Title: Entanglement renormalization and gauge symmetry

Date: May 25, 2010 02:45 PM

URL: http://pirsa.org/10050066

Abstract: A lattice gauge theory is described by a redundantly large vector space that is subject to local constraints, and it can be regarded as the low energy limit of a lattice model with a local symmetry. I will describe a coarse-graining scheme capable of exactly preserving local symmetries. The approach results in a variational ansatz for the ground state(s) and low energy excitations of a lattice gauge theory. This ansatz has built-in local symmetries, which are exploited to significantly reduce simulation costs. I will describe benchmark results in the context of Kitaev’s toric code with a magnetic field or, equivalently, Z2 lattice gauge theory, for lattices with up to 16 x 16 sites (16^2 x 2 = 512 spins) on a torus.

Emergence and Entanglement
Perimeter Institute
25th-30th of May 2010

Entanglement Renormalization and Gauge Symmetry

Guifre Vidal

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

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Collaboration with Luca Tagliacozzo

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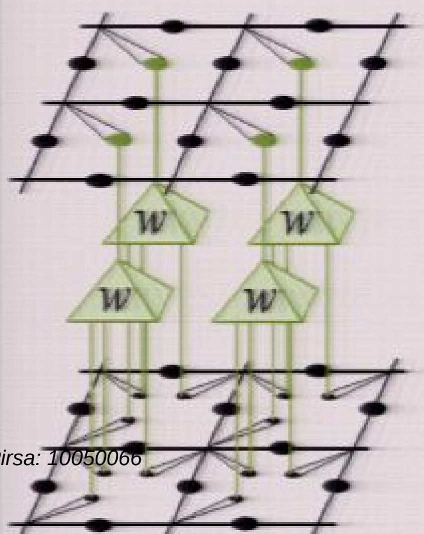
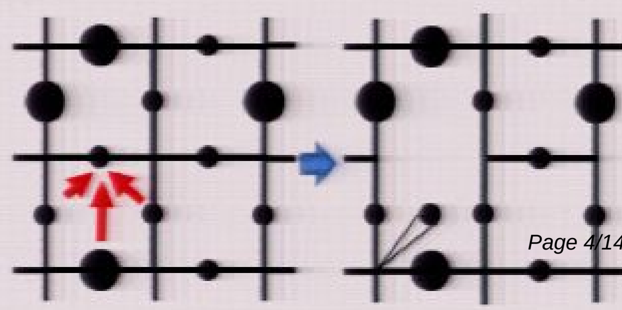
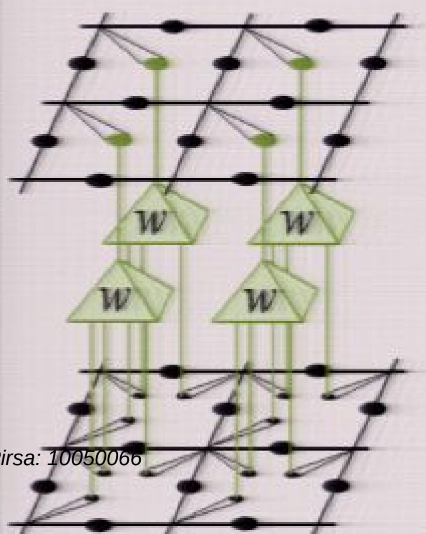


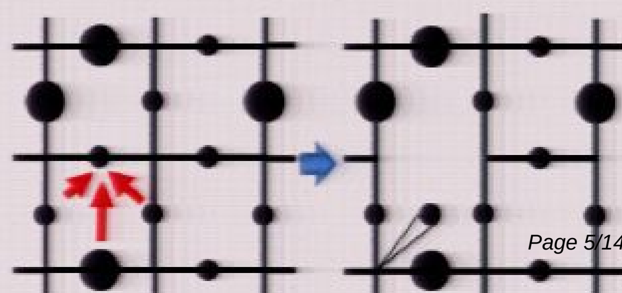


Entanglement Renormalization and Gauge Symmetry

How to coarse-grain:

- a quantum spin model with a local symmetry
  e.g. toric code with magnetic field h_x





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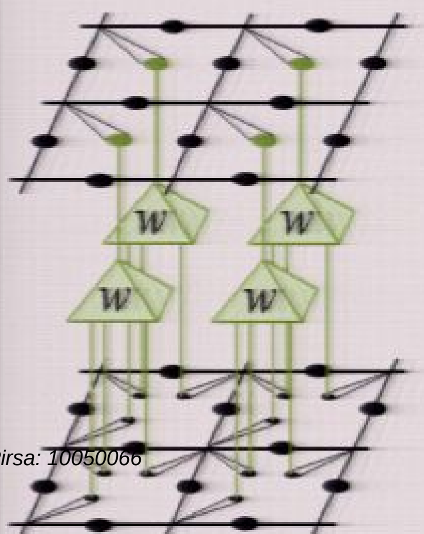

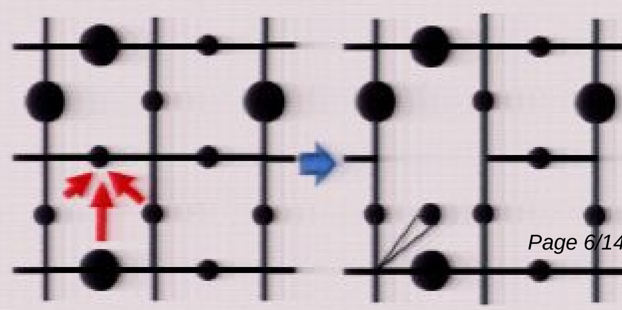

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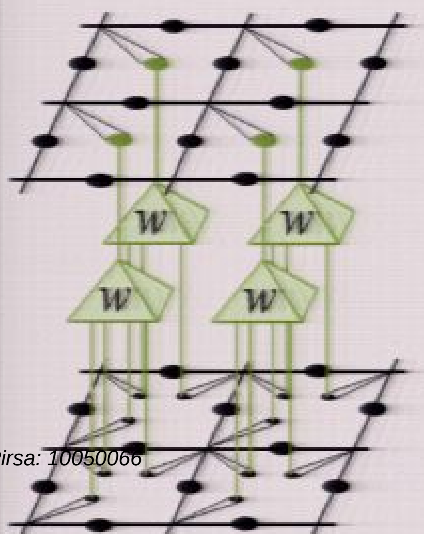
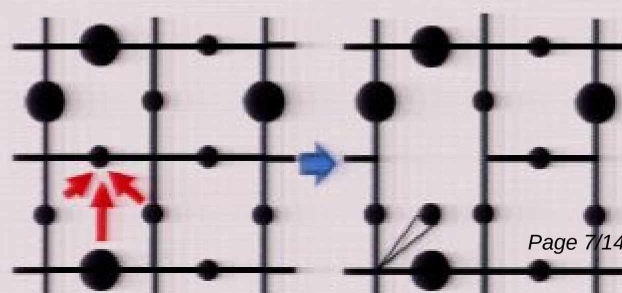


➡ tensor network variational ansatz for ground state(s)



Entanglement Renormalization and Gauge Symmetry

Outline:

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- Introduction

entanglement renormalization

local symmetry ⇔ gauge theory

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  exact transformation (CNOTs)
  numerical transformation

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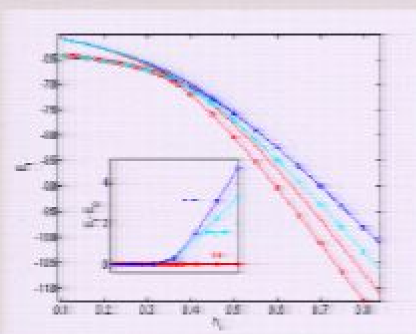
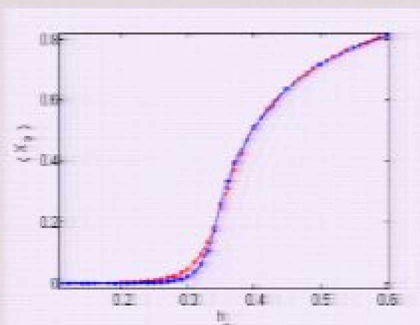
  exact transformation (CNOTs)
  numerical transformation

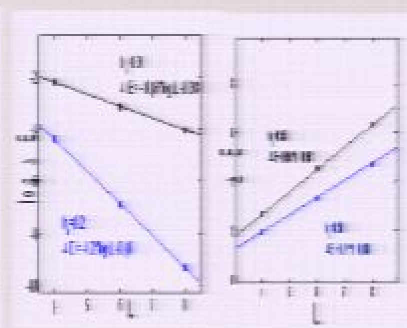
- Benchmark results (toric code & Z_2 LGT)



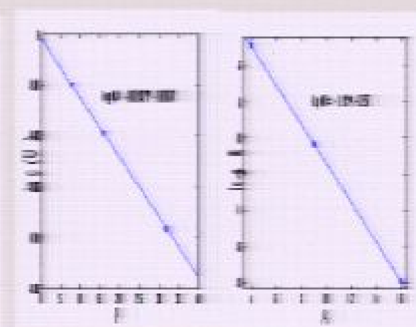
energy gaps within each topological sector



non-local order parameter



gaps between topological sectors



Wilson loops

Entanglement Renormalization

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quantum spin system

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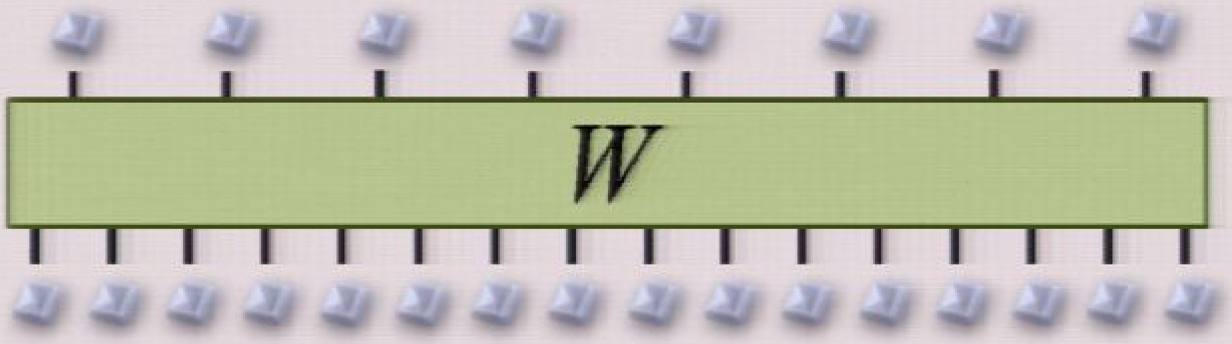
quantum spin system

- Coarse-graining transformation



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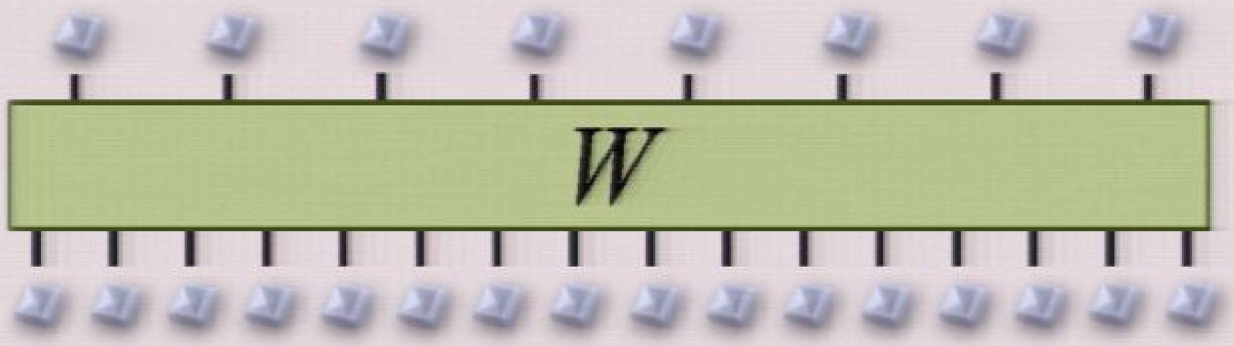

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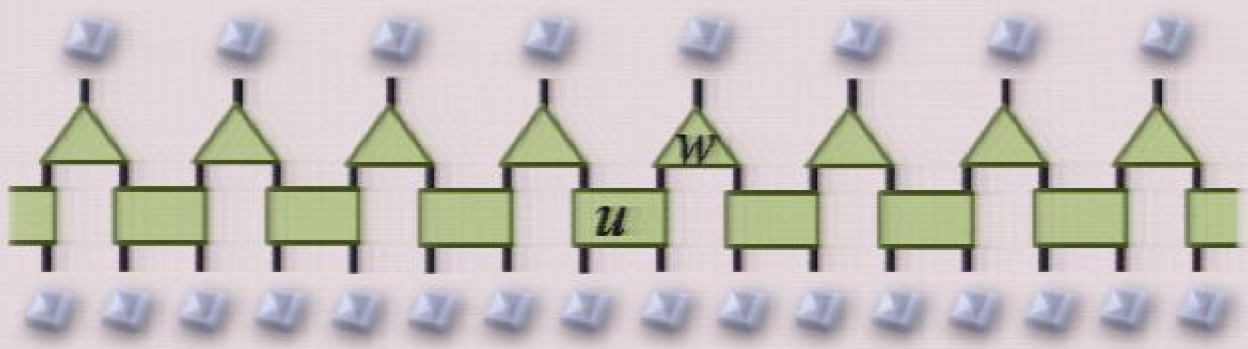

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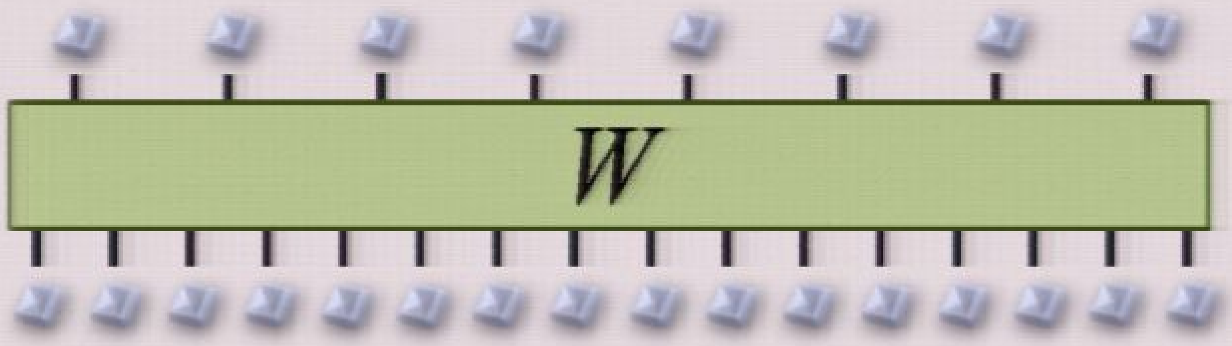

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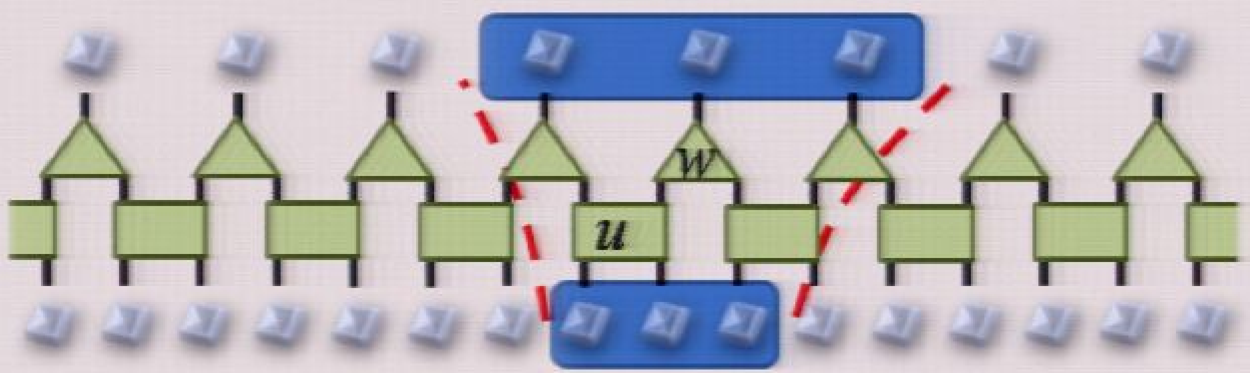

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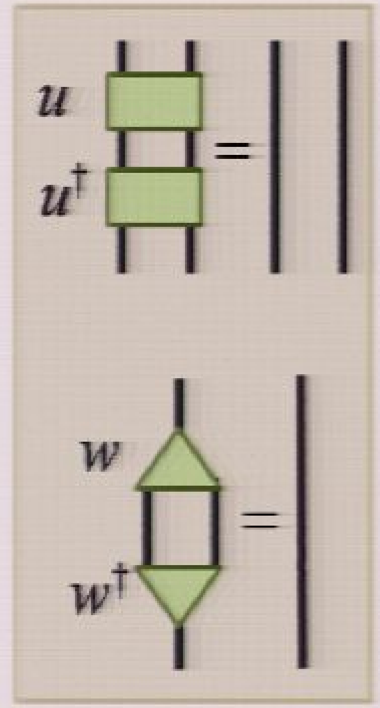

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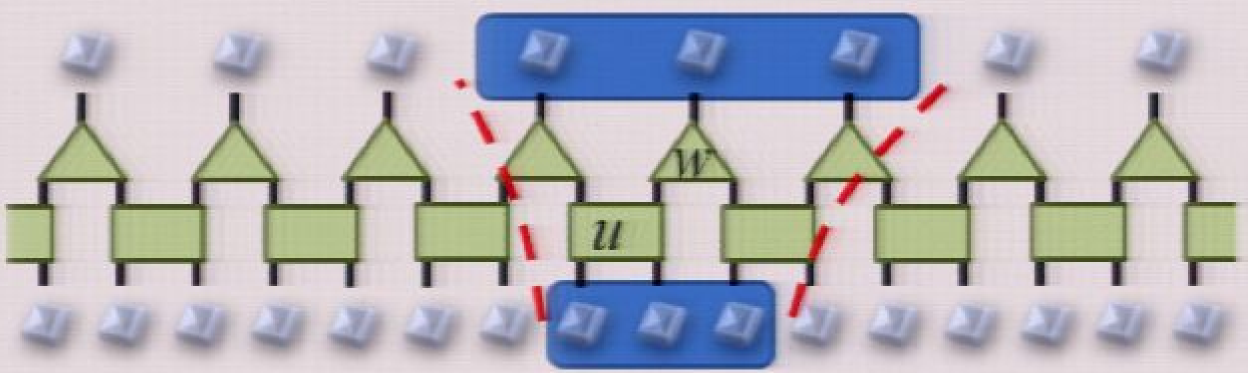

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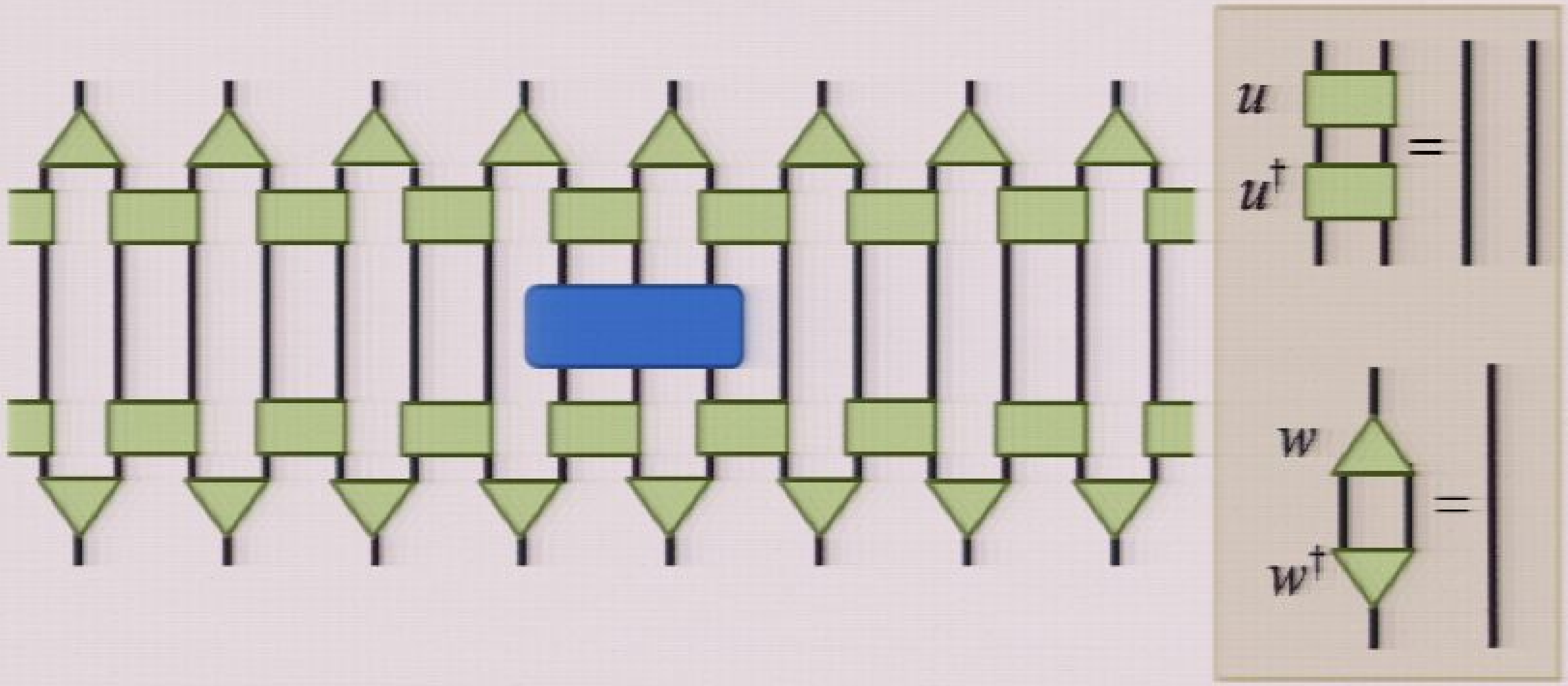

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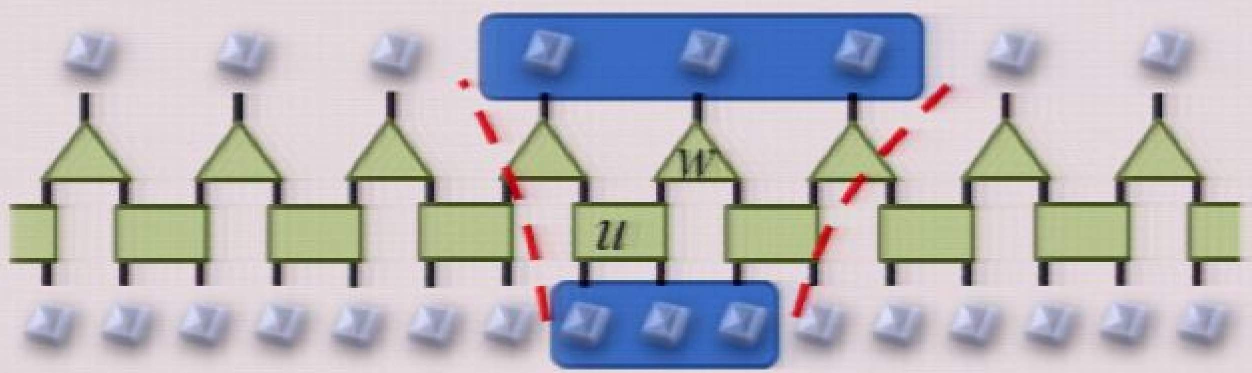

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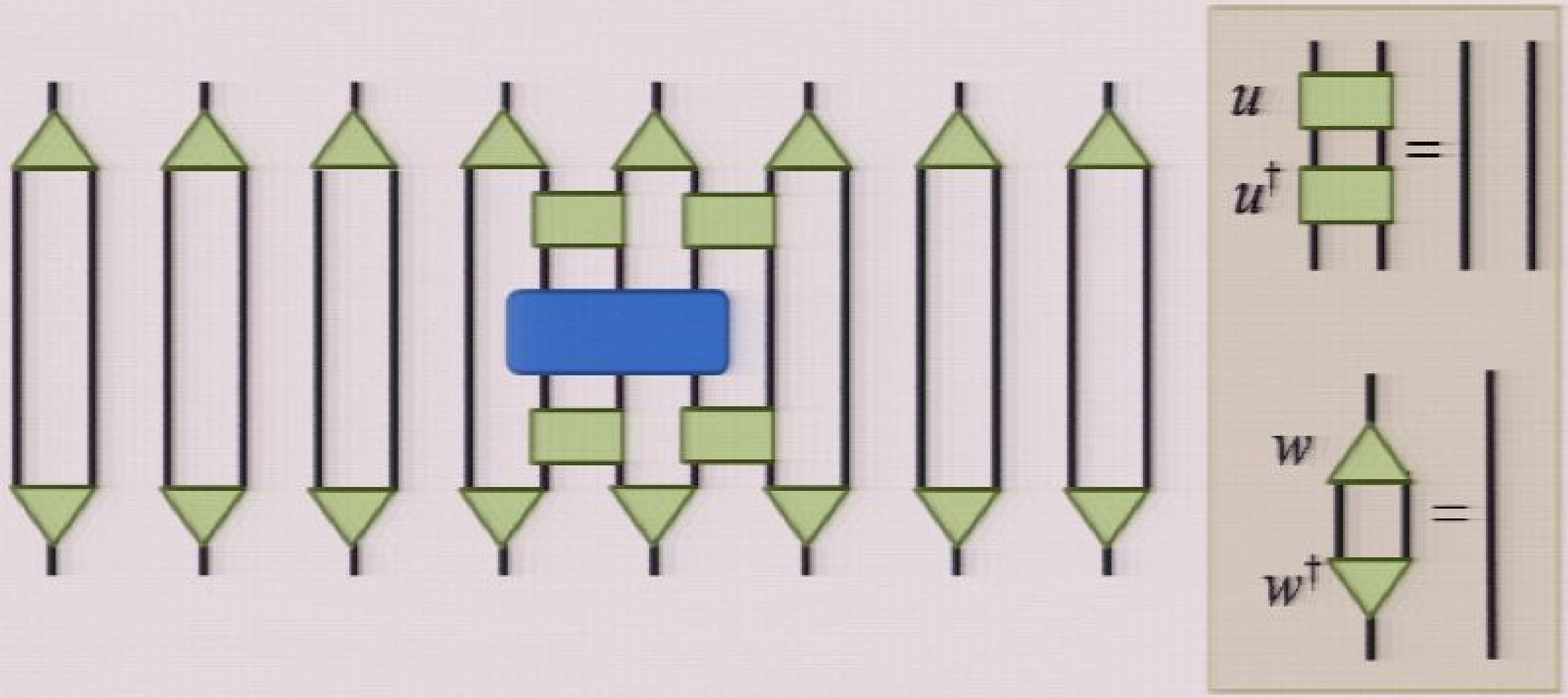

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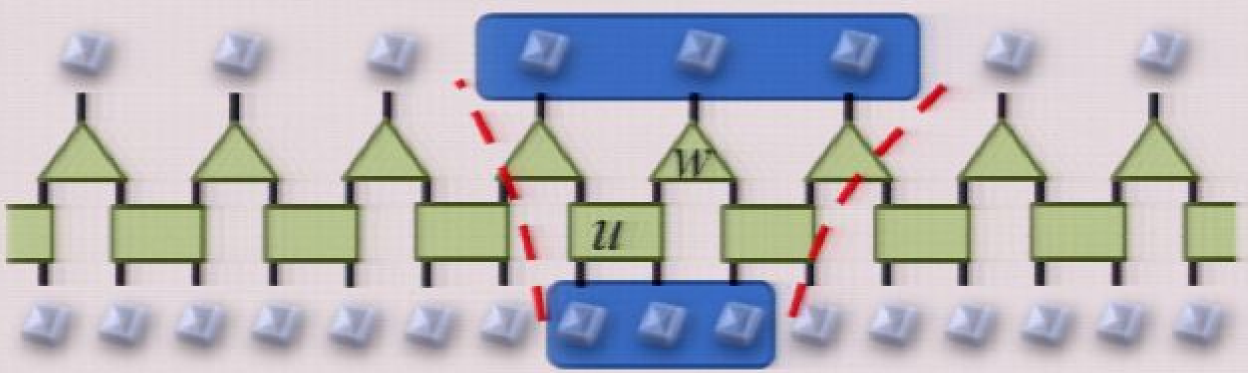

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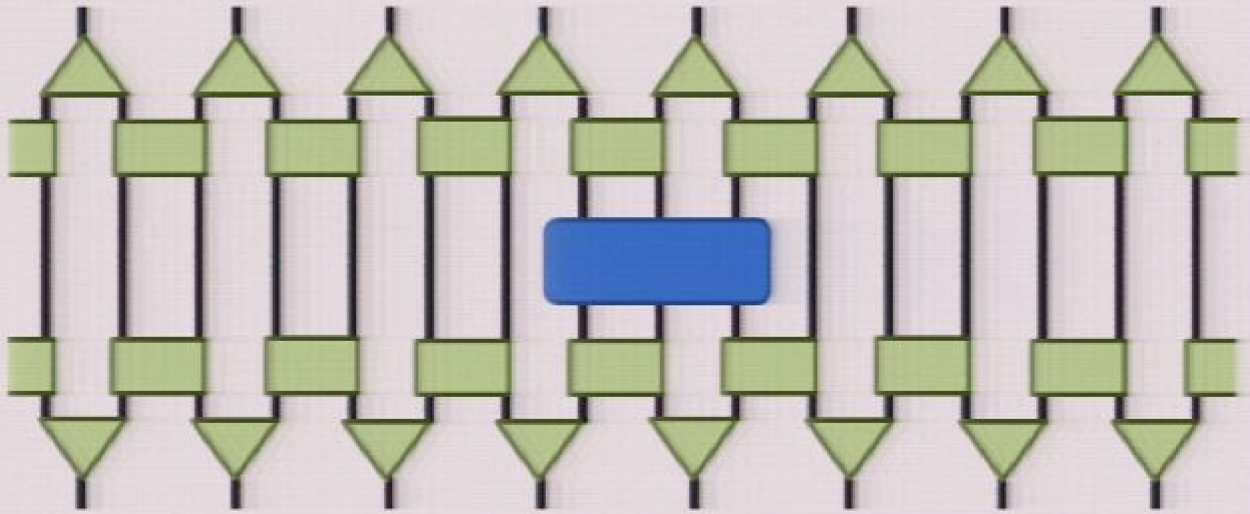

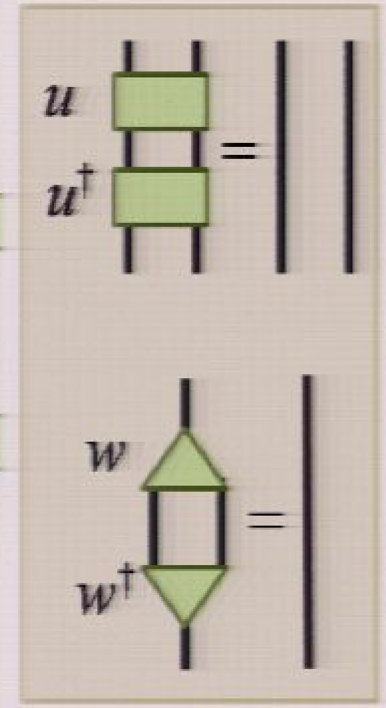

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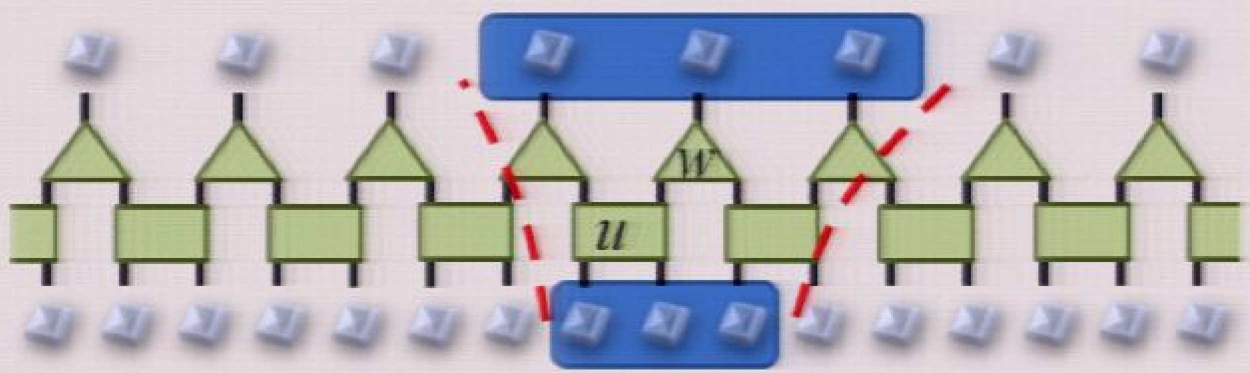

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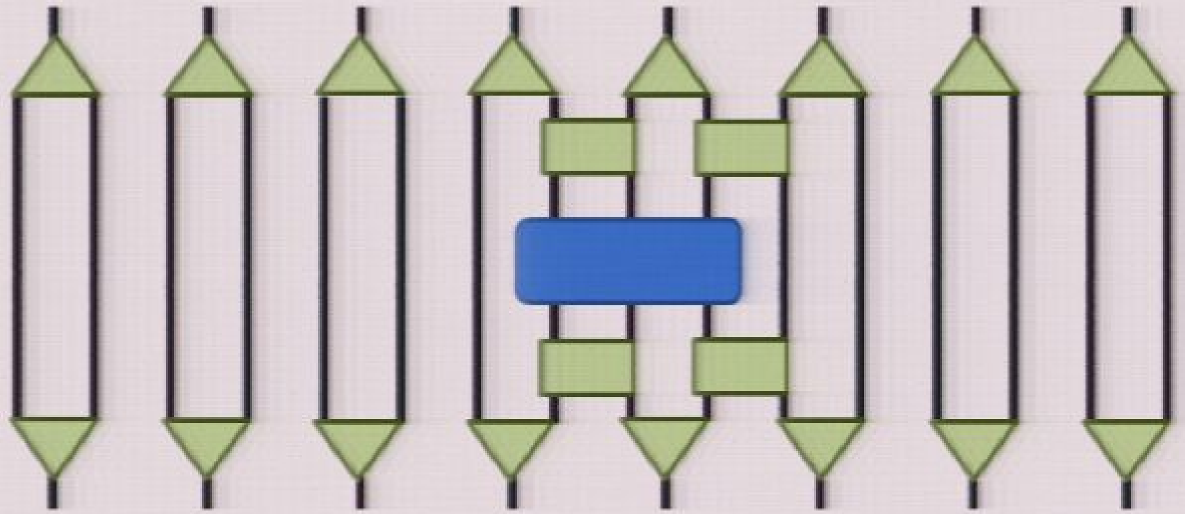

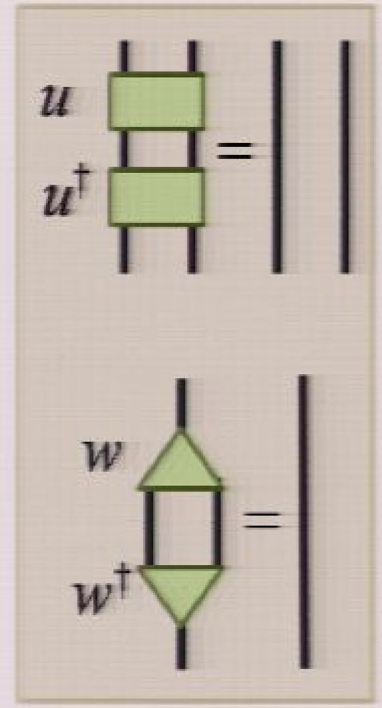

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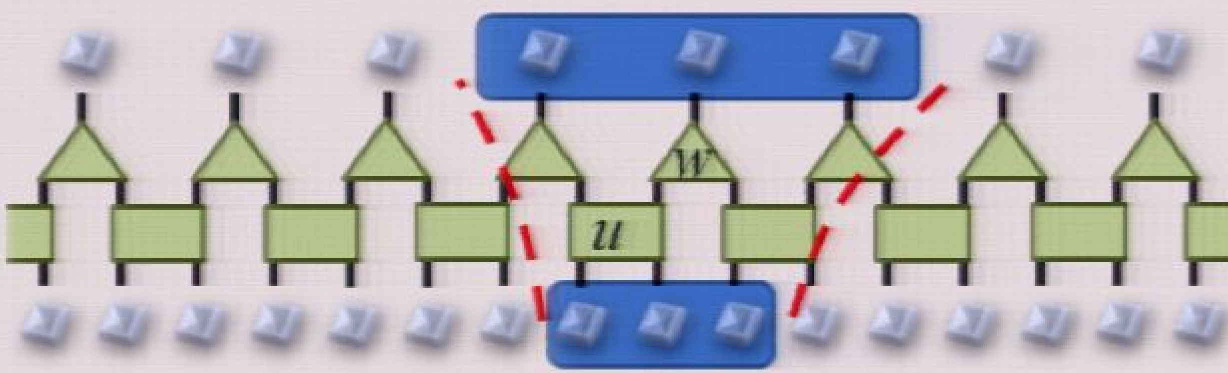

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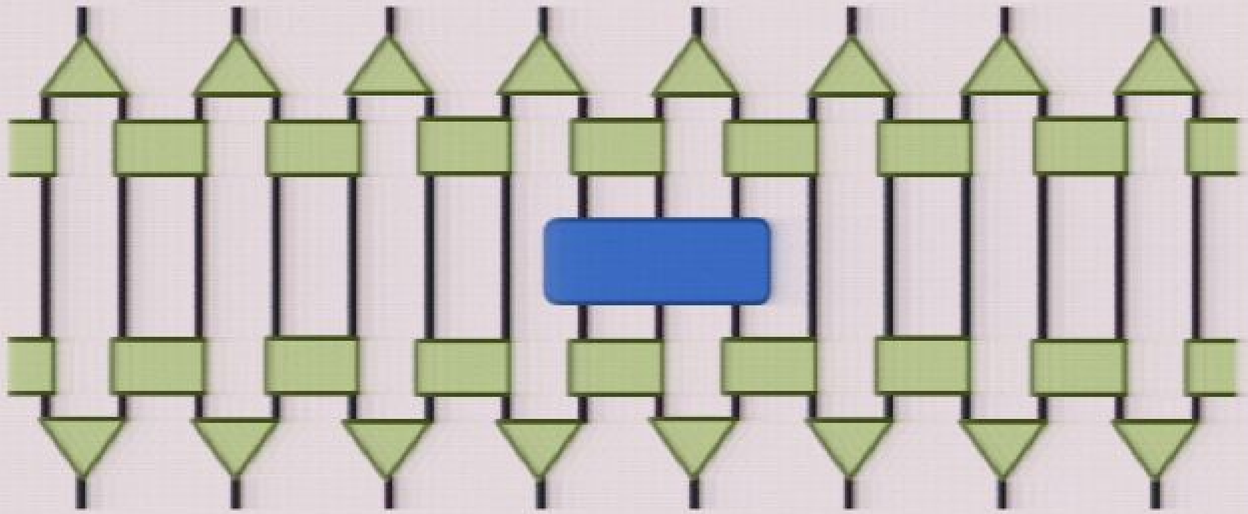

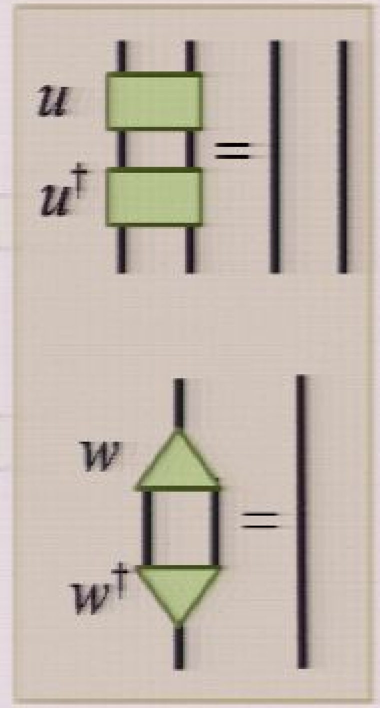

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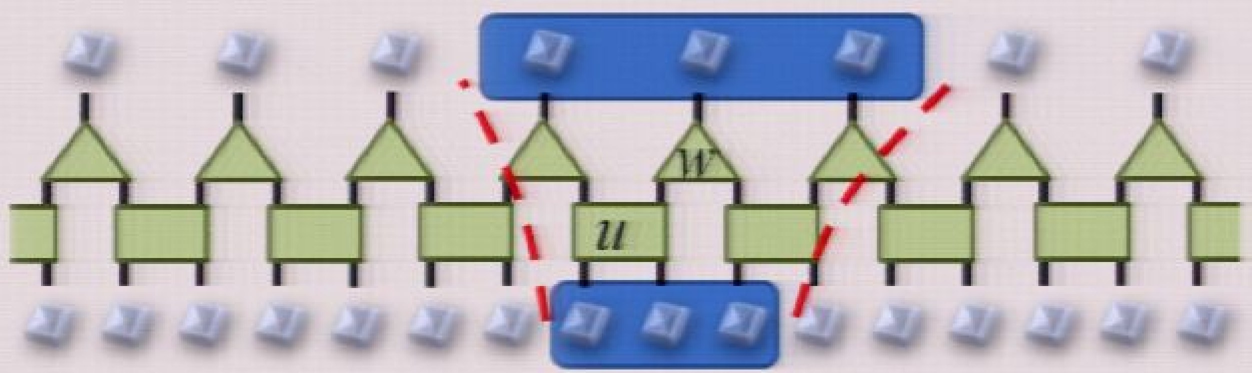

