Title: Quantum Phases, Wave Function Renormalization, and Tensor Product States

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Abstract: tba

Quantum Phases , Wave Function Renormalization, and Tensor Product States

Xie Chen, MIT
Joint work with
Zheng-Cheng Gu and Xiao-Gang Wen

Quantum Phases and Local Unitary Transformation

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- States are in the same gapped phase if and only if they are related by local unitary transformations Hastings & Wen, 05, Bravyi, Hastings, Michalakis, 10

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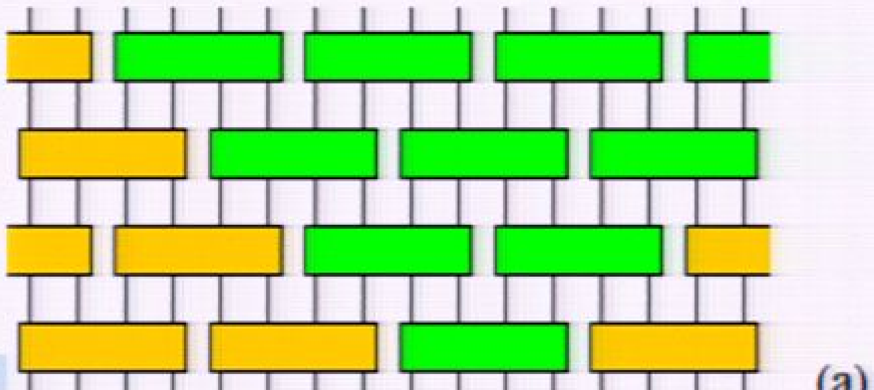

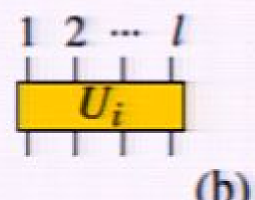

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- Away from fixed point, any state should be connected to fixed point state with a local unitary transformation

Local Unitary Transformation -- Renormalization Flow

- Local unitary circuit can be used to remove local entanglement

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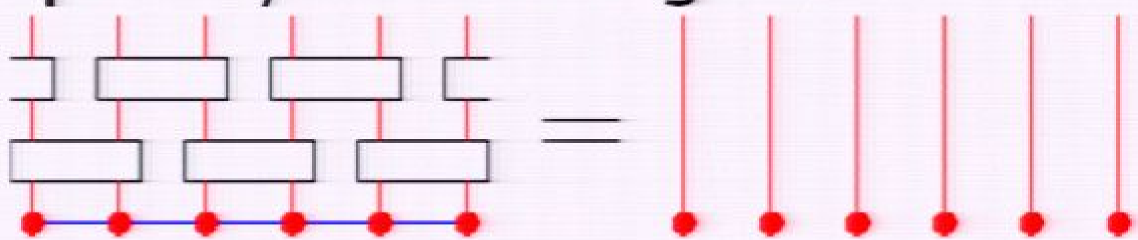

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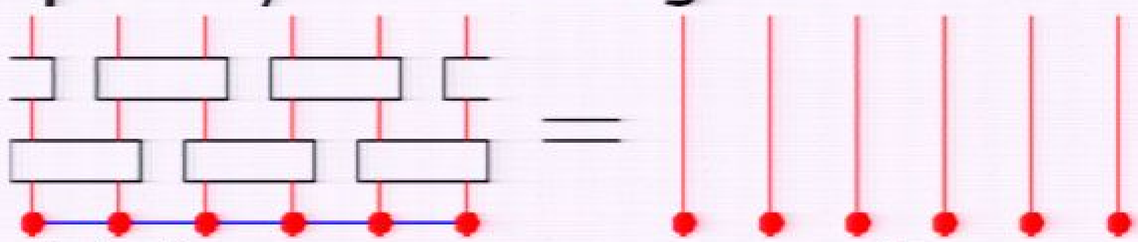

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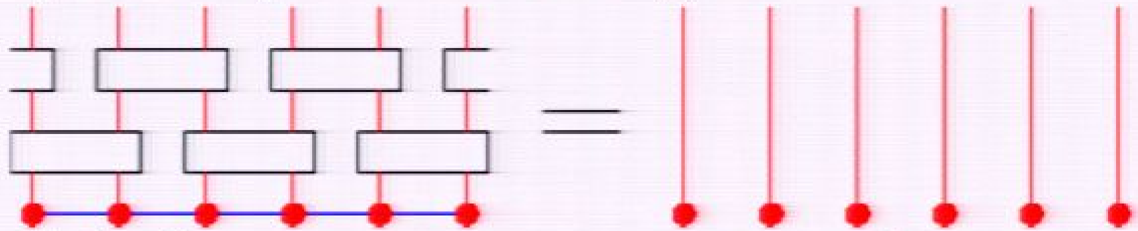

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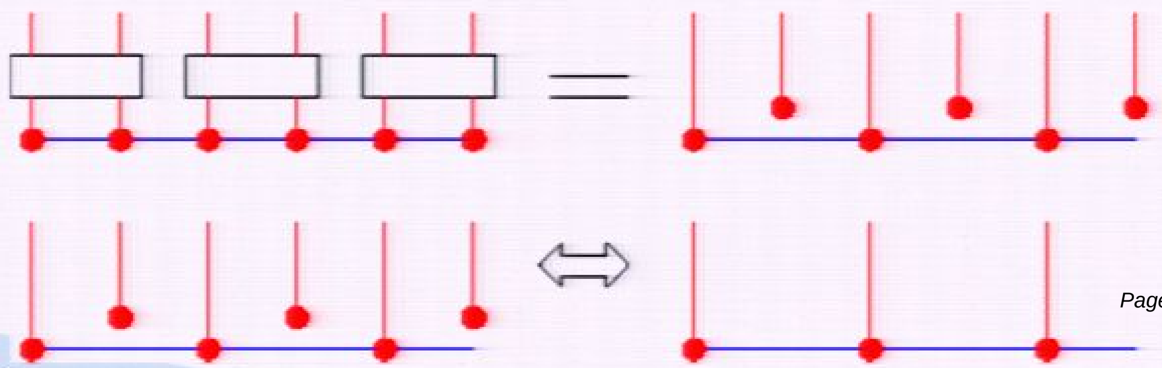

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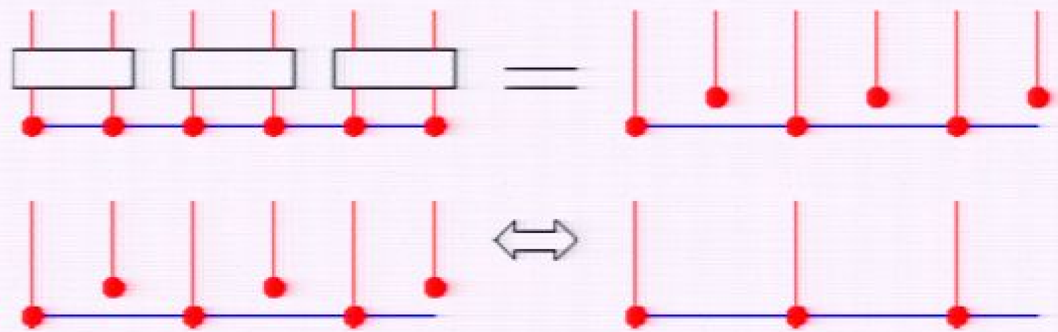

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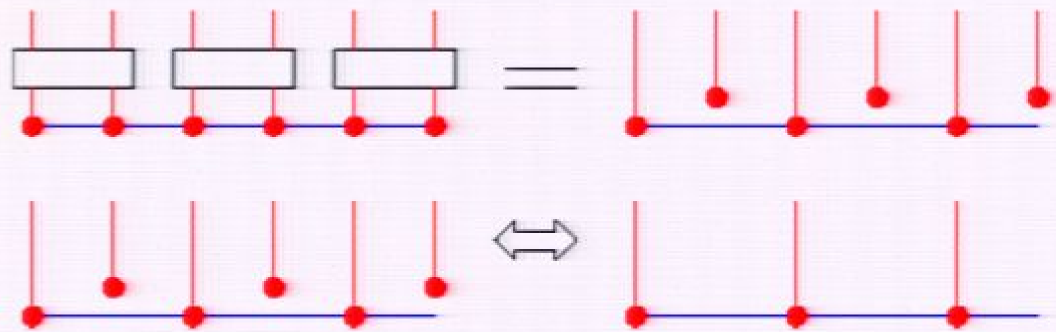

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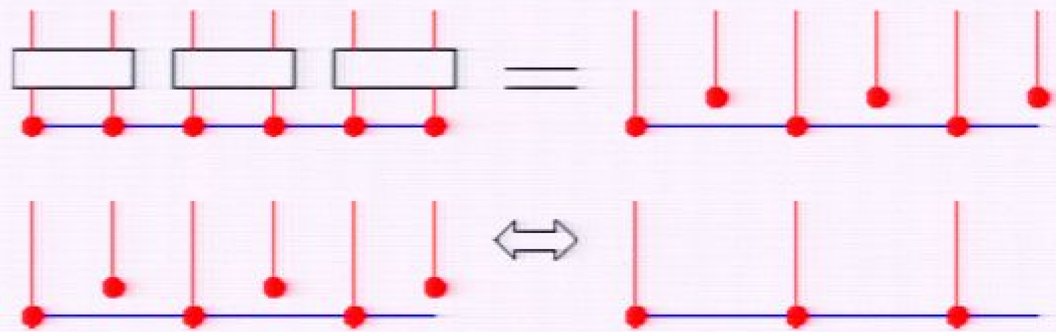

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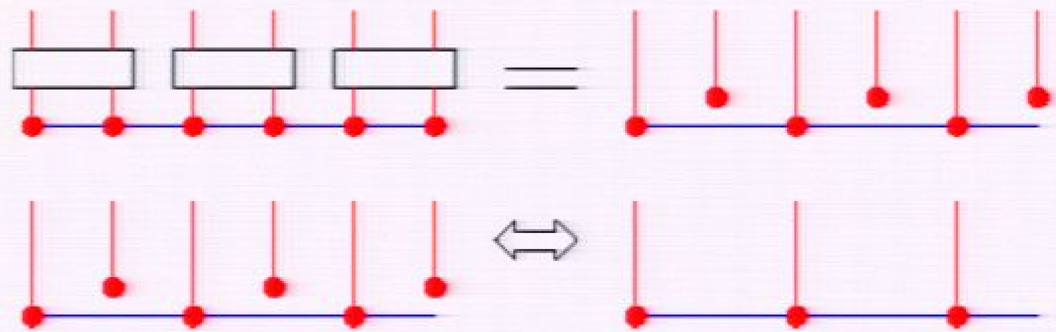
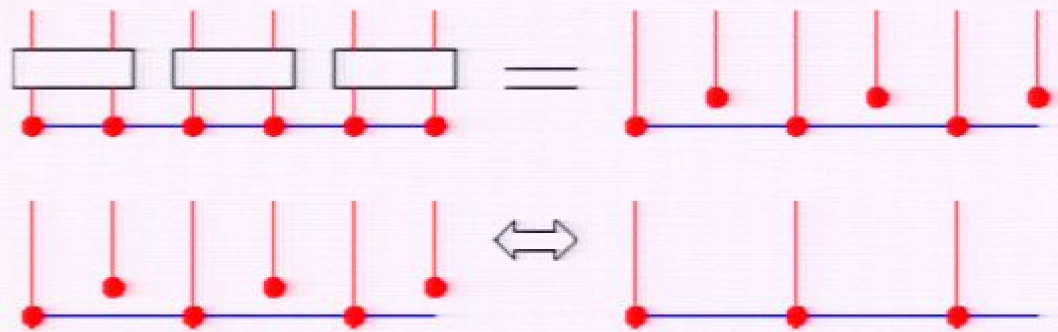


Local Unitary Transformation -- Renormalization Flow

Local unitary operations

Remove decoupled degrees of freedom



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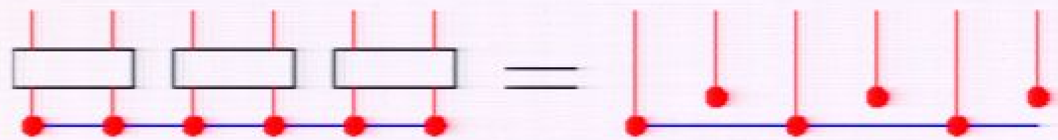

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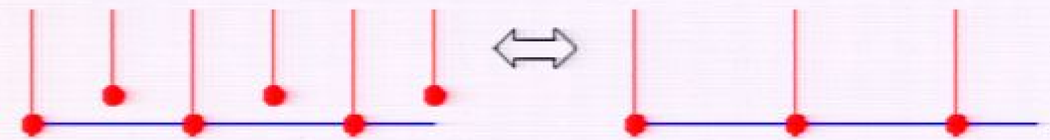

Equivalence relation between systems defined on different Hilbert space

Local Unitary Transformation -- Renormalization Flow

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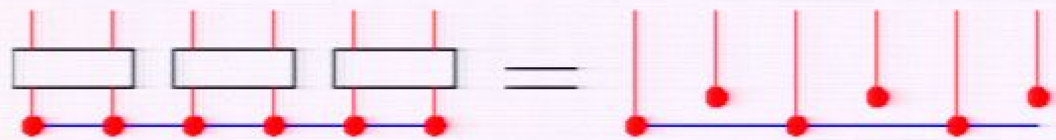

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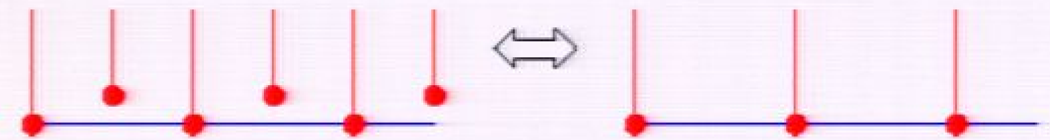

Equivalence relation between systems defined on different Hilbert space

Instead of $\Psi(0) \sim \Psi(1) \Leftrightarrow \Psi(1)=U\ \Psi(0)$

We have $\Psi(0) \sim \Psi(1) \Leftrightarrow \Psi(1)=U\ (\Psi(0)\otimes|0000\rangle)$

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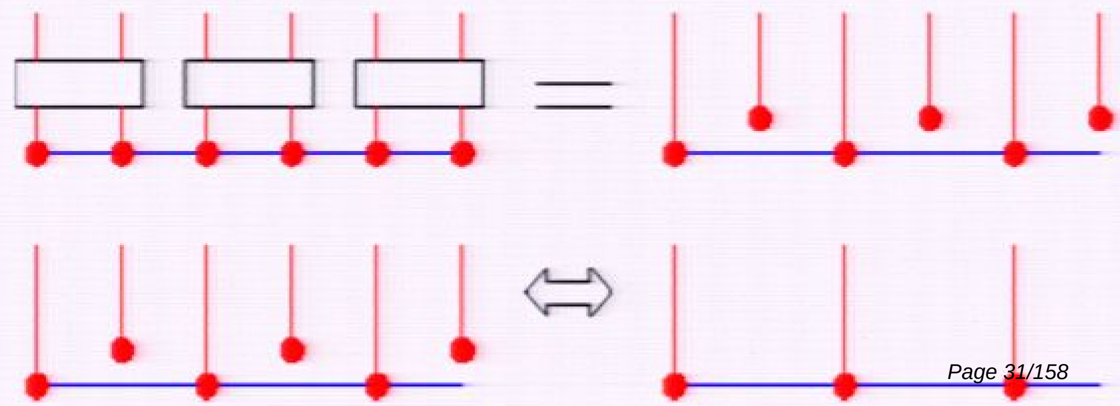

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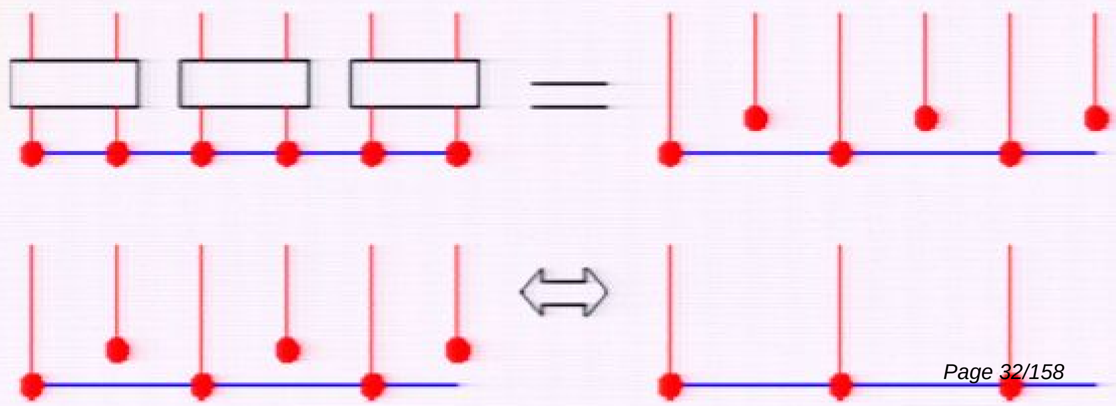

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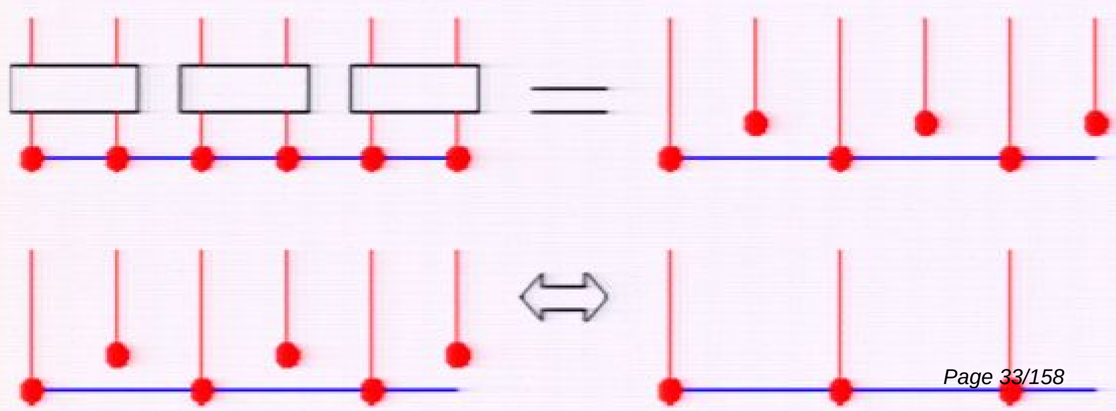
Decoupled degrees of freedom carry trivial quantum number



