Title: Entanglement and Fluctuations in Many-Body Quantum Systems

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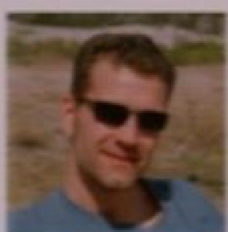
Abstract: Many one dimensional random quantum systems exhibit infinite randomness phases, such as the random singlet phase of the spin-1/2 Heisenberg model. These phases are typically the result of destabilizing systems described by a conformal field theory with disorder. Interestingly, entanglement entropy in 1d infinite randomness phases also exhibits a universal log scaling with length. In my talk I will touch upon calculating the entanglement entropy for inifinite-randomness phases, as well as describe the exotic infinite randomness phases realized in chains of non-abelian anyon chains. It was speculated that the entanglement entropy of an infinite-randomness phase is associated with the direction of RG flow, just as the c-theorem dictates the direction of RG flows for CFT's. I will also show that the entanglement entropy in disordered non-abelian chains provide the only known counter example.

Entanglement & (number particle/spin) Fluctuations

arXiv:1002.0825

Francis Song — Yale University



Stephan Rachel — Yale University

Nicolas Laflorencie — LPS Orsay, Paris

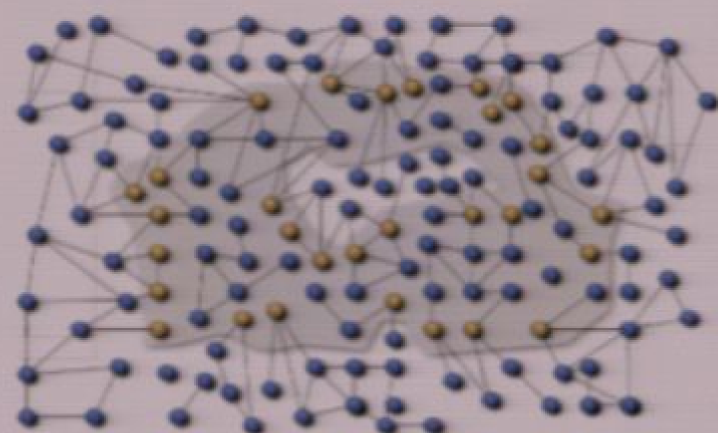
classical:
entropy quantifies uncertainty (lack of information)

quantum mechanics:
entropy > 0 without an objective lack of information

non-degenerate
pure ground state

$$\rho_0 = |\psi\rangle\langle\psi|$$

$$\Rightarrow S(\rho_0) = 0$$



(Eisert *et al.*, RMP, 2010)

shaded region A
remainder B

$$\rho_A = \mathrm{tr}_B(\rho)$$

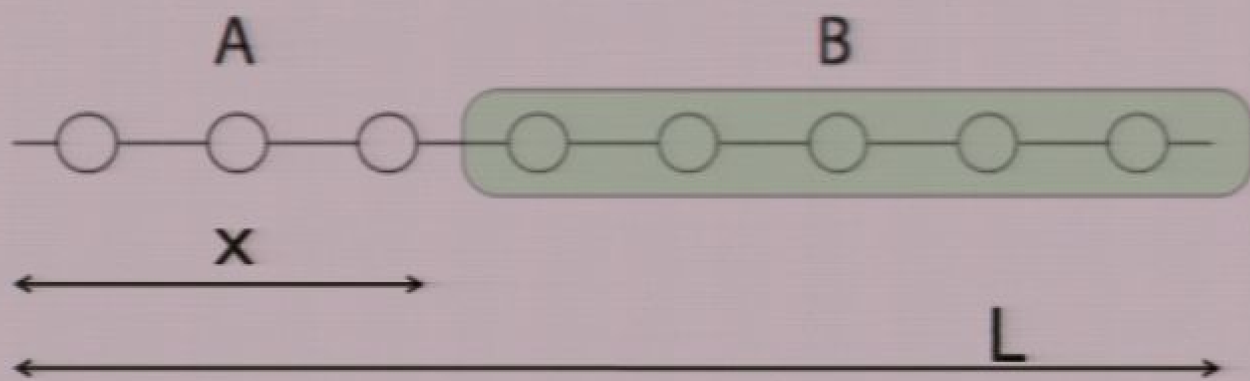
$$\Rightarrow S(\rho_A) \neq 0$$

Entanglement
Entropy

von Neumann entropy

$$S(\rho) = -\mathrm{tr}(\rho \log_2 \rho)$$

entanglement entropy in one dimension



Given a pure state $|\psi\rangle$, the reduced density matrix is

$$\hat{\rho}_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|.$$

Entanglement entropy (von Neumann entropy):

$$\mathcal{S}_A = -\mathrm{Tr}\ \hat{\rho}_A \log \hat{\rho}_A.$$

Definition is extremely general.

entanglement entropy

Conformal field theory (CFT)

$$\mathcal{S}(x) = \frac{c}{3} \ln \frac{x}{a}, \qquad c = \text{central charge.}$$

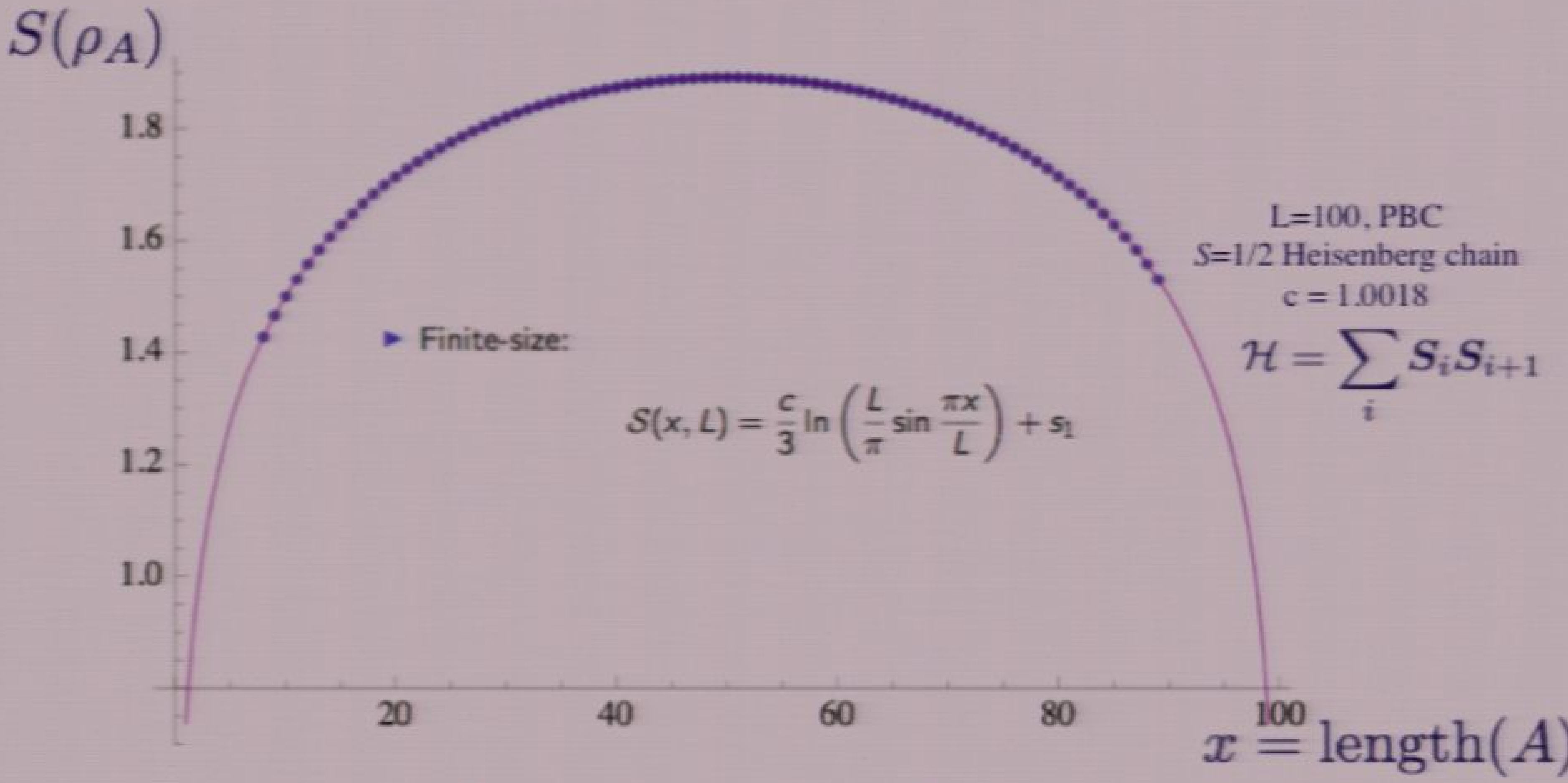
- Finite-size:

$$\mathcal{S}(x, L) = \frac{c}{3} \ln \left(\frac{L}{\pi} \sin \frac{\pi x}{L} \right) + s_1.$$

- Similar expression for finite temperature, free boundaries.

Are there observables that behave similarly?
Yes. Suggestions: QPC with free fermion (Klich & Levitov), fractional quantum Hall (Hsu et al.) wires.

entanglement entropy



L=100, PBC

S=1/2 Heisenberg chain

c = 1.0018

$$\mathcal{H} = \sum_i \boldsymbol{S}_i \boldsymbol{S}_{i+1}$$

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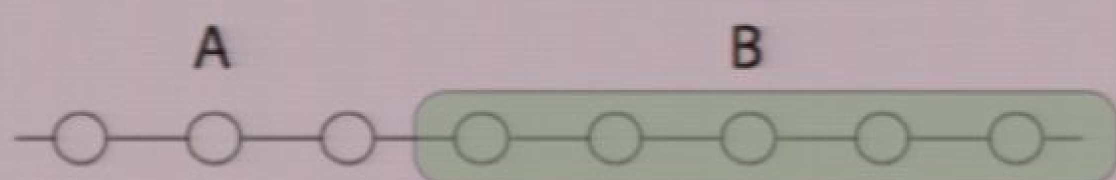
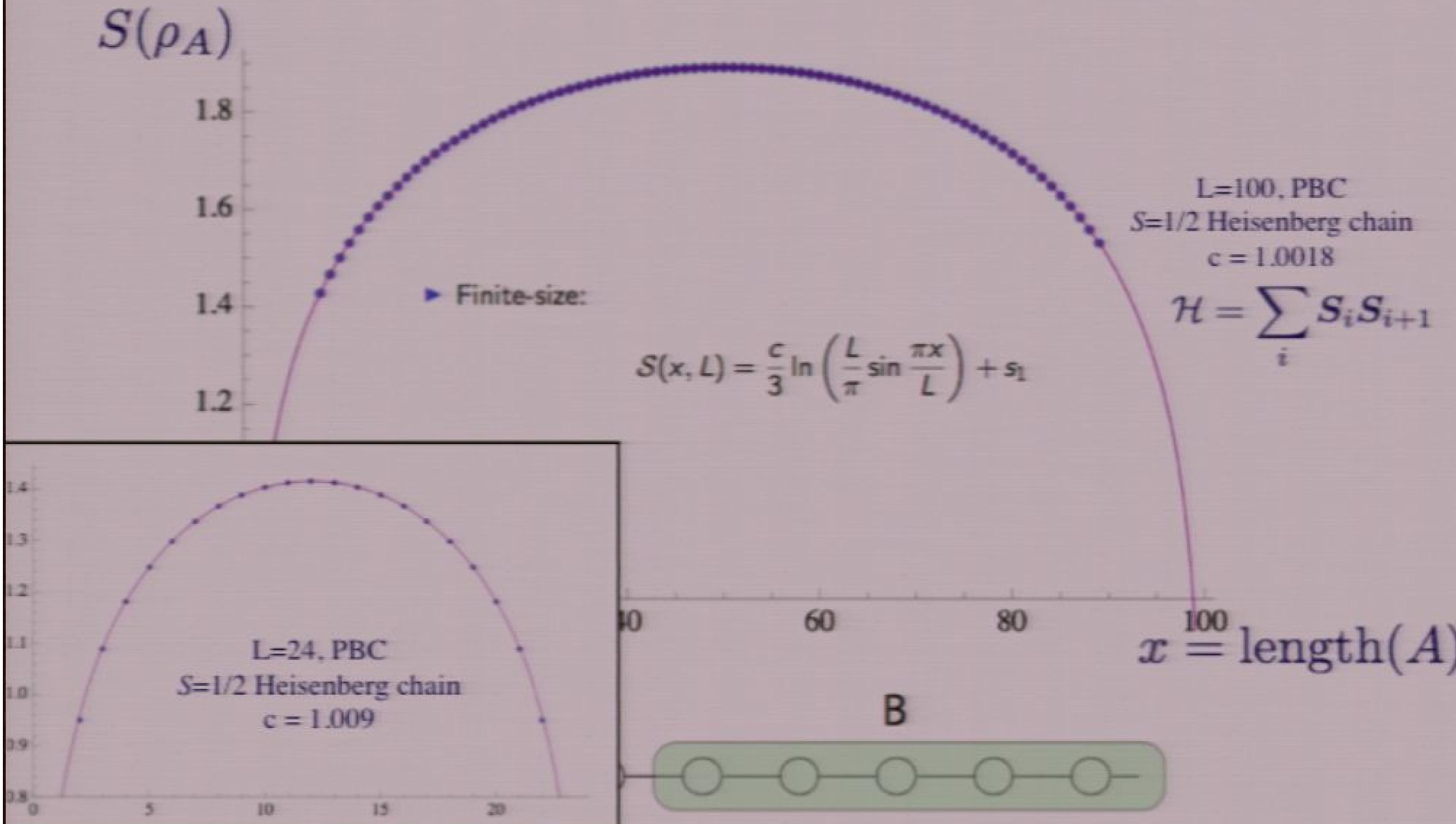


entanglement entropy



entanglement entropy

Conformal field theory (CFT) (Calabrese & Cardy):

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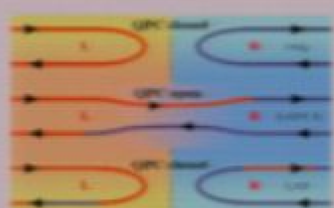

QPC, free fermions: Current fluctuations are Gaussian with a log. Variance:
$C_2 = (1/\pi^2) \ln|t_1 - t_0|$ & $S \sim 1/3. \ln|t_1 - t_0|$

Definitions

$$\hat{N}_A \rightarrow \hat{S}_z = \sum_{i \in A} \hat{S}_z^i$$

Number (spin) fluctuations:

$$\mathcal{F}_A = \langle (\hat{N}_A - \langle \hat{N}_A \rangle)^2 \rangle.$$

- Symmetric (cf. entanglement entropy):

$$\mathcal{F}_A = \mathcal{F}_B.$$

- Zero for a product state

$$\mathcal{F}_A = 0 \text{ if } |\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle.$$

- Luttinger liquids:

$$\pi^2 \mathcal{F}(x) = \frac{1}{\pi^2} \langle [\phi(x) - \phi(0)]^2 \rangle \sim K \ln \frac{x}{a}.$$

=1 free fermions (Klich-Levitov)
=v for fractional QH edges
(Fradkin et al)

See also Klich, Silva & Refael

Outline:

-Fluctuations in one dimensional CFTs: scales as Entropy
More general: open boundary, sub-leading corrections
- Gapped systems
- But Counter-examples
- Fluctuations vs. Entanglement entropy in higher d:
area law or not?