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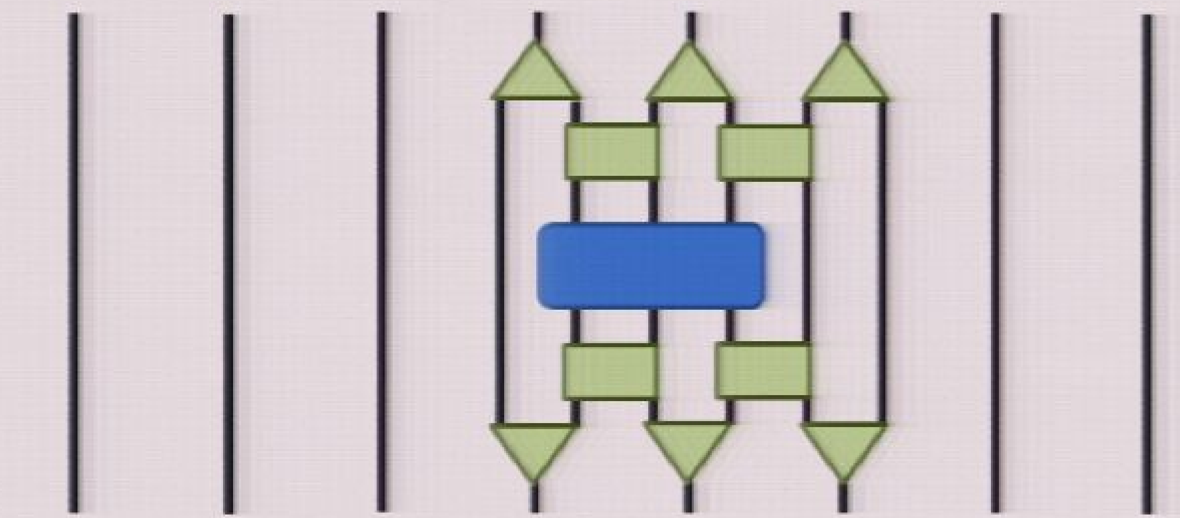

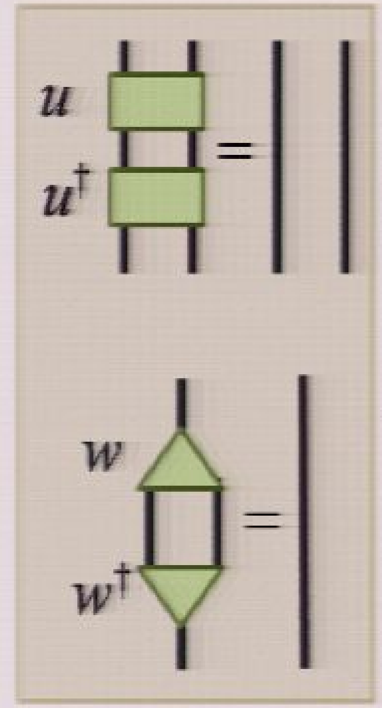

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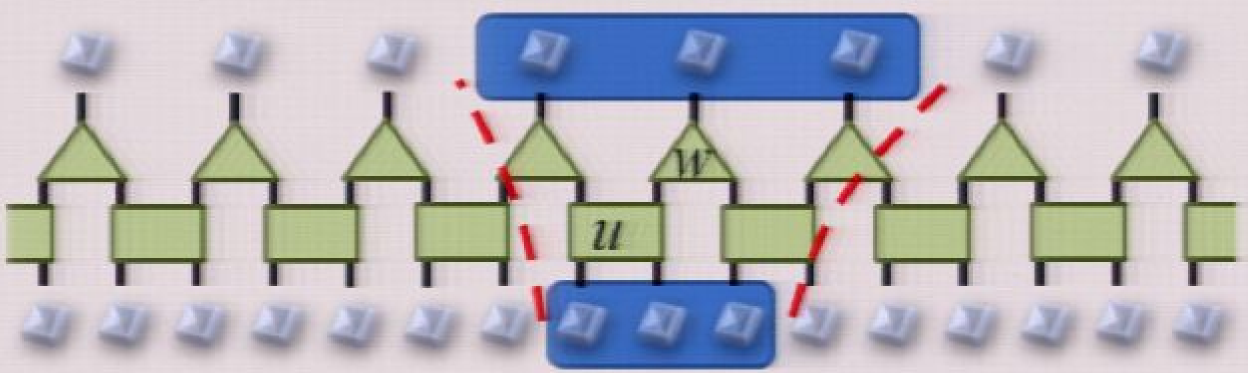

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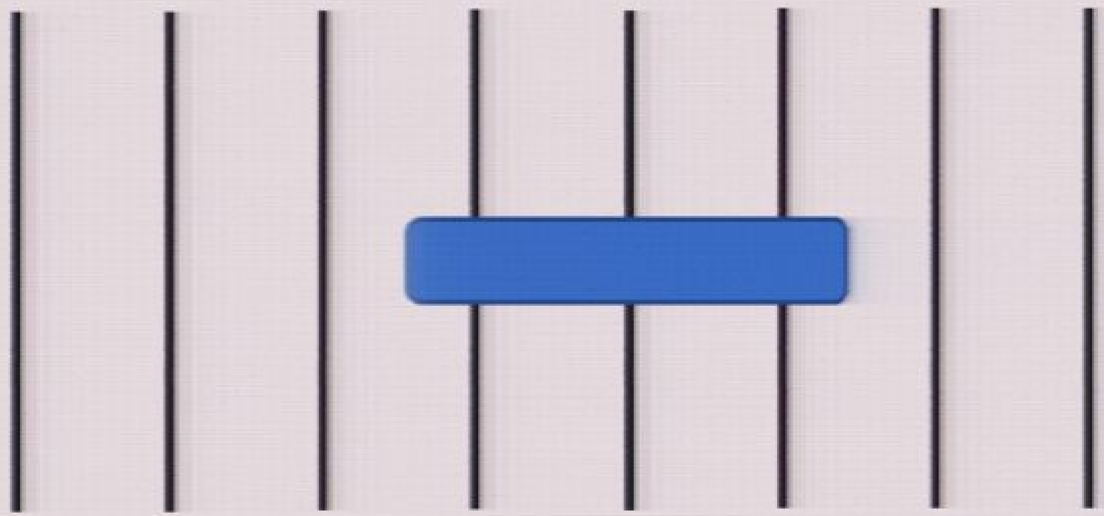

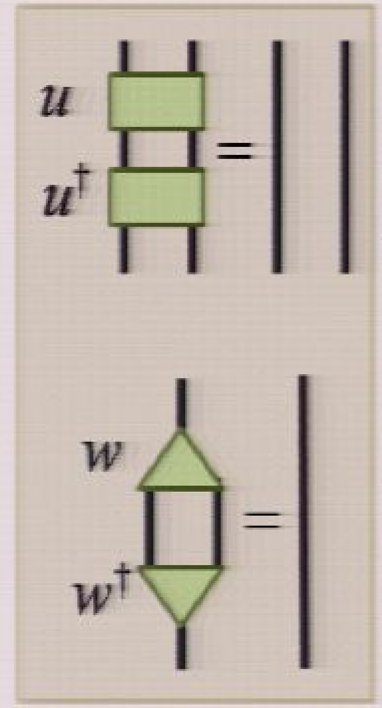

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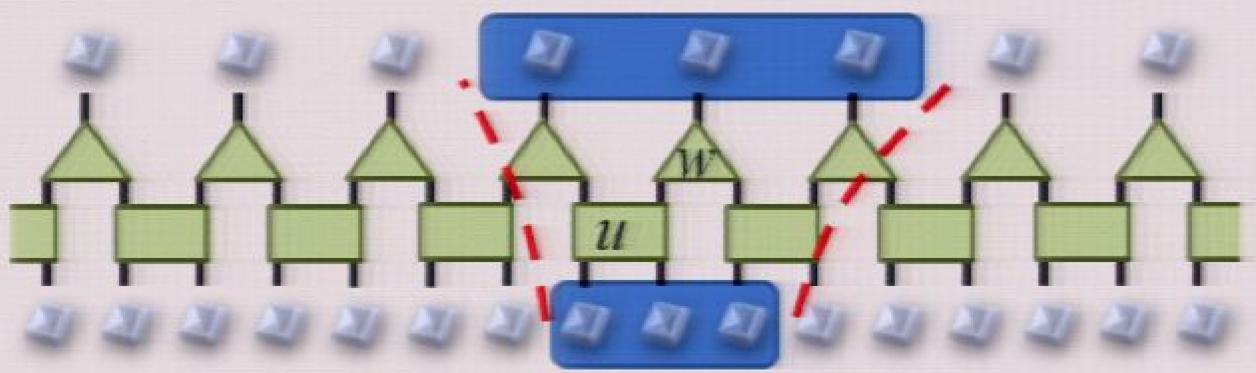

Entanglement Renormalization

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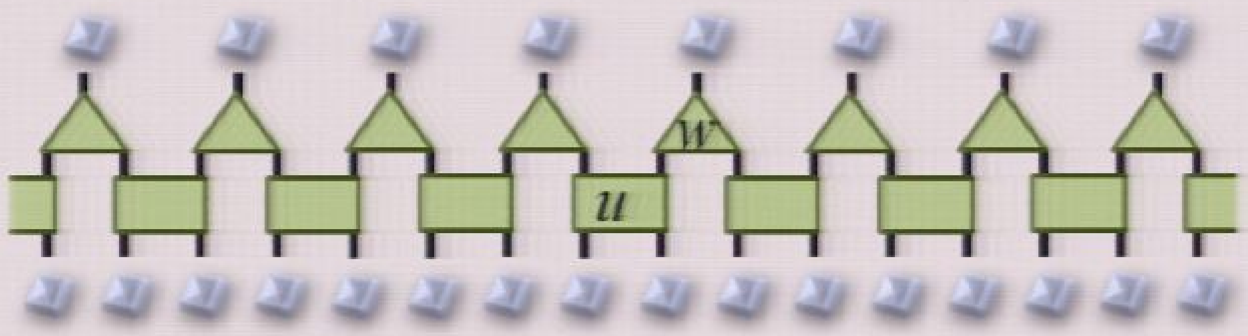

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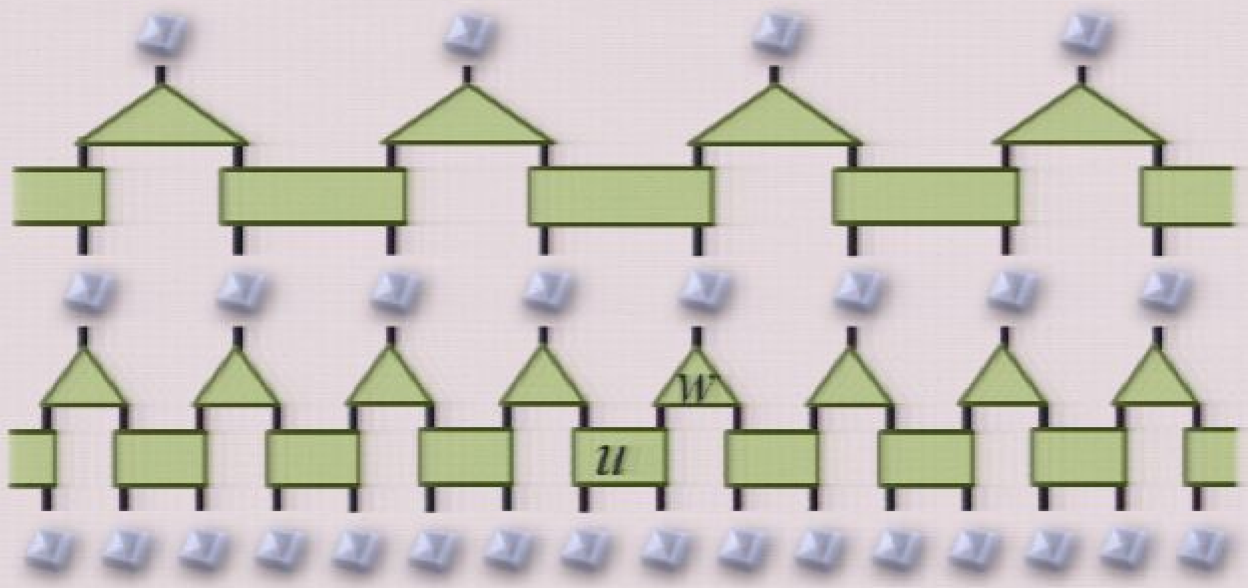

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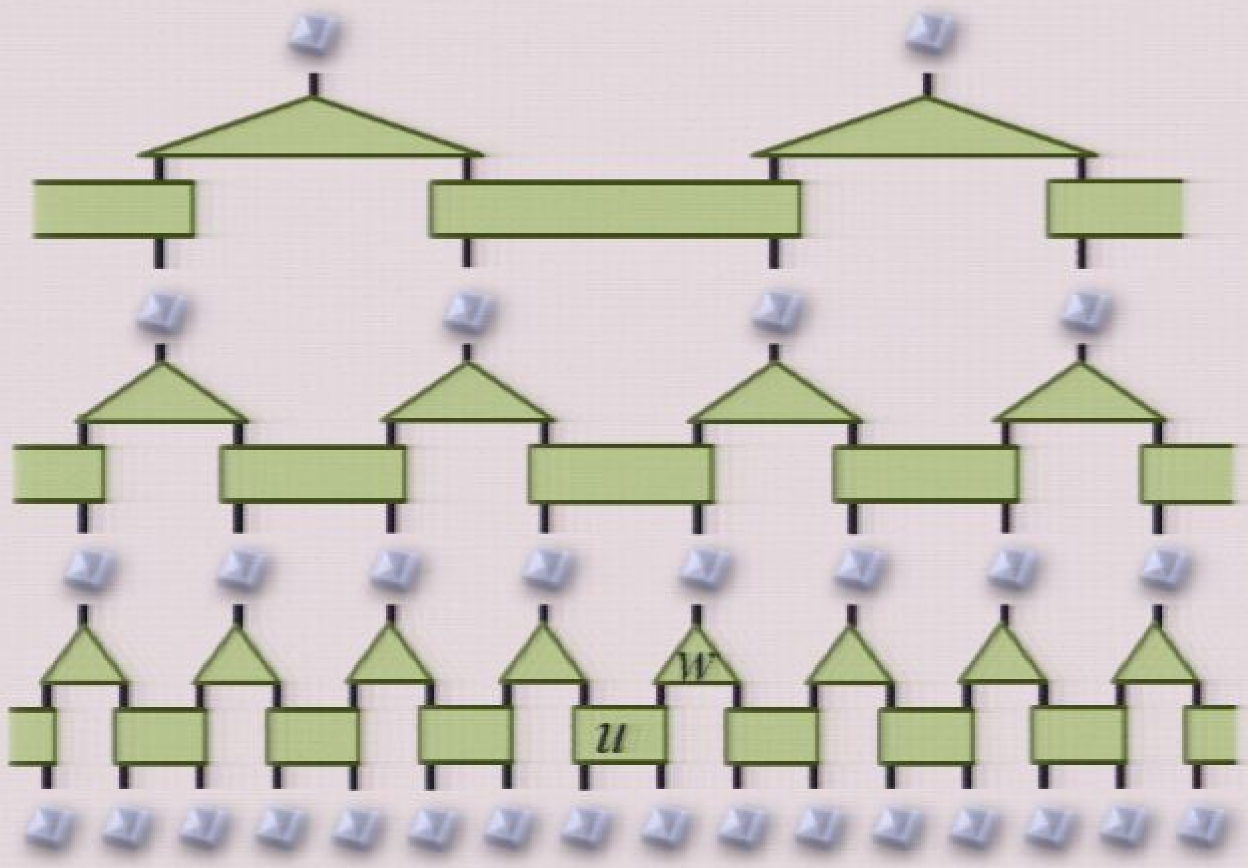

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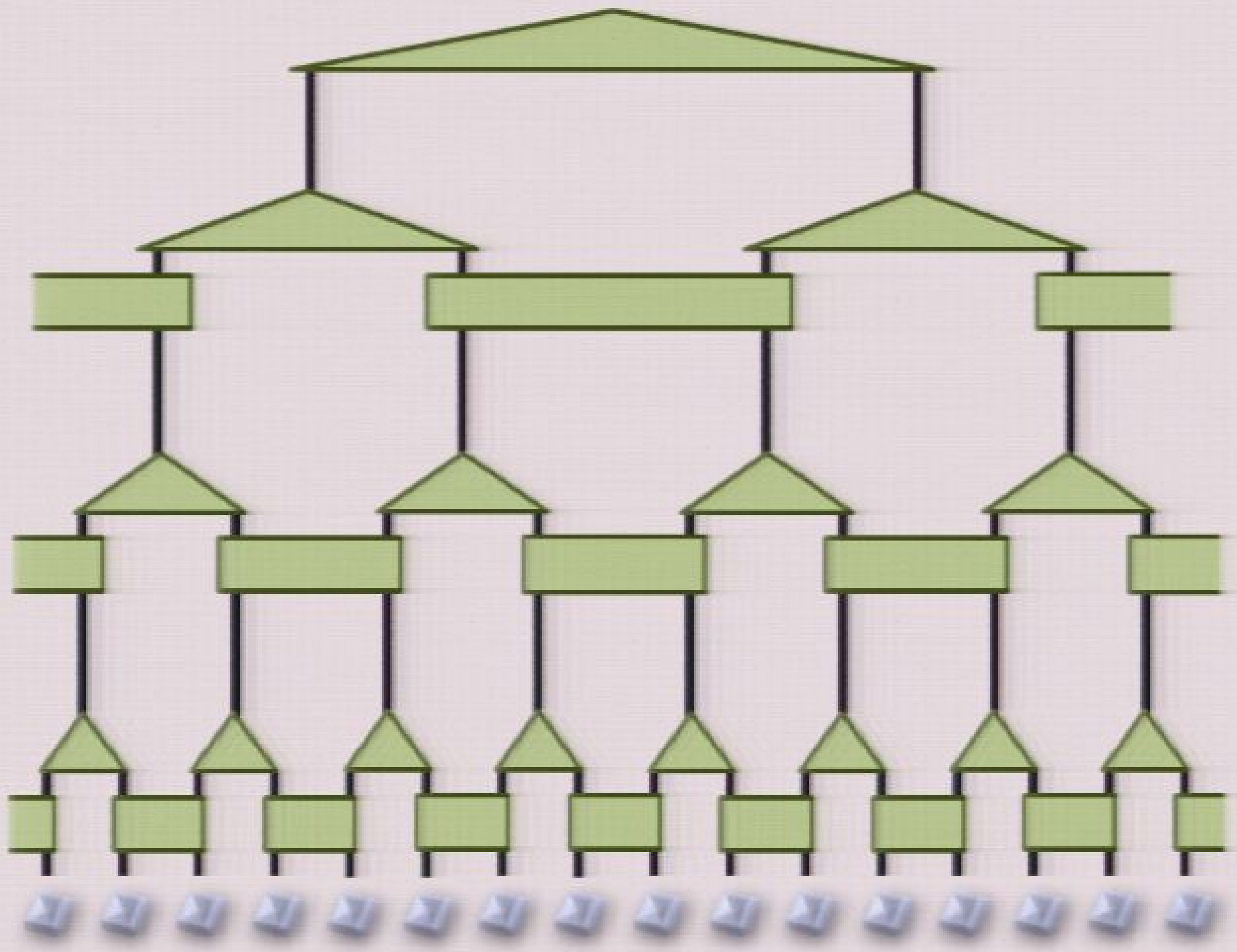
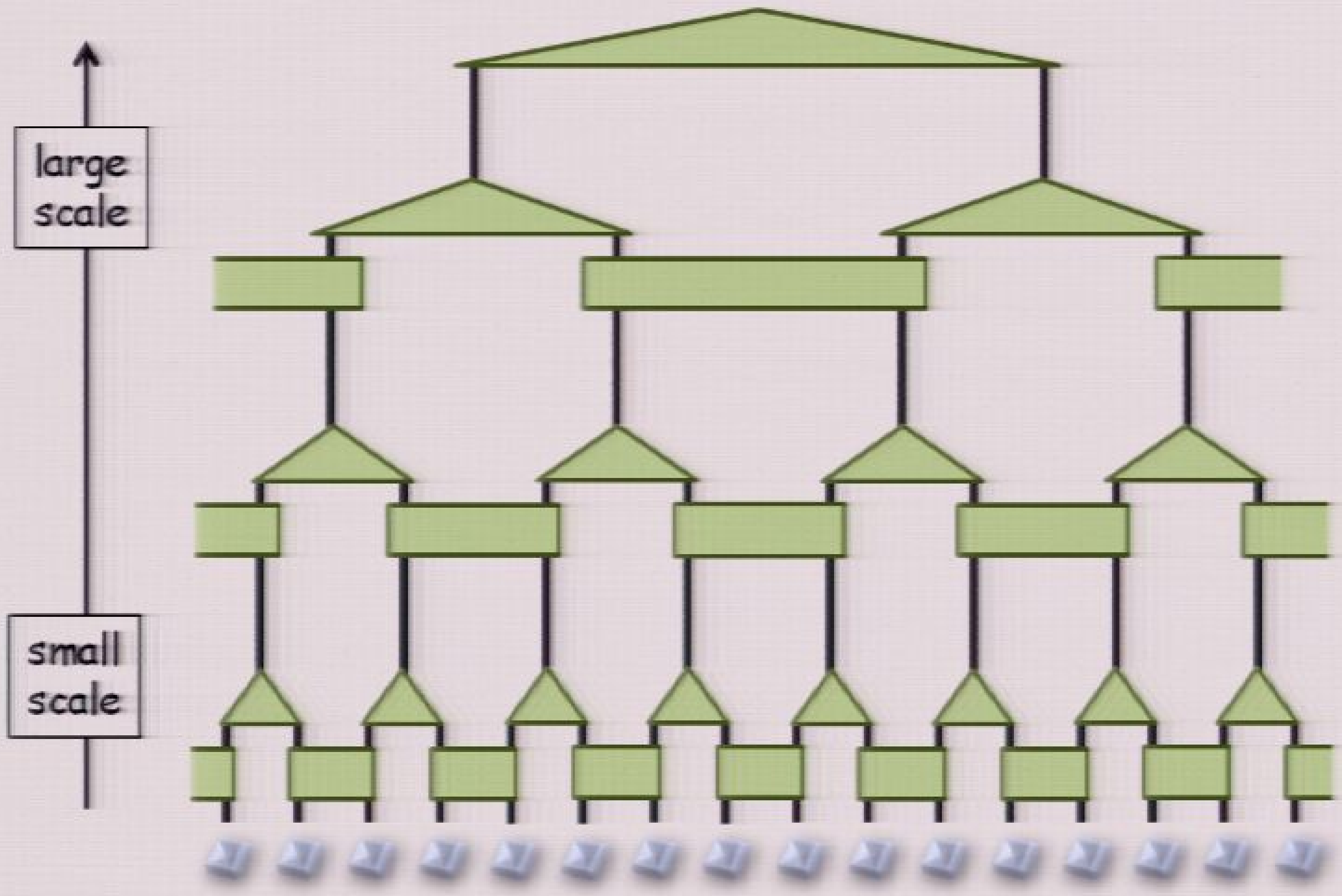


Entanglement Renormalization

• tensor network ansatz <u>with built-in coarse-graining</u>



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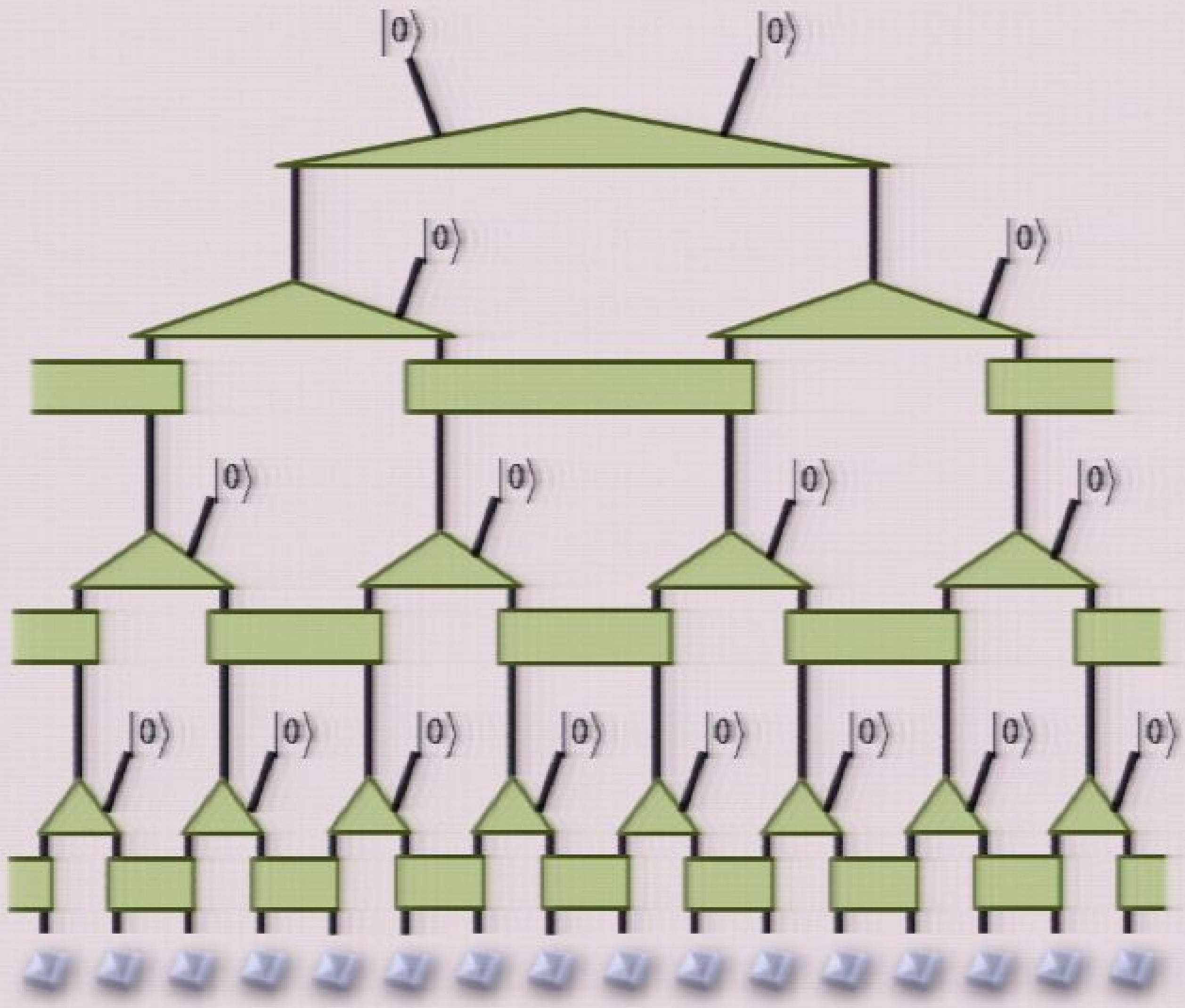

Entanglement Renormalization

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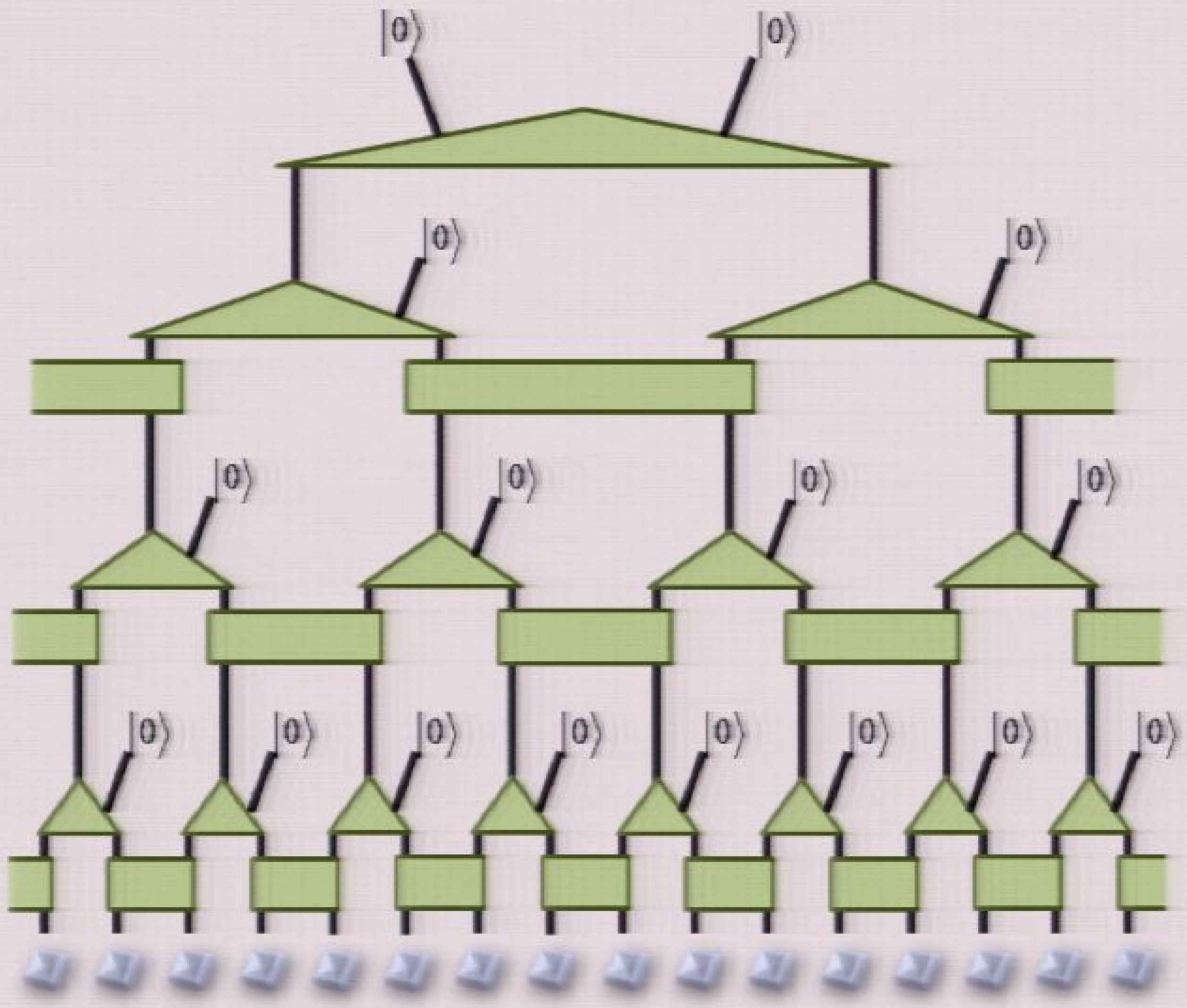

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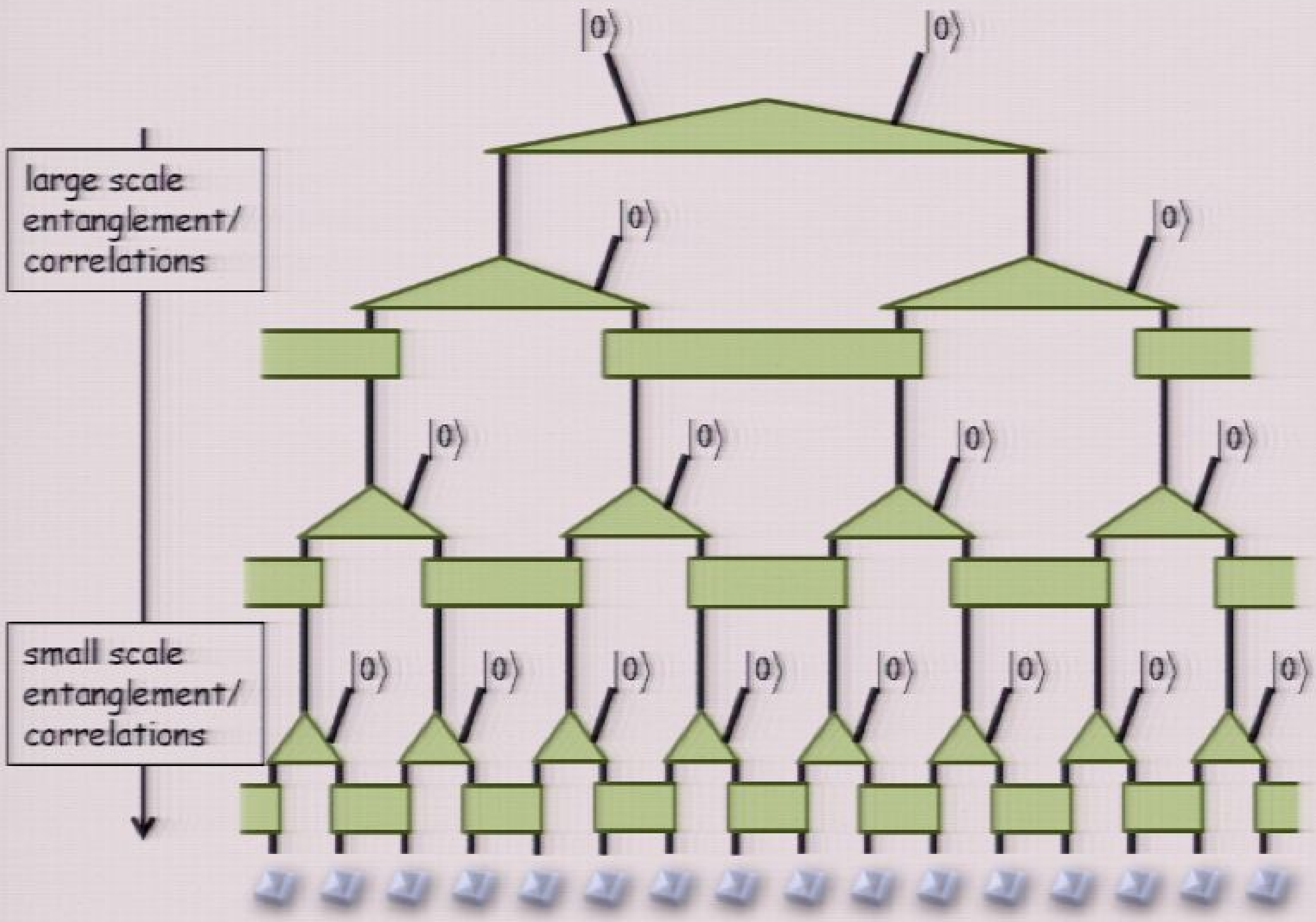

Entanglement Renormalization

Examples of application in 2D lattices (beyond quantum Monte Carlo):

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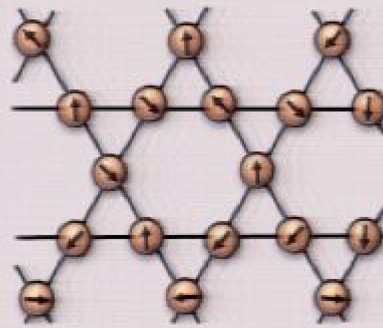
Heisenberg antiferromagnet on Kagome lattice

Glen Evenbly, G. Vidal, PRL 104, 187203 (2010)



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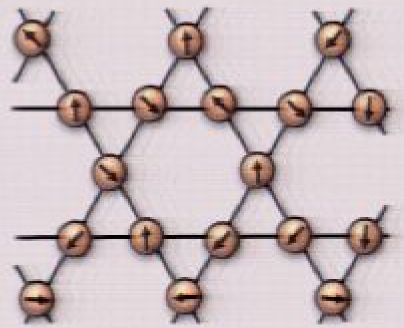
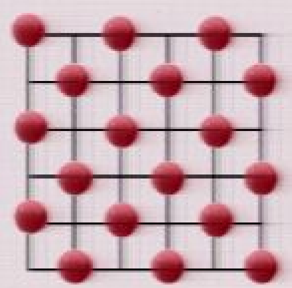


- interacting fermions

Interacting fermions on square lattice

Philippe Corboz, G. Vidal, PRB 80, 165129 (2009)



Entanglement Renormalization

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S. Singh, R. Pfeifer,
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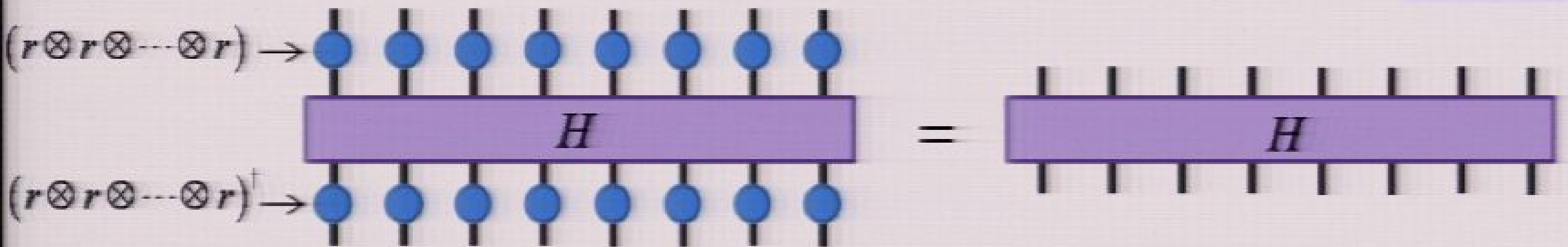

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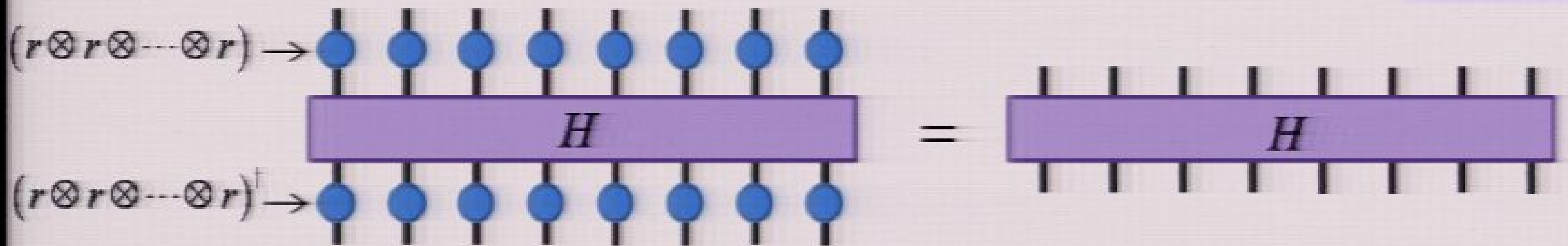
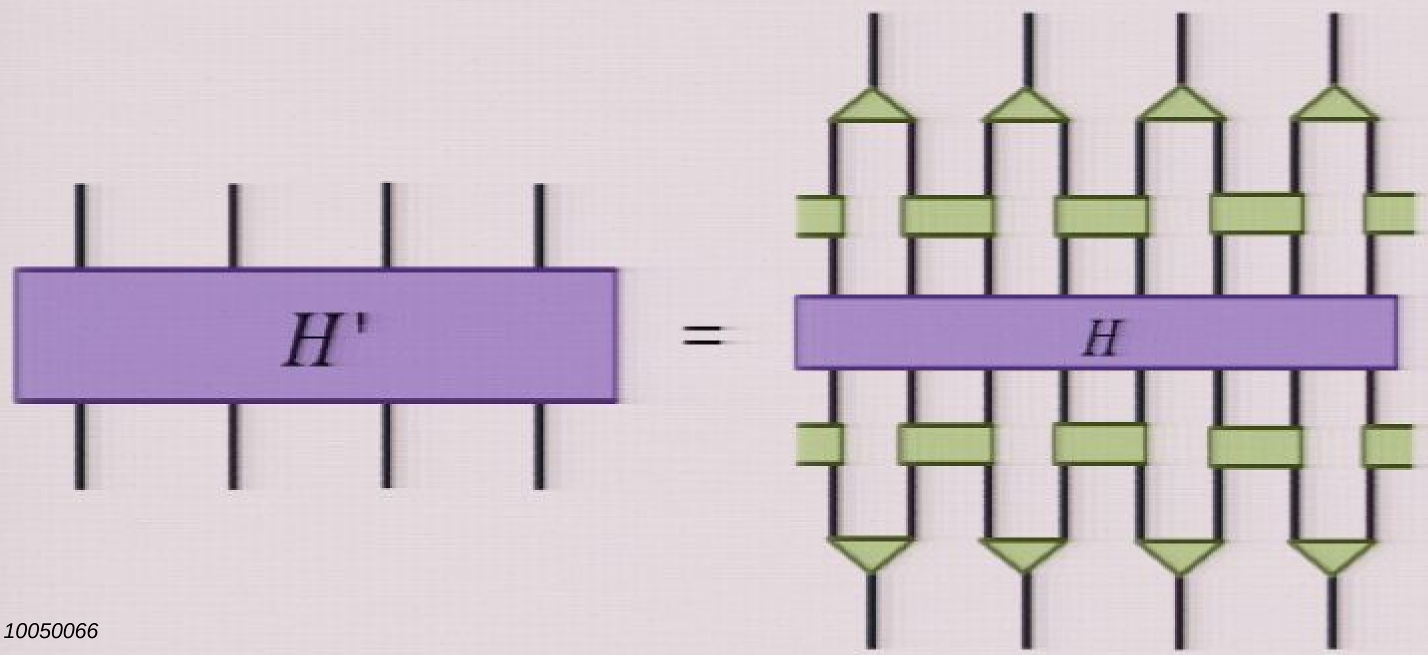