Quantum Phases

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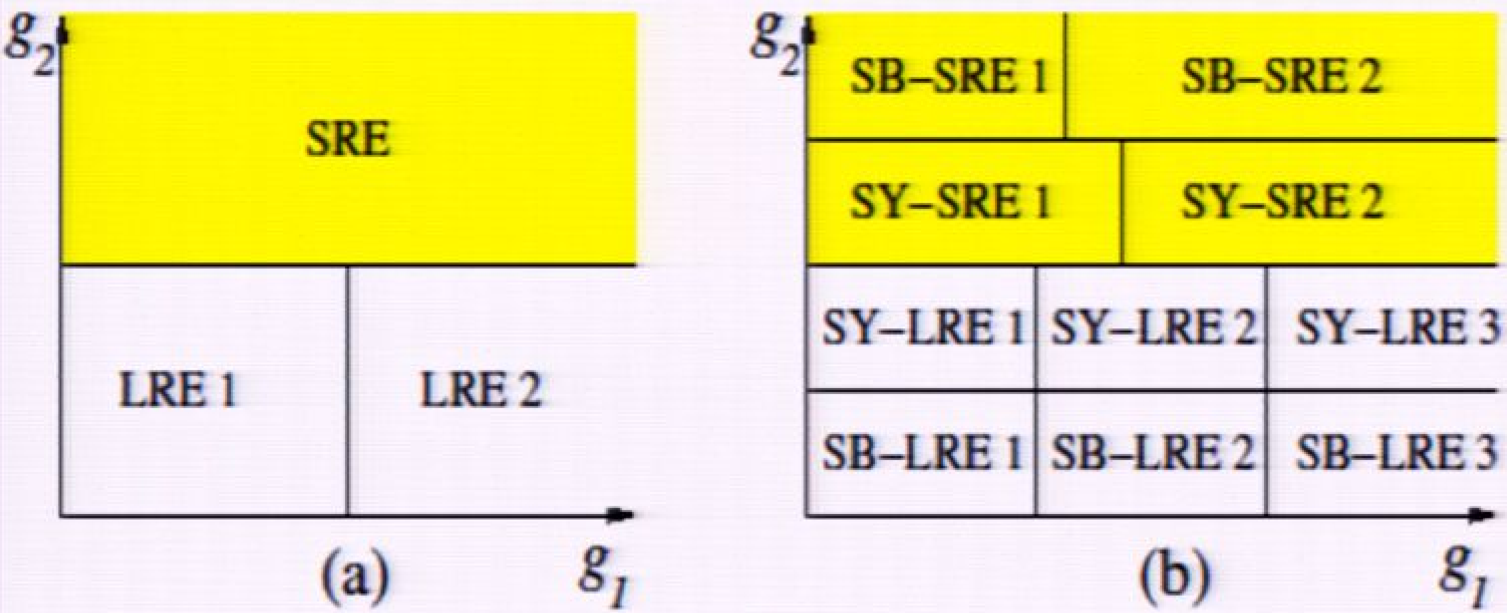

Without Symmetry

With Symmetry

Quantum Phases

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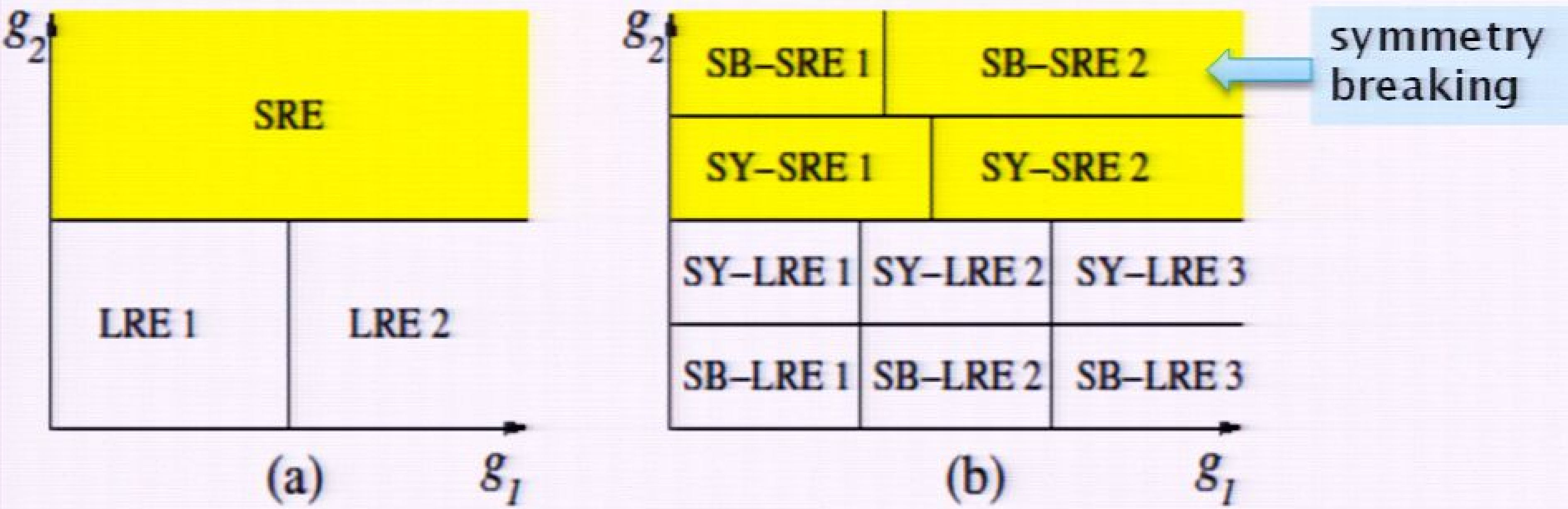

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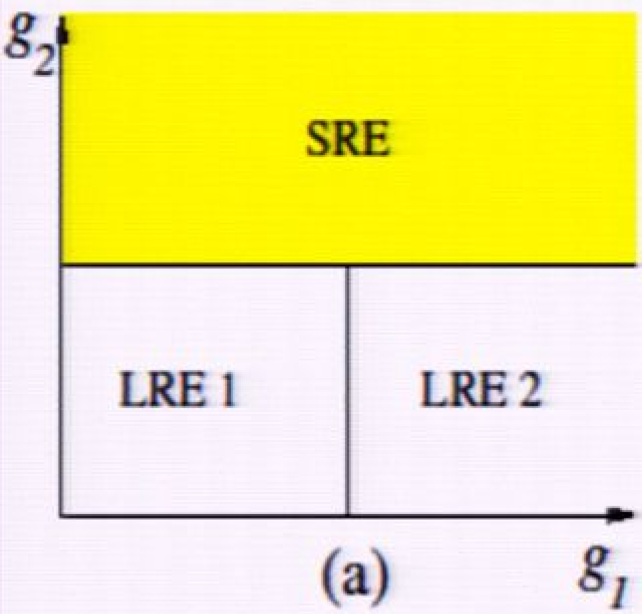

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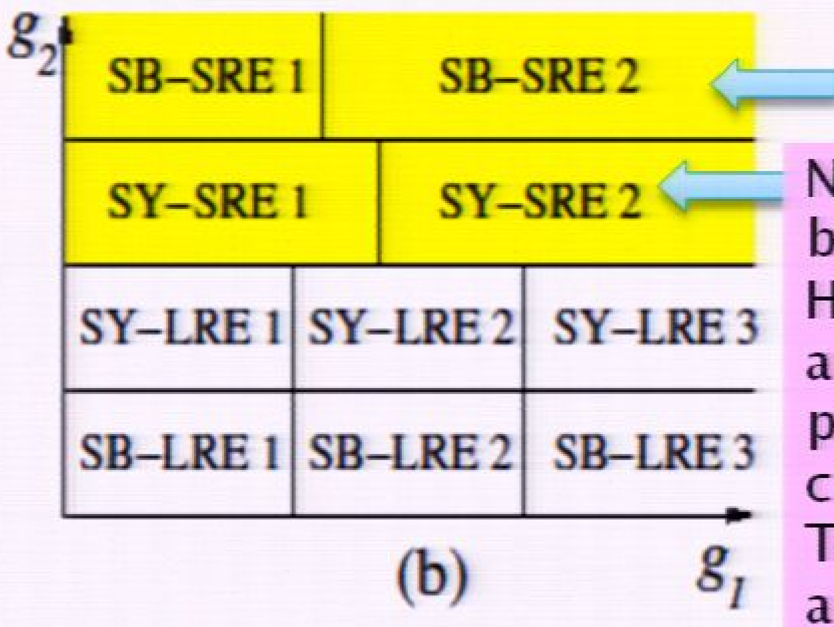

With Symmetry

symmetry breaking

No symmetry breaking: Haldane phase and Sz=0 phase of spin 1 chain Topological and Band insulator

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- FIND the local unitary transformation that does the job!

Results in short

- Found the right local unitary transformation for any state (1D,2D) based on tensor product states

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- No topological order in 1D: every state can be mapped to product state via local unitary transformation
- With symmetry, there are different phases in 1D (at least 2 with Parity only)
- Can determine whether, a state has topological order, or a state is in symmetry breaking phase or not.

Matrix Product States

Matrix Product States

$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} tTr\left(A^{i_1} A^{i_2} \ldots A^{i_N}\right) |i_1 i_2 \ldots i_N\rangle$$

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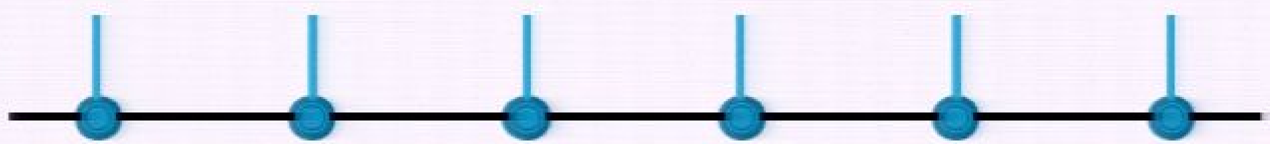

❖ Matrix product states approximate ground states of 1D gapped system well

❖ Consider translational invariant, finite dimensional matrices

Matrix Product States

- Local Unitary Operation

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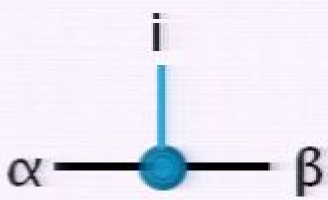
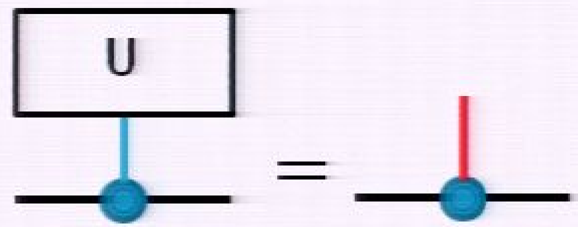


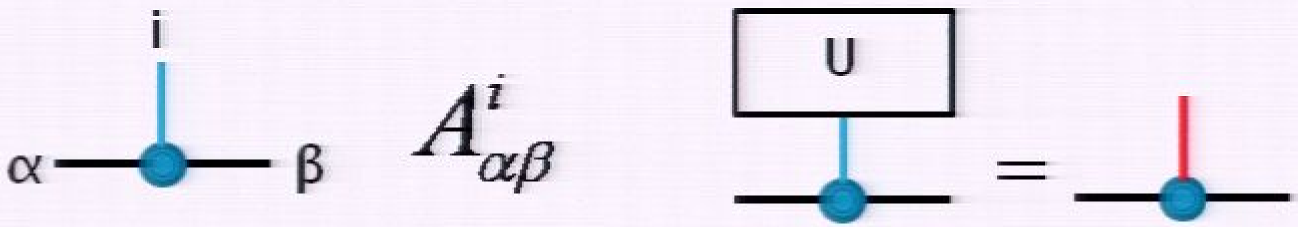
$A^{i}_{\alpha\beta}$



Reduce the range of the physical index

Matrix Product States

- Local Unitary Operation



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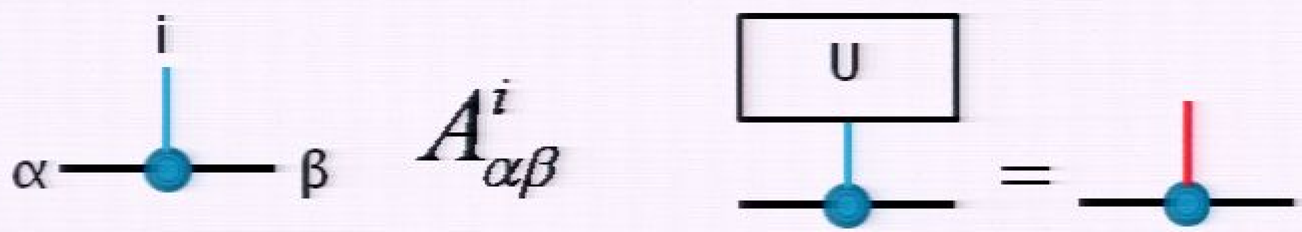

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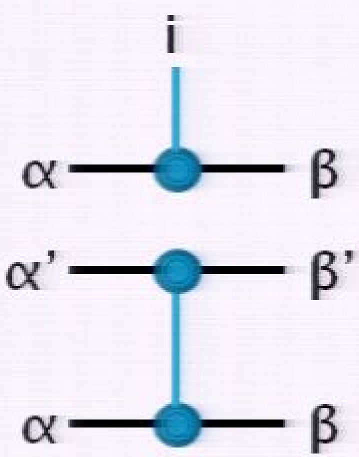

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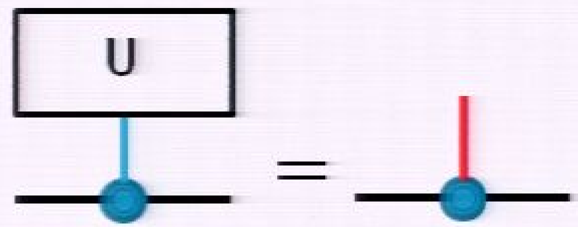

Reduce the range of the physical index

Double tensor $\mathbb{T}_{\alpha'\beta',\alpha\beta} = \sum_{i} (A^{i}_{\alpha'\beta'})^{*} \times A^{i}_{\alpha\beta}$

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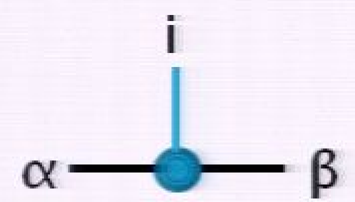

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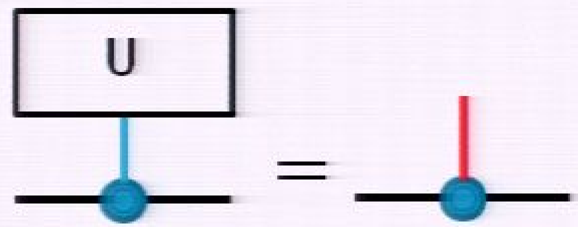

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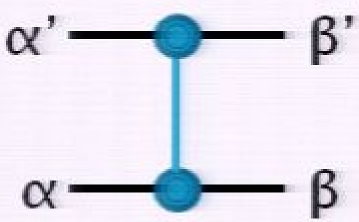

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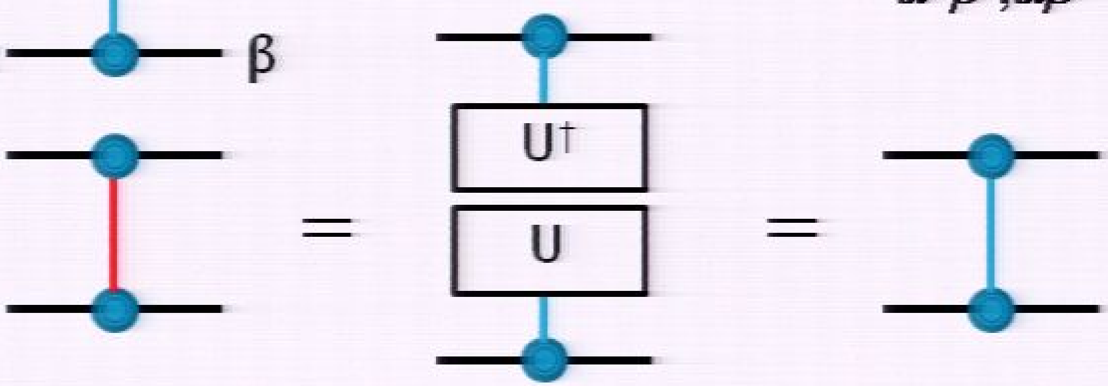

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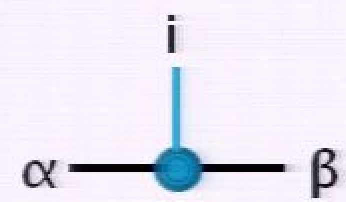

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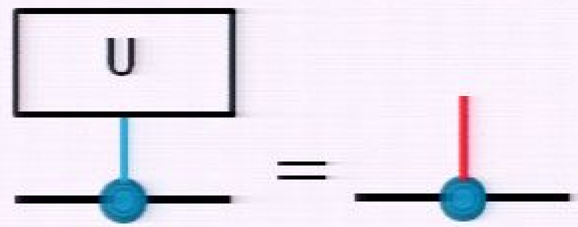

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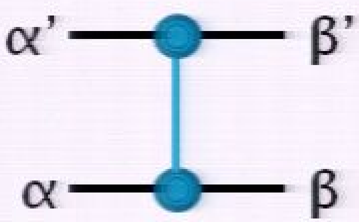

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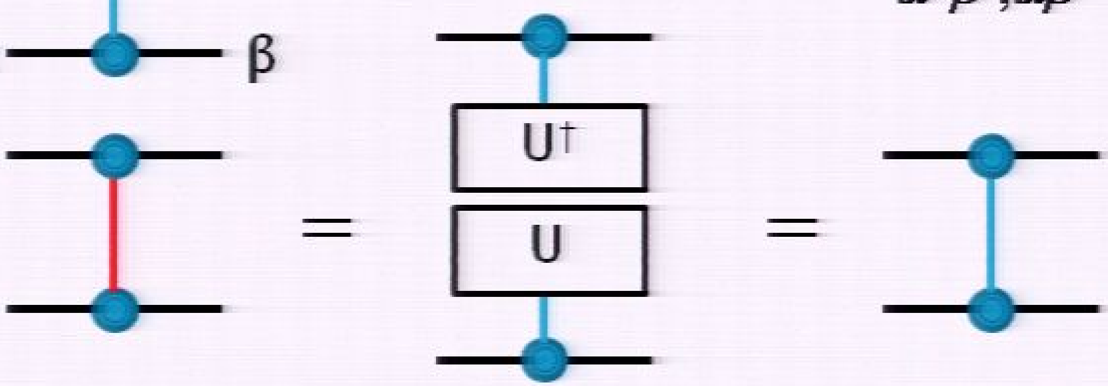

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m<=n

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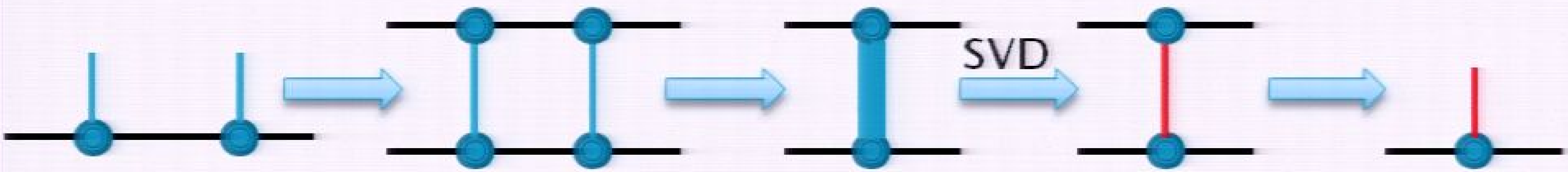

