

Title: Epistemology and the laws of nature

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Abstract:

EPISTEMOLOGY AND THE LAWS OF NATURE

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NASA-ARC-03-0667

ROADMAP

1) *Shortcomings of current impossibility results concerning laws of Nature*

2) *Knowledge operators and their shortcoming*

3) *Formalize mathematical structure shared by observation and prediction: inference devices*

4) *Elementary properties of inference devices*

COMPUTATION AND PHYSICS

1) *Impossibility results of Moore, Pour-El and Richards, etc. rely on uncountable number of states of universe.*

- *What if universe is countable, or even finite?*
- *What if there exist oracles, so Halting theorem (the basis of those results) is irrelevant?*

2) *Impossibility results of Lloyd rely on current model of laws of physics (e.g., no superluminal travel).*

- *What if laws are actually different?*

COMPUTATION AND PHYSICS

3) *To apply Godel's incompleteness theorem presumes physical laws are "written in predicate logic"*

- *What if universe is written in a different language?*
- *What if there are no "laws" at all, just a huge list of events, which just happen to appear to have patterns?*
- *What if Godel-style intuitionism is correct?*

What if our models are wrong?

COMPUTATION AND PHYSICS

3) *To apply Godel's incompleteness theorem presumes physical laws are "written in predicate logic"*

- *What if universe is written in a different language?*
- *What if there are no "laws" at all, just a huge list of events, which just happen to appear to have patterns?*
- *What if Godel-style intuitionism is correct?*

What if our model is wrong?

*Is there some model more fundamental,
(almost) impossible not to accept?*

ROADMAP

1) *Shortcomings of current impossibility results concerning laws of Nature*

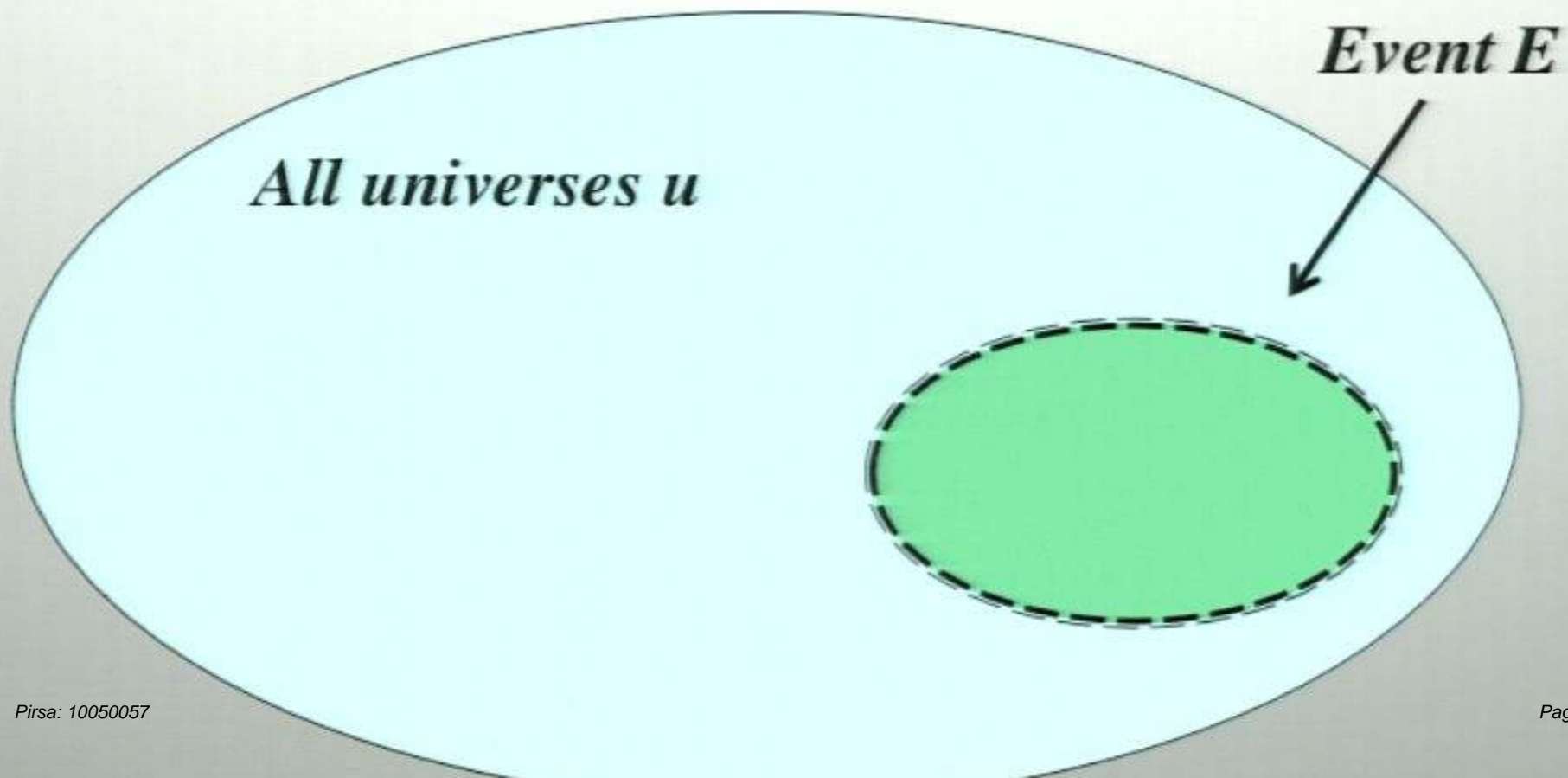
2) *Knowledge operators and their shortcoming*

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4) *Elementary properties of inference devices*

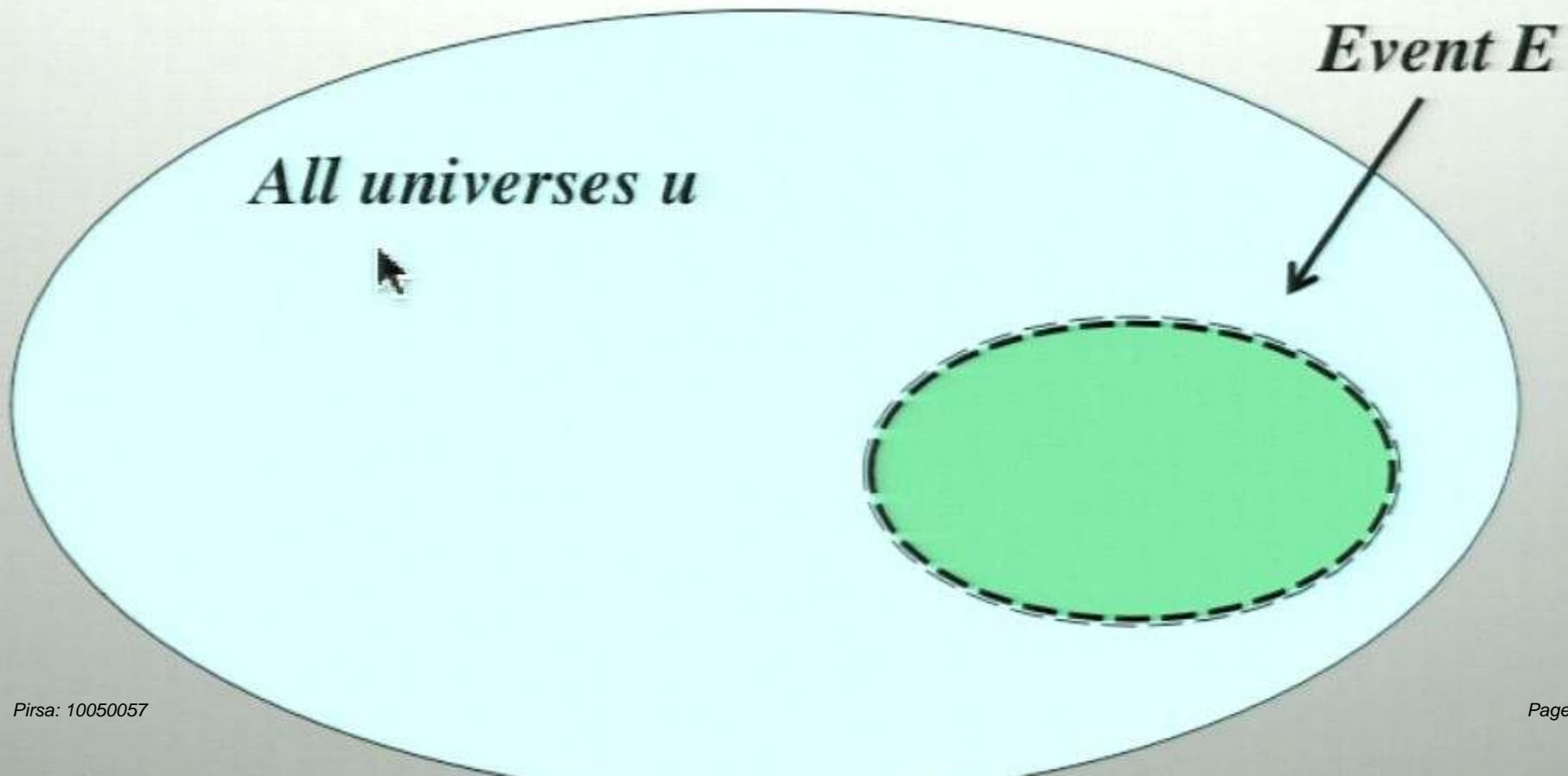
KNOWLEDGE OPERATORS

- 1) *Laws are patterns, relating “events I know”*
- 2) *What does it mean to “know” an event?*



KNOWLEDGE OPERATORS

- 1) *Laws are patterns, relating “events I know”*
- 2) *What does it mean to “know” an event?*



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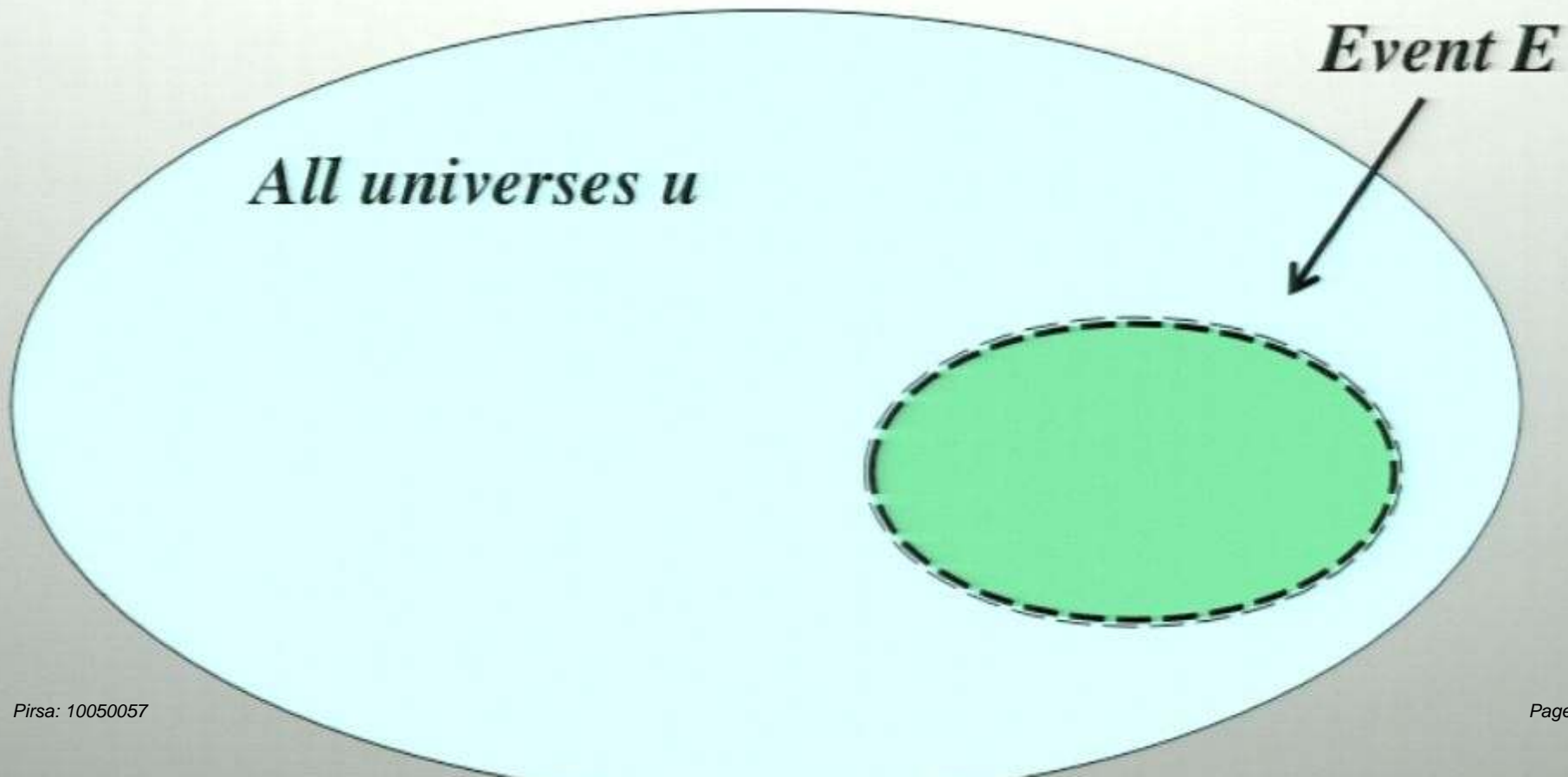


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