Title: Spin Liquid and Topological Insulator in Frustrated Magnets

Date: May 28, 2010 02:45 PM

URL: http://pirsa.org/10050085

Abstract: Recently several proposals are made for possible spin liquid and topological insulator phases in frustrated magnets. I will review some of these efforts and present some new results. Implications to real materials will also be made.

# Spin Liquid and Topological Insulator in Frustrated Magnets

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Perimeter Institute, Waterloo, May 28, 2010

# Outline

☆ Prelude: Frustrated Magnets/Interactions

☆ Spin Liquid

1. Theory of Spin Liquid
2. Experimental Race for Spin Liquid
3. $Na_4Ir_3O_8$: 3D Spin Liquid

☆ Topological Insulator

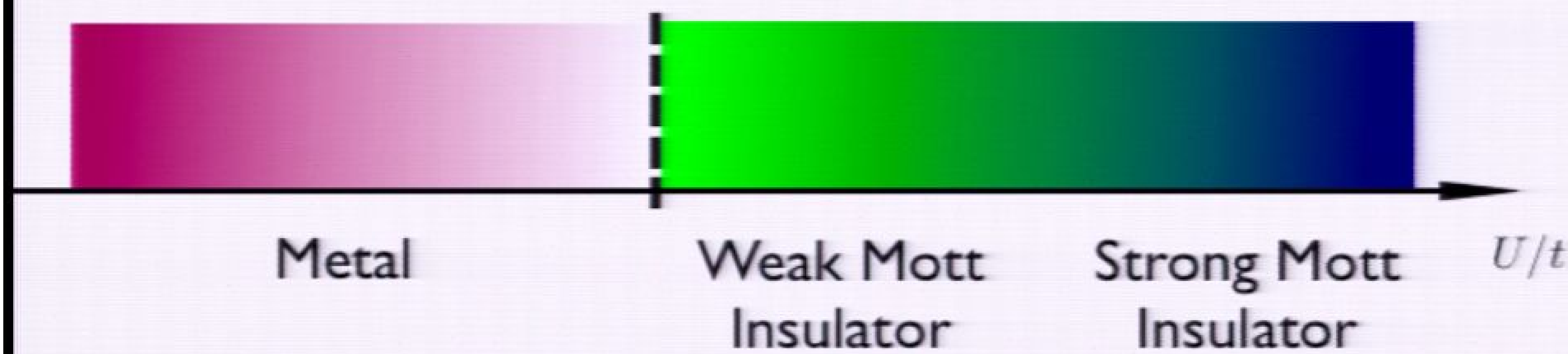
1. Theory of Topological Band Insulator
2. Towards Topological Mott Insulator
3. $A_2Ir_2O_7$: 3D Topological Insulator

# Prelude:
# Frustrated Magnets

# Mott Insulators

## Hubbard Model

$$H = \sum_{ij,\alpha} t_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + U \sum_{i} (n_i - \bar{n})^2$$




Insulators with an odd number of electrons per unit cell

# Mott Insulators

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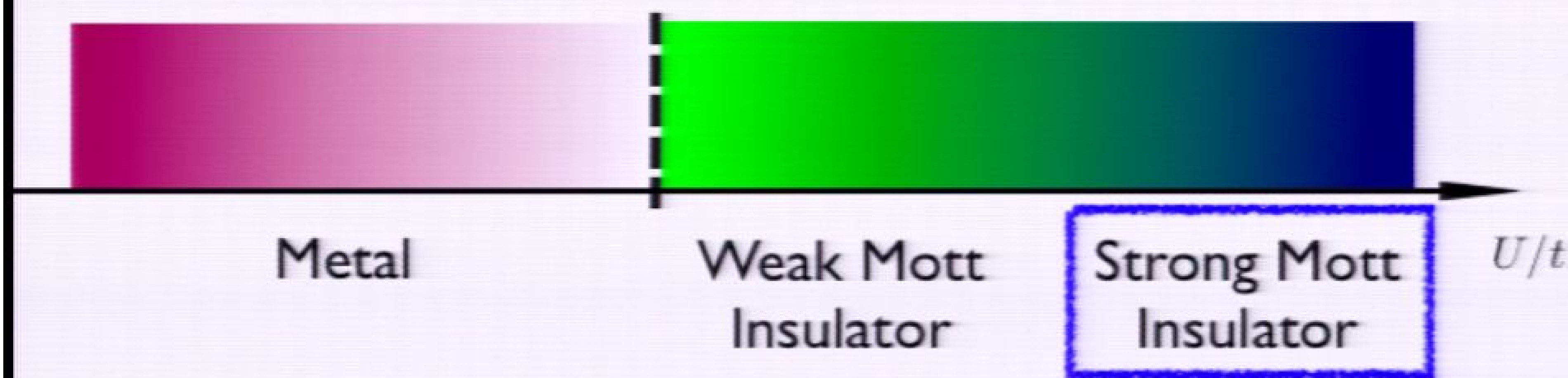
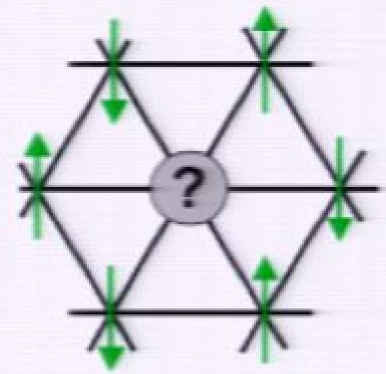


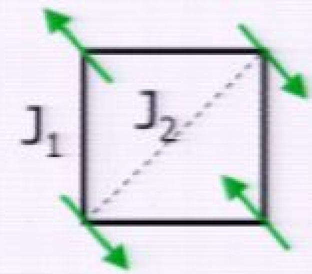

$U/t \gg 1$ $\quad H_{\rm eff} = \sum_{ij} J_{ij}\, \mathbf{S}_i \cdot \mathbf{S}_j \quad J \sim t^2/U$

# Geometric Frustration

The arrangement of spins on a lattice precludes (fully) satisfying all interactions at the same time




Large degeneracy of the (classical) ground state manifold $\sim e^{\alpha N}$




Consequence:

No energy scale of its own; any perturbation is strong

# Mott Insulators

## Hubbard Model

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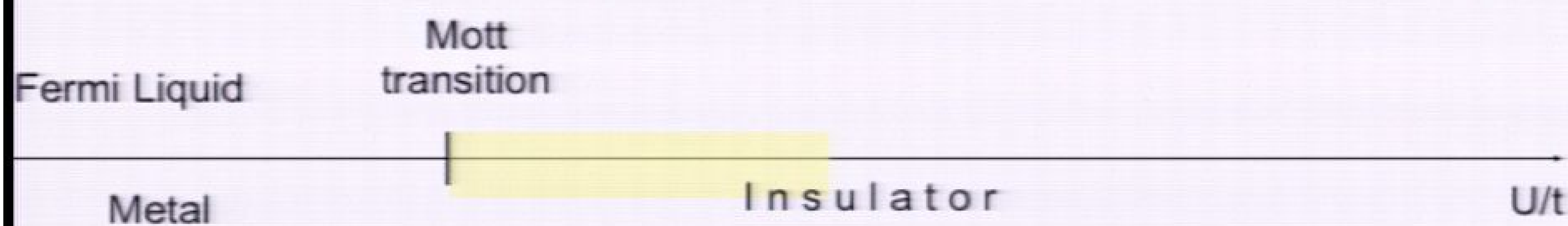
Virtual charge fluctuations are important !

# Importance of charge fluctuations



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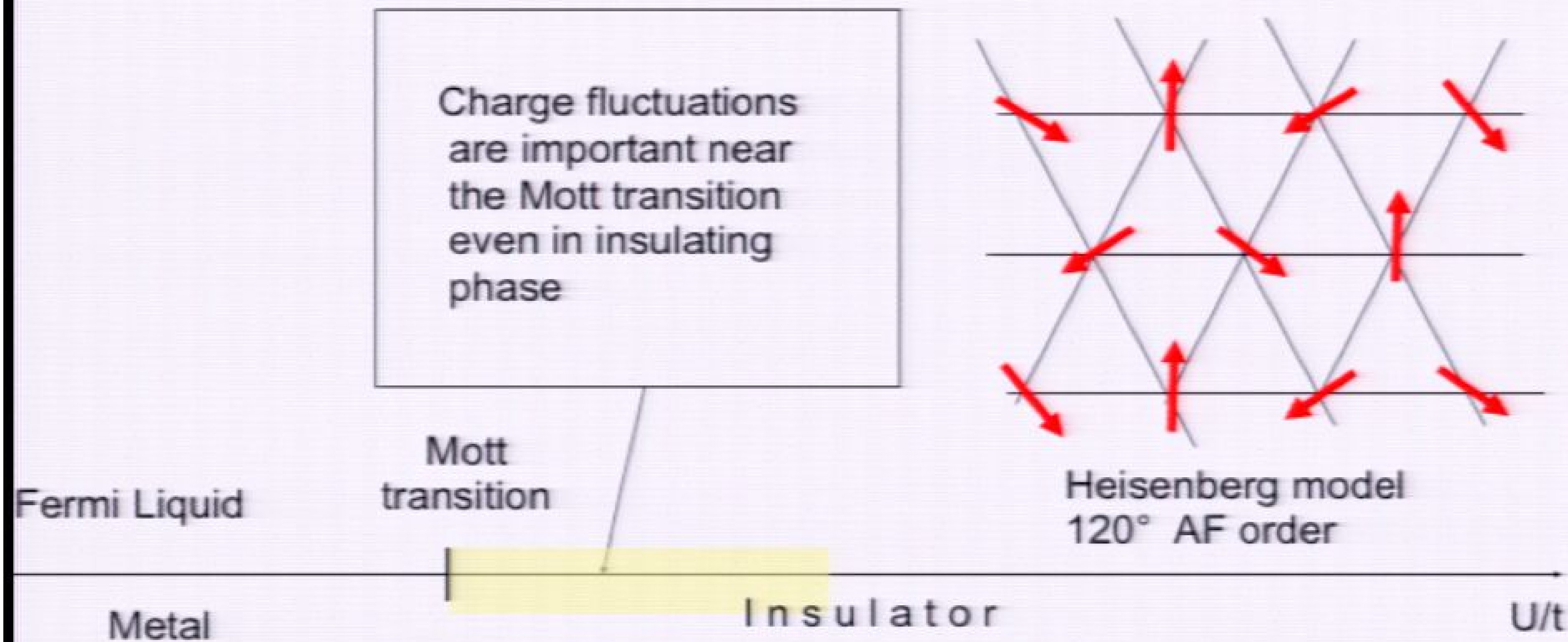
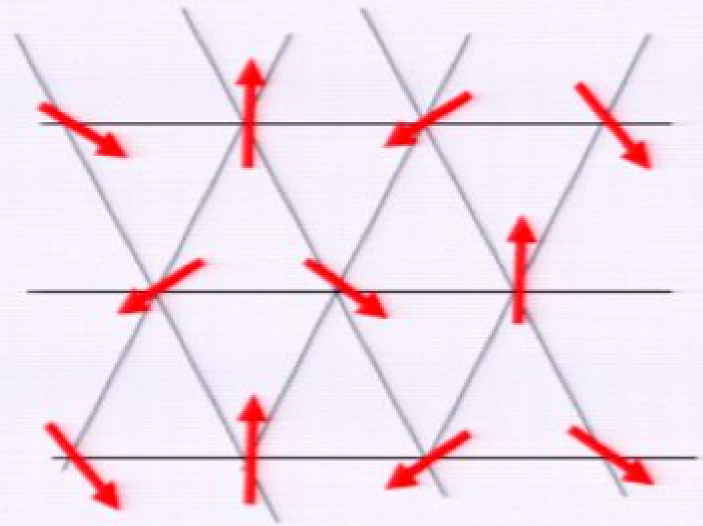


# Importance of charge fluctuations

Charge fluctuations are important near the Mott transition even in insulating phase



Heisenberg model
120° AF order

Fermi Liquid

Mott transition

Metal

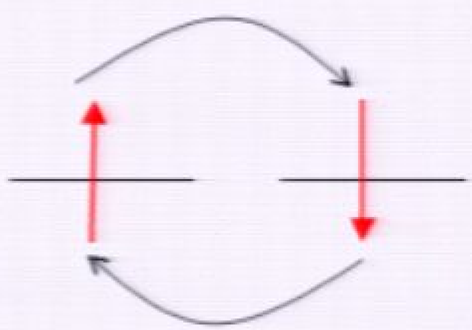
Insulator

U/t

ada (2003)

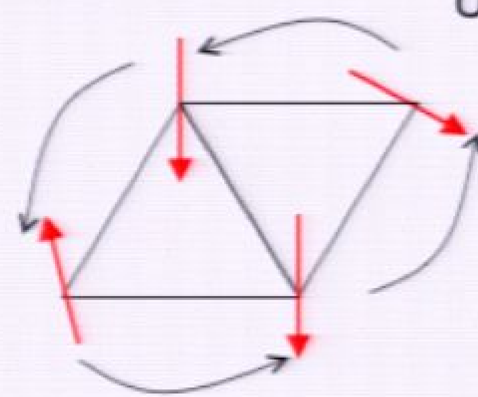
trunich (2005)

S. Lee and P. A. Lee (2005)



$J \sim t^2/U$

+



+

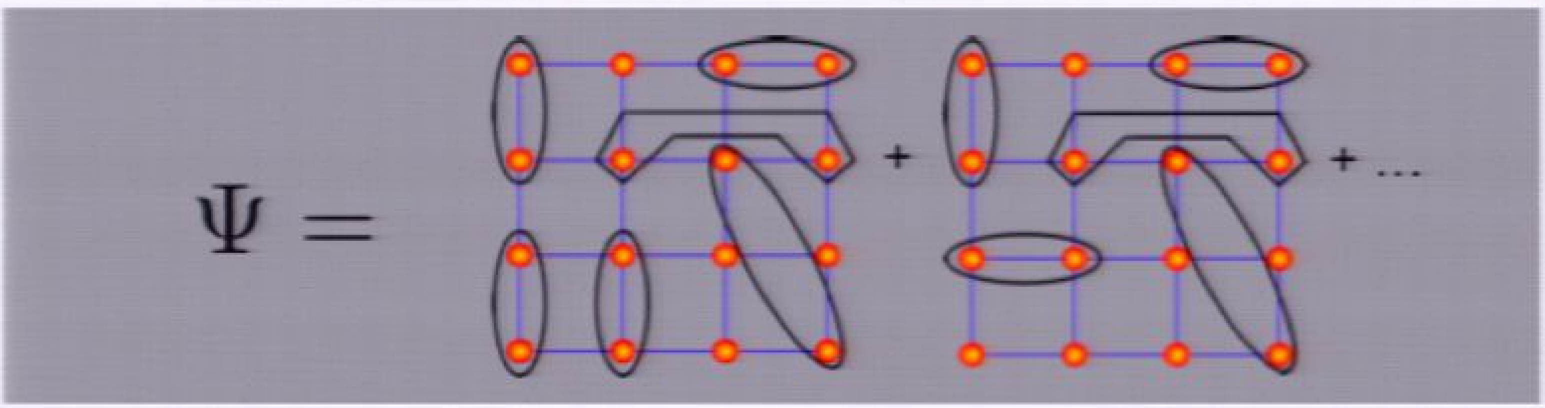
$J' \sim t^4/U^3$

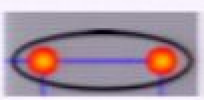
# Ultimate Frustration

## Disorder by Disorder: "Strong" Quantum Fluctuations

Novel Quantum State at T=0 without a broken symmetry - Quantum Spin Liquid

P. W. Anderson 73, 87



$$= |\mathrm{VB}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Superposition of valence bond configurations

# Quantum Spin Liquid

# Slave Particle Approach

## Fermionic representation of the spin operator

$$\vec{S}_i = \frac{1}{2} f^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} f_{i\beta} \quad \text{with the constraint} \quad \sum_\alpha f^\dagger_{i\alpha} f_{i\alpha} = 1 \quad \alpha, \beta = \{\uparrow, \downarrow\}$$

## Mean-Field Theory

# Slave Particle Approach

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$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$ $\longrightarrow$ fermion-fermion interaction

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$\chi_{ij} = \langle f^\dagger_{i\alpha} f_{j\alpha} \rangle$ fermion "kinetic" energy dynamically generated

$\Delta_{ij} = \langle \epsilon_{\alpha\beta} f_{i\alpha} f_{j\beta} \rangle$ possible pairing correlation

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$H_{MF} = J \sum_{ij} \left[ \chi_{ij} f^{\dagger}_{j\alpha} f_{i\alpha} + \Delta_{ij} \epsilon_{\alpha\beta} f^{\dagger}_{j\beta} f^{\dagger}_{i\alpha} + h.c \right]$ The constraint is only imposed on average

# Slave Particle Approach

## Projected Wave Function Approach

$$\Psi = P_G \Psi_{MF}$$

Impose the constraint exactly

Project out unphysical Hilbert space in
the mean-field ground states

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# Gauge Theory

X. G. Wen, S. Sachdev,
M. P. A. Fisher, T. Senthil,
P. A. Lee, N. Nagaosa .......

In the spin liquid phases, the fermionic spinons are liberated (deconfined) and become emergent excitations

✔ When $\Delta_{ij} = 0$ $\quad H_{\rm eff} = J \sum_{ij} |\chi_{ij}| e^{i a_{ij}} f^{\dagger}_{i\alpha} f_{j\alpha}$ is U(1) gauge invariant

$$f_{i\alpha} \to e^{i\theta_i} f_{i\alpha} \qquad a_{ij} \to a_{ij} + \theta_i - \theta_j$$

The resulting theory is a U(1) gauge theory with fermionic spinons

U(1) Spin Liquid

✔ When $\Delta_{ij} \neq 0$ Only $a_{ij} = 0, \pi$ is allowed: Z2 gauge theory

Z2 Spin Liquid

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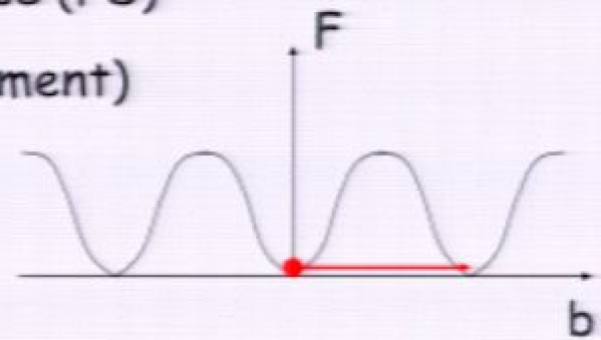
✔ U(1) Spin Liquid

Unpaired spinons - gapped, Dirac or Fermi surface (FS)

Spinons must be gapless in d=2 (to avoid confinement)

$\mathcal{L} \sim \mathbf{e}^2 + \cos(\nabla \times \mathbf{a})$




Strongly-coupled gauge theory:

FS in d=2: S. S. Lee, M. Metlitski, S. Sachdev, J. McGreevy, H. Liu, T. Senthil .......

