Title: An isotropic to anisotropic transition in a fractional quantum Hall state

Date: May 29, 2010 02:00 PM

URL: http://pirsa.org/10050090

Abstract: I describe a novel abelian gauge theory in 2+1 dimensions which has surprising theoretical and phenomenological features. The theory has a vanishing coefficient for the square of the electric field e_i^2, characteristic of a quantum critical point with dynamical critical exponent $z=2$, and a level-k Chern-Simons coupling, which is marginal at this critical point. For $k=0$, this theory is dual to a free $z=2$ scalar field theory describing a quantum Lifshitz transition, but $k \neq 0$ renders the scalar description non-local. The $k \neq 0$ theory exhibits properties intermediate between the (topological) pure Chern-Simons theory and the scalar theory. For instance, the Chern-Simons term does not make the gauge field massive. Nevertheless, there are chiral edge modes when the theory is placed on a space with boundary, and a non-trivial ground state degeneracy k^g when it is placed on a finite-size Riemann surface of genus g. The coefficient of e_i^2 is the only relevant coupling; it tunes the system through a quantum phase transition between an isotropic fractional quantum Hall state and an anisotropic fractional quantum Hall state. I describe zero-temperature transport coefficients in both phases and at the critical point, and comment briefly on the relevance of the results to recent experiments.

A Novel Abelian Gauge Theory and New Quantum Hall Transitions

Shamit Kachru (KITP & Stanford)

Based (in part) on arXiv : 1004.3570 written with:

Michael Mulligan (MIT)
Chetan Nayak (Microsoft Station Q / UCSB)

OUTLINE:

I. Introduction, with three motivations.

II. Some “topological” frippery.

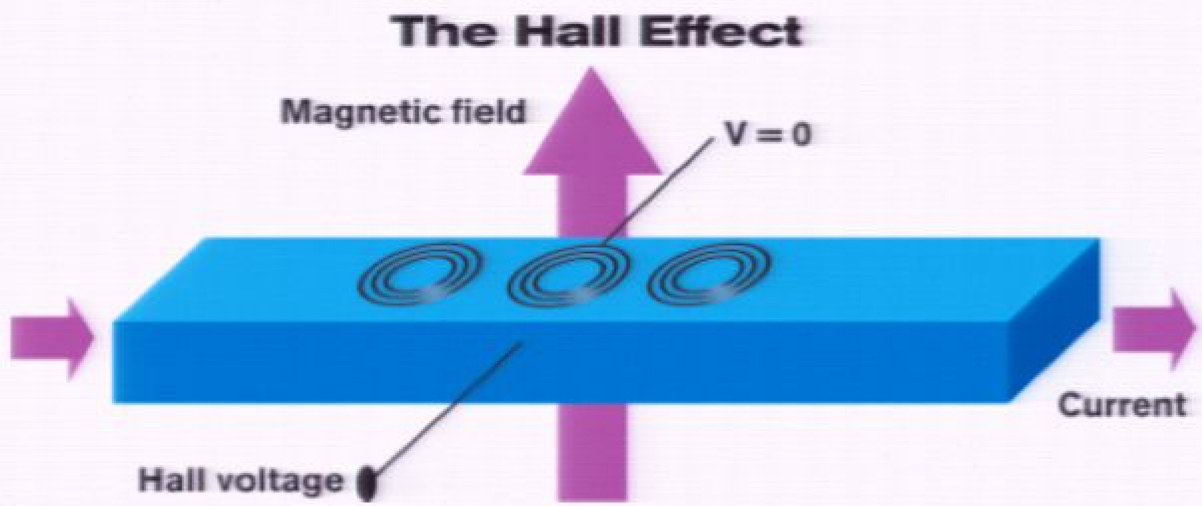
III. The anisotropic phase : basic facts.

IV. Conductivities, compressibilities, & all that.

V. Conclusion (where, inevitably, gravity duals are mentioned).

his talk has a few different motivations. Here are three:

I.



The physics of 2D electron gas in a (large) background magnetic field has been a rich, fruitful playground for both experimentalists and theorists. Famously, it is the home of the quantum Hall (and fractional quantum Hall) effect.

While if you are clever you can guess the ground state wavefunction for the system and derive much interesting physics in that way  , there is also a systematic

effective field theory approach to understanding such 2D electron gases.

Zhang, Hansson, Kivelson; Frohlich, Zee

1) The essential physics is 2+1 dimensional.

2) The electromagnetic current is conserved:

$$\partial_\mu J^\mu = 0$$

3) We are interested in the physics at long distances and large time (= small wave number, low frequency).

4) P and T are broken by the external magnetic field.

2+1 dimensions, we can write the conserved current as the curl of a vector potential:

$$J_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\rho}\partial_\nu a_\rho.$$

This vector enjoys a gauge symmetry under which the current remains invariant.

low, we apply standard logic: we expect the low-energy action to be governed by the lowest dimension operator(s) we can write down, consistent with the symmetries of the problem. The result is:

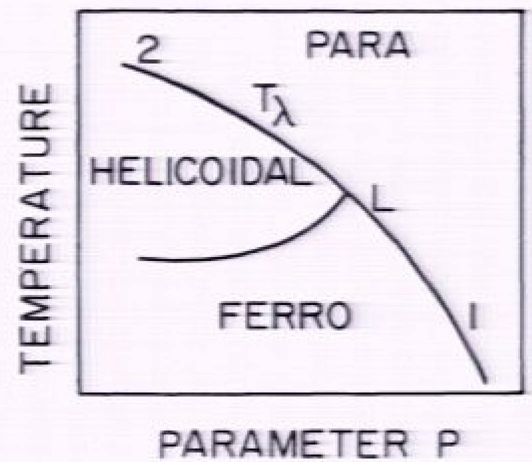
$$\mathcal{L} = \frac{k}{4\pi}\epsilon_{\mu\nu\rho}a_\mu\partial_\nu a_\rho + \frac{1}{e^2\Lambda}f_{\mu\nu}f^{\mu\nu} + \cdots$$

The theory is a Chern-Simons theory at low energies.

Using elementary reasoning starting from the abelian hern-Simons theory, the phenomenology of the simplest odd-denominator filling fractions in the FQHE can be well explained.

Natural question: The irrelevant Maxwell perturbation ere does not change the physics in the deep IR. A priori, one might wonder: are there other extensions of the Chern-Simons theory that we can imagine, that would ange observable properties in the IR? Might they arise in real systems?

2.



