Title: Cluster expansions and the stability of topological phases

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Abstract: Anyons are a special kind of excitations which are allowed in two dimensional systems, along with fermions and bosons. The topological nature of braiding of non-abelian anyons may allow a realization of quantum computing gates which is immune to noise. While the insensitivity of the such systems to a localized noise source is a built-in feature, an issue of great importance is more subtle: the robustness to slight deformations of the amiltonian describing the phase by perturbations which are locally tiny but are spread over through the entire system. Such will always arise if the realization of the Hamiltonian in a particular system is not quite perfect. The subject of the talk will be a proof of such stability, and the cluster expansion representation of deformed topological states.

Cluster expansions and the stability of topological phases

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Acknowledgments:
Michael Freedman (Station Q, UCSB)
Matthew B. Hastings (Station Q, UCSB)

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- The question of stability

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- Bound on irreducible linked clusters
- Toric code: Thin Torus
- Toric code: stability to abelian perturbations.
- How to obtain the cluster expansions (Yarotsky method)

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Anyons appear as quasi-particles of two dimensional systems.

- Fractional Quantum Hall (Moore-Read, Read-Rezayi…)
- **Lattice** systems (Kitaev's toric code/hexagonal lattice, Levin-Wen models, Ioffe's model, Freedman-Nayak-Shtengel…)

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Energy gap is essential to protect anyons for applications

large systems are needed to have many anyons and move them in arbitrary paths

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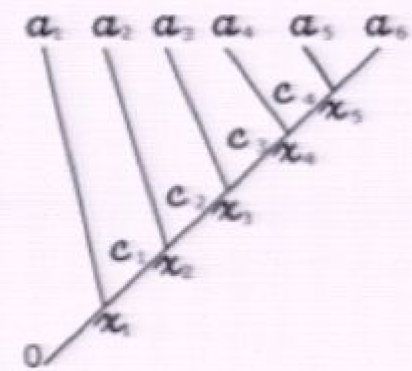
Ability depends on stability

General anyonic systems are defined by a set of charges, augmented by fusion and braiding rules.

Location of particles alone is not enough to specify the state. Need to keep track of history.

Hilbert space is spanned by "histories" of generating particles from the ground state then fusing and braiding them.

Pictorially a state may be written as: $|x_1\, a_1\, c_1\, x_2\, a_2 \ldots\rangle \propto$



Anyon Lattice model = Anyon model, Hilbert space spanned by diagrams where anyons constrained to have final locations on lattice points.

Canonical example: The Toric Code (Kitaev96)

$$H_{TC} = \sum_{+} P_{+} + \sum_{[]} P_{[]}$$

$$P_{+} = \tfrac{1}{2}(1 - \prod_{j \in star} \sigma_j^x)$$

$$P_{[]} = \tfrac{1}{2}(1 - \prod_{j \in plaquette} \sigma_j^z)$$

$$magnetic: \quad X_C |vac> \quad , \quad X_C = \prod_{a \in C} \sigma_a^x$$

$$electric: \quad Z_C |vac> \quad , \quad Z_C = \prod_{a \in C} \sigma_a^z$$

- Ground state is a loop gas
- Excitations are ends of strings
- Degeneracy: 2^genus
- Other examples: Levin Wen etc. (Freedman, Nayak Shtengel), etc..

The Toric Code

$$H_{TC} = \sum_{+} P_{+} + \sum_{[]} P_{[]}$$

$$P_{+} = \tfrac{1}{2}(1 - \prod_{j \in star} \sigma_j^x)$$ Even spins point in x direction