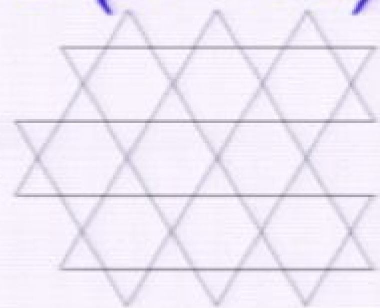
✔ Z2 Spin Liquid

Paired spinons

T > 0 Ising transition in d=3

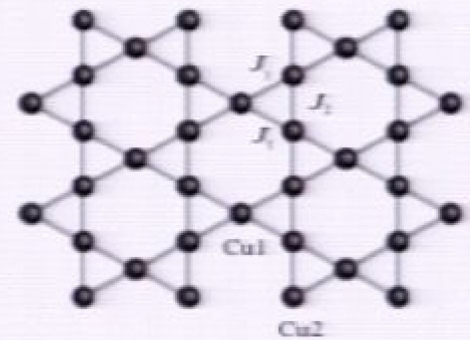
# Race for Spin Liquid

# Search for Quantum Spin Liquid (S=1/2)



**Herbertsmithite** "Ideal" Kagome lattice

$ZnCu_3(OH)_6Cl_2$ D. G. Nocera, Y. S. Lee

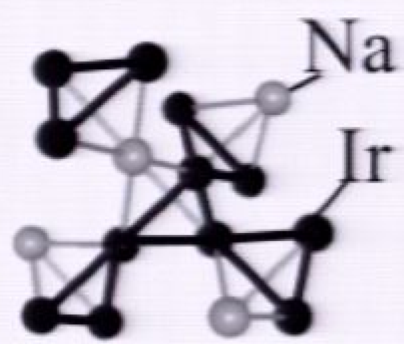



**Volborthite** Distorted Kagome lattice

$Cu_3V_2O_7(OH)_2 . 2H_2O$ Z. Hiori

$\kappa$-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ Triangular Lattice; near Mott transition

K. Kanoda




**Hyper-Kagome** $Na_4Ir_3O_8$

H. Takagi

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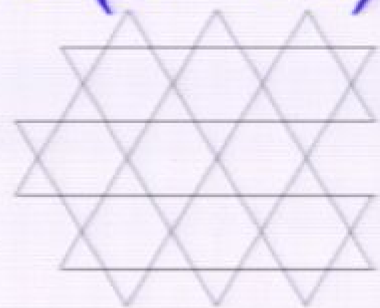

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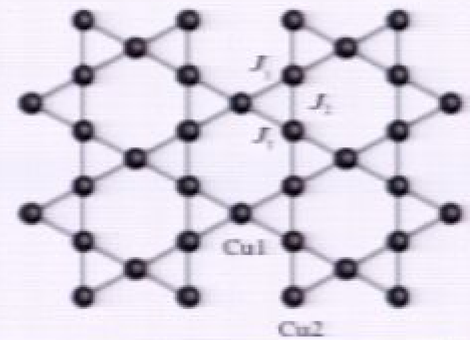


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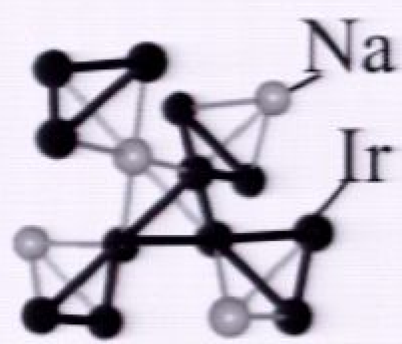
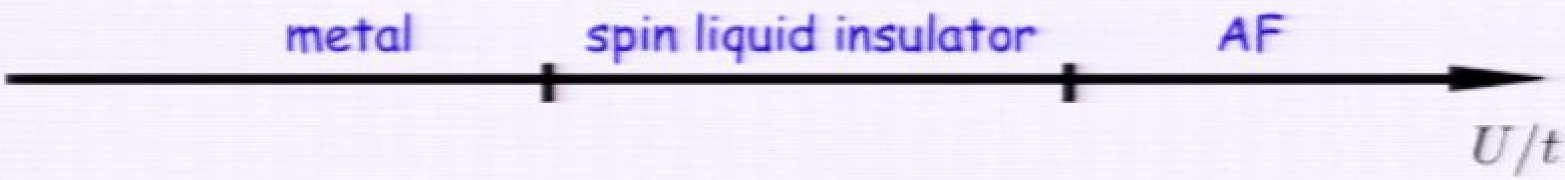


# Weak Mott Insulator

$\kappa$-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ Triangular Lattice; near Mott transition

K. Kanoda

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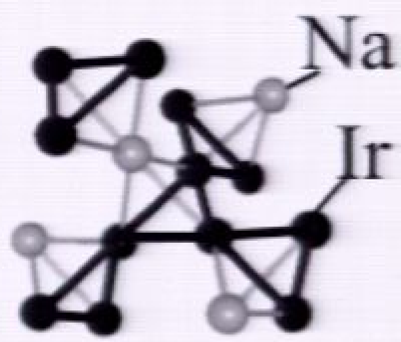
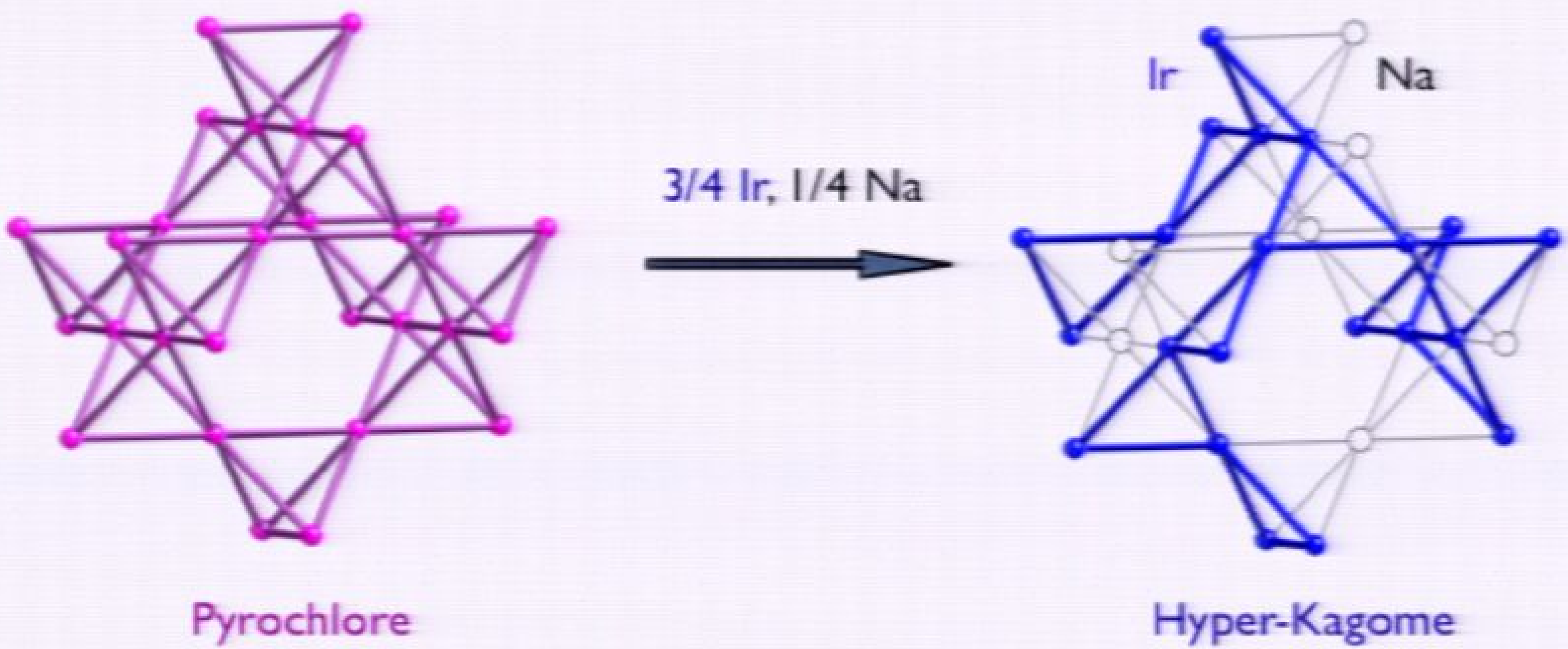



# Hyper-Kagome Lattice: $Na_4Ir_3O_8$

# Three-dimensional Frustrated Magnet

$Na_4Ir_3O_8$ has a Hyper-Kagome sublattice of Ir ions



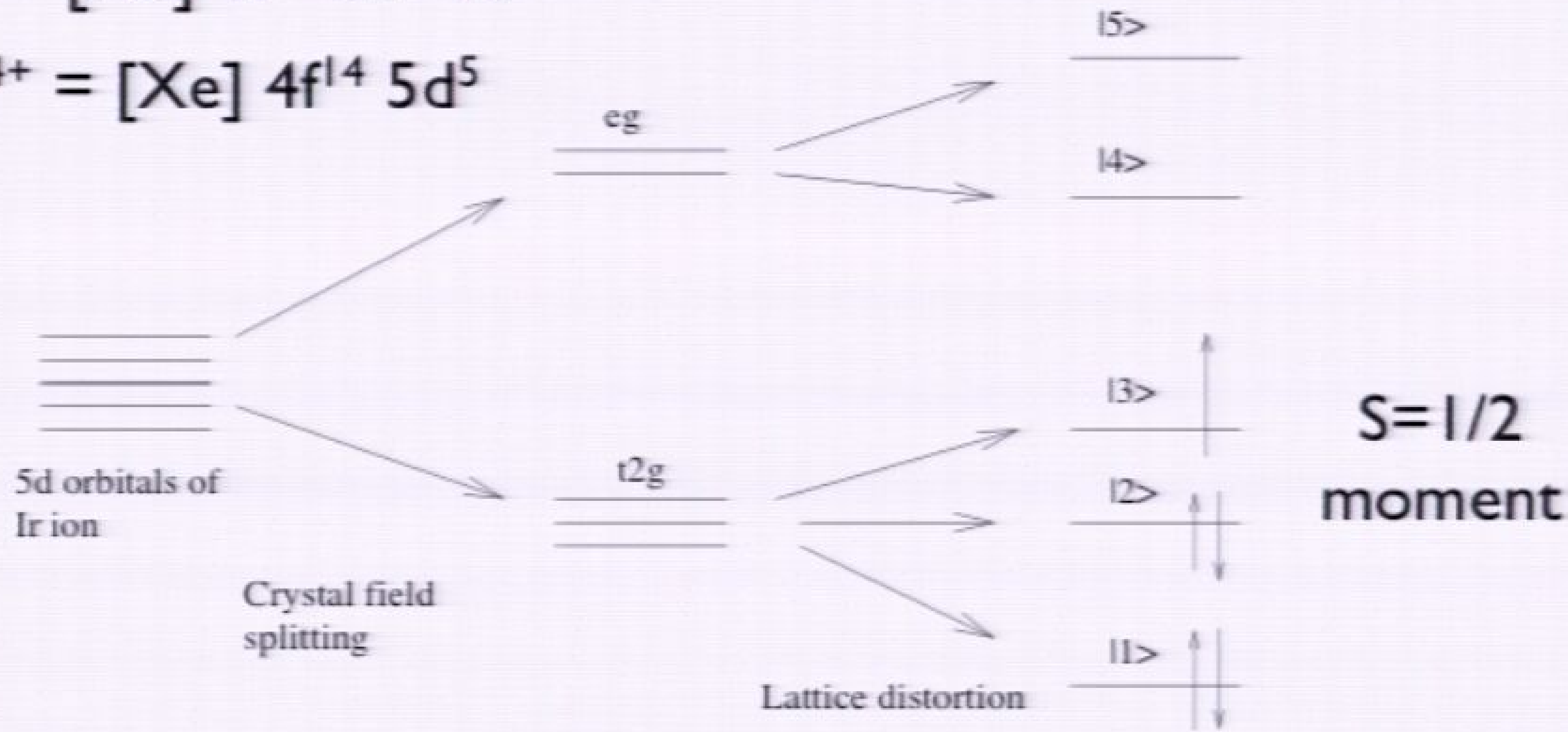

$Ir^{4+}$ ($5d^5$) carries "S=1/2" moment ?

Y. Okamoto, M. Nohara, H. Agura-Katrori, and H. Takagi, PRL 99, 137207 (2007)

# Small spin-orbit coupling limit

$Ir = [Xe]\ 4f^{14}\ 5d^7\ 6s^2$

$Ir^{4+} = [Xe]\ 4f^{14}\ 5d^5$

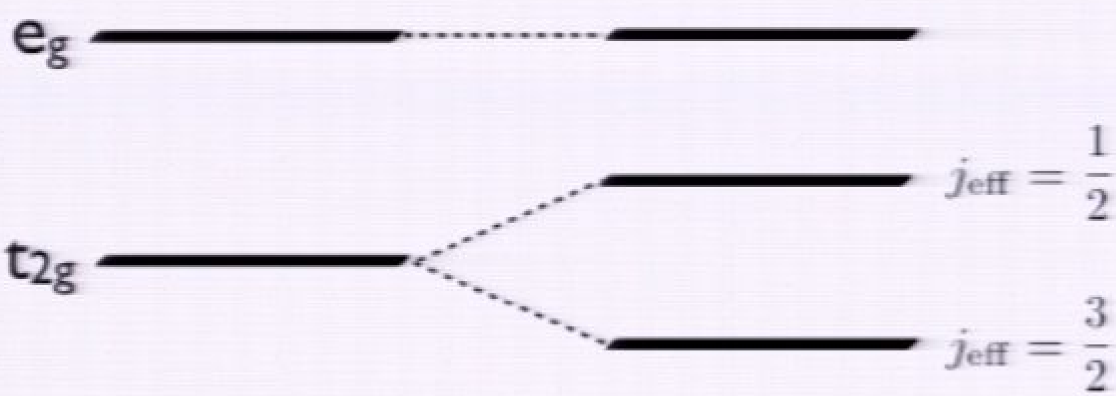



Without Spin-Orbit Coupling

# 5d orbitals of $Ir^{4+}$: large spin-orbit coupling

$Ir^{4+}$ = [Xe] $4f^{14}\,5d^5$ $\qquad H_{\rm so} = \lambda_{so} \mathbf{L}_i \cdot \mathbf{S}_i$

$e_g$ - $t_{2g}$ splitting $\gg \lambda_{\rm so} \gg$ splitting within $t_{2g}$




$$|\uparrow_j\rangle \;=\; \frac{1}{\sqrt{3}}\left(i|xz,\downarrow_s\rangle + |yz,\downarrow_s\rangle + |xy,\uparrow_s\rangle\right)$$
$$|\downarrow_j\rangle \;=\; -\frac{1}{\sqrt{3}}\left(i|xz,\uparrow_s\rangle - |yz,\uparrow_s\rangle + |xy,\downarrow_s\rangle\right)$$

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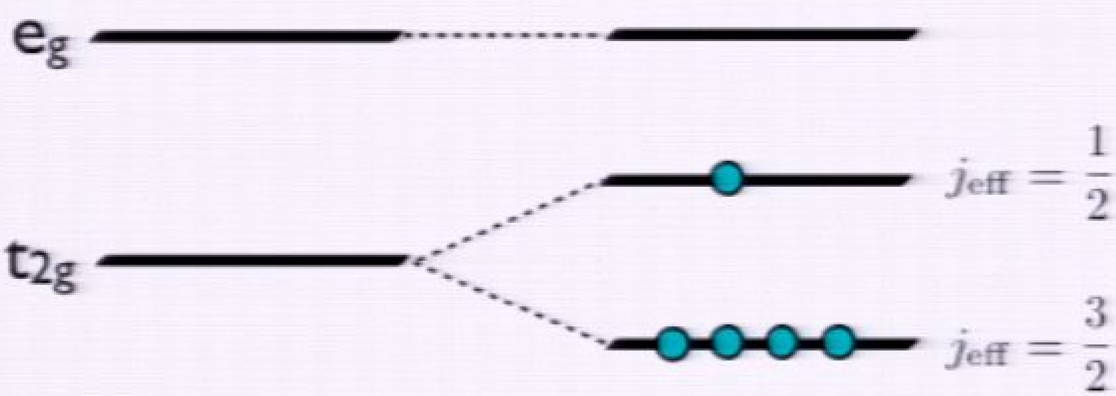


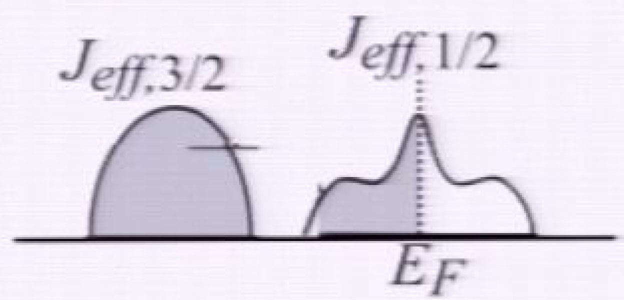


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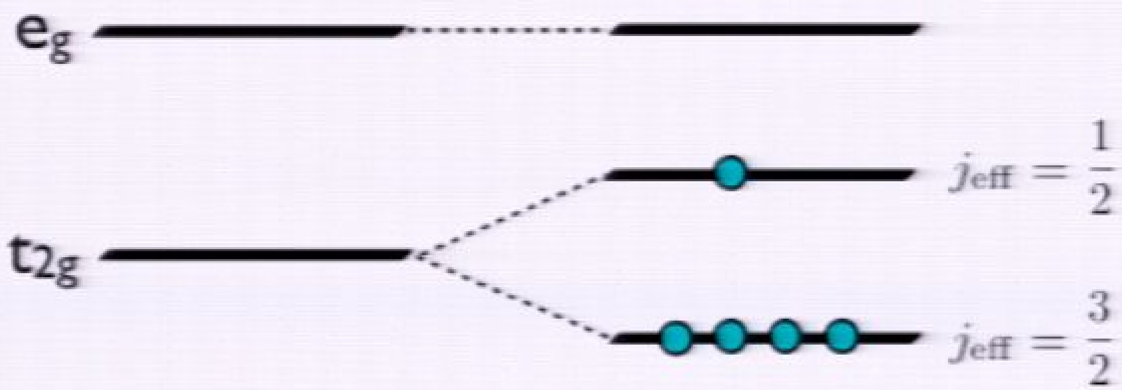


