Title: Holographic description of quantum field theory

Date: May 29, 2010 09:45 AM

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Abstract: The AdS/CFT correspondence has opened the door to understand a class of strongly coupled quantum field theories. Although the original correspondence has been conjectured based on string theory, it is possible that the underlying principle is more general, and a wider class of quantum field theories can be understood through holographic descriptions. In this talk, I will discuss about a prescription to construct holographic theories for general quantum field theories. As an example, I will present a holographic dual theory for the D-dimensional O(N) vector model. The phase transition and critical behaviors of the model are reproduced through the holographic theory.

Anti de-Sitter Space/Conformal Field Theory (AdS/CFT) correspondence

[Maldacena; Gubser-Klebanov-Polyakov; Witten]

$$Z[J(x)] = \int D\phi(x) e^{-S[\phi]-\int dx J\phi}$$

D-dimension

$$= \int D \text{ ``}J(x,z)\text{''}\, e^{-S'[J(x,z)]}\Big|_{J(x,0)=J(x)}$$

(D+1)-dimension

- D-dim theory is dual to (D+1)-dim theory
- The additional direction corresponds to redundant energy scale
- Bulk space-time is not gauge invariant
- Emergence of classical space-time in large N limit
- In strong coupling limit, the bulk theory has a local description
- A few concrete examples : N=4 SU(N) in d=4, …

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Dictionary in AdS/CFT

Low energy

(D+1)-dim anti-de Sitter space

$$\hat{O}(x) \;\rightarrow\; J_O(x,z)$$

$$\Delta_O \;\rightarrow\; m_O$$

z

High energy

x

D –dim flat space

RG and AdS/CFT

Saddle point equation of motions in the bulk correspond
o beta function(al)s of boundary field theory [Verlinde]

Low energy

EOM $\rightarrow$ RG flow

$$J(x,z) \;=\; J(x,0)$$

Z

$$S_0 + \int dx J(x) O(x)$$

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D –dim flat space

No known general prescription to derive holographic theory from quantum field theory

his talk :

Q. Can we derive (fully quantum) holographic theory for general quantum field theory?

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Zero dimensional field theory :

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Z[J] &= \int d\Phi \; e^{-(S_M+S_J)}, \\
S_M[\Phi] &= M^2\Phi^2, \\
S_J[\Phi] &= \sum_{n=1}^{\infty} J_n\Phi^n.
\end{aligned}
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S_M : bare action with mass M

S_J : deformation with source J

Step 1 : introduce an auxiliary field

$$Z[J] = \mu \int d\Phi d\tilde{\Phi}\; e^{-(S_M + S_J + \boxed{\mu^2 \tilde{\Phi}^2})},$$

`low energy' field

$$\Phi = \phi + \tilde{\phi},$$

`high energy' field

$$\tilde{\Phi} = A\phi + B\tilde{\phi}$$

[Polchinski (84)]

$$Z[J] = \int d\phi d\tilde{\phi}\; e^{-(S_J[\phi+\tilde{\phi}] + M'^2\phi^2 + m'^2\tilde{\phi}^2)},$$

$$M'^2 = M^2 e^{2\alpha dz}$$

$$m'^2 = \frac{M^2}{2\alpha dz}.$$

infinitesimally small parameter

positive constant

Step 2 : rescale the fields

$$\phi \to e^{-\alpha dz}\phi, \qquad \tilde{\phi} \to e^{-\alpha dz}\tilde{\phi}$$

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Step 3 : expand the action in the power of low energy field

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S_j[\phi + \tilde{\phi}] \;=\; & S_j[\tilde{\phi}] \\
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& + j_4\phi^4.
\end{aligned}
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- Usually, one integrates out high energy modes to obtain an effective action for the low energy mode
- Instead, we interpret high energy field as fluctuating sources for the low energy field

