Title: Epistemology and the laws of nature

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Abstract:

EPISTEMOLOGY AND THE LAWS OF NATURE

David H. Wolpert

NASA Ames Research Center

David.H.Wolpert@nasa.gov

http://ti.arc.nasa.gov/people/dhw/

Walter Read

Mathematics Department, CS Fresno

NASA-ARC-03-0667

ROADMAP

1) *Shortcomings of current impossibility results concerning laws of Nature*

↓

2) *Knowledge operators and their shortcoming*

↓

3) *Formalize mathematical structure shared by observation and prediction:* inference devices

↓

4) *Elementary properties of inference devices*

COMPUTATION AND PHYSICS

1) Impossibility results of Moore, Pour-El and Richards, etc. rely on uncountable number of states of universe.

- What if universe is countable, or even finite?
- What if there exist oracles, so Halting theorem (the basis of those results) is irrelevant?

2) Impossibility results of Lloyd rely on current model of laws of physics (e.g., no superluminal travel).

- What if laws are actually different?

COMPUTATION AND PHYSICS

3) *To apply Godel's incompleteness theorem presumes physical laws are "written in predicate logic"*

- *What if universe is written in a different language?*
- *What if there are no "laws" at all, just a huge list of events, which just happen to appear to have patterns?*
- *What if Godel-style intuitionism is correct?*

What if our models are wrong?

COMPUTATION AND PHYSICS

3) *To apply Godel's incompleteness theorem presumes physical laws are "written in predicate logic"*

- *What if universe is written in a different language?*
- *What if there are no "laws" at all, just a huge list of events, which just happen to appear to have patterns?*
- *What if Godel-style intuitionism is correct?*

What if our model is wrong?

Is there some model more fundamental, (almost) impossible not to accept?

ROADMAP

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3) *Formalize mathematical structure shared by observation and prediction:* inference devices

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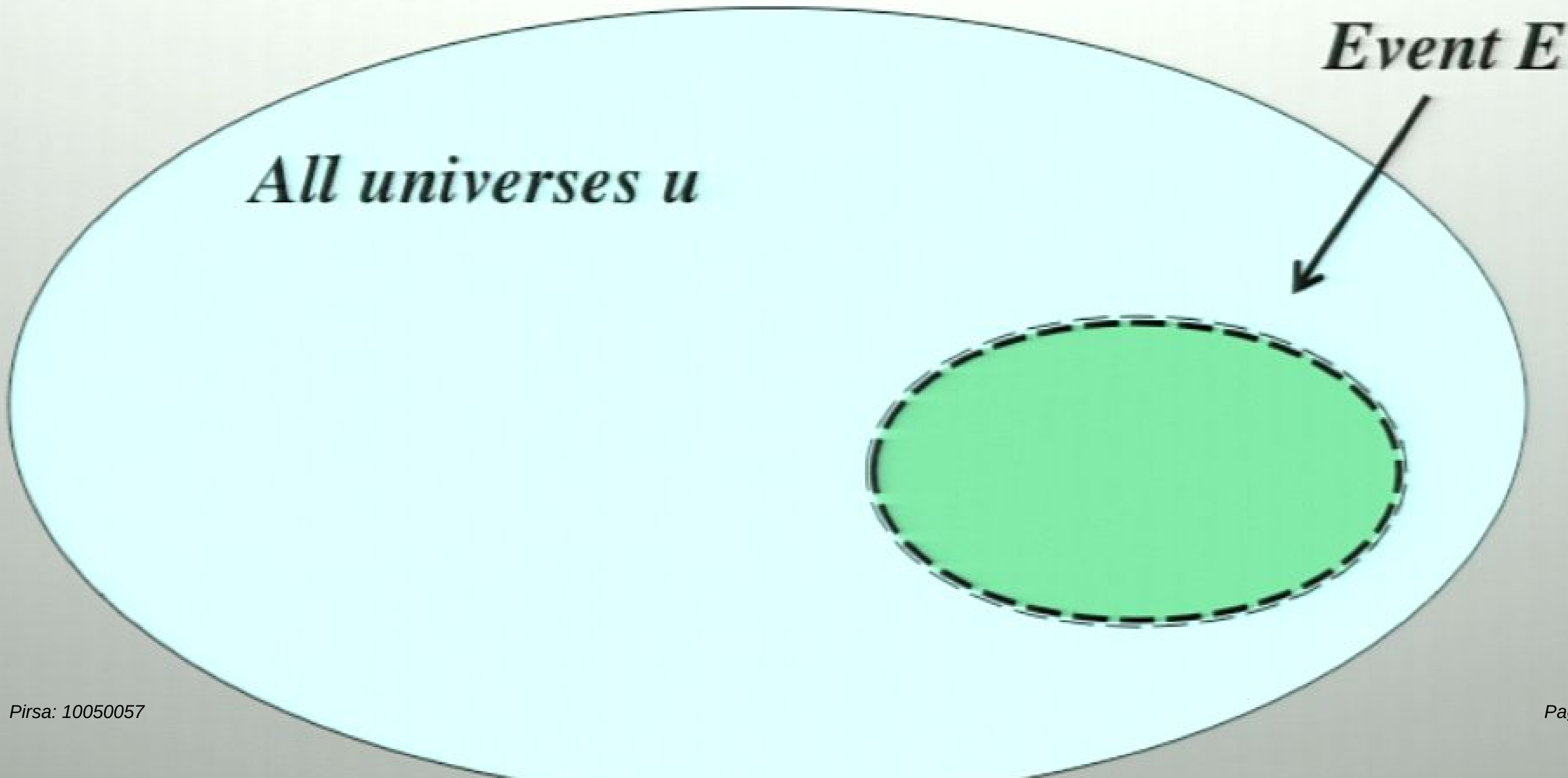
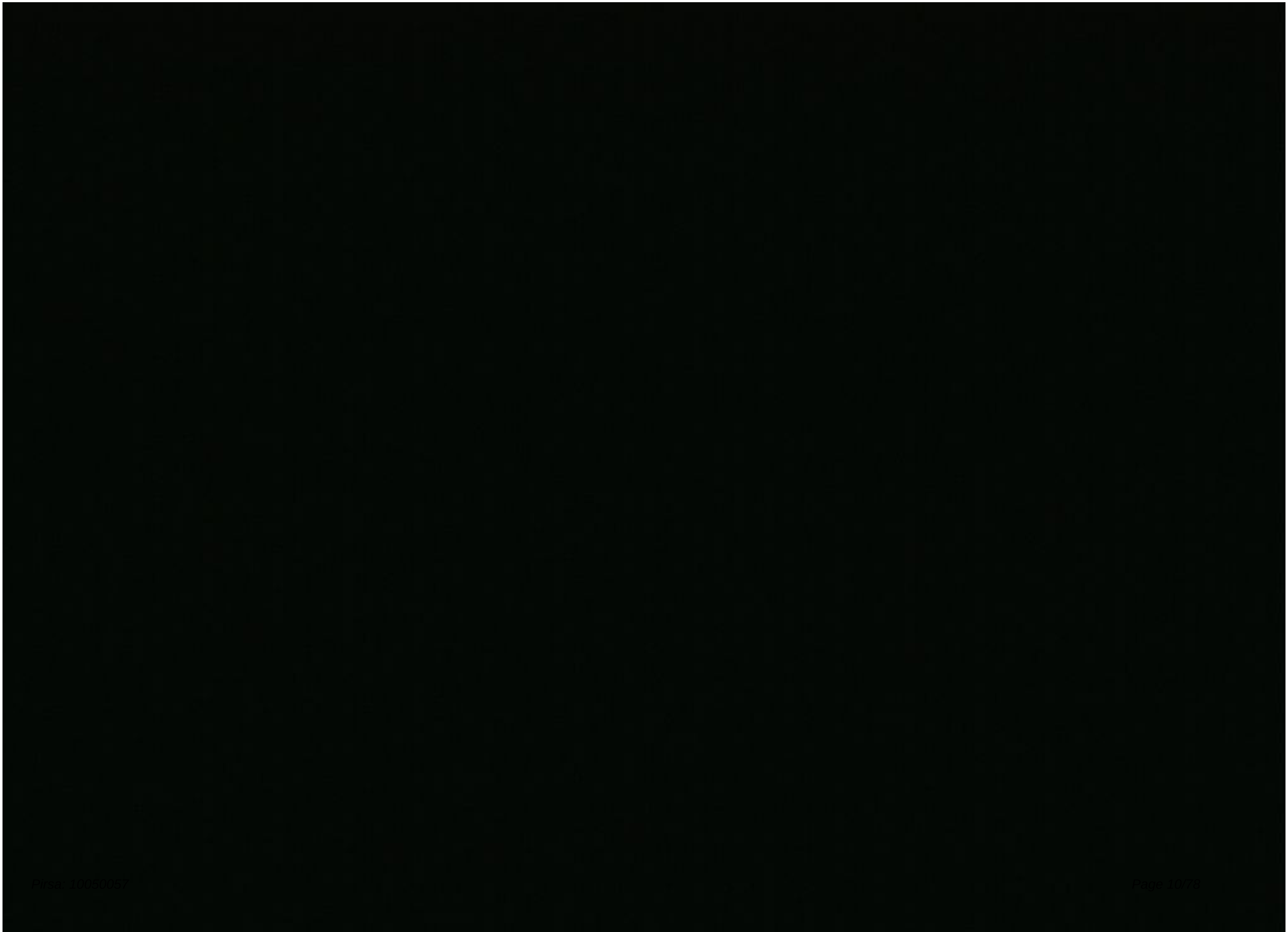
4) *Elementary properties of inference devices*

KNOWLEDGE OPERATORS

1) Laws are patterns, relating "events I know"

2) What does it mean to "know" an event?





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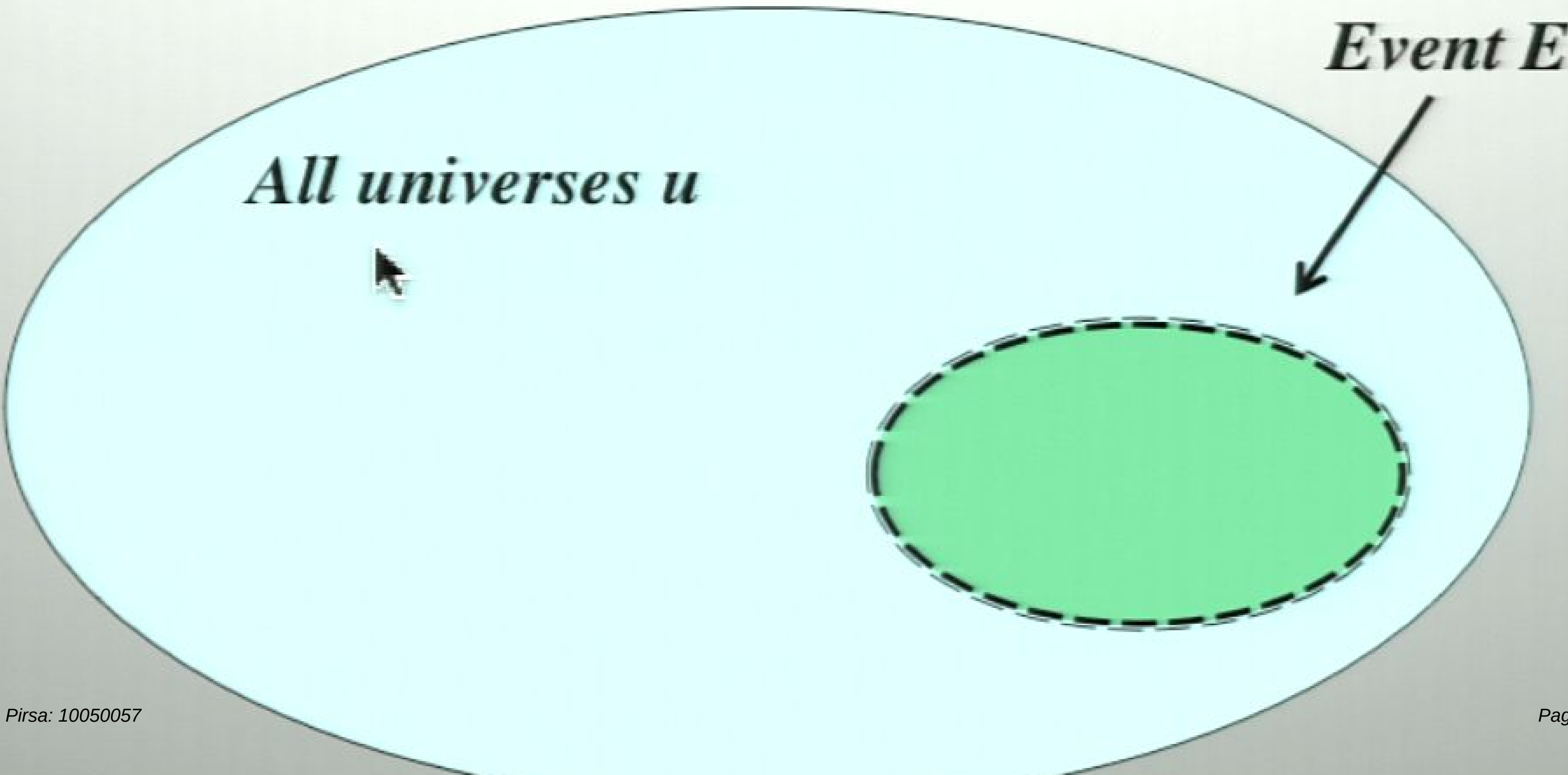

