Title: Entanglement entropy and infinite randomness fixed points in disordered magnetic and non-abelian quasi-particle chains

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Abstract: Many one dimensional random quantum systems exhibit infinite randomness phases, such as the random singlet phase of the spin-1/2 Heisenberg model. These phases are typically the result of destabilizing systems described by a conformal field theory with disorder. Interestingly, entanglement entropy in 1d infinite randomness phases also exhibits a universal log scaling with length. In my talk I will touch upon calculating the entanglement entropy for inifinite-randomness phases, as well as describe the exotic infinite randomness phases realized in chains of non-abelian anyon chains. It was speculated that the entanglement entropy of an infinite-randomness phase is associated with the direction of RG flow, just as the c-theorem dictates the direction of RG flows for CFT's. I will also show that the entanglement entropy in disordered non-abelian chains provide the only known counter example.

Random spin chains: Abelian, non-Abelian, and non-linear

Gil Refael (Caltech)

Non-Abelian collaborators:

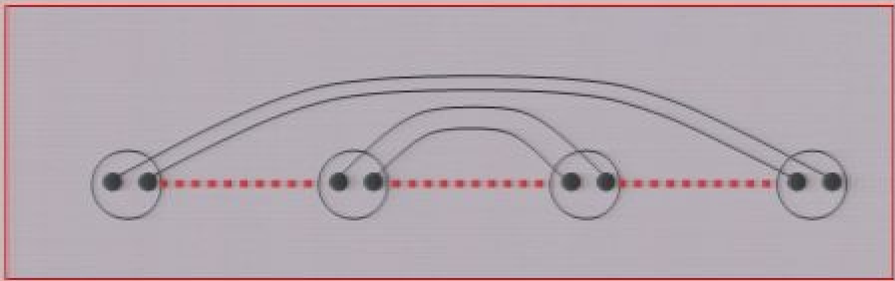
Lukasz Fidkowski (Caltech)

Joel Moore (UC Berkeley)

Nick Bonsteel (FSU)

Undergraduate forced labor:

Paraj Titum (IIT Kanpur), Han Hsuan Lin (MIT)



Funding:
Packard Foundation,
Sloan Foundation

Outline

- Disordered Heisenberg models.

  Random singlet and infinite randomness fixed points.

- Entanglement entropy in disordered spin chains

- Random Non-Abelian spin chains:

  - Fibonacci anyons
  - SU(2)k anyons

Disordered Heisenberg Chains

Heisenberg spin chains

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Solution methods rely on translational invariance:

- Bethe ansatz
- Bosonization

Spin chains with randomness

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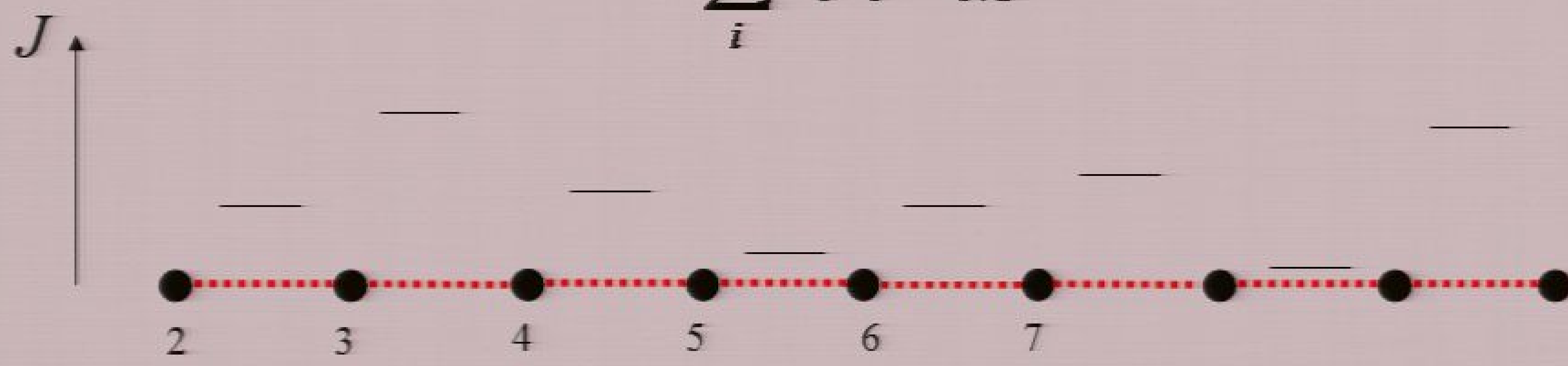

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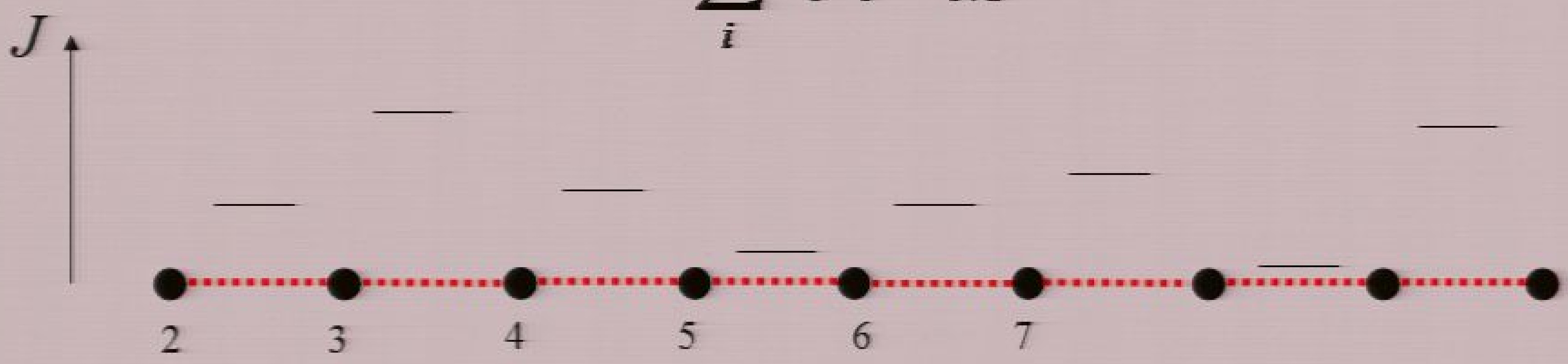

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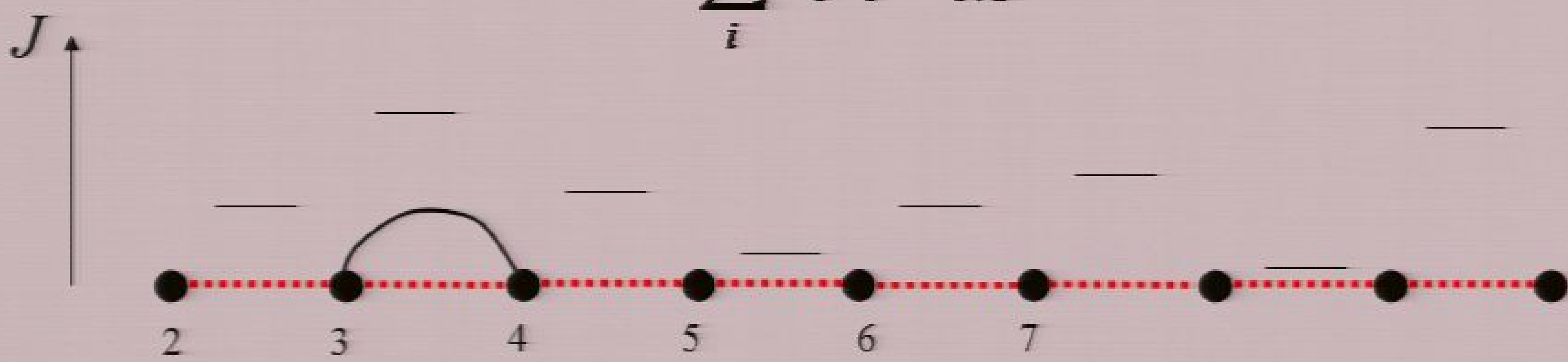

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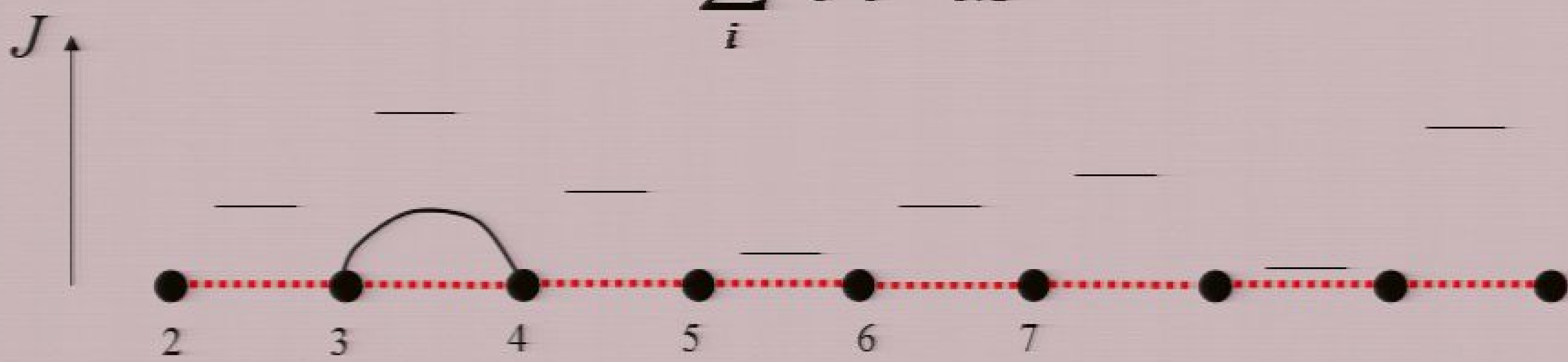

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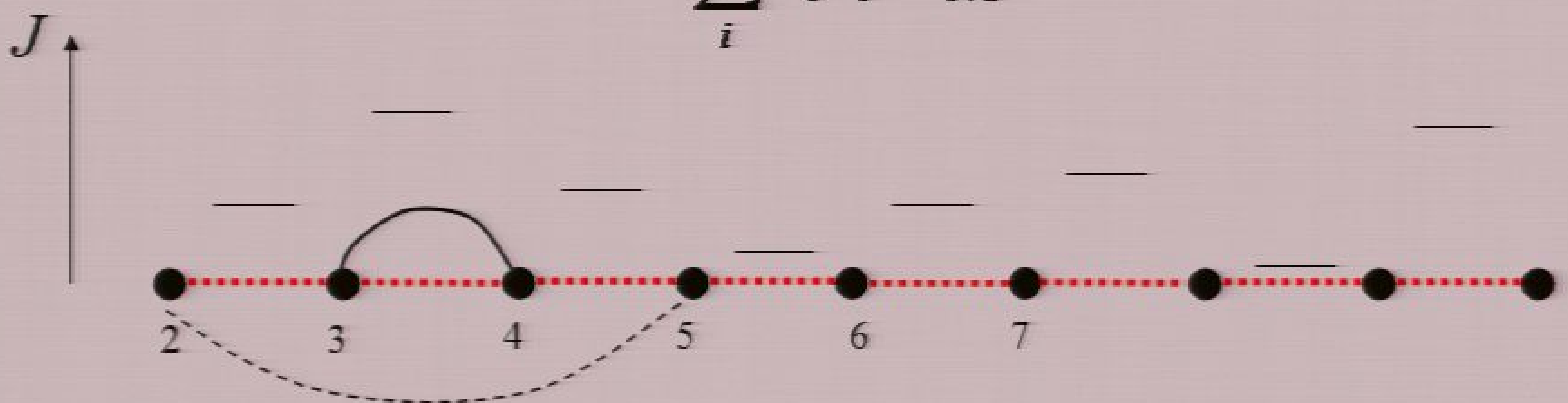

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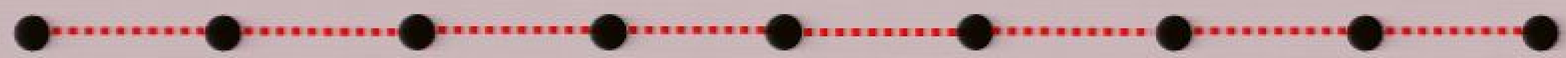
Ma, Dasgupta, Hu (1979)

Random Singlet Phase

[D.S. Fisher (1994), Bhatt, Lee (1982)]



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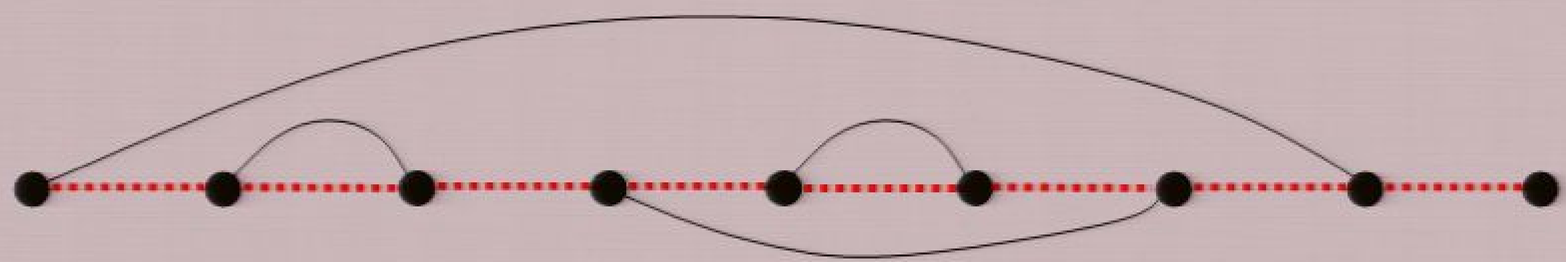

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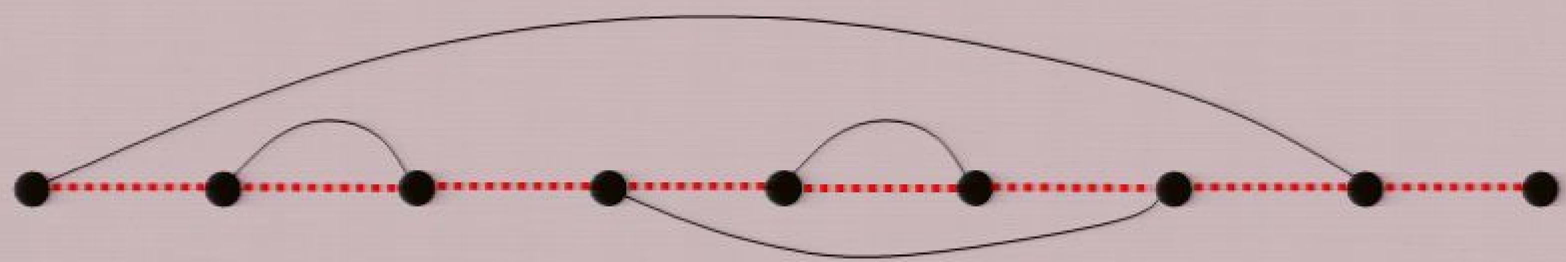

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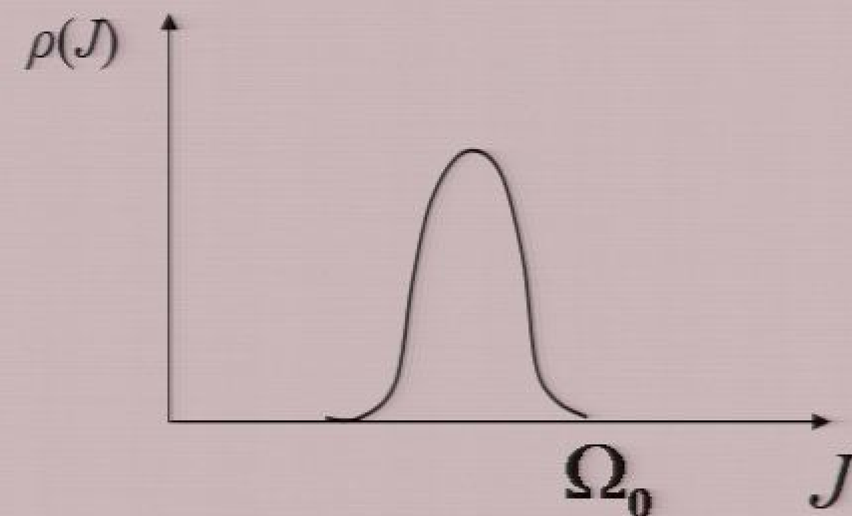

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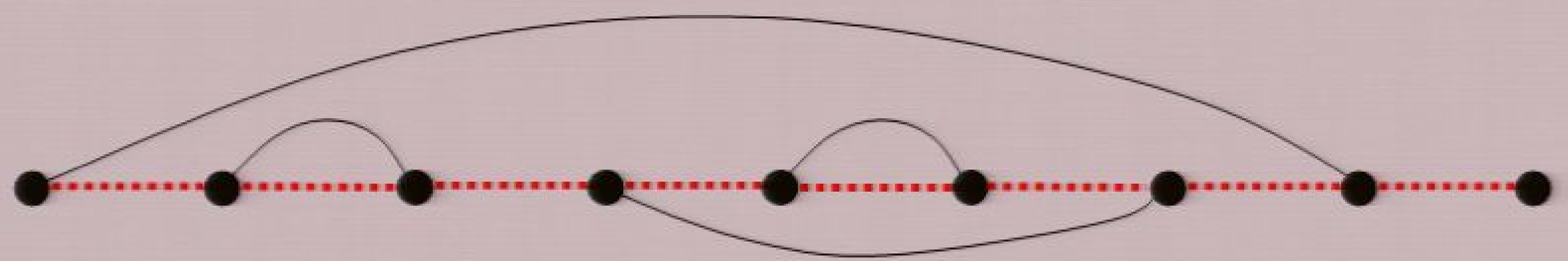

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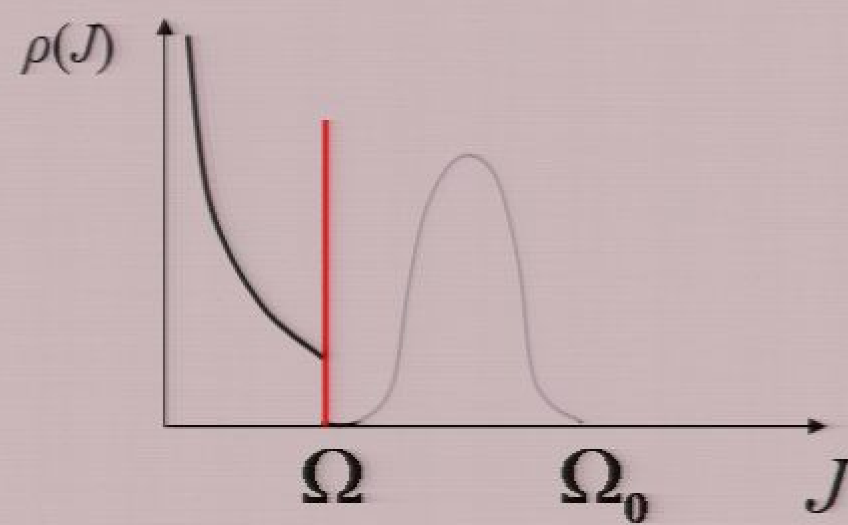

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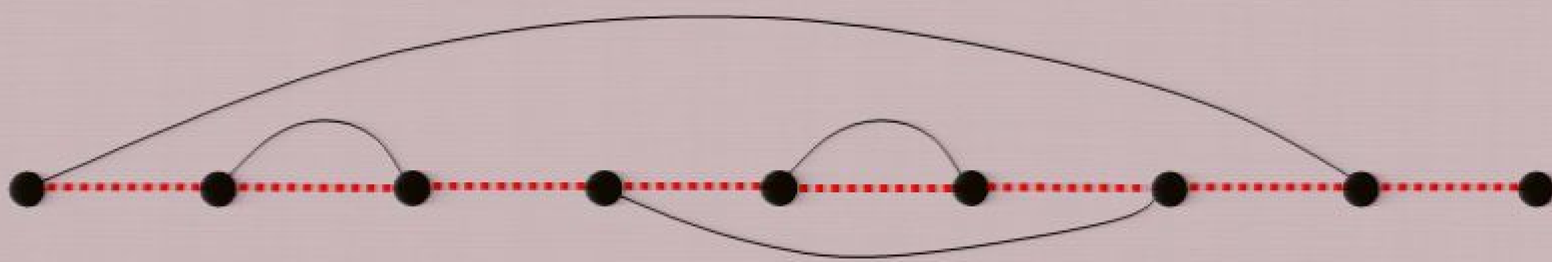

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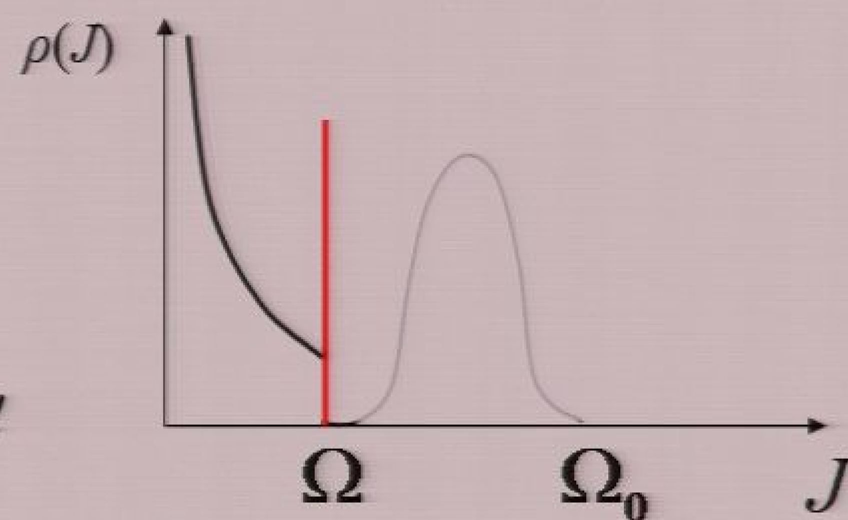

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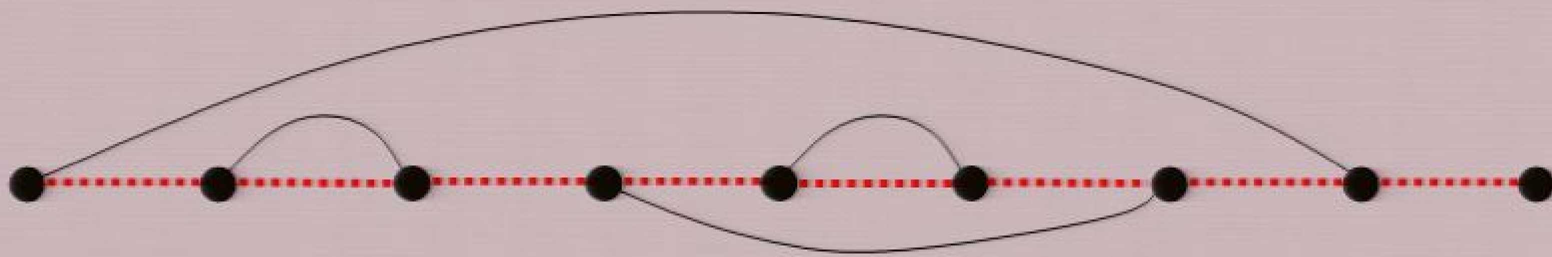

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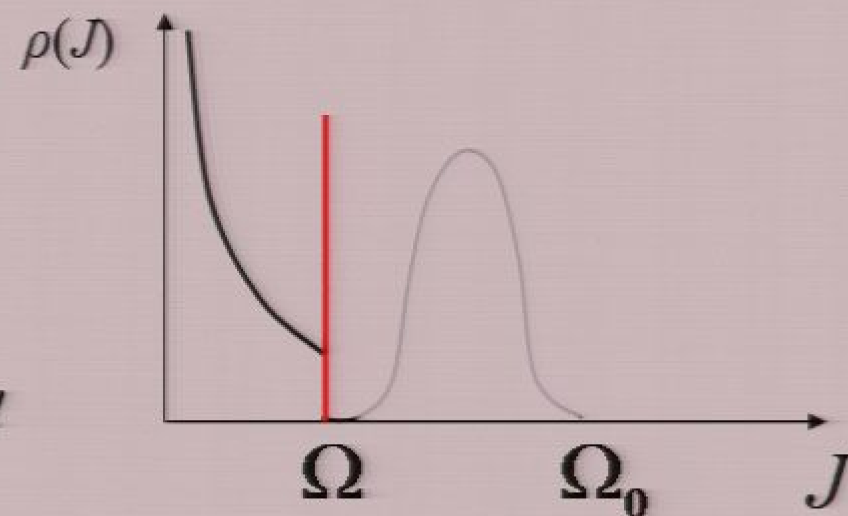

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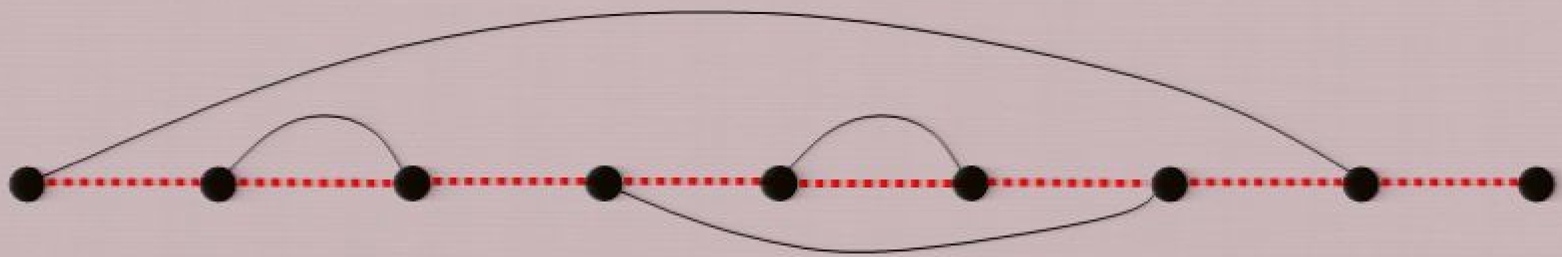

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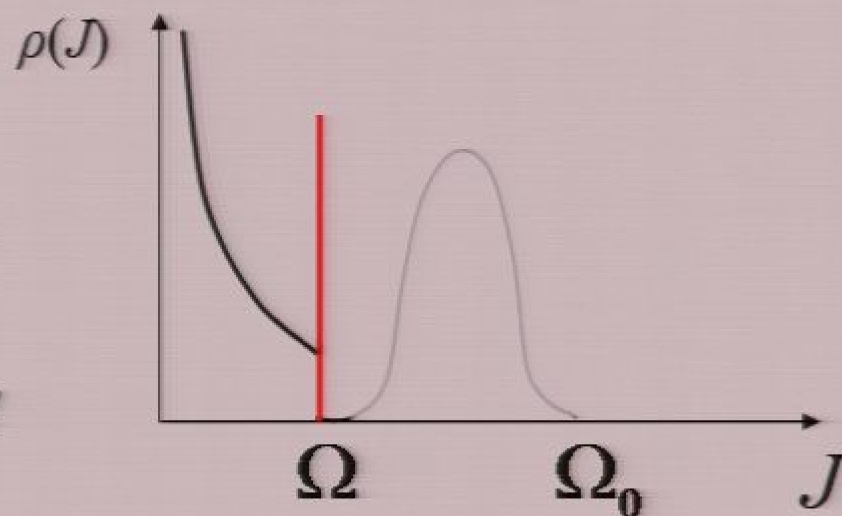

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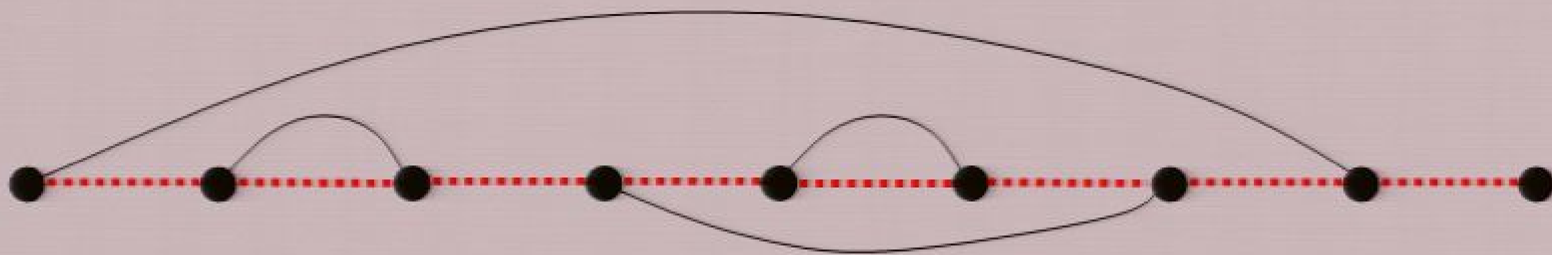

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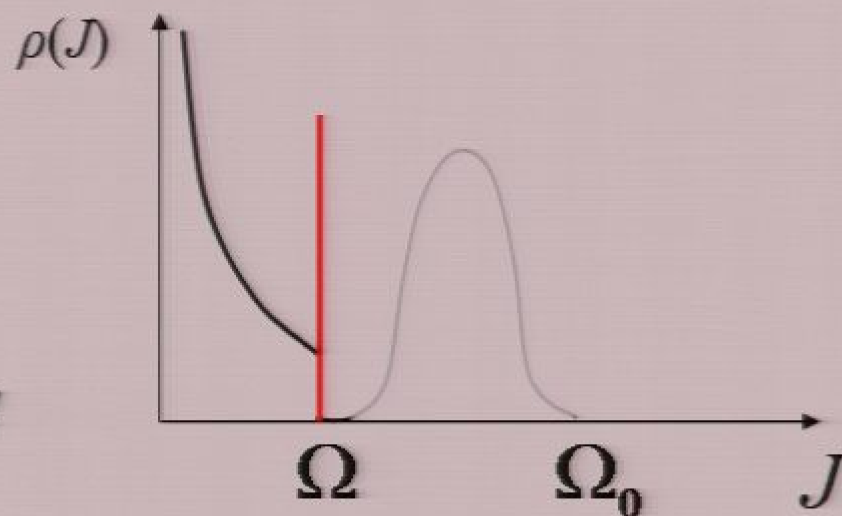