IR fixed points with non-trivial dynamical scaling

$$x \to \lambda x, \quad t \to \lambda^z t$$

re quite common in condensed matter systems (where
orentz invariance is not a particularly natural symmetry).

n especially simple theory, which arises at critical points
in many toy models of spin systems, is:

$$\mathcal{L} = \frac{1}{2}\int\ dt\ d^2x\ \left((\partial_t\phi)^2 - \kappa^2(\nabla^2\phi)^2\right)\ .$$

his is a free scalar field theory with a line of fixed points, l with dynamical exponent z=2. It can be dualized to an abelian gauge theory:

$$S = \frac{1}{g^2}\int dt\, d^2x \Big[e_i\partial_t a_i + a_t\partial_i e_i - H[e,a]\Big],$$

$$H[e,a] = \frac{\kappa^2}{2}(\partial_i e_j)^2 + \frac{1}{2}b^2,$$

$$b = \epsilon_{ij}\partial_i a_j\,.$$

Vith the gauge field scaling inversely to the coordinates, this also enjoys z=2 scaling, of course.

is theory has three potentially interesting perturbations. Two have been studied in the literature:

One can perturb by the more standard quadratic in the electric field (which is clearly relevant):

$$\Delta\mathcal{L} \sim e_i^2 .$$

One can perturb by a quartic in the electric field, which is naively marginal:

$$\Delta\mathcal{L} \sim (e_i^2)^2 \ .$$

Vishwanath, Balents, Senthil

/ith the sign of the perturbation that leaves the potential bounded below, this operator is marginally irrelevant.

* Finally, one can consider the theory with action:

$$S = \int dt\, d^2x \left[\frac{1}{g^2}\Big(e_i \partial_t a_i + a_t \partial_i e_i - H[e,a] \Big) + \frac{k}{4\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right].$$

; formally solving for E and reinserting (valid at non-zero momenta):

$$S = \frac{1}{2} \int dt\, d^2x \left[\frac{1}{g^2}\Big(\kappa^2 (\partial_i e_j)^2 - b^2 \Big) + \frac{k}{2\pi} \epsilon_{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right],$$

$$e_i = \left(\frac{1}{\kappa^2}\right) \frac{\partial_i a_t - \partial_t a_i}{\nabla^2}, \quad b = \partial_1 a_2 - \partial_2 a_1.$$

This might be called the abelian "Lifshitz-Chern-Simons" theory. The Chern-Simons term looks like a marginal perturbation" of the z=2 critical theory, and vice-versa.

is theory might be expected to arise if one combines the rcumstances that normally give rise to z=2 scaling, with P d T violating background fields. It cannot be mapped (by a local transformation) into a scalar theory.

enceforth, I will take the attitude that since this theory is simple extension of two different physical theories that ive played an important role in characterizing interesting phases of matter, and since it is eminently tractable, we should investigate its physics.

timately, the usefulness of this theory will be determined y whether its phase structure and transport properties match those of observed systems. In fact, our third motivation (arising, in part, as we finished!) comes from experiment.

3. Weird goings on at higher filling fraction

r some time, the Eisenstein and Tsui labs have been doing interesting experiments with 2D electron systems in lower magnetic fields, where higher Landau levels are occupied.

With B-fields and electron densities in the vicinity of:

$$B \sim 5T,\ n_s \sim 10^{11}\mathrm{cm}^{-2}$$

$$\nu = \frac{hn_s}{eB} = 9/2, 11/2, 13/2, 15/2$$

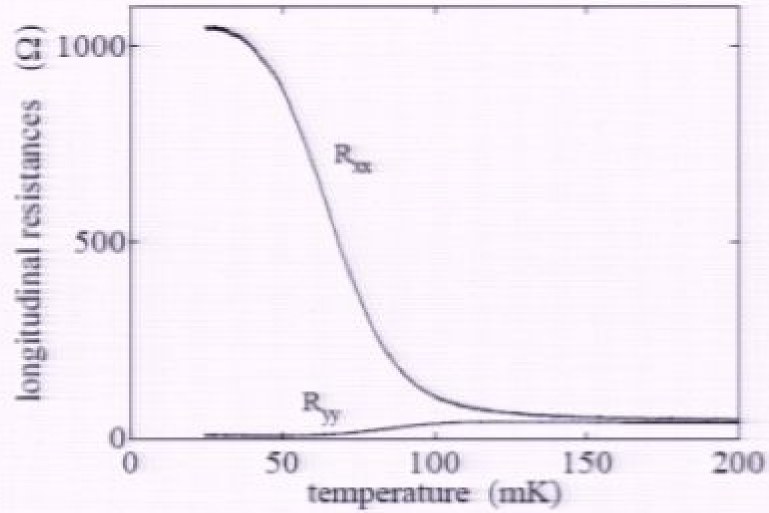
Lilly et al found evidence for new anisotropic states

$$\sigma_{xx}(T) \neq \sigma_{yy}(T)$$

$$\sigma_{xy} \text{ unquantized}$$

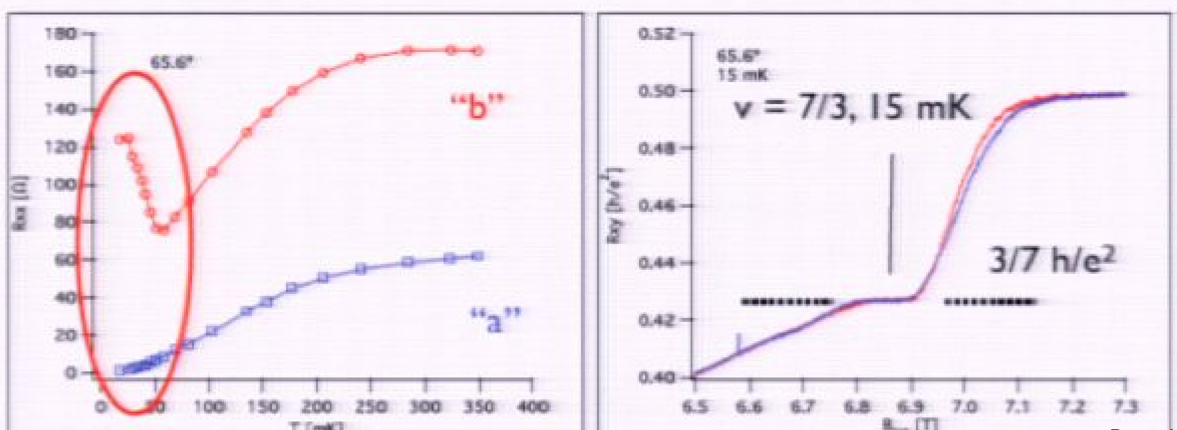
c.f. literature on Nematic phases, Fradkin et al, arXiv : 0910.4166