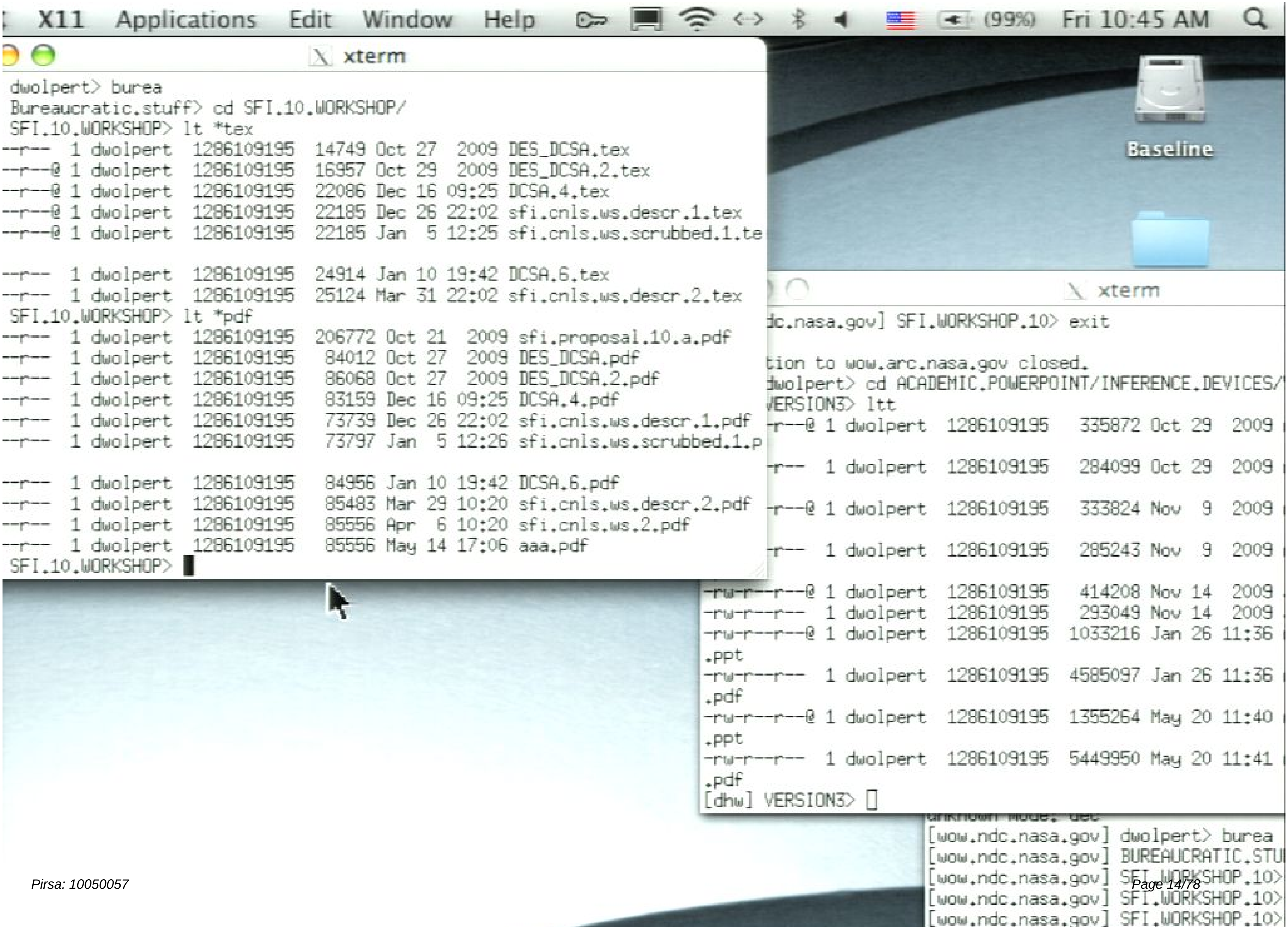
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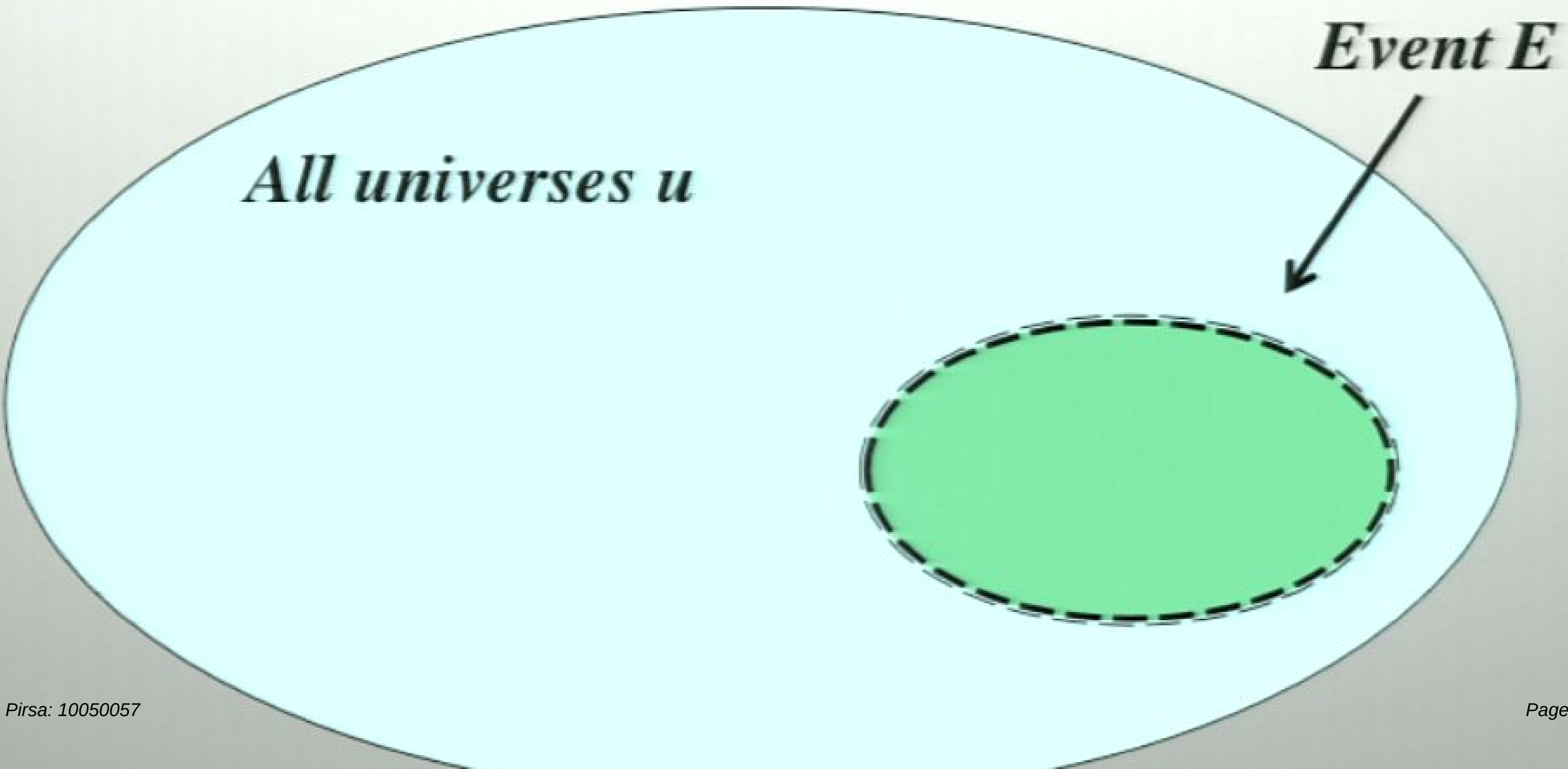
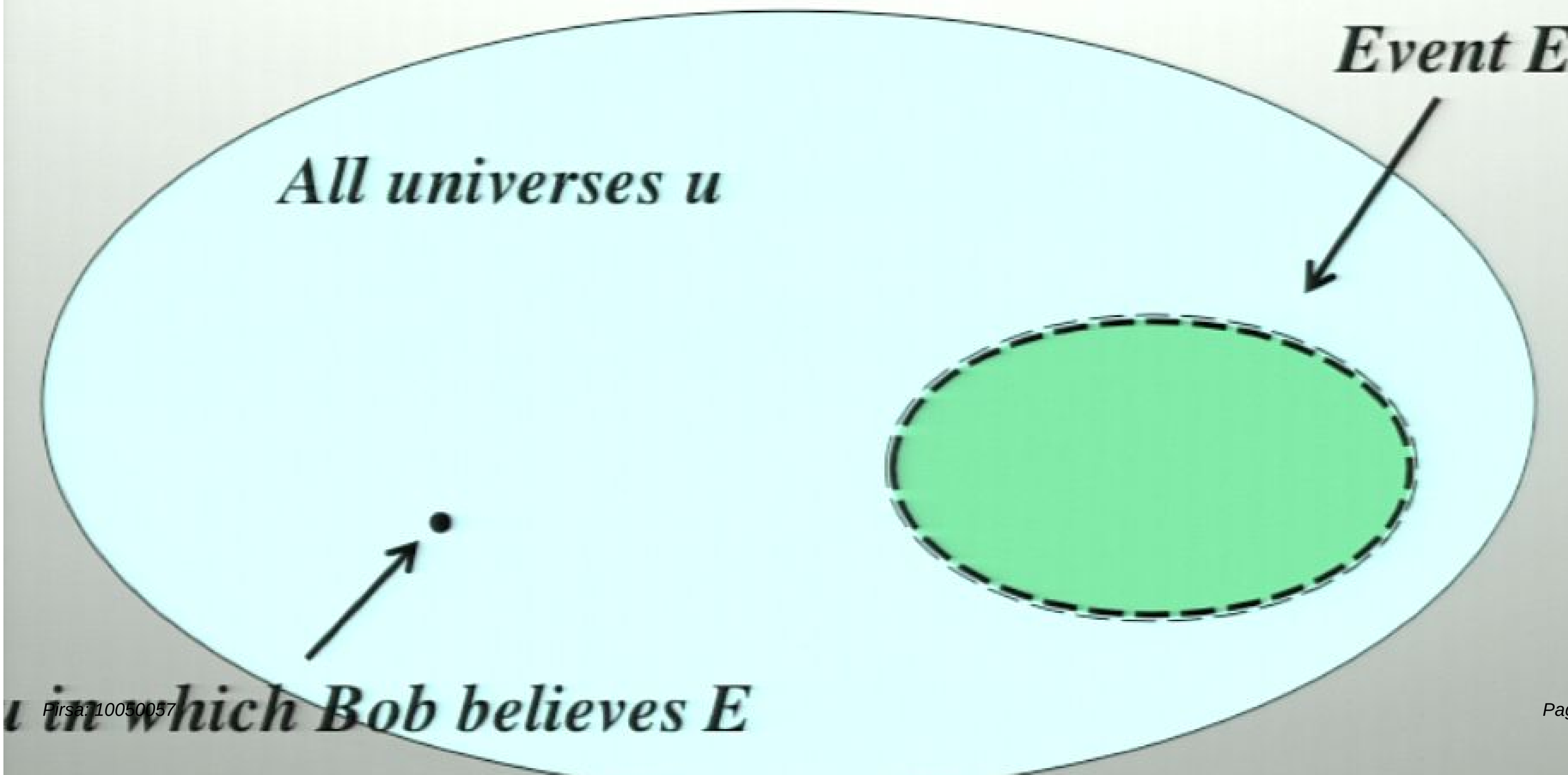


KNOWLEDGE OPERATORS

1) At indicated u, Bob has belief that he's in E.

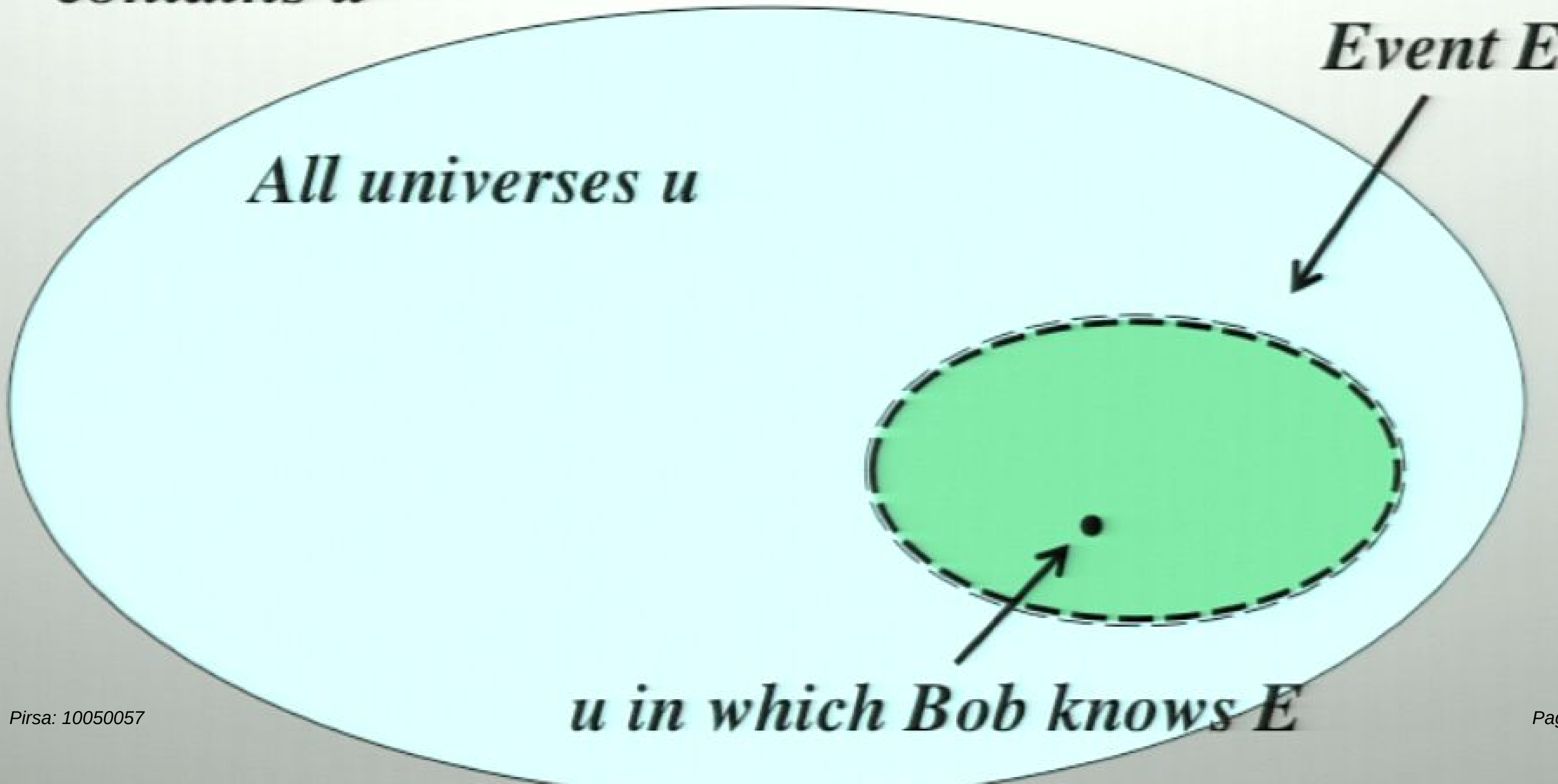
2) Belief is a function from u to subsets of $\{u\}$