- Preserving a global symmetry during coarse-graining



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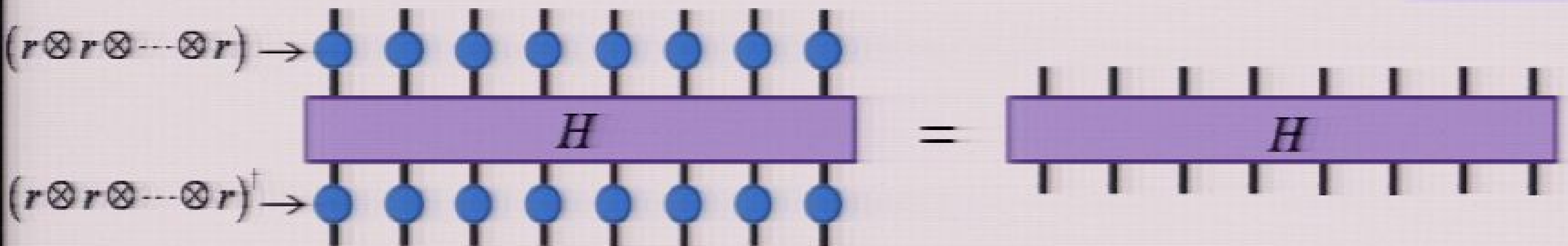

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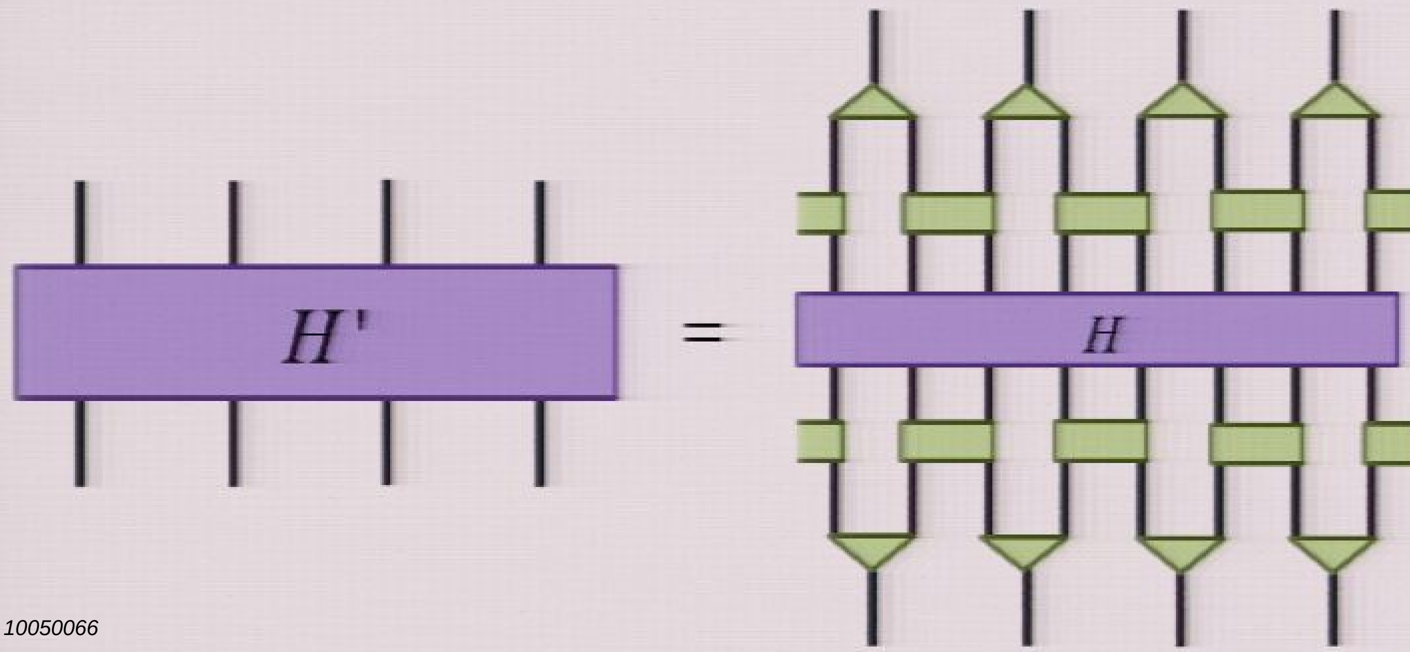

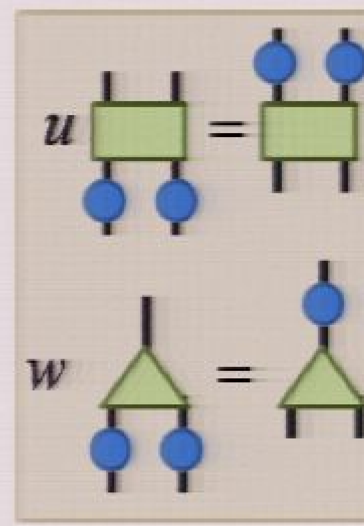

symmetric tensors

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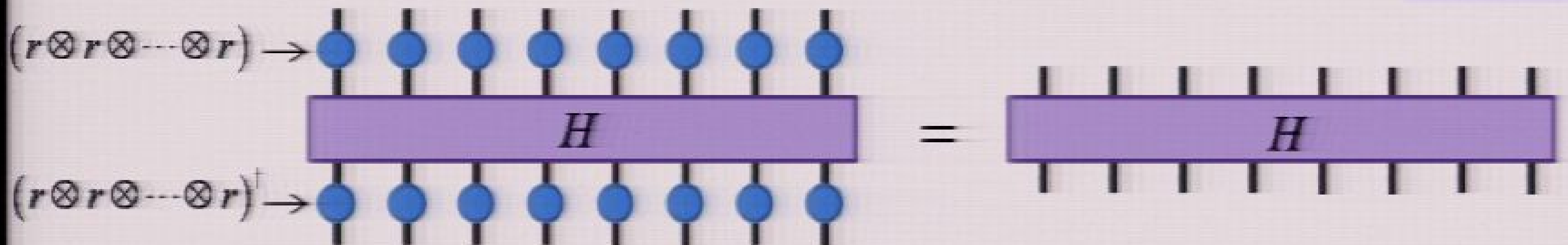

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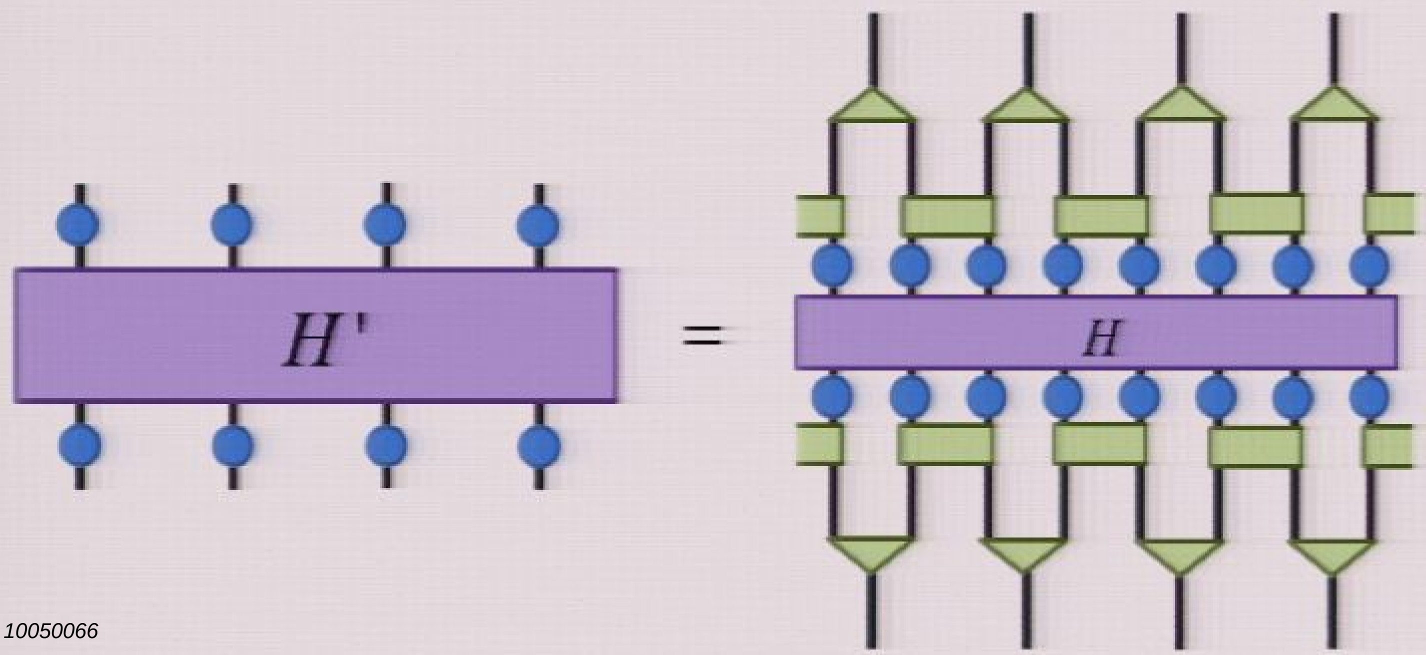

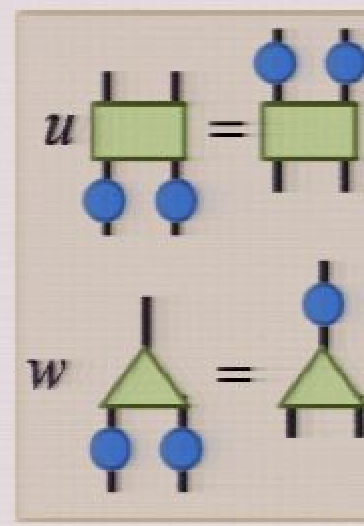

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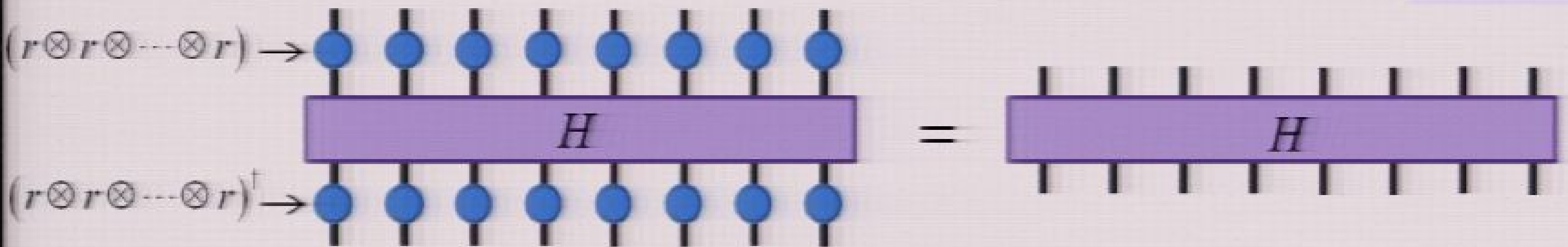

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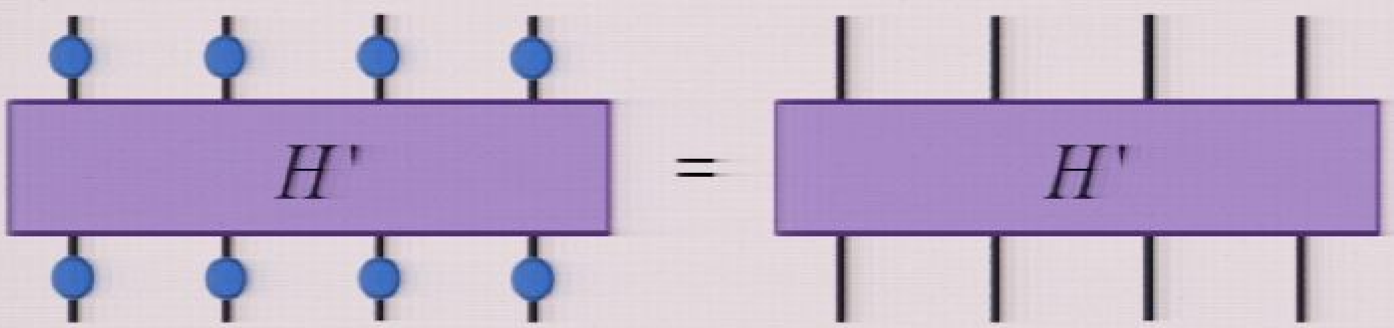
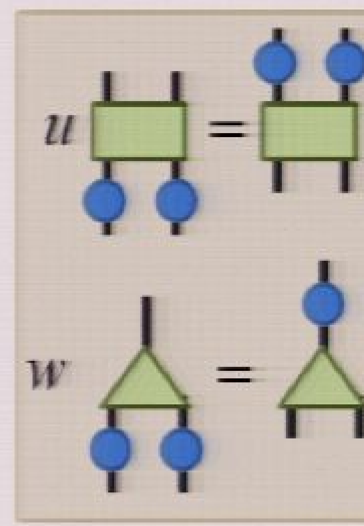




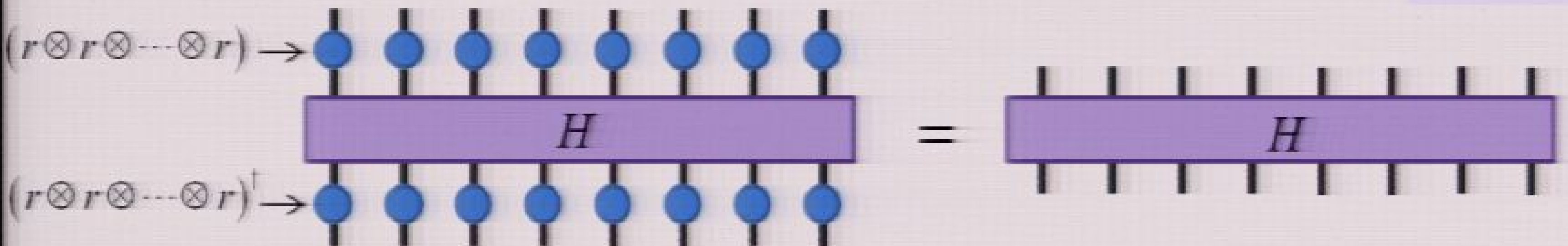
symmetric tensors

Entanglement Renormalization

S. Singh, R. Pfeifer, G. Vidal, arXiv:0907.2994

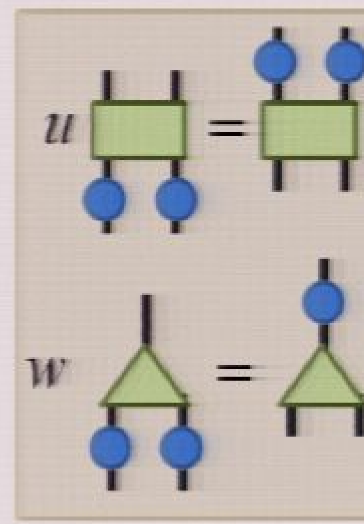
- Global symmetry

$$H \xrightarrow{(r\otimes r\otimes\cdots\otimes r)} (r\otimes r\otimes\cdots\otimes r) H (r\otimes r\otimes\cdots\otimes r)^{\dagger} = H$$



- Preserving a global symmetry during coarse-graining

Advantages of using symmetry tensors:



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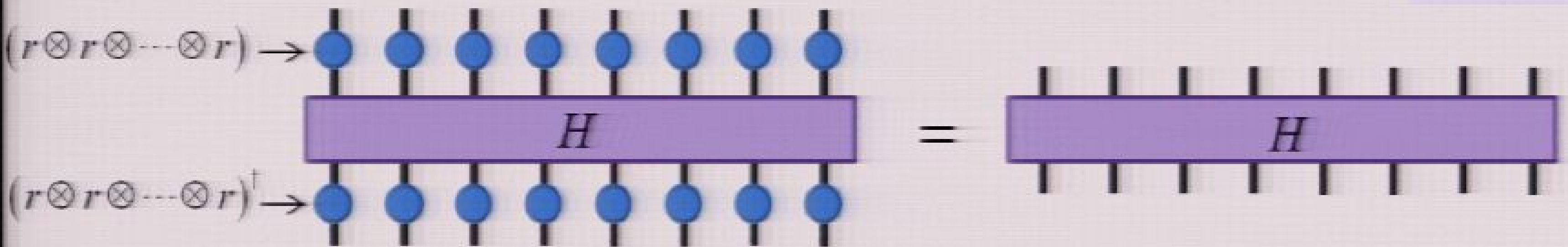
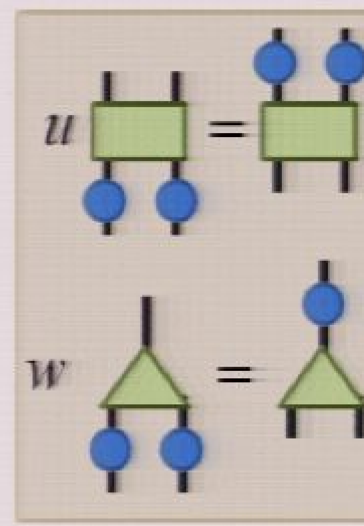


- Preserving a global symmetry during coarse-graining

Advantages of using symmetry tensors:

- exact preservation of symmetry
- target specific symmetry sectors



symmetric tensors

Entanglement Renormalization

- What about local symmetries ?

Luca Tagliacozzo,
G. Vidal, in preparation

Entanglement Renormalization

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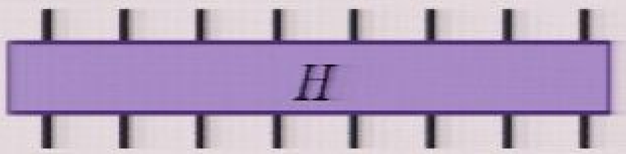

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Entanglement Renormalization

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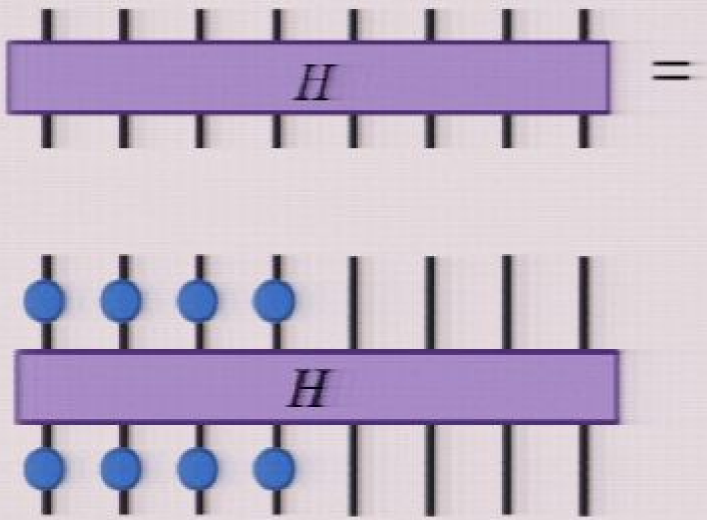

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Entanglement Renormalization

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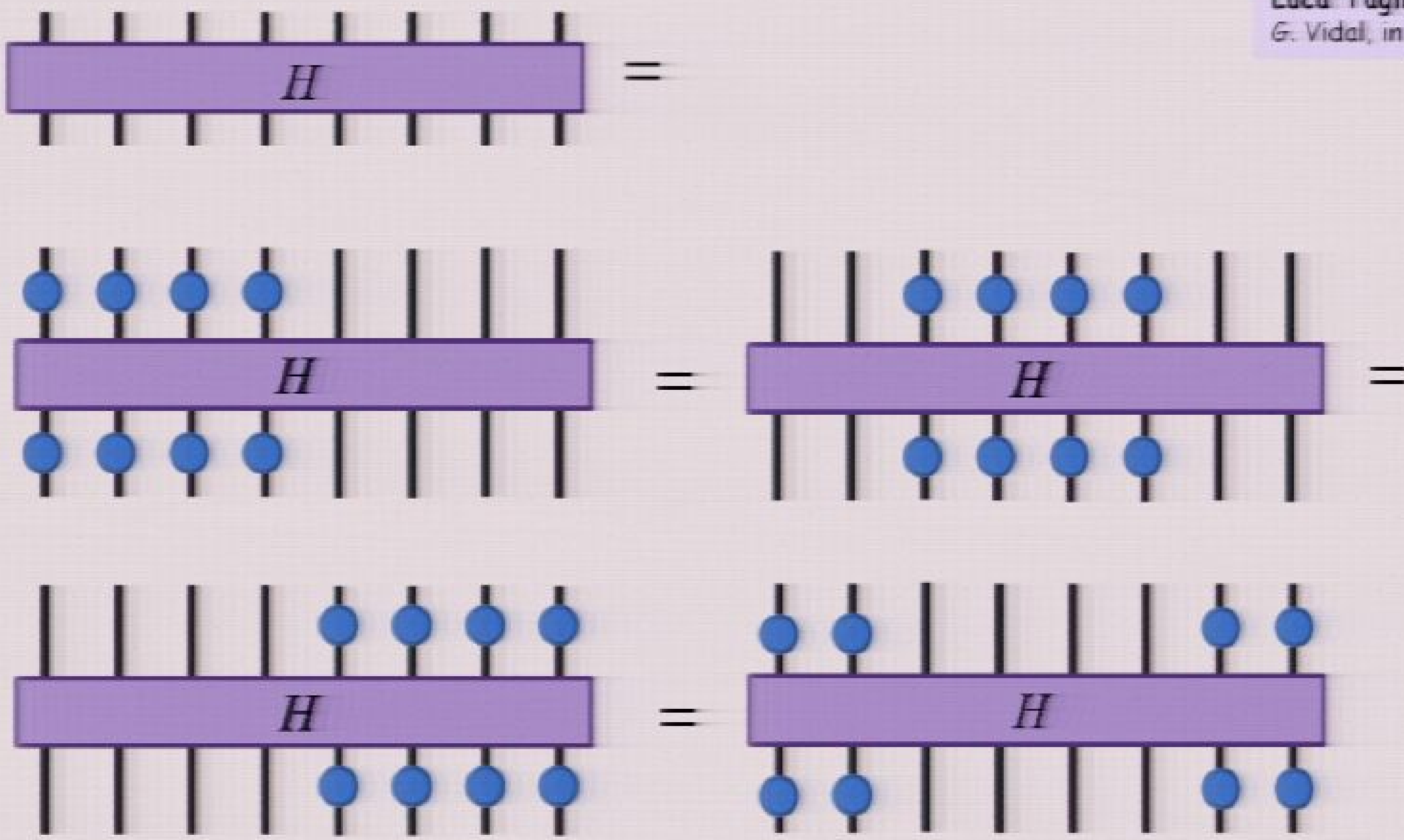

Local symmetry and gauge theory

Toric code A. Kitaev, Annals Phys. (2003)

Local symmetry and gauge theory

Toric code

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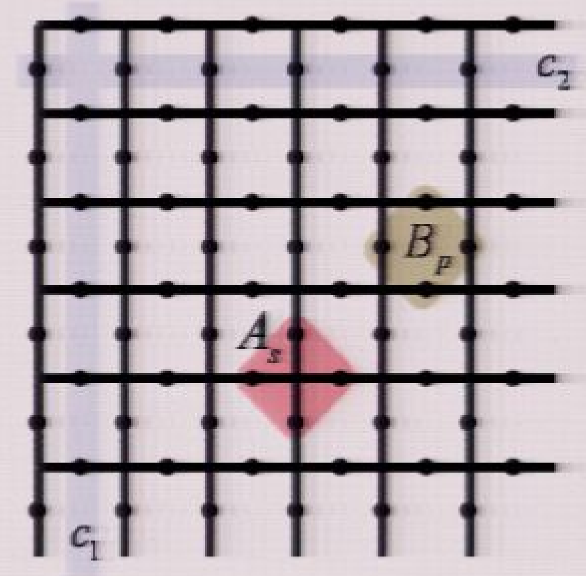
$L \times L$ lattice $\mathcal{L}$ on a torus (PBC)



Local symmetry and gauge theory

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$$H_{\mathrm{TC}} = -J_e \sum_s A_s - J_m \sum_p B_p$$

$$A_s \equiv \prod_{j \in s} \sigma_j^x \qquad B_p \equiv \prod_{j \in p} \sigma_j^z$$

star operators   plaquette operators

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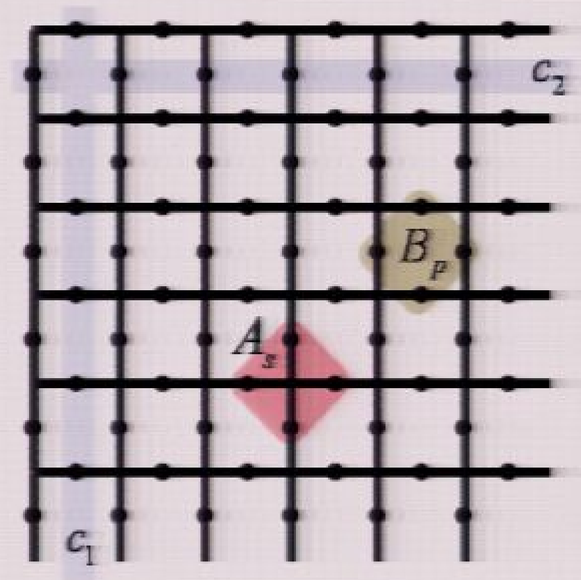

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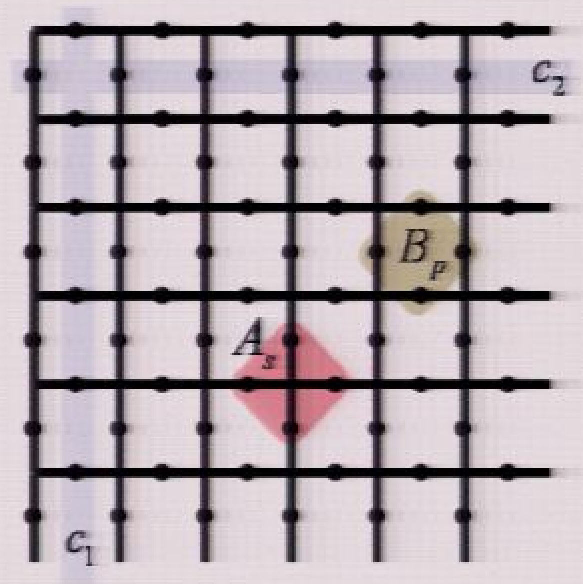

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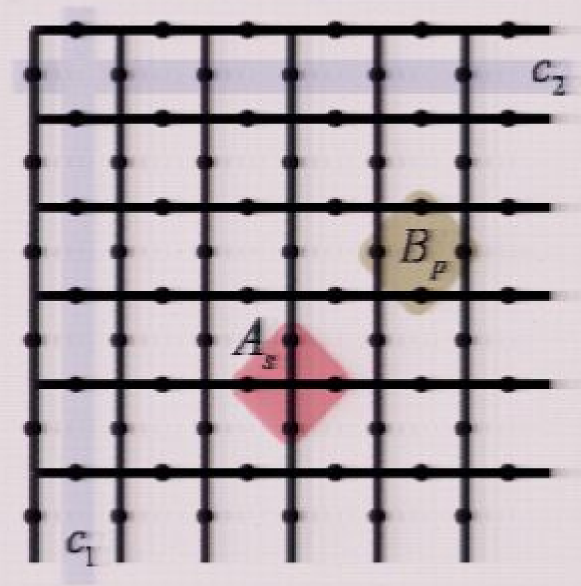

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$$B_p |\xi\rangle = |\xi\rangle \quad \forall p \in \mathcal{L}$$

four-fold degenerate

$\Rightarrow$ $|\Psi_{++}\rangle$ $|\Psi_{-+}\rangle$ $|\Psi_{+-}\rangle$ $|\Psi_{--}\rangle$

symmetry: non-contractible loops

$$X_1 \equiv \prod_{j \in c_1} \sigma_j^x \qquad [H_{\rm TC}, X_1] = 0$$

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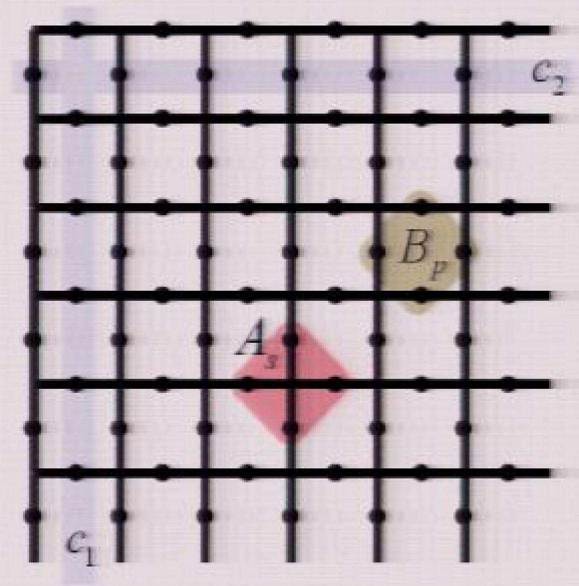

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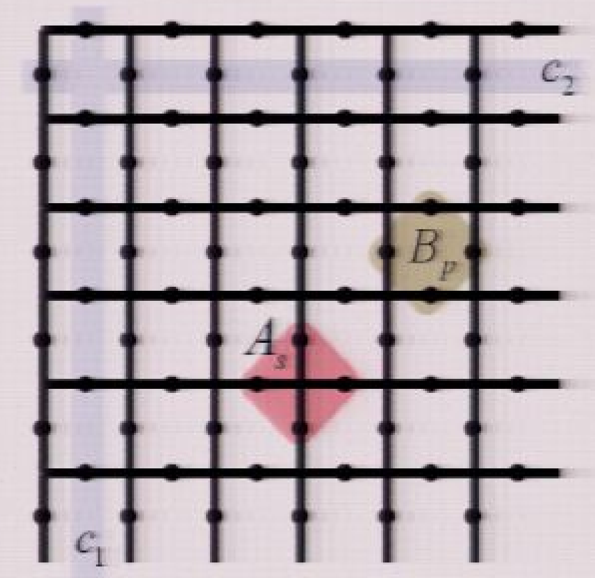
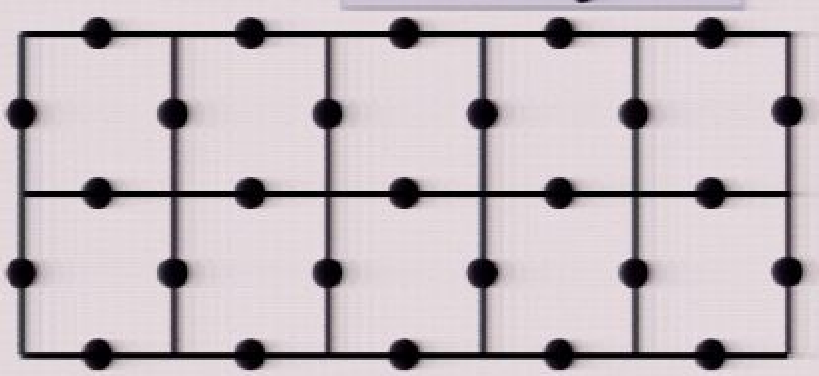


- Excitations?

energy $\sim J_e$

$A_s|\xi\rangle = -|\xi\rangle$ e

electric charges



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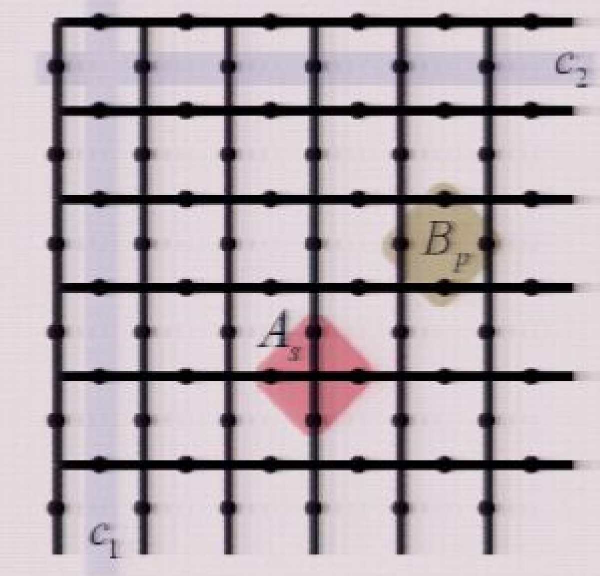

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energy $\sim J_e$

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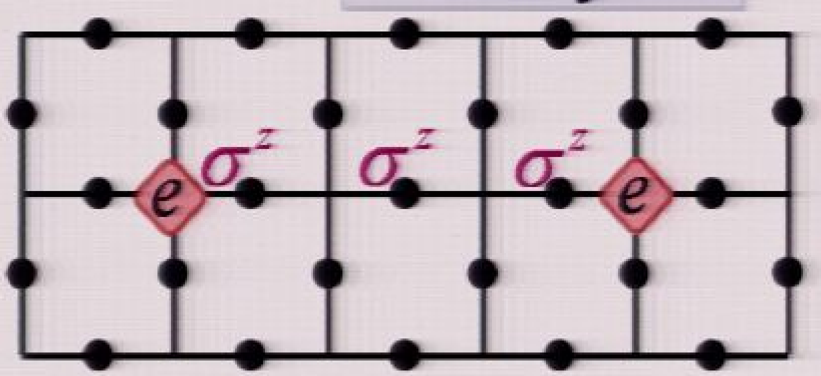
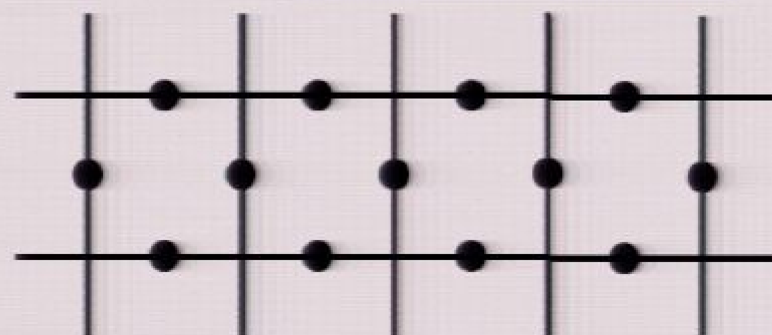
magnetic vortices





Local symmetry and gauge theory

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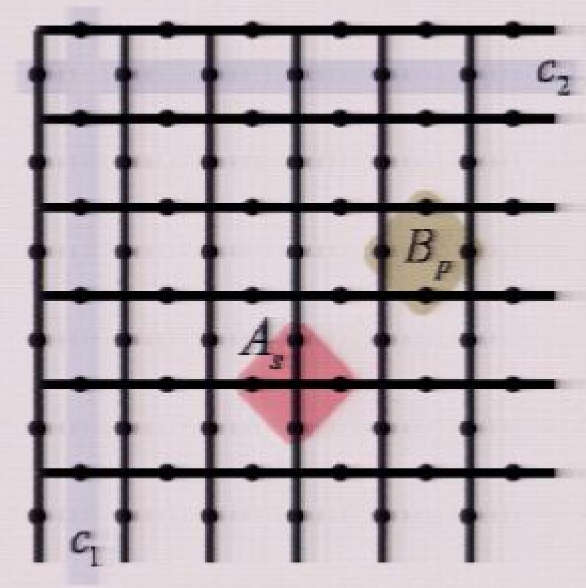

Deformation of toric code: S. Trebst et al., PRL 98, 070602 (2007)

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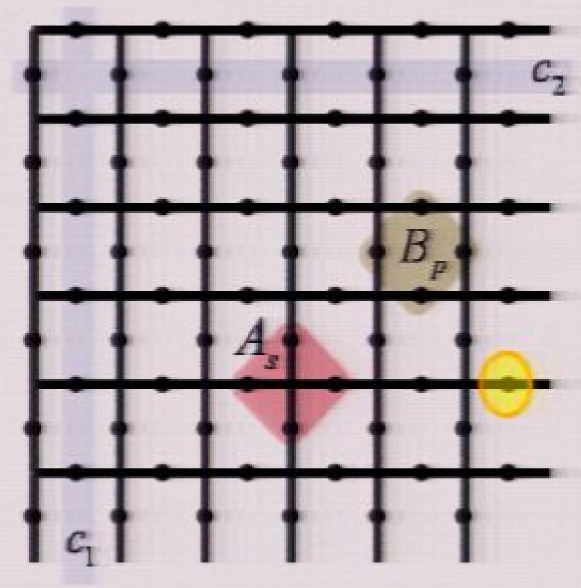

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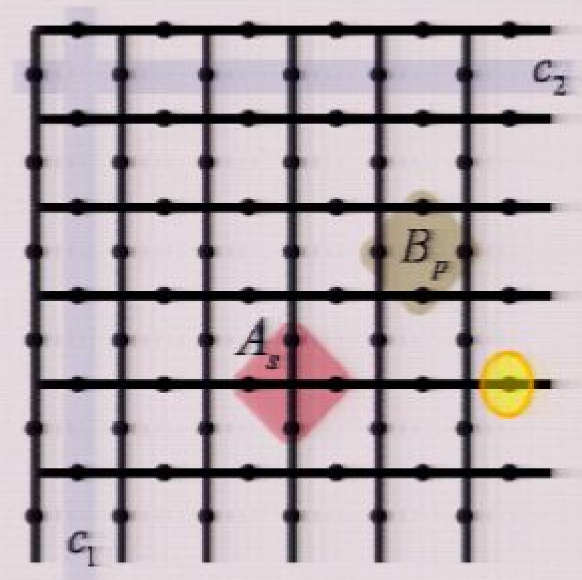

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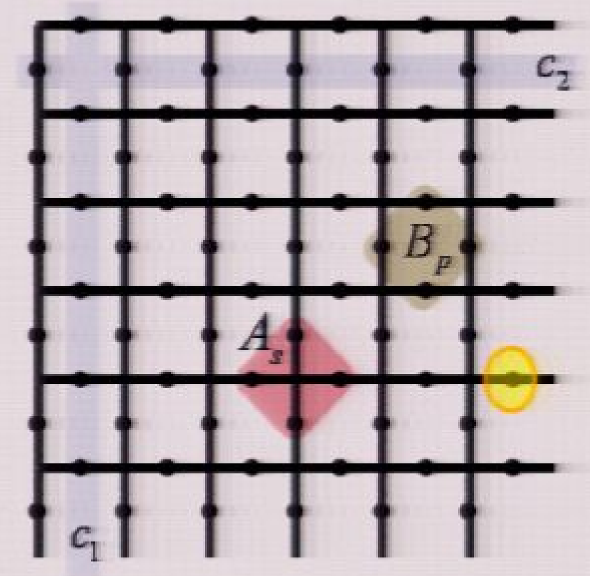

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➡ still four topological sectors

$$|\Psi_{++}\rangle \quad |\Psi_{-+}\rangle \quad |\Psi_{+-}\rangle \quad |\Psi_{--}\rangle$$

near degenerate for small h_x

Local symmetry and gauge theory

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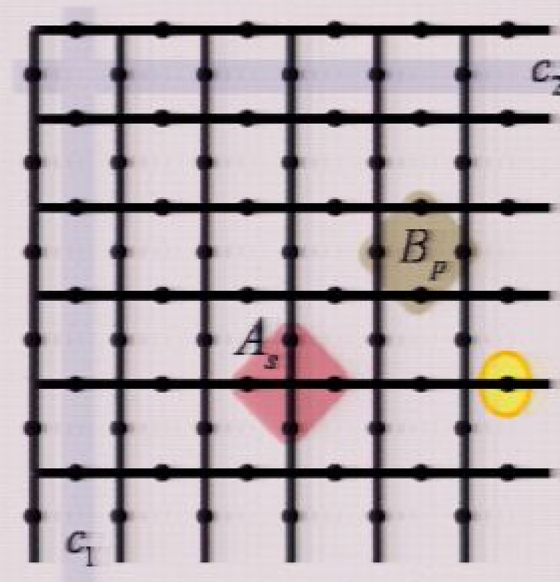

Local symmetry and gauge theory

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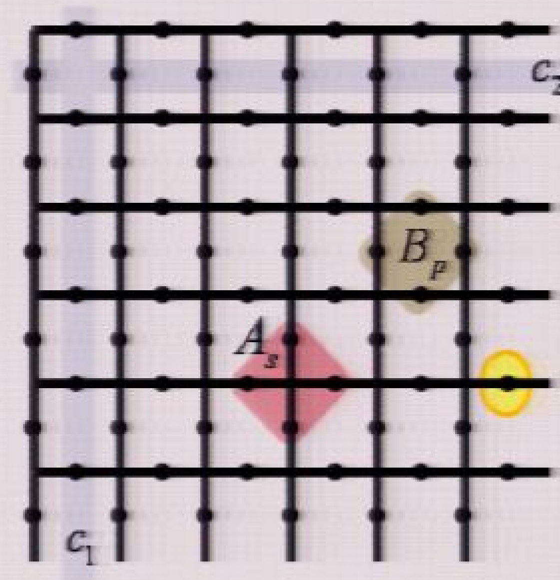
$L \times L$ lattice $\mathcal{L}$ on a torus (PBC)



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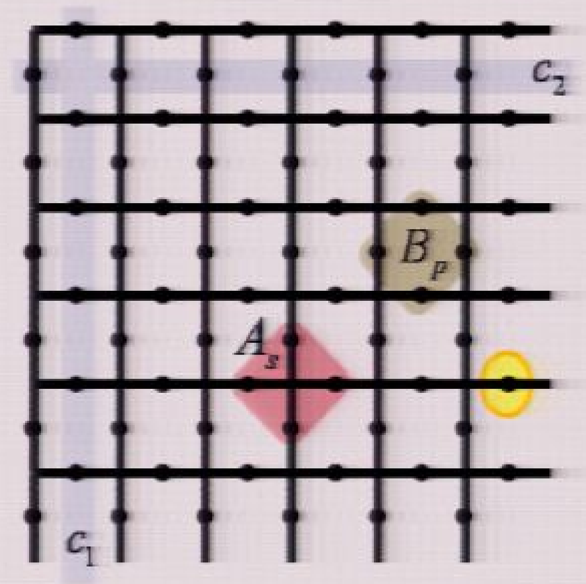

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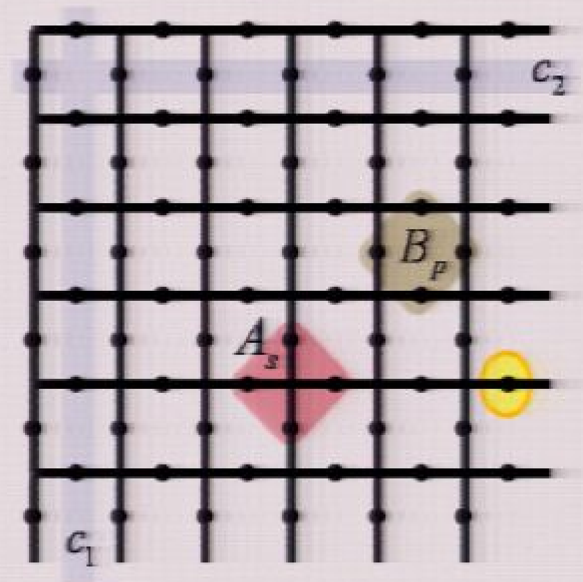

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Can we incorporate this local symmetry into a coarse-graining scheme?