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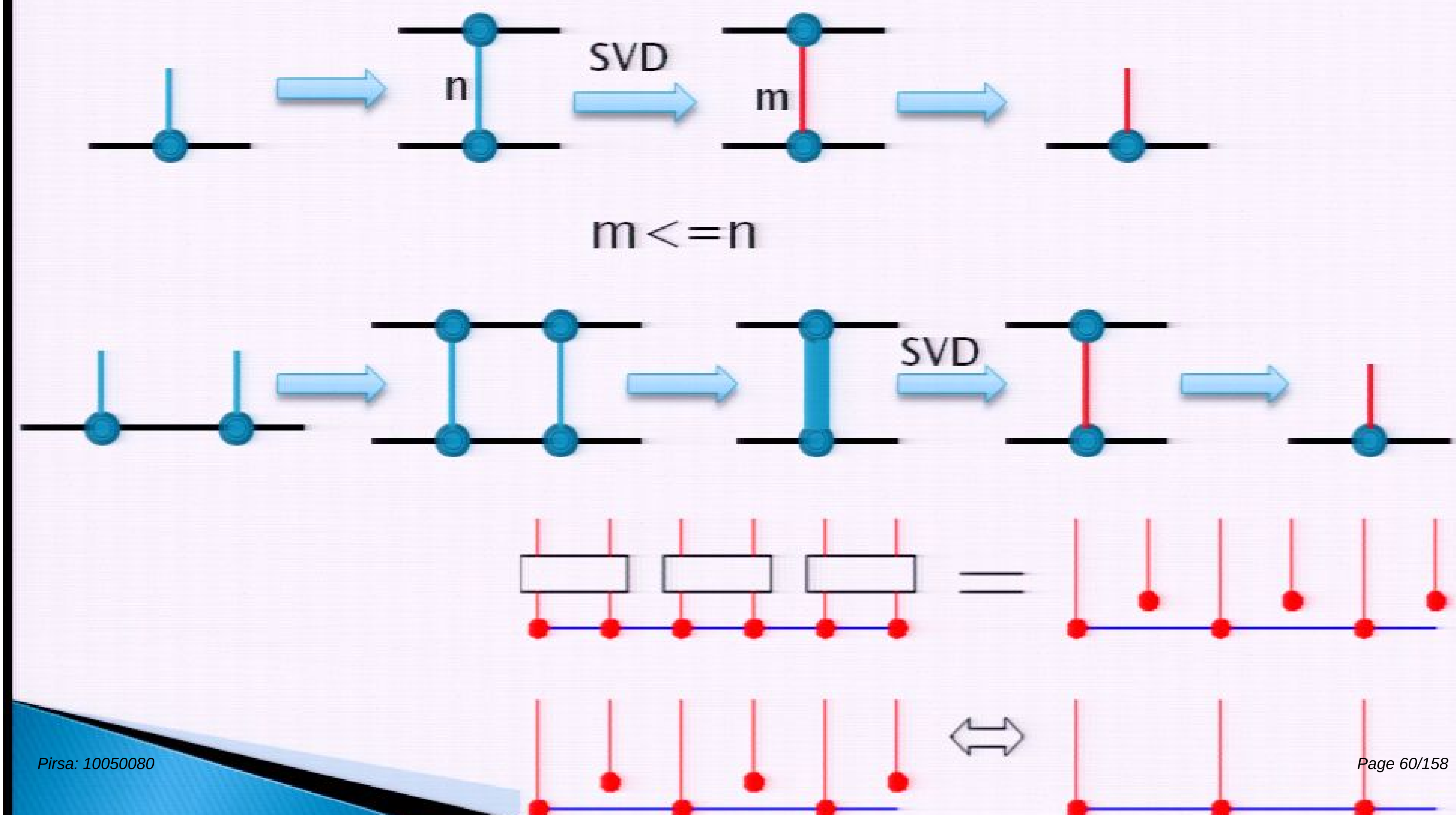

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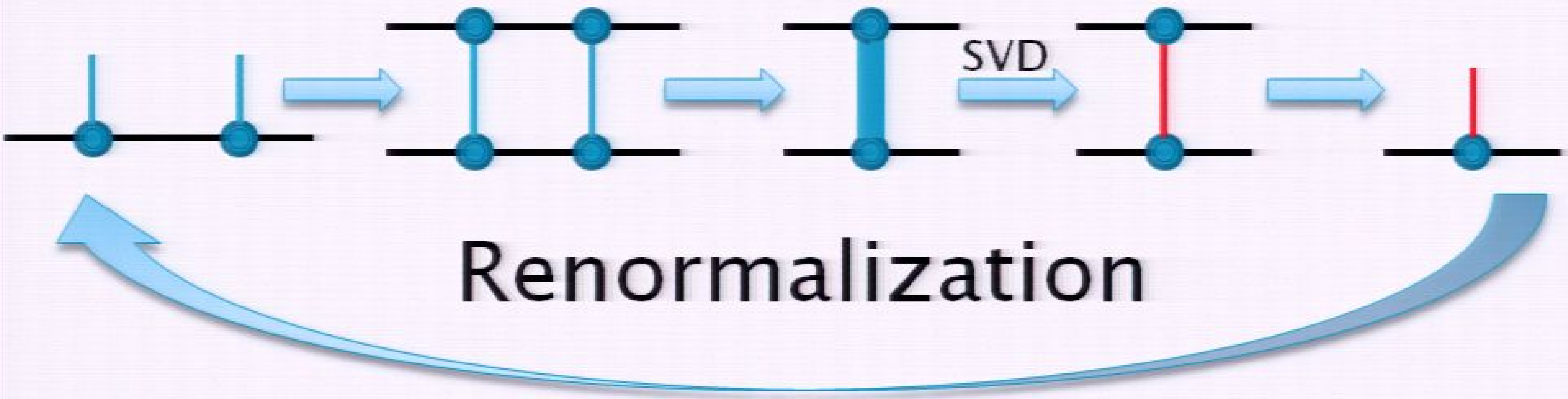

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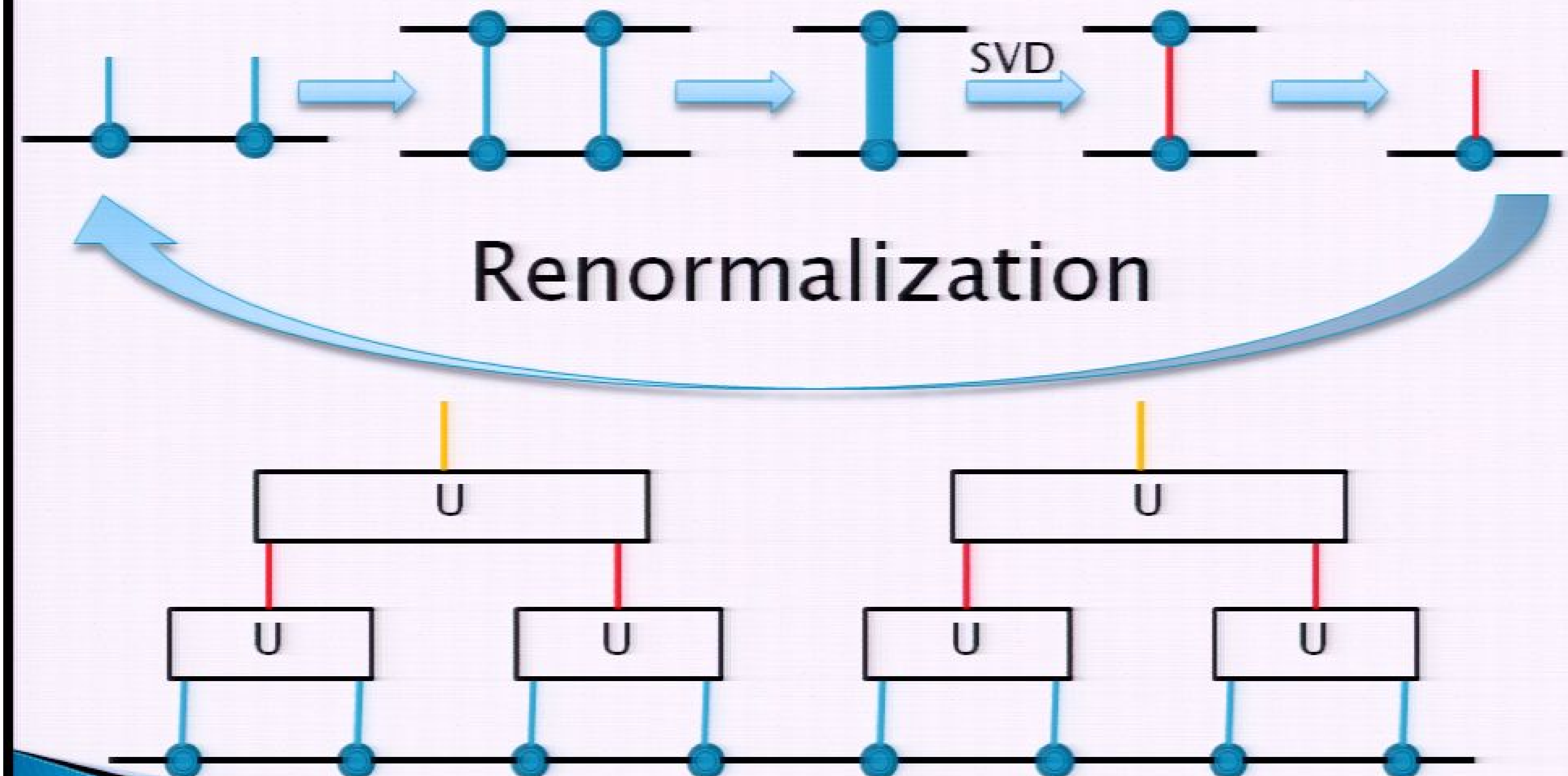

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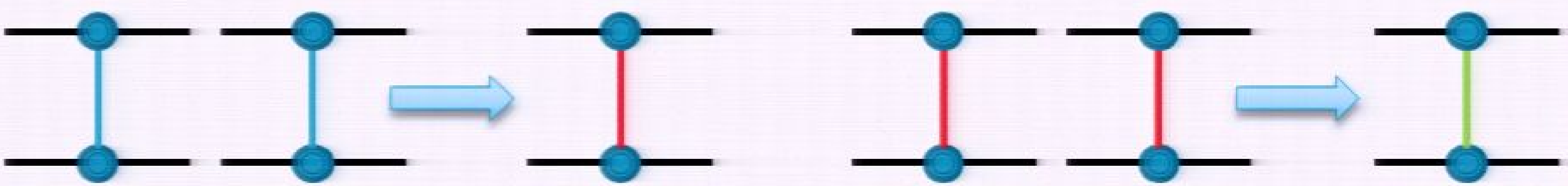

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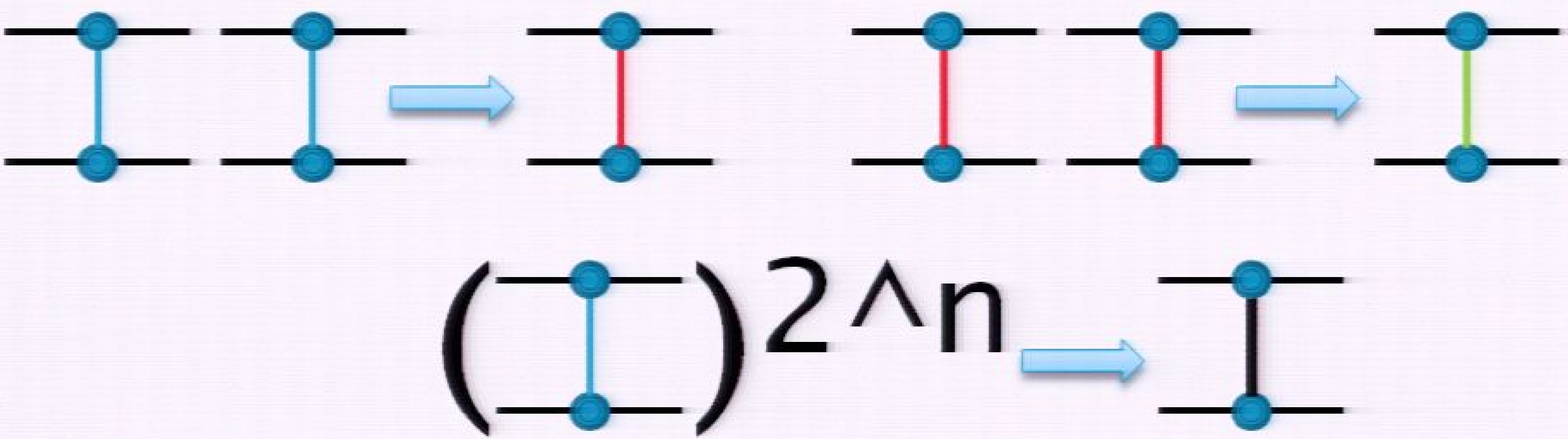

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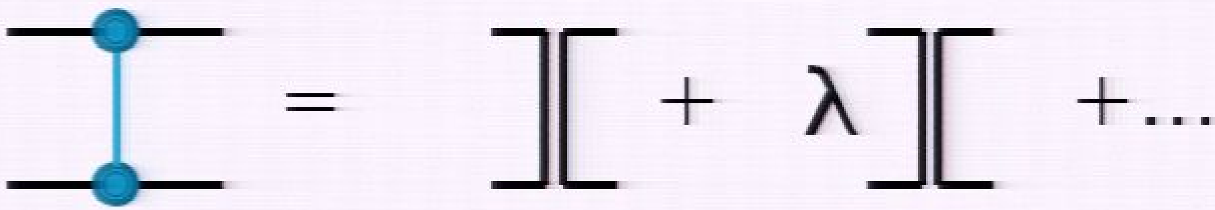

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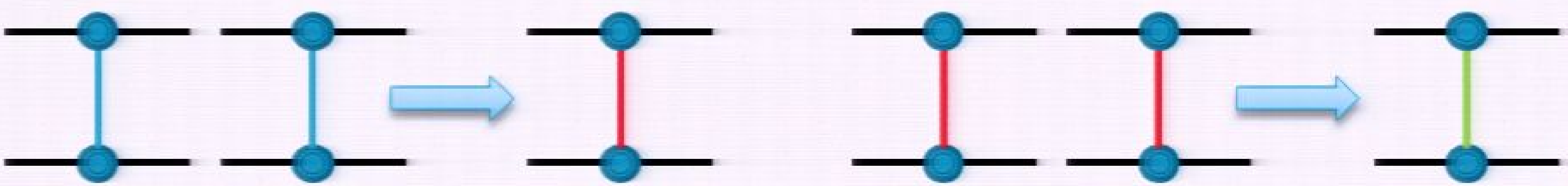

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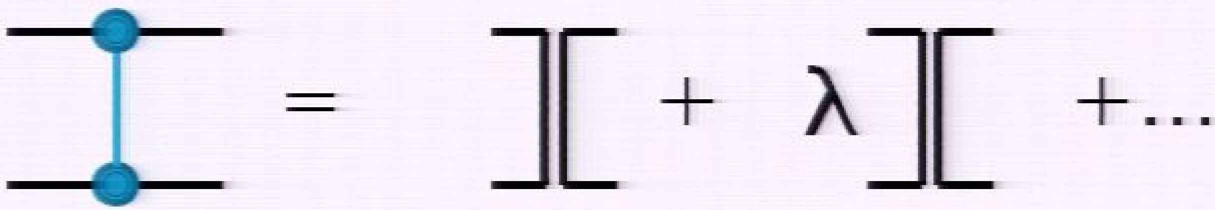

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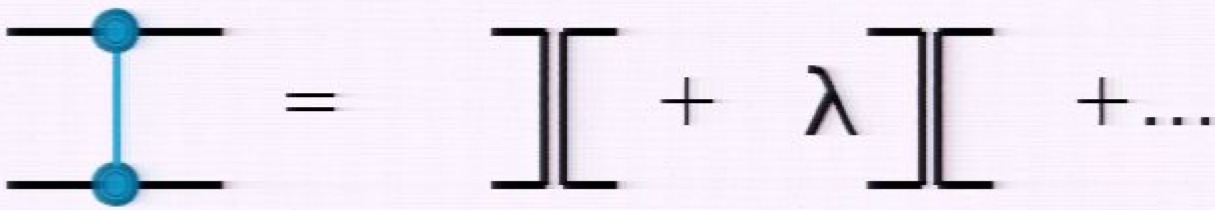

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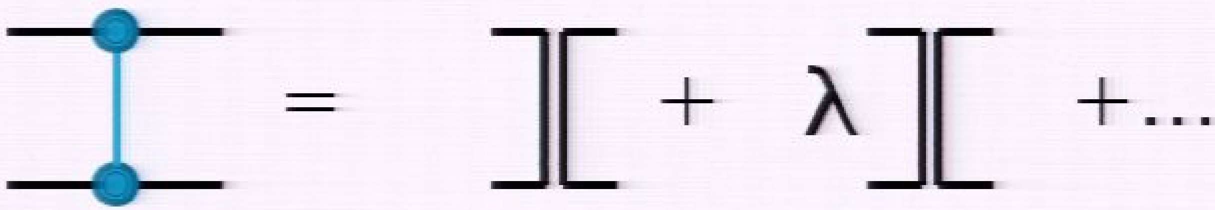

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Finite correlation length: $\lambda<1$

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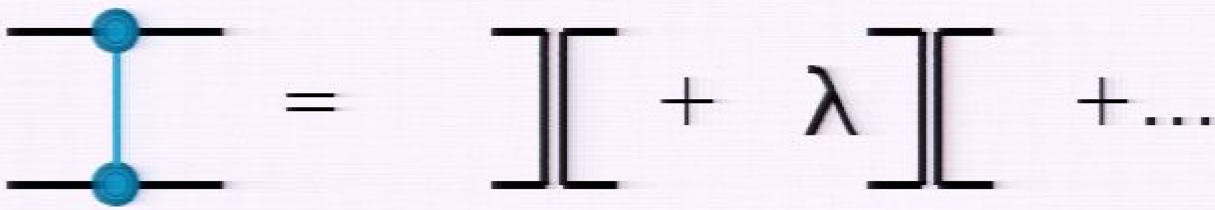

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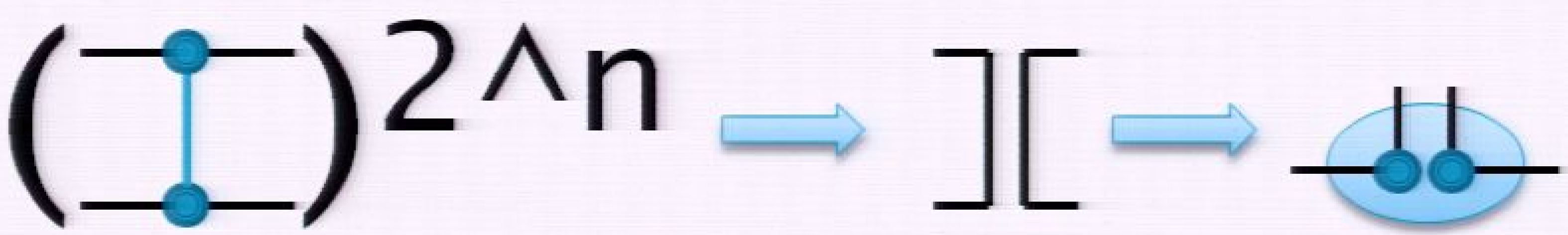

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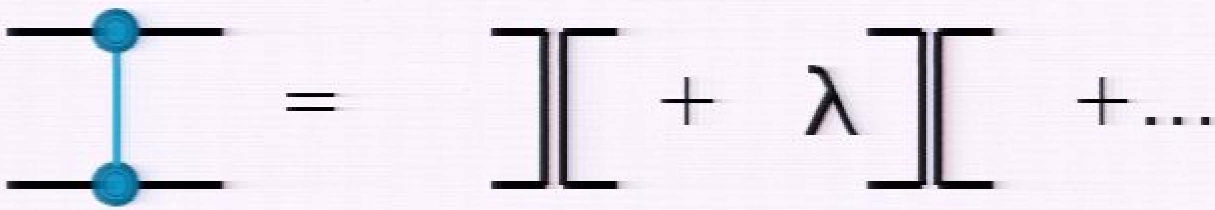

