Title: A particle physicist's perspective on topological insulators

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Abstract: The theory of topological insulators will be reviewed in terms familiar to particle theorists.

A particle physicist's perspective on topological insulators.

Andreas Karch

PI, May 30, 2010

based on: "Fractional topological insulators in three dimensions",
by J.Maciejko, X.-L. Qi, AK, S. Zhang, arxiv:1004.3628

Outline

- Low energy description of generic insulator
- Properties of the low energy theory
- Microscopic Example in a continuum field theory and the ABJ anomaly.
- Fractional Topological insulators in 3+1 dimensions.

Low energy effective theory.

What is the low energy description of a generic, time reversal invariant insulator?

Insulator = gapped spectrum

Low energy DOFs: only Maxwell field.

Task: Write down the most general action for E and B, with up to two derivatives, consistent with symmetries.

Low energy effective action.

Low energy DOFs: only Maxwell field.

$$S_0 = \int d^3x dt L_0 = \frac{1}{8\pi} \int d^3x dt \left(\epsilon \vec{E}^2 - \frac{1}{\mu} \vec{B}^2 \right).$$

Permittivity and Permeability.

Rotations allow one extra term.

$$S_\theta = \frac{\theta}{2\pi}\frac{\alpha}{16\pi}\int d^3x\,dt\,\epsilon_{\mu\nu\rho\tau}F^{\mu\nu}F^{\rho\tau} = \frac{\theta}{2\pi}\frac{\alpha}{4\pi}\int d^3x\,dt\,\partial^\mu(\epsilon_{\mu\nu\rho\sigma}A^\nu\partial^\rho A^\tau)$$

$$= \frac{\theta\alpha}{4\pi}\int d^3x\,dt(E\cdot B)$$

But: Under time reversal E is even, B odd

So naively the most general description of a time reversal invariant insulator does not allow for a theta term

Flux Quantization.

Dirac Quantization of magnetic charge:

$$g = n\frac{e}{2\alpha}$$

Implies quantization of magnetic flux!

$$\int_S F = g$$

On any Euclidean closed 4-manifold M:

$$\frac{\alpha}{32\pi^2}\int_M d^4x F_{\mu\nu}F_{\sigma\tau}\epsilon^{\mu\nu\sigma\tau} = N \in \mathbf{Z}$$

Flux Quantization.

- Partition function

$$Z(\theta) = \exp\left\{ i\frac{\alpha\theta}{32\pi^2} \int_M d^4x F_{\mu\nu} F_{\sigma\tau} \epsilon^{\mu\nu\sigma\tau} \right\} = e^{iN\theta}$$

- is periodic in θ→θ+2π (Abelian version of the "Θ vacuum" (Callan, Dashen, Gross 1976, Jackiw&Rebbi, 1976))
- θ is time-reversal odd
- →time-reversal invariant insulator can have θ=0 or π
- Z_2 classification

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Topological Insulators

Low energy description of a T-invariant insulator described by 3 parameters: ε, μ, and:

$\theta = 0$	Topologically trivial insulators
$\theta = \pi$	Topologically non-trivial insulators

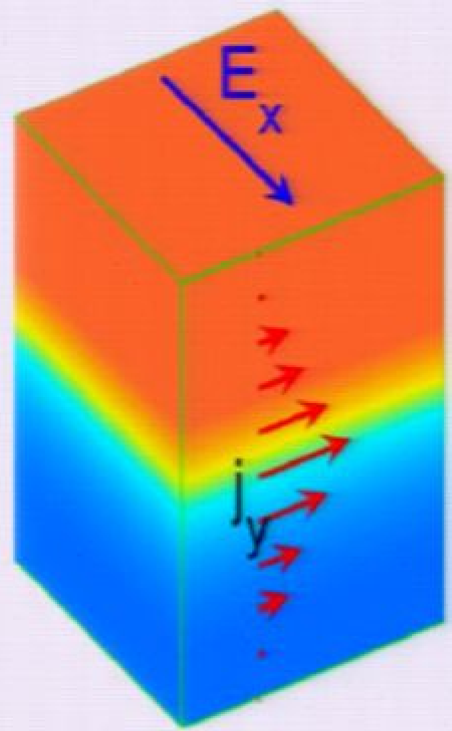
Physical Consequences.

Theta term is total derivative. In the absence of interface can be absorbed by redefining E and B. But:

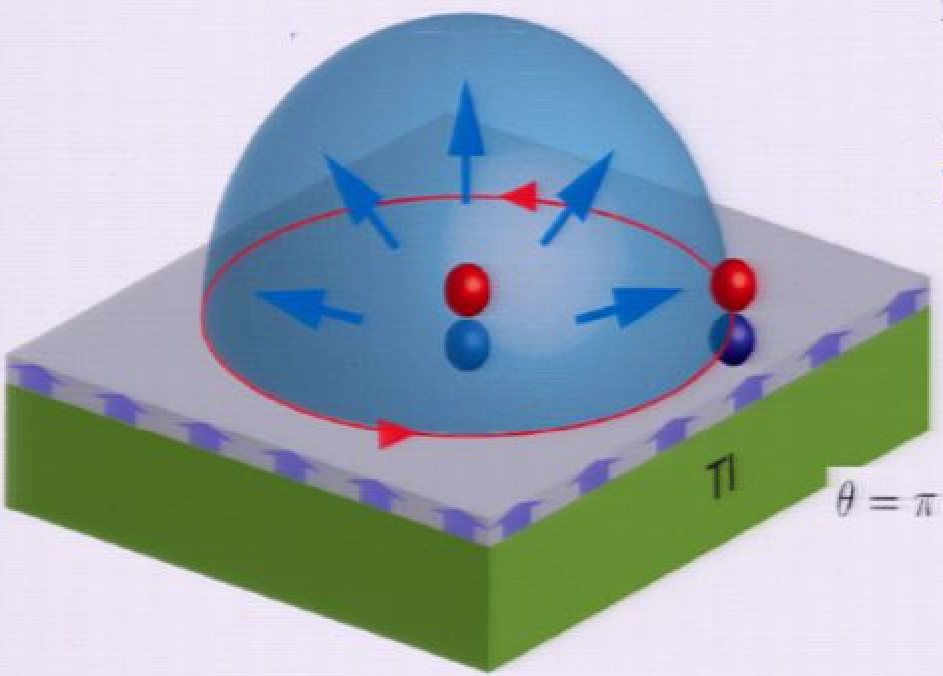
Boundary of topological insulator
= domain wall of θ

$$j^\mu = \frac{e^2}{h}\frac{1}{2\pi}\epsilon^{\mu\nu\sigma\tau}\partial_\nu\theta\partial_\sigma A_\tau$$

$$\Rightarrow \sigma_H = \frac{e^2}{h}\frac{\Delta\theta}{2\pi} = \frac{e^2}{h}\left(\frac{1}{2}+n\right)$$



