields, long-range interactions and free models
ee Frank's talk)

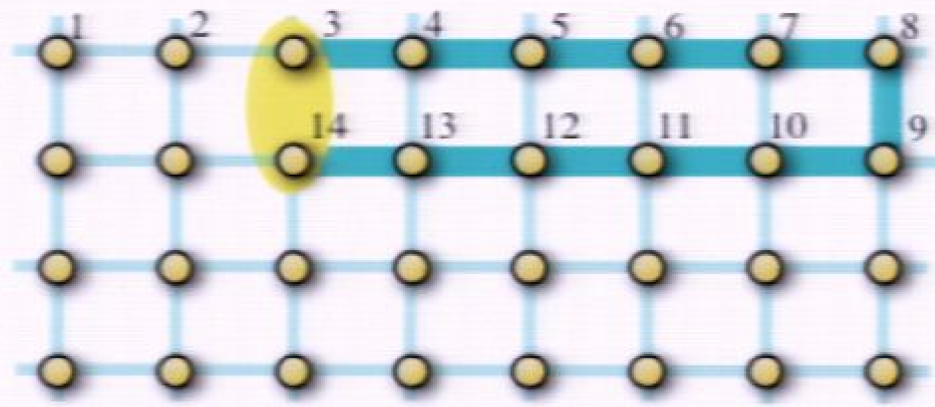
ensor networks for **fermionic** models (Hubbard/spinless models)?



$$H = -\sum_{\langle i,j\rangle,\sigma} (f^\dagger_{i,\sigma} f_{j,\sigma} + h.c.) + U \sum_i n_{i,\uparrow} n_{i,\downarrow} - \mu \sum_{i,\sigma} n_{i,\sigma}$$

roblem: Naive mapping from fermionic model to spin models creates
on-local strings under Jordan-Wigner rendering contraction inefficient

$$J_O(f^\dagger_{O_j} f_{O_k}) = \sigma^-_j \otimes \bigotimes_{j<l<k} \sigma^z_l \otimes \sigma^+_k$$



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lating of interacting fermions with entanglement renormalization, Corboz, Evenbly, Verstraete, Vidal, *Phys Rev A* **81**, 010303 (2010)

Role of parity: Causal cone of MERA is the same for fermions as for spins

- **Overcome fixed order of modes:**
  - Jordan-Wigner transformations **local** in space and time: e.g., for given order $\mathfrak{m} : \{1, \ldots, |m|\} \to m$, $\mathfrak{n} : \{1, \ldots, |m|\} \to n$ in spin representation

$$J_{\mathfrak{n},\mathfrak{m}}(A) = \sum_{n,m} |n)\,{}_{\mathfrak{n}}\langle n|A|m\rangle_{\mathfrak{m}}(m|$$

$$|m\rangle_{\mathfrak{m}} \in \mathcal{F}_m,\ |n\rangle_{\mathfrak{n}} \in \mathcal{F}_n,\ |m) \in (\mathbb{C}^2)^{\otimes|m|},\ |n) \in (\mathbb{C}^2)^{\otimes|n|}$$

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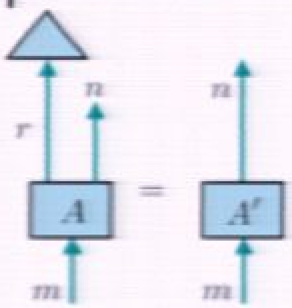
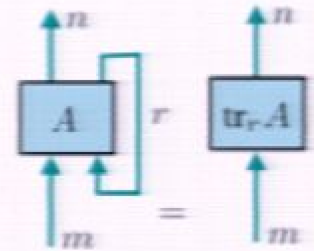
Set of complete **fermionic contraction** rules:



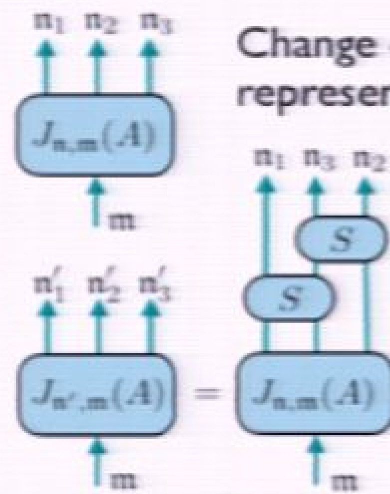
Partial projection

$(\ldots|J_{\mathbf{n},\mathbf{m}}(A')|m) = (r', n|J_{\mathbf{a},\mathbf{m}}(A)|m)$



