Title: How to make a low-dimensional thermally stable quantum memory

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Abstract: I will discuss the question of thermal stability of a passive quantum memory, or finite-temperature topological order, in two or three spatial dimensions. We will analyze the criteria for thermal stability. We will present new results on Majorana fermion codes and a new extension of the 2D surface code to three dimensions.

How to make a quantum memory

or: can we have finite-temperature topological order in low-dimensional systems

Barbara M. Terhal
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Bravyi, Leemhuis, Terhal (2010)
Topological order in an exactly solvable 3D spin model

Bravyi & Terhal (2008)
A no-go theorem for a two-dimensional self-correcting quantum memory based on stabilizer codes

Chesi, Loss, Bravyi, Terhal (2009)
Thermodynamic stability criteria for a quantum memory based on stabilizer and subsystem codes

Outline

- Motivation: making a quantum memory
- Loss of topological order at non-zero temperature.
- A 3D spin model: extension of the 2D surface code with qubits with membrane-like logical operators.

Two Approaches

	Fault-Tolerant QC	Gap-based topological QC	Finite-T topological QC
Error correction (EC)	pervasive	none	Only upon read-out
Protection (logical error rate p_e)	p_e arb. small below threshold $O(10^{-3})$	$p_e \sim \exp(-\Delta/T)$	p_e arb. small at low enough $T < T_c$
Dimensionality	2D+extensive I/O for EC	2D+ controls for moving anyons	> 2D or 2D long-range interactions
Advantage	"Lego Approach"	Physics approach, no micro-engineering?	Hands-free error control...
Drawback	Have to be below threshold, EC engineering and I/O challenge,fast classical decoding	Error-rate fixed	3D? or higher D → impossible. EC infrastructure still needed.

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Topological Order (Wen, Kitaev)

states ϕ in degenerate ground-space and any local operator V

| V | ϕ>=C(V) independent of ϕ (*)

There will not be a local order parameter

ural toy models for topological order:
antum error-correction codes defined on lattices with code
ance d growing with system size:
holds for any operator V with support on less than d spins.

und-space degeneracy is exponentially (in d) insensitive to
turbations of the Hamiltonian and when d grows with system
we get a zero-temperature topological phase.

e recent rigorous proofs by Bravyi-Hastings-Michalakis
0)

Non-Local Order Parameter

agine ground-space encodes a single qubit.

non-local logical (spin) operators **S**=(**X**,**Y**,**Z**) we have that | **X** | ϕ> does depend on ϕ (whether ϕ represents the logical **0** **1** etc.).

s order parameter **Z** is not robust since `virtual' or thermal itations (errors) can flip its sign. For models based on antum error-correcting codes, one can define a -local order parameter as the error-corrected logical erator. Consider the action of finding defects and annihilating m and then measuring **Z**, this can be captured in an ervable $\mathbf{Z}_{ec}$. This is the same (but much more involved since it uantum! as taking the average magnetization (doing error-correction by taking a majority vote).

Topological Order at T > 0

Choosing a topologically-ordered ground-space exponentially suppresses dephasing.
Assume that Hamiltonian has constant gap Δ above the ground-space. (e.g. immediate for stabilizer codes, string-net Levin-Wen models).

What happens with order-parameter at non-zero temperature?

We initialize the system of n spins in the logical state, say, $|\mathbf{0}\rangle$ ($\mathbf{Z}(t=0)=1$) and bring the system in contact with a heat-bath at temperature T and time t measure $\mathbf{Z}_{ec}(t)$ (which includes error-correction). How quickly does $\mathbf{Z}$ decay? Relaxation time $\tau_{qmem} \geqslant C\, n^{-1} \exp(\Delta/T)$

(can be shown explicitly for stabilizer codes)

Topological Order at T > 0

Having only a gap, one would expect $\tau_{qmem} \sim C\, n^{-1} \exp(\Delta/T)$
Thus in the thermodynamic limit $n \rightarrow \infty$, one needs to go to lower temperature $T=O(1/\log n)$ to preserve the memory.

