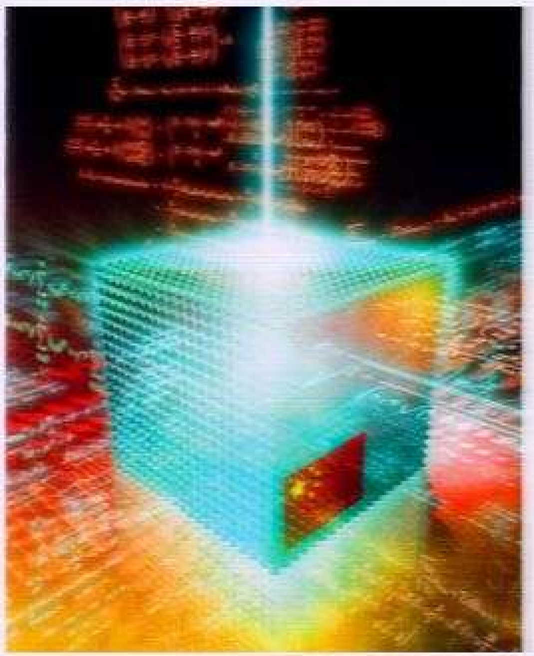
Title: Description of many-body systems using MPS, PEPS, and other families of states

Date: May 25, 2010 09:45 AM

URL: http://pirsa.org/10050093

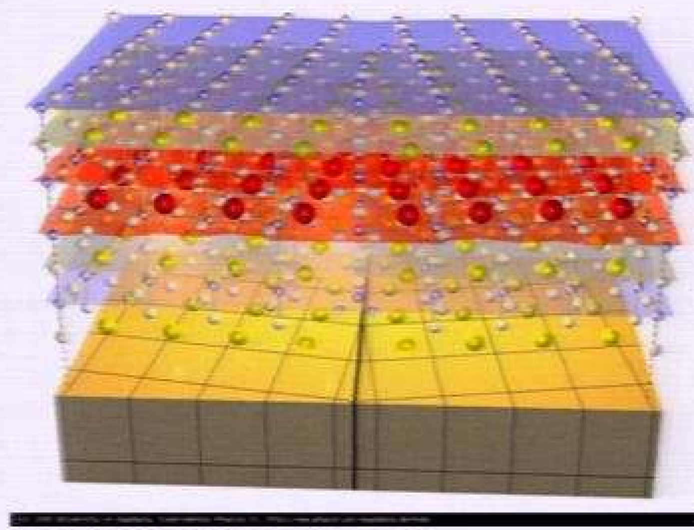
Abstract: Matrix Product States (MPS) and their higher dimensional extensions, the Projected Entangled-Pair States (PEPS) can efficiently describe the ground and thermal states of interacting systems with short-range interactions. We will describe some mathematical properties of this families of states, as well as possible extensions. Work in collaboration with N. Schuch, D. Perez-Garcia, M. Sanz, M. Wolf, F. Verstraete and G. Sierra.

Description of many-body states using MPS and PEPS



J. IGNACIO CIRAC

MPQ
Max-Planck-Institut für Quantenoptik



Emergence and Entanglement,
Perimeter Institute, Waterloo, May 2010

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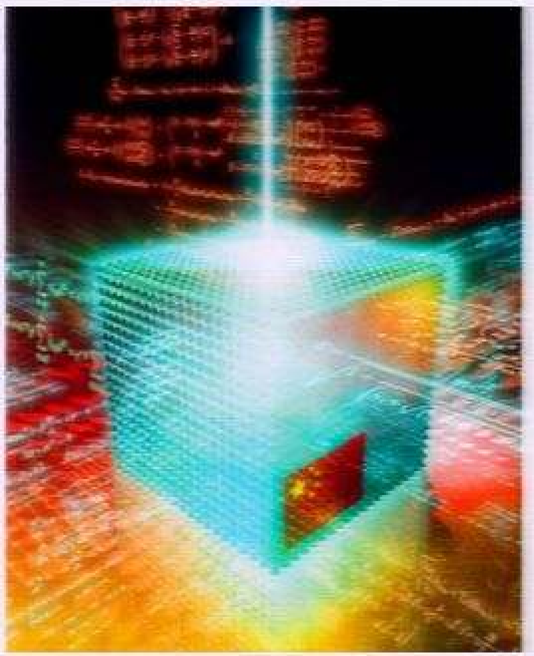

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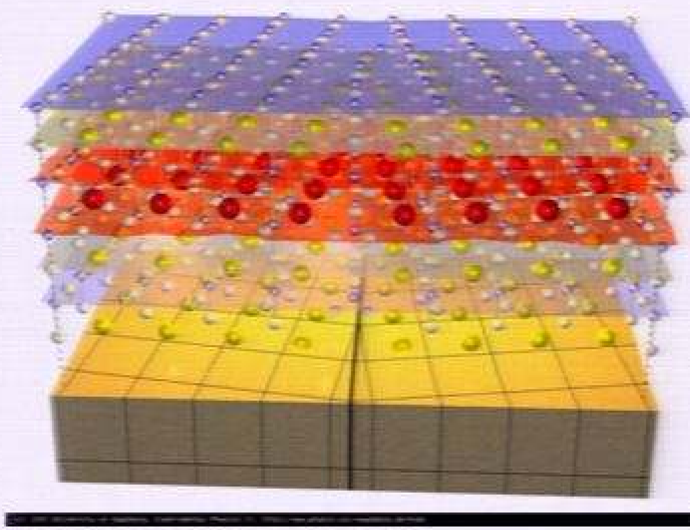

Emergence and Entanglement,
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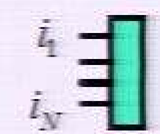
MANY-BODY PROBLEMS

- N spins:

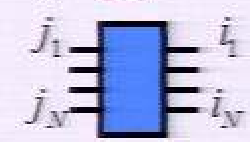
  - States and observables can be written in terms of tensors

$$|\Psi\rangle = \sum_i c^{i_1,\dots,i_N} \, | i_1,\dots,i_N\rangle$$



$c^{i_1,\dots,i_N}$

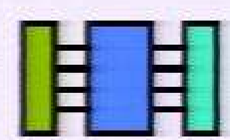
$$X = \sum_{i,j} X^{j_1,\dots,j_N}_{i_1,\dots,i_N} \, | j_1,\dots,j_N\rangle\langle i_1,\dots,i_N |$$



$X^{j_1,\dots,j_N}_{i_1,\dots,i_N}$

  - Expectation values are tensor contractions:

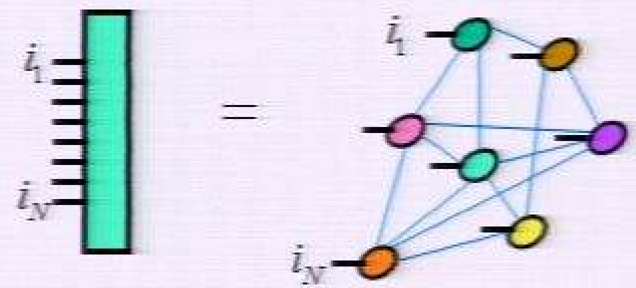
$$\langle\Psi | X | \Psi\rangle = \sum_{i,j} c^*_{j_1,\dots,j_N} X^{j_1,\dots,j_N}_{i_1,\dots,i_N} c^{i_1,\dots,i_N}$$



TENSOR NETWORKS

- Rewrite tensors in terms of smaller tensors:

STATES:



Tensor network states

or

Tensor product states

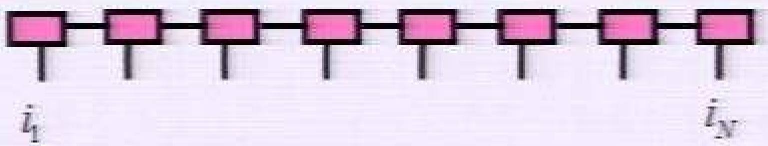
OBSERVABLES: similarly

- Why? Efficient description:

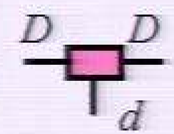
d^N $\quad\quad$ NdD^{rank}

EXAMPLES

1. Matrix Product States (MPS):



$$c^{i_1,i_2,\ldots} = \mathrm{tr}\left[A^{i_1} A^{i_2} \ldots A^{i_N} \right]$$

$A^{i}_{\alpha\beta}$ 

- Tensors are different at different sites.
- Translationally invariant: all tensors are equal.

