Title: Gapless Spin Liquids in Two Dimensions

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Abstract: Many crystalline materials predicted by band theory to be metals are insulators due to strong electron interactions. Both experiment and theory suggest that such Mott-insulators can exhibit exotic gapless spin-liquid ground states, having no magnetic or any other order. Such “critical spin liquids” will possess power law spin correlations which oscillate at various wavevectors. In a sub-class dubbed “Spin Bose-Metals” the singularities reside along surfaces in momentum space, analogous to a Fermi surface but without long-lived quasiparticle excitations. I will describe recent theoretical progress in accessing such states via controlled numerical and analytical studies on quasi-1d model systems.

# Gapless Spin Liquids in Two dimensions

MPA Fisher (with O. Motrunich, D. Sheng and Matt Block)

Perimeter Institute 05/27/10

ocus - Quantum Phases of 2d electrons (spins) with
***nergent*** rather than broken ***symmetry***

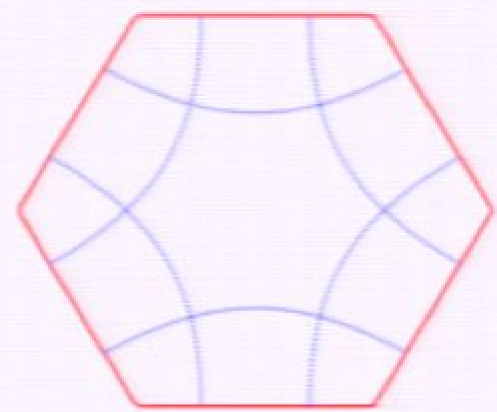
utline:

Quantum theory of solids; Metals and Band Insulators

Mott insulators

Spin liquids

***Gapless*** spin liquids ⬅

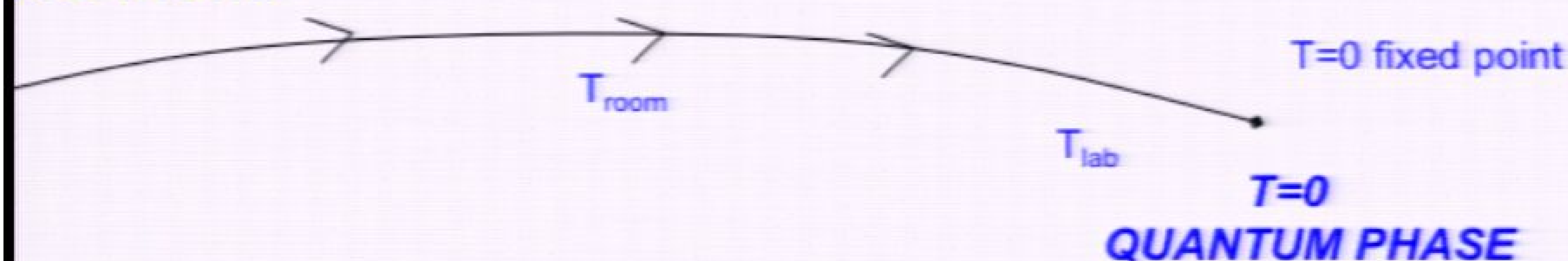


# “Simplicity” of Electrons in solids

eparation of energy scales for the electrons;

netic energy
nd Coulomb energy: $E_{KE}$ , $E_{coul} >> T_{room} >> T_{lab}$

## RG Flows



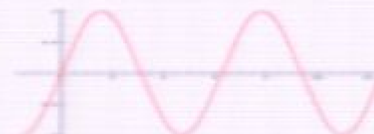
# Quantum Theory of Solids: 2 dominant phases

ld number of electrons/cell ⟶ Metals

en number of electrons/cell ⟶ Band Insulators

# Band Theory: Metals versus insulators

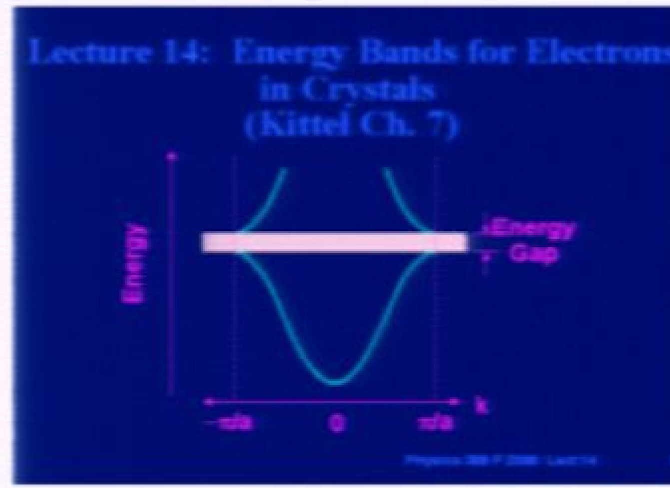
$$H = \sum_j \frac{\mathbf{p}_j^2}{2m} + \sum_i V(\mathbf{r}_i)$$



Energy Bands

***Band insulators***: Filled bands

***Metals***: Partially filled highest energy band

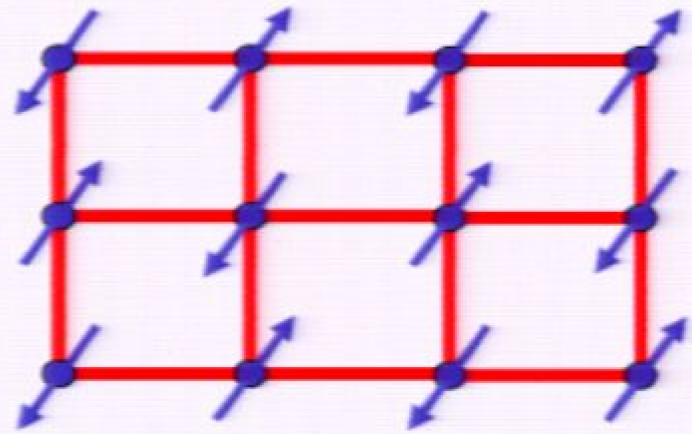



Even number of electrons/cell - (usually) a band insulator

Odd number per cell - always a metal

# But most d and f shell crystals with odd number of electrons are NOT metals

ue to Coulomb repulsion
lectrons gets stuck on atoms



'Mott Insulators"

Mott Insulators:
Insulating materials with odd number of electrons/unit cell



Sir Neville Mott

# Quantum Phases of Electrons

ld number of electrons/cell
om atomic s or p orbitals) → Metal

en number of electrons/cell → Band Insulator

number of electrons/cell
ic d or f orbitals) → Mott insulator

# Spin Physics in Mott insulators

Toy model - Hubbard with one electron/site $\mathcal{H} = -t\sum_{\langle ij\rangle}[c^\dagger_{i\alpha}c_{j\alpha} + h.c.] + U\sum_i n_{i\uparrow}n_{i\downarrow}$

For U>>t electrons get self localized - residual s=1/2 operator per site

Generalized Heisenberg spin model: $H_{spin} = \sum_{ij} J_{ij}\vec{S}_i \cdot \vec{S}_j + ...$

***Global symmetry: SU(2) spin symmetry***

***Discrete lattice point/space group***

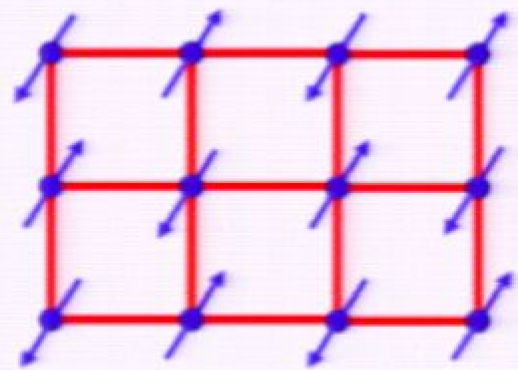
# Symmetry breaking in Mott insulators

Mott Insulator ➡ Unit cell doubling ("Band Insulator")

Symmetry breaking

2 electrons/cell

Ex: 2d square Lattice AFM



# Quantum Phases of Electrons

ld number of electrons/cell
om atomic s or p orbitals) → Metal

en number of electrons/cell → Band Insulator

number of electrons/cell
ic d or f orbitals) → Mott insulator

Mott insulator → Symmetry breaking eg AFM

Mott insulator → ???

# 2d Spin liquids

## *Mott insulators with no broken symmetries*

Theorem (Matt Hastings, 2005): Mott insulators on an L by L torus have a low energy excitation with $(E_1\text{-}E_0) < \ln(L)/L$

Remarkable implication: 2d spin liquids come in two flavors

1) Topological Spin Liquids
2) Gapless Spin liquids

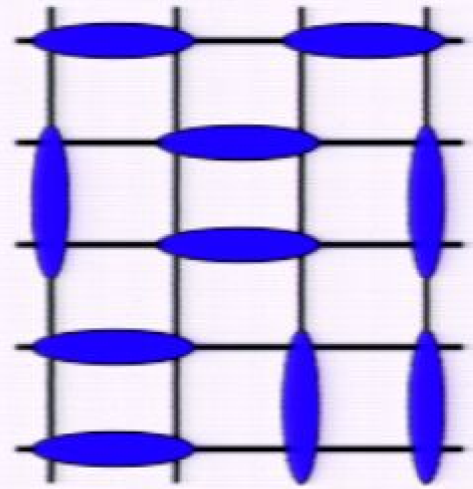
Topological Spin Liquids

Spin Liquids
***o broken***
***mmetries)***

Gapless Spin Liquids
(***emergent symmetries)***

# Topological Spin Liquids

RVB state (Anderson)



ppological Spin liquids are time reversal invariant analogs
f the Fractional Quantum Hall effect states

***Gap to all bulk excitations***

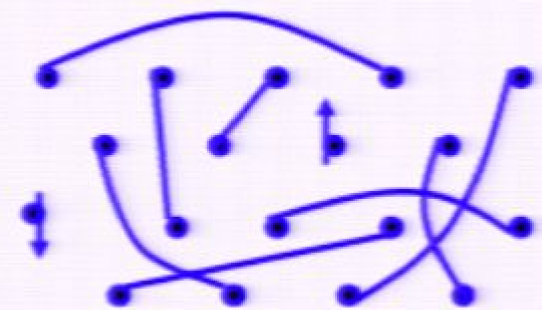
Ground state degeneracies on a torus

"Particle" excitations with fractional quantum numbers, eg spinon

Simplest is short-ranged RVB, $Z_2$ Gauge structure

No example (yet) of a physically reasonable SU(2) spin model with a topological spin liquid ground state

# Gapless Spin liquids



Valence bonds on all length scales

***Stable gapless phases*** with no broken symmetries

no free particle description

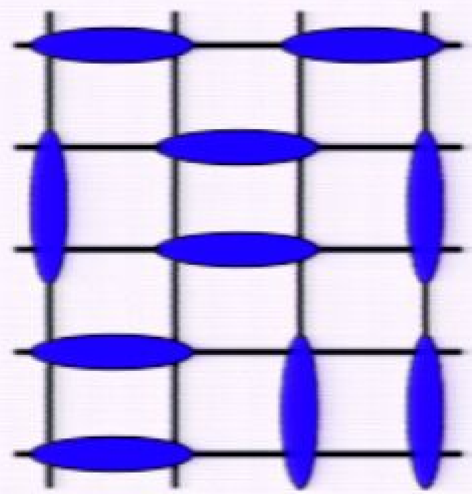
Power-law correlations with anomalous exponents

***Emergent symmetries*** at low energies

Lattice scale physics manifest in IR

# Topological Spin Liquids

RVB state (Anderson)



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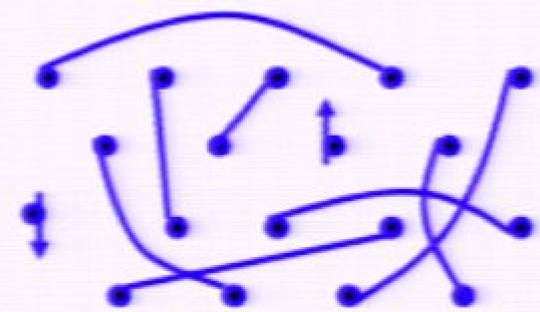

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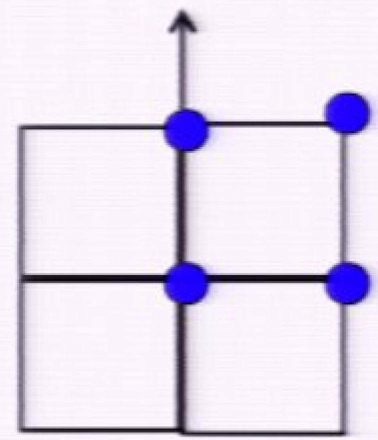
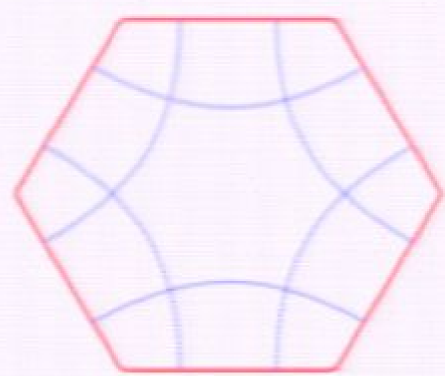
# 2 classes of gapless SL's



***Algebraic Spin Liquids***

Power law correlations at a
***finite set of discrete momenta***

***"Spin Bose-Metals"***



Spin correlation functions singular along
***surfaces in momentum space***

***"Bose Surfaces"***

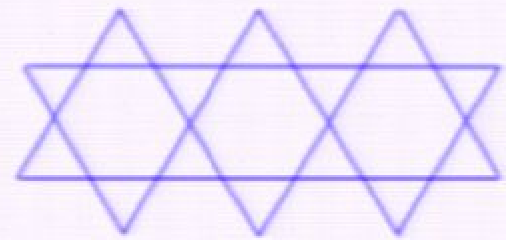
# 2d Spin liquids

- Spin Liquids
  - → Topological Spin Liquids
  - → Gapless Spin Liquids
    - → Algebraic SL
    - → Spin Bose-Metal

# Access gapless spin liquids?

**) Algebraic spin liquids**

Kagome lattice AFM



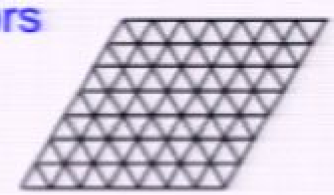
- Frustration
- low spin (s=1/2)
- low coordination number (Kagome lattice)

- Volborthite $Cu_3V_2O_7(OH)_2\ 2H_2O$ $Cu^{2+}$ s=1/2
- Herbertsmithite $ZnCu_3(OH)_6Cl_2$ $Cu^{2+}$ s=1/2 ,

**.) Spin Bose-Metals**

- Quasi-itinerancy
- "Weak" Mott insulator
- Small charge gap, comparable to J

Triangular lattice based
Organic Mott insulators



# Theoretical route to Gapless SL's: Slave-fermions

Generalized Heisenberg s=1/2 model

$$\mathcal{H} = J \sum_{\langle rr' \rangle} S_r \cdot S_{r'} + \cdots$$

Fermionic representation of spin-1/2

$$\mathbf{S}_i = f_i^\dagger \frac{\boldsymbol{\sigma}}{2} f_i; \qquad f_{i\alpha}^\dagger f_{i\alpha} = 1;$$

General "Hartree-Fock" in the singlet channel

$$\mathcal{H}_{\text{trial}} = -\sum_{ij} t_{ij} f_{i\alpha}^\dagger f_{j\alpha}$$

$\rightarrow |\Psi_0\rangle \rightarrow |\Psi_{\text{spin}}\rangle = P_G(|\Psi_0\rangle)$

free fermion determinant — spins — Gutzwiller projection

# Access gapless spin liquids?

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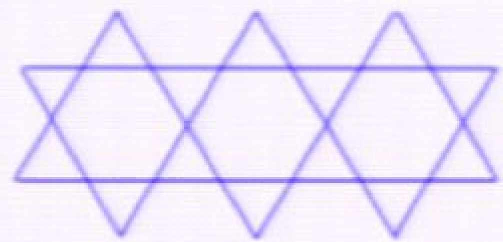
Kagome lattice AFM



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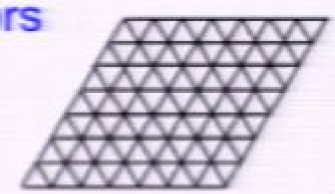

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free fermion determinant

spins

Gutzwiller projection

# Gutzwiller-projected "Spinon" determinant

$$P_G \left( |\text{Fermi Sea}\rangle = \prod_{k<k_F} c^\dagger_{k\uparrow} c^\dagger_{k\downarrow} |vac\rangle \right)$$

$$= a|(\uparrow)\ (\uparrow)\ (\downarrow)\ (\uparrow)\ (\downarrow)\ (\uparrow)\rangle + b|(\uparrow\downarrow)\ (0)\ (\downarrow)\ (\uparrow)\ (\uparrow)\ (\downarrow)\rangle$$

$$+c|(\uparrow)\ (\downarrow)\ (\downarrow)\ (\uparrow\downarrow)\ (\uparrow)\ (0)\rangle + d|(\downarrow)\ (\uparrow)\ (\uparrow)\ (\downarrow)\ (\uparrow)\ (\downarrow)\rangle + \dots$$

real-space configurations

Arrive at a spin wavefunction

# Gauge Theory

auge redundancy:

$$f_{i\alpha} \longrightarrow e^{i\theta_i} f_{i\alpha}$$ leaves spin invariant $$\vec{S}_i = f^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

ed gauge field to eliminate redundancy

$$\mathcal{H} = -t \sum_{\langle ij \rangle} e^{a_{ij}} f^\dagger_{i\alpha} f_{j\alpha} + \mathcal{H}_a$$