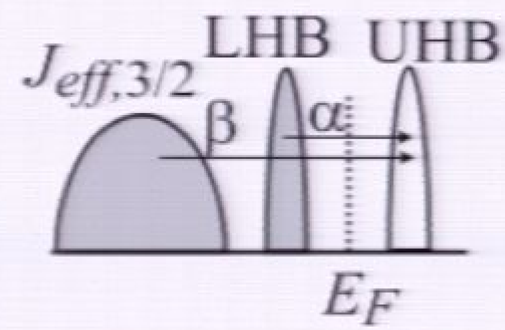


Half-filled $j_{\text{eff}} = \frac{1}{2}$ band

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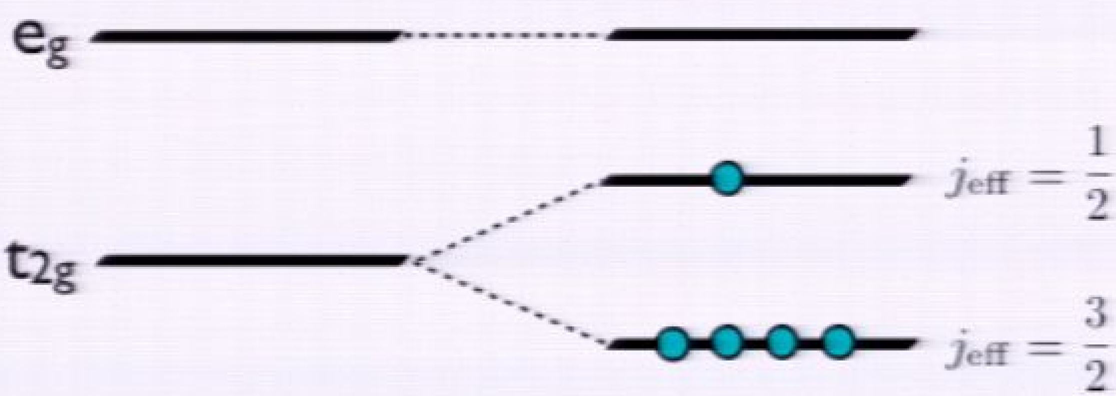


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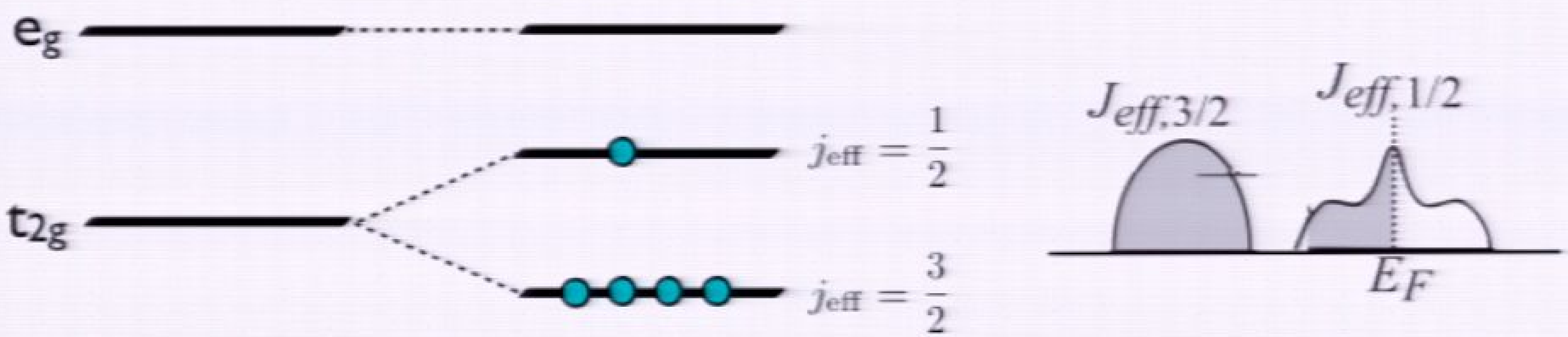


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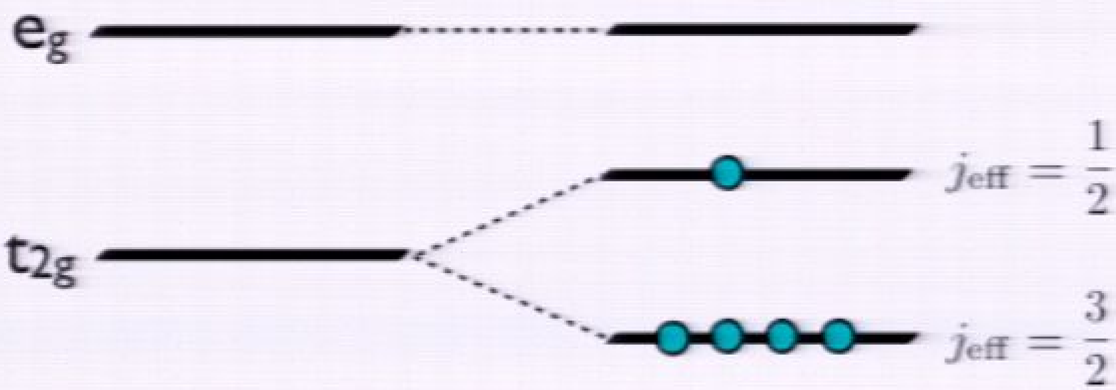


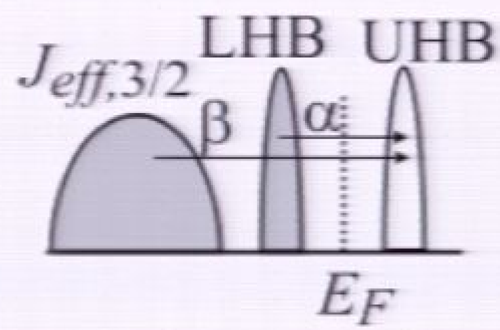


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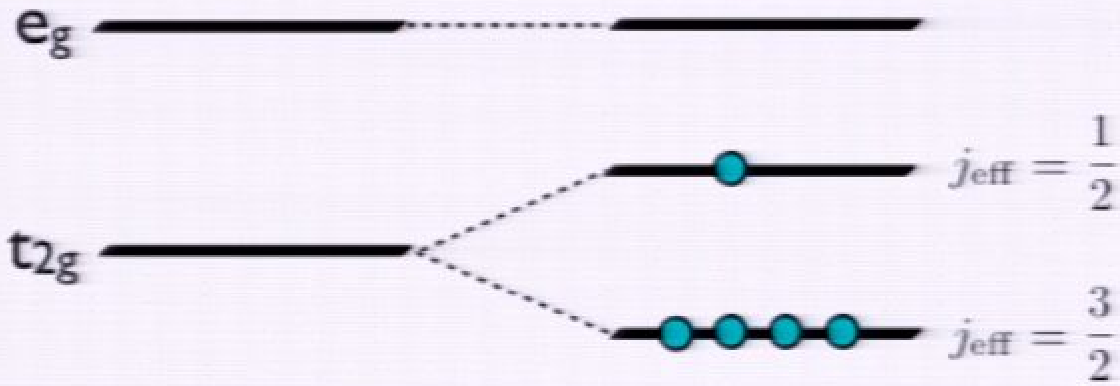


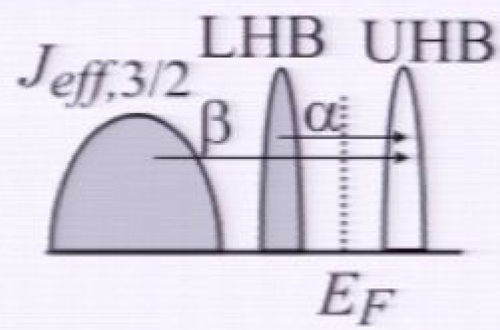


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Spin-orbit coupling enhances the correlation effect

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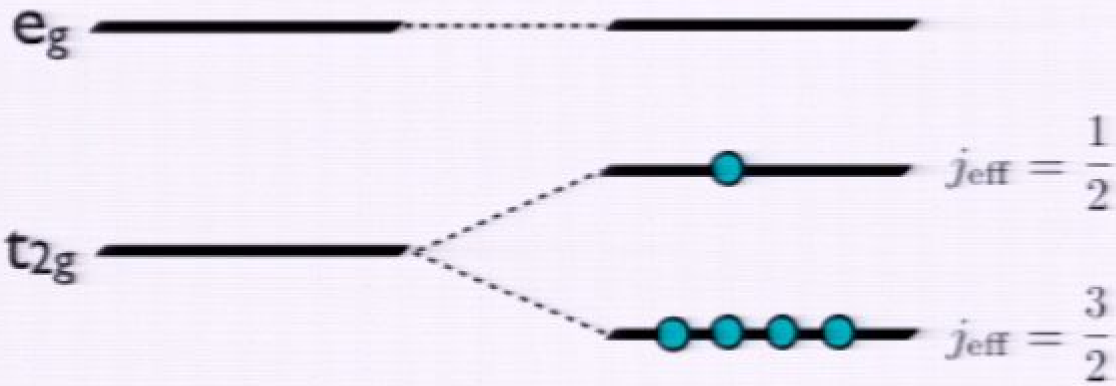
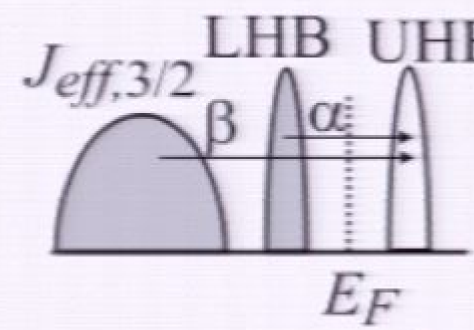


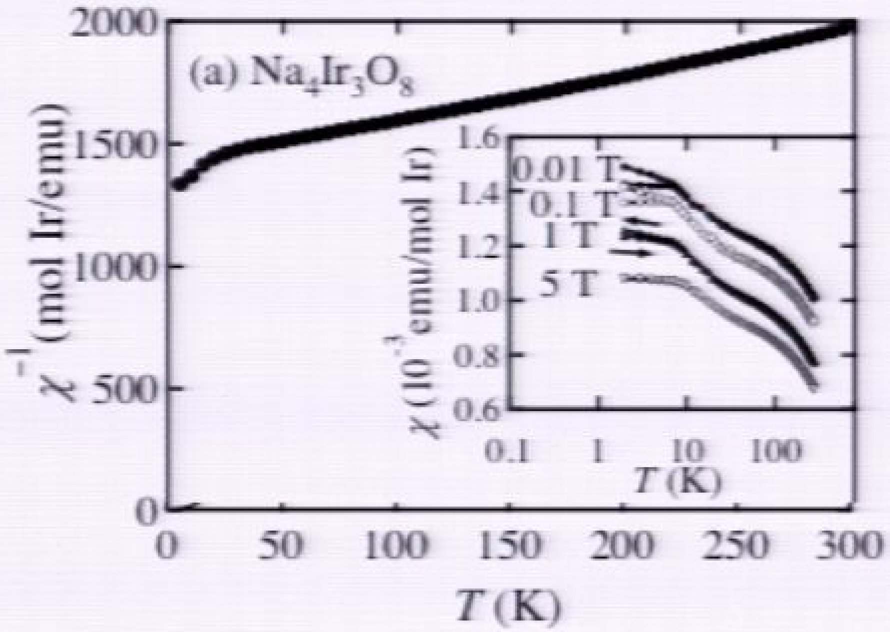




Half-filled $j_{\rm eff} = \frac{1}{2}$ band

Spin-orbit coupling enhances the correlation effect
(effective bandwidth reduced)

# Inverse Spin Susceptibility; Strong Spin Frustration

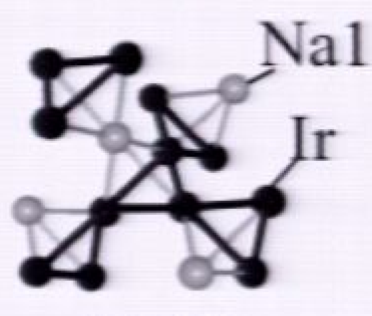
Y. Okamoto, M. Nohara, H. Agura-Katrori, and H. Takagi, PRL 99, 137207 (2007)




Curie-Weiss fit

$\Theta_{CW} = -650K$

No magnetic ordering down to $|\Theta_{CW}|/300$

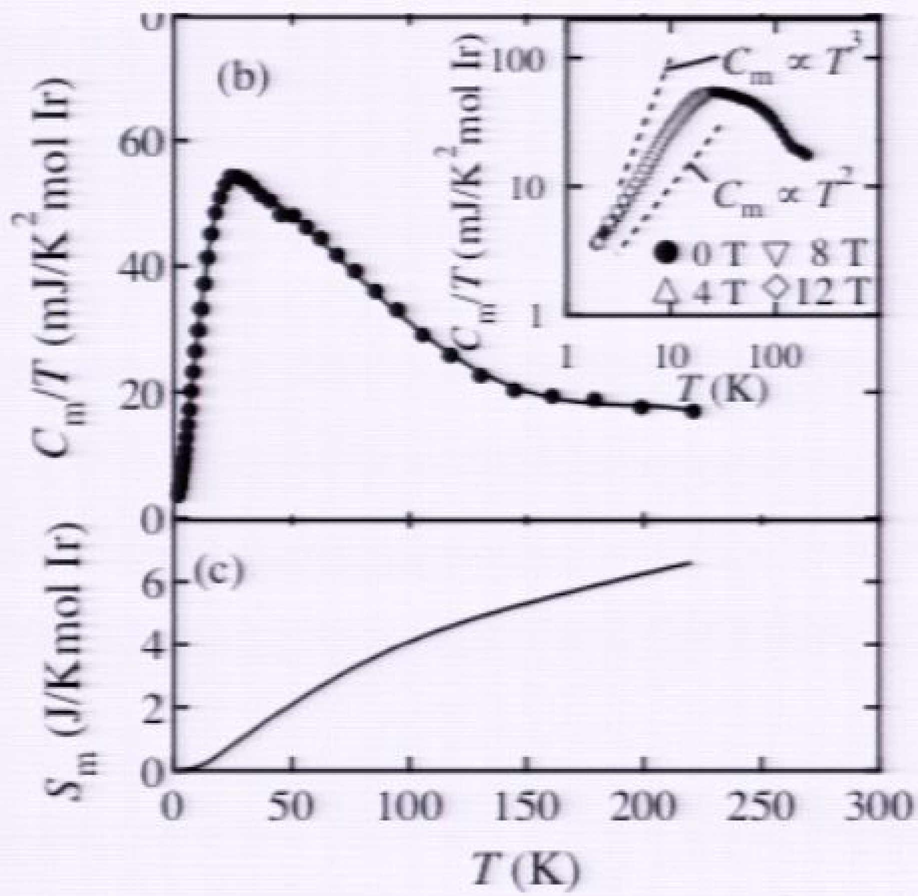



$P4_132$

$\chi(T) \sim$ constant

# Specific Heat; Low Energy Excitations ?

Y. Okamoto, M. Nohara, H. Agura-Katrori, and H. Takagi, PRL 99, 137207 (2007)

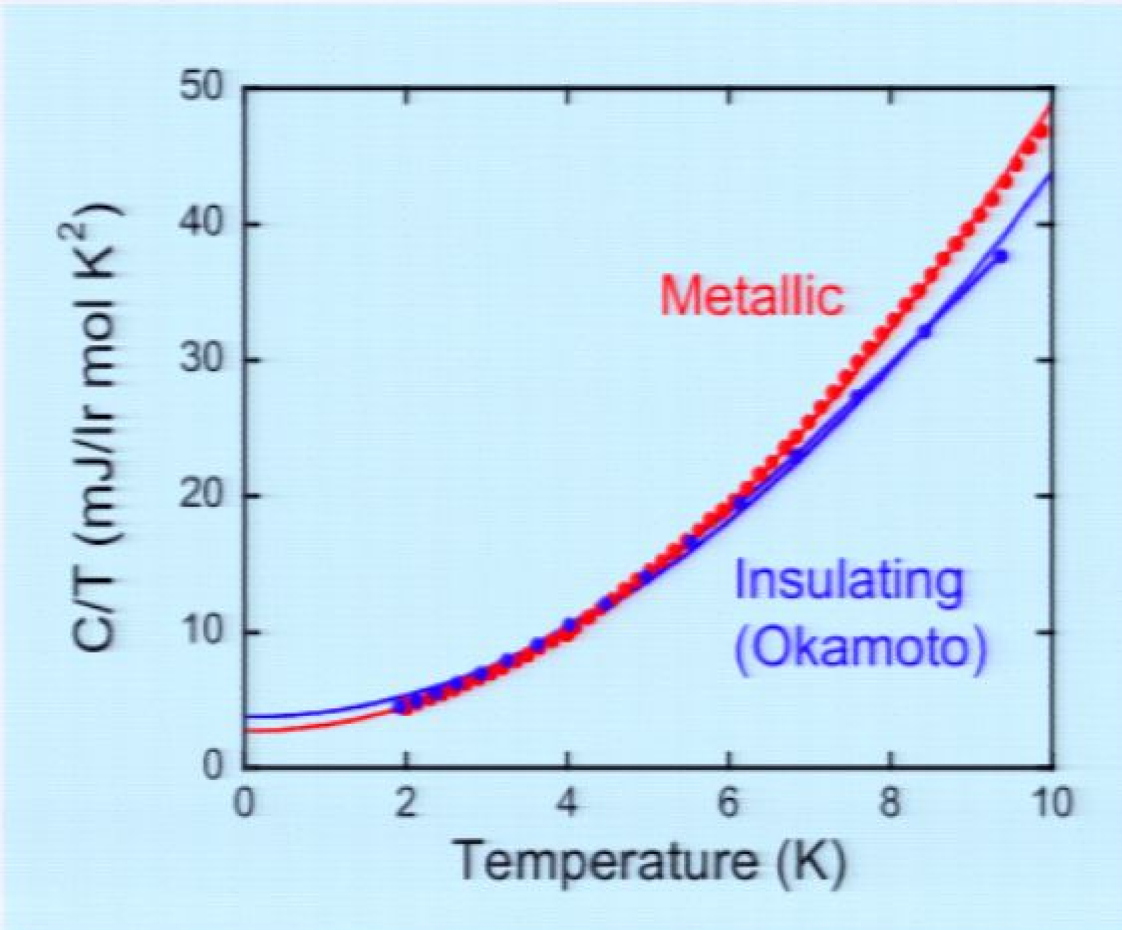



No Magnetic Ordering

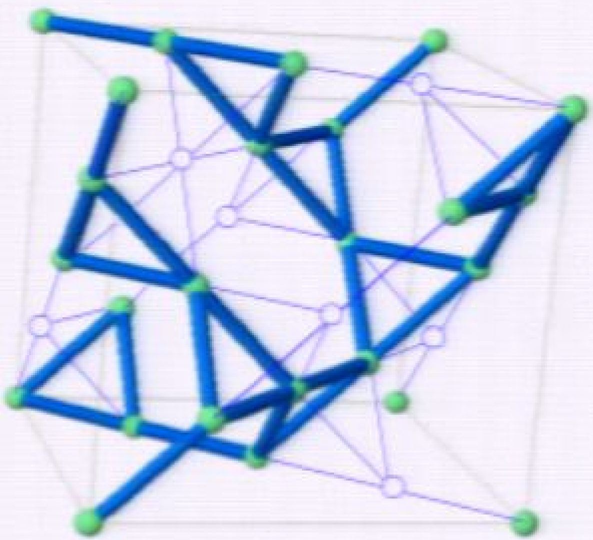
$$C(T) \sim T, T^2$$

Gapless Excitations in an Insulator ?