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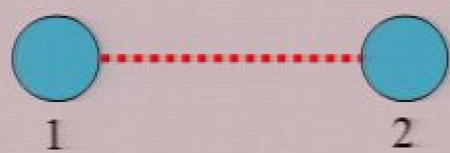

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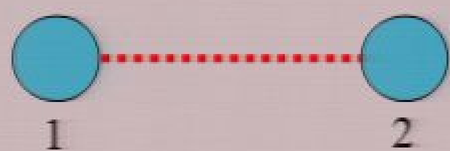

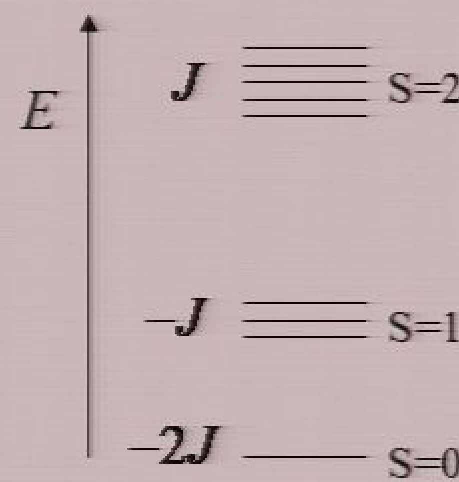

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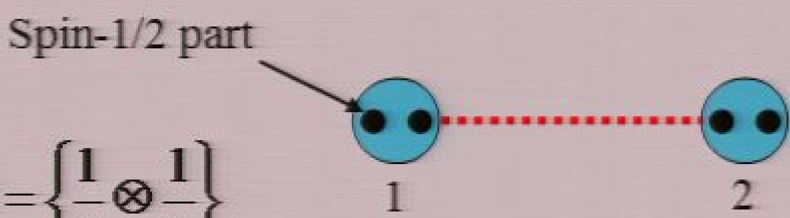

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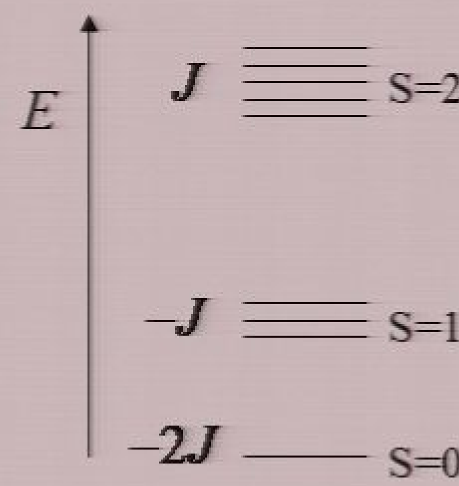

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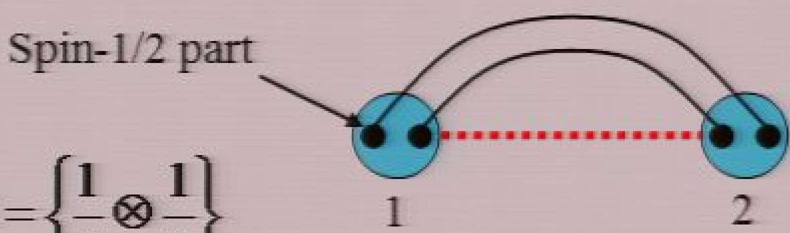

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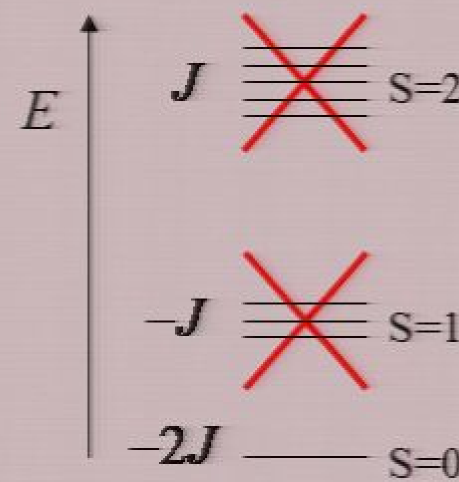

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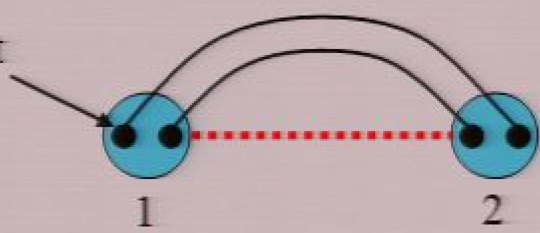
Spin-1/2 part



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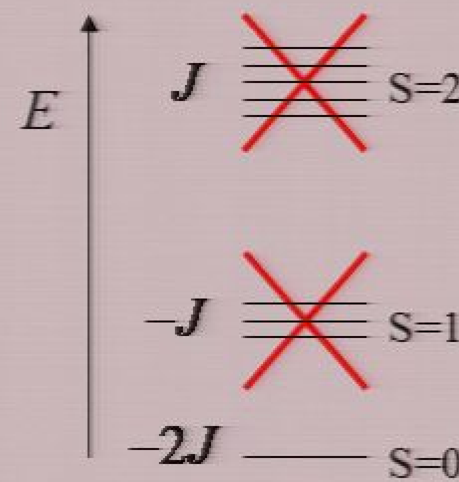

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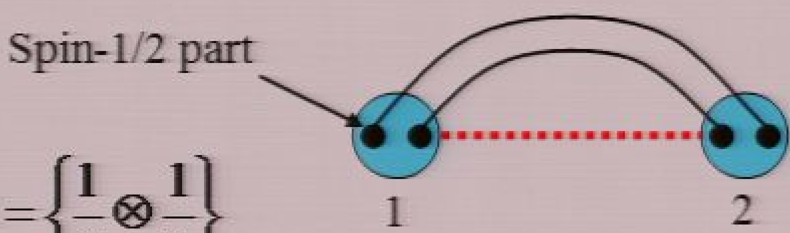

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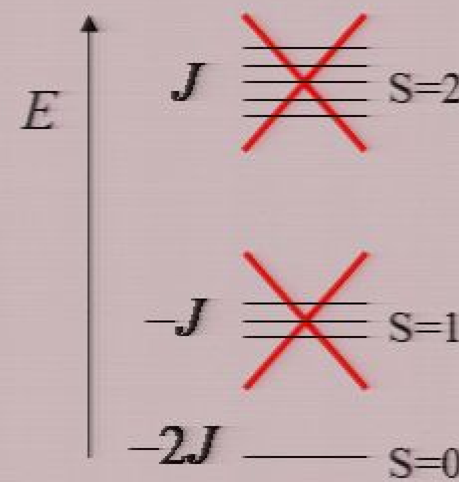

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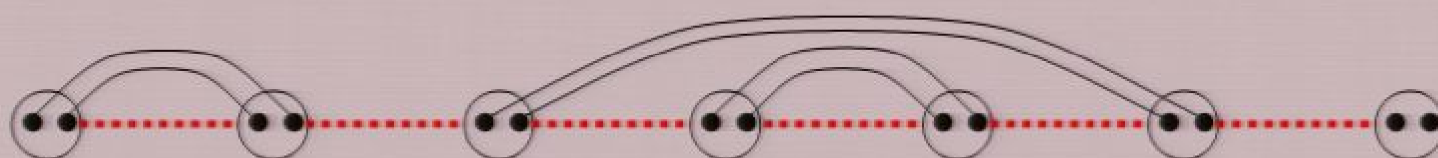

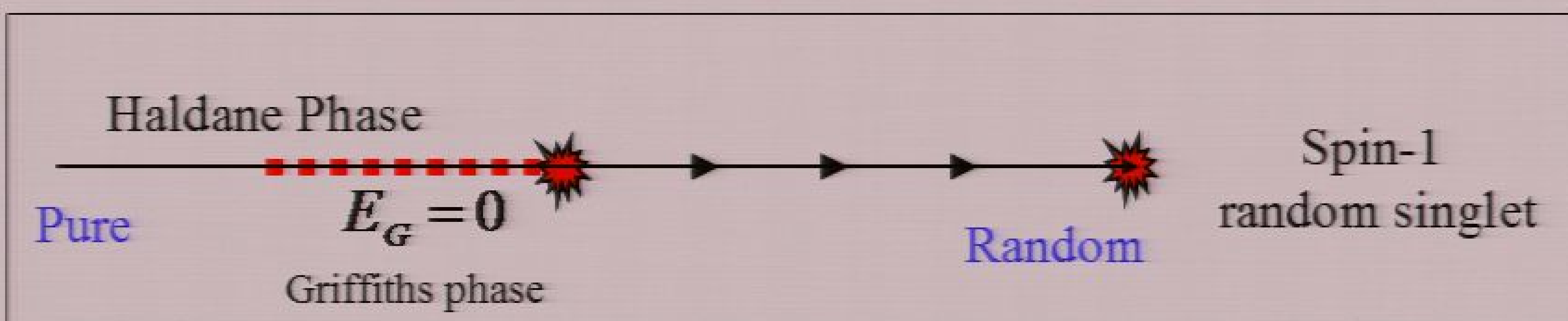

Monthus, Golinelli, Jolicoeur (1998), Hyman, Yang (1997).

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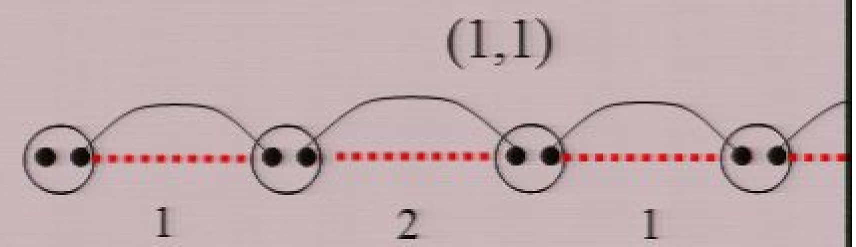

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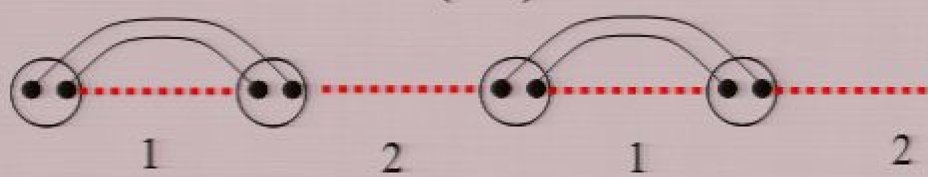
(2,0)



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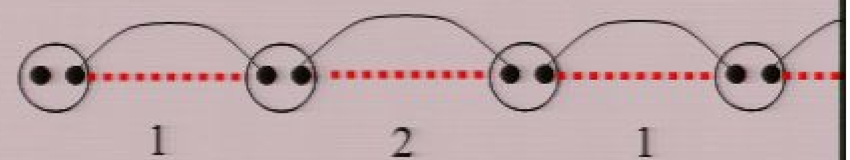

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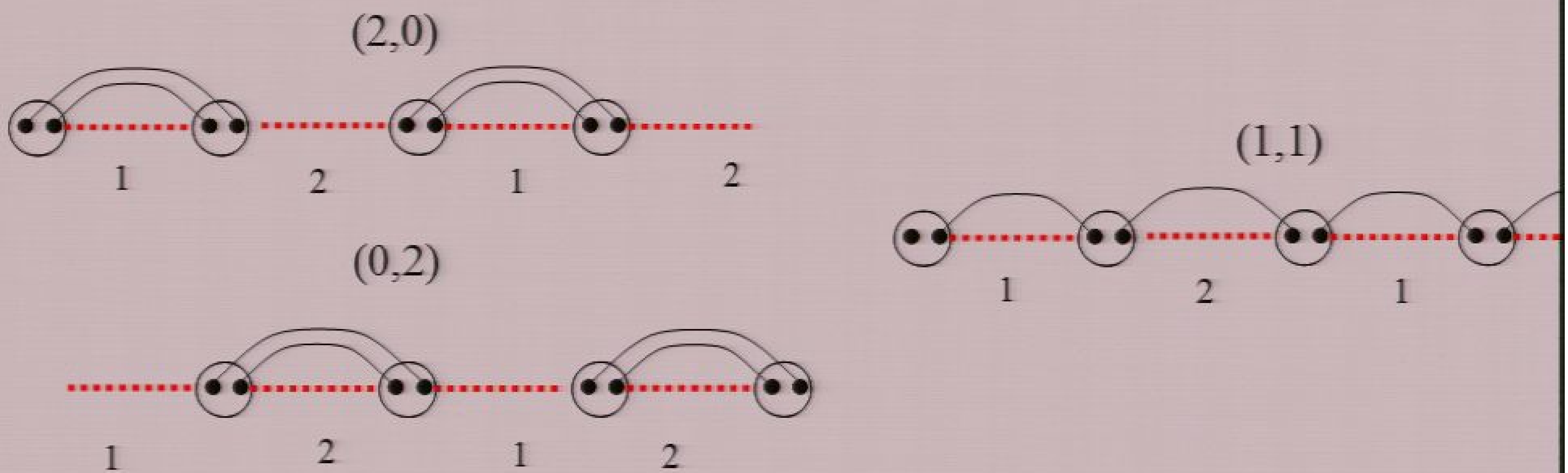

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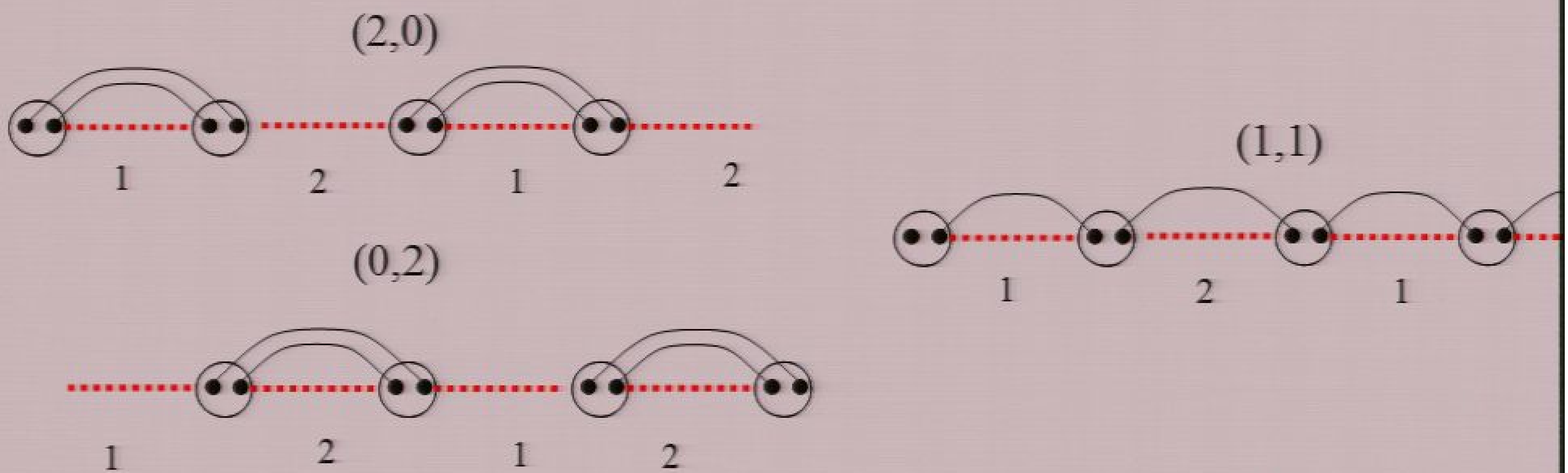

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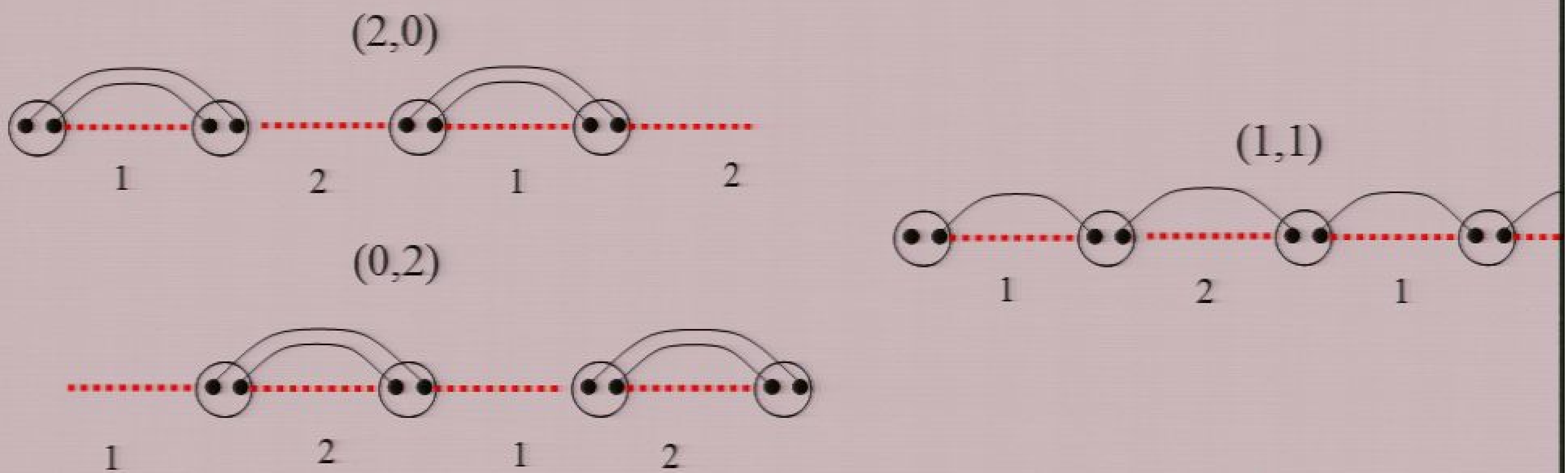

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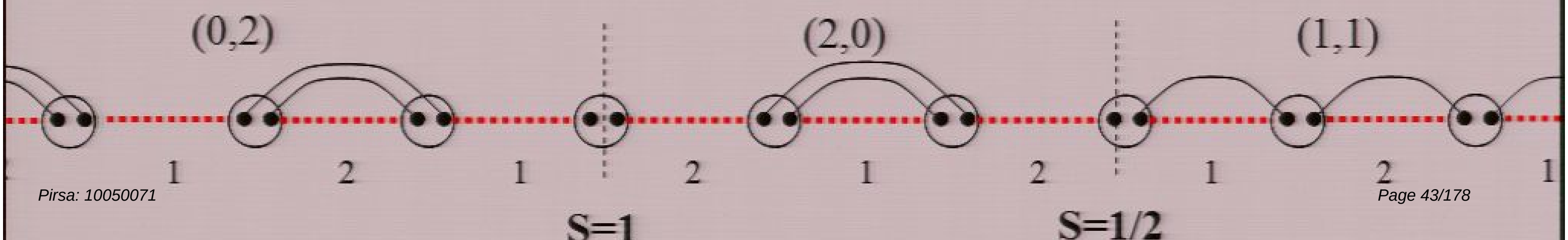

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Compare to $\chi = 1$ for random-singlet phase

- *In general, **spin S**:*

$$\psi = 1/(2S+1)$$

$$\chi = 2S$$

Entanglement entropy in Random Heisenberg chains

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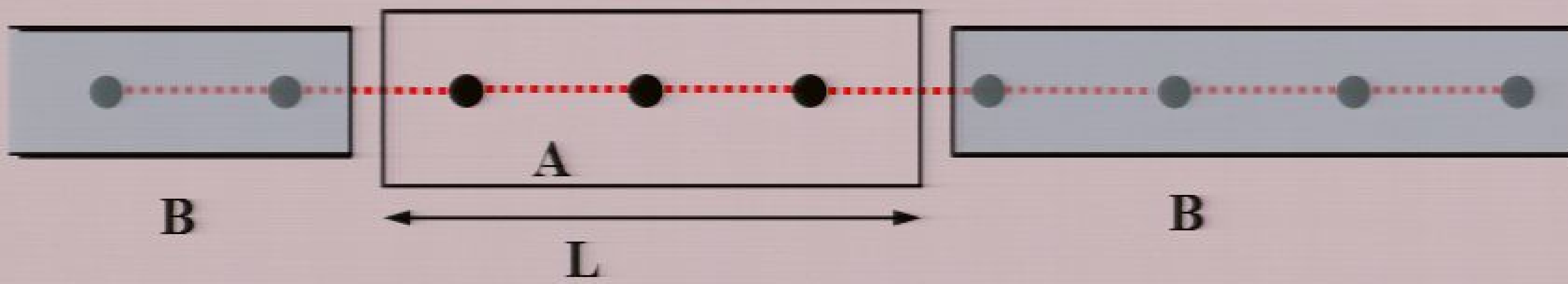

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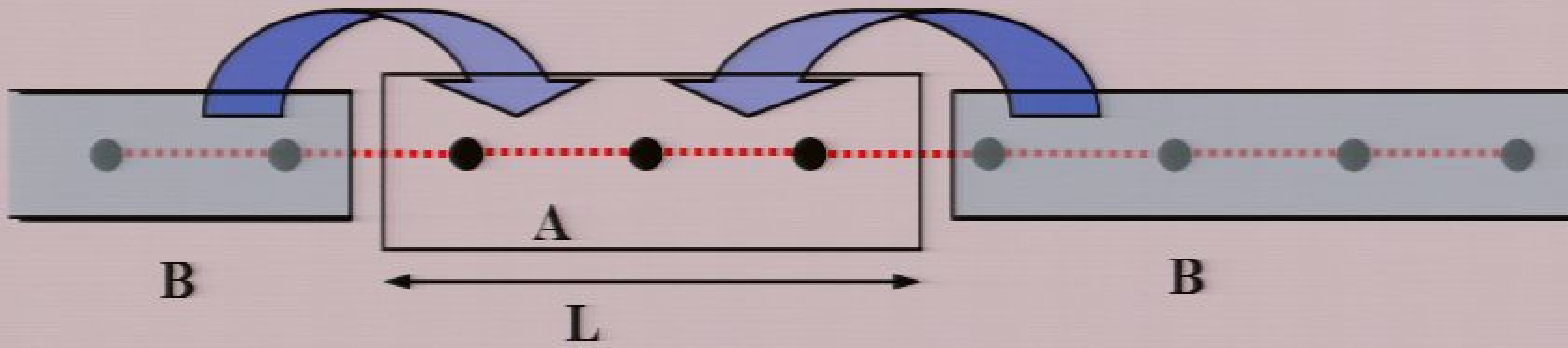

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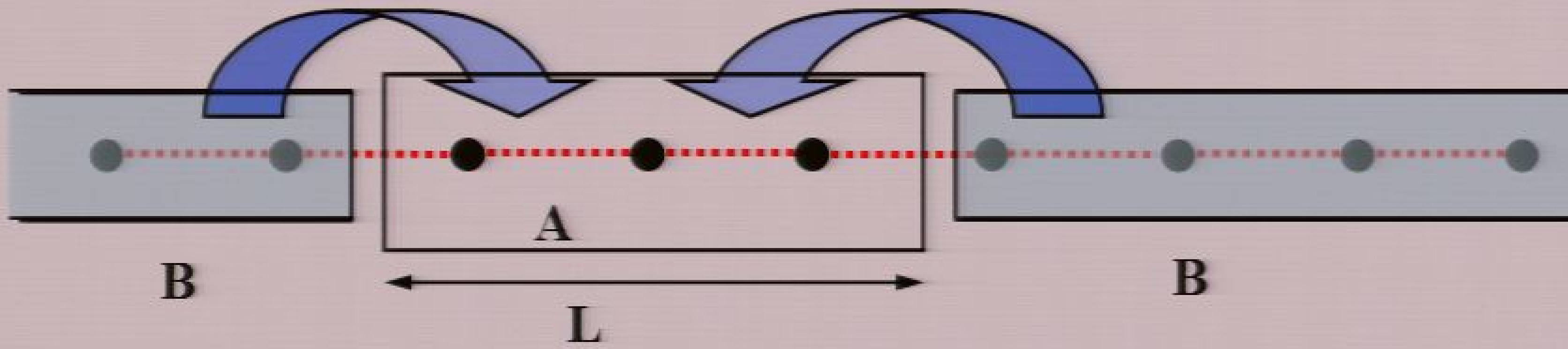

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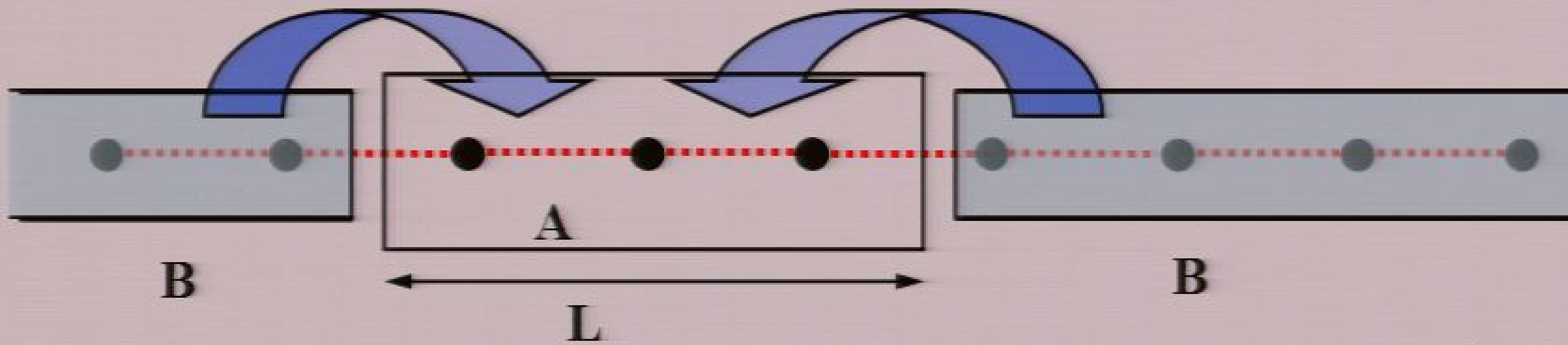

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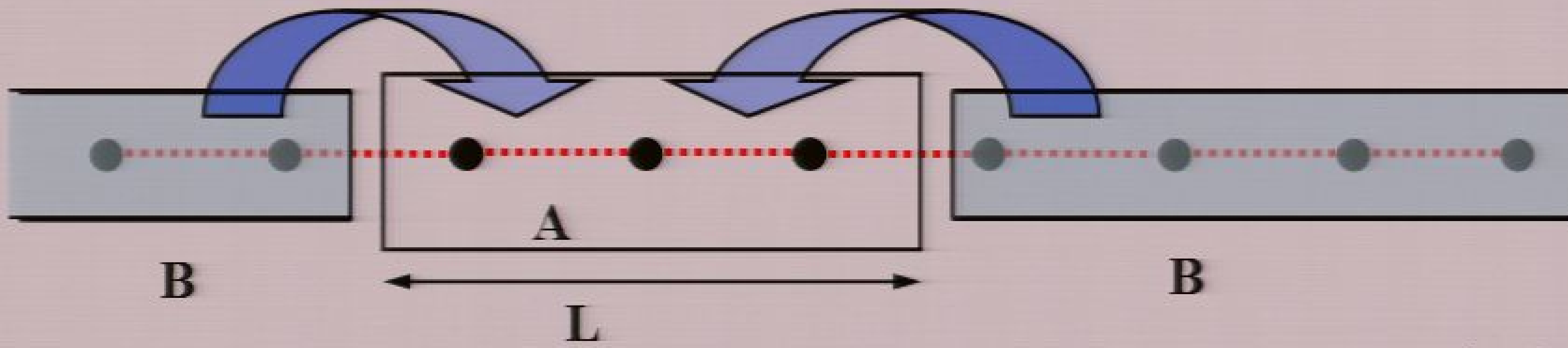

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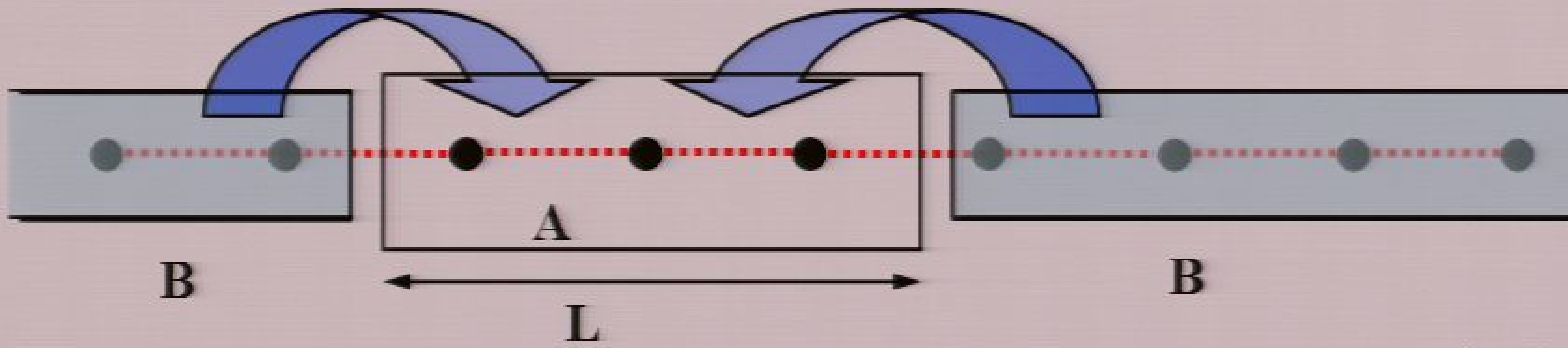