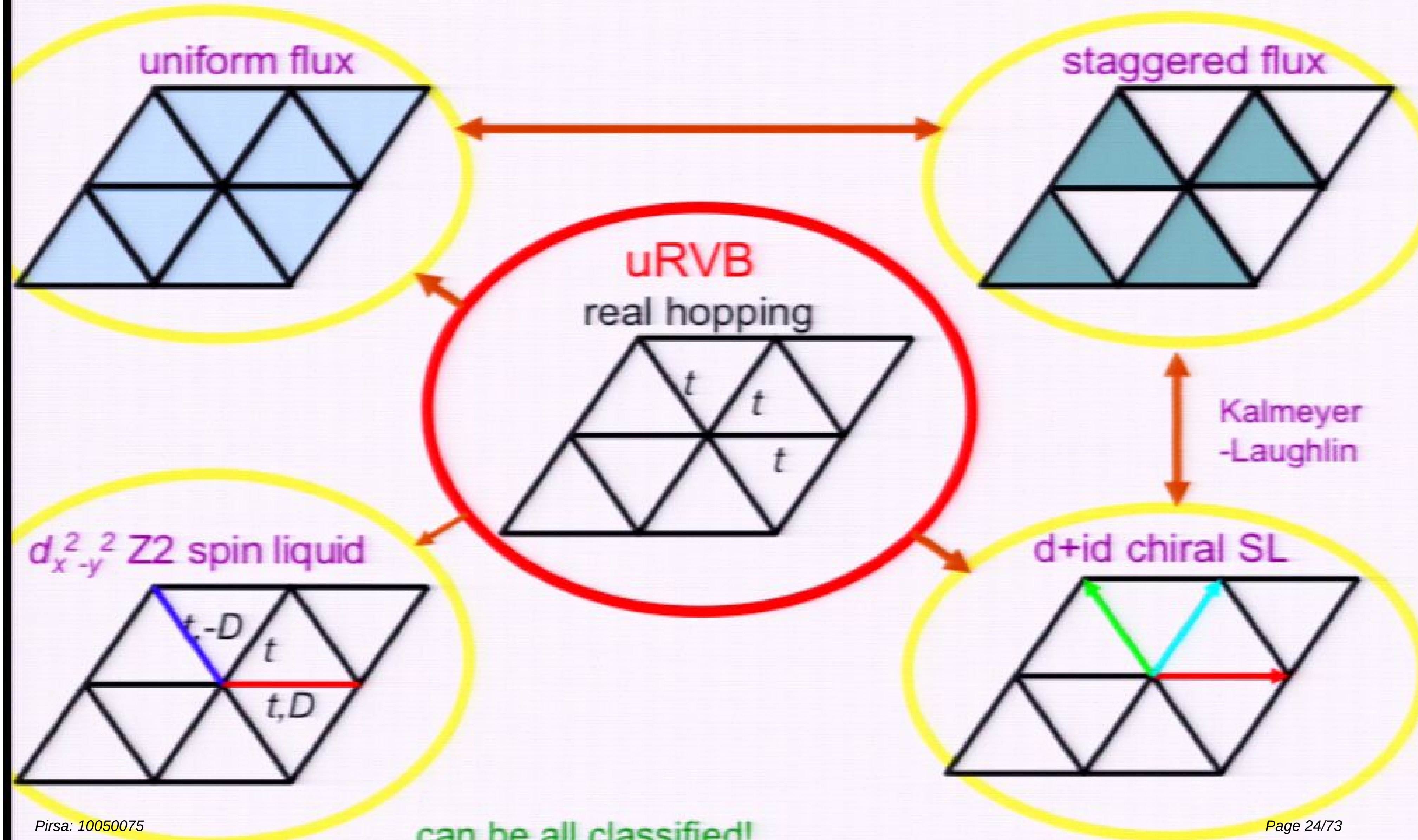
lust treat $a_{ij}$ as a dynamical variable

$$\mathcal{H}_a = h \sum_{\langle ij \rangle} e^2_{ij} - K \sum_{squares} \cos(\nabla \times \vec{a})$$

$$(\nabla \cdot \vec{e})_i + f^\dagger_i f_i = 1$$

***Fermionic spinons minimally coupled to compact U(1) gauge field;***

# Examples of "fermionic" spin liquids




can be all classified!

Wen 2001; Zhou and Wen 2002

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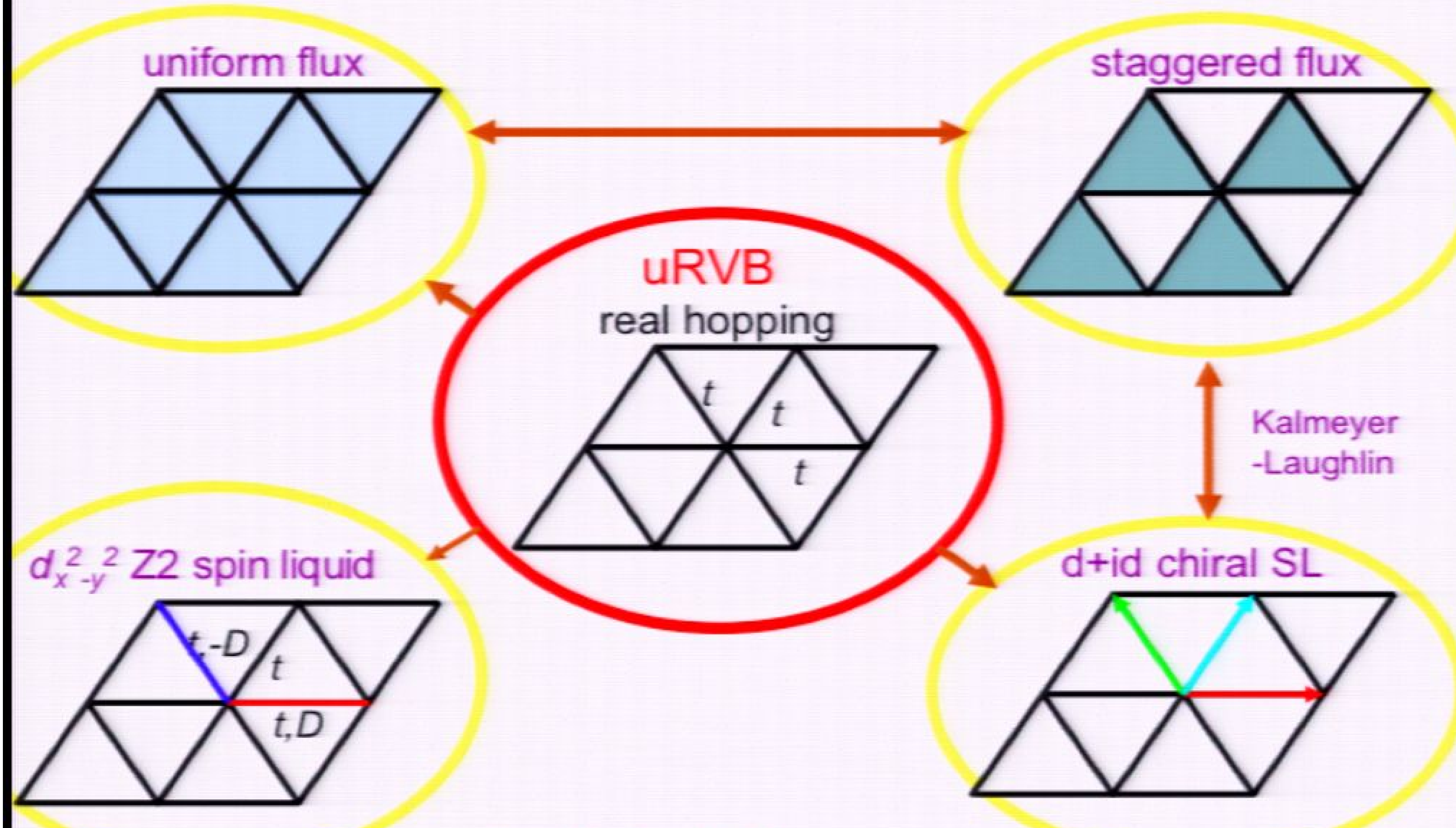


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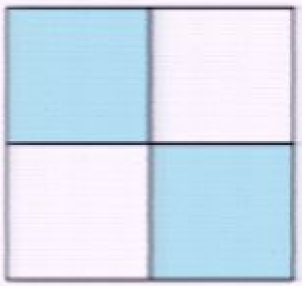
Wen 2001; Zhou and Wen 2002

# Algebraic Spin Liquid (example)

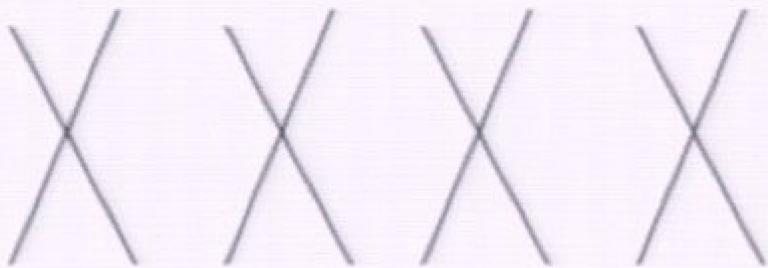
***Staggered flux state on 2d square lattice***



Mean field Hamiltonian:

$$\mathcal{H}_{\rm sF}^0 = -\sum_{r\in A}\sum_{r' \text{ NN } r}\{[it+(-1)^{(r_y-r_y')}\Delta]f_{r\alpha}^\dagger f_{r'\alpha}+\text{H.c.}\},$$

Band structure has relativistic dispersion
with four 2-component Dirac fermions



Effective field theory is non-compact QED3

$$\mathcal{L}_E = \bar{\Psi}[-i\gamma^\mu(\partial_\mu + ia_\mu)]\Psi + \frac{1}{2e^2}\sum_\mu(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + \cdots,$$

Note: can argue that the monopoles are irrelevant due to massless Fermions,
cf Polyakov confinement argument for pure compact U(1) gauge theory)

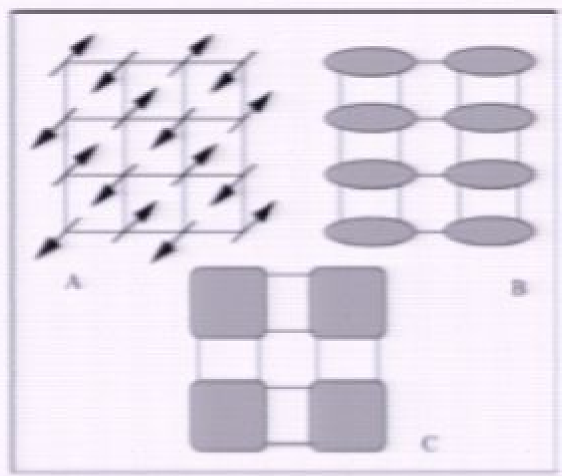
# Emergent symmetry in Algebraic spin liquid

Spin Hamiltonian has ***global SU(2)*** spin symmetry

$$\mathcal{H} = J \sum_{\langle rr' \rangle} \mathbf{S}_r \cdot \mathbf{S}_{r'} + \cdots$$

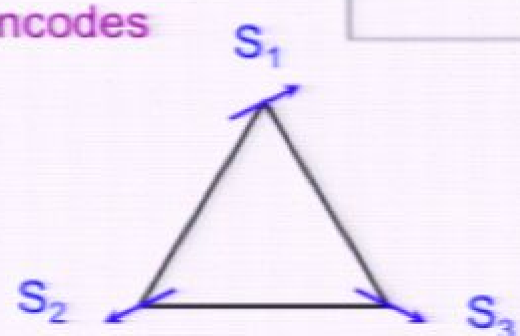
Low energy effective field theory is non-compact QED3 with ***SU(4) flavor symmetry and U(1) flux conservation symmetry***

$$\mathcal{L}_E = \bar{\Psi}[-i\gamma^\mu(\partial_\mu + ia_\mu)]\Psi + \frac{1}{2e^2}\sum_\mu (\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + \cdots,$$




The SU(4) symmetry encodes slowly varying competing order parameters

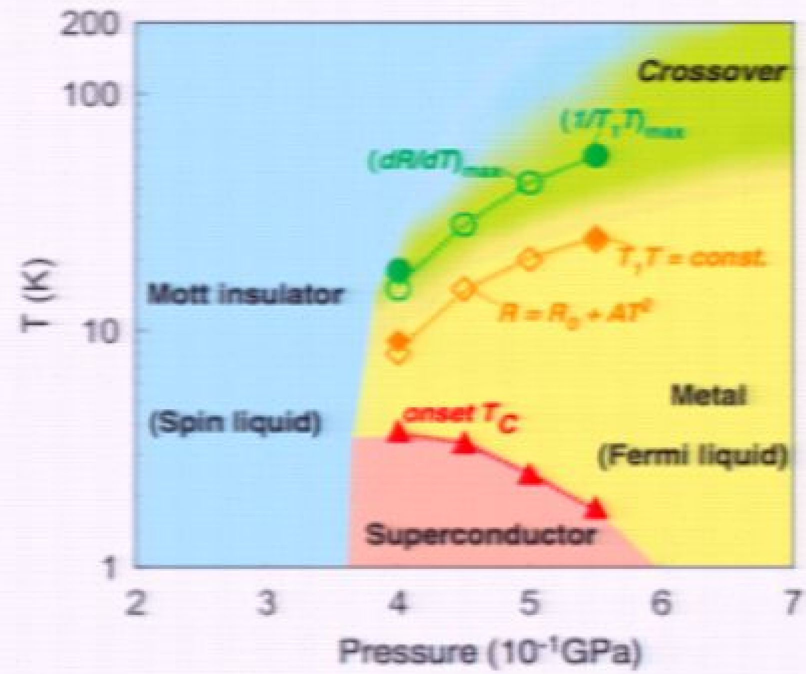
The U(1) flux conservation symmetry encodes a conserved spin chirality




$$\nabla \times a \sim \vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3)$$

# Candidate Spin Bose-Metal: k-$(ET)_2Cu_2(CN)_3$

Kanoda et. al. PRL 91, 177001 (2005)

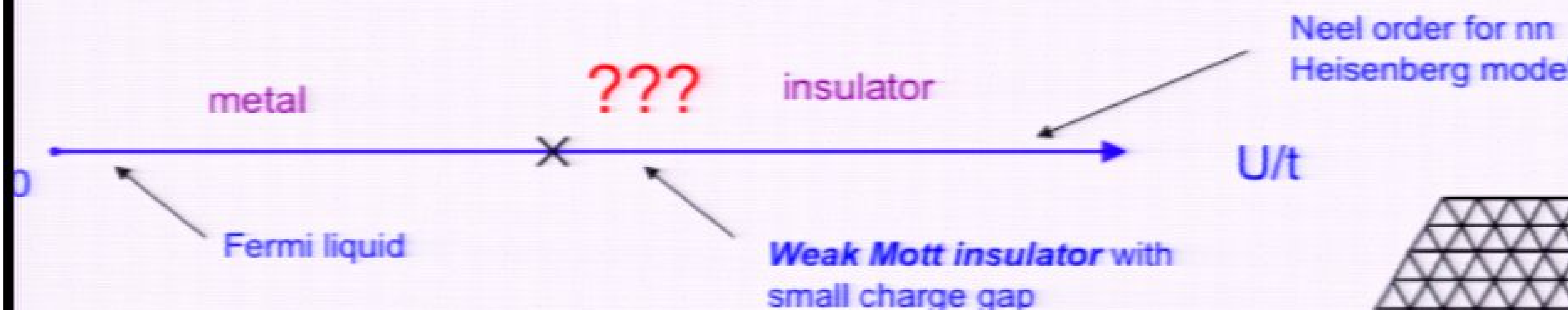


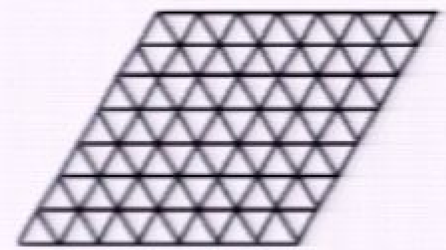

Modelled as triangular Hubbard at half filling
Weak Mott insulator - metal under pressure
No magnetic order down to 20mK ~ $10^{-4}$ J
Large entropy – more than in a metal
"Metallic" specific heat, C~T

Motrunich (2005) , S. Lee and P.A. Lee (2005)
suggested spin liquid with "spinon Fermi surface"

