

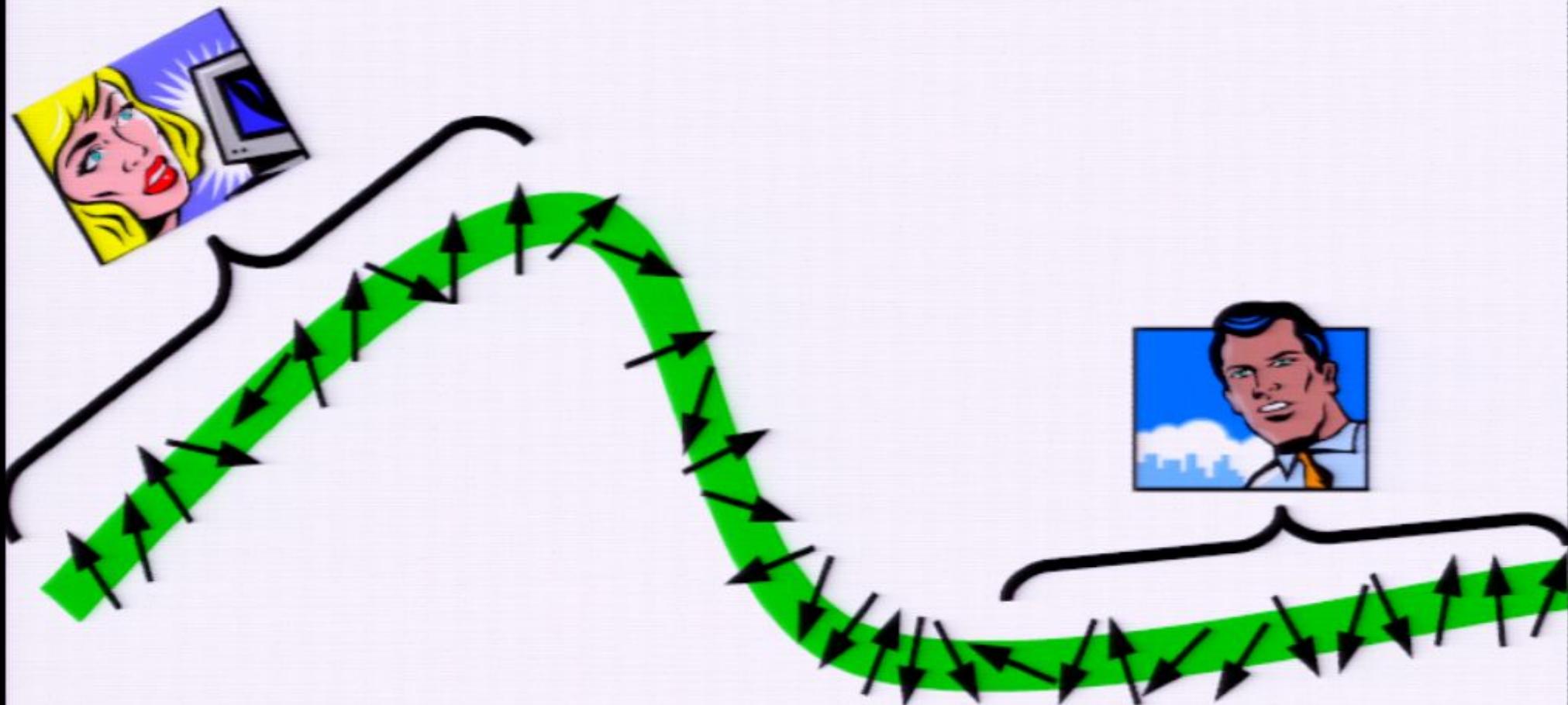
Title: Entanglement across a separation in spin chains: statics & dynamics

Date: May 25, 2010 04:45 PM

URL: <http://pirsa.org/10050068>

Abstract: In this talk, I will present two schemes which would result in substantial entanglement between distant individual spins of a spin chain. One relies on a global quench of the couplings of a spin chain, while the other relies on a bond quenching at one end. Both of the schemes result in substantial entanglement between the ends of a chain so that such chains could be used as a quantum wire to connect quantum registers. I will also examine the resource of entanglement already existing between separated parts of a many-body system at criticality as the size of the parts and their separation is varied. This form of entanglement displays an interesting scale invariance.

Entanglement “between” separated systems  
is *in principle* a resource for QIP:



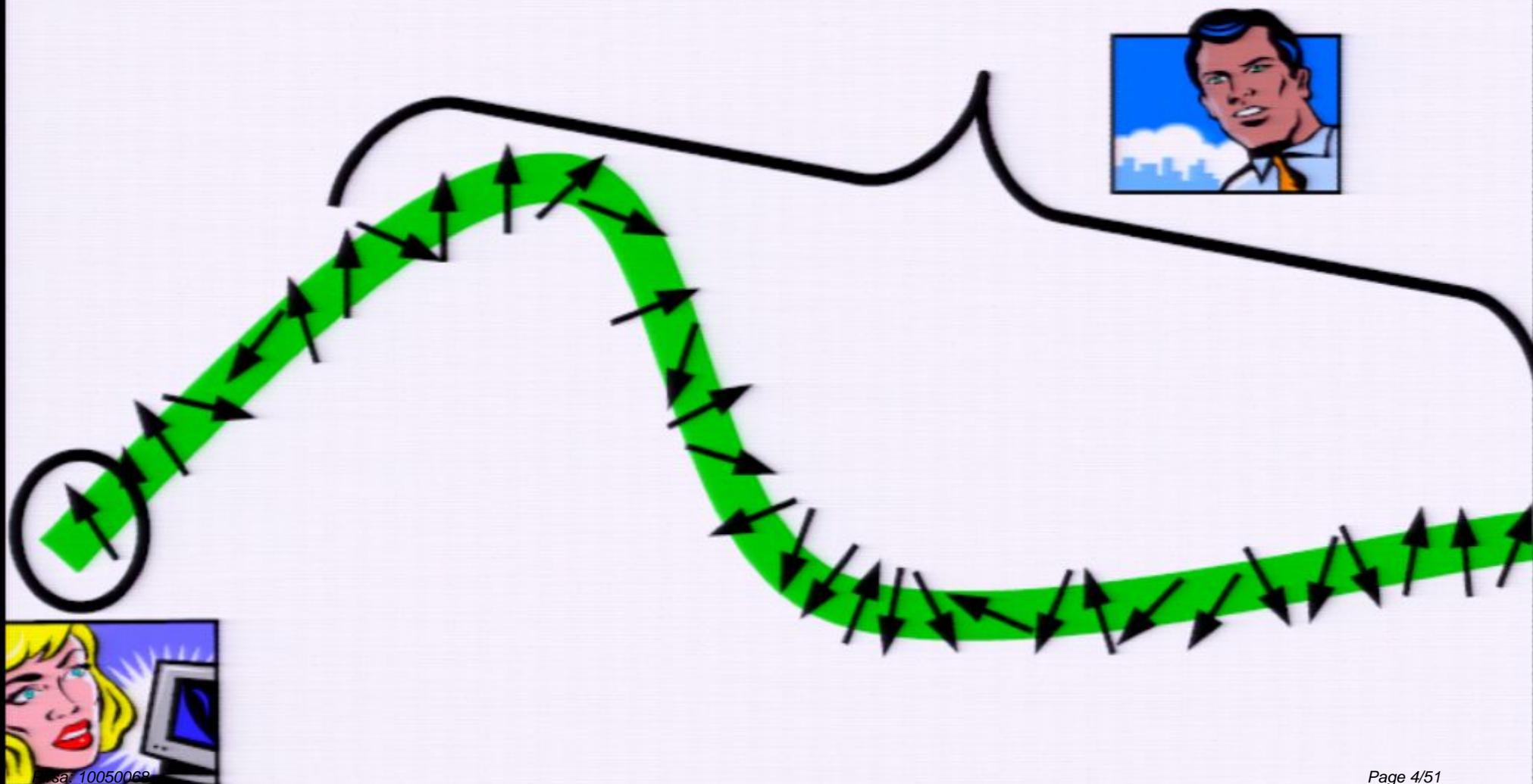
But processing may be required to bring  
it to an useful form

Entanglement “between” separated systems  
is *in principle* a resource for QIP

We will also study:

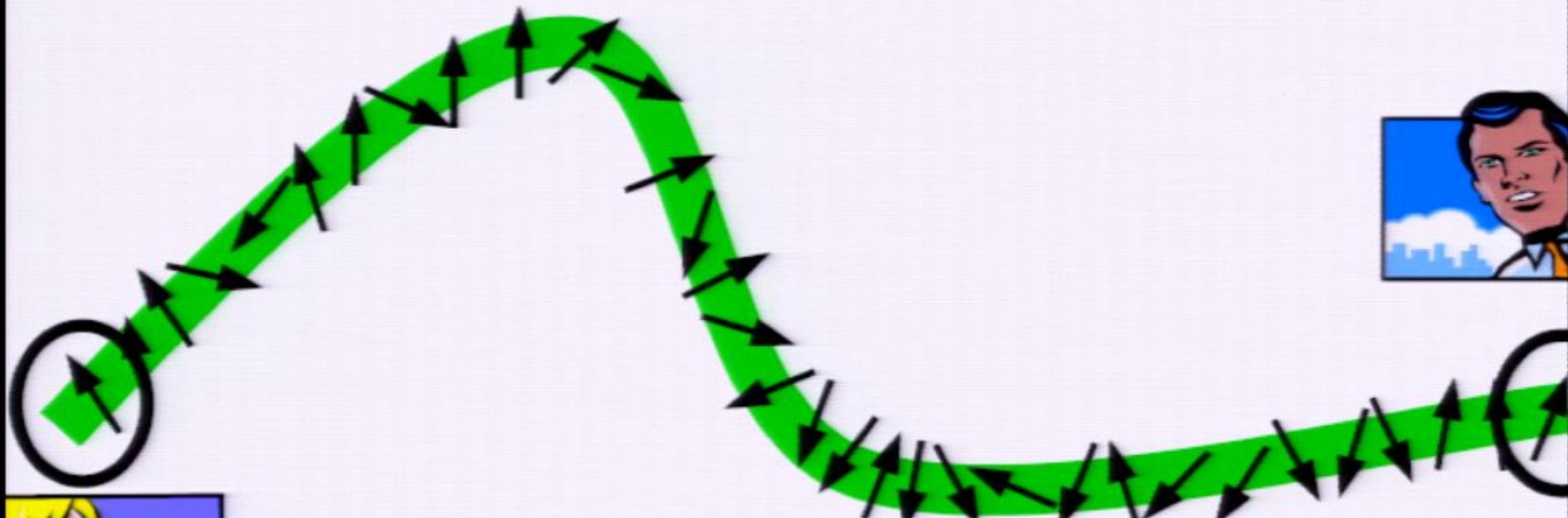


# Entanglement “between” separated systems is *in principle* a resource for QIP



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*Can be converted to:*



Two much studied forms of bi-partite entanglement:

**Between individual spins:** e.g. Arnesen, Bose, Vedral, 2000,  
Osborne & Nielsen 2002, Osterloh et al 2002 etc



*Extremely short ranged*



**Mostly uncharted territory**

(only exception we could find:  
Audenaert, Eisert, Plenio, Werner, 2002)

**Between complementary parts:** e.g. Vidal et. al. 2003,

Calabrese & Cardy; Korepin; Peschel etc.



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Noncomplementarity Blocks



**Mostly uncharted territory**

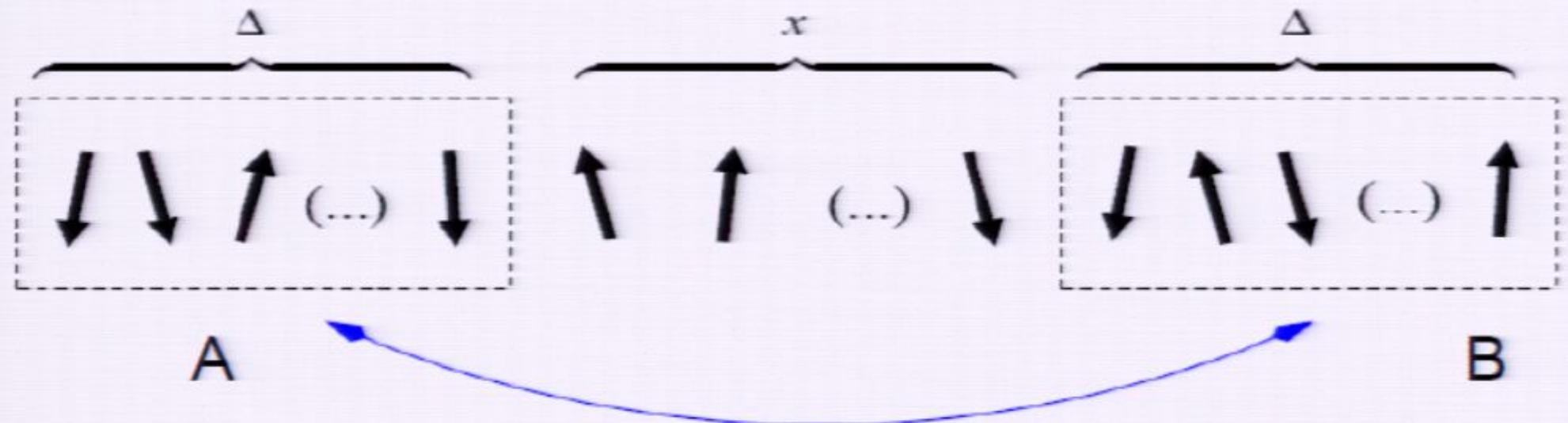
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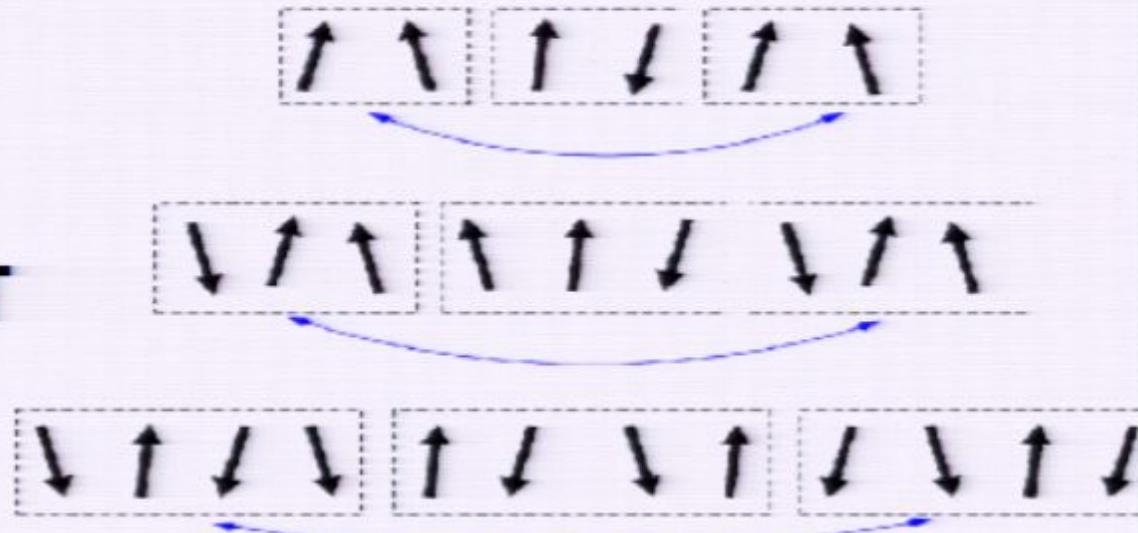


## Entanglement between separated blocks

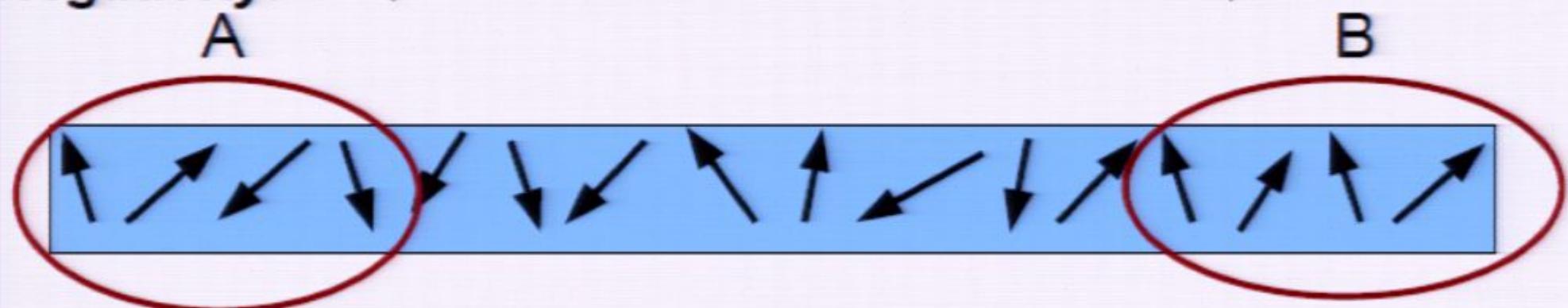


## Scale invariant at QPT

(e.g. in contrast with von Neumann entropy)



Entanglement “**between**” two arbitrary dimensional systems in a mixed state – the only computable measure we know is the **Negativity**: (Eisert 2001, Vidal and Werner 2002)



One first has to find the partial transpose  $\rho^{T_A}$  defined as  
 $\langle i_A, j_B | \rho^{T_A} | k_A, l_B \rangle \equiv \langle k_A, j_B | \rho | i_A, l_B \rangle$   
then Negativity  $\boxed{\mathcal{N}(\rho) = \sum_i |a_i| - 1}$ , where  $a_i$  are the eigenvalues of  $\rho^{T_A}$ .

Usually much more difficult to compute for a given many-body system than the concurrence or entropy

## General XY model with anisotropy

$$H = - \sum_{k=1}^{N-1} \left( \frac{1+\gamma}{2} \sigma_k^x \sigma_{k+1}^x + \frac{1-\gamma}{2} \sigma_k^y \sigma_{k+1}^y \right) - \sum_{k=1}^N \lambda \sigma_k^z$$

$(\gamma = 1, \lambda = 1)$

Ising universality

$(\gamma = 0.5, \lambda = 1)$

Ising universality

$(\gamma = 0, \lambda = 0)$

XX model

0.12

0.1

0.08

0.06

0.04

0.02

0

## Scale Invariance

0.5

-0.4

-0.3

-0.2

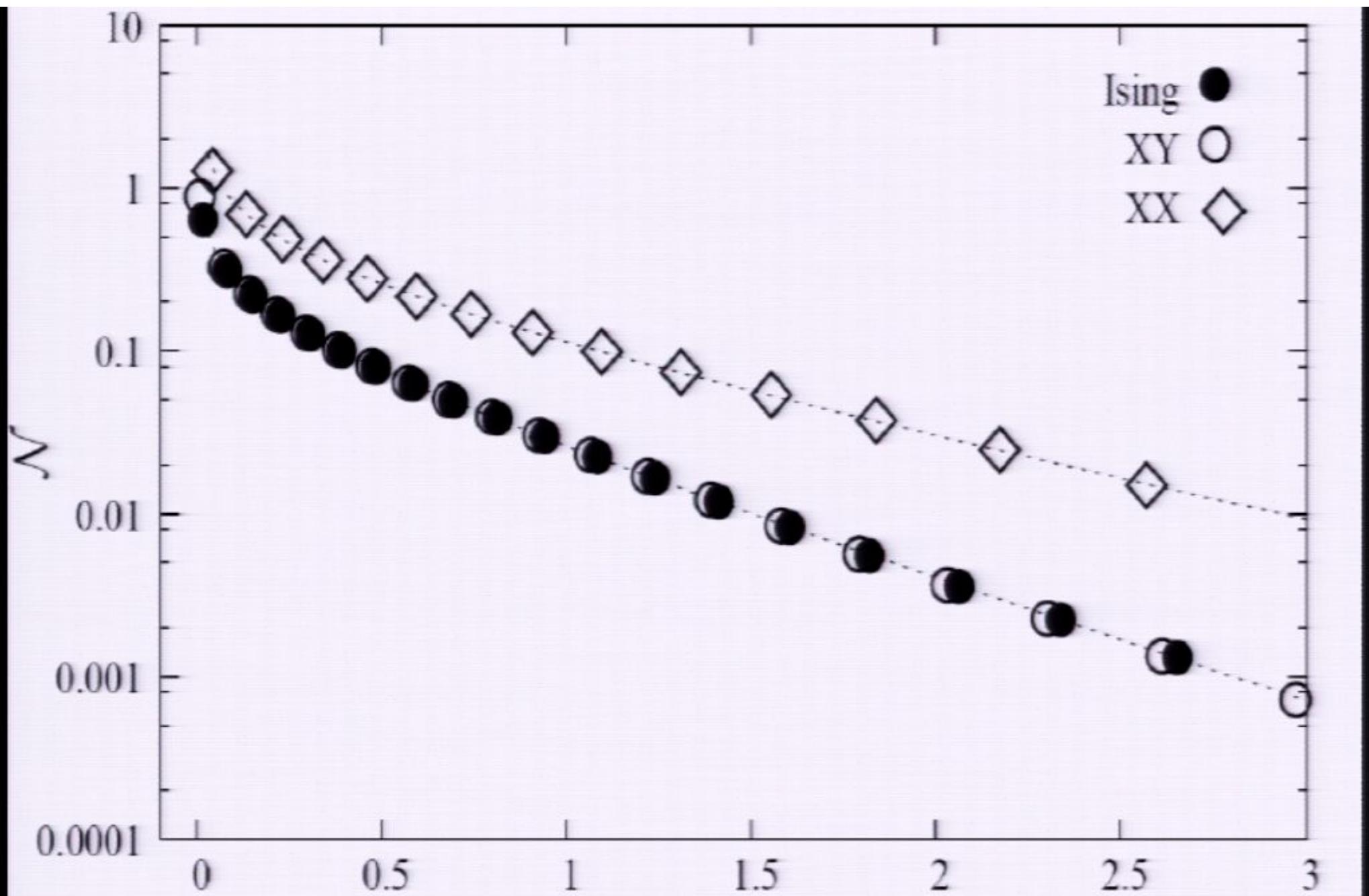
-0.1

0

0.1

$\lambda - \lambda_c$





Scale Invariance: Entanglement is function of

$$\mu = \frac{x}{\Delta} \text{ only}$$

The fitted ansatz:

Polynomial decay of correlations?