KNOWLEDGE OPERATORS

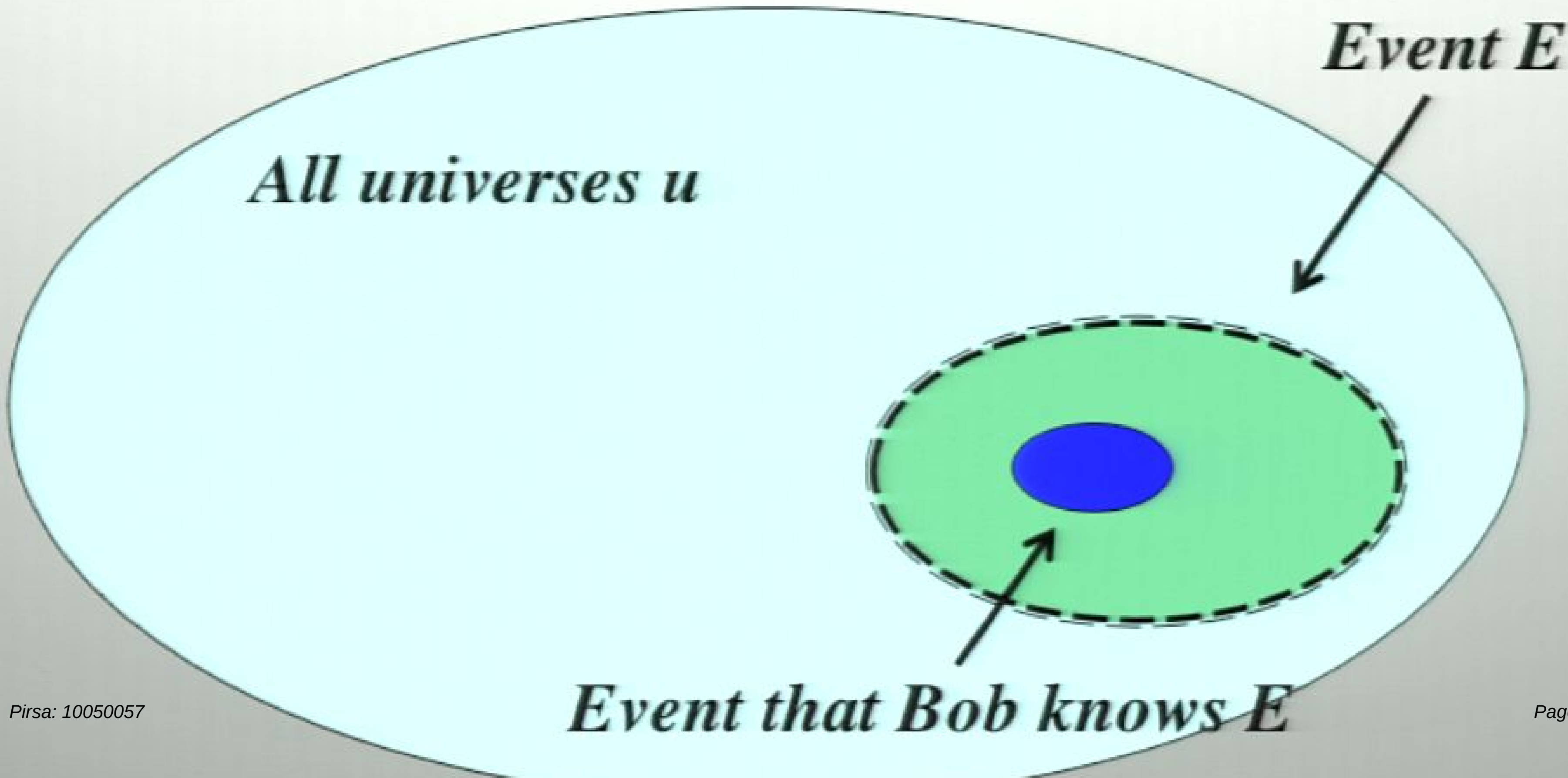
1) *At indicated u, Bob* **knows** *he's in E.*

2) ***Knowledge is a belief function where the image of u contains u***



KNOWLEDGE OPERATORS

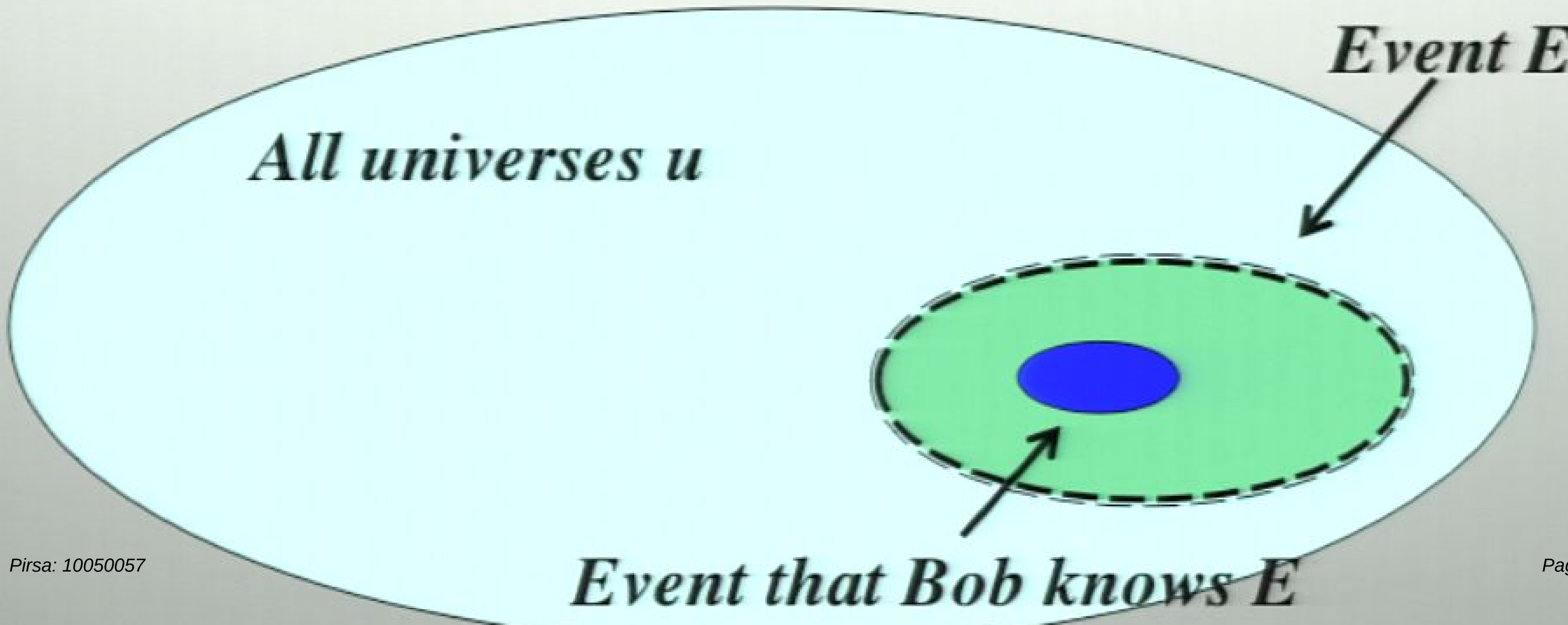
1) Throughout blue region, Bob knows he's in E.
(At other u in E, Bob instead knows some E' that overlaps E.)



KNOWLEDGE OPERATORS

1) *Need* **physical manifestation** *of such knowledge, if Bob's having it involves the laws of nature.*

2) *So need Bob to be able to* **physically answer** *questions about what he knows.*



KNOWLEDGE OPERATORS

1) What does it means for Bob to be able to physically answer questions about what he knows?

2) To formalize this, analyze physical phenomena where Bob knows an event.

3) These are phenomena where information <u>outside</u> Bob gets <u>inside</u> Bob.

4) Examples:

- Observation
- Prediction
- Memory
- Control

ROADMAP

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EXAMPLE OF PHYSICAL KNOWLDEGE: OBSERVATION

1)Present a stylized example of observation.

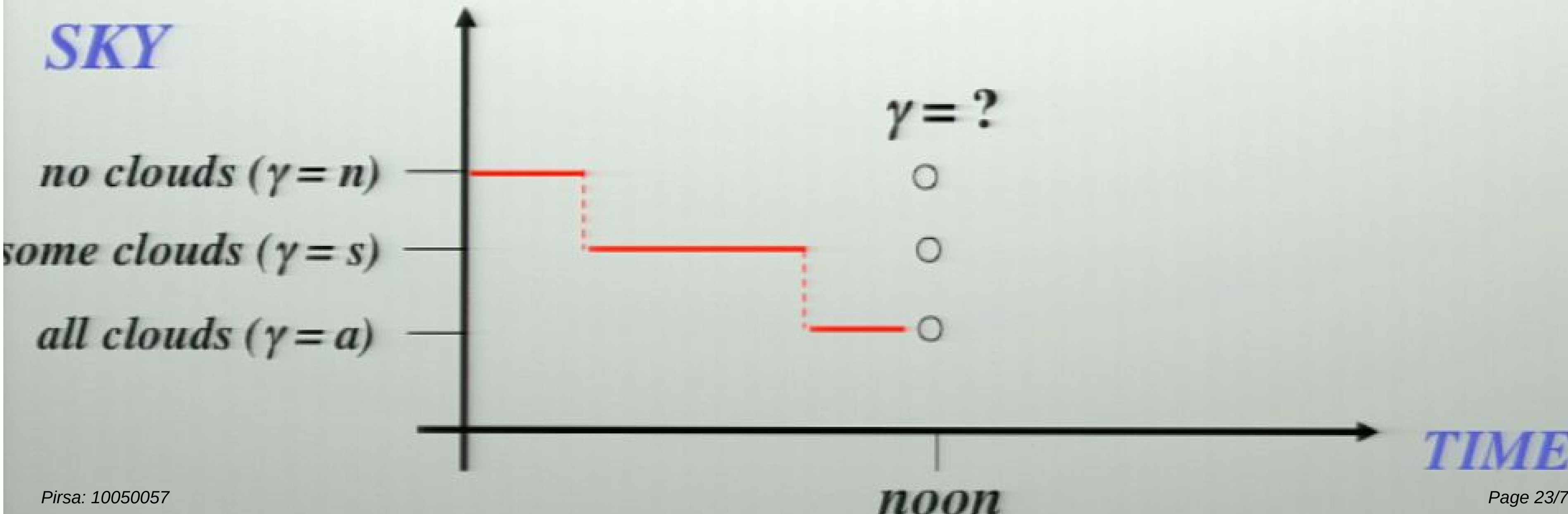
2)Emphasize features of that example found in all "observations"

3)Why those features are always found in observations:

Without those features, the observation conveys no semantic information

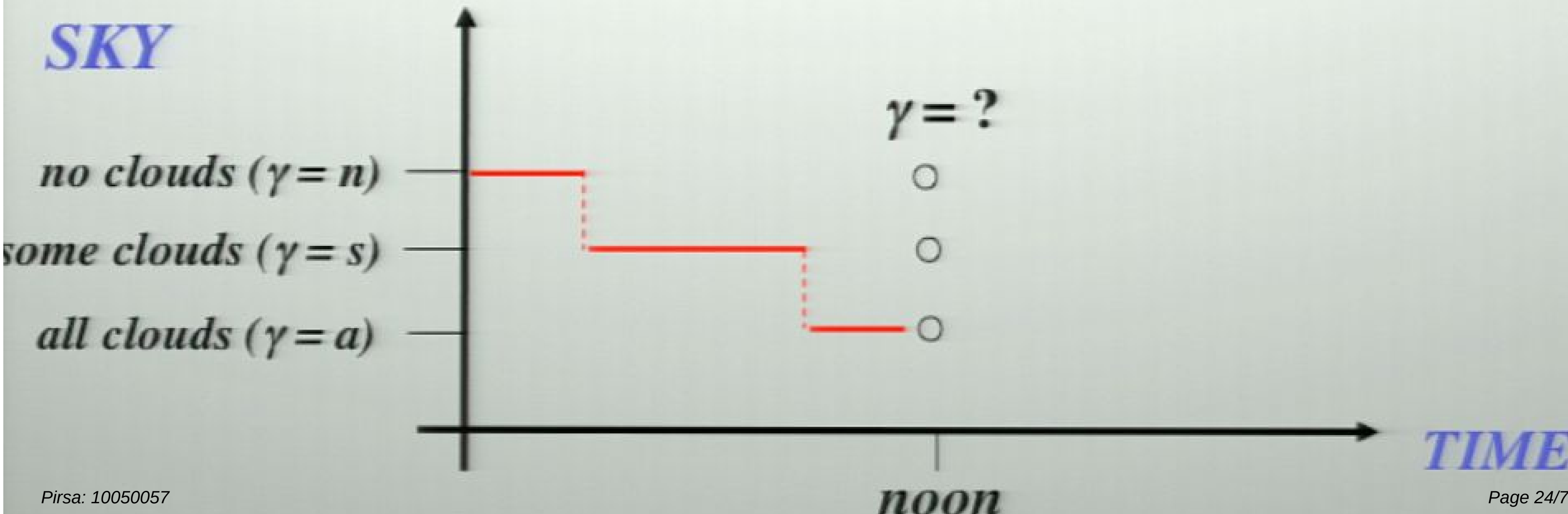
OBSERVATION

- *Want to observe γ, state of sky at noon tomorrow*



OBSERVATION

- *Bob claims to be able to make that observation*



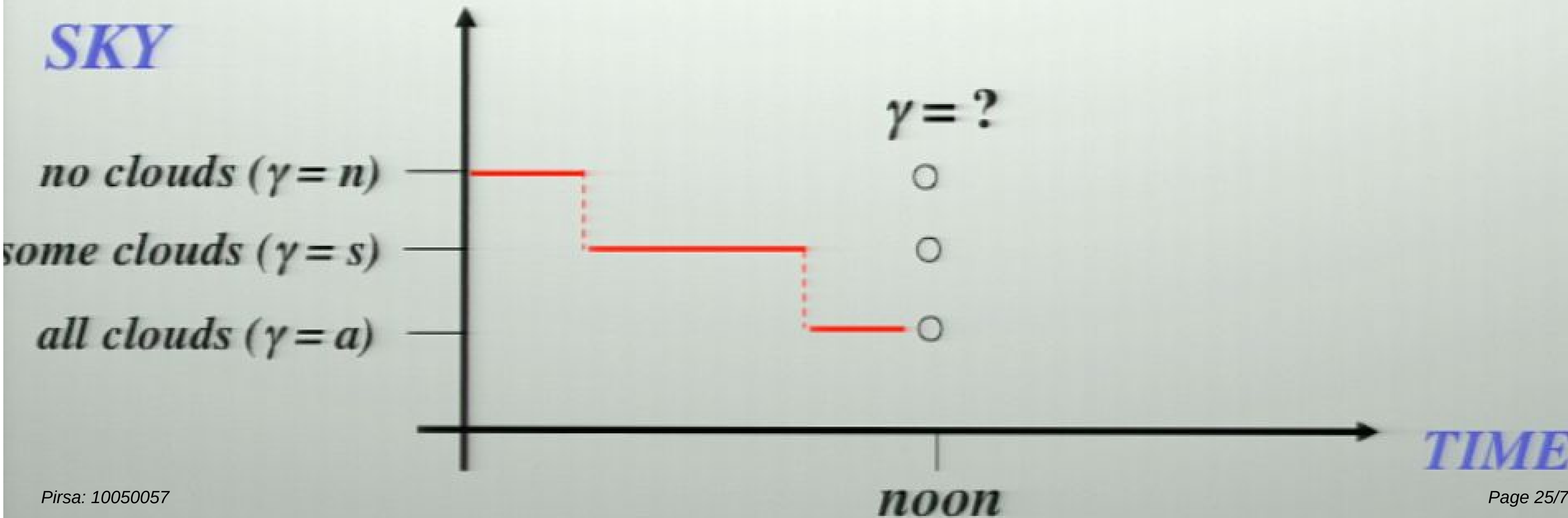
OBSERVATION

If Bob's claim is true, he can correctly answer three questions that could be posed to him:

i) *Does* $\gamma =$ *'n'? (Yes / no)*

ii) *Does* $\gamma =$ *'s'? (Yes / no)*

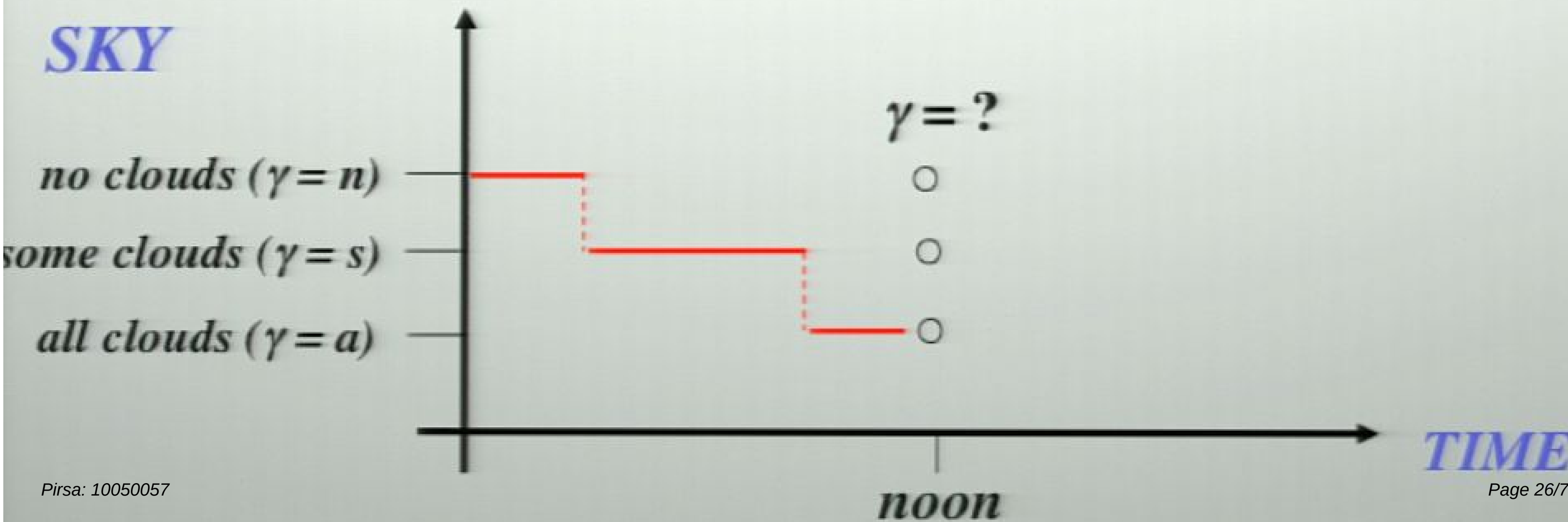
iii) *Does* $\gamma =$ *'a'? (Yes / no)*



OBSERVATION

What does this mean physically?