# Hubbard on triangular lattice

At half filling
$$\hat{H}_{\text{Hubbard}} = -t \sum_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



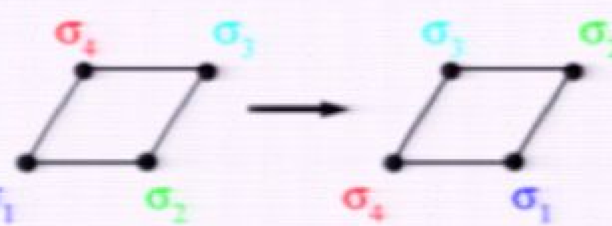



Weak Mott Insulator --> spin model with ***ring exchange***

$$\hat{H}_{\text{eff}} = \frac{2t^2}{U} \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{20t^4}{U^3} \sum_{\text{rhombi}} (P_{ijkl} + \text{h.c.}) + \ldots$$

Ring exchange mimics charge fluctuations

$P_{1234}$:

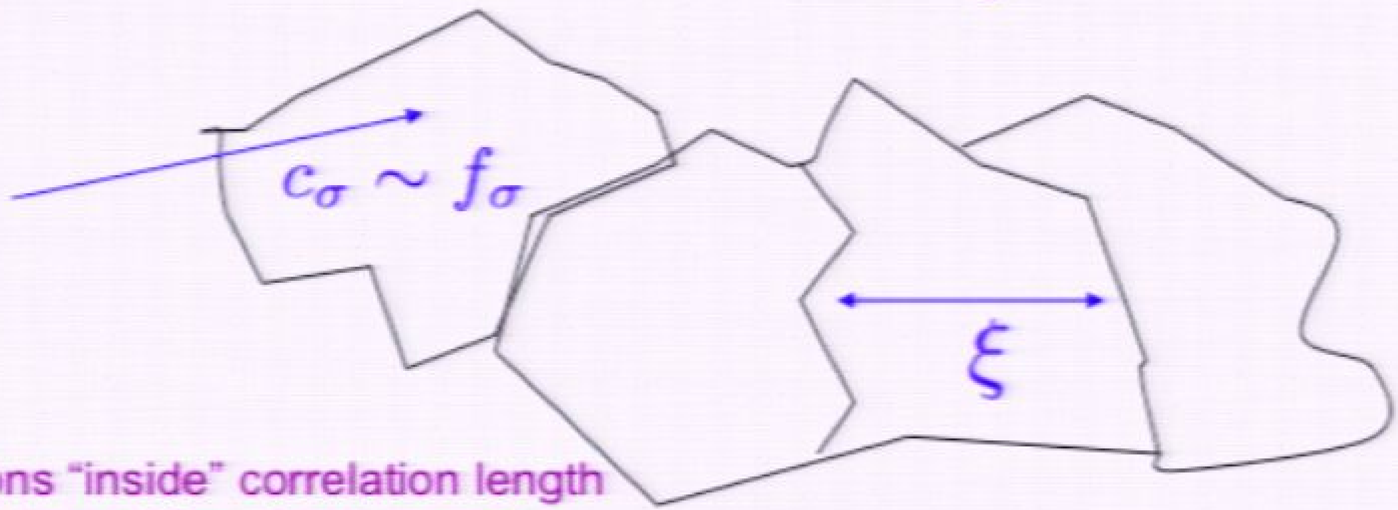


# Weak Mott insulator: Which spin liquid?

Motrunich (2005)

Long charge correlation length, $\langle c_\sigma(x) c_\sigma^\dagger(0) \rangle \sim e^{-x/\xi}$ $\xi >> a$

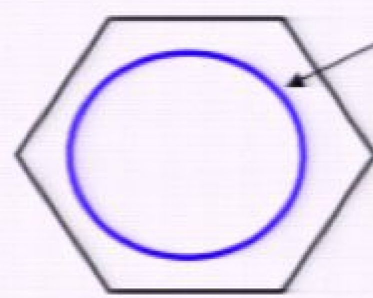
de correlation region
trons do not "know"
are insulating




ess: ***Spin*** correlations "inside" correlation length
semble" spin correlations of free fermion metal,
cillating at $2k_F$

$$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \cos(2k_F x)/x^\alpha$$

propriate spin liquid:



Spinon fermi surface is not physical in the spin model

***utzwiller projected Fermi sea***

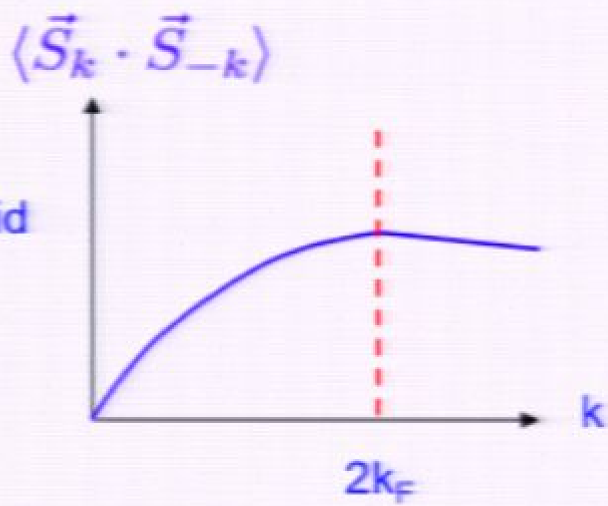


***"Spin Bose-Metal")***

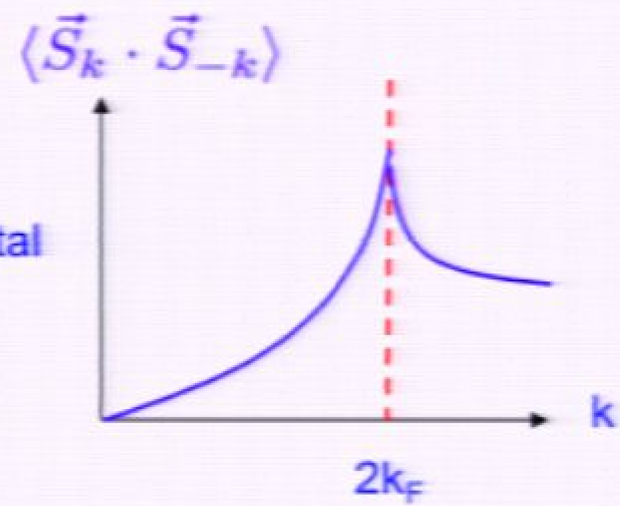
# Phenomenology of Spin Bose-Metal
# (from Gutzwiller wf and Gauge theory)

ingular spin structure factor at "$2k_F$" in Spin Bose-Metal
more singular than in Fermi liquid metal)

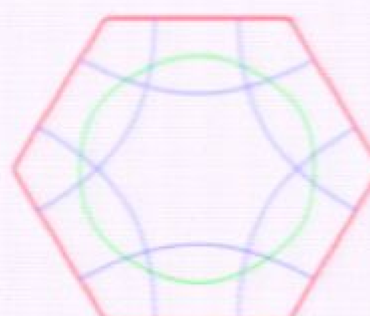
ni liquid




Spin Bose-metal




$2k_F$ "Bose surface" in
triangular lattice Spin Bose-Metal



# Is projected Fermi sea a good caricature of Triangular ring model ground state?

$$\hat{H}_{\text{ring}} = J_2 \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + J_4 \sum_{\text{rhombi}} (P_{ijkl} + \text{h.c.})$$

Variational Monte Carlo analysis suggests it might be for $J_4/J_2 > 0.3$
(O. Motrunich - 2005)

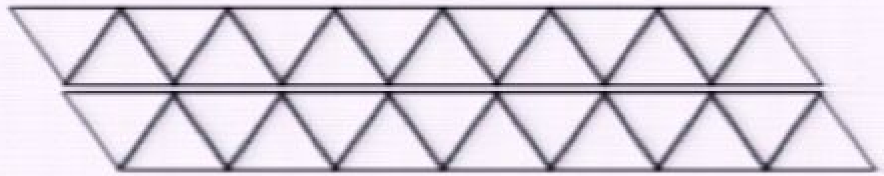
***A theoretical quandary***: Triangular ring model is intractable

- Exact diagonalization: so small,
- Quantum Monte Carlo - sign problem
- Variational Monte Carlo - biased
- DMRG - problematic in 2d

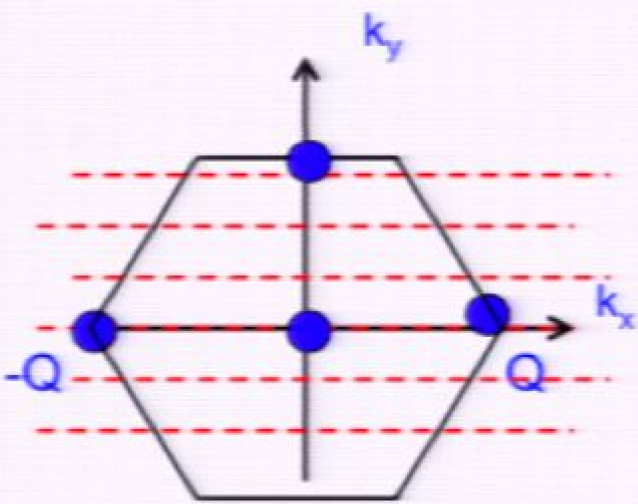
?????

# Quasi-1d route to Spin Bose-Metal

Triangular strips:

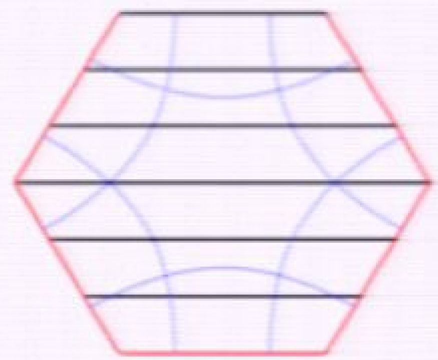


Algebraic Spin liquid




Few gapless 1d modes

Spin Bose-Metal

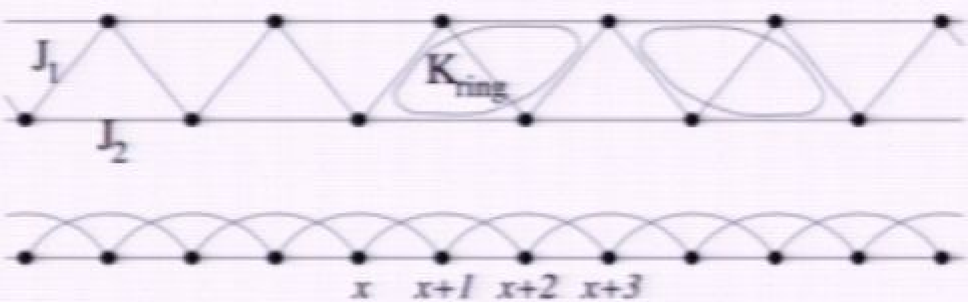


Fingerprint of 2d singular surface - many gapless 1d modes, of order N

***New spin liquid phases on quasi-1d strips, each a descendent of a 2d Spin Bose-Metal***

# 2-leg zigzag strip

$$\mathcal{H}_\Delta = J\sum_{\langle ij\rangle}\vec{S}_i\cdot\vec{S}_j + K\sum_{\langle 1234\rangle}\left[\mathcal{P}_{1234}+\mathcal{P}_{1234}^{-1}\right]$$

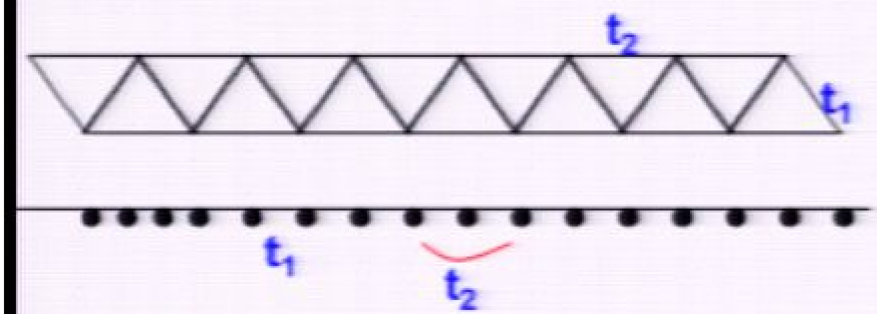



Analysis of $J_1$-$J_2$-K model on zigzag strip

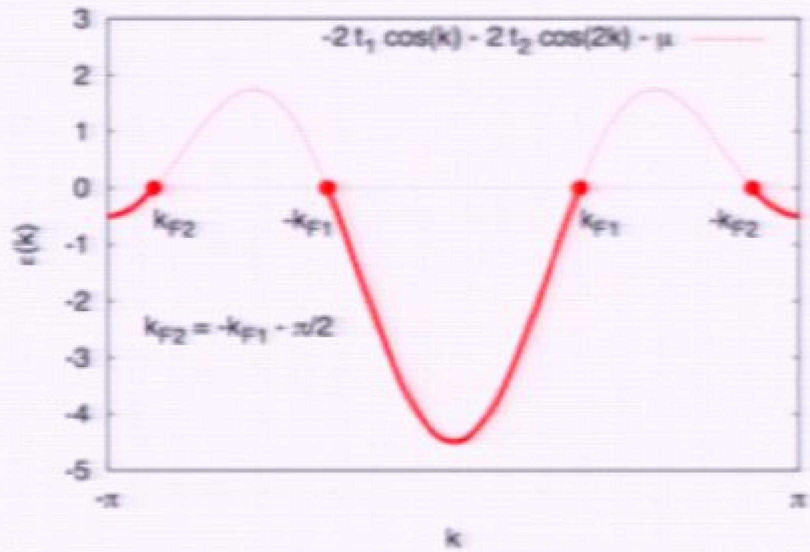
D. Sheng, O. Motrunich, MPAF
PRB (2009)

Variational Monte Carlo of Gutzwiller wavefunctions
DMRG
Bosonization of gauge theory

# Gutzwiller Wavefunction on zigzag




$$\mathcal{H}_{\rm trial} = -\sum_{ij} t_{ij} f^{\dagger}_{i\alpha} f_{j\alpha}$$




$\rightarrow$ $|\Psi_0\rangle$ $\rightarrow$ $|\Psi_{\rm spin}\rangle = P_G(|\Psi_0\rangle)$

fermion determinant — spins — Gutzwiller projection

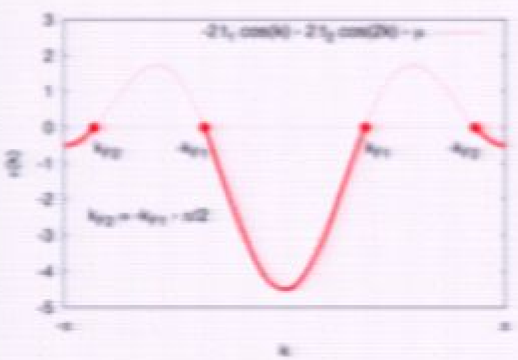
Spinon band structure

***Single Variational parameter:*** $t_2/t_1$ or $k_{F2}$

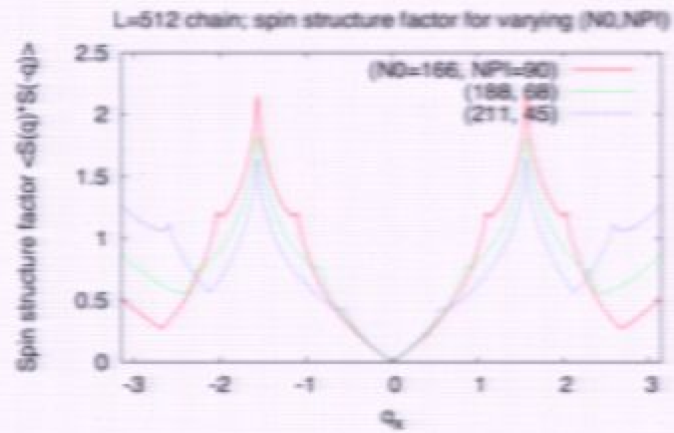
($k_{F1}+k_{F2}$ = pi/2)

# Gutzwiller wf; SU(4) Exponents?

utzwiller wf; 2 Fermi sea's



Spin structure factor for L=512




Power law spin correlator

$$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \cos(\pi x/2)|x|^{-\alpha}$$