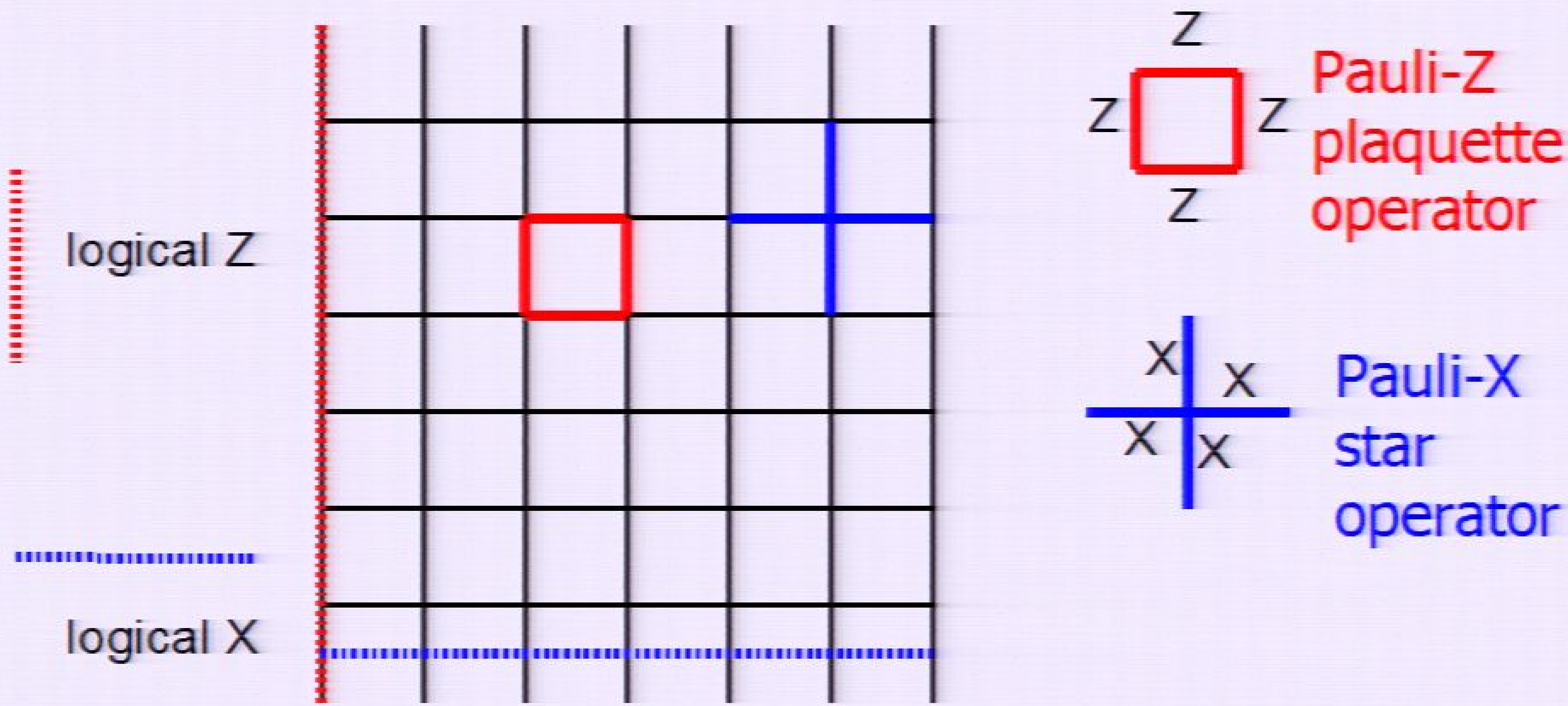
Gap does capture the distinction between the 1D and 2D Ising model.

In 2D Ising model there are macroscopic energy barriers between the logical |**0**> and the logical |**1**>.

How about the surface or toric codes, are there energy barriers between different logical states preventing thermalization?

Surface code example

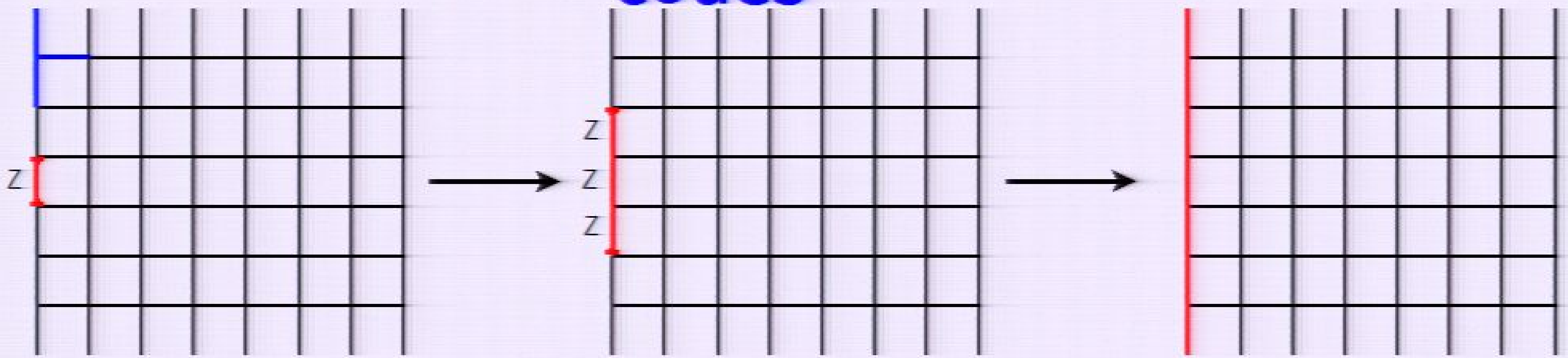
n qubits on edges. Here n=85, L=7.



× L lattice. Number of qubits is $L^2 + (L-1)^2$.

oundspace: states with eigenvalue 1 with respect to plaquette and star erators.