- Klich-Levitov formula for free fermions relating entropy to the full set of cumulants of charge statistics
(in collaboration with H. F. Song, C. Flindt & I. Klich; work in progress)

Conserved U(1) current **(fixed total number of particles or spin)**

U(1) currents are described by a massless free boson

CFT + U(1)

- Generating function:

$$M_A(\lambda) = \langle e^{i\lambda(\hat{N}_A - \langle \hat{N}_A \rangle)} \rangle = \left(\frac{x}{a}\right)^{-g\lambda^2/(2\pi^2)}.$$

$$\pi^2 \mathcal{F}_A = -\pi^2 M_A''(0) = g \ln \frac{x}{a}.$$

- g fixed heuristically by comparing to finite-T:

$$\pi^2 \mathcal{F}(x, \beta) = g \ln \left(\frac{\beta}{\pi a} \sinh \frac{\pi x}{\beta} \right).$$

Compare to $\mathcal{F}(x \gg \beta, \beta) \sim \kappa x / \beta$:

$$g = \pi v \kappa.$$

High T: sub-system A in grand canonical ensemble in equilibrium with a thermal bath

Relation between Entanglement Entropy and Fluctuations

$$\frac{\mathcal{S}(x)}{\pi^2 \mathcal{F}(x)} \sim \frac{c}{3\pi v \kappa}, \qquad x \gg a.$$

Simple interpretation for previous results.

$\pi v \kappa = K$ for Luttinger theories

Also Valid for gapless spin chains…

Sub-leading corrections & boundaries?

For spin systems, the compressibility turns

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XX Model – Periodic Boundary Conditions

- Hamiltonian:

$$H_{XX} = \sum_i (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y).$$

Free fermions, and therefore solvable.

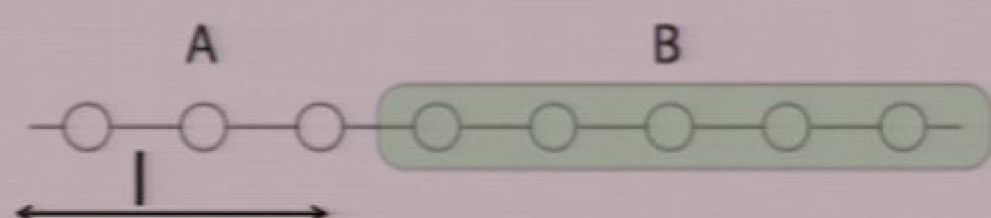
- Entanglement entropy (Jin & Korepin, 2004):

$$\mathcal{S}(\ell, L) = \frac{c}{3}\log_2 \ell + s_1, \qquad s_1 \simeq 1.047.$$

- Spin fluctuations:

$$\pi^2 \mathcal{F}(\ell, L) = \ln \ell + f_1 + O(\ell^{-2}), \qquad f_1 = 1 + \gamma + \ln 2.$$

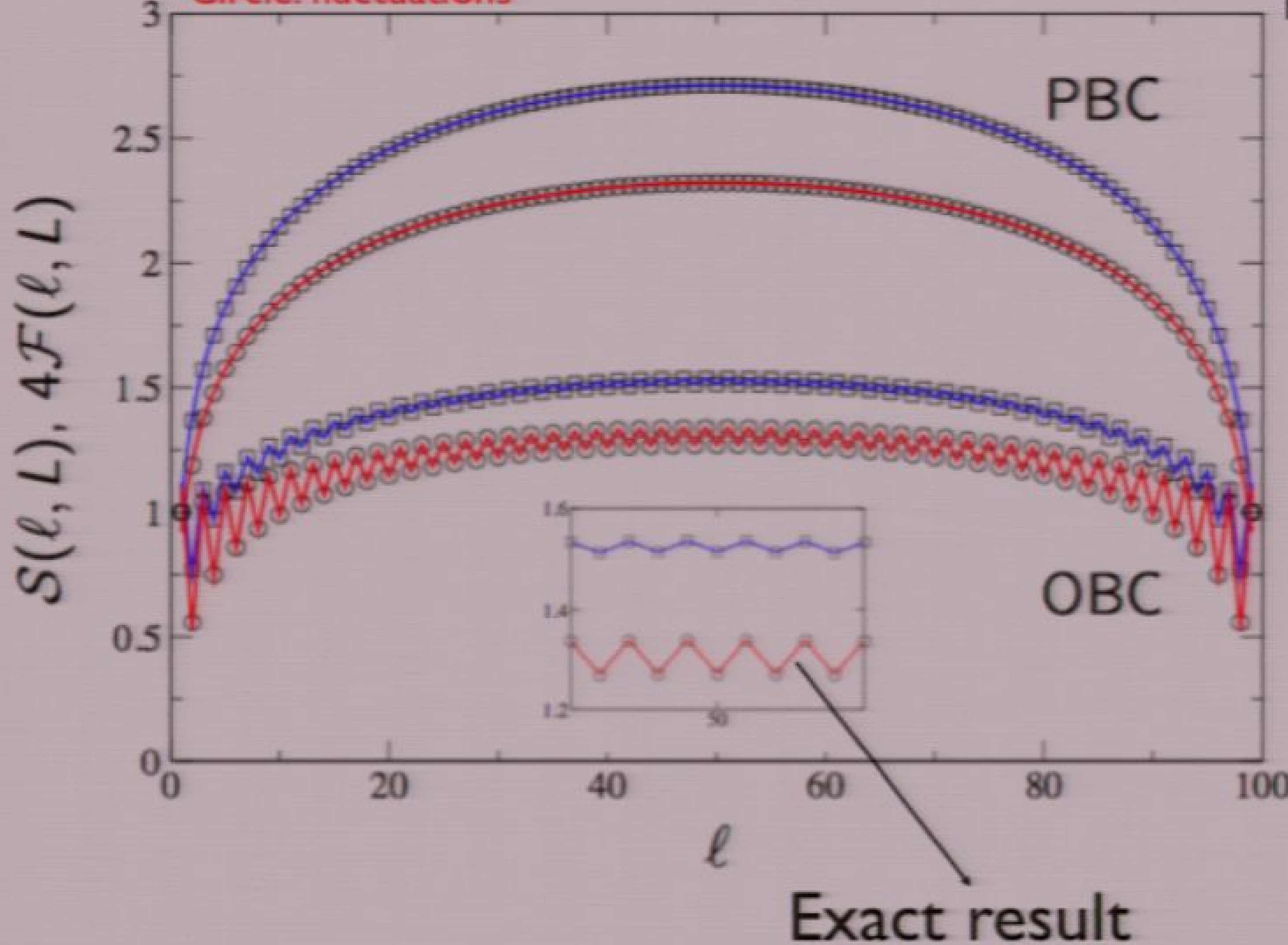
γ is Euler's constant.



XX Model

Exact diagonalization
L=100
+
Exact Results

Square: entropy
Circle: fluctuations



XX Model – Open Boundary Conditions

- Entanglement entropy

$$\mathcal{S}_{\text{OBC}}(\ell, L) = \frac{1}{2}\mathcal{S}_{\text{PBC}}(2\ell, L) + a_1 \frac{1}{(2\ell)} - a_2 \frac{(-1)^\ell}{(2\ell)}.$$

(Laflorencie, Sorensen, Chang & Affleck, PRL 2006)

- Spin fluctuations:

$$\mathcal{F}_{\text{OBC}}(\ell, L) = \frac{1}{2}\mathcal{F}_{\text{PBC}}(2\ell, L) + \frac{1}{2\pi^2}\frac{1}{(2\ell)} - \left[\frac{\ln(2\ell) + \gamma + \ln 2}{\pi^2}\right]\frac{(-1)^\ell}{(2\ell)}$$

Friedel like contribution

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L=100
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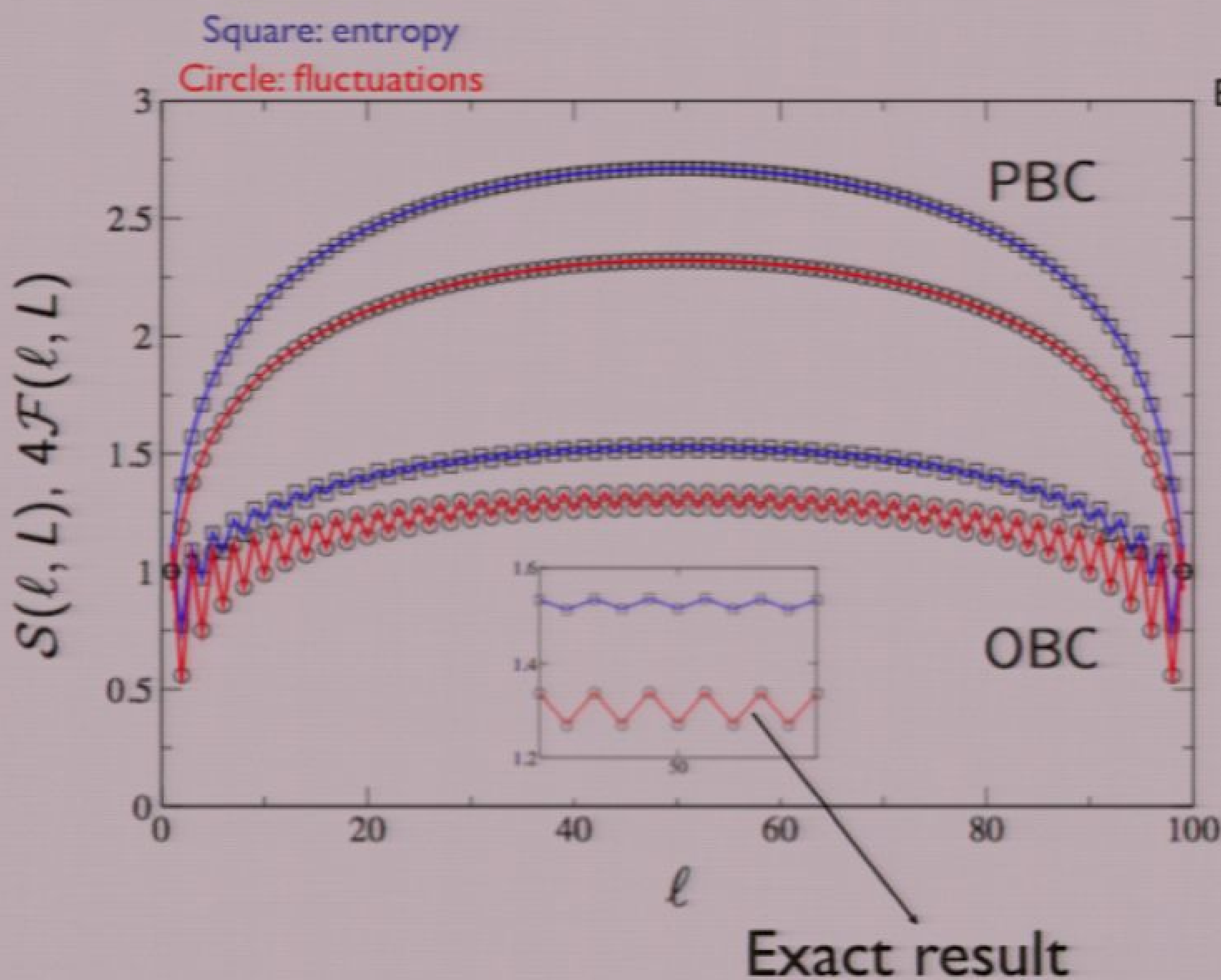

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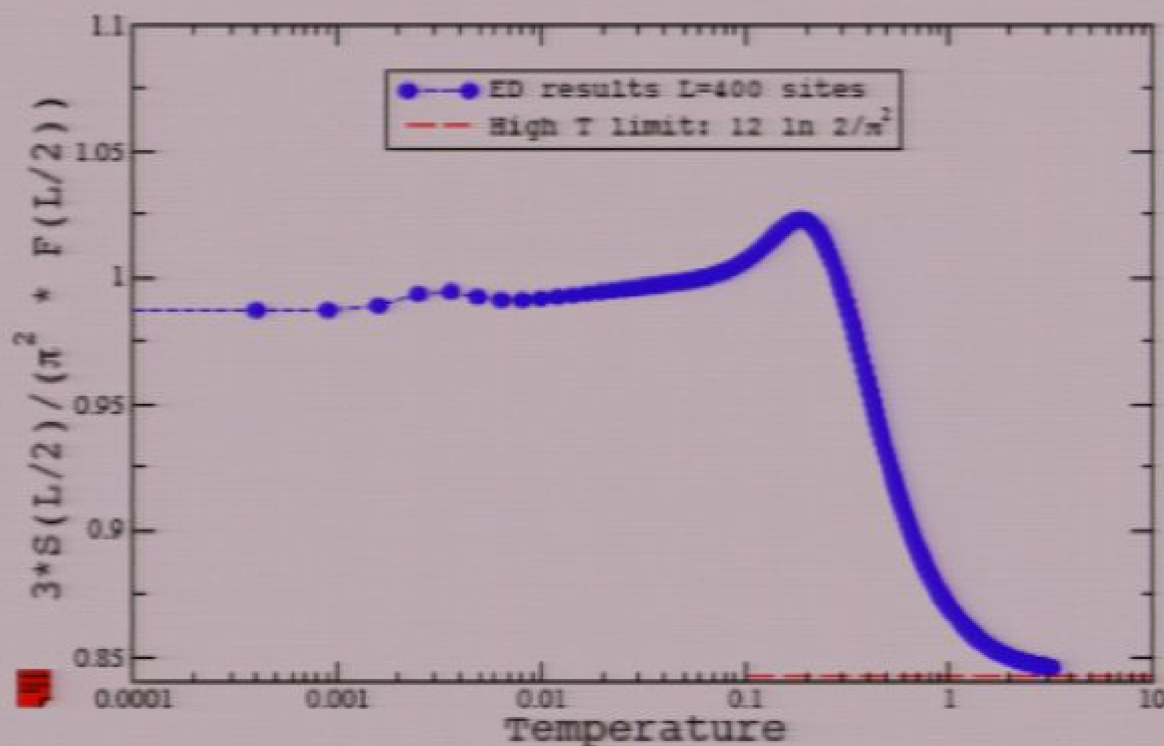
Friedel like contribution

Finite Temperature

x=l=L/2
and J=1



RSP: S=(ln2/3)Lnx (Refael-Moore)
F=(1/12)Lnx (from Hoyos et al, 2007)

High T: S and F approach L.ln 2/2 and L/8
(same ratio as **random singlet phase**)

Finite T & Exact Diagonalization for periodic

XXZ Model (Luttinger Theory)

- Hamiltonian:

$$H_{XXZ} = \sum_i (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z).$$

  Gapless for $|\Delta| \leq 1$. $\Delta = -1$ is *not* conformal.