Is the T=0 Ground State a Spin Liquid ?


C/T (mJ/Ir mol K$^2$)
Temperature (K)
Metallic
Insulating
(Okamoto)
0
10
20
30
40
50
0
2
4
6
8
10

# Xiao-Gang's Notebook in 2020 ...



$$= (N_{ijk}, F^{ijm,\alpha\beta}_{kln,\chi\delta}, P^{kj,\alpha\beta}_{i})$$

# Theory of Spin Liquid with Gapless Fermionic Spinons

Mean field theory + Projection $\chi_{ij} = \langle f^{\dagger}_{i\alpha} f_{j\alpha} \rangle \quad \Delta_{ij} = \langle \epsilon_{\alpha\beta} f_{i\alpha} f_{j\beta} \rangle$

the lowest energy state has uniform $\chi_{ij}$ and $\Delta_{ij} = 0$

The ground state energy after projection = - 0.41 J per site

c.f. exact diagonalizaiton on 12-site cluster = - 0.42 J per site

The resulting spin liquid has a "spinon fermi surface"

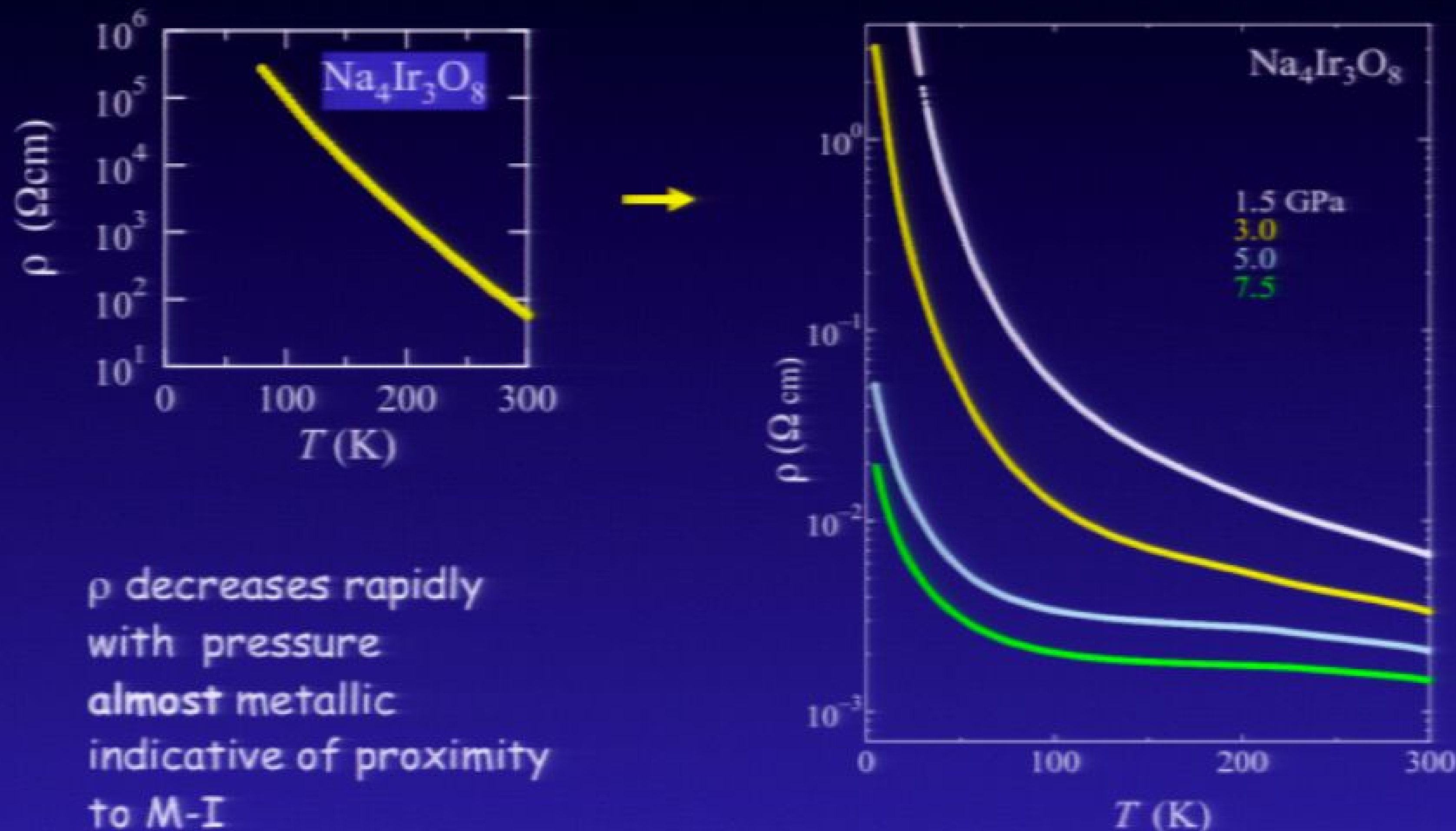
(Electron-like and Hole-like Pockets)

$C(T) \sim T$ $\qquad\qquad$ $\chi(T) \sim$ constant

M. J. Lawler, , A. Paramekanti, Y. B. Kim and L. Balents, PRL 101, 197202 (2008)

# Weak Mottness

# Itinerancy and stabilization of spin liquid




ρ decreases rapidly
with pressure
**almost** metallic
indicative of proximity
to M-I

# Transition from a Gapless Spin-liquid to a Metal (Half-filling)

## Slave-rotor field theory

$$c_{i\alpha} = f_{i\alpha} e^{i\theta_i} \qquad \phi_i = e^{i\theta_i}: \text{ rotor field}$$

$f_{i\sigma}$ and $\phi_i$ interacting with a U(1) gauge field

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In the spin-liquid insulator: The rotor (charge) degree of freedom is gapped

$U > U_c$ $\langle \phi_i \rangle = 0$

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The quasiparticle weight is proportional to $|\langle \phi_i \rangle|^2$

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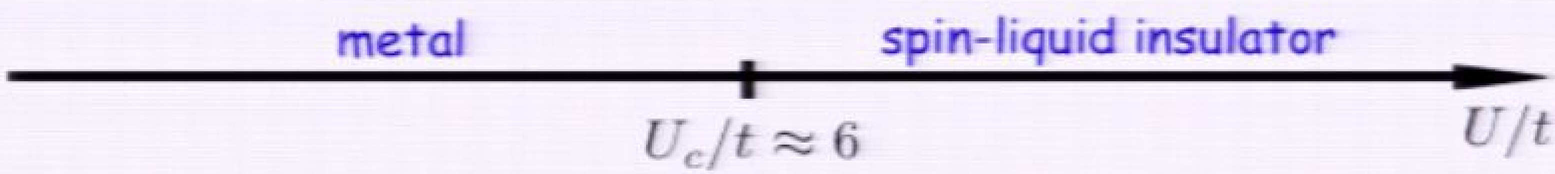
Mean-field theory on the hyper-kagome lattice



# Effective field theory

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$$f_{i\sigma} \longrightarrow \psi_\sigma(\mathbf{r}) \qquad\qquad \phi_i \longrightarrow \phi(\mathbf{r})$$

$$\mathcal{L} \;=\; \mathcal{L}_b + \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_{bf}$$

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$$\mathcal{L}_f = \psi_\sigma^\dagger(\partial_\tau - ia_0 - \mu_f)\psi_\sigma + \frac{1}{2m_f}|(\nabla - i\boldsymbol{a})\psi_\sigma|^2$$

$$\mathcal{L}_{bf} = \lambda|\psi_\sigma|^2|\phi|^2$$

# Mean-Field Insulator-Metal Transition

Control Parameter $m^2 \propto U - U_c$

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# Gauge Field Fluctuations

The spinon and rotor fields are interacting with
an overdamped U(1) gauge field.

$f \sim \epsilon \mathbf{e}^2 + \mu^{-1} \mathbf{b}^2$ The free energy density of "electromagnetic" field

$$\mathcal{L}_g = \sum_{i\nu,\mathbf{q}} \epsilon(i\nu,\mathbf{q}) |\mathbf{e}(i\nu,\mathbf{q})|^2 + \mu^{-1}(i\nu,\mathbf{q}) |\mathbf{b}(i\nu,\mathbf{q})|^2$$

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$$D_T^{-1} = \epsilon(i\nu,\mathbf{q})\nu^2 + \mu^{-1}(i\nu,\mathbf{q})q^2$$

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$\epsilon(i\nu,\mathbf{q}) = \epsilon_f(i\nu,\mathbf{q}) + \epsilon_b(i\nu,\mathbf{q})$ 'dielectric' function

$\mu^{-1}(i\nu,\mathbf{q}) = \mu_f^{-1}(i\nu,\mathbf{q}) + \mu_b^{-1}(i\nu,\mathbf{q})$ 'permeability' function

# Gauge Field Fluctuations

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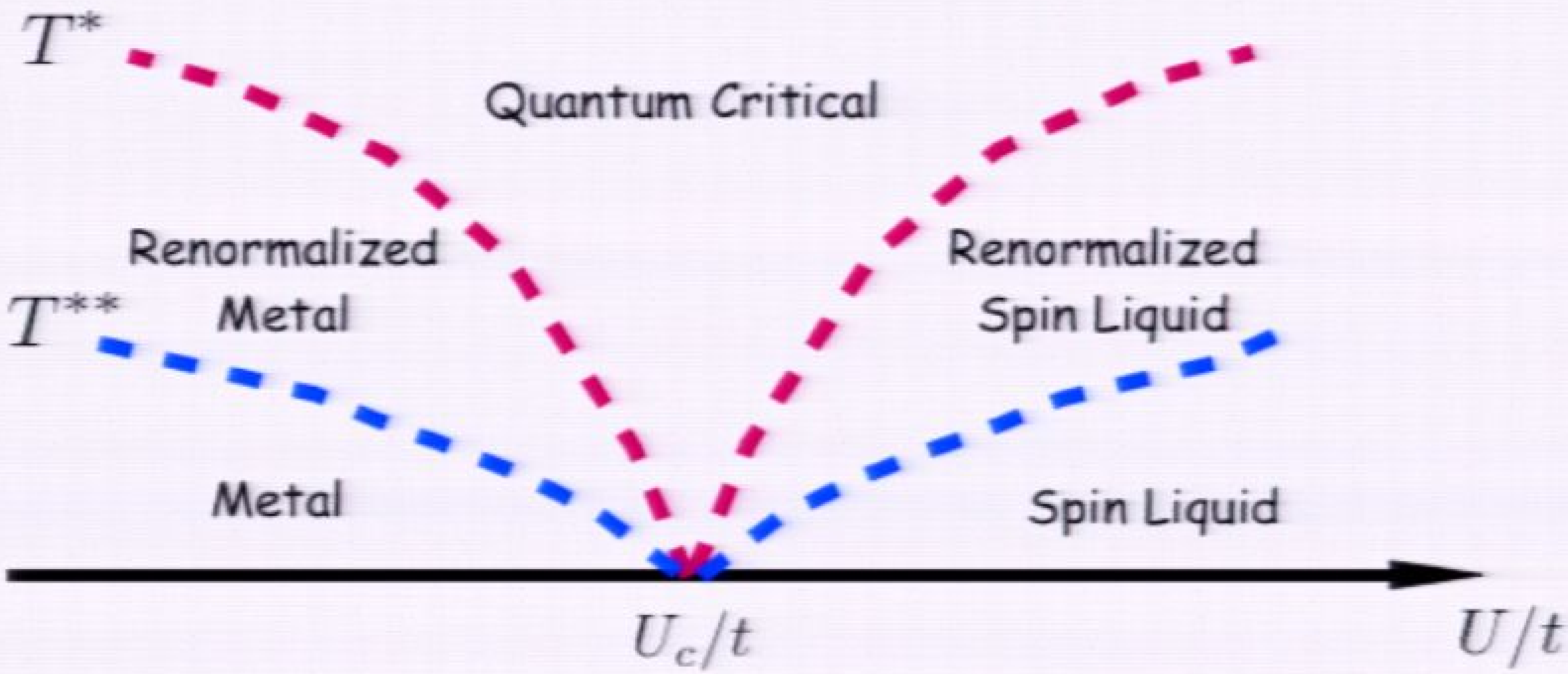
$\mu^{-1} = \text{constant}$ $\quad m^2 > 0 \ (U > U_c)$

$\mu^{-1} \approx \mu_b^{-1} \approx \tilde{\chi}\ln(1/q)$ $\quad m^2 = 0 \ (U = U_c)$

$\mu^{-1} \approx \mu_b^{-1} \propto 1/q^2$ $\quad m^2 < 0 \ (U < U_c)$

# Finite Temperature Phase Diagram

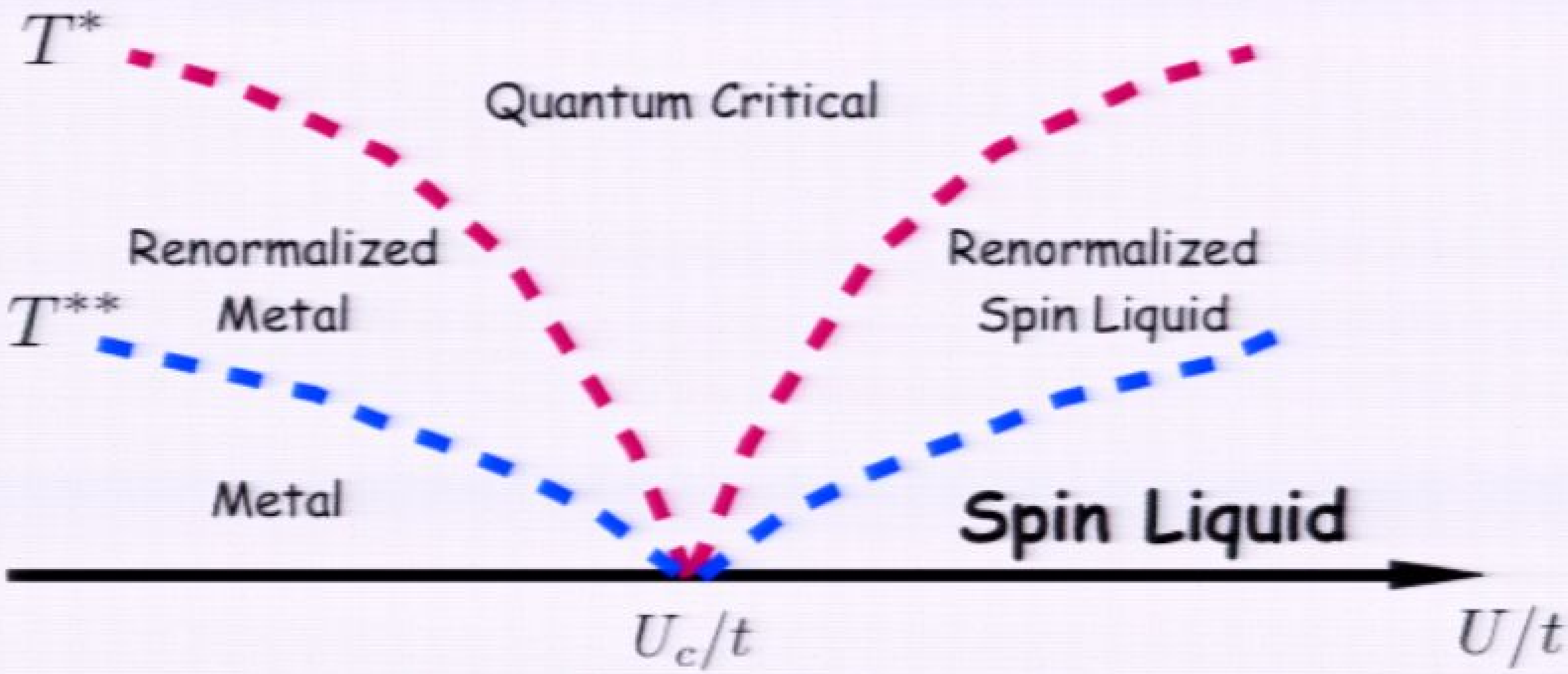



$T^* \propto |U - U_c|^{1/2}$ crossover in charge transport (rotor)
z=1

$T^{**} \propto |U - U_c|^{3/2}$ crossover in entropy (spinon + gauge field system);
z=3

D. Podolsky, A. Paramekanti, Y. B. Kim and T. Senthil, PRL 102, 186401 (2009)

# Finite Temperature Phase Diagram

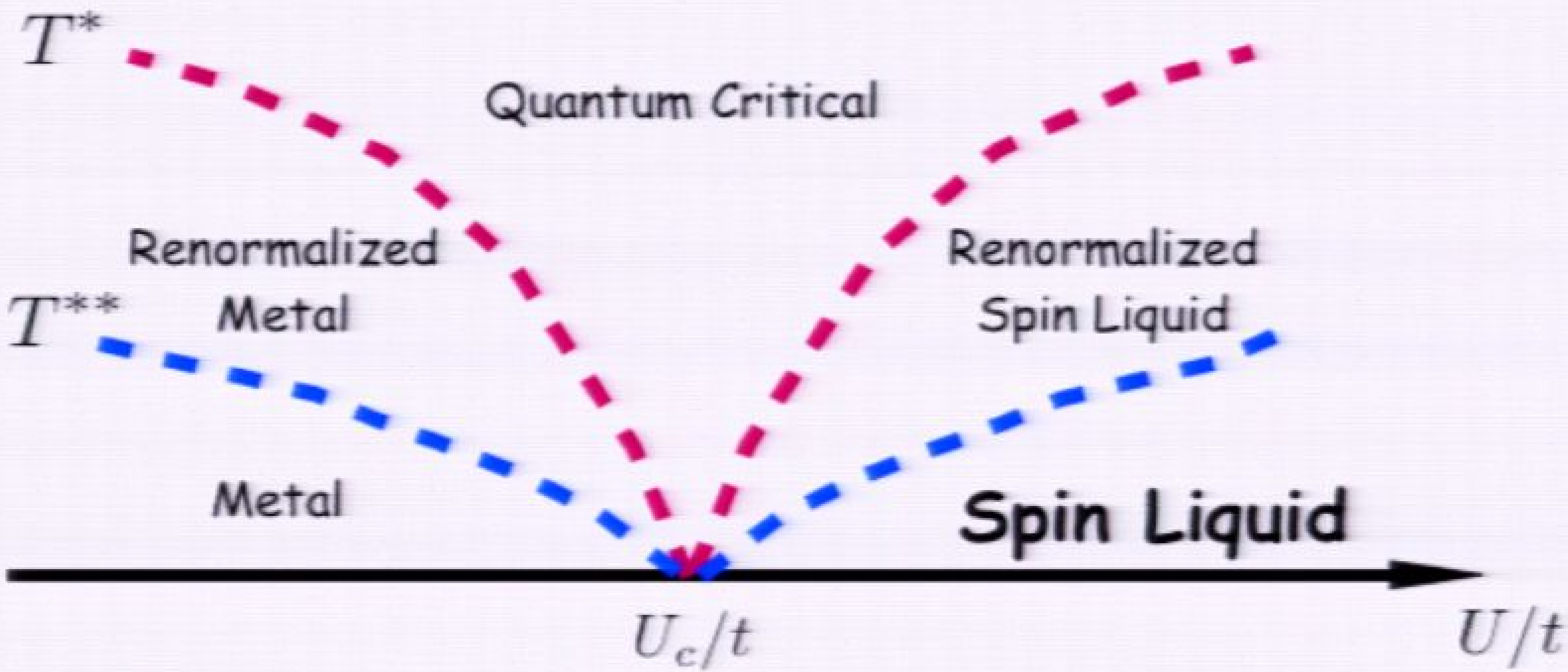



Spin Liquid Phase

$$\Sigma_f \sim \omega \ln \frac{1}{|\omega|} + i\frac{\pi}{2}|\omega| \qquad \mathrm{Im}\,\Sigma_f \propto |\omega|$$

Spinons are not so well defined ...