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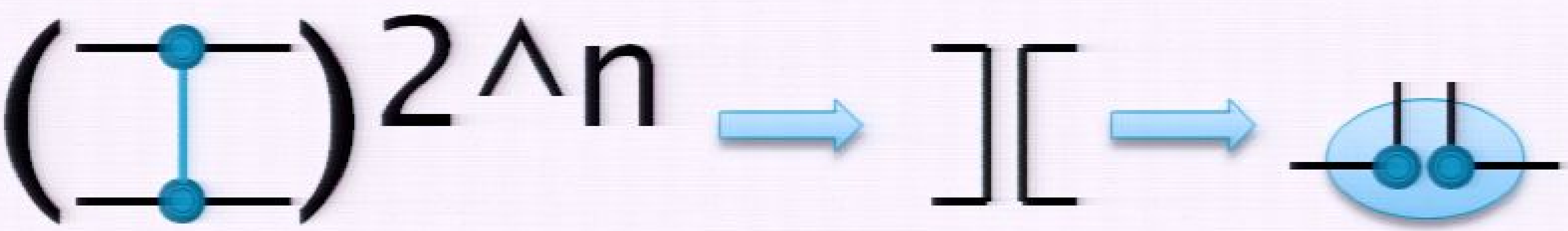
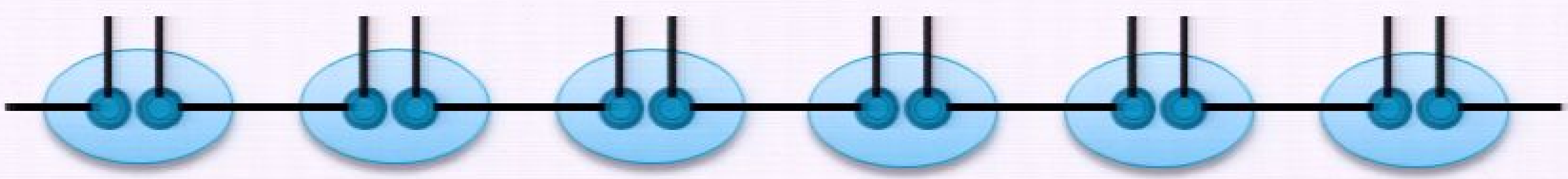
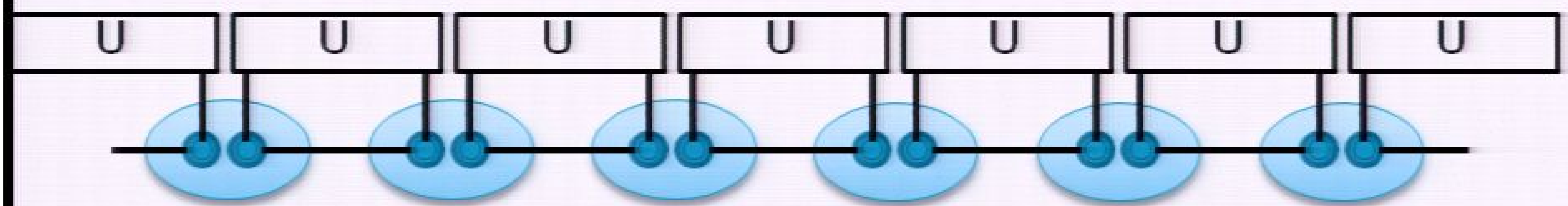




$\eta_0|00\rangle+\eta_1|11\rangle+\ldots$

Matrix Product States
--No Topological Order



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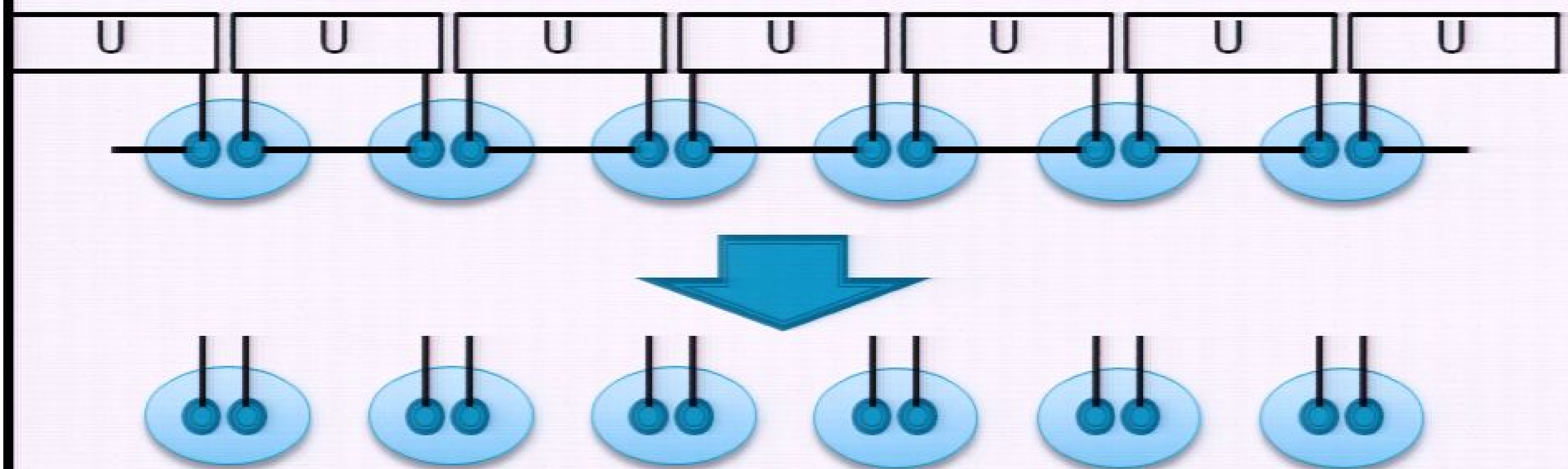

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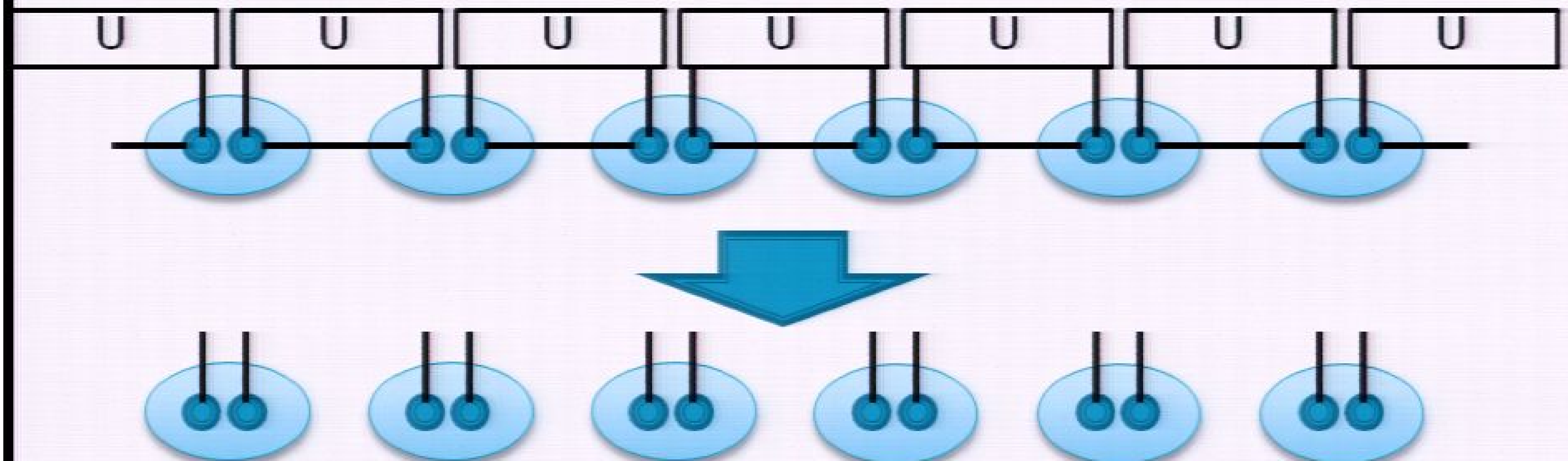

❖ Every finitely correlated matrix product state can be mapped to product states with local unitary transformation

❖ No topological order in 1D

Matrix Product States
--Symmetry

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- What if we have symmetry?
  Short Answer: There will be more phases

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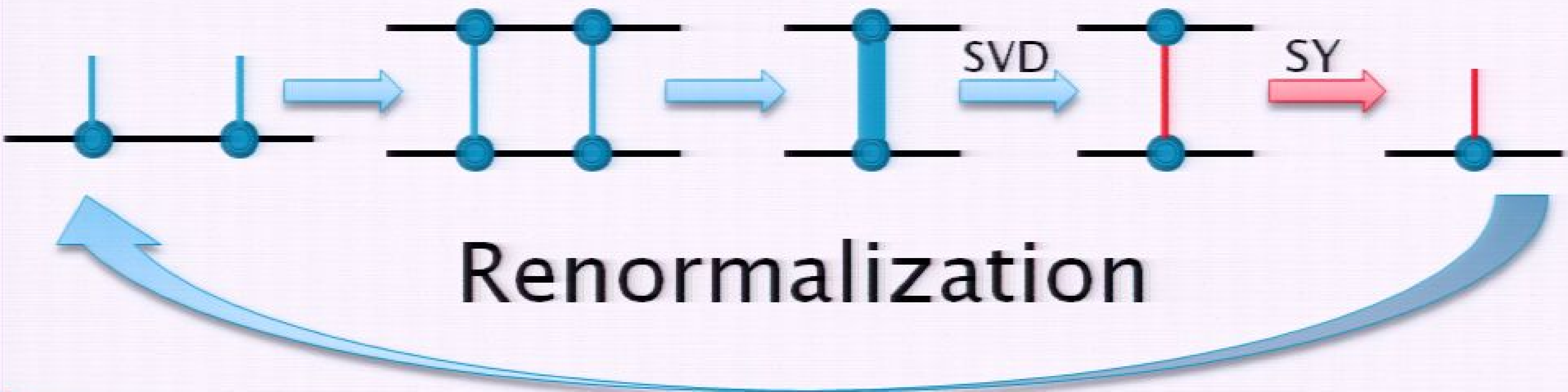

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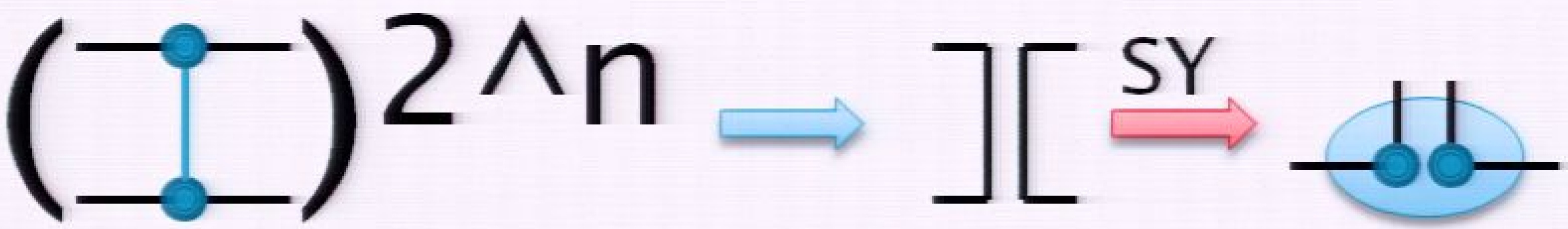
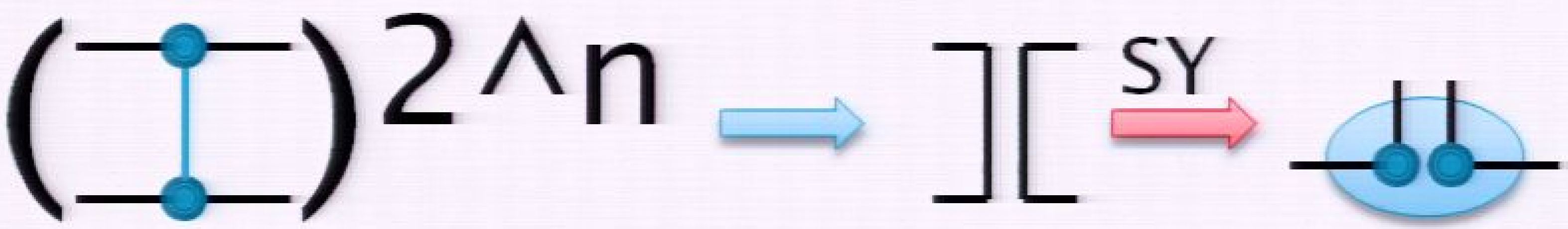
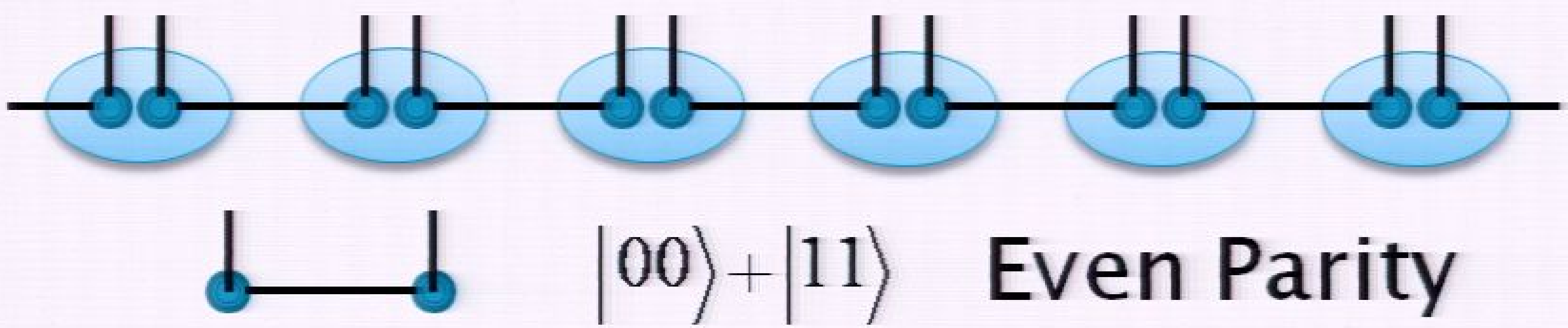


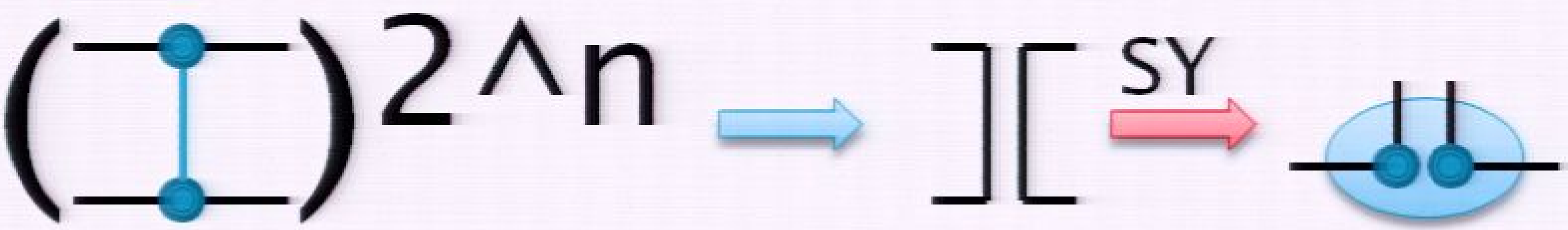
Matrix Product States --Symmetry

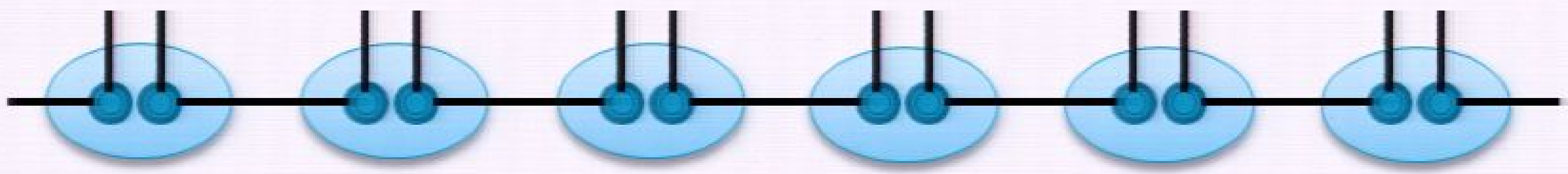


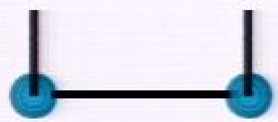


$|00\rangle + |11\rangle$ **Even Parity**

Matrix Product States
--Symmetry







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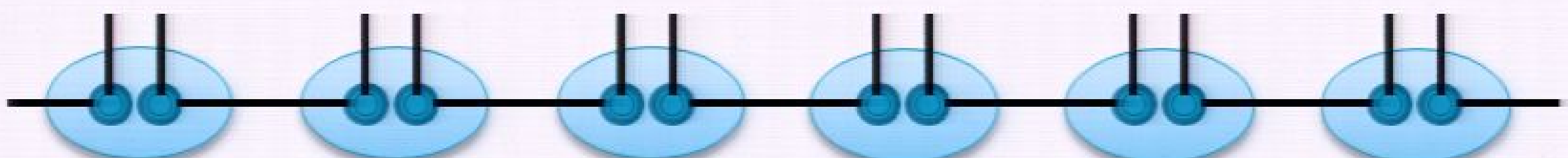
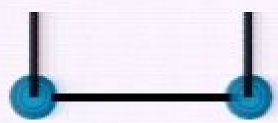
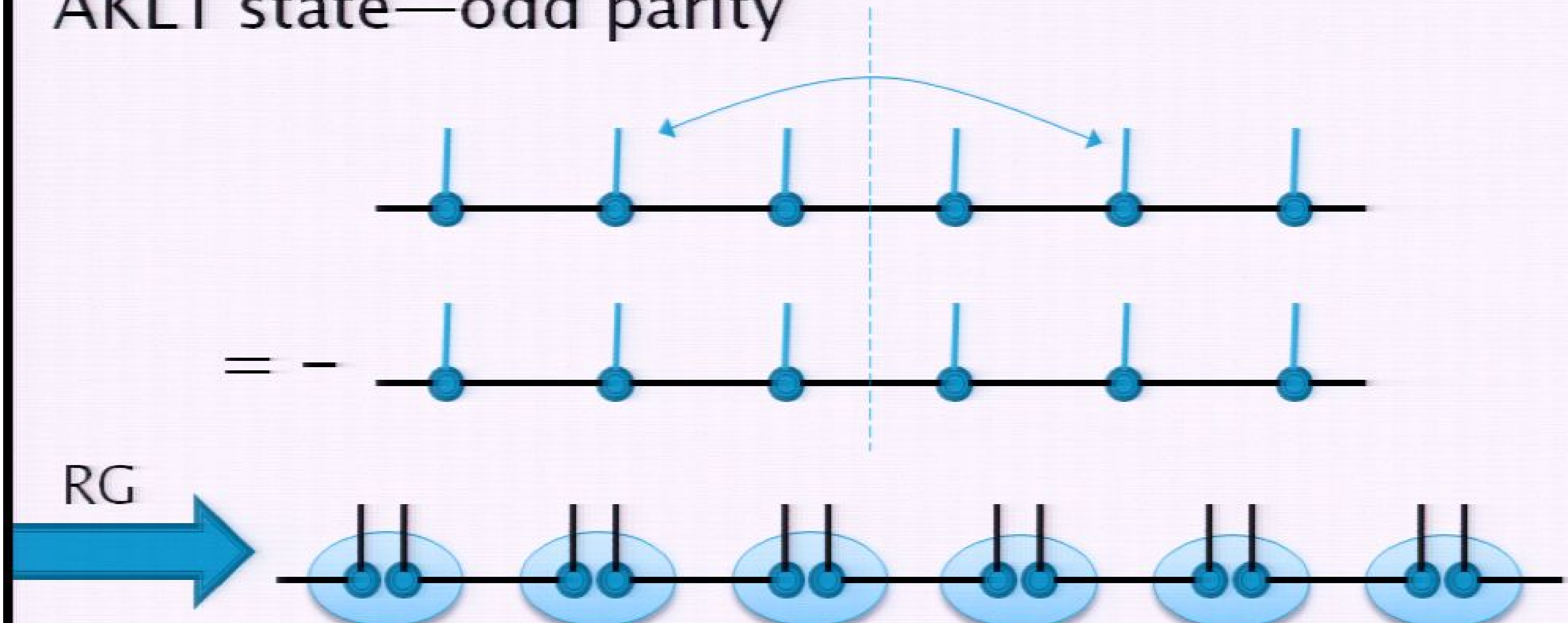