Step 4 : decouple low energy field from high energy field

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Step 6 : repeat the steps 1-5

$$Z[J] = \int \Pi_{k=1}^{R} \Pi_{n=1}^{4} (DJ_n^{(k+1)} DP_n^{(k)}) e^{-S[J^{(k)},P^{(k)}]} Z[J^{(R+1)}],$$

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1D Path integral of source fields J and conjugate fields P for partition function

Initial value of J fixed by the original couplings

Non-trivial solution for Z[J]

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\begin{aligned}
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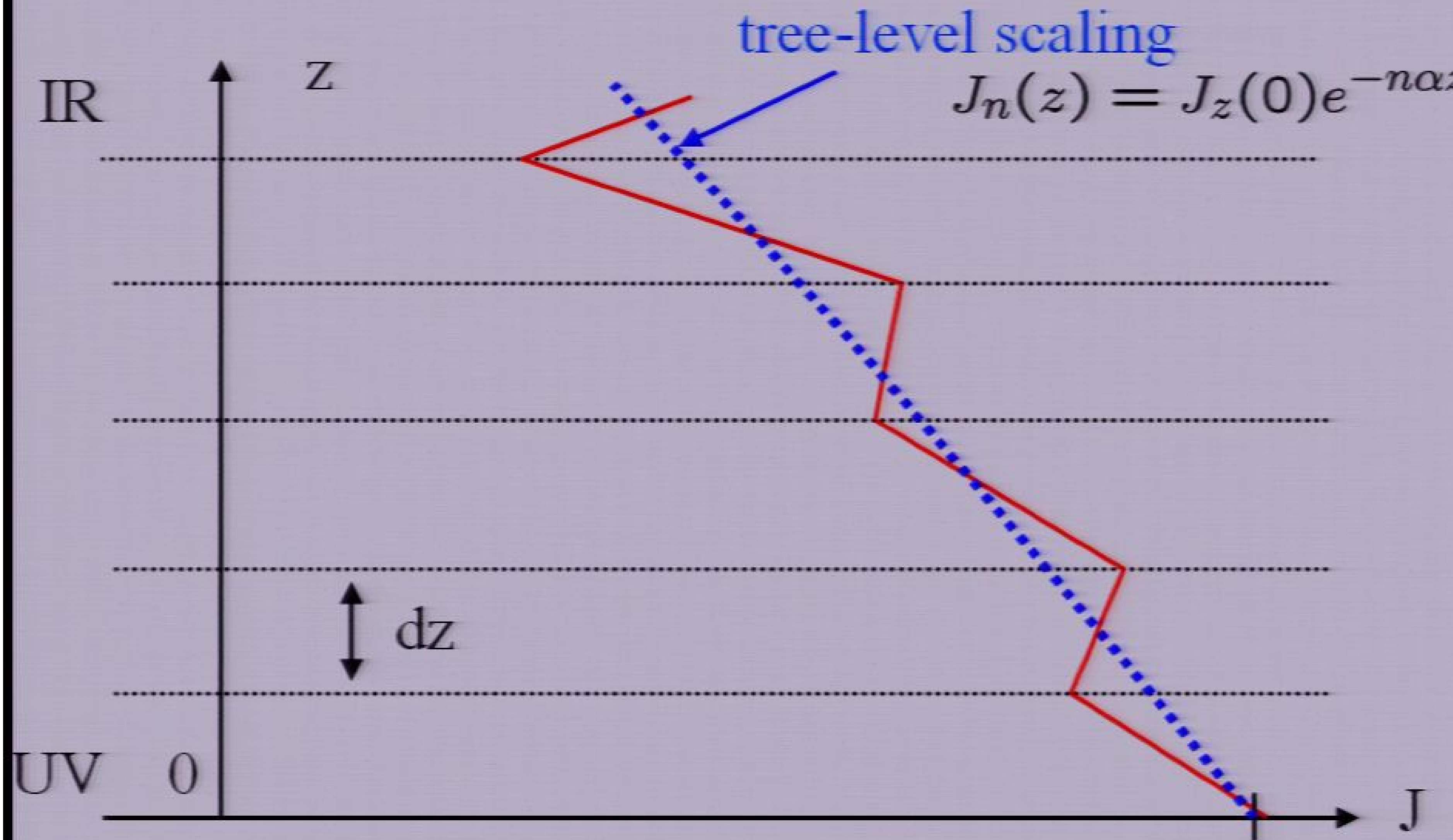
Functional integral over coupling fields