Local symmetry and gauge theory

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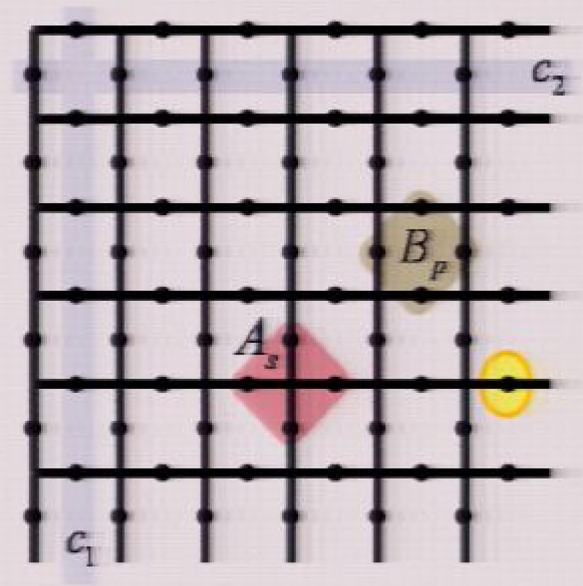

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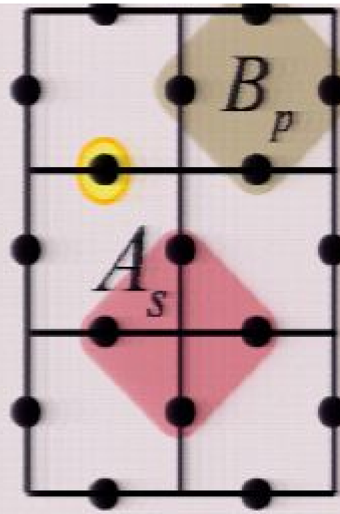
Can we incorporate this local symmetry into a coarse-graining scheme?

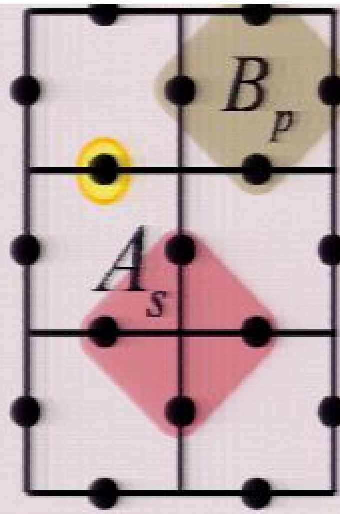
- exact preservation of local symmetry
- significant reduction in simulation costs

Local symmetry and gauge theory



$$\mathbb{V}_{\mathrm{TC}} \cong \left(\mathbb{C}_2\right)^{\otimes 2L^2} \qquad H_{\mathrm{TC}}^{x} = -J_e \sum_s A_s - J_m \sum_p B_p - h_x \sum_j \sigma_j^x$$

Local symmetry and gauge theory



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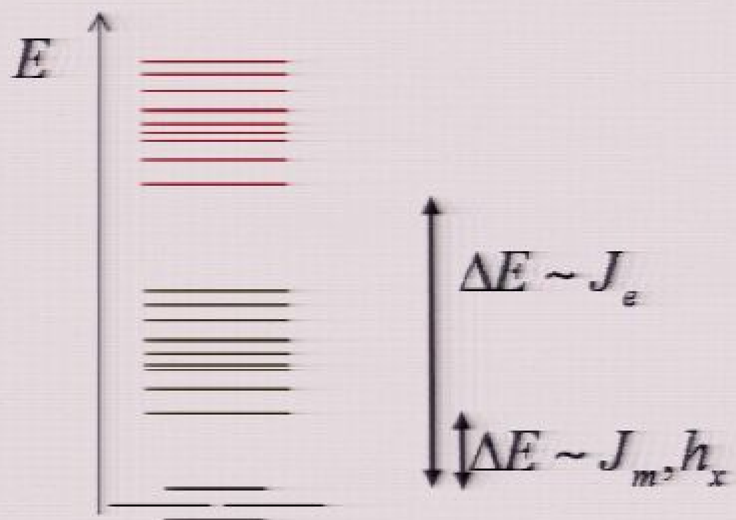

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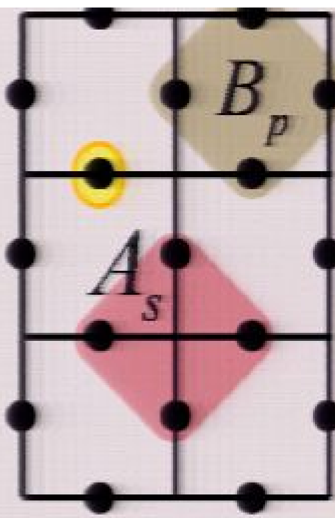

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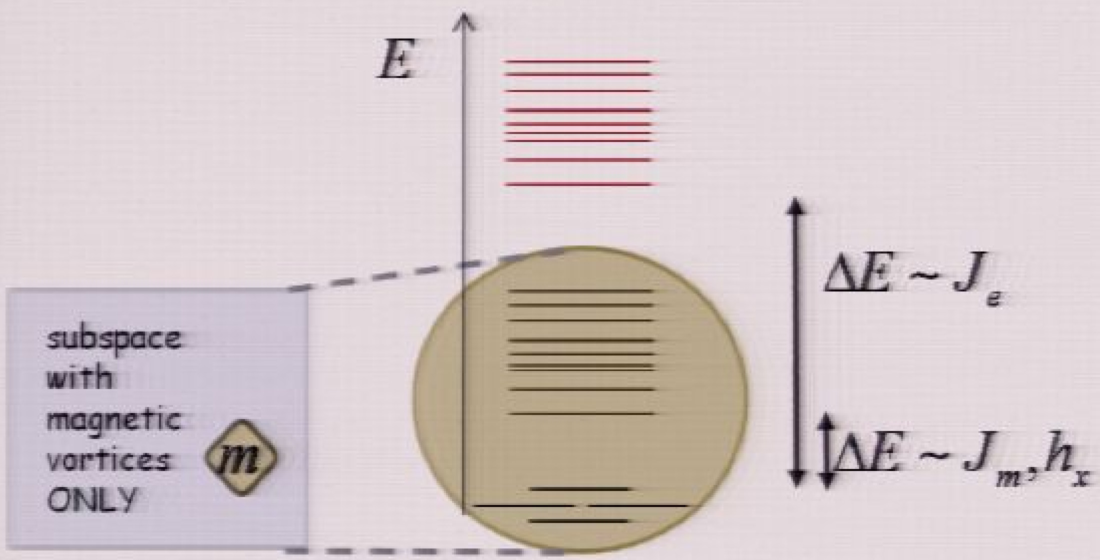

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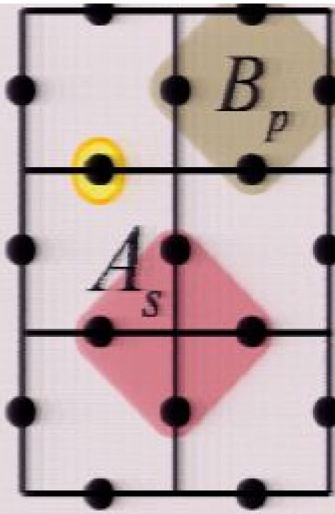

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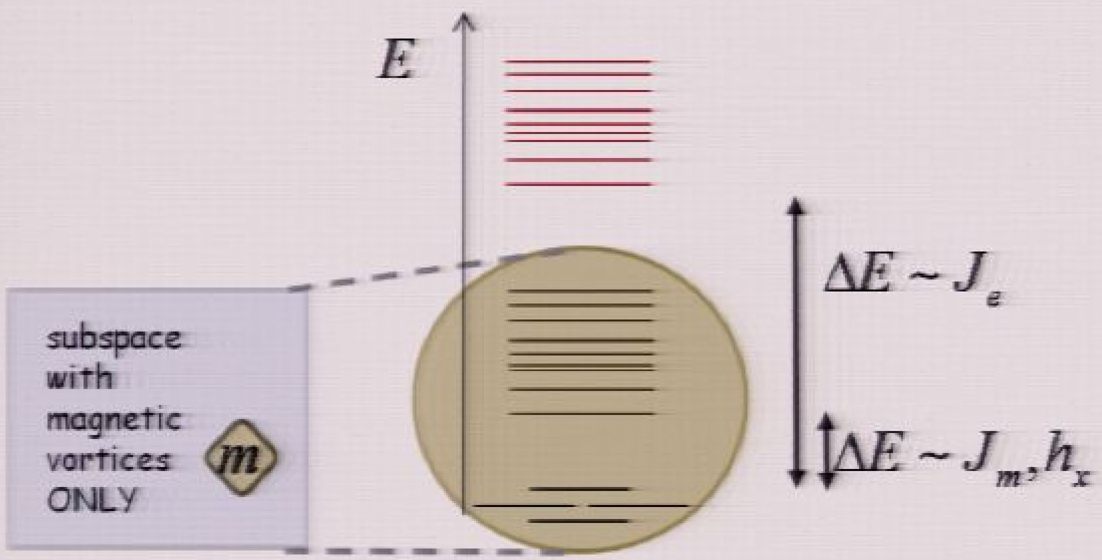


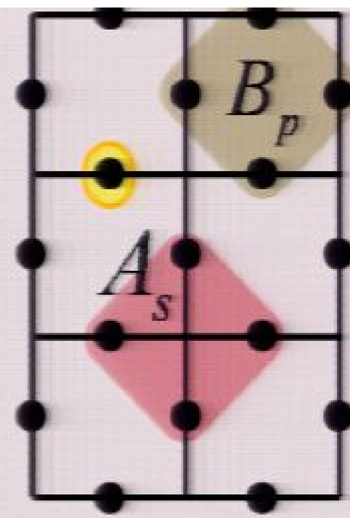
All states fulfill

$$A_s |\xi\rangle = |\xi\rangle \quad \forall s \in \mathcal{L}$$

low energy equivalent to Z_2 Lattice Gauge Theory

F. Wegner, J. Math. Phys. (1971)

Local symmetry and gauge theory



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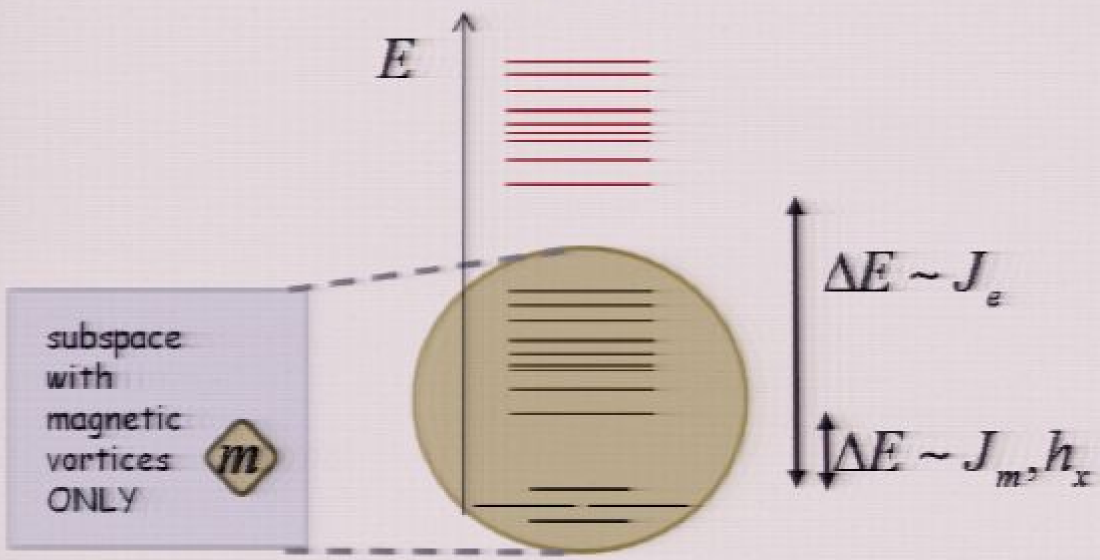

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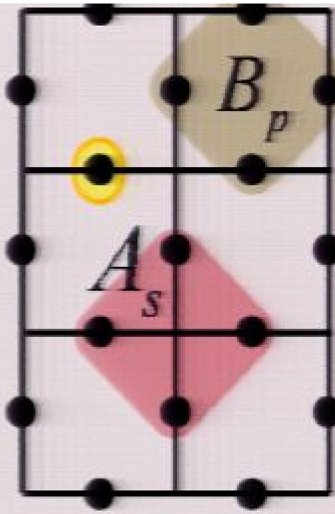
Local symmetry and gauge theory



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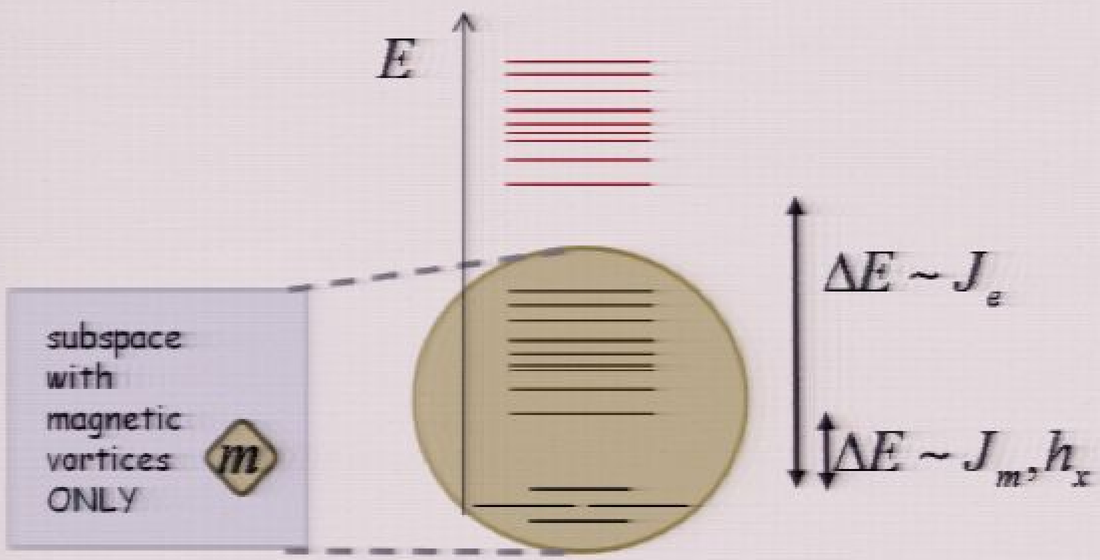

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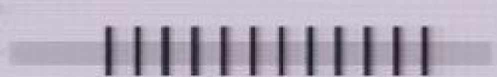
Symmetries? $[H_{\mathrm{LGT}}, X_1] = 0$

$[H_{\mathrm{LGT}}, X_2] = 0$

Coarse-graining scheme

without local symmetry

$\mathcal{L}$

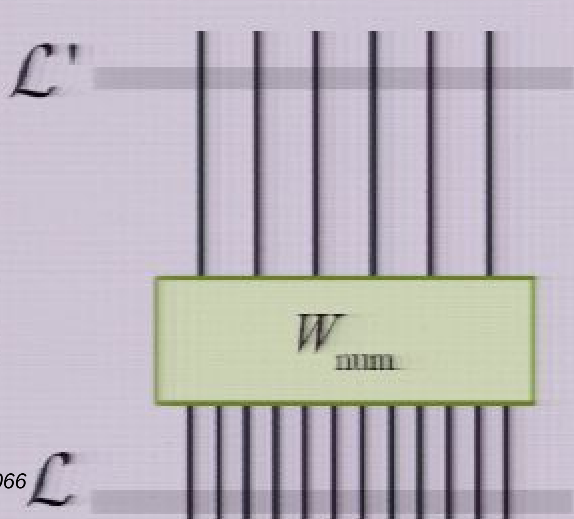


with local symmetry

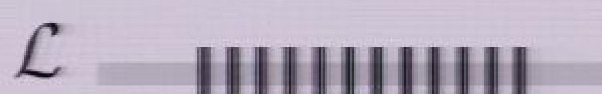
$\mathcal{L}$

Coarse-graining scheme

without local symmetry



with local symmetry



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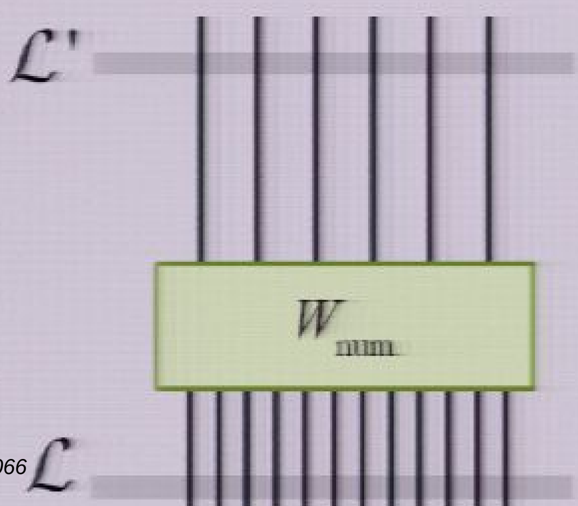
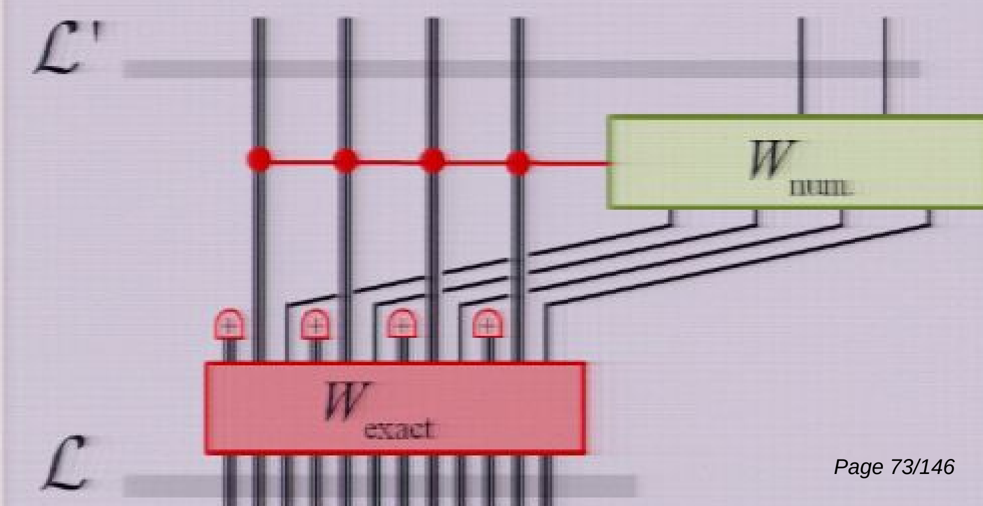


with local symmetry



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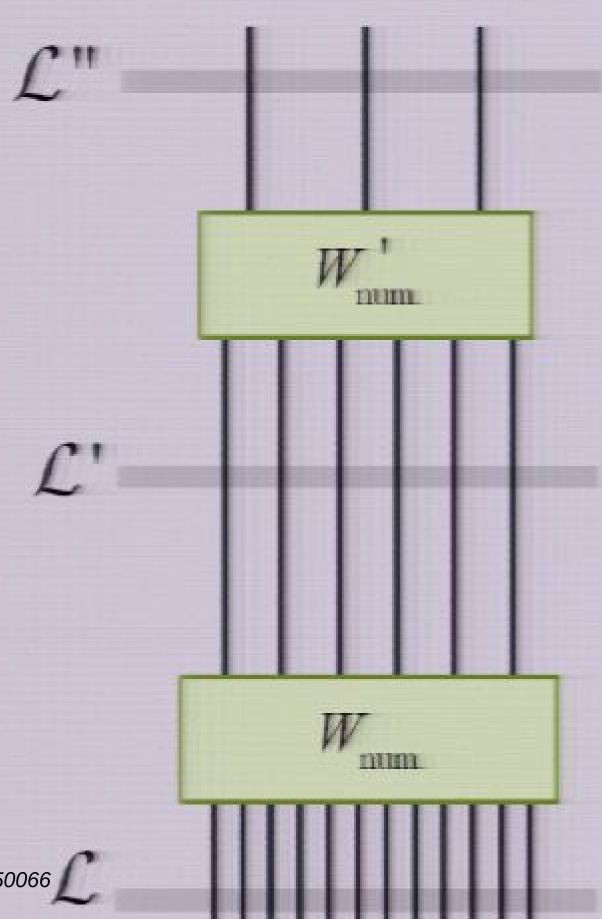

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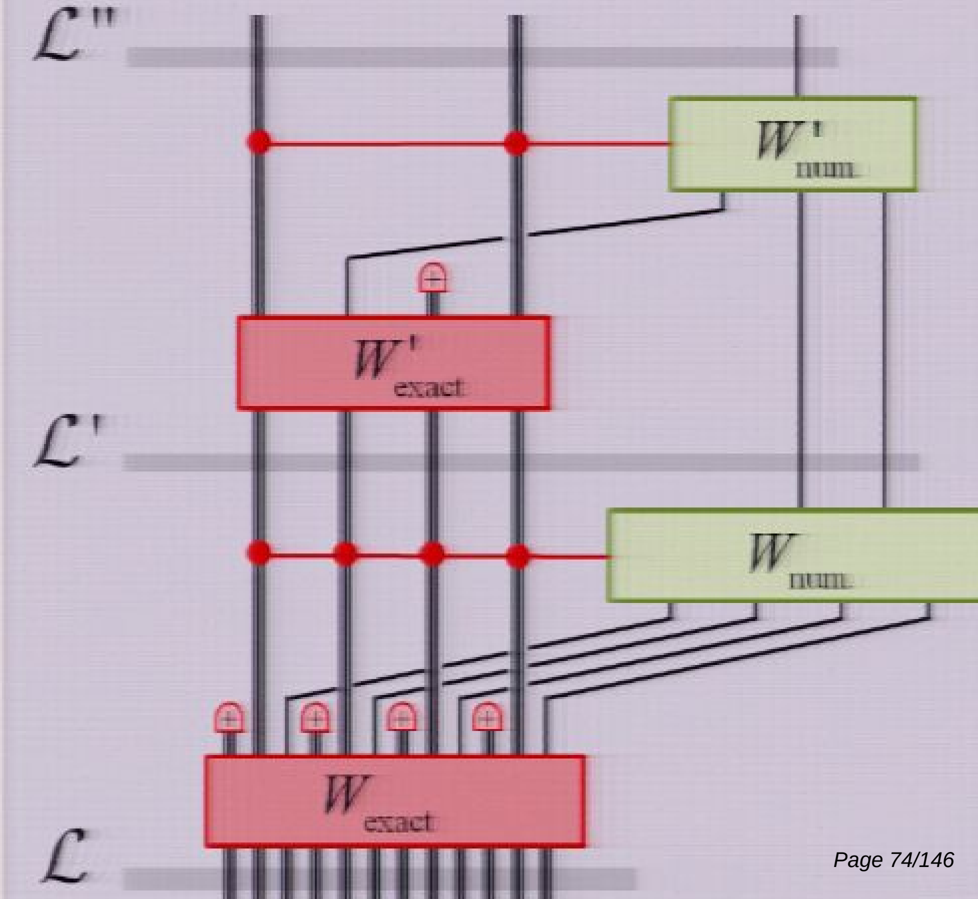

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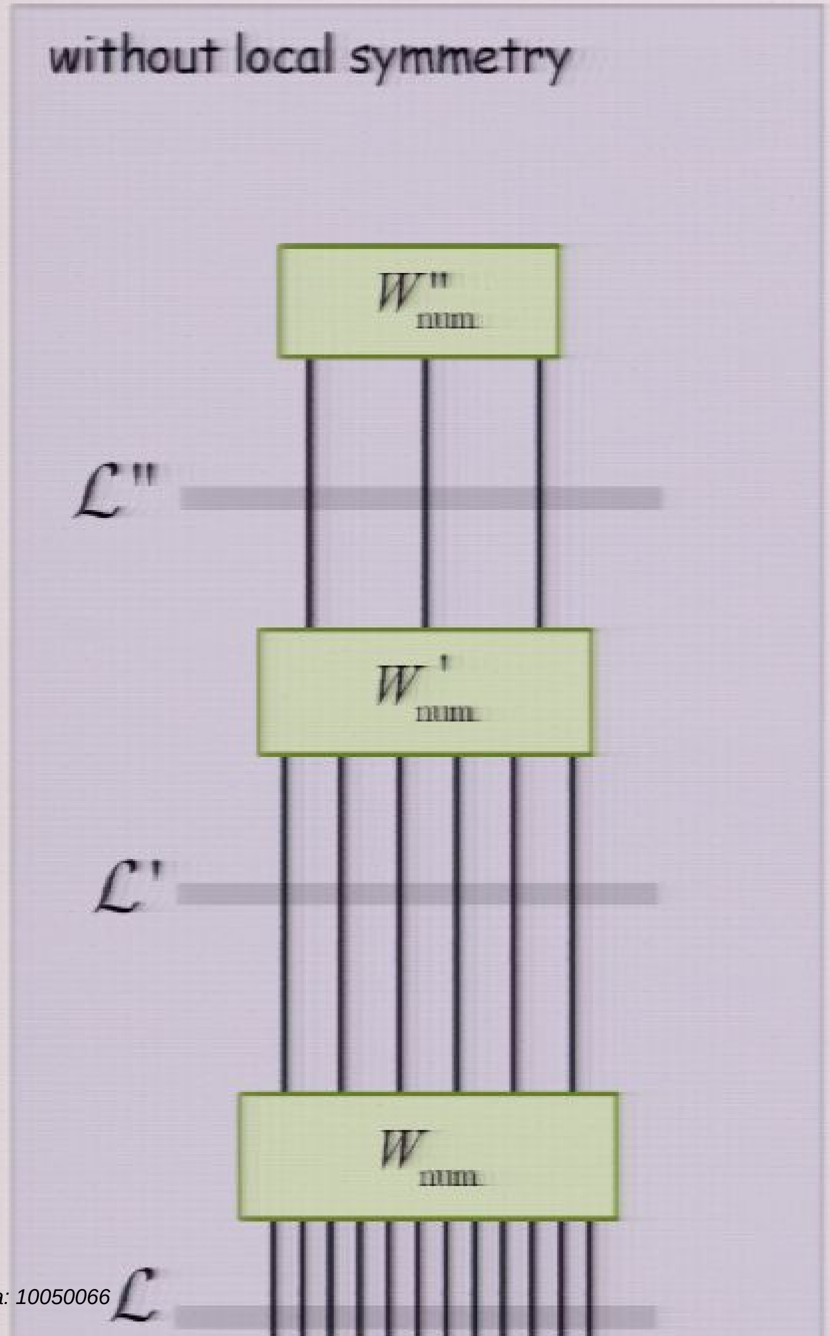
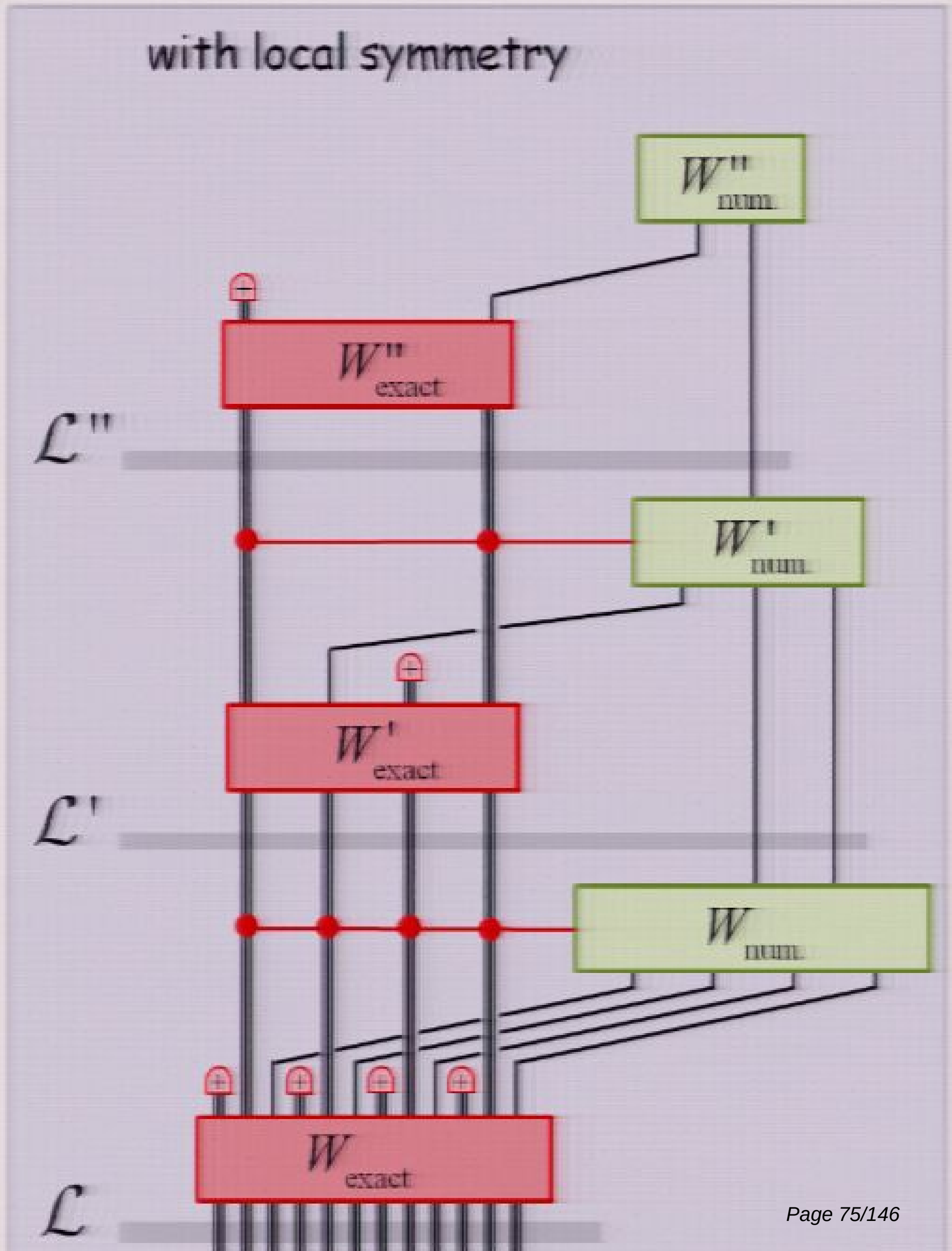
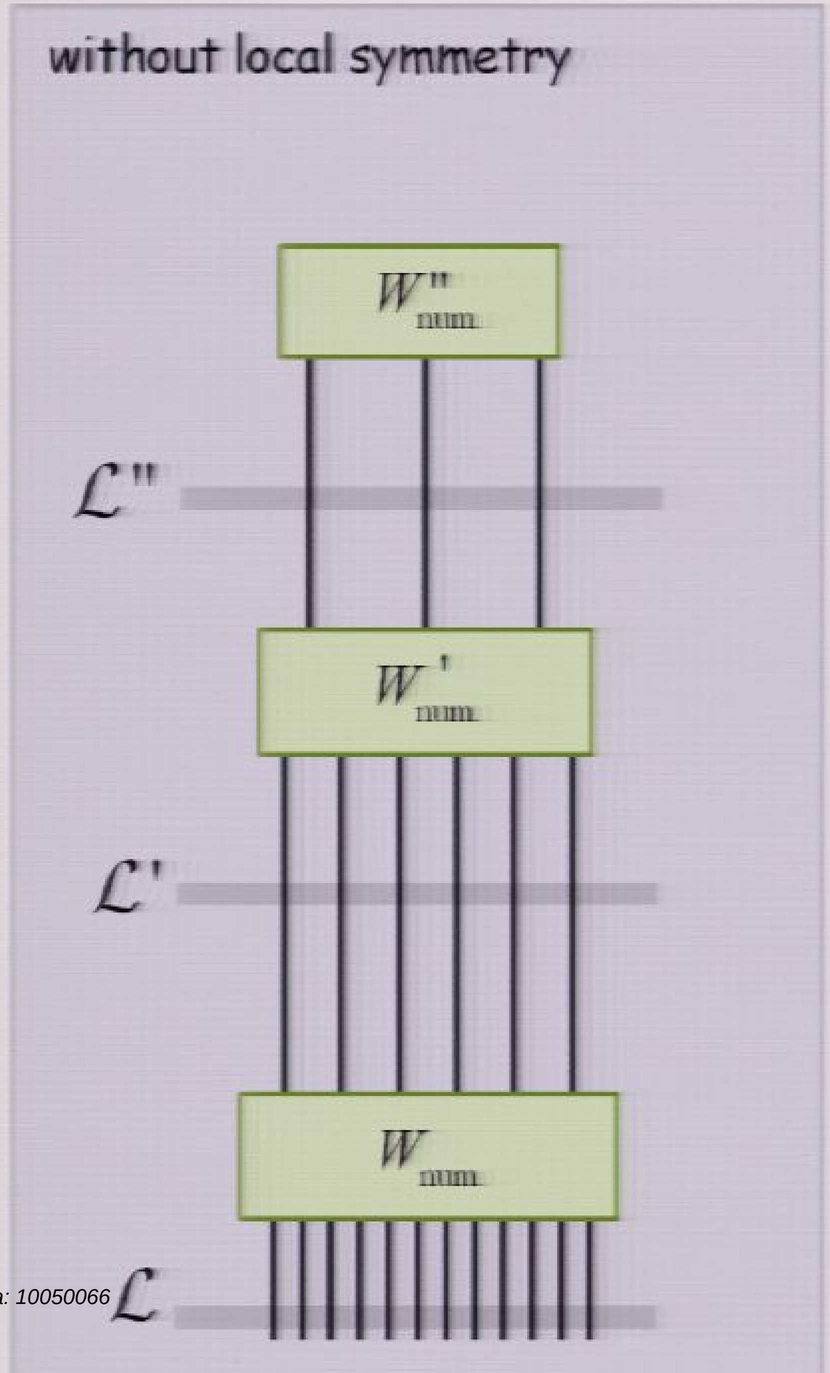


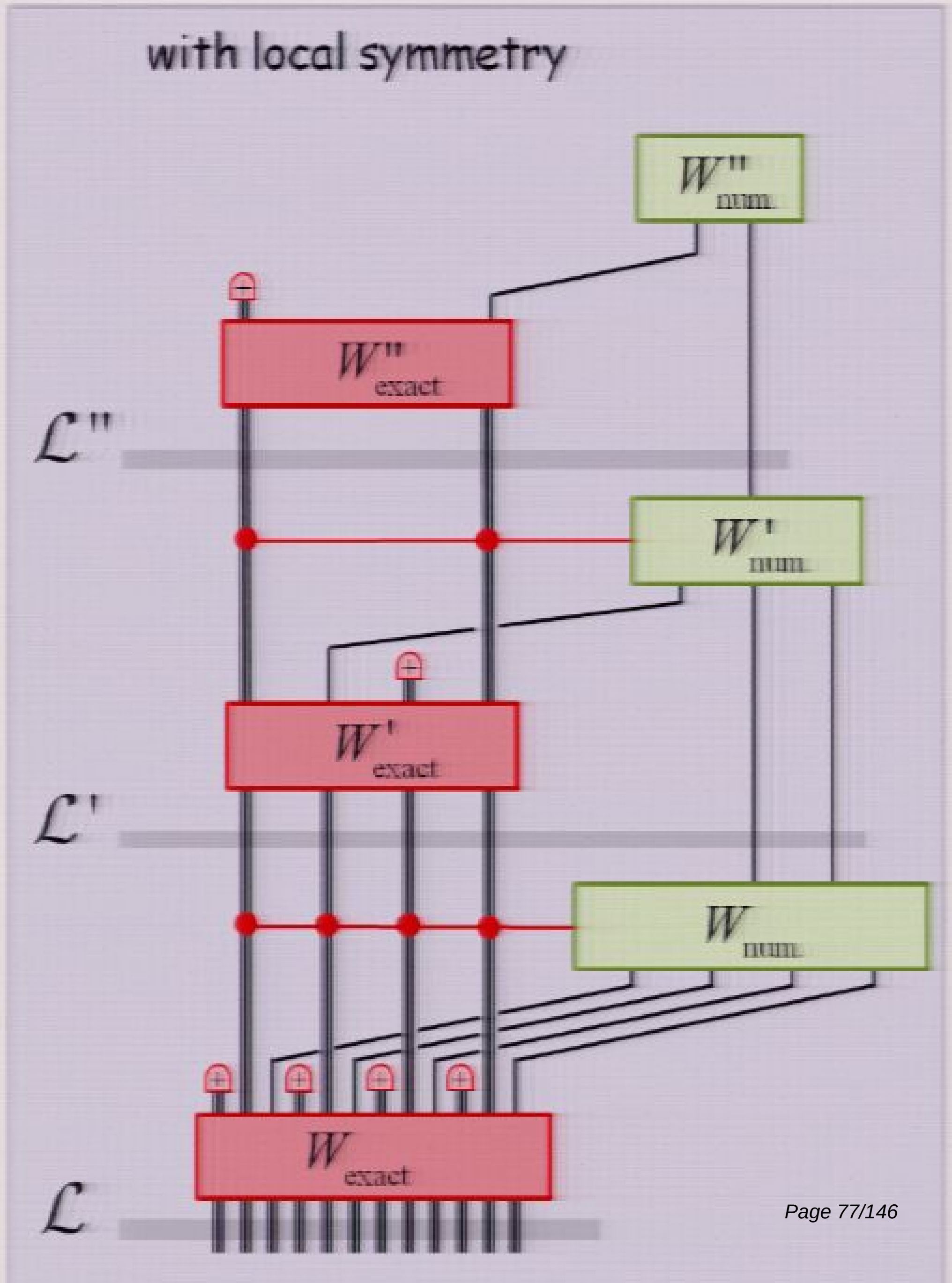


Coarse-graining scheme

- Exact transformation W_{exact} sequence of CNOTs & projections on states $|+\rangle$

Coarse-graining scheme





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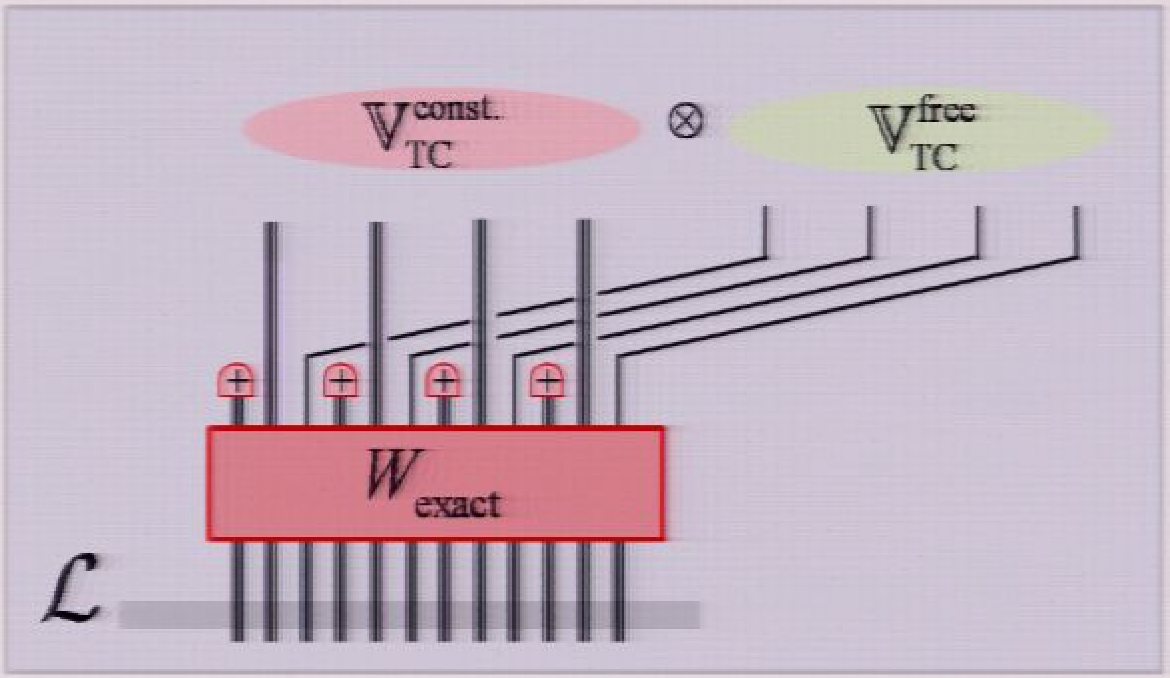