- Spin-spin correlation function:

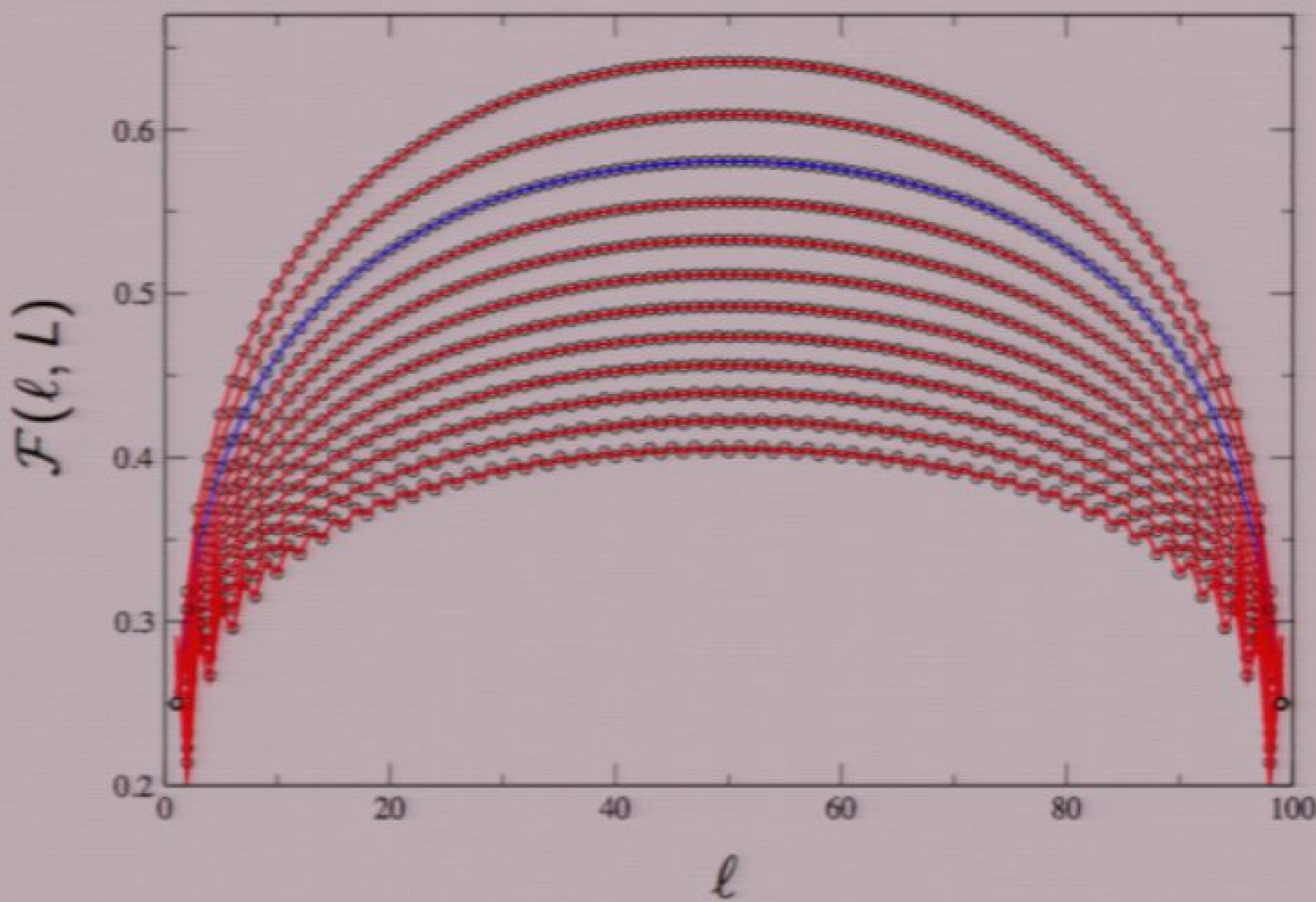
$$\langle \hat{S}_{i+r}^z \hat{S}_i^z \rangle - \langle \hat{S}_{i+r}^z \rangle \langle \hat{S}_i^z \rangle = -\frac{K}{2\pi^2}\frac{1}{r^2} + \frac{2A_2}{\pi^2}\frac{(-1)^r}{r^{2K}} + \dots.$$

- Fluctuations:

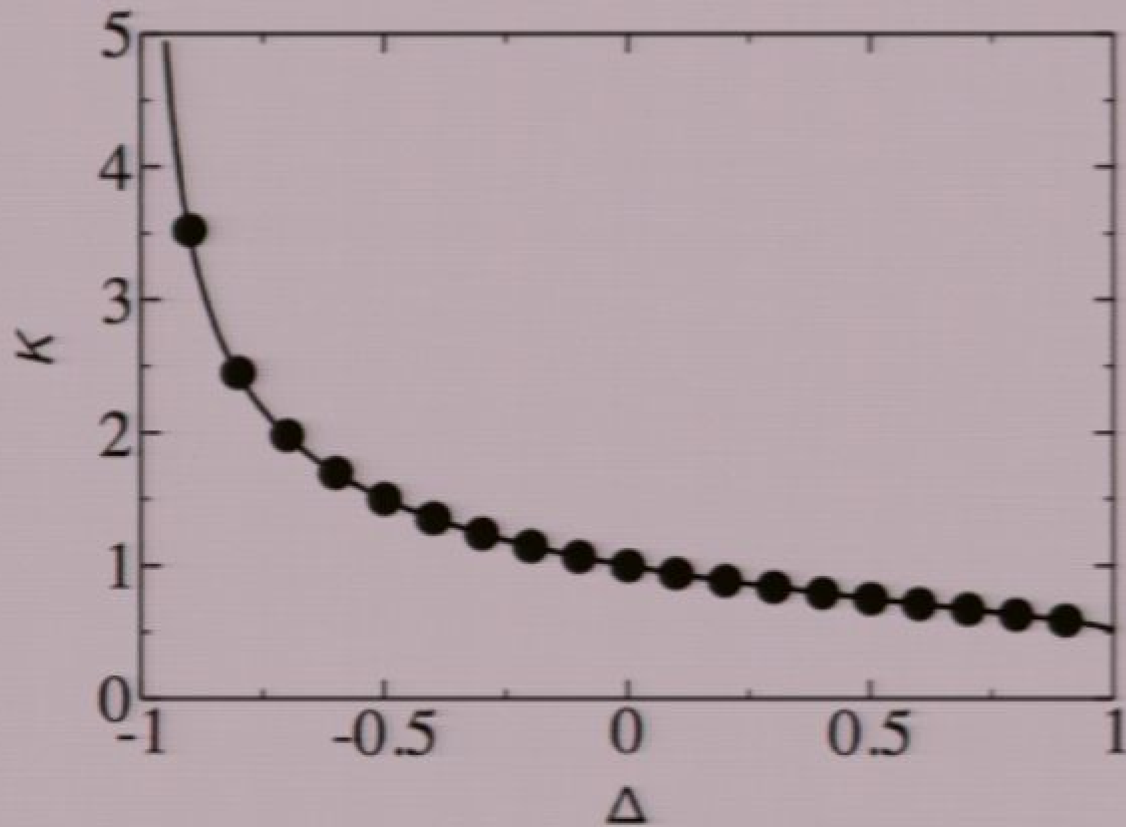
$$\pi^2 \mathcal{F}_{XXZ}(\ell) = K \ln \ell + f_2 - A_2 \frac{(-1)^\ell}{\ell^{2K}} + O(\ell^{-2}).$$

$$\mathcal{F}(l) = \sum_{i,j=1,\dots,l} [\langle \hat{S}_i^z \hat{S}_j^z \rangle - \langle \hat{S}_i^z \rangle \langle \hat{S}_j \rangle]$$

XXZ Model - Fluctuations (L=100)



XXZ Model - Luttinger Parameter



from Bethe ansatz: $K = \dfrac{1}{2(1 - (\cos^{-1}\Delta)/\pi)}$

data points from fluctuations

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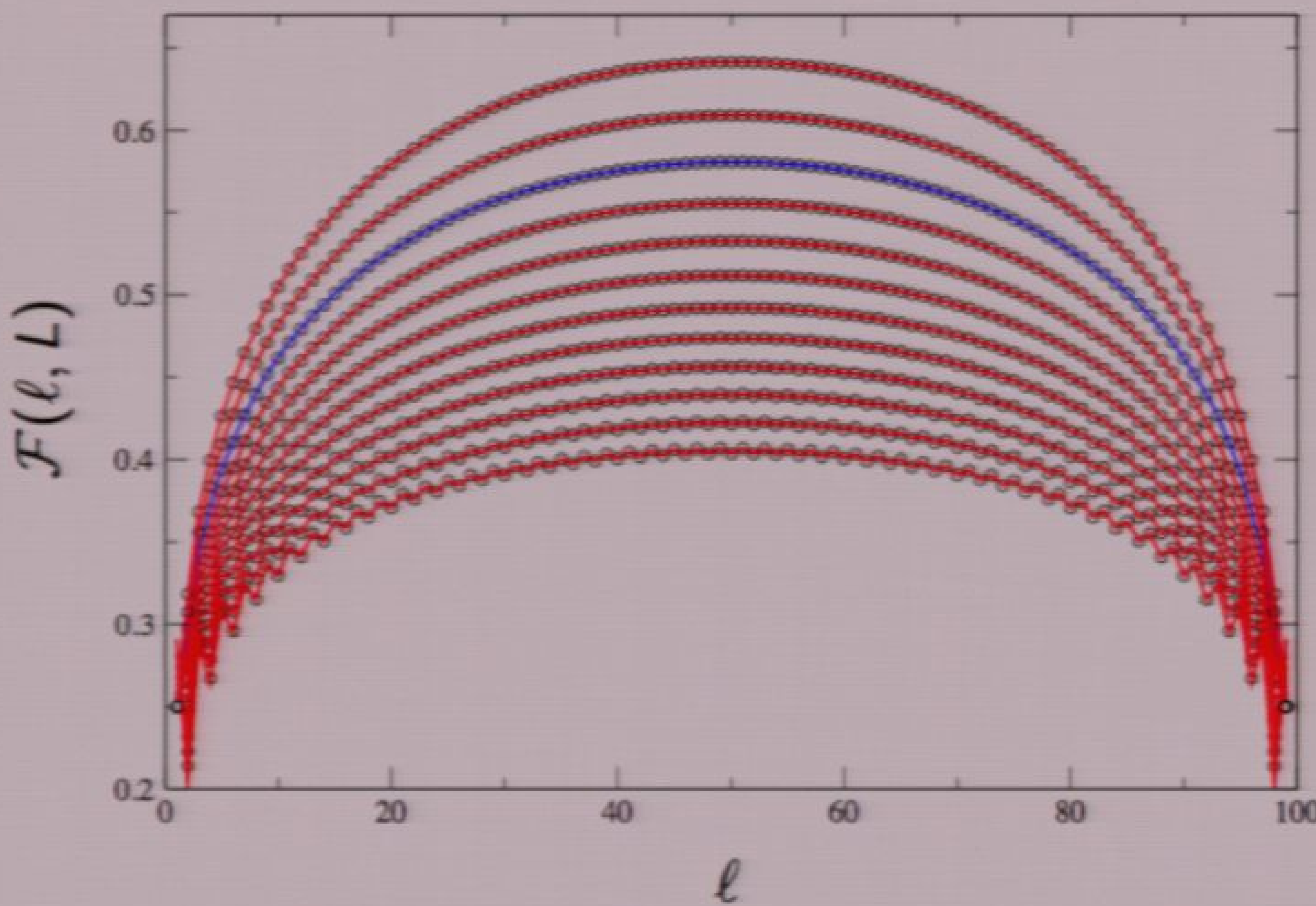