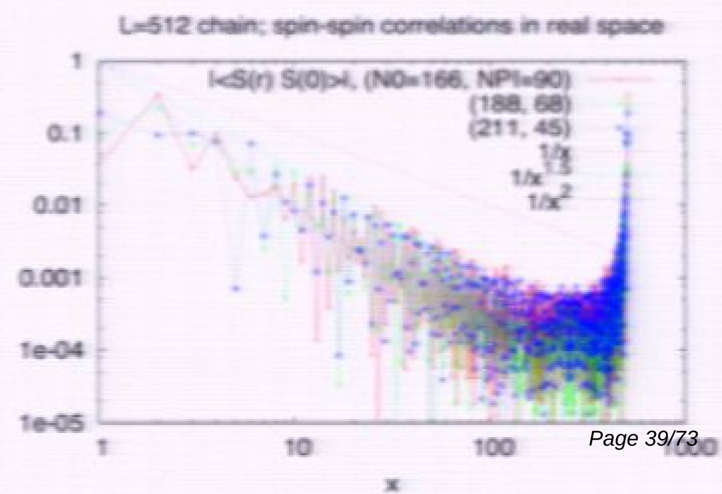
Exponent consistent with SU(4) spin chain

$$\alpha \approx 3/2 = \alpha_{SU(4)}$$

$$\alpha_{SU(N)} = 2 - (2/N)$$

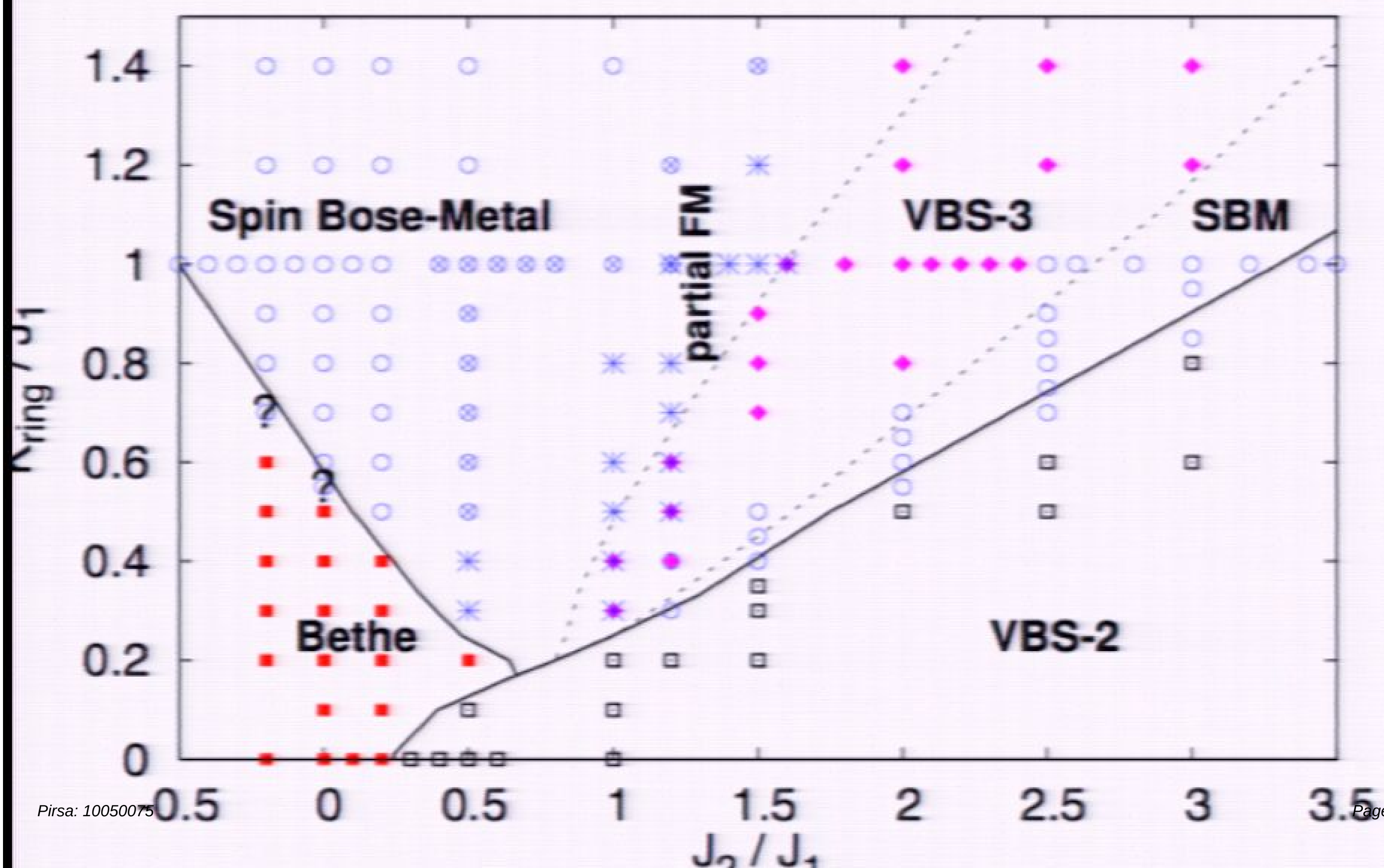
Gutzwiller wf doesn't "know" about 2 different spinon velocities

Analytic progress possible??
(Schur polynomials? Matrix product states?)

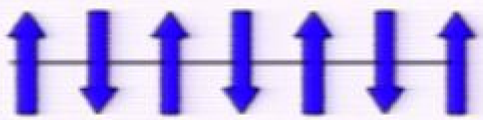



(cf Haldane-Shastry SU(2) chain with exact Gutzwiller Fermi sea ground state)

# DMRG Phase diagram of zigzag ring model



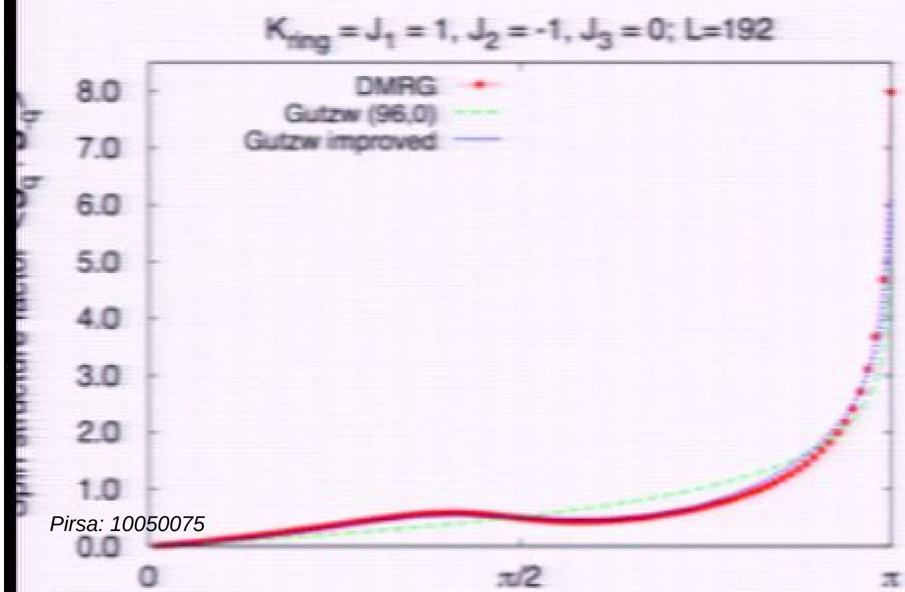
# Bethe chain and VBS-2 States

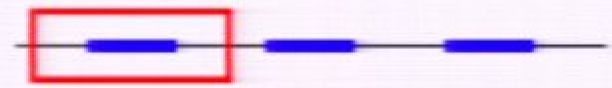


Bethe chain state; "1d analog of Neel state"

$$\langle \vec{S}_x \cdot \vec{S}_0 \rangle \sim (-1)^x / x$$

Spin structure factor



— = $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Valence Bond solid (VBS-2 )

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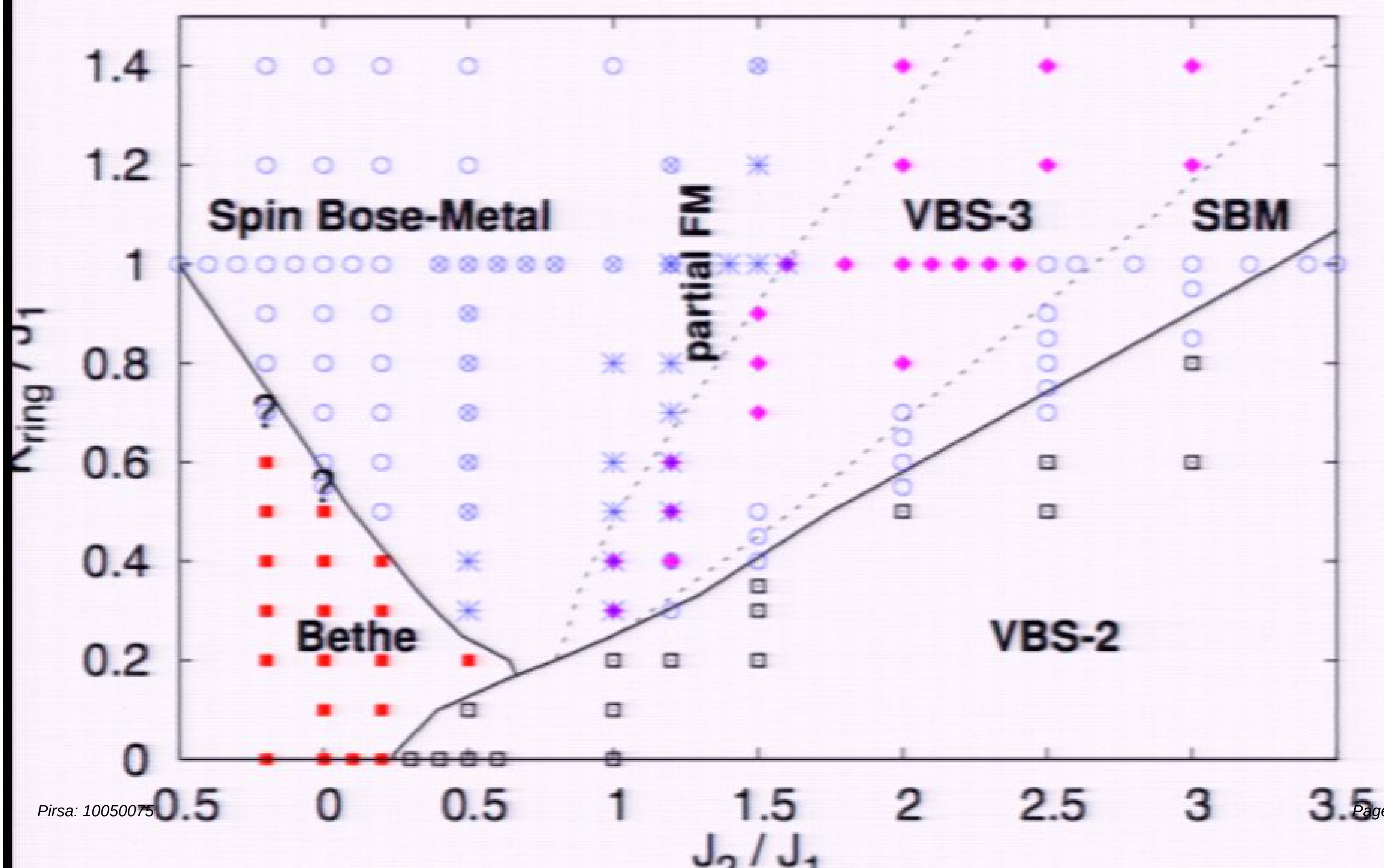

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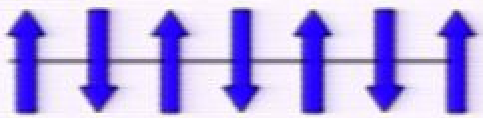

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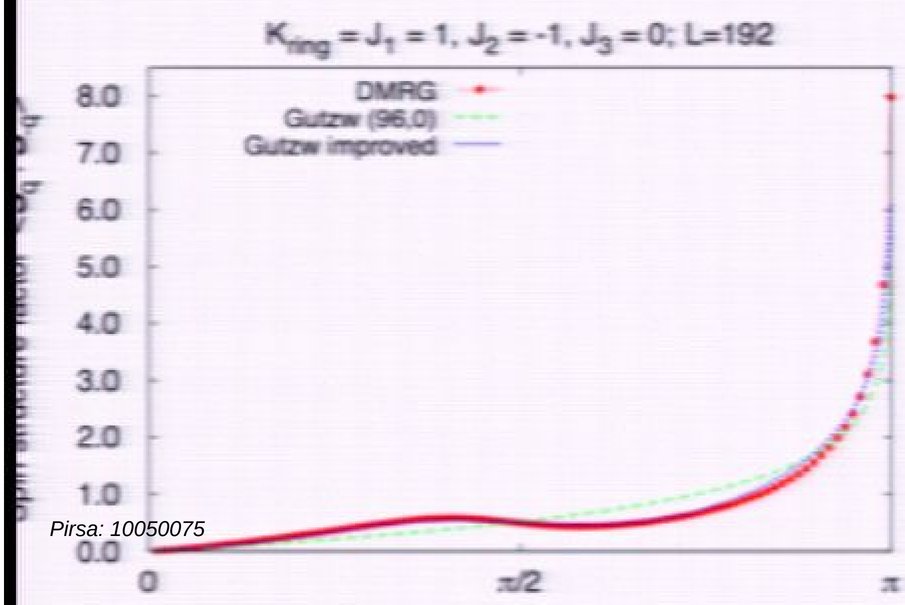


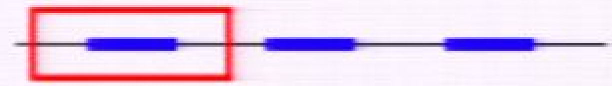

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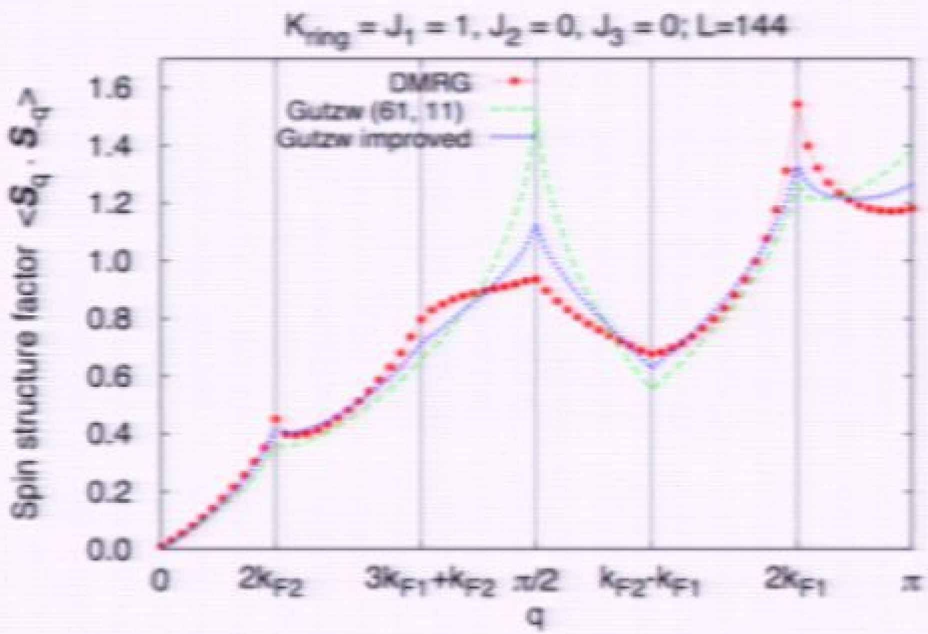
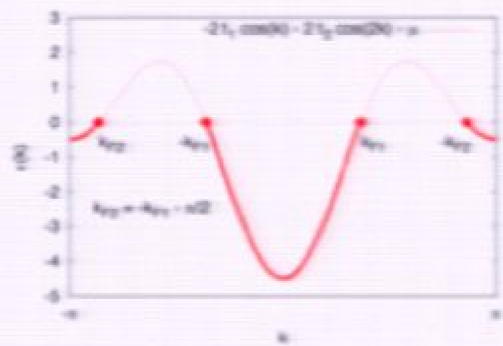
***Singularities in momentum space locate the "Bose" surface (points in 1d)***

$\langle \vec{S}_k \cdot \vec{S}_{-k} \rangle$




Gutzwiller improved has 2 variational parameters)