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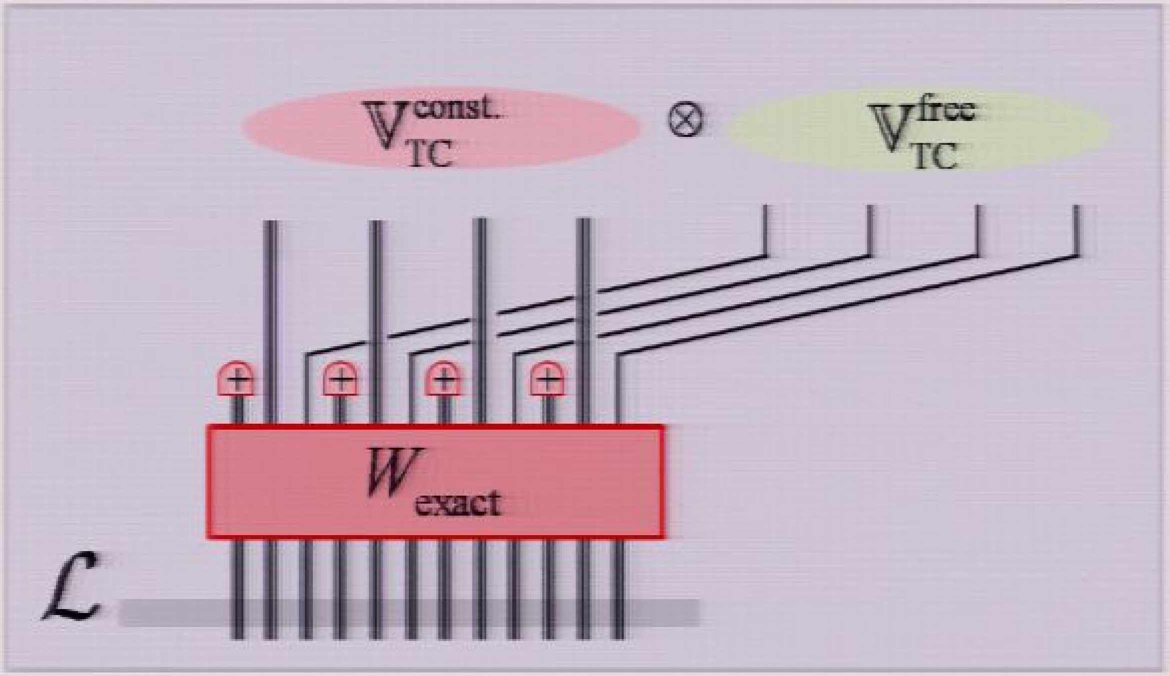


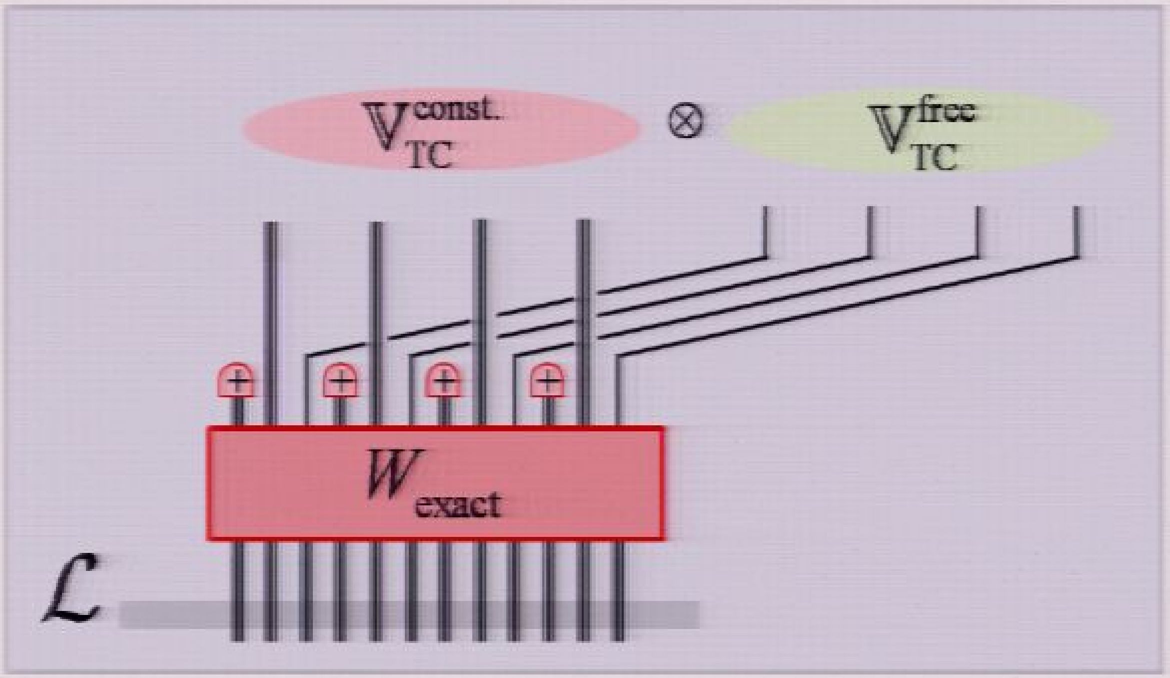
$$\mathbb{V}_{\text{TC}} \cong (\mathbb{C}_2)^{\otimes 2L^2}$$

$\Rightarrow$ $\mathbb{V}_{\text{TC}}^{\text{constrained}} \otimes \mathbb{V}_{\text{TC}}^{\text{product}} \otimes \mathbb{V}_{\text{TC}}^{\text{free}}$

CNOTs

Coarse-graining scheme

- Exact transformation W_{exact} sequence of CNOTs & projections on states $|+\rangle$



$\mathbb{V}_{TC} \cong (\mathbb{C}_2)^{\otimes 2L^2}$

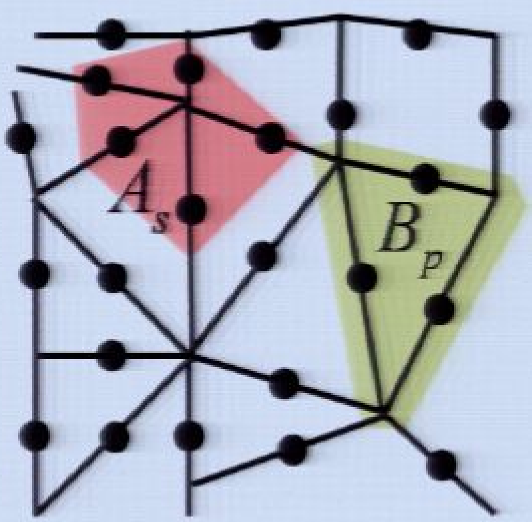
CNOTs ➡ $\mathbb{V}_{TC}^{constrained} \otimes \mathbb{V}_{TC}^{product} \otimes \mathbb{V}_{TC}^{free}$

$|+\rangle$ projections ➡ $\mathbb{V}_{TC}^{constrained} \otimes \mathbb{V}_{TC}^{free}$

Coarse-graining scheme

- Exact transformation W_{exact} sequence of CNOTs & projections on states $|+\rangle$

toric code on irregular lattice



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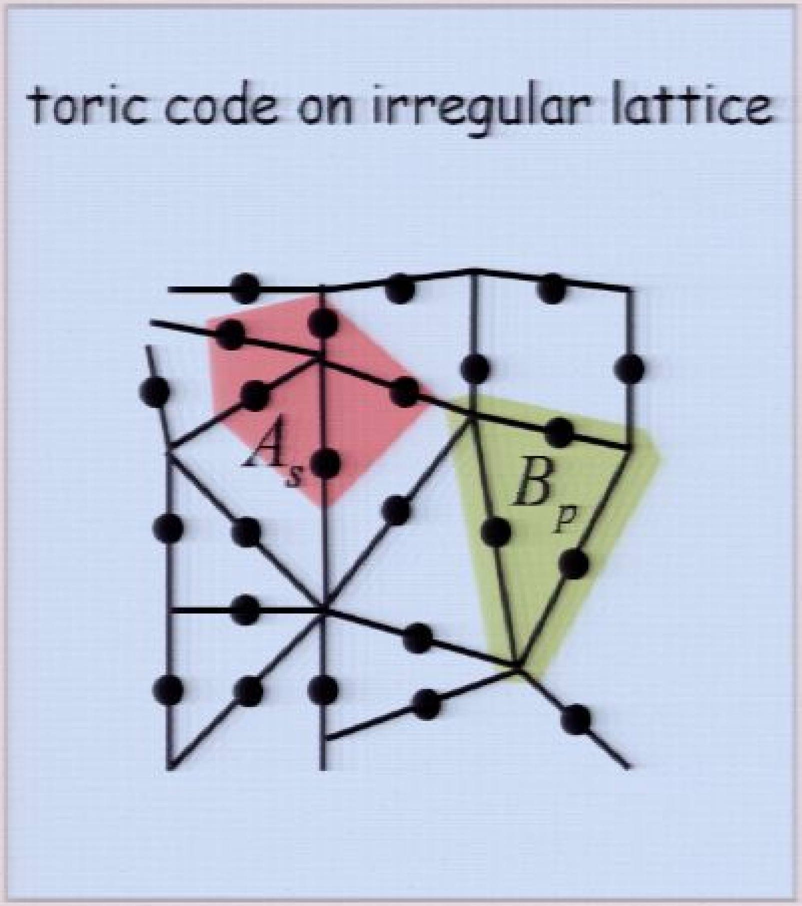

stabilizer formalism and basic moves

D. Gottesman, Ph.D. thesis, 1997

E. Dennis et al., J. Math. Phys. 43, 4452-4505 (2002)

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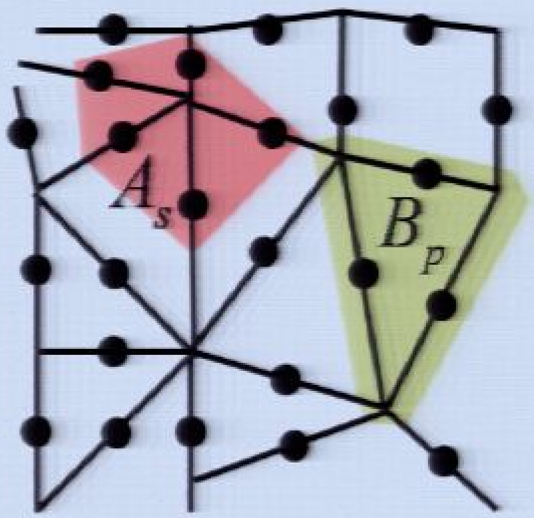

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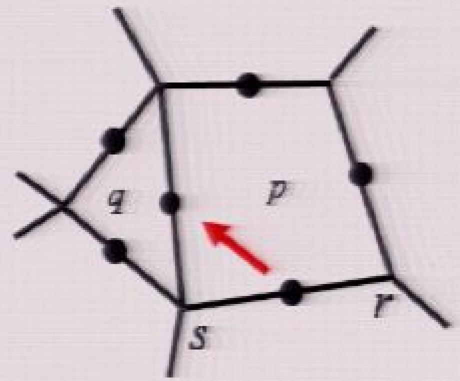

↑ = CNOT

Coarse-graining scheme

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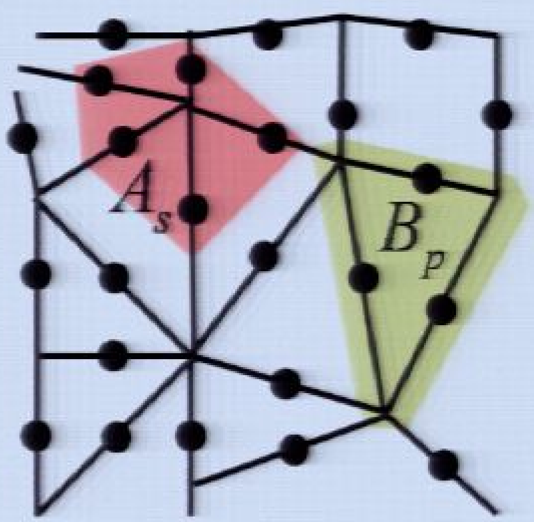

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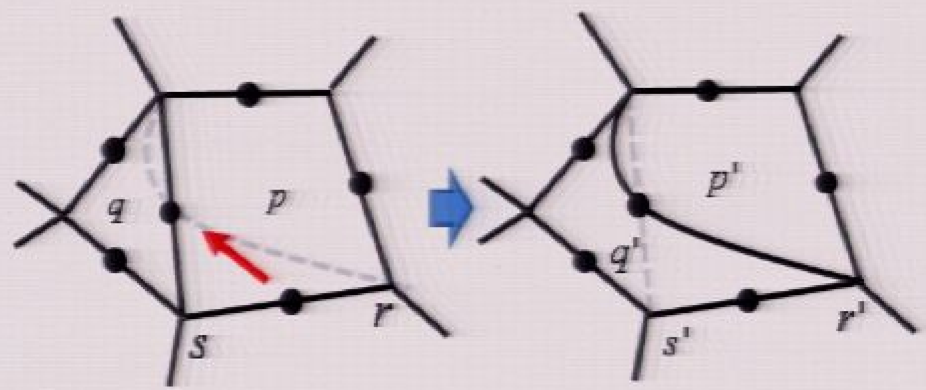

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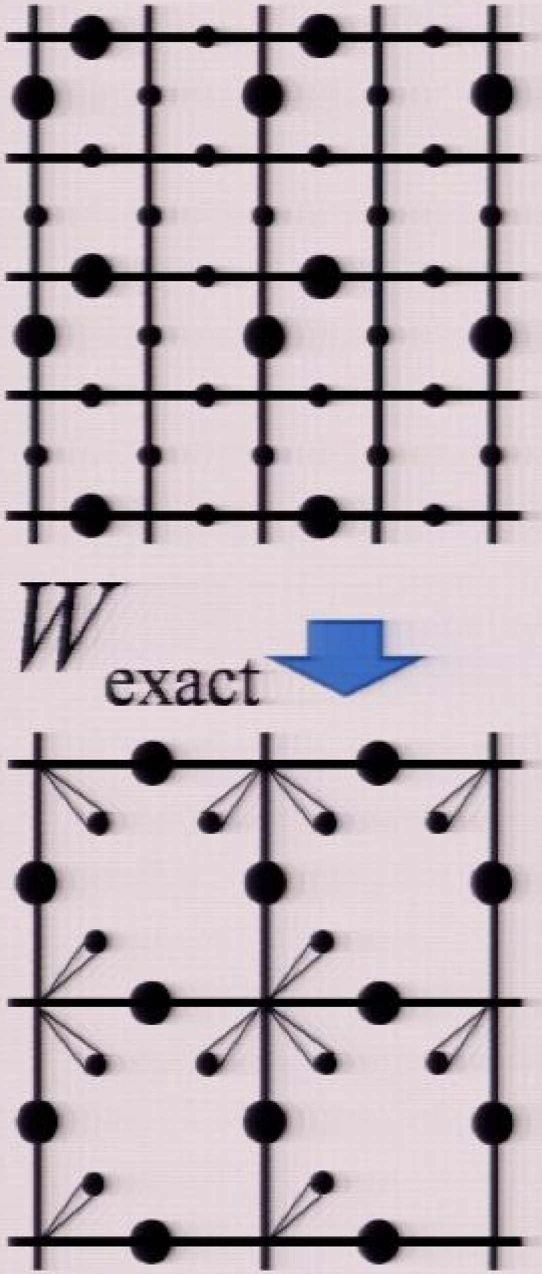

sequence of CNOTs
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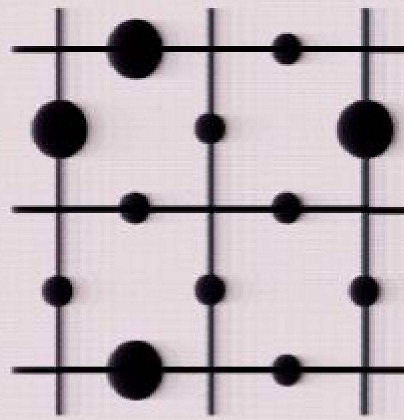

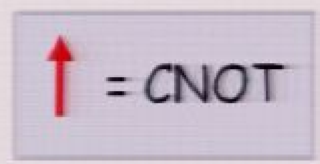

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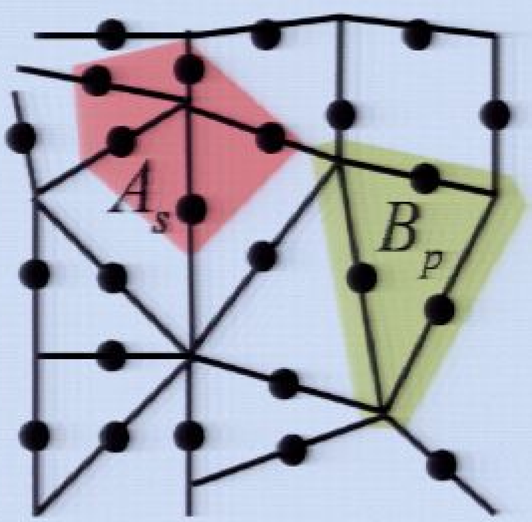

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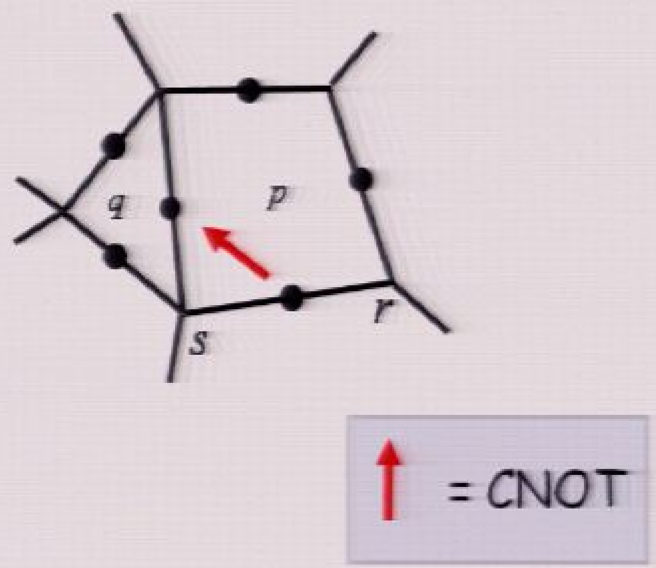

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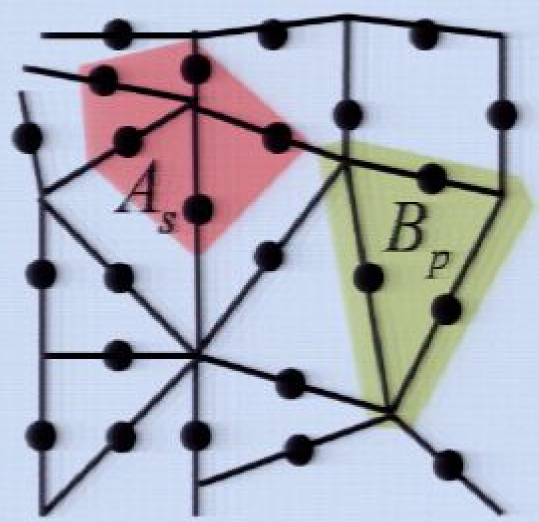

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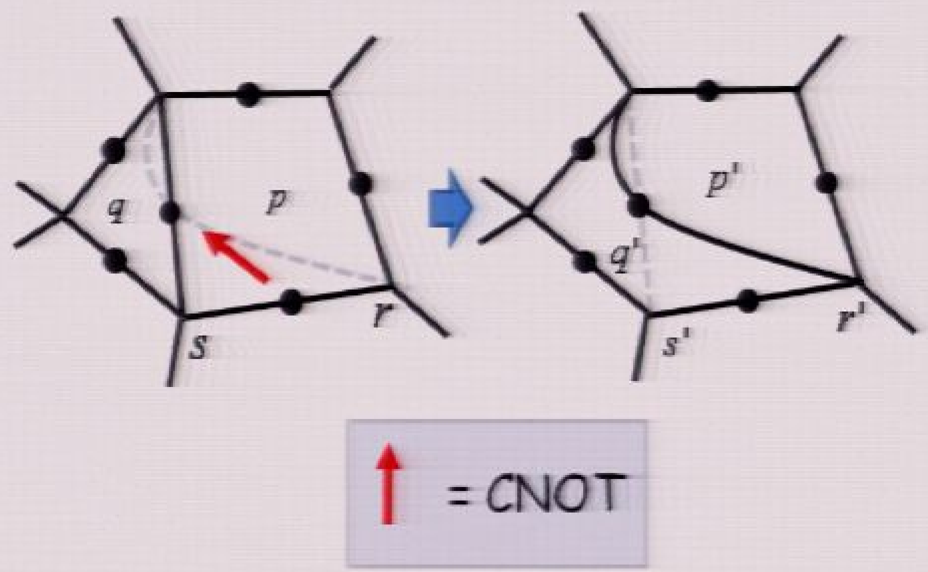

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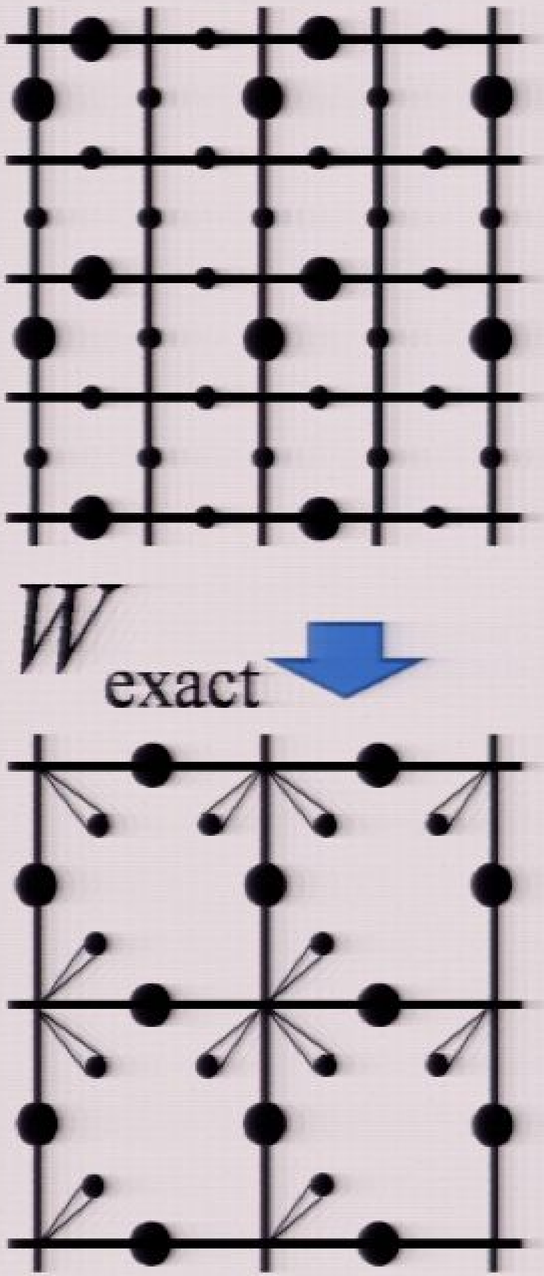

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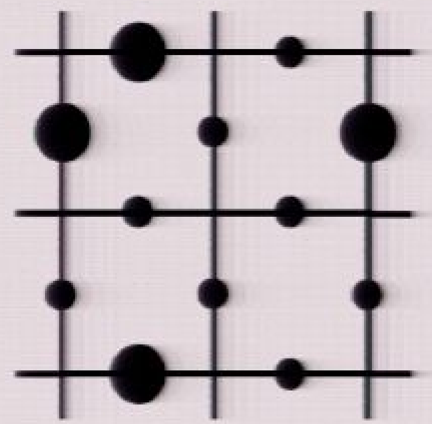

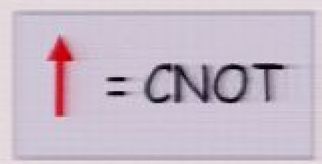

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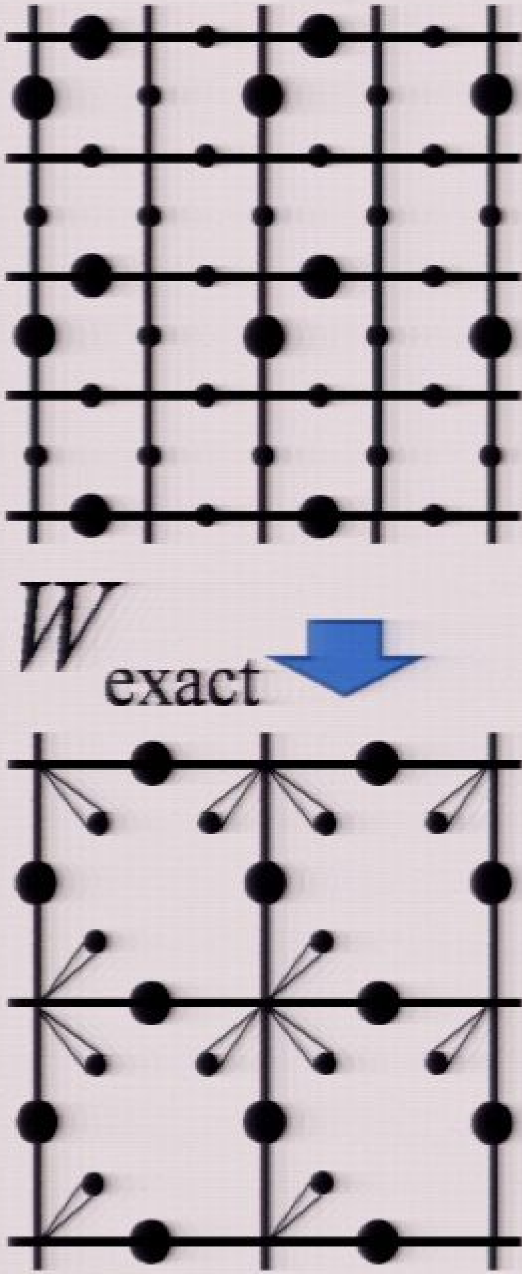

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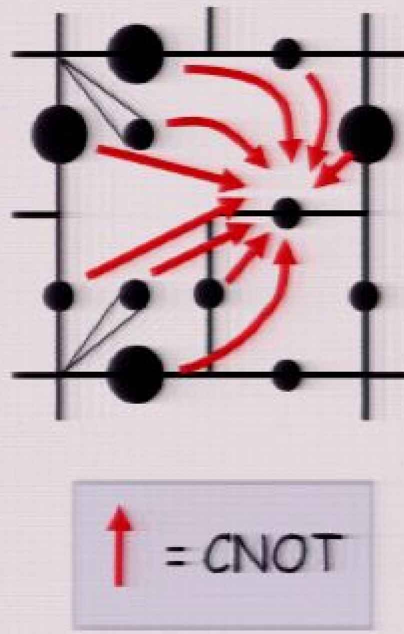

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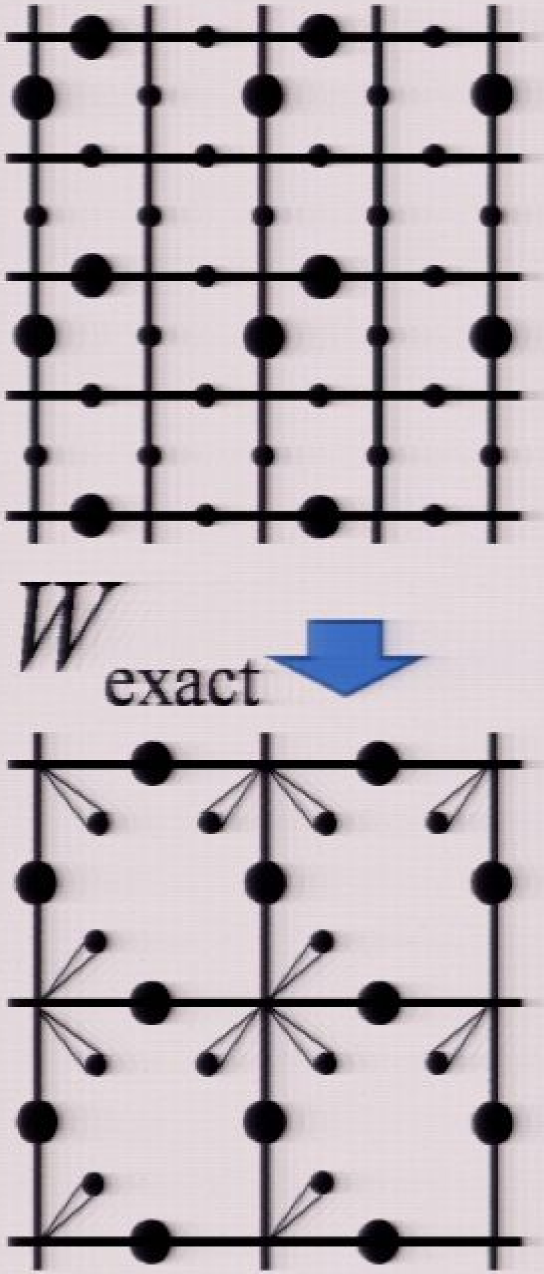

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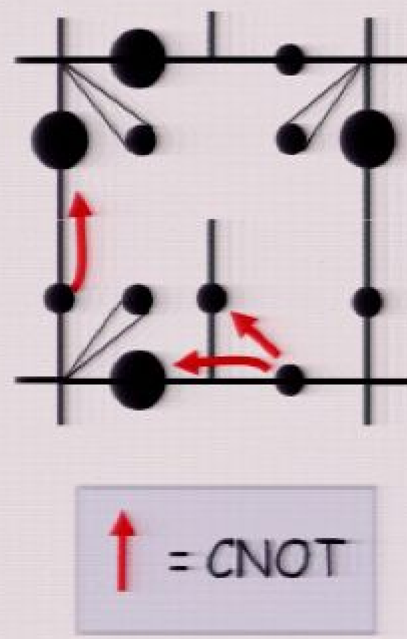

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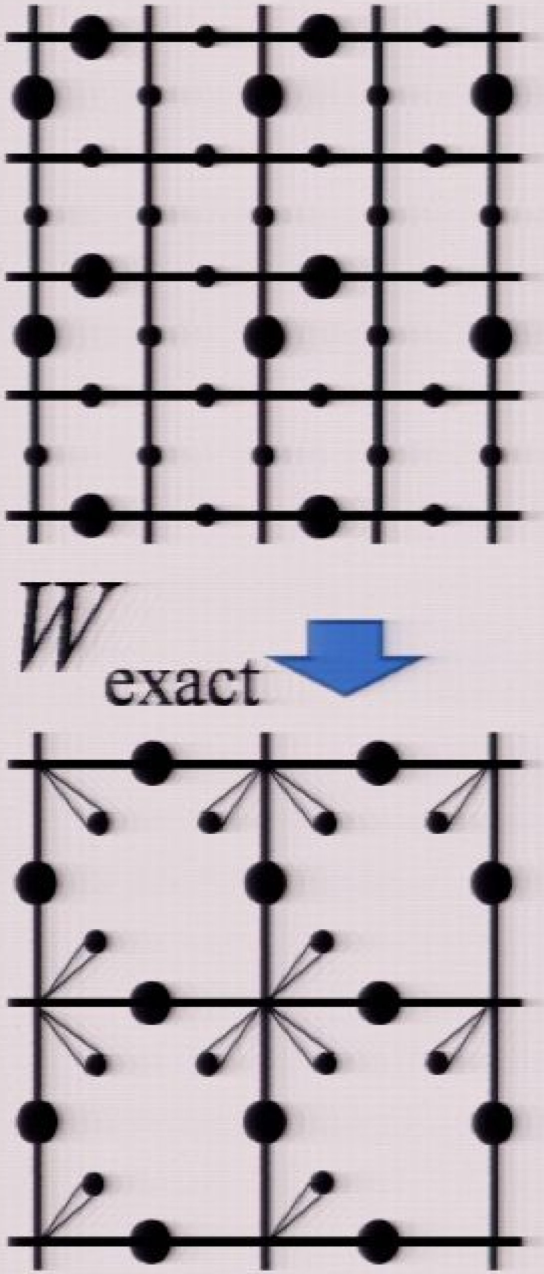

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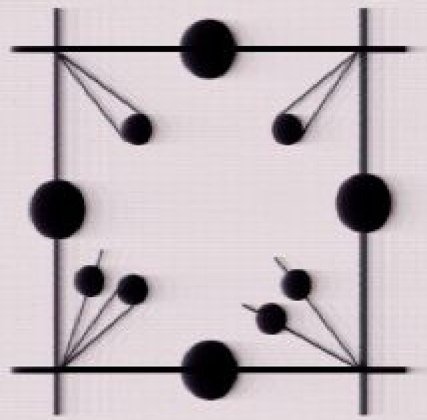

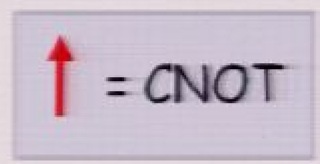

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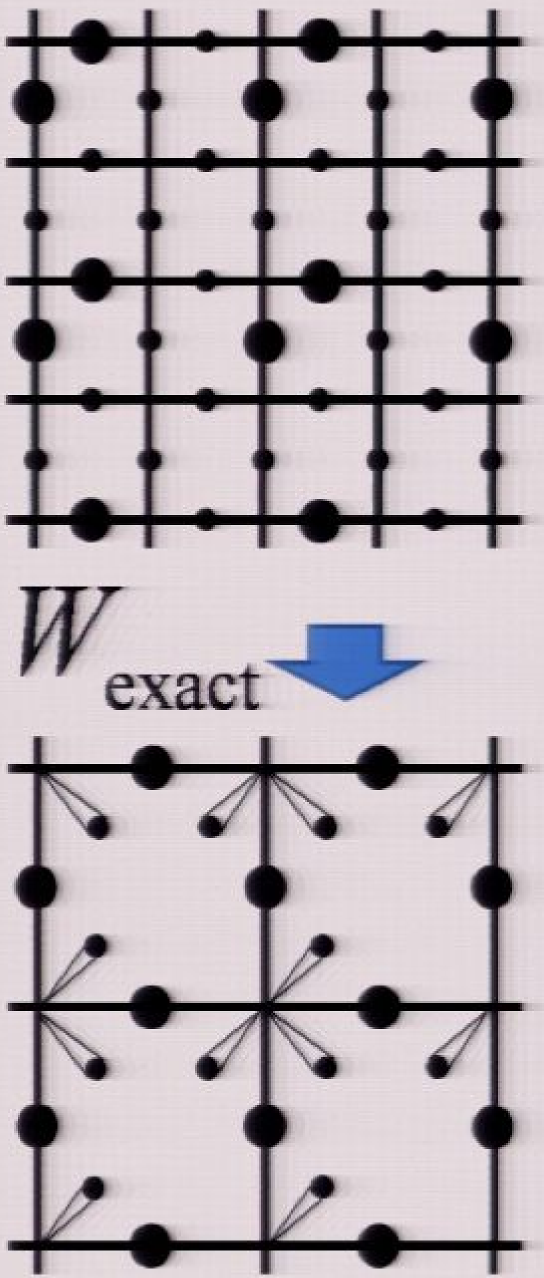

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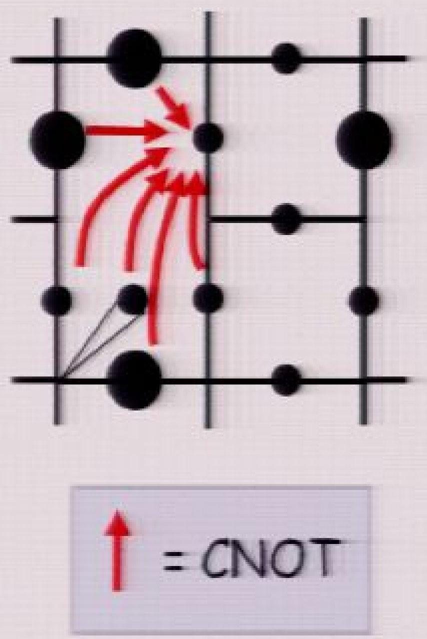

Coarse-graining scheme

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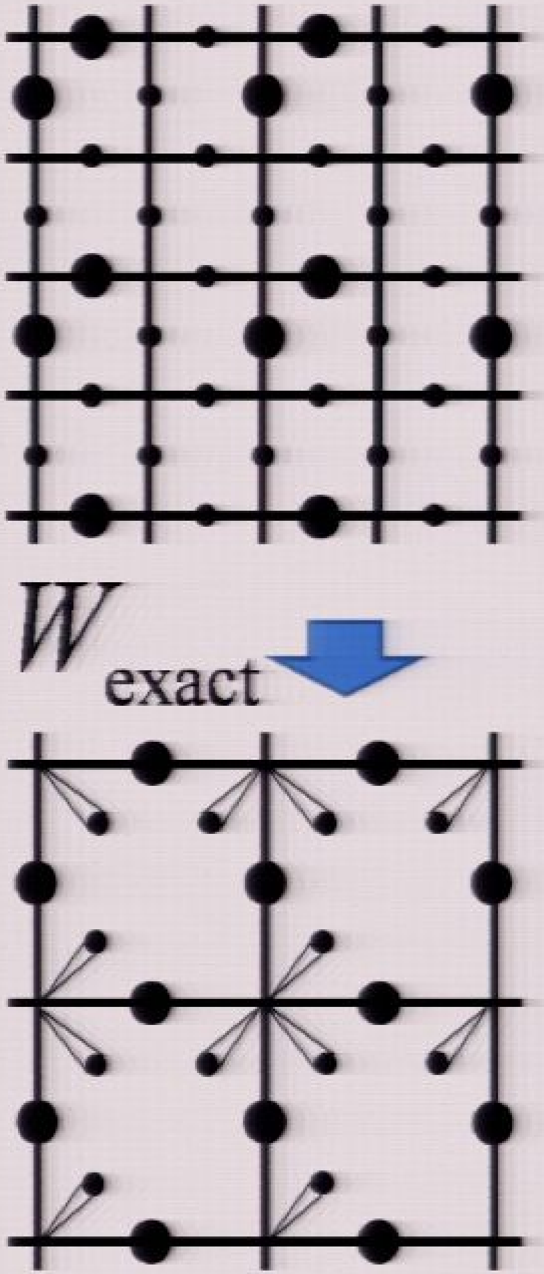

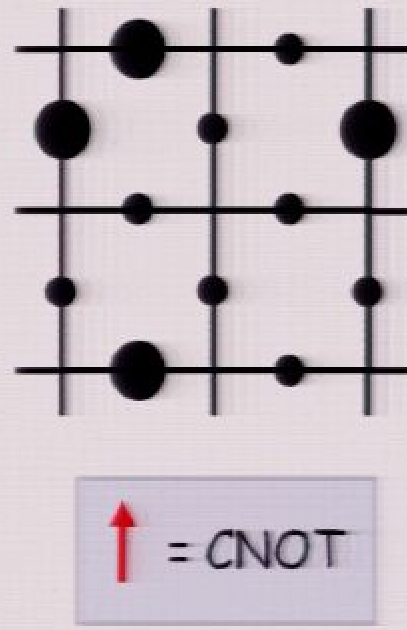

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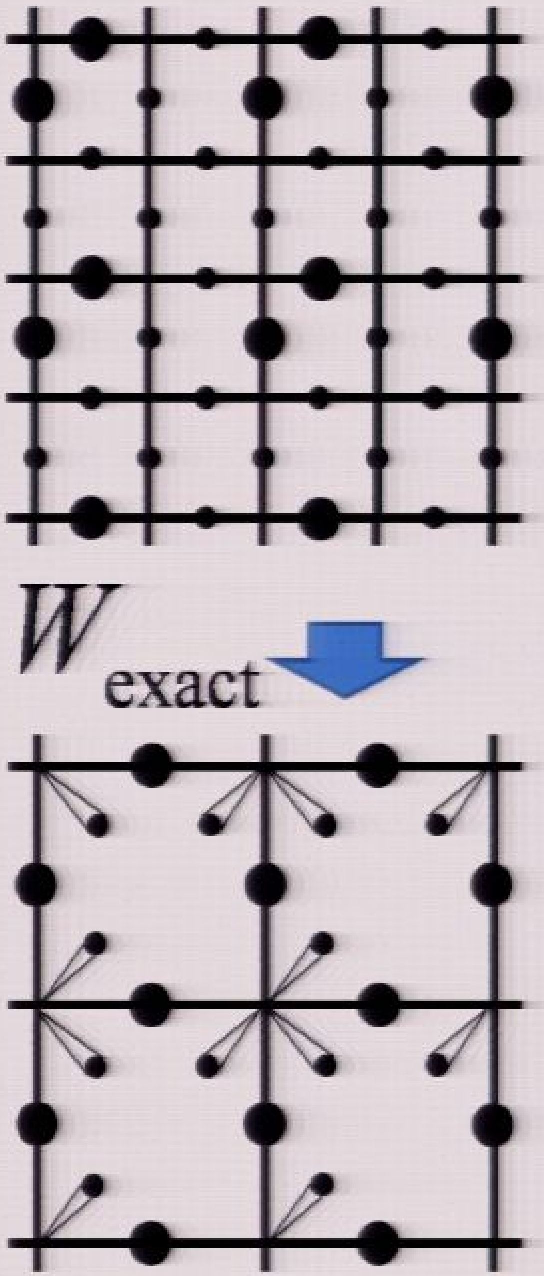

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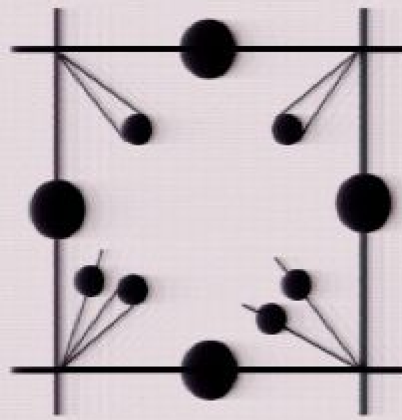