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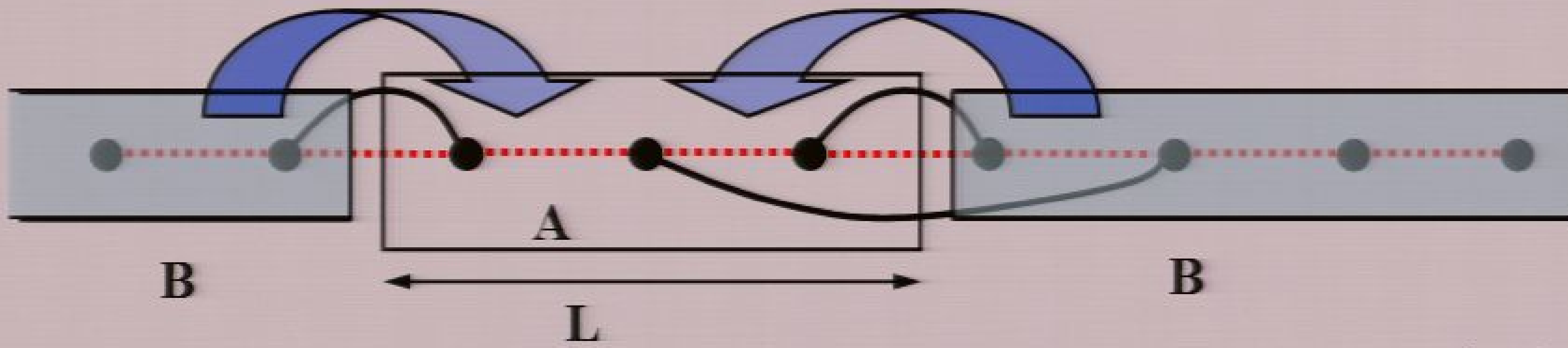

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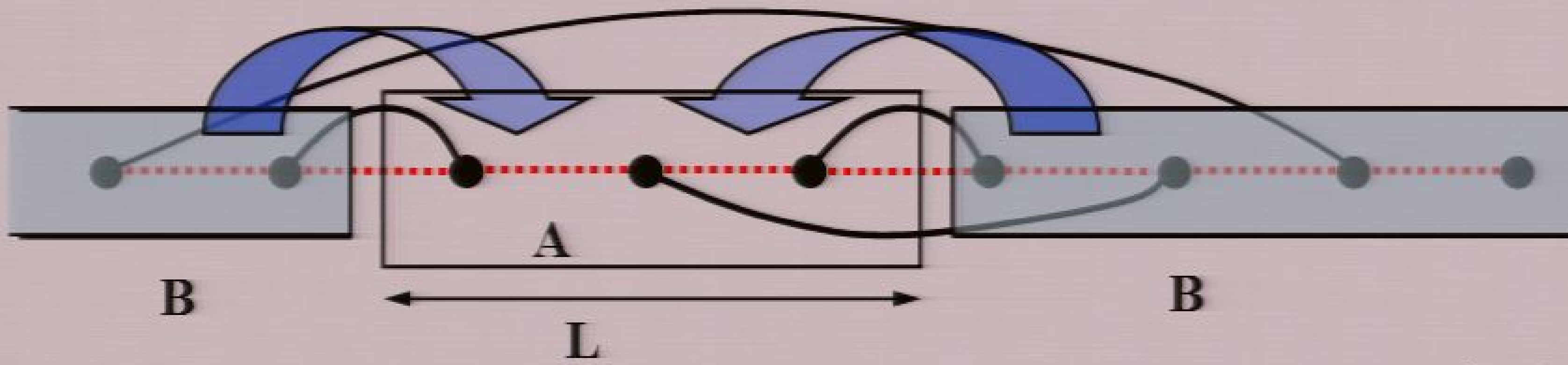

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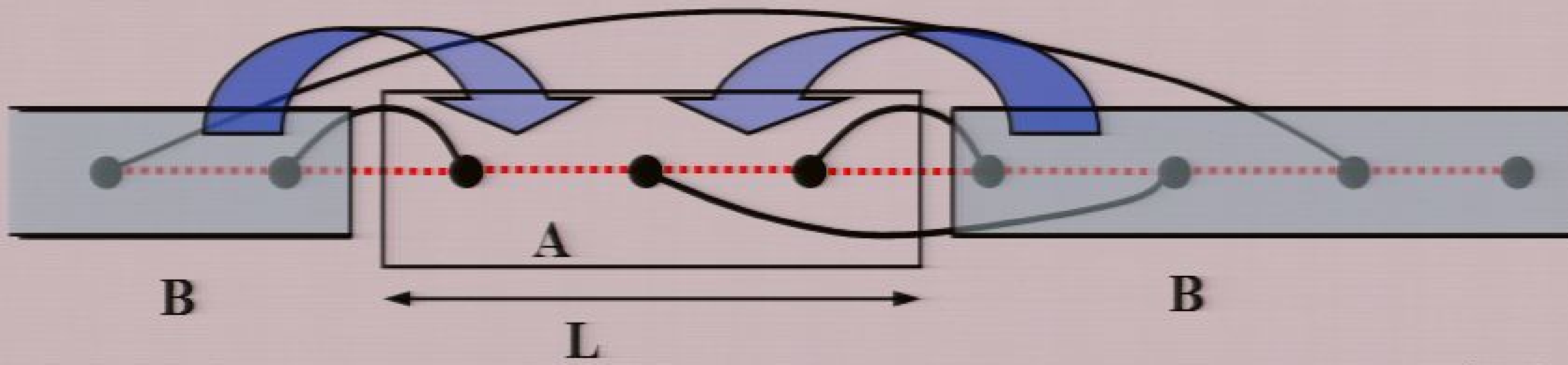

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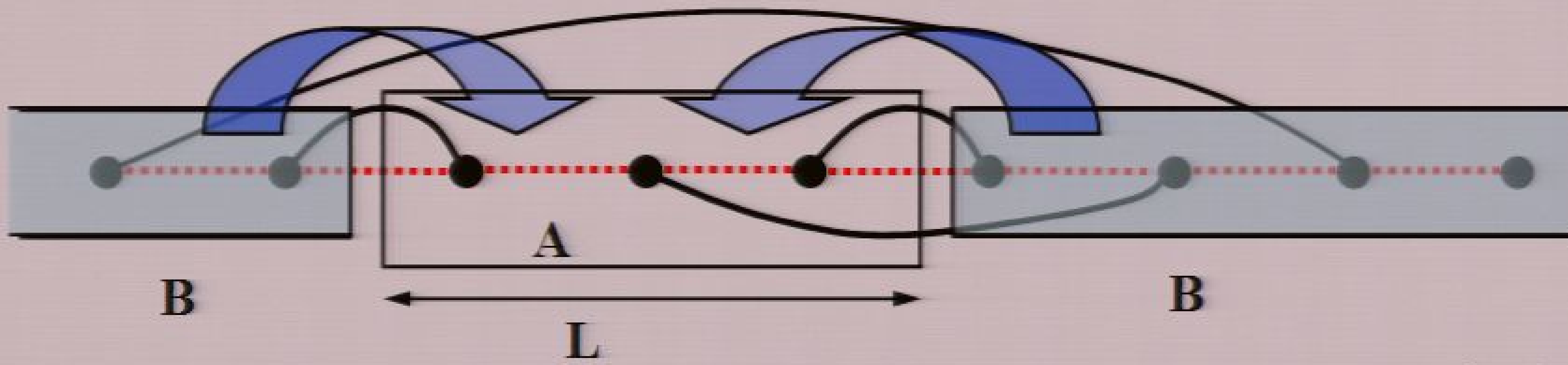
$$E_L = -Tr_A \rho_A \log_2 \rho_A = N_{singlet}$$

$N_{singlet}$ = number of singlets entering region A.

Engtanglement entropy in the Heisenberg model



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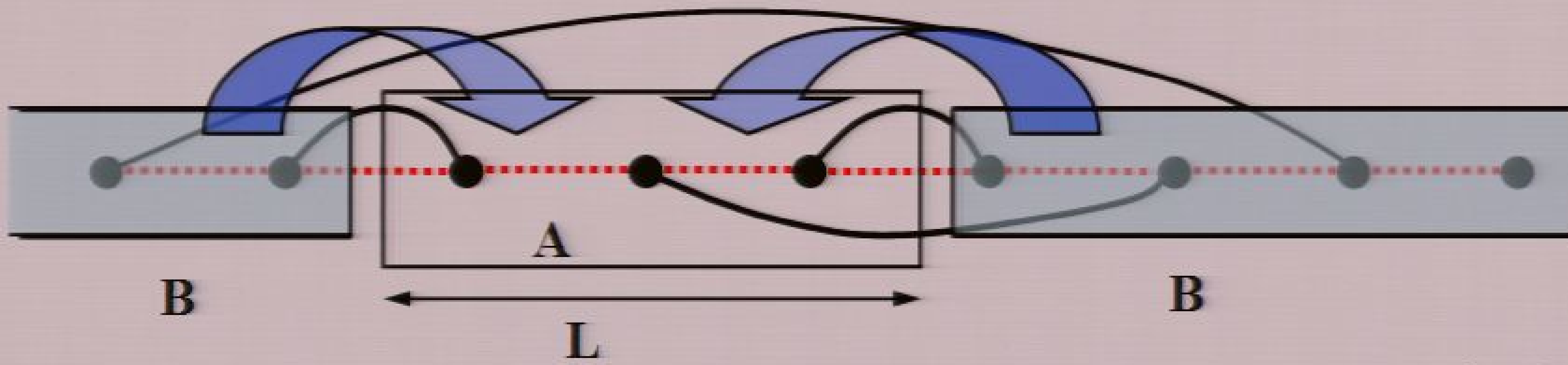

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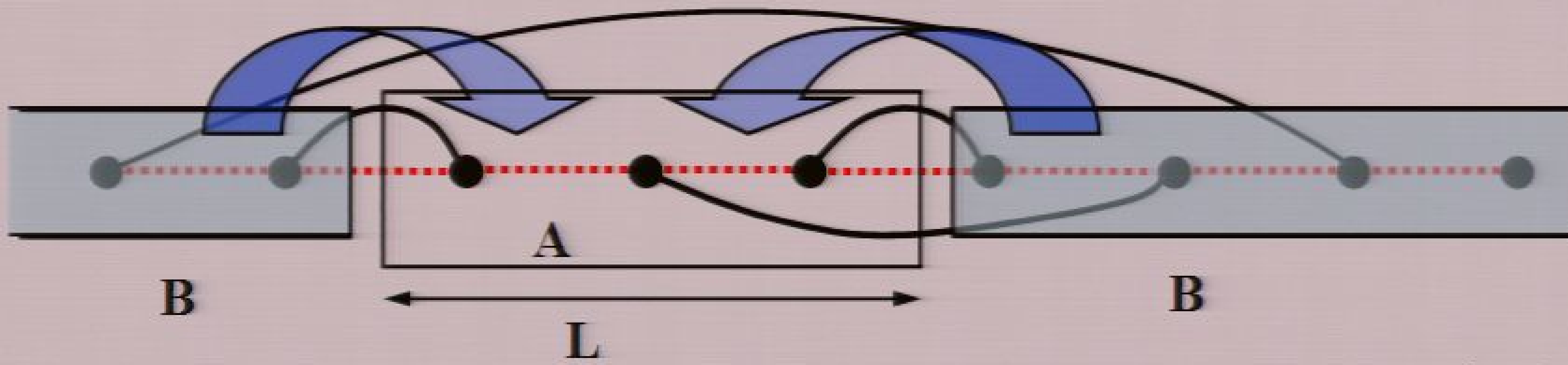

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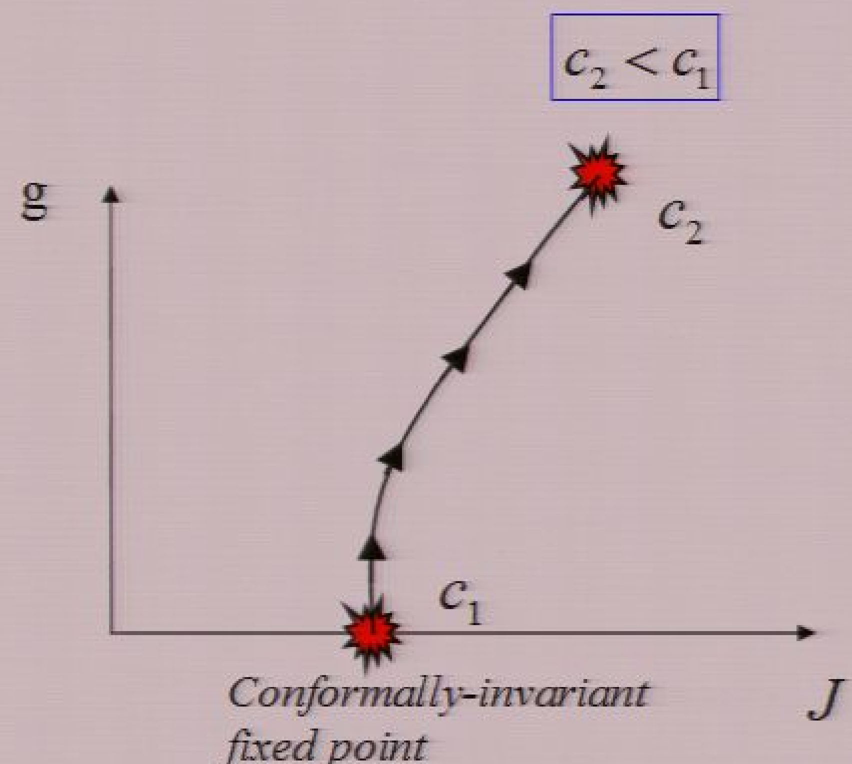
• Generally: $c_{RS} = \ln D$ $\quad D$ – dimension of local Hilbert space

(Bonsteel, Yang, 2006)

Generalized c-theorem?

• The central charge always decreases along RG flows. (Zamolodchikov, 1986)



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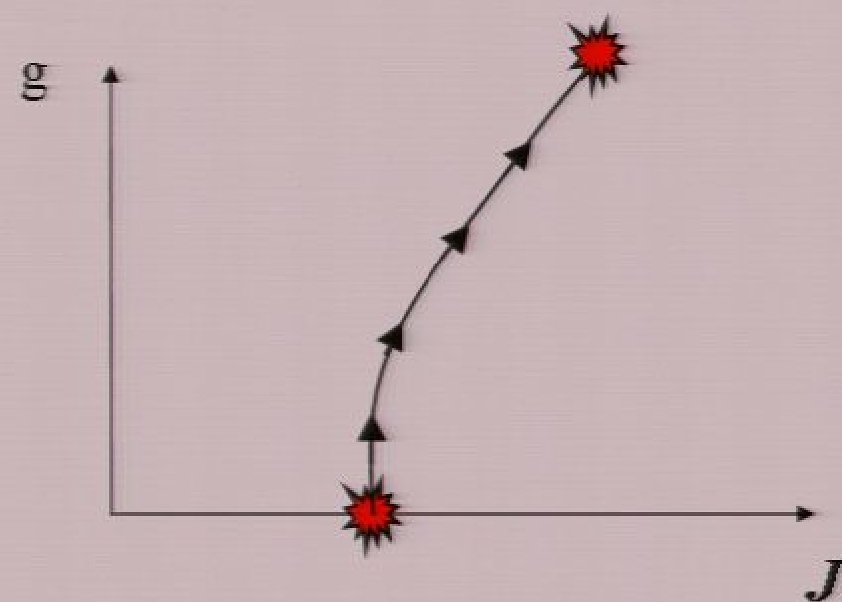

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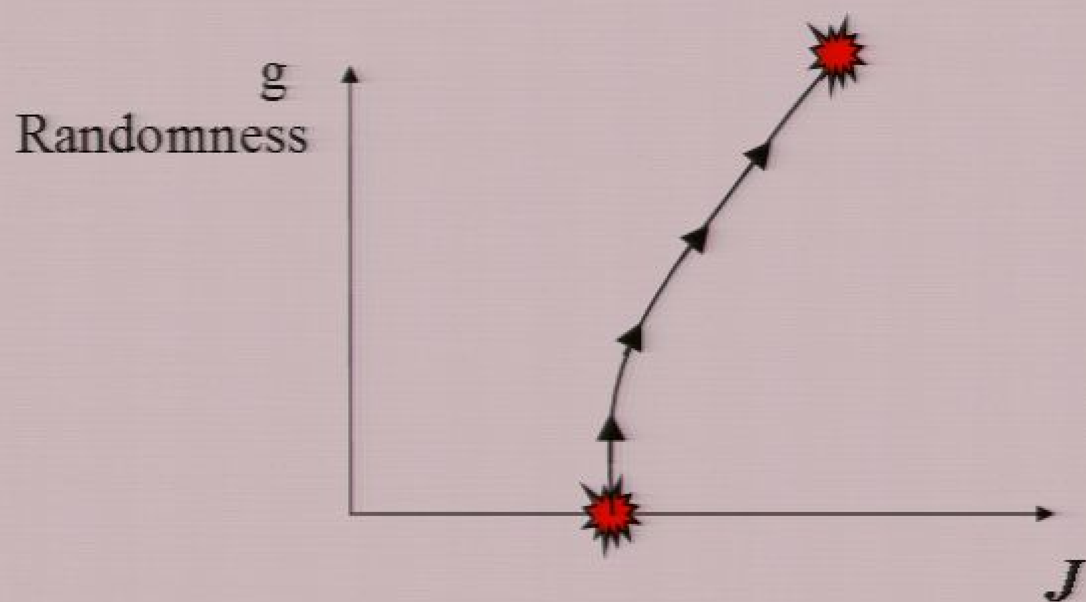

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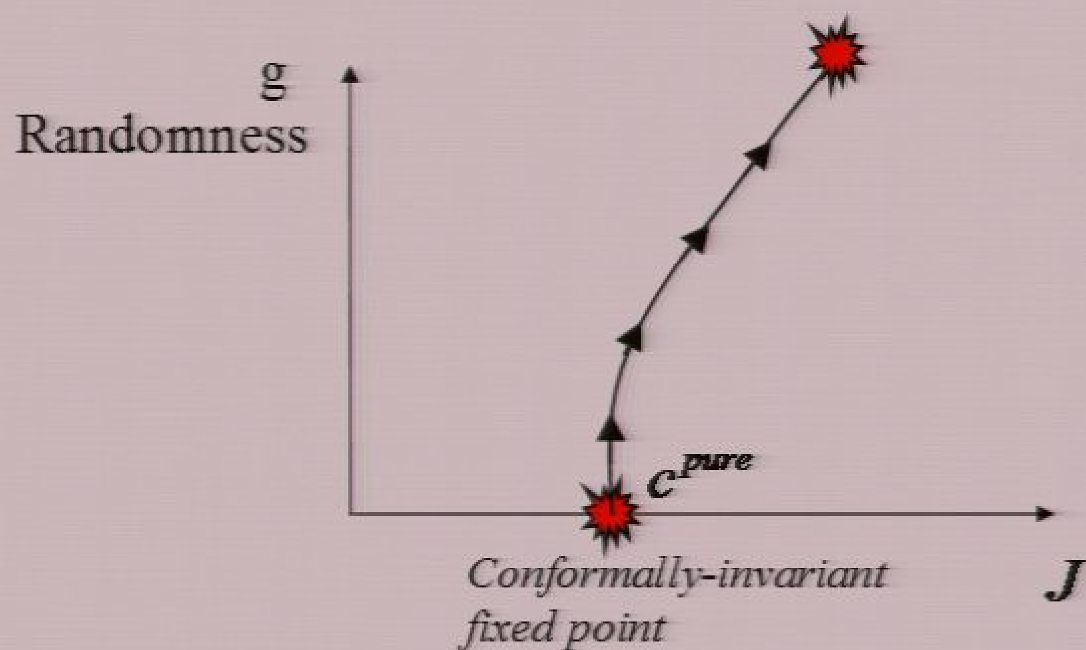

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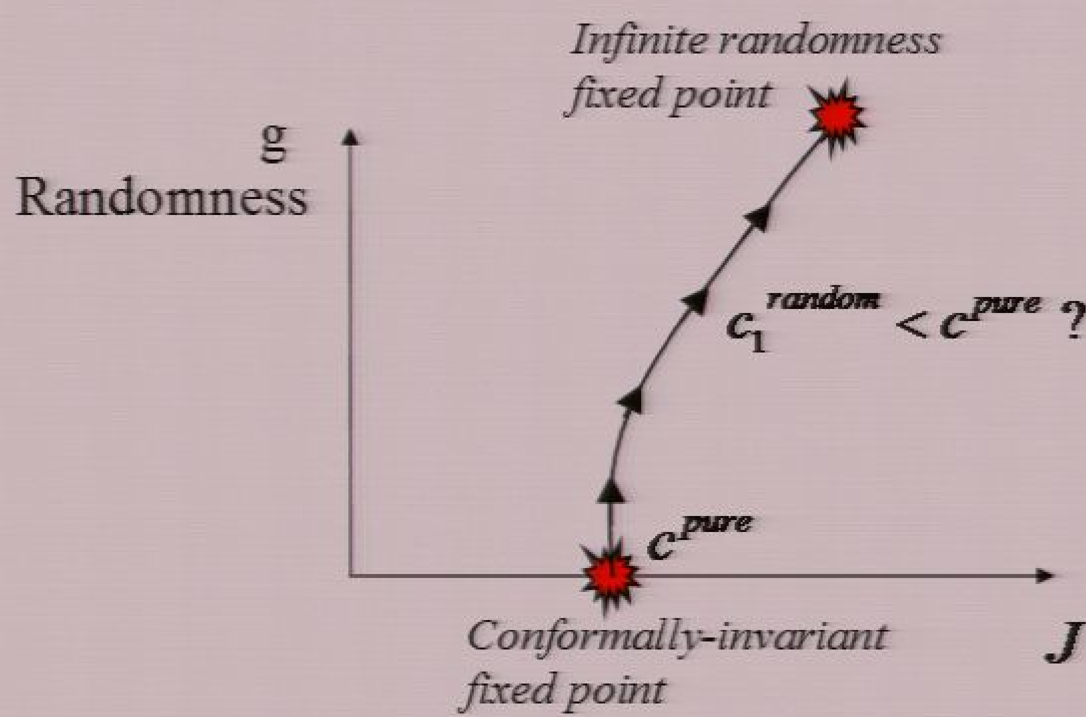

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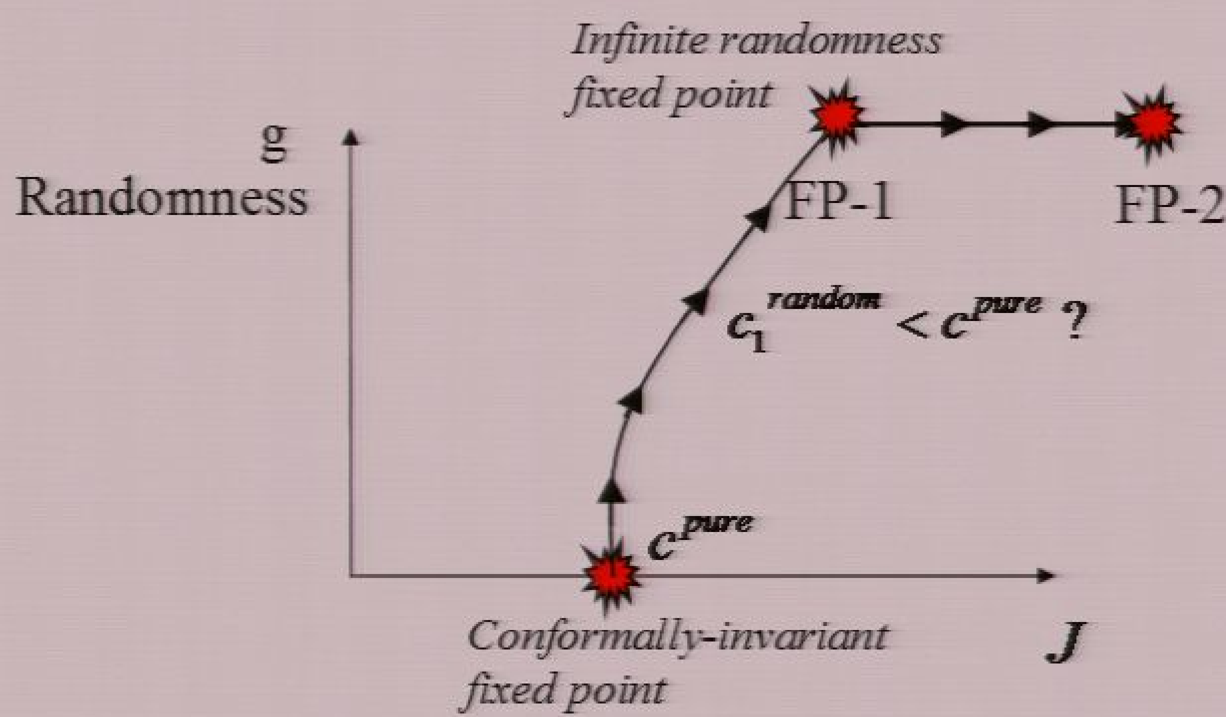

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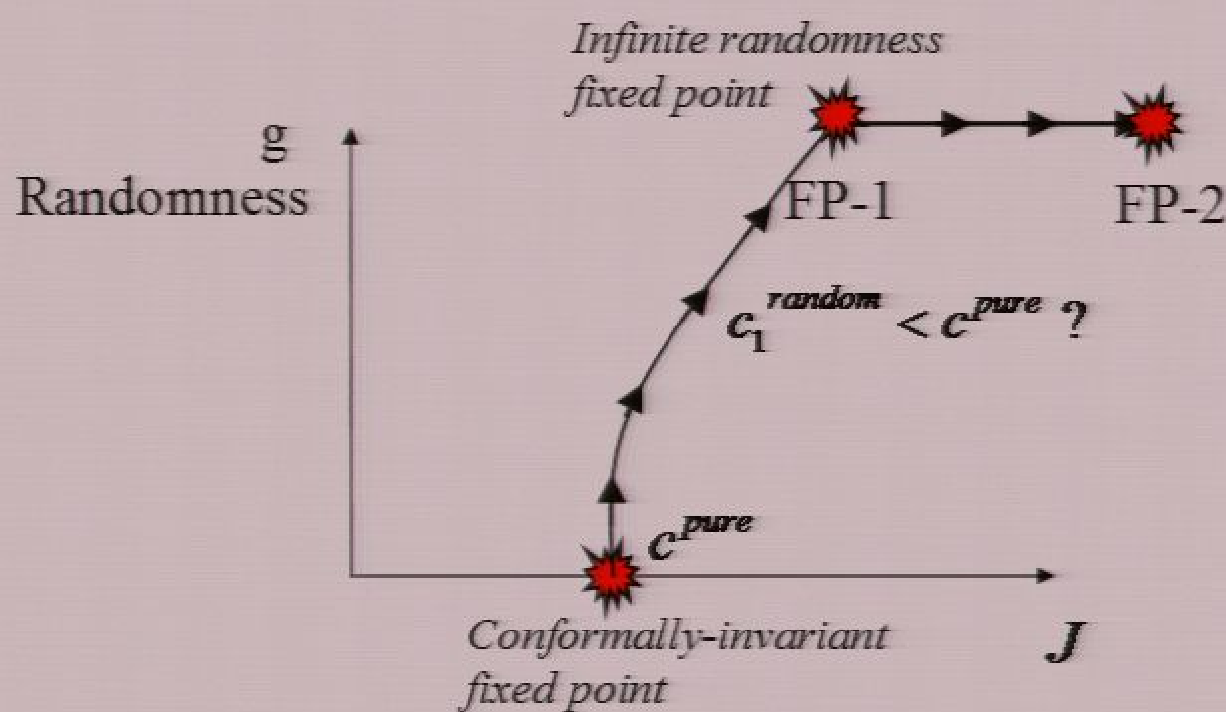

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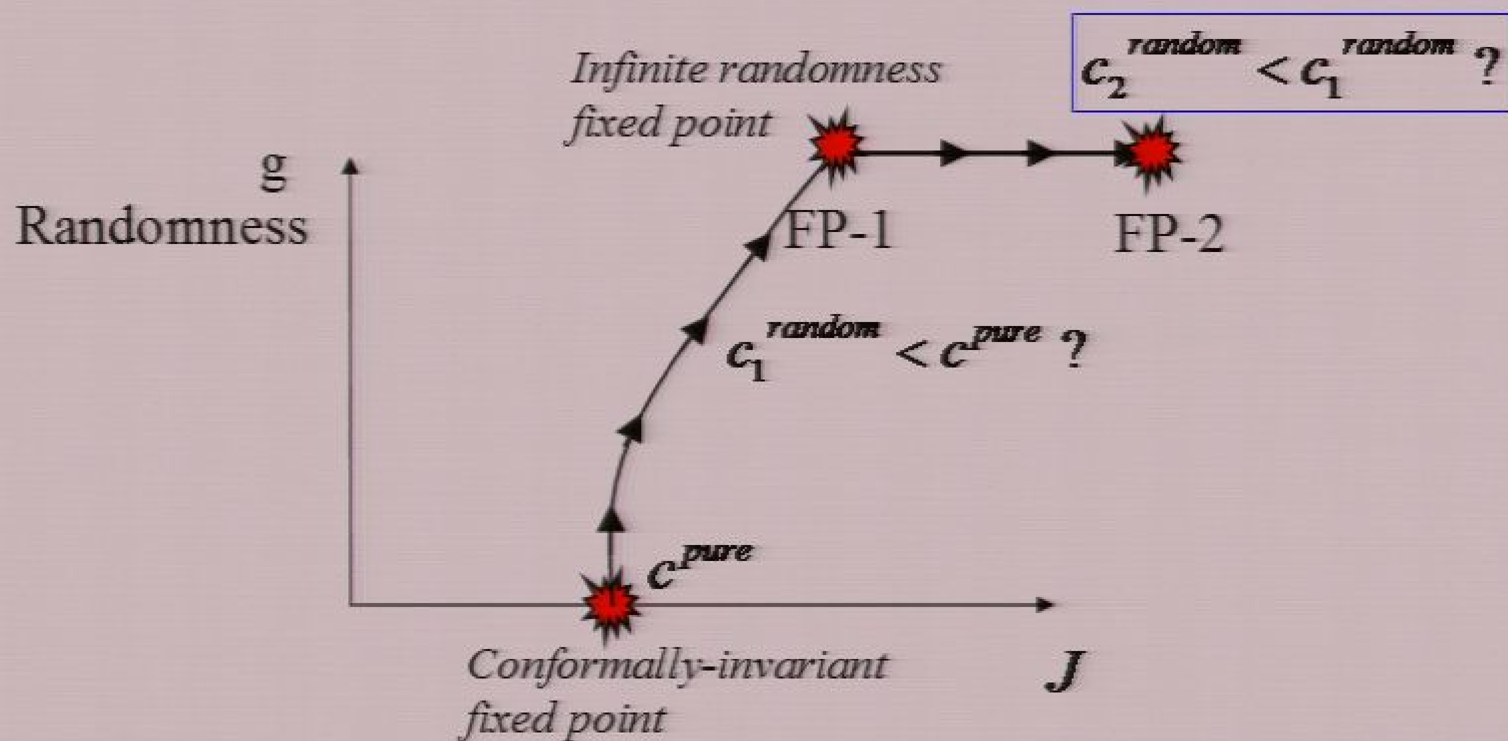