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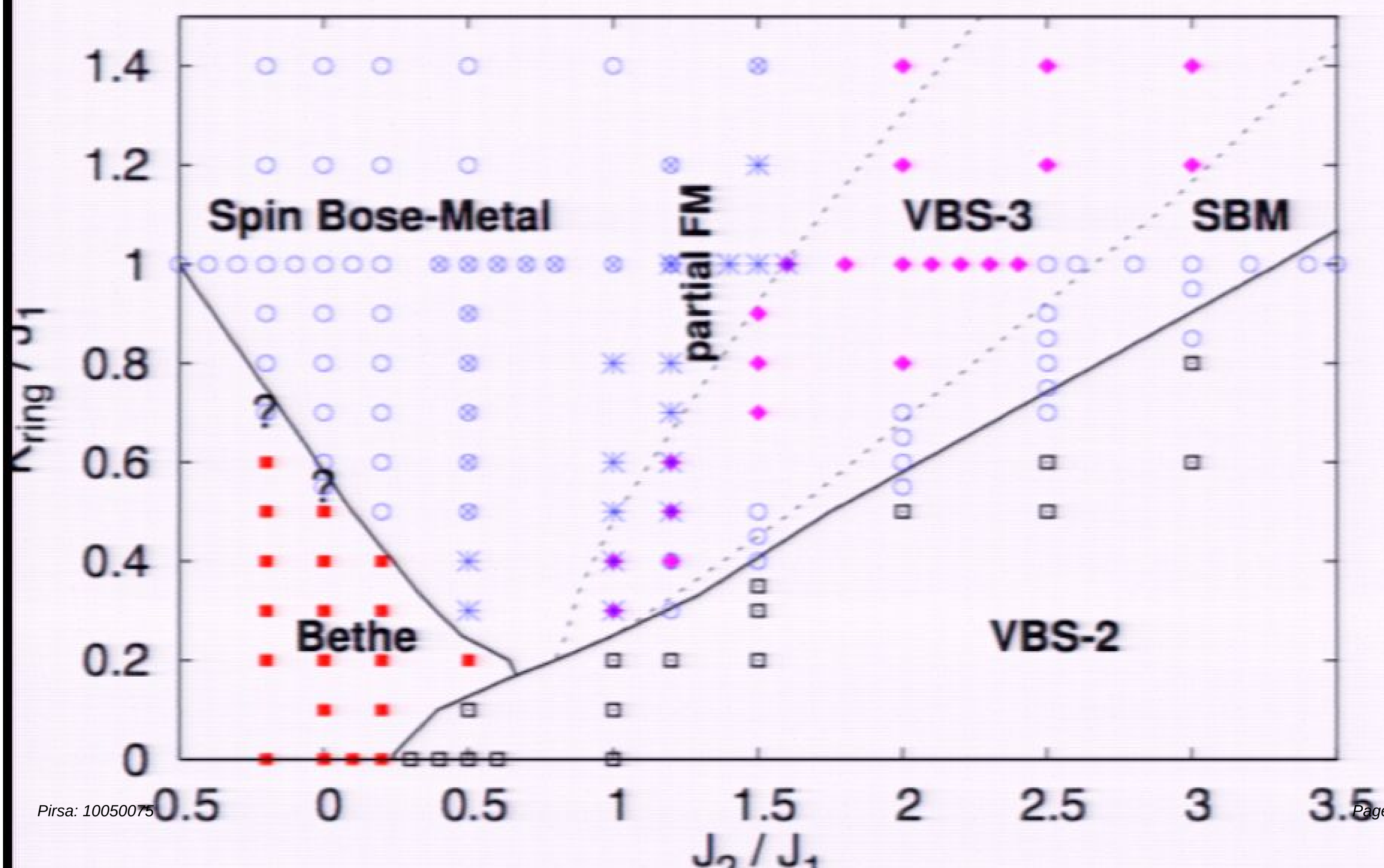

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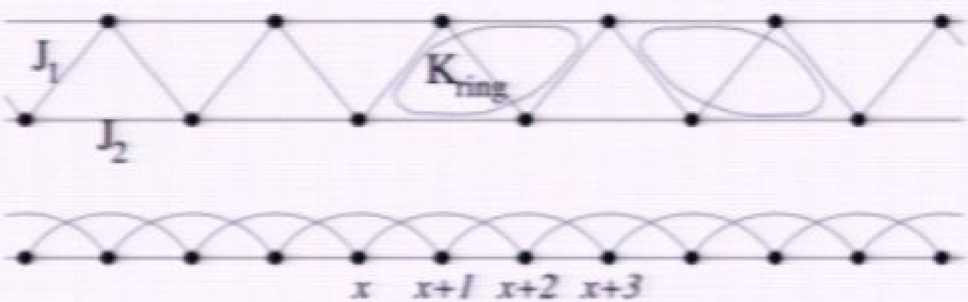


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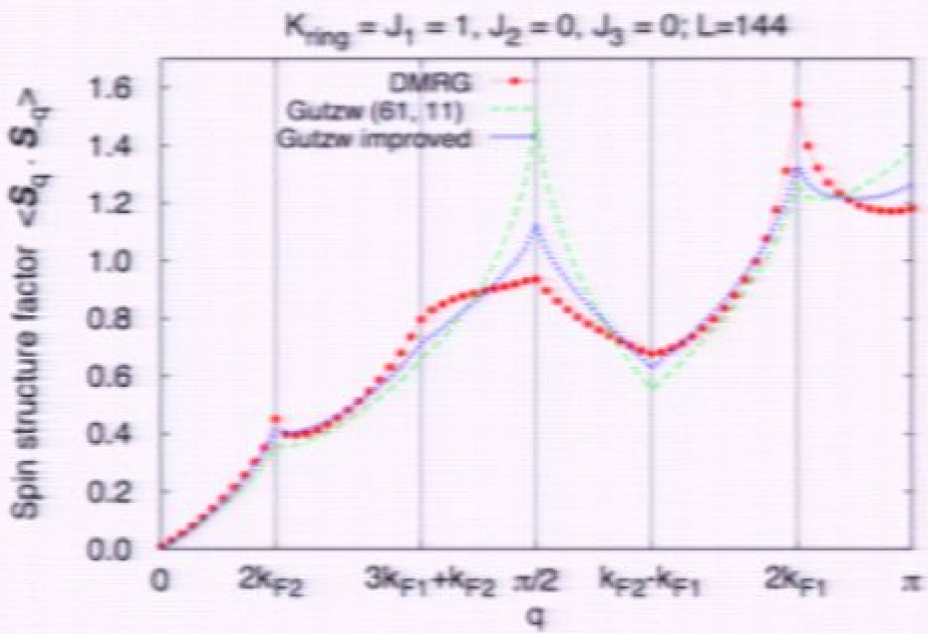


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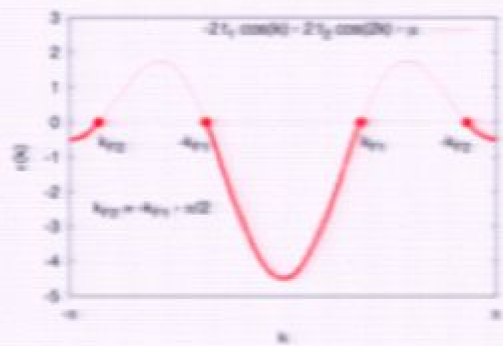

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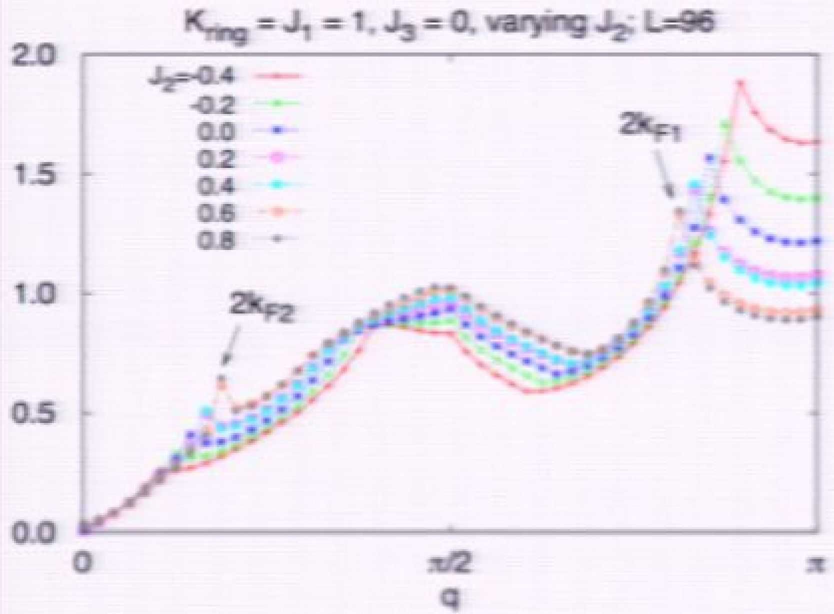
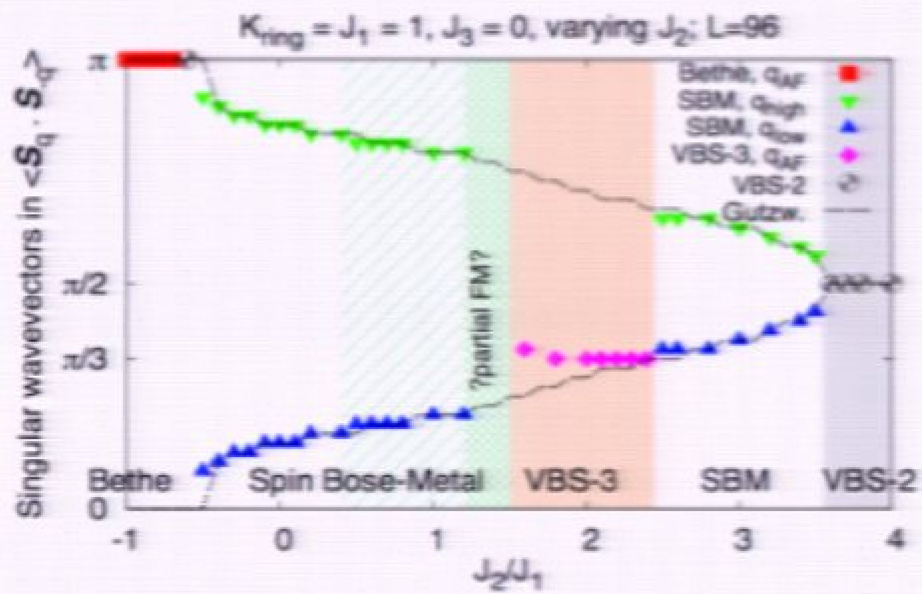
DMRG



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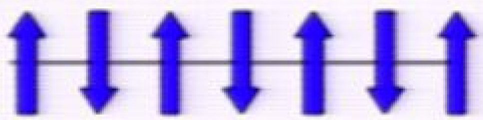

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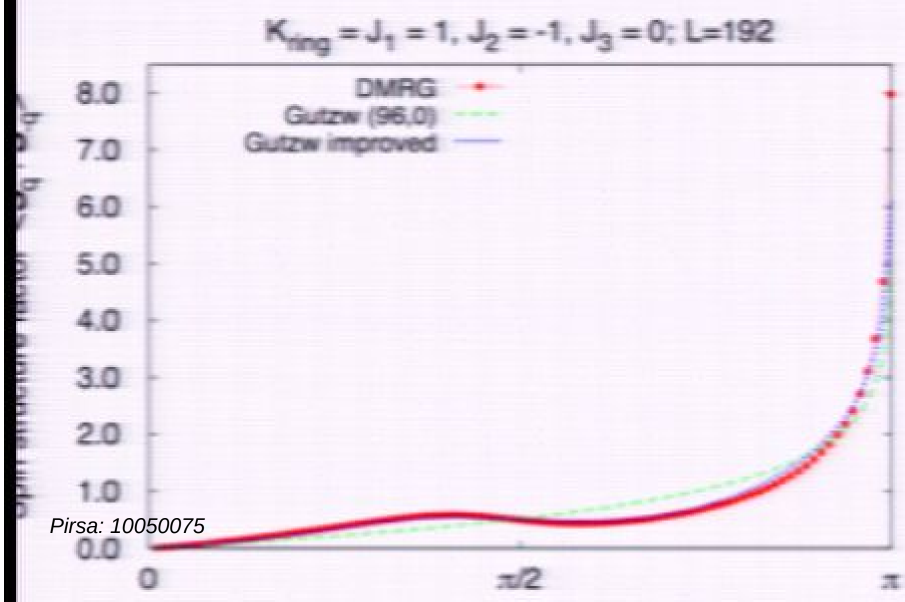


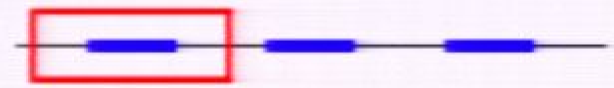

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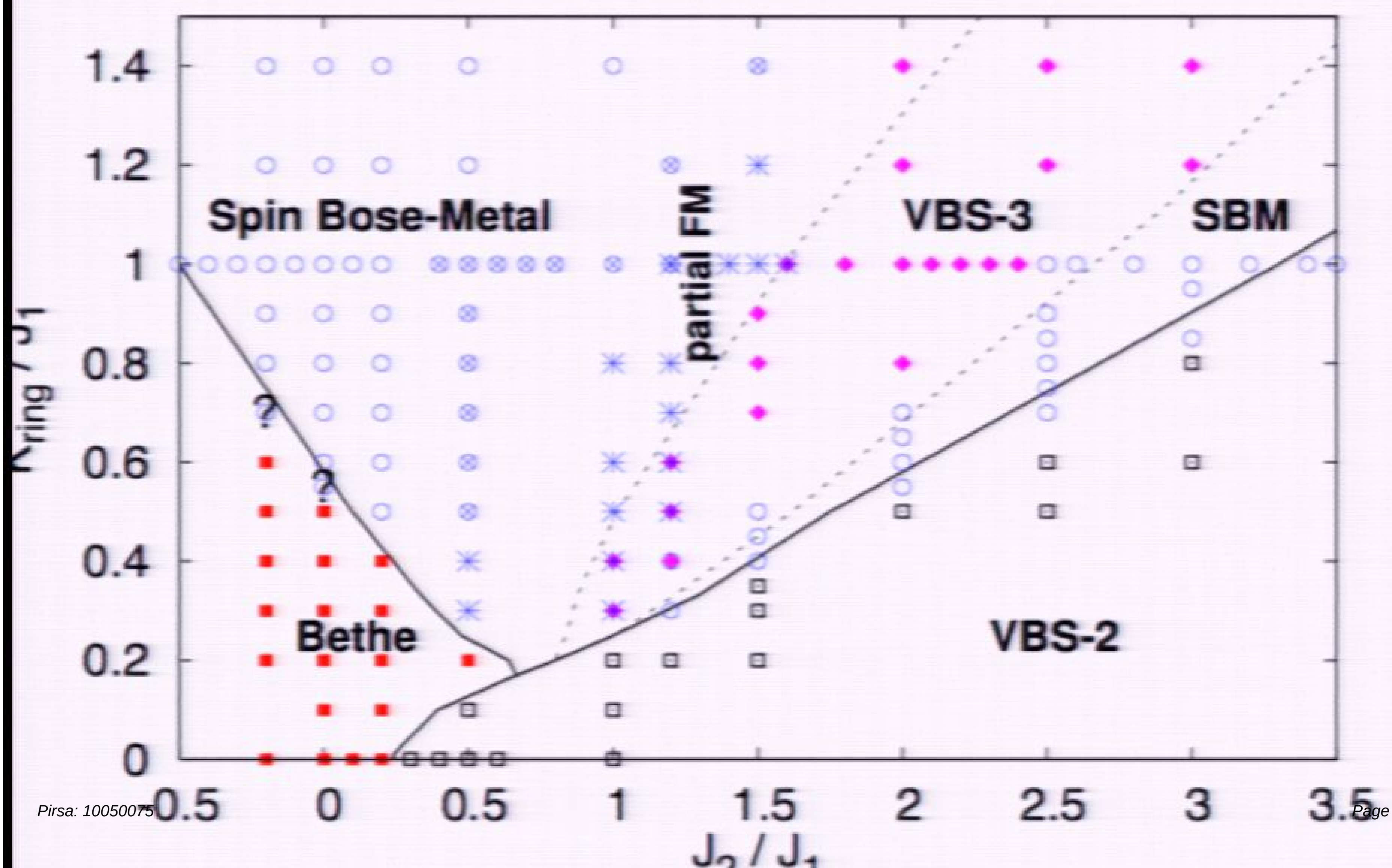

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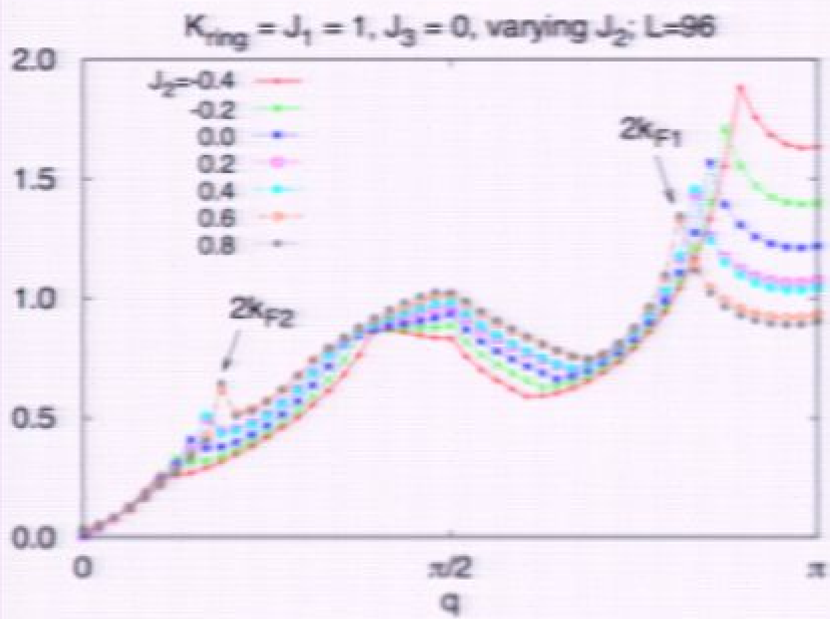
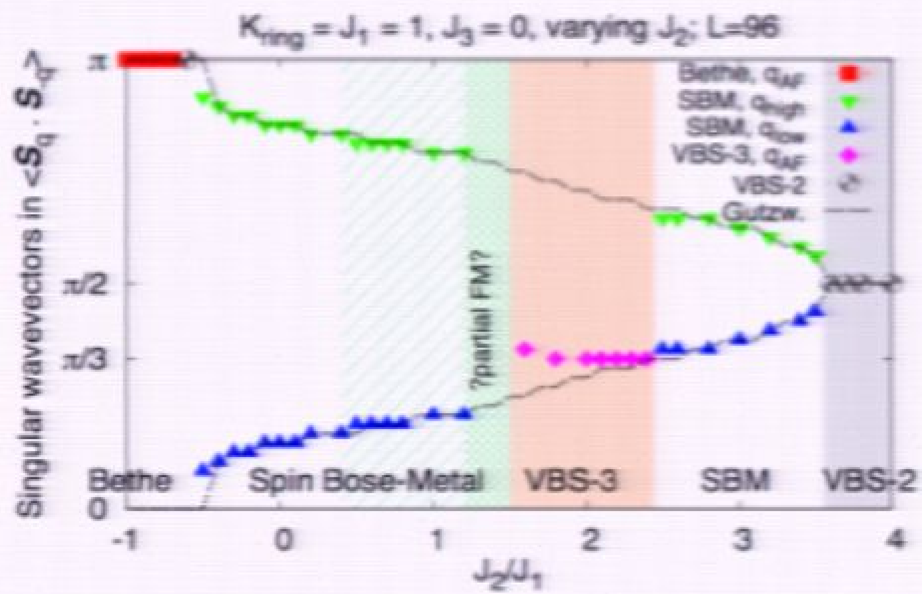