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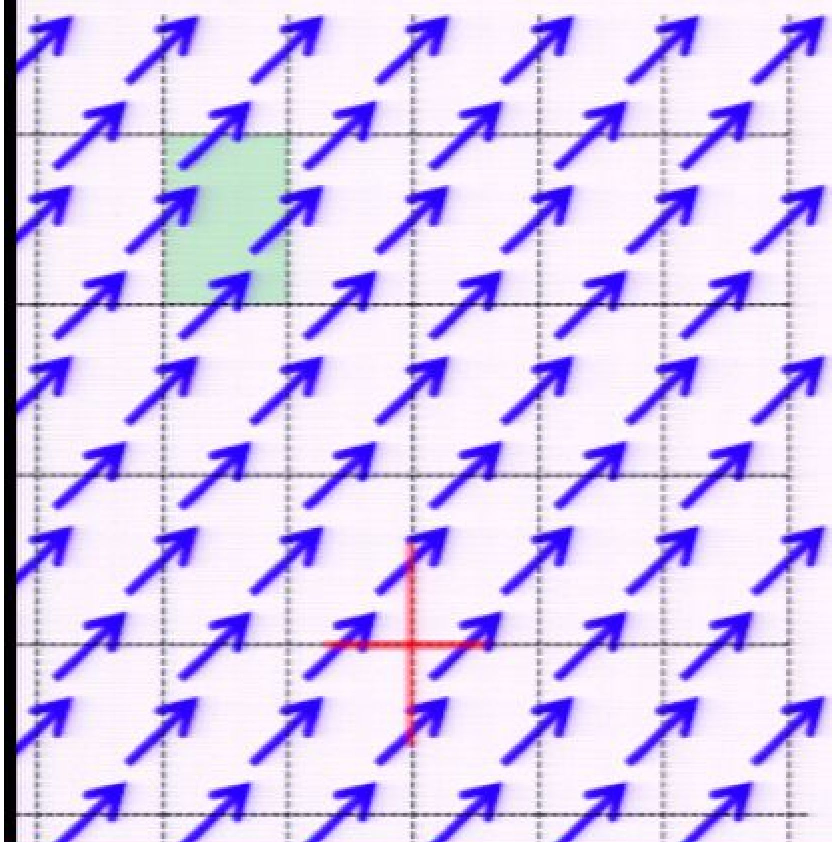

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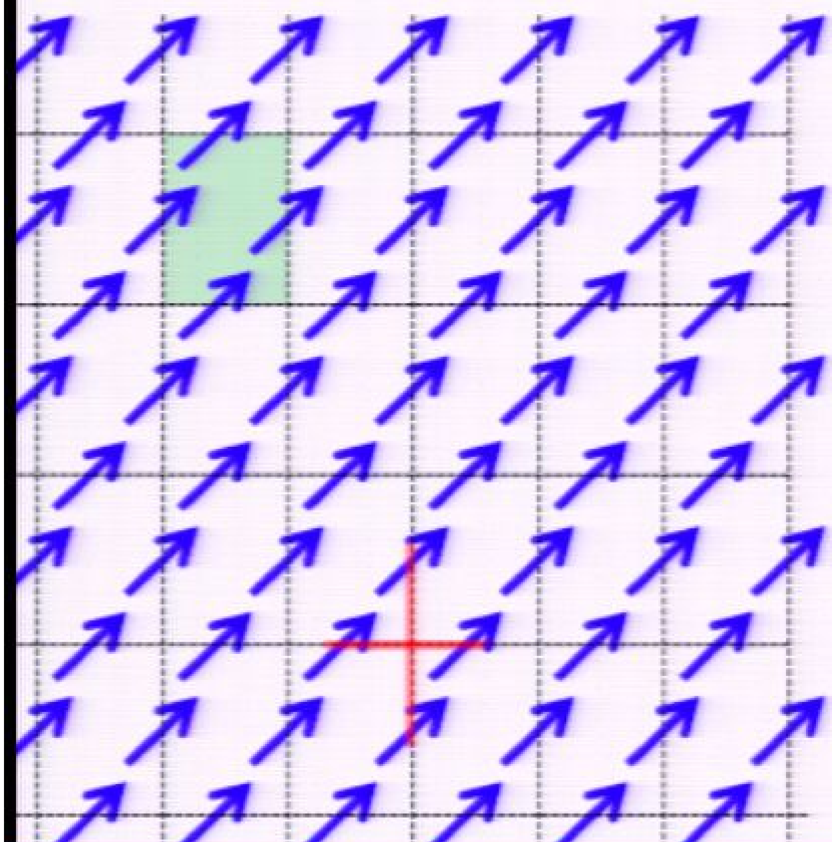