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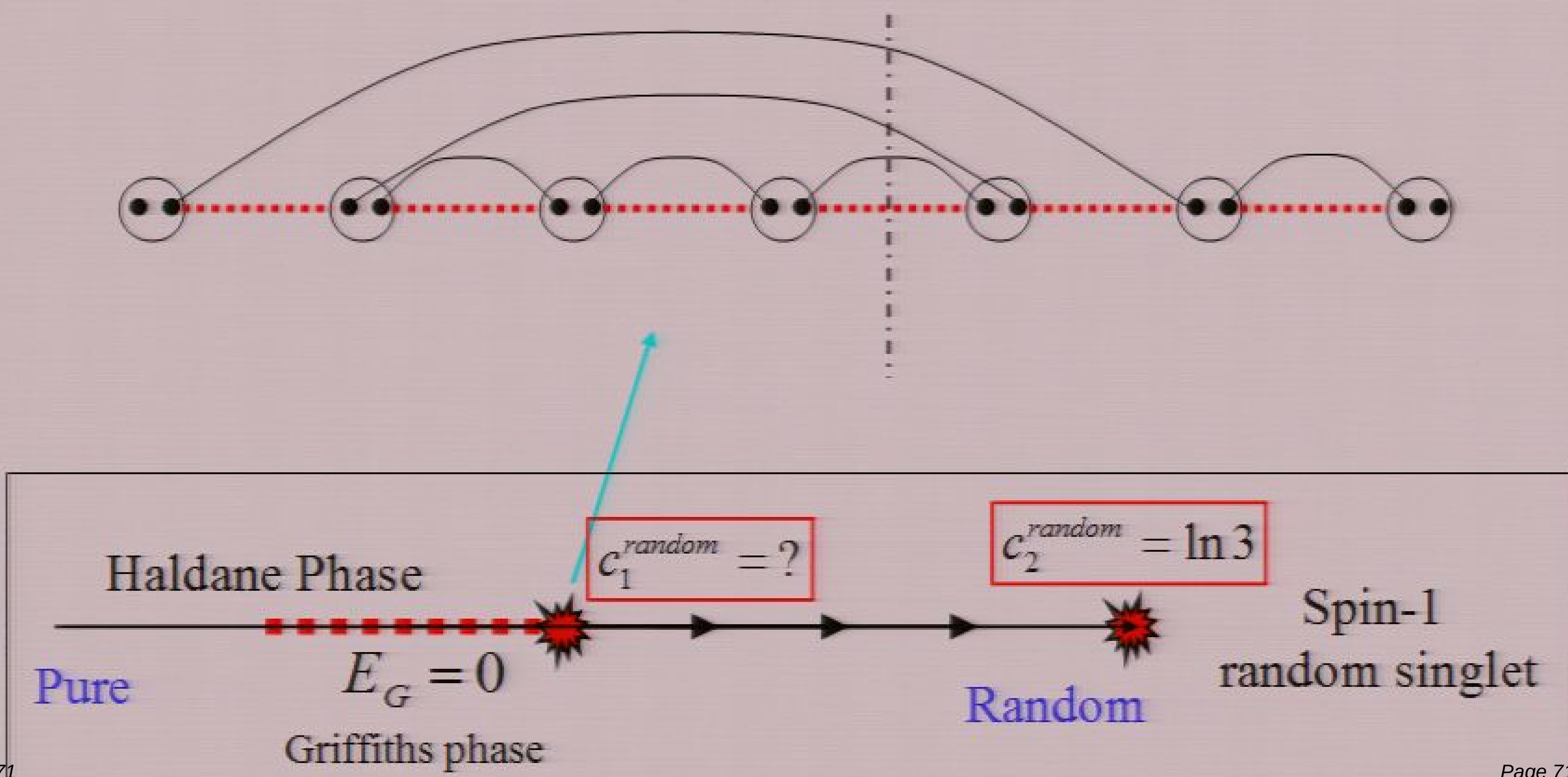
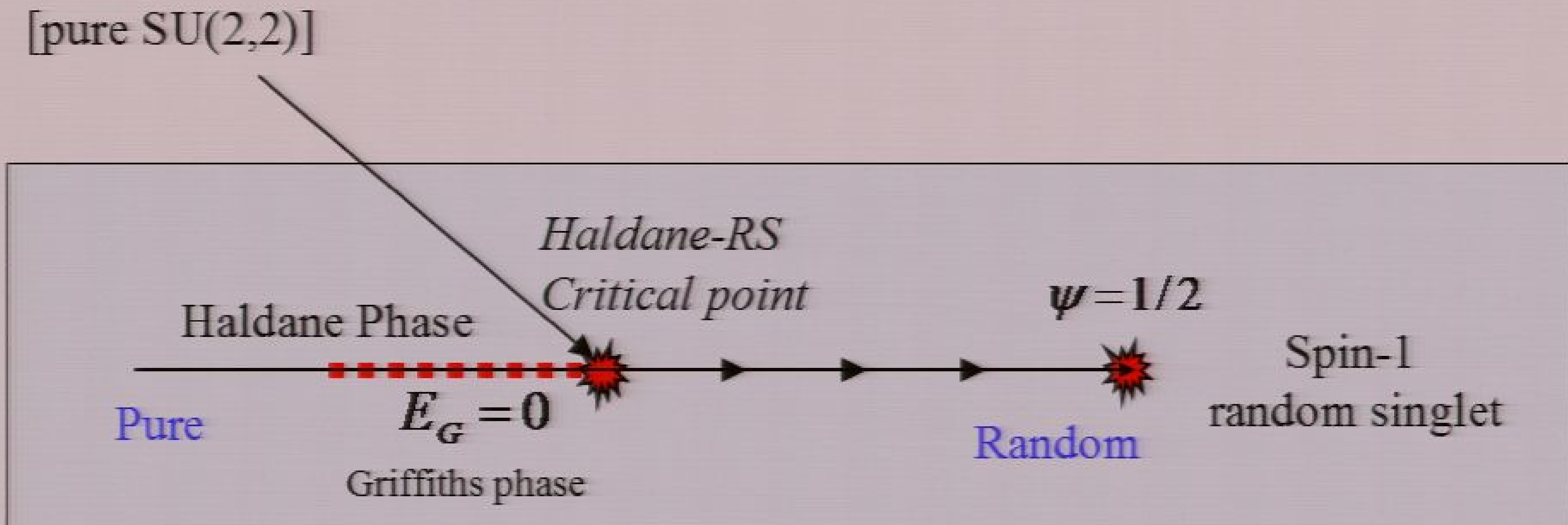
Spin-1 "SU(2,2)" critical point

- Complex spin configurations:



Spin-1 Heisenberg results

- Lenghty calculation gives:



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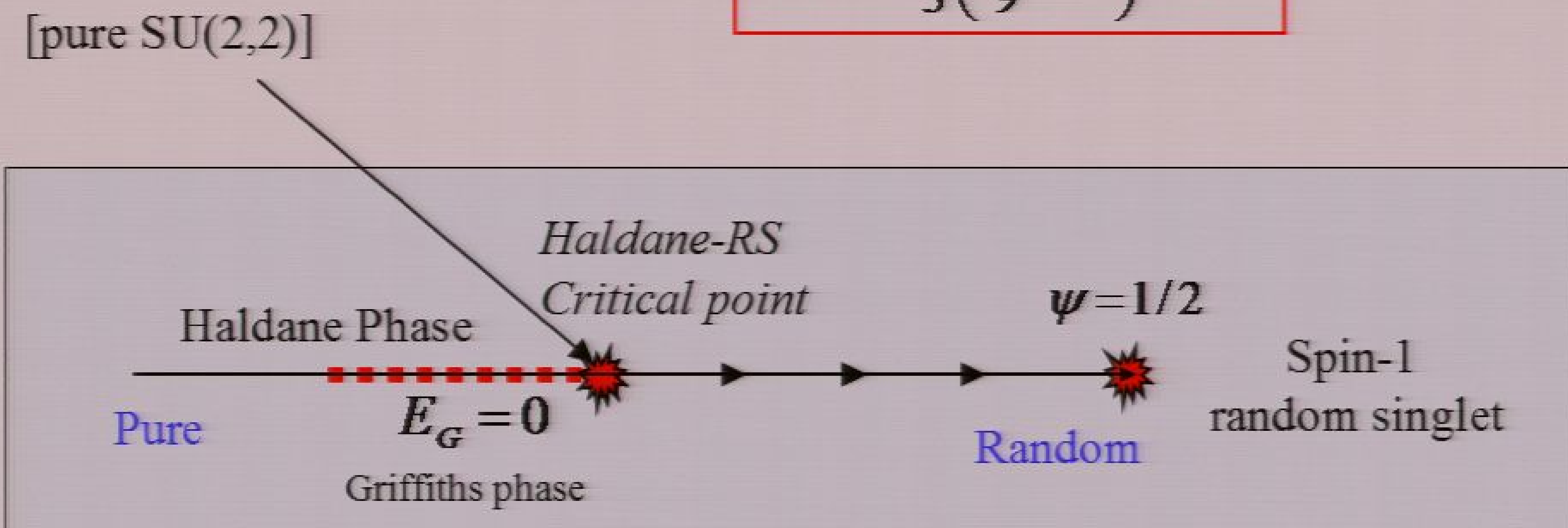
$$E_{H-RS} \approx \frac{1}{3}\left(\frac{16}{9}\ln 2\right)\log_2 L$$



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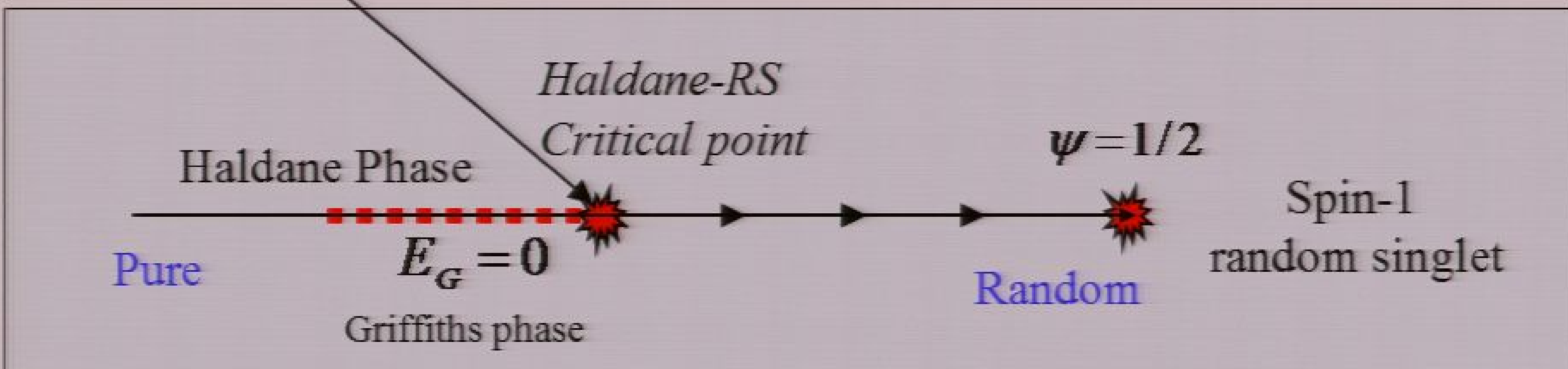

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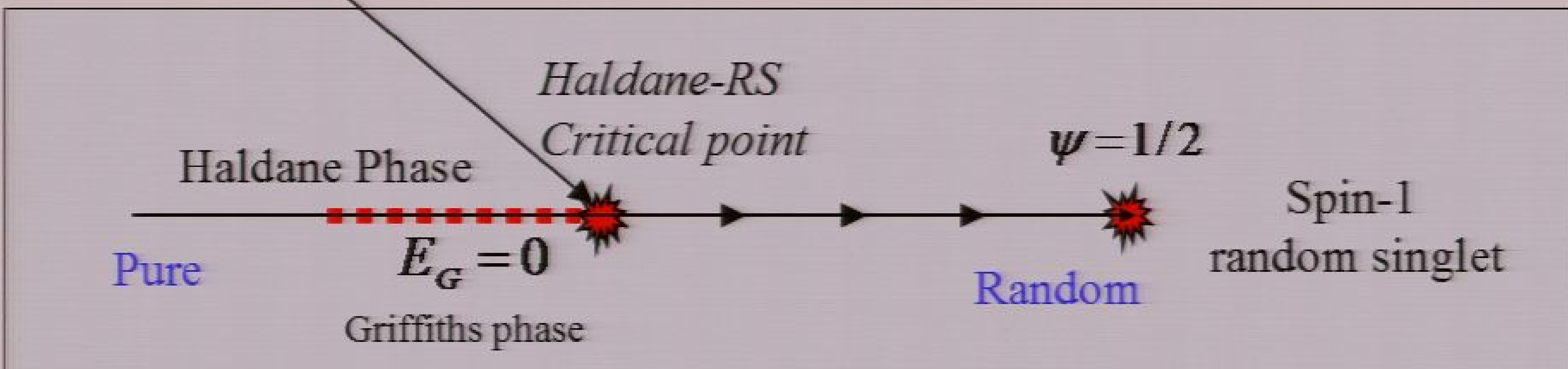

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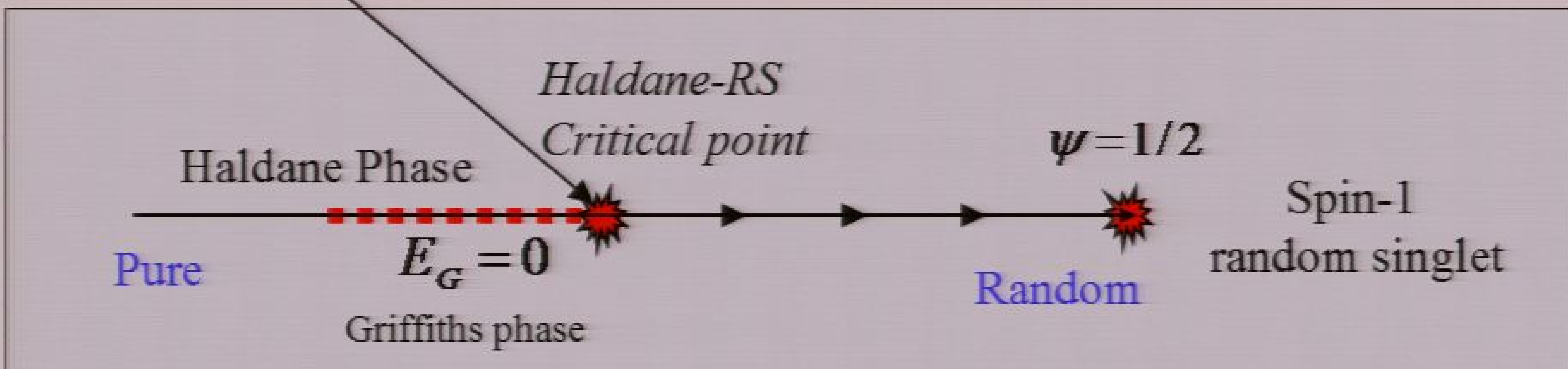

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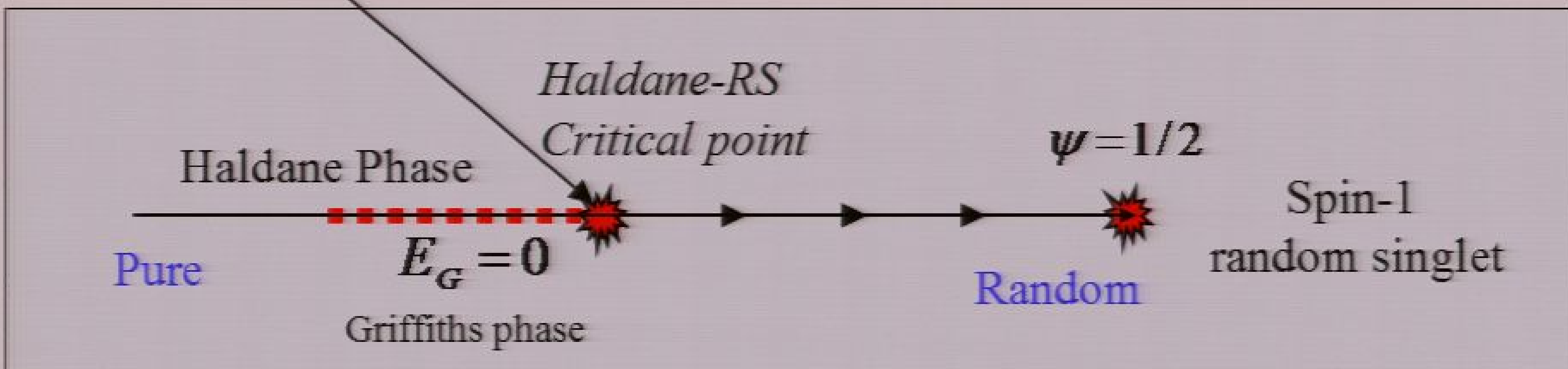

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Random Non-abelian Spin Chains

Other spin systems II: random sign Heisenberg models

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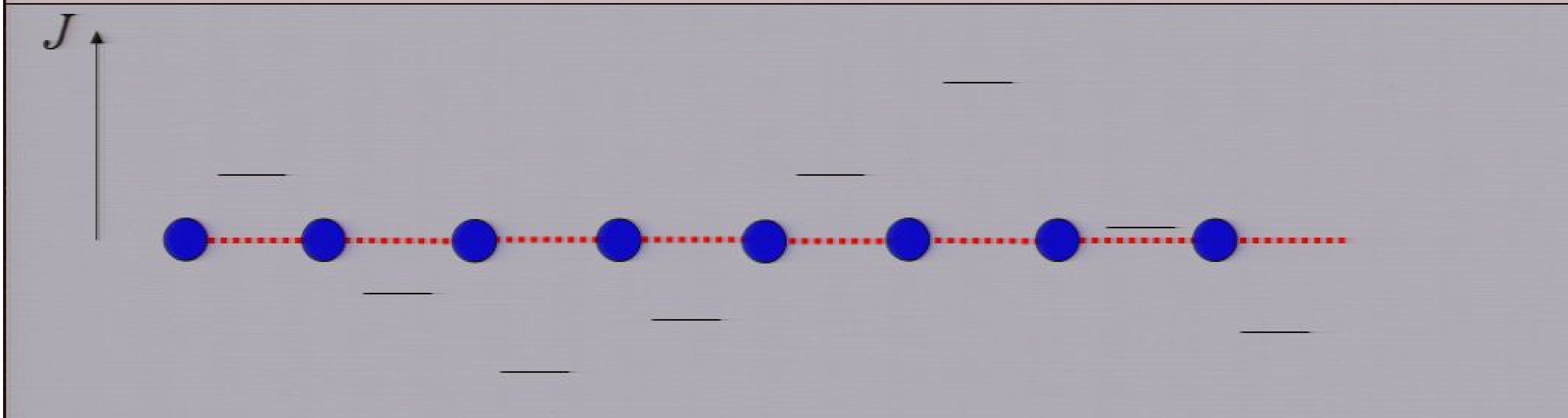

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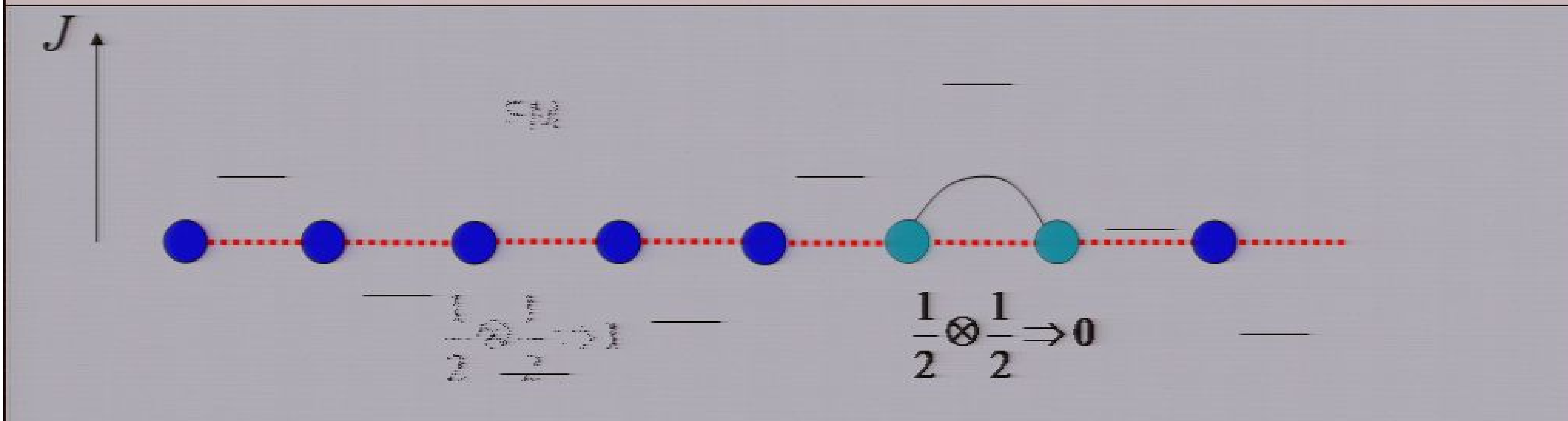

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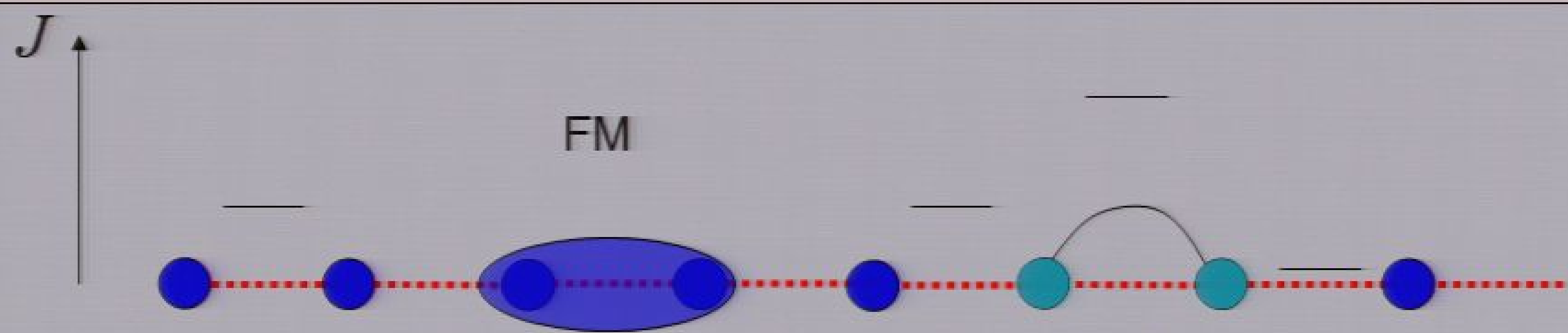

$$\frac{1}{2}\otimes\frac{1}{2}\Rightarrow 1 \qquad \frac{1}{2}\otimes\frac{1}{2}\Rightarrow 0$$

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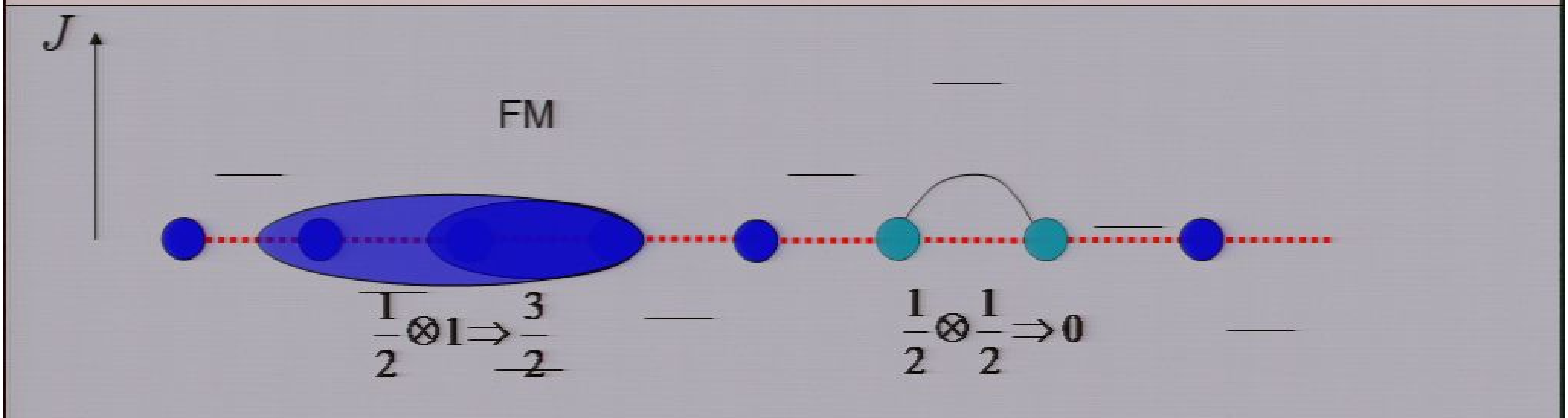

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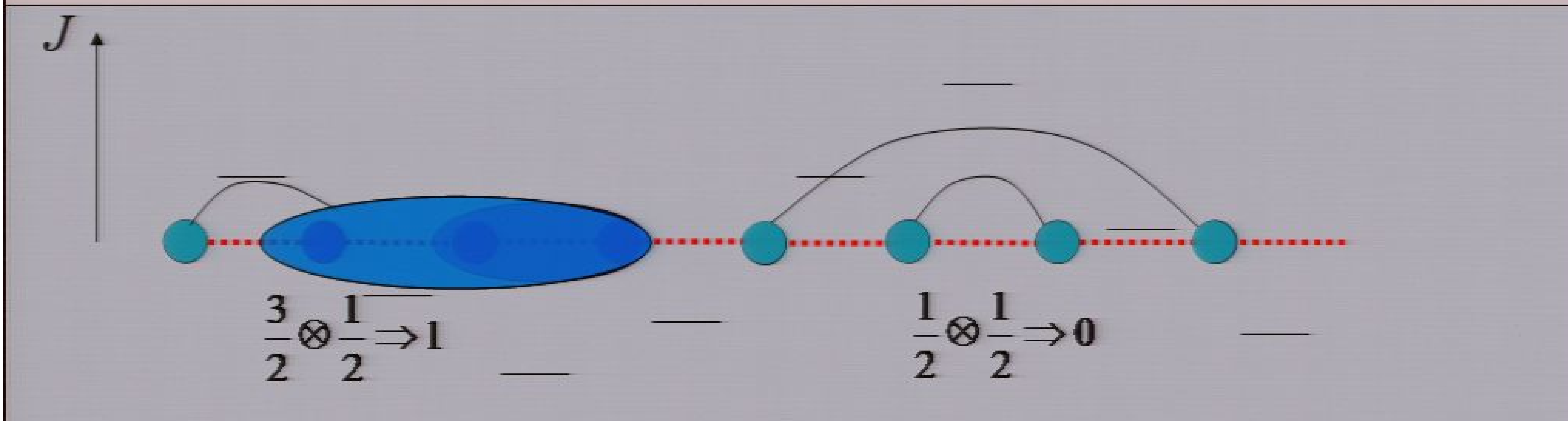

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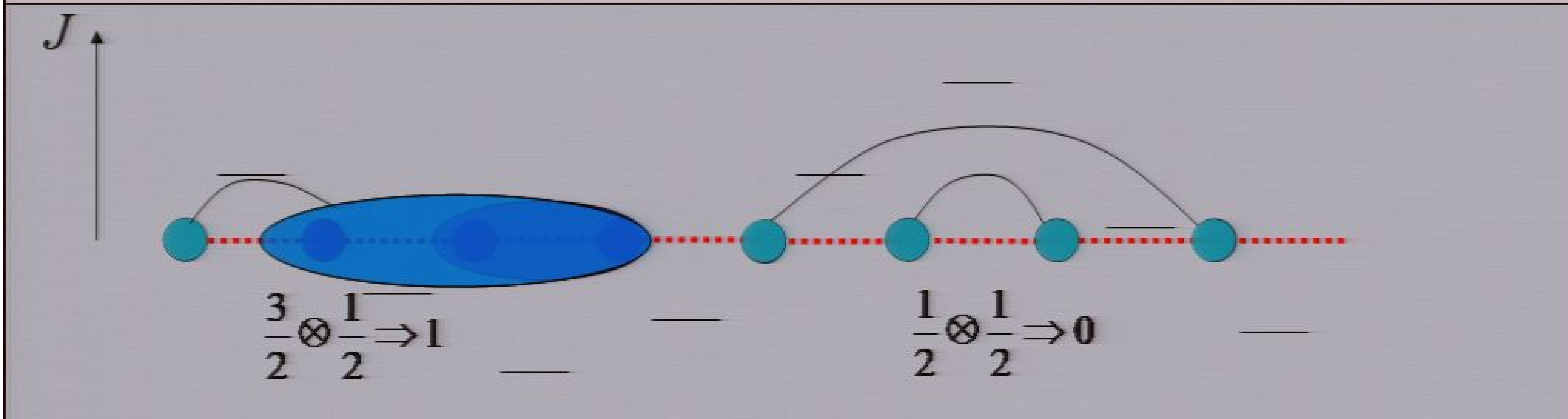