Constant energy barrier for 2D surface codes



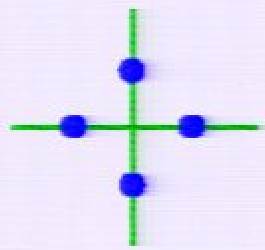
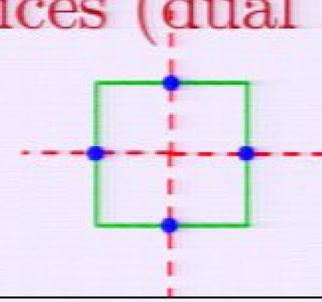
1. A Z error occurs. This costs energy $O(1)$. "Two defects are created"
2. More Z errors along the line happen. This costs no additional energy. "Defects travel in opposite direction without force holding them together"
3. Line of Z errors reaches boundary: a logical error is created. "Defects annihilate"
4. Thus a logical error can be created with constant energy cost, not scaling with L. Chamon-Castelnovo (2007), Iblisdir et al. (2008): At nonzero temperature, in the thermodynamic limit, topological order is lost.

Surface Code Family on L x L x .. lattice

	Qubits	X-stabilizers	Z-stabilizers

3D surface code has distance $d \geq cL$ (logical **Z** is string, logical **X** is membrane). 4D surface code has distance $d \geq cL^2$ (all membrane).

Surface Code Family on L x L x .. lattice

	Qubits	X-stabilizers	Z-stabilizers
D	links		vertices (dual lattice) 

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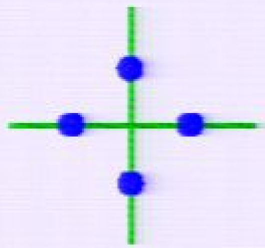
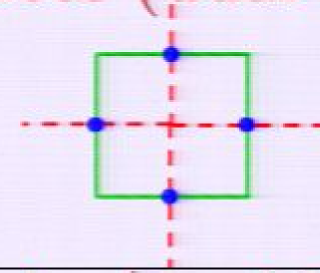
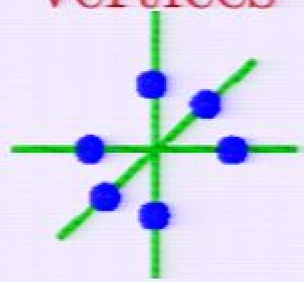
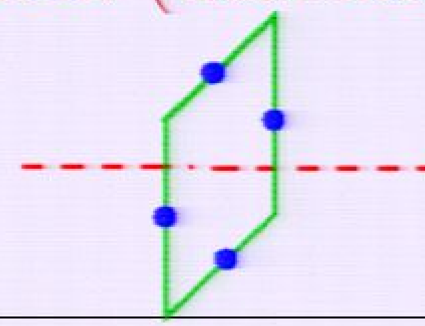
D	links		vertices (dual lattice) 
D	links	vertices 	links (dual lattice) 

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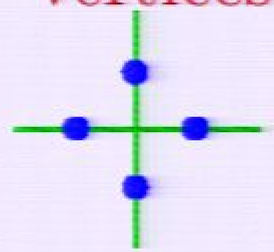
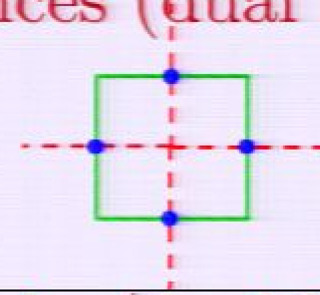
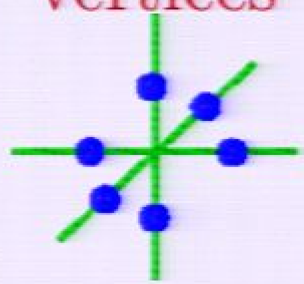
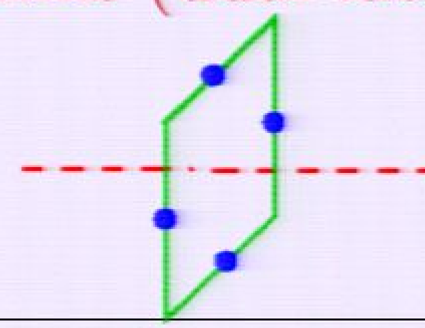
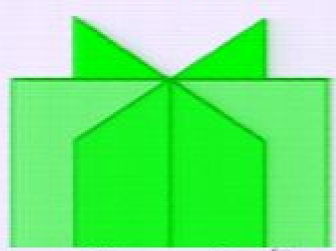
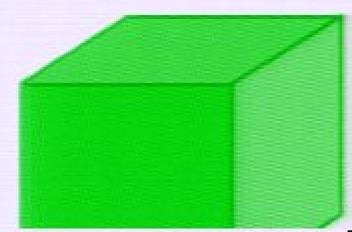
D	links	vertices 	vertices (dual lattice) 
D	links	vertices 	links (dual lattice) 
D	plaquettes	links 	links (dual lattice) 

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brane), 4D surface code has distance $d \geq cL^2$ (all membrane).

Topological Order at finite T?