Boundary value : fixed by UV coupling

Physical meaning α

$$L = i(\partial_z J_n + n\alpha J_n)P_n + \frac{\alpha}{2M^2}(iJ_1 + 2P_1J_2 + 3P_2J_3 + 4P_3J_4)^2$$

$$L = \dot{J}_n P_n + \boxed{\alpha} H$$

α : rate high energy modes are eliminated

('speed' of RG flow)

Can be made to be z-dependent α(z)

Interpret z as time : α(z) becomes the lapse function $\sqrt{g_{zz}}$

α(z) is NOT integrated : H=0 NOT imposed

$l = \int_0^\infty \alpha(z)dz$ total proper 'time' is fixed to be infinite

The theory can be viewed as 1D gravitational theory with fixed size along the z-direction

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1D gravitation theory

$$S[J,P] = \int_0^\infty dz[i(\partial_z J_n + n\alpha J_n)P_n + \frac{\alpha}{2M^2}(iJ_1 + 2P_1J_2 + 3P_2J_3 + 4P_3J_4)^2$$

The theory is local in the `bulk'

Only one set of independent mode because the original theory has only one propagating mode

Freedom to choose independent fields (say, J_3, P_3)

Physical meaning α

$$L = i(\partial_z J_n + n\alpha J_n)P_n + \frac{\alpha}{2M^2}(iJ_1 + 2P_1 J_2 + 3P_2 J_3 + 4P_3 J_4)^2$$

$$L = \dot{J}_n P_n + \boxed{\alpha} H$$

α : rate high energy modes are eliminated

('speed' of RG flow)

Can be made to be z-dependent α(z)

Interpret z as time : α(z) becomes the lapse function $\sqrt{g_{zz}}$

α(z) is NOT integrated : H=0 NOT imposed

$l = \int_0^\infty \alpha(z) dz$ total proper 'time' is fixed to be infinite

The theory can be viewed as 1D gravitational theory

with fixed size along the z-direction

1D gravitation theory

$$S[J,P] = \int_0^\infty dz[i(\partial_z J_n + n\alpha J_n)P_n + \frac{\alpha}{2M^2}(iJ_1 + 2P_1J_2 + 3P_2J_3 + 4P_3J_4)^2$$

The theory is local in the `bulk'

Only one set of independent mode because the original theory has only one propagating mode

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S_M : quadratic action with UV cut-off M