$|01\rangle - |10\rangle$ **Odd Parity**

Matrix Product States --Symmetry

AKLT state—odd parity



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Matrix Product States
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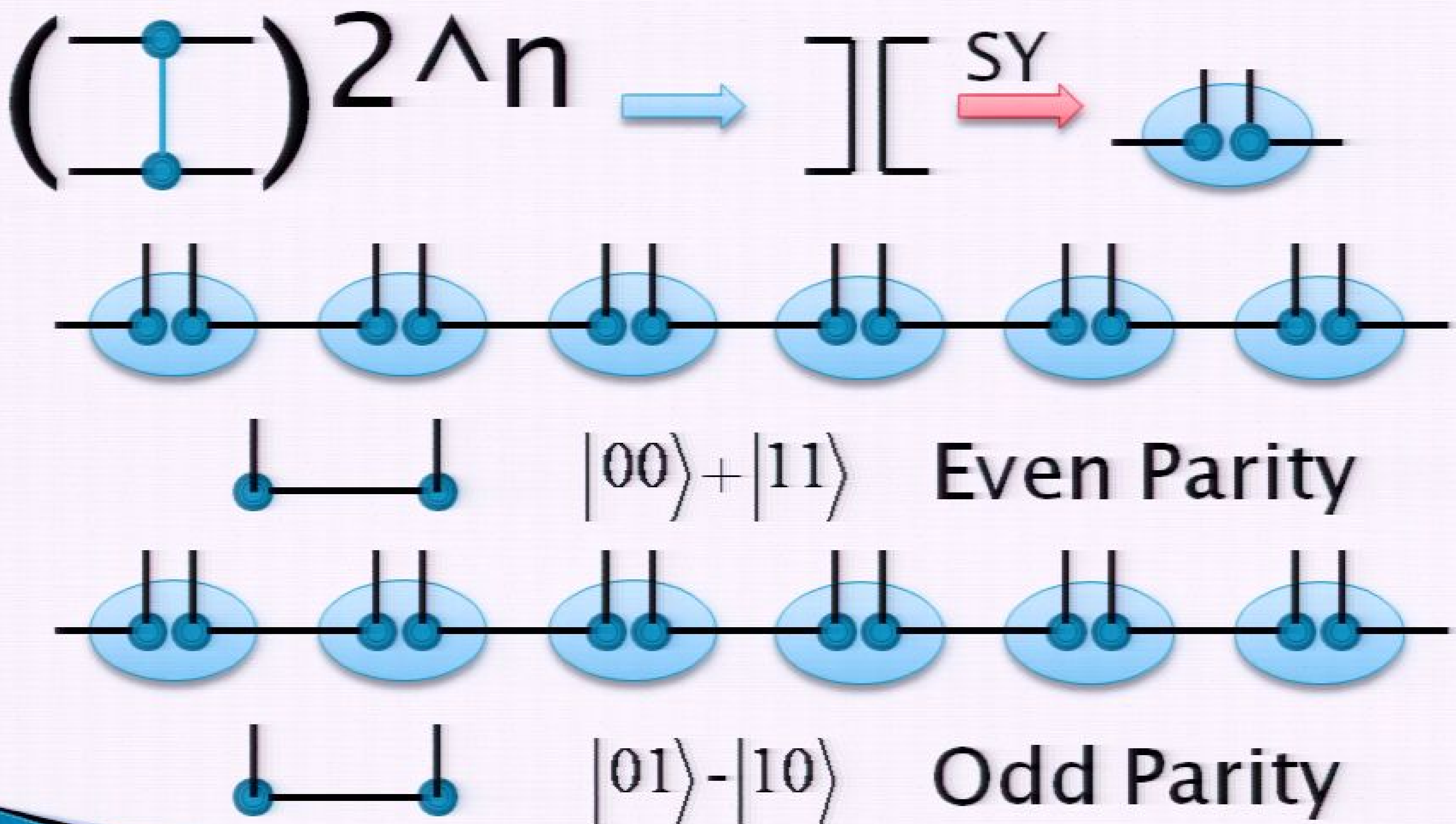

Matrix Product States --Symmetry

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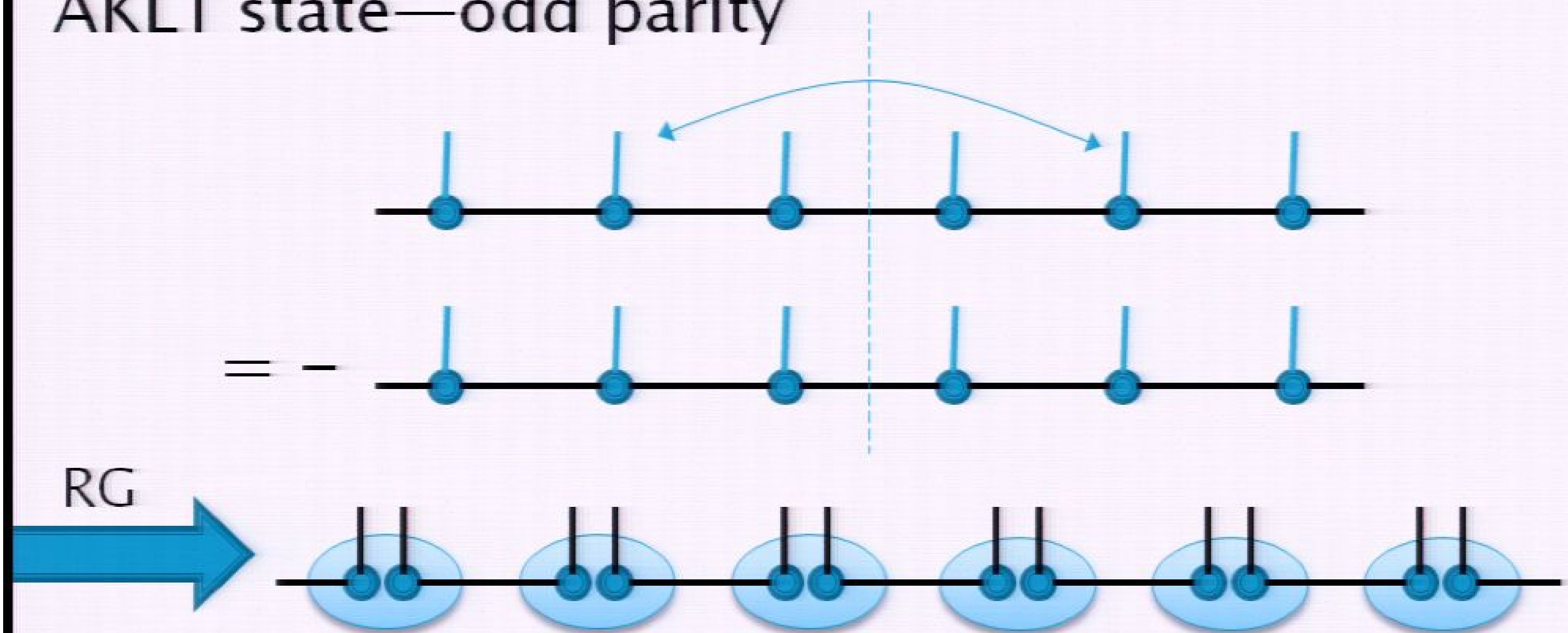

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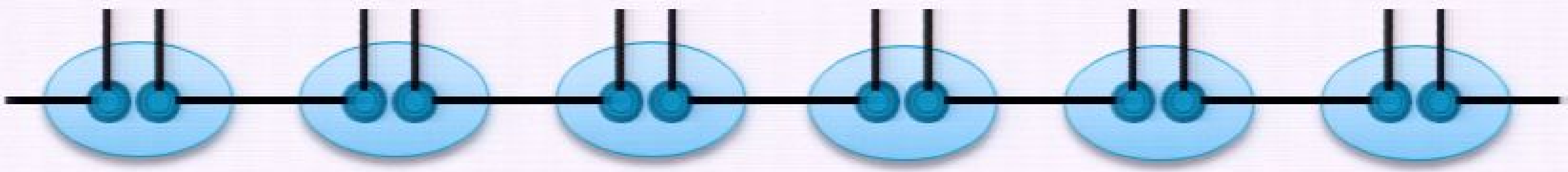
Matrix Product States --Symmetry



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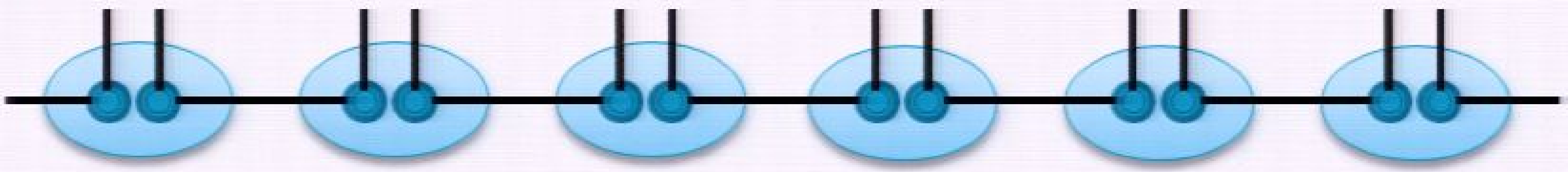

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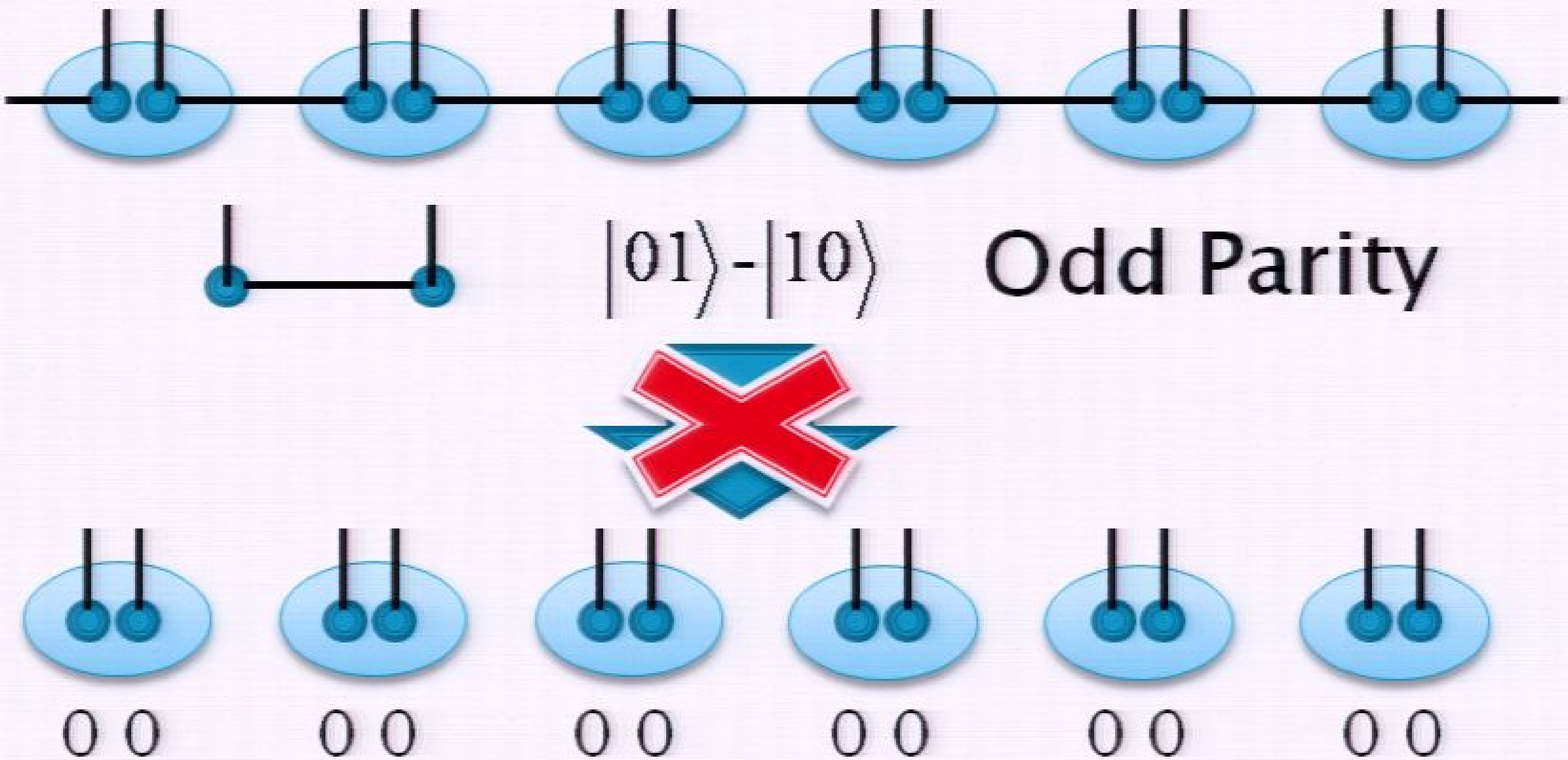

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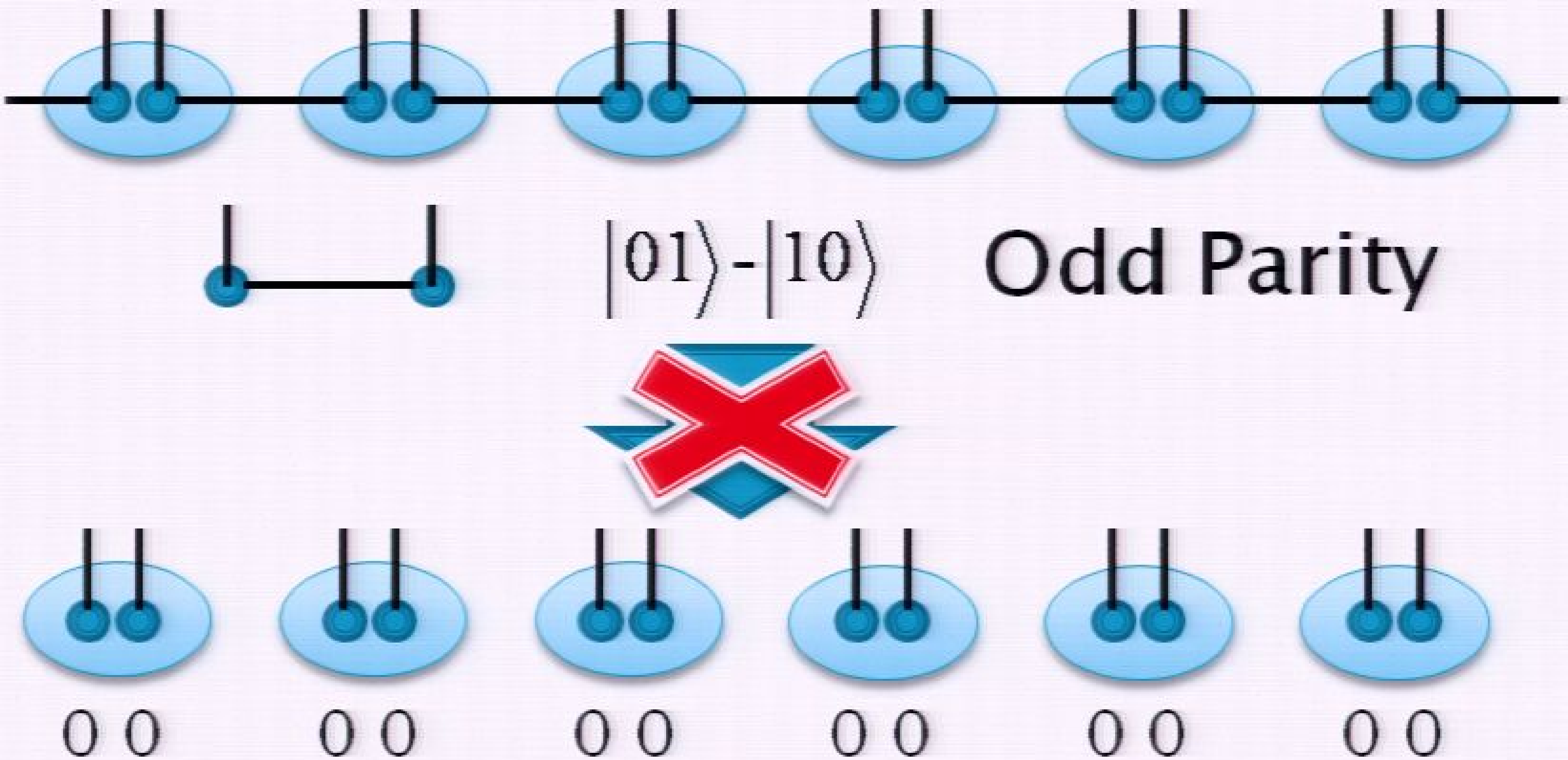

Haldane phase is a different phase from the trivial phase if parity symmetry is preserved

Tensor Product State
-- Renormalization Flow

- 1D no topological order

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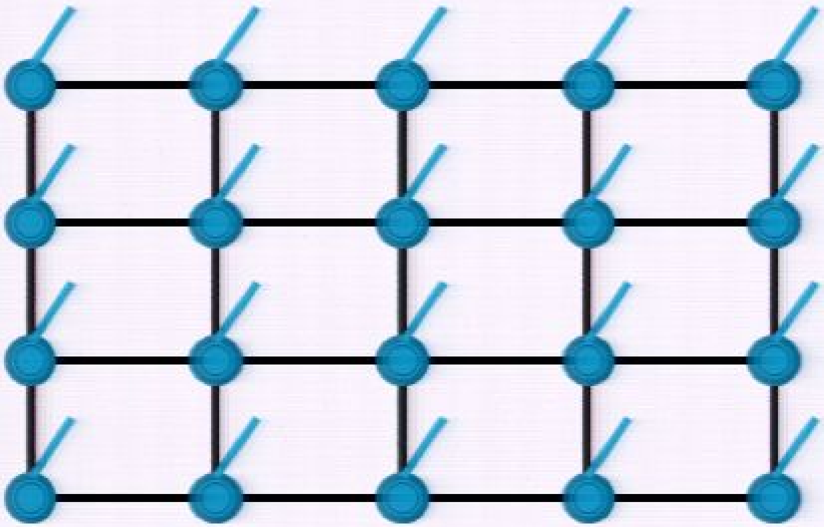
Tensor Product State
-- Renormalization Flow

- 1D no topological order
- Provide framework for classifying 1D phases with symmetry
- 2D?
- A variety of topological order described by fixed point states
- How to generate an RG flow that flows a generic state to the simple fixed points? How to remove local entanglement while retaining global entanglement?