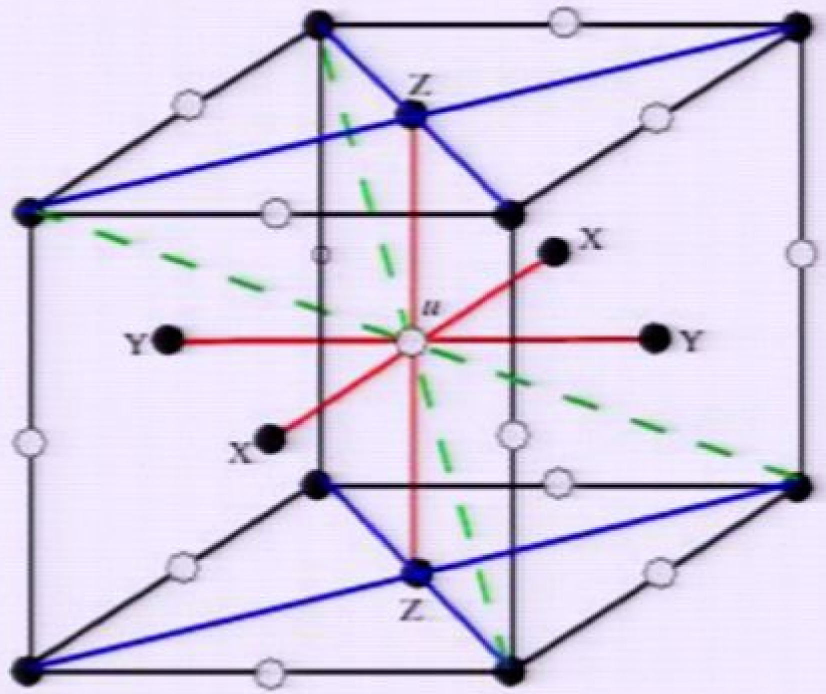
Dennis-Kitaev-Landahl-Preskill (2001) proposed 4D surface code ($O(L^4)$ qubits) which has distance scaling with L^2 and energy barriers of all logical (membrane-like) operators scale as L (Alicki-Horodeckis (2008): relaxation time τ for 4D surface code is $e^{O(L)}$ /poly(L) below a critical temperature T_c)

The 3D surface code is stable against X excitations, but behaves similarly as 2D surface code for Z excitations.
Can we get a thermally stable model in 2D or 3D?
(with bounded strength, local interactions between finite local degrees of freedom).
Almost certainly not in 2D (no-go theorems by Bravyi-Terhal).
3D models: is there a duality between string-like and membrane-like operators?

Answer: not always. Model with qubits with membrane-like logical operators (and pinned monopoles which are the end-points of membranes).

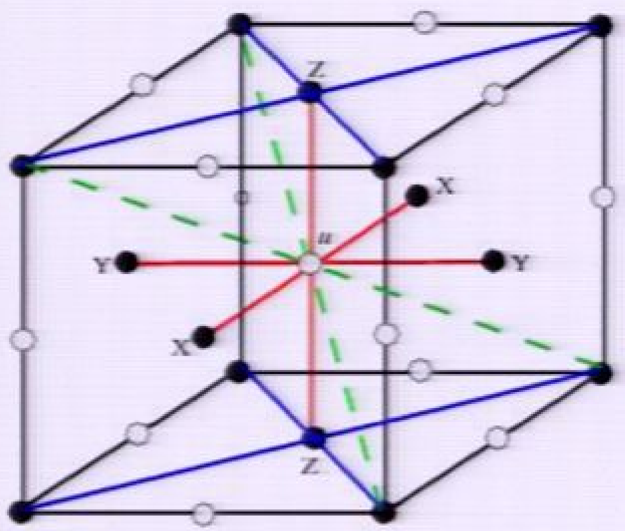
Unit of the Lattice

Claudio Chamon, Quantum Glassiness, PRL 94, 040402 (2005)



Qubits live on FCC lattice (Λ_{even}). Terms in Hamiltonian (depicted in red, aka stabilizer generators) are centered on sites in Λ_{odd}. Blue lines are examples of face diagonals. Green dotted lines are examples of body diagonals.

Ground-space degeneracy



Let lattice $\Lambda = Z_{L_x} \times Z_{L_y} \times Z_{L_z}$ and $L_x = 2p_x, L_y = 2p_y, L_z = 2p_z$.

Hamiltonian $H = -\sum_u S_u$ for $u \in \Lambda_{odd}$. S_u is the 6-spin operator with
envalues ± 1.

Theorem: Ground-space degeneracy of H is 2^{4g} where $g = gcd(p_x, p_y, p_z)$.

Focus on mutually co-prime, odd, $p_x, p_y, p_z \rightarrow$ four qubits in ground-space.

Four logical operators

$\Lambda_{even} = \Lambda_{000} \cup \Lambda_{110} \cup \Lambda_{011} \cup \Lambda_{101}.$

with $\Lambda_{abc} = \{(i,j,k)\,|\, i \bmod 2 = a, j \bmod 2 = b, k \bmod 2 = c\}$

Each of these four logical qubits lives on a different lattice.