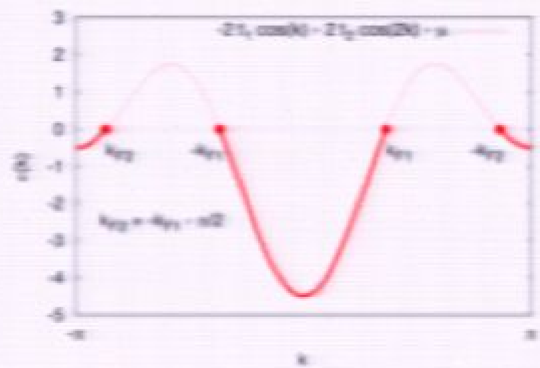



# Entanglement in SBM? Quasi-1d Gauge Theory

ize about two
points,
ize and integrate
uge field

$$f_\alpha(x) = \sum_{a,P} e^{iPk_{Fa}x} f_{Pa\alpha}$$

$$f_{Pa\alpha} \sim e^{i(\varphi_{a\alpha} + P\theta_{a\alpha})}$$



***ixed-point" theory of zigzag Spin Bose-Metal***

$$\mathcal{L}_{s\ell} = \mathcal{L}_\sigma + \mathcal{L}_\chi$$

Two gapless spin modes

$$\mathcal{L}_\sigma = \frac{1}{2\pi} \sum_{a=1,2} \left[ \frac{1}{v_a} (\partial_\tau \theta_{a\sigma})^2 + v_a (\partial_x \theta_{a\sigma})^2 \right]$$

Gapless spin-chirality mode

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$$\chi = \vec{S}_{x-1} \cdot [\vec{S}_x \times \vec{S}_{x+1}] \qquad \chi \sim \partial_x \varphi_\chi$$

mergent global symmetries: SU(2)xSU(2) and U(1) Spin chirality

***3 Gapless Boson modes – central charge c=3***

# Measure c=3 with DMRG? Entanglement Entropy

$$S(X, L) = \frac{c}{3} \log\left(\frac{L}{\pi} \sin\frac{\pi X}{L}\right) + A$$

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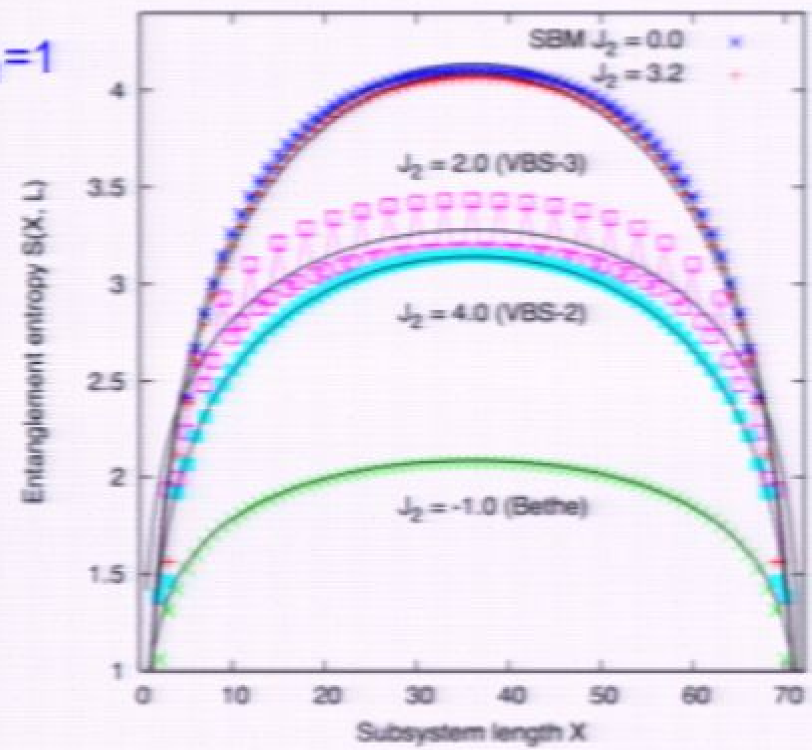
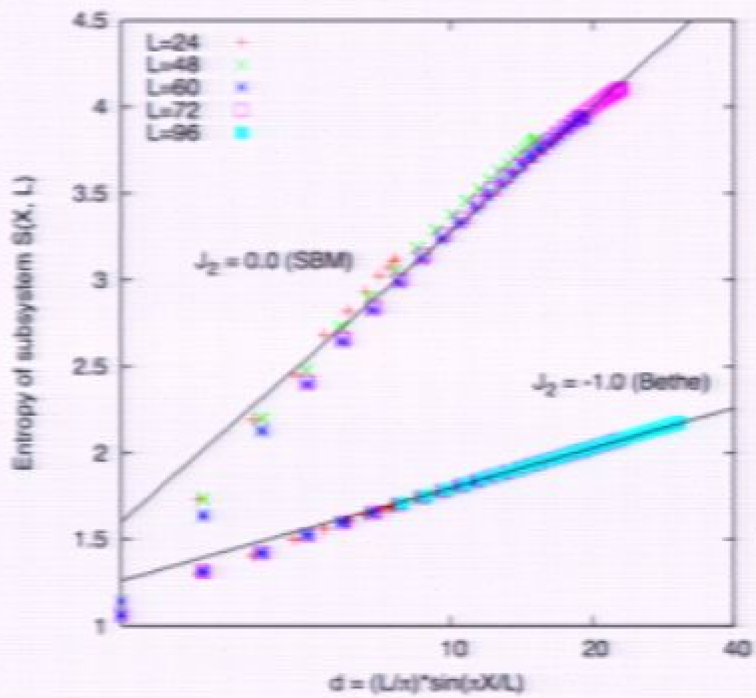
(VBS-2 c~2; VBS-3 c~1.5)

$J_1$=1






Entanglement entropy in SBM consistent with c=3 for 3 gapless Boson modes!

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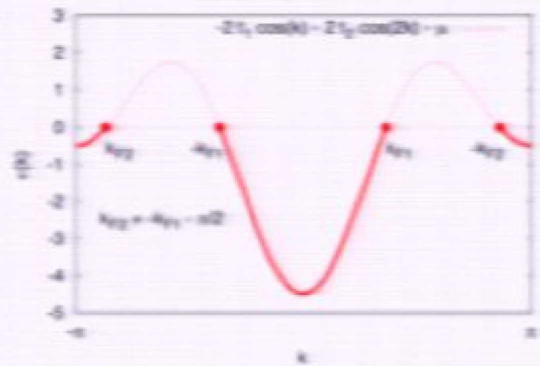

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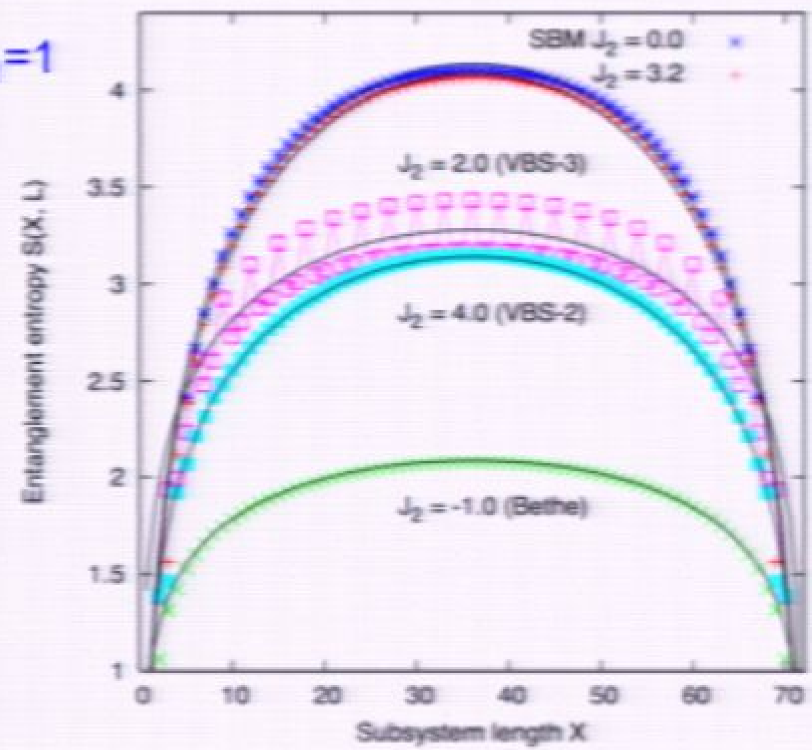


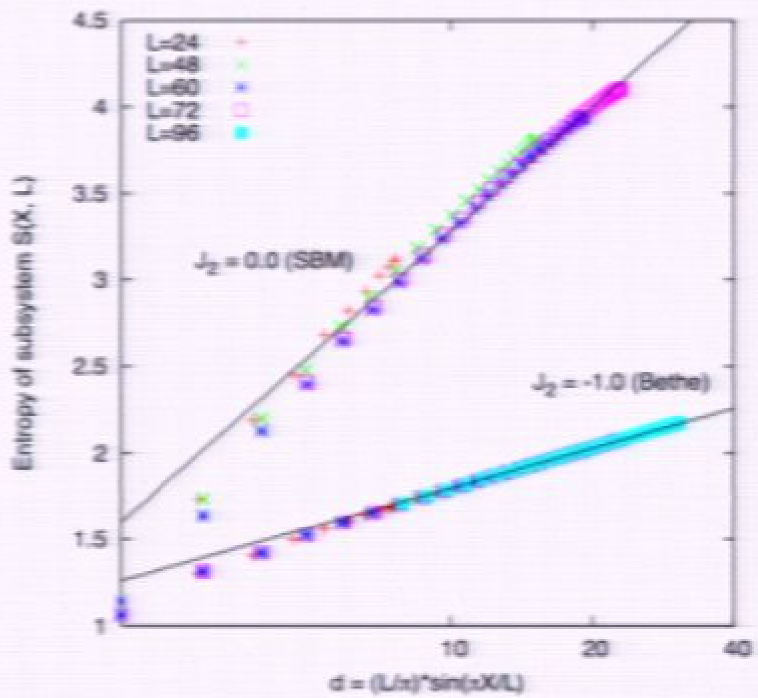





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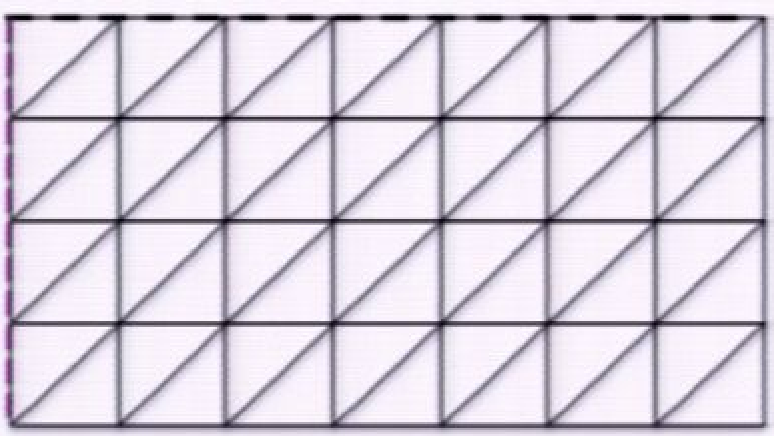
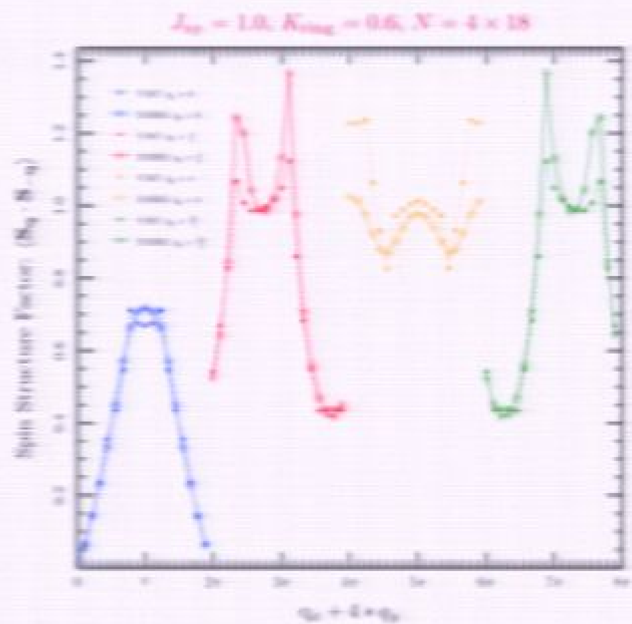
# Phase Diagram (preliminary); 4-leg Triangular Ladder



Singlets along the “rungs” for K=0

Spin structure factor shows singularities consistent with a Spin-Bose-Metal, (ie. 3-band Spinon-Fermi-Sea wavefunction) with 5 gapless modes, ie. c=5

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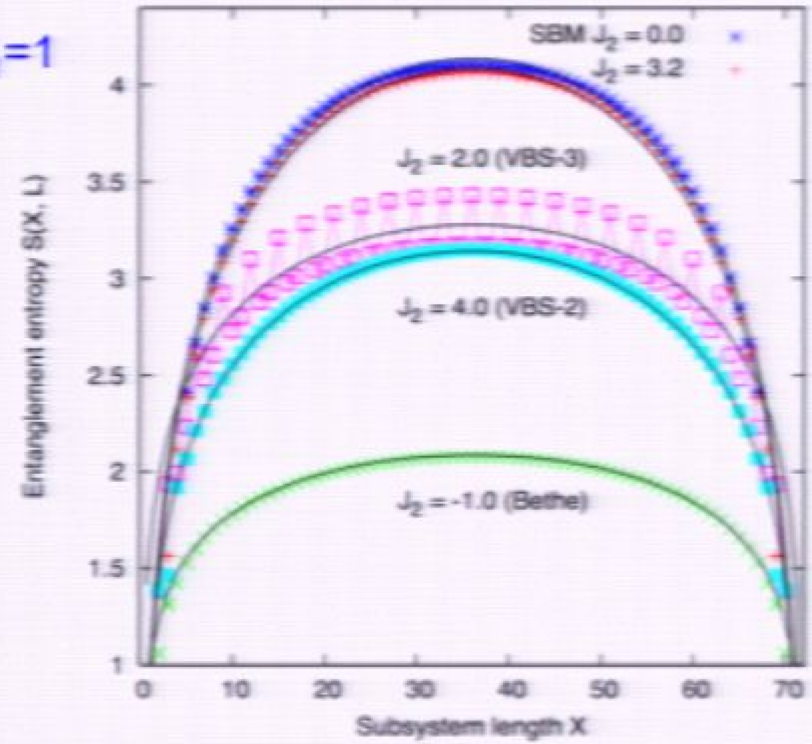