Affleck I Kennedy T Lieb E H and Tasaki H 1987 *Phys. Rev. Lett.* **59** 799.
Fannes M Nachtergaele B and Werner R F 1992 *Comm. Math. Phys.* **144** 443
Klümper A Schadschneider A and Zittartz J 1993 *Europhys. Lett.* **24** 293

DMRG:

White S R 1992 *Phys. Rev. Lett.* **69** 2863
Östlund S and Rommer S 1995 *Phys. Rev. Lett.* **75** 3537

QIT:

Vidal, Phys. Rev. Lett. 91, 147902 (2003)
Verstraete, Martin-Delgado, Cirac, Phys. Rev. Lett. 92, 087201 (2004)

EXAMPLES

1. Matrix Product States (MPS):

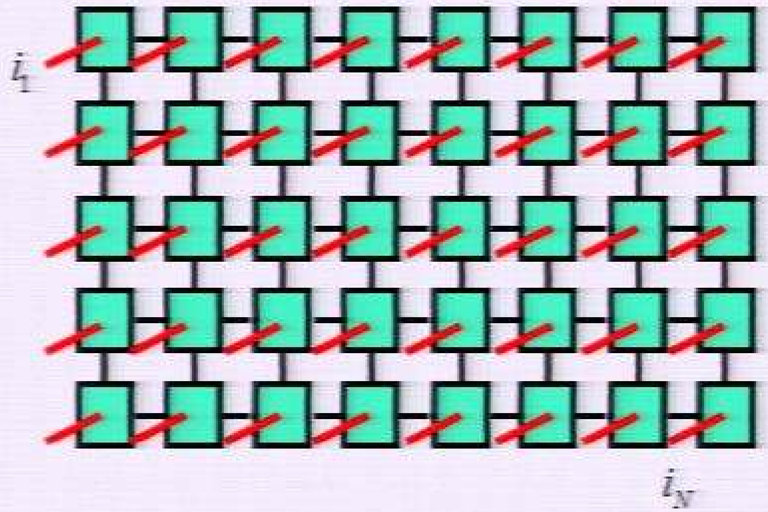
- Efficient description of ground states of 1D systems
  - Area-law and logarithmic violations:

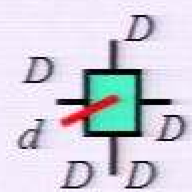
    Verstraete F and Cirac J I 2006 *Phys. Rev. B* **73** 094423
  - Gapped Hamiltonians:

    Hastings M B 2007 *J. Stat. Phys.* P08024

EXAMPLES

2. Projected Entangled-Pairs (PEPS):





Affleck I Kennedy T Lieb E H and Tasaki H 1987 *Phys. Rev. Lett.* **59** 799.

DMRG: (matrix product vertex ansatz)

Sierra G and Martin-Delgado M A *Preprint:* cond-mat/9811170

Nishino T Hieida Y Okunishi K Maeshima N and Akutsu Y 2001 *Prog. Theor. Phys.* **105** 409

QIT:

Verstraete F and Cirac J I 2004 *Phys. Rev. A* **70** 060302

EFFICIENT DESCIPTIONS:

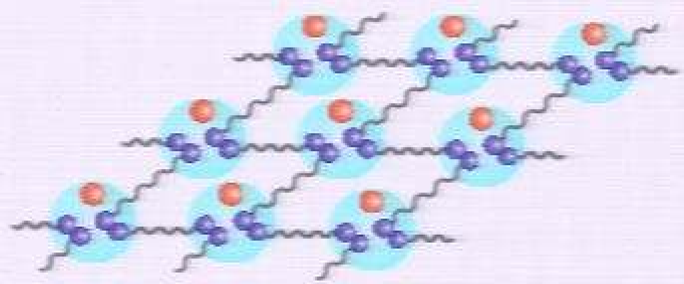
Verstraete F and Cirac J I *Preprint* arXiv:cond-mat/0407066

EXAMPLES

MPQ

2. Projected Entangled-Pairs (PEPS):

- Valence-bond construction:



- Efficient description of low energy states of higher dimensions
  - Thermal states, arbitrary temperature:

    Hastings M B 2007 *Phys. Rev. B* **76** 035114

MPS and PEPS: efficient descriptions in thermal equilibrium (spins in a lattice, with short-rangle interactions)

EXAMPLES

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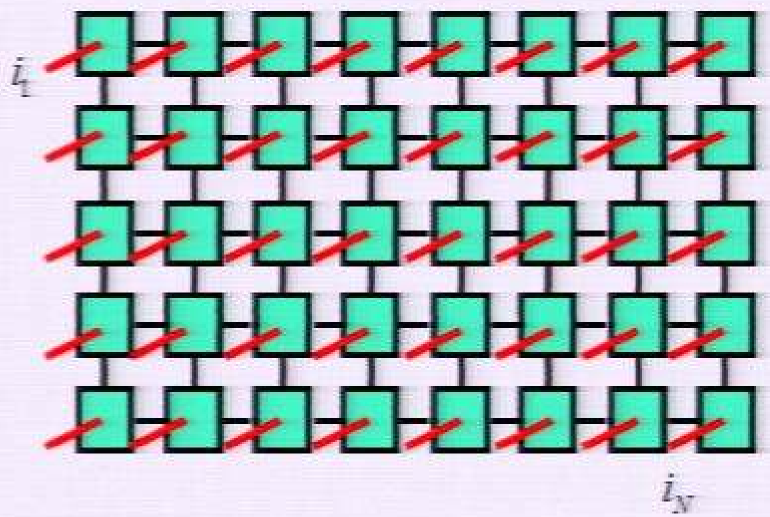

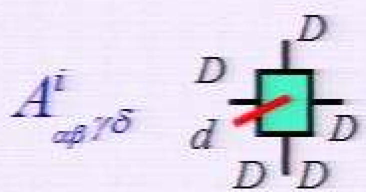

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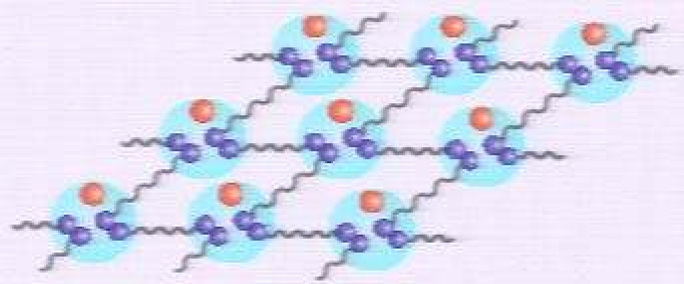