- *Restrict attention to universes where Bob and the sky exist; Bob considers one of the three binary questions; then observes γ; then gives his honest answer to that question.*



OBSERVATION

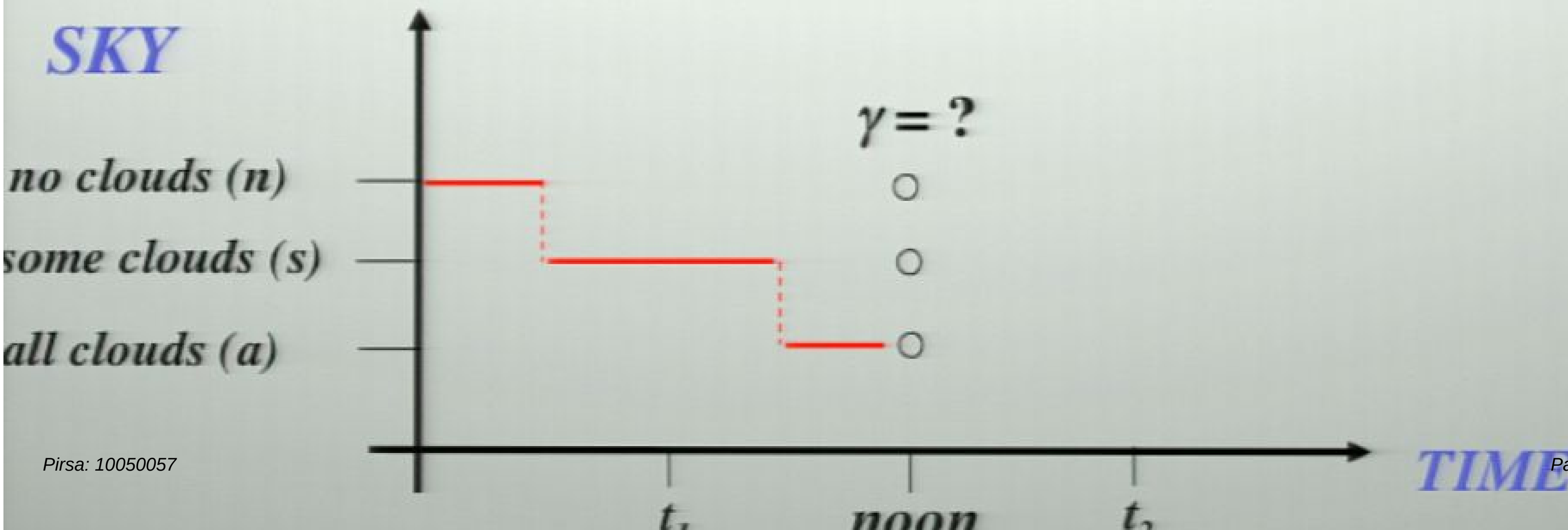
Bob can observe γ if for each of the three questions, q,

The universe has property x_q:

"At some t_1 < noon Bob considers q"

$$\Rightarrow$$

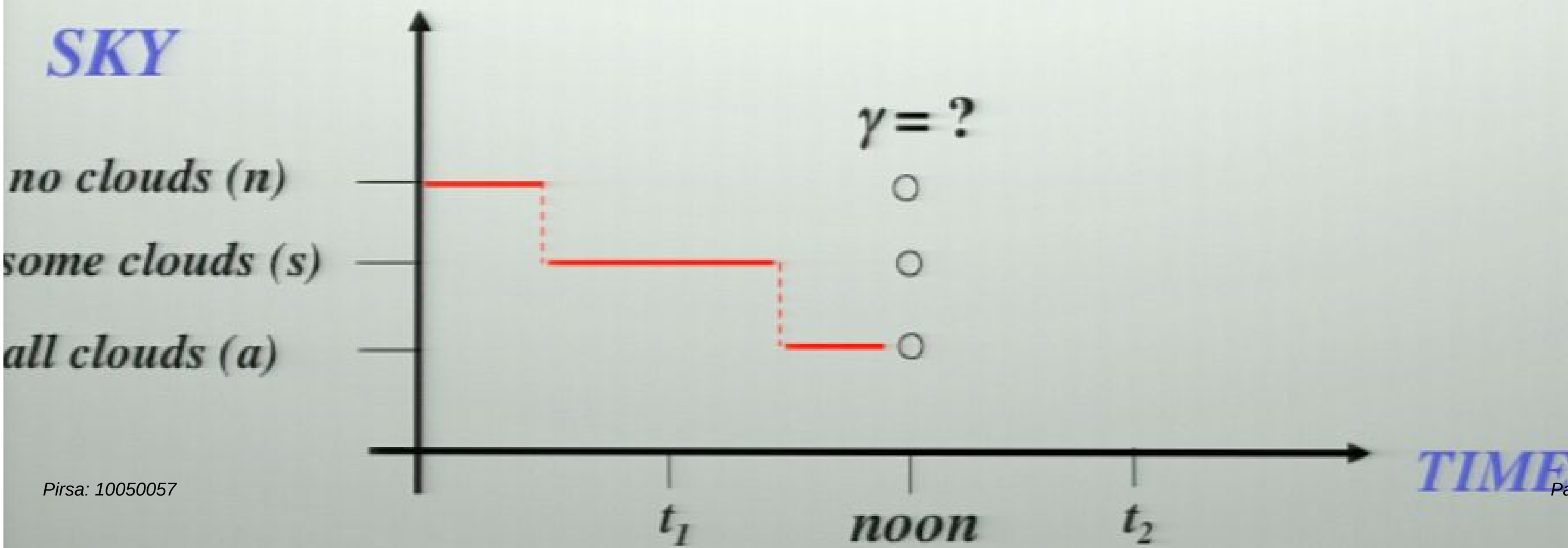
y, the binary answer Bob gives at some t_2 > noon, equals correct answer to q



OBSERVATION

U = {all universe-histories consistent with physics, in which Bob and the sky exist; at t_1 Bob considers a q; then observes γ; then gives honest answer to that q}

State of sky at noon is fixed by $u \in U$, the actual universe-history. So $\gamma = \Gamma(u)$ for some function Γ



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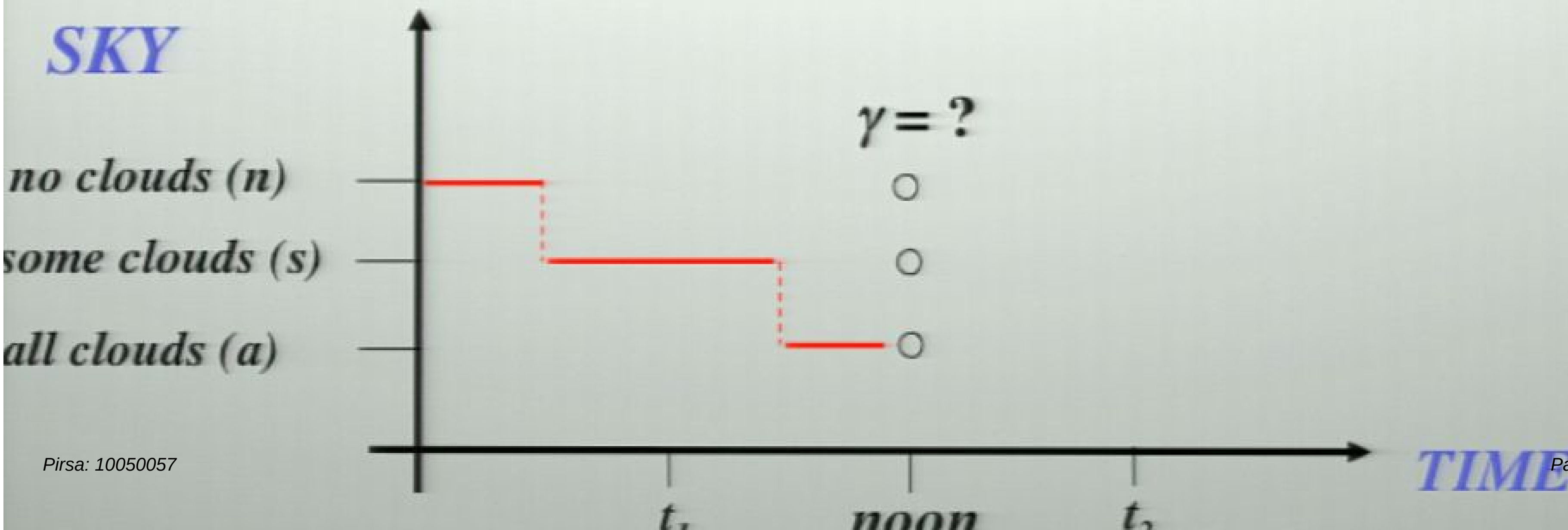

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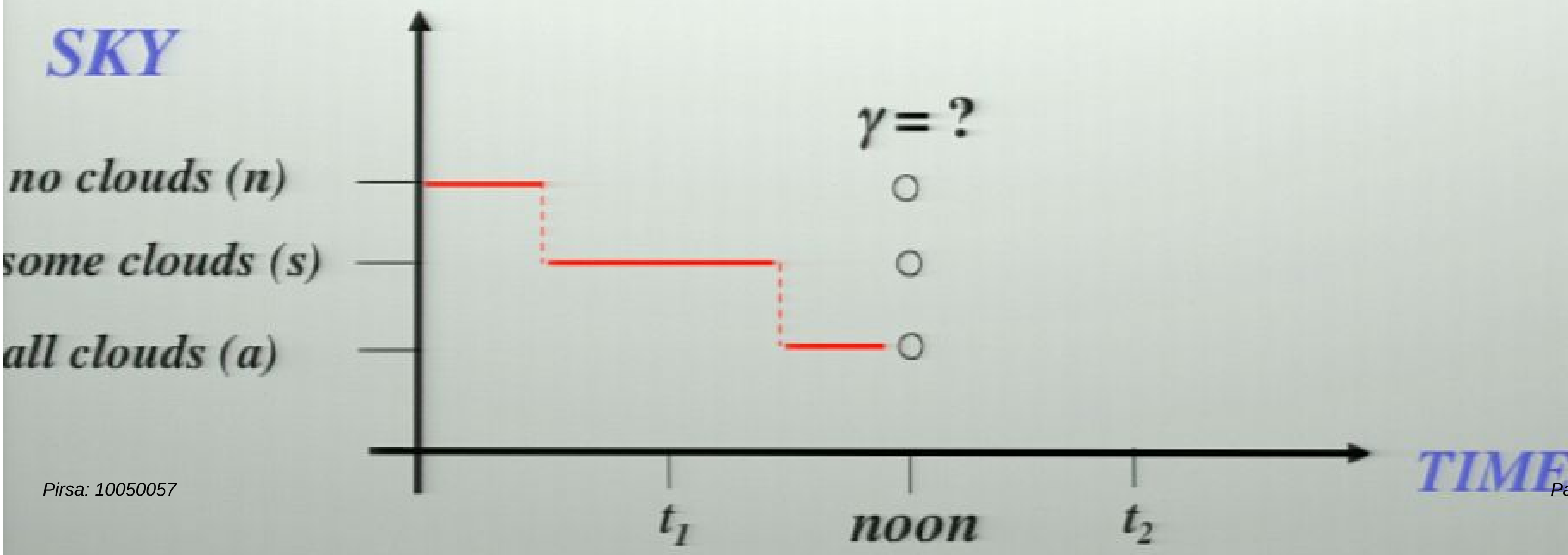
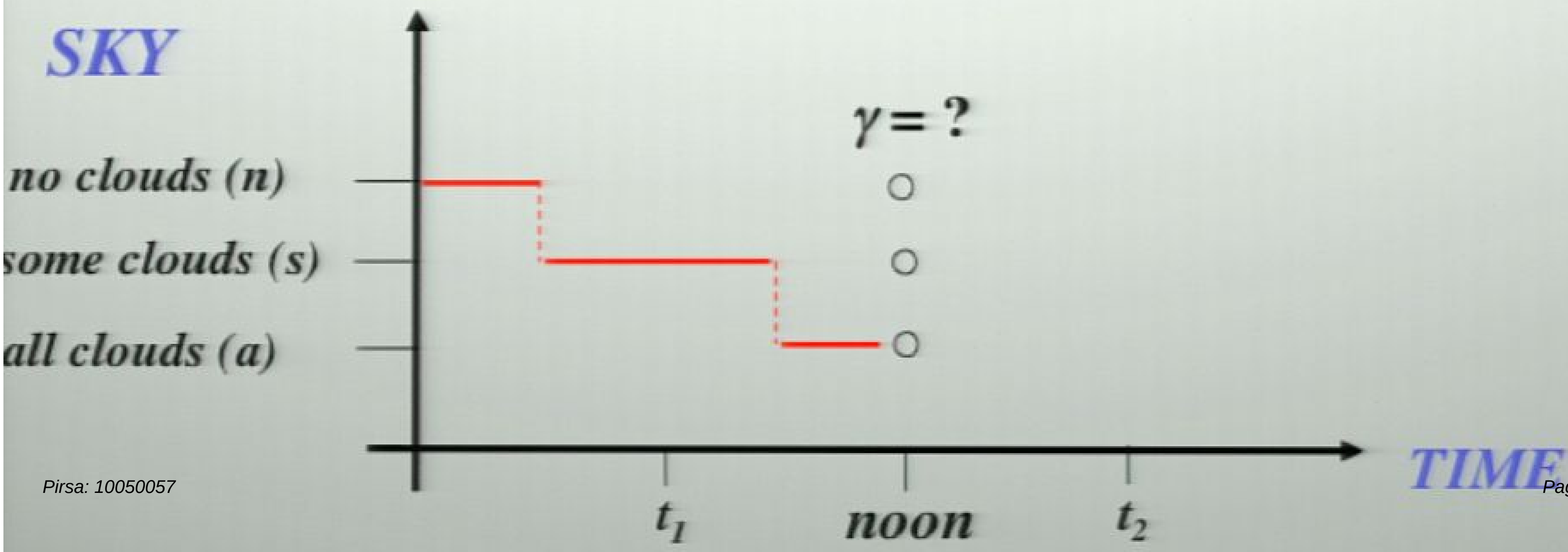