S_J : deformation with sources

$J_{ab..}$: flavor dependent sources

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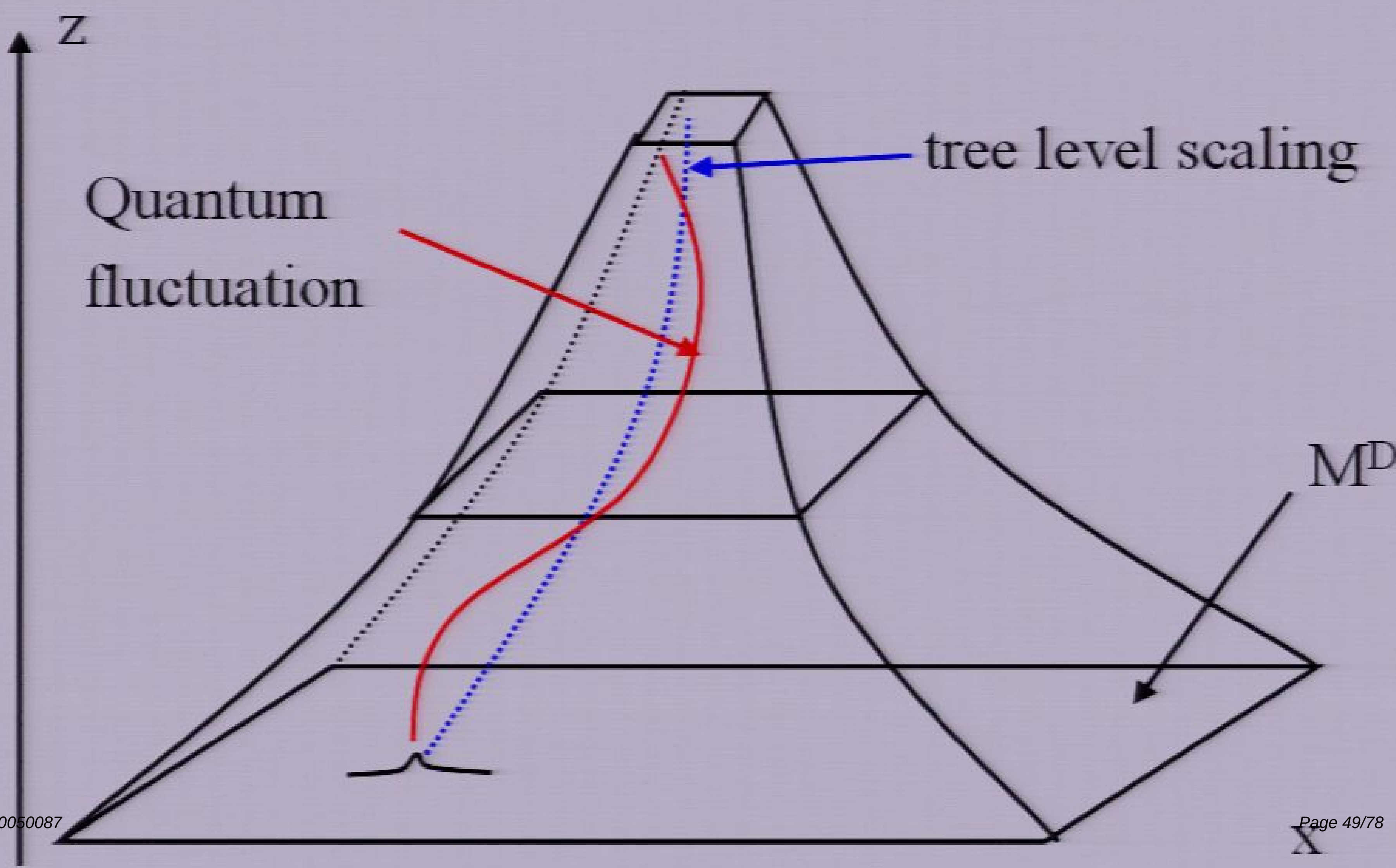
Bulk Space : AdS



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(D+1)- dimensional local quantum theory

Path integral of source fields J and conjugate fields P on $M^D * [0,\infty)$

Boundary value of J fixed by the original couplings

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Action is manifestly proportional to N

Singlet fields become classical

Correlation function of singlet fields can be computed

from saddle point solutions of the bulk action :
reproduce the known critical exponents, e.g.

[ϕ2]=2 (D=3)

Critical exponent

Turn on x-dependent sources at UV boundary :

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IR boundary condition

Bulk solution with UV boundary condition :

$$f_a(\mathbf{p}, z) \;=\; y_a(\mathbf{p}) \left[1 - \frac{K\left(pe^{\alpha z}/M\right)}{K\left(p/M\right)} \right]$$

One arbitrary parameter should be fixed by additional boundary condition.