Magnetic Monopoles in TI



prediction: mirror charge of an electron is a **magnetic monopole**

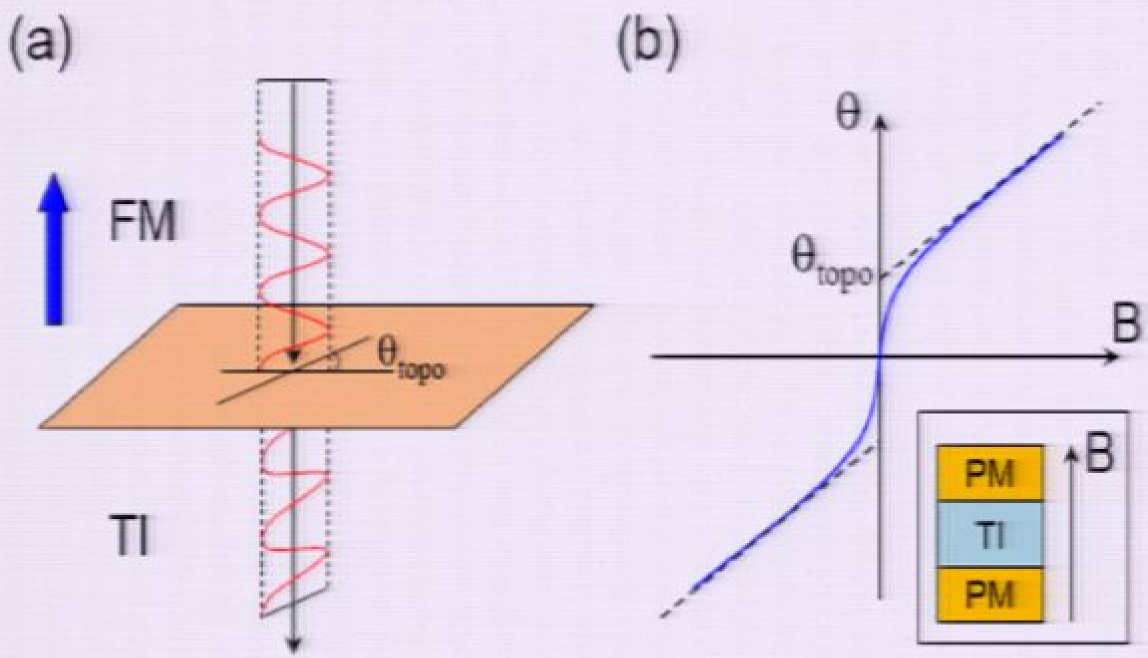
first pointed out by Sikivie, re-obtained in the TI context by Qi et. al.

Compact expression from requiring **E&M duality covariance** (AK).

$$g = \frac{\alpha\theta/2\pi}{1+\alpha^2\theta^2/4\pi^2} q$$

(for $\mu=\mu'$, $\varepsilon=\varepsilon'$)

Faraday and Kerr Effect



change in θ

B independent contribution to Kerr/Faraday (Qi et al)

(Polarization of transmitted and

$$\theta_{\rm topo} = \arctan \frac{2\alpha\Delta}{\sqrt{\epsilon/\mu} + \sqrt{\epsilon'/\mu'}}$$

A Microscopic Model

A microscopic model: Massive Dirac Fermion.

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu - M)\psi$$

Time Reversal: $M \longrightarrow M^*$

Time reversal system has real mass.

Two options: positive or negative.

Chiral rotation and ABJ anomaly.

Massless theory invariant under chiral rotations:

$$\psi \to e^{-i\phi\gamma_5/2}\psi$$

Symmetry of massive theory if mass transforms:

$$M \to e^{i\phi} M$$

Phase can be rotated away! Chose M positive.

Chiral rotation and ABJ anomaly.

But in the quantum theory chiral rotation is anomalous. Measure transforms.

$$\Delta\mathcal{L} = C\alpha\frac{\phi}{32\pi^2}\,\mathrm{tr}\;\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$C = \sum_{fields} q^2 = 1\cdot 1^2 = 1$$

Single field with unit charge.

$$\theta \to \theta - C\phi$$

Chiral rotation and ABJ anomaly.

$$\theta \to \theta - C\phi$$

Axial rotation with $\Phi=\pi$:

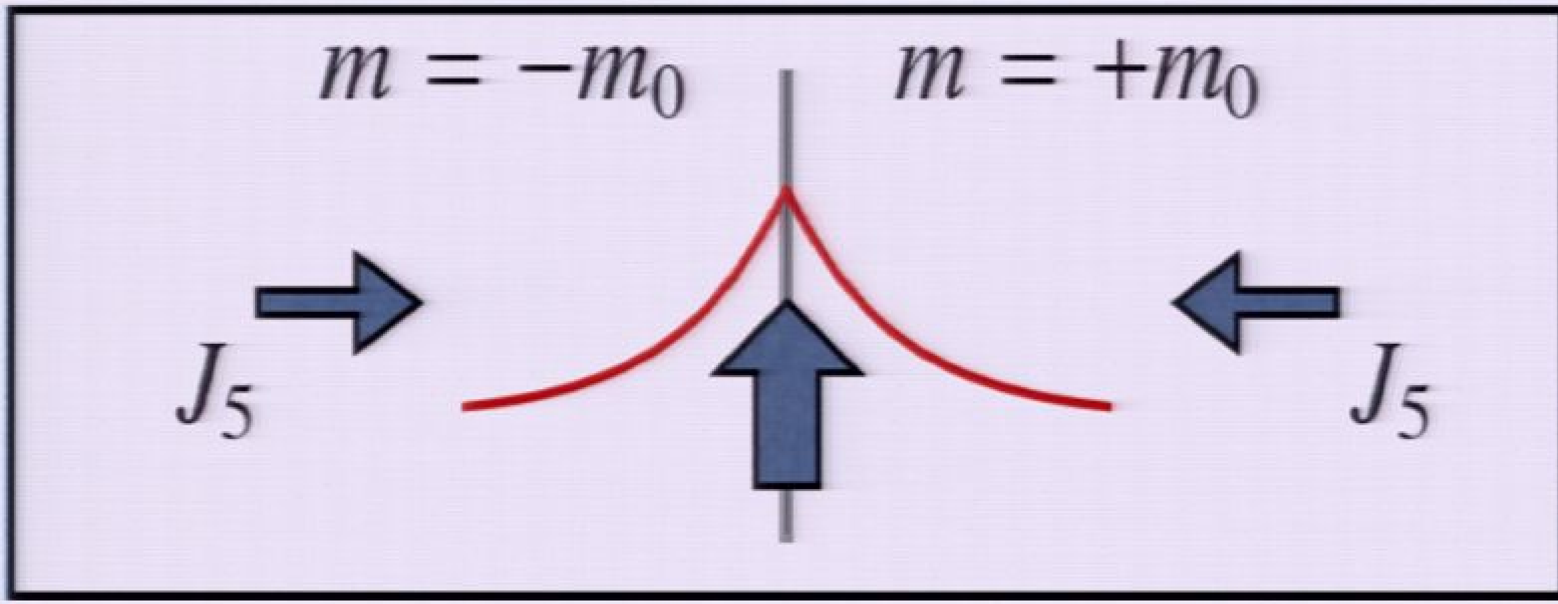
- Rotates real negative mass into positive mass.
- Generates $\theta=\pi$!

Positive mass = Trivial Insulator.

Negative mass = Topological Insulator.

Localized Zero Mode on Interface.

Domain Wall has localized zero mode!



Kaplan's Domain Wall Fermion

Domain Wall = TI/non-TI Interface

Generalizations:

Bulk topological insulator → Boundary theory With anomaly

4+1	Callan-Harvey, Kaplan	Z	axial anomaly	3+1
3+1	TRI topological insulator	Z_2	parity anomaly	2+1
2+1	Quantum Hall	Z	chiral anomaly	1+1

A Lattice Realization.

How to get $\theta = \pi$ from non-interacting electrons in periodic potential (Band-Insulator)?

Topology of Band Structure!

Define Z_2 valued topological invariant of bandstructure to distinguish trivial ("positive mass") from topologically non-trivial ("negative mass").

TKKN for topological insulator.

Multi-Band-Berry-Connection.

(Qi, Hughes, Zhang)

$$\theta \equiv 2\pi P_3(\theta) = \frac{1}{16\pi^2}\int d^3\mathbf{k}\epsilon^{ijk}\mathrm{Tr}\{[f_{ij}(\mathbf{k}) - \frac{2}{3}ia_i(\mathbf{k})\cdot a_j(\mathbf{k})]\cdot a_k(\mathbf{k})\}$$

$$f_{ij}^{\alpha\beta} = \partial_i a_j^{\alpha\beta} - \partial_j a_i^{\alpha\beta} + i\,[a_i, a_j]^{\alpha\beta},$$

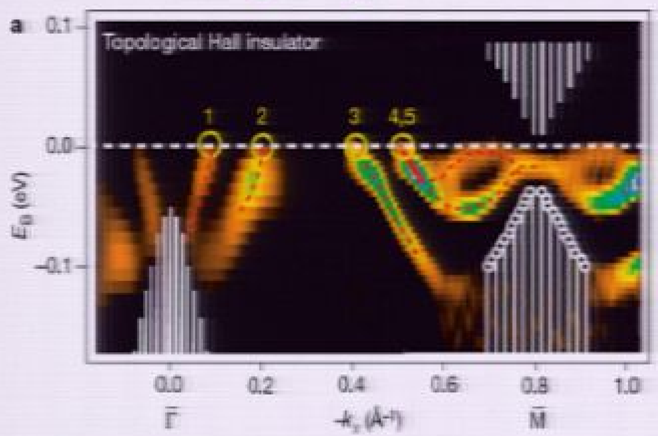
$$a_i^{\alpha\beta}(\mathbf{k}) = -i\,\langle\alpha,\mathbf{k}|\,\frac{\partial}{\partial k_i}\,|\beta,\mathbf{k}\rangle$$

θ=0	Vacuum, ...
θ=π	$Bi_{1-x}Sb_x$, Bi2Se3, Bi2Te3, Sb2Te3

predicted: TlBi(Sb)Te(Se, S) , LaPtBi etc (Heusler compounds)

Experimentally found Zero Modes.