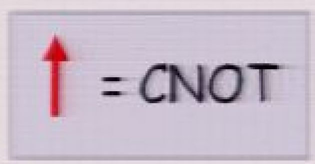

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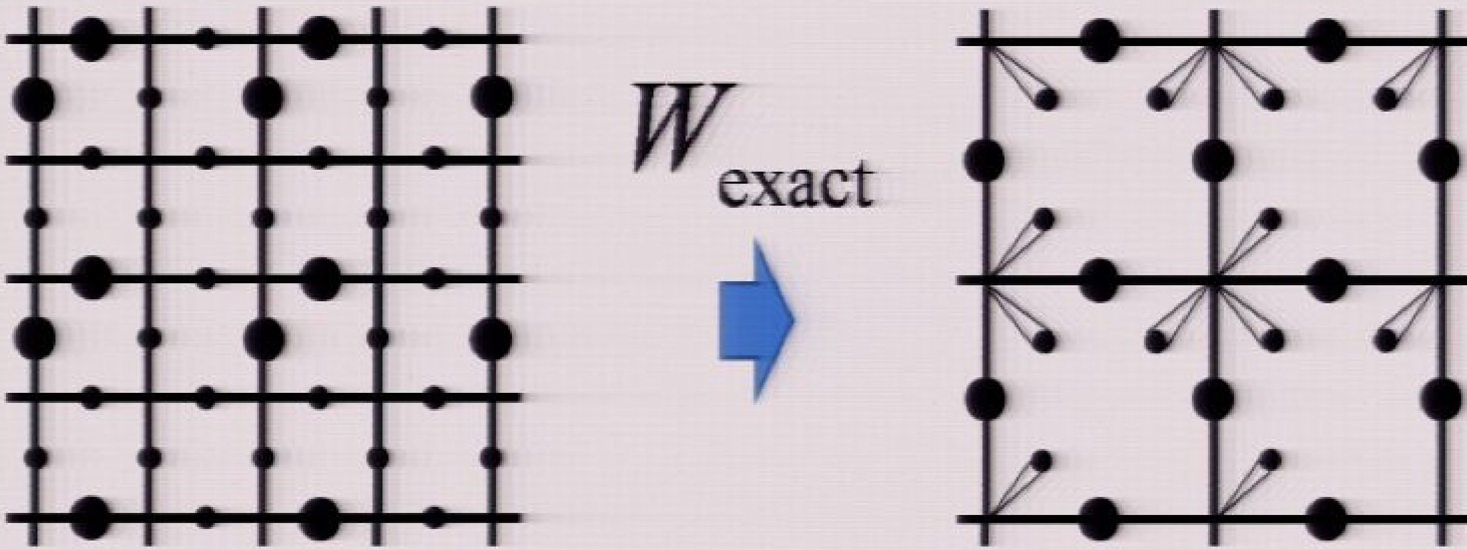

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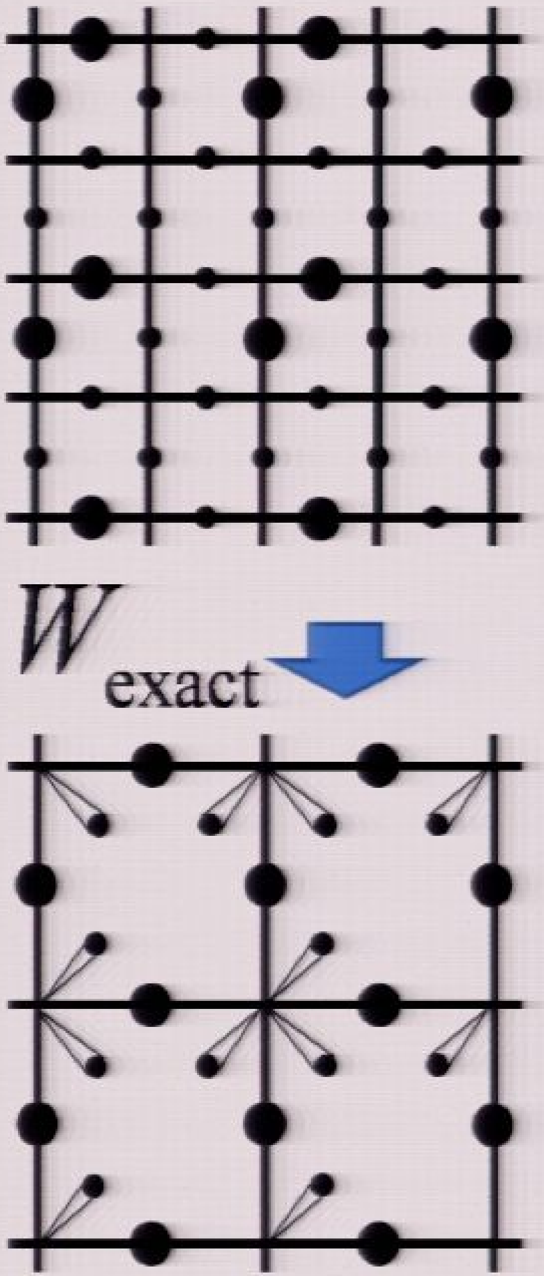

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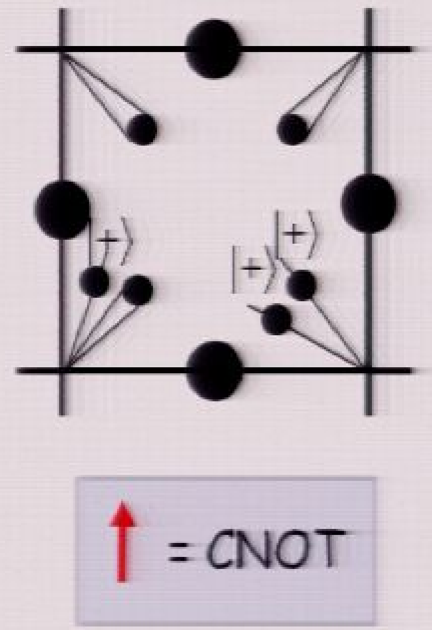

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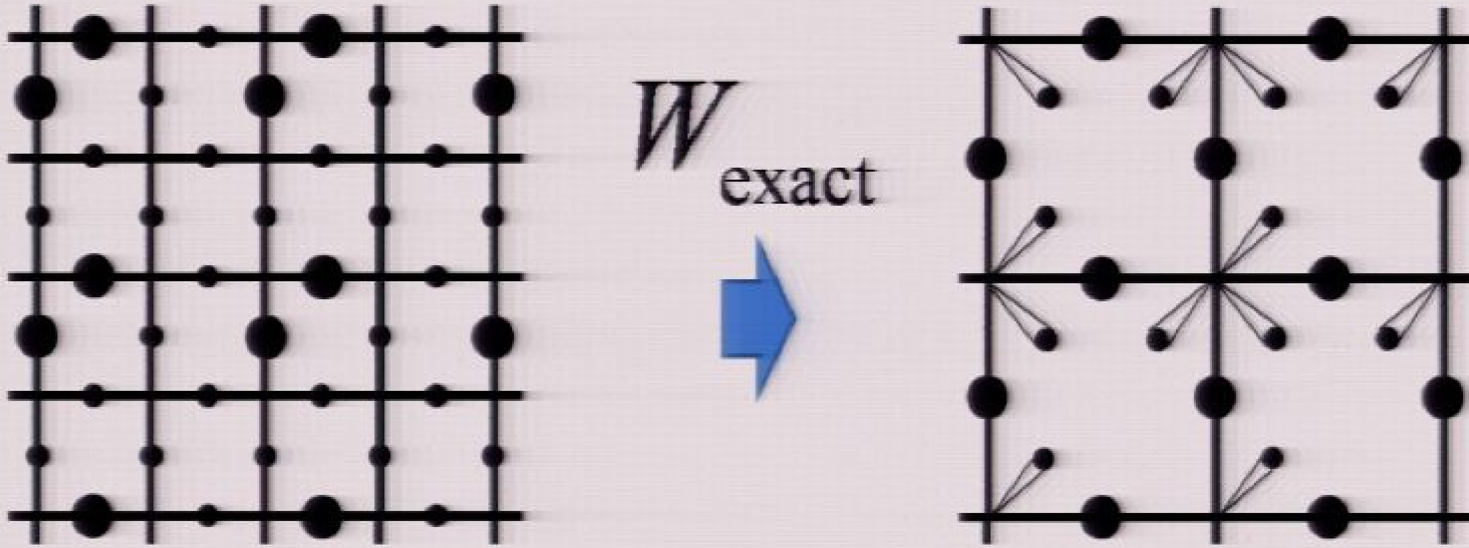

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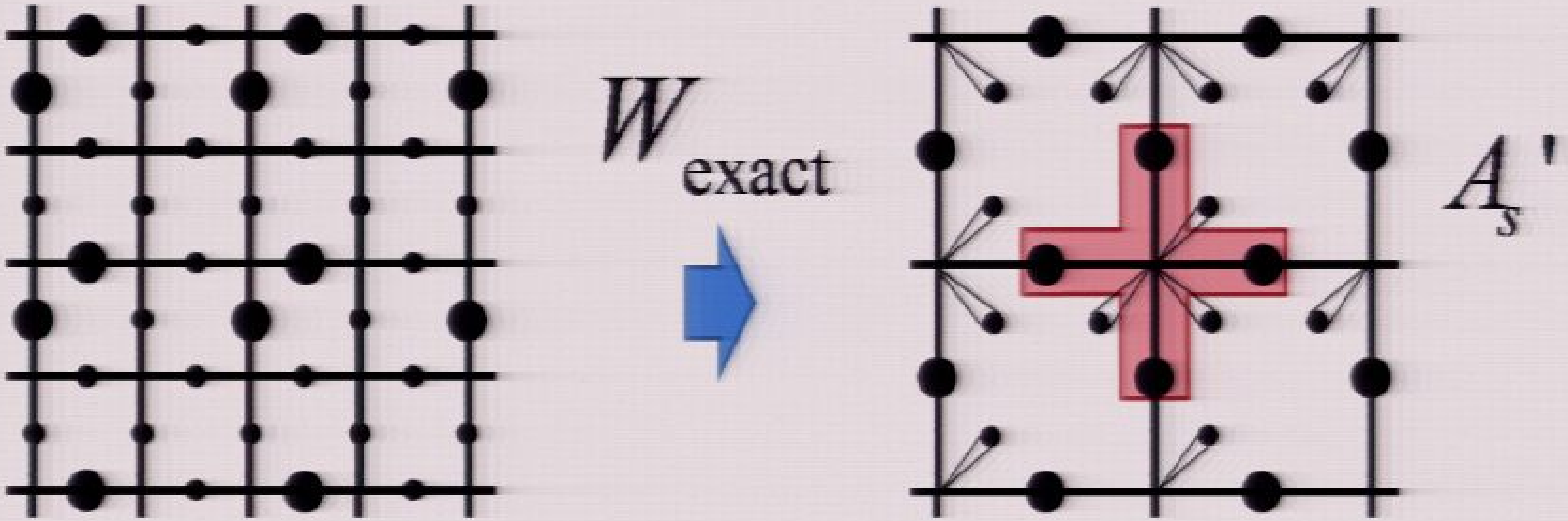

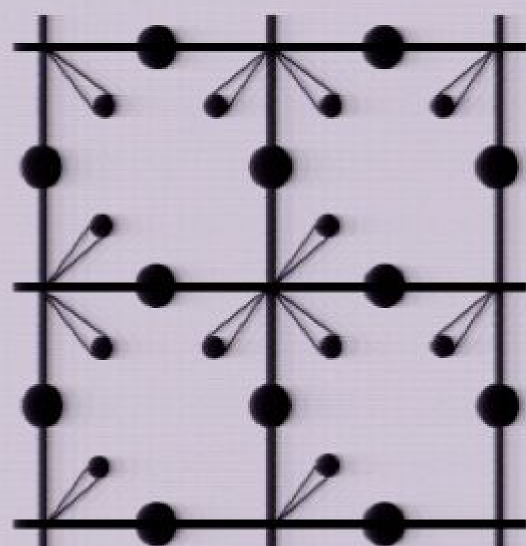

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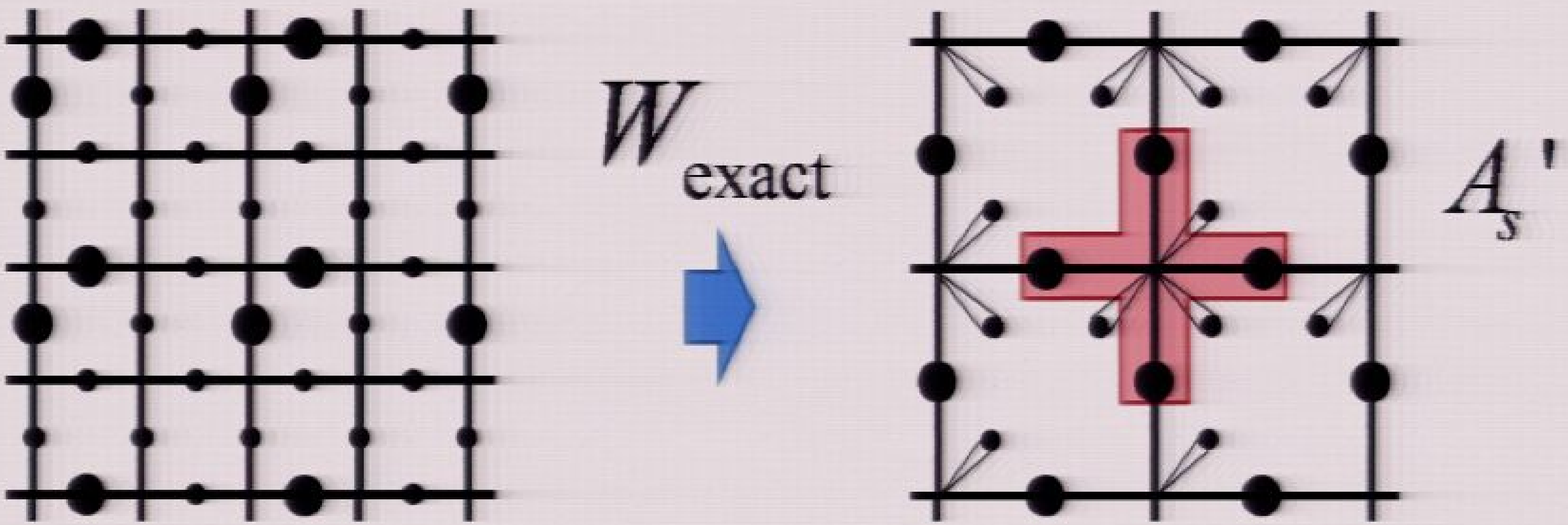

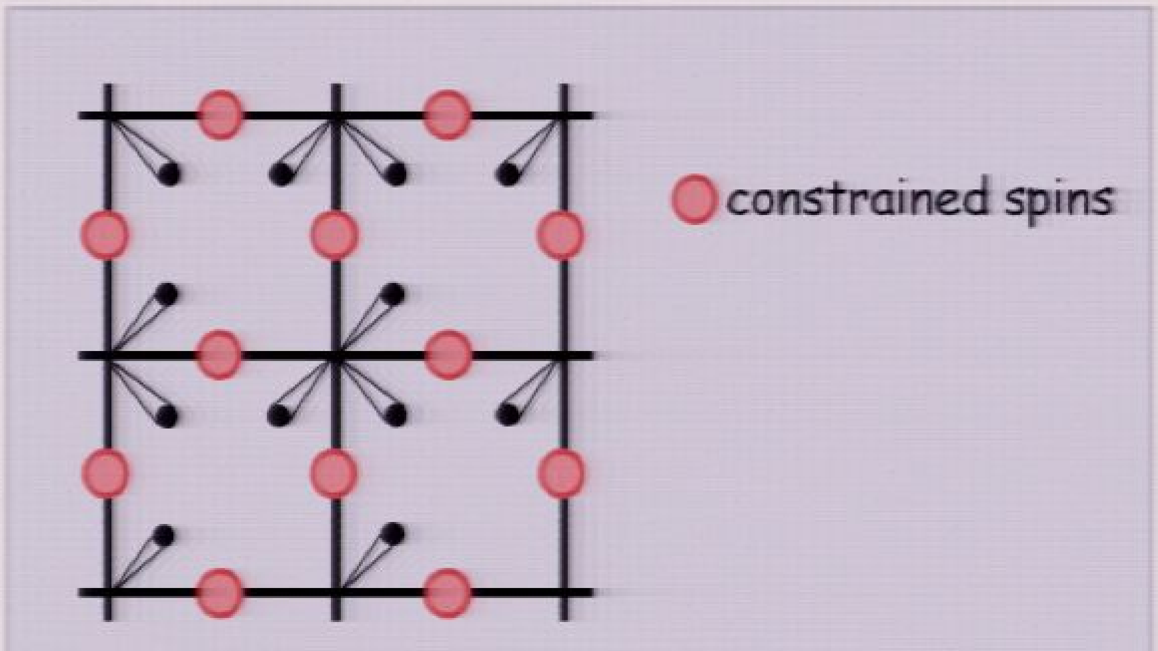

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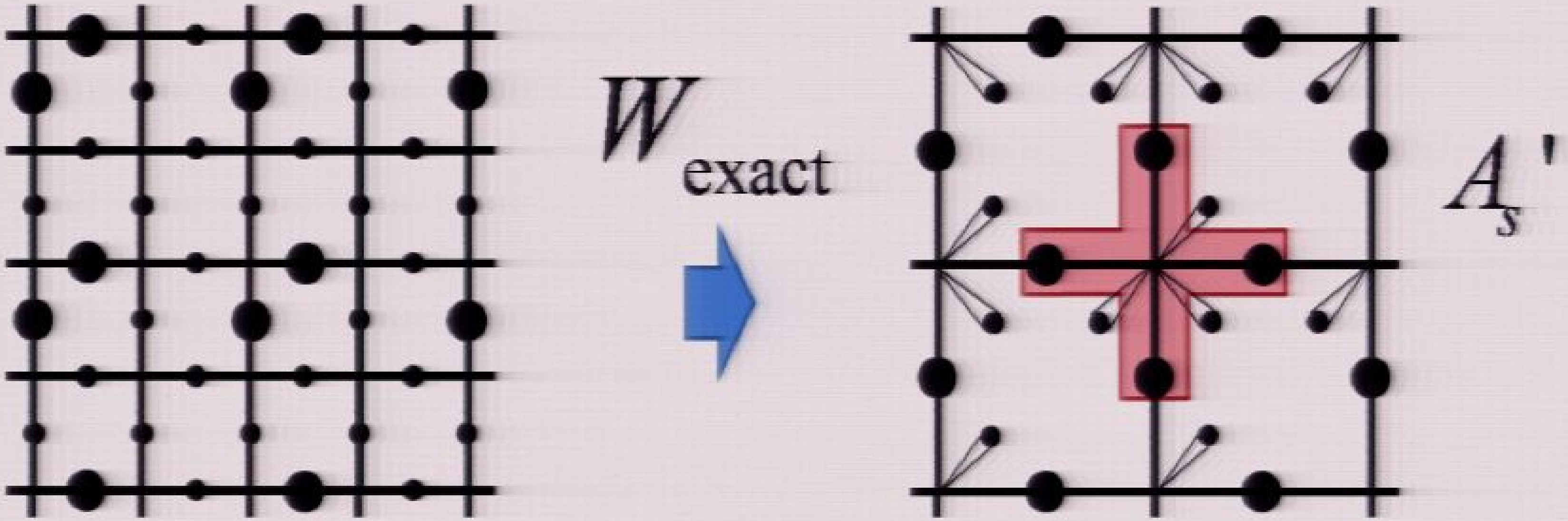

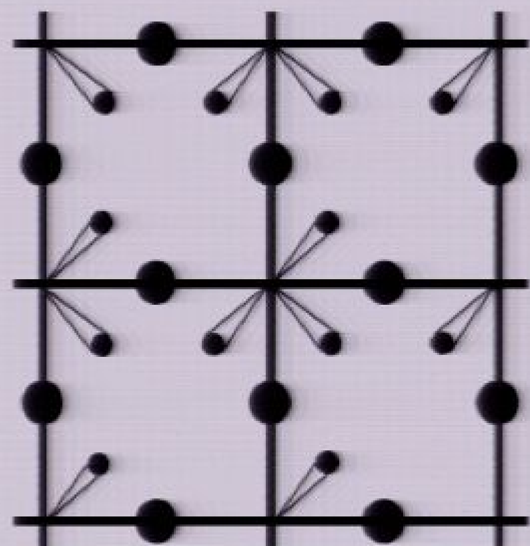

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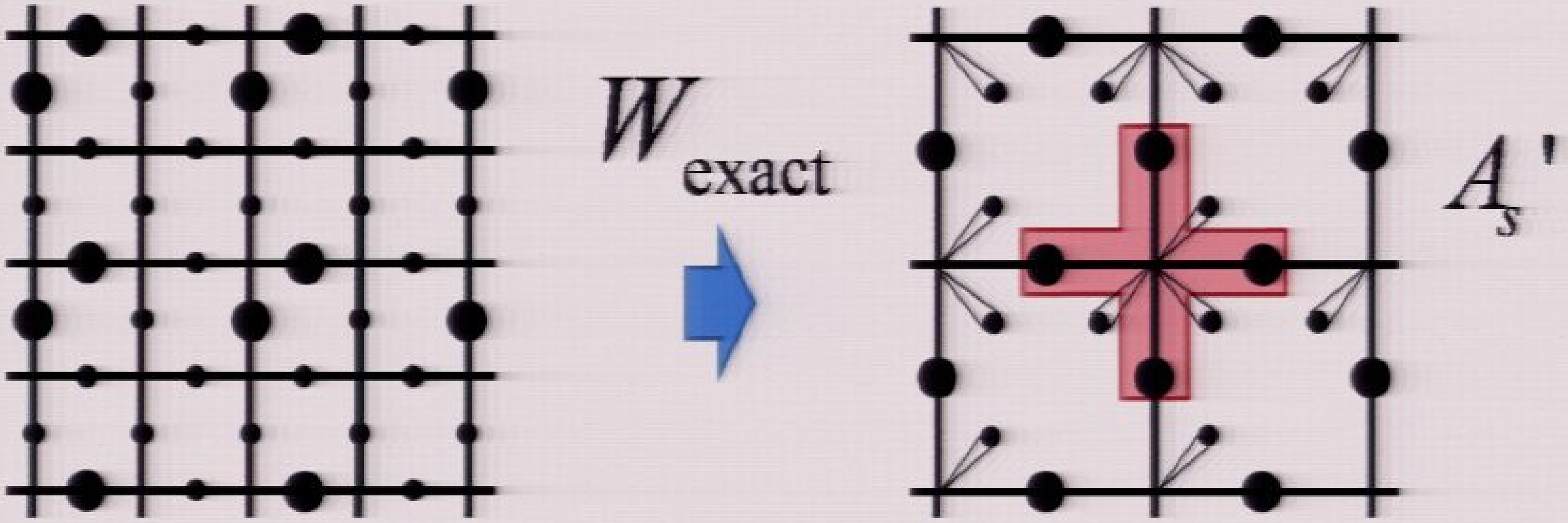
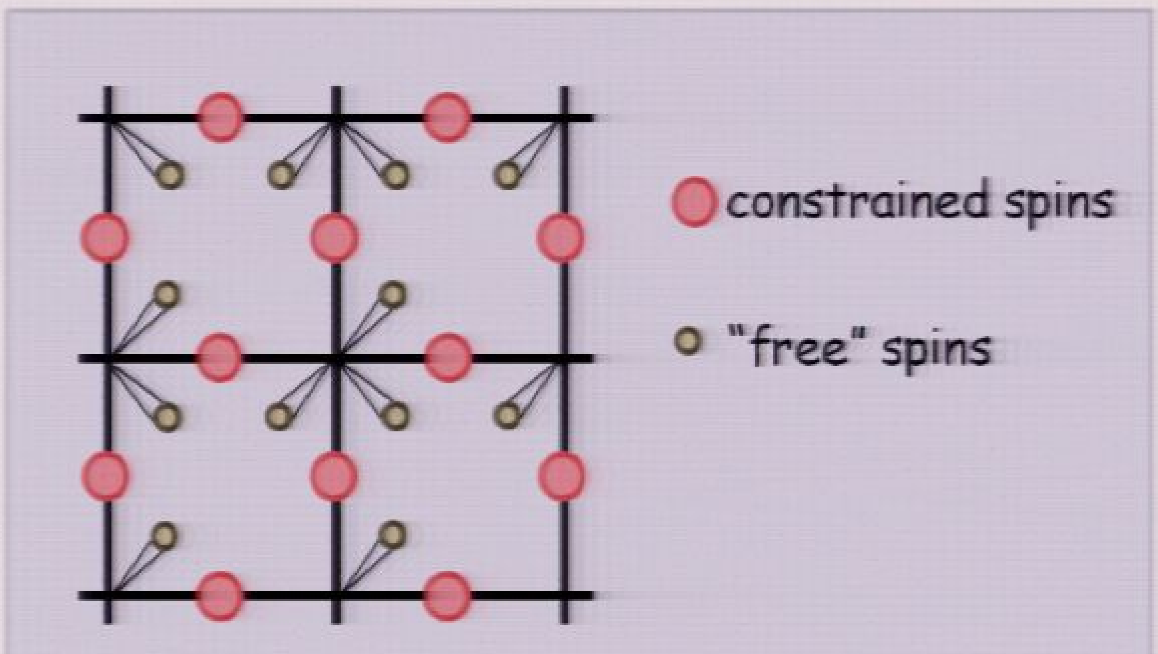




Coarse-graining scheme

$$H_{\mathrm{TC}}^{\mathrm{x}} = -J_e \sum_s A_s - J_m \sum_p B_p - h_x \sum_j \sigma_j^x$$

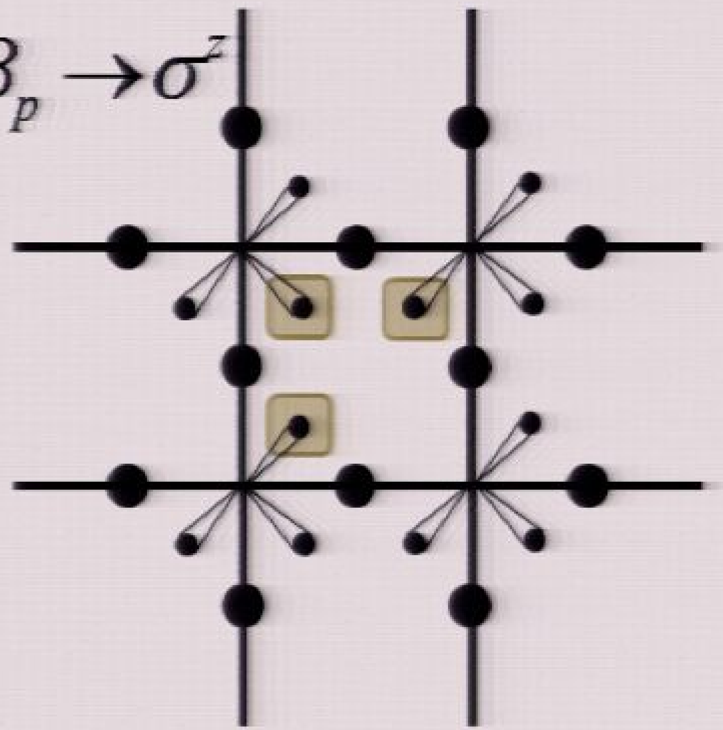
- How do plaquette operators transform under W_{exact} ?

Coarse-graining scheme

$$H_{TC}^{x} = -J_e \sum_s A_s - J_m \sum_p B_p - h_x \sum_j \sigma_j^x$$

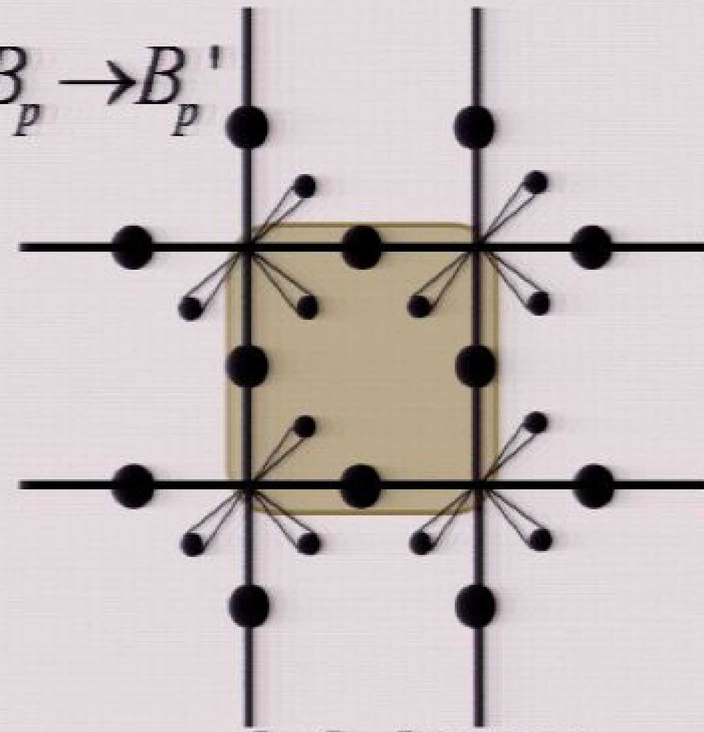
- How do plaquette operators transform under W_{exact}?

$B_p \rightarrow \sigma^z$



σ^z magnetic field

$B_p \rightarrow B_p'$

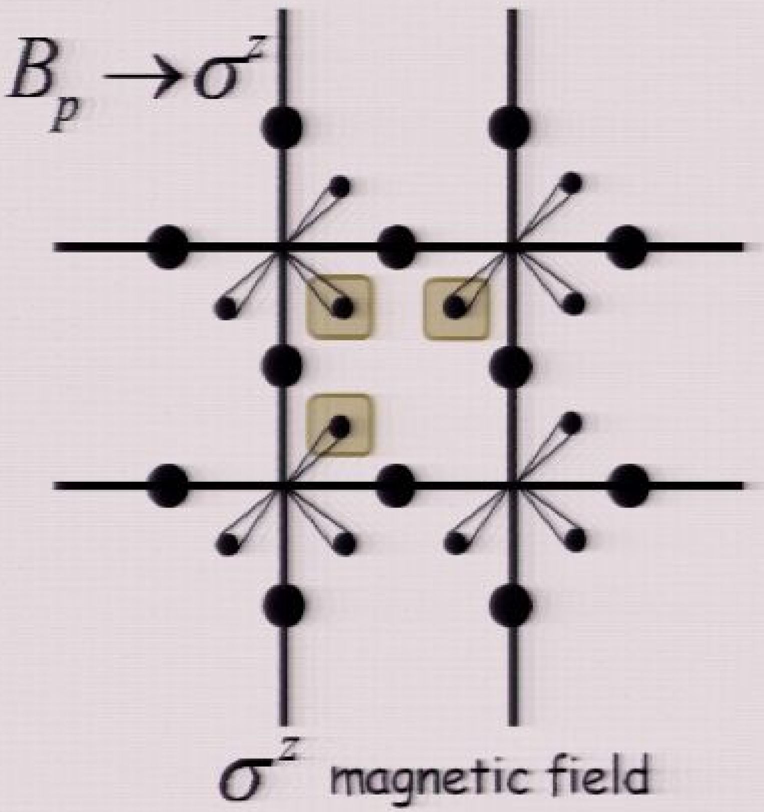


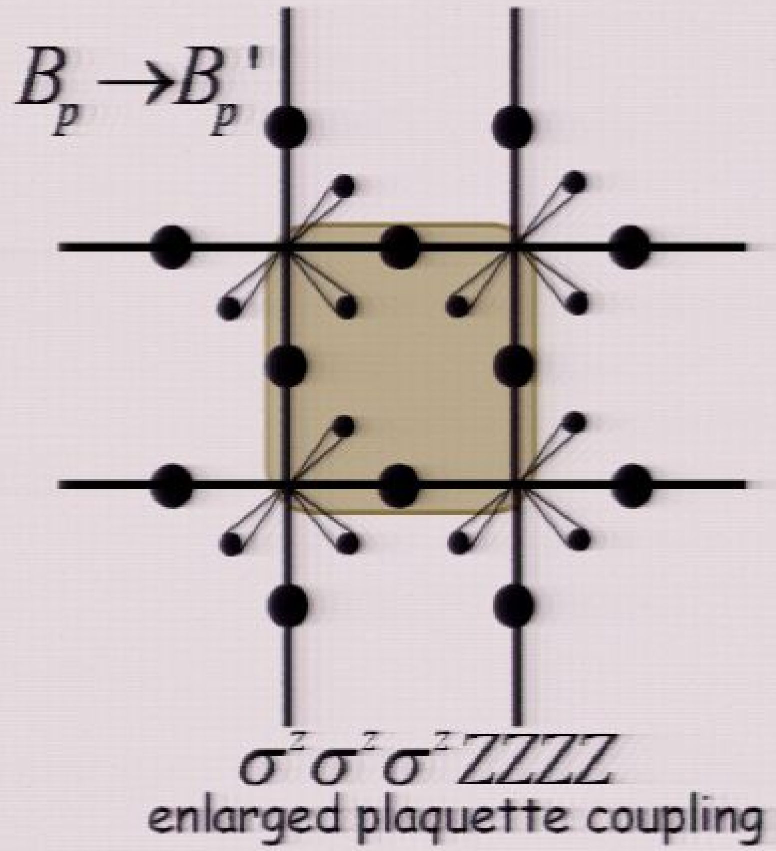
$\sigma^z \sigma^z \sigma^z ZZZZ$
enlarged plaquette coupling

Coarse-graining scheme

$$H^x_{\mathrm{TC}} = -J_e \sum_s A_s - J_m \sum_p B_p - h_x \sum_j \sigma^x_j$$

- How do plaquette operators transform under W_{exact}?





local terms!

Coarse-graining scheme

$$H_{\mathrm{TC}}^{x} = -J_e \sum_s A_s - J_m \sum_p B_p - h_x \sum_j \sigma_j^x$$

- How does the magnetic field transform under W_{exact} ?

Coarse-graining scheme

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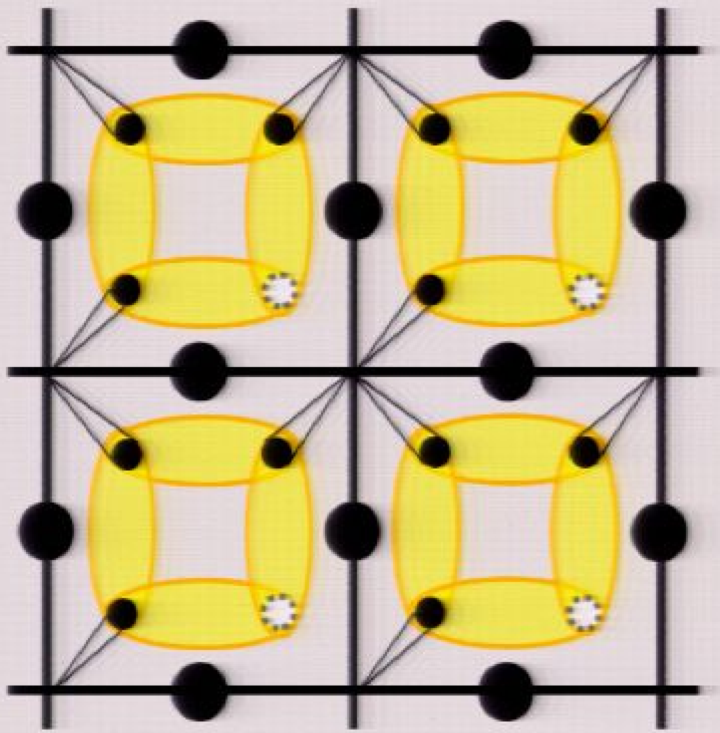

$\sigma^x \sigma^x$ Ising couplings

Coarse-graining scheme

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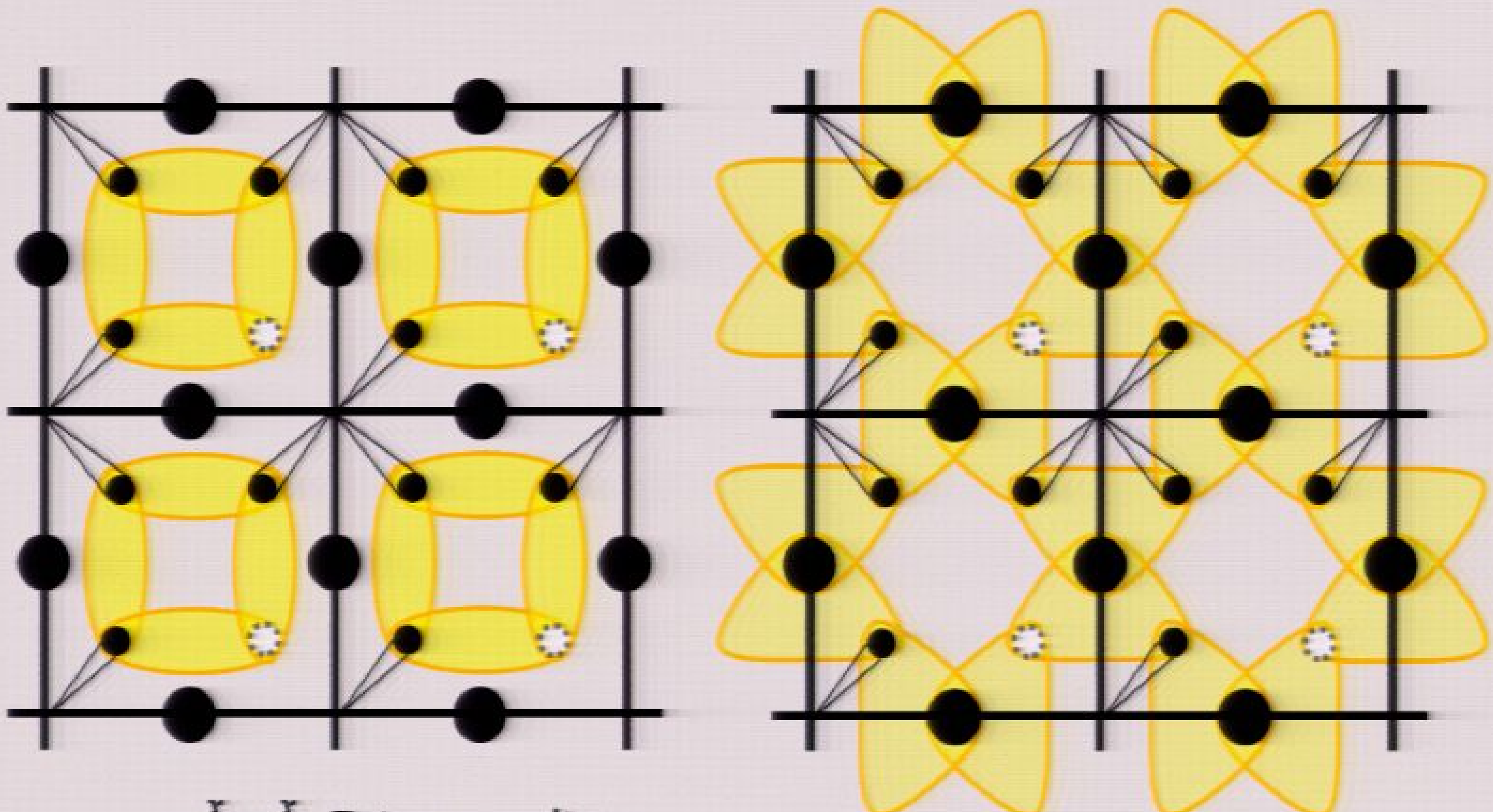
- How does the magnetic field transform under W_{exact} ?