Logical operators $\bar{\sigma}^x_{abc}, \bar{\sigma}^y_{abc}, \bar{\sigma}^z_{abc}$ are half-filled planes, orthogonal to x, y, z-
ction:

$$\bar{\sigma}^x_{abc} \equiv \prod_{i=a} \prod_{j=b \pmod 2} \prod_{k=c \pmod 2} X_{i,j,k},$$

$$\bar{\sigma}^y_{abc} \equiv \prod_{i=a \pmod 2} \prod_{j=b} \prod_{k=c \pmod 2} Y_{i,j,k},$$

$$\bar{\sigma}^z_{abc} \equiv \prod_{i=a \pmod 2} \prod_{j=b \pmod 2} \prod_{k=c} X_{i,j,k}.$$

Should be Z

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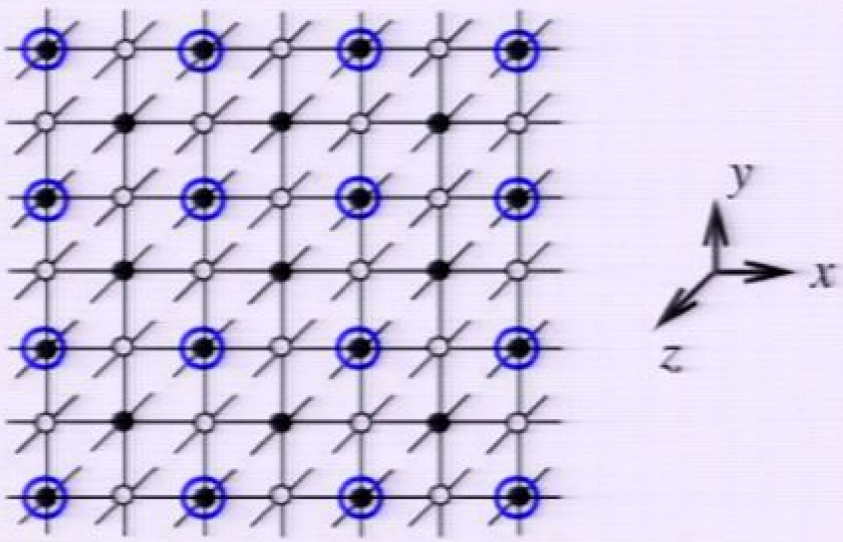
Half-filled membrane

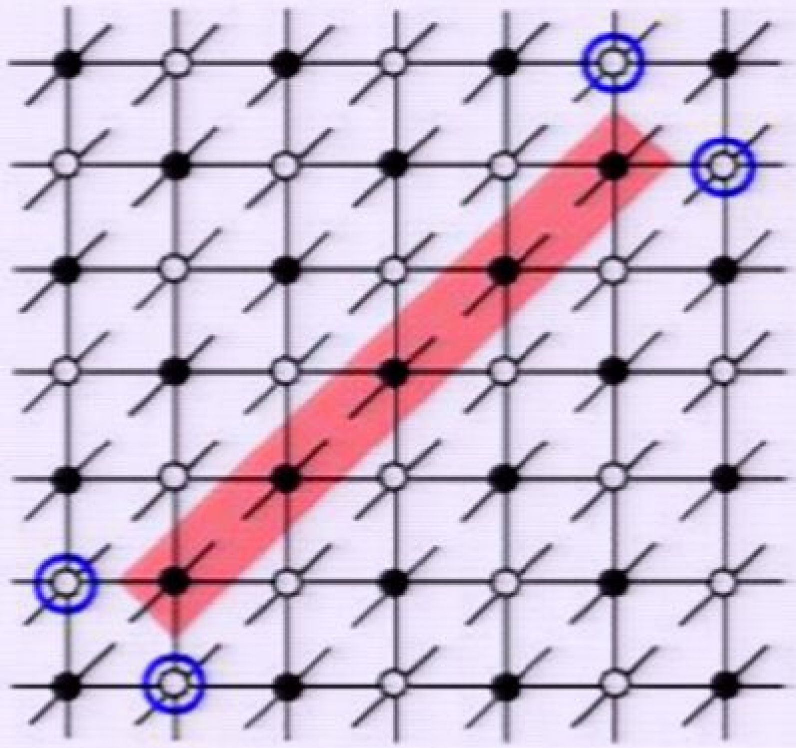


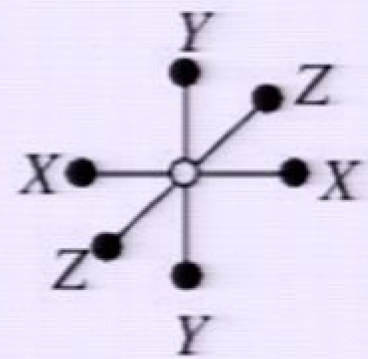
re 12: (Color Online) A half-filled membrane operator $\bar{\sigma}^z$ lying in some plane orthogonal to the s has Z-operators on the circled blue qubits which have a fixed parity of the x- and y-coordinates.

ut are we sure that no product of these operators multiplied by ements S_u (which act trivially on ground-space) is string-like?