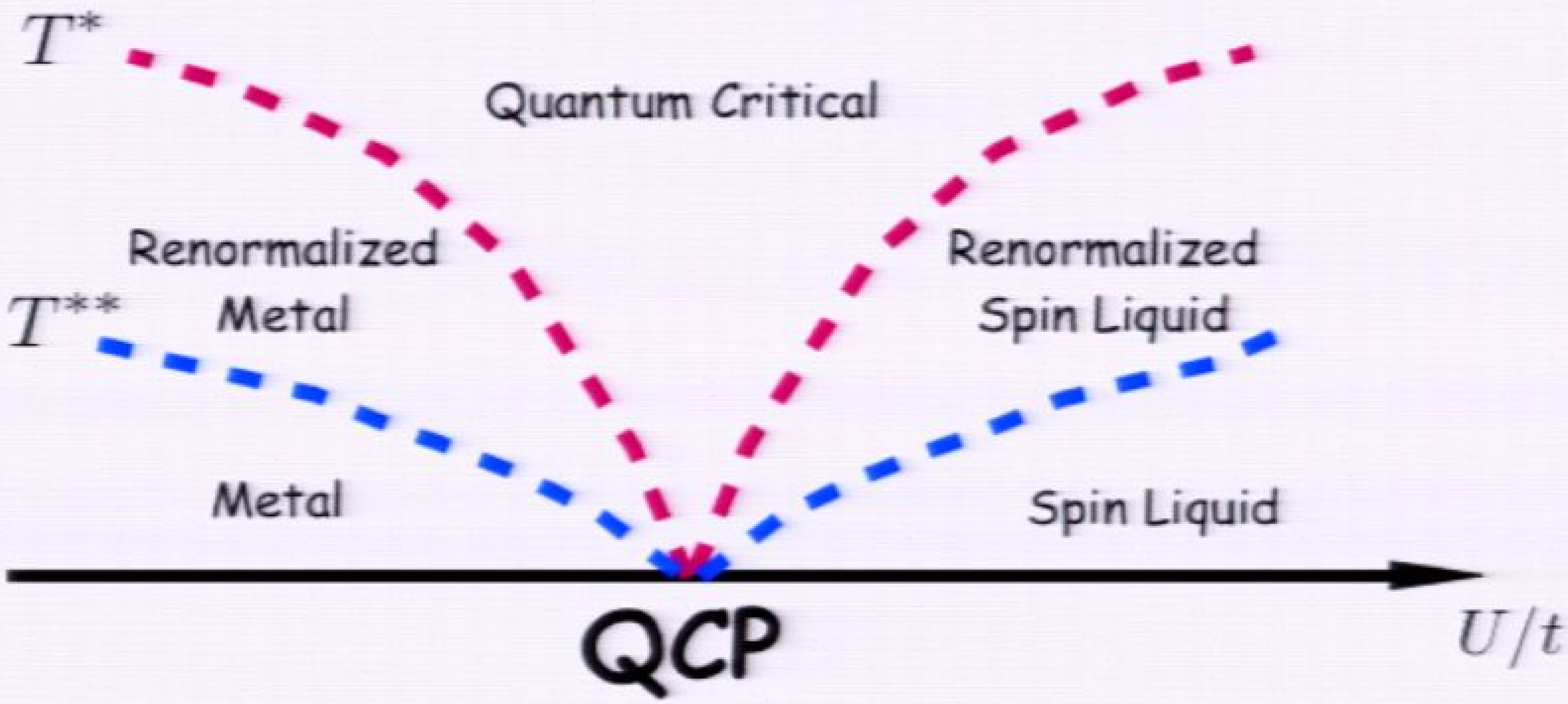
# Finite Temperature Phase Diagram




Spin Liquid Phase

$$C(T) \sim T \ln(1/T)$$

# Finite Temperature Phase Diagram

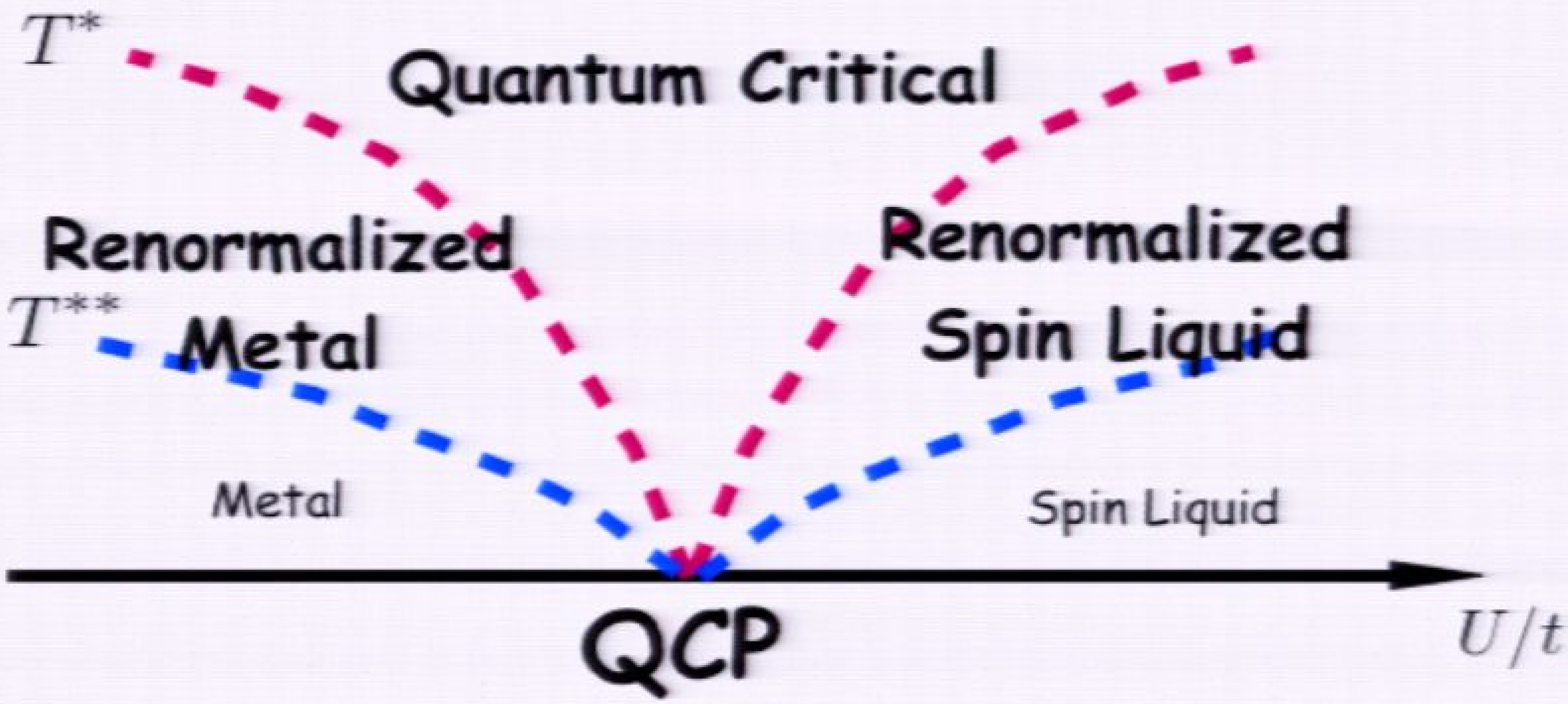



Quantum Critical Point

$$\Sigma_f \sim \omega \ln\ln \frac{1}{|\omega|} + i\frac{\pi}{2}\frac{|\omega|}{\ln\frac{1}{|\omega|}} \qquad \mathrm{Im}\,\Sigma_f \propto \frac{|\omega|}{\ln\frac{1}{|\omega|}} \ll |\omega|$$

Spinons are better defined at the critical point !

# Finite Temperature Phase Diagram

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Possibly more relevant to experiments

D. Podolsky, A. Paramekanti, Y. B. Kim and T. Senthil, PRL 102, 186401 (2009)

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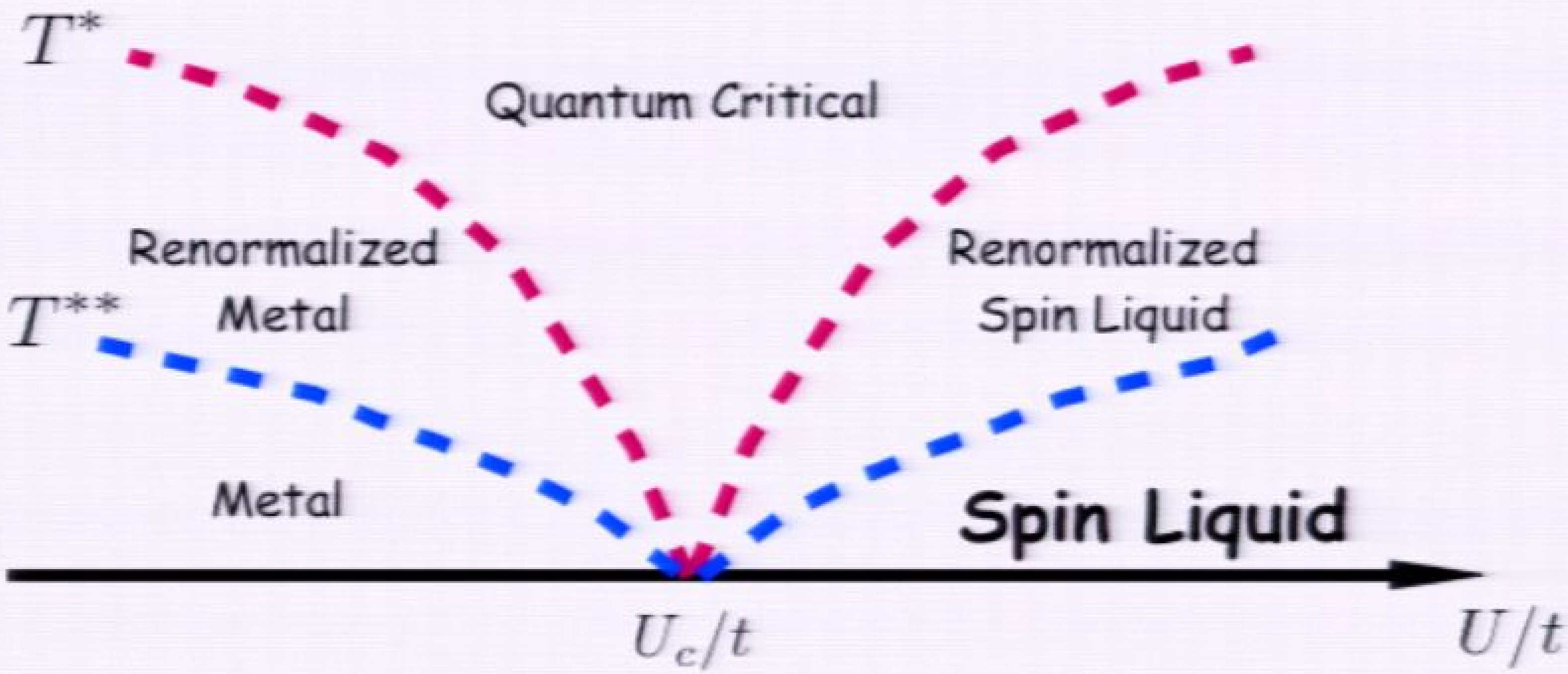


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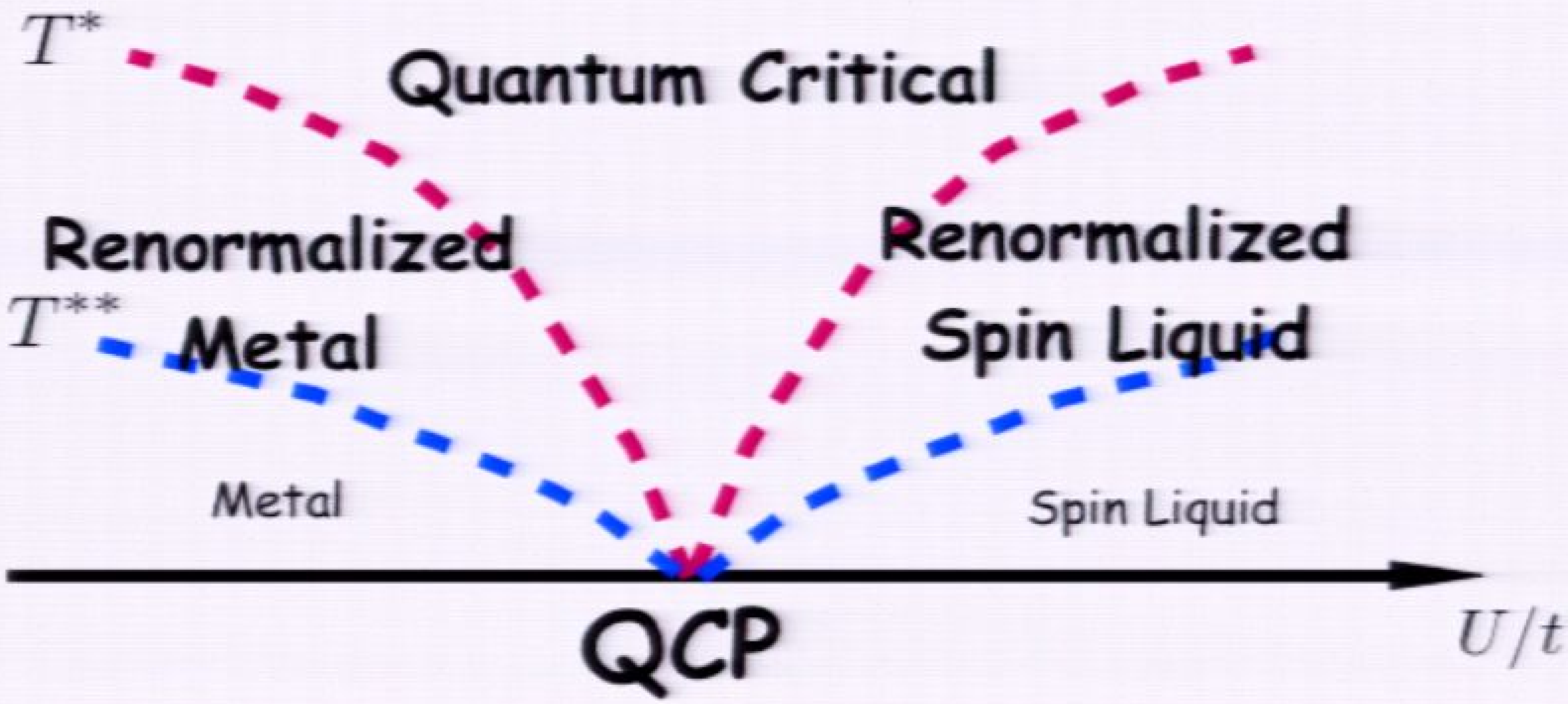


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Possibly more relevant to experiments

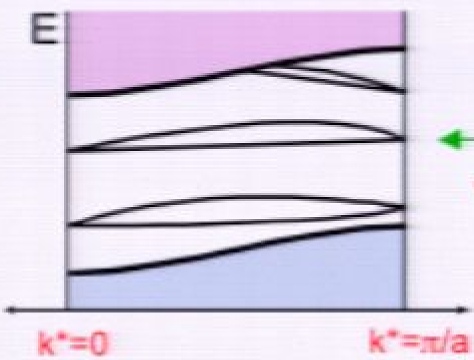
D. Podolsky, A. Paramekanti, Y. B. Kim and T. Senthil, PRL 102, 186401 (2009)

# Topological Insulator

# Topological Insulator

2D time reversal invariant band structure have
a Z2 topological invariant

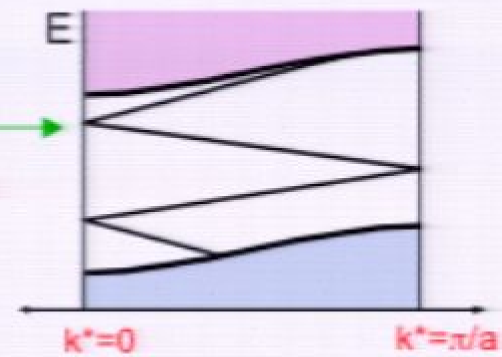
ν=0 : Conventional Insulator




Edge States

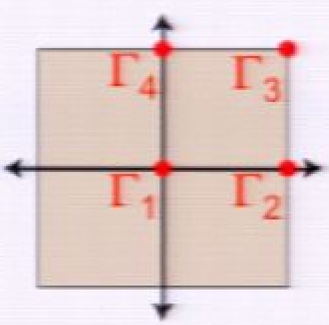
Kramers degenerate at
time reversal invariant momenta
$k^* = -k^* + G$

ν=1 : Topological Insulator




nversion (P) Symmetry : determined by Parity of occupied 2D Bloch states at $\Gamma_{1,2,3,4}$

Bulk
Brillouin
Zone




$$P|\psi_n(\Gamma_i)\rangle = \xi_n(\Gamma_i)|\psi_n(\Gamma_i)\rangle$$
$$\xi_n(\Gamma_i) = \pm 1$$

$$(-1)^{\nu} = \prod_{i=1}^{4}\prod_{n}\xi_{2n}(\Gamma_i)$$

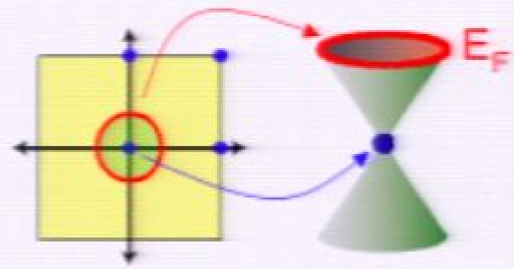
C. L. Kane, E. Mele, L. Fu     B. A. Bernevig, T. L. Hughes, X.-L. Qi, S. C. Zhang

# 3D Topological Insulator

In 3D there are 4 $Z_2$ invariants: $(\nu_0 ; \nu_1\nu_2\nu_3)$ characterizing the bulk. These determine how surface states connect.

