

Title: Unitary Equilibrations: Temporal and Hilbert Space Typicality Loschmidt Echo

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Abstract: Closed quantum systems evolve unitarily and therefore cannot converge in a strong sense to an equilibrium state starting out from a generic pure state. Nevertheless for large system size one observes temporal typicality. Namely, for the overwhelming majority of the time instants, the statistics of observables is practically indistinguishable from an effective equilibrium one. In this talk we will discuss the Loschmidt echo (LE) to study this sort of unitary equilibration after a quench. In particular we will address the issue of typicality with respect to the initial state preparation and the influence of quantum criticality on the long-time probability distribution of LE.

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Temporal and Hilbert Space Typicality of Loschmidt Echo



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Lorenzo Campos Venuti (ISI)

Paolo Zanardi (USC)

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Hey wait a sec: Equilibration of a finite closed quantum system?!? What are you talking about dude???

- **Unitary Evolution** \implies no non-trivial fixed points for $t = \infty$
I.e., no strong (norm) convergence

$$\lim_{t \rightarrow \infty} U(t) |\Psi\rangle = |\Psi\rangle_\infty \implies |\Psi\rangle = |\Psi\rangle_\infty$$

- **Finite size** \implies **Point spectrum** $\implies A(t) = \text{measurable quantity is a quasi-periodic function}$ \implies no $t = \infty$ limit (quasi-returns/revivals) \implies not even weak op convergence

$$A(t) = \sum_{p=1}^D A_p \exp(i\omega_p t) \implies \forall \varepsilon > 0 \exists T(\varepsilon, D) / |A(T) - A(0)| \leq \varepsilon$$

Unitary equilibrations will have to be a different kind of convergence....

Loschmidt Echo:

$$L(t) = |\langle \Psi | \exp(-iHt) | \Psi \rangle|^2$$

$$= \text{Tr}[|\Psi\rangle\langle\Psi| e^{-iHt} |\Psi\rangle\langle\Psi| e^{iHt}] = \langle \rho_\Psi(t) \rangle_\Psi = \sum_{n,m} p_n p_m \exp[-it(E_n - E_m)]$$

$$H = \sum_n E_n |n\rangle\langle n|$$

Spectral resolution

$$p_n := |\langle \Psi | n \rangle|^2$$

Probability distribution(s)

Different Time-Scales & Characteristic quantities

- Relaxation Time (to get to a small value by dephasing and oscillate around)
- Revivals Time (signal strikes back due to re-phasing)

Q1: how all these depend on $H, |\Psi\rangle$, and system size?

Q2: how the global statistical features of $L(t)$ depend on $H, |\Psi\rangle$ and system size N ?