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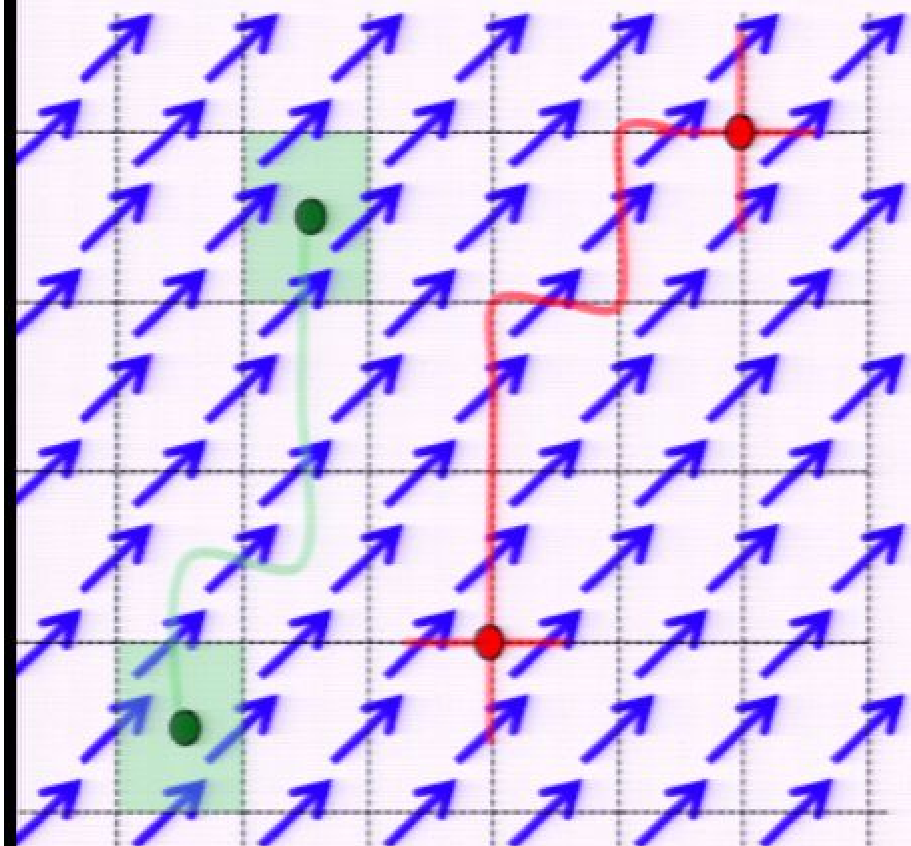

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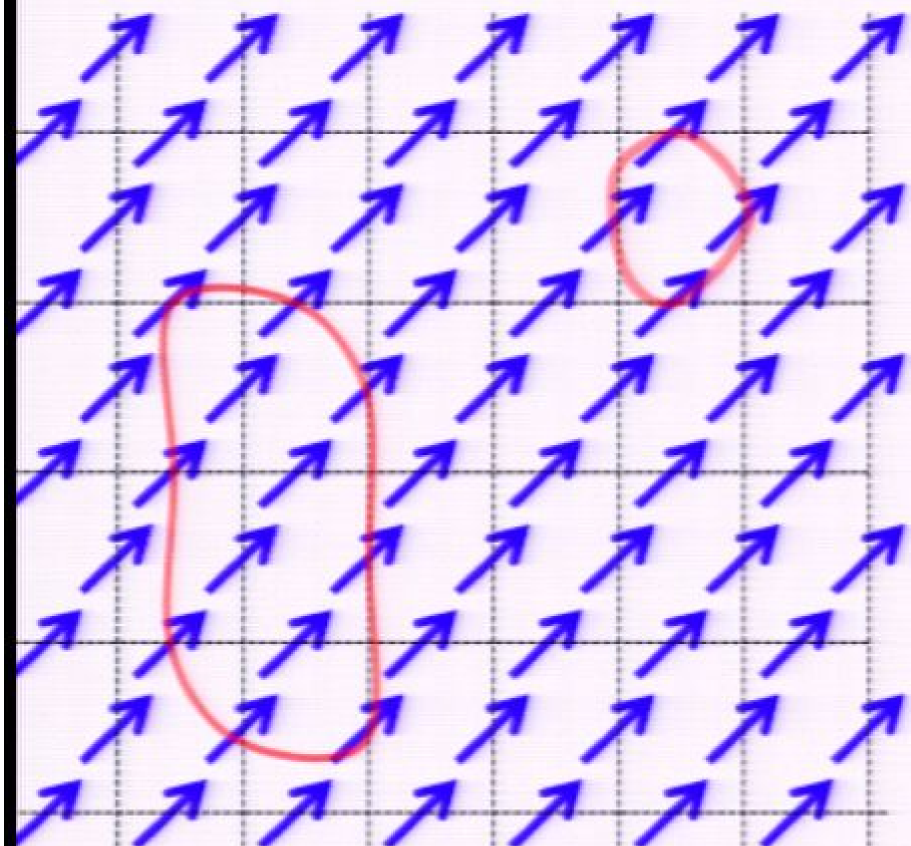
magnetic: $X_C \mid vac>$, $X_C = \prod_{a \in C} \sigma_a^x$

electric: $Z_C \mid vac>$, $Z_C = \prod_{a \in C} \sigma_a^z$

Note that the red and green trajectories do not commute!

Ground state is a loop gas:



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We can construc ground state as:

$$|gs> = \prod_{plaquettes} (1 - P_{[]}) \, | x - polarized > \sim$$

$$\prod_{plaquettes} (1 + \prod_{j \in plaquette} \sigma_j^z) \, | x - polarized >$$

Flips spins on boundary of plaquette

Ground state superposition of loops. Degeneracy if non-trivial loops exist.