No, there are in fact strings...

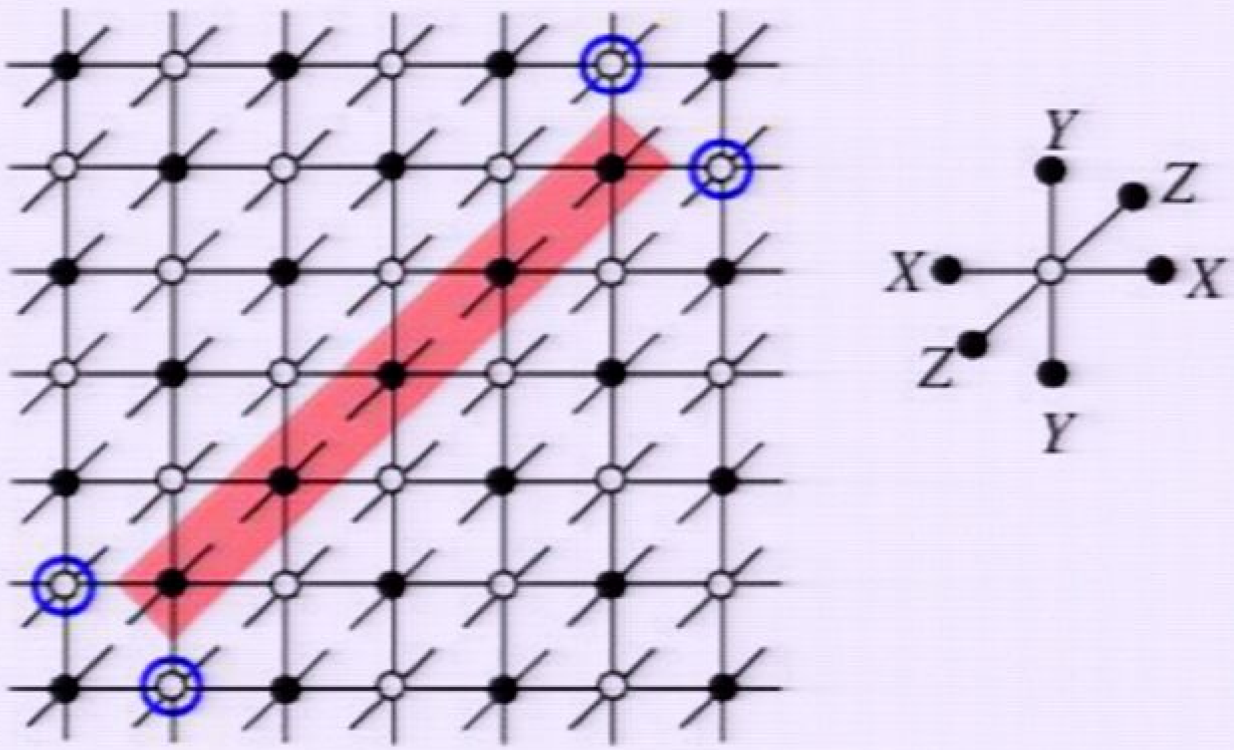
Rigid string





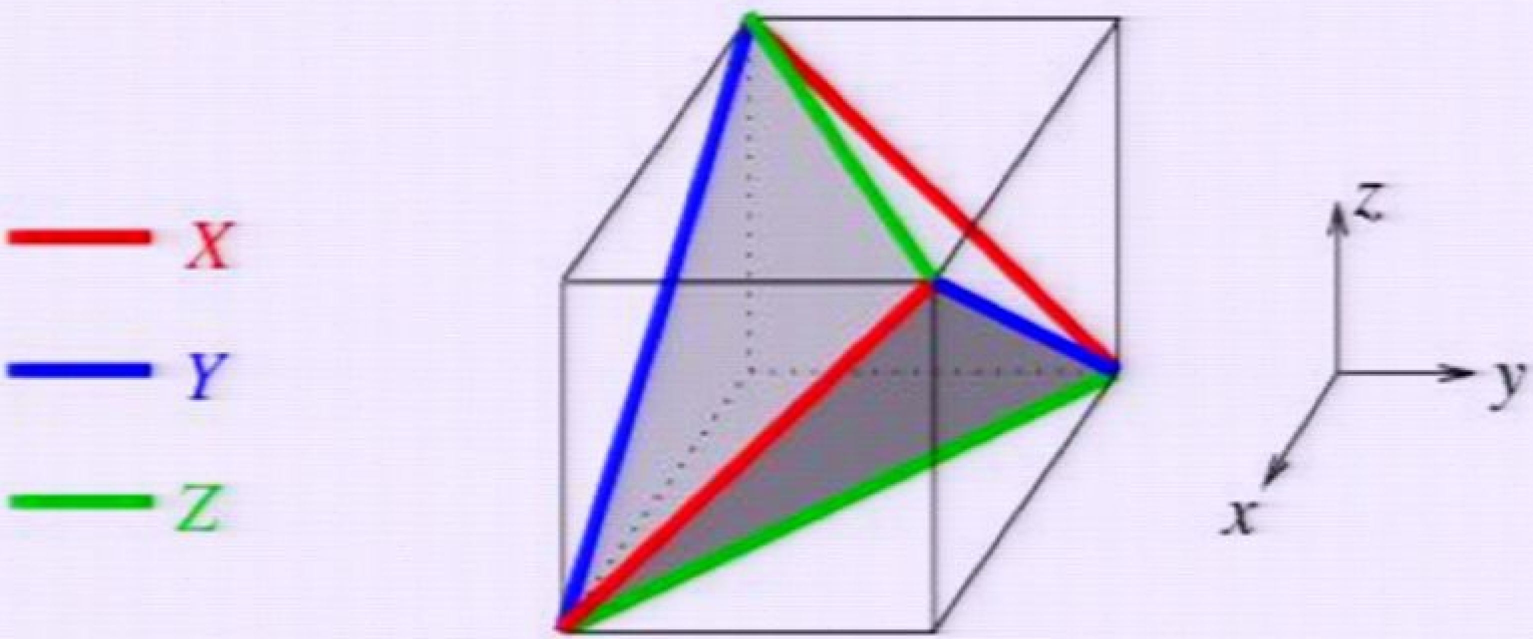
erator is a string of Zs in pink area and anti-commutes
n terms centered on blue circles.
plane-terms terms in Hamiltonian act as toric code on the plane.
ms centered above and below plane act a single Z operators on
qubits in the plane.