Hasan group, *Nature* 2008



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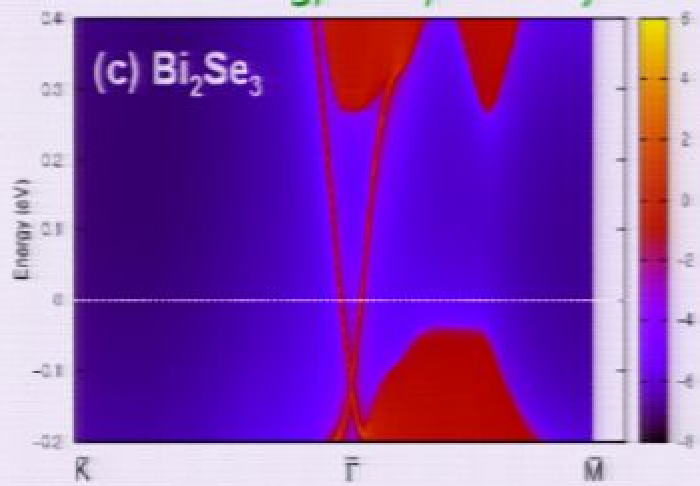
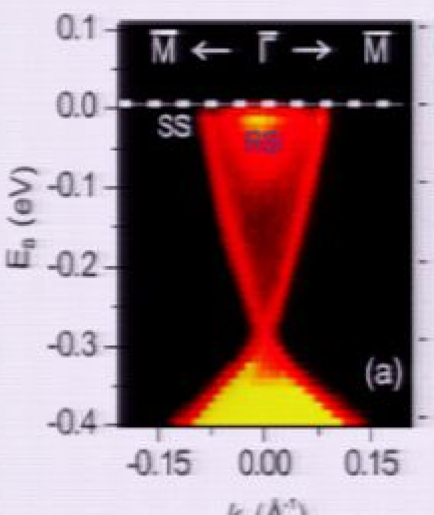
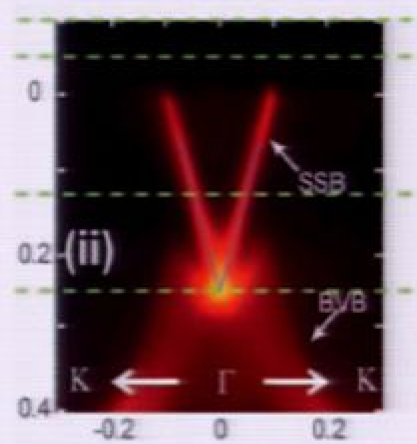




Hasan group, Bi_2Se_3 *Nat Phys* 2009



Bi_2Te_3 Chen et al *Science* 2009

Summary of Strategy:

Low Energy Effective Theory:

Dirac Quantization $\longrightarrow$ $\theta = \text{Integer} \cdot \pi$

Microscopic Model:

ABJ anomaly $\longrightarrow$ $\theta/\pi = \sum (\text{charge})^2$

Connection to Experiment:

Band Topology $\longrightarrow$ $\theta = \text{QHZ-invariant}$

IR

UV

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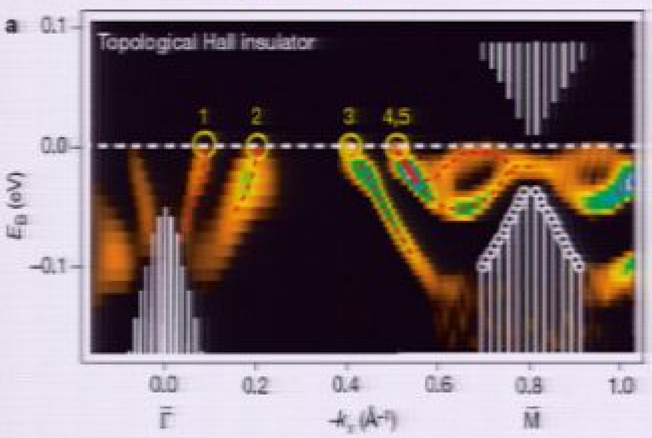

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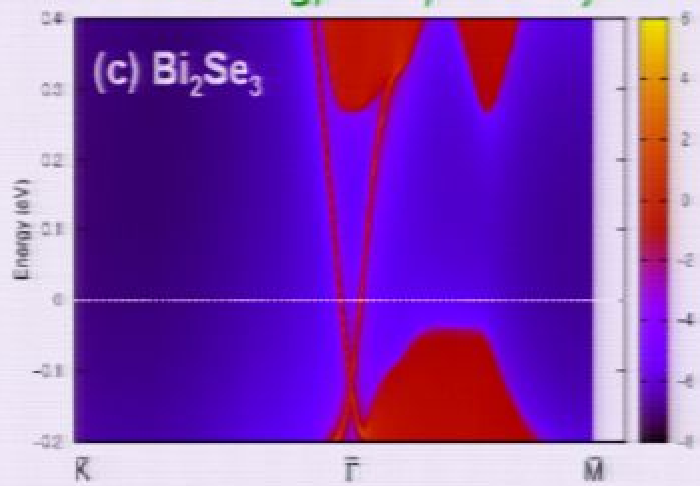

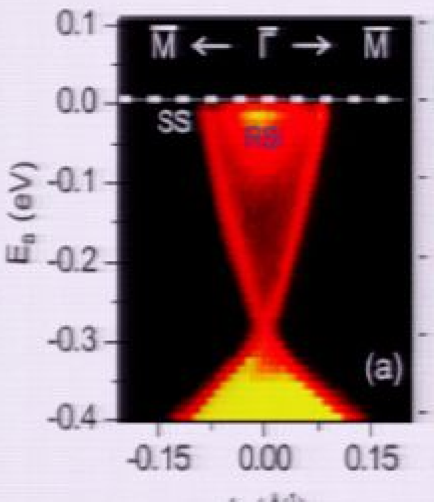

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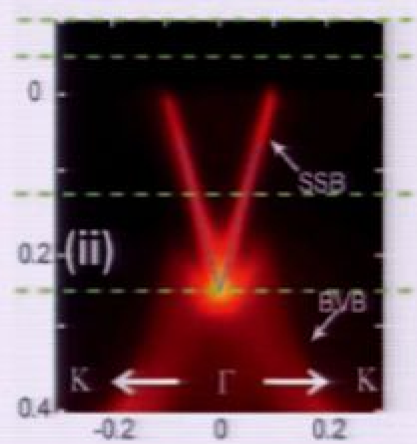

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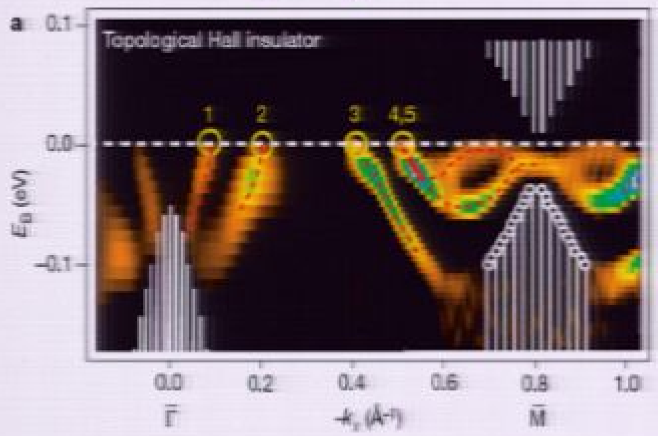