Higher and higher moments form

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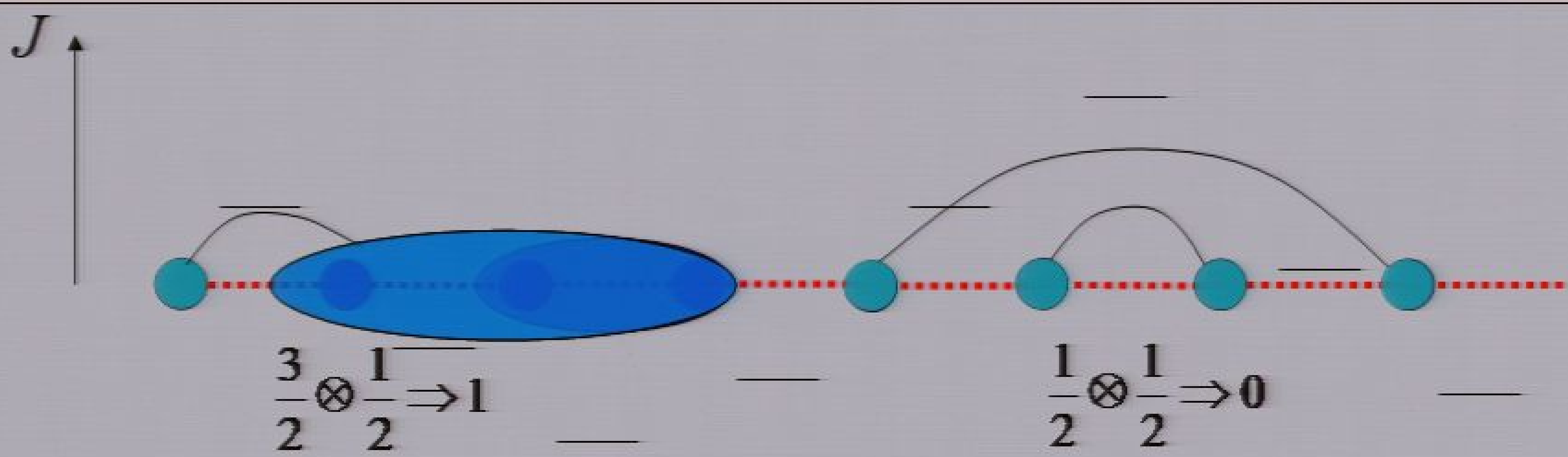

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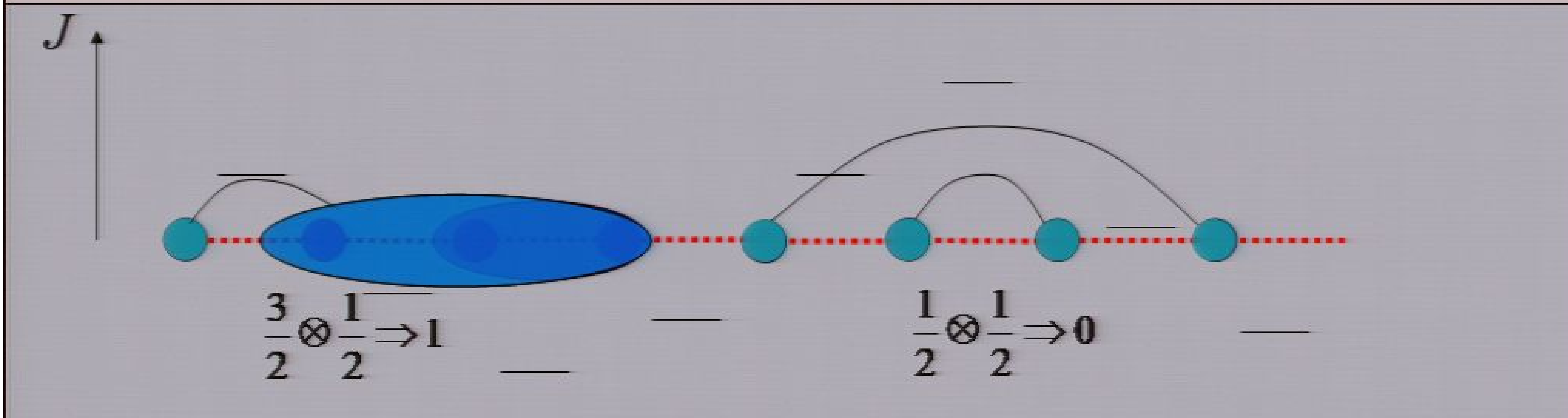

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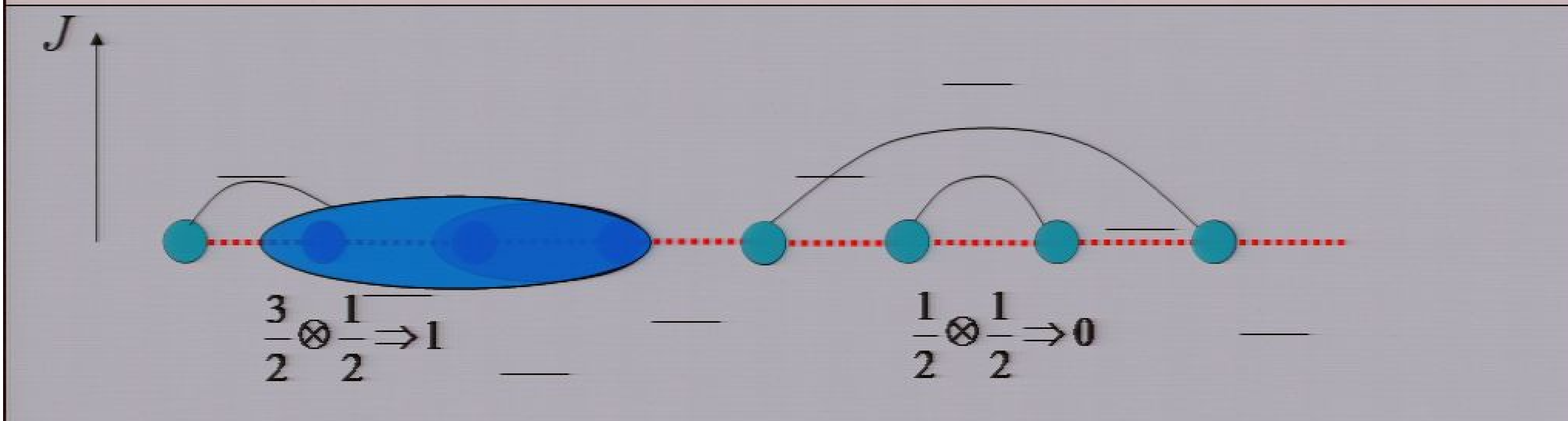

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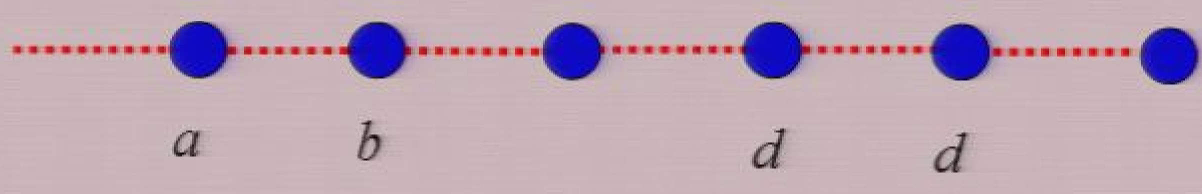
E. Westerberg, A. Furusaki, M. Sigrist, and P. A. Lee (1995)

Other spin systems III:
Non-abelian spin chains

Fidkowski, Refael, Bonesteel, Moore (2008)



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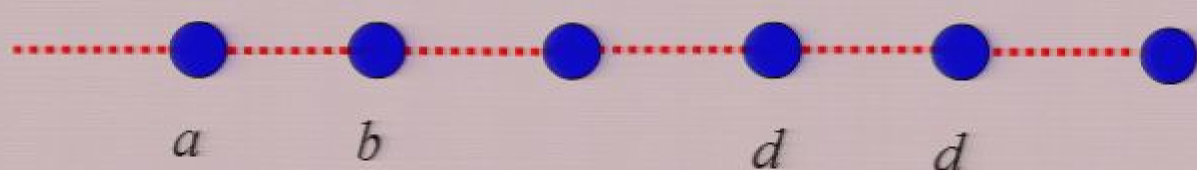

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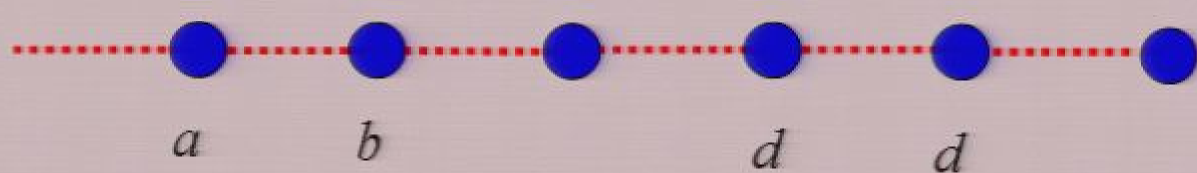

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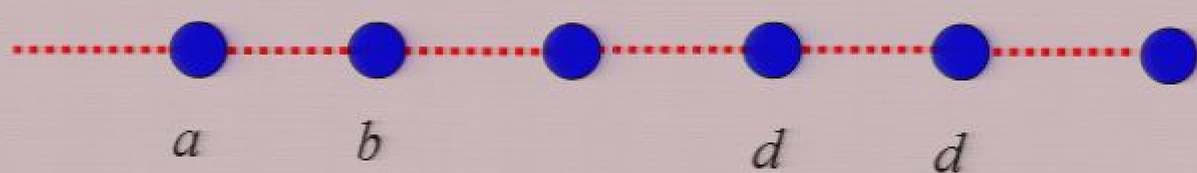

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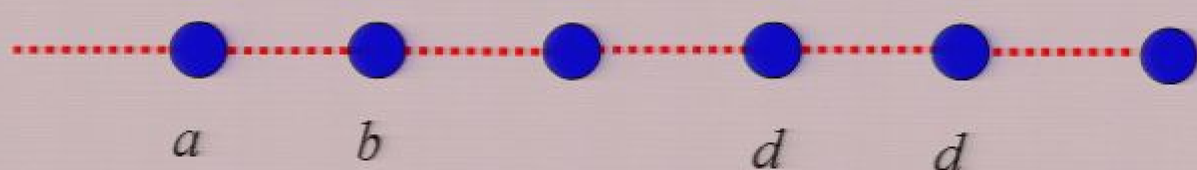

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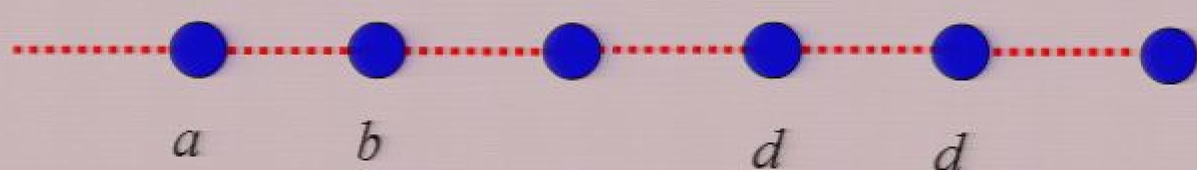

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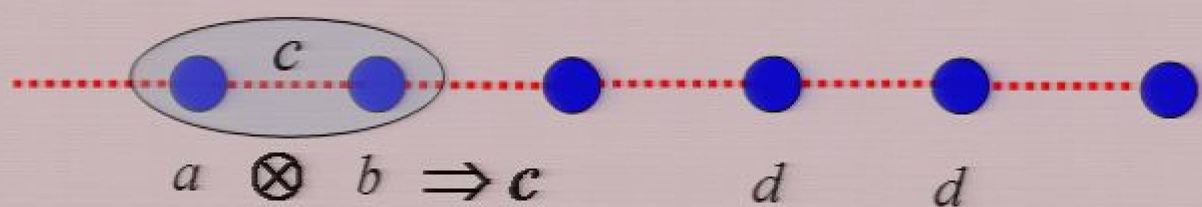

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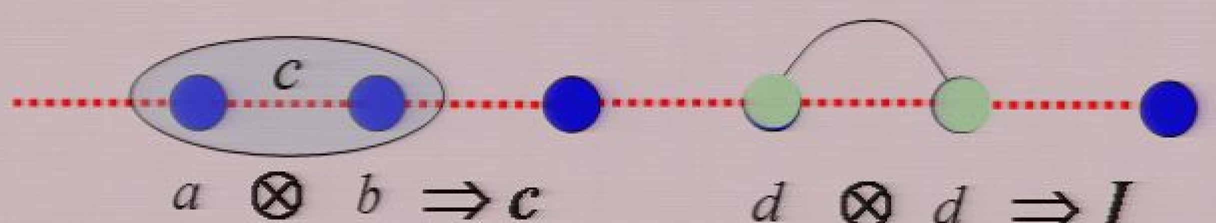

$\nu = 12/5$ Quantum Hall state: Fibonacci anyons

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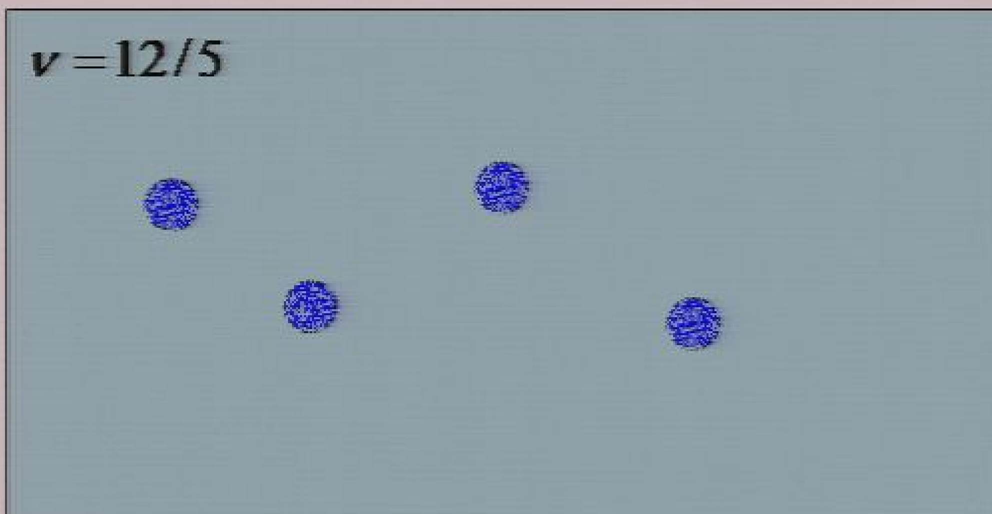

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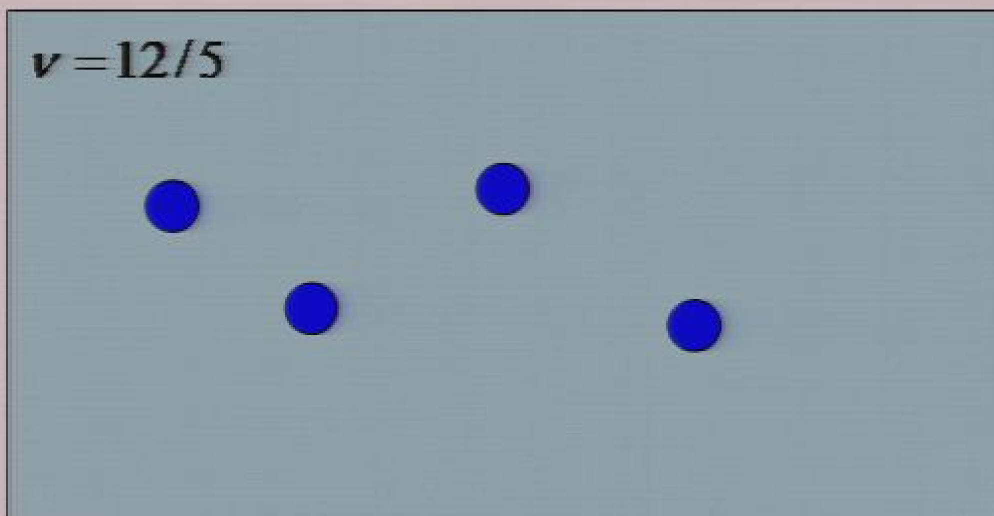

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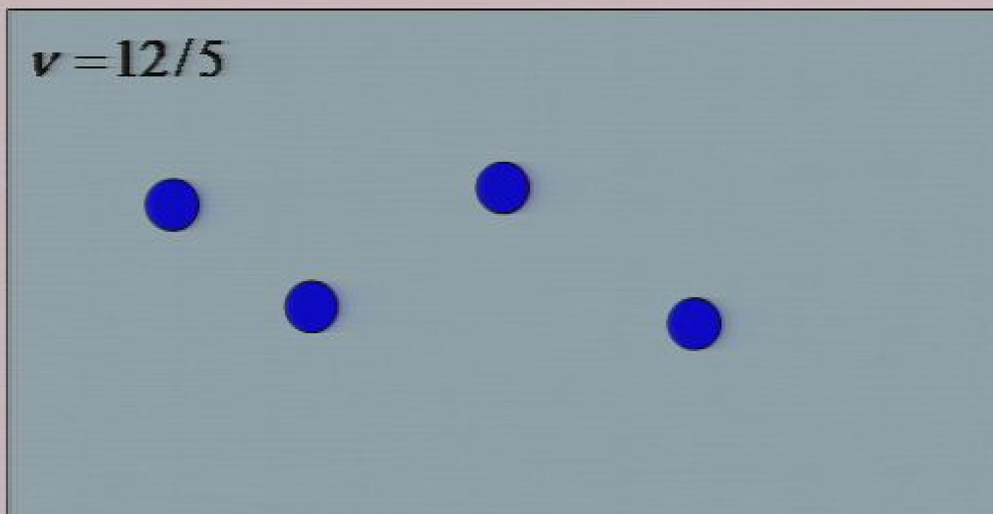

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$$\tau \otimes \tau = Id \oplus \tau$$

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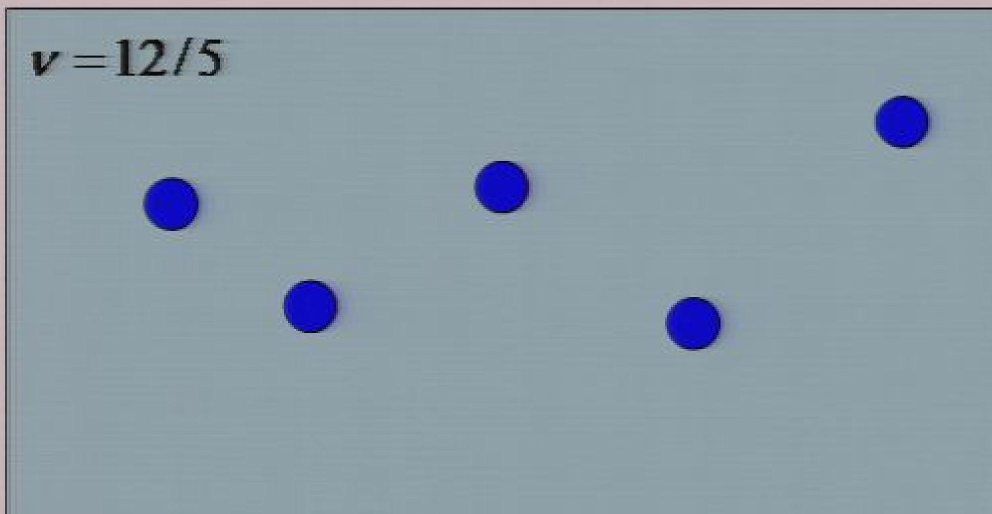

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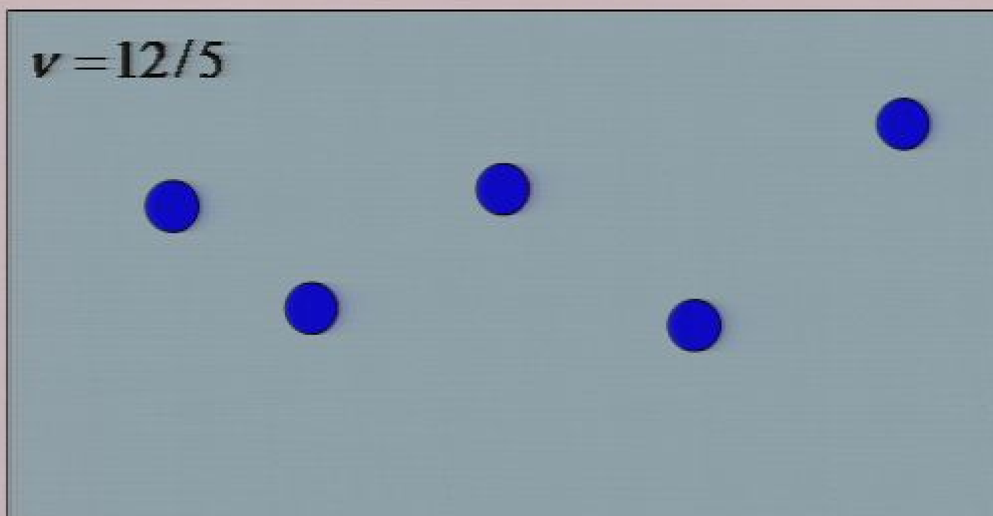

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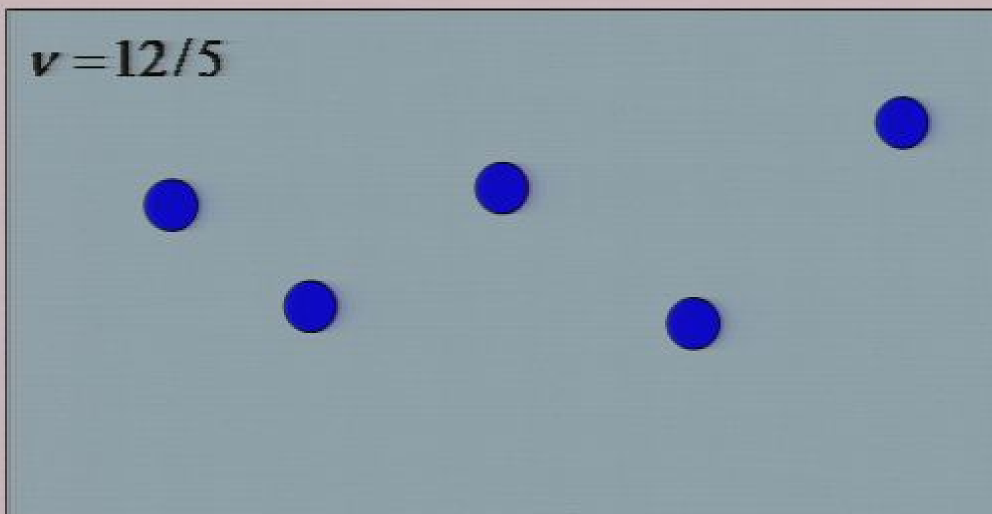

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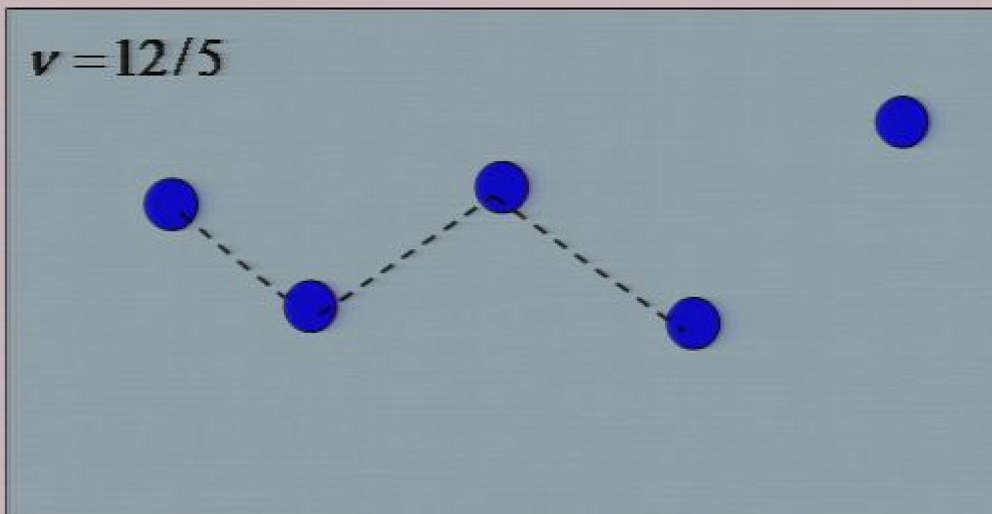

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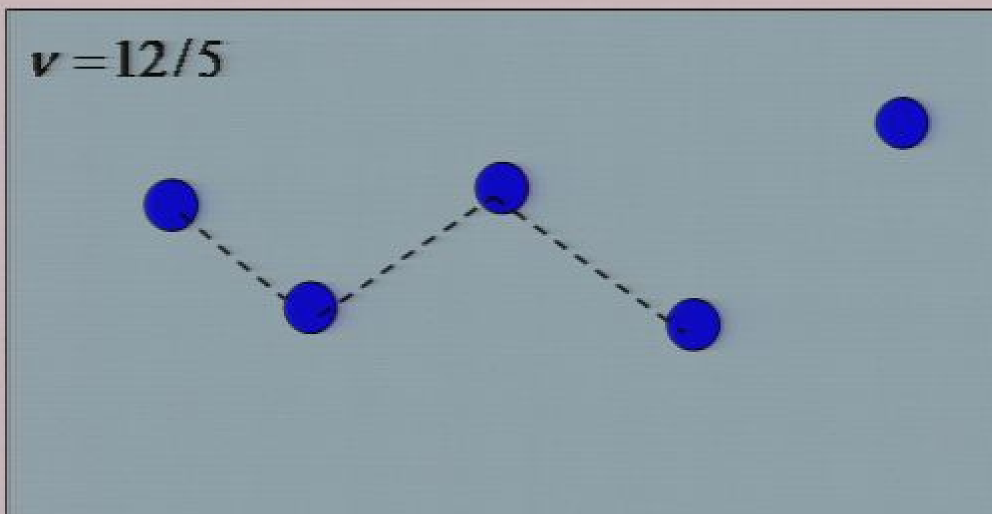

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Ground state?

Low energy properties?

Diagramatic representation

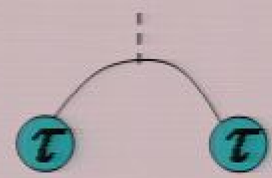
Diagramatic representation

- Propagation lines:

Id (zero chage)	- - - - -	τ	——

- Expectation values:

$$\langle \tau\otimes\tau = Id \,|\, \tau\otimes\tau = Id \rangle =$$



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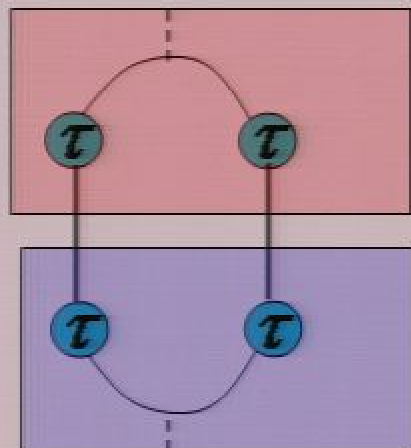

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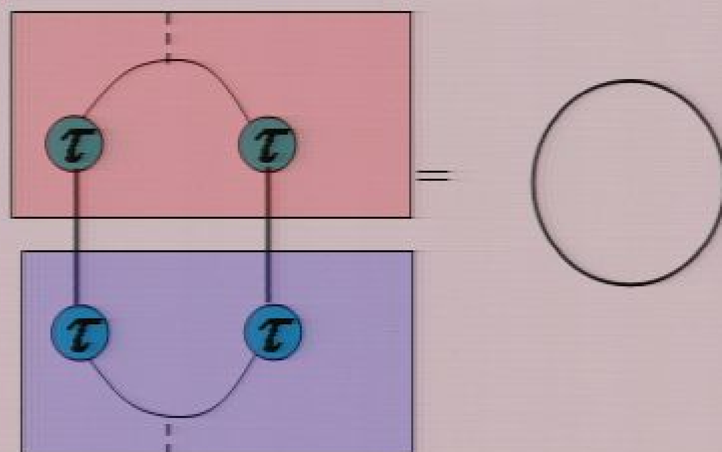

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$$H = -\sum_i J_i \hat{P}^{(Id)}_{i,i+1}$$ (sum of projections)

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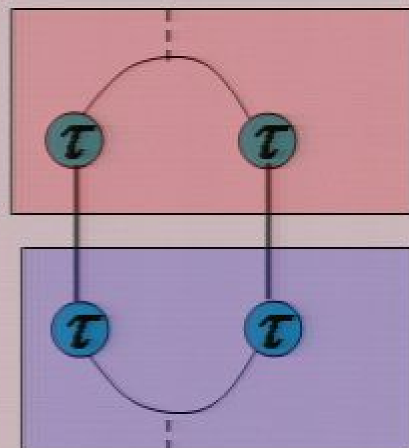

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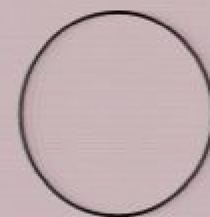

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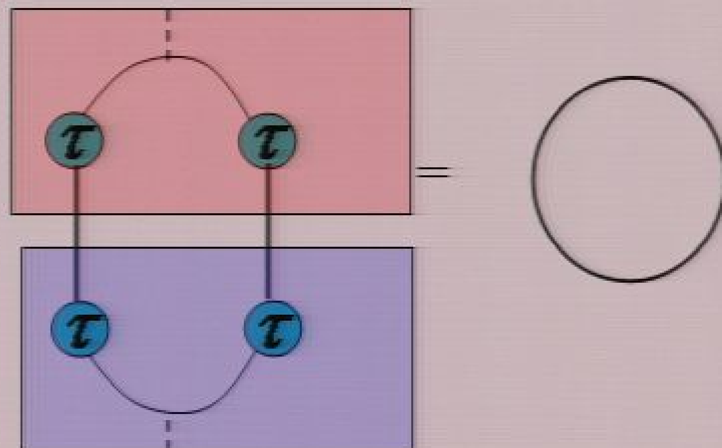

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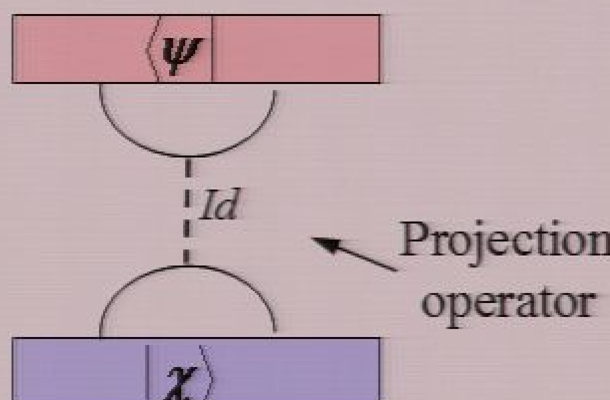