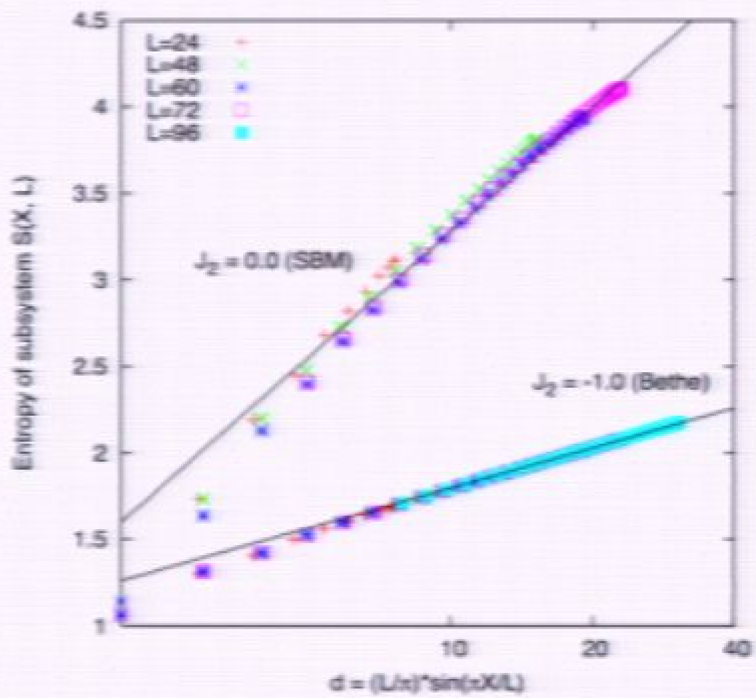




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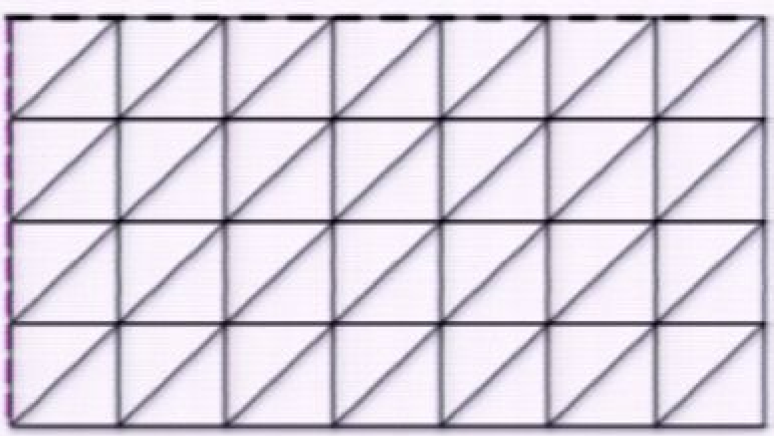

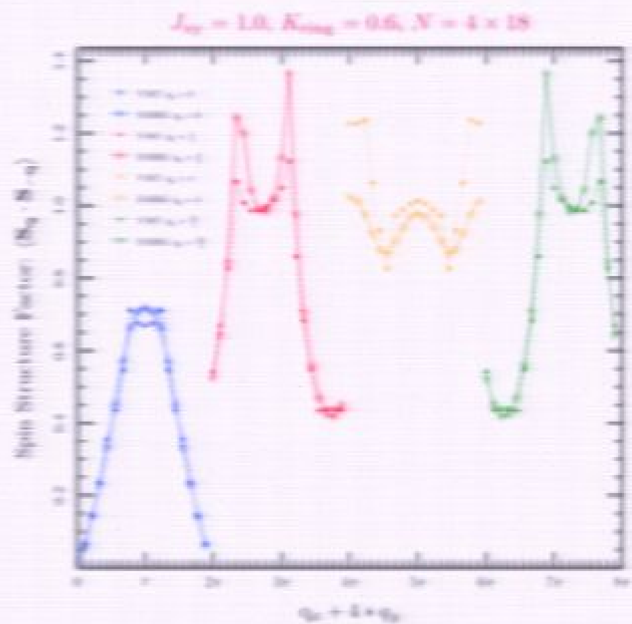

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# Entanglement Entropy for SBM on N-leg ladder

For length L segment on N-leg ladder expect

$$S_N = \frac{c_N}{3}\log(L/a) + A \qquad\qquad c_N \sim N$$

For 2d Spin Bose-Metal expectation is that L by L region has entanglement entropy

$$S_{2d}(L) \sim L\log(L/a)$$

***2d Spin Bose-Metal as entangled as a 2d Fermi liquid***

# Summary on 2d Spin liquids

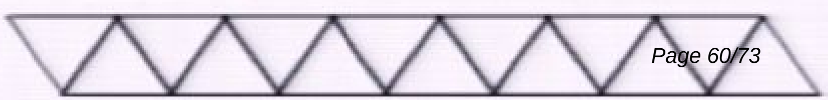
- ***Spin liquids*** - Mott insulators with no broken symmetries
- Two classes of spin liquids - ***Topological and Gapless***
- ***Gapless spin liquids are stable quantum phases with emergent symmetries***
  (lattice scale physics manifest in the IR)
- ***Algebraic spin liquids*** can have large global emergent symmetries, eg SU(4) x U(1) flux
- ***Spin Bose-Metals*** have ***Singular Bose surfaces*** in momentum space

## Challenge:

2d Gapless spin liquids are highly entangled states
with no free particle description,

New non-perturbative approaches are needed –
tensor product states useful??

Quasi-1d offers one route to Spin Bose-Metals



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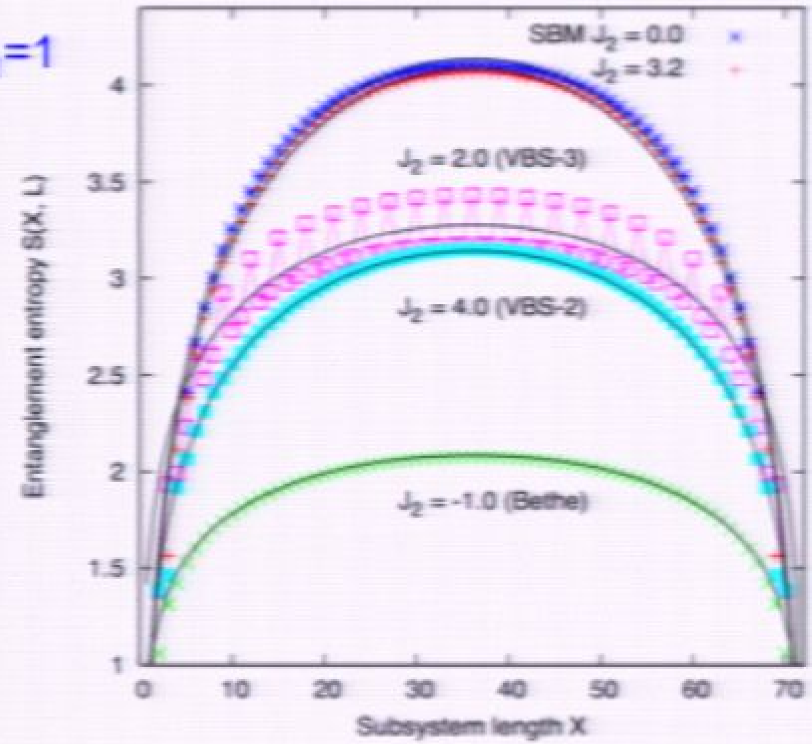


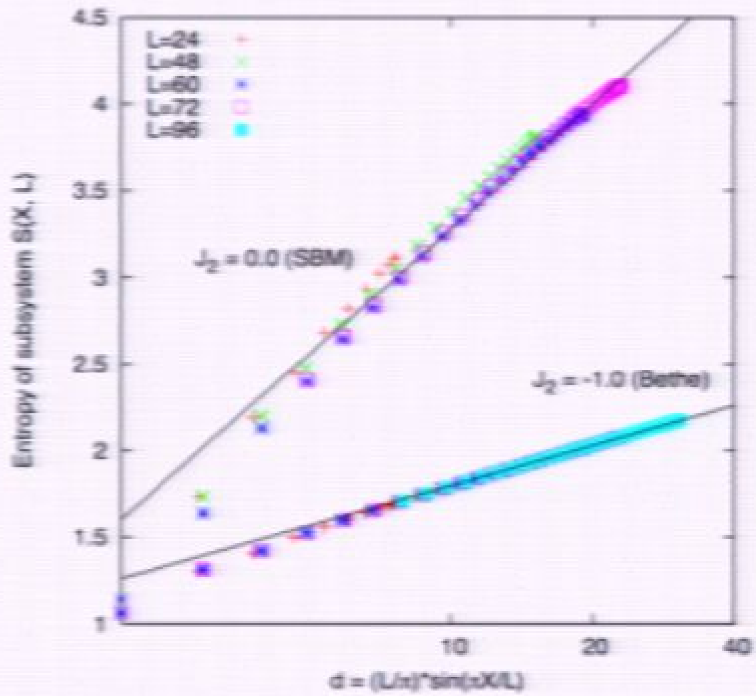




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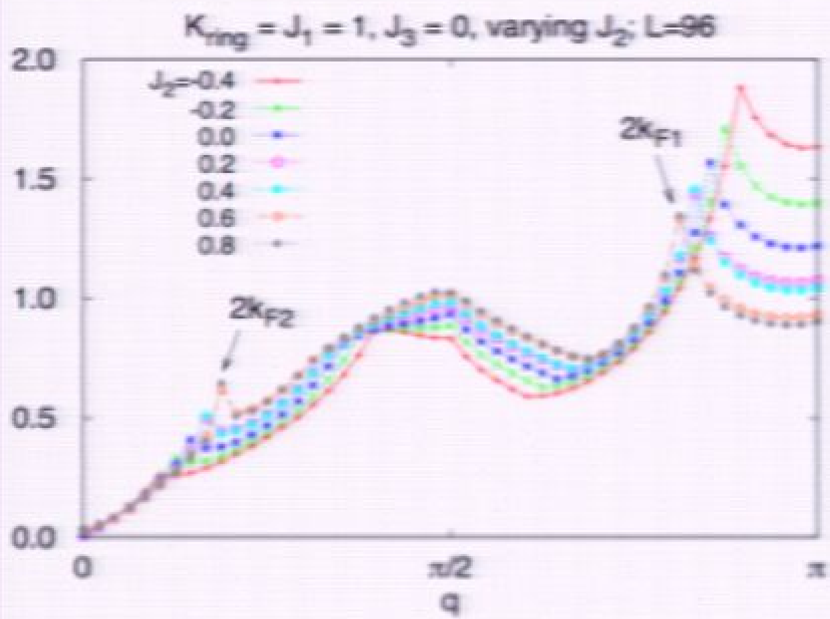


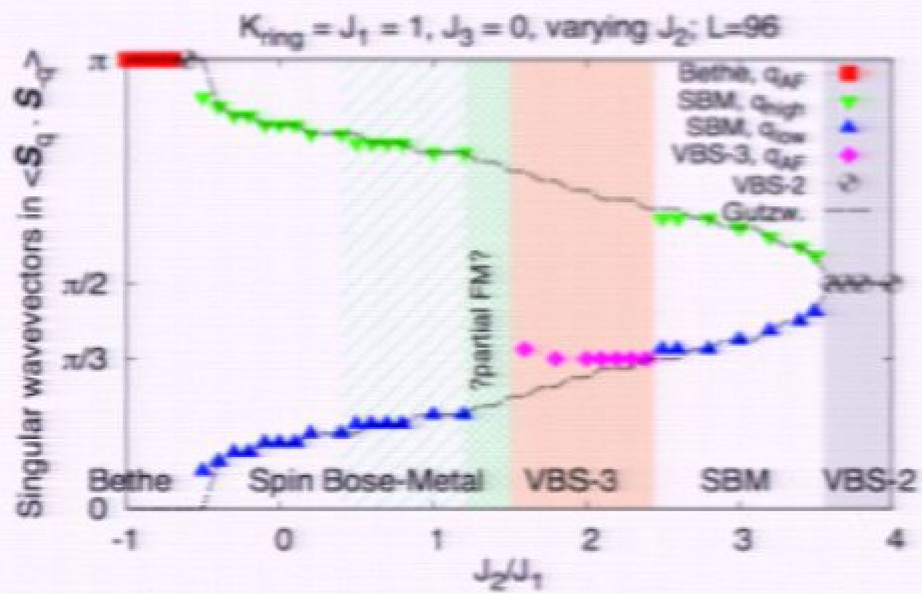

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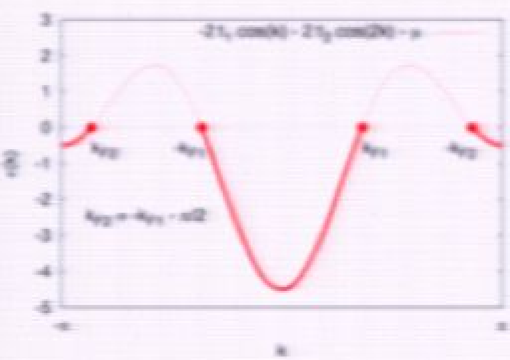
utzwiller wf; 2 Fermi sea's



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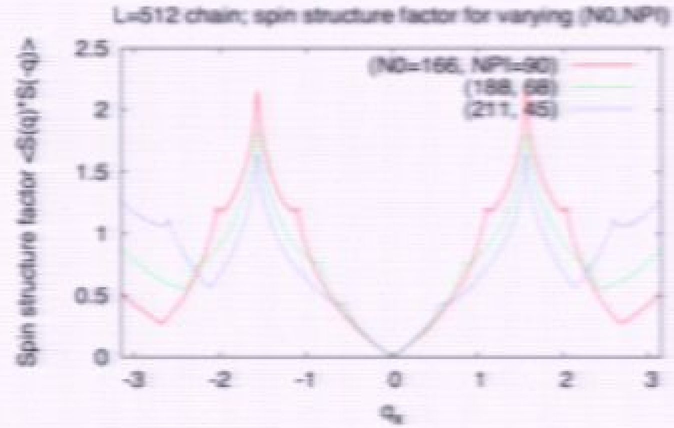


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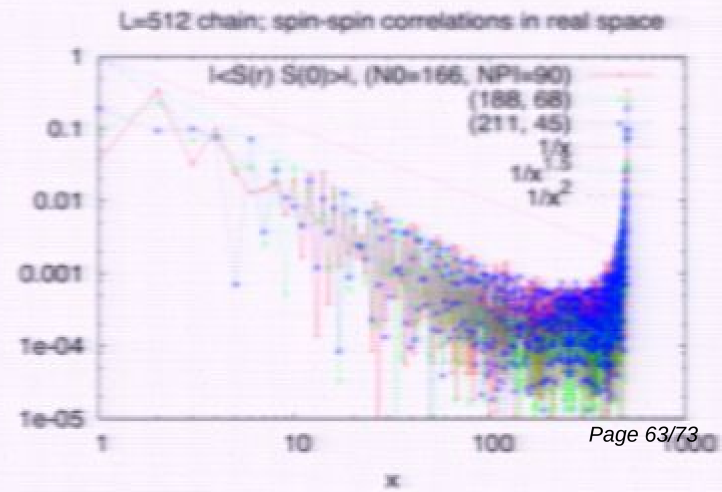
$$\alpha \approx 3/2 = \alpha_{SU(4)}$$

$$\alpha_{SU(N)} = 2 - (2/N)$$

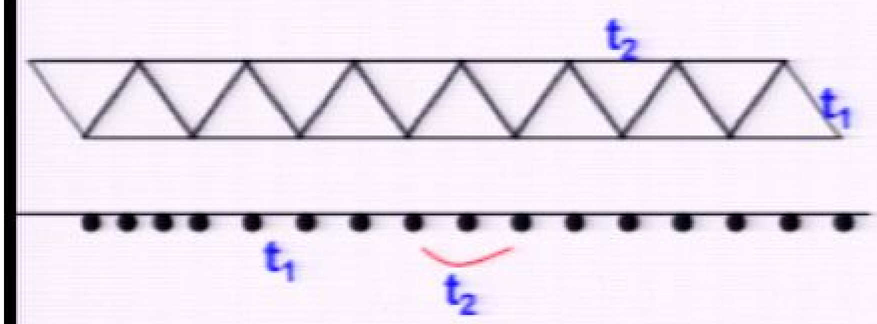



Gutzwiller wf doesn't "know" about 2 different spinon velocities

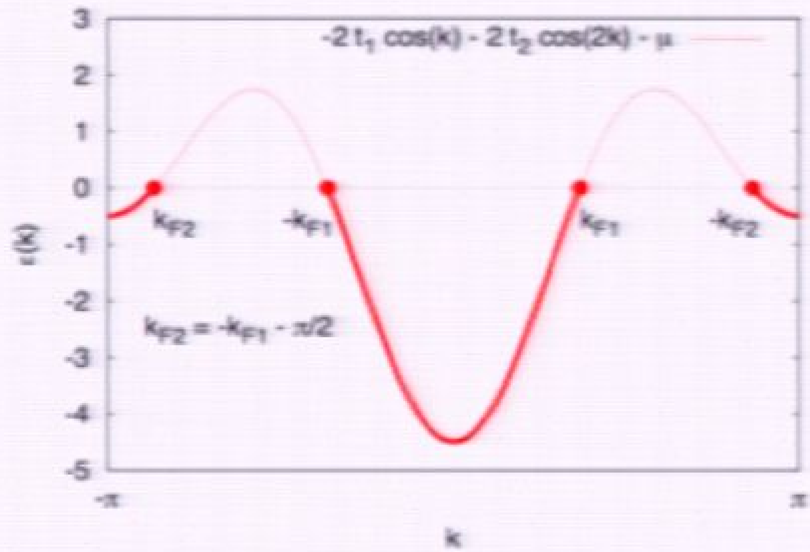
Analytic progress possible?? (Schur polynomials? Matrix product states?)