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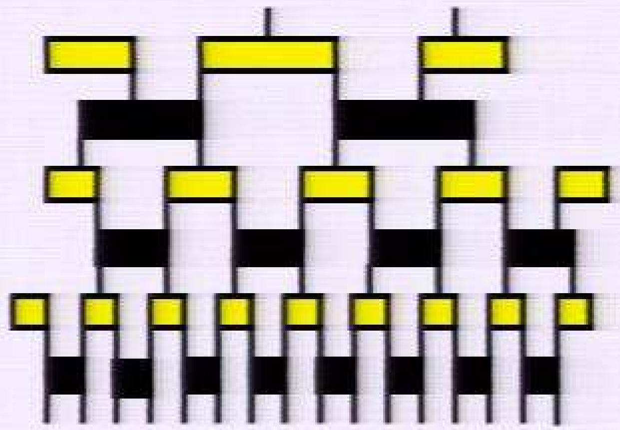
MPS and PEPS: efficient descriptions in thermal equilibrium (spins in a lattice, with short-rangle interactions)

EXAMPLES

3. Multi-scale renormalization ansatz (MERA):

Vidal G 2007 *Phys. Rev. Lett.* **99** 220405
Vidal G 2008 *Phys. Rev. Lett.* **101** 110501

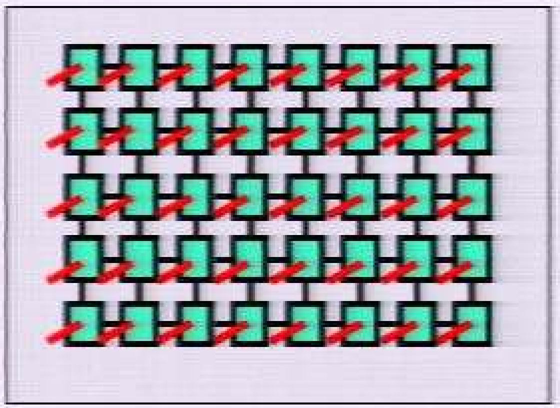


- Specially suited for critical systems

MPS & PEPS

Numerical algorithms

Convergence



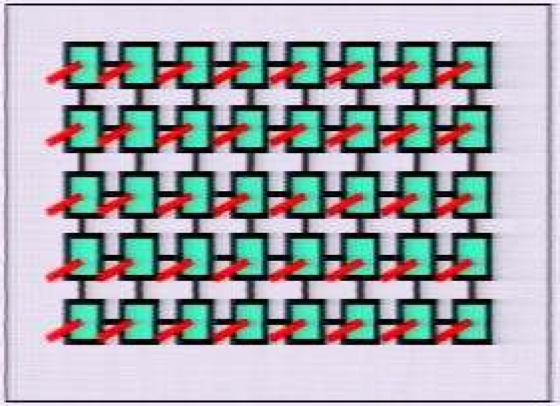
Physical properties

Other systems

MPS & PEPS

Numerical algorithms

Time-evolution: thermalization



Convergence

1 D systems

Physical properties

Symmetries
Topology

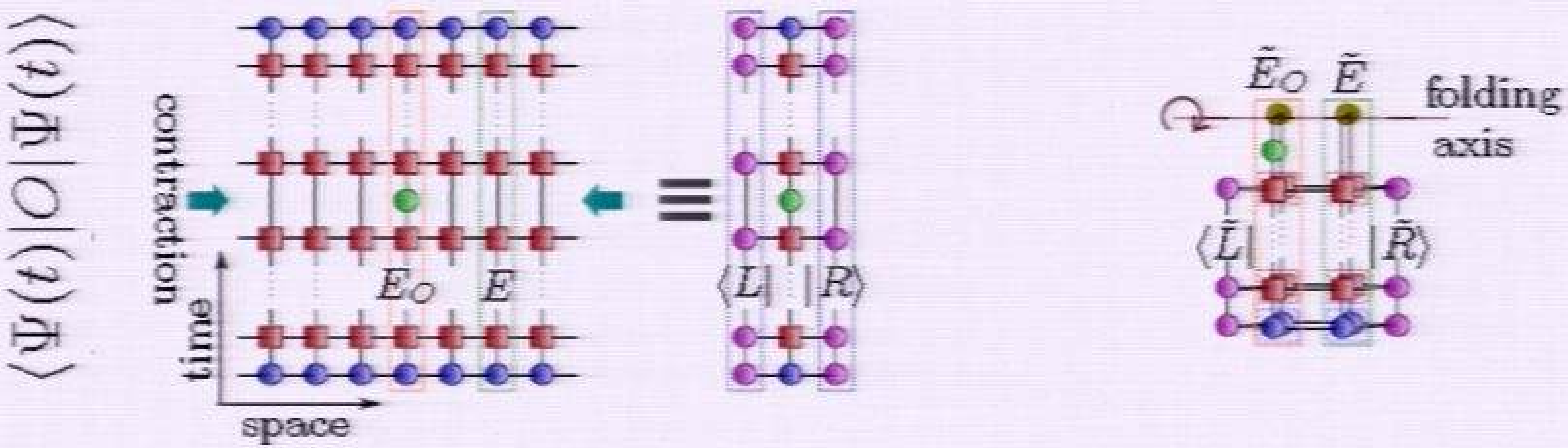
Other systems

Fermions
Critical
Continuous

ALGORITHMS

MPS: time evolution

Bañuls, Hastings, Verstraete, Cirac, Phys. Rev. Lett. 102, 240603 (2009)



One can reach longer times for the time evolution, and thus study other processes.

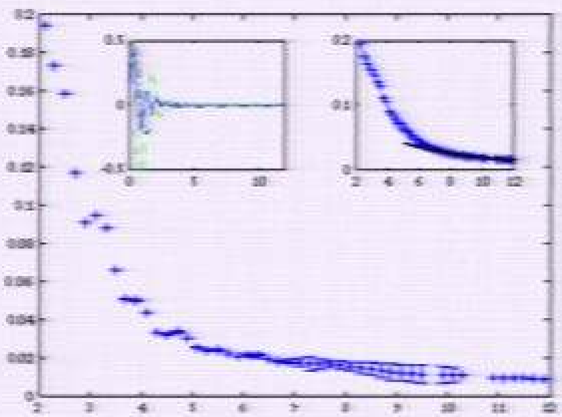
ALGORITHMS

Thermalization:

Bañuls, Hastings, Cirac, submitted

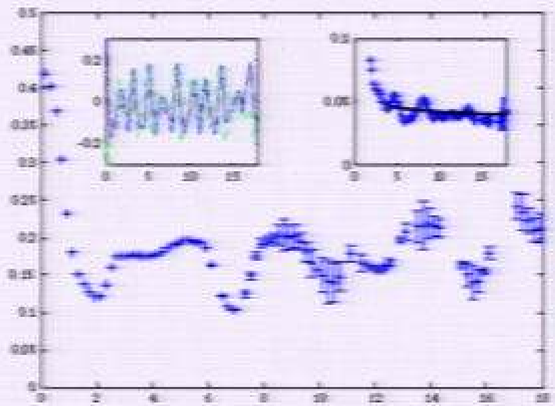
$$H = -\sum_i \sigma_z^{[i]} \otimes \sigma_z^{[i+1]} - g \sum_i \sigma_x^{[i]} - h \sum_i \sigma_z^{[i]}.$$