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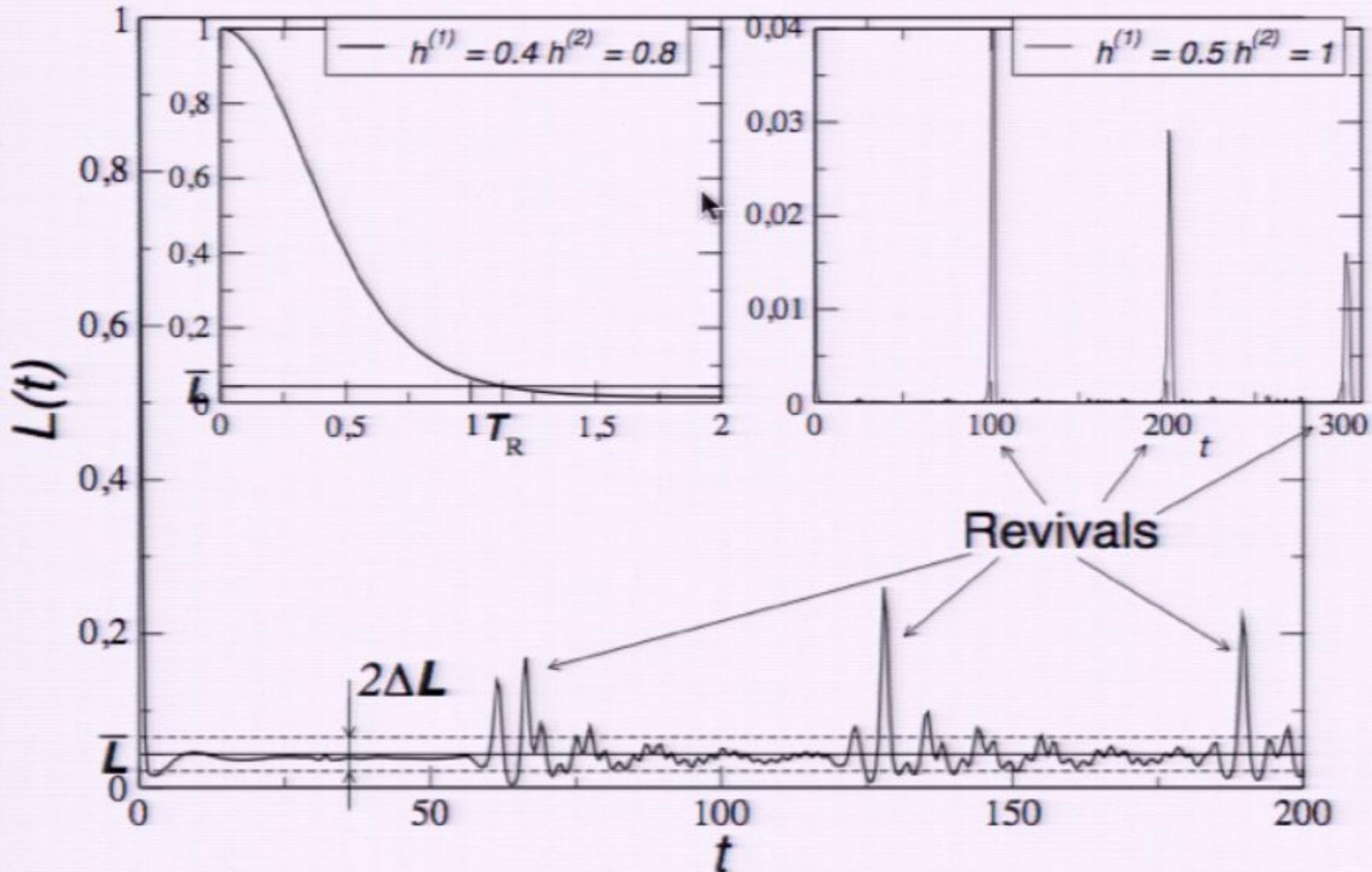
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Typical Time Pattern of $L(t)$