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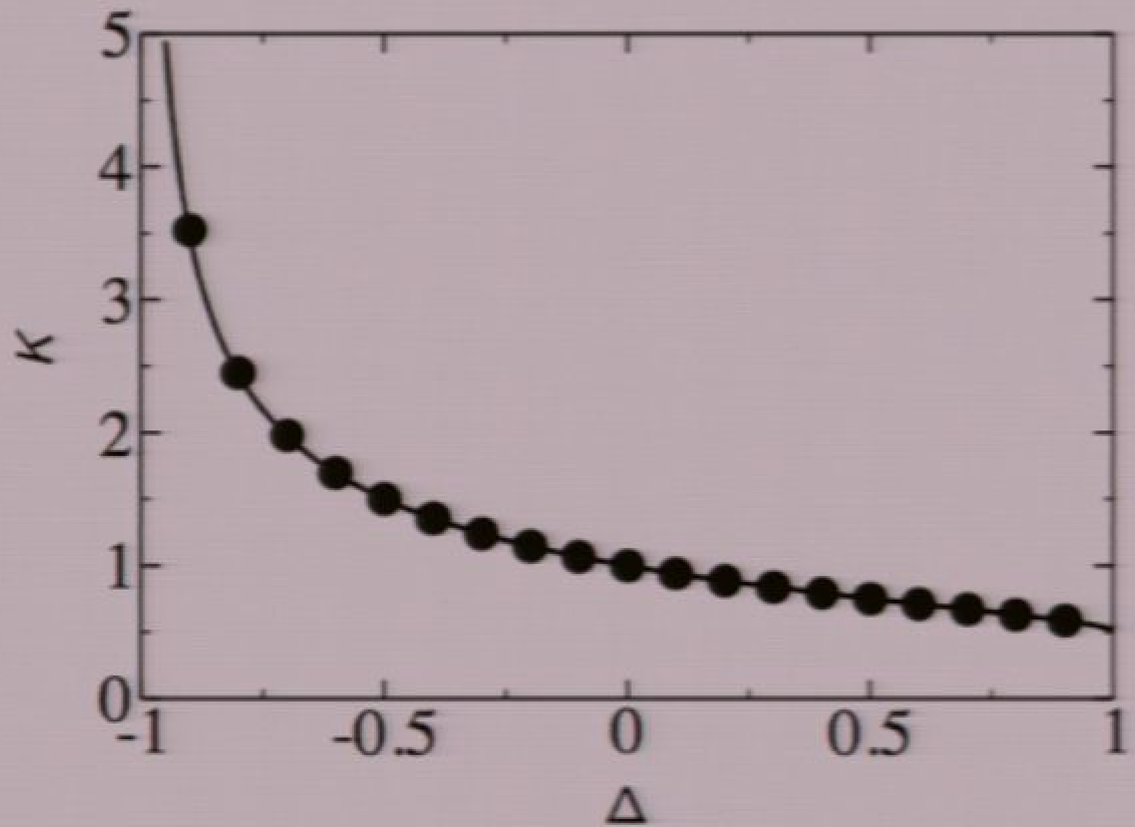

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but: also described by SU(2) WZW CFT

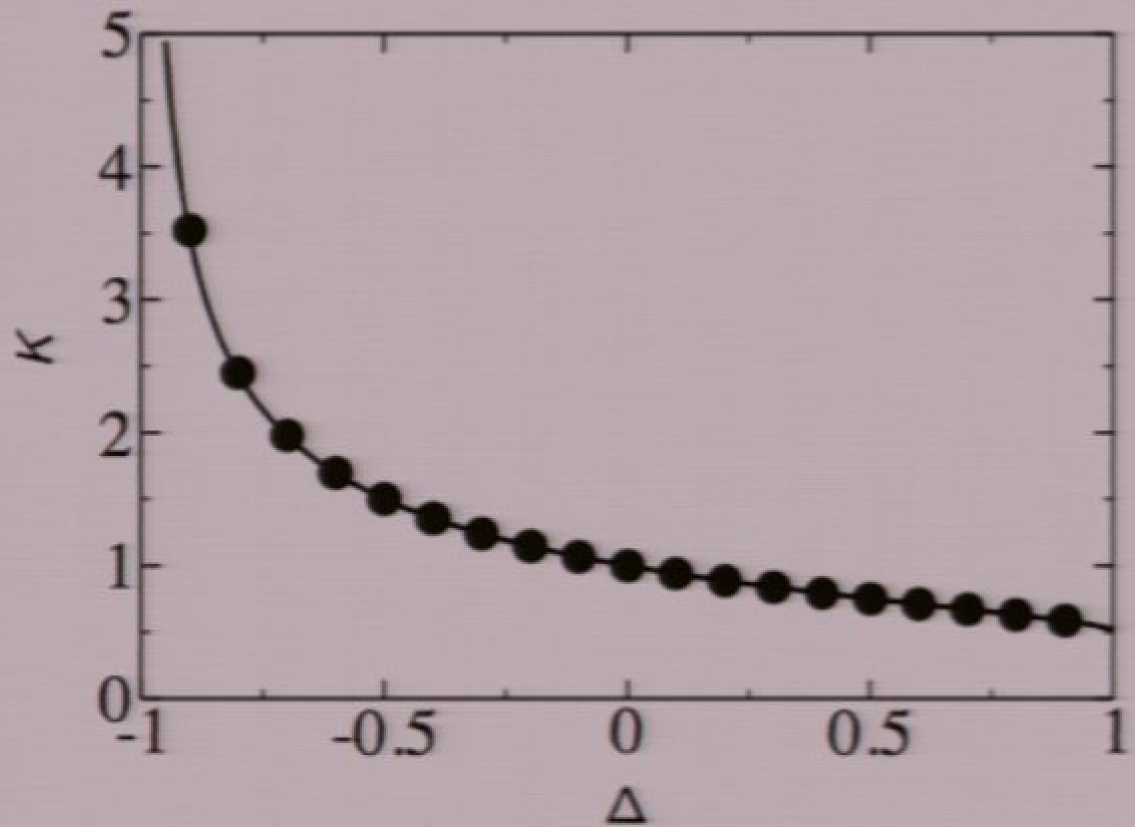
Haldane-Shastry model: long-range $1/r^2$ interactions with spin-spin correlation function

$$\langle \hat{S}^z_{i+r}\hat{S}^z_i\rangle - \langle \hat{S}^z_{i+r}\rangle\langle \hat{S}^z_i\rangle = \frac{1}{4}(-1)^r \frac{\mathrm{Si}(\pi r)}{\pi r}, \qquad \mathrm{Si}(x) = \int_0^x dt\, \frac{\sin t}{t}$$

and fluctuations

$$\mathcal{F}_{\mathrm{HS}}(\ell) = \frac{1}{2\pi^2}\ln \ell + f_3 - \frac{(-1)^\ell}{16\ell} + O(\ell^{-2}), \qquad f_3 \simeq 0.197.$$

XXZ Model - Luttinger Parameter



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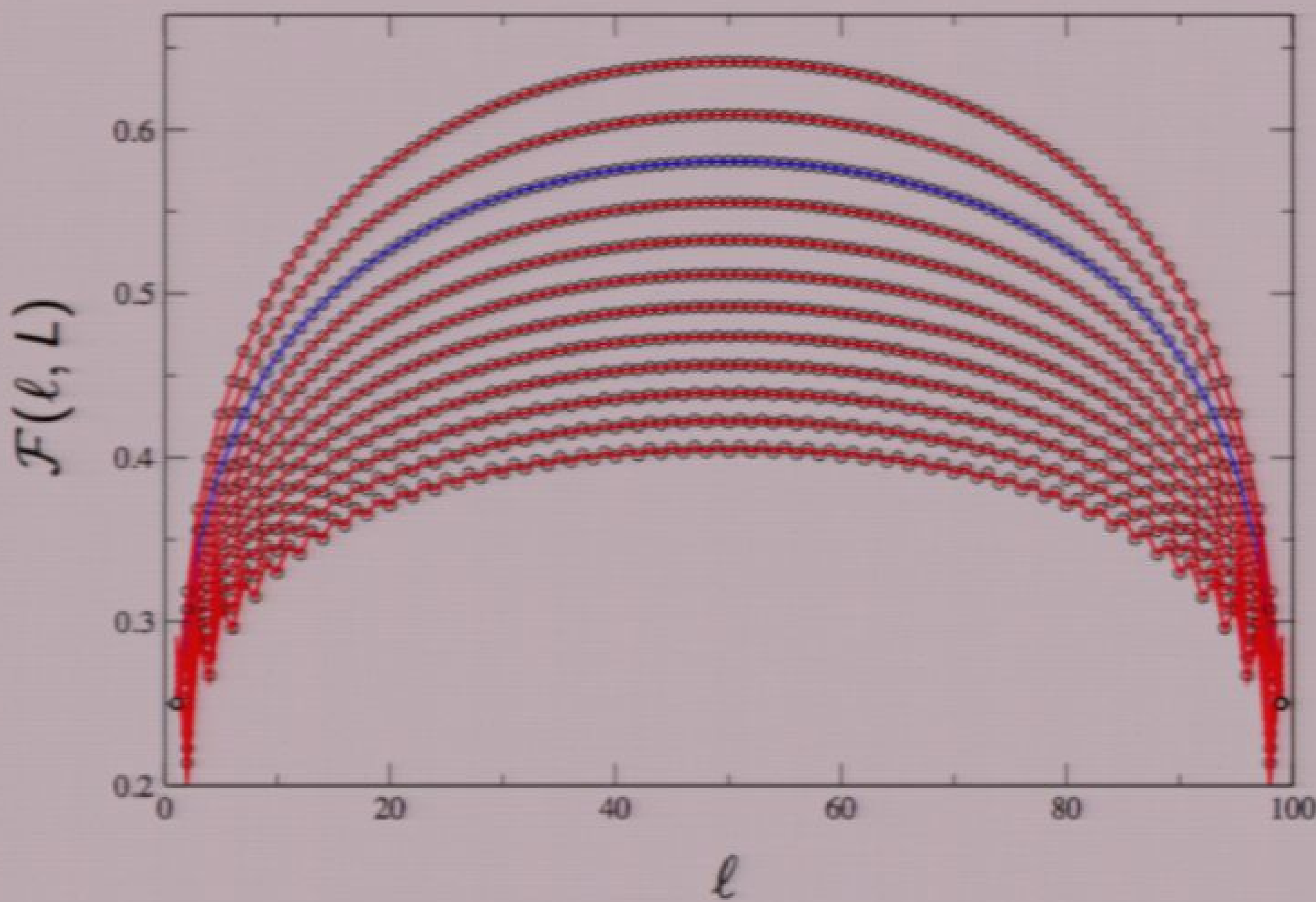

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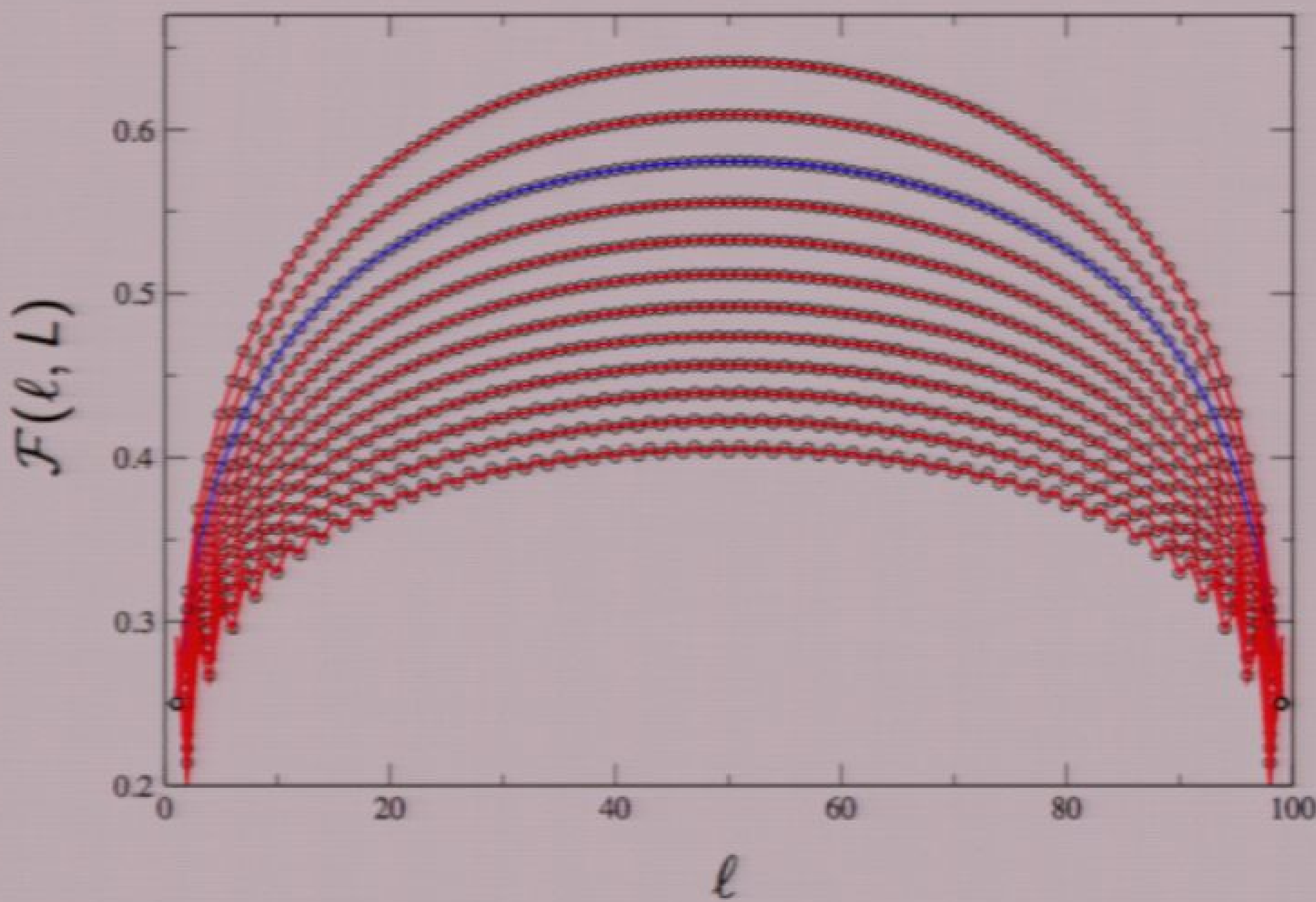