Strong thermalization



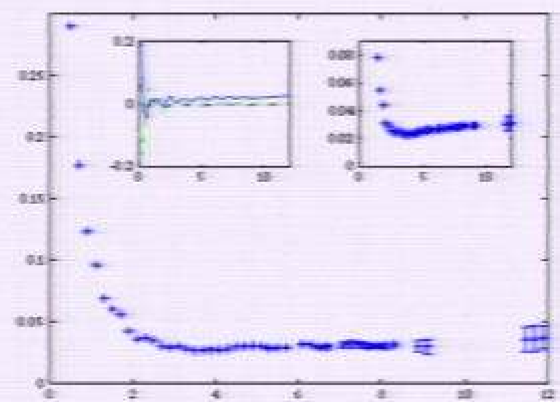
$|0\rangle + i|1\rangle$

Weak thermalization



$|1\rangle$

No thermalization



$|0\rangle + |1\rangle$

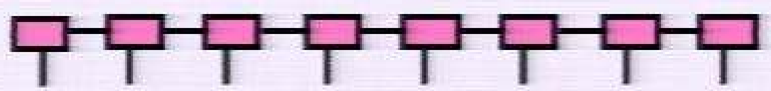
CONVERGENCE

Schuch and Cirac, arXiv:0910.4264
Aharanov, Ahad, Irani, arXiv:0910.5055

MPS as ground states:

- Given a 1D short-range Hamiltonian for N spins of dimension d.

  Find the MPS with bond dimension D which minimizes energy within some prescribed error, ε.



- Result: $t \propto (N^{2d} / \varepsilon)^{6D^2}$

  NP-hard:

  J. Eisert, Phys. Rev. Lett. **97**, 260501

  Schuch N Cirac J I and Verstraete F 2008 *Phys. Rev. Lett.* **100** 250501

CONVERGENCE

Schuch and Cirac, arXiv:0910.4264
Aharanov, Ahad, Irani, arXiv:0910.5055

MPS as ground states:

- Idea: dynamical programming
- Consequences:
  - Energy density: $|e(\infty) - e(N)| = \mathrm{poly}(1/N)$
  - Gapped Hamiltonians: $D = \mathrm{poly}(\log(N/\delta))$
  - To obtain accuracy δ:

$$t \propto (1/\delta)^{\mathrm{poly}[\log(1/\delta)]}$$

OTHER SYSTEMS

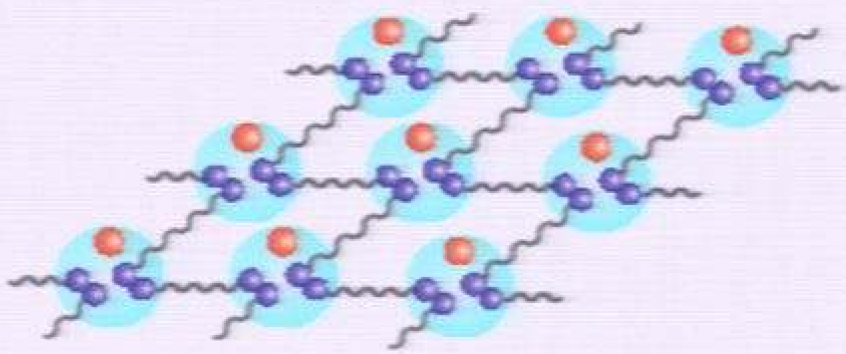
Fermionic PEPS:

Kraus, Schuch, Verstraete, Cirac, arXiv:0904.4667

Fermionic MERA:

Corboz, Evenbly, Verstraete, Vidal, arXiv:0904.4151
Pineda, Barthel, Eisert, arXiv:0905.0669

- Idea:

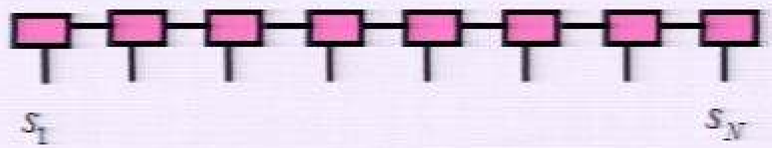


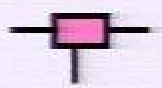
- We also give a Hamiltonian for which an fPEPS is ground state.

OTHER SYSTEMS

Infinite MPS:

Cirac and Sierra, arXiv: 0911.3029





$$V_s(\alpha, z_n) = :e^{i\sqrt{\alpha} s \varphi(z_n)}:$$

vertex operator

$\phi(z)$ chiral free boson field

z: complex number

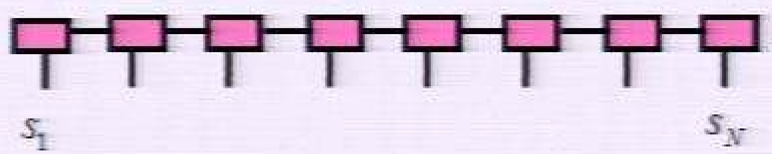
$$c_{s_1 .. s_N}(\alpha, z_1, \ldots, z_N) = \langle V_{s_1}(\alpha, z_1) \ldots V_{s_N}(\alpha, z_N) \rangle_{\mathrm{vac}}$$

- Critical systems.
- Exact ground state of inhomogeneous Haldane-Shastry

OTHER SYSTEMS

Continuous MPS:

Verstraete and Cirac, arXiv: 1002.1824

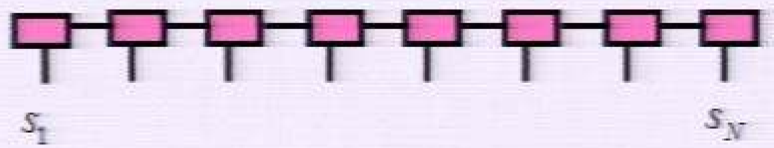


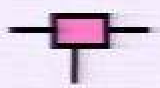
$$|\chi\rangle = \mathrm{Tr}_{aux}\left[\mathcal{P}e^{\int_0^L dx[Q(x)\otimes\mathbb{1}+R(x)\otimes\hat{\psi}^\dagger(x)]}\right]|\Omega\rangle$$

OTHER SYSTEMS

Infinite MPS:

Cirac and Sierra, arXiv: 0911.3029





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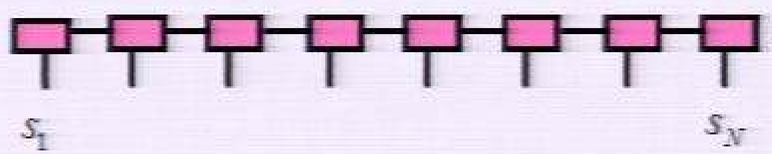

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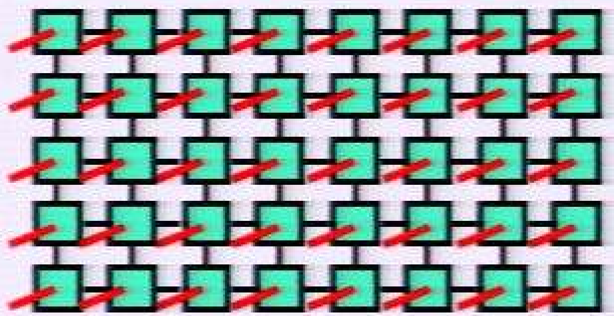
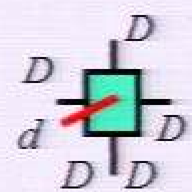
PHYSICAL PROPERTIES