How to close a rigid string



Closing string by winding in plane, since lattice dimensions are in-commensurate, we get a fully-filled membrane operator, a product of two half-filled membrane logical operators.

How to close a rigid string in 3D



trahedron operator W(T) for tetrahedron T.
(T) is supported on its edges and W(T)~$\Pi_{u \in T}$ S_u
. W(T) is trivial and can detect a single excitation in its interior.

5/27/2010 How can we close rigid strings in 3D non-trivially?...

Action of terms on <111>-bilayers

$\Sigma_\alpha = \{(i,j,k) \mid i+j+k=\alpha\}$

Bilayer $\Sigma_0 \cup \Sigma_2$

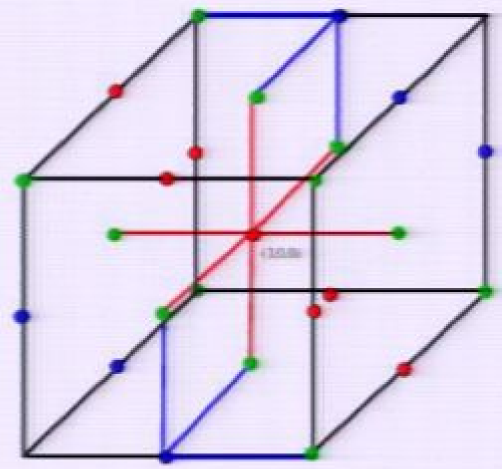


Figure 6: (Color Online) If the (red) stabilizer at the center has coordinates (1,0,0), then the green qubits represent qubits in the bilayer $\Sigma_0 \cup \Sigma_2$. The (red) stabilizers in the plane Σ_1 can be viewed as six-body hexagonal plaquettes on this bilayer. The (blue) stabilizers in the planes Σ_{-1} and Σ_3 each touch