Monogamy of entanglement?

$$\mathcal{N}(\rho) \sim \mu^{-h} e^{-\alpha\mu}$$

*Interpolation formula*

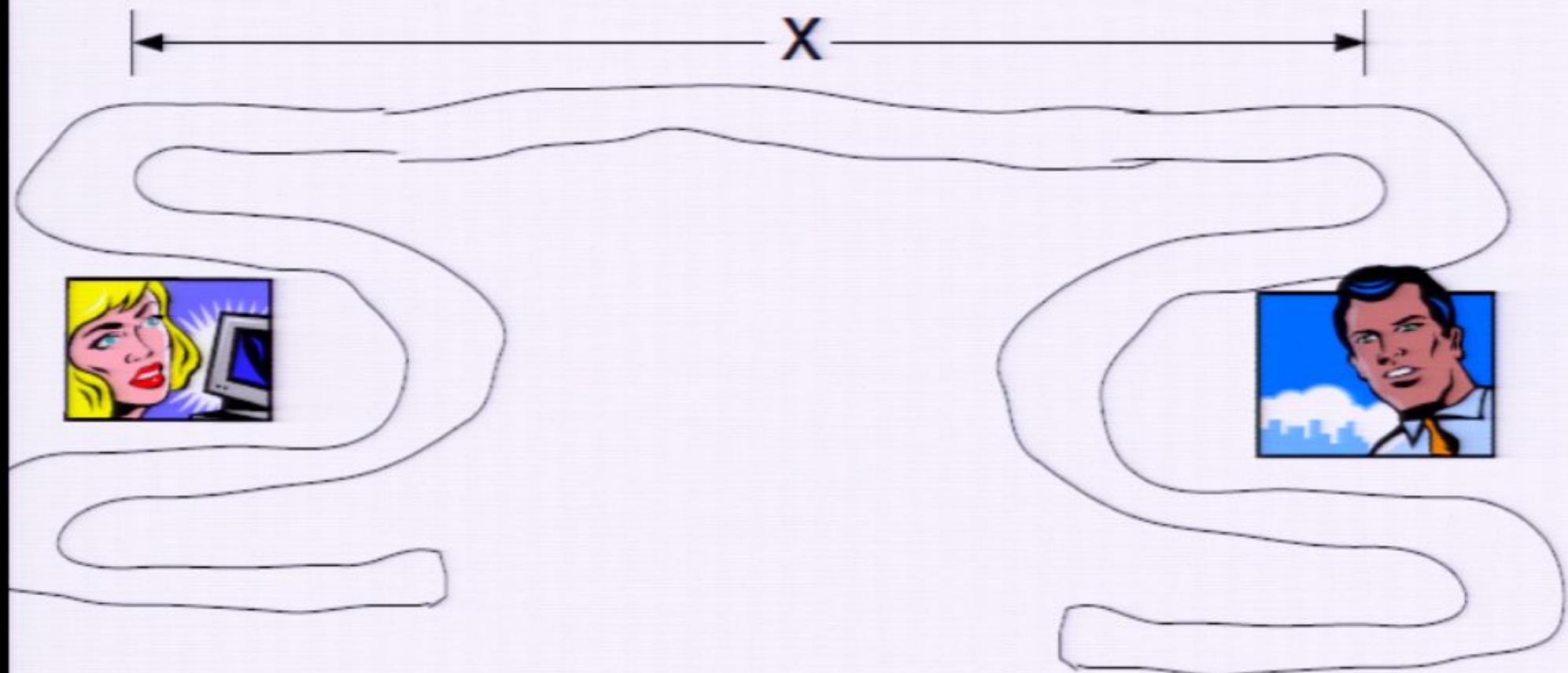
$$h = 0.47, \alpha = 0.99 \quad \text{xx}$$

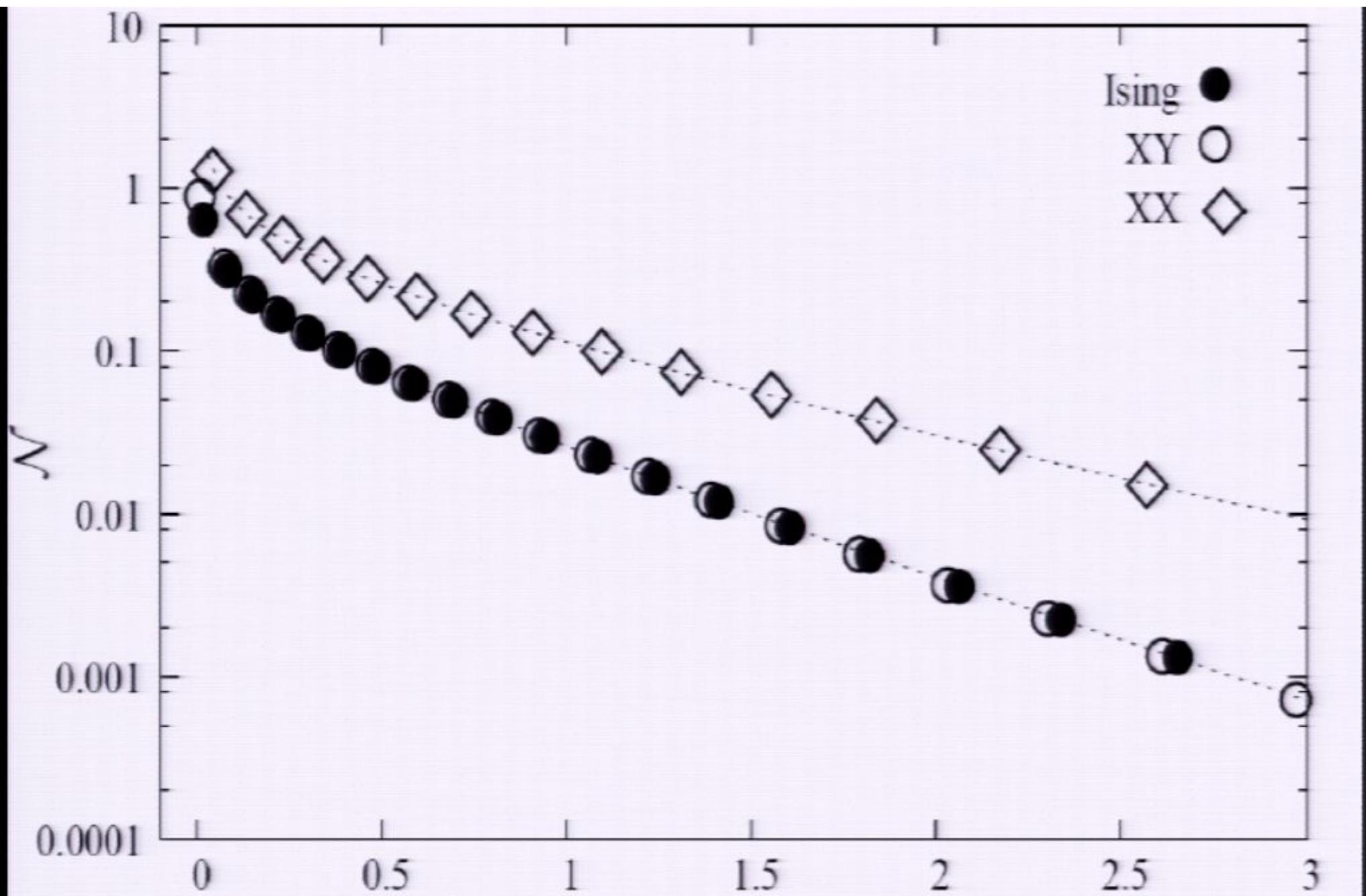
$$h = 0.38, \alpha = 1.68 \quad \text{Ising}$$

**Open qs:** How to relate to known critical exponents

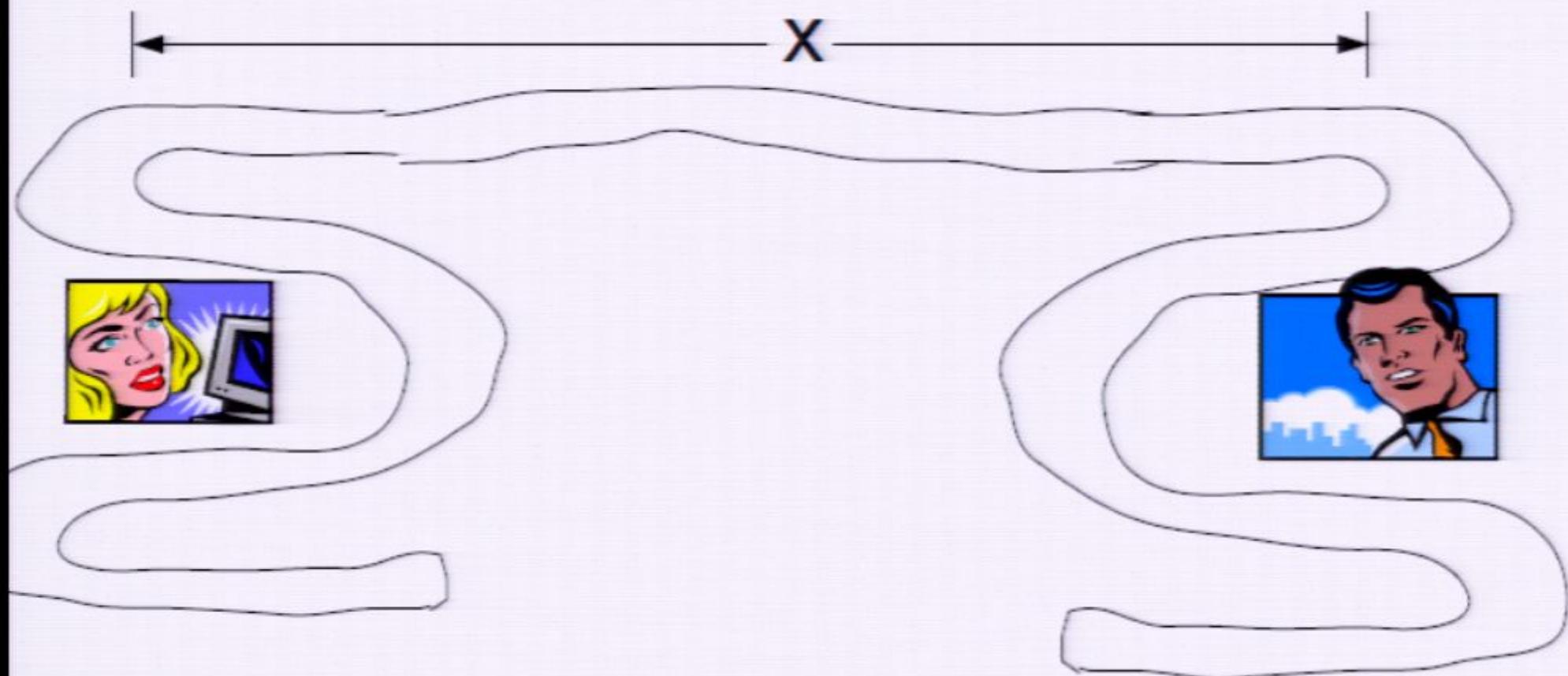
Vichterich, Molina-Vilaplana, Bose, Phys. Rev. A 80, 10304(R) (2009). Also see related work: Marcovitch et al PRB 009  
Page 13/51

To share a good amount of entanglement across a given separation  $x$ , Alice and Bob should access regions as large as  $x$  ...



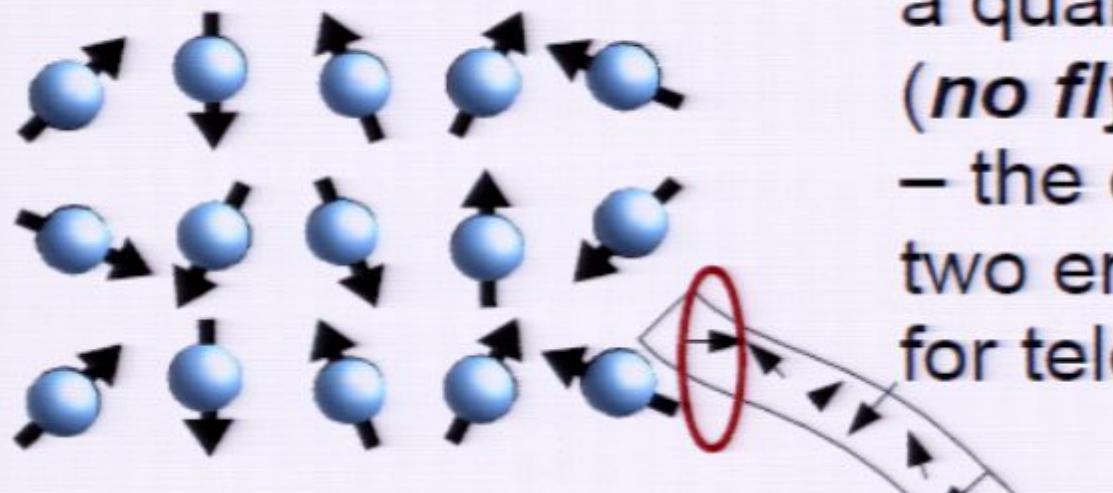


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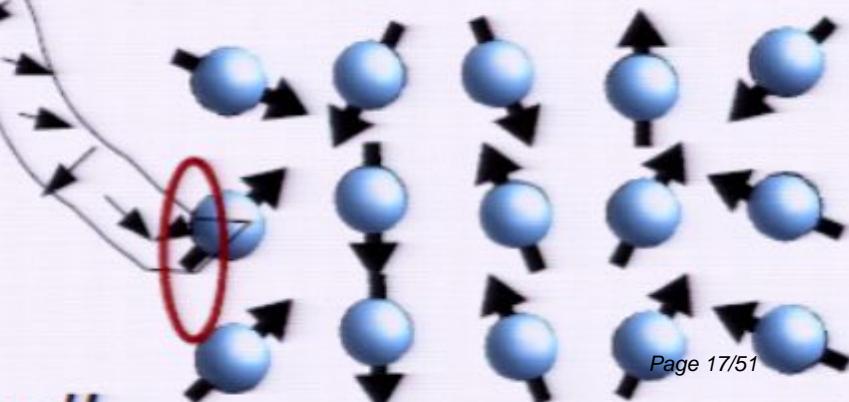
It would be more readily useful if two individual spins at opposite ends were entangled to high degree?

## Quantum Register 1



Using a many-body system as a quantum wire  
**(no flying qubits;**  
– the entanglement between two end spins usable for teleportation):

## Quantum Register



Note: Entanglement in many-body systems are normally *notoriously short ranged!*

Can non-equilibrium dynamics induced by a quench help?

**Quench** is a change of the Hamiltonian. Can that be used to generate substantial entanglement between the two individual spins at the ends?

Studied already in the context of block entanglement:



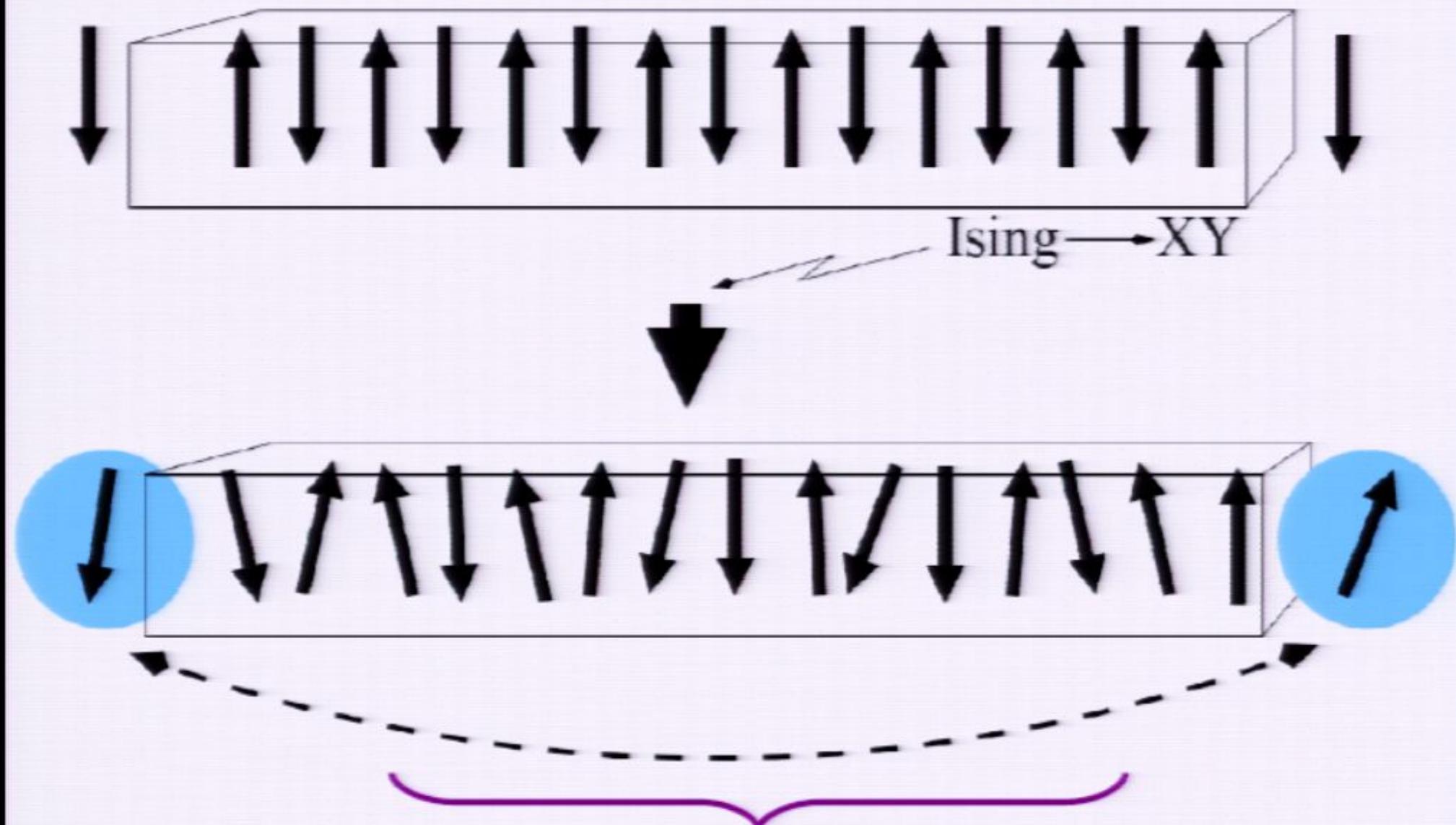
P. Calabrese and J. Cardy, J. Stat. Mech. P04010 (2005).