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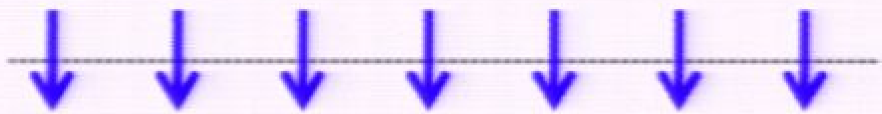
- Argument not valid for translational invariant perturbations in thermodynamic limit!

Example of what can go wrong (1):

pin chain

$$= |\uparrow_0\rangle\langle\uparrow_0| - \sum_i \left(|\uparrow_i \uparrow_{i+1}\rangle\langle\uparrow_i \uparrow_{i+1}| + |\downarrow_i \downarrow_{i+1}\rangle\langle\downarrow_i \downarrow_{i+1}| \right)$$

nique Ground state : All spins pointing down. Gap = 1



dd perturbation : $V = -\frac{1}{N} \sum_i |\uparrow_i\rangle\langle\uparrow_i|$

infinitesimally very small when $N \to \infty$

ut : now the splitting of all up and all down is 1/N

The example above has two features:

1) No translational invariance
2) Spectrum breaking not too bad – still finite g.s. degeneracy separated by gap
3) Perturbation considered is “correlated”

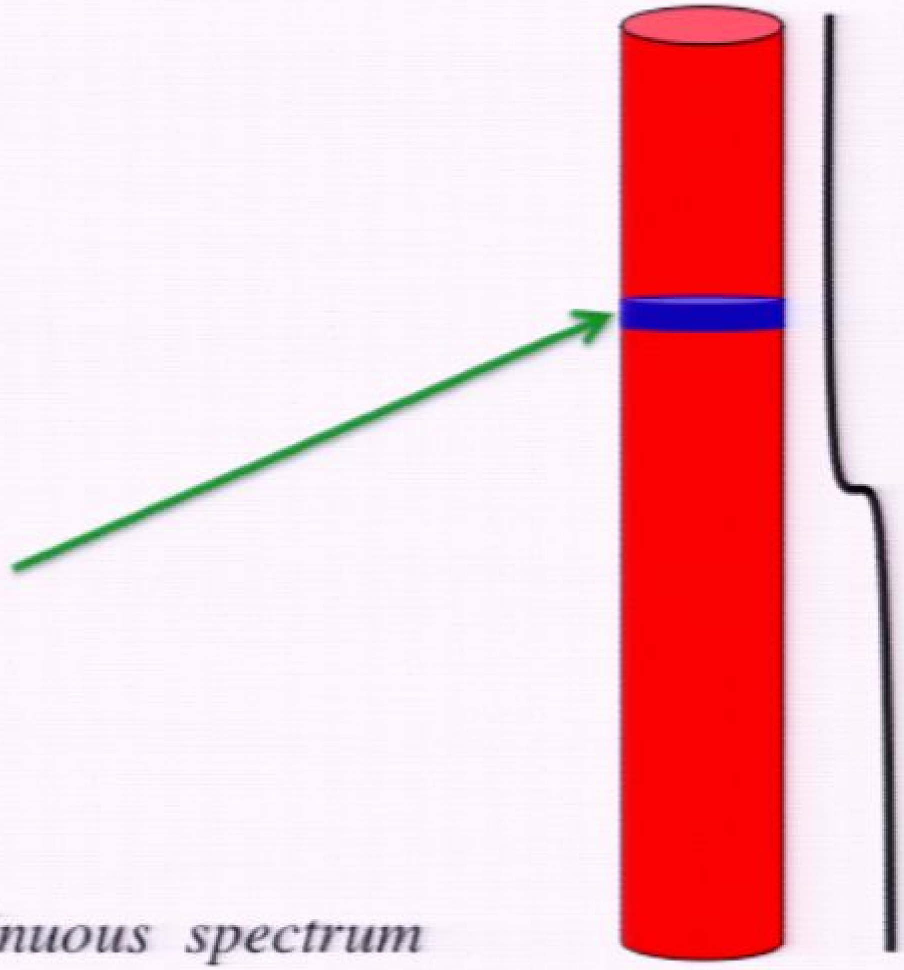
Maybe translationaly invariant systems cannot be destroyed?