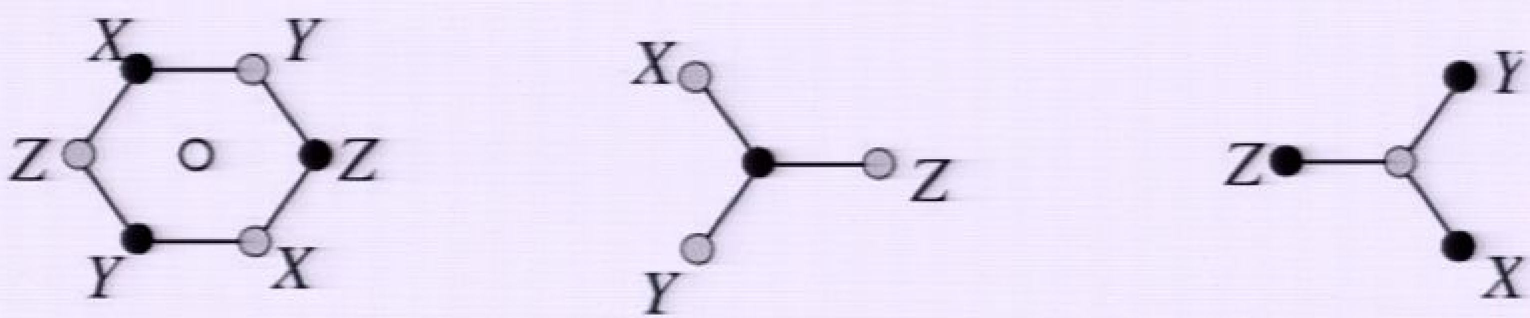
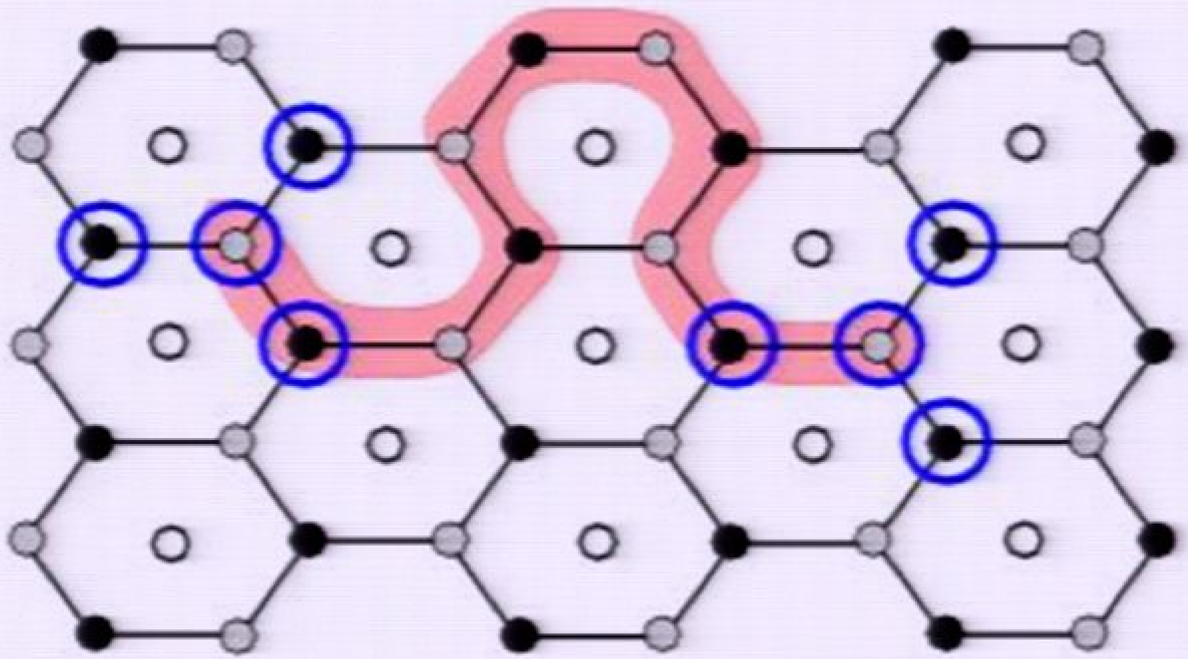


Figure 8: Plaquette and star operators.

Flexible strings in <111>-bilayer



ially-closed flexible strings are products of hexagonal
quette operators S_u between two <111>-layers of qubits.
ntrivially-closed flexible strings (characterized by
 parities of winding numbers) are products of logical operators.

Our Claims

e spin model has

wo good qubits for which all logical operators are mbrane-like (associated with non-trivially closed rigid strings)

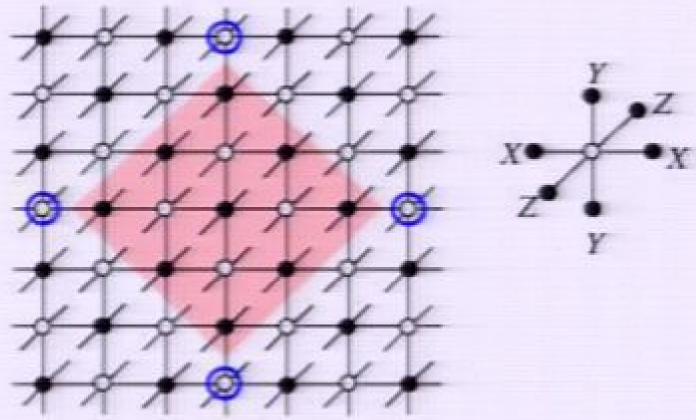
wo other bad qubits where one of the logical operator for each bit is string-like (is non-trivially closed flexible string) and the er is membrane-like.

ates the idea that if one logical operator, say $\mathbf{Z}_1$, is mbrane-like than the logical operator $\mathbf{X}_1$ must be string or nt-like.

ermal stability? A non-trivially closed rigid string still has) energy barrier, can be made by diagonally vibrating ends of ngs which combine....so this does not solve quantum memory problem.......but....

dio Chamon,
ntum Glassiness,
94, 040402 (2005)

Monopoles



onopole is a single excitation.
toric code, monopole is the
l-point of a string and can be moved.
n our model, we can prove that an isolated monopole is
end-point of a membrane ⇒

1. We cannot move a monopole by local operators,
y are pinned.

2. Creation of isolated monopoles above ground-space requires either a low-energy but very specific sequence of local errors. Glassiness in the creation of isolated monopoles. Isolated monopoles are low-lying excitations above ground-space but unlikely to be obtained by thermalization.

5/27/2010

[redacted]