G. De Chiara, S. Montangero, P. Calabrese, and R. Fazio, J. Stat. Mech. P03001 (2006).

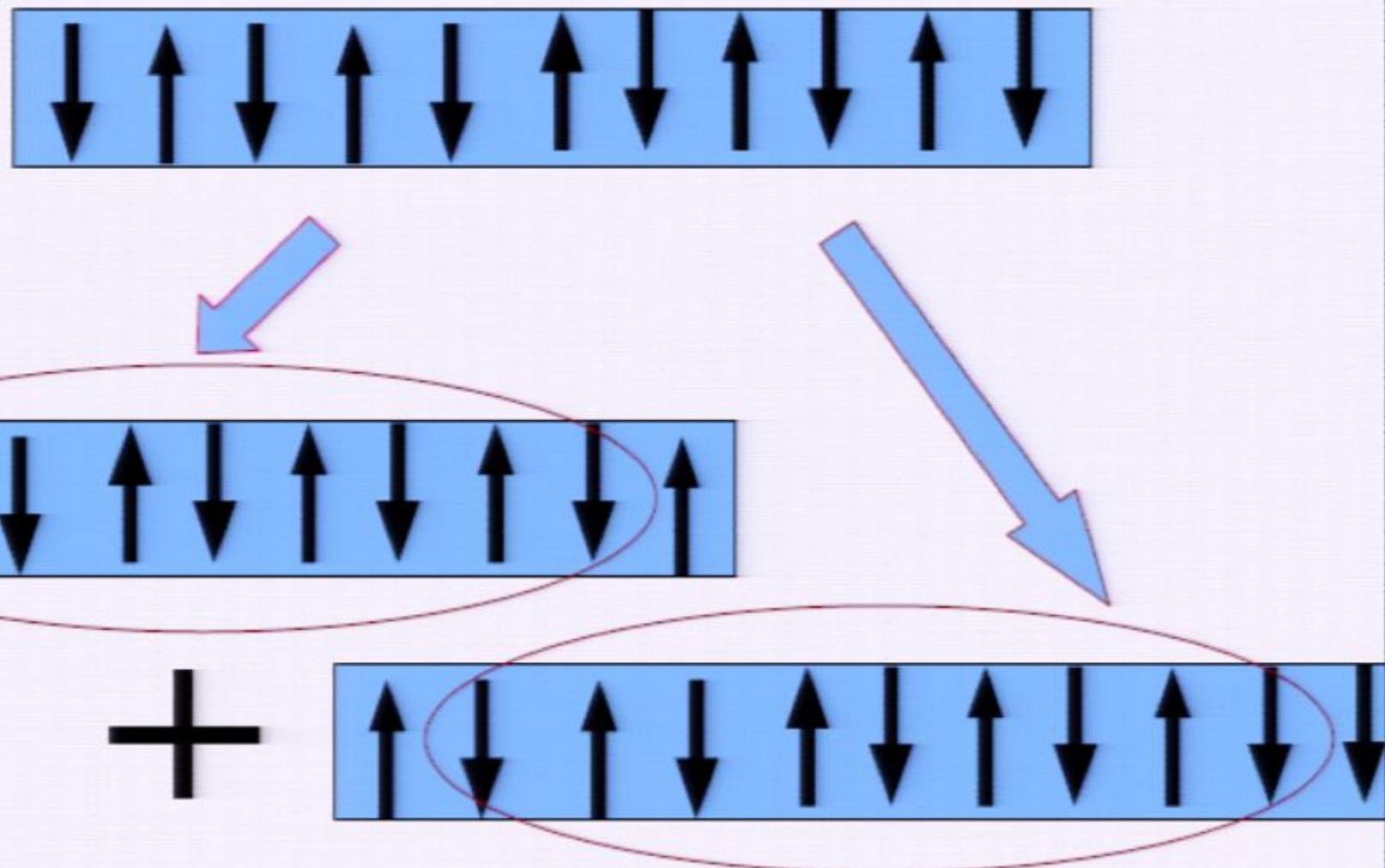
*Does not guarantee that individual spins at the ends will be significantly entangled.*

# Quantum Communication Resource from Quench

Hannu Wichterich & Sougato Bose, Phys. Rev. A 79, 060302(R) (2009)



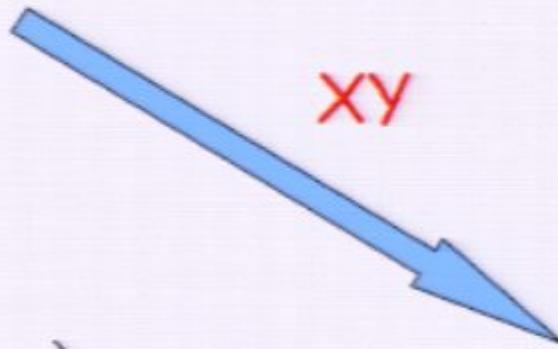
## Required configurations for entanglement:



$$H = \sum_{k=1}^{N-1} \frac{J}{2} (\sigma_k^x \sigma_{k+1}^x + \sigma_k^y \sigma_{k+1}^y + \Delta \sigma_k^z \sigma_{k+1}^z)$$

Quench:  $\Delta_1 \rightarrow \infty$  to  $\Delta_2 = 0$

 Ising

 XY

$$\rho_0 = \frac{1}{2} (|\mathcal{N}_1\rangle\langle\mathcal{N}_1| + |\mathcal{N}_2\rangle\langle\mathcal{N}_2|) \longrightarrow ?$$

where

$$|\mathcal{N}_1\rangle \equiv |\downarrow_1, \uparrow_2, \downarrow_3, \dots\rangle$$

$$|\mathcal{N}_2\rangle \equiv |\uparrow_1, \downarrow_2, \uparrow_3, \dots\rangle$$

Starting from a mixed state

Mapped to free fermions:  $c_k^\dagger \equiv \left( \prod_{l=1}^{k-1} -\sigma_l^z \right) \sigma_k^+$

Components of the density matrix at any time:

$$\langle \downarrow \uparrow | \rho_{1,N} | \uparrow \downarrow \rangle = \frac{1}{2} ((-1)^{M+1} \langle c_N^\dagger c_1 \rangle_1 + c.c.)$$

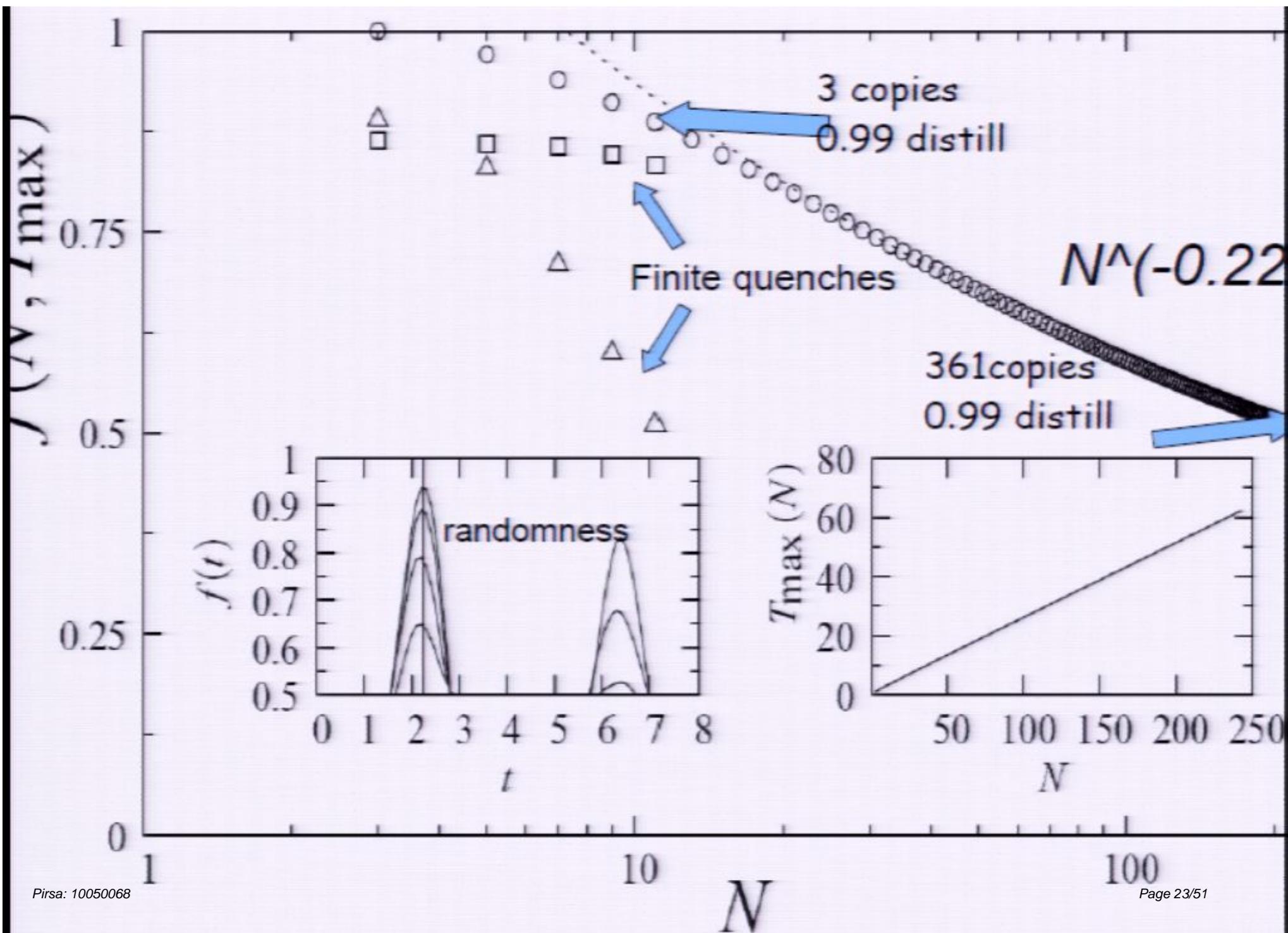
$$\langle c_i^\dagger(t) c_j(t) \rangle = \sum_{k,l=1}^N f_{i,k}(t) f_{j,l}^*(t) \langle c_k^\dagger(0) c_l(0) \rangle$$

Free fermion amplitudes

$$\langle c_k^\dagger(0) c_l(0) \rangle_1 = \delta_{k,l} \delta_{k,2m}, \quad (m = 1, 2, \dots, M)$$

Odd chain only:

$$\rho_{1,N} \simeq f |\psi^+\rangle \langle \psi^+| + \frac{(1-f)}{2} (|\uparrow, \uparrow\rangle \langle \uparrow, \uparrow| + |\downarrow, \downarrow\rangle \langle \downarrow, \downarrow|)$$



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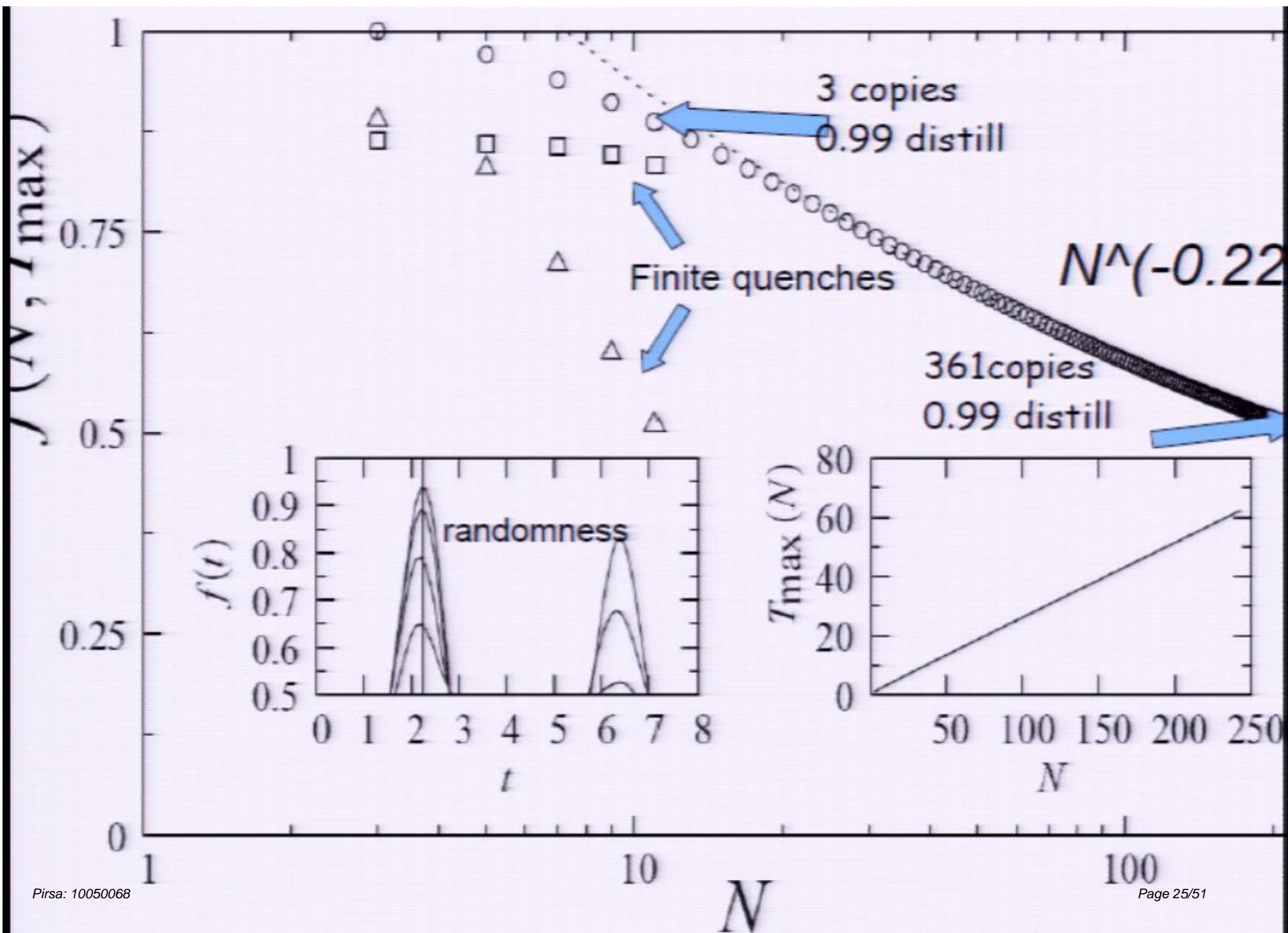
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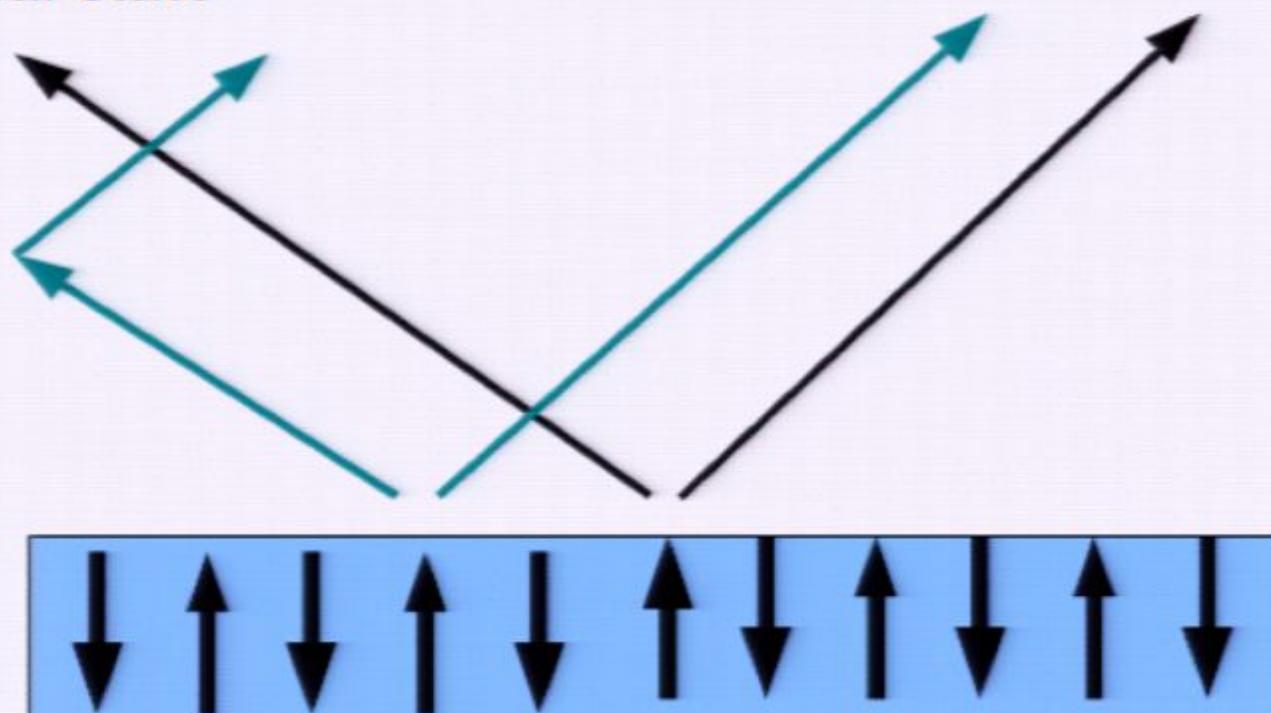
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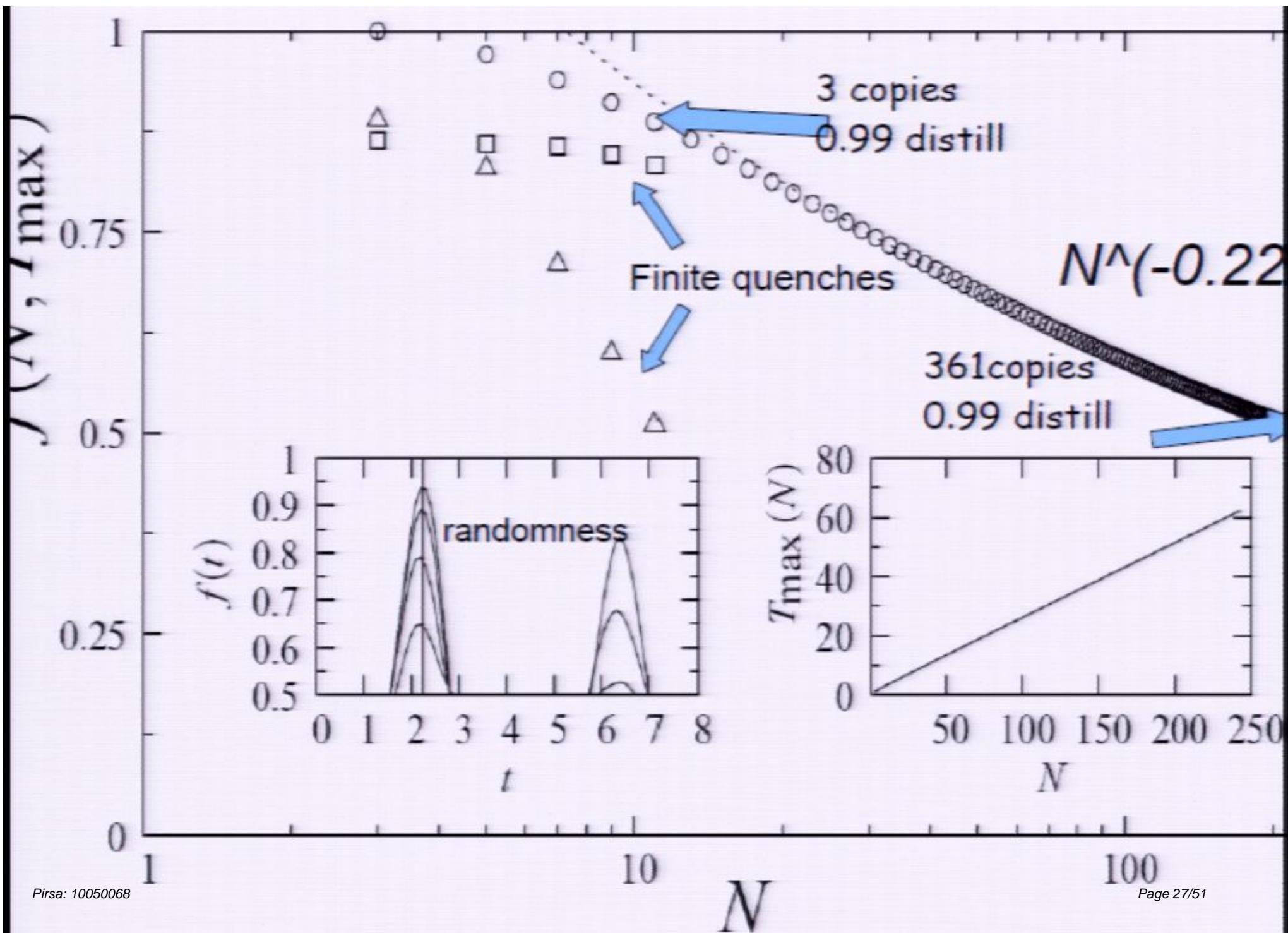