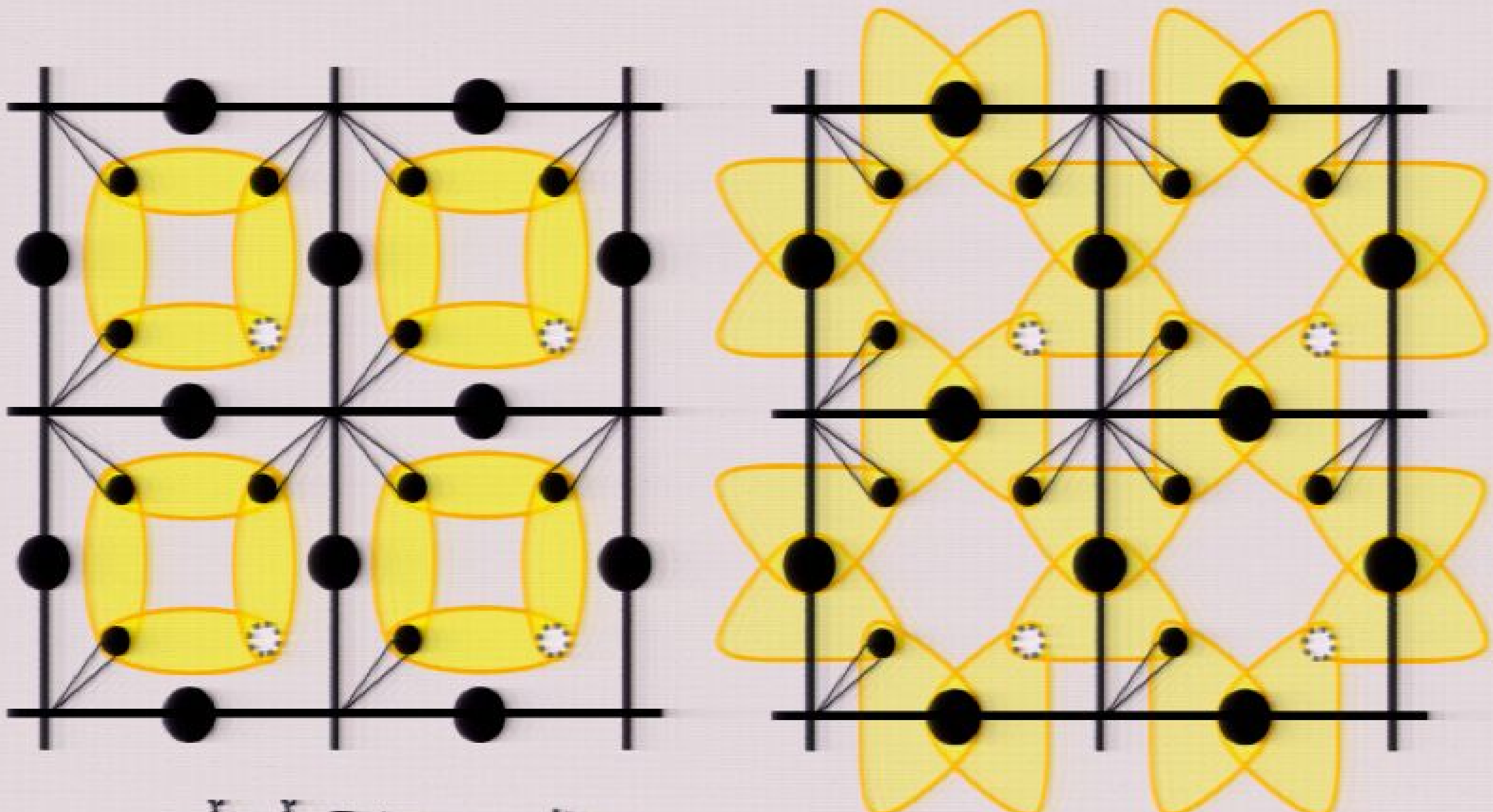
$\sigma^x \sigma^x$ Ising couplings

$\sigma^x X \sigma^x$ "gauge-matter" couplings

Coarse-graining scheme

$$H_{\mathrm{TC}}^{x} = -J_e \sum_s A_s - J_m \sum_p B_p - h_x \sum_j \sigma_j^x$$

- How does the magnetic field transform under W_{exact} ?



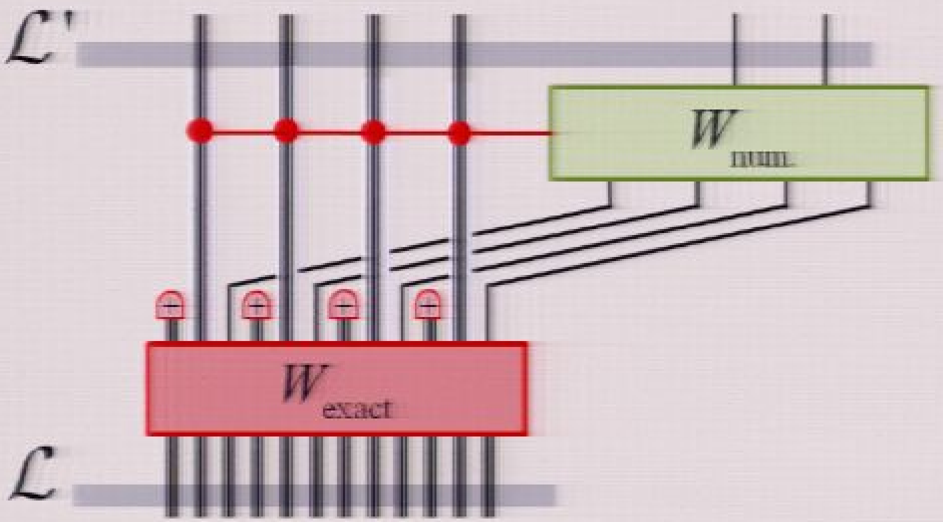
$\sigma^x \sigma^x$ Ising couplings

$\sigma^x X \sigma^x$ "gauge-matter" couplings

local terms!

Coarse-graining scheme

- Exact transformation W_{exact}
- Numerical transformation W_{num}



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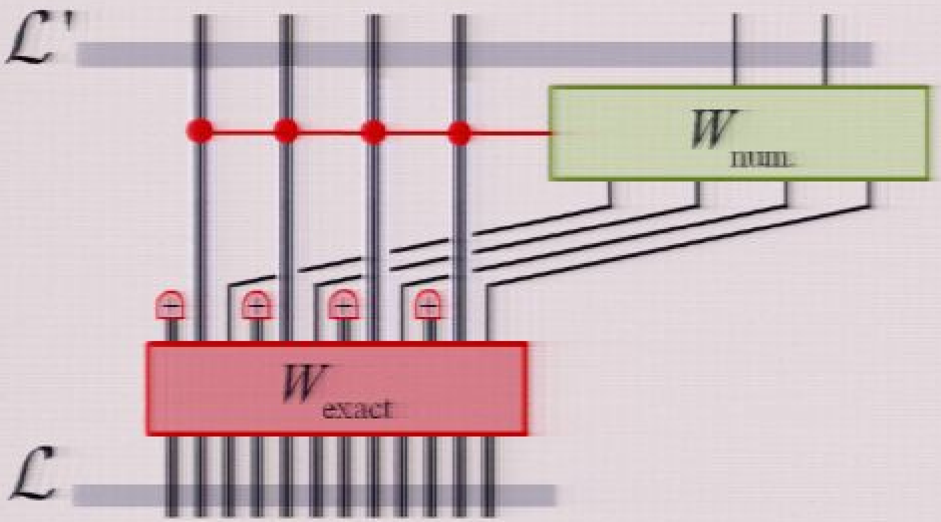
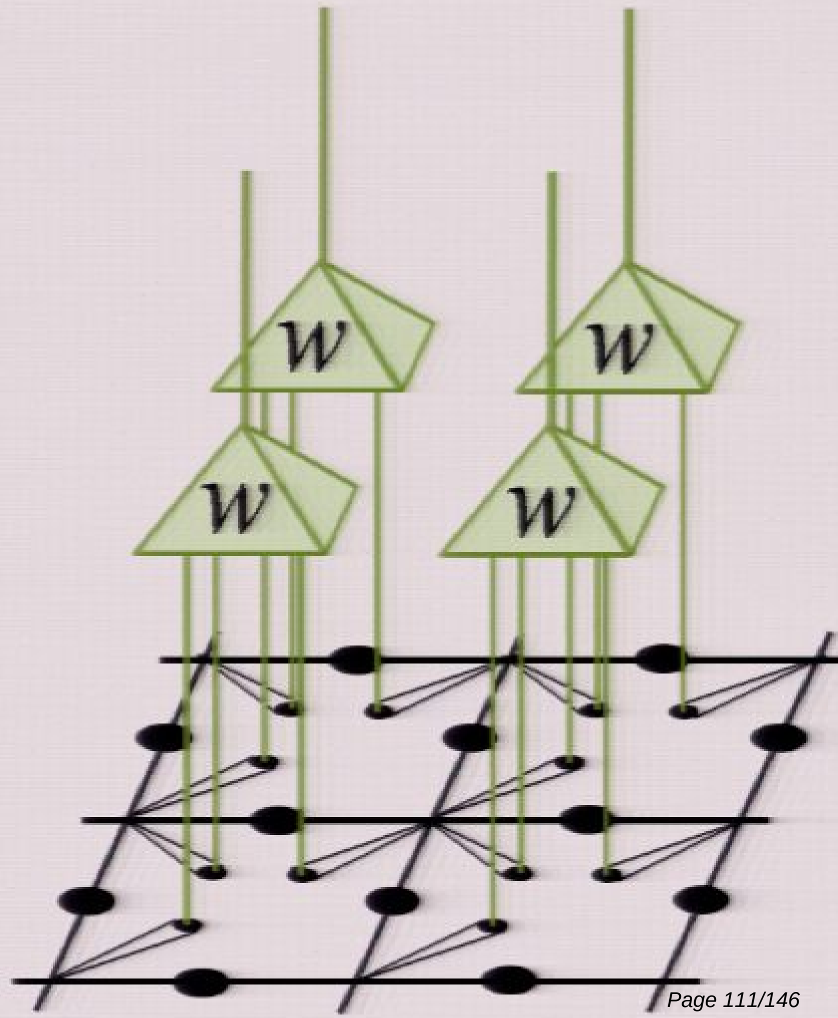


In this work, for simplicity:



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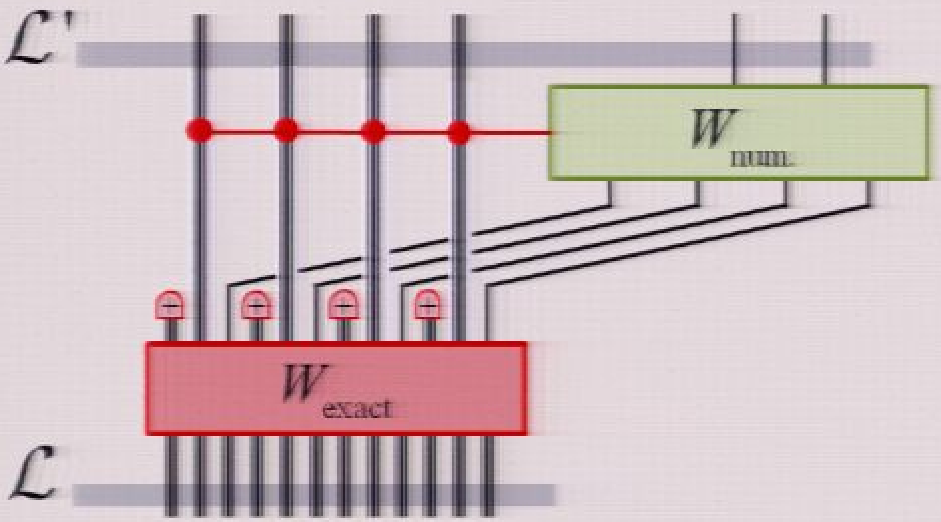

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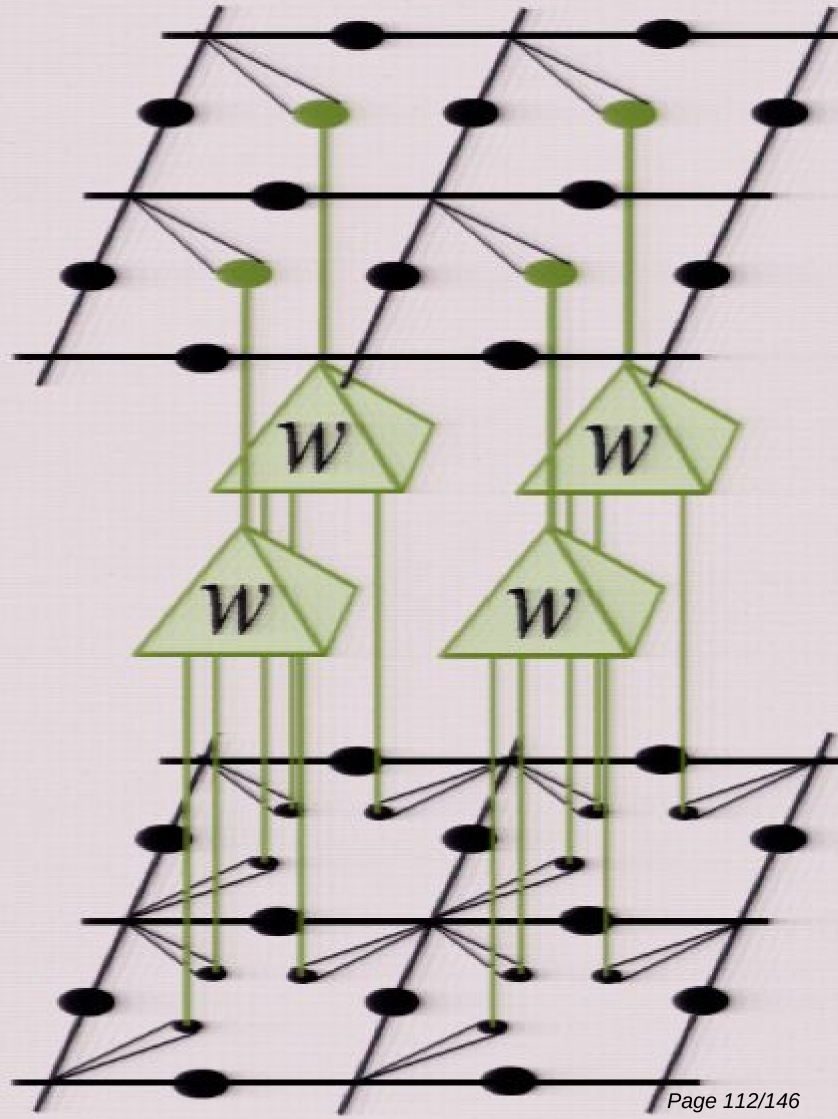
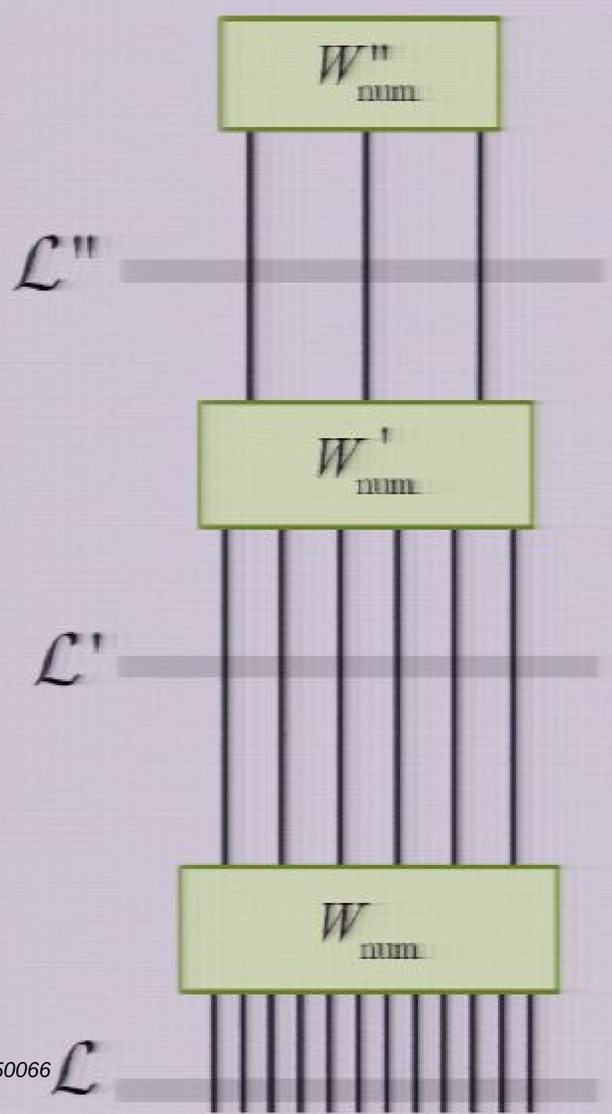


Coarse-graining scheme

variational ansatz
(without local symmetry)



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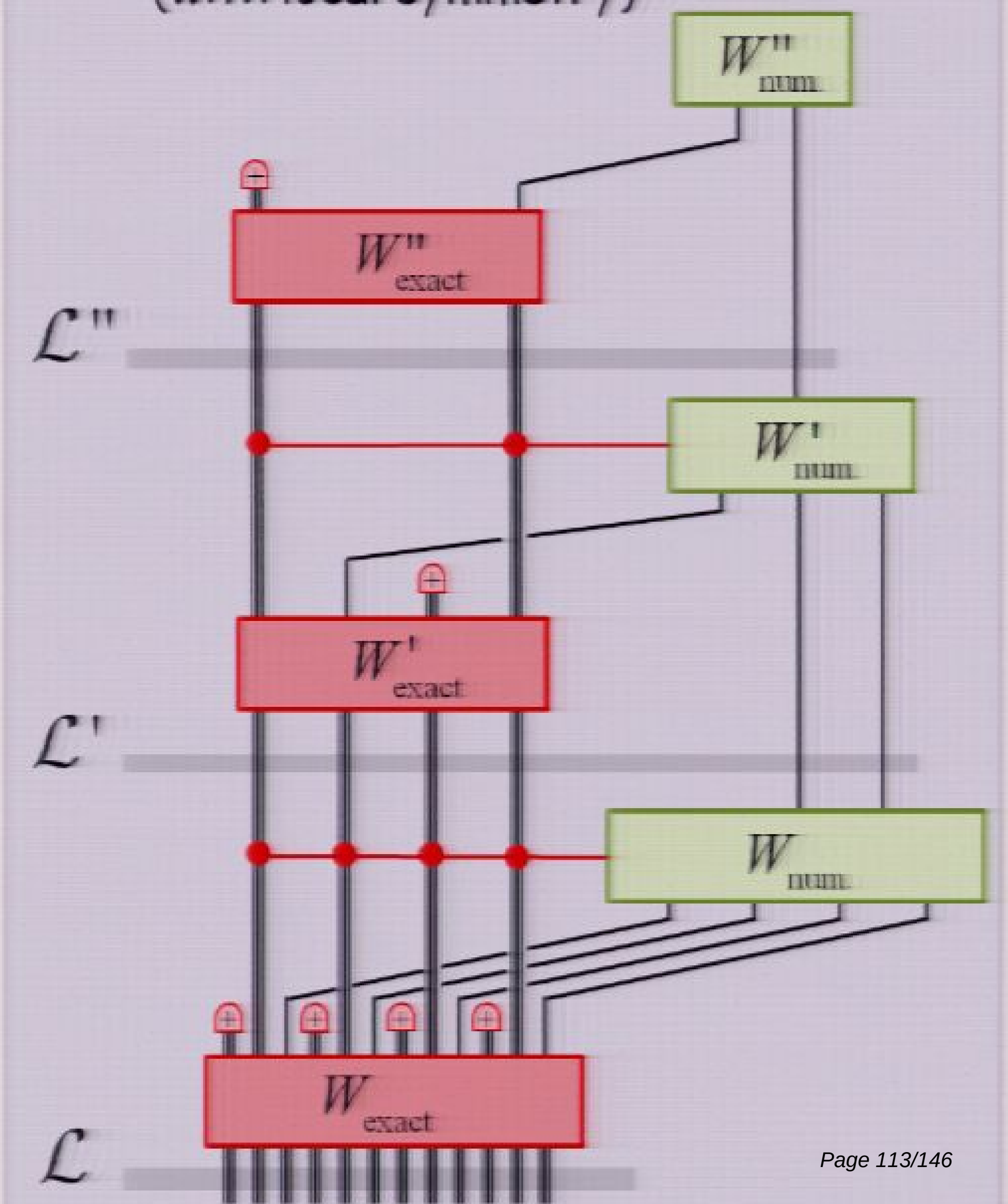

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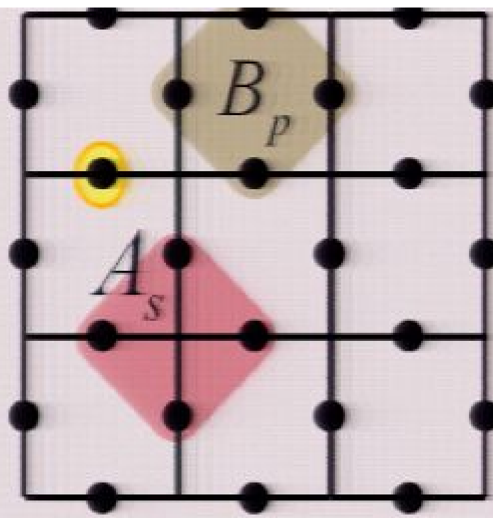
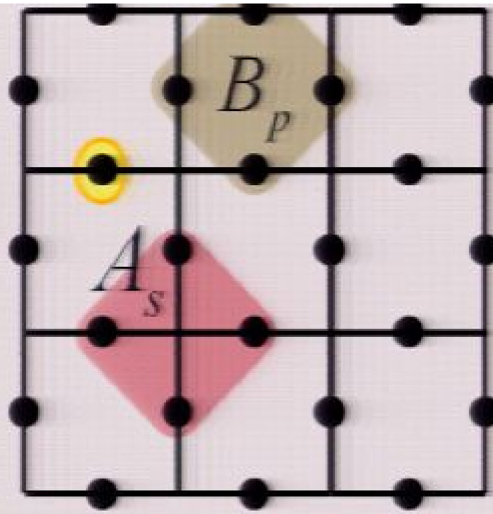


Coarse-graining scheme

$$H_{\mathrm{TC}}^{x} = -J_e \sum_s A_s - J_m \sum_p B_p - h_x \sum_j \sigma_j^x \qquad J_e \gg J_m, h_x$$



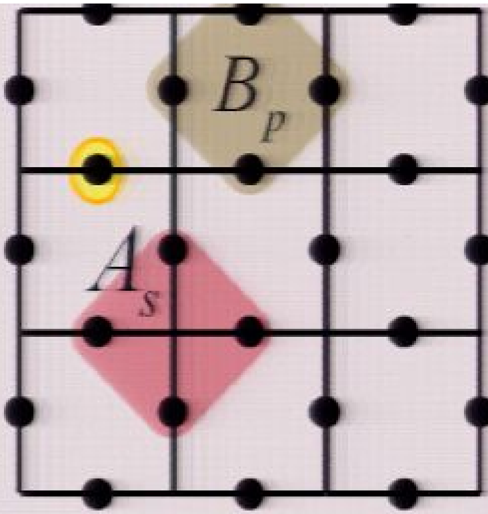
one parameter

h_x / J_m

$h_x \ll J_m$ ——————— $h_x \gg J_m$

- Extreme cases?

Coarse-graining scheme



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"free" spins are close to a product state

$|0\rangle|0\rangle|0\rangle\cdots|0\rangle$ unentangled -> cheap

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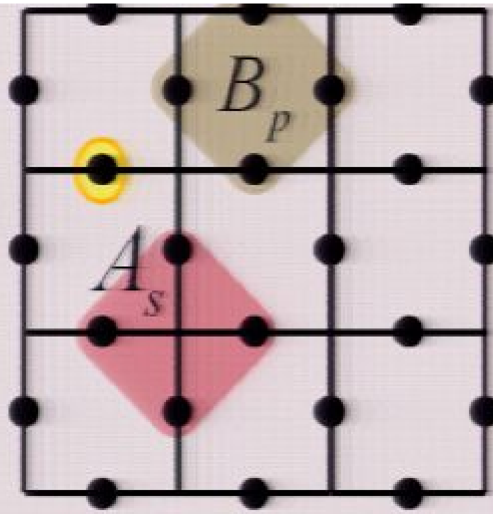

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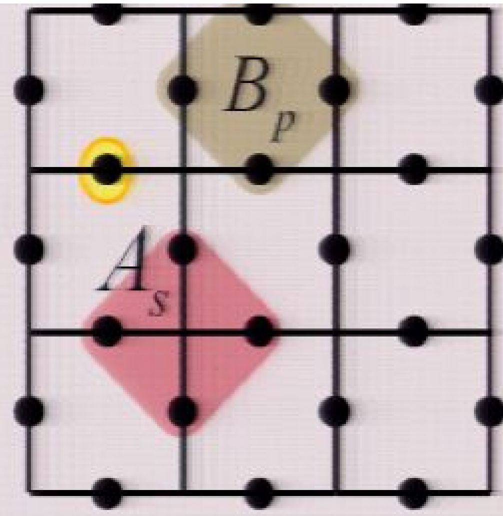

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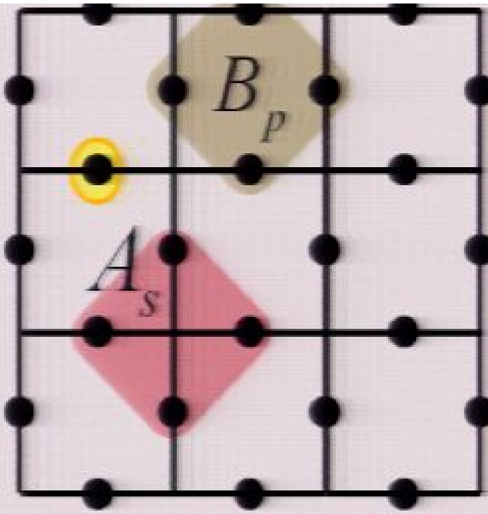

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We recover toric code

M. Aguado, G. Vidal,
PRL 100, 070404 (2008)

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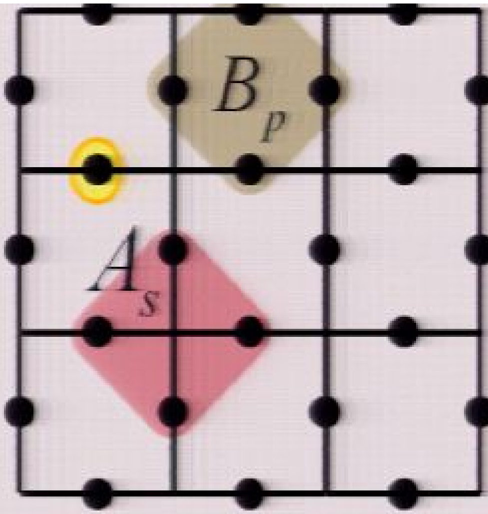

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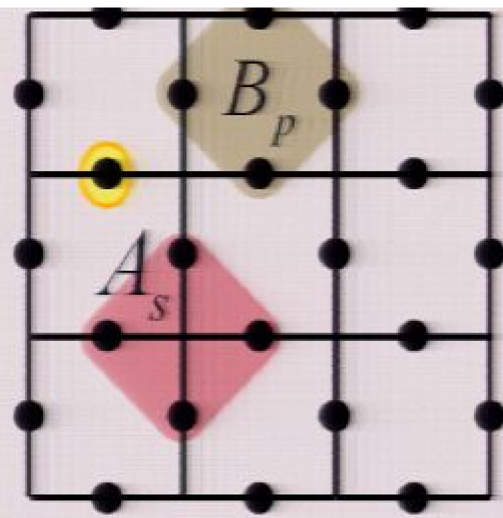

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$h_x \sim J_m$ $\qquad$ "free" spins are entangled with themselves and with constrained spins

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$$(J_z = 1) \quad H_{\mathrm{LGT}} = -\sum_p B_p - h_x \sum_j \sigma_j^x$$

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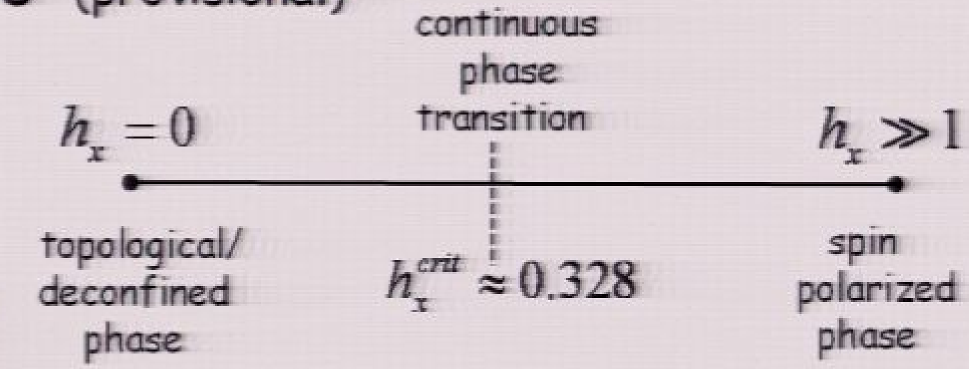

Lattice sizes:

4×4	(=32 spins)	6×6	(=72 spins)
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From optimized ansatz one can extract many properties:

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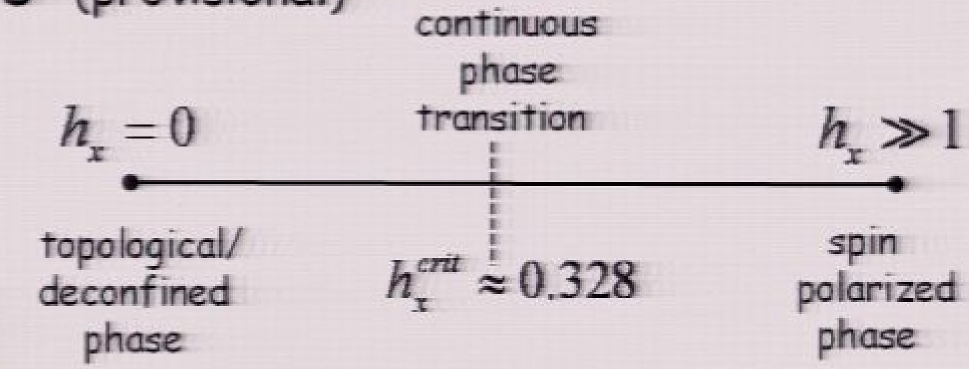

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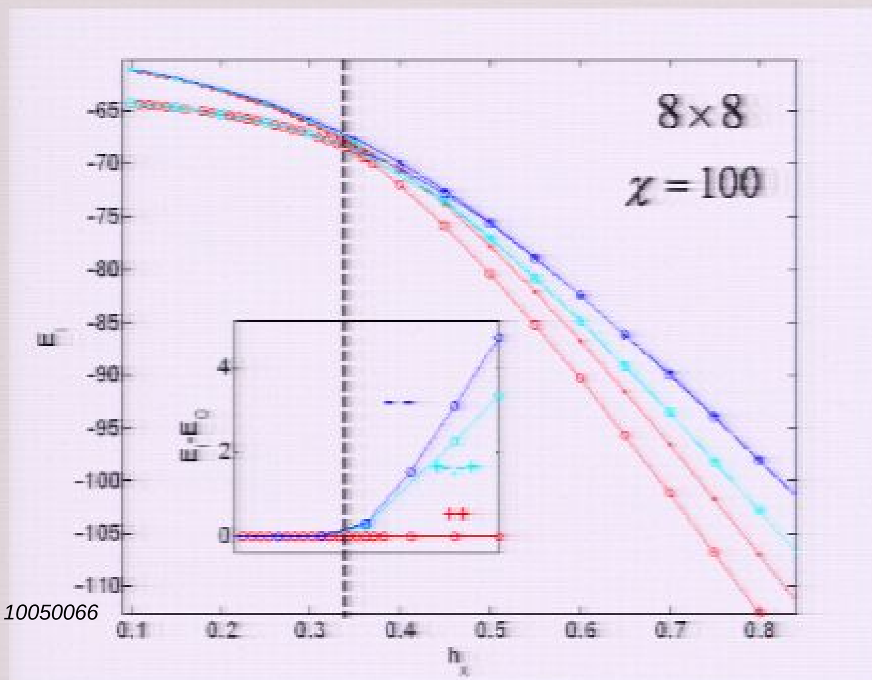

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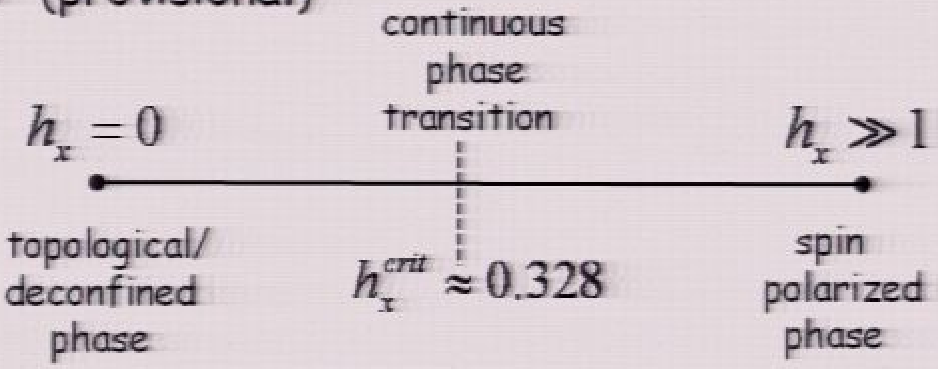

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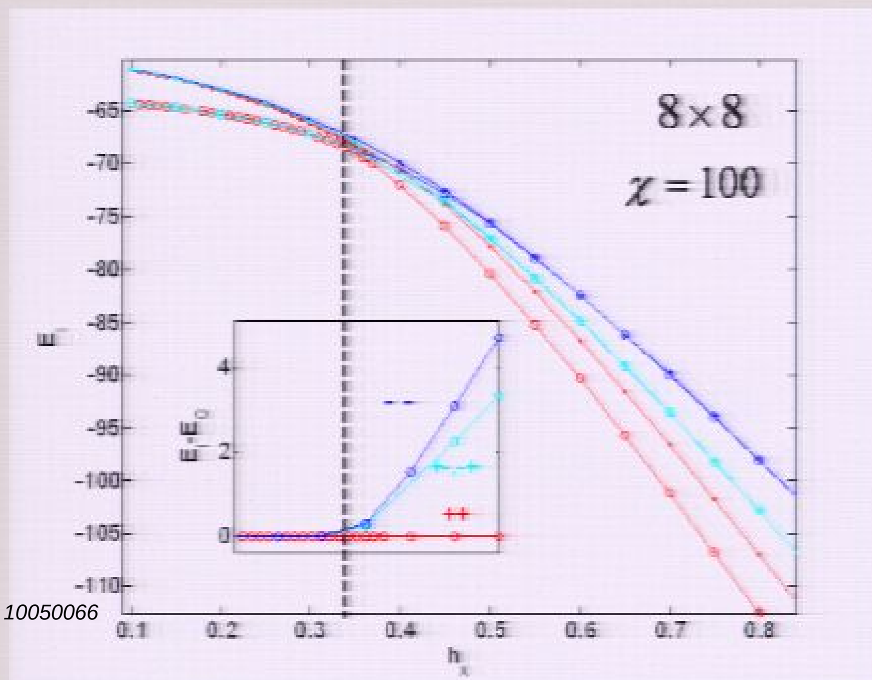

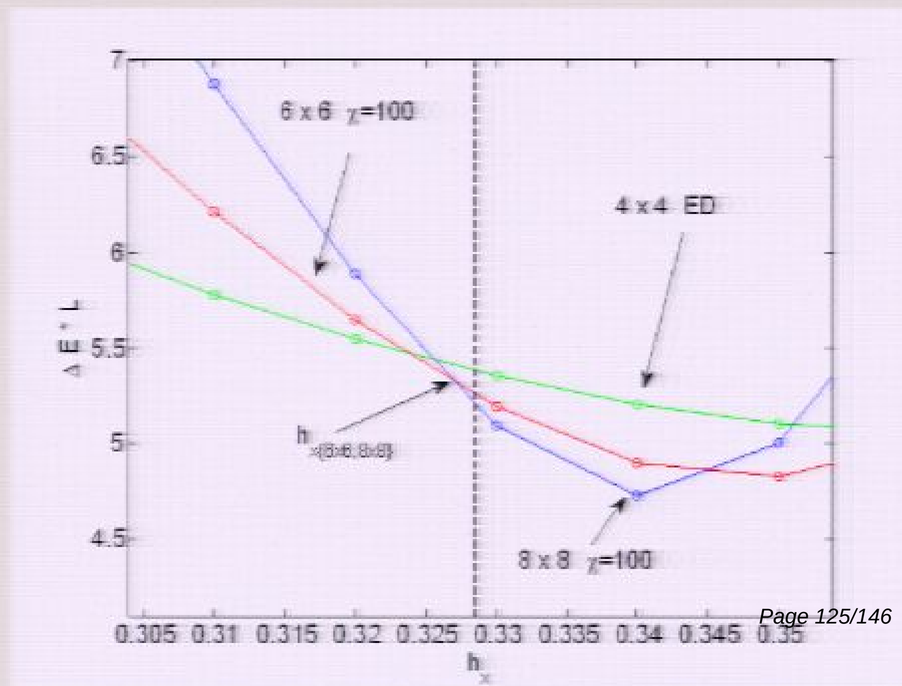

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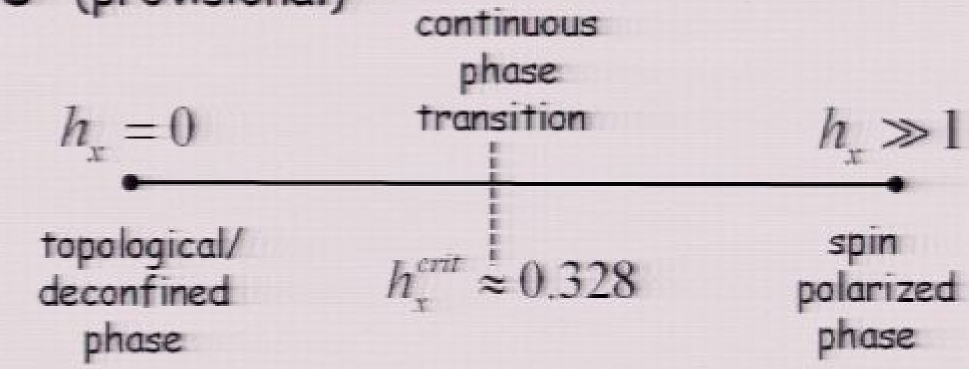

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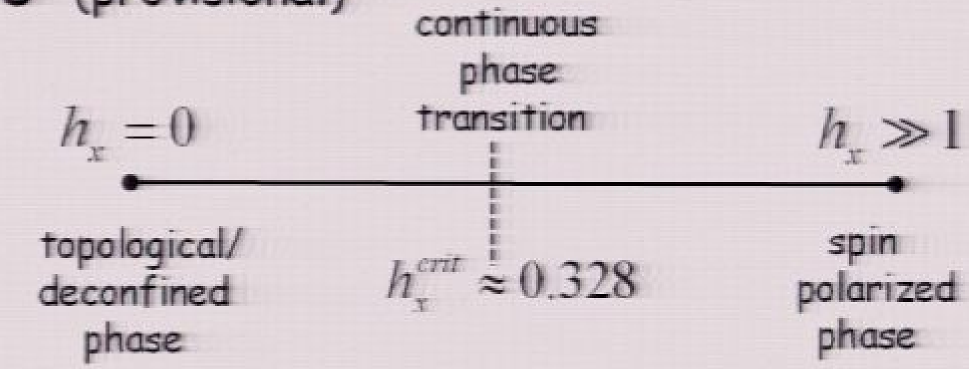
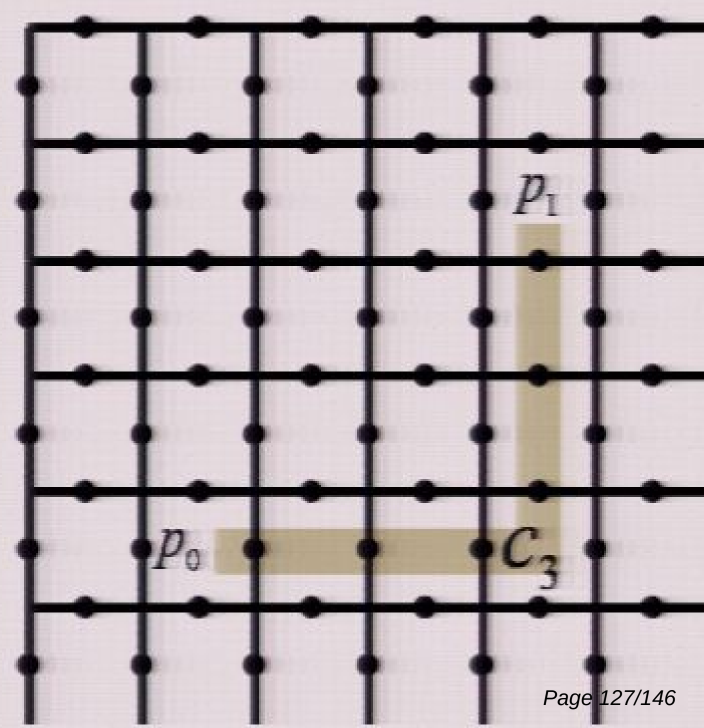


- Non-local order parameter $\langle X_3 \rangle$

$$X_3 \equiv \prod_{j \in c_3} \sigma_j^x$$



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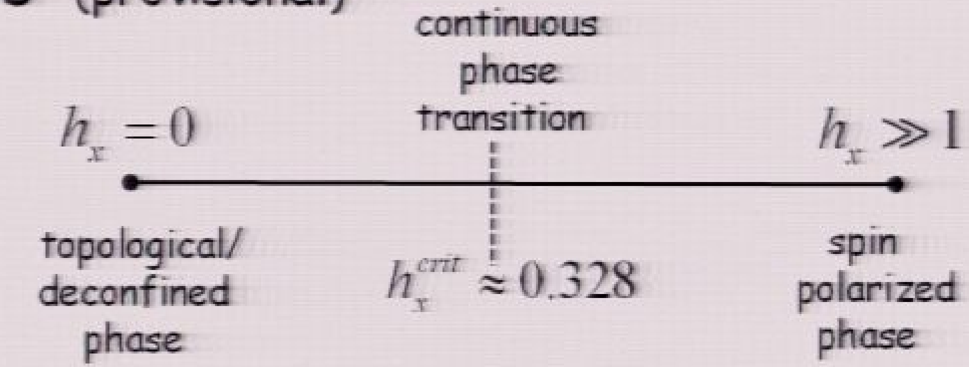
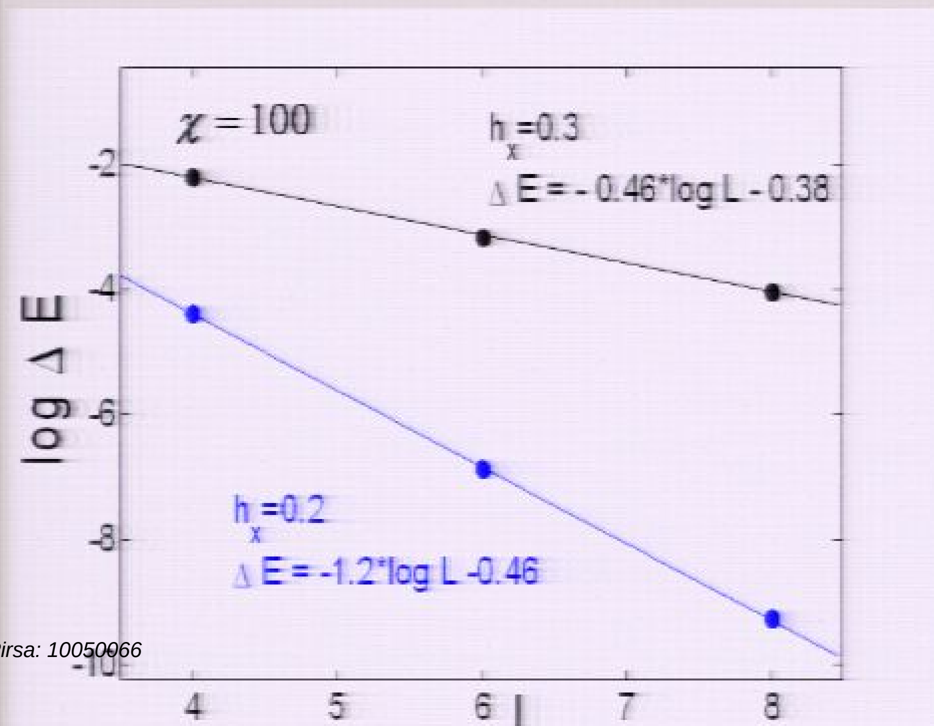