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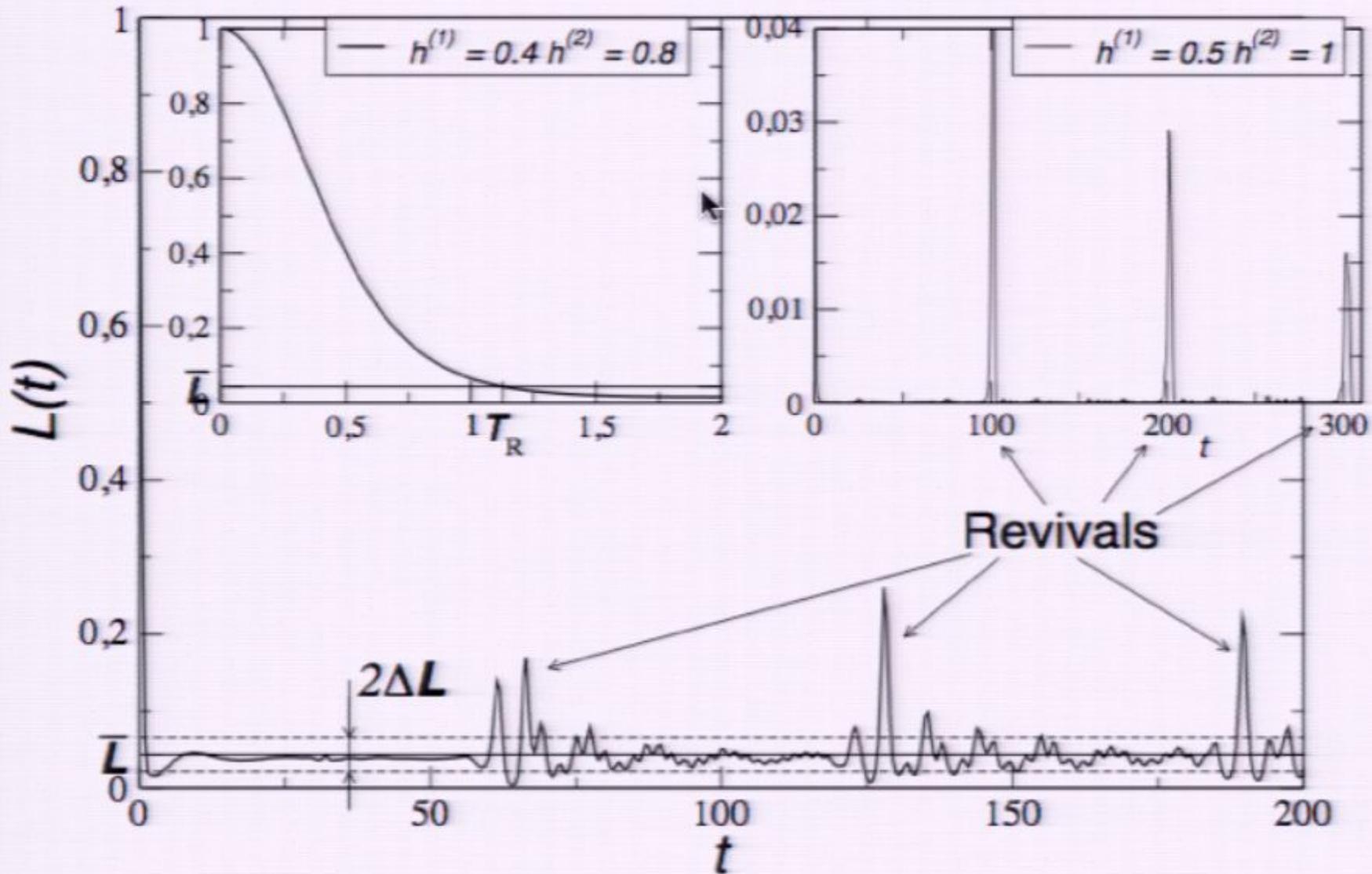
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For a *given* initial state L-echo is a RV over the time line $[0, \infty)$ with Prob

$$\mu_\infty(A) := \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \chi_A(t) dt \quad \chi_A \text{ Characteristic function of } A \subset [0, \infty)$$

Probability distribution of L-echo

$$P(y = L(t)) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \delta(y - L(t)) = \langle \delta(L - L(t)) \rangle_t$$

Goal: study $P(y)$ to extract global information about the Equilibration process

$$\varepsilon > 0 \Rightarrow \lim_{N \rightarrow \infty} \mu_\infty \{t / |L_N(t) - \langle L_N(t) \rangle_t| \geq \varepsilon\} = 0$$

Prob of (arbitrarily small) fluctuations goes to zero for large system size N (*measure concentration phenomenon*)

1 Moments of $P(y)$

$$\kappa_n := \int y^n P(y) dy = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T L^k(t) dt$$

Mean:

$$\begin{aligned} \kappa_1 &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \langle \rho_\Psi, \rho_\Psi(t) \rangle dt = \sum_{m,n} \langle \rho_\Psi, \Pi_n \rho_\Psi, \Pi_m \rangle \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T e^{-i(E_n - E_m)t} dt \\ &= \sum_n \langle \rho_\Psi, \Pi_n \rho_\Psi, \Pi_n \rangle = \langle \rho_\Psi, D_1(\rho_\Psi) \rangle = \langle D_1(\rho_\Psi), D_1(\rho_\Psi) \rangle = \text{Tr} D_1(\rho_\Psi)^2 \end{aligned}$$



Long time average of $L(t)$ is the **purity** of the time-average density matrix

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T e^{-iHt} \rho_\Psi e^{iHt} dt = \sum_n \Pi_n \rho_\Psi, \Pi_n =: D_1(\rho_\Psi)$$

Remark D_1 Is a **projection** on the algebra of the fixed points
Of the (Heisenberg) time-evolution generated by H

Question: How about the other moments e.g., variance and initial state dependence? Are there "typical" values?