Translationally invariant (on a torus):



$A^i_{\alpha\beta\gamma\delta}$



All physical properties are encapsulated in this tensor

Local symmetries:

$$|\Psi\rangle \rightarrow u^{\otimes N} |\Psi\rangle = e^{iN\theta} |\Psi\rangle \Longrightarrow A^i_{\alpha\beta\gamma\delta} \rightarrow ?$$

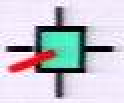
Global symmetries:

Topological properties:

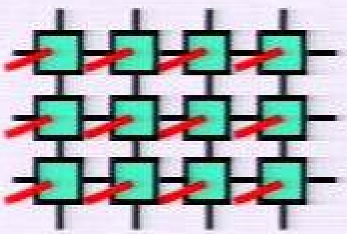
$$\Longrightarrow A^i_{\alpha\beta\gamma\delta} ?$$

PHYSICAL PROPERTIES

Injectivity:



For $d < D^4$, $\{A^1_{\alpha\beta\gamma\delta}, \ldots, A^d_{\alpha\beta\gamma\delta}\}$ does not span $T_{DxDxDxD}$



Typically, it will

Applications:

1D: injective iff unique ground states of local FF Hamiltonian

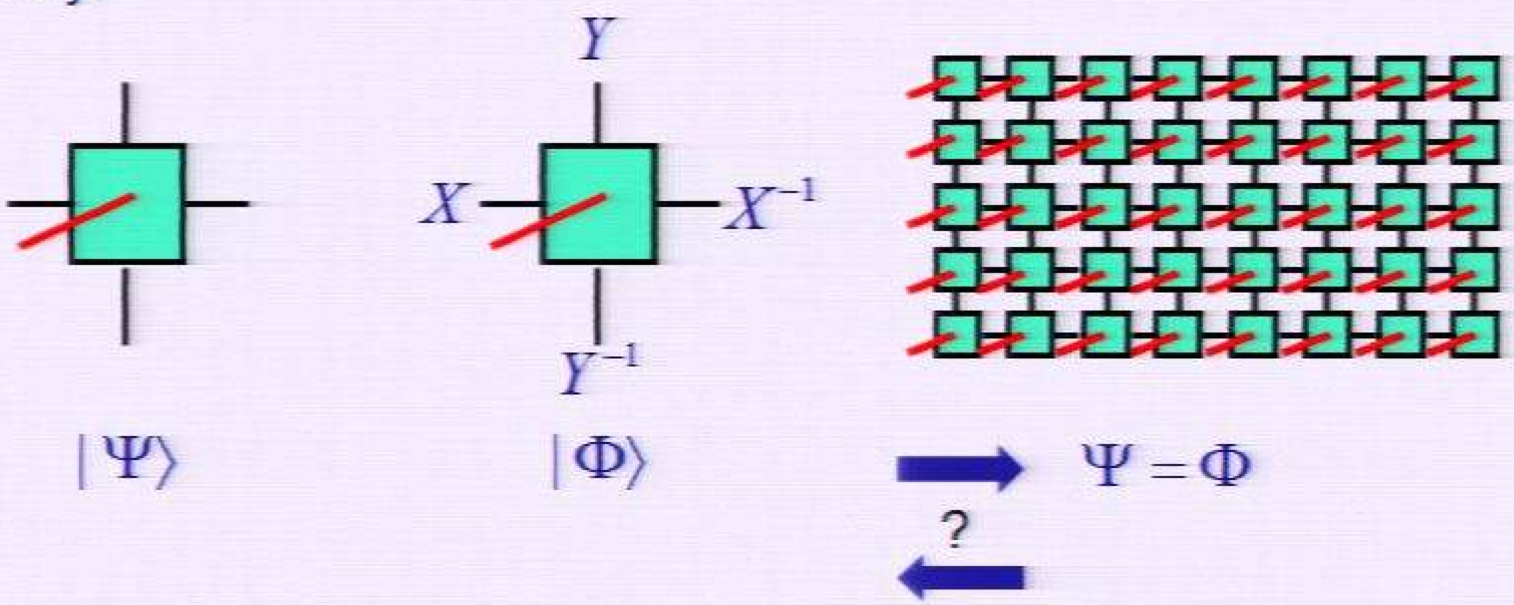
Fannes M Nachtergaele B and Werner R F 1992 *Comm. Math. Phys.* **144** 443

2D: similar

D. Pérez-García, F. Verstraete, J.I. Cirac and M.M. Wolf, Quant. Inf. Comp. 8, 0650-0663 (2008).

PHYSICAL PROPERTIES

Gauge symmetry:



1D: Pérez-Garcia, Verstraete, Cirac, Wolf, Q. Info. Comp. 7 401 (2007)

2D: Pérez-Garcia, Sanz, González, Wolf, Cirac, arXiv:0908.1674

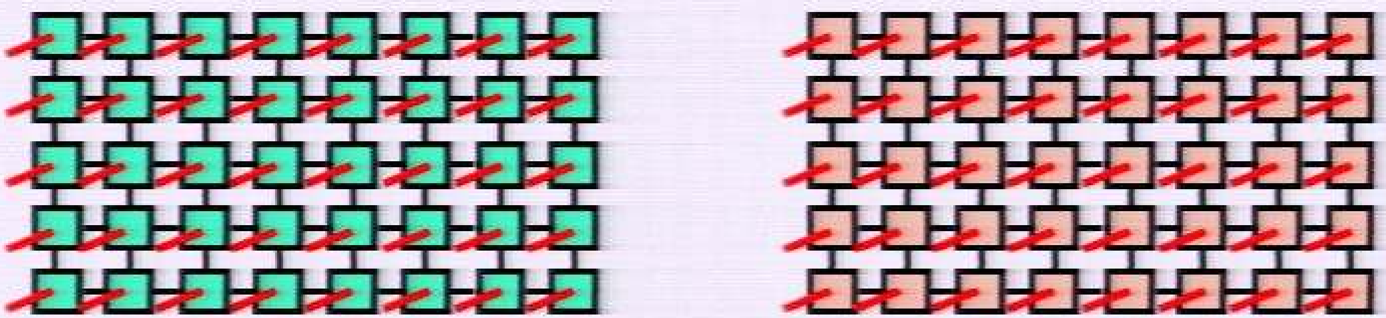
Applications:

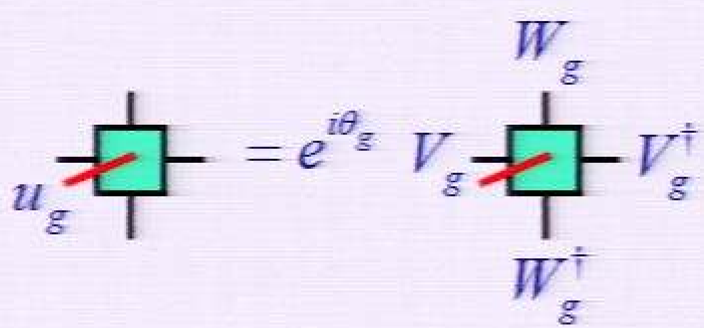
1. Standard form
2. Symmetries

PHYSICAL PROPERTIES

Local symmetries:

$$u^{\otimes N} \,|\Psi\rangle = e^{iN\theta} \,|\Psi\rangle$$



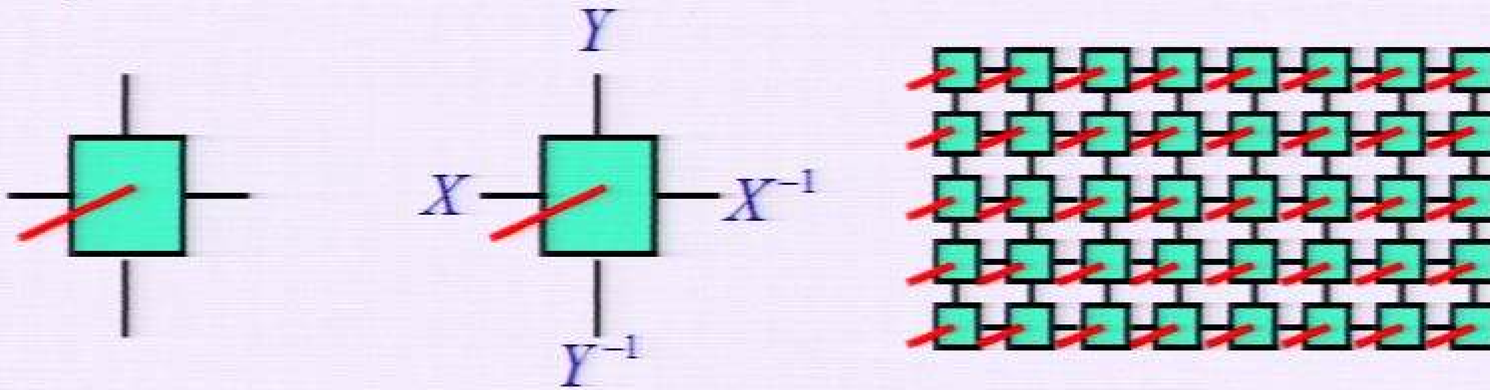


Representations of the same group, G

PHYSICAL PROPERTIES

MPQ

Gauge symmetry:



$|\Psi\rangle$ $|\Phi\rangle$

$\Longrightarrow$ $\Psi = \Phi$

?

$\Longleftarrow$

1D: Pérez-Garcia, Verstraete, Cirac, Wolf, Q. Info. Comp. 7 401 (2007)

2D: Pérez-Garcia, Sanz, González, Wolf, Cirac, arXiv:0908.1674

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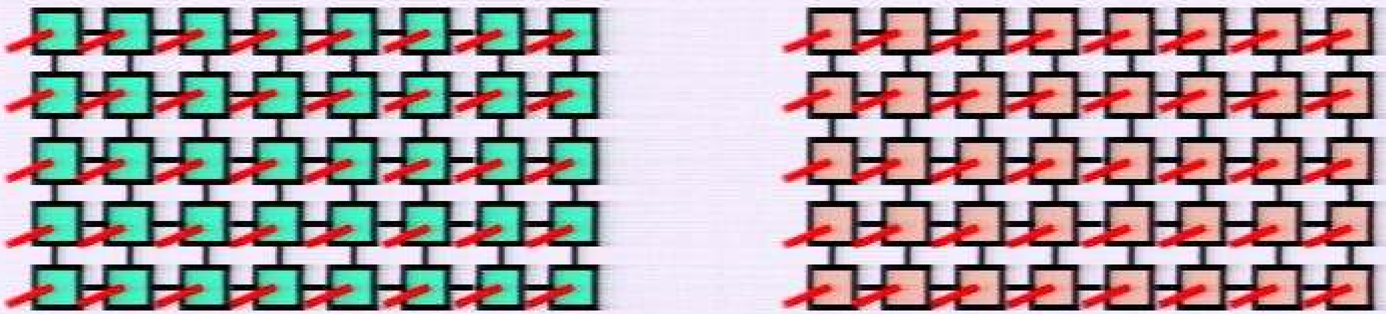
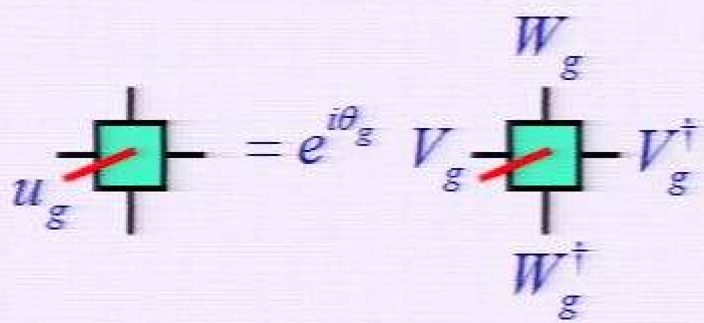




Representations of the same group, G

PHYSICAL PROPERTIES

Applications: Lieb-Schultz-Mattis like theorems:

1D: E. Lieb, T. Schultz and D. Mattis, Ann. Phys. **16**, 4 (1961).

M. Oshikawa, M. Yamanaka and I. Affleck, Phys. Rev. Lett. **78**, 1984 (1997).

2D: M.B. Hastings, Phys.Rev. B69 (2004) 104431;

- If a PEPS, representing spins J, is injective and invariant under U(1), then J-m is an integer, where m is the magnetization per particle.
- If the state is invariant under SU(2) and J is semi-integer, then It cannot be injective.

Pérez-Garcia, Sanz, González, Wolf, Cirac, arXiv:0908.1674

- Other applications: string orders.

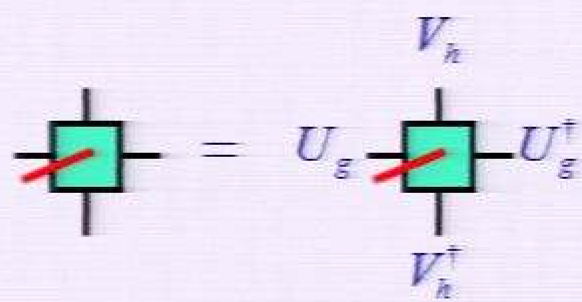
D. Perez-Garcia, M. Wolf, M. Sanz, F. Verstraete, and J. Cirac, Phys. Rev. Lett. **100**, 167202 (2008), arXiv:0802.0447.

- Symmetries in numerical algorithms

S. Singh, H. Q. Zhou, and G. Vidal, (2007), cond-mat/0701427.

PHYSICAL PROPERTIES

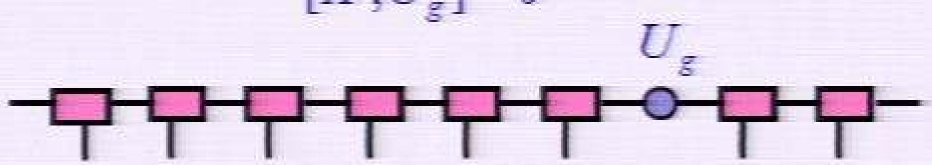
Not injective (degeneracy):



$g,h \in G$

- For non-trivial representation, we have degeneracy.
- 1D: complete characterization of non-injective MPS.