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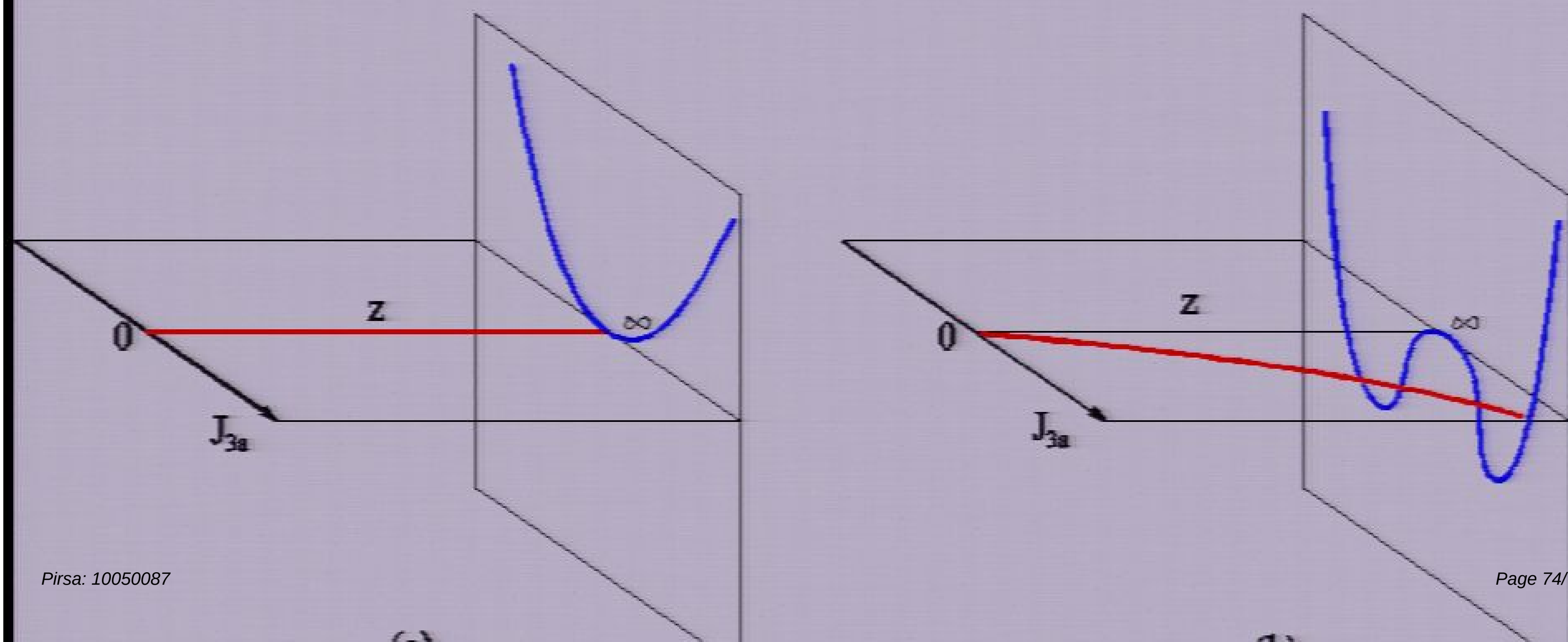
Spontaneous symmetry breaking

- Non-singlet conjugate field spontaneously develop expectation value at IR

$P_{3a} \sim < \phi^2 \phi_a > = 0$ $\qquad$ $P_{3a} \sim < \phi^2 \phi_a > \neq 0$



Physical Locality

- In O(N) vector model, one has to keep non-singlet fields in the bulk to maintain the locality : theory become non-local if one integrate out non-singlet modes [higher spin gauge theory (Klebanov, Polyakov)]
- In more strongly coupled theories (matrix models), there will be fewer light bulk fields and the holographic description may become more useful