Explanation: A number of sources cooperatively give the same entangled state at  $\sim T/2$

Left & right components become equidistant from respective ends

Each fermion evolves independently – does not see each other except for statistics --- whose effects are also cancelled here by the special initial state





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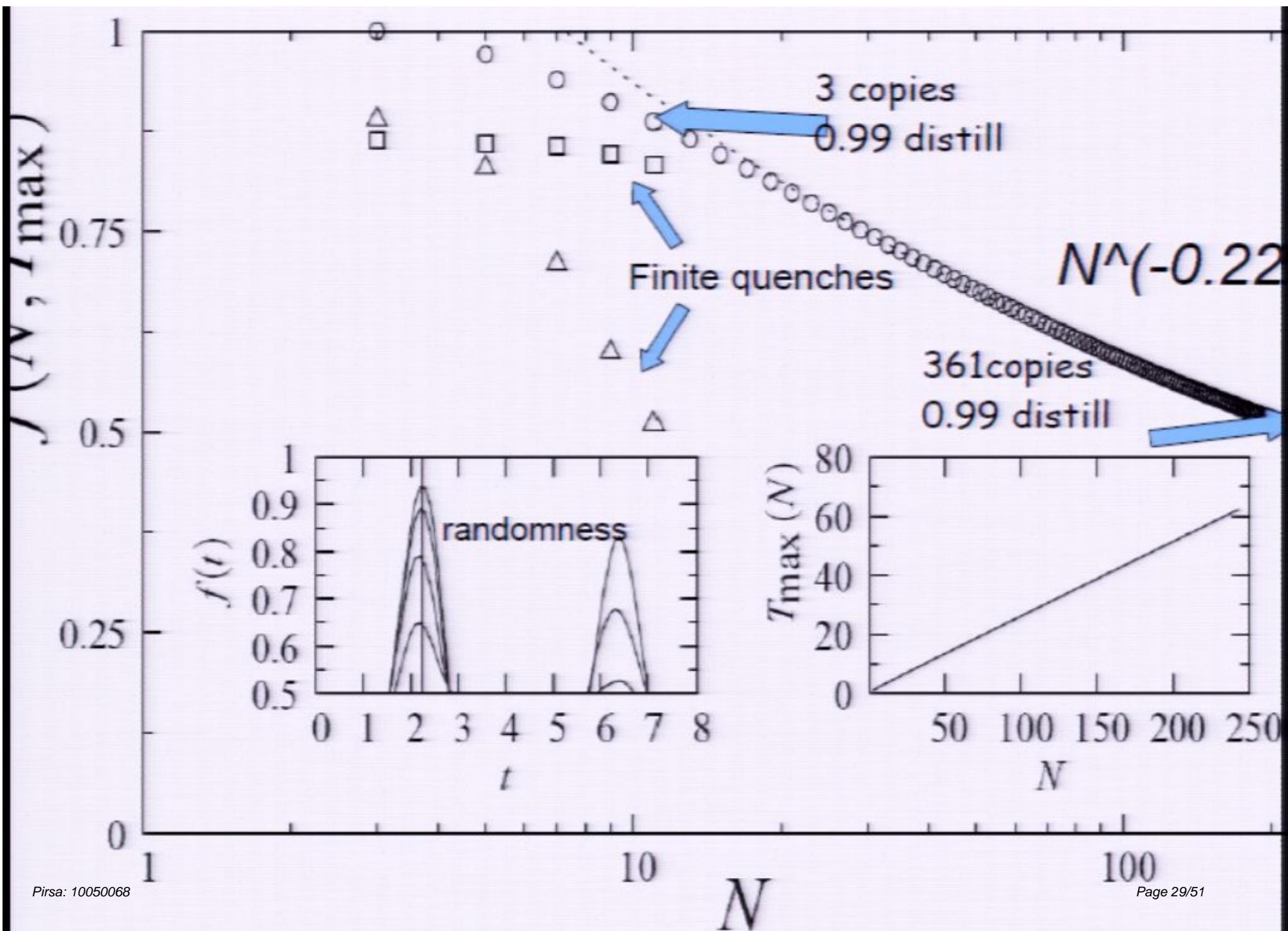
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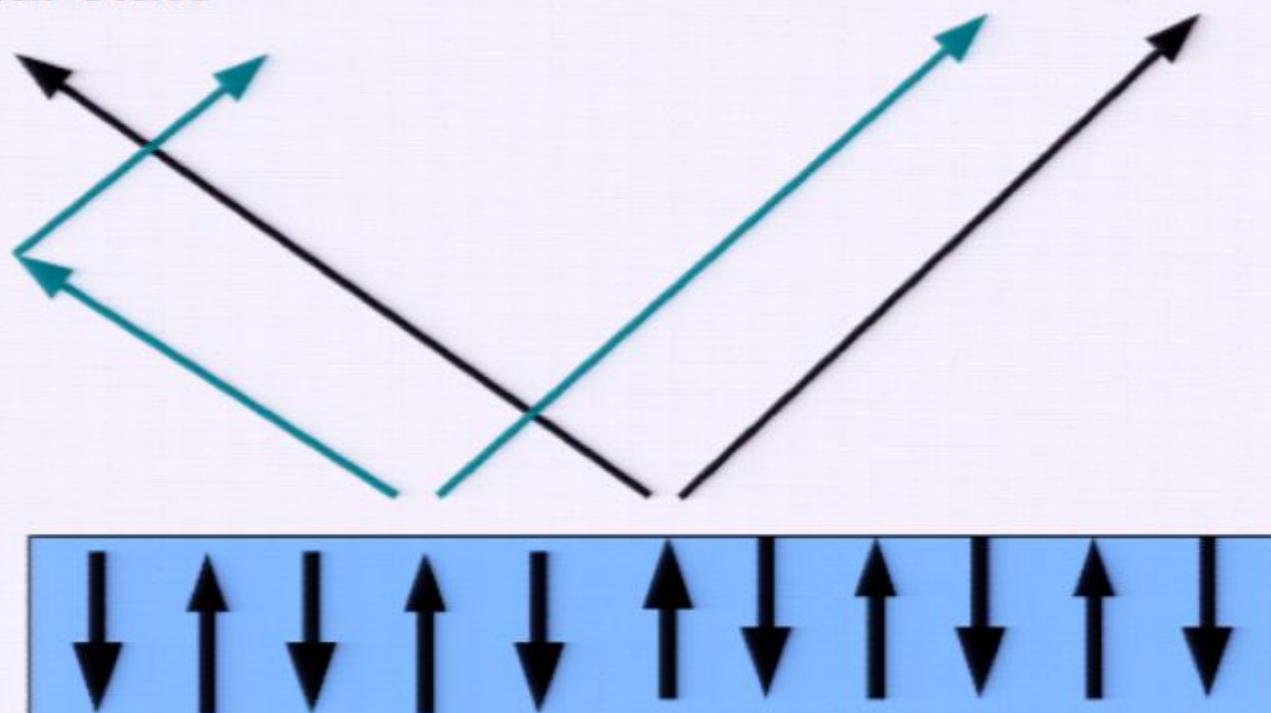
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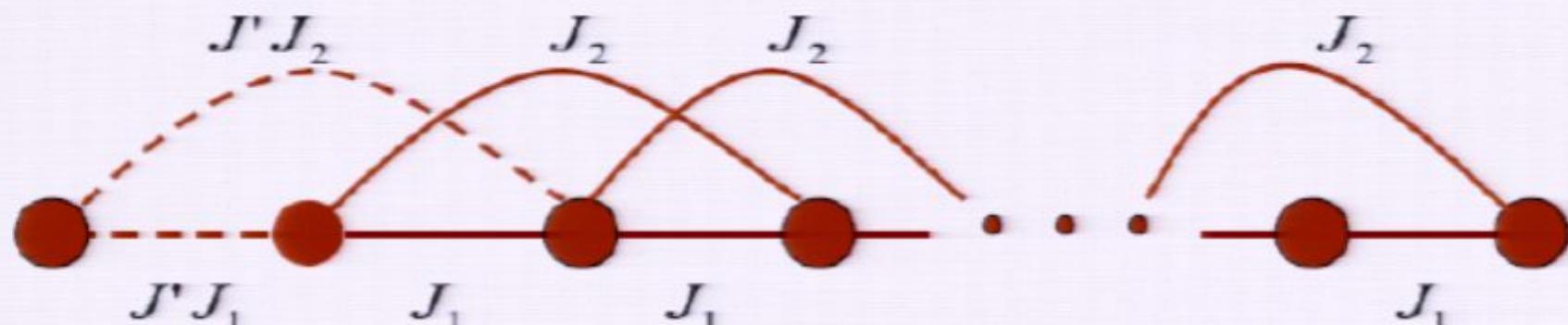


Can one get a distance *independent* entanglement of a high amount between the ends of a spin chain?

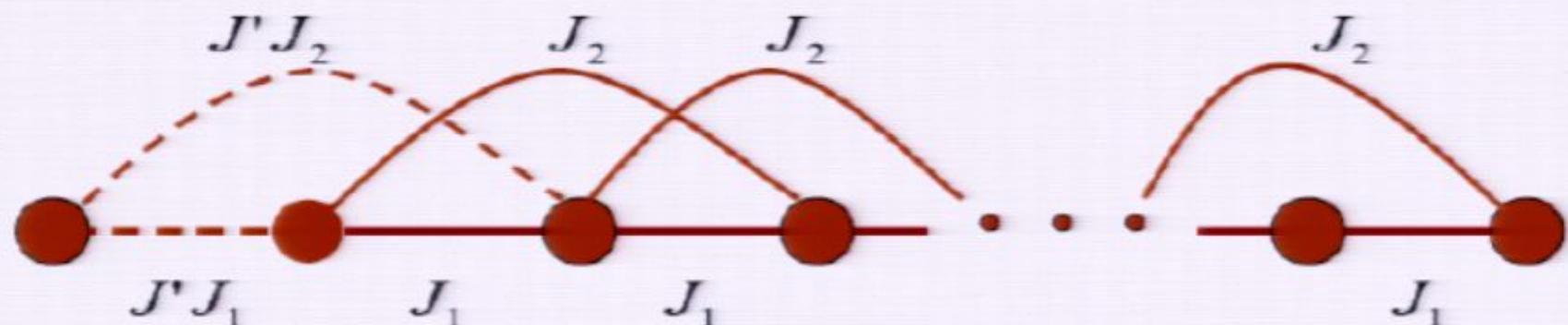
One possibility is to have very weakly coupled end spins (Campos-Venuti *et al* 2006) --- this is not robust to temperature.

Another possibility with stronger, but nonuniform couplings: To exploit the Kondo cloud in a **Kondo spin chain** (Affleck and co-workers)

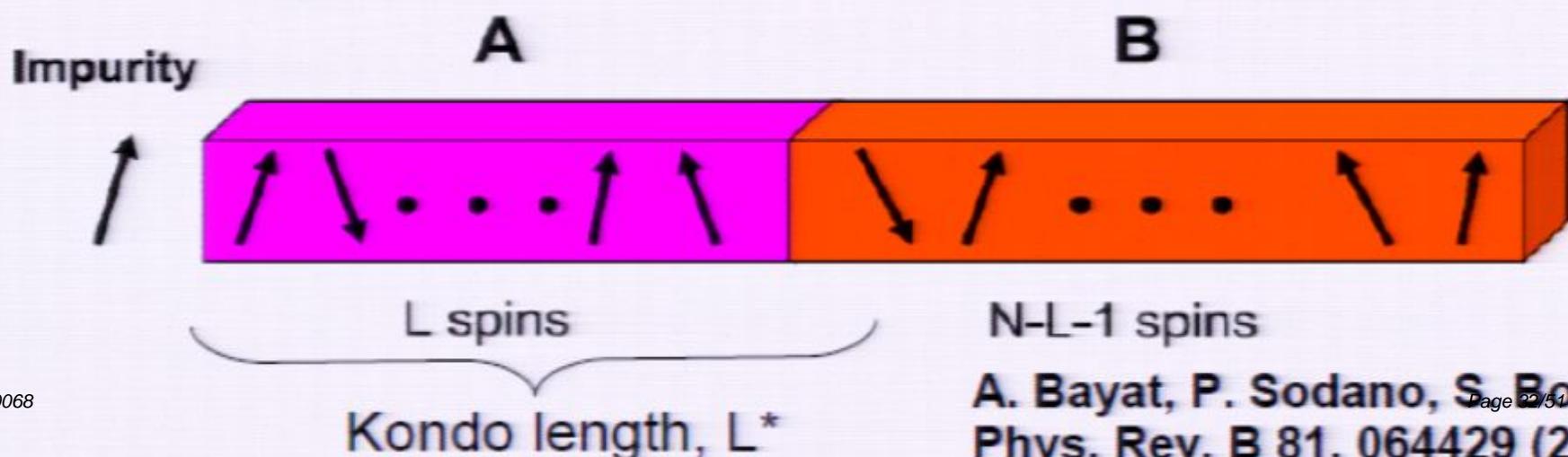
(a)

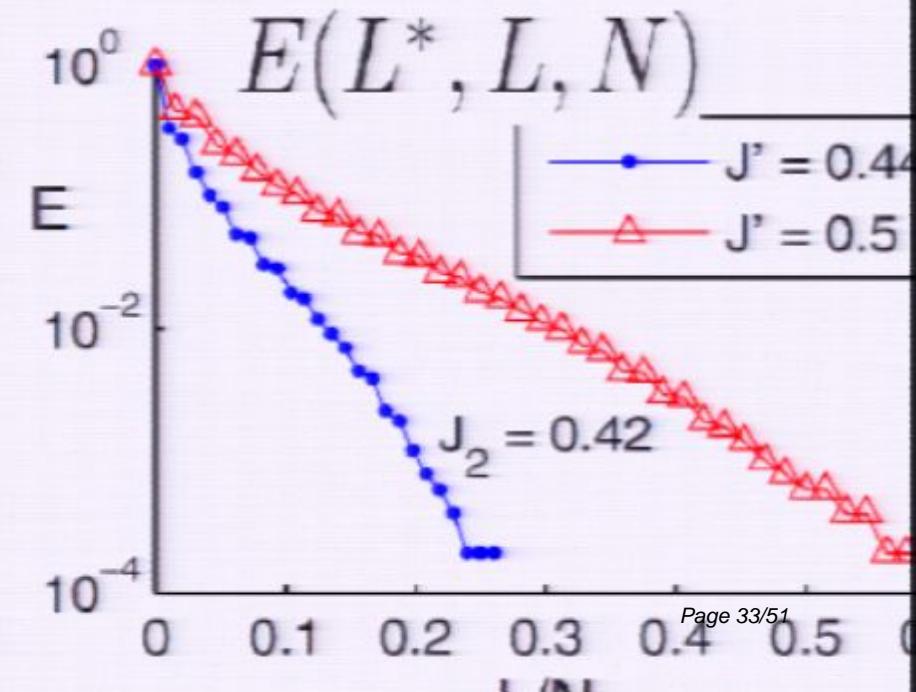
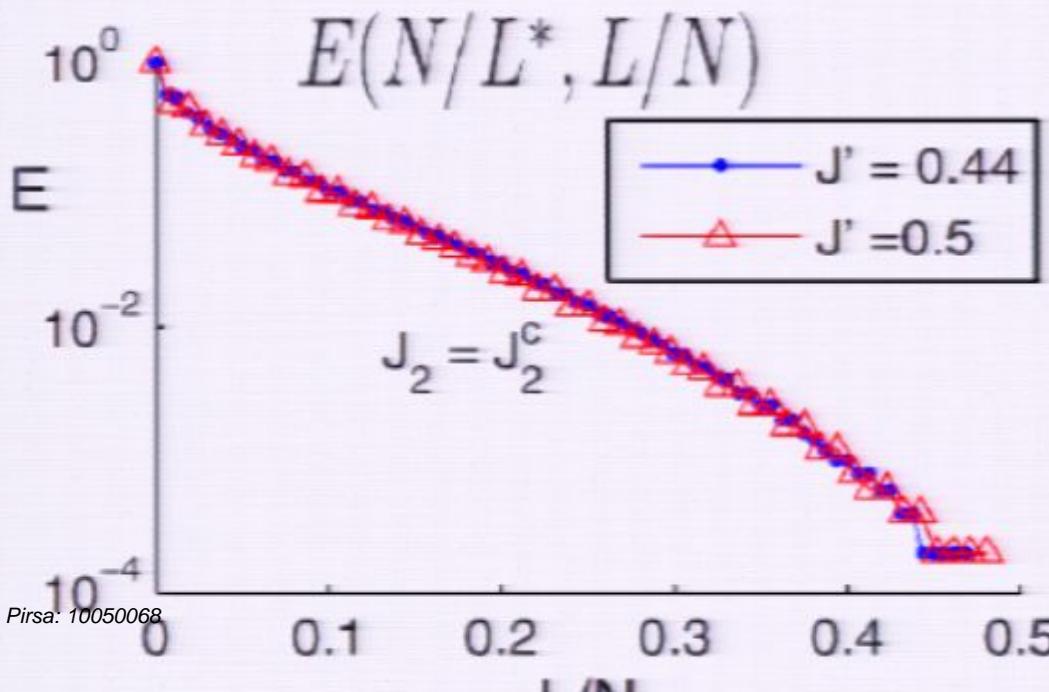
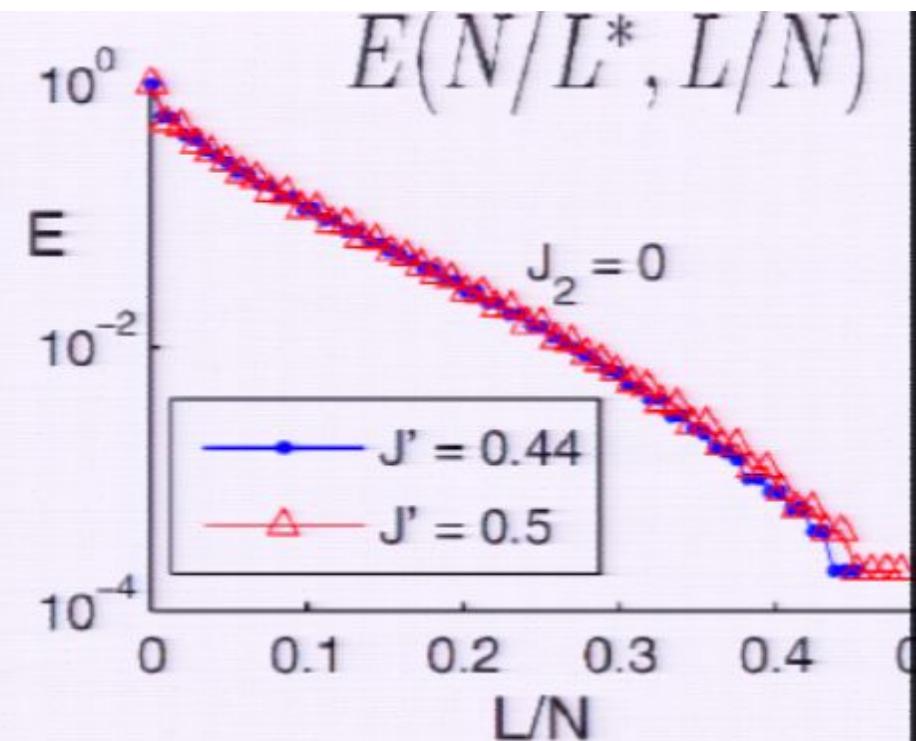
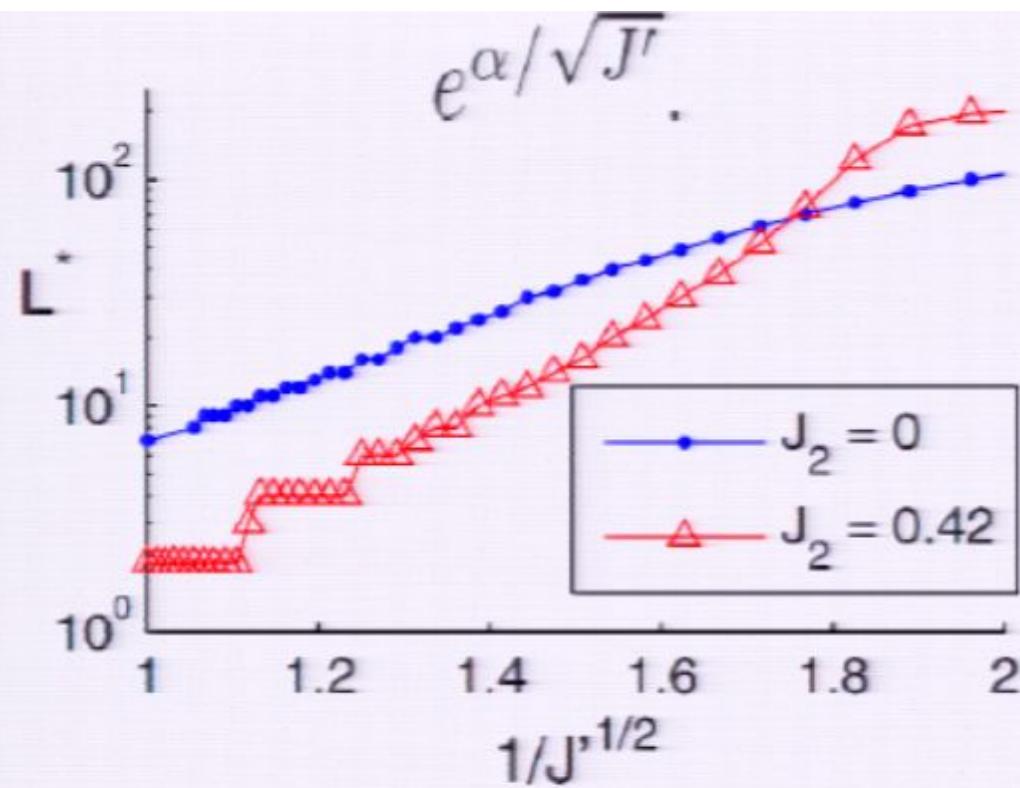