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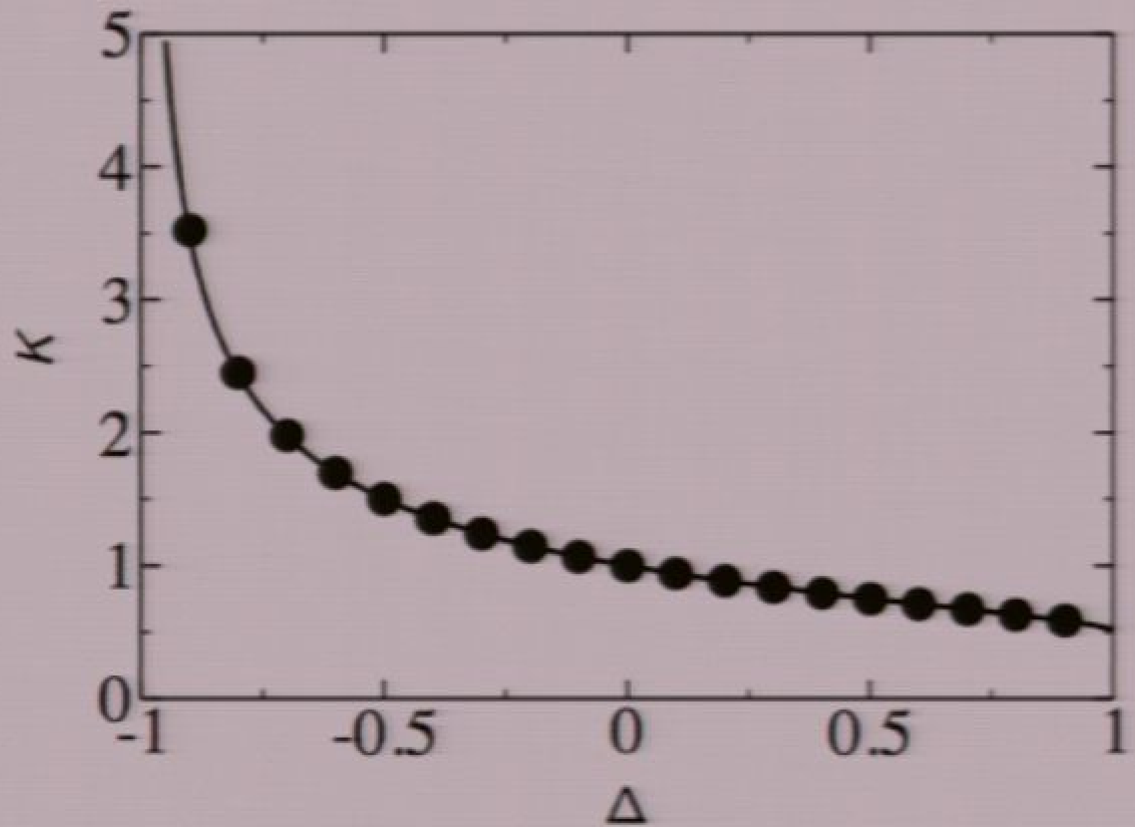

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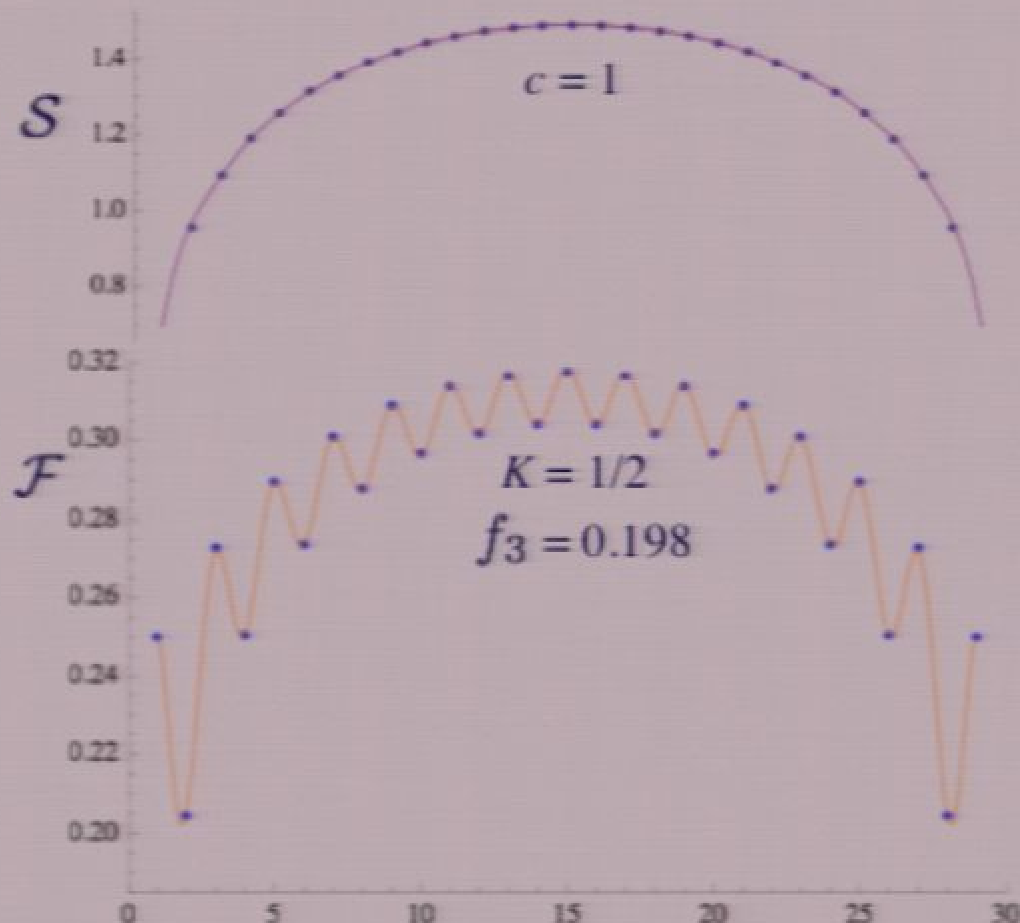


Haldane-Shastry chain: exact results

Other spin chains

- Spin-s Takhtajan-Babujian chain: exactly solvable, $k = 2s$, $\mathrm{SU}(2)_k \mathrm{WZW}$

$$\pi v \chi = s = \frac{k}{2}$$

Spin-$s = 3/2$ Heisenberg model, $k = 1$ vs. Spin-$s = 3/2$ TB model, $k = 3$

we really measure k

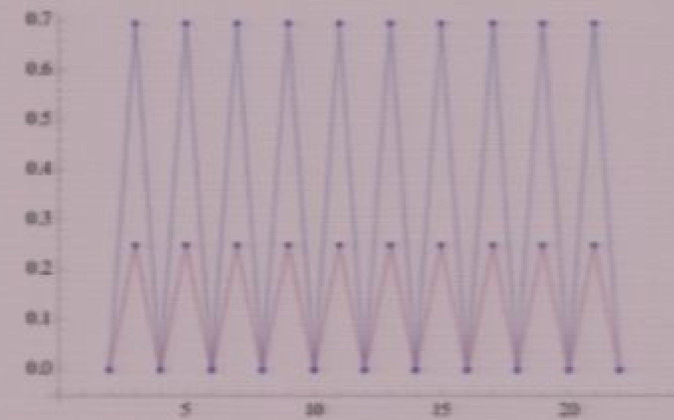
- spin 1 Uimin-Sutherland model: both entanglement entropy and fluctuations oscillate with period 3 because of SU(3) symmetry

- SU(n) generalization of TB chain: Andrei-Johannesson model, $\mathrm{SU}(n)_k \mathrm{WZW}$

$$\pi v \chi = \frac{k(n^2 - 1)}{6}$$

- Similar generalizations for $\mathrm{SP}(n)$ and $\mathrm{SO}(n)$ symmetry

Other spin chains (beyond CFT)



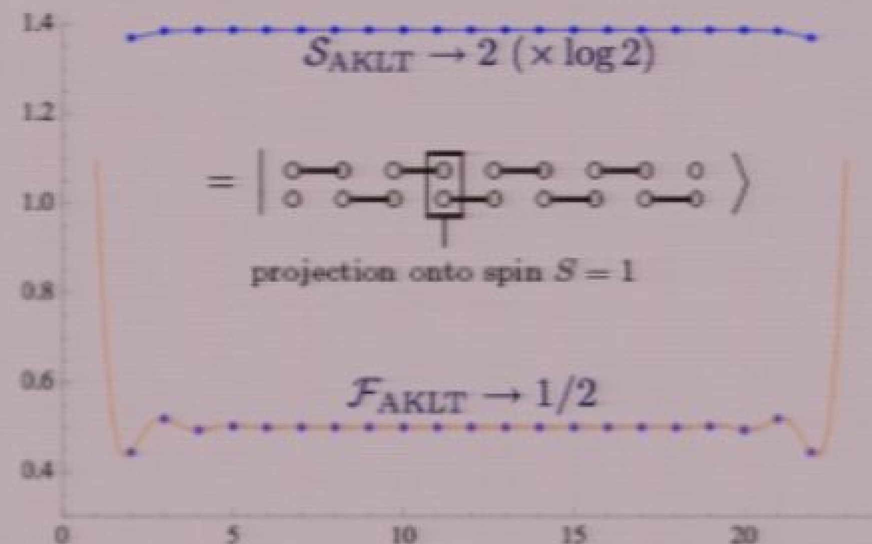
❖ Dimer phase (e.g. Majumdar Ghosh model)

❖ Gapped phase (e.g. AKLT model)

$$\mathcal{F}_{\mathrm{AKLT}}(\ell) = \frac{1}{2}\left(1-(-1)^{\ell}e^{-\ell/\xi}\right)$$

$$\mathcal{S}_{\mathrm{AKLT}}(\ell) = 2\left(1-\frac{e^{-2\ell/\xi}}{2\ln 2}\right)$$

(From Katsura et al, 2007)

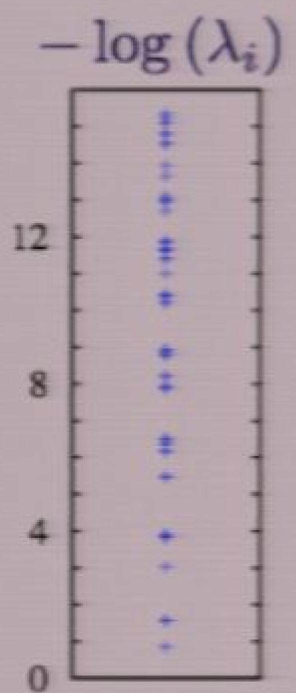


❖ $\mathcal{S}/\mathcal{F} \to \text{const.}$

Affleck-Kennedy-Lieb-Tasaki (AKLT) model

More general? Not so obvious…

- Calculate spin-spin correlations and sum them up (analytically and numerically (DMRG, QMC, ...))
- Availability of reduced density matrix?



$$\mathcal{S} = -\sum_i \lambda_i \log \lambda_i$$

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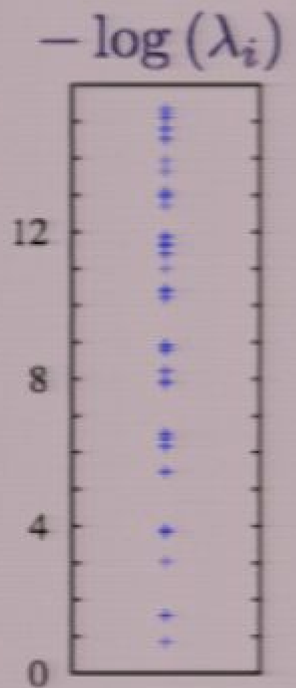
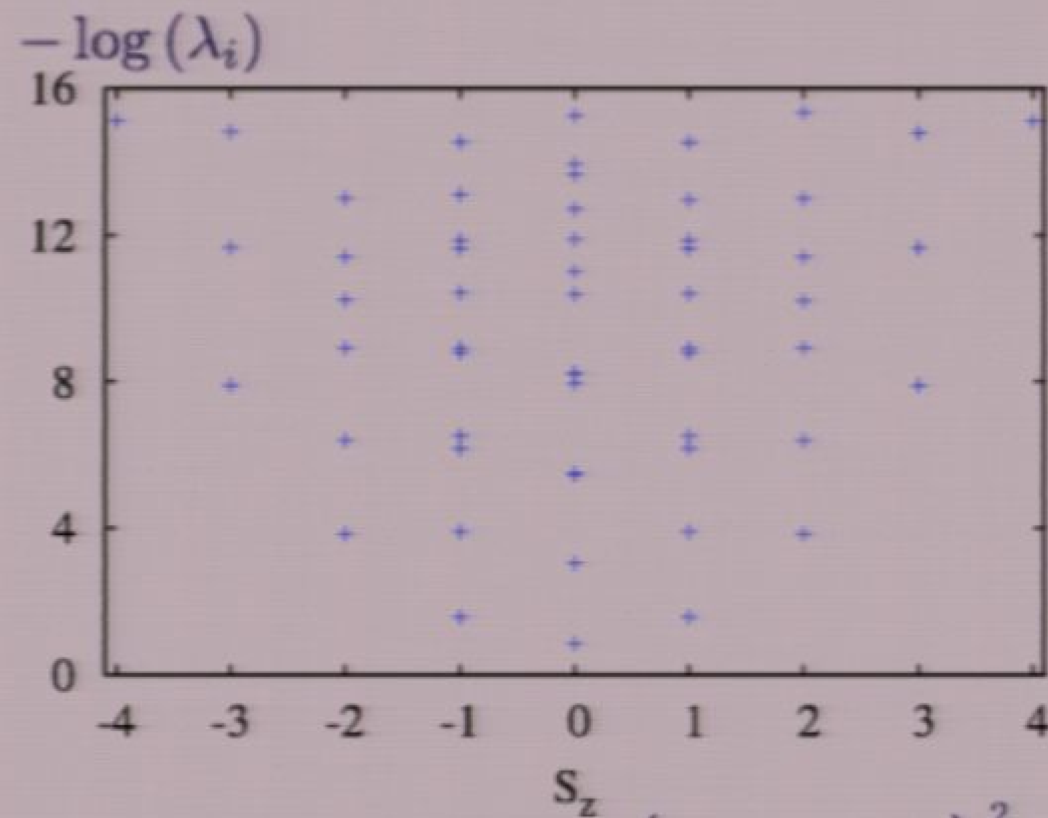
- Availability of reduced density matrix? Quantum number? --> DMRG!





$$\mathcal{S} = -\sum_i \lambda_i \log \lambda_i$$

$$\mathcal{F} = \sum_{S_z} S_z^2 \sum_i \lambda_i^{S_z} - \left(\sum_{S_z} S_z \sum_i \lambda_i^{S_z} \right)^2$$

Counter-Example: Ideal Bose gas

Binomial distribution:

$$\rho_A(n) = \binom{N}{n} p^n (1-p)^{N-n}, \qquad n = 0, \ldots, N.$$

$$\langle n \rangle = N p$$

$$F = \langle n^2 \rangle - \langle n \rangle^2 = N p (1-p)$$

$$S(N) = -\sum_{n=0}^{\infty} \rho_A(n) \log_2 \rho_A(n)$$

This gives: $S \sim \ln F^{1/2}$

It is also important that there is a conserved quantity:
total number of particles or spin

In particular, spin fluctuations do not scale logarithmically
in the quantum Ising chain

What about 2D ?