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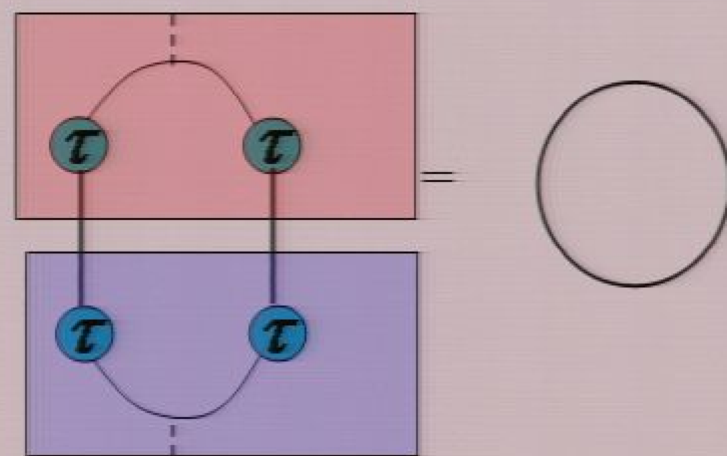

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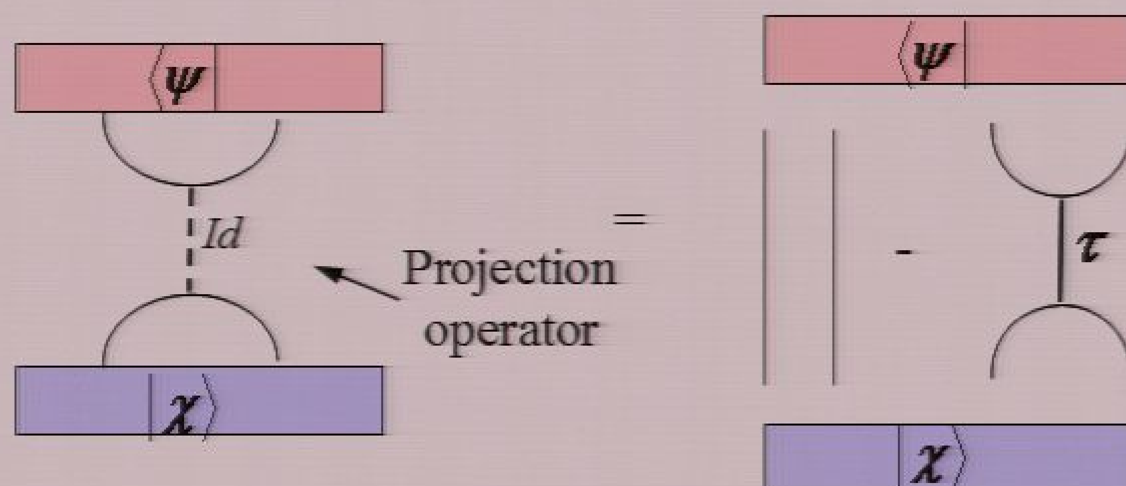

Random Fibonacci chain

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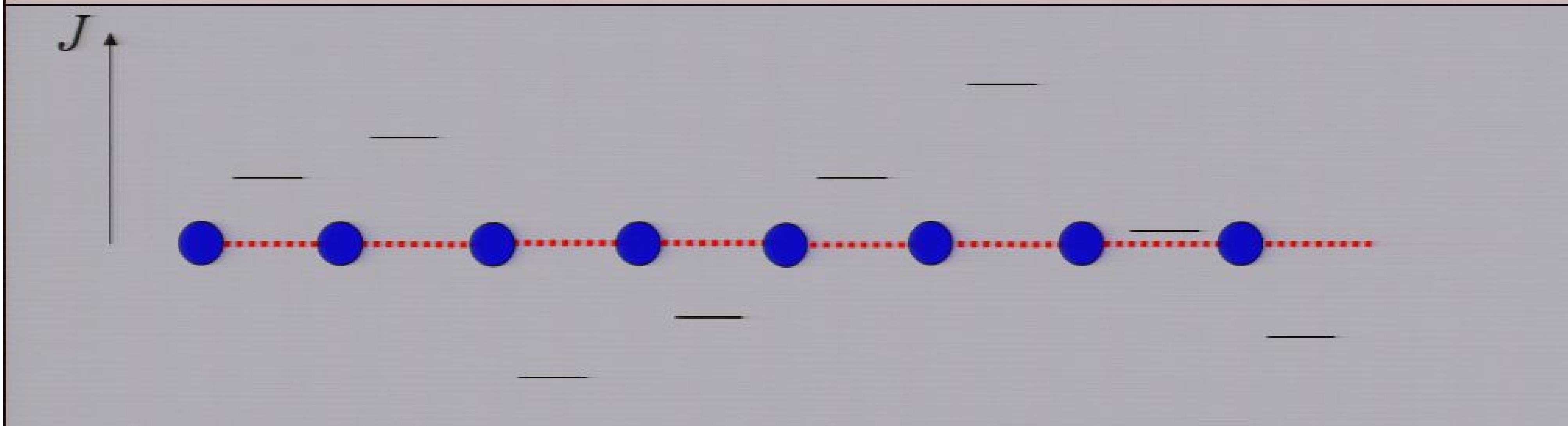

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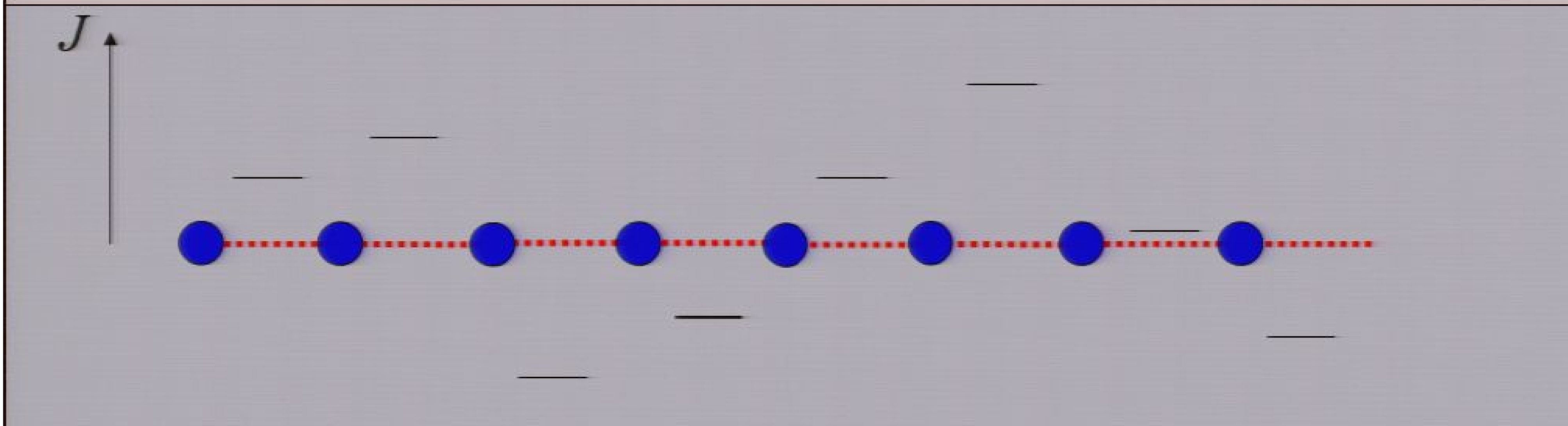

- Ma-Dasgupta singlet channel decimation – same as before.

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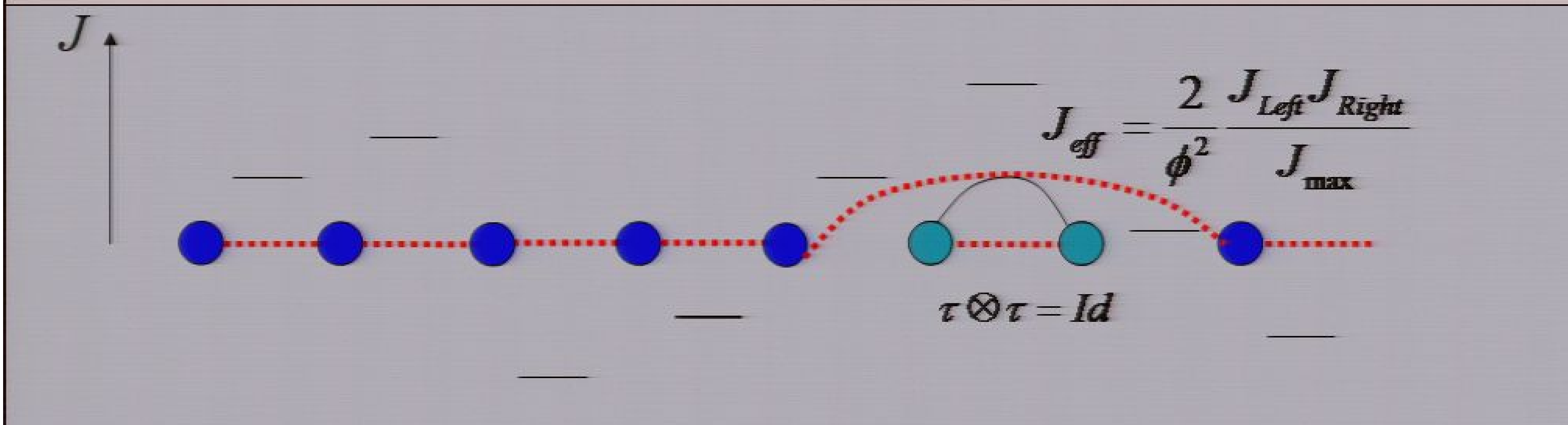

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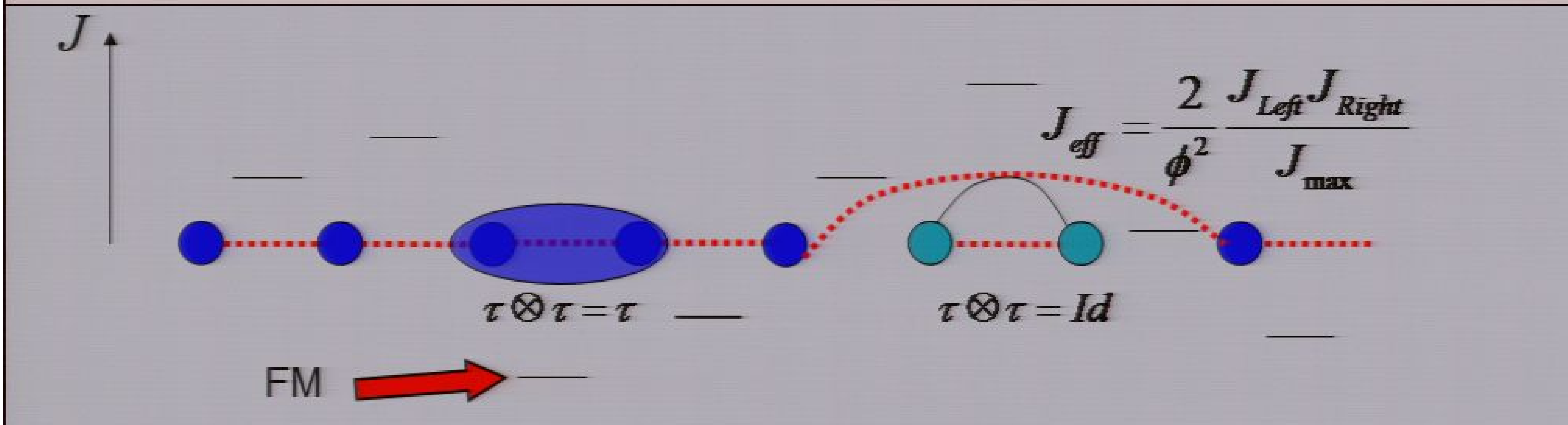

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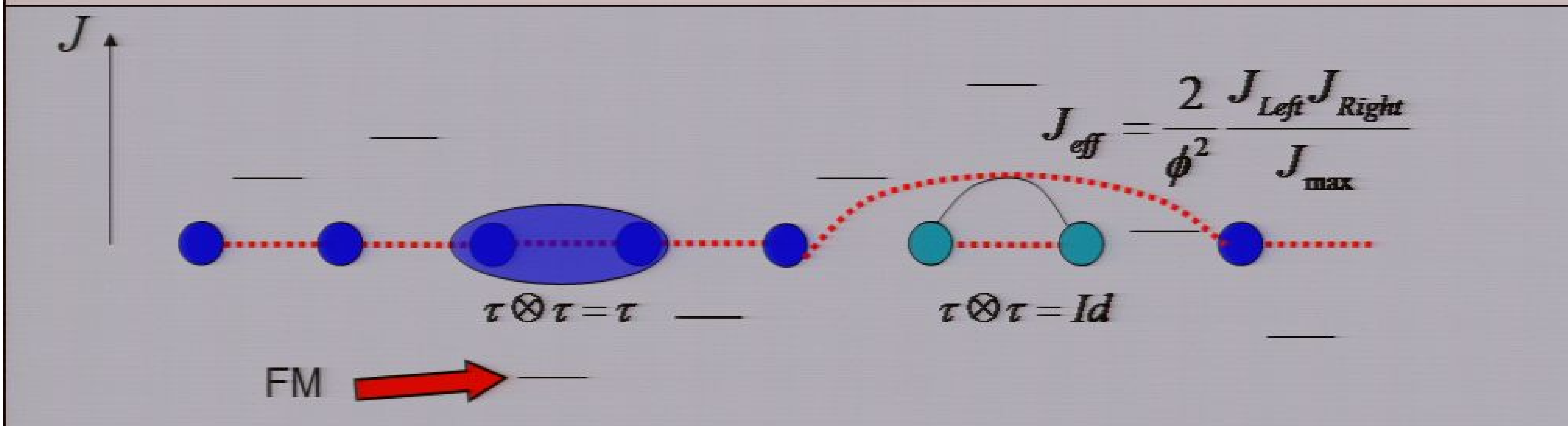

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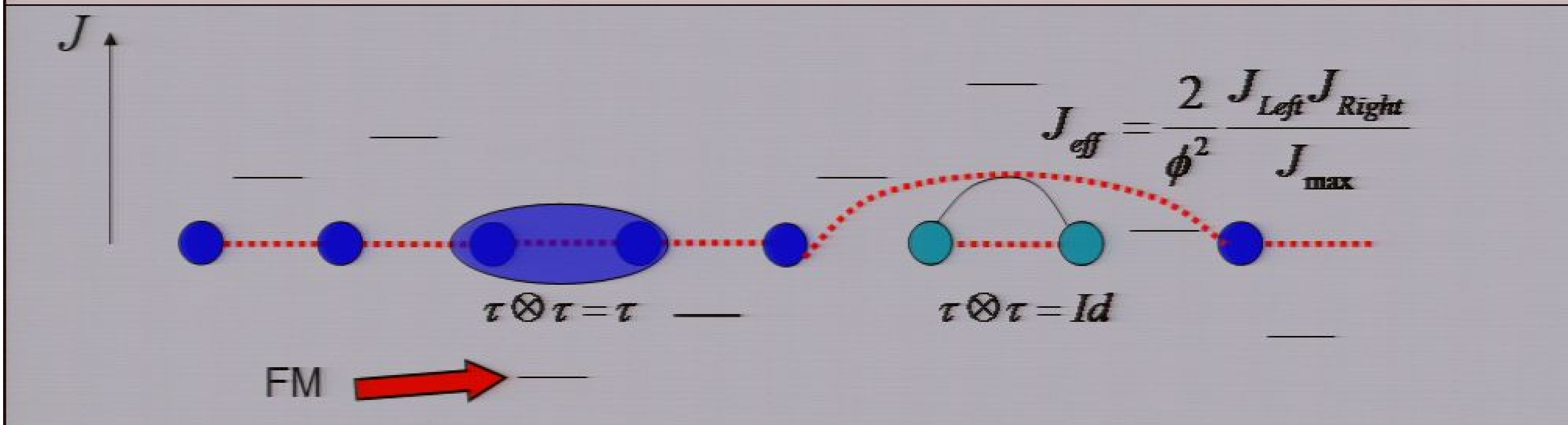

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$$-J_{eff} \cdot P^{(Id)}_{(12),3}$$

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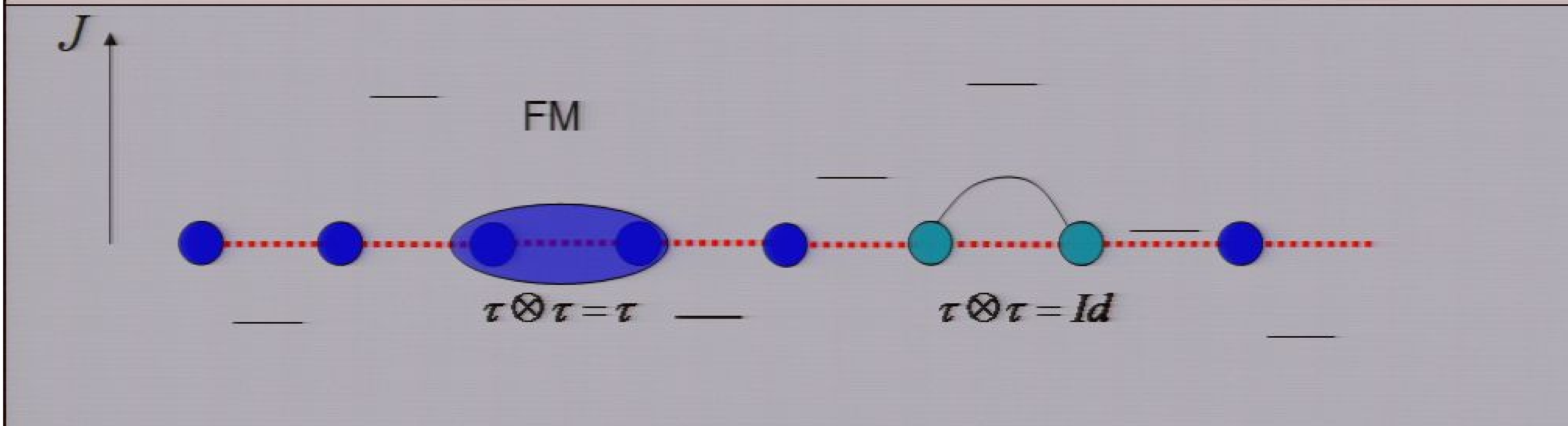

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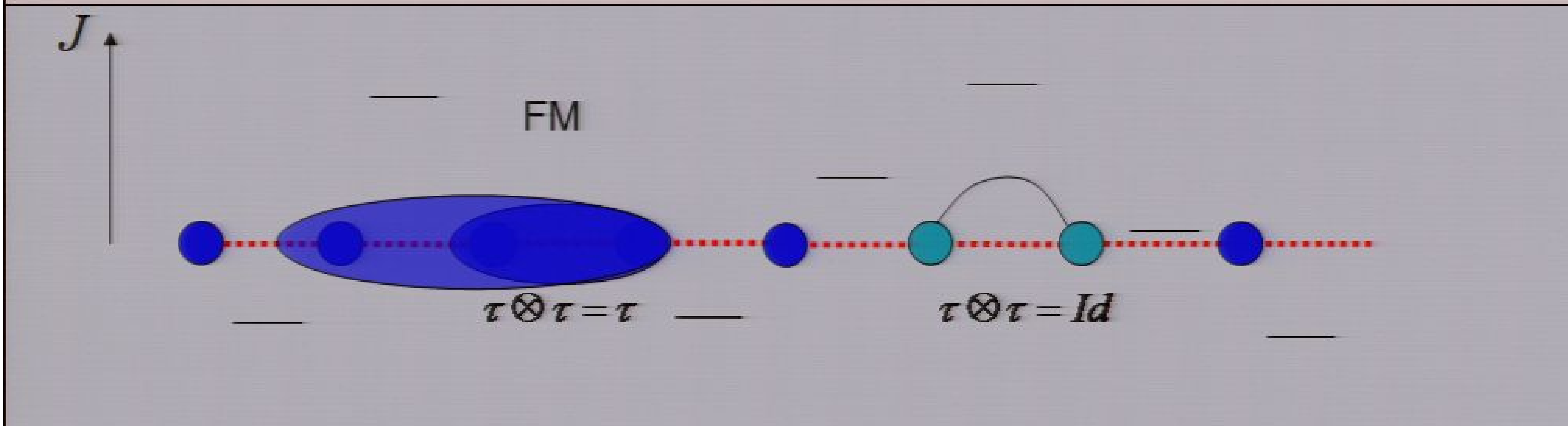

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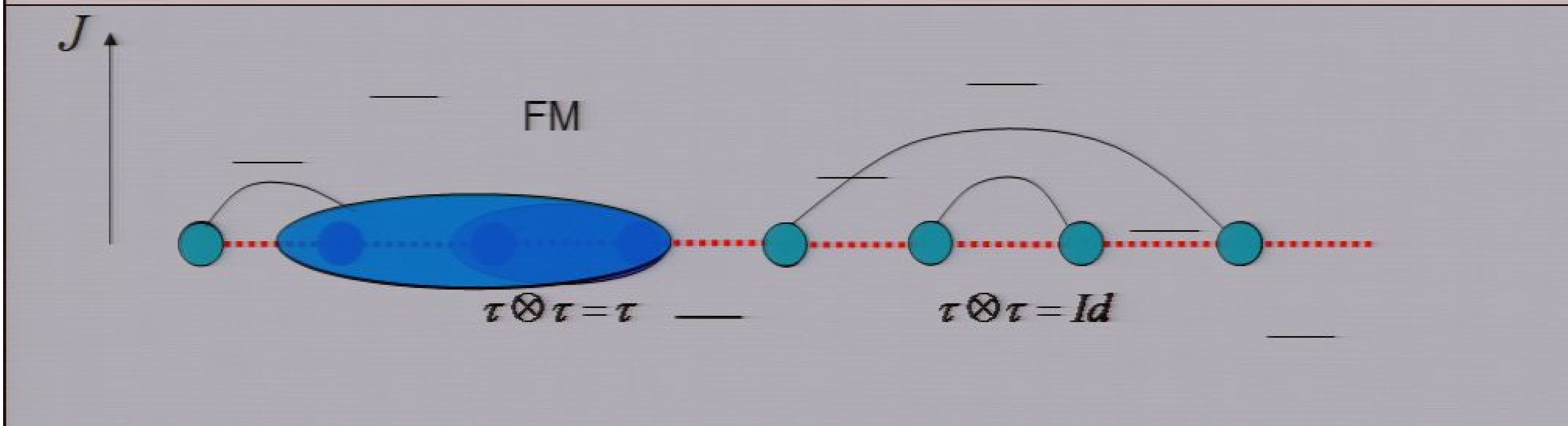

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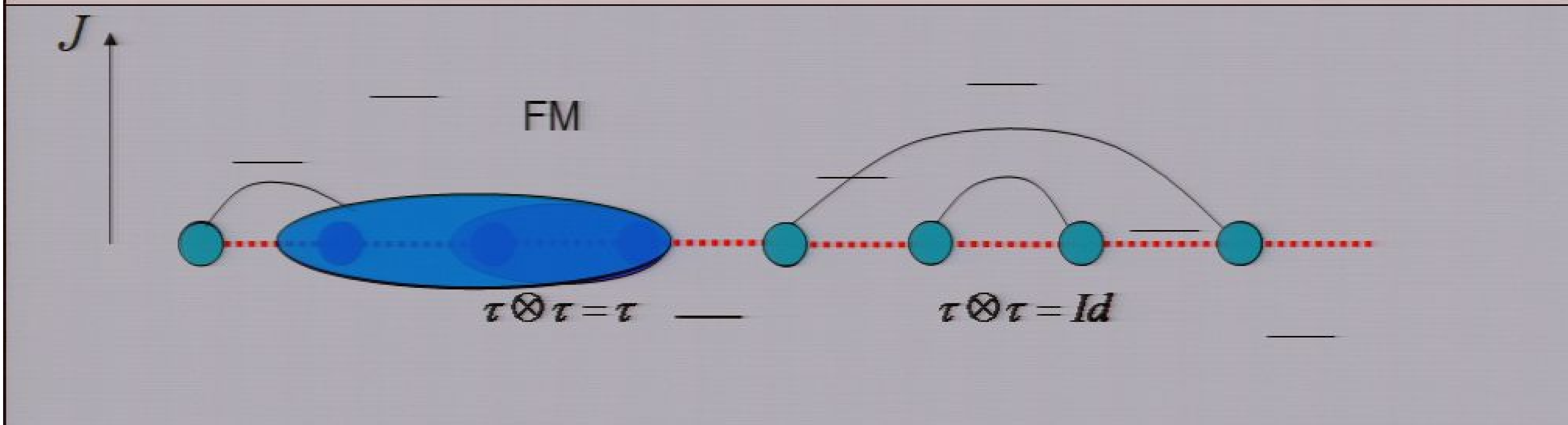

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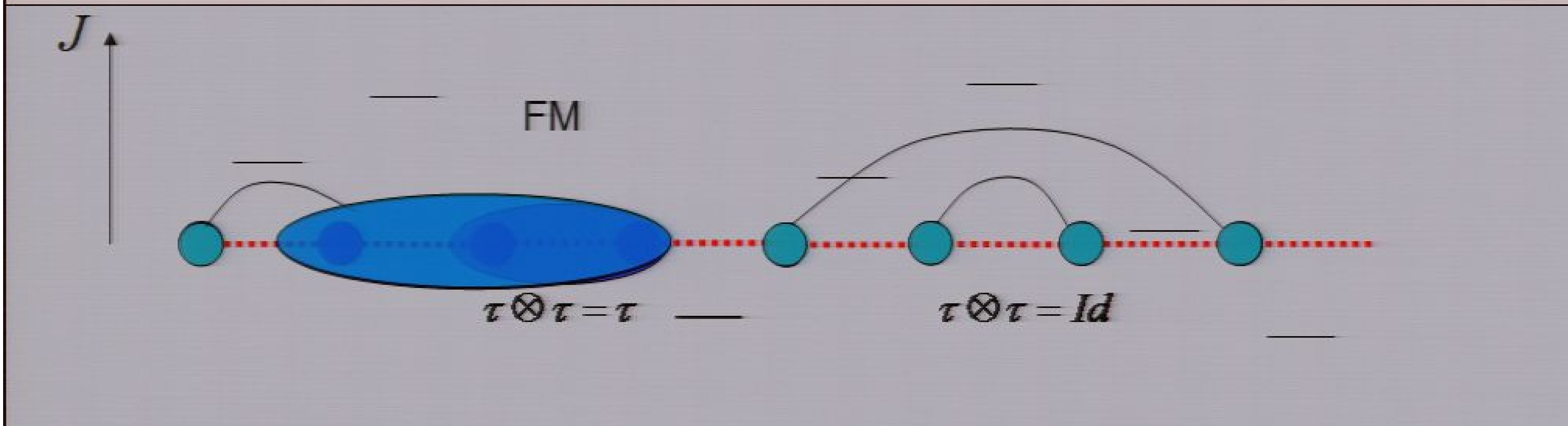

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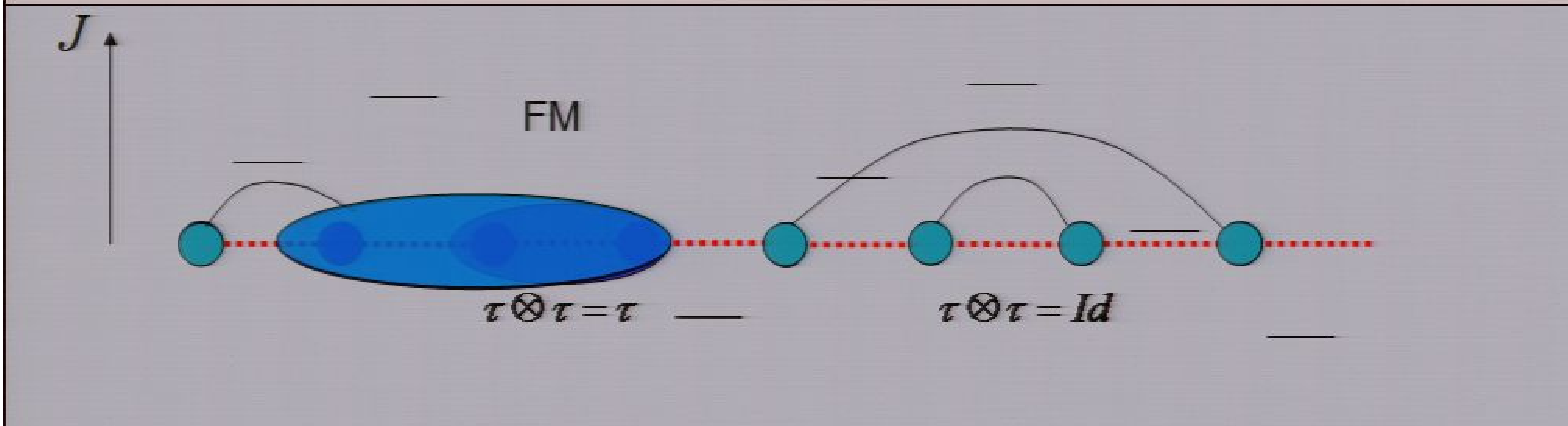

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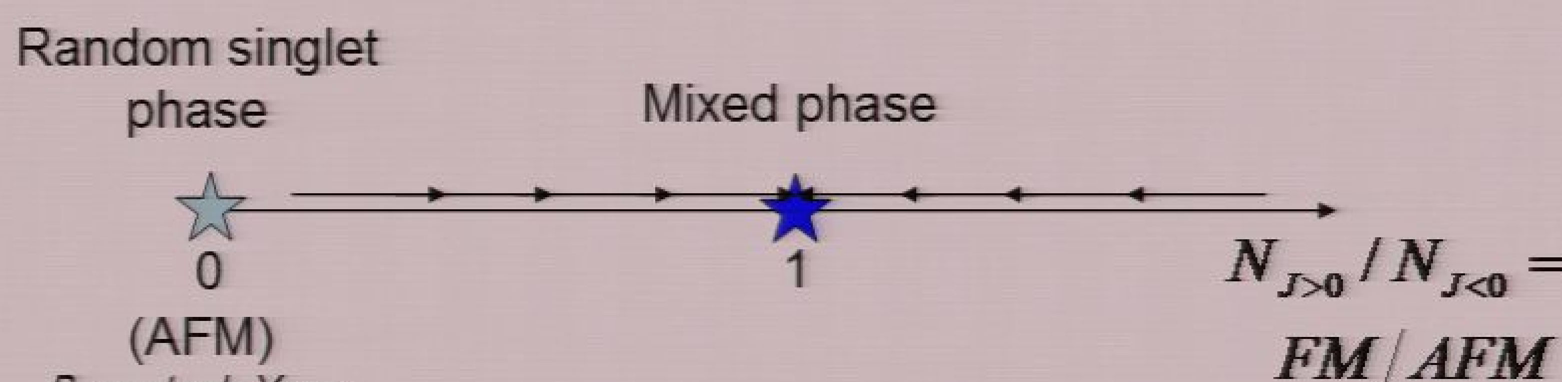