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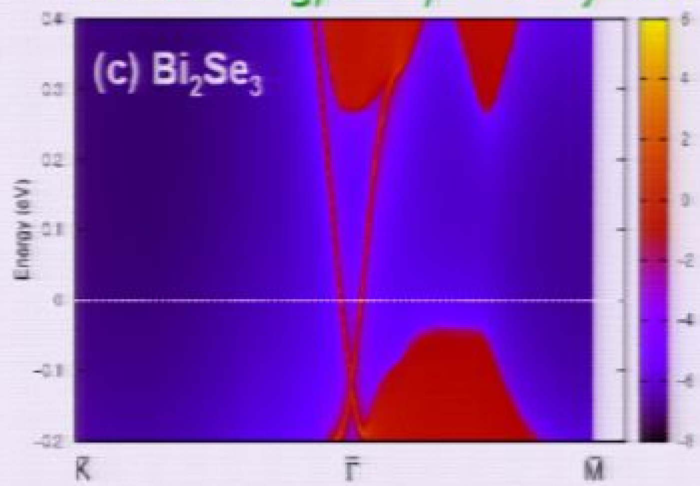

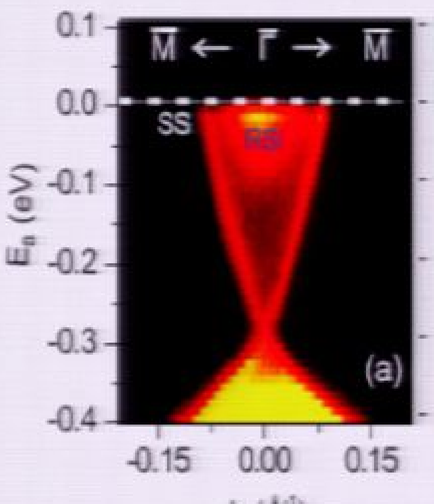

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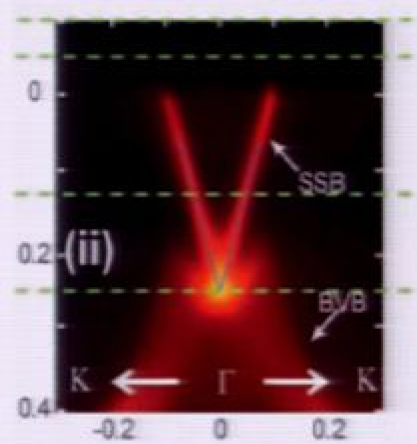

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Fractional Topological Insulators?

Recall from Quantum Hall physics:

electron

fractionalizes into m partons

e^- interactions →

(m odd for fermions)

$$\sigma_{xy} = n\frac{e^2}{h}$$

$$\sigma_{xy} = \frac{n}{m}\frac{e^2}{h}$$

Quantum Hall

Fractional Quantum Hall

Fractional Topological Insulators?

TI = half of an integer quantum hall state on the surface

expect: fractional TI = half a fractional QHS
Hall quantum = half of 1/odd integer.

Can we get this from charge fractionalization?

Partons.

Microscopic Model:

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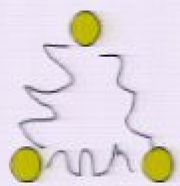
e^- =



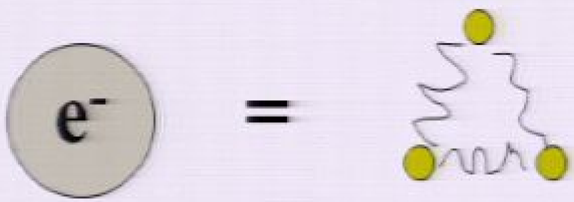
electron breaks up into m partons.

$$\theta / \pi = \sum (\text{charge})^2 = m \cdot \left(\frac{1}{m}\right)^2 = \frac{1}{m}$$

(m odd so e^- is fermion)

(if partons form a TI – have negative mass)

Partons.



To ensure that partons confine into electron add "statistical" gauge field

Simplest model: SU(3) with N_f=1

Partons.

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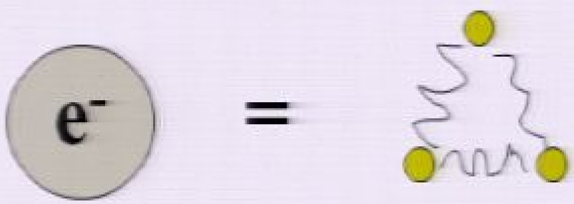

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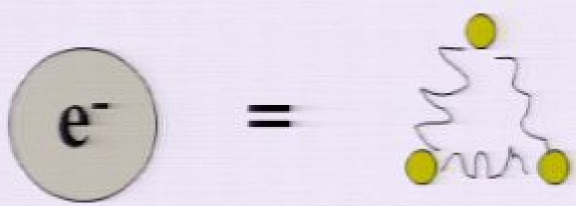

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Relativistic Partons.

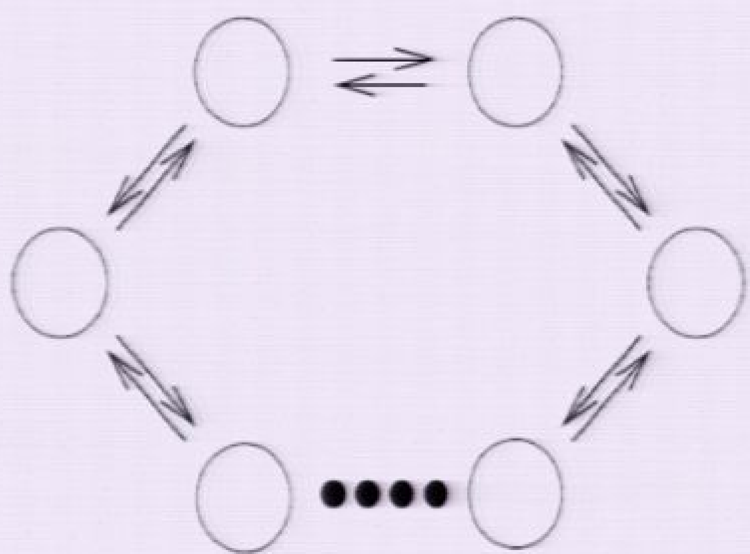
In a relativistic field theory the electron=baryon has spin m/2 in the SU(m) model.