Facts:

M. Wolf, 2006;
U. Schollwock et al 2006

- entanglement entropy: $\sim \ell \log \ell$ for free fermions (Gioev & Klich, 2006)
- valence bond entropy: $\sim \ell \log \ell$ (for critical systems) (Alet *et al.*, 2007)
- entanglement entropy for gapped models: AREA law $\sim \ell$

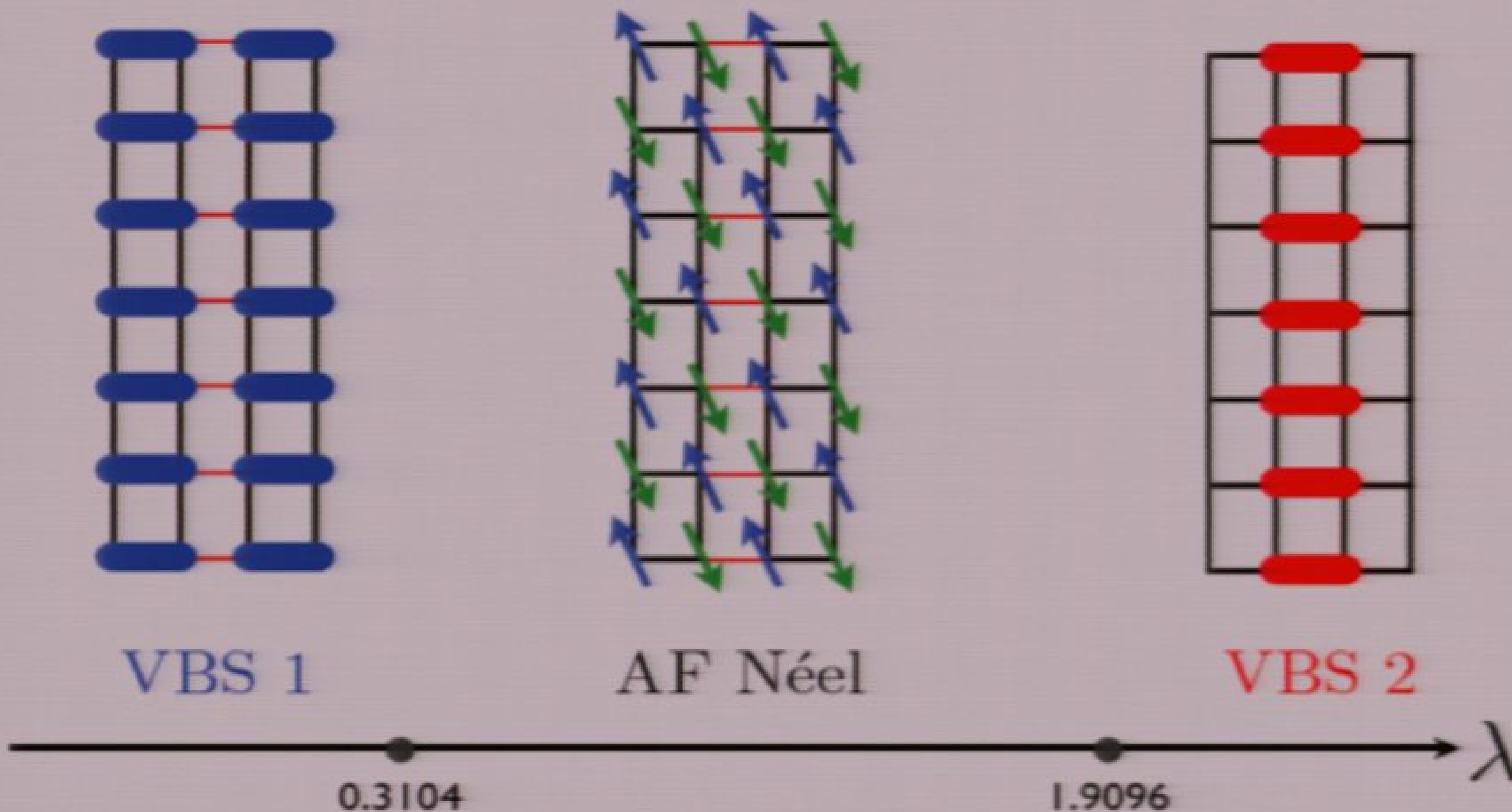
(see also Eisert *et al.*, RMP, 2010)

- fluctuations: $\sim \ell \log \ell$ for free fermions (Gioev & Klich, 2006)

Simple understanding (B. Swingle, 2010): Fermi surface patching

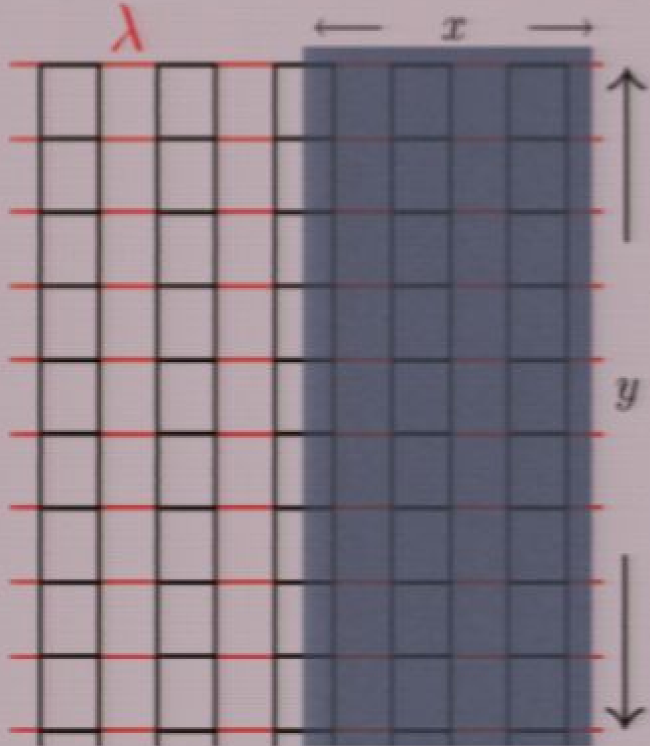
Can be extended to the “Fermi liquid” fixed point

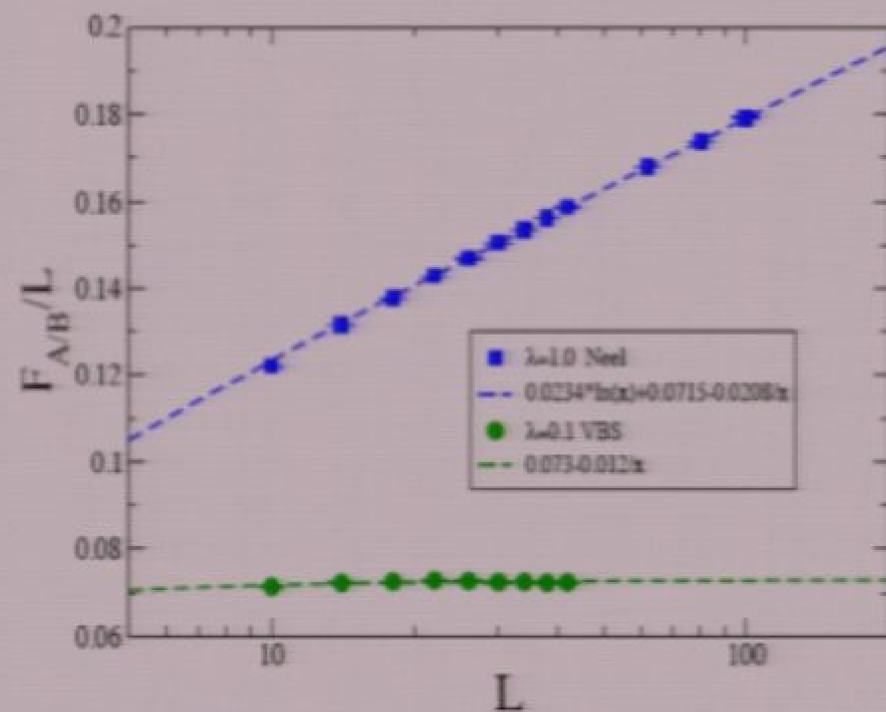
Coupled Ladder SPIN Systems



Fluctuations

x=L/2 & y=L





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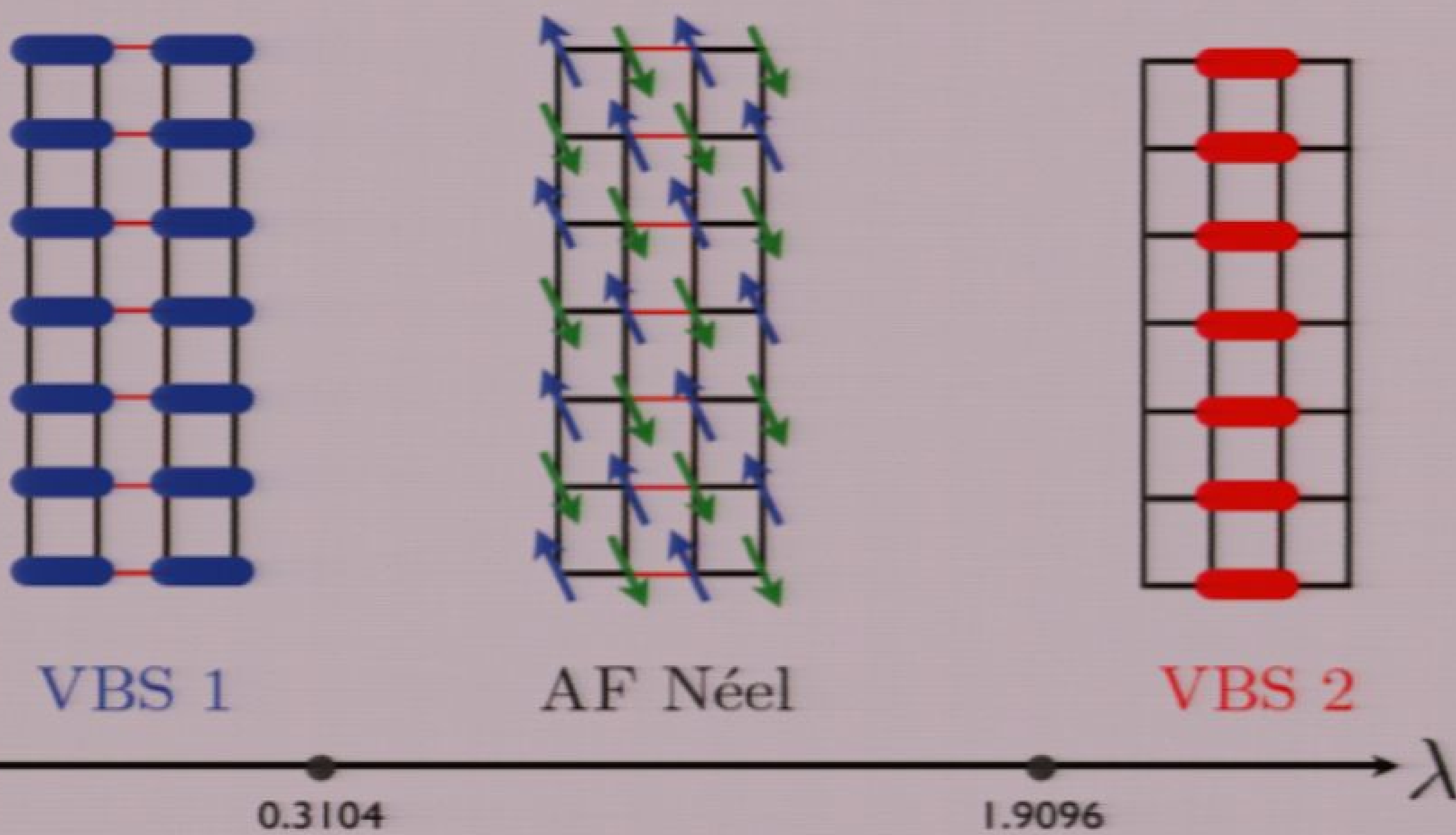
$F_{A/B}(L) = a_0 + a_1 L + a_2 L \ln L$ (AF Neel) (Attempt: spin wave theory)