Partial trace

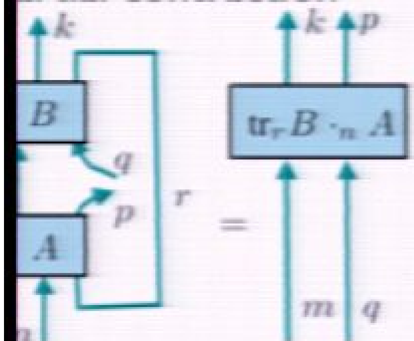
$(n|J_{\mathbf{n},\mathbf{m}}(\mathrm{tr}_r A)|m) = (n, r|J_{\mathbf{b},\mathbf{a}}(A)|m, r)$



Change order of spin representations

$S = |0,0)(0,0| - |1,1)(1,1|$
$+ |0,1)(1,0| + |1,0)(0,1|$

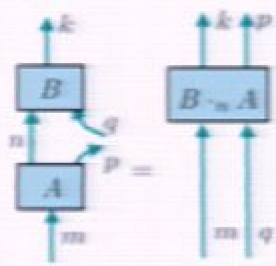
artial contraction



$(k,p|J_{\mathbf{a},\mathbf{b}}(C)|m,q)$

$\ldots^{r(p+q)} \cdot (k,r|B|n,q)(n,p|A|m,r)$

Partial multiplication



$(k,p|B \cdot_n A|m,q) = (-1)^{\bar{p}\bar{q}}(k|B|n,q)(n,p|A|m)$

- **Observation 6:** Fermionic tensor networks can be contracted with the same efficiency (at most small constant overhead) compared to spin models

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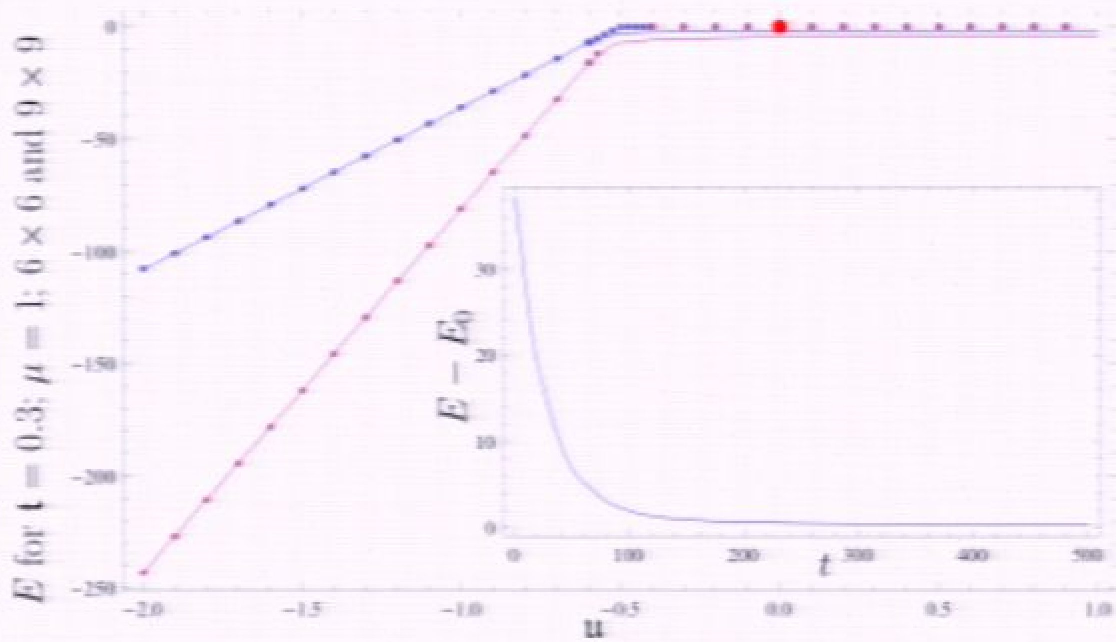
Now careful **numerical benchmarking** (MERA)

- Benchmarking, comparison with exact diagonalization on 25 fermions, free models
- Promising results, tricky to get large bond dimension, good for small hopping



$$H = \mathrm{t}\sum_{\langle j,k\rangle}(f_j f_k + h.c.) + \sum_j f_j f_j + \mathrm{u}\sum_{\langle j,k\rangle} f_j f_j f_k f_k$$

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This talk: Grasping quantum many-body systems in terms of tensor networks