First one needs to understand the structure of entanglement in the ground state of the *spin chain Kondo model*. (Affleck et. al have already found that the “*impurity entanglement entropy*” witnesses it. But *Negativity* is needed to know by “how much” the impurity is entangled with a block in the spin chain)

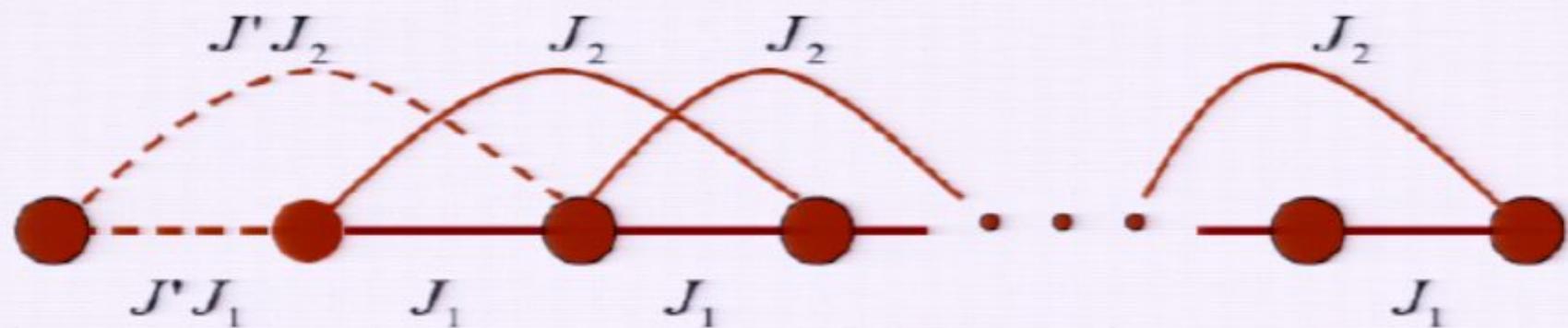


The model  $J_2 < 0.23$  is gapless Kondo; After that dimer

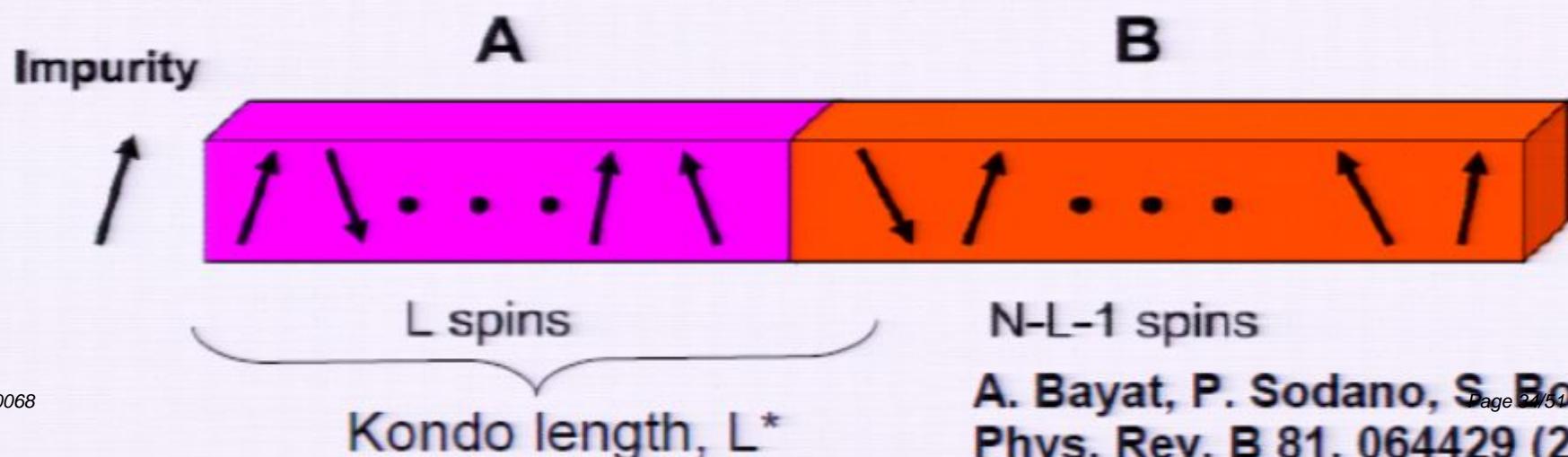


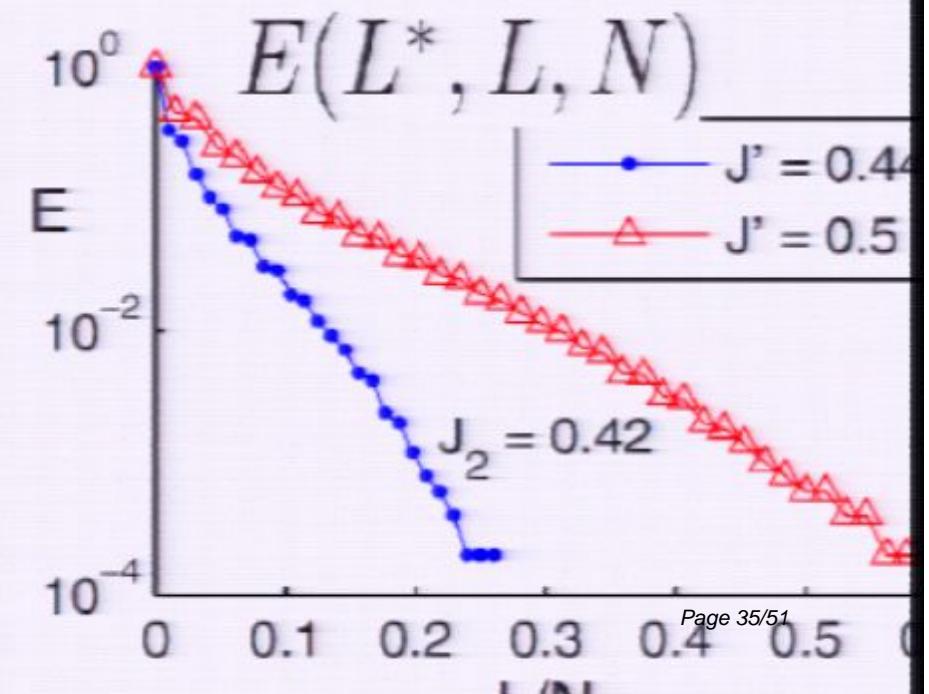
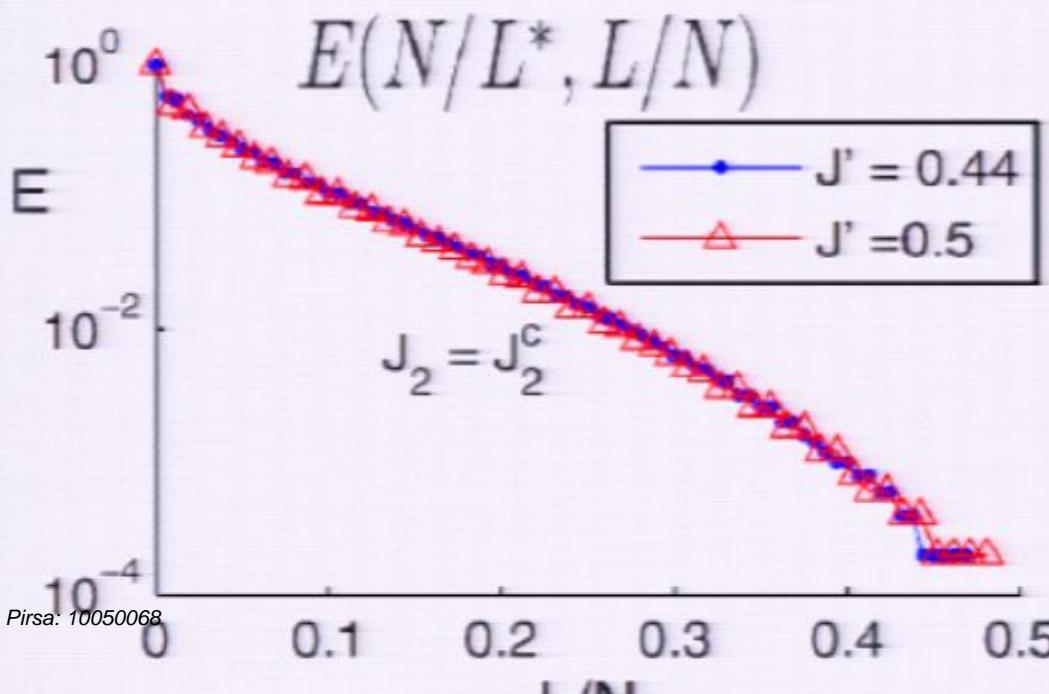
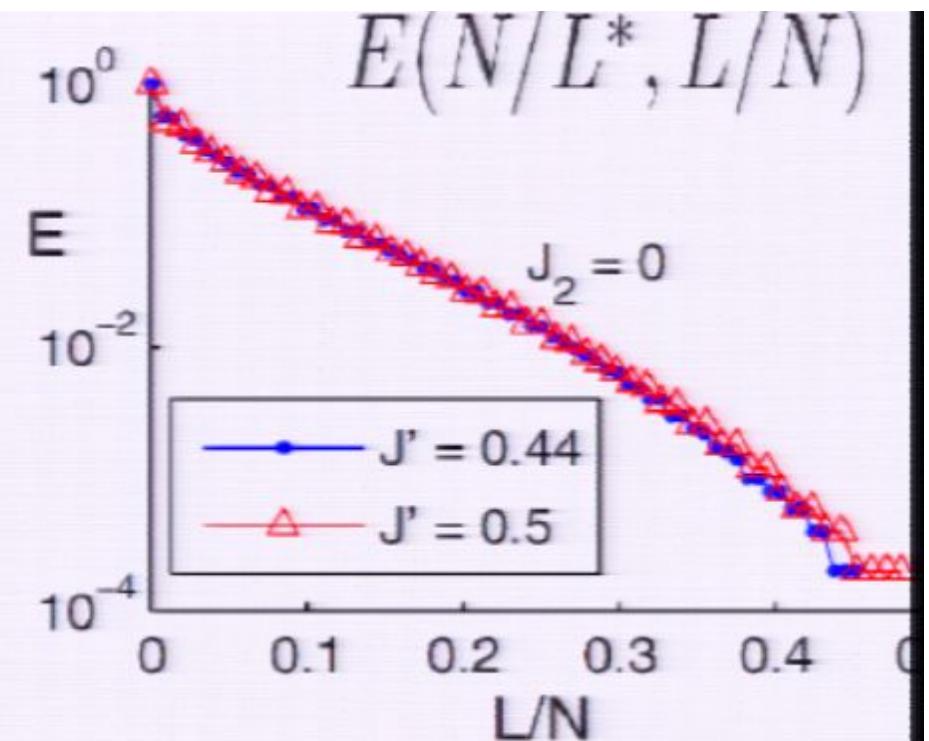
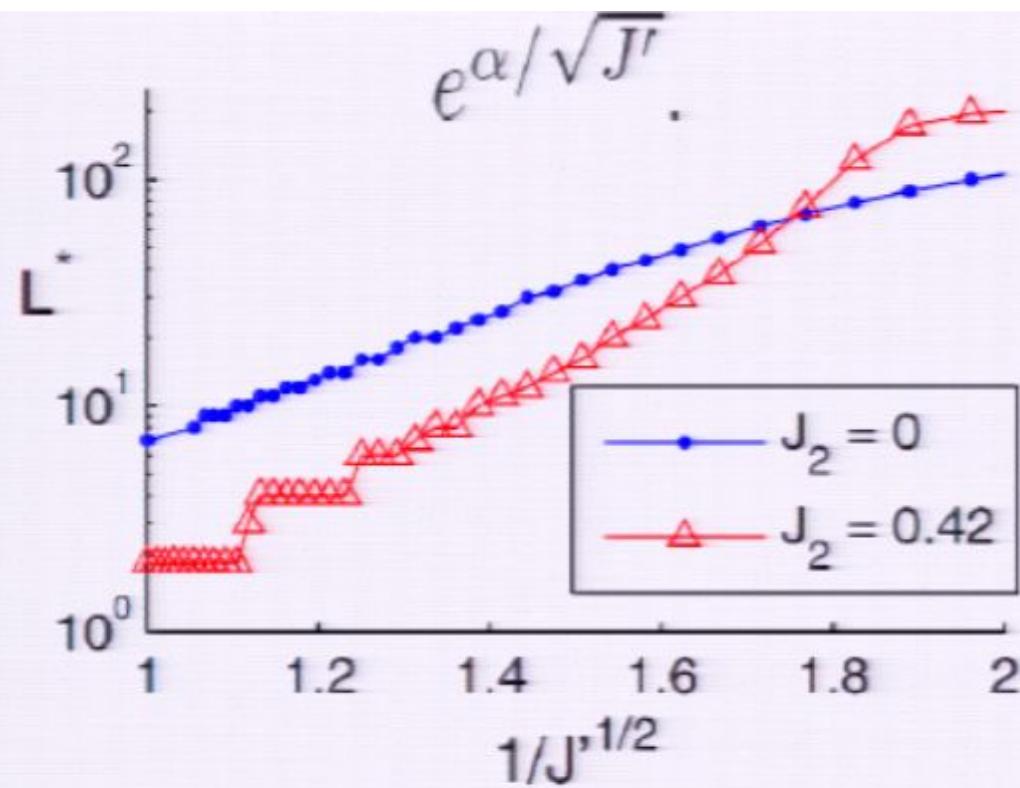


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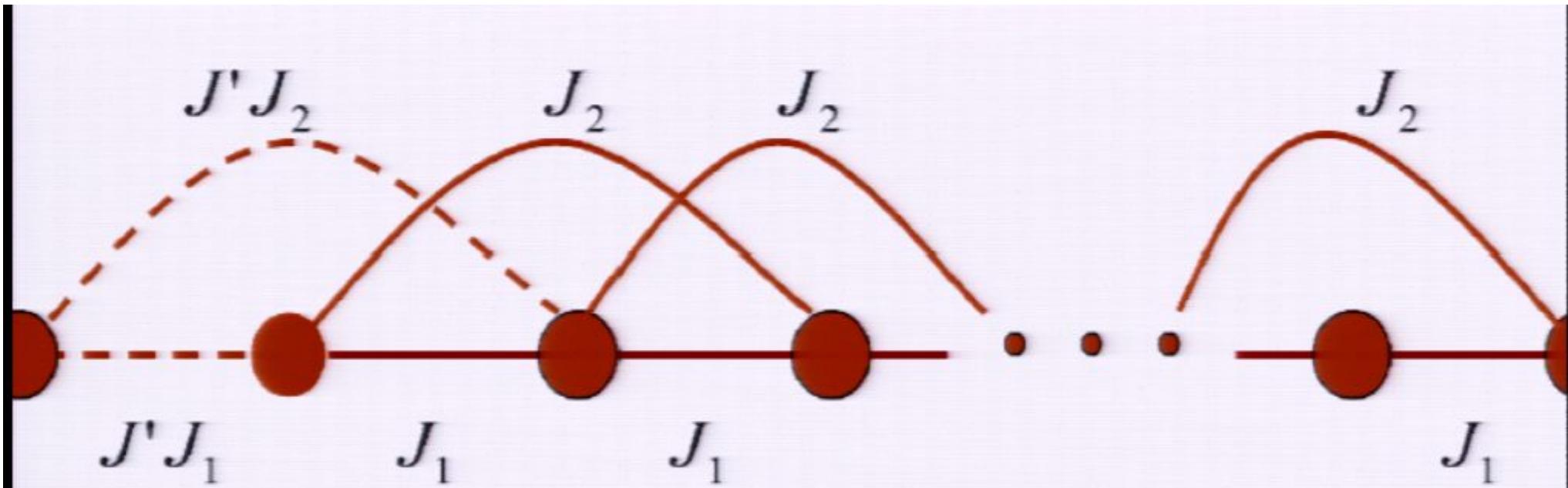




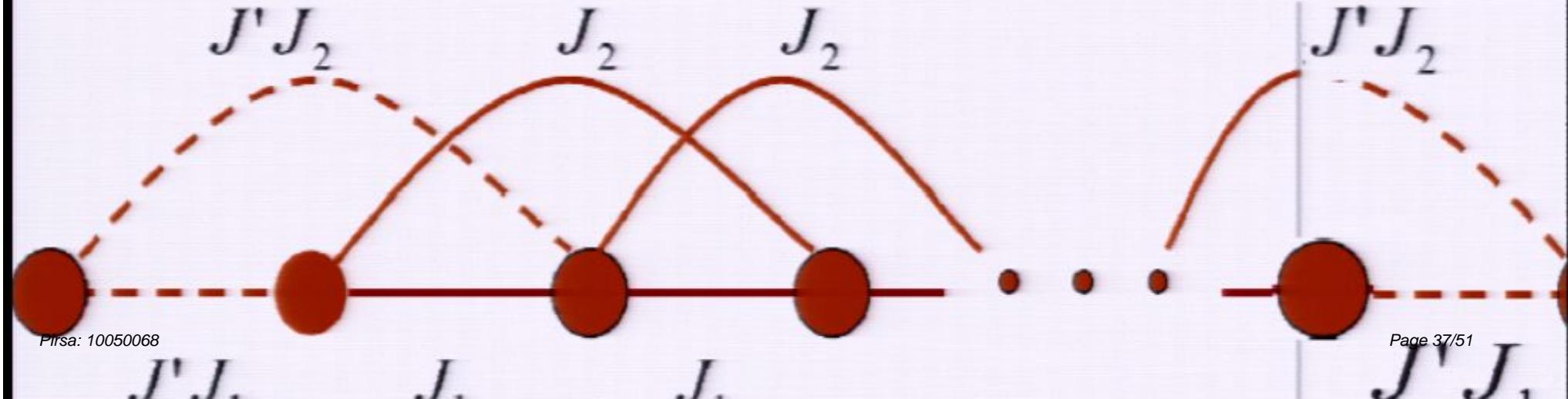
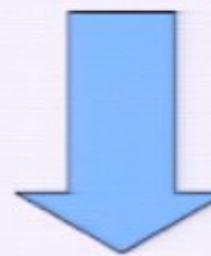
Ground state structure in the Kondo phase:

$$GS_K\rangle = \sum_i \alpha_i \frac{|\uparrow\rangle|L_i^\uparrow(J')\rangle - |\downarrow\rangle|L_i^\downarrow(J')\rangle}{\sqrt{2}} \otimes |R_i(J')\rangle$$