OBSERVATION

U = {all universe-histories consistent with physics, in which Bob and the sky exist; Bob considers a q; then observes γ; then gives honest answer to that q}

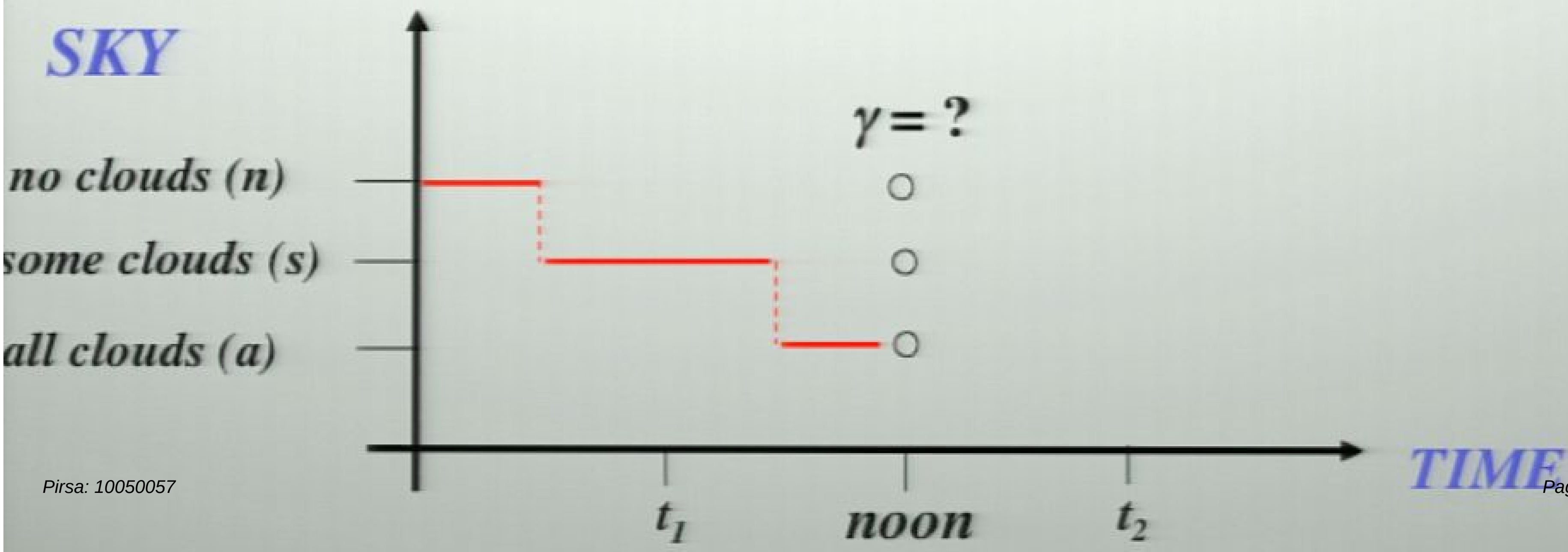
The question Bob considers at t_1 is given by the actual universe-history $u \in U$, so $x = X(u)$ for some function X



OBSERVATION

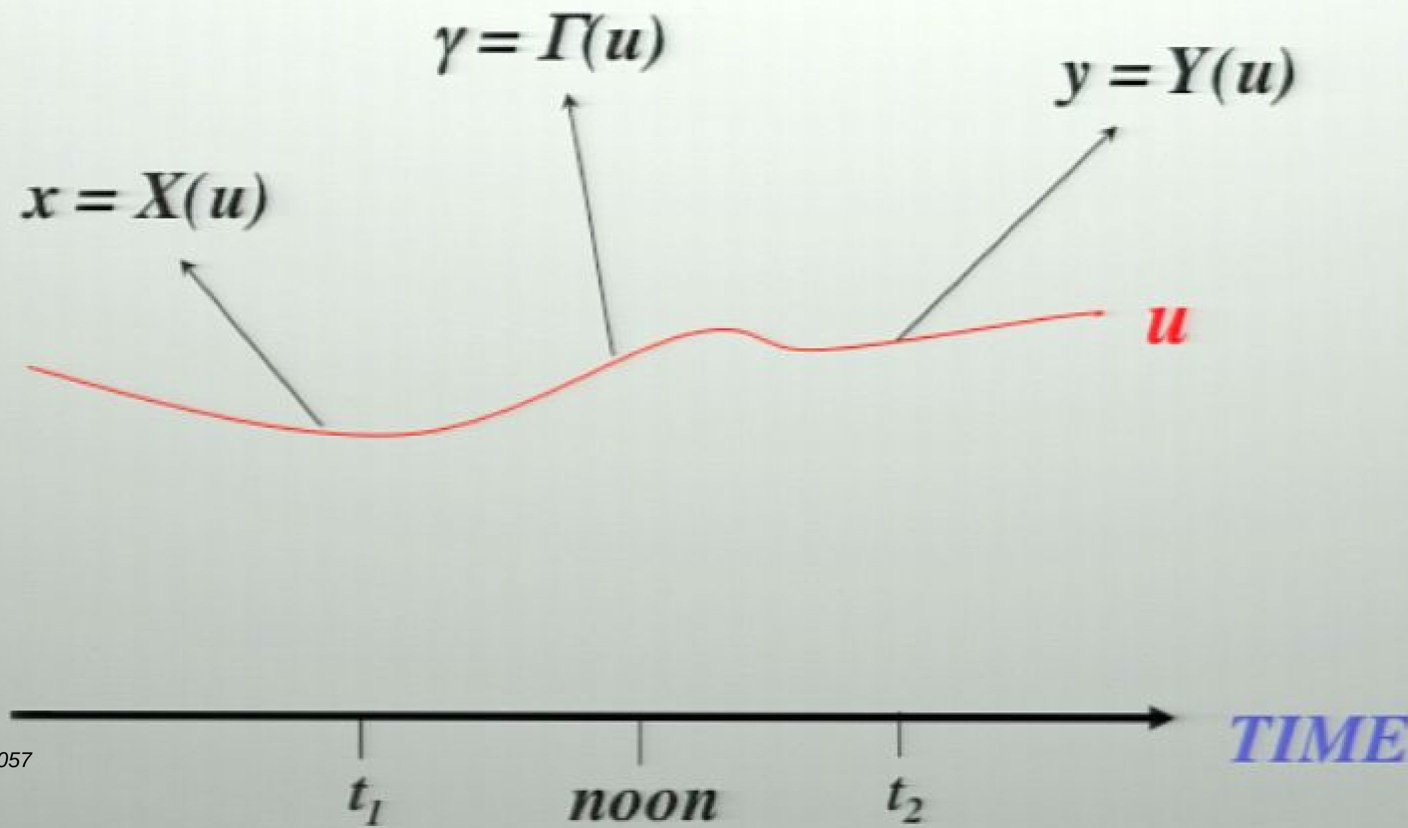
U = {all universe-histories consistent with physics, in which Bob and the sky exist; Bob considers a q; then observes γ; then gives honest answer to that q}

Bob's answer at t_2 is given by the actual universe-history $u \in U$, so $y = Y(u)$ for some function Y



OBSERVATION

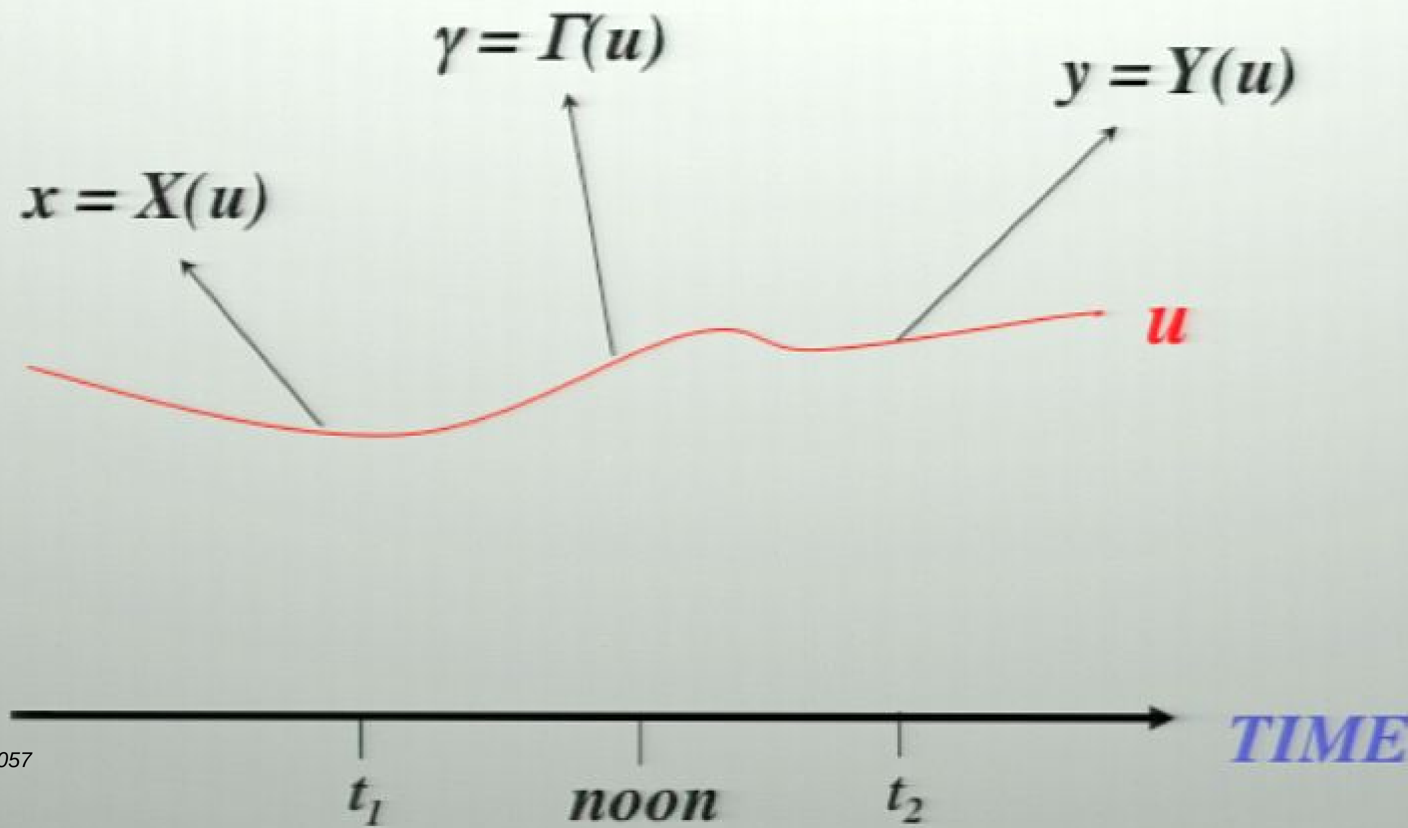
- $\gamma \in \{\text{'n'}, \text{'s'}, \text{'a'}\}$ = sky at noon = $\Gamma(u)$
- x = (what q Bob considers at t_1) = $X(u)$
- y = (Bob's answer at t_2) = $Y(u)$



OBSERVATION

So:

- For each of three binary questions q_γ,
- $\exists\, x$ such that
- $X(u) = x \;\; \Rightarrow \;\; Y(u) = q_\gamma(\Gamma(u))$

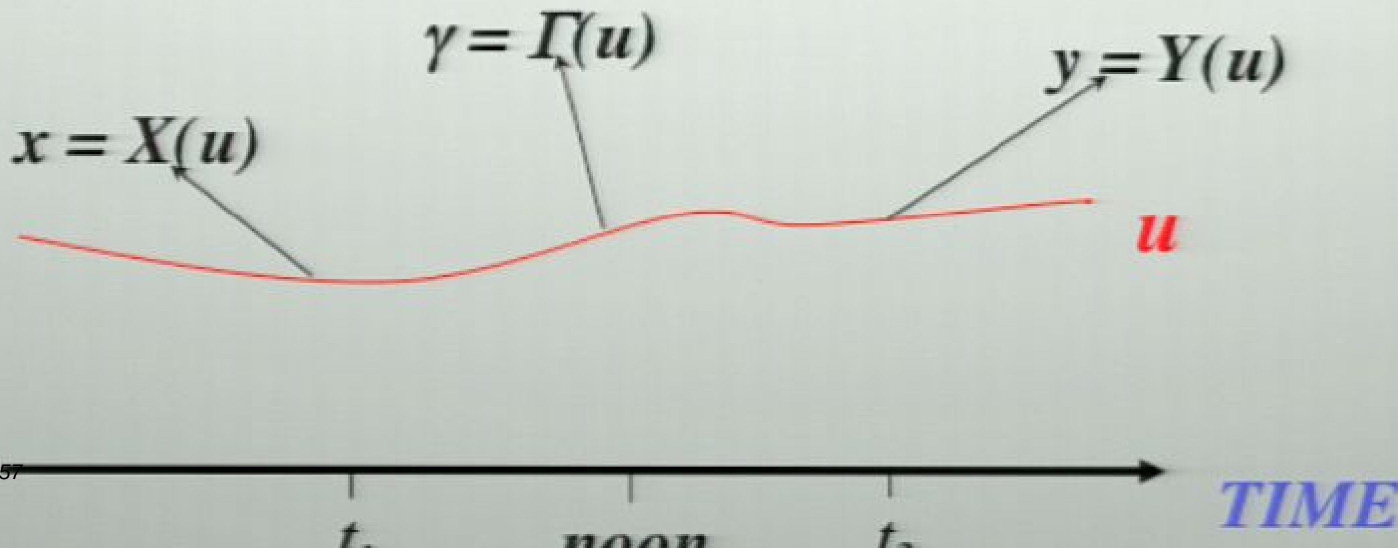


OBSERVATION

- *Nothing about observation* **process;** *all about what it means to* **successfully observe.**
- *The 'what' of observation, not the 'how'.*

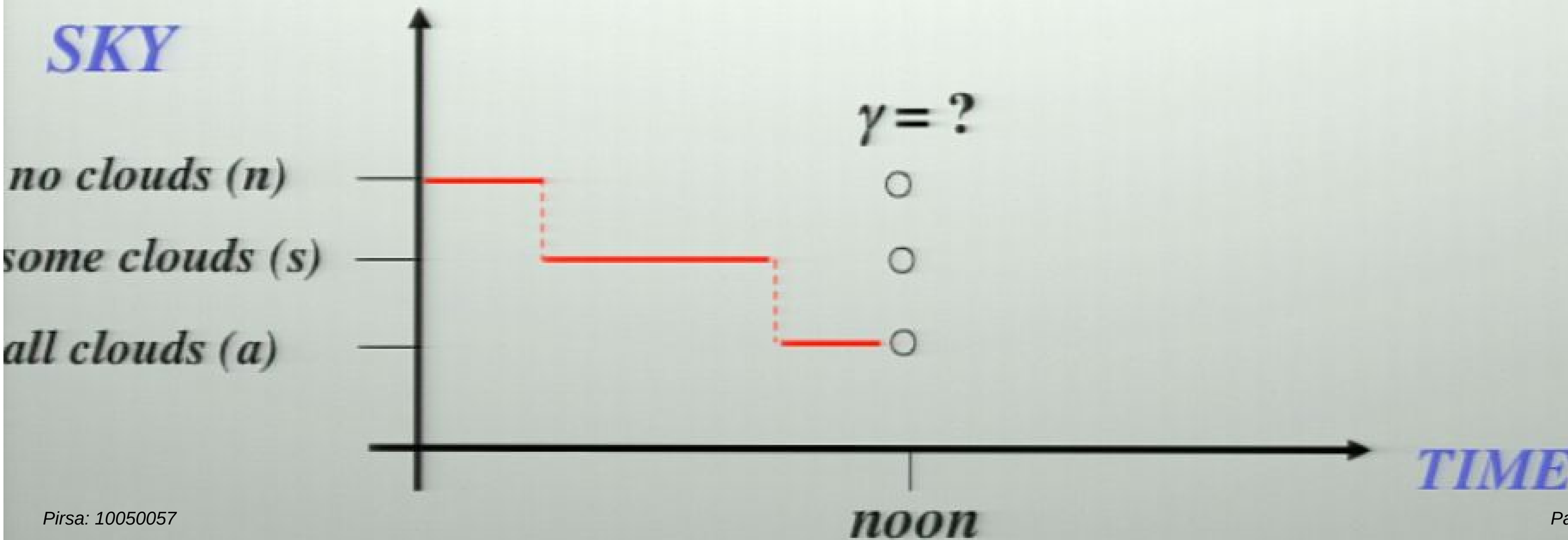
For each of three binary questions q_γ,

$$\exists\, x \text{ such that } X(u) = x \quad \Rightarrow \quad Y(u) = q_\gamma(\Gamma(u))$$



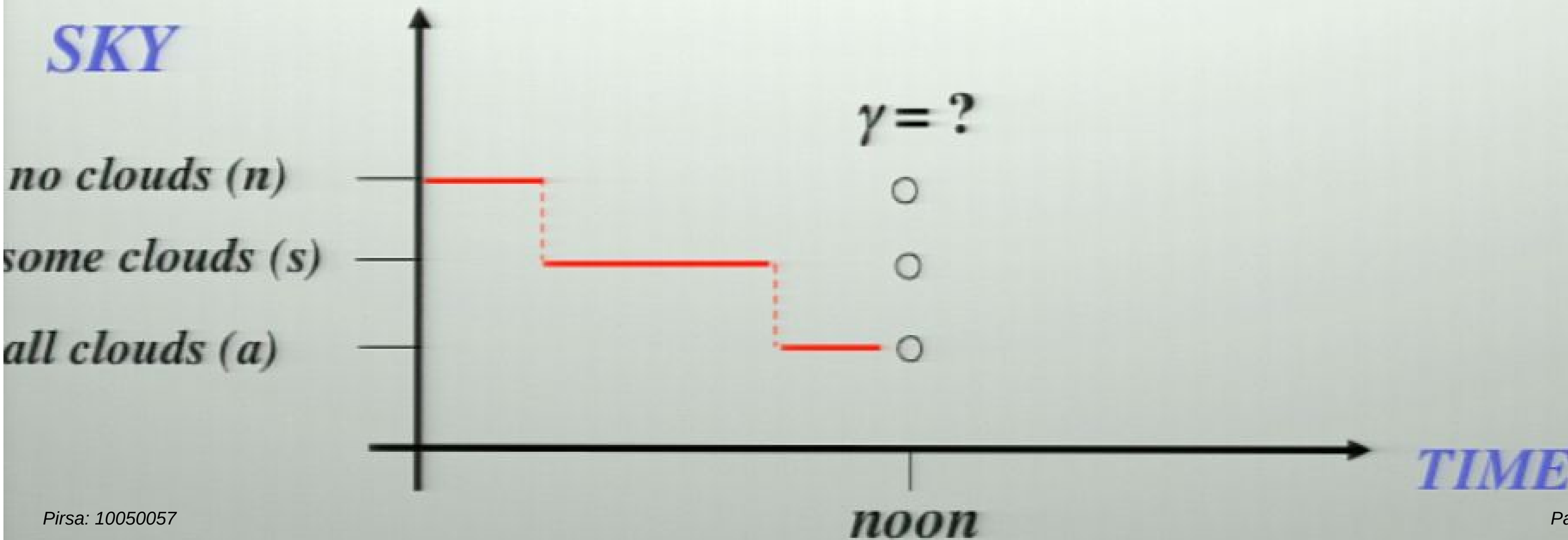
PREDICTION

- *Want to predict γ, state of sky at noon tomorrow*



PREDICTION

- *Bob claims to have a laptop that he can program to make that prediction*



PREDICTION

If Bob's claim is true, he will be able to correctly answer three questions:

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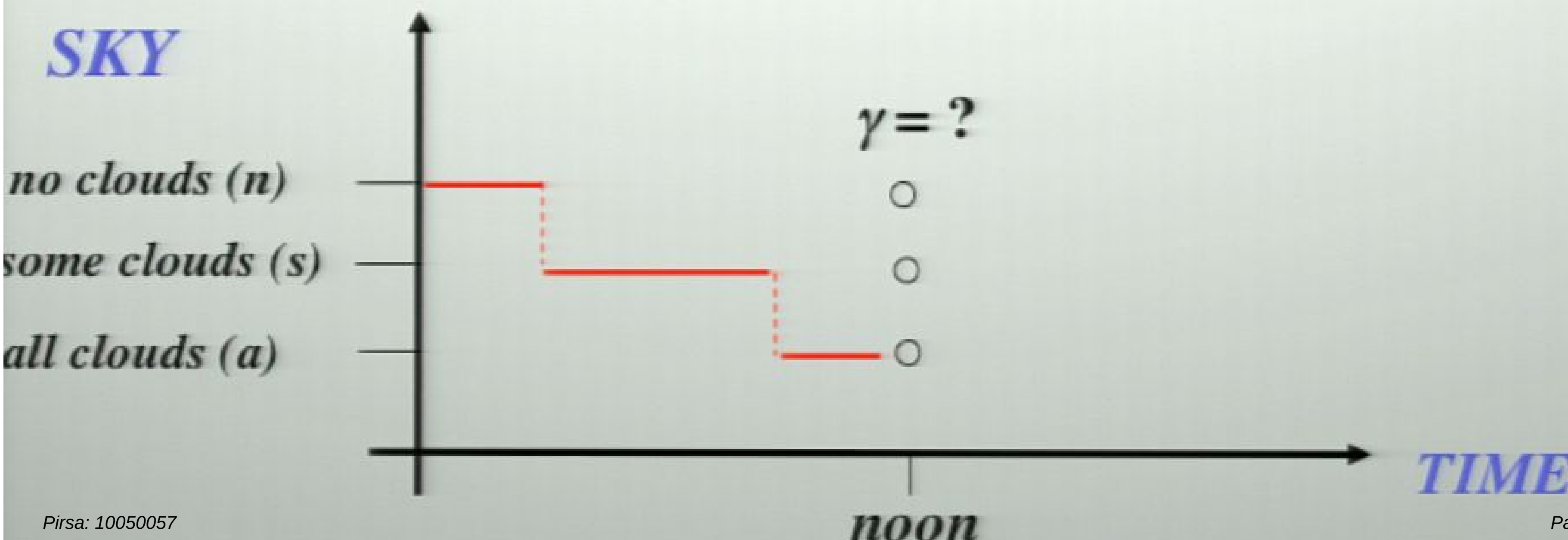

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Bob can predict γ if for each of the three questions, q,

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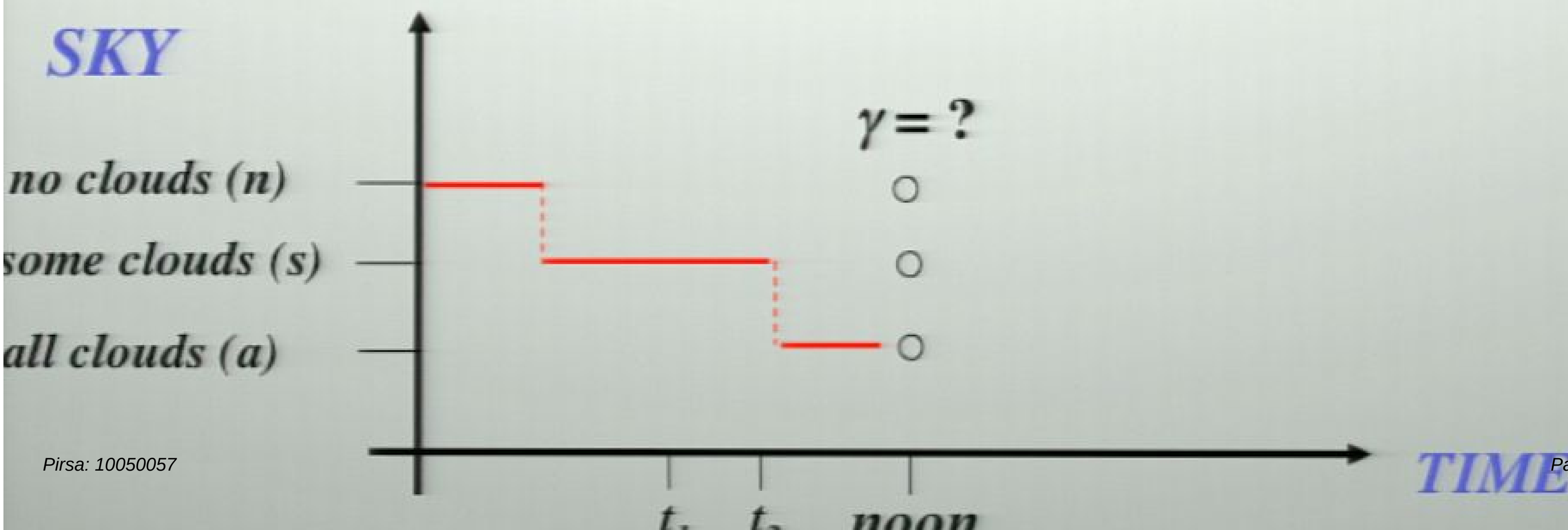

PREDICTION

- $\gamma \in \{$'n', 's', 'a'$\}$ = sky at noon = $\Gamma(u)$
- x = (laptop program at t_1) = $X(u)$
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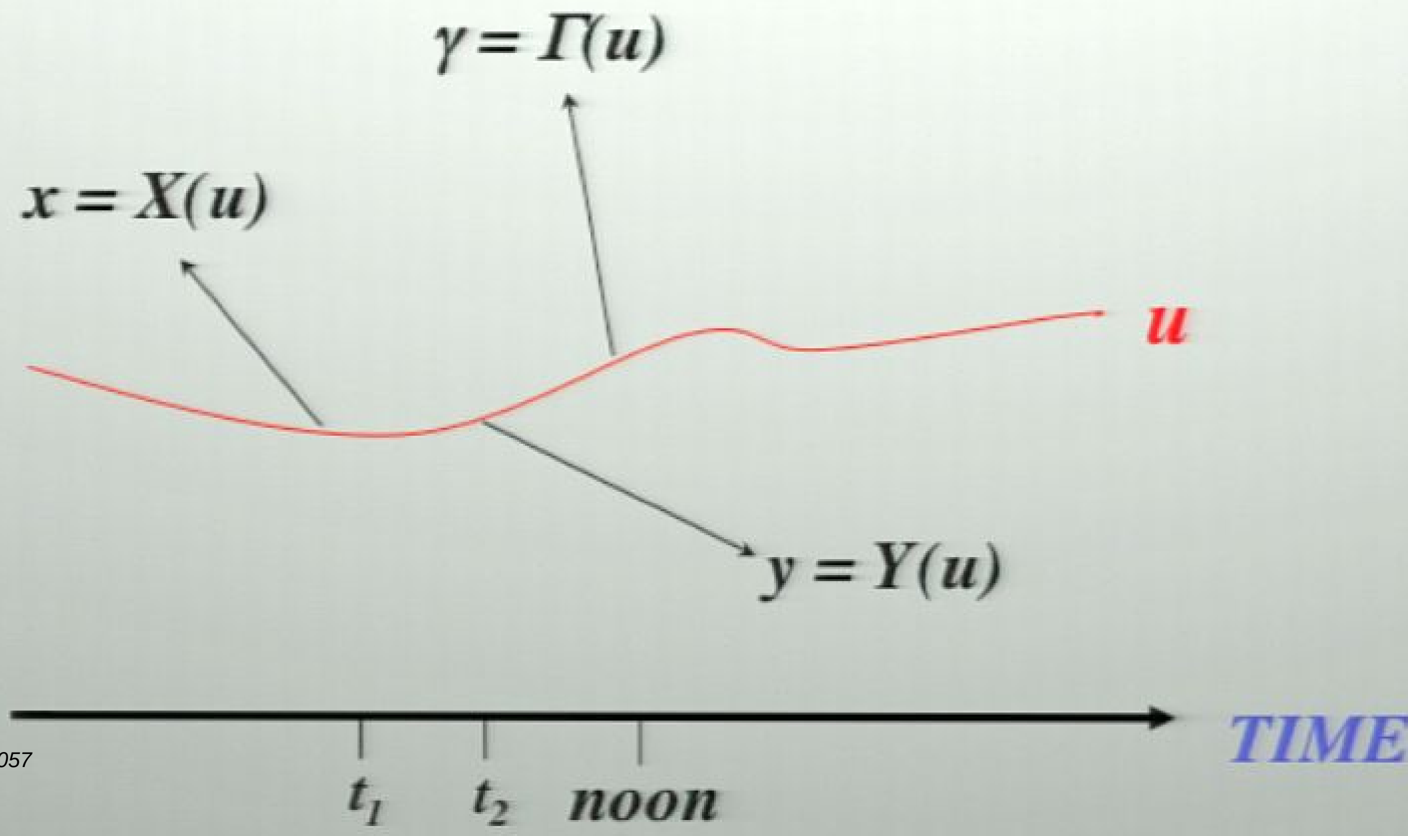

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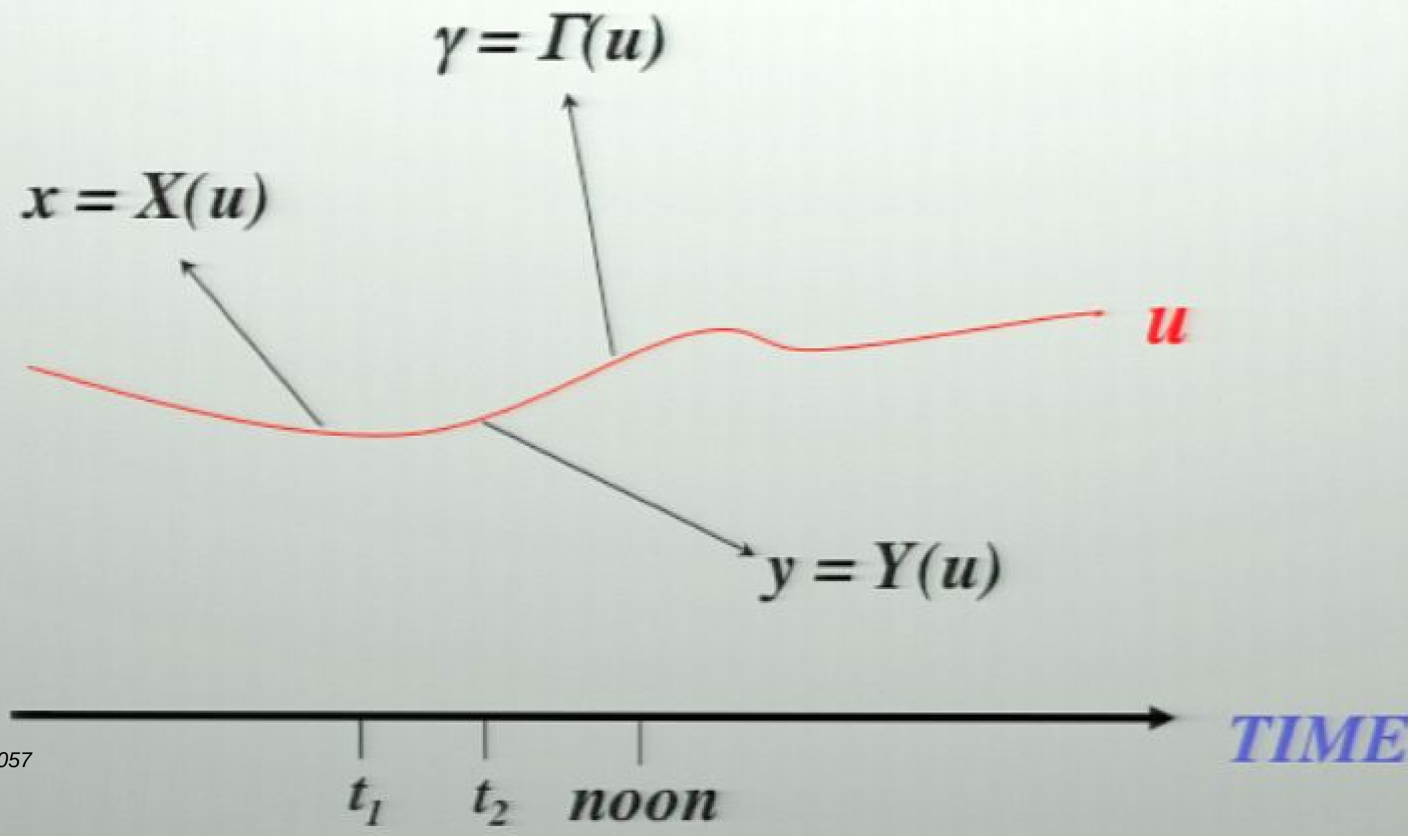

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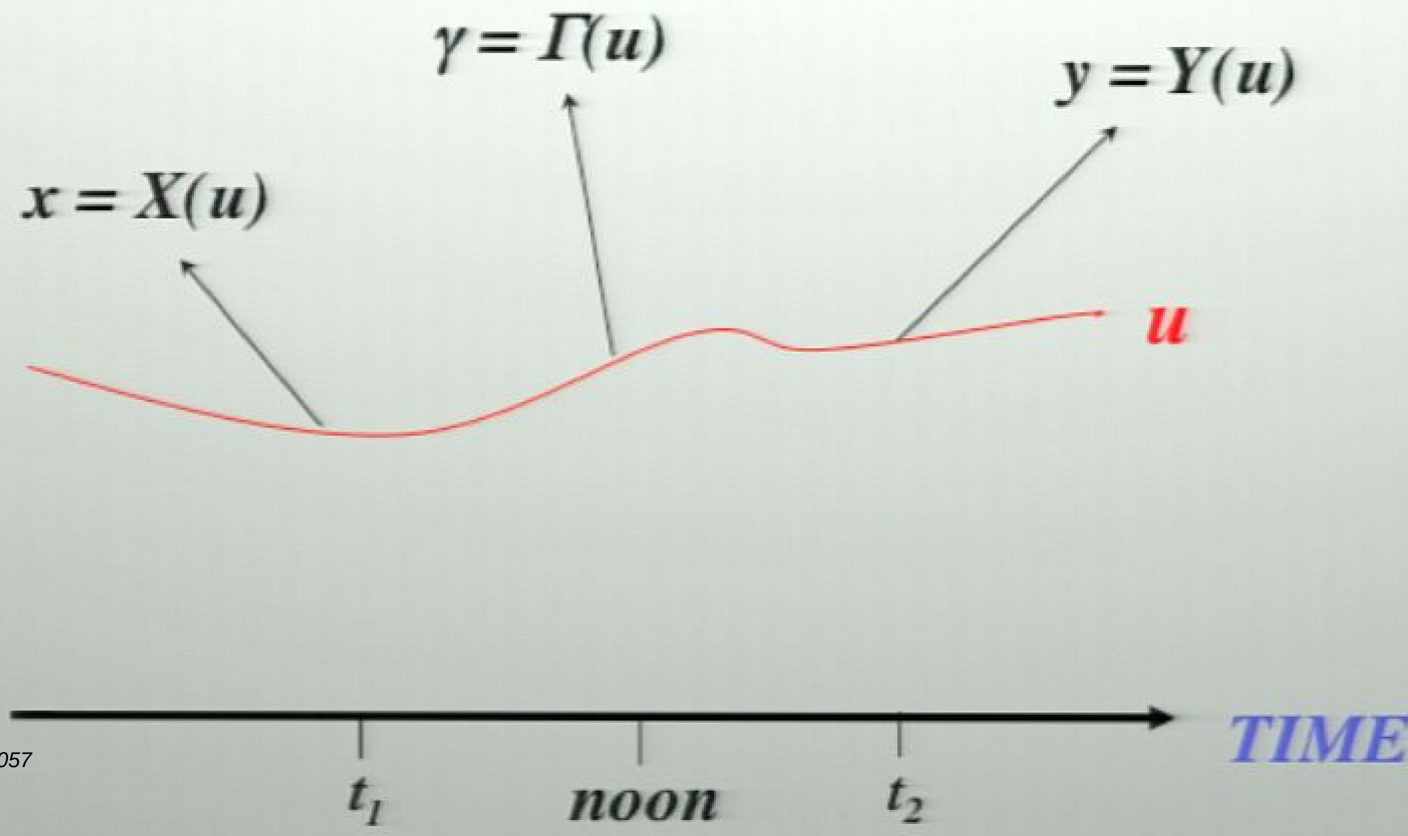

KNOWLEDGE

- *More generally, if at some time, "Bob knows the state of the sky at noon", γ, then he can answer three questions:*
  - *i) Does γ = 'n'? (Yes / no)*
  - *ii) Does γ = 's'? (Yes / no)*
  - *iii) Does γ = 'a'? (Yes / no)*

- *Note no chronological ordering. Just*
  X (what question Bob considers),
  Y (his answer),
  Γ (the sky's actual state at noon),
  - *all functions of $u \in U$*

OBSERVATION

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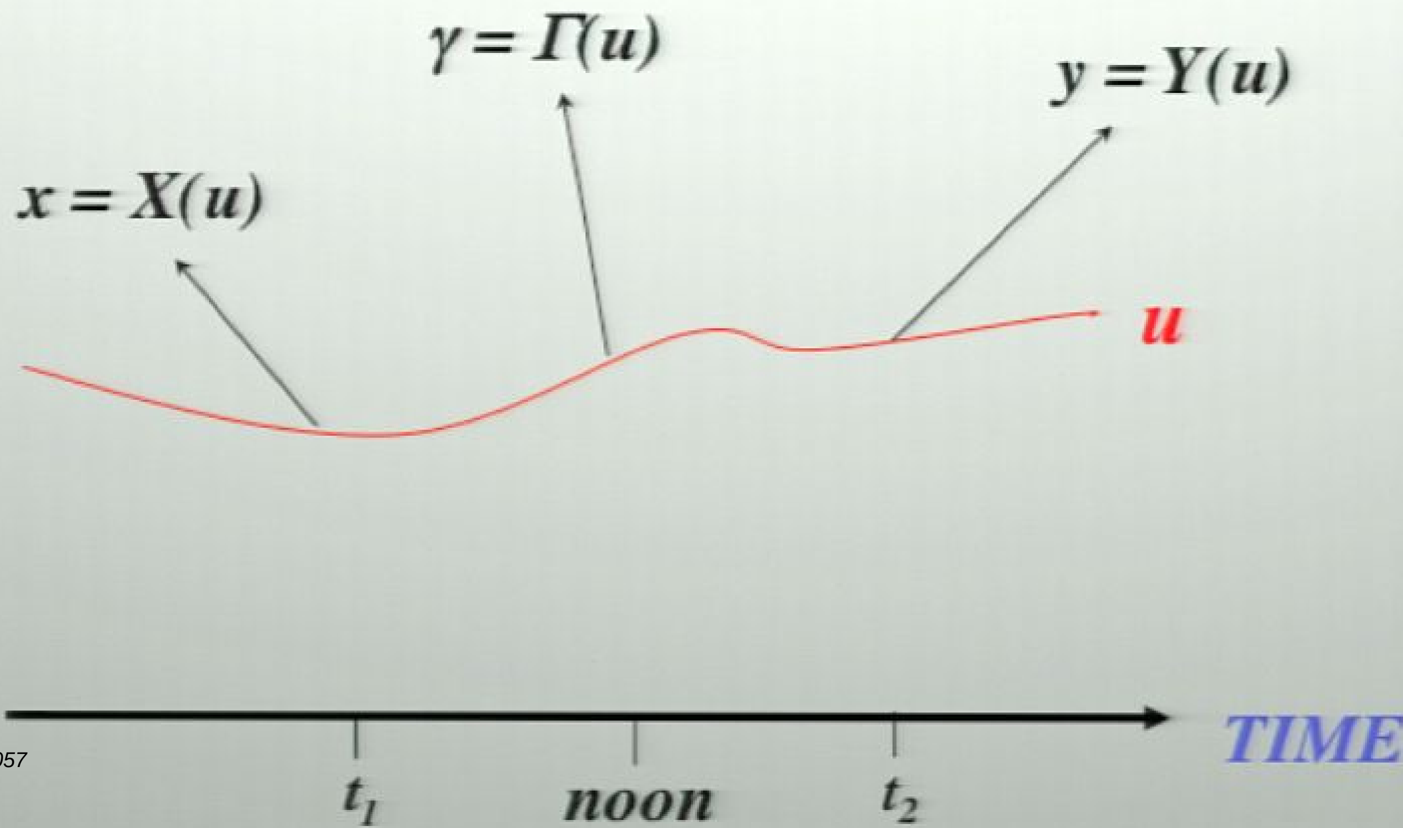

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  - all functions of $u \in U$

INFERENCE DEVICES

- *An inference device is any two functions (X, Y) over U, where range of Y is binary.*

- *An inference device (X, Y) (weakly) infers a function Γ over U iff*

$$\forall \gamma \text{ in } \Gamma\text{'s range},$$
$$\exists x \text{ such that } X(u) = x \Rightarrow Y(u) = q_\gamma(\Gamma(u))$$

- *A necessary condition to say that (X, Y) "observes", "predicts", or "knows" Γ is that (X, Y) weakly infers Γ.*

- *No claims of sufficiency; observation, prediction, knowledge, etc. involve much more than just weak inference.*

- *But even requiring weak inference restricts observation, prediction, and knowledge.*

INFERENCE DEVICES

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- *An inference device* (X, Y) (weakly) infers *a function* Γ *over* U *iff*

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- *Note the use of counterfactual questions, asking whether* $\Gamma(u)$ *has a value that it does not have. Analogous to the use of intervention to define causality in Bayes nets.*

- *Contrast inference with Aumann-style "knowledge operators"*

INFERENCE DEVICES

- ***Advantages of using binary questions:***

  i) Formalism doesn't change if range of Γ changes

  ii) Device never need give value $\Gamma(u)$, only confirm/reject suggested $\Gamma(u)$'s. (Cf. computational complexity)

  iii) Formalizes semantic information (contrast Shannon)

INFERENCE DEVICES

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INFERENCE DEVICES TERMINOLOGY

1) **Setup** *function X over U*

2) **Conclusion** *binary-valued function Y over U*

3) "$(X, Y) > \Gamma$" *means* (X, Y) *weakly infers* Γ

INFERENCE DEVICES AND THE LAWS OF NATURE

1) A reality is a space U, a set of devices defined over U, and a set of functions the devices might infer.

2) So a reality is a quadruple, $(U, \{X_j, Y_j\}, \{\Gamma_i\})$.

3) As far as any device in a reality is concerned, *U is irrelevant*. It's only the inference graph relating the sets $\{X_j, Y_j\}$ and $\{\Gamma_i\}$ that matter:

The laws of Nature are patterns in the inference graph of a reality

ROADMAP

1) *Shortcomings of current impossibility results concerning laws of Nature*

↓

2) *Knowledge operators and their shortcoming*

↓

3) *Formalize mathematical structure shared by observation and prediction:* <u>inference devices</u>

↓

4) *Elementary properties of inference devices*

ELEMENTARY PROPERTIES OF INFERENCE

1) **Inference need not be transitive:**

   $(X_1, Y_1) > Y_2$ **and** $(X_2, Y_2) > Y_3$ **does not mean** $(X_1, Y_1) > Y_3$

2) **For any** Γ, $\exists$ **a device that infers** Γ**.**

3) **For any device,** $\exists$ **a** Γ **it does not infer. (*Impossibility result*)**
   - ***Intuition*: X ~ initial configuration of a Turing machine.**

     Y (a bit) ~ whether Turing machine halts or not.

     So apply Halting theorem-style reasoning

IMPLICATIONS OF IMPOSSIBILITY RESULT

1) For any simulator, there is always a prediction that cannot be guaranteed correct.

- *Laplace was wrong.*
- *Results of Pour-El et al., Fredkin et al., Moore, etc. are far narrower than this general impossibility*

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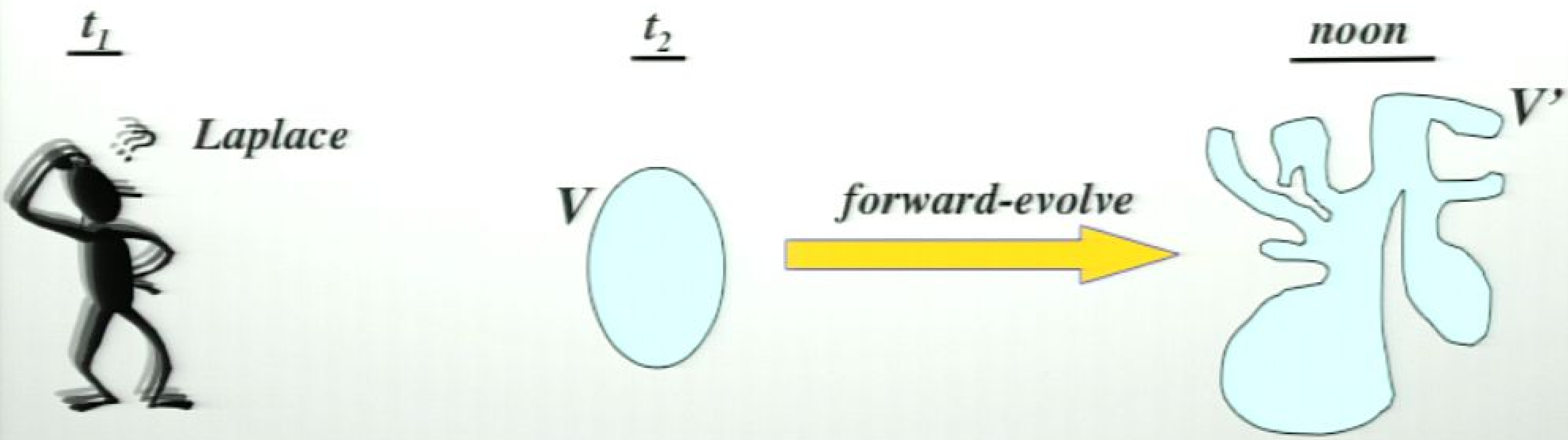

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Trivially, Laplace's answer is wrong

INFERENCE RELATIONS BETWEEN DEVICES

- **Often not interested in inference of arbitrary functions, but rather inference relation among a set of devices.**

I) Two devices (X, Y), (X', Y') are *pairwise distinguishable* iff every pair (x, x') occurs for some u

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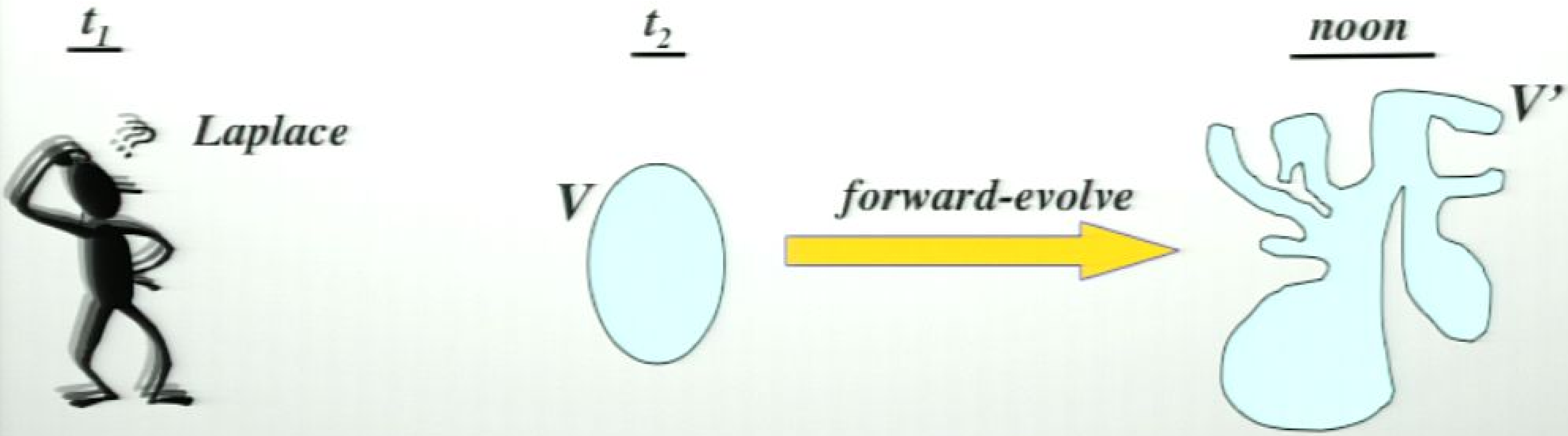

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3) If all pairs of devices from $\{C_i\}$ are pairwise distinguishable, $\exists$ at most one $k : C_k > C_j \ \forall j \neq k$. "*Monotheism*" theorem.

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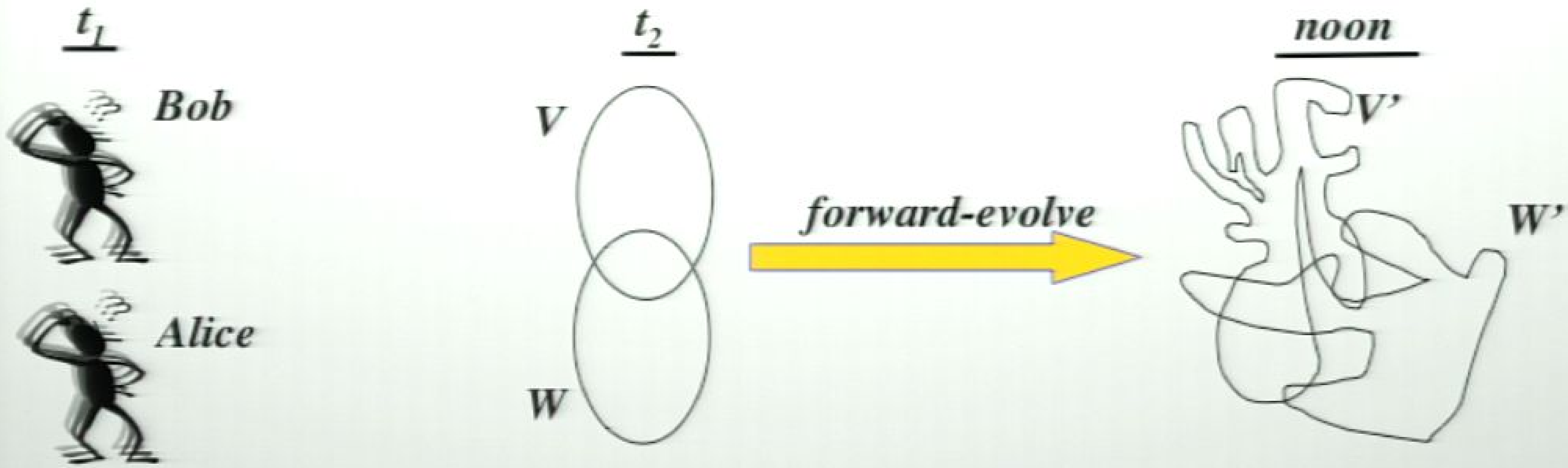

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INFERENCE KNOWLEDGE AND BOOLEAN ALGEBRA

Knowledge defined in terms of weak inference obeys many of the properties of Boolean algebra:

) (X, Y) may know A, or may know ~A, but not both.

) If (X, Y) knows event A, and knows event A ⇒ B, then B is true.

- However (X, Y) need not know B; no problem of knowing all truths via deduction.

) If (X, Y) knows A ⇒ B and (X, Y) knows B ⇒ C, then (X, Y) knows A ⇒ C.

) If (X, Y) knows A, then (X, Y) knows event "(X, Y) knows A".

STOCHASTIC INFERENCE

- **What changes if there is probability measure P over U?**

1) Given a function Γ and device C = (X, Y), C infers Γ with *covariance accuracy*

$$\varepsilon(C,\Gamma) = \frac{\sum_{q_L} \max_x \left[E_P(Y q_L(\Gamma) \mid x) \right]}{|\Gamma(U)|}$$

2) Can't instead use mutual information; that only captures *syntactic* content of distributions, not *semantic* content.

EXAMPLE OF STOCHASTIC INFERENCE RESULT

) **For any probability distribution P over U,**

$$\varepsilon((X,Y),\Gamma) \geq (2-n)\ \frac{\max_x\left[E_P(Y\,|\,x)\right]}{n}$$

where $n = |\Gamma(U)|$

) **For any probability distribution P over U, there exists two devices (X_1, Y_1), (X_2, Y_2) where X_1 and X_2 are distinguishable, but both $\varepsilon((X_1, Y_1), Y_2)$ and $\varepsilon((X_1, Y_1), Y_2)$ are arbitrarily close to one;**

Second Laplace impossibility theorem is "barely true"

HOWEVER RELATED RESULTS ARE QUITE STRONG

1) Let C_1 and C_2 be two devices, where:
   i) Both $X_1(U)$ and $X_2(U)$ are the binaries;
   ii) $C_1 > C_2$ with accuracy ε_1, and $C_2 > C_1$ with accuracy ε_2.
   iii) $P(X_1 = -1) = \alpha$, and $P(X_2 = -1) = \beta$

2) Define H as the four-dimensional unit open hypercube, and
   i) $\forall z \in H$, $k(z) = z_1 + z_4 - z_2 - z_3$;
   ii) $\forall z \in H$, $m(z) = z_2 - z_4$;
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4) E.g., for $\alpha = \beta = 1/2$, $\varepsilon_1\varepsilon_2 \leq 1/4$.

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INFERENCE DEVICES AND THE LAWS OF NATURE

1) A reality is a space U, a set of devices defined over U, and a set of functions the devices might infer.

2) So a reality is a quadruple, $(U, \{X_j, Y_j\}, \{\Gamma_i\})$.

3) As far as any device in a reality is concerned, *U is irrelevant*. It's only the inference graph relating the sets $\{X_j, Y_j\}$ and $\{\Gamma_i\}$ that matter:

The laws of Nature are patterns in the inference graph of a reality

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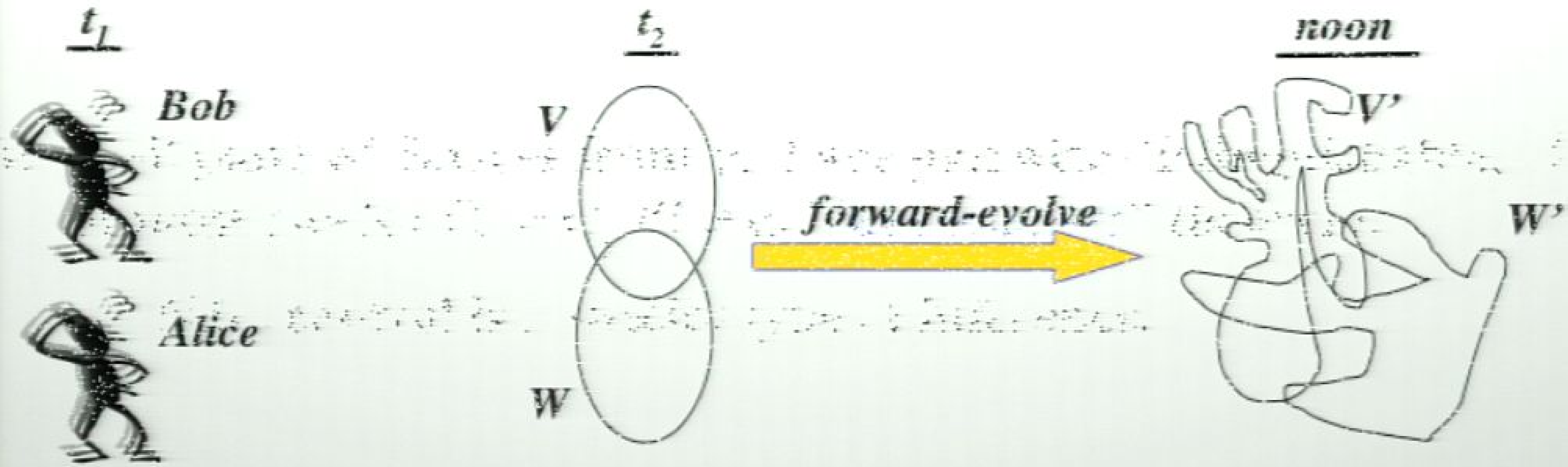

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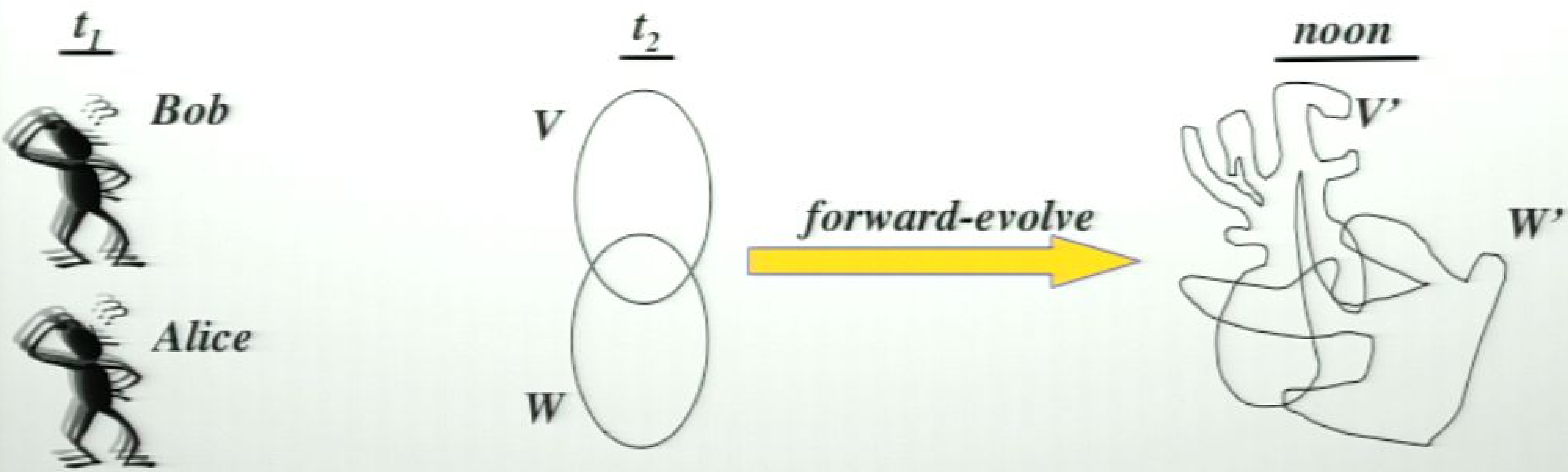

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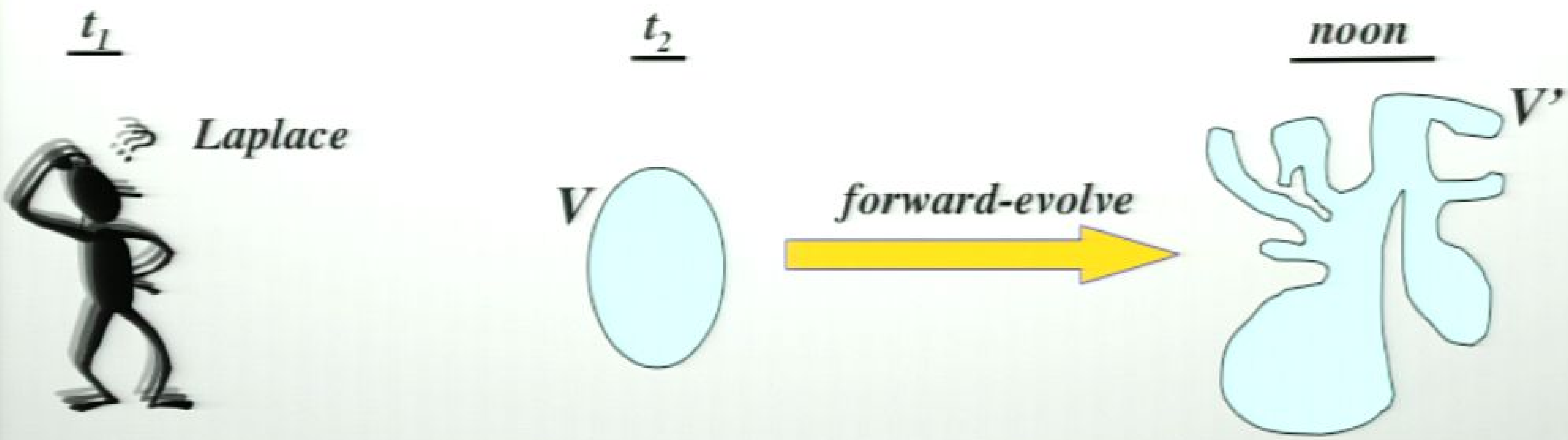

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