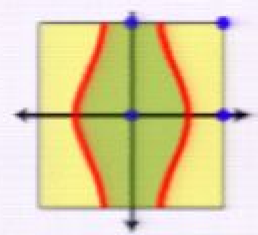
$\nu_0 = 1$ : Strong Topological Insulator

Fermi surface encloses odd number of Dirac points




$\nu_0 = 0$ : Weak Topological Insulator

Fermi surface encloses even number of Dirac points



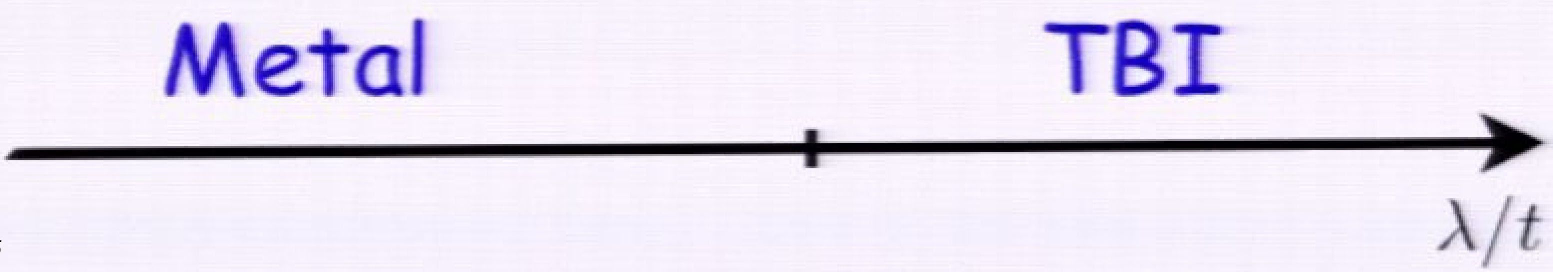
L. Fu, C. L. Kane

J. E. Moore, L. Balents

# Towards
# Topological Mott Insulator

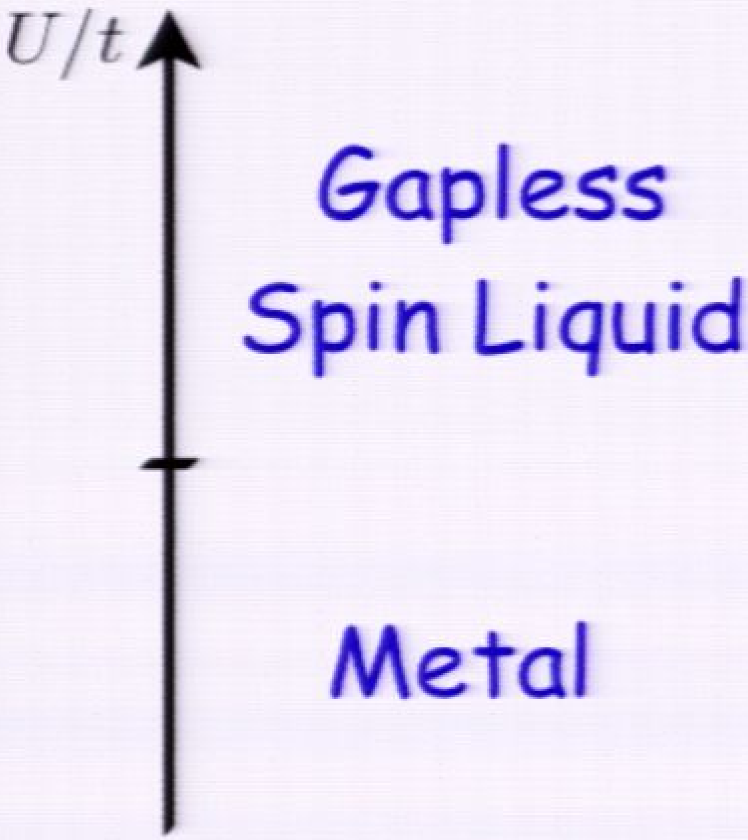
# What about interaction effect ?

Assume strong spin-orbit coupling $\lambda$ leads to a topological insulator in the absence of U



# What about interaction effect ?

Effect of interaction U in with $\lambda = 0$



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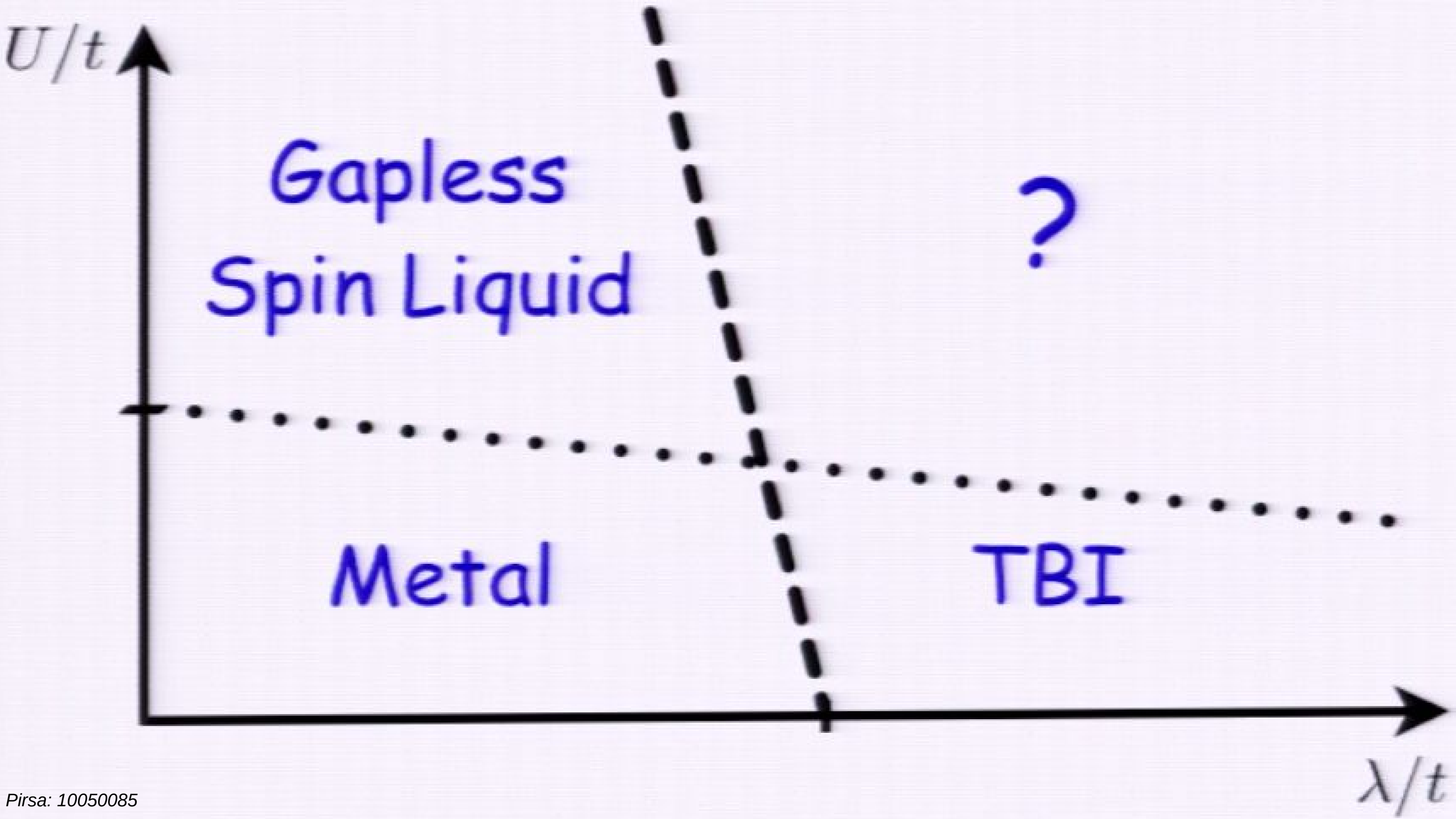
Effect of interaction U ?



# Slave Rotor Field Theory

$c_{i\alpha} = f_{i\alpha} e^{i\theta_i}$ $\phi_i = e^{i\theta_i}$: rotor field

$f_{i\sigma}$ and $\phi_i$ interacting with a U(1) gauge field

For sufficiently large $\lambda/t$

Topological Insulator: The rotor (charge) degree of freedom coherent

$U < U_c$ $\langle \phi_i \rangle \neq 0$

Topological Mott Insuator: The rotor (charge) degree of freedom is gapped

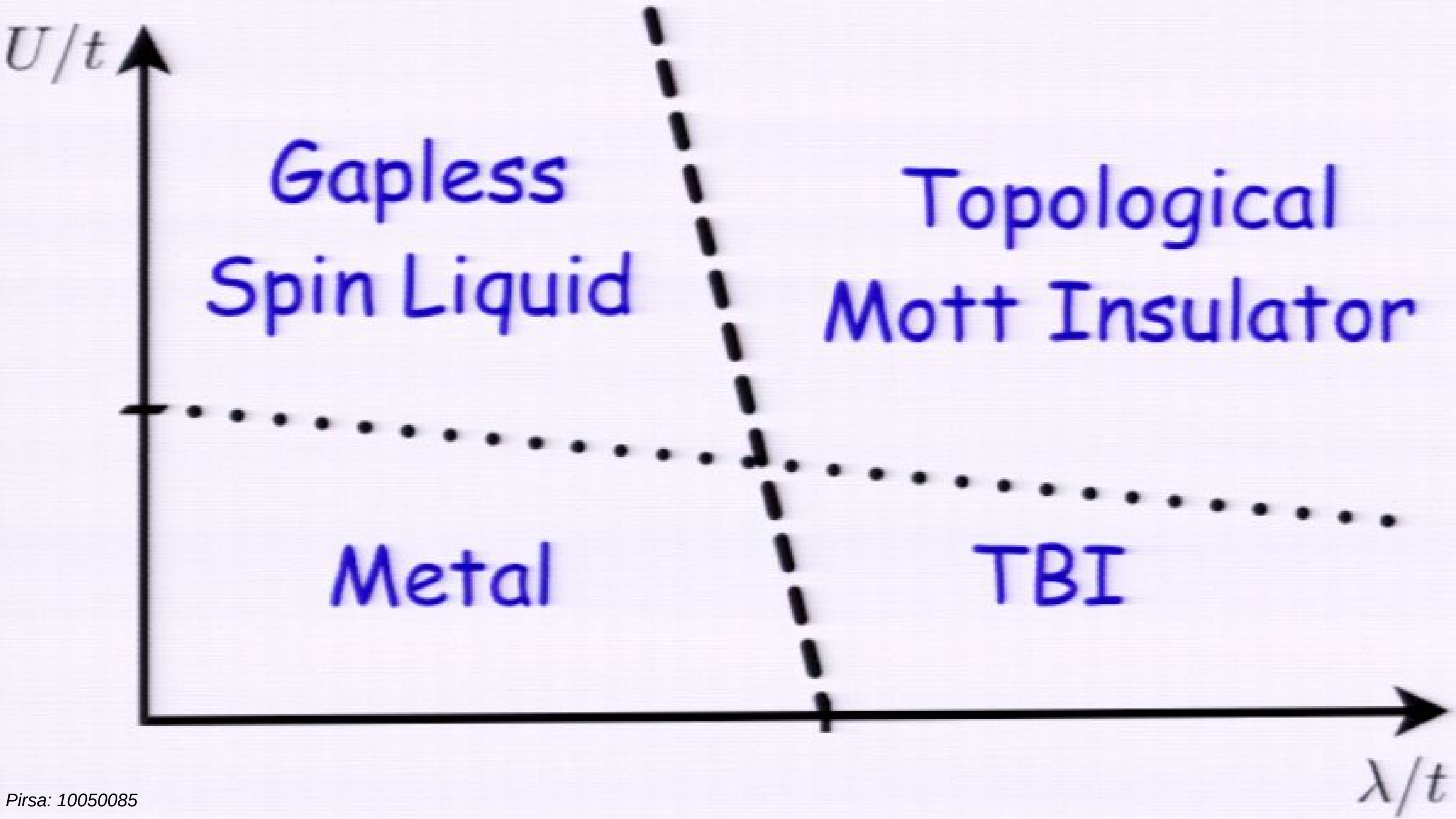
$U > U_c$ $\langle \phi_i \rangle = 0$

The spinon band structure is topological; interacting with U(1) gauge field

Such phases are unstable in 2D, but possible in 3D

# What about interaction effect ?

Effect of interaction U ?



# 5d transition metal (Ir) oxides as a playground for SO physics

arge U

3d U~2-10eV

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn |
| Y | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd |
| Lu | Hf | Ta | W | Re | Os | **Ir** | Pt | Au | Hg |

amll U

ysics of correlated electrons
transition metal oxides

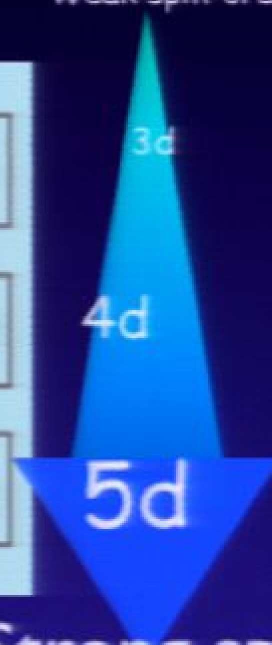
5d U ~0.5 eV

U too weak to enjoy U??

# 5d transition metal (Ir) oxides as a playground for SO physics

Large U    3d U~2-10eV    Weak spin-orbit coupling

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn |
| Y | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd |
| Lu | Hf | Ta | W | Re | Os | Ir | Pt | Au | Hg |




Strong spin-orbit coupling

Samll U    $\Delta_{SO}$ ~0.5eV

hysics of correlated electrons
transition metal oxides

5d U ~0.5 eV
U too weak to enjoy U??

- transition metal oxides d-only character
- interplay of U and $\Delta_{SO}$
- synchrotron x-ray powerful

# d transition metal (Ir) oxides
# as a playground for SO physics

arge U

3d U~2-10eV

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sc | Ti | V | Cr | Mn | Fe | Co | Ni | Cu | Zn |
| Y | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd |
| Lu | Hf | Ta | W | Re | Os | **Ir** | Pt | Au | Hg |


Strong spin-orbit coupling

amll U

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5d U ~0.5 eV
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- interplay of U and $\Delta_{SO}$
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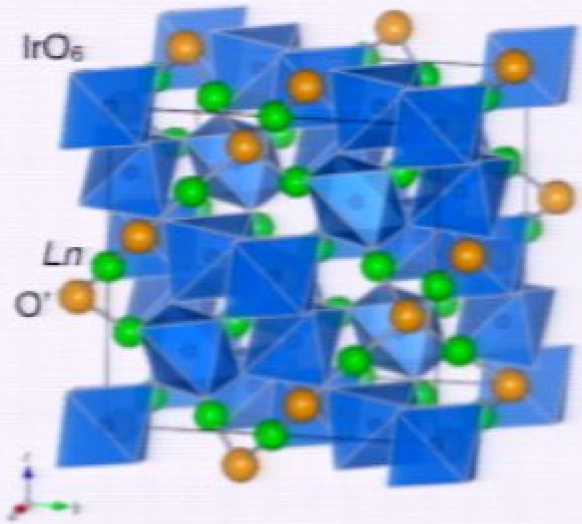
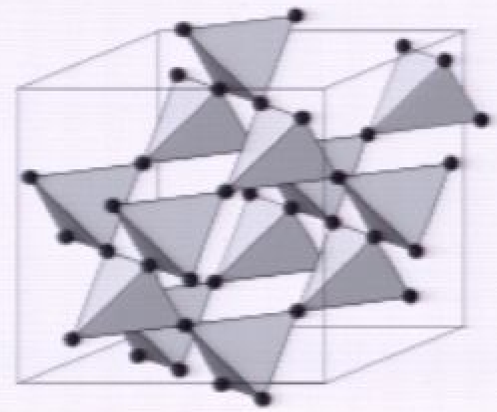
# Pyrochlore Iridates:
# $A_2Ir_2O_7$

# Pyrochlore Iridates $A_2Ir_2O_7$



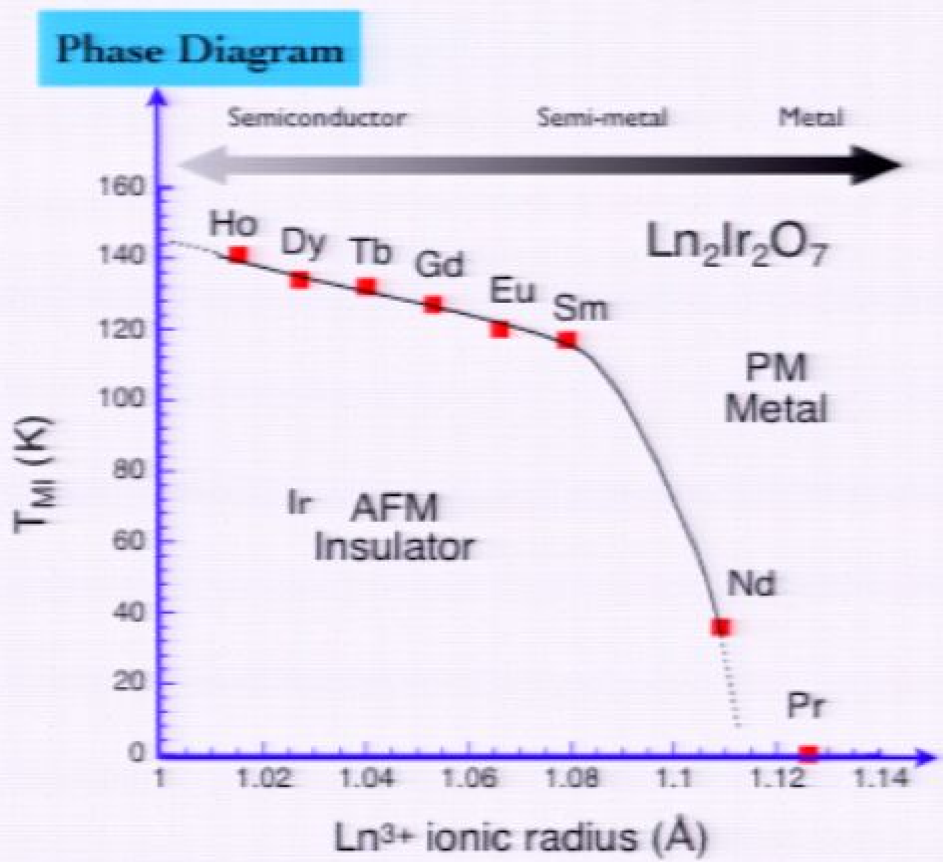
A=Ln and Ir reside in the inter-penetrating two pyrochlore lattices (cubic, FCC Bavais lattice)