Bonesteel, Yang

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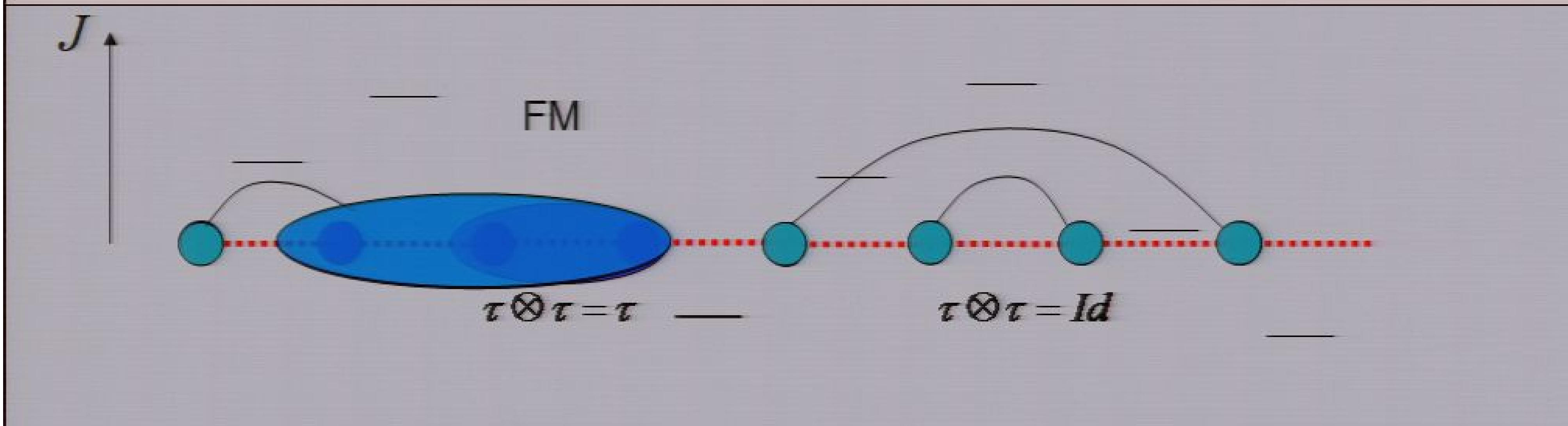

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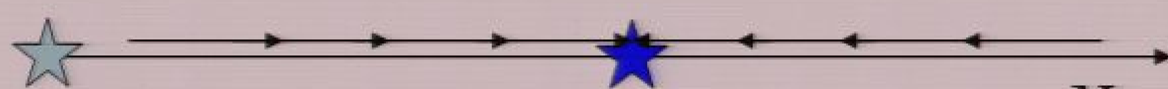

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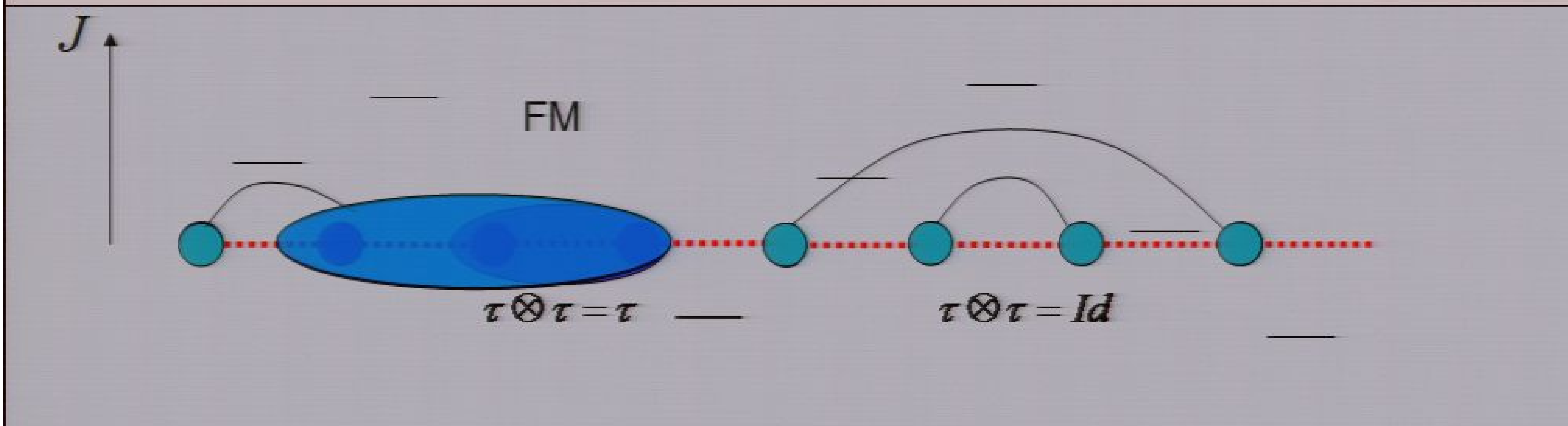
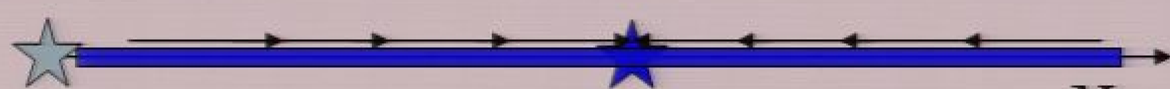


- **<u>Phase diagram:</u>**

Random singlet phase

Mixed phase



0 (AFM)

Bonesteel, Yang

1

$\psi = 1/3$ $\chi = 2$

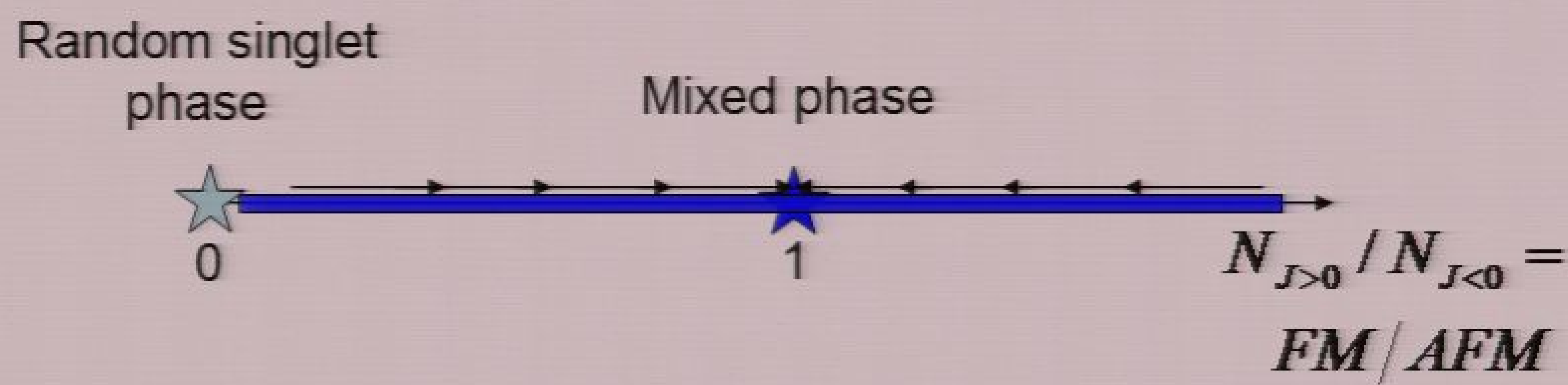
$N_{J>0} / N_{J<0} = FM / AFM$

Same universality class as **Spin-1** Haldane-RS

Universal entanglement in random **Fibonacci chain**

$$E_L = \frac{1}{3} c_{eff} \cdot \log_2 L$$

- **Phase diagram:**



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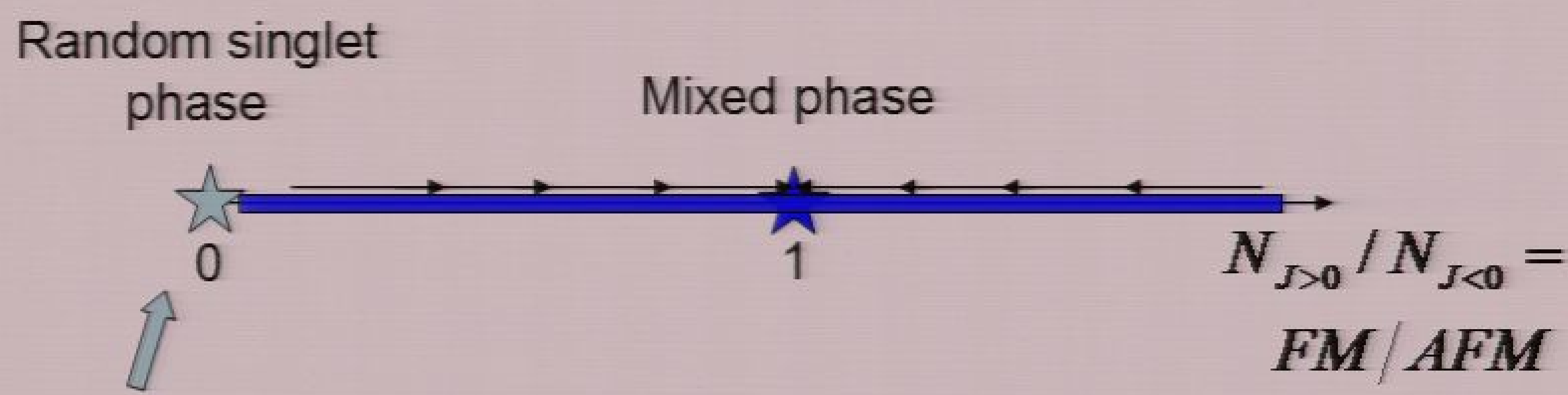

$$E_L^{Random} = \frac{1}{3} \ln \phi \cdot \log_2 L$$

$$\ln \phi = 0.481$$

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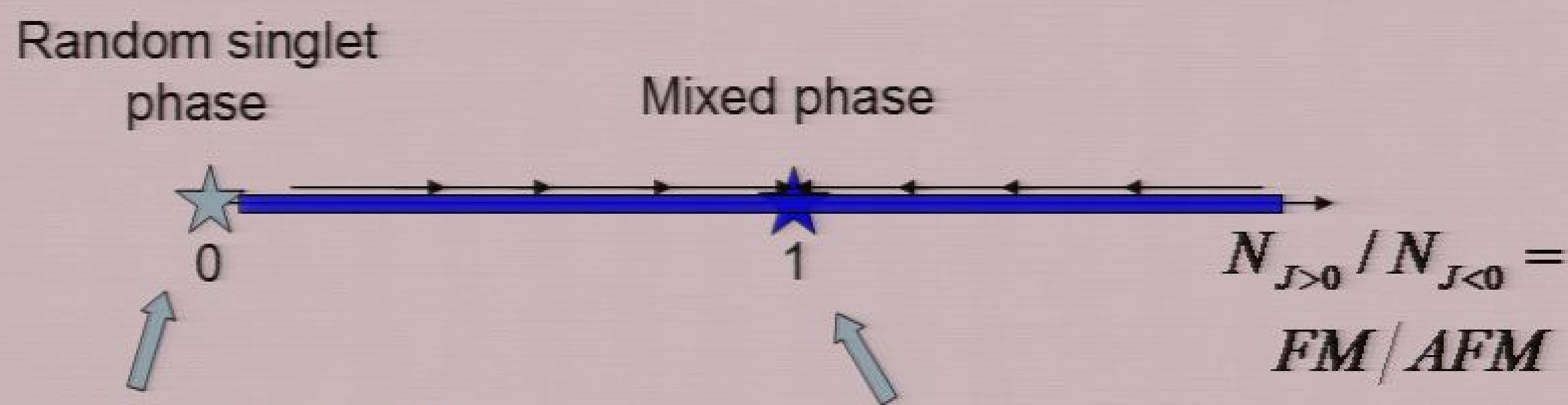

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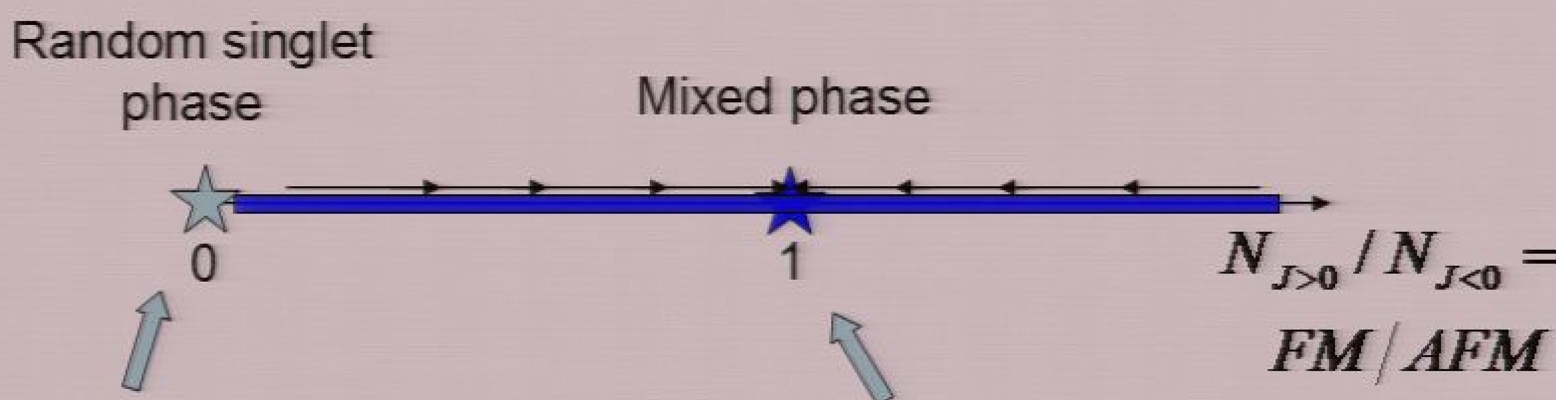

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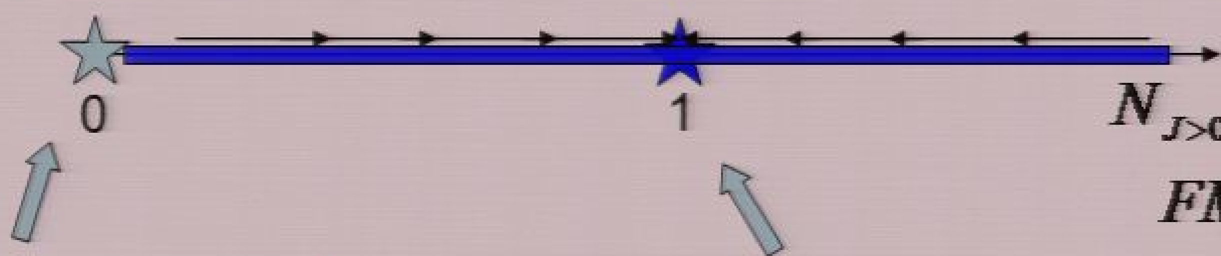

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- **No random-to-random c-theorem.**

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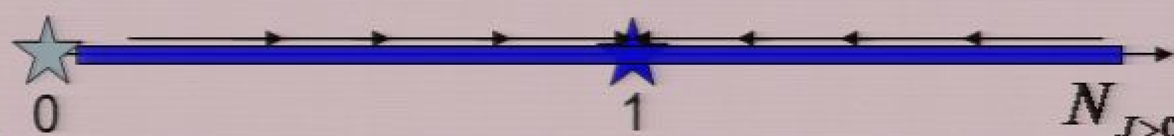

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- Pure-to-random c-theorem? ***Still open.***

General class of non-Abelian anyons: SU(2)k

(w/ Lukasz Fidkowski,
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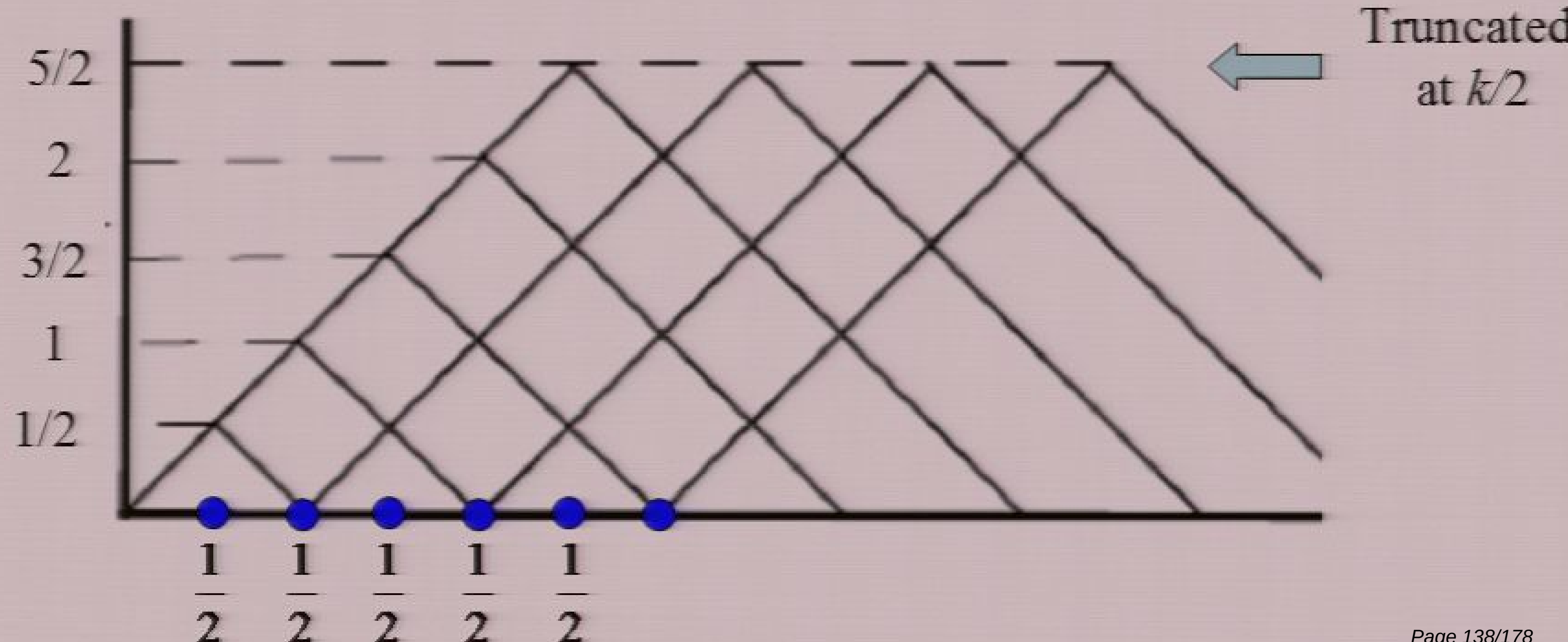

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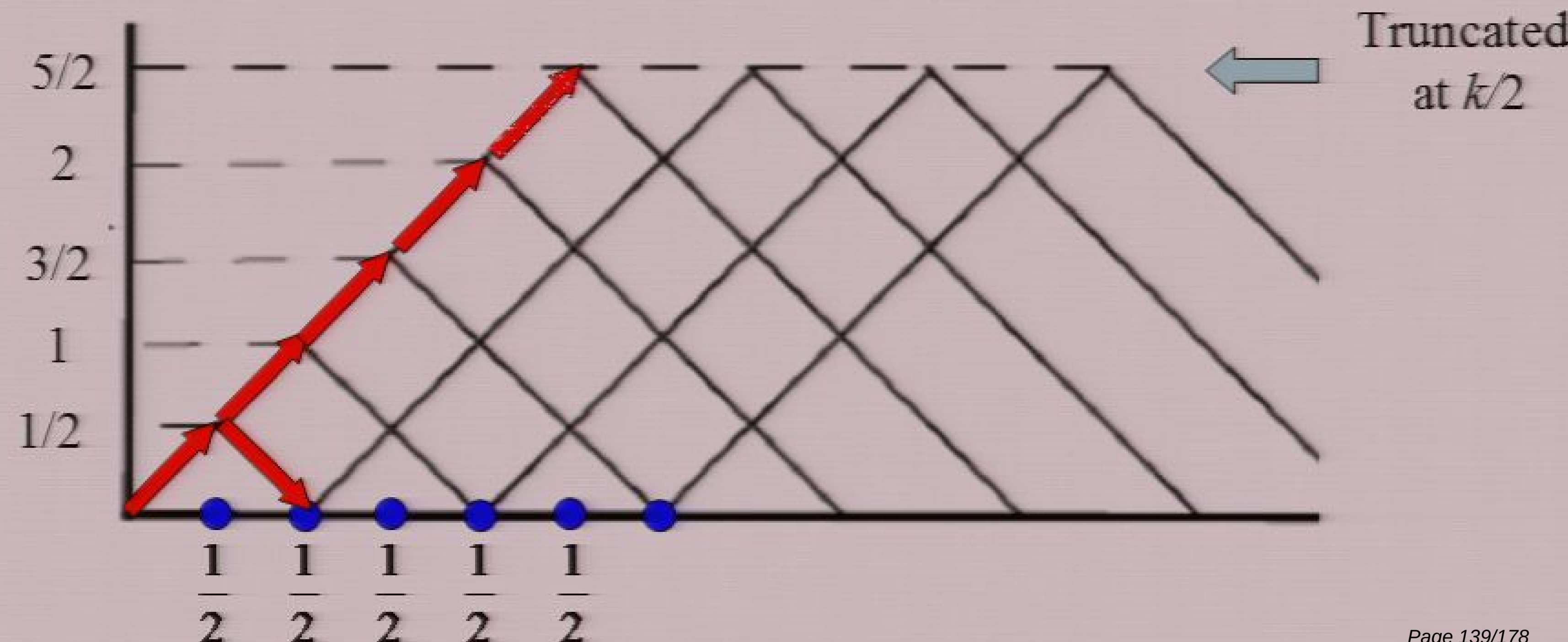

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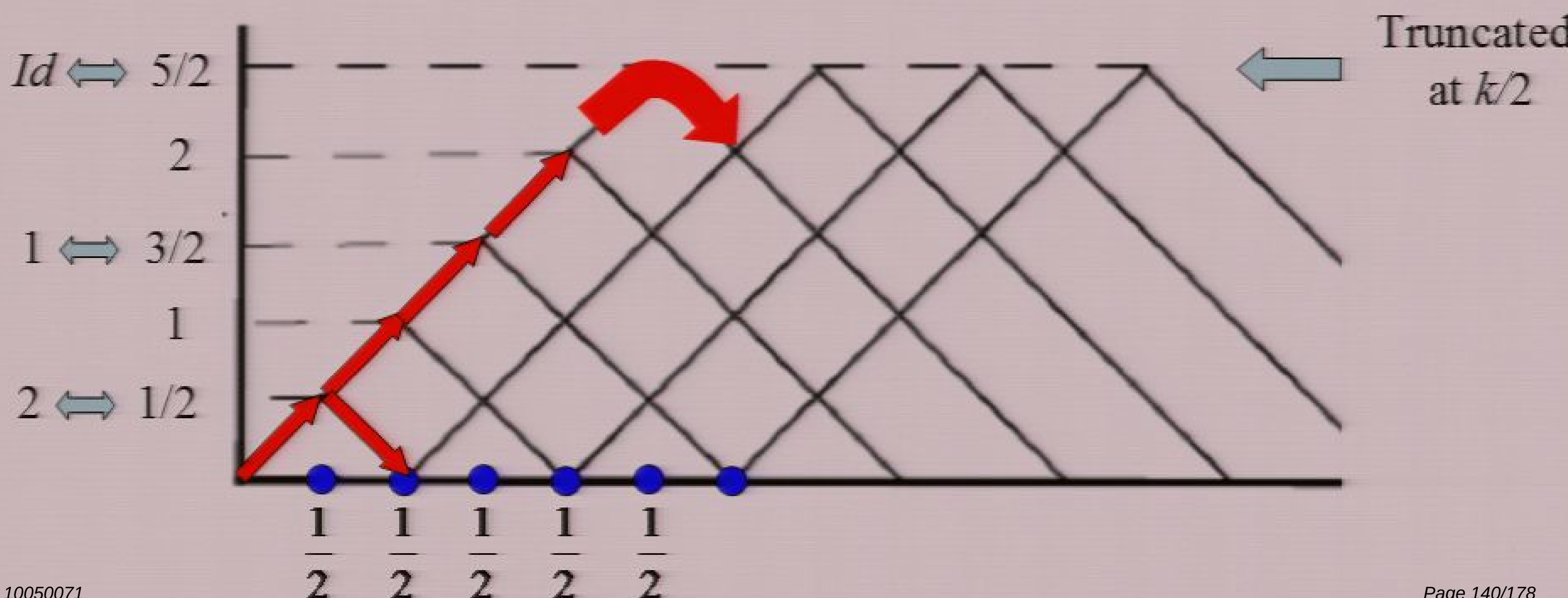

General class of non-Abelian anyons: SU(2)k

(w/ Lukasz Fidkowski,
and SURF students Paraj Titum, Han-Hsuan Lin)

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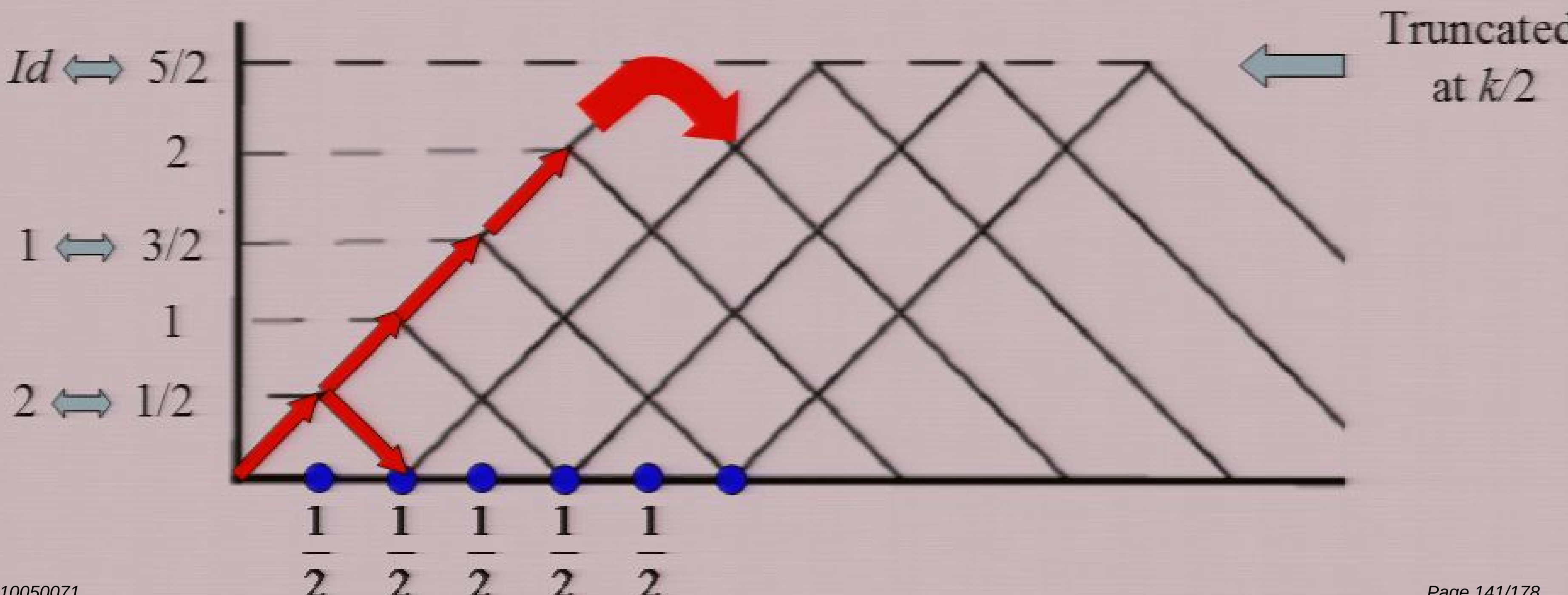

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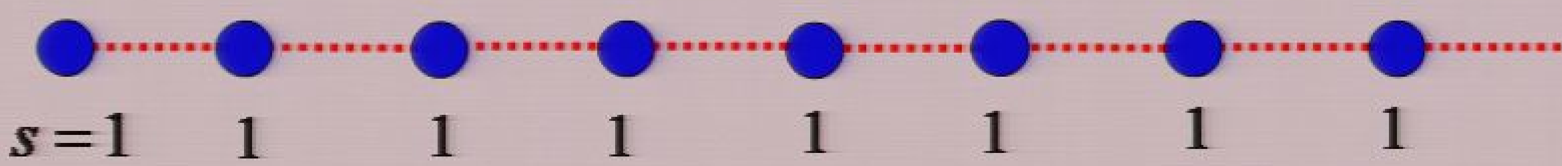

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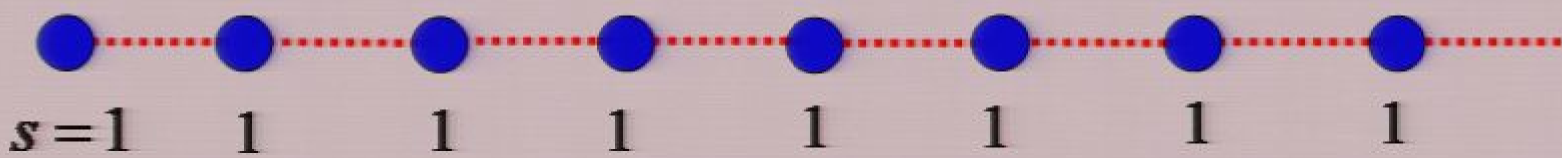

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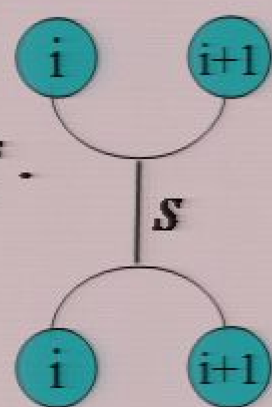