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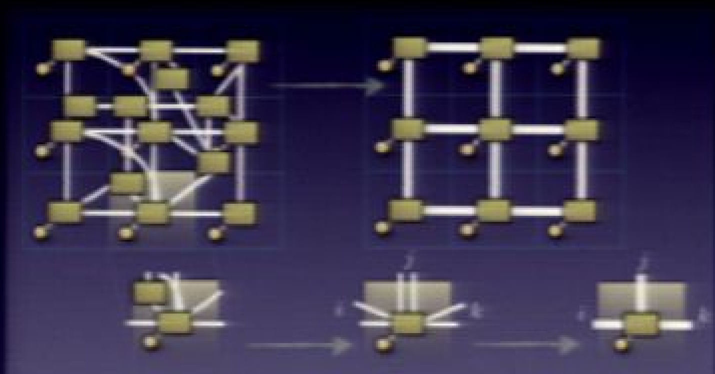
Relating MERA and PEPS



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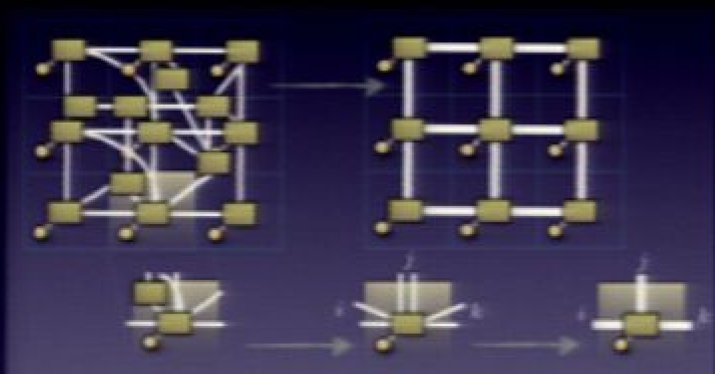
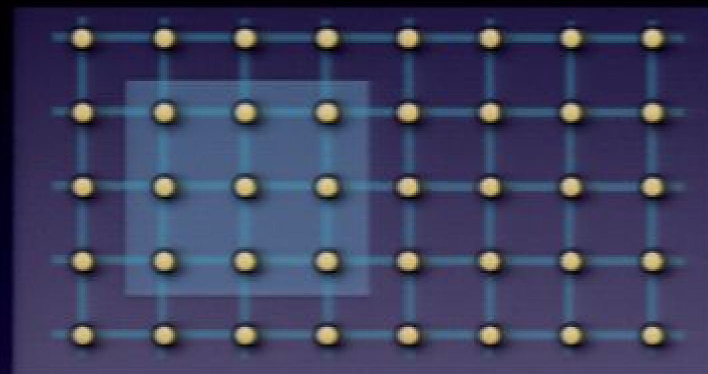


2. Solving frustration-free spin systems



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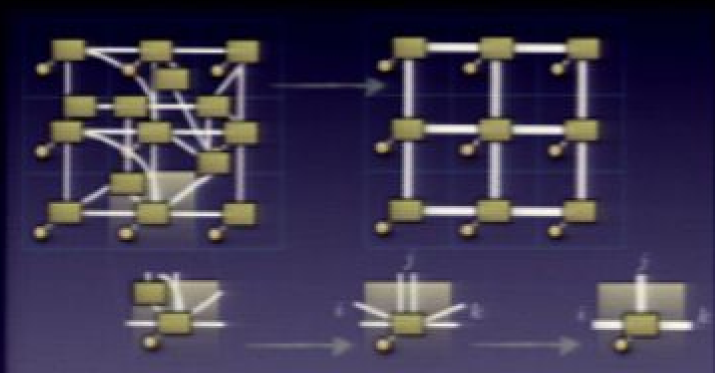

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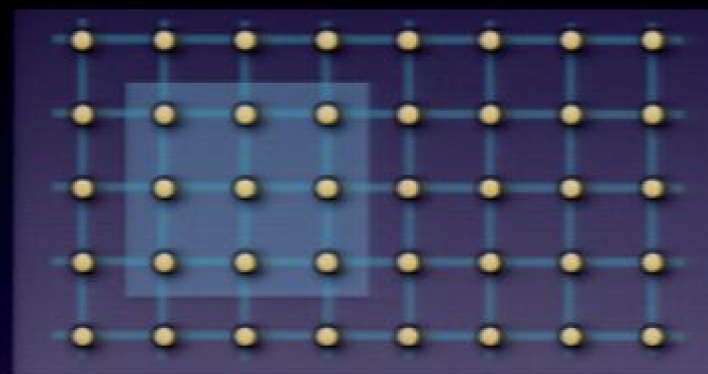

A glimpse of simulating fermionic systems