( [illegible]e-Shastry SU(2) chain with exact Gutzwiller Fermi sea ground state)

# Gutzwiller Wavefunction on zigzag




$$\mathcal{H}_{\rm trial} = -\sum_{ij} t_{ij} f^\dagger_{i\alpha} f_{j\alpha}$$




→ $|\Psi_0\rangle$ → $|\Psi_{\rm spin}\rangle = P_G(|\Psi_0\rangle)$

fermion determinant — spins — Gutzwiller projection

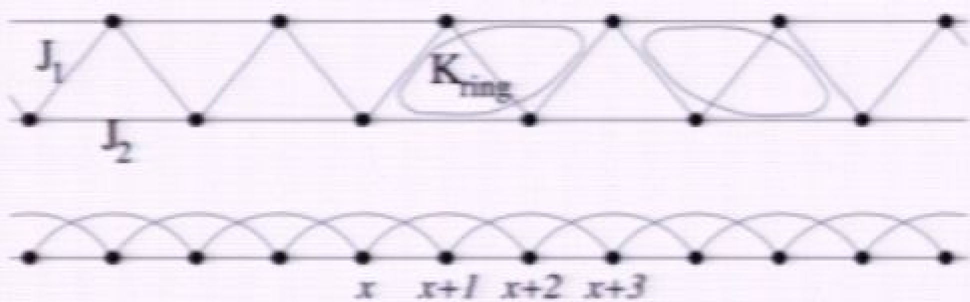
Spinon band structure

***Single Variational parameter:*** $t_2/t_1$ or $k_{F2}$

$(k_{F1}+k_{F2} = pi/2)$

# 2-leg zigzag strip

$$\mathcal{H}_\Delta = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle 1234 \rangle} [\mathcal{P}_{1234} + \mathcal{P}_{1234}^{-1}]$$




Analysis of $J_1$-$J_2$-K model on zigzag strip

D. Sheng, O. Motrunich, MPAF
PRB (2009)

Variational Monte Carlo of Gutzwiller wavefunctions
DMRG
Bosonization of gauge theory

# Is projected Fermi sea a good caricature of Triangular ring model ground state?

$$\hat{H}_{\text{ring}} = J_2 \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + J_4 \sum_{\text{rhombi}} (P_{ijkl} + \text{h.c.})$$

Variational Monte Carlo analysis suggests it might be for $J_4/J_2 > 0.3$
(O. Motrunich - 2005)

***A theoretical quandary***: Triangular ring model is intractable

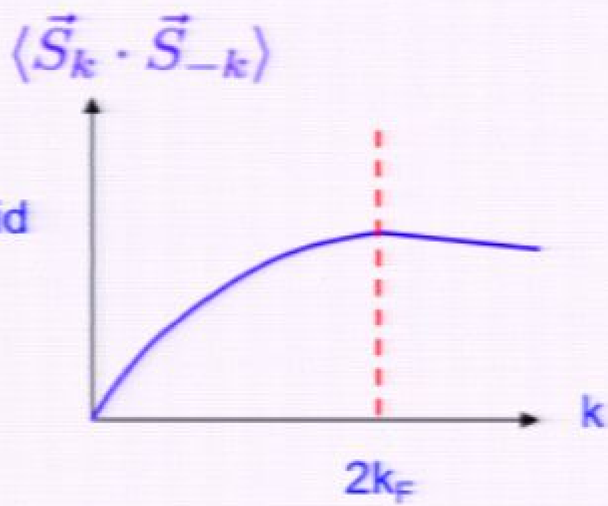
- Exact diagonalization: so small,
- Quantum Monte Carlo - sign problem
- Variational Monte Carlo - biased
- DMRG - problematic in 2d

?????

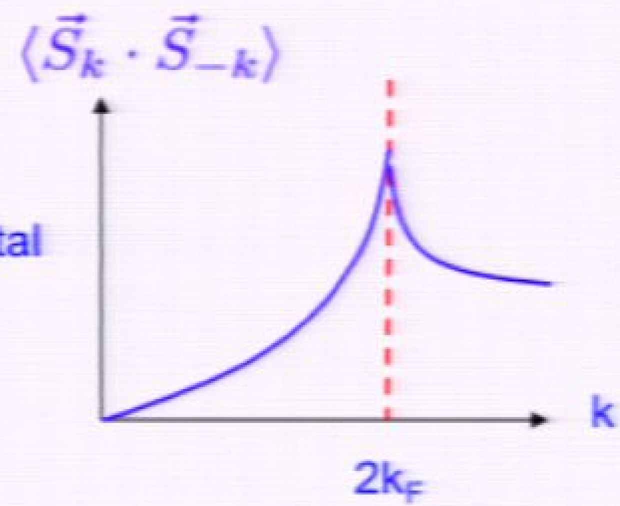
# Phenomenology of Spin Bose-Metal (from Gutzwiller wf and Gauge theory)

ingular spin structure factor at "$2k_F$" in Spin Bose-Metal
more singular than in Fermi liquid metal)

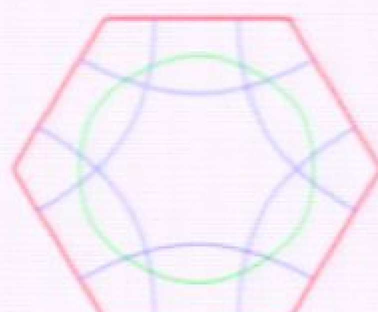
mi liquid




Spin Bose-metal



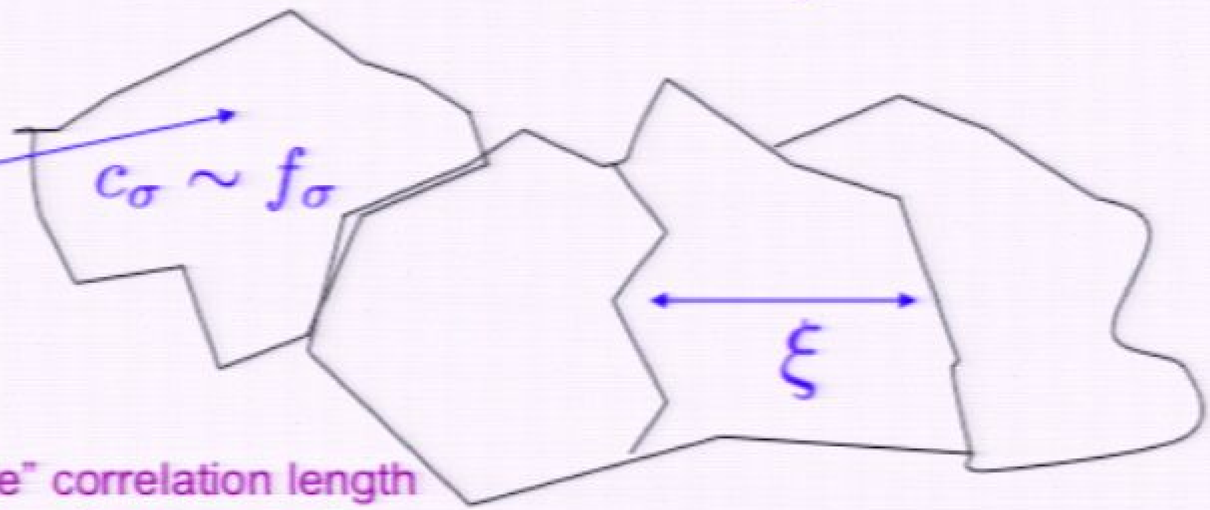

$2k_F$ "Bose surface" in
triangular lattice Spin Bose-Metal



# Weak Mott insulator: Which spin liquid?

Motrunich (2005)

Long charge correlation length, $\langle c_\sigma(x) c_\sigma^\dagger(0) \rangle \sim e^{-x/\xi}$ $\xi >> a$

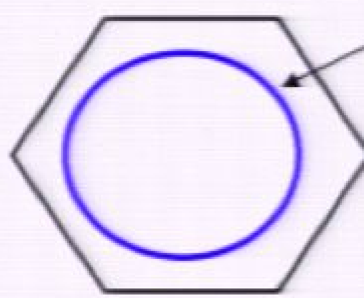



de correlation region
trons do not "know"
are insulating

ess: ***Spin*** correlations "inside" correlation length
semble" spin correlations of free fermion metal,
cillating at $2k_F$

$$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \cos(2k_F x)/x^\alpha$$

propriate spin liquid:



Spinon fermi surface is not physical in the spin model

***utzwiller projected Fermi sea***
***"Spin Bose-Metal")***

# Is projected Fermi sea a good caricature of Triangular ring model ground state?

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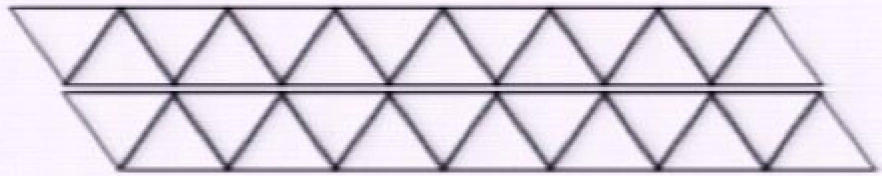
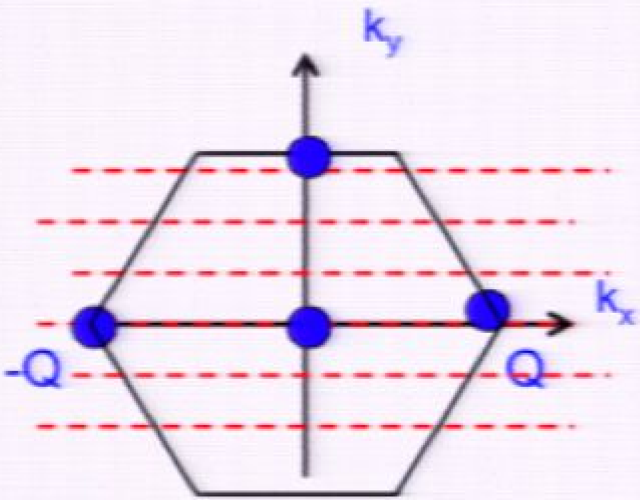
?????

# Quasi-1d route to Spin Bose-Metal

Triangular strips:

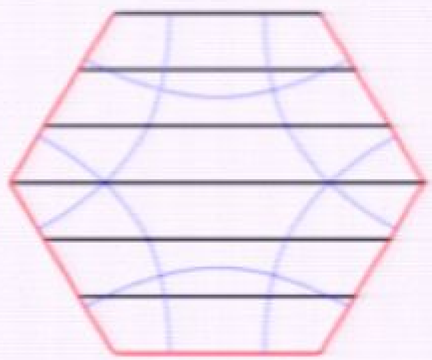


Algebraic Spin liquid




Few gapless 1d modes

Spin Bose-Metal

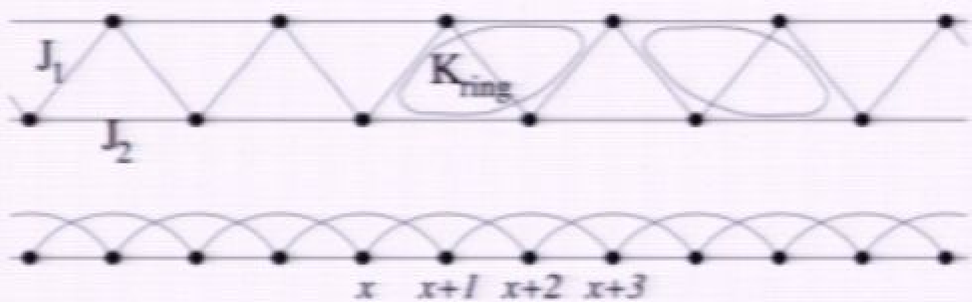


Fingerprint of 2d singular surface - many gapless 1d modes, of order N

***New spin liquid phases on quasi-1d strips, each a descendent of a 2d Spin Bose-Metal***

# 2-leg zigzag strip

$$\mathcal{H}_\Delta = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle 1234 \rangle} [\mathcal{P}_{1234} + \mathcal{P}_{1234}^{-1}]$$

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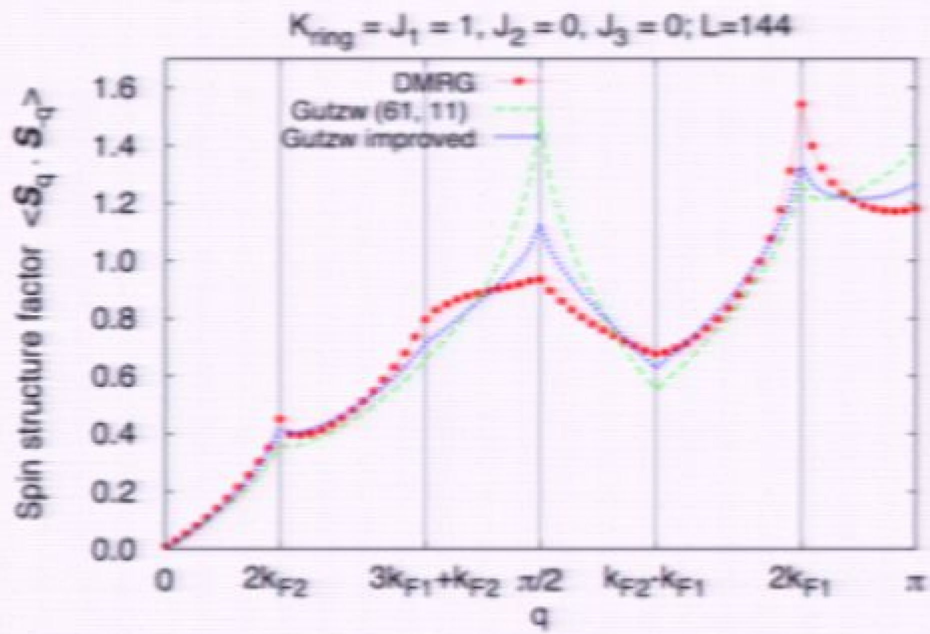
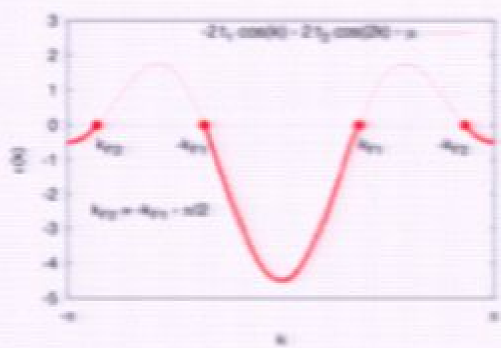
# Spin Bose-Metal: Spin Structure Factor

***Singularities in momentum space locate the "Bose" surface (points in 1d)***

$\langle \vec{S}_k \cdot \vec{S}_{-k} \rangle$




Gutzwiller improved has 2 variational parameters)



***ngular momenta can be identified with $2k_{F1}$, $2k_{F2}$***

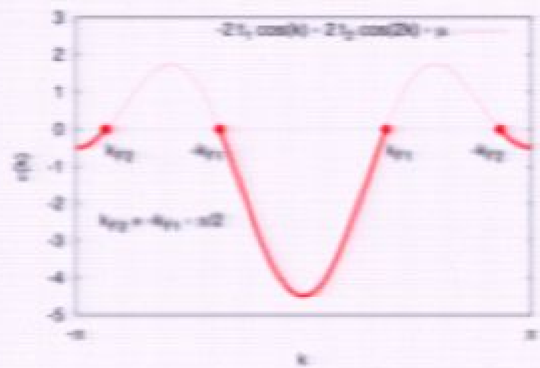
***hich enter into Gutzwiller wavefunction!***

# Entanglement in SBM?
# Quasi-1d Gauge Theory

ize about two
points,
ize and integrate
uge field

$$f_\alpha(x) = \sum_{a,P} e^{iPk_{Fa}x} f_{Pa\alpha}$$

$$f_{Pa\alpha} \sim e^{i(\varphi_{a\alpha}+P\theta_{a\alpha})}$$



***ixed-point" theory of zigzag Spin Bose-Metal***

$$\mathcal{L}_{s\ell} = \mathcal{L}_\sigma + \mathcal{L}_\chi$$

Two gapless spin modes

$$\mathcal{L}_\sigma = \frac{1}{2\pi}\sum_{a=1,2}\left[\frac{1}{v_a}(\partial_\tau\theta_{a\sigma})^2 + v_a(\partial_x\theta_{a\sigma})^2\right]$$

Gapless spin-chirality mode

$$\mathcal{L}_\chi = \frac{1}{2\pi g}\left[\frac{1}{v}(\partial_\tau\theta_\chi)^2 + v(\partial_x\theta_\chi)^2\right]$$

$$\chi = \vec{S}_{x-1}\cdot[\vec{S}_x \times \vec{S}_{x+1}] \qquad \chi \sim \partial_x\varphi_\chi$$

mergent global symmetries: SU(2)xSU(2) and U(1) Spin chirality

***3 Gapless Boson modes – central charge c=3***