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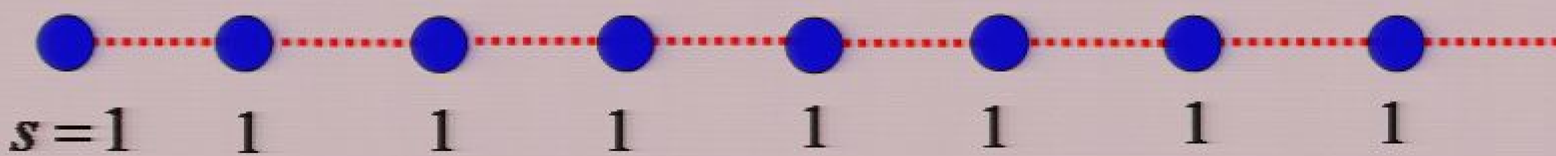

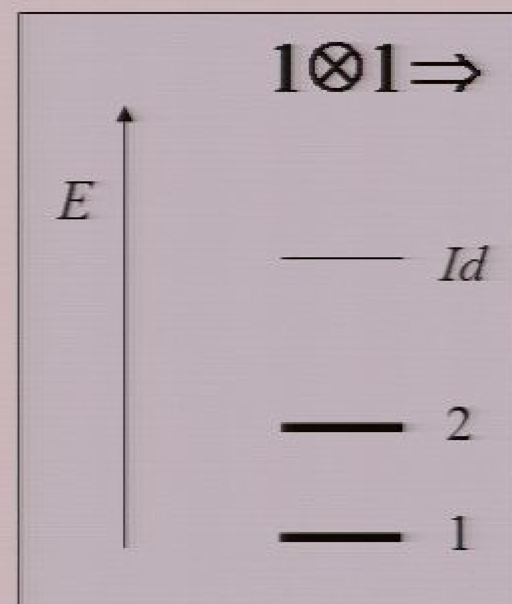

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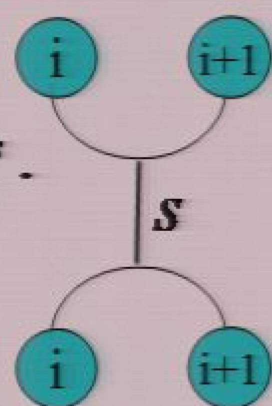

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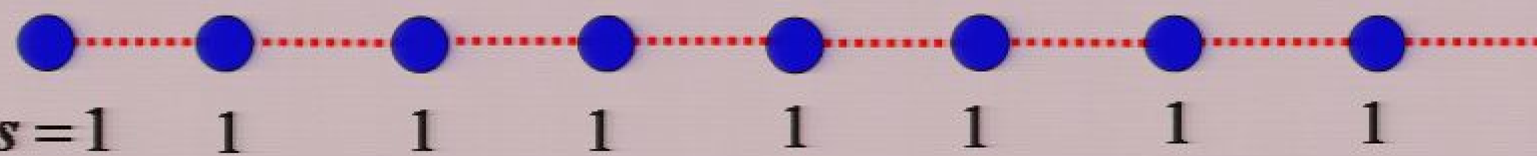

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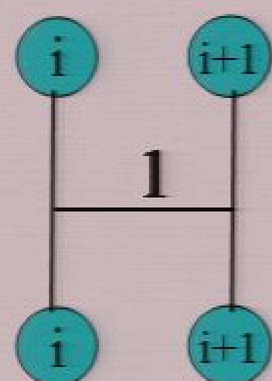

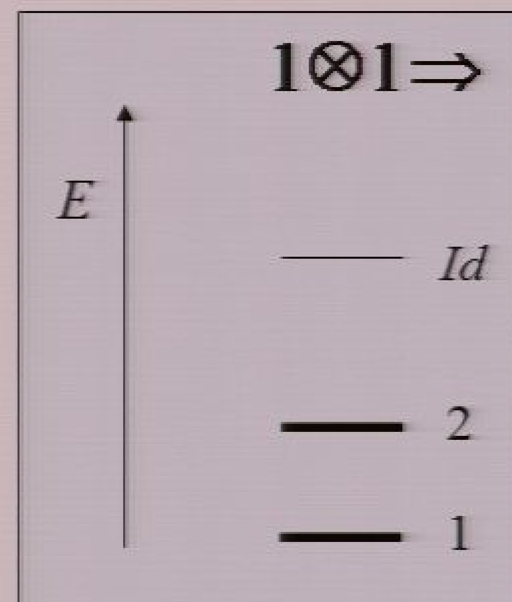

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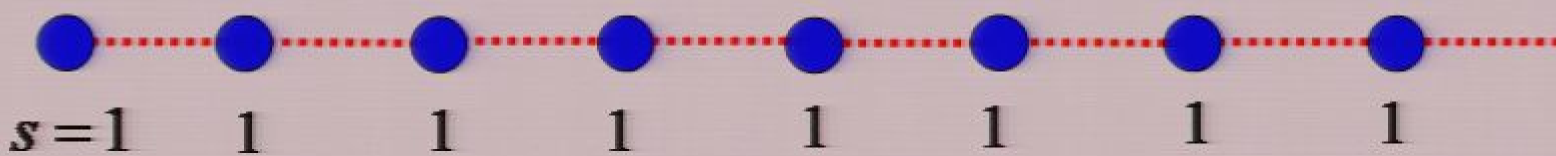

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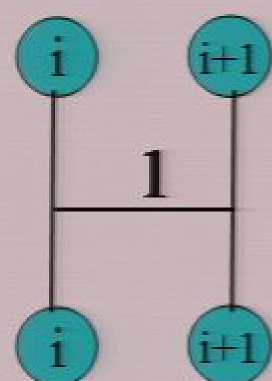

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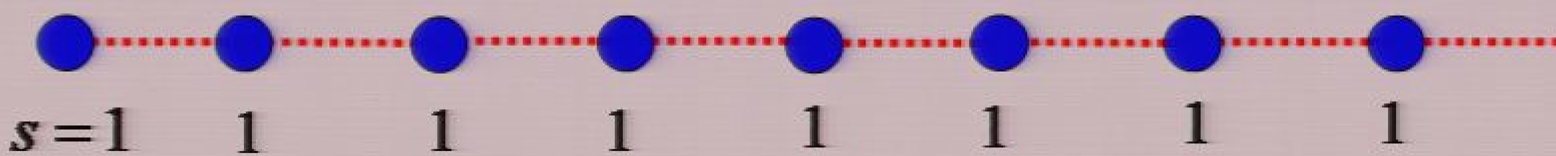

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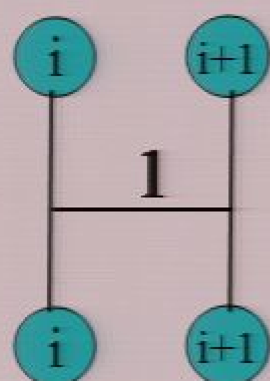

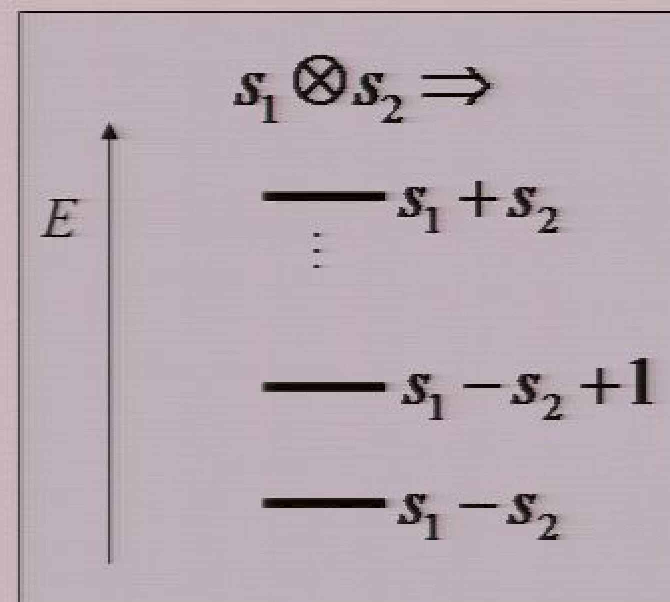

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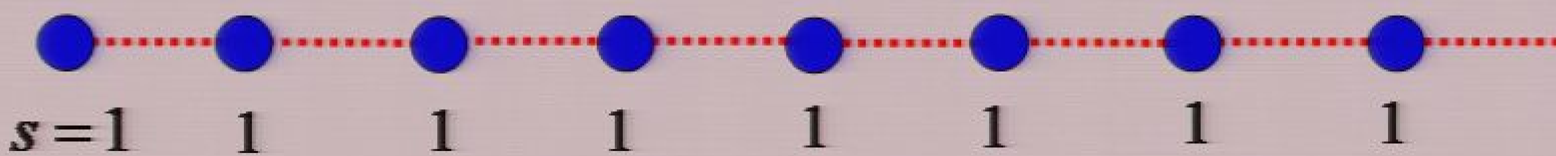

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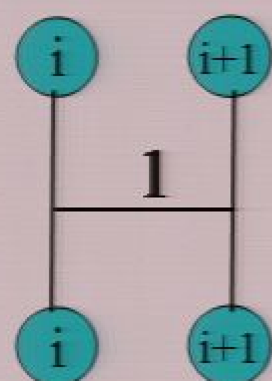

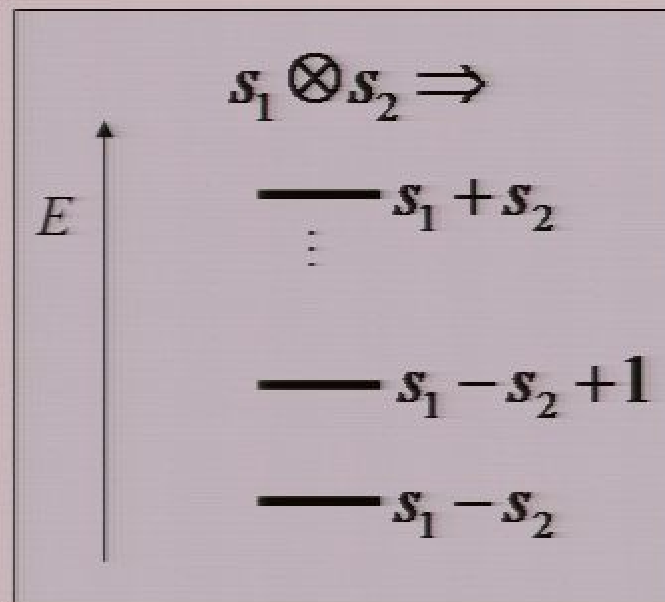

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- All *Fixed Points* are ***Stable***! ⇨ Represent stable phases.
  (Contrary to DH points in abelian spins)

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- Domain walls: $(d_1, 2S - d_1) \mid (d_2, 2S - d_2)$ ➡ $S_{eff} = \frac{1}{2} | d_1 - d_2 |$

<u>k=5:</u>   <u>Domains 0-4</u>

Spin-2:

Non-abelian: k=5

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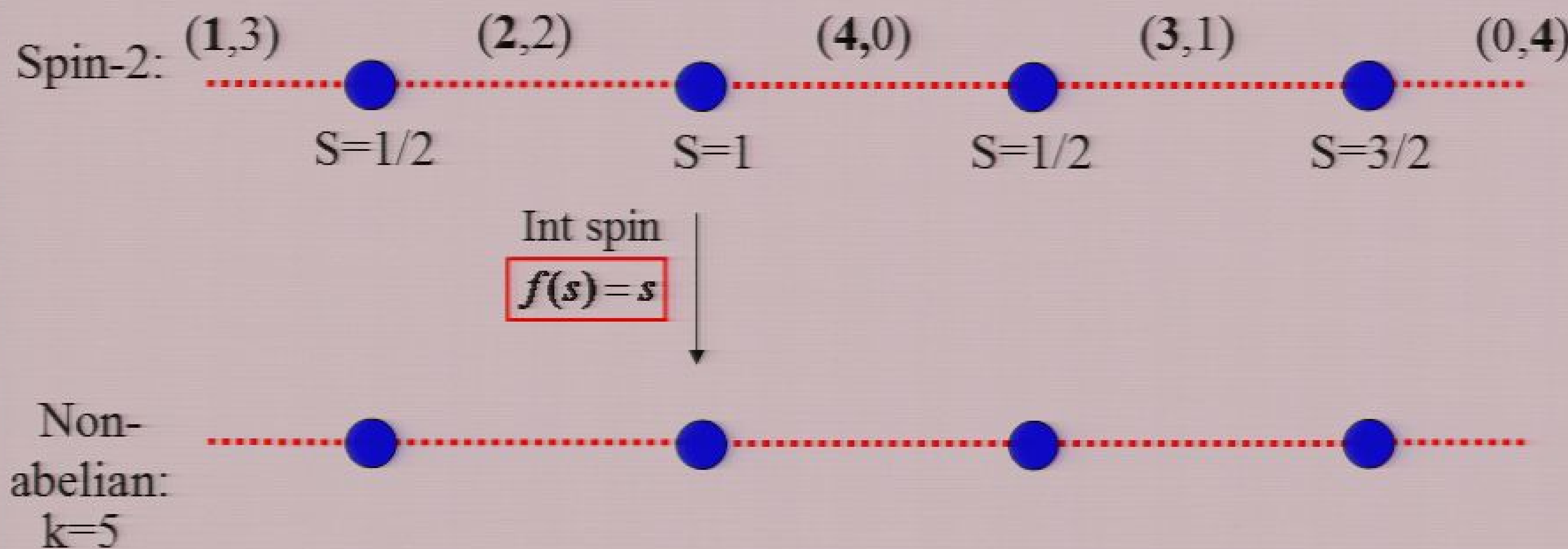

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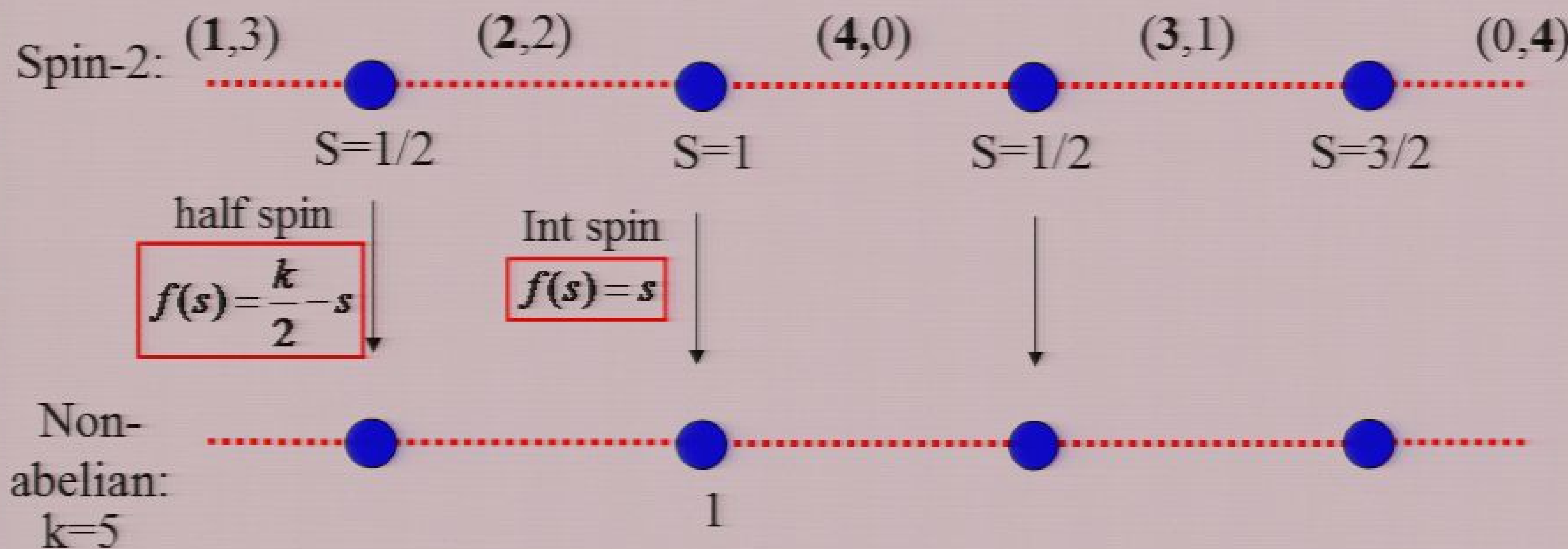

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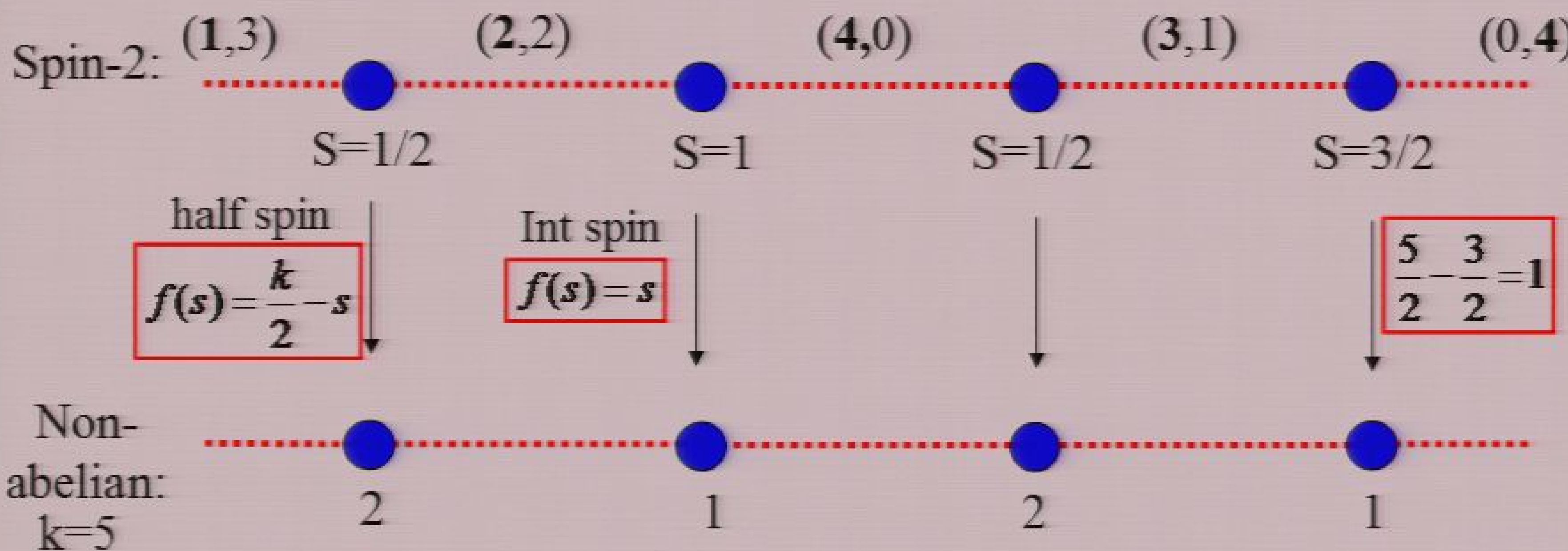

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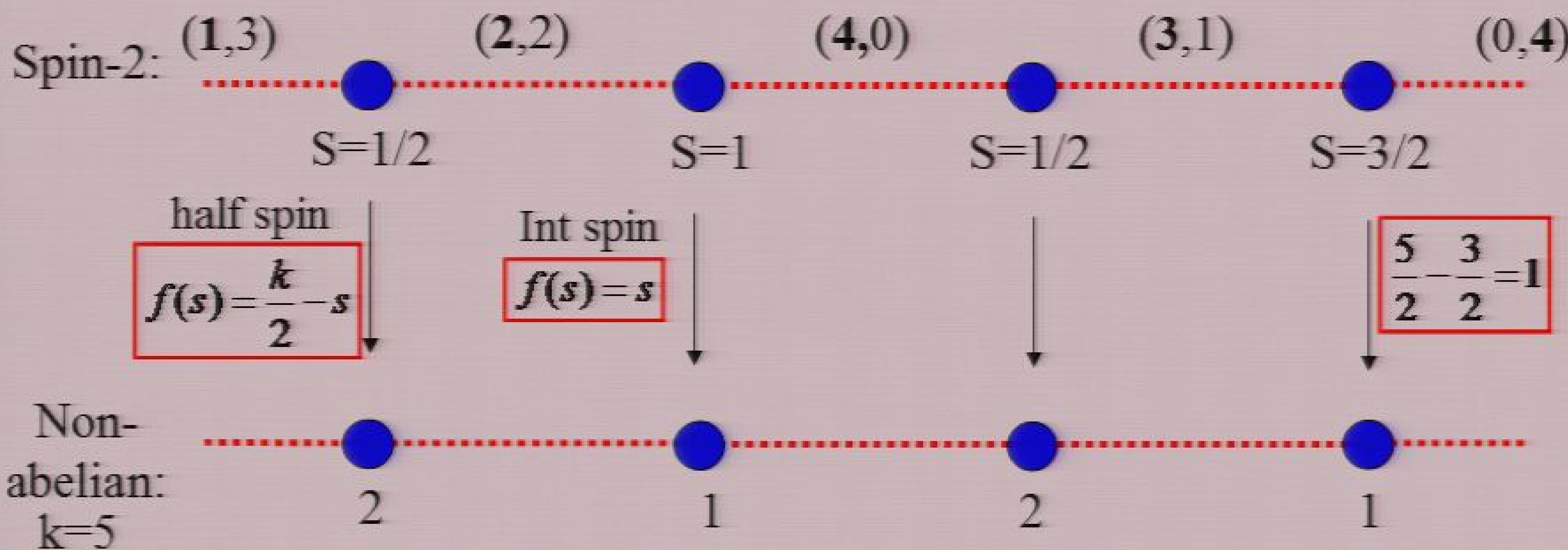

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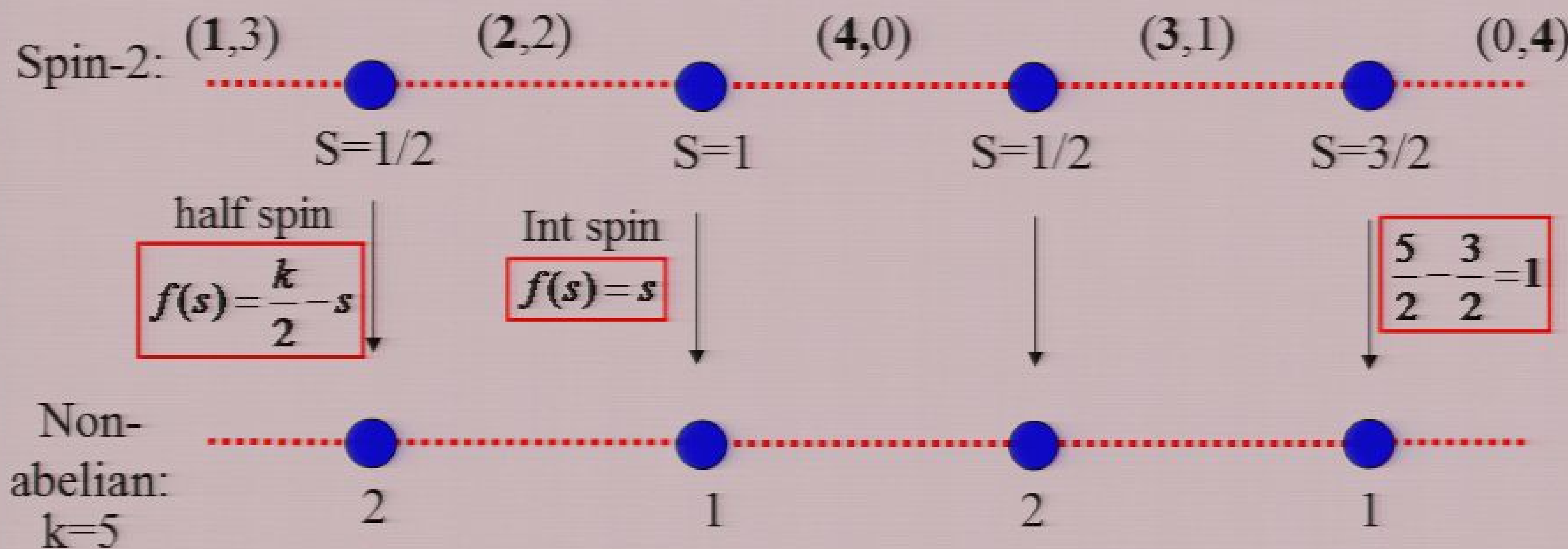

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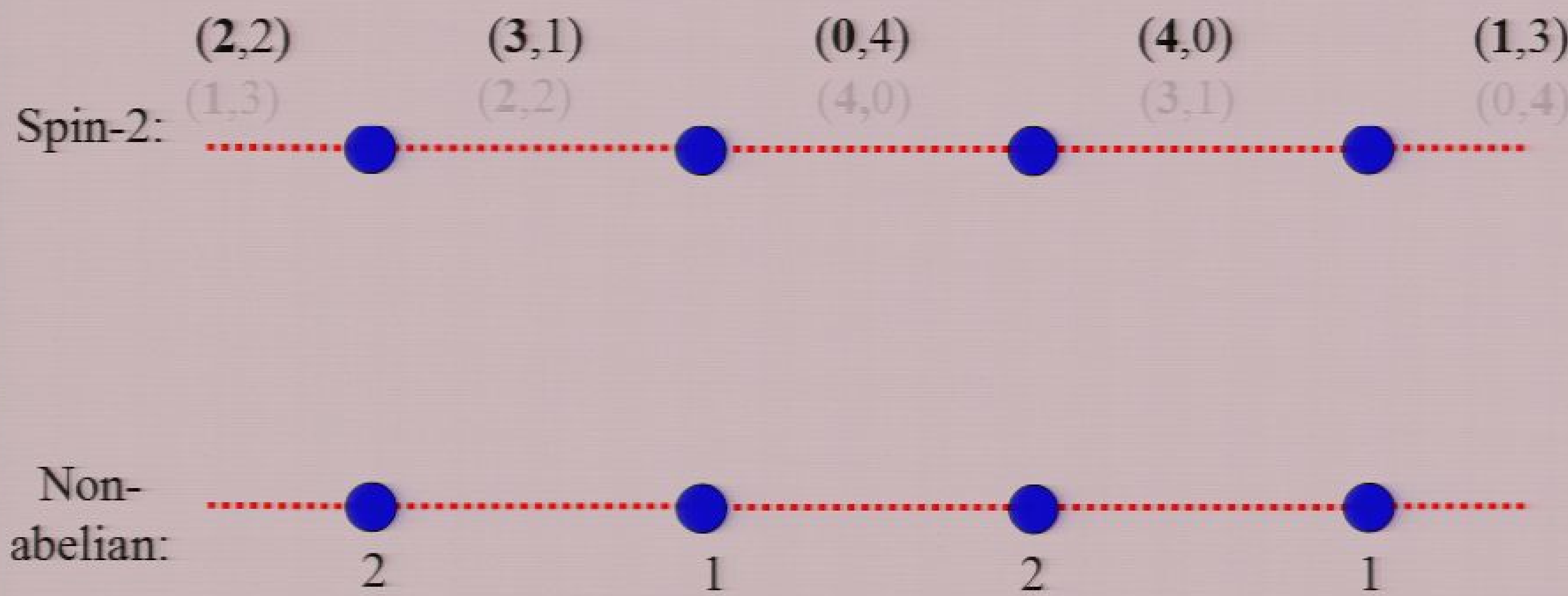

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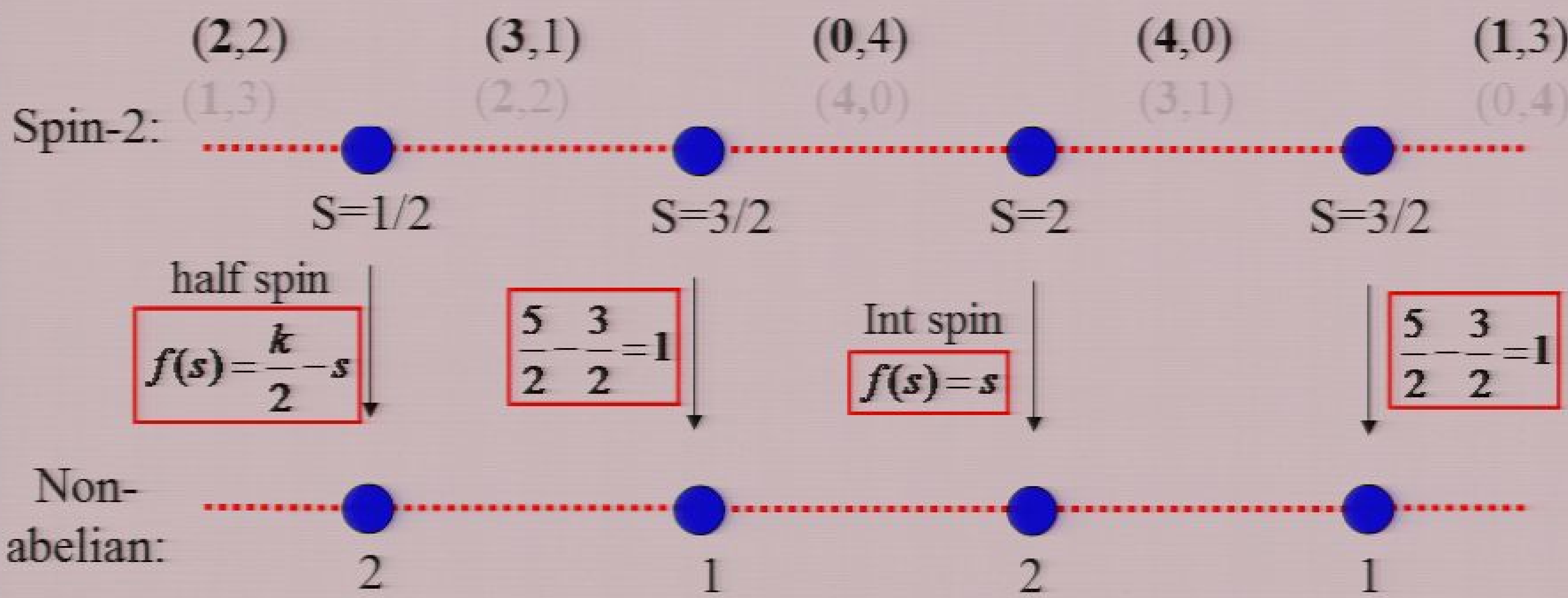

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No relevant perturbations of the non-Abelian permutation symmetric IR points!

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Engtanglement and CFT's central charge