Not a problem in non-relativistic context.

Alternative: Quiver Model

Still need m partons to be gauge invariant.

Baryons of any spin possible.



General Parton model.

$$U(1)_{\text{em}} \times \prod_{f=1}^{N_f} U(N_c^{(f)})/U(1)$$

$$\downarrow$$

$$\sigma_{H,s} = \frac{p}{q}\frac{e^2}{2h}, \quad p, q \text{ odd}$$

Important dynamical question.

Is the gauge theory in a confining or deconfining phase?

We need: deconfined! Favors abelian models.
(or extra neutral matter, e.g.
N=4 SYM with N=2 massive hyper)

Gapless modes present; charged fields all gapped.

How to make a fractional TI?

Need: Strong electron/electron interactions

(so electrons can potentially fractionalize)

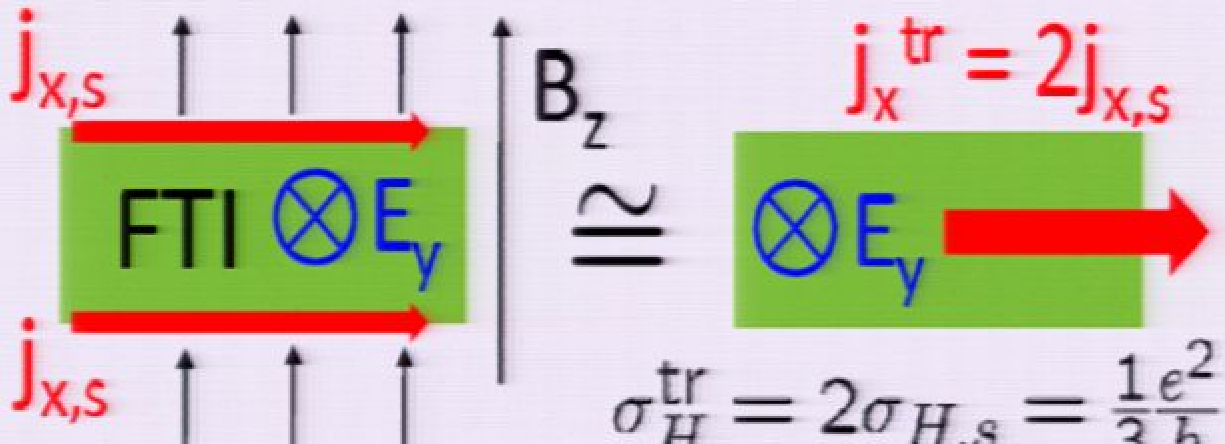
Strong spin/orbit coupling

(so partons can form topological insulator)

How can one tell if a given material is a fractional TI (in theory/in practice)?

Transport Measurements

$$\sigma_{H,s} = \frac{1}{6}\frac{e^2}{h}$$



$$\sigma_H^{tr} = 2\sigma_{H,s} = \frac{1}{3}\frac{e^2}{h}$$

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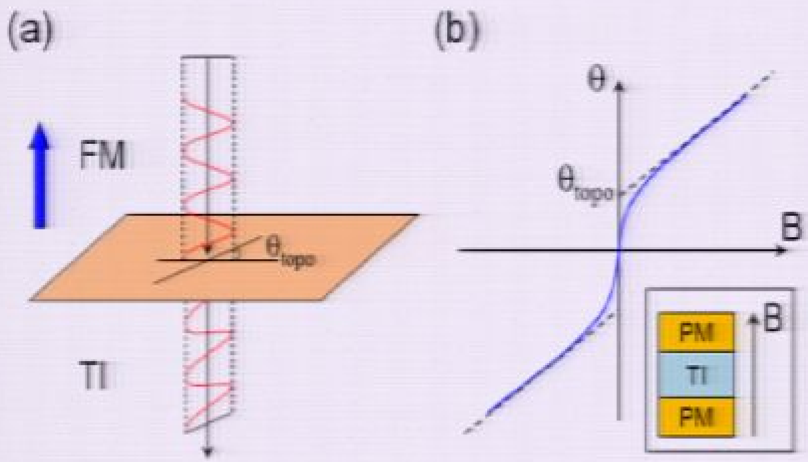
Transport sees only sum of Hall currents.

Add to zero – can be aligned by external B.

Faraday and Kerr Effect



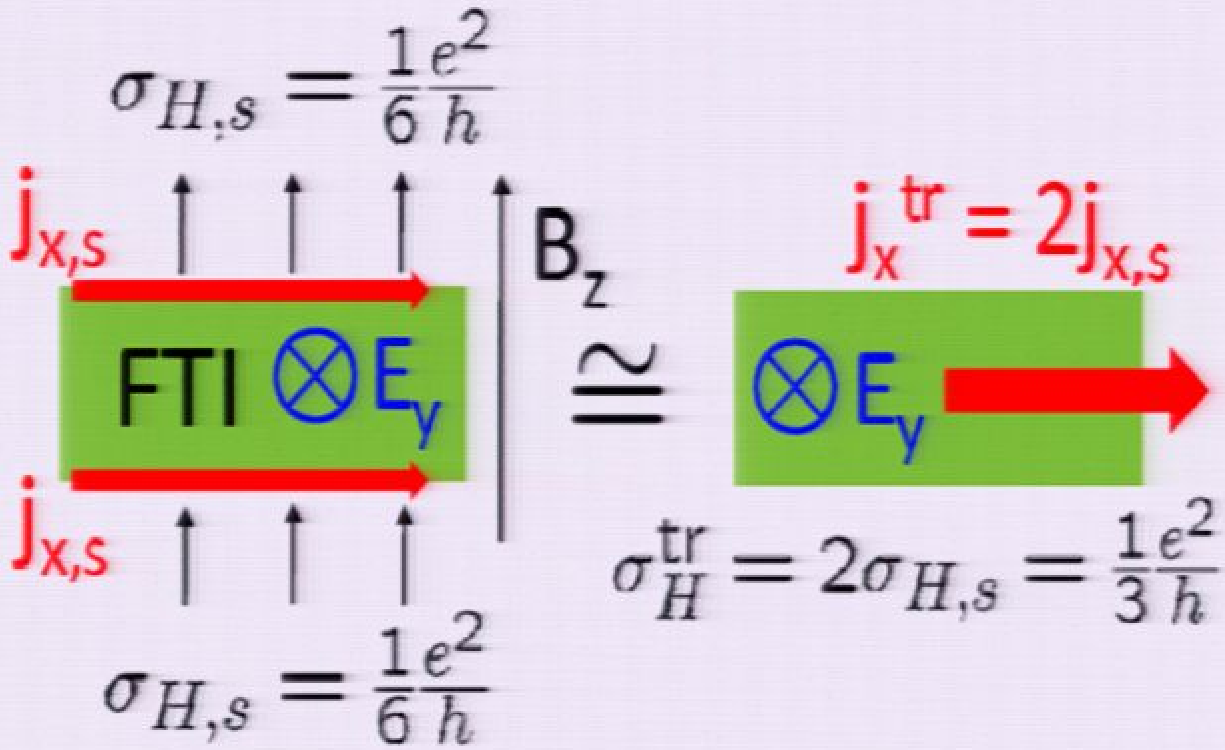
Kerr rotation sees only one interface.