- Gaps between topological sectors

$h_x < h_x^{crit}$ exponentially suppressed $\Delta E \propto \mathrm{e}^{-L}$



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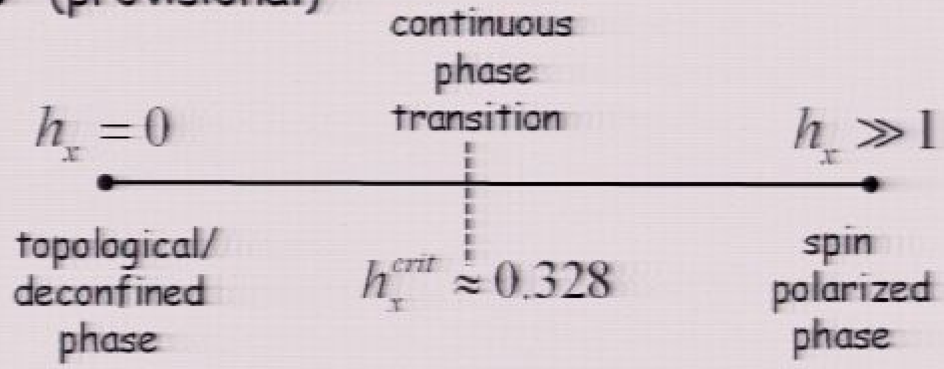

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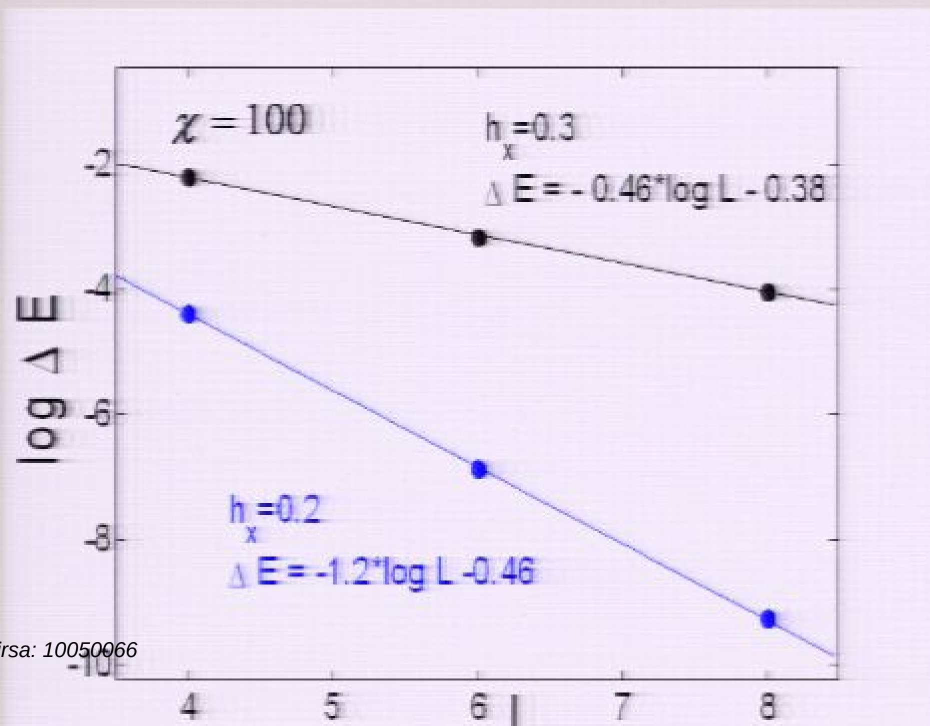

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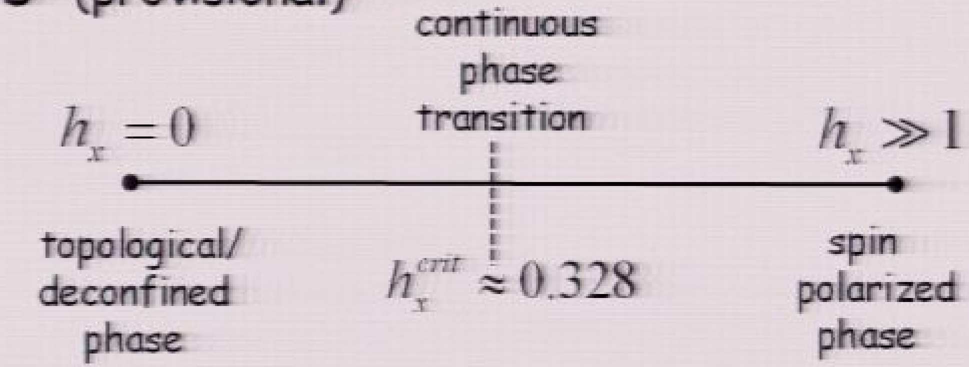

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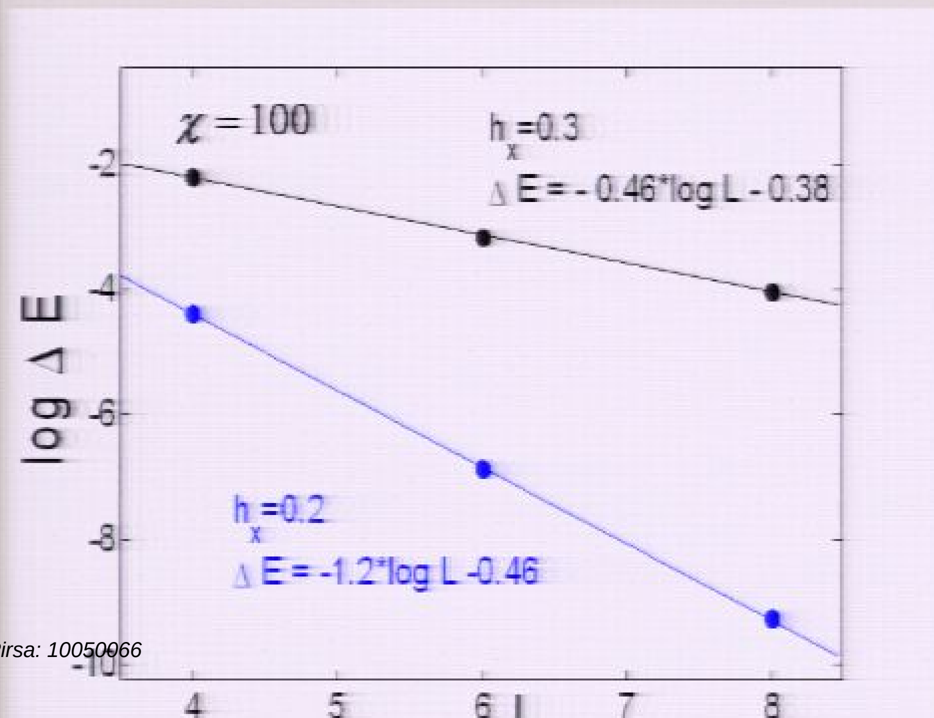

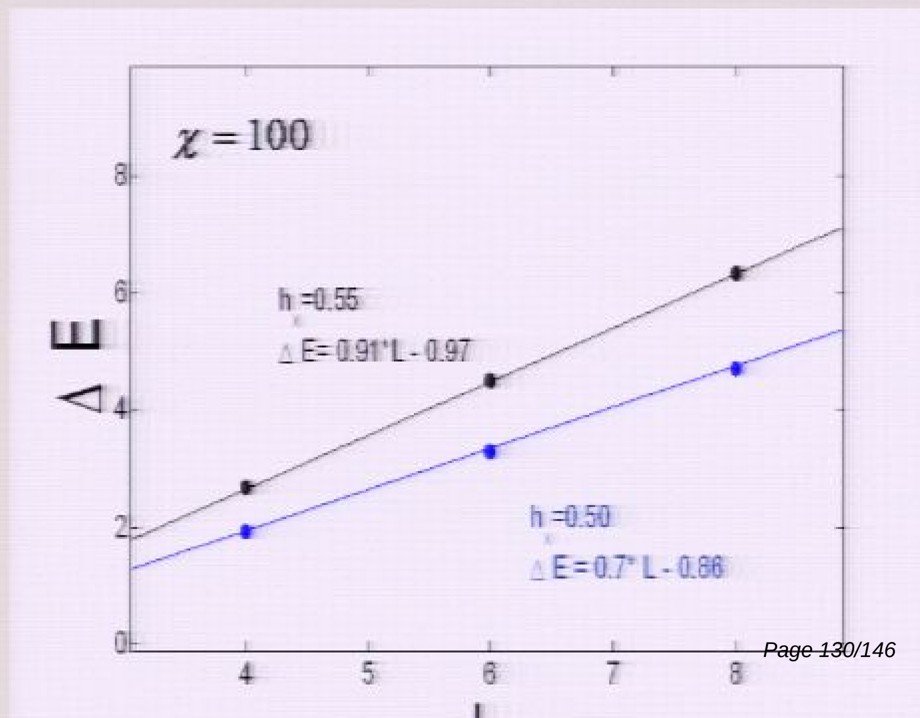

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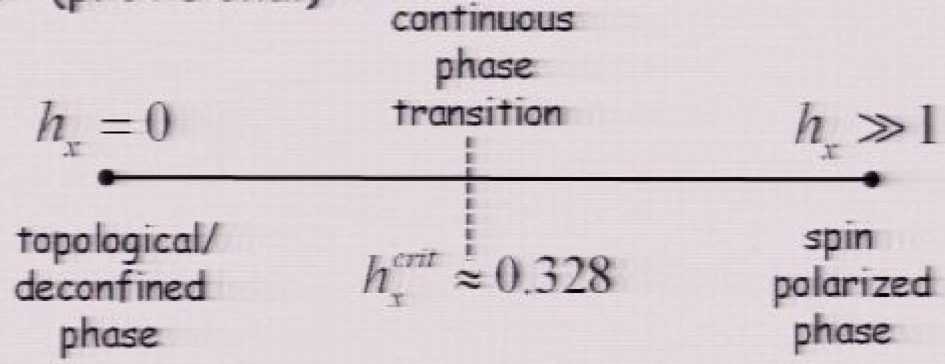

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$h_x < h_x^{crit}$ perimeter law $W \propto \mathrm{e}^{-P(l)}$

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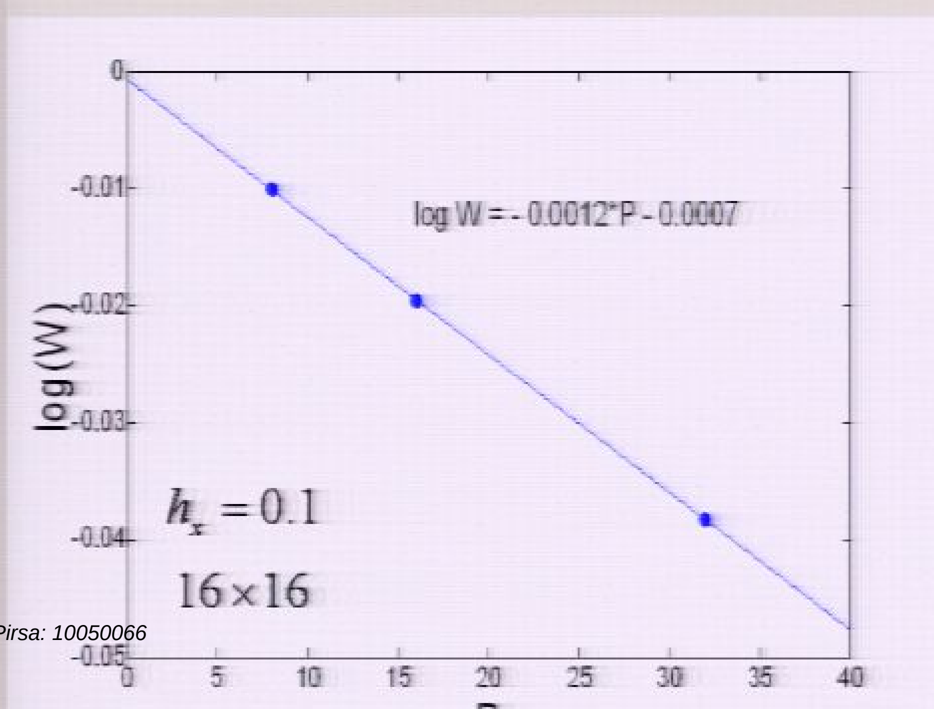

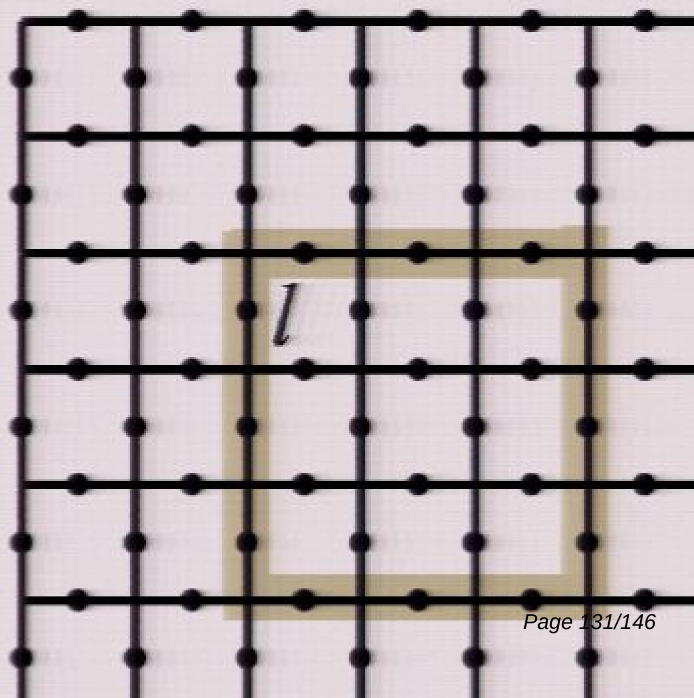

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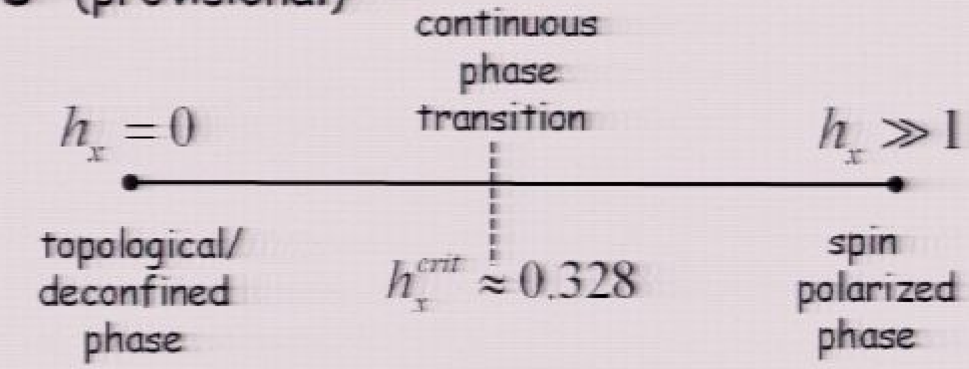

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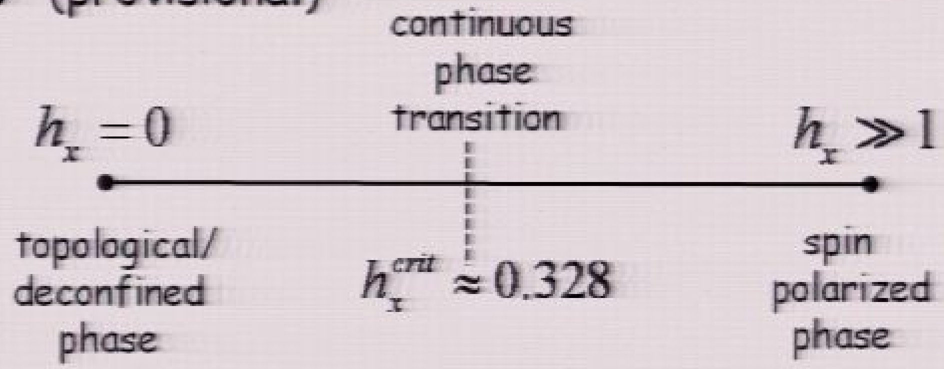

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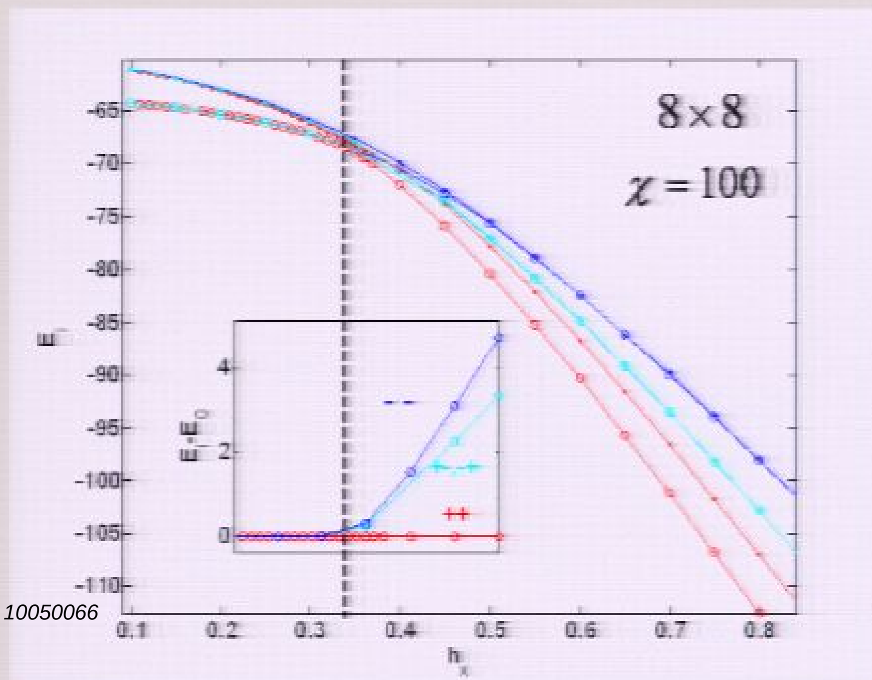

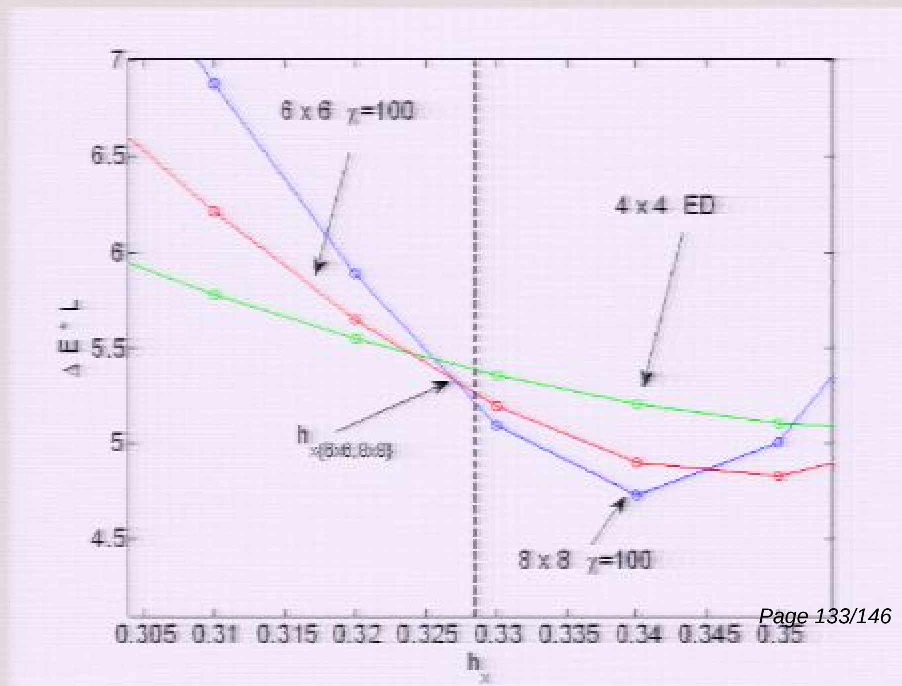

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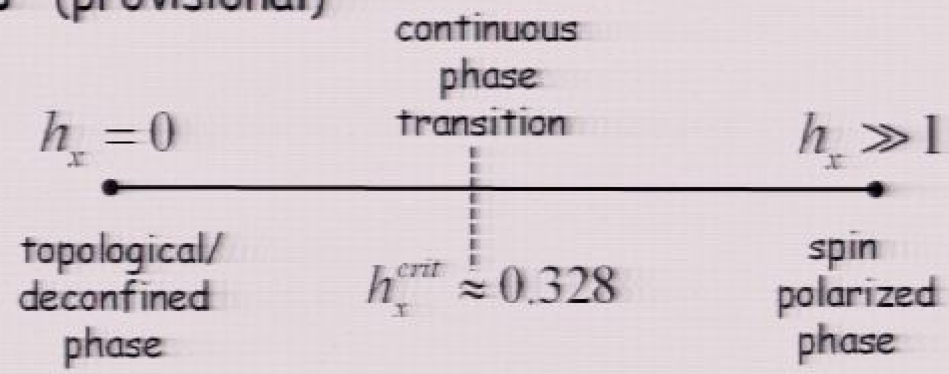

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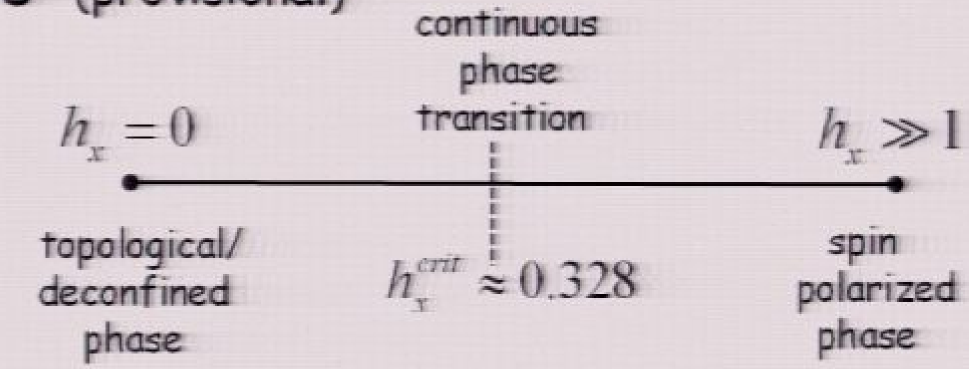

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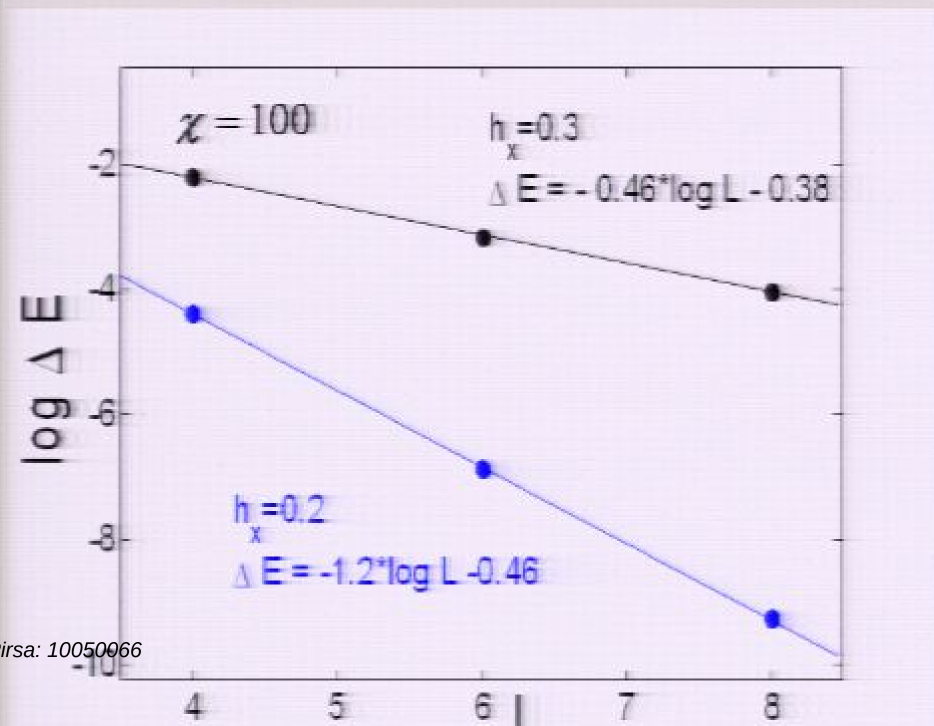

Benchmark results (provisional)

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$h_x = 0$ — continuous phase transition — $h_x \gg 1$

topological/ deconfined phase — $h_x^{crit} \approx 0.328$ — spin polarized phase

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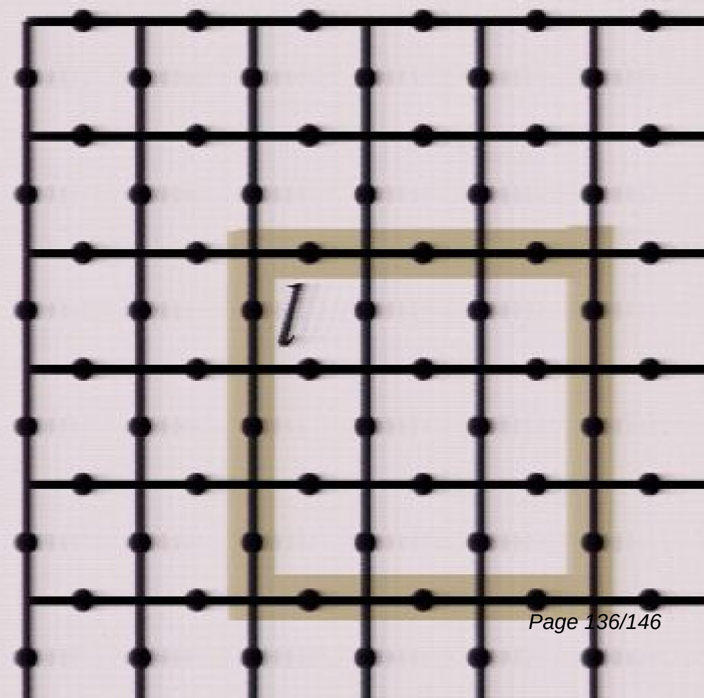
$$W \equiv \left\langle \prod_{j \in l} \sigma_j^Z \right\rangle$$



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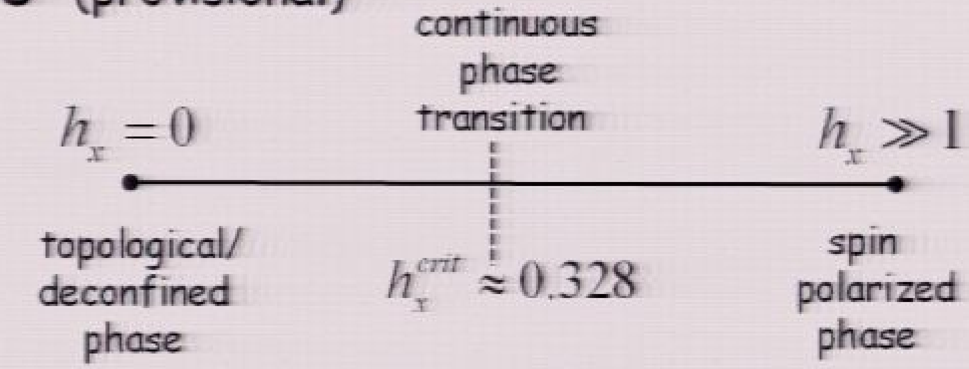

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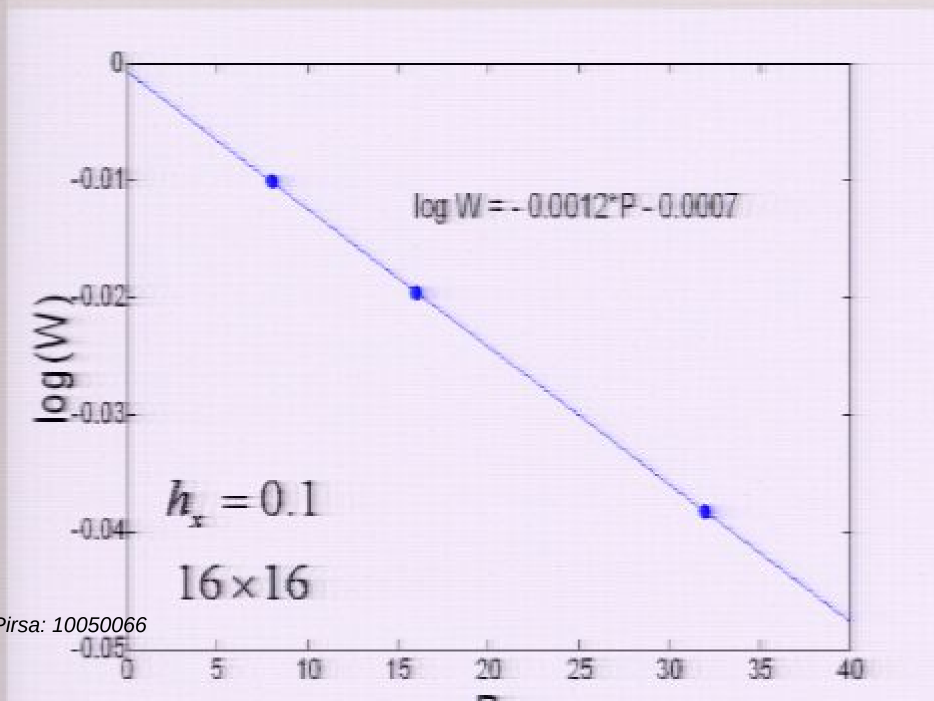
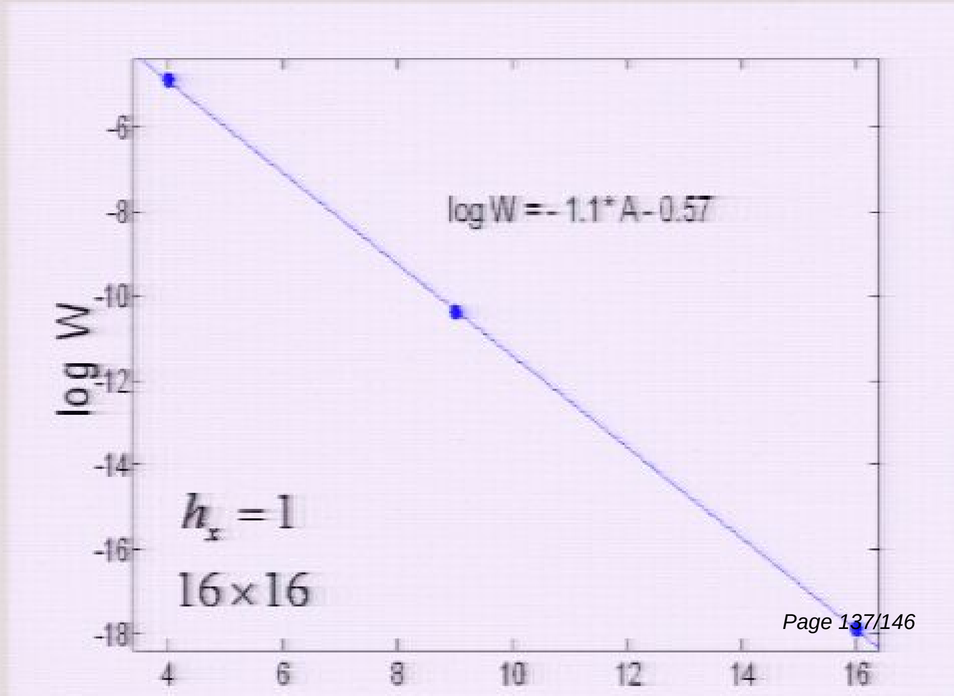
$h_x > h_x^{crit}$ area law $W \propto e^{-A(l)}$





Entanglement Renormalization and Gauge Symmetry

- Summary

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entanglement renormalization	local symmetry	⇔	gauge theory

- Coarse-graining scheme

  exact transformation (CNOTs)
  numerical transformation

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Work in progress

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Work in progress

- add *numerical disentanglers*: infinite systems, recover RG fixed points
- toric code + arbitrary perturbation (no local symmetry)

Entanglement Renormalization and Gauge Symmetry

- Summary

entanglement renormalization	local symmetry	⇔	gauge theory

- Coarse-graining scheme

  exact transformation (CNOTs)
  numerical transformation

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M. Aguado, G. Vidal,
PRL 100, 070404 (2008)

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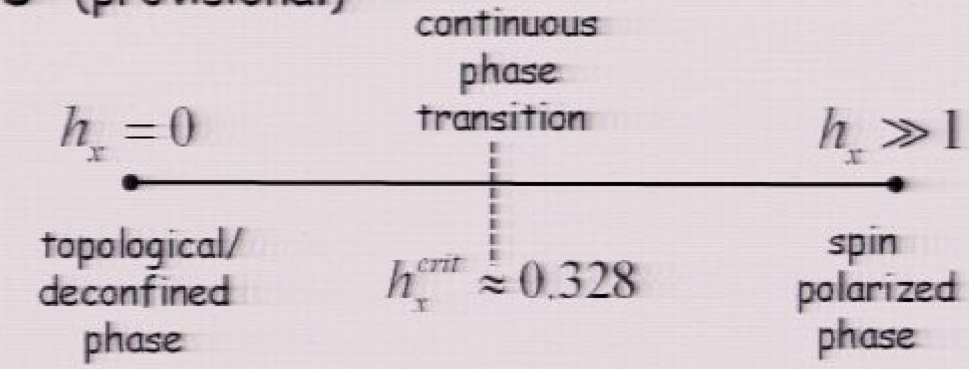
exact transformation:

- **other groups** (non-abelian and/or continuous) — M. Aguado, G. Vidal, PRL 100, 070404 (2008)
- **string-nets models (Levin & Wen)** — R. Koenig, B. Reichardt, G. Vidal, PRB 79, 195123 (2009)

Benchmark results (provisional)

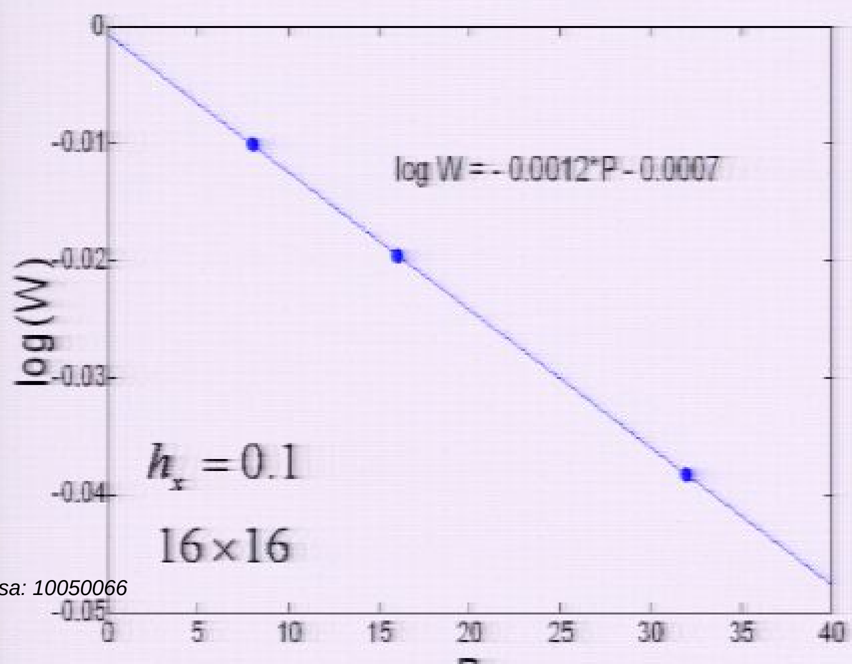
$$(J_m = 1) \quad H_{\mathrm{LGT}} = -\sum_p B_p - h_x \sum_j \sigma_j^x$$

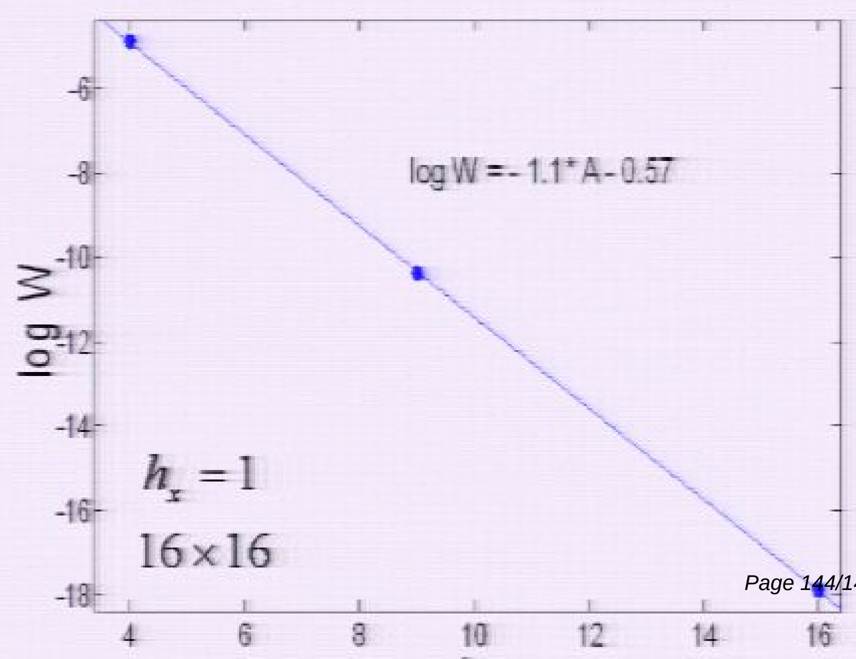


- Wilson loops

$h_x < h_x^{crit}$ perimeter law $W \propto e^{-P(l)}$

$h_x > h_x^{crit}$ area law $W \propto e^{-A(l)}$





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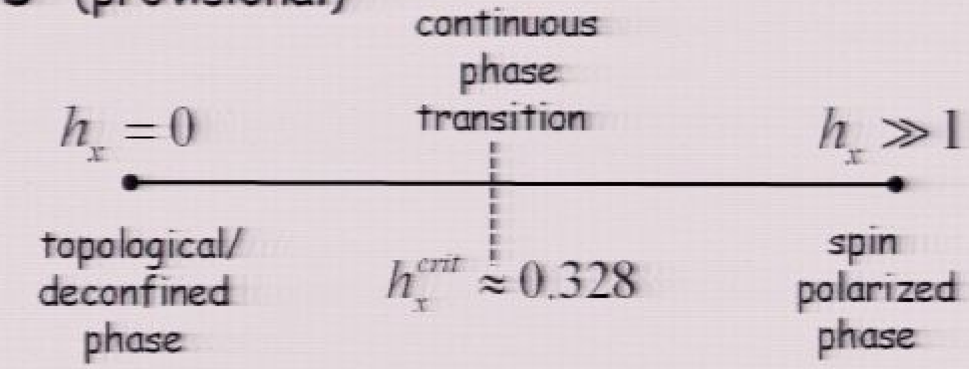

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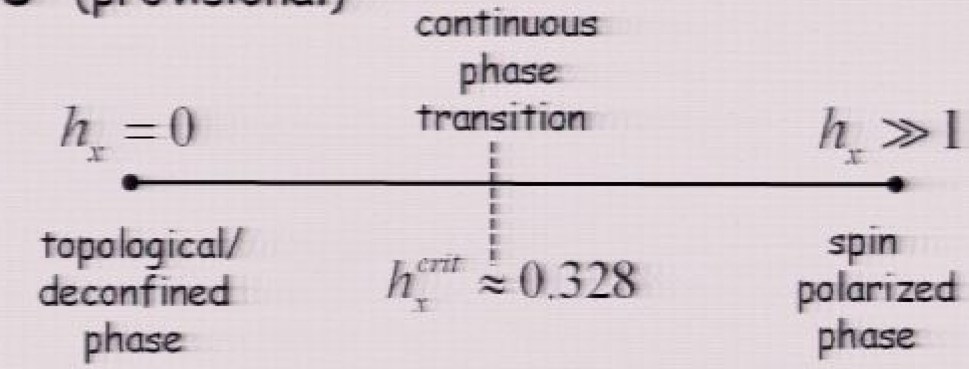

- Gaps between topological sectors

$h_x < h_x^{crit}$ exponentially suppressed $\Delta E \propto e^{-L}$

$h_x > h_x^{crit}$ linear scaling $\Delta E \propto L$