Example of what can go wrong (2): Thin Toric Code

Torus of length 2L

$$H = H_{TC} - \tfrac{2}{L}\sum_{m=1}^{L}(l_m + 1) - \tfrac{2}{L}\sum_{m=L}^{2L}(l_m + 1)$$

l_m loop around short direction at m



Favors a "kink" at m = L

Cost of moving it is $1/L \quad \Rightarrow \quad Continuous\ \ spectrum$

formation of disturbances may be favored in some cases, like in Peierls transition

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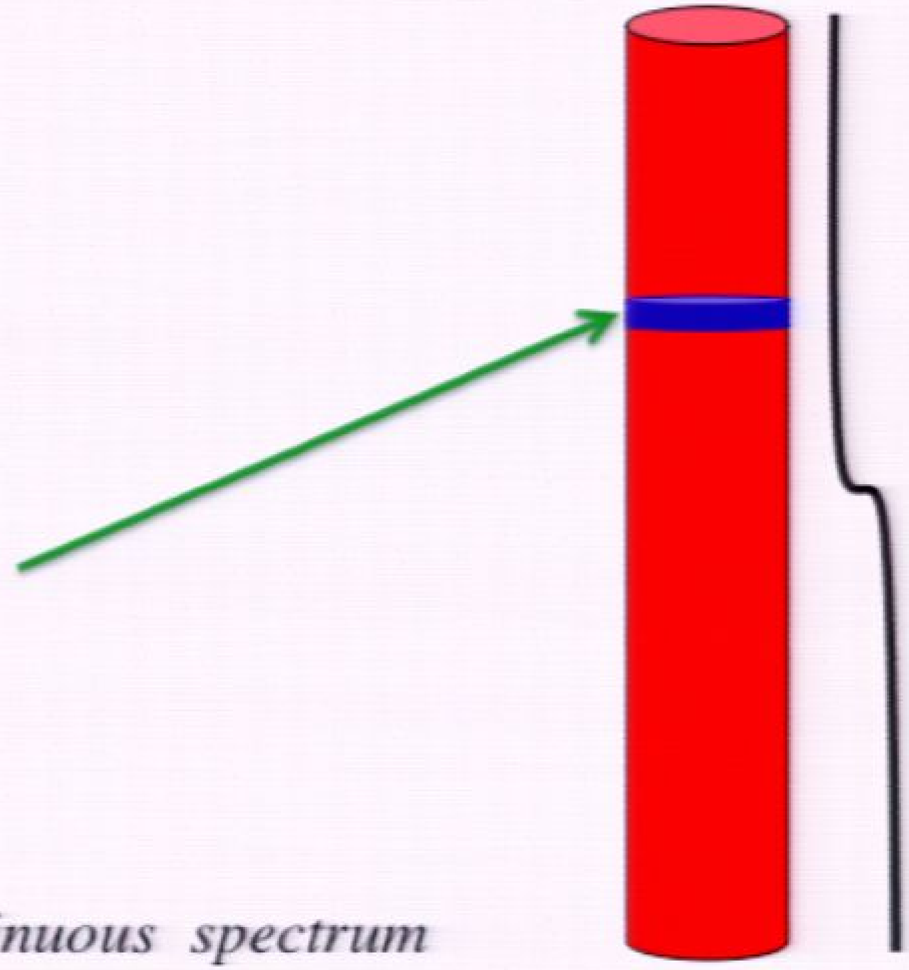

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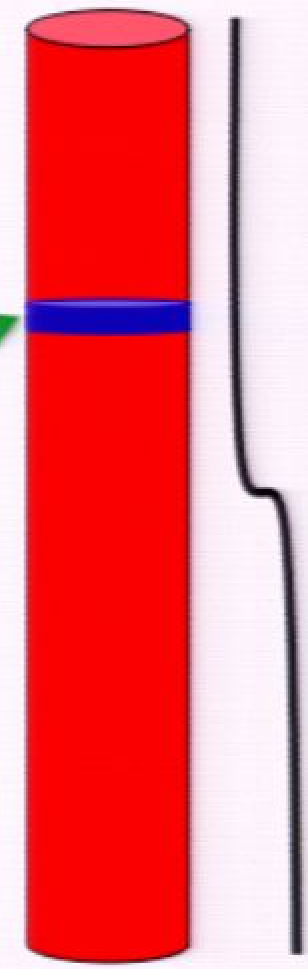

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Quantum perturbation of classical states:

Quantum perturbation of classical states:

- Cluster expansions:
- Pirogov-Sinai theory (for classical systems satisfying a Peierls condition, and finite number of ground states)
- Datta, Rodriguez, Frohlich (96)…Yarotsky (04)

Unperturbed Hamiltonian:

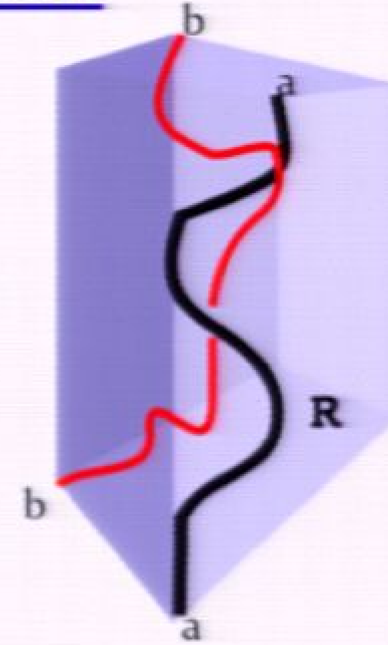
$$Hamiltonian: \ H_0 = \sum_{i \in \Lambda} h_i$$

$$h_i \ \ are \ \ not \ \ kinetic: \ h_i | \chi_1, a_1, c_1, \chi_2, a_2, c_2, \ldots \chi_{N-1}, a_{N-1}, a_N > =$$
$$h(a_i) | \chi_1, a_1, c_1, \chi_2, a_2, c_2, \ldots \chi_{N-1}, a_{N-1}, a_N >$$

$$Note \ \ [h_i, h_j] = 0$$
$$h(vac) = 0 \qquad h_m \geq h(a \neq vac) \geq 1$$

H0 is may be thought of as set of constraints. h(a) is the penalty for creating excitation a

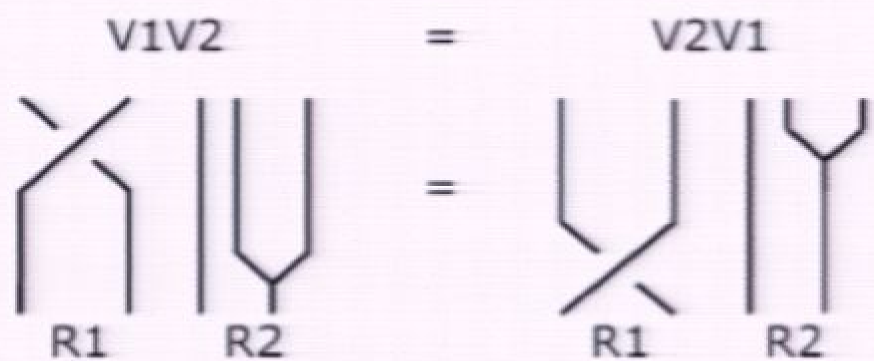
Properties of perturbations which we consider:



Perturbations:

V_i *local operators,* $\|V_i\| \leq 1$ *can be expressed as fusions and braids in a finite region* R_i

if $R_i \cap R_j = \emptyset$ *for* V_i, V_j *then they are commuting:*



Stability Result:

Assume that the ground state of an anyonic lattice H_0 is unique.

And $\|V_i\| < 1$, are local perturbations.

Then for any $\beta < 1/6$

$$H = H_0 + \beta \sum V_i$$

has a gap $\gamma > c(\beta)$, where estimates are independent on the volume of the lattice.

Results for toric code:

1) Thin torus limit is not stable can destroy gap if L_2 is kept fixed while sending L_1 to infinity.

2) Toric code with z - pertubrations stable if ratio of directions is not exponential, when taking thermodynamic limits.

Strategy:

If there is a gap:

$$Z(N) = \langle \psi | e^{-NH} | \psi \rangle = \sum_n |\langle n | \psi \rangle|^2 e^{-NE_n}$$

$$\xrightarrow{N \to \infty} C_1 e^{-NE_0} + e^{-NE_0} o(e^{-\gamma N})$$

so

$$\log(Z(N)) = c - E_0 N + o(e^{-\gamma N})$$

Compute log Z(N) using cluster expansions.

assume c are certain "polymers" in space time: $\Lambda \times \mathrm{N}$ so that

$$Z(N) = \sum_{Clusters} \omega(c)$$

polymers are compatible if $\omega(c_1 \cup c_2) = \omega(c_1)\ \omega(c_2)$

then:

$$\log(\mathrm{Z(N)}) = \sum_{\mathrm{X}} \omega(X)$$

where X is a "cluster": polymers $\mathrm{c_k}$ and multiplicities $\mathrm{n_k}$

$$\omega(X) = \prod \frac{\omega(c_k)^{n_k}}{n_k!}\ a(X)$$

$a(X)$ *is a combinatorial factor*

General strategy of proofs using cluster expansion

(For relatively simple proof: Bovier&Zhradnik00)

Basic strategy needs following ingredients:

$\omega(c) < e^{-a\ |c|}$

number of irreducible clusters of size L touching a point x is bounded by e^{bL}

$\Rightarrow$ for large c, sum of polymer clusters touching a point is convergent, independent of system size

After these are established we can proceed as:

$$\log(Z(N)) = \sum_{l(X)<N} \omega(X) + \sum_{l(X)=N} \omega(X) =$$

$$\sum_{\text{X cluster starting at t=0}} (l(x) - N)\, \omega(X) - \sum_{\substack{\text{X cluster starting at t=0} \\ l(X)>N}} (l(x) - N)\, \omega(X) =$$

$$\sum_{\text{X cluster starting at t=0}} l(x)\, \omega(X) - N \sum_{\text{X cluster starting at t=0}} \omega(X) -$$

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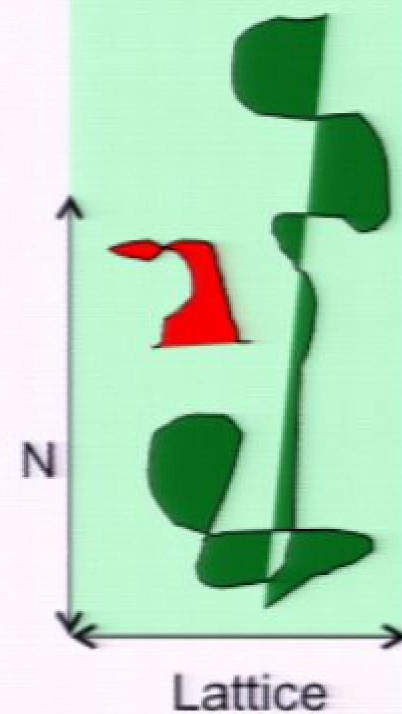
$Const$ $\qquad$ $o(e^{-gap\ N})$ $\qquad$ E_0



) number of irreducible clusters of size L touching a point x

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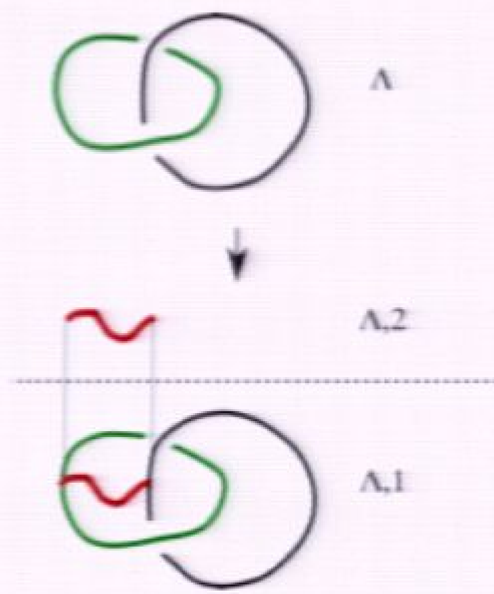
Major difference from usual theory of lattice gases is

that the appropriate condition for the clusters to be compatible

i.e. $\omega(c_1 \cup c_2) = \omega(c_1)\,\omega(c_2)$

if c_1, c_2 are "non-linking" rather than "non touching"



umber of linked clusters of size

L is much larger than connected cluster of size L

To show that the bound is still exponential we count

ompare to connected clusters of larger size :

Cluster expansions (Yarotsky approach):

Given a set of sites $L \subset I \subset \Lambda$, define $\mathrm{H}_{\mathrm{L,I}} = \sum_{i \in \bar{I}} h_i + \beta \sum_{i \in L} V_i$

$$T_I = e^{-\sum_{i \notin \bar{I}} h_i} \sum_{L \subset I} (-1)^{|I|-|L|} e^{-H_{L,I}}$$

Note by the inclusion - exclusion principle:

$$\mathrm{f}(\Lambda) = \sum_{I \subset \Lambda} \sum_{L \subset I} (-1)^{|I|-|L|} f(L)$$