Tensor Product State
-- Renormalization Flow



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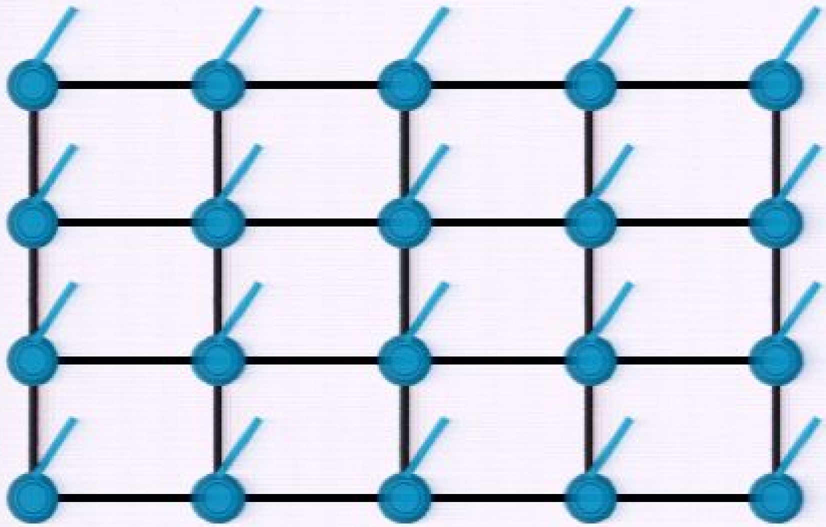
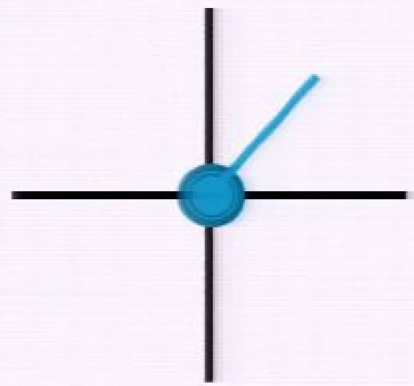
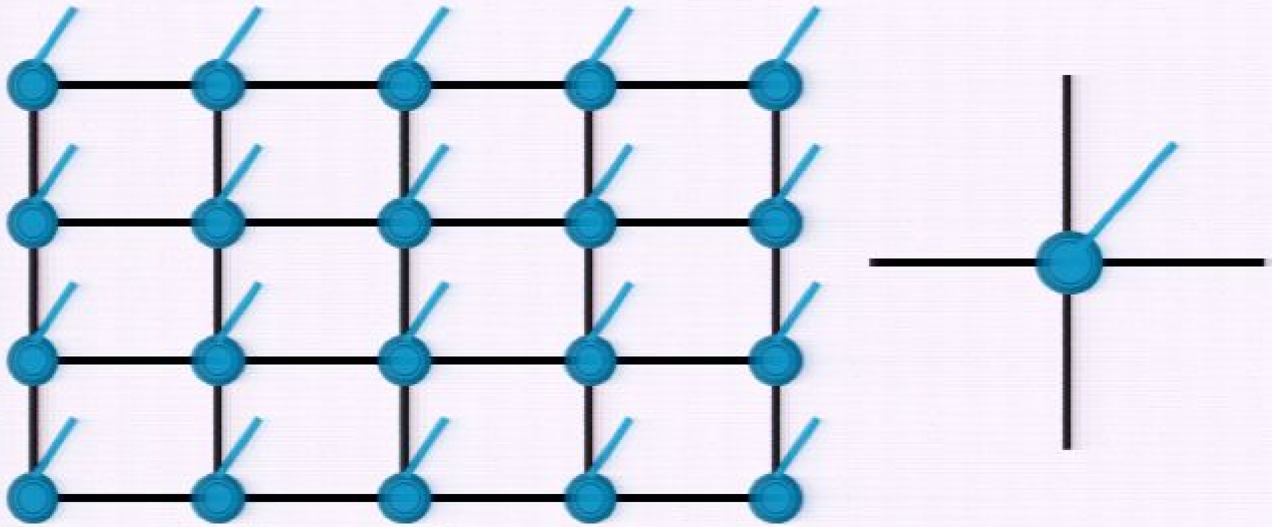




Local tensor contains complete information about entanglement of the state

Tensor Product State -- Renormalization Flow



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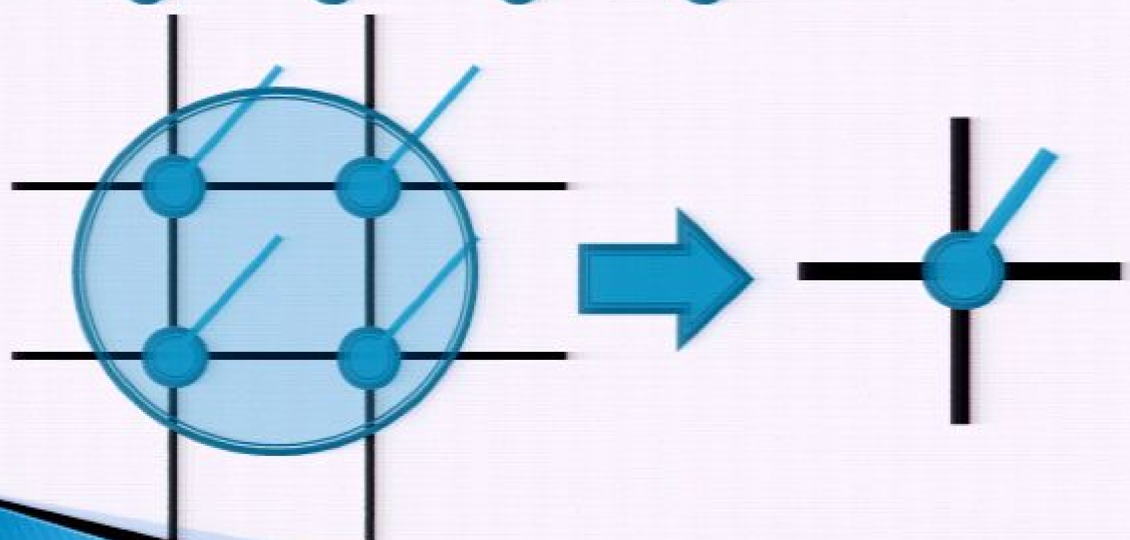

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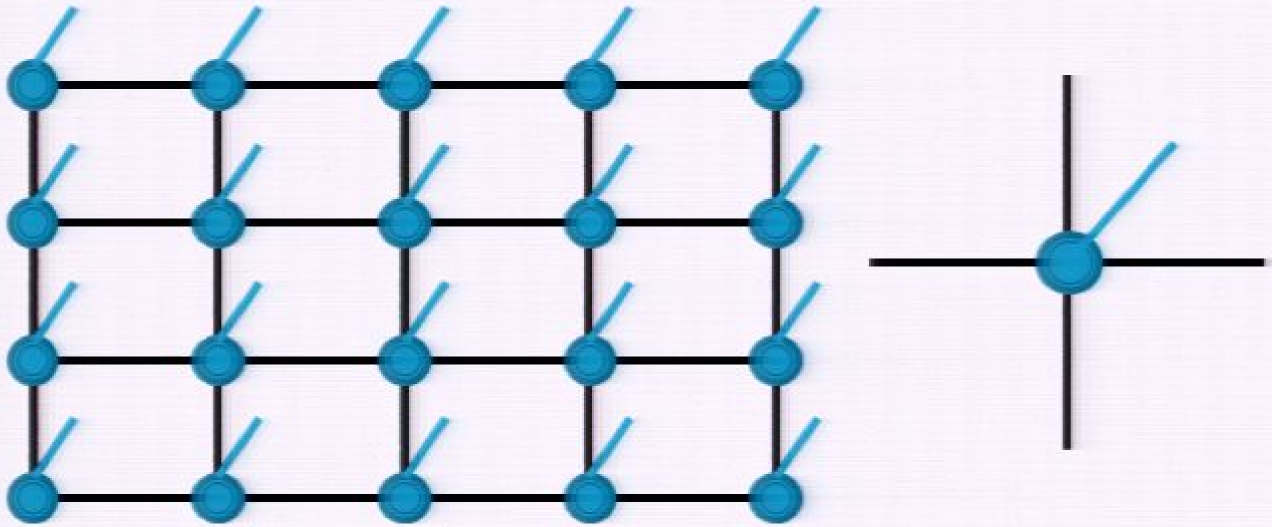

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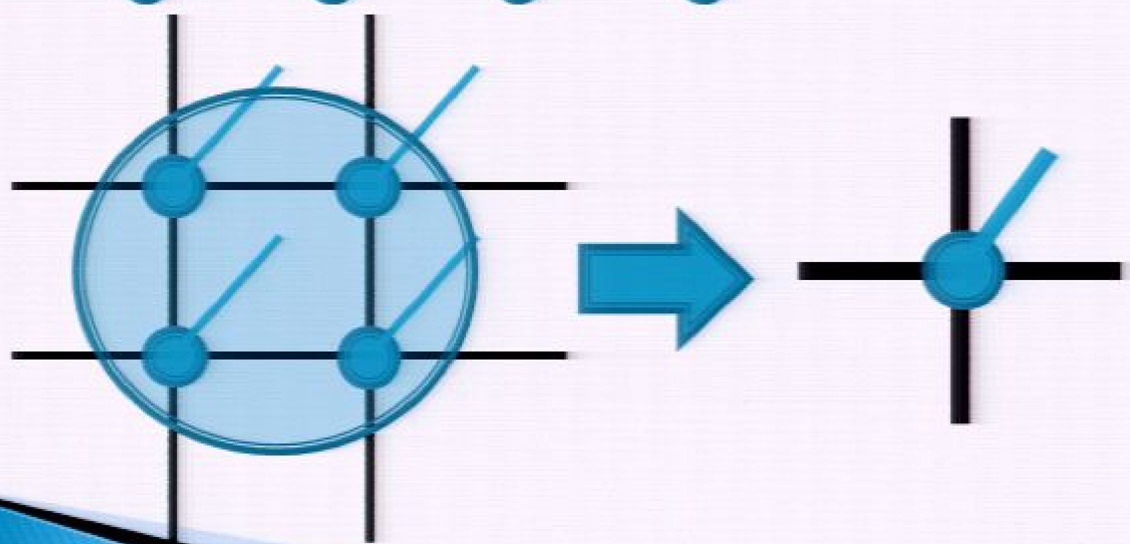

- Link dimension, physical dimension grow exponentially
- Need to find a way to reduce them

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- Identify the entanglement structure and necessary tensor structure to represent it

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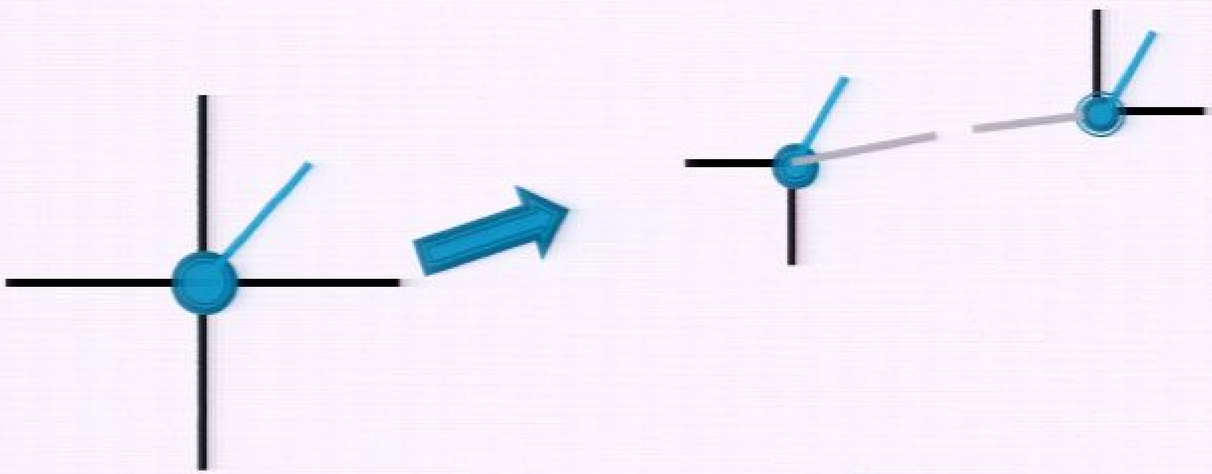

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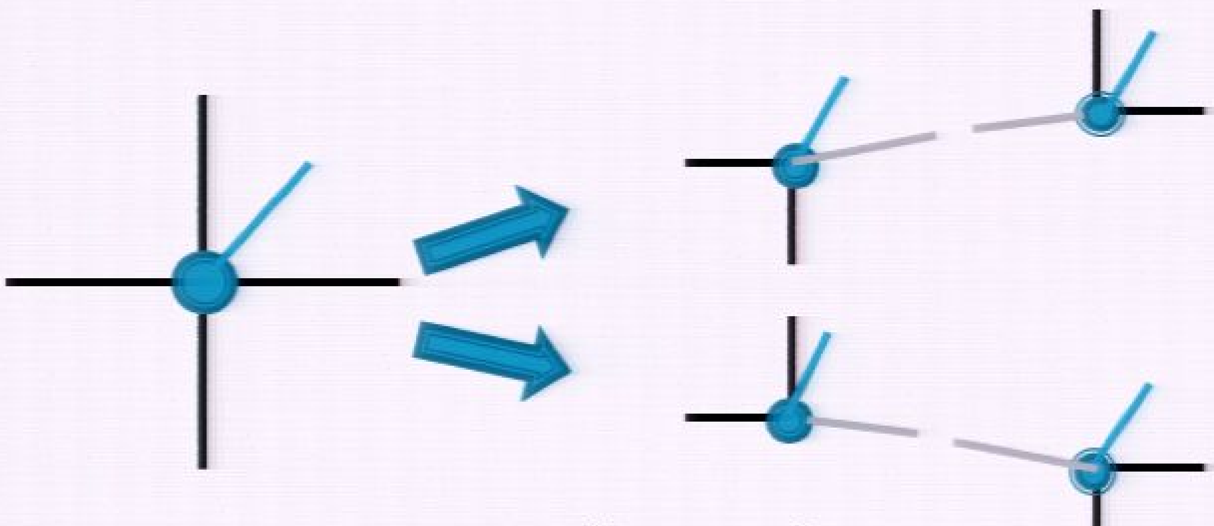

- Non-zero singular values represent necessary entanglement structure; Retain only the non-zero dimensions

Tensor Product State
-- Renormalization Flow

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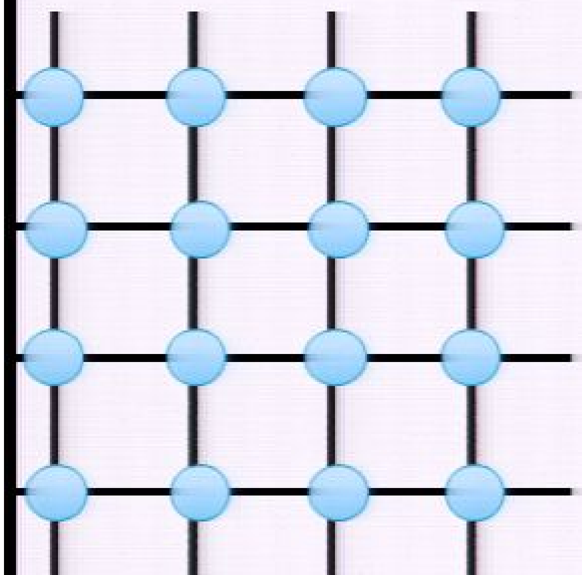

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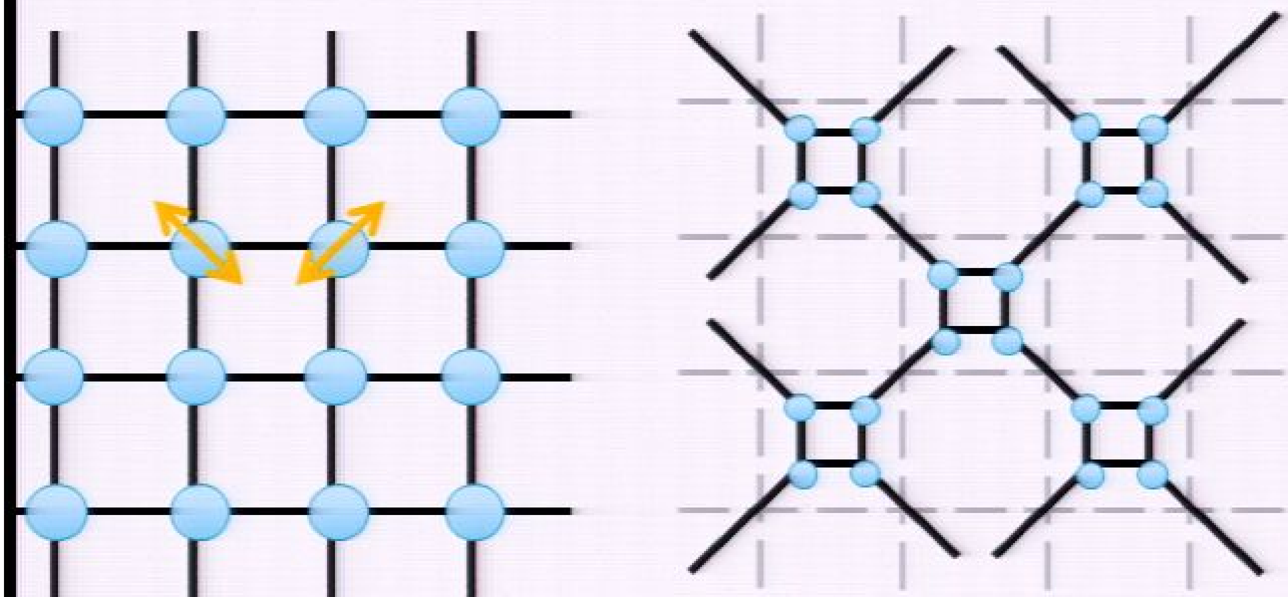

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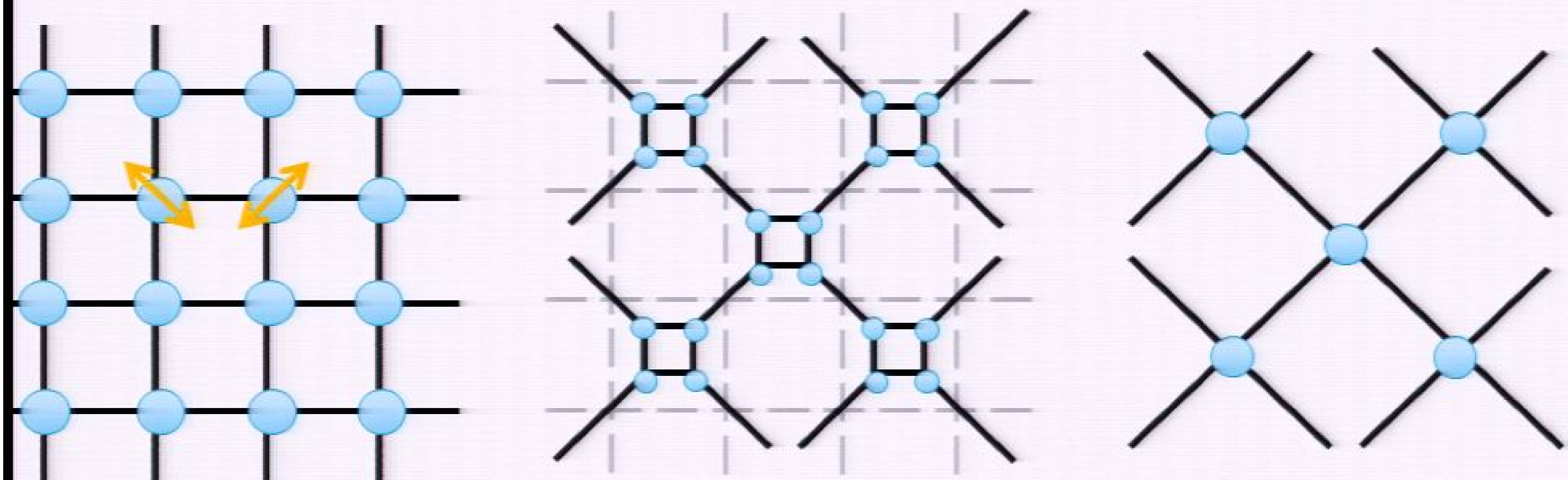
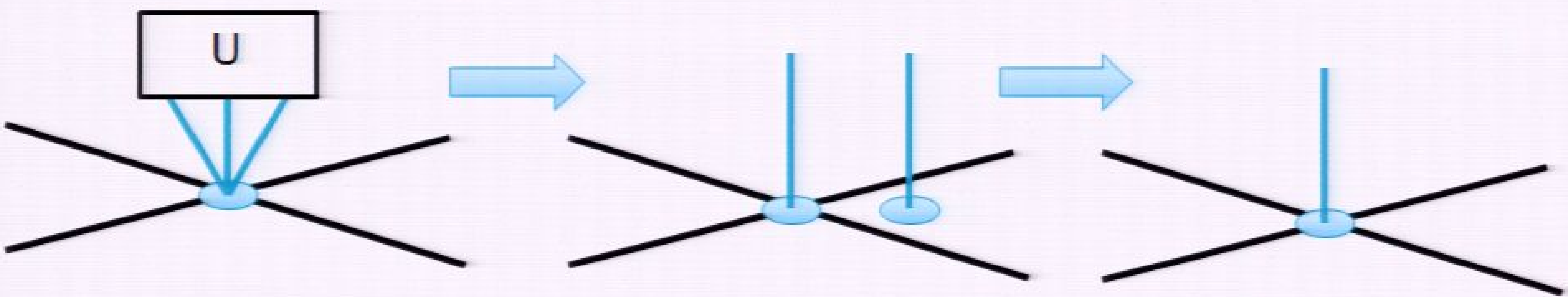