Entropy would scale as L/2

Coupled Ladder SPIN Systems



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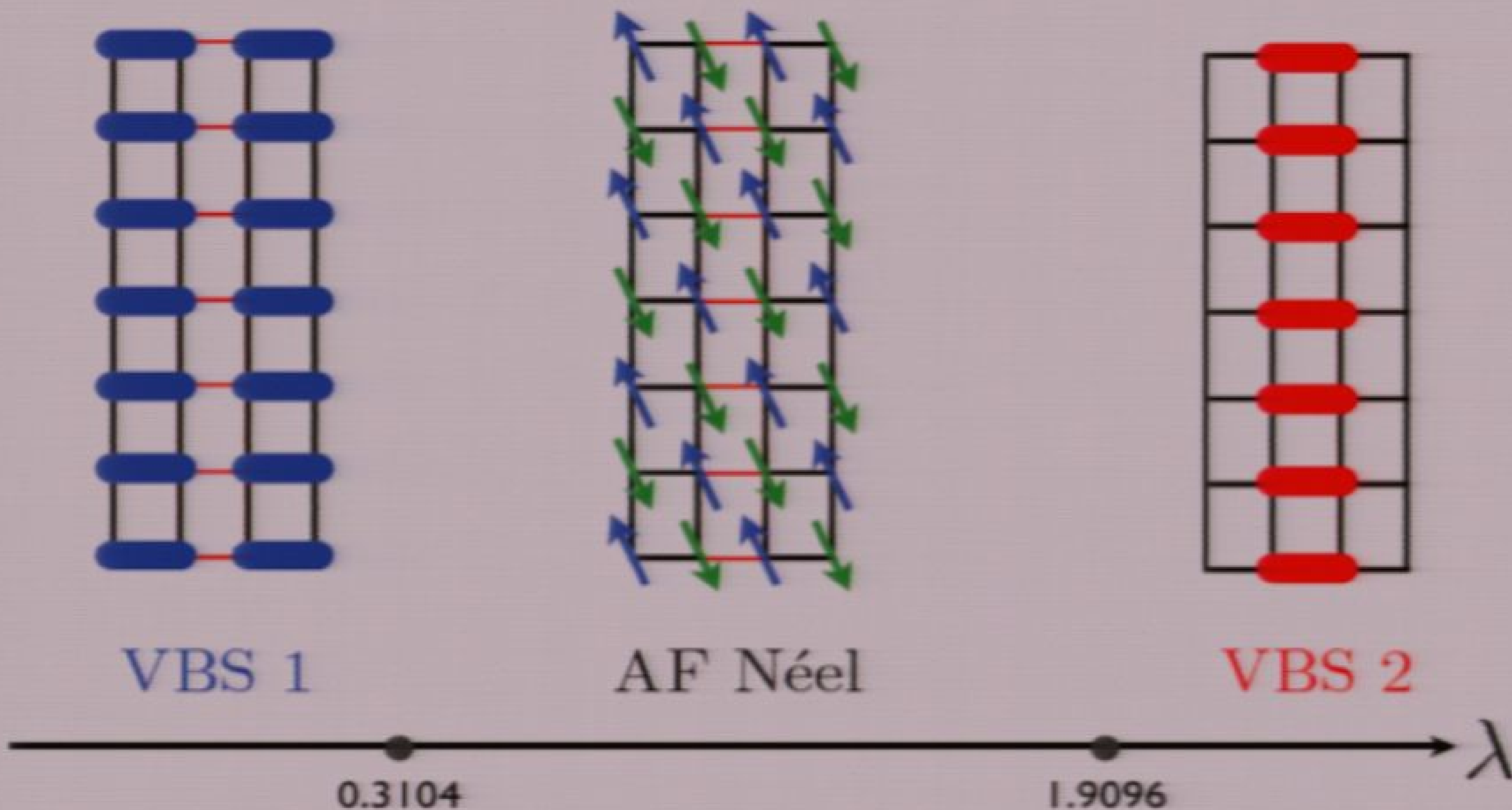

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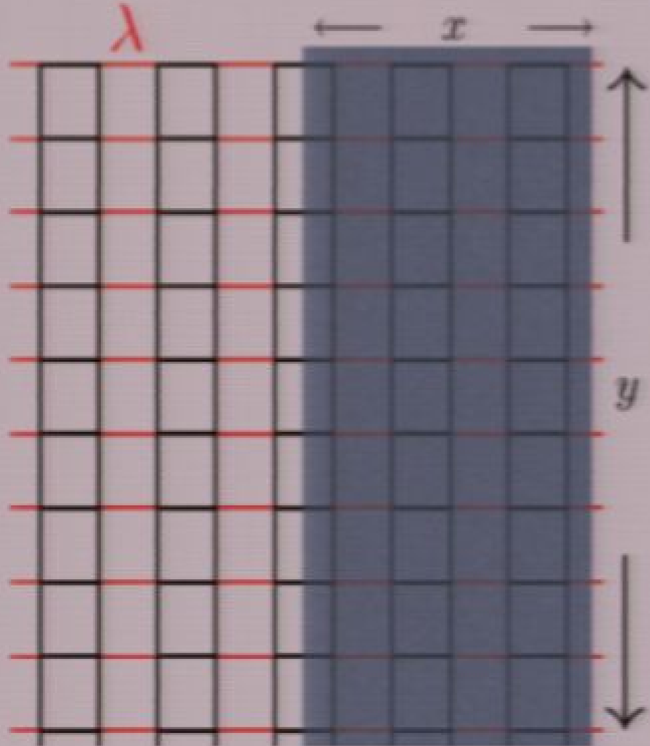

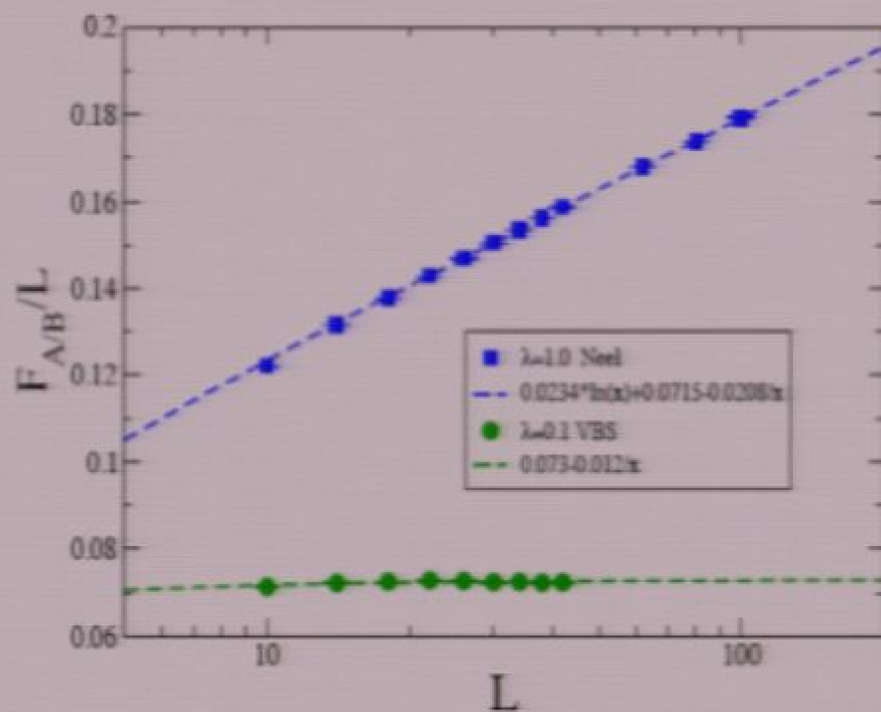

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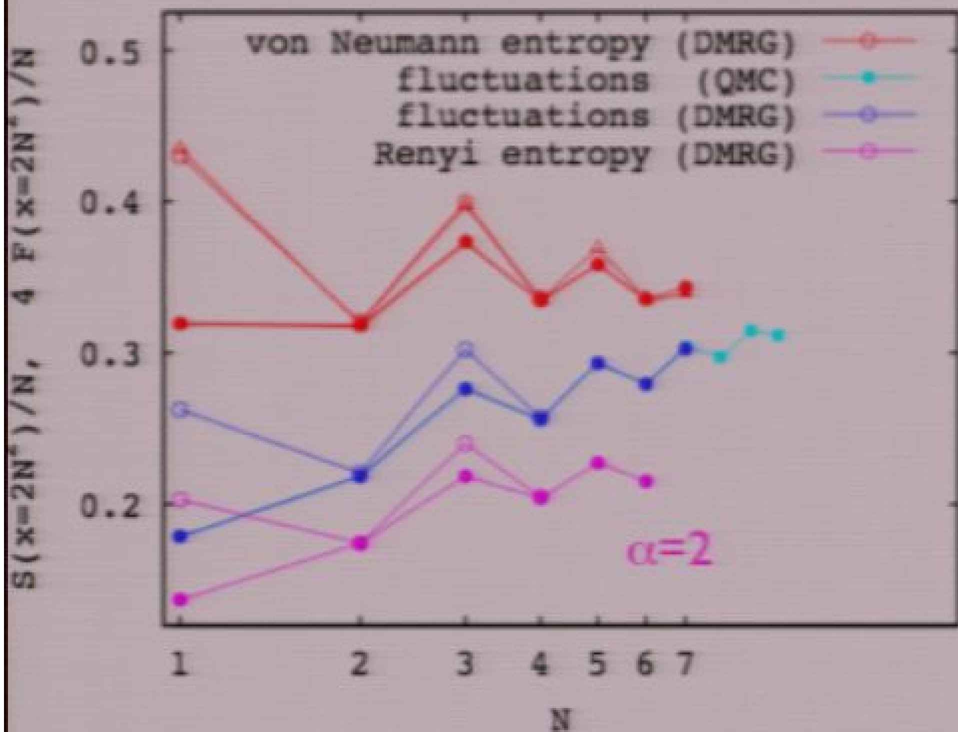
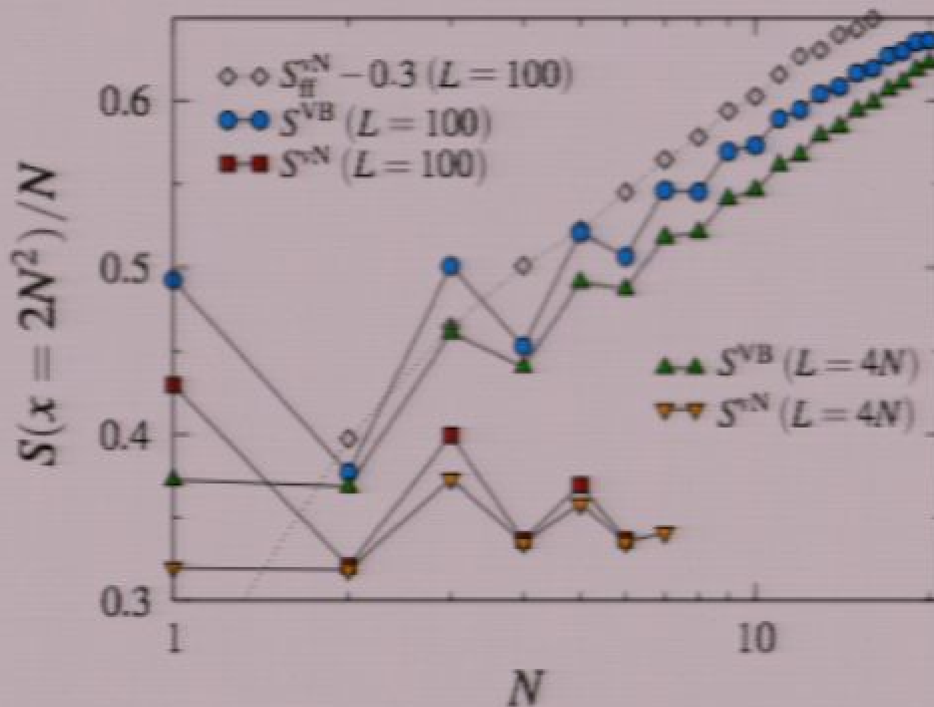
open symbols = N x 100 ladder
closed symbols = N x 4N ladder



A. Kallin, I. Gonzalez, M. B. Hastings, R. G. Melko, PRL 2009



$$S_\alpha(\rho) = \frac{1}{1-\alpha} \log \mathrm{tr} \rho^\alpha \quad \xrightarrow{\text{CFT}} \quad \frac{c}{6}\left(1 + \frac{1}{\alpha}\right) \log \frac{x}{a}$$

Renyi entropies

Fluctuations

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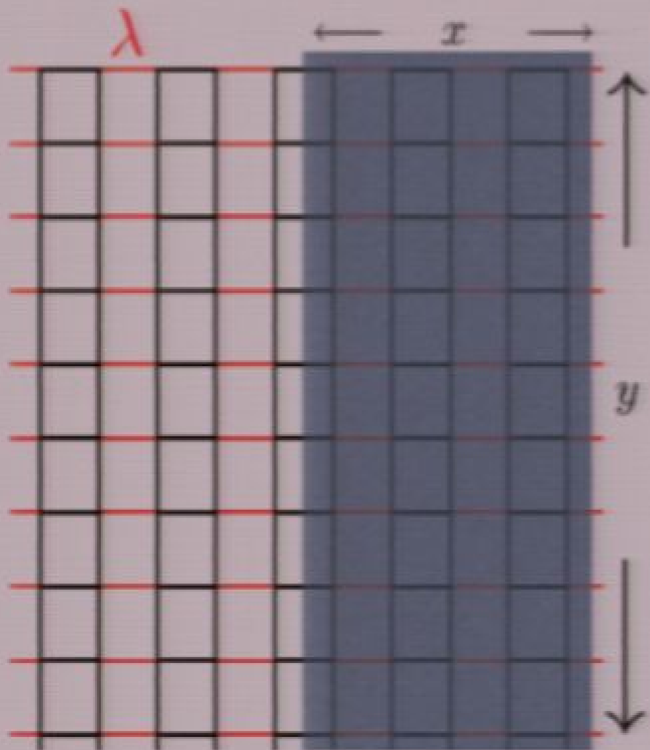

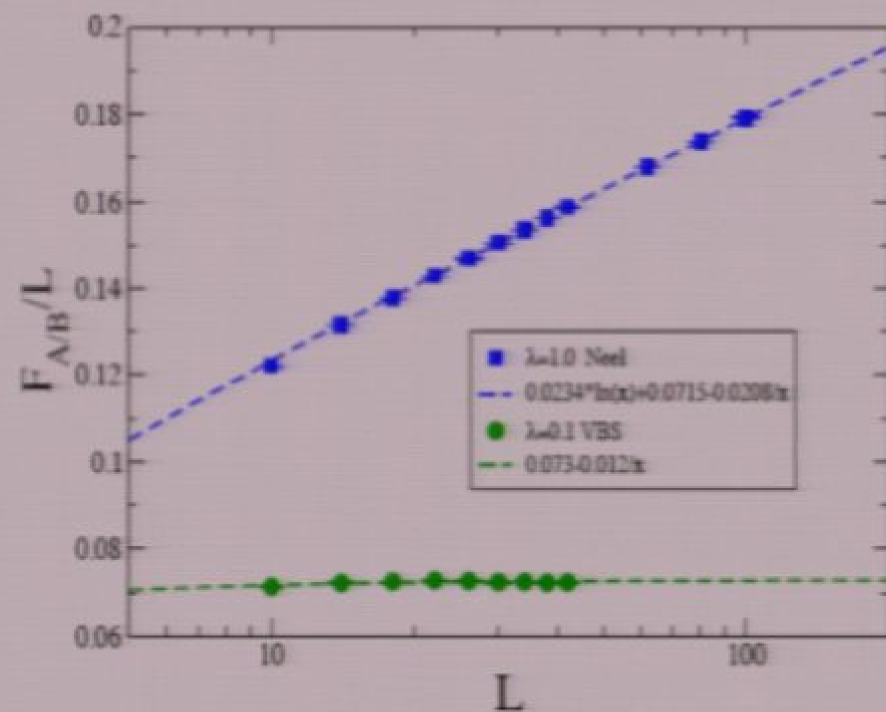

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Conclusion

Similarity between entanglement entropy and fluctuations

in 1D CFTs and beyond (gapped systems).