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$$\kappa_n(\psi) = \text{Tr}[D_n^{\otimes 2}(S) |\psi\rangle\langle\psi|^{\otimes 2n}]$$

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Dephasing CP-map of the n -copies Hamiltonian, S is a swap in $(\text{Hilb}^{\otimes n})^{\otimes 2}$

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$d = \dim(\text{Hilb}) \implies \overline{\kappa_1(\psi)}^\psi$ exp (in M) small
 = (positivity) \implies exp (in M) state-space concentration of $\overline{\kappa_n(\psi)}^\psi$ around $\overline{\kappa_n(\psi)}^\psi$

Remark We assumed NO DEGENERACY, in general bounded above by $\overline{\kappa_1(\psi)}^\psi = \frac{1+d \sum_j (d_j/d)}{d+1}$

Remark $|\kappa_n(\psi) - \kappa_n(\phi)| \leq \|D_n^{\otimes 2}(S)\|_\infty \| |\psi\rangle\langle\psi|^{\otimes 2n} - |\phi\rangle\langle\phi|^{\otimes 2n} \|_1 \leq 4n \| |\psi\rangle - |\phi\rangle \|$

All $\kappa_n(t)$ moments are Lipschitz functions on the unit sphere of Hilb

\implies **Levi's Lemma** implies exp (in d) concentration around $\overline{\kappa_n(\psi)}^\psi$

$$\sigma^2(L) := \kappa_2 - \kappa_1^2 = \sum_{i \neq j} p_i^2 p_j^2 = \kappa_1^2 - \text{tr}(\rho_{eq}^4) \leq \kappa_1^2$$

Remark: non-resonance assumed

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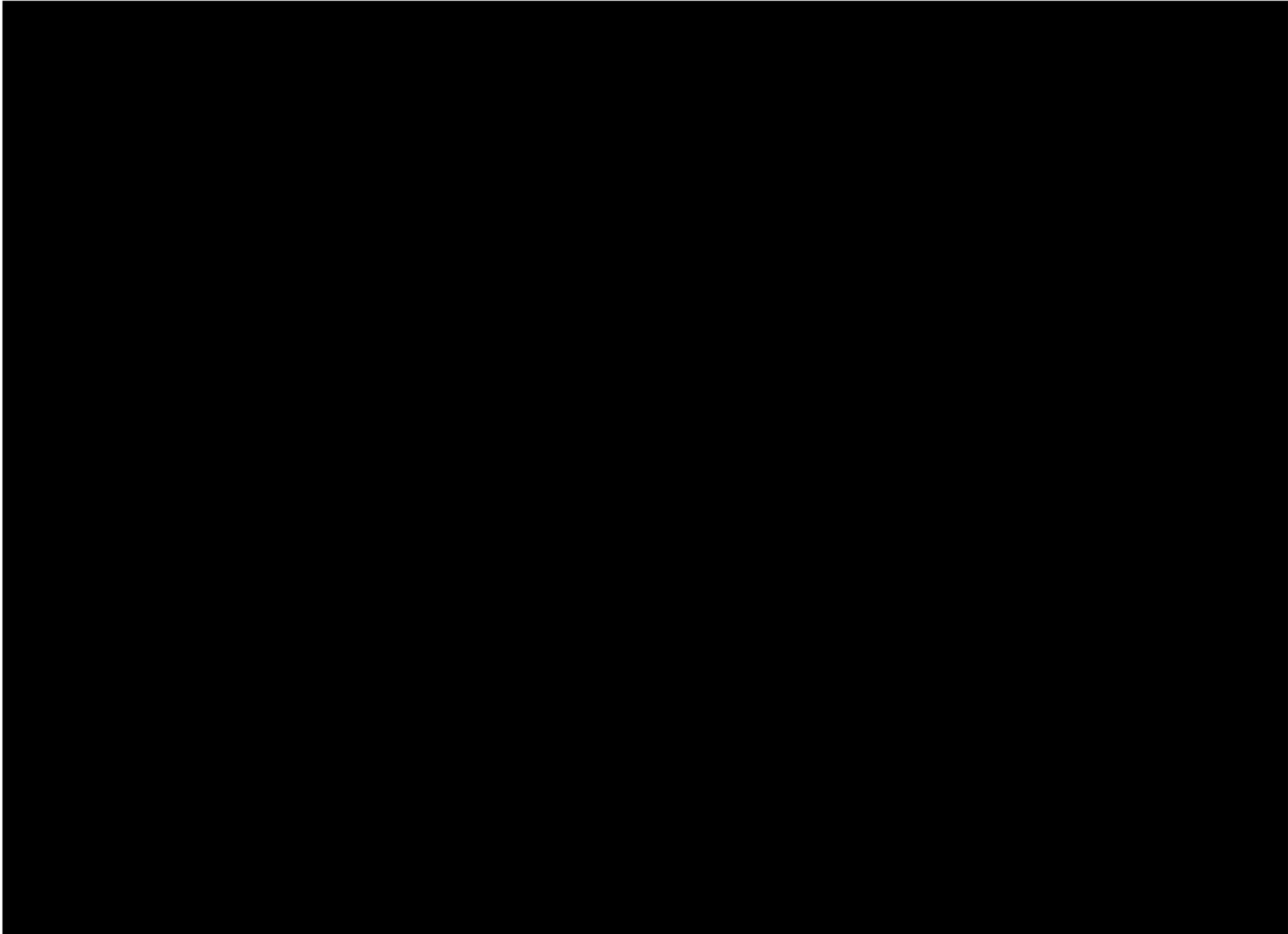