Directly measures change in θ.

$$\theta_{\text{topo}} = \arctan \frac{2\alpha\Delta}{\sqrt{\epsilon/\mu} + \sqrt{\epsilon'/\mu'}}$$

← change in θ

Transport Measurements



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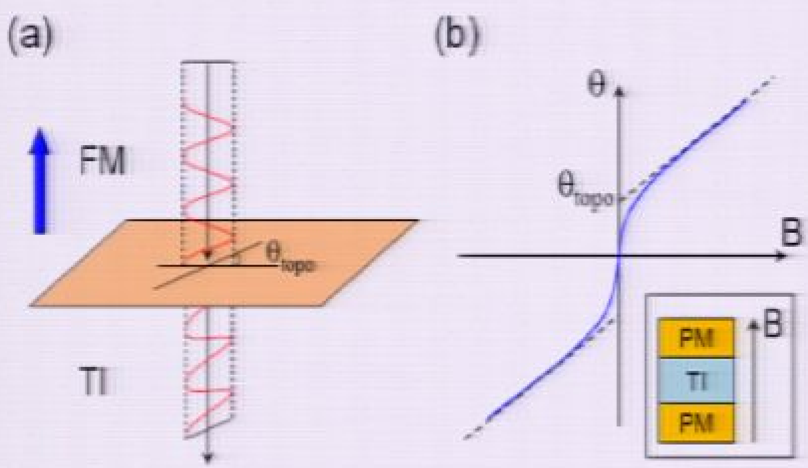

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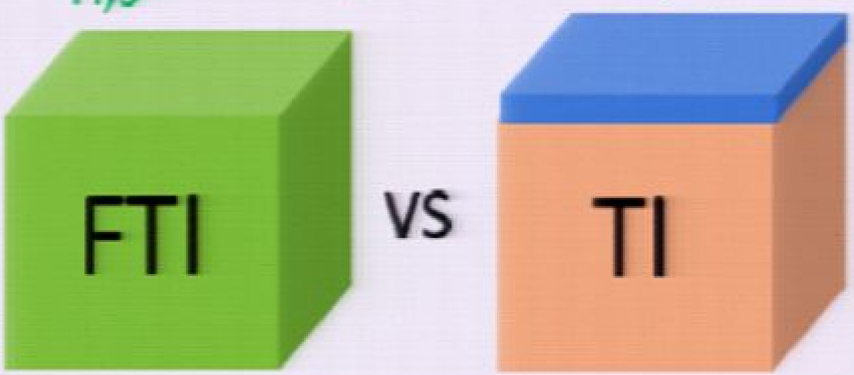
Fractional TI or Surface FQHE?

$\sigma_{H,s} = -e^2/6h$ $\quad \nu=1/3$ FQH, $\sigma_H = e^2/3h$



$$+$$

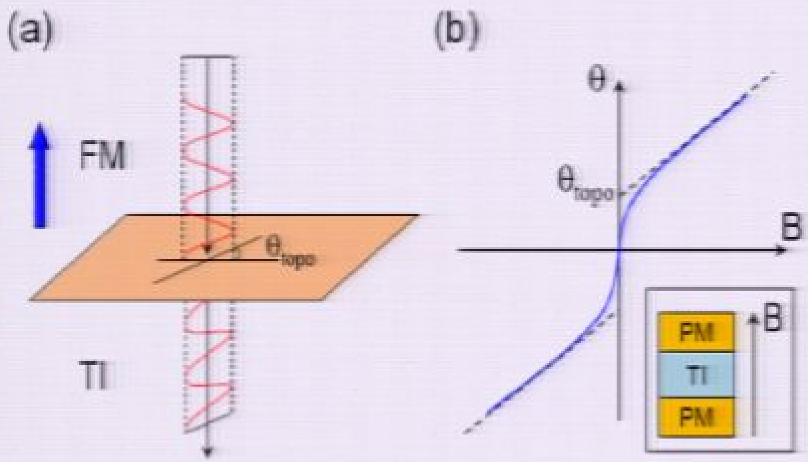
$$\sigma_{H,s} = -e^2/2h$$

$$\Downarrow$$

$$\sigma_{H,s}^{tot} = -e^2/6h$$

Can be in principle be distinguished.