$[A^i, U_g] = 0$

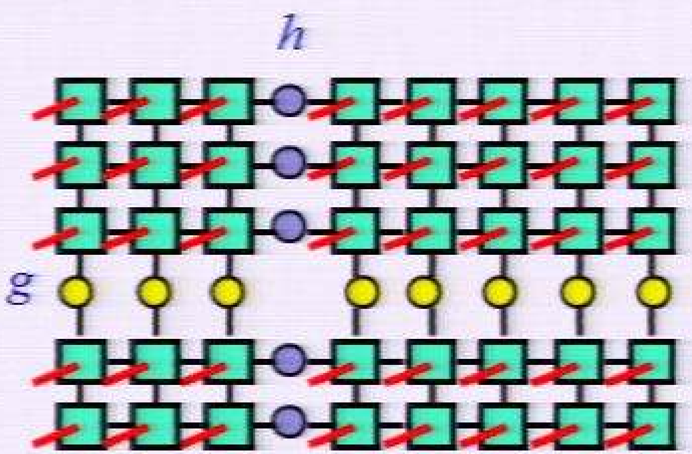


degeneracy = # Conjugacy classes

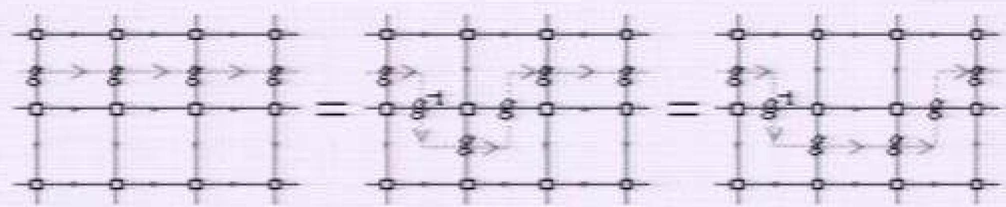
PHYSICAL PROPERTIES

Not injective (degeneracy):

2D: Partial caracterization of non-injective PEPS
Degeneracy = Number of pair-conjugacy classes of G



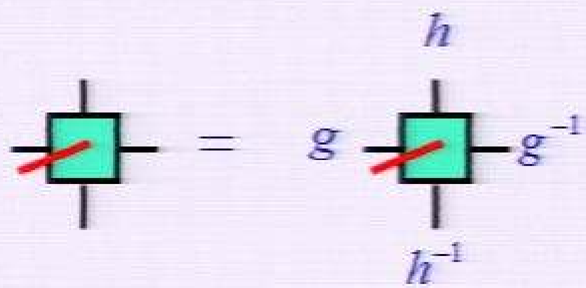
$[h, g] = 0$



- A's are isometries: we have „access to the virtual particles"

PHYSICAL PROPERTIES

Topology:



$g,h \in G$

If A is also an isometry and U the (left) regula represeantation:

- Ground states are locally indistinguishable.
- Wilson loops: moving in the ground state subspace.
- Topological entropy.
- Commuting parent Hamiltonian.
- Fixed points of a renormalization group procedure.
- Anyonic excitations (abelian or non-abelian depending on G)

PHYSICAL PROPERTIES

Topology:

Other models:

- Hopf algebras, tensor categories

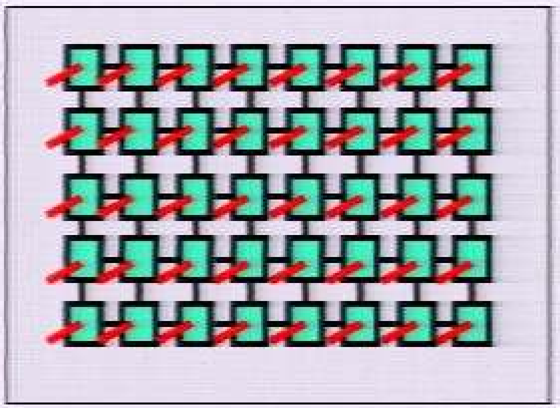
  M. A. Levin and X.-G. Wen, Phys.Rev. B **71**, 045110 (2005)
  Aguado, Burschaper, Christiandl, private comm.

- Ferminic PEPS
- Continous PEPS

SUMMARY

Numerical algorithms

M.C. Banuls (MPQ)
M. Hastings (UCSB)
F. Verstraete (Vienna)



Convergence

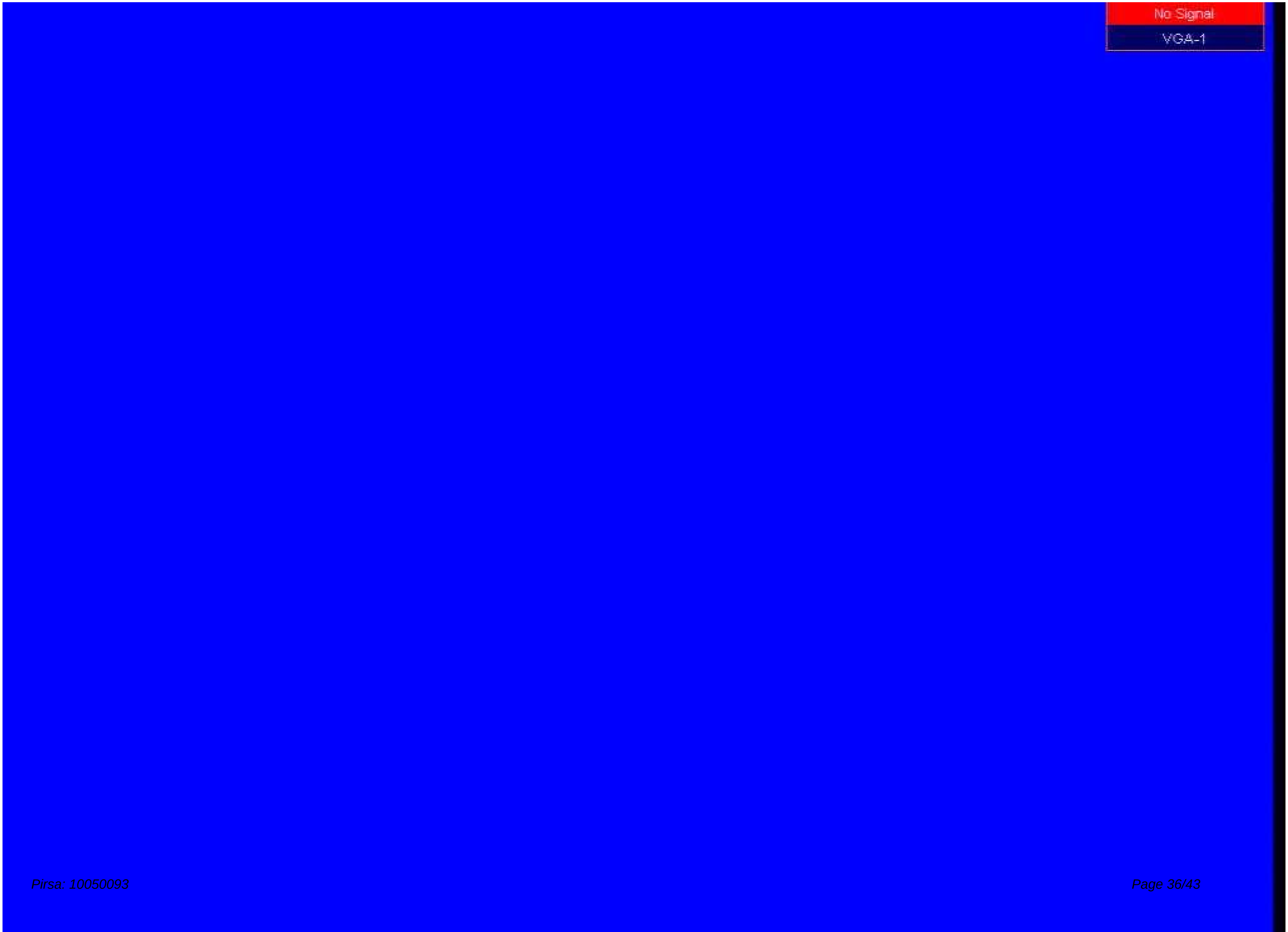
N. Schuch (CALTECH)