Bayat, Sodano, Bose, Phys. Rev. B 81, 064429 (2010)



Single local quench



## A single local quench for distant entanglement:

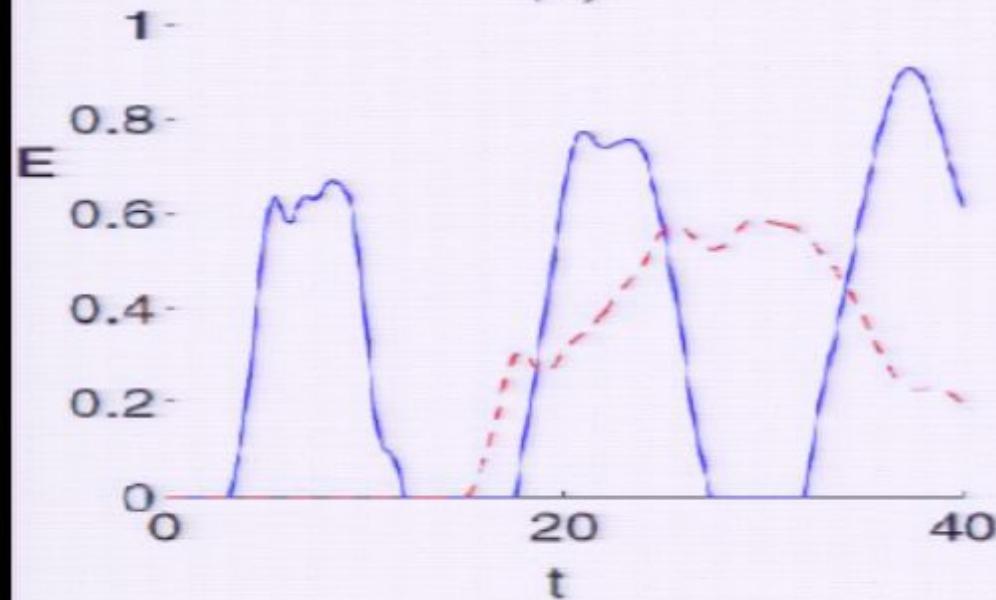
$$H_I = J' (J_1 \sigma_1 \cdot \sigma_2 + J_2 \sigma_1 \cdot \sigma_3) + J_1 \sum_{i=2}^{N-2} \sigma_i \cdot \sigma_{i+1} + J_2 \sum_{i=2}^{N-3} \sigma_i \cdot \sigma_{i+2}$$



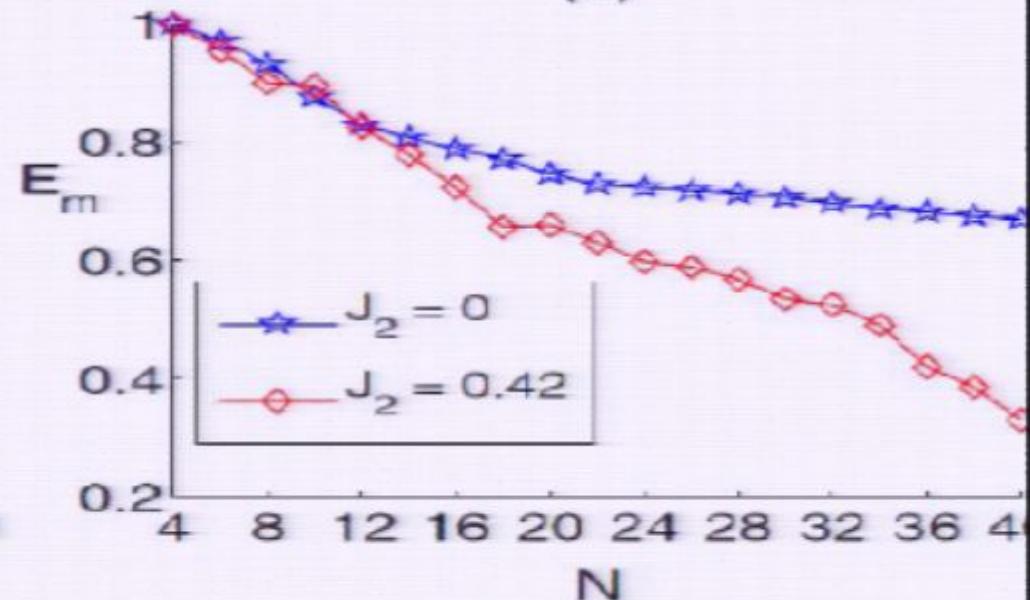
$$H_F = J' (J_1 \sigma_1 \cdot \sigma_2 + J_2 \sigma_1 \cdot \sigma_3 + J_1 \sigma_{N-1} \cdot \sigma_N + J_2 \sigma_{N-2} \cdot \sigma_N) + J_1 \sum_{i=2}^{N-2} \sigma_i \cdot \sigma_{i+1} + J_2 \sum_{i=2}^{N-3} \sigma_i \cdot \sigma_{i+2}. \quad (2)$$

$$|\psi(t)\rangle = e^{-iH_F t} |GS_I\rangle$$

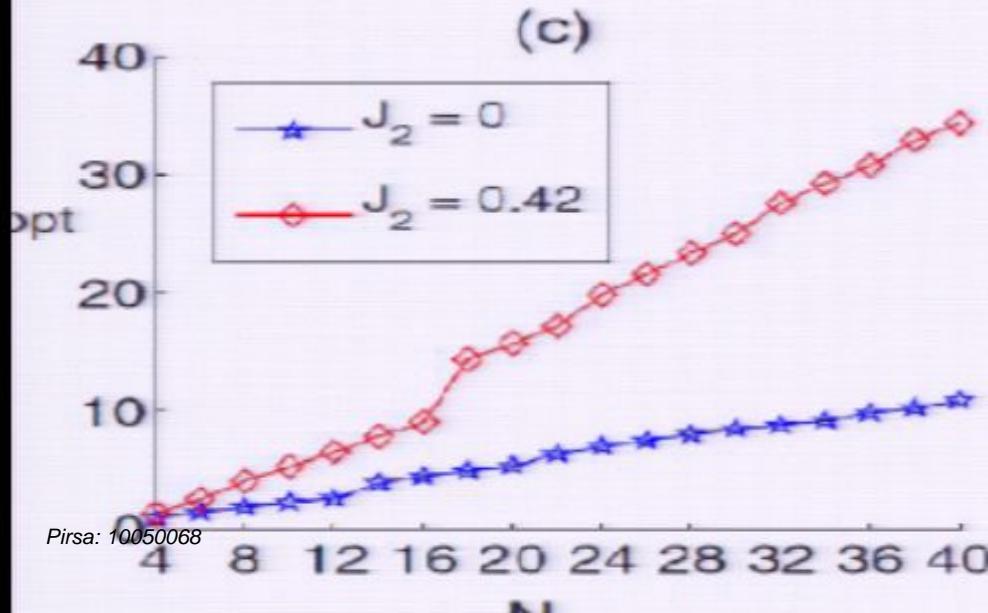
(a)



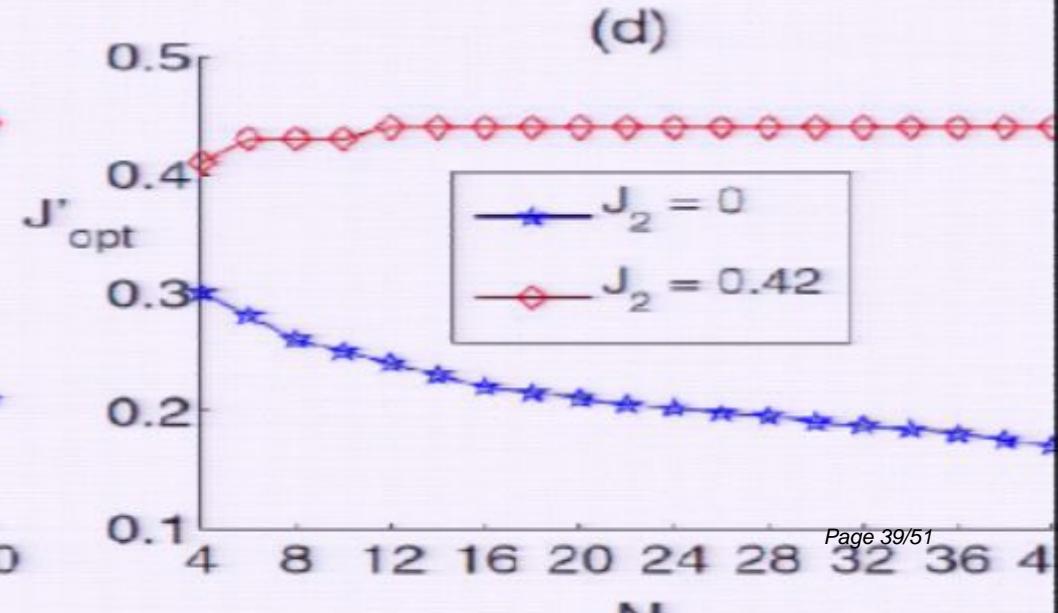
(b)



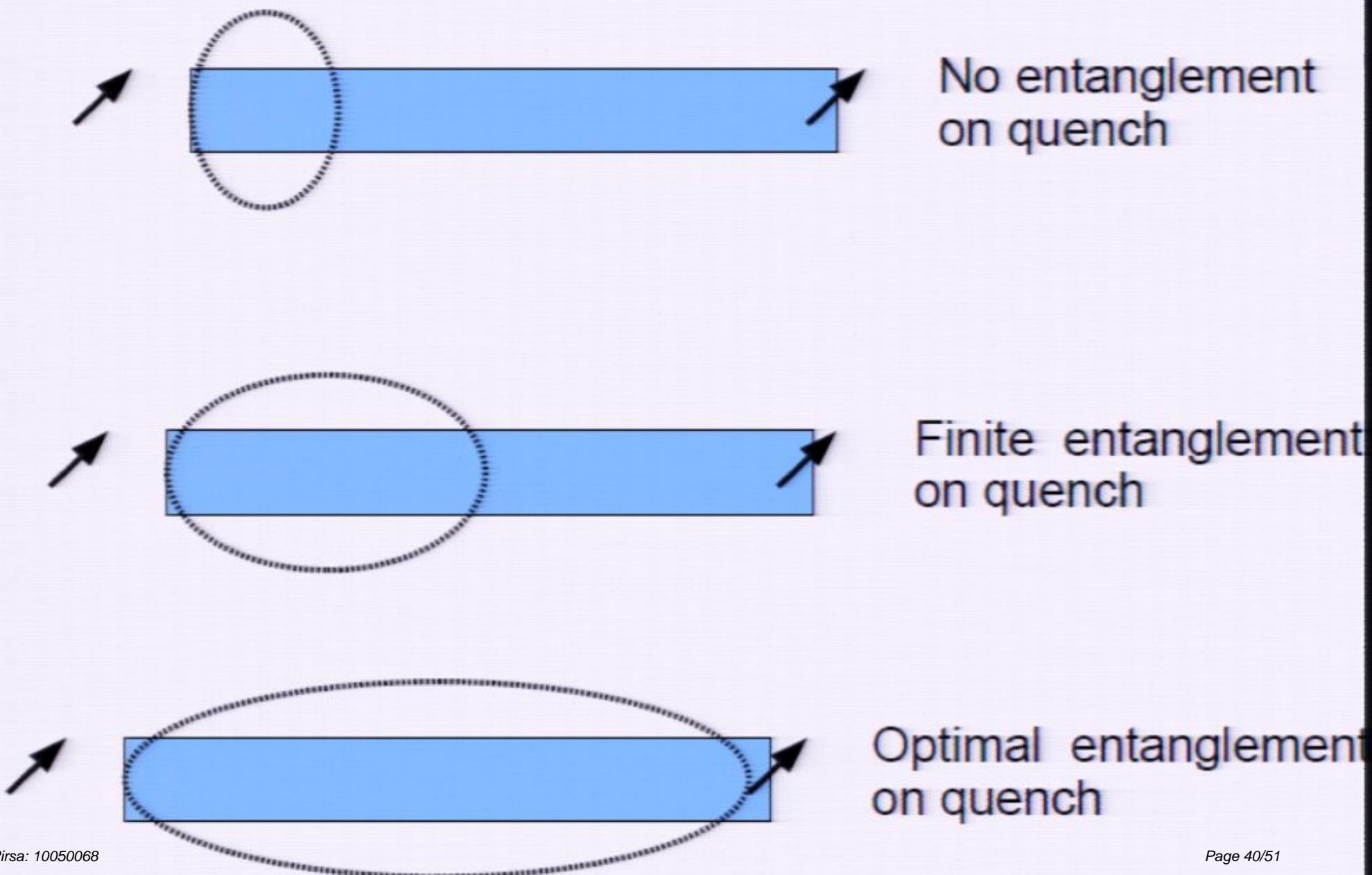
(c)



(d)