Tensor Product State
-- Renormalization Flow

- Remove unnecessary physical degrees of freedom according to the entanglement structure by applying a local unitary operation



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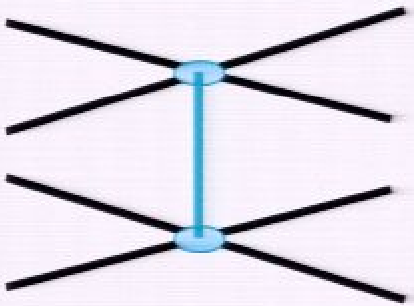

Double tensor

$$\mathbb{T}_{\alpha'\beta'\gamma'\delta',\alpha\beta\gamma\delta} = \sum_i (T^i_{\alpha'\beta'\gamma'\delta'})^* \times T^i_{\alpha\beta\gamma\delta}$$

Tensor Product State
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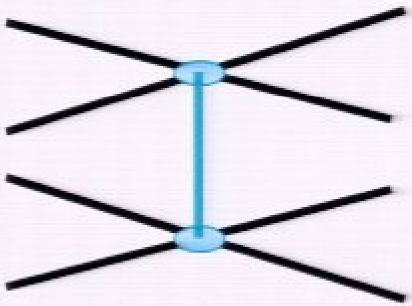

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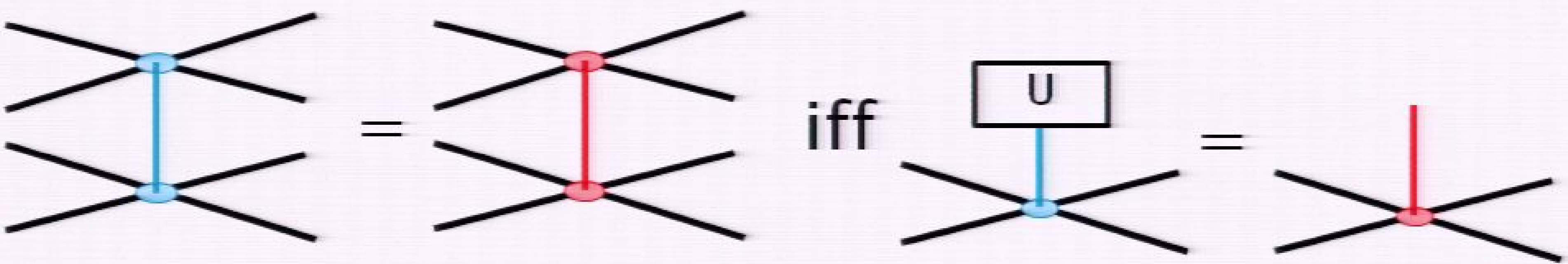
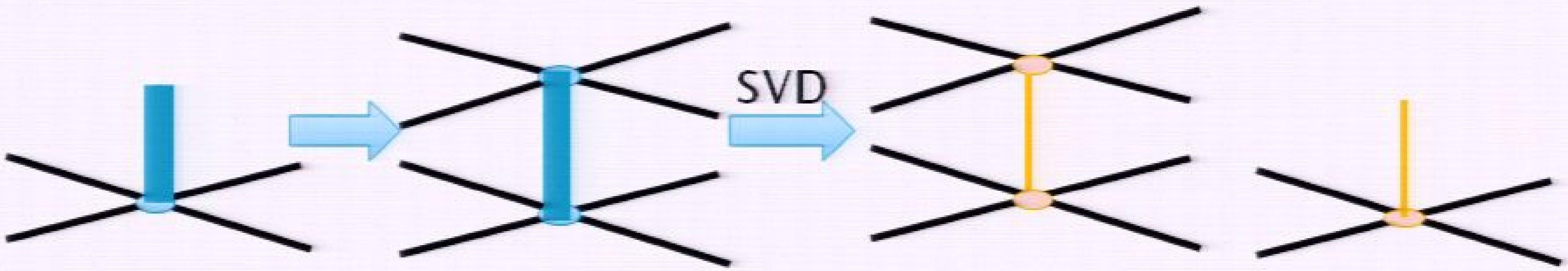


Tensor Product State
-- Renormalization Flow

- To obtain the optimal U



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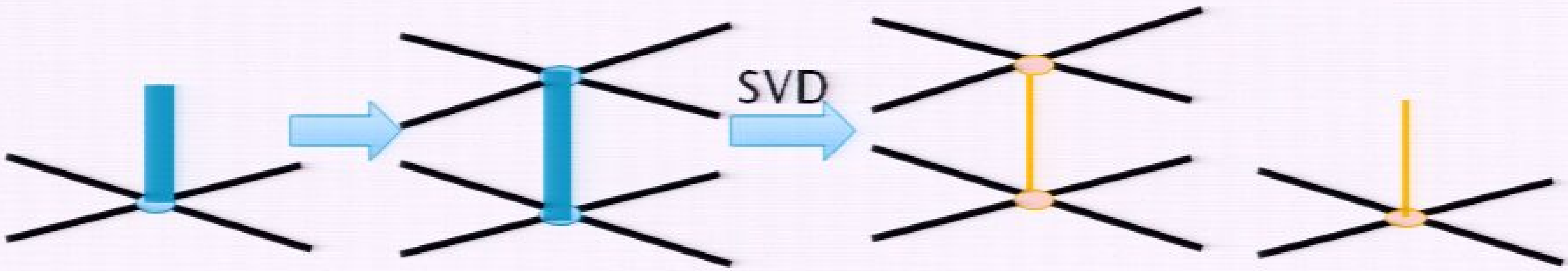

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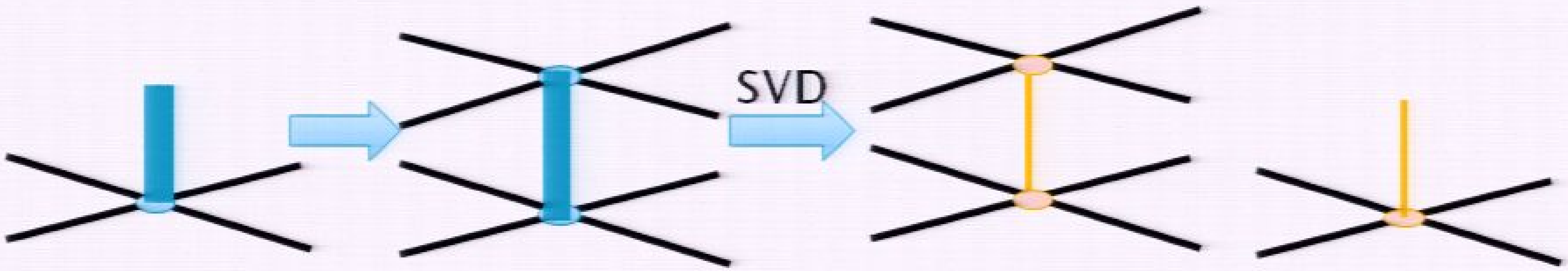

- Retain only the non-zero physical dimensions

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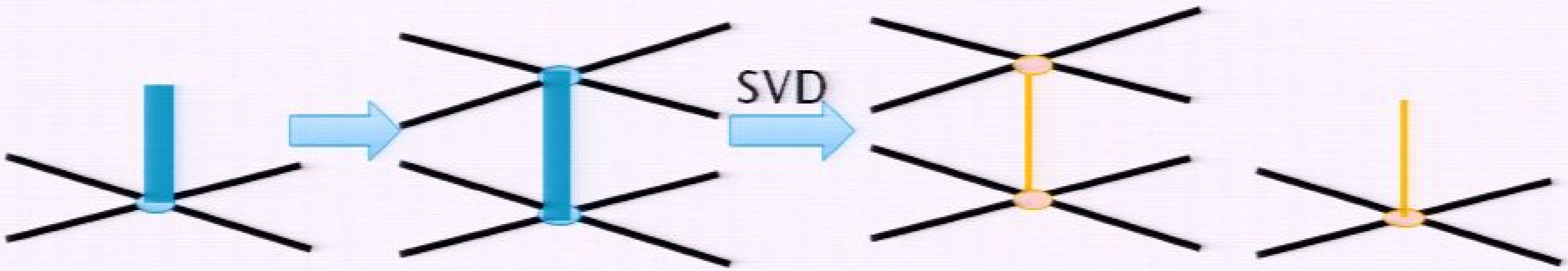

- Retain only the non-zero physical dimensions
- Complete one round of renormalization(one layer of unitary transformation).

Tensor Product State
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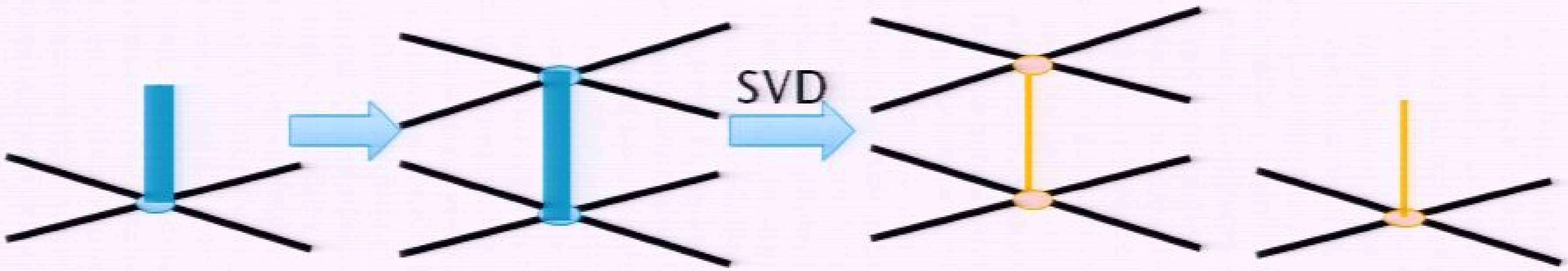

- Retain only the non-zero physical dimensions
- Complete one round of renormalization(one layer of unitary transformation).
- Continue to the next round, until the tensor flows to fixed point

Application: topological order

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- Renormalization flow → Classification of Topological Order

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- Fixed point tensor gives an efficient labeling of topological order
- For tensors not at the fixed points, applying the renormalization flow, we can determine the topological order of the state, by studying the fixed point it flows to

Example: Toric Code

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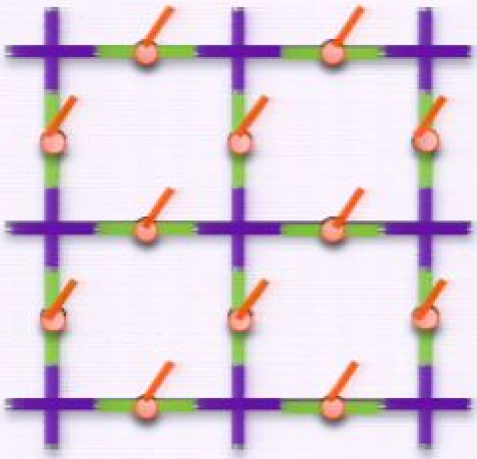

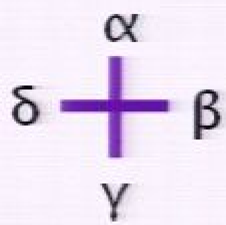

$= 1$, if $\alpha+\beta+\gamma+\delta$ is even

$= 0$, if $\alpha+\beta+\gamma+\delta$ is odd

$= 1$, only when $k=\mu=\nu=0$ or 1

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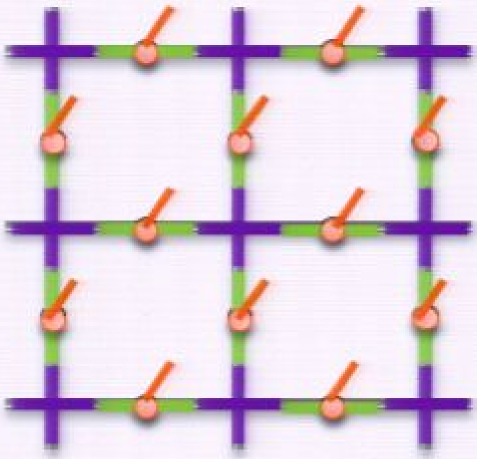

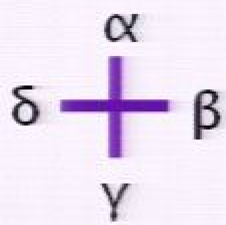

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- Vary the tensor a little bit which corresponds to local perturbation of the Hamiltonian and apply the Renormalization algorithm

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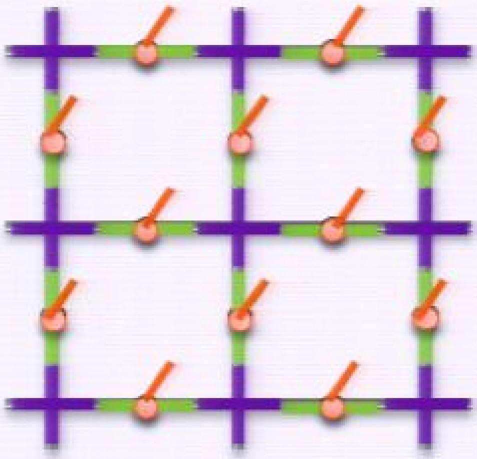

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- Fixed point tensor
- Vary the tensor a little bit which corresponds to local perturbation of the Hamiltonian and apply the Renormalization algorithm
- The varied tensor always flows back to toric code $\Rightarrow$ toric code is stable against local perturbation

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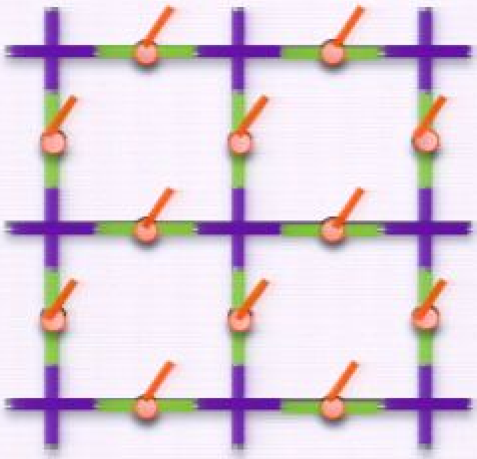

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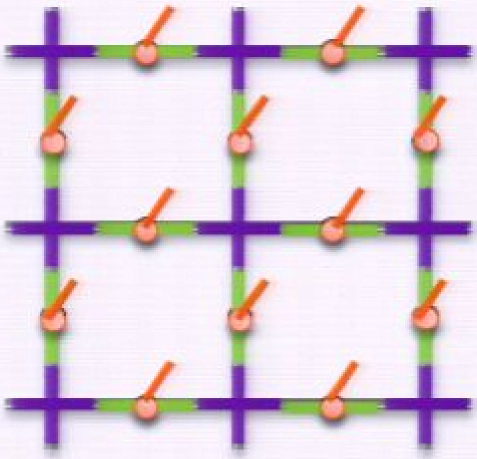

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- $g=1$, toric code, fixed point

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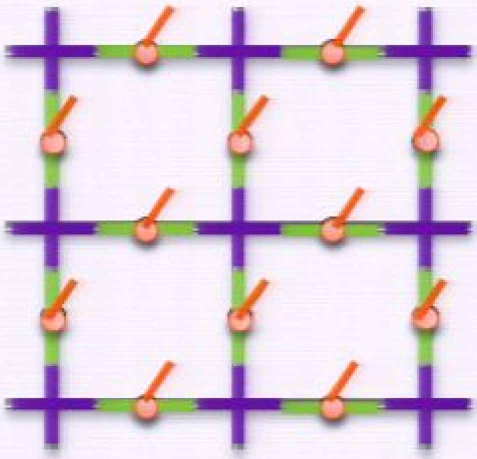

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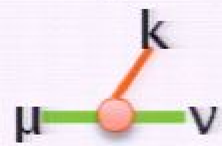

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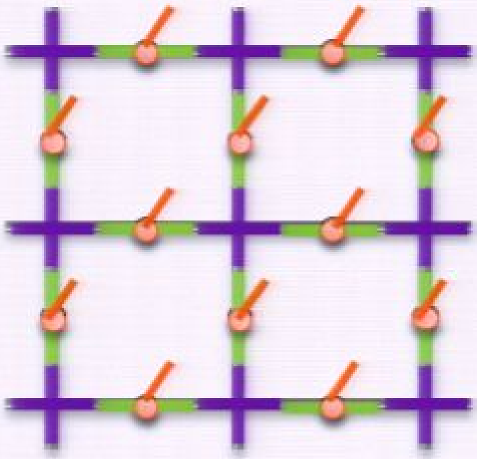
Example: Toric Code



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- $g=1$, toric code, fixed point
- $g=0$, all $|0\rangle$ product state, fixed point
- Phase transition across $g_c=0.804$

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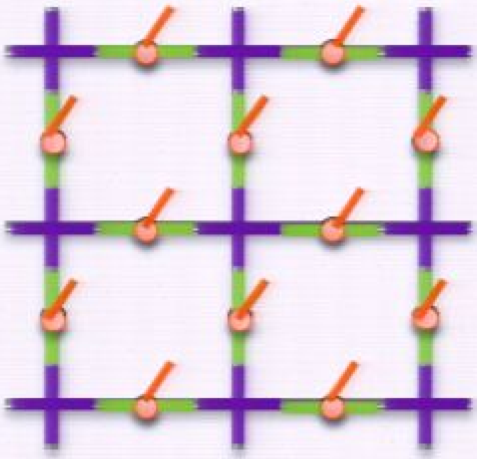

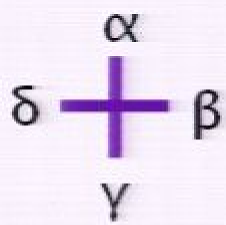

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- Phase transition across $g_c=0.804$
- $g>g_c$, flows to toric code

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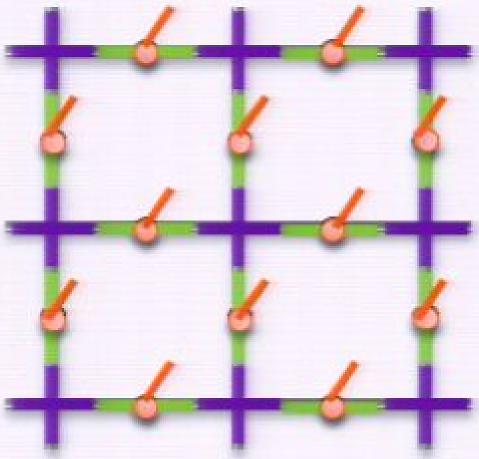

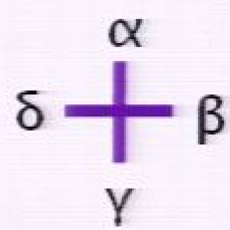

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$= 1$, when $k=\mu=\nu=0$

$= g$, when $k=\mu=\nu=1$

- $g=1$, toric code, fixed point
- $g=0$, all $|0\rangle$ product state, fixed point
- Phase transition across $g_c = 0.804$
- $g>g_c$, flows to toric code
- $g<g_c$, flows to product state