A=Ln local moments may be important only at low temperatures

D. Yanagishima and Y. Maeno, JPSJ 70, 2880 (2001)

# Metal-Insulator Transition

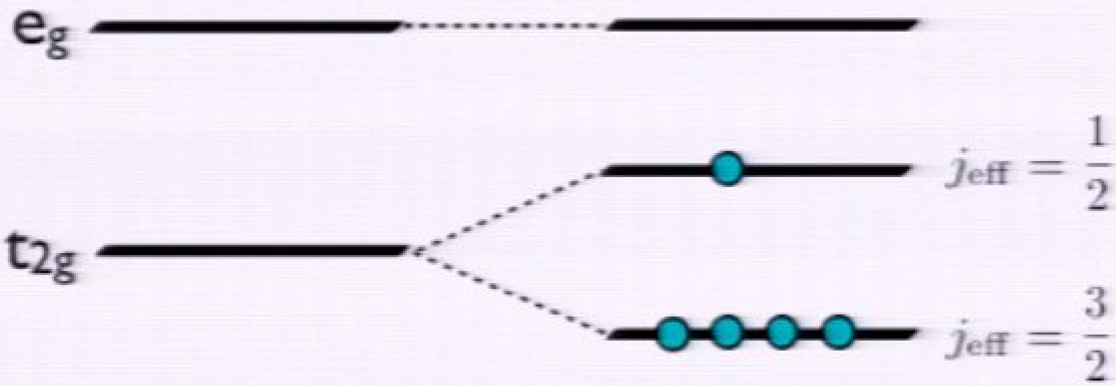
K. Matsuhira et al
JPSJ 76, 043706 2007

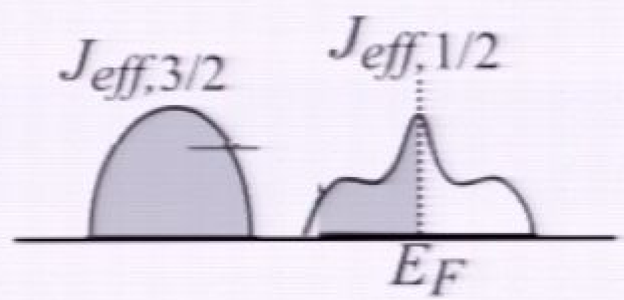


# 5d orbitals of $Ir^{4+}$: large spin-orbit coupling

$Ir^{4+}$ = [Xe] $4f^{14}\ 5d^{5}$ $\qquad H_{so} = \lambda_{so}\mathbf{L}_i \cdot \mathbf{S}_i$

$e_g$ - $t_{2g}$ splitting $\gg \lambda_{so} \gg$ splitting within $t_{2g}$



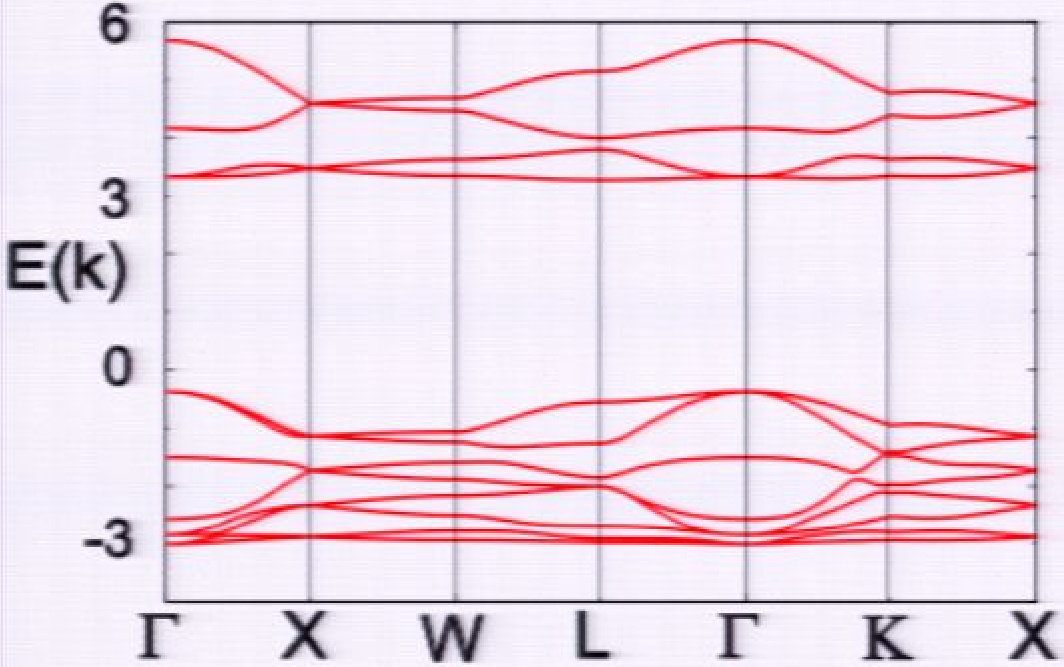



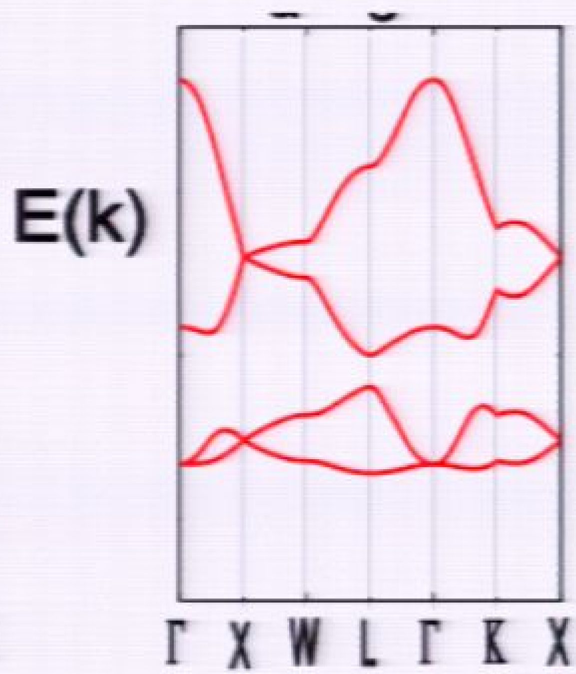

Half-filled $j_{\rm eff} = \frac{1}{2}$ band

$$|\uparrow_j\rangle = \frac{1}{\sqrt{3}}(i|xz,\downarrow_s\rangle + |yz,\downarrow_s\rangle + |xy,\uparrow_s\rangle)$$

$$|\downarrow_j\rangle = -\frac{1}{\sqrt{3}}(i|xz,\uparrow_s\rangle - |yz,\uparrow_s\rangle + |xy,\downarrow_s\rangle)$$

# Band Structure with Spin-Orbit at U=0




$\lambda = 4t$

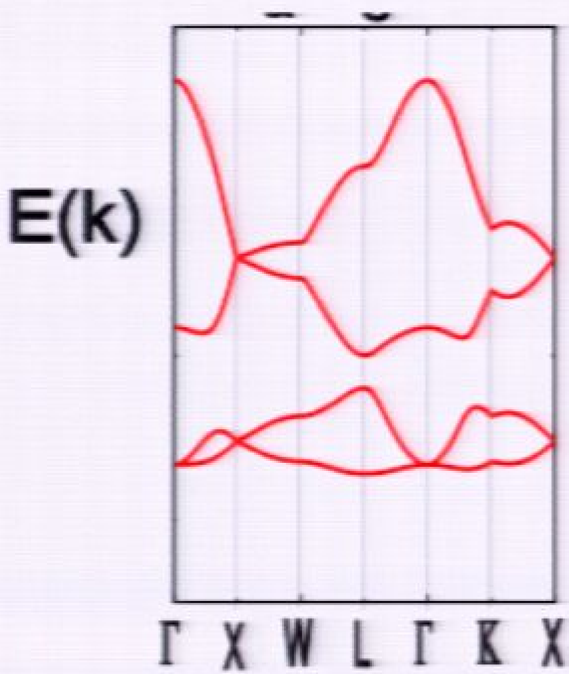
12 (=3 x 4) degenerate bands or 24 bands

20 (=5 x 4) d-electrons

D. A. Pesin and L. Balents, 2009

Half-filled j=1/2 bands

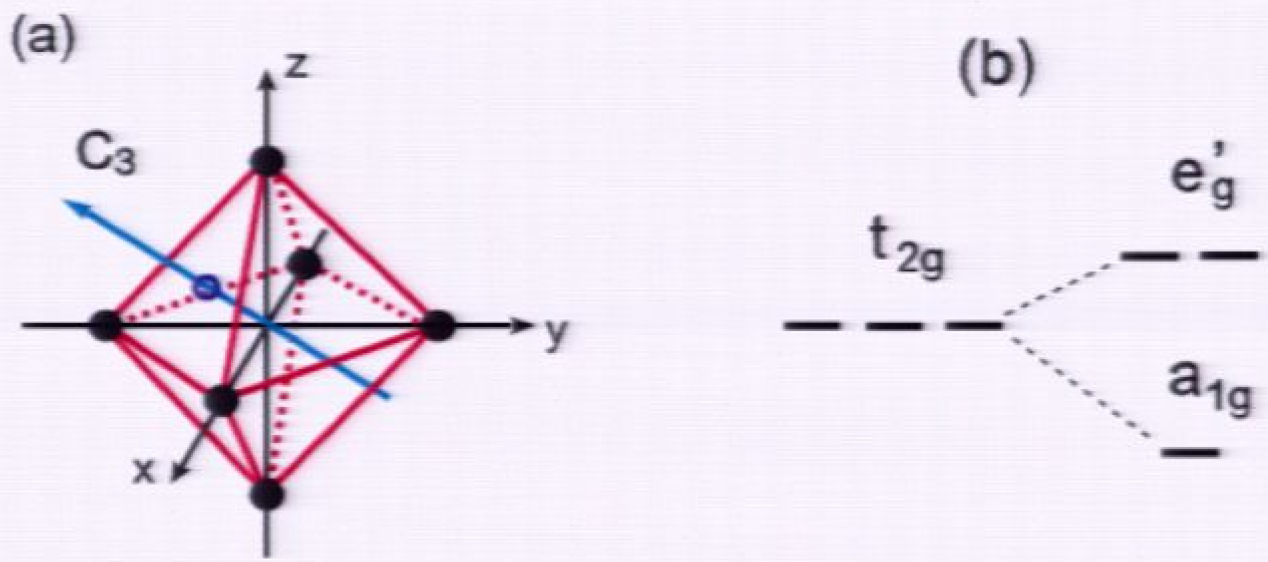
# Z2 Topological Index: $(\nu;\nu_1\nu_2\nu_3)$




$$(\nu;\nu_1\nu_2\nu_3) = (1;000)$$

Strong Topological Insulator !

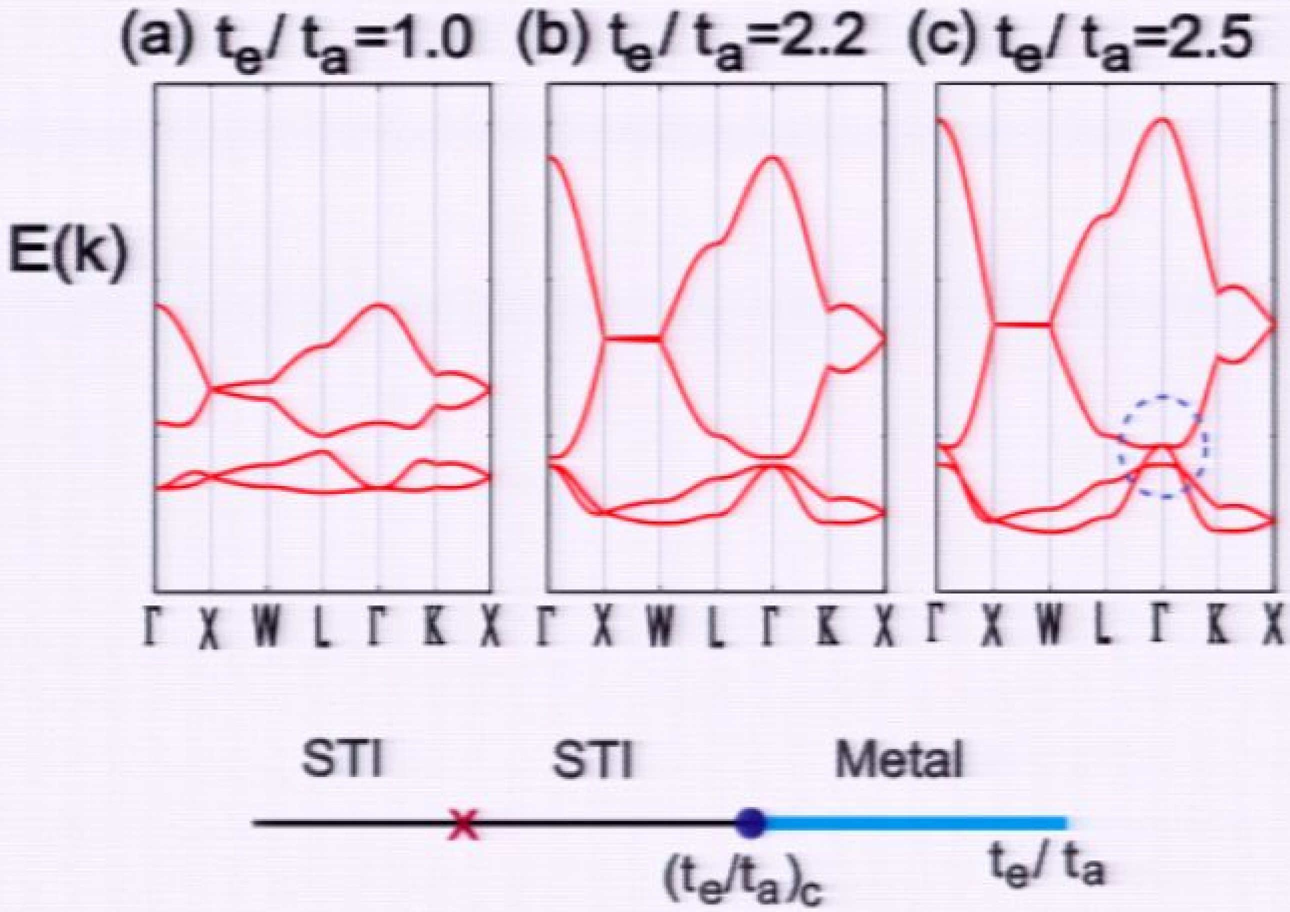
# Too Good to be True ?




compression or elongation of the surrounding oxygen octahedra along a C3 symmetry axis

Trigonal Crystal Field at the Ir site

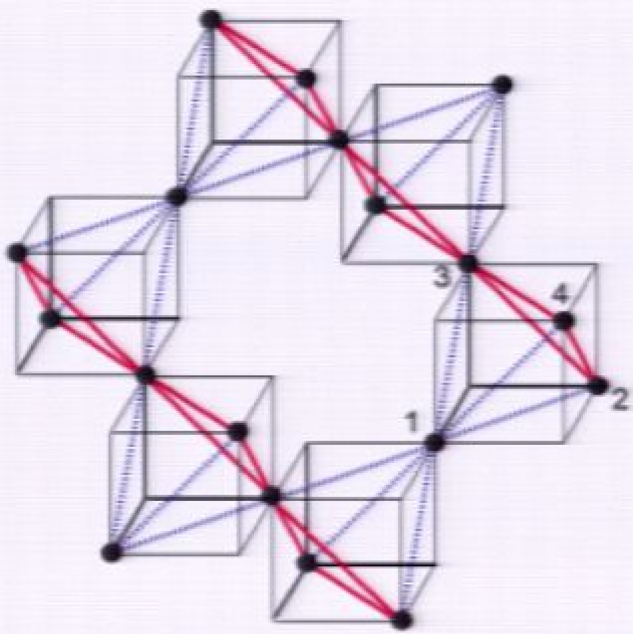
# Too Good to be True ?