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$$f(t) = \langle \psi | U_t(\rho_\psi) | \psi \rangle$$

$$\underline{A(t) = \langle \psi | A(t) | \psi \rangle}$$

$$\overline{(A(t) - \overline{A(t)})^2} \leq \|A\|_\infty^2 \overline{L(t)}^t$$

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Q: Can we do better? E.g., *exp in d* concentration?

A: yes we can! $t \rightarrow \alpha := (E_1 t, \dots, E_d t) \in T^d \rightarrow |\sum p_n e^{i\alpha_n}|^2$

HP: energies rationally independent \implies motion on the d -torus ergodic
 \implies Temporal averages = phase-space averages

$$|L(\alpha) - L(\beta)| \leq 2 \sum p_n |e^{i\alpha_n} - e^{i\beta_n}| \leq \sum p_n |\alpha_n - \beta_n| =: 4\pi D(\alpha, \beta)$$

L is Lipschitz on the d -torus with metric $D \implies$
known measure concentration phenomenon!

$$c = (128\pi^2)$$

$$\mu_\infty \{t / |L_N(t) - \langle L_N(t) \rangle_t| \geq \varepsilon\} = \Pr\{\alpha \in T^d / |L(\alpha) - \langle L \rangle_\alpha| \geq \varepsilon\} \leq \exp\left(-\frac{c\varepsilon^2}{\sum p_n^4}\right)$$

Remark The rate of meas-conc is the inv of purity of the dephased States I.e., mean of $L \implies$ Typically order $d = \text{epx}(N)$, as promised...

Far from typicality: Small Quenches

$S_{lin} := 1 - \kappa_1$ = measures how initial state fails to be a quenched Hamiltonian Eigenstate. For $H(\text{quench})$ close to $H(0)$ we expect it to be small....

$$\begin{aligned} H_0 |\Psi_0\rangle &= E_0 |\Psi_0\rangle \\ H &= H_0 + V \\ \|V\| &= o(\epsilon) \end{aligned}$$

Ground State of an initial Hamiltonian
 Quench-Ham = init-Ham + perturbation

$p_n = |\langle \Psi_n^{\text{Quench}} | \Psi_0 \rangle|^2$ Distribution on the eigenbasis of $H = H_0 + V$

$p_0 \approx 1 - \chi_F = o(1)$ **GS Fidelity:** leading term! $p_{n \neq 0} \approx \frac{|\langle \Psi_0 | V | \Psi_n \rangle|^2}{(E_n - E_0)^2} = o(\epsilon^2)$

$$S_{lin} = 1 - \text{Tr}[D_1(\rho_{\Psi_0})^2] = 1 - \langle L(t) \rangle_t = 1 - \sum_n p_n^2 \approx 1 - p_0^2 \approx 2\chi_F$$

The linear entropy of the dephased state for a small quench
 Is given by the *fidelity susceptibility*: a well-known object!

$$\sigma^2(L) \approx p_0(1 - p_0) \approx \chi_F$$

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Quench-Ham = *init-Ham* + *perturbation*

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Distribution on the eigenbasis of $H = H_0 + V$

$$p_0 \approx 1 - \chi_F = o(1) \quad \text{GS Fidelity: leading term!} \quad p_{n \neq 0} \approx \frac{|\langle \Psi_0 | V | \Psi_n \rangle|^2}{(E_n - E_0)^2} = o(\epsilon^2)$$

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