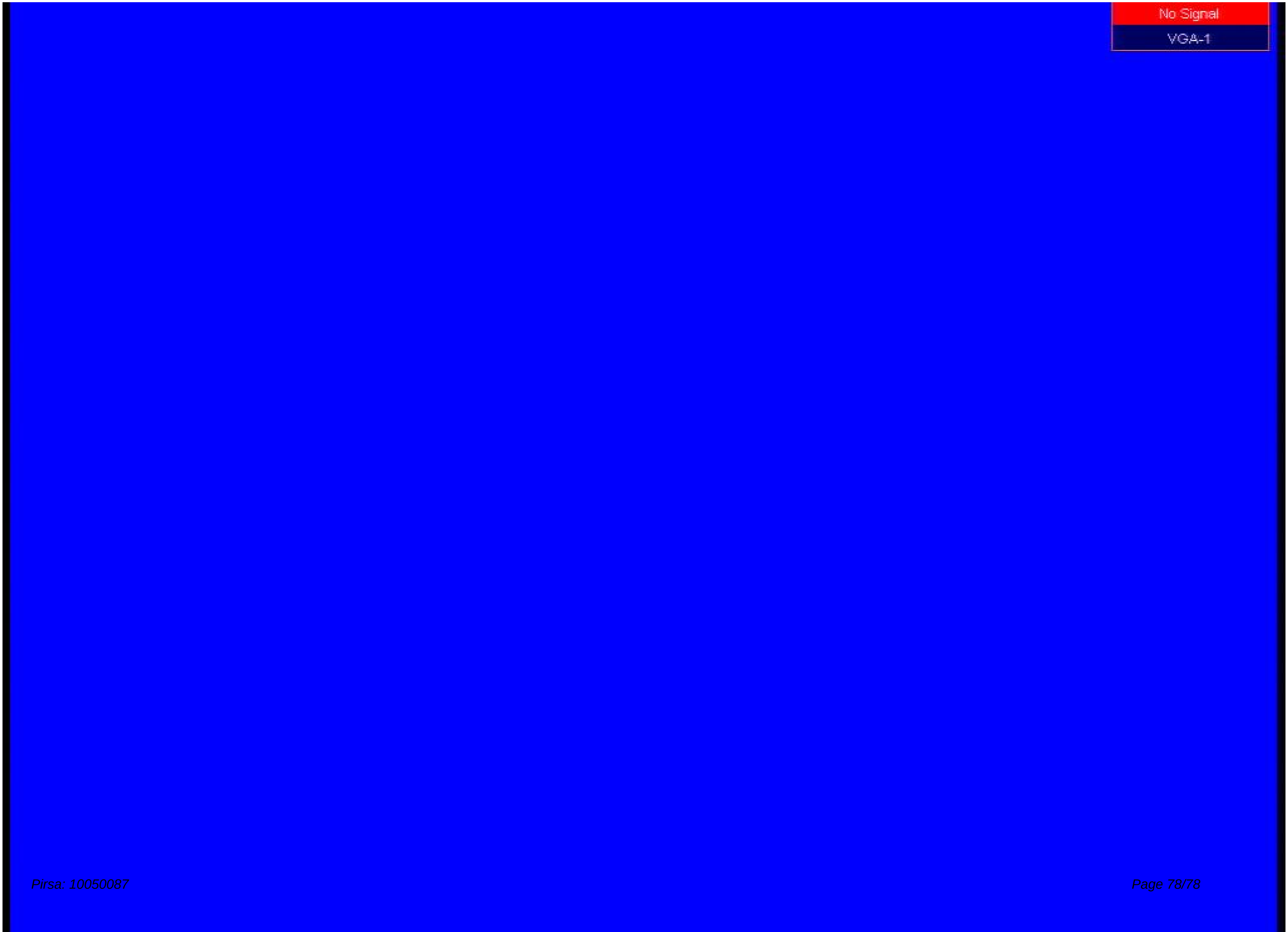
Summary

- General D-dimensional QFT can be mapped into (D+1)-dimensional quantum theory for fluctuating source fields

- Field theory predictions reproduced for O(N) model

Future directions

- Application the method to more strongly coupled systems (matrix models, non-Fermi liquids,…)
- What bulk theories are dual to local boundary theories ? Classification of strongly interacting field theories using holographic description
- Fully covariant formalism
- …….


No Signal
VGA-1