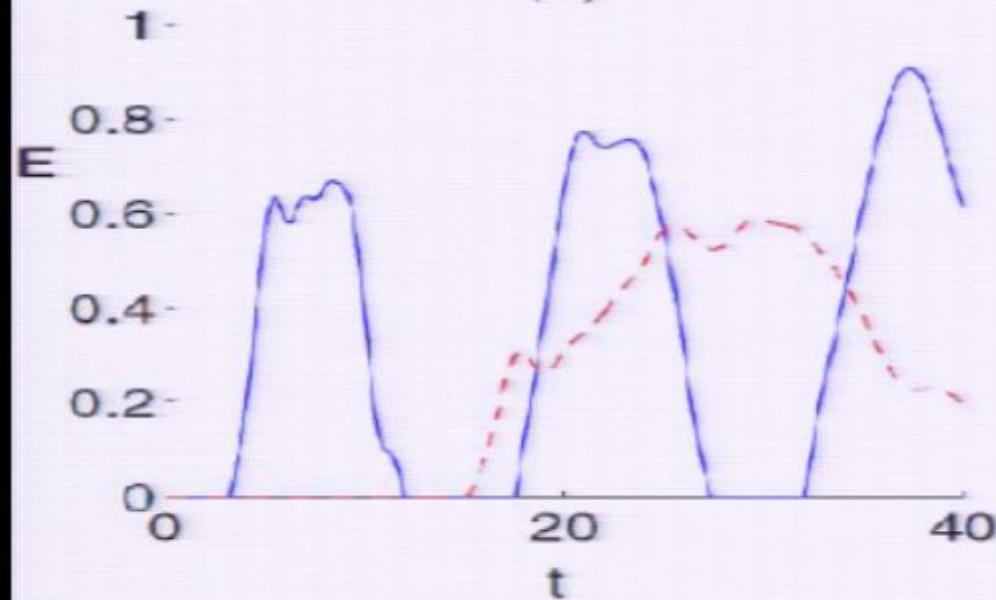


## Interpretation in terms of Kondo Cloud

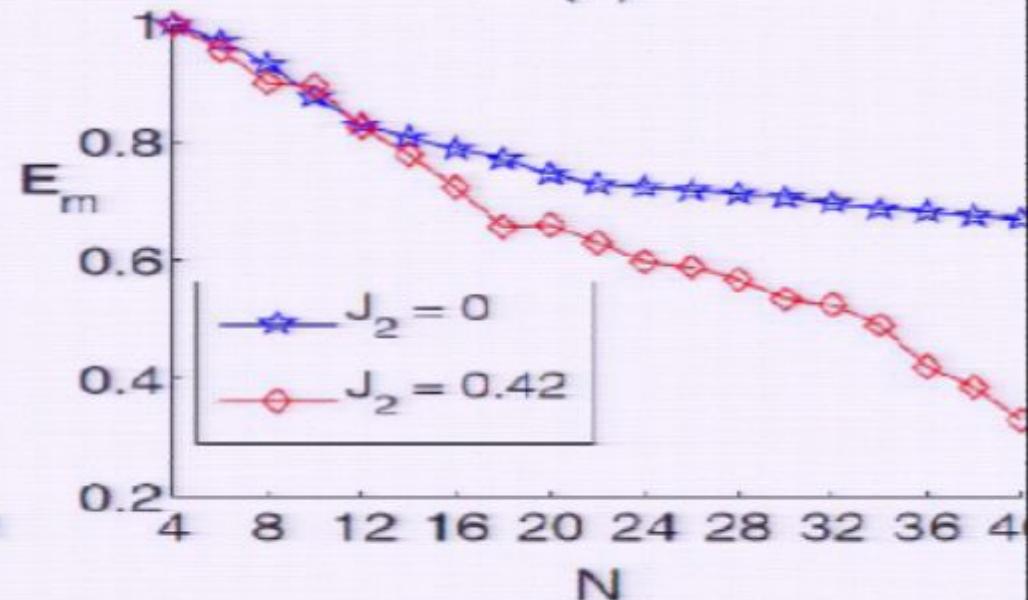


$$|\psi(t)\rangle = e^{-iH_F t} |GS_I\rangle$$

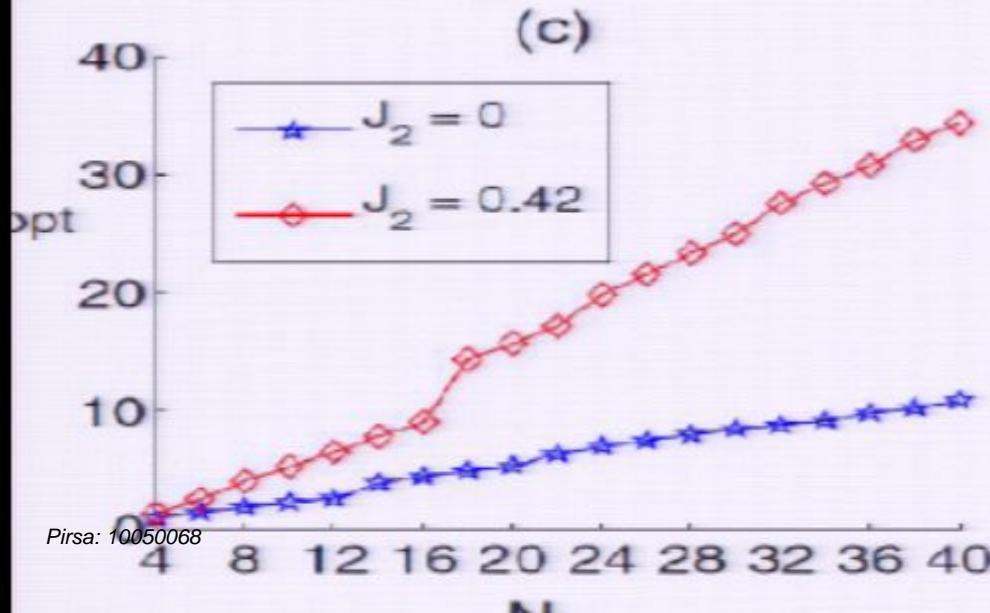
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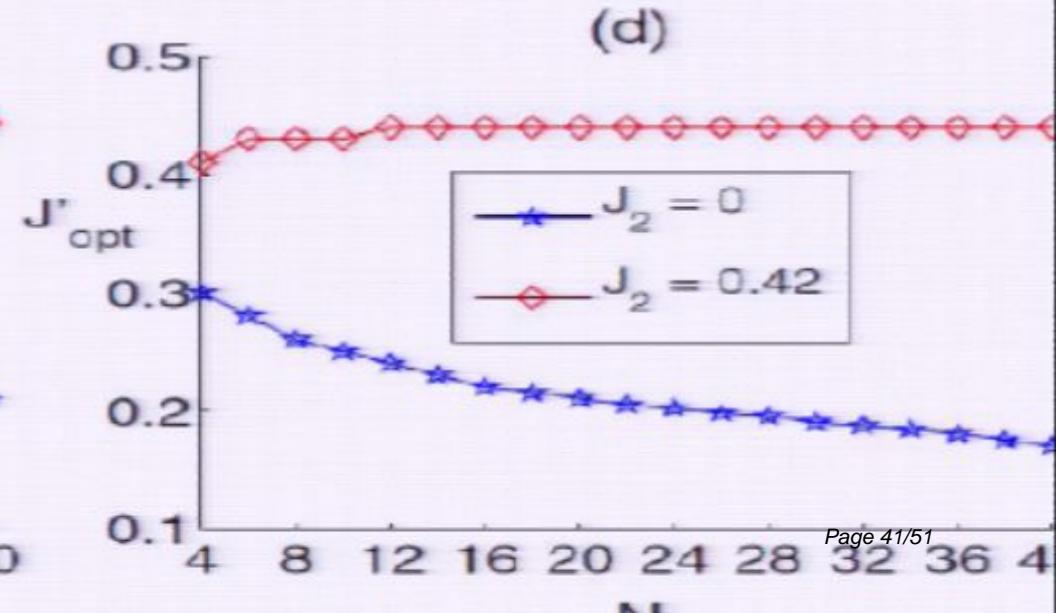
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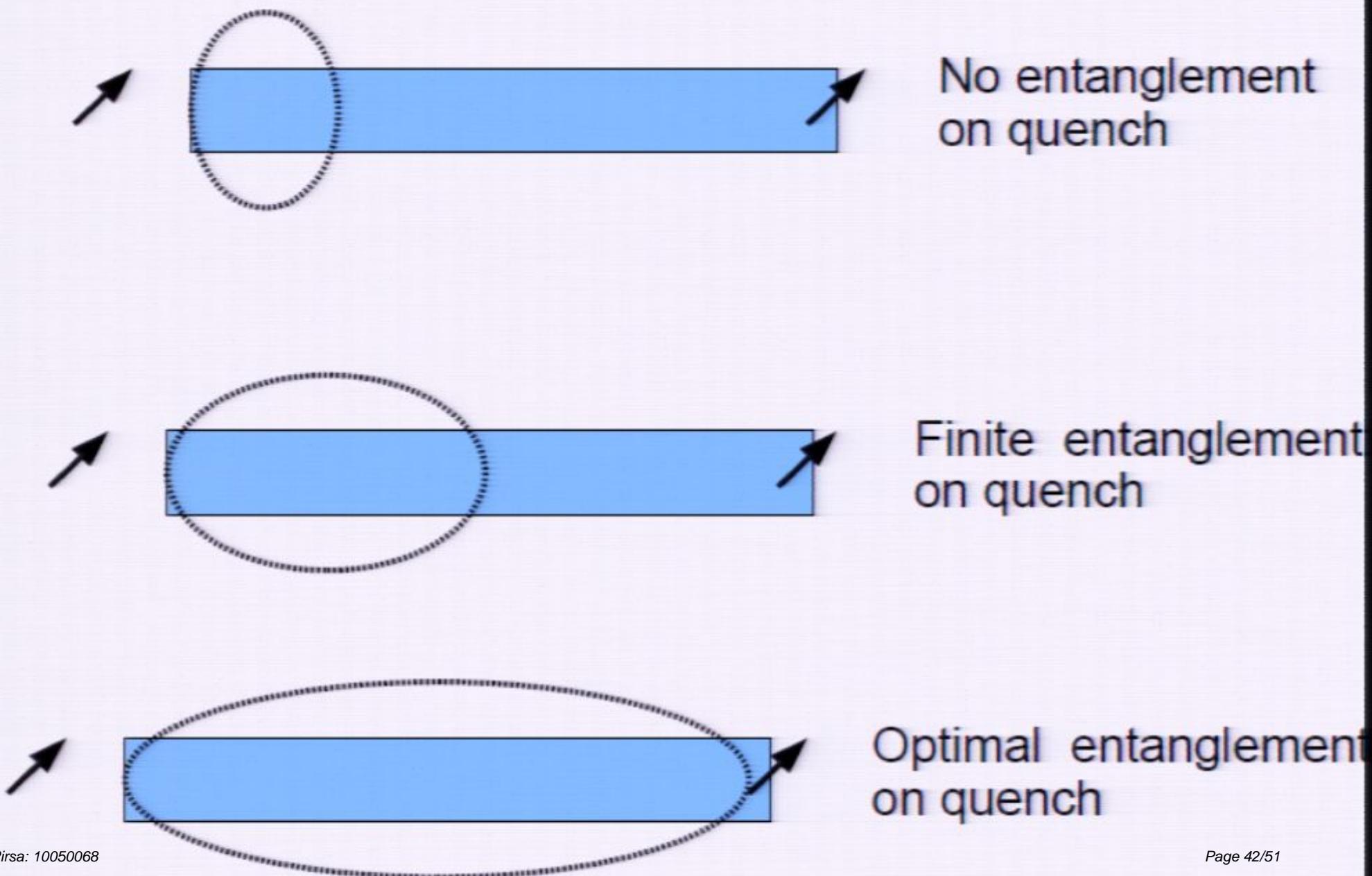
(c)



(d)



## Interpretation in terms of Kondo Cloud



**An explanation: Only two states involved  
in the dynamics and a interference between them**

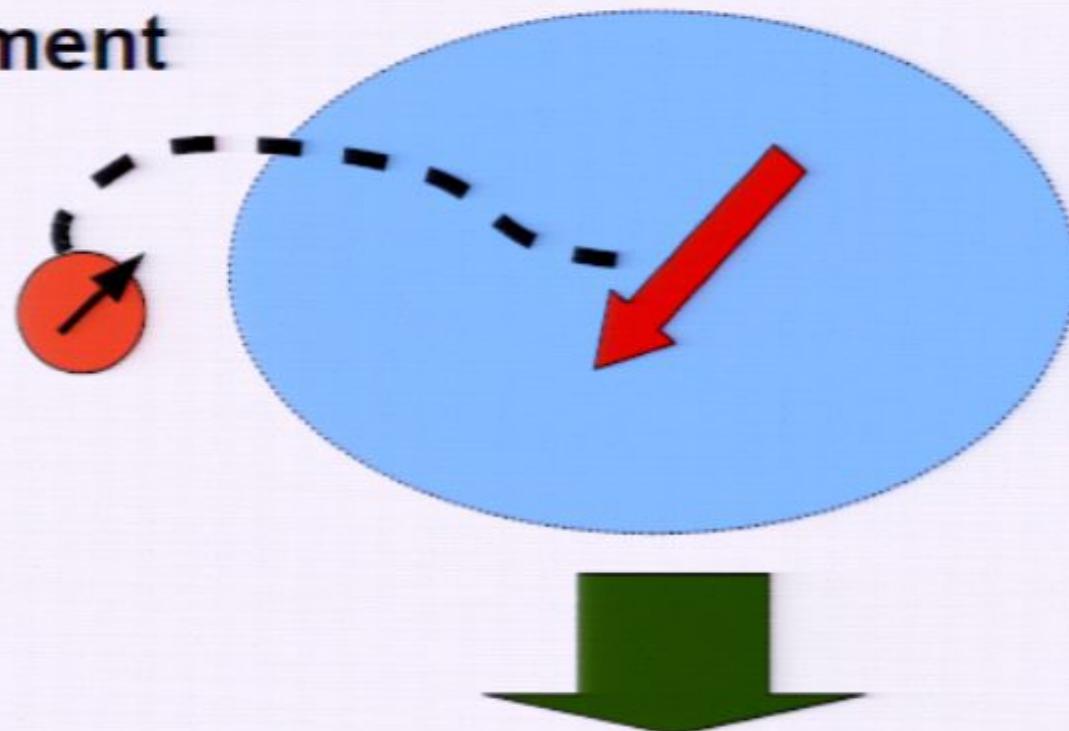
$$|E_1\rangle = \alpha_1 |\psi^-\rangle_{1N} |\phi^-\rangle_b + \beta_1 (|\psi^+\rangle_{1N} |\phi^{00}\rangle_b + |\psi^+\rangle_{1N} |\phi^{11}\rangle_b - |\psi^-\rangle_{1N} |\phi^+\rangle_b),$$

$$|E_2\rangle = \alpha_2 |\psi^-\rangle_{1N} |\phi^-\rangle_b - \beta_2 (|\psi^+\rangle_{1N} |\phi^{00}\rangle_b + |\psi^+\rangle_{1N} |\phi^{11}\rangle_b - |\psi^-\rangle_{1N} |\phi^+\rangle_b).$$

Conversion of useless natural entanglement to an useful form

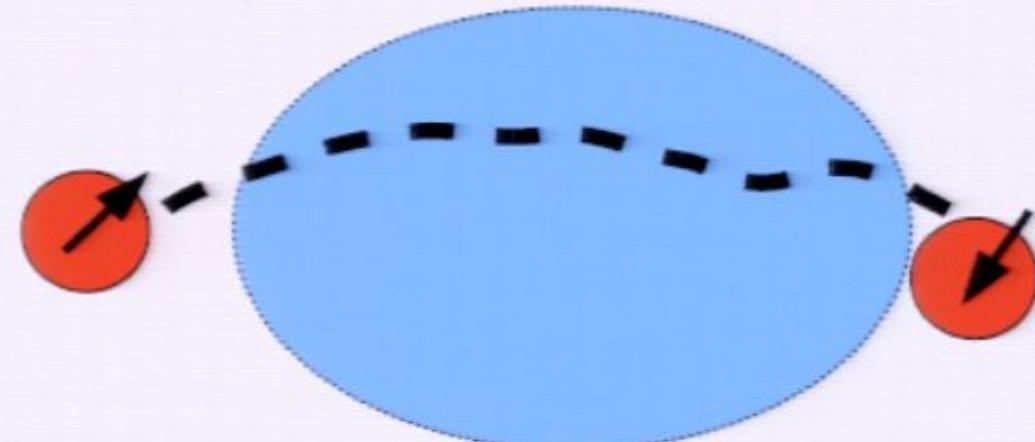
## Kondo entanglement

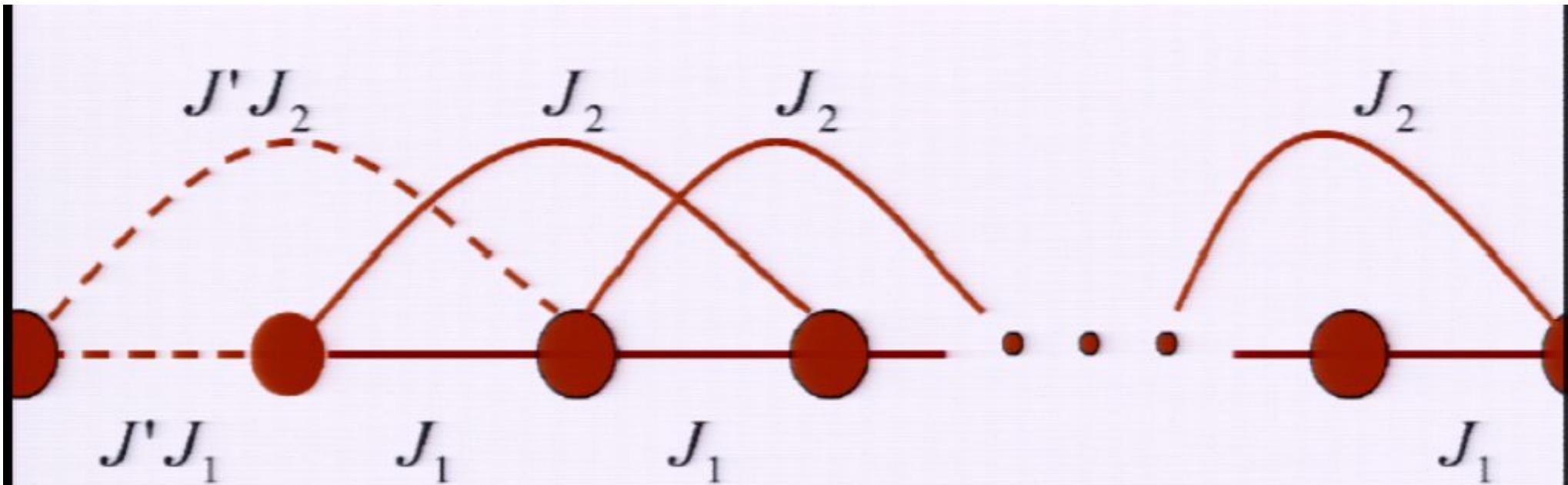
Useless  
Entanglement



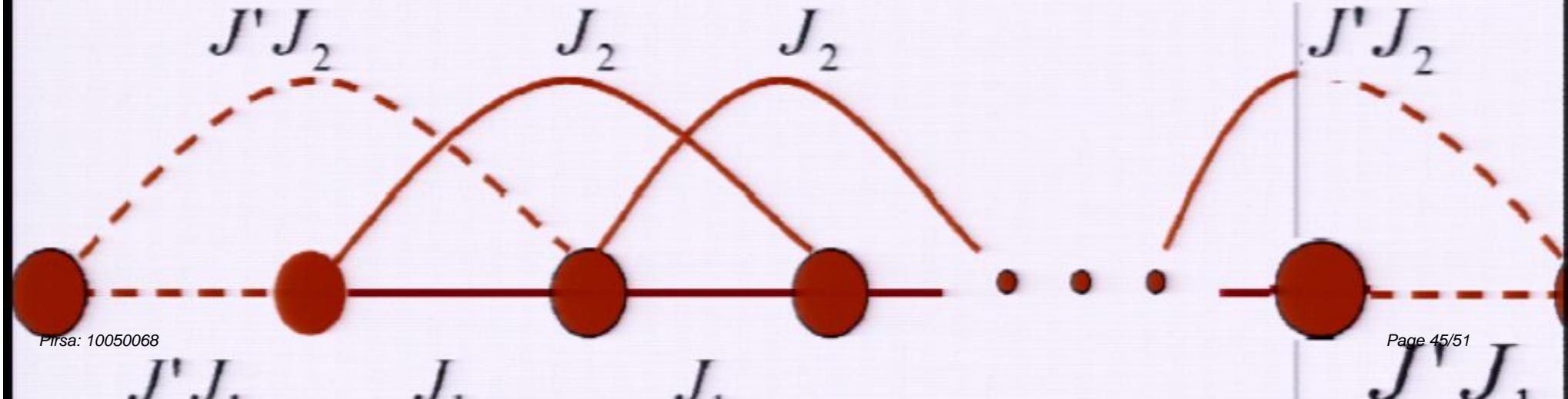
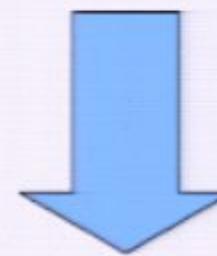
After Quench

Useful Entanglement

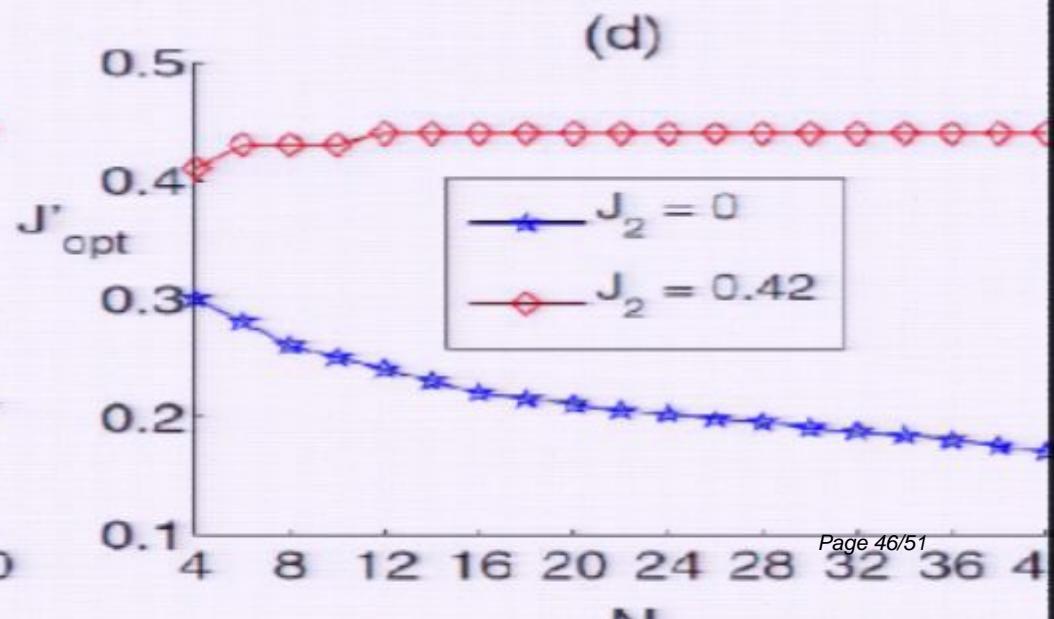
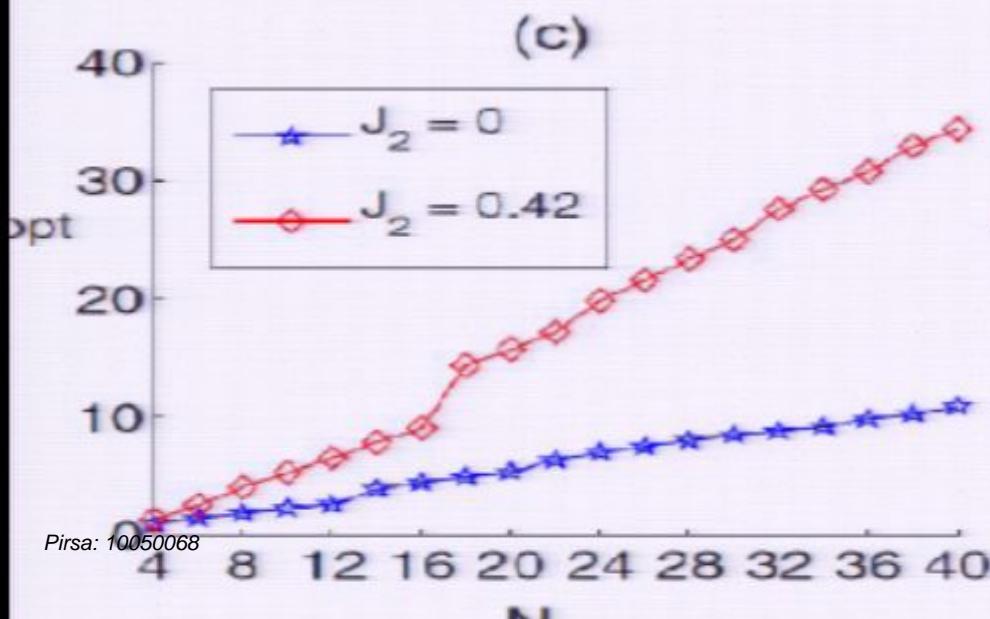
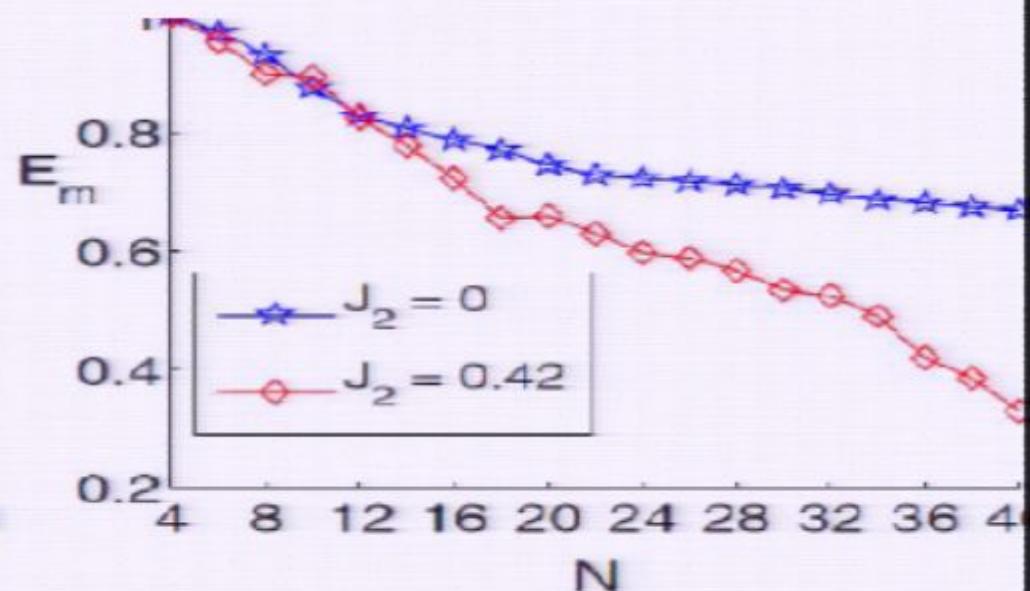
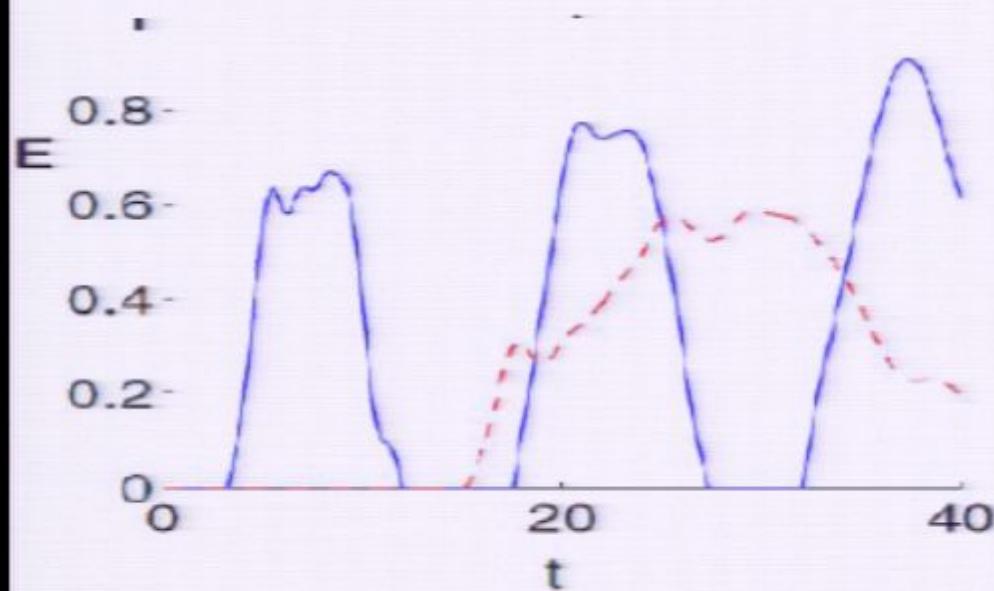