Not the same thing, entropy is more universal.

But, fluctuations can be measured.

Fluctuations can also be accessed in QMC for $d > 1$.

arXiv:1002.0825

Measuring Entanglement Entropy for **free** fermions

Klich-Levitov 2009, PRL

$$S = \frac{1}{\pi}\int_{-\infty}^{\infty} du\, \frac{u}{\cosh^2 u}\mathrm{Im}\left[\ln\chi\left(-2i\left(u+i\frac{\pi}{2}\right)\right)\right]$$

$$\ln\chi(\lambda) = \sum_{m=1}^{\infty}\frac{(i\lambda)^m}{m!}C_m$$

and hence $S = \sum_{m=1}^{\infty}(a_m/m!)C_m$ with

$$a_m = \frac{2^m}{\pi}\int_{-\infty}^{\infty} du\, \frac{u}{\cosh^2 u}\mathrm{Im}\left[\left(u+i\frac{\pi}{2}\right)^m\right]$$

= 0 for m odd

= $(2\pi)^m|B_m|$ for m even

This formula works for Gaussian fluctuations (QPC, wire) $S=(\pi^2/3)C_2$

Counter-example: a single fermion (spin 1/2), S=ln 2

$$\ln\chi(\lambda) = \ln\cos\frac{\lambda}{2}$$

$$C_m = \frac{2^m-1}{m}B_m,$$

$$|C_m| \sim \frac{2^m}{m}\frac{2m!}{(2\pi)^m} = \frac{2(m-1)!}{\pi^m} \to \infty.$$

Another Attempt: (ongoing work)

Start: Relate entanglement entropy & Factorial Cumulants $\chi_X(\lambda) = \lambda^X$

$$\mathcal{S} = \sum_{n=1}^{\infty} \frac{A_n}{n}$$

$$A_n = (-1)^{n+1}\left[\frac{F_n}{(n-1)!} + \frac{F_{n+1}}{n!}\right] + \sum_{k=0}^{n} \binom{n}{k} \frac{F_{k+1}}{k!}.$$

Then, relate Factorial Cumulants to ordinary Cumulants

$$\mathcal{S} = \sum_{n=2}^{M+1} a_M(n) C_n$$

$$a_M(n) = \sum_{k=n-1}^{M} \frac{1}{kk!}\left[(-1)^{k-1} s(k,n-1) + s(k+1,n)\binom{M}{k}\right]$$

Numerical evidence

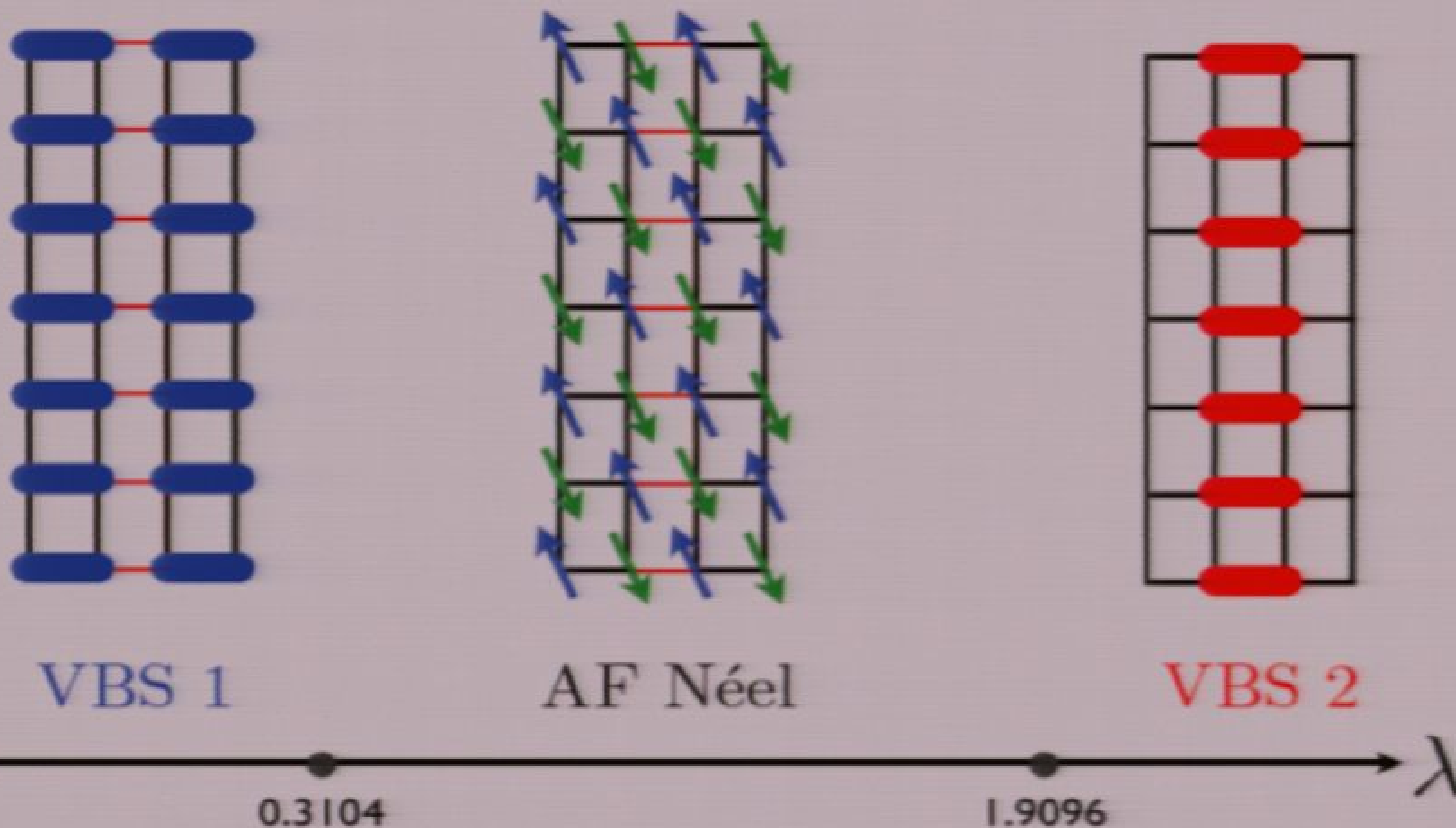
$a_M(n) = 0$ for odd n

$a_\infty(n) = 2\zeta(n) = \frac{(2\pi)^n}{n!}|B_n|$

s(n,m) are the signed Stirling numbers of the first kind

in collaboration with H. F. Song, C. Flindt &

Coupled Ladder SPIN Systems



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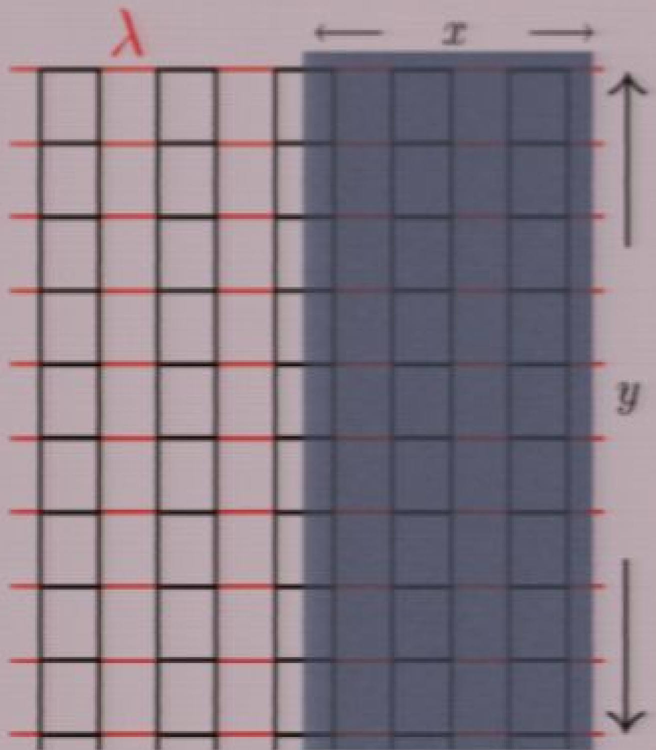

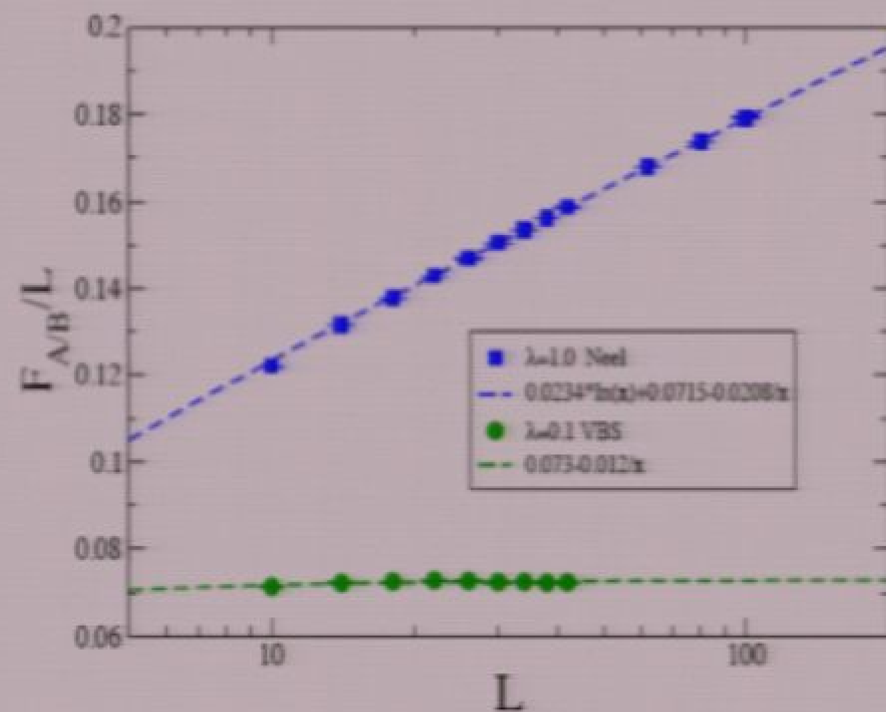

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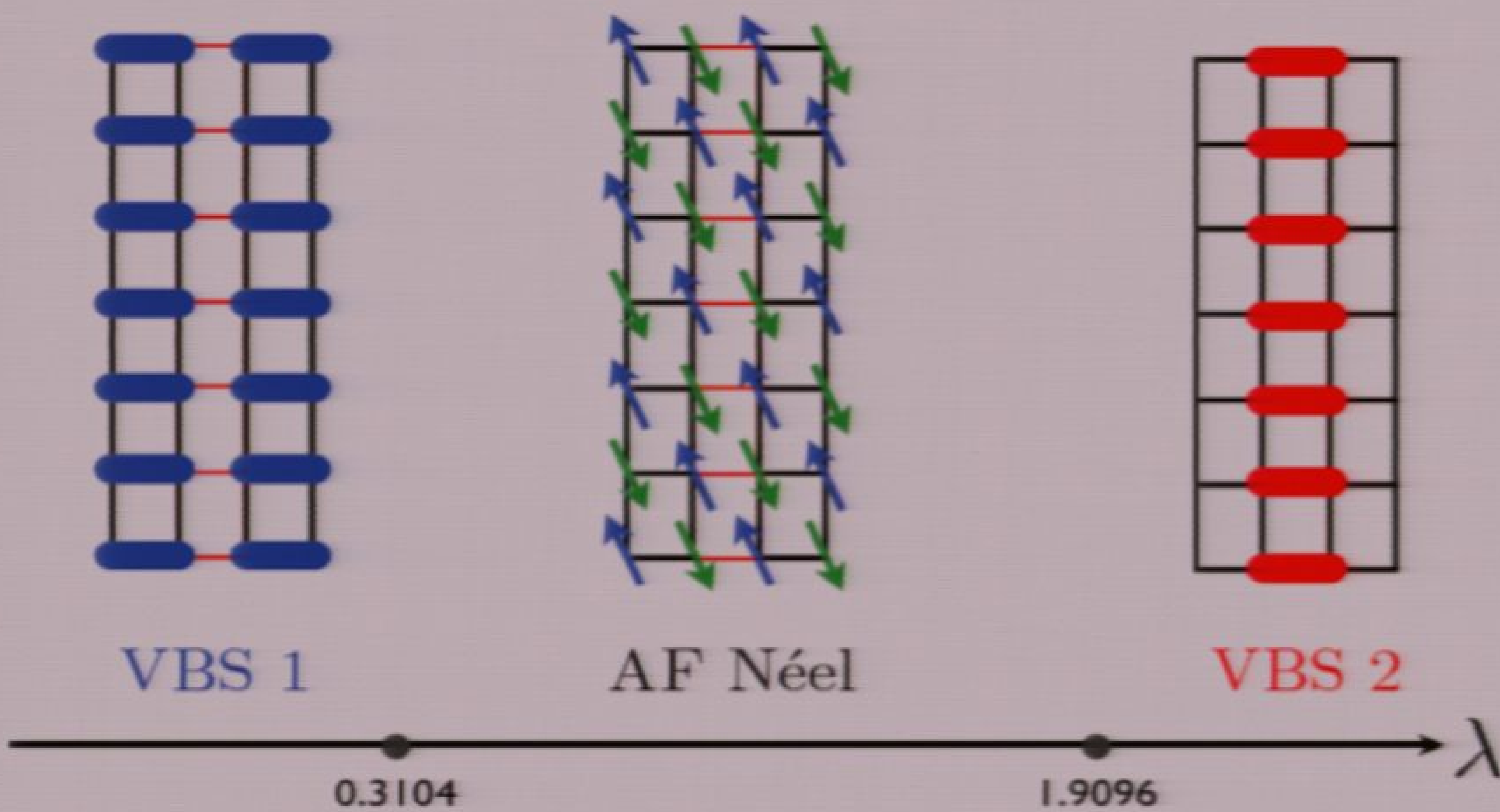

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