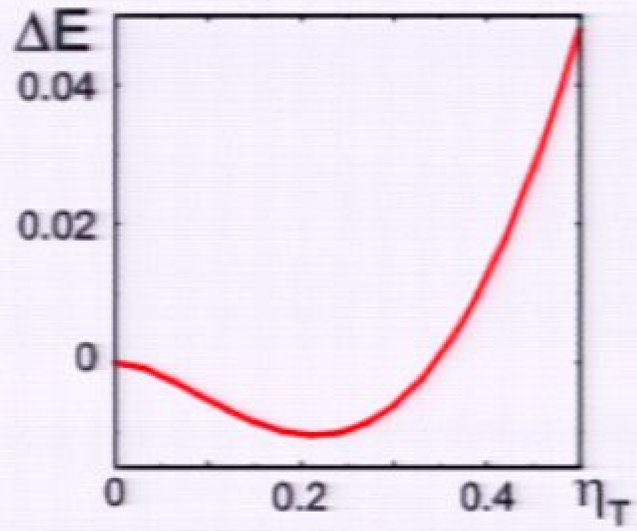

B. -J. Yang and Y. B. Kim, 2010

# Saving Topological Insulator

## Electron-Lattice Coupling



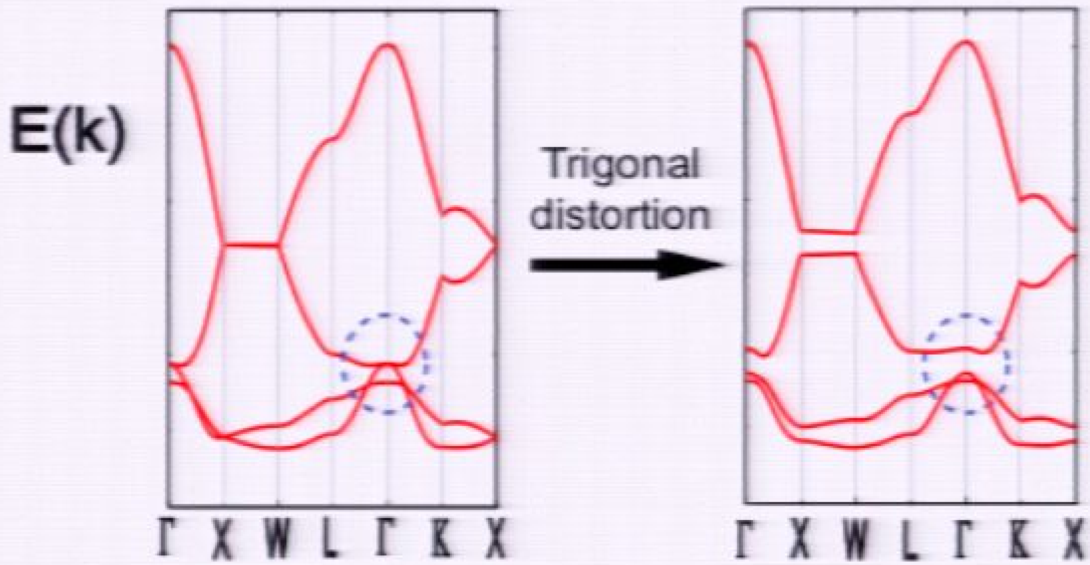

Trigonal Lattice Distortion of the Ir pyrochlore lattice




Distortion along the [111] direction: Inversion symmetry intact

B. -J. Yang and Y. B. Kim, 2010

# Resurrection of Topological Insulator



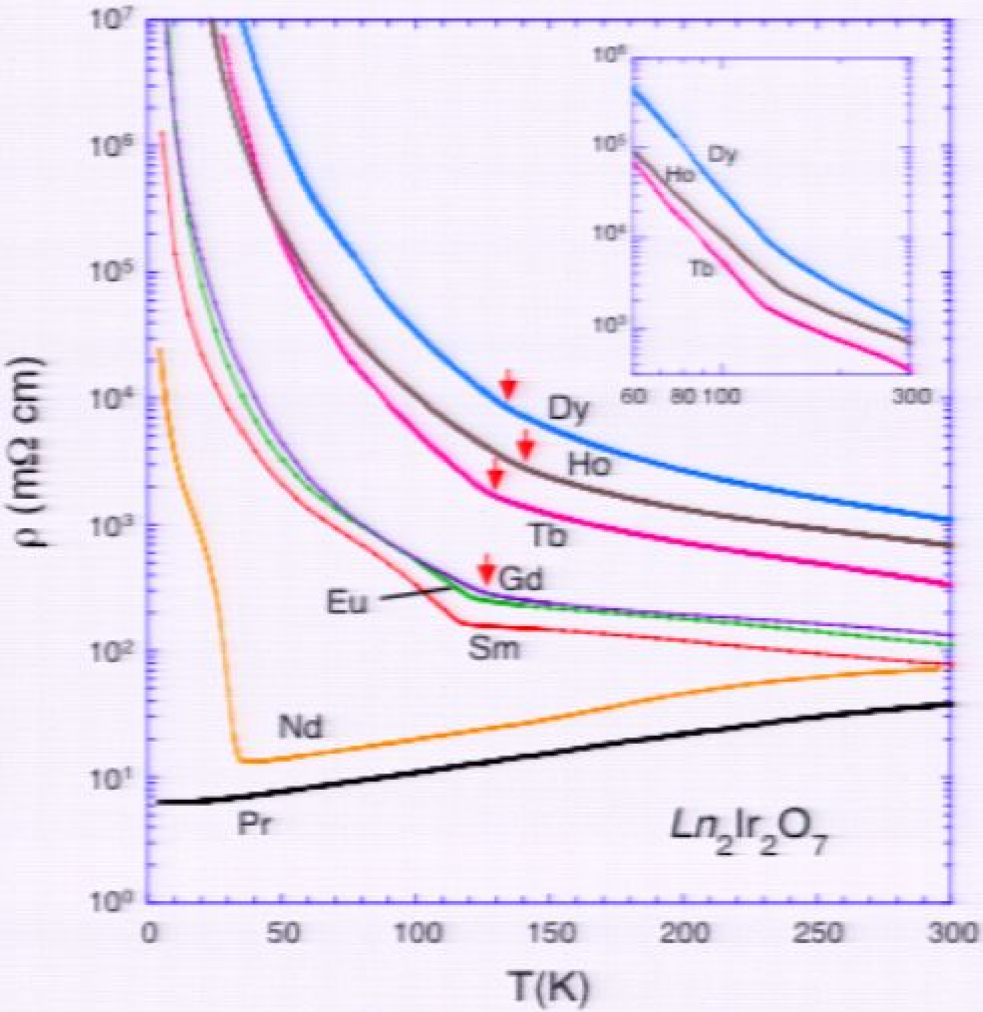

$$(\nu;\nu_1\nu_2\nu_3) = (1;000)$$

Resurrection of Strong Topological Insulator !

Apply uniaxial pressure along the [111] direction

B. -J. Yang and Y. B. Kim, 2010

# Resurrection of Topological Insulator ?

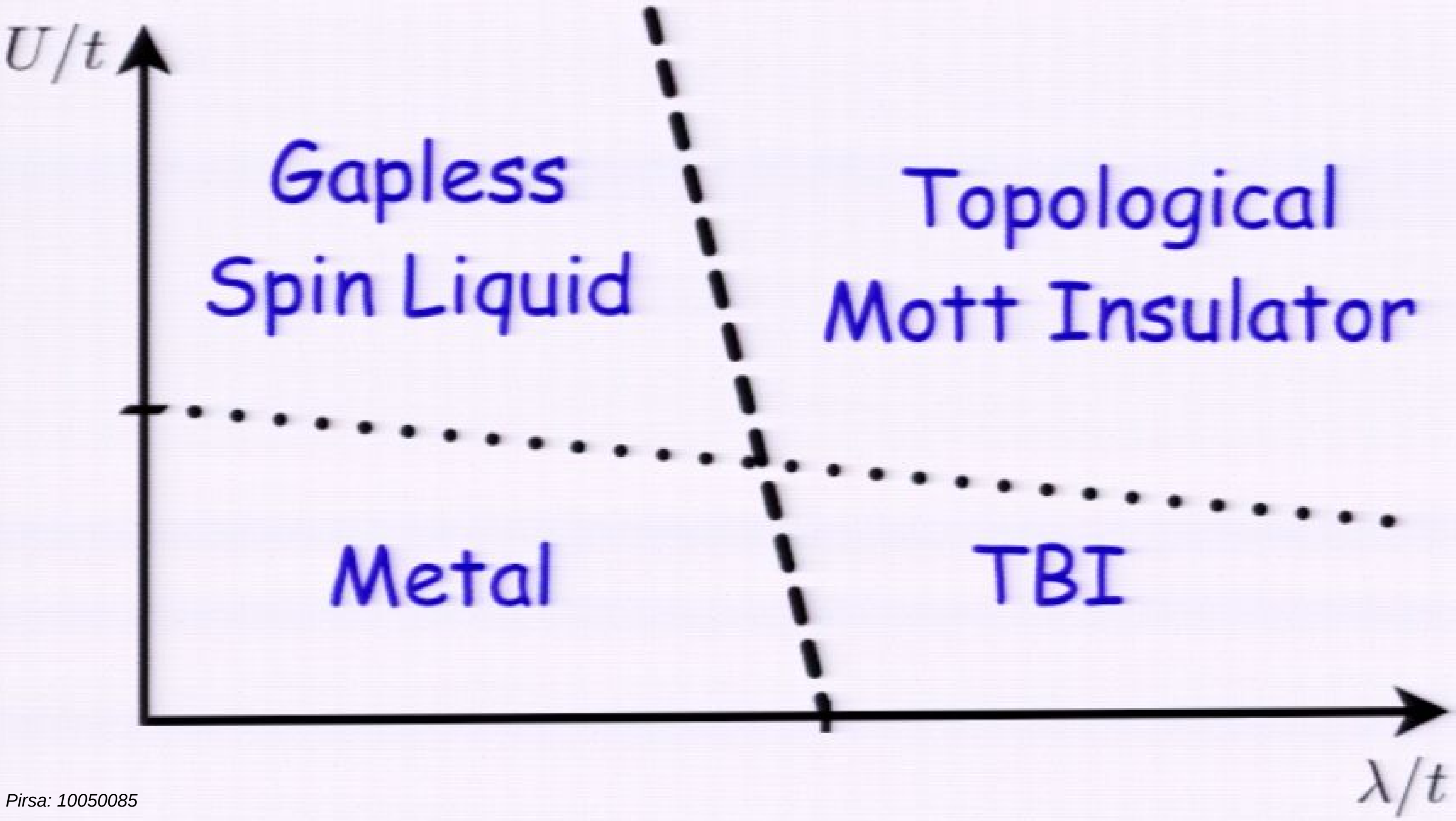


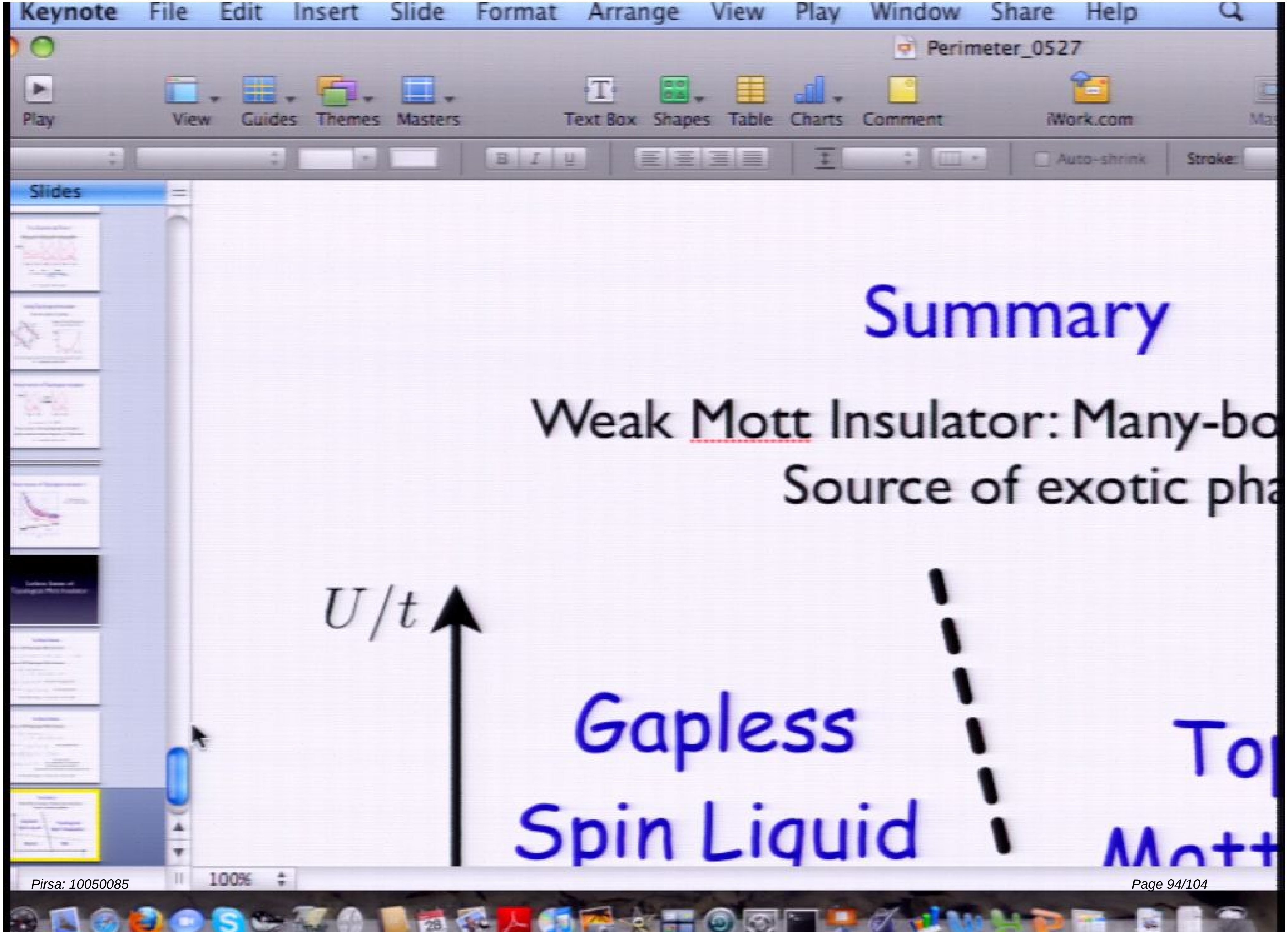

K. Matsuhira et al
JPSJ 76, 043706 2007

# Surface States of Topological Mott Insulator

# Summary

Weak Mott Insulator: Many-body frustration
Source of exotic phases

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Perimeter_0527
Play View Guides Themes Masters Text Box Shapes Table Charts Comment iWork.com
Slides
Summary
Weak Mott Insulator: Many-bo
Source of exotic pha
$U/t$
Gapless
Spin Liquid
To
100%

# Surface States

## Surface of 3D Topological Band Insulat

$$H = \int d^3x\, \psi^\dagger(x)[-i\boldsymbol{\sigma}\cdot(\hat{z}\wedge\nabla) - \mu]\psi(x)$$

## Surface of 3D Topological Mott Insulat

$$\mathcal{L} = \psi^\dagger\Big\{\partial_\tau + igA_0(\tau, \boldsymbol{x}, 0) - \mu$$

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$\mathcal{L}_0[A] \; = \; (1/g_0)^2 \int d^3x dz F^2$ Emergent U(1) gauge field

$\mathcal{D}(q_\mu, n_z) = \dfrac{1}{\chi_z \left(\frac{\pi n_z}{L}\right)^2 + \chi q^2 + \gamma \frac{|\nu|}{q}}$ 3D U(1) gauge field

W. Witczak-Krempa, T. P. Choy, and Y. B. Kim, 2010

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3D U(1) gauge field

$$\hat{\Sigma}(p_\mu) = \begin{pmatrix} \Sigma_{\uparrow\uparrow} & \Sigma_{\uparrow\downarrow} \\ \Sigma_{\downarrow\uparrow} & \Sigma_{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} 1 & -ie^{i\theta_p} \\ ie^{-i\theta_p} & 1 \end{pmatrix} \Sigma_{\uparrow\uparrow}(\omega)$$

$$\Sigma_{\uparrow\uparrow}(\omega) = -i\, \lambda_3 \omega \log \frac{\mu}{|\omega|}$$

U(1) spin liquid:
more stable/better-controlled
spinon Fermi surface state ?
(compared to the usual 2D U(1) spin liquid with FS)

W. Witczak-Krempa, T. P. Choy, and Y. B. Kim, 2010

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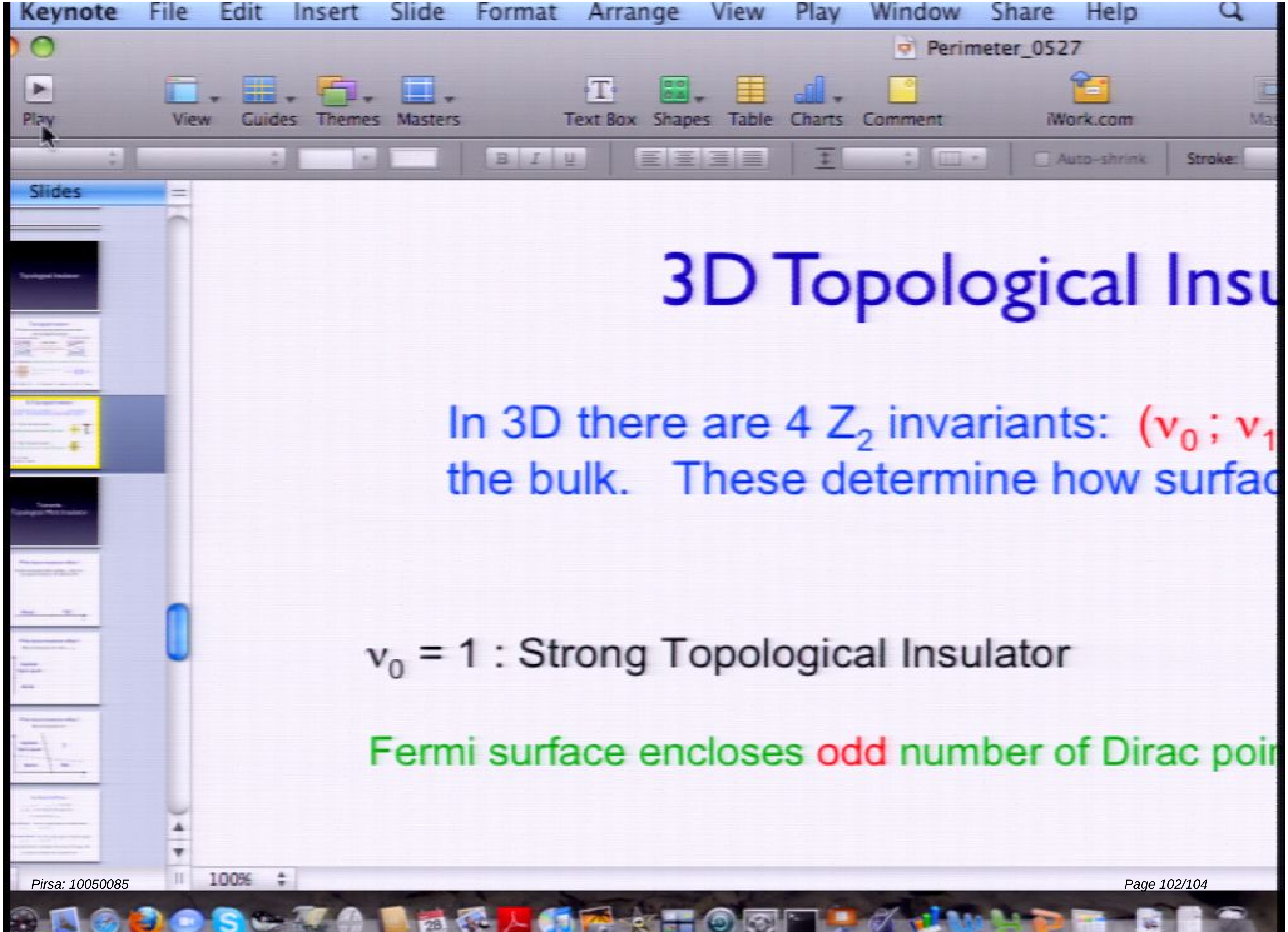
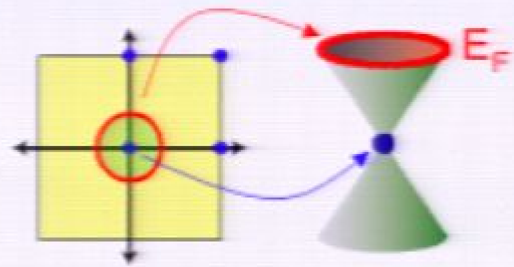

3D Topological Insu
In 3D there are 4 $Z_2$ invariants: $(\nu_0 ; \nu_1$
the bulk. These determine how surfac
$\nu_0$ = 1 : Strong Topological Insulator
Fermi surface encloses odd number of Dirac poi

# 3D Topological Insulator

In 3D there are 4 $Z_2$ invariants: $(\nu_0 ; \nu_1\nu_2\nu_3)$ characterizing the bulk. These determine how surface states connect.

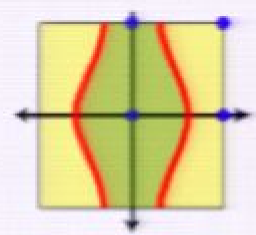
$\nu_0 = 1$ : Strong Topological Insulator

Fermi surface encloses odd number of Dirac points




$\nu_0 = 0$ : Weak Topological Insulator

Fermi surface encloses even number of Dirac points



L. Fu, C. L. Kane

J. E. Moore, L. Balents

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