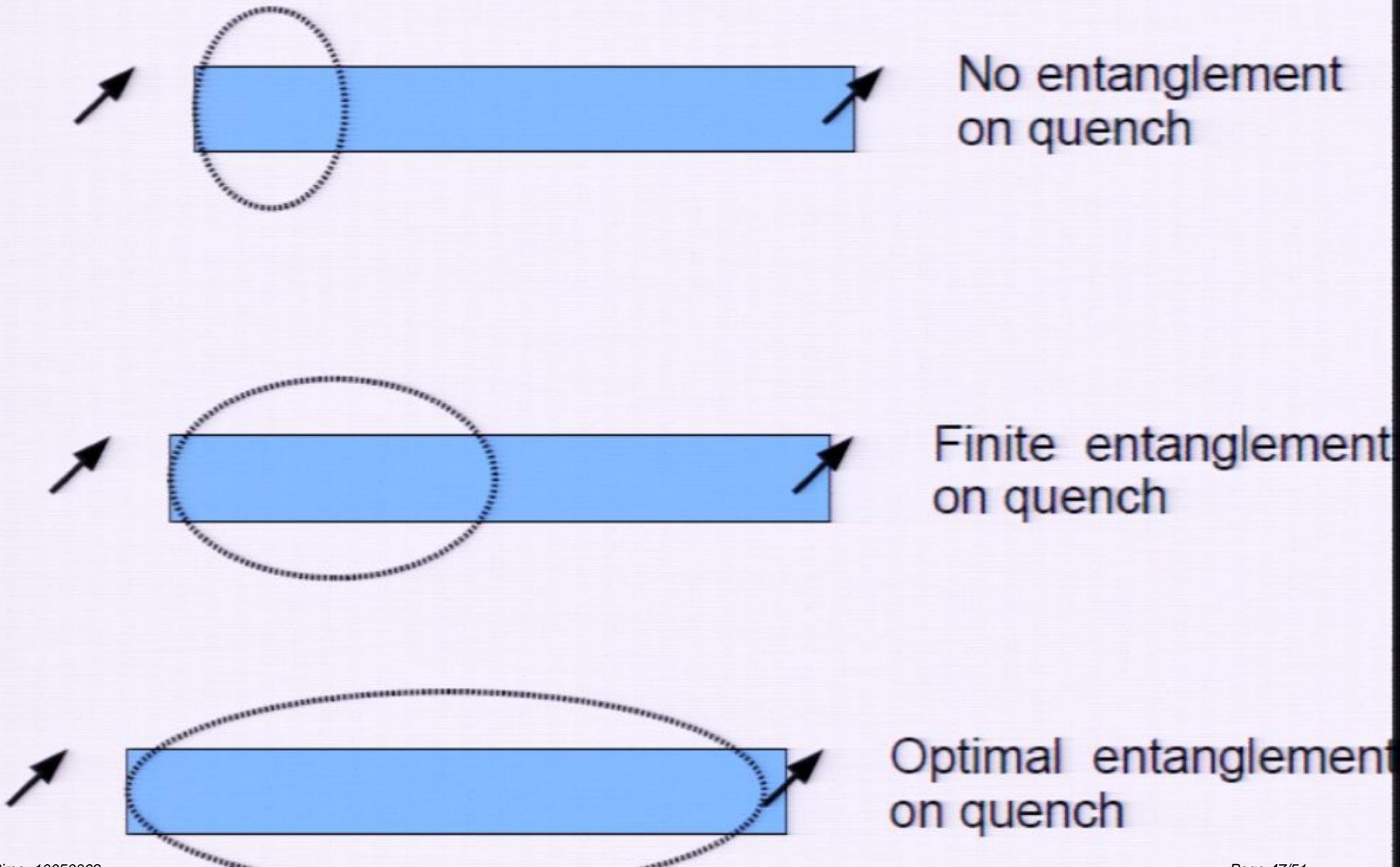
Single local quench



## A single local quench for distant entanglement:



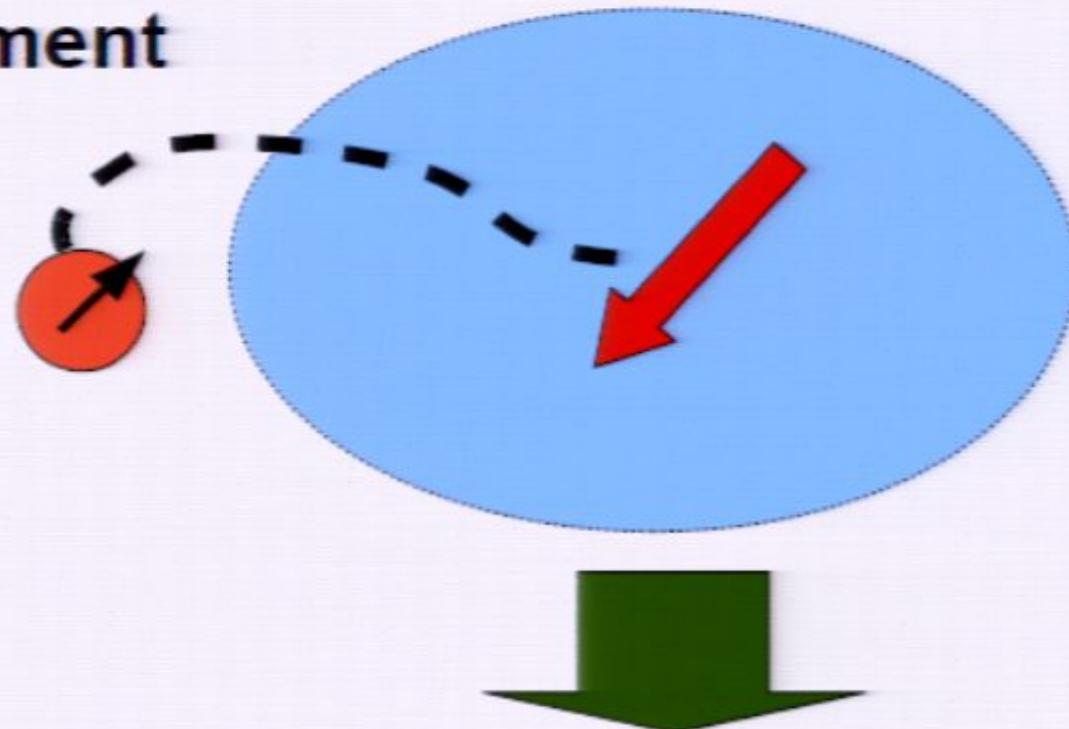
## Interpretation in terms of Kondo Cloud



Conversion of useless natural entanglement to an useful form

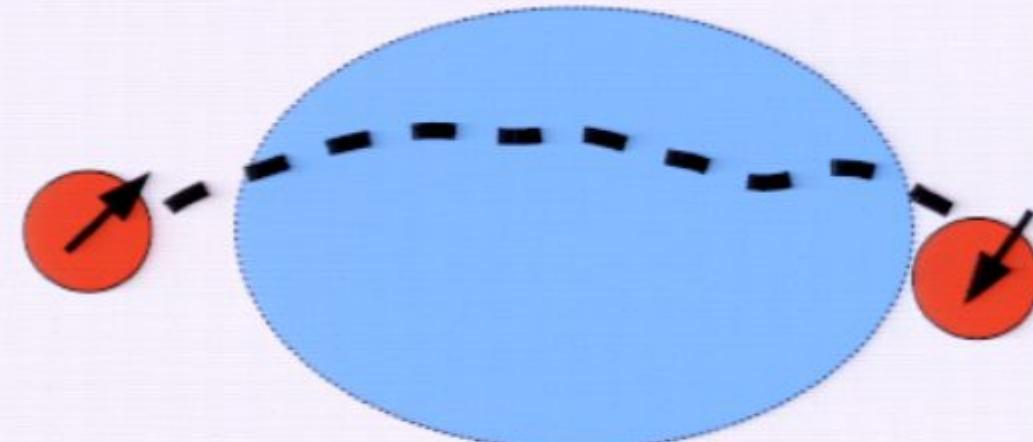
## Kondo entanglement

Useless  
Entanglement

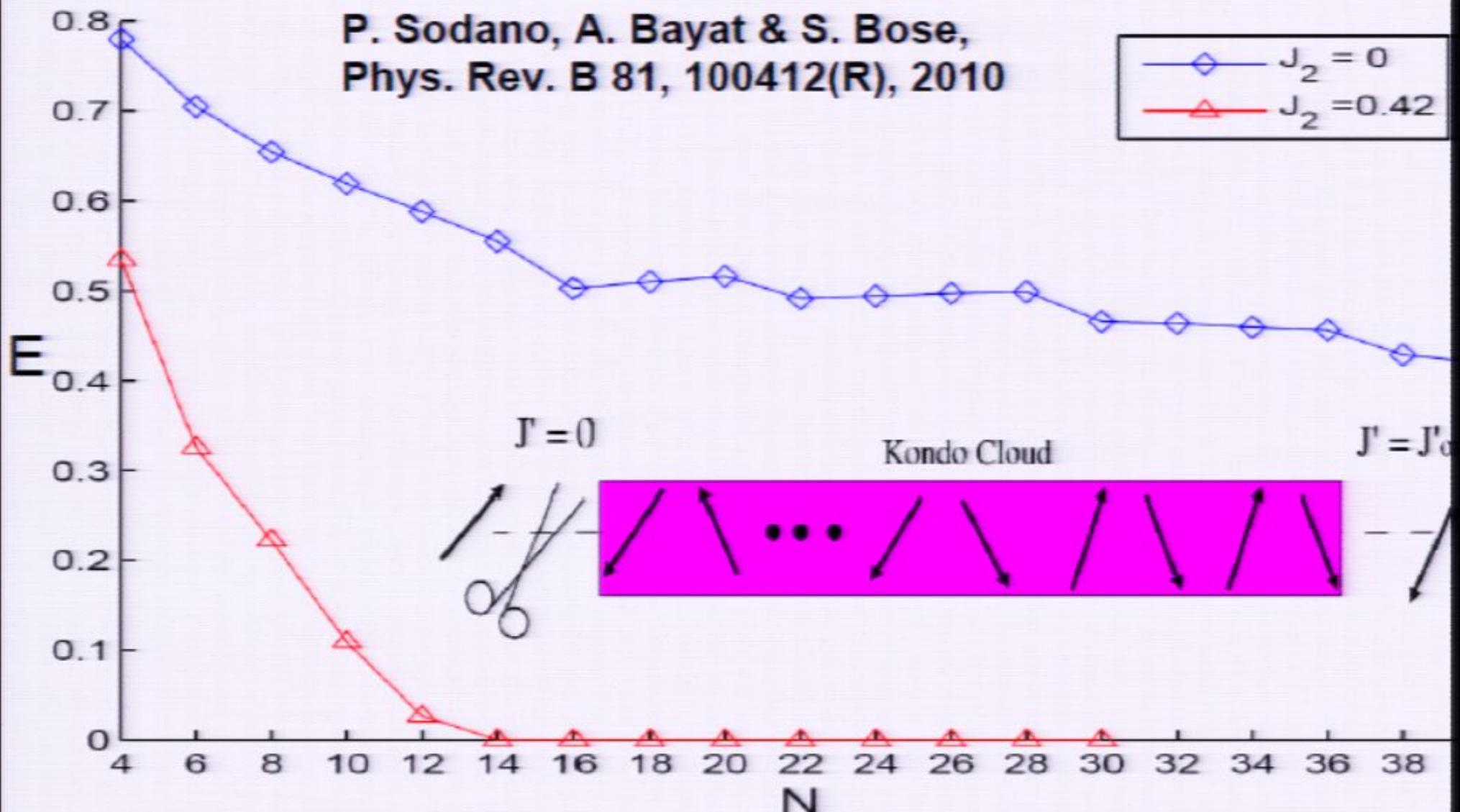


After Quench

Useful Entanglement



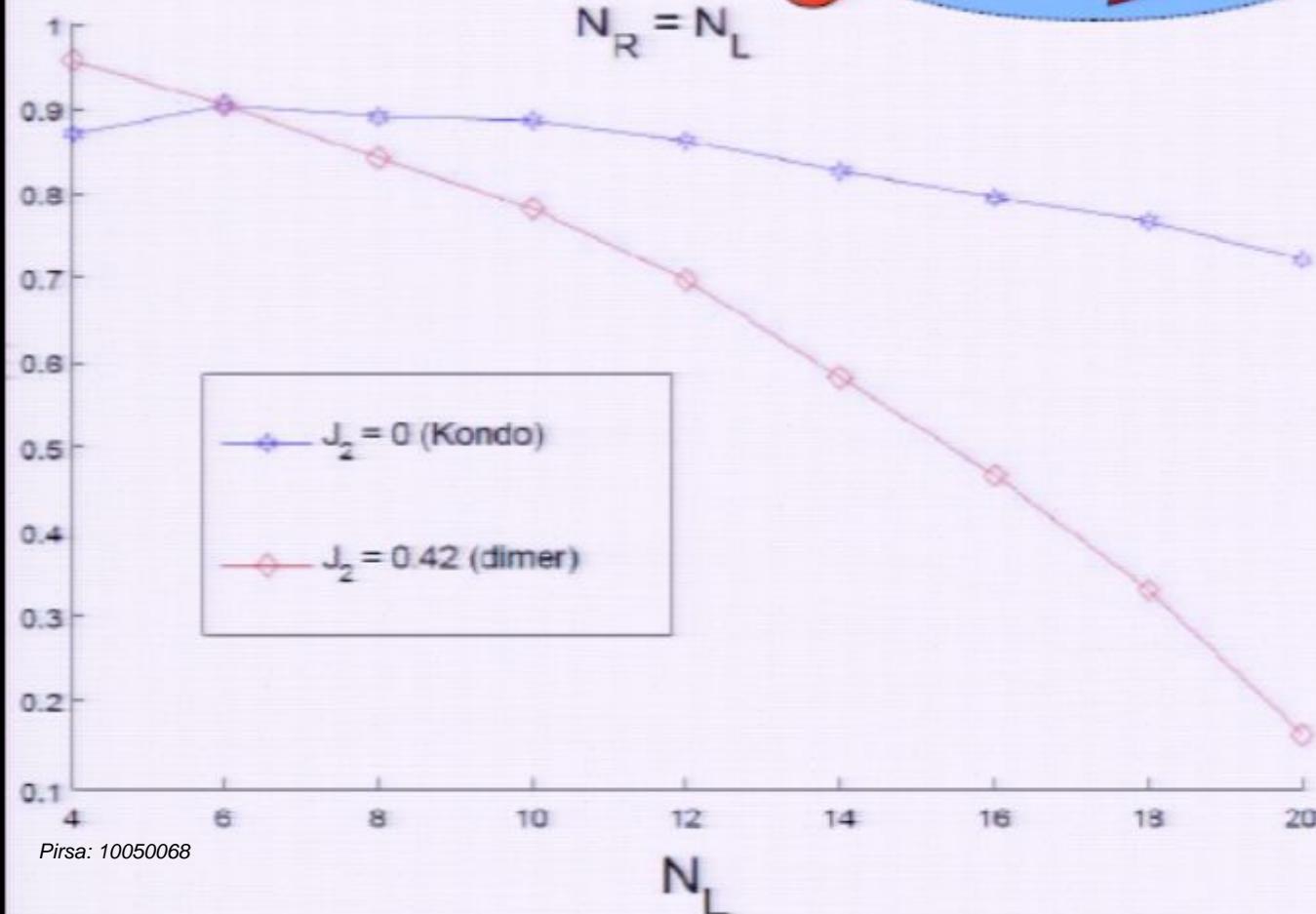
Is it truly the cloud's entanglement which is going over?



Detection of the entanglement of spin & cloud in Kondo systems, as well as the cloud

## Kondo Routers (unpublished):

(with Bayat & Sodano)



$N_L$

We have investigated what kind of **long-range entanglement** exists and/or what kind of long range entanglement can be created by **non-equilibrium dynamics** in spin chain systems.

Analytics/justf for non-compl. blocks

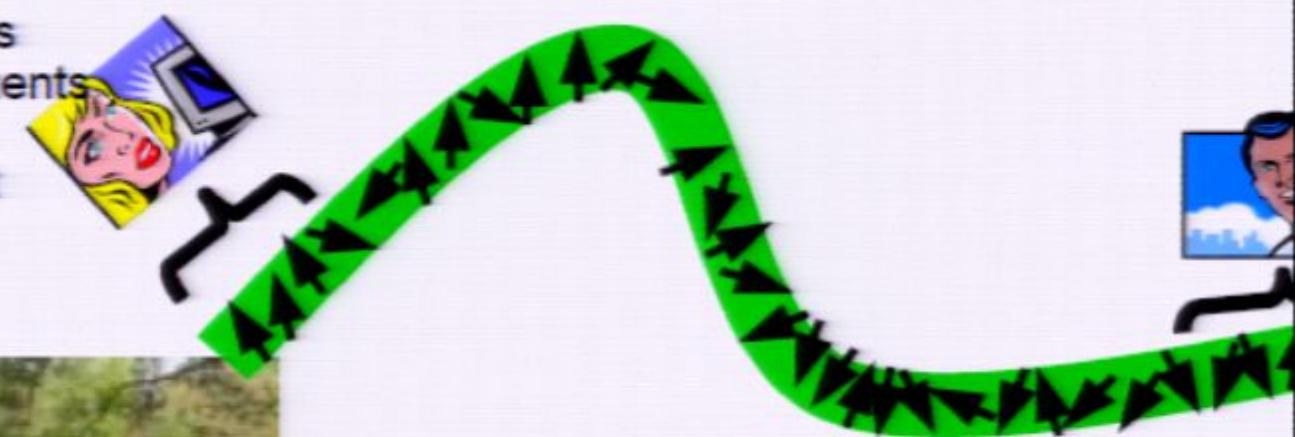
Relating the above to critical exponents

Measuring such entanglement

Is the Kondo quench entanglement

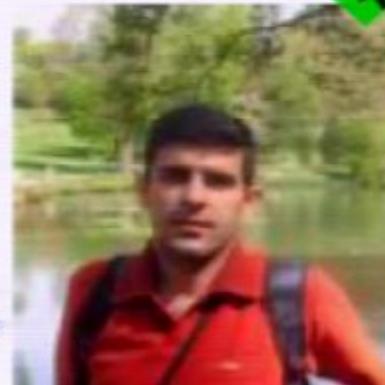
truly distance independent?

Physical Implementations?



## Collaborators:

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Pasquale  
Sodano



Hannu Wichterich



Javi Molina



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