Faraday and Kerr Effect



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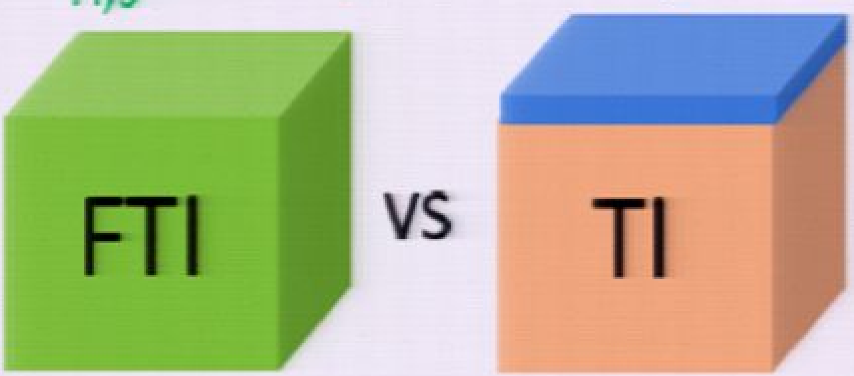

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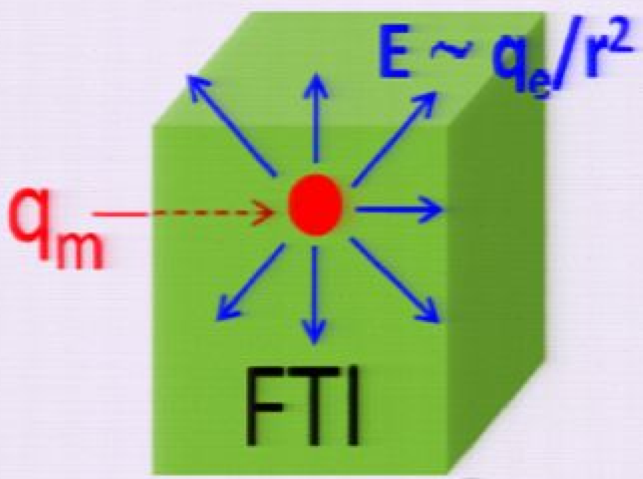
Witten Effect for fractional TI.



$$q_e = \frac{\theta_{\text{eff}} e^2}{8\pi^2} q_m$$

Experimental Protocol:.

- **Catch a monopole from cosmic rays**
- **Insert it into bulk material**
- **Measure electric flux to determine charge**

Ground State Degeneracy.

On non-trivial 3 manifold groundstate can be degenerate.

Different for fractional TI and TI + fractional QH

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Ground State Degeneracy in QH.

Review of QH: $$\sigma_{xy} = \frac{1}{m}\frac{e^2}{h}$$

Effective Description: CS of level m

Equation of motion: $$F_{\mu\nu} = 0$$

Degrees of freedom: flat connections.

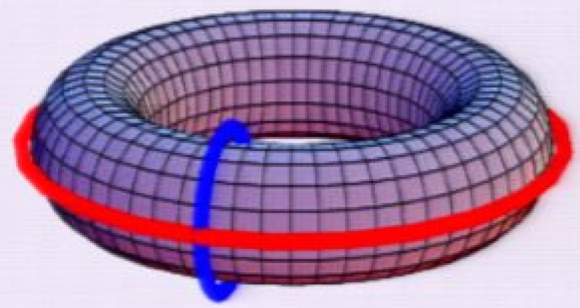
Ground State Degeneracy in QH.

Degrees of freedom: flat connections.

Trivial on R^2

On the torus: Wilson lines

x and y



Wilson lines periodic: x and y= positions on a torus.

Ground State Degeneracy in QH.

$$S_{CS} = m\int A\wedge dA = m\int y\dot{x}$$

CS terms $=$ Magnetic field

Groundstates: Particle on a torus with m units of magnetic flux.

$$d_{GS} = m$$

Ground State Degeneracy in QH.

genus g Riemann surface:

2g non-contractible cycles
→ 2g Wilson lines
→ particle on 2g-torus with
m units of magnetic flux.

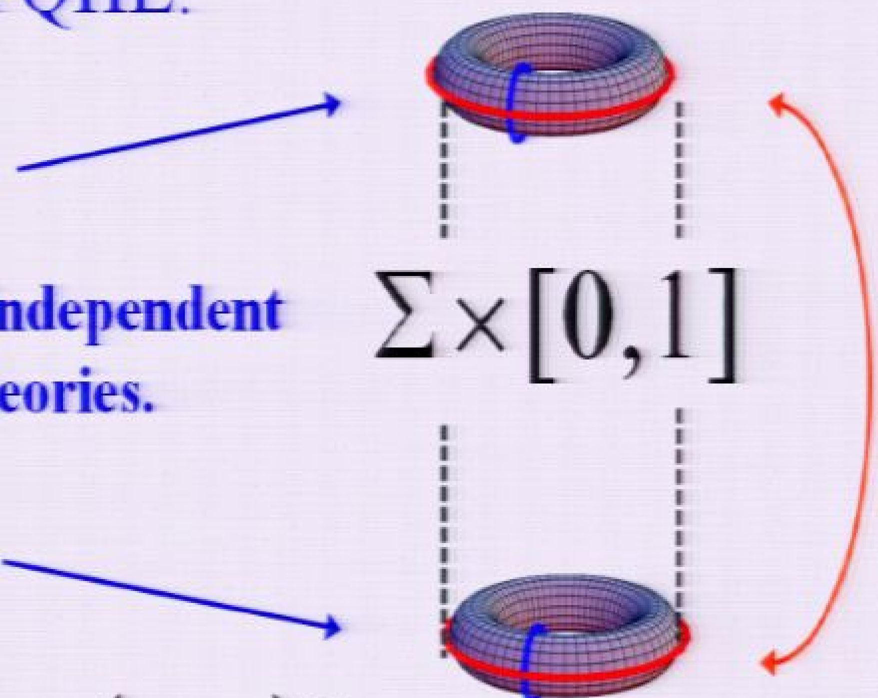
$$d_{GS} = m^g$$

FTI versus TI+FQHE.

TI+FQHE:

FTI:

Two independent CS theories.



Linked by $F^2 = 0$ constraint for vacuum.

$$d_{GS} = \left(m^g\right)^2$$

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T3.

Groundstate trivial on T3.

But: Swingle, Barkeshli, McGreevy, Senthil:

No fractional θ unless groundstate on T3 is degenerate!

Assumes gap for all excitations (not just charged). Avoided in deconfined phase due to color magnetic flux!

Summary.

- Effective field theory for fractional TI can be constructed
- Basic ingredient: fractionalization
- Effective θ follows from anomaly/Dirac quantization
- Requires strong LS coupling and strong interactions.
- Experimental signatures: transport + Kerr/Faraday