Freedom in the fixed point tensor

Example: Toric Code

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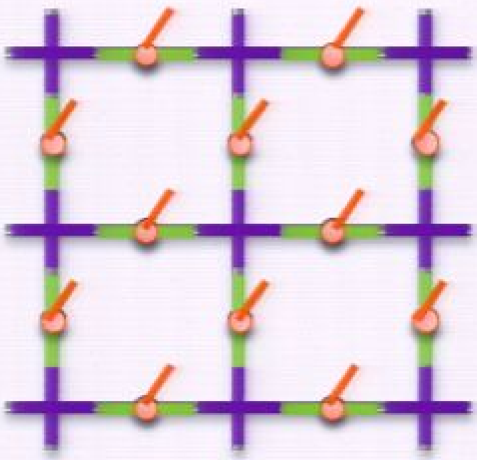

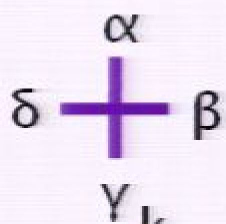

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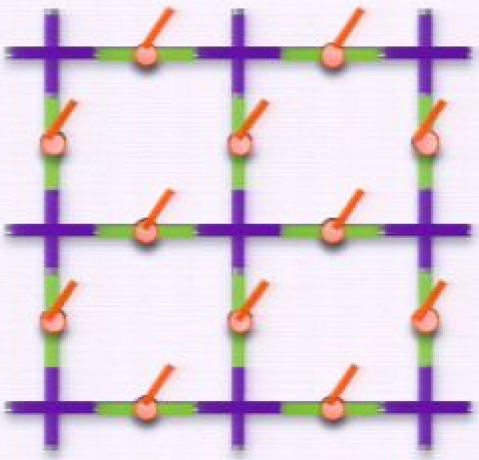

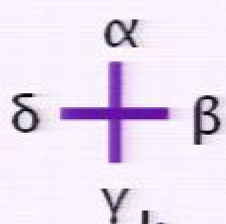

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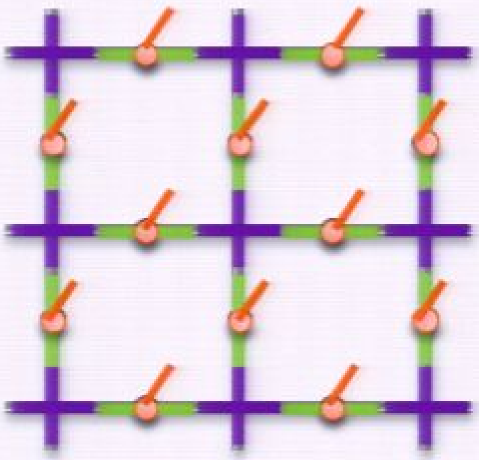

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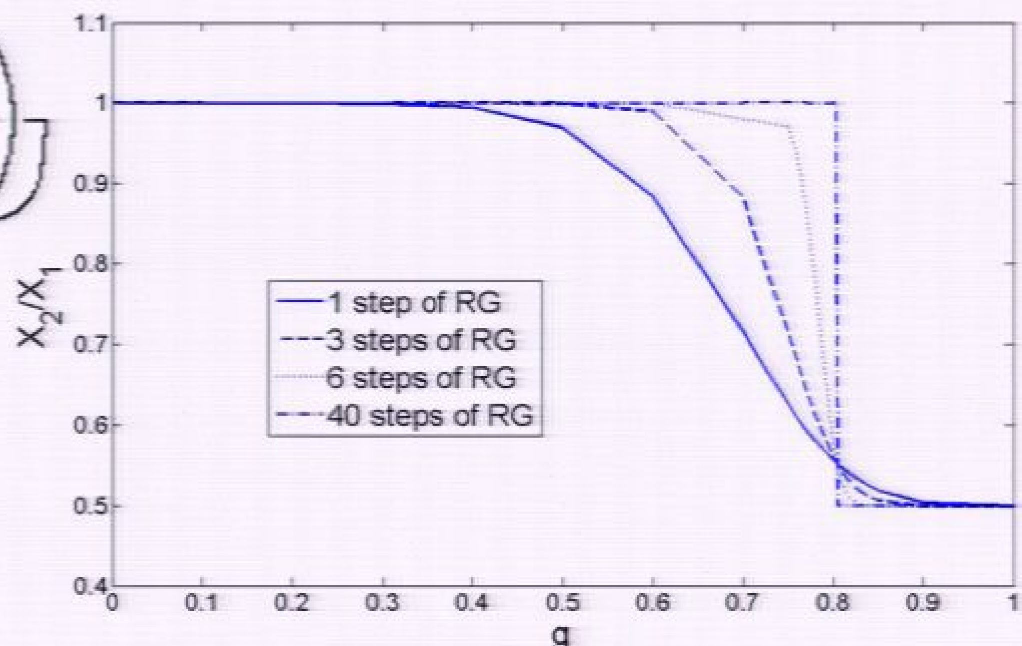

Application: symmetry breaking

Application: symmetry breaking

- Such a renormalization procedure can also be applied to symmetry breaking phase and phase transitions

Application: symmetry breaking

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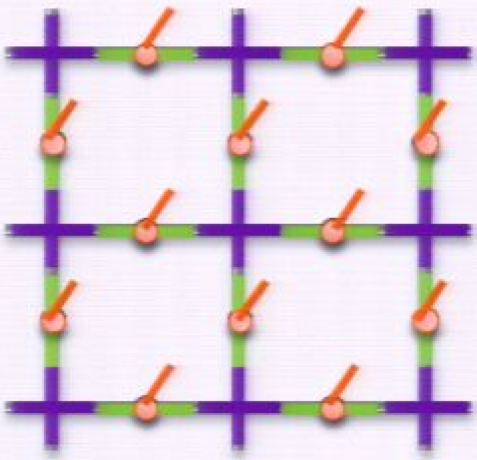

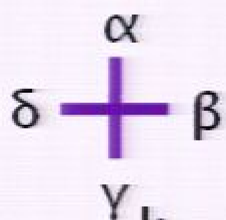

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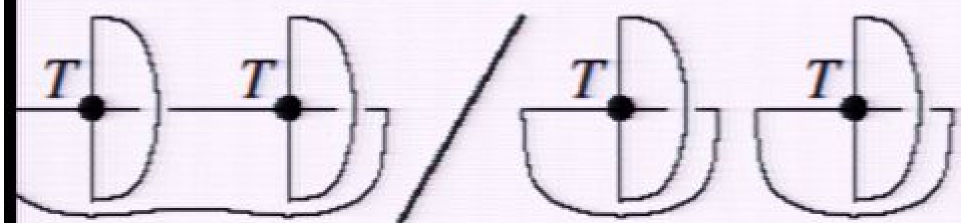

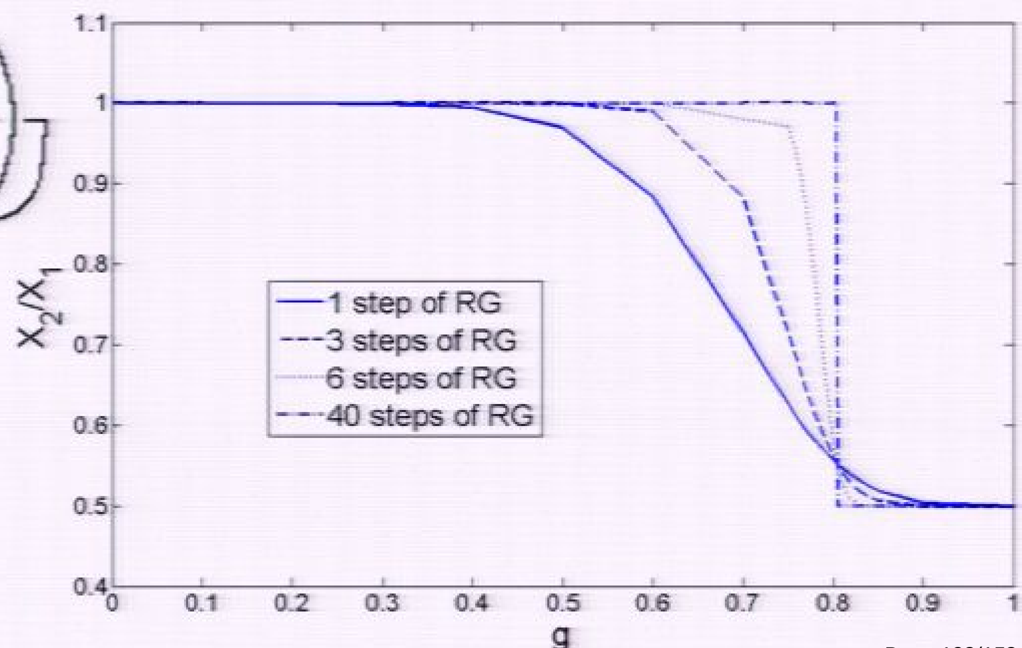

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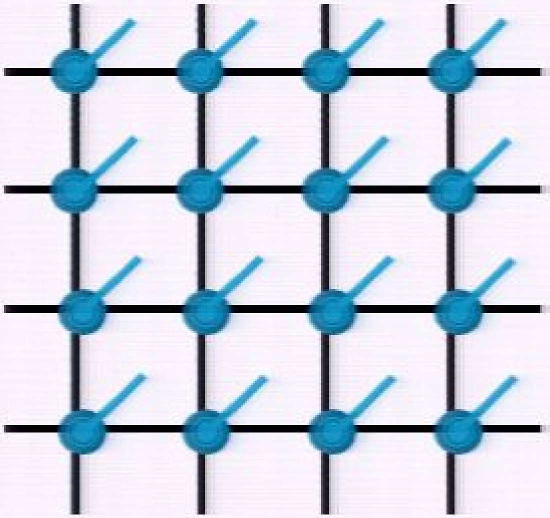
- Suppose we have obtained a symmetric TPS description of ground state and want to determine whether it belongs to the symmetry breaking phase or non-breaking phase
- Apply the algorithm and make sure that the symmetry is carefully preserved.

Example: Transverse Ising Model



$$H = \sum_{i,j} Z_i Z_j + h \sum_i X_i$$

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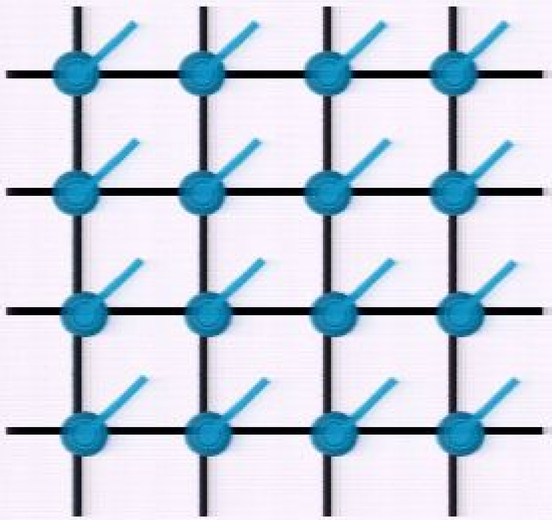

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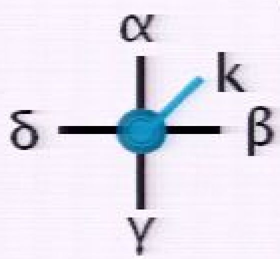

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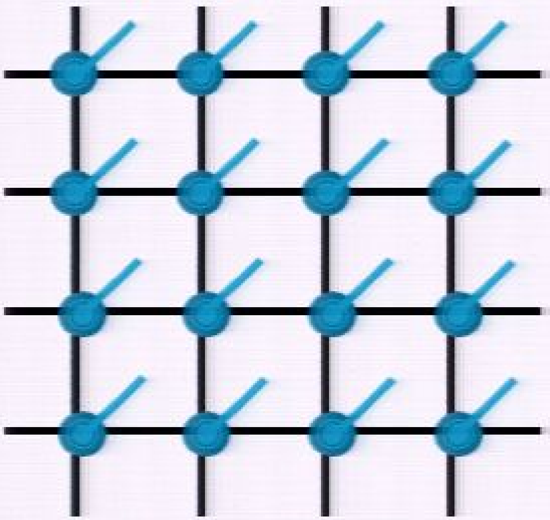

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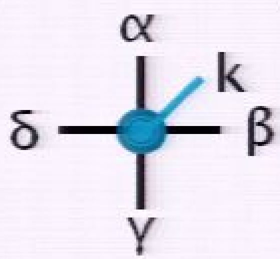

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- The TPS is symmetric under XX…X for any λ
- when λ=0, |00…>+|11…1>, ground state for h=0, symmetry breaking phase, tensor is a fixed point
- when λ=1, |++…+>, ground state for h=∞, symmetric phase, tensor is a fixed point
- phase transition at λ= λc=0.358
- For λ< λc, tensor flow to λ=0, for λ> λc, tensor flow to λ=1

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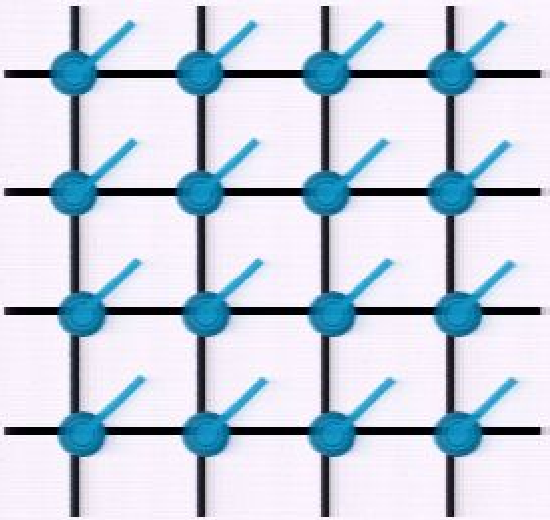

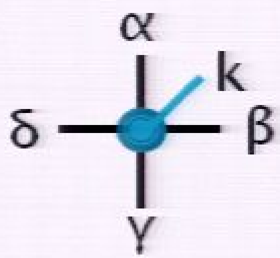

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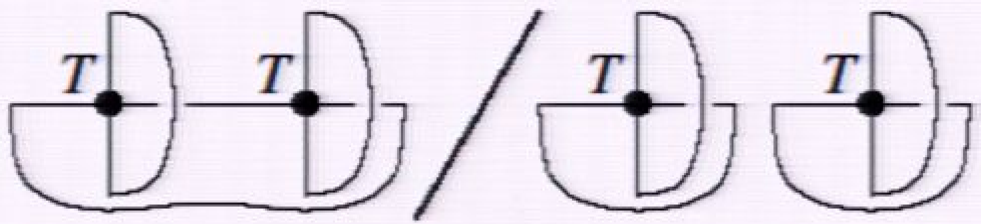

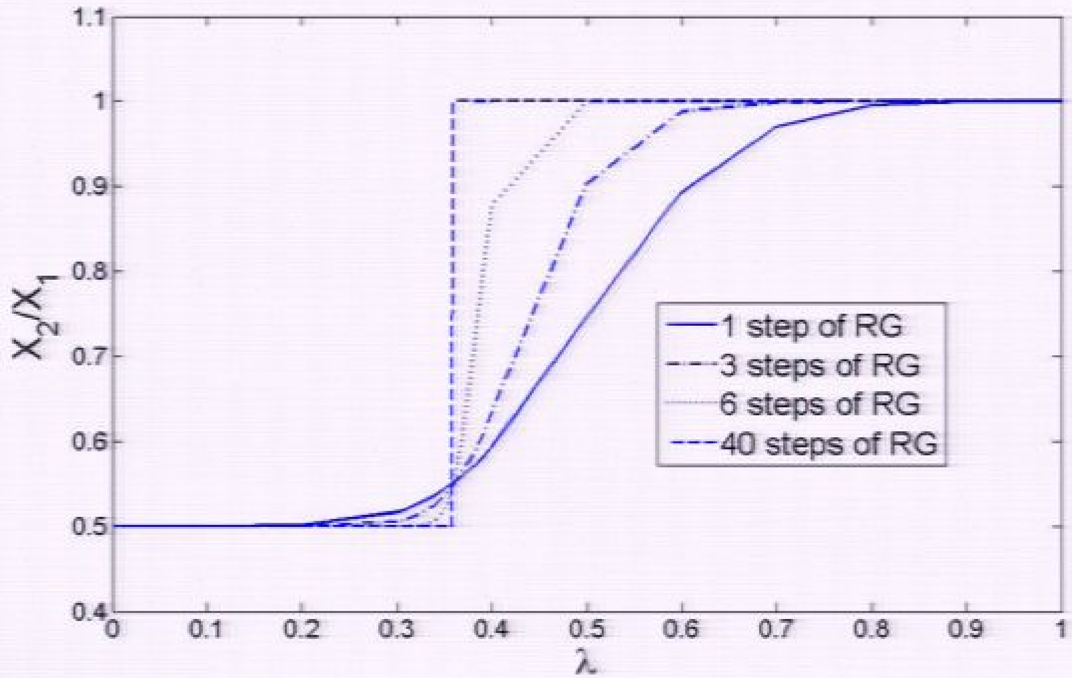

Structure of Fixed Point Tensor

- Fixed point tensor in symmetry breaking phase

  Direct sum of two parts: symmetry breaking

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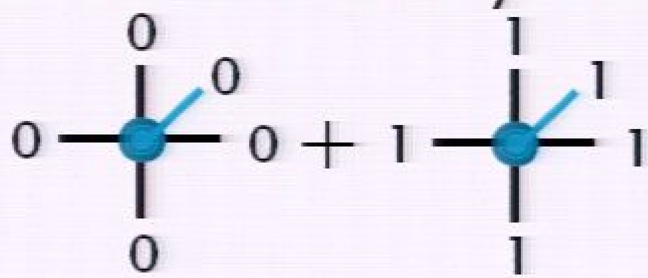

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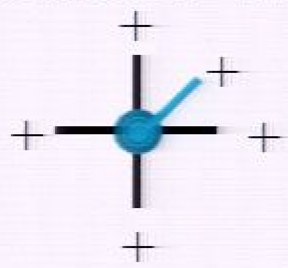

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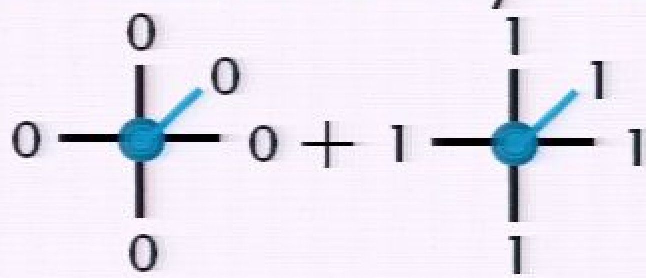

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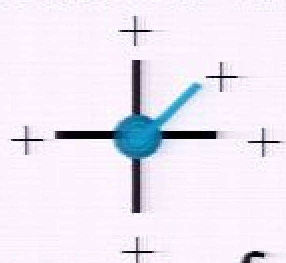

- Fixed point tensor for topological ordered phase

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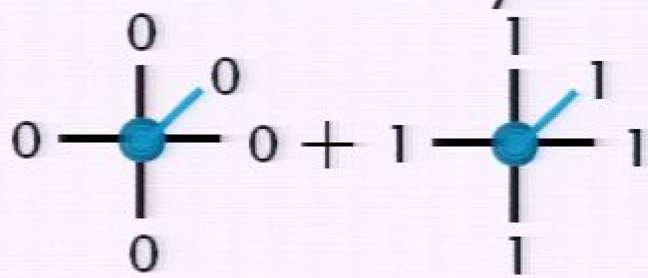

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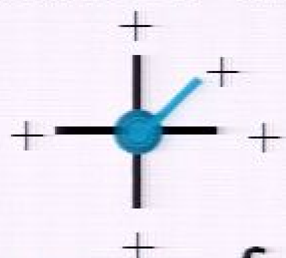

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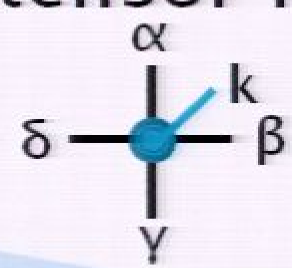

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- Various topological order in 2D
- Identifies phase transition point in for topological and symmetry breaking phase transitions