so

$$\mathrm{e}^{-\mathrm{H}} = \sum_{I \subset \Lambda} T_I$$

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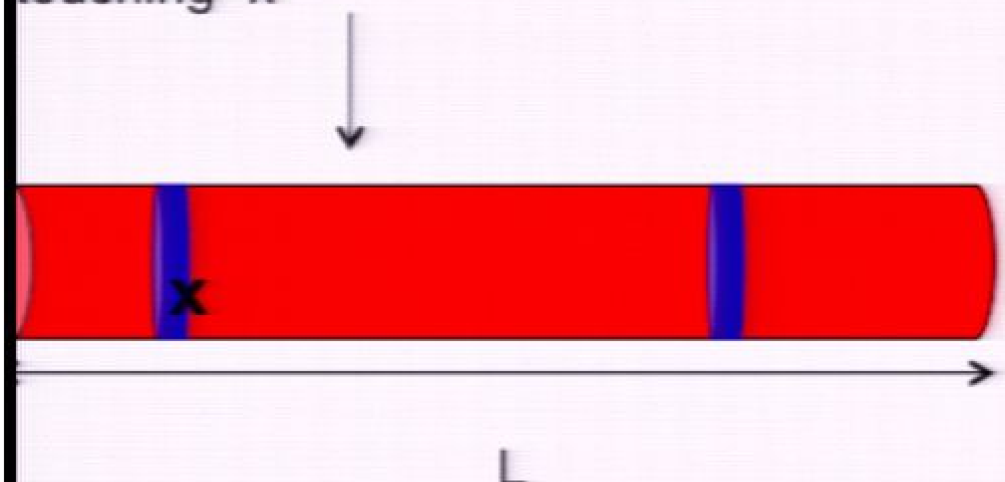
What fails in the torus case?

This is an irreducible cluster touching “x”



Number of such clusters touching “x” of
size two loops
scales like L
three loops: L^2 etc…

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- Non trivial loops may be correlated when **arbitrary far apart**

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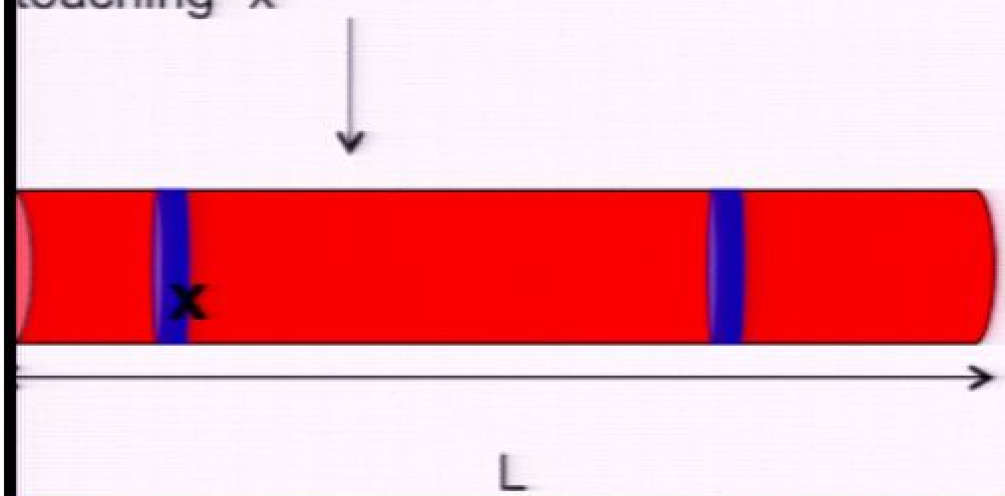

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→ Too many irreducibles. More careful analysis needed

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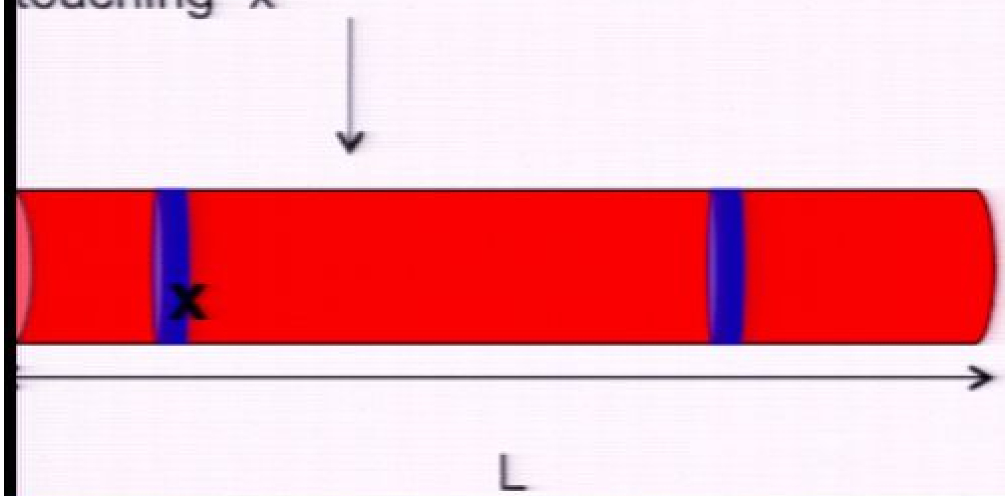

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- Useful information on the new, deformed ground state. Any use for numerics?