Physical properties

D. Perez-Garcia (Madrid)
N. Schuch (CALTECH)
M. Wolf (NBI)
M. Sanz (MPQ)
F. Verstraete (Vienna)

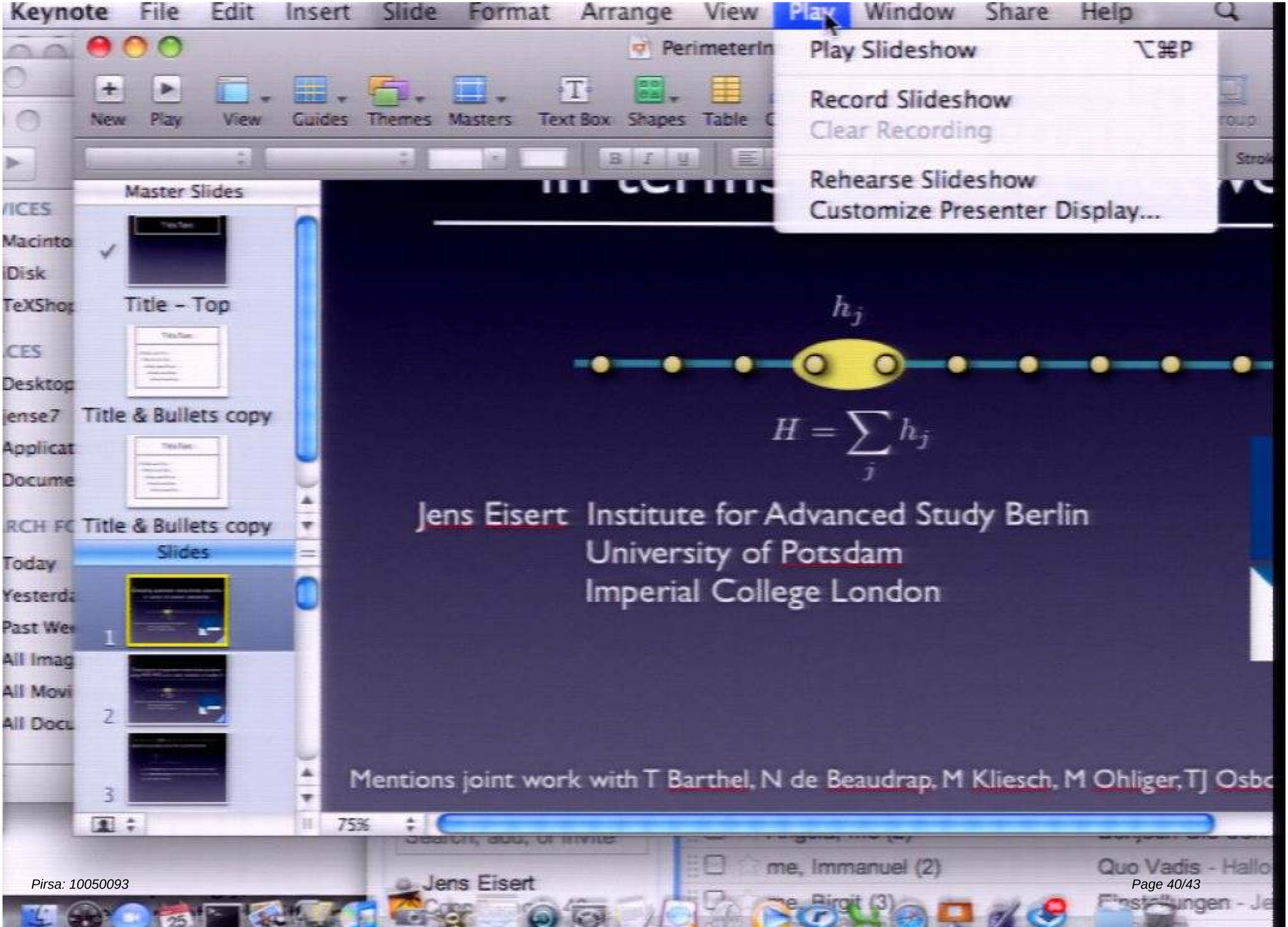
Other systems

F. Verstraete (Vienna)
C. Kraus (MPQ)
N. Schuch (CALTECH)
G. Sierra (Madrid)


No Signal
VGA-1

[redacted]

No Signal
VGA-1


Keynote File Edit Insert Slide Format Arrange View Play Window Share Help
Play Slideshow ⌥⌘P
Record Slideshow
Clear Recording
Rehearse Slideshow
Customize Presenter Display...
New Play View Guides Themes Masters Text Box Shapes Table
Master Slides
Title – Top
Title & Bullets copy
Title & Bullets copy
Slides
h_j
$H = \sum_j h_j$
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University of Potsdam
Imperial College London
Mentions joint work with T Barthel, N de Beaudrap, M Kliesch, M Ohliger, TJ Osbo
75%

Grasping quantum many-body systems in terms of tensor networks

h_j

$$H = \sum_j h_j$$

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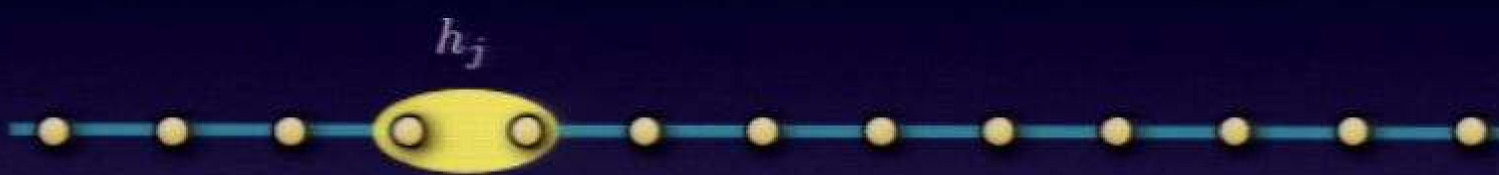
University of Potsdam
Imperial College London

tions joint work with T Barthel, N de Beaudrap, M Kliesch, M Ohliger, TJ Osborne, C Pineda, F Verstraete

Description of quantum many-body systems sing MPS, PEPS, and other families of states II



$$H = \sum_j h_j$$

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tions joint work with T Barthel, N de Beaudrap, M Kliesch, M Ohliger, TJ Osborne, C Pineda, F Verstraete

Quantum many-body system with a local Hamiltonian

$$H = \sum_j h_j$$

$$h_j = \mathbb{I}_{1,\ldots,j-1} \otimes h \otimes \mathbb{I}_{j+2,\ldots,n}$$

- Given **lattice** (chain, cubic lattice, triangular lattice, some graph)
- One **quantum degree** of freedom (spin, boson, fermion) per site
- Finite-ranged interaction