Lilly et al '99, at
filling fraction 9/2



Iore recently (March '10), Xia et al have given dramatic
evidence of a new phase (the "quantum Hall nematic")
ith anisotropic transport & quantized Hall conductance:

duced by in-plane B-field
along the red direction

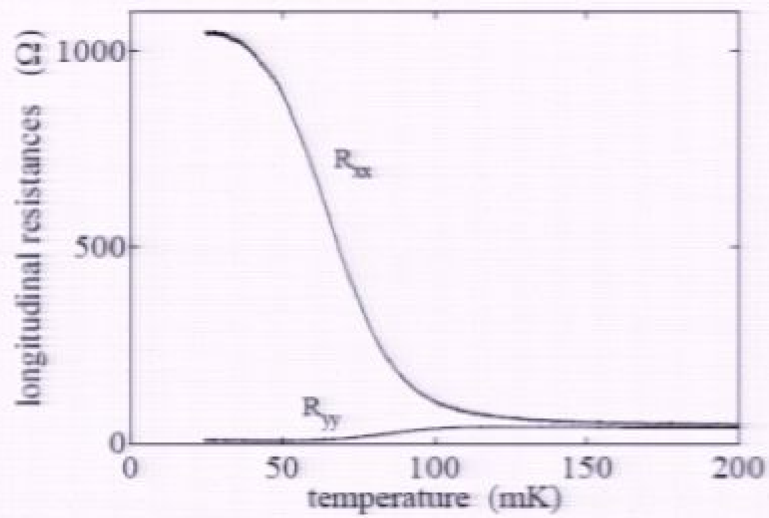


Since the "normal" quantum Hall phases are well xplained by Chern-Simons theory, and since the Lifshitz-hern-Simons theory has a single relevant e_i^2 perturbation that carries it between Maxwell-Chern-Simons and nisotropic phases (through a quantum critical point), it is at least plausible that it may play some role in the phenomena seen in these experiments.

n any case, hopefully our three motivations will convince you to tolerate a simple talk by a string theorist for the next 40 minutes.

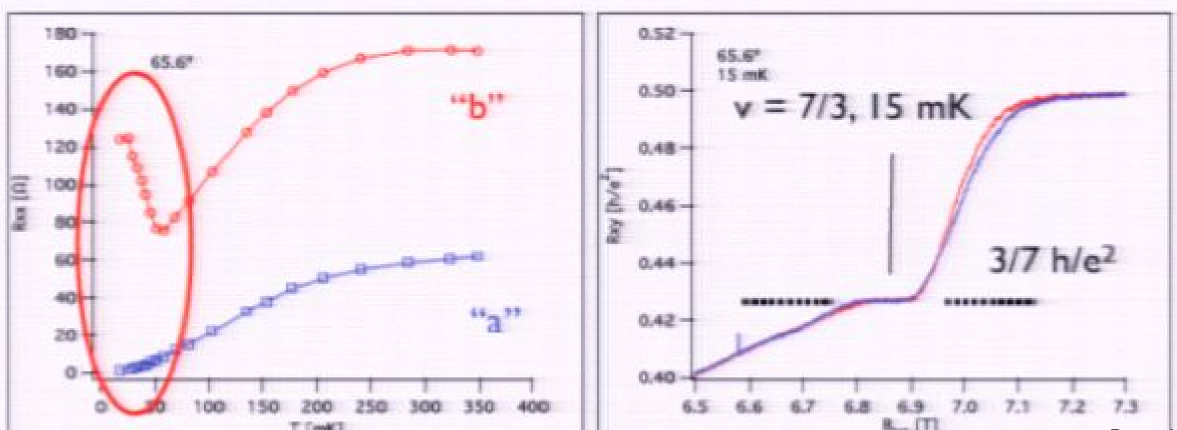
efore solving for the transport properties in the phases of the LCS theory, we begin by exploring some more formal theoretical issues.

Lilly et al '99, at filling fraction 9/2



ore recently (March '10), Xia et al have given dramatic evidence of a new phase (the "quantum Hall nematic") ith anisotropic transport & quantized Hall conductance:

duced by in-plane B-field along the red direction



Since the "normal" quantum Hall phases are well xplained by Chern-Simons theory, and since the Lifshitz-hern-Simons theory has a single relevant e_i^2 perturbation that carries it between Maxwell-Chern-Simons and nisotropic phases (through a quantum critical point), it is at least plausible that it may play some role in the phenomena seen in these experiments.

n any case, hopefully our three motivations will convince you to tolerate a simple talk by a string theorist for the next 40 minutes.

efore solving for the transport properties in the phases of the LCS theory, we begin by exploring some more formal theoretical issues.

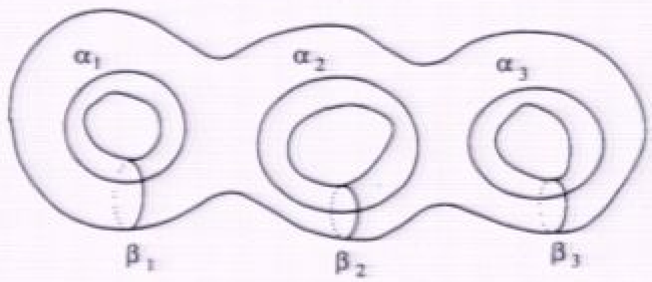
eresting "topological" properties of the (L)CS theory

The Chern-Simons theory is (famously) a topological field theory, whose physical observables are correlated with interesting mathematical features of the space on which it lives (e.g. the Jones polynomials). Witten

Studies of "topologically ordered" states in condensed matter (gapped models with degenerate ground states which are not distinguished by a simple Landau order parameter) have taken on a life of their own.

t Chern-Simons theory is still the canonical example, and its two most interesting properties in this regard are:

I: The existence of (almost) degenerate ground states, when the theory is formulated on a Riemann surface of genus g.



Wen;
Wen, Niu

In searching for ground states, one can restrict to zero-momentum modes of the gauge field (assuming the vacuum doesn't spontaneously break translation invariance).

In Maxwell-Chern-Simons theory, this yields a quantum mechanics problem with (on a torus):

$$S = \int dt \left[\frac{1}{2r\Lambda} (\partial_t a_i)^2 + k(a_1 \partial_t a_2 - a_2 \partial_t a_1) \right]$$

In the case of e.g. the torus, we can reduce this to a problem involving the two Wilson-line zero modes.

hese Wilson lines themselves live on a (dual) torus, and are governed by the action:

$$S = \int dt \ k\pi(\dot{x}y - \dot{y}x) + \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$$

vhere the mass term is generated by the Maxwell term).

is Lagrangian describes a charged particle moving on the ual torus, in the presence of a magnetic field of order k. he quantum mechanics of such a particle was studied by Haldane and Rezayi.

With

$$H = \frac{1}{2m}\left(-(\partial_x - ia_x)^2 - (\partial_y - ia_y)^2\right)$$

$$a_x = 0, a_y = Bx = 2\pi k x$$

hey find that the ground state is k-fold degenerate. The wave functions of the ground states are given by:

$$\Psi_l(x,y) = \left(\sum_n e^{2\pi(x+iy)(nk+l) - \frac{(nk+l)^2}{k}\pi}\right) e^{-2\pi^2 B x^2}, \quad l = 0, 1, ..., k-1$$

(This thing is a theta function)

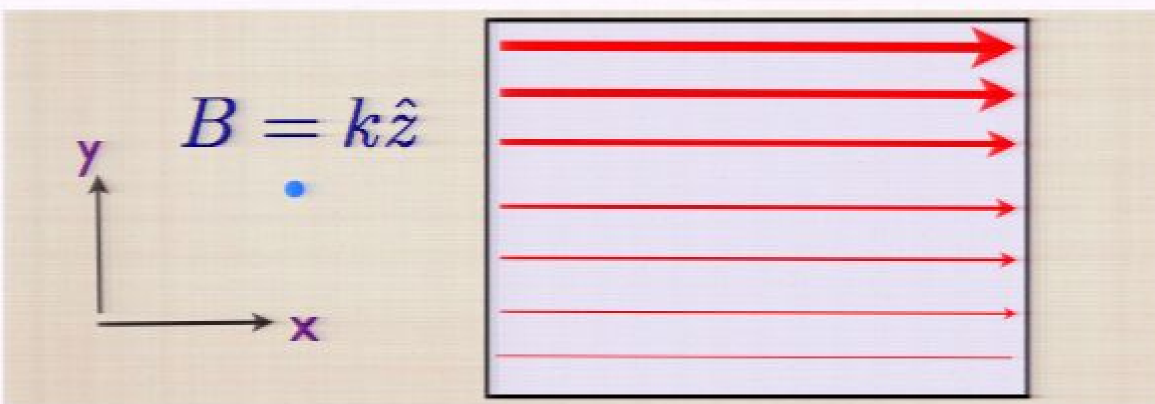
This result isn't so fancy. The system separates into Landau levels, spaced by the cutoff. The wavefunctions are roughly plane waves in one direction

$$\sim e^{ip_x x}$$

and harmonic oscillator modes

$$\sim e^{-k(y-p_x/k)^2}$$

centered about the line $y = p_x/k$ in the other.



Only a certain number of momentum modes fit on the compact space, yielding k modes for each value of the harmonic oscilltor principal quantum number. The same analysis generalizes to higher genus, with theta functions again playing a starring role (but the basic physics being that of Landau levels).

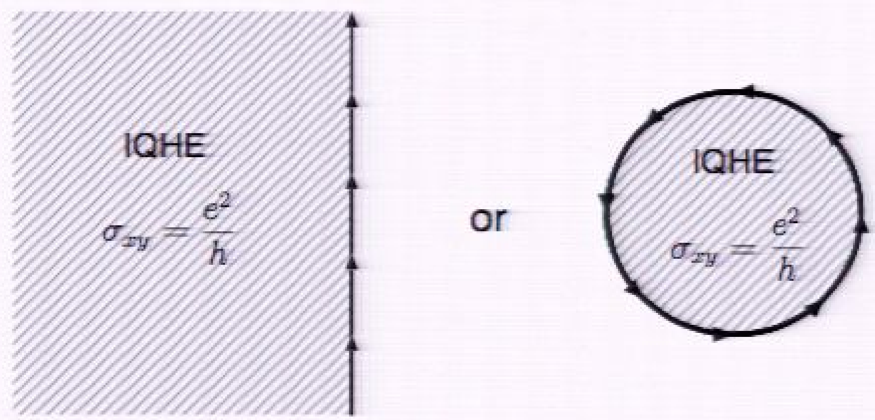
$$\text{Result}: k^g \text{ degenerate ground states}$$

More precisely, on a system with finite size L, the splitting of these ground states goes like

$$\Delta E \sim \mathrm{Exp}(-Lm)$$

where m is related to the gap and the mass of the lightest quasiparticle excitations.

: The existence of chiral edge modes, when the theory is placed on a space with boundary:



is is more or less self-explanatory. The derivation of the istence of the edge modes most elegantly proceeds from anomaly considerations:

Vhen formulated on a space with boundary, the gauge-variation of the Chern-Simons action yields:

$$\delta A = \partial f \rightarrow$$

$$\delta S \sim \int_{\partial \Sigma_3} f \wedge F \ .$$

For the level k theory, this lack of gauge invariance can fixed by adding a boundary chiral boson (which for k=1 can also be described as a chiral fermion):

$$S = \frac{k}{4\pi} \int dt dx \ \partial_x \phi (i \partial_t - \partial_x) \phi .$$

Such "edge modes" are characteristic of topological ases, and in fact play a central role in some recent efforts to classify topological insulators. Ryu, Schnyder, Furusaki, Ludwig; Kitaev

oth the chiral edge modes and the degenerate ground ates of these topologically ordered systems, have been implicitly assumed to be closely related to the gapped nature of the bulk theory.

One of the interesting aspects of the abelian Lifshitz-Chern-Simons theory is that it exhibits both the chiral edge modes and the ground-state degeneracy, while manifesting a gapless critical theory in the bulk. (This in contrast to Maxwell-Chern-Simons, which is massive).

Deser, Jackiw, Templeton

The persistence of the edge modes is clear.

Their presence is still required to cancel a non-vanishing auge variation when the LCS theory is placed on a space with boundary.

Perturbation theory in the Lifshitz gauge couplings could in principle perturb the Lagrangian for the chiral edge modes, but this purely chiral theory has no relevant erturbations. It is thus robust under the addition of the Lifshitz terms, with small gauge couplings.

The ground state degeneracy persists for the following intuitive reason. Let's consider the theory with a regulating (marginally irrelevant) perturbation:

$$S = \int dt \, \left(e_i \partial_t a_i - \lambda (e_i^2)^2 + \frac{k}{4\pi}(a_2 \partial_t a_1 - a_1 \partial_t a_2) \right)$$

The associated Hamiltonian takes the form:

$$H = \frac{\lambda}{L^2} \Big(\sum_i (\Pi_i - \frac{k}{4\pi} \epsilon_{ij} a_j)^2 \Big)^2$$

(where L is the length scale of the torus). This is the square of the old Hamiltonian! The degeneracies are e same, but the spacing between levels is now quadratic.

When formulated on a space with boundary, the gauge-variation of the Chern-Simons action yields:

$$\delta A = \partial f \to$$

$$\delta S \sim \int_{\partial\Sigma_3} f \wedge F \ .$$

For the level k theory, this lack of gauge invariance can be fixed by adding a boundary chiral boson (which for k=1 can also be described as a chiral fermion):

$$S = \frac{k}{4\pi} \int dtdx \ \partial_x \phi (i\partial_t - \partial_x)\phi.$$

Only a certain number of momentum modes fit on the compact space, yielding k modes for each value of the armonic oscilltor principal quantum number. The same nalysis generalizes to higher genus, with theta functions ain playing a starring role (but the basic physics being that of Landau levels).

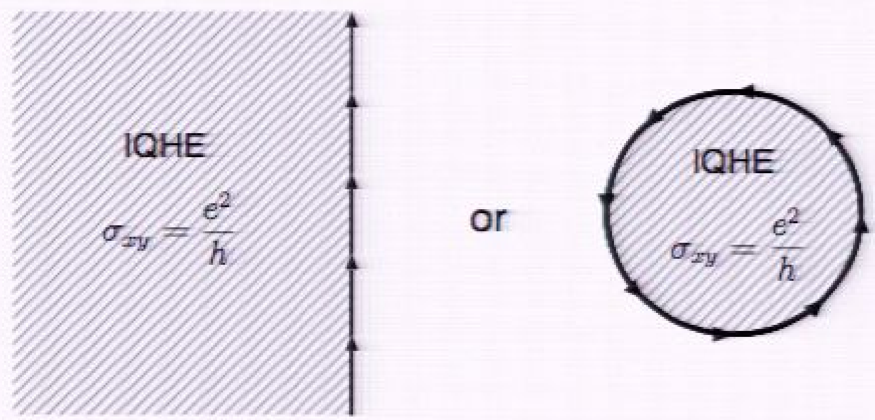
$$\text{Result}: k^g \text{ degenerate ground states}$$

lore precisely, on a system with finite size L, the splitting of these ground states goes like

$$\Delta E \sim \mathrm{Exp}(-Lm)$$

here m is related to the gap and the mass of the lightest quasiparticle excitations.

: The existence of chiral edge modes, when the theory is placed on a space with boundary:



is is more or less self-explanatory. The derivation of the istence of the edge modes most elegantly proceeds from anomaly considerations:

Only a certain number of momentum modes fit on the compact space, yielding k modes for each value of the armonic oscilltor principal quantum number. The same nalysis generalizes to higher genus, with theta functions ain playing a starring role (but the basic physics being that of Landau levels).

$$\text{Result} : k^g \text{ degenerate ground states}$$

lore precisely, on a system with finite size L, the splitting of these ground states goes like

$$\Delta E \sim \mathrm{Exp}(-Lm)$$

here m is related to the gap and the mass of the lightest quasiparticle excitations.

When formulated on a space with boundary, the gauge-variation of the Chern-Simons action yields:

$$\delta A = \partial f \rightarrow$$

$$\delta S \sim \int_{\partial \Sigma_3} f \wedge F \ .$$

For the level k theory, this lack of gauge invariance can be fixed by adding a boundary chiral boson (which for k=1 can also be described as a chiral fermion):

$$S = \frac{k}{4\pi} \int dt dx \ \partial_x \phi (i \partial_t - \partial_x) \phi.$$

The persistence of the edge modes is clear.

Their presence is still required to cancel a non-vanishing uge variation when the LCS theory is placed on a space with boundary.

Perturbation theory in the Lifshitz gauge couplings could in principle perturb the Lagrangian for the chiral edge modes, but this purely chiral theory has no relevant erturbations. It is thus robust under the addition of the Lifshitz terms, with small gauge couplings.

The persistence of the edge modes is clear.

Their presence is still required to cancel a non-vanishing uge variation when the LCS theory is placed on a space with boundary.

Perturbation theory in the Lifshitz gauge couplings could in principle perturb the Lagrangian for the chiral edge modes, but this purely chiral theory has no relevant erturbations. It is thus robust under the addition of the Lifshitz terms, with small gauge couplings.

The ground state degeneracy persists for the following intuitive reason. Let's consider the theory with a regulating (marginally irrelevant) perturbation:

$$S = \int dt \, \left(e_i \partial_t a_i - \lambda (e_i^2)^2 + \frac{k}{4\pi}(a_2 \partial_t a_1 - a_1 \partial_t a_2) \right)$$

The associated Hamiltonian takes the form:

$$H = \frac{\lambda}{L^2}\left(\sum_i (\Pi_i - \frac{k}{4\pi}\epsilon_{ij} a_j)^2\right)^2$$

(where L is the length scale of the torus). This is the square of the old Hamiltonian! The degeneracies are the same, but the spacing between levels is now quadratic.

The Anisotropic Phase

n the phase diagram of

$$S = \int dt\, d^2x \left[\frac{1}{g^2} \Big(e_i \partial_t a_i + a_t \partial_i e_i - \frac{r}{2} e_i^2 - \frac{\kappa^2}{2} (\partial_i e_j)^2 - \frac{\lambda}{4} (e_i^2)^2 - \frac{1}{2} b^2 \Big) + \frac{k}{4\pi} \epsilon_{\mu\nu\rho} a_\mu \partial_\nu a_\rho \right]$$

here are three regions:

$$r > 0 \;:\; \text{Maxwell} - \text{Chern} - \text{Simons theory}$$

$$r = 0 \;:\; \text{Critical theory (LCS fixed point)}$$

$$r < 0 \;:\; \text{Anisotropic phase}$$

he latter two are new and interesting. Lets first discuss e third, then compute transport coefficients in all three.

Clearly for r < 0, the electric field will develop an expectation value, breaking SO(2) rotational symmetry:

$$g^2 H[e,a] = -\frac{|r|}{2} e_i^2 + \frac{\kappa^2}{2}(\partial_i e_j)^2 + \frac{\lambda}{4}(e_i^2)^2 + b^2$$

$$e_i^2 = \frac{|r|}{\lambda},\ a_i = 0.$$

Without loss of generality, we are free to choose:

$$e_i = (\sqrt{|r|/\lambda} + \tilde{e}_x, e_y)$$

Re-expanding, we find:

$$S = \int dt\, d^2x \Big[\frac{1}{g^2} \Big(\sqrt{|r|/\lambda}\, \partial_t a_x + \tilde{e}_x \partial_t a_x - |r| \tilde{e}_x^2 - (\partial_y a_x - \partial_x a_y)^2 + a_t \partial_i e_i + e_y \partial_t a_y - \frac{\kappa^2}{2} (\partial_i e_y)^2 \Big) + \frac{k}{4\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \Big].$$

The Anisotropic Phase

n the phase diagram of

$$S=\int dt\,d^2x\left[\frac{1}{g^2}\left(e_i\partial_t a_i+a_t\partial_i e_i-\frac{r}{2}e_i^2-\frac{\kappa^2}{2}(\partial_i e_j)^2-\frac{\lambda}{4}(e_i^2)^2-\frac{1}{2}b^2\right)+\frac{k}{4\pi}\epsilon_{\mu\nu\rho}a_\mu\partial_\nu a_\rho\right]$$

there are three regions:

$$r>0\ :\ \text{Maxwell} - \text{Chern} - \text{Simons theory}$$

$$r=0\ :\ \text{Critical theory (LCS fixed point)}$$

$$r<0\ :\ \text{Anisotropic phase}$$

The latter two are new and interesting. Lets first discuss e third, then compute transport coefficients in all three.

Clearly for r < 0, the electric field will develop an expectation value, breaking SO(2) rotational symmetry:

$$g^2 H[e,a] = -\frac{|r|}{2} e_i^2 + \frac{\kappa^2}{2}(\partial_i e_j)^2 + \frac{\lambda}{4}(e_i^2)^2 + b^2$$

$$e_i^2 = \frac{|r|}{\lambda},\ a_i = 0.$$

Without loss of generality, we are free to choose:

$$e_i = (\sqrt{|r|/\lambda} + \tilde{e}_x, e_y)$$

Re-expanding, we find:

$$S = \int dt\, d^2x \left[\frac{1}{g^2} \left(\sqrt{|r|/\lambda}\, \partial_t a_x + \tilde{e}_x \partial_t a_x - |r| \tilde{e}_x^2 - (\partial_y a_x - \partial_x a_y)^2 + a_t \partial_i e_i \right.\right.$$
$$\left.\left. + e_y \partial_t a_y - \frac{\kappa^2}{2}(\partial_i e_y)^2 \right) + \frac{k}{4\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right].$$

The low energy theory is gapless (due to the Goldstone boson) and has z=1. The propagators are given by:

$$\langle e_x(-i\omega_n,-\mathbf{p})\, e_x(i\omega_n,\mathbf{p})\rangle = \frac{g^2(p_y^2+\kappa^2 g^4\tilde{k}^2 p^2)}{\omega_n^2+g^2 r p_y^2+\kappa^2 p^2(g^6\tilde{k}^2 r+p_x{}^2)}$$

$$\langle e_y(-i\omega_n,-\mathbf{p})\, e_y(i\omega_n,\mathbf{p})\rangle = \frac{g^2(g^6\tilde{k}^2 r+p_x^2)}{\omega_n^2+g^2 r p_y^2+\kappa^2 p^2(g^6\tilde{k}^2 r+p_x{}^2)}$$

$\tilde{k} \equiv k/2\pi$, and $\tilde{\kappa}^2 = \kappa^2(1+\kappa^2 g^4\tilde{k}^2)$

All you should glean from this is mess is that there is a gapless Goldstone mode, with:

$$\omega^2 = (\kappa^2 g^6\tilde{k}^2 r)p_x^2 + (g^2 r+\kappa^2 g^6\tilde{k}^2 r)p_y^2 + \kappa^2 p^2 p_x{}^2$$

The last term is subdominant at low energy. The Goldstone mode has anisotropic velocities (and even anisotropic scaling if k=0, where the last term matters).

Absence of exact rotation symmetry?

Even in absence of the spontaneous breaking, rotation ymmetry won't be an exact symmetry of most relevant systems. It will be broken by the underlying crystalline lattice, and by disorder.

e studied both. To model lattice symmetry breaking, we consider a Hamiltonian:

$$g^2 H[e,a] = -\frac{|r|}{2}(e_x^2 + (1-\alpha^2)\, e_y^2) + \frac{\kappa^2}{2}(\partial_i e_j)^2 + \frac{\lambda}{4}(e_i^2)^2 + b^2$$

e-expanding about the symmetry-breaking vev, we find:

$$H[e,a] = |r|\tilde{e}_x^2 + |r|\alpha^2 e_y^2 + b^2 + \mathcal{O}(e^3)\ .$$

here is now a quadratic term for both legs of the e-field, and integrating it out, we find:

$$S = \int dt\, d^2x \Big[\frac{1}{g^2} \Big(\sqrt{|r|/\lambda}\, (\partial_t a_x - \partial_x a_t) + \frac{1}{|r|} (\partial_t a_x - \partial_x a_t)^2 + \frac{1}{|r|\alpha^2} (\partial_t a_y - \partial_y a_t)^2$$

$$- (\partial_y a_x - \partial_x a_y)^2 \Big) + \frac{k}{4\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \Big].$$

his is now an anisotropic Maxwell-Chern-Simons theory with a gap. So in particular, we expect a well quantized Hall conductance, but also that finite temperature and equency transport will be anisotropic. It turns out that these states, the anisotropy reflects itself in anisotropic dielectric constants (imaginary parts of the longitudinal conductivity); as in the Hall states, there is no real longitudinal conductivity.

Finally, we can consider the addition of disorder. The simplest way to do this, is to change some of the couplings in the action, to random functions of x. One simple illustrative choice is:

$$S = \int dt\, d^2x \left[\frac{1}{g^2} \left(e_i \partial_t a_i + a_t \partial_i e_i - \frac{1}{2} r e_i^2 - \frac{1}{2} r_x(\mathbf{x}) e_x^2 - \frac{1}{2} r_y(\mathbf{x}) e_y^2 + \ldots \right]. \right.$$

Here, the random parts of the r-coupling are assumed to be Gaussian white-noise correlated with zero mean and variances given by distinct parameters:

$$\overline{r_i(\mathbf{x})} = 0\,, \qquad \overline{r_i(\mathbf{x}) r_j(\mathbf{x}')} = W_i \delta_{ij}\, \delta(\mathbf{x} - \mathbf{x}').$$

One can then compute disorder-averaged correlators using the replica trick. I won't talk about this here. The only important point, for us, will be that the disorder will here is now a quadratic term for both legs of the e-field, and integrating it out, we find:

$$S = \int dt\, d^2x \left[\frac{1}{g^2} \left(\sqrt{|r|/\lambda}\,(\partial_t a_x - \partial_x a_t) + \frac{1}{|r|}(\partial_t a_x - \partial_x a_t)^2 + \frac{1}{|r|\alpha^2}(\partial_t a_y - \partial_y a_t)^2 \right.\right.$$

$$\left.\left. - (\partial_y a_x - \partial_x a_y)^2 \right) + \frac{k}{4\pi}\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right].$$

his is now an anisotropic Maxwell-Chern-Simons theory vith a gap. So in particular, we expect a well quantized Hall conductance, but also that finite temperature and equency transport will be anisotropic. It turns out that these states, the anisotropy reflects itself in anisotropic dielectric constants (imaginary parts of the longitudinal conductivity); as in the Hall states, there is no real longitudinal conductivity.

Finally, we can consider the addition of disorder. The simplest way to do this, is to change some of the couplings in the action, to random functions of x. One simple illustrative choice is:

$$S = \int dt\, d^2x \left[\frac{1}{g^2} \left(e_i \partial_t a_i + a_t \partial_i e_i - \frac{1}{2} r e_i^2 - \frac{1}{2} r_x(\mathbf{x}) e_x^2 - \frac{1}{2} r_y(\mathbf{x}) e_y^2 + \ldots \right].$$

lere, the random parts of the r-coupling are assumed to e Gaussian white-noise correlated with zero mean and variances given by distinct parameters:

$$\overline{r_i(\mathbf{x})} = 0\,, \qquad \overline{r_i(\mathbf{x}) r_j(\mathbf{x}')} = W_i \delta_{ij}\, \delta(\mathbf{x} - \mathbf{x}').$$

ne can then compute disorder-averaged correlators using the replica trick. I won't talk about this here. The only important point, for us, will be that the disorder will contribute to self energies for the LCS modes:

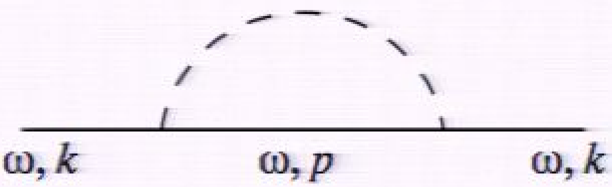


Fig. 1. Lowest order diagram contributing to the e_y self-energy.

These will give rise to real longitudinal conductivity (c.f. adding scattering time in the Drude model).

So to summarize:

* Systems with explicit SO(2)-breaking are expected to exhibit anisotropic imaginary longitudinal conductivities, but well-quantized Hall conductivity.

* The addition of disorder should give rise to real longitudinal components, which remain anisotropic.

*When the system is SO(2) invariant to an excellent approximation, there is in addition a Goldstone boson, hich will have a significant effect on transport properties.

Response Functions @ and near the critical point

Now, we couple our current to an external electromagnetic field

$$\delta L = \int dt d^2x \; J^\mu A_\mu.$$

Because the action is quadratic in the emergent gauge eld, we can integrate it out. The action for the external gauge field then takes the form:

$$S_{\text{eff}} = \frac{1}{2}\sum_n \int d^2p \; A_\mu(-i\omega_n, -\mathbf{p}) K_{\mu\nu}(i\omega_n, \mathbf{p}) A_\nu(i\omega_n, \mathbf{p})$$

The conductivity is then obtained from the K matrix according to

$$\sigma_{jk}(\omega) = \frac{1}{i\omega + \delta} K_{jk}(\omega + i\delta, \mathbf{p} = 0)$$

But since

$$J_\mu = \frac{1}{2\pi} \epsilon_{\mu\nu\rho} \partial_\nu a_\rho$$

we can easily compute the transport properties using the propagator in the various phases:

$$K_{\mu\nu}(i\omega_n, \mathbf{p}) = \left(\frac{1}{2\pi}\right)^2 \epsilon_{\mu\alpha\beta} \epsilon_{\nu\lambda\rho} p_\alpha p_\lambda \left\langle a_\beta(-i\omega_n, -\mathbf{p})\, a_\rho(i\omega_n, \mathbf{p})\right\rangle$$

Warm-up: r > 0 phase

Here, we find:

$$\sigma_{xx}(\omega) = \frac{i}{2\pi k} \cdot \frac{\omega/g^4\tilde{k}r}{1-(\omega/g^4\tilde{k}r)^2}$$

$$\sigma_{xy}(\omega) = \frac{1}{2\pi k} \cdot \frac{1}{1-(\omega/g^4\tilde{k}r)^2}$$

while the compressibility is:

$$lim_{p\to 0}\, lim_{\omega\to 0}\, K_{00}(\omega,p) \sim lim_{p\to 0}\, p^2 = 0\ .$$

At low frequency, this gives the expected DC response of the quantum Hall system. The massive pole is one of the gapped excitations above the degenerate ground states.

Moving right along, to the r=0 phase:

$$\mathrm{Re}\,\sigma_{xx}(\omega) = \mathrm{Im}\,\sigma_{xx}(\omega) = 0$$

$$\mathrm{Re}\,\sigma_{xy}(\omega) = \mathrm{Im}\,\sigma_{xy}(\omega) = 0$$

he critical theory has identically vanishing conductivities - insulates at any frequency! As the gap from the quantum all phase collapses, the system becomes more insulating. he reason is that the critical theory does not couple to a spatially-homogeneous electric field, regardless of the requency (except at frequency precisely equal to zero).

he finds that the critical point has a finite compressibility:

$$K = \frac{g^2}{8\pi^2(1+\kappa^2\tilde{k}^2)}$$

Transport in the anisotropic phase:

Recall that in this phase, the Hamiltonian looks like (if the sample is rotationally symmetric):

$$H[e,a] = |r|\tilde{e}_x^2 + \frac{\kappa^2}{2}(\partial_i e_y)^2 + b^2 + \mathcal{O}(e^3),$$

i.e., an ungodly mix of the critical and Hall phases. The leading transport properties reflect this:

$$\sigma_{xx}(\omega) = 0$$

$$\sigma_{yy}(\omega) = \frac{g^4 r}{8\pi^2}\delta(\omega) + \frac{ig^4 r}{8\pi^2\omega}$$

$$\sigma_{xy}(\omega) = 0$$

The Goldstone mode makes the system an anisotropic insulator/superconductor hybrid! Likely, further effects smooth out the delta function, and the insulating behavior.

there is some in-built intrinsic anisotropy in the sample, hen the transport becomes instead characteristic of an anisotropic quantum Hall state:

$$\sigma_{xx}(\omega) = \alpha \frac{i}{2\pi k} \cdot \frac{\omega/g^4\tilde{k}r\alpha}{1-(\omega/g^4\tilde{k}r\alpha)^2}$$

$$\sigma_{yy}(\omega) = \frac{1}{\alpha} \cdot \frac{i}{2\pi k} \cdot \frac{\omega/g^4\tilde{k}r\alpha}{1-(\omega/g^4\tilde{k}r\alpha)^2}$$

$$\sigma_{xy}(\omega) = \frac{1}{2\pi k} \cdot \frac{1}{1-(\omega/g^4\tilde{k}r\alpha)^2}$$

with vanishing compressibility).

his is getting pretty close to the recent experiments, but ere is still no real longitudinal conductivity, while Xia et al measure anisotropic non-zero answers there.

Finally, including disorder, we find:

$$\sigma_{xx}(\omega) = \frac{1}{8\pi^2}\frac{ig^4\omega}{g^8\tilde{k}^2 r + i(\omega\tau)^2}$$

$$\sigma_{yy}(\omega) = \frac{1}{8\pi^2}\frac{g^4 r\,\omega\,\tau^2}{g^8\tilde{k}^2 r + i(\omega\tau)^2}$$

$$\sigma_{xy}(\omega) = \frac{1}{2\pi k}\cdot\frac{1}{1 + i(\omega\tau)^2/g^8\tilde{k}^2 r}$$

This is the relevant component ofW because the disorder is relevant in the direction transverse to the e-field condensate

$$\frac{1}{\tau^2} = \frac{W_y g^8\tilde{k}^2 r}{2v_x v_y}$$

This now shows great qualitative similarity to the new phase seen in the Eisenstein lab! There is well quantized Hall conductivity and real, anisotropic longitudinal conductivity.

CONCLUSION

o conclude, I will discuss two related topics. One is the bject of reasonably comprehensive study right now, and the other is almost totally neglected.

. You have heard, by now, of the metrics dual to e.g. 2+1 dimensional conformal field theories:

$$ds^2 = r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2}$$

Maldacena

But in many real condensed matter systems, orentz invariance is not a particularly natural symmetry (since it is always broken by the UV theory).

why should we expect the "accidental" SO(2,1) which acts there at fixed r to rotate space and time into one another? In general, we shouldn't. A simple way to geometrize scale invariant but non-Lorentz invariant metrics, which still enjoy rotational symmetry and space-time translation symmetries, is the Lifshitz metric:

$$ds^2 = -r^{2z}dt^2 + r^2(dx^2 + dy^2) + \frac{dr^2}{r^2}$$

S.K., Liu, Mulligan

Solutions supporting this metric do not arise in pure nstein gravity with negative cosmological constant, but do arise in simple cousins like:

$$S = \int d^4x\sqrt{-g}\,(R - 2\Lambda) - \frac{1}{2}\int \left(\frac{1}{e^2}F_{(2)} \wedge *F_{(2)} + F_{(3)} \wedge *F_{(3)}\right) - c\int B_{(2)} \wedge F_{(2)}.$$

Probably more importantly, this metric also arises to
characterize "emergent" infra-red fixed points in gravity studies of duals to toy CM systems, including:

(Backreacted) probe fermions in AdS, whose physics gives rise to fascinating non-Fermi liquids. Faulkner, Liu, McGreevy, Vegh

Maxwell-Dilaton black holes in AdS Goldstein, S.K., Prakash, Trivedi

Gravity duals of simple models which capture linear resistivity, as a toy of the "strange metallic phase" Hartnoll, Polchinski, Silverstein, Tong

The "Lifshitz-like" space-times are one simple class of solutions that emerge in the IR of spacetimes which [ch]aracterize CFTs with non-trivial charge density. I believe many remain yet to be found. But more about these, in a dual picture:

why should we expect the "accidental" SO(2,1) which acts there at fixed r to rotate space and time into one another? In general, we shouldn't. A simple way to geometrize scale invariant but non-Lorentz invariant metrics, which still enjoy rotational symmetry and space-time translation symmetries, is the Lifshitz metric:

$$ds^2 = -r^{2z}dt^2 + r^2(dx^2 + dy^2) + \frac{dr^2}{r^2}$$

S.K., Liu, Mulligan

Solutions supporting this metric do not arise in pure Einstein gravity with negative cosmological constant, but do arise in simple cousins like:

$$S = \int d^4x\sqrt{-g}\,(R - 2\Lambda) - \frac{1}{2}\int \left(\frac{1}{e^2}F_{(2)} \wedge *F_{(2)} + F_{(3)} \wedge *F_{(3)}\right) - c\int B_{(2)} \wedge F_{(2)}.$$

Probably more importantly, this metric also arises to
haracterize "emergent" infra-red fixed points in gravity
studies of duals to toy CM systems, including:

(Backreacted) probe fermions in AdS, whose physics gives rise to fascinating non-Fermi liquids. Faulkner, Liu, McGreevy, Vegh

Maxwell-Dilaton black holes in AdS Goldstein, S.K., Prakash, Trivedi

Gravity duals of simple models which capture linear resistivity, as a toy of the "strange metallic phase" Hartnoll, Polchinski, Silverstein, Tong

The "Lifshitz-like" space-times are one simple class of solutions that emerge in the IR of spacetimes which aracterize CFTs with non-trivial charge density. I believe nany remain yet to be found. But more about these, in a dual picture:

In the abelian case, the "Lifshitz" gauge theory without Chern-Simons term is free, and has vanishing beta nctions. In the non-Abelian case, it is already non-trivial, so a first step involves study of this theory in isolation.

The natural generalization of our abelian theory has an action of the rough form:

$$S[A_0, A_i, E_i] = \int d^2x d\tau \operatorname{Tr}\Big(\frac{1}{g_1^2}(E_i \partial_\tau A_i + A_0 D_i E_i) - \frac{1}{2g_2^2}(D_j E_i)^2 + \frac{1}{2g_3^2} B^2 + i\frac{\kappa}{2}\epsilon_{ij}[E_i, E_j]B + \lambda [E_i, E_j]^2\Big),$$

where:

$$D_i V = \partial_i V - i[A_i, V] = (\partial_i V^a + f^{abc} A_i^b V^c) t^a,$$

$$B(A) = \partial_1 A_2 - \partial_2 A_1 - i[A_1, A_2] = (\partial_1 A_2^a - \partial_2 A_1^a + f^{abc} A_1^b A_2^c) t^a$$

The gauge-couplings in this theory do run, and the current indications are:

S.K., Mulligan, Nayak, to appear

Some of the couplings run strong, and some run weak, in he deep infrared. I.e., this theory is a pure gauge theory which is not asymptotically free in three space-time dimensions.

It has novel possible instabilities about the origin, which are however non-perturbative in nature.

herefore, finding non-trivial fixed points (which could, in some suitable limit, have gravity duals); or studying the oupling to Chern-Simons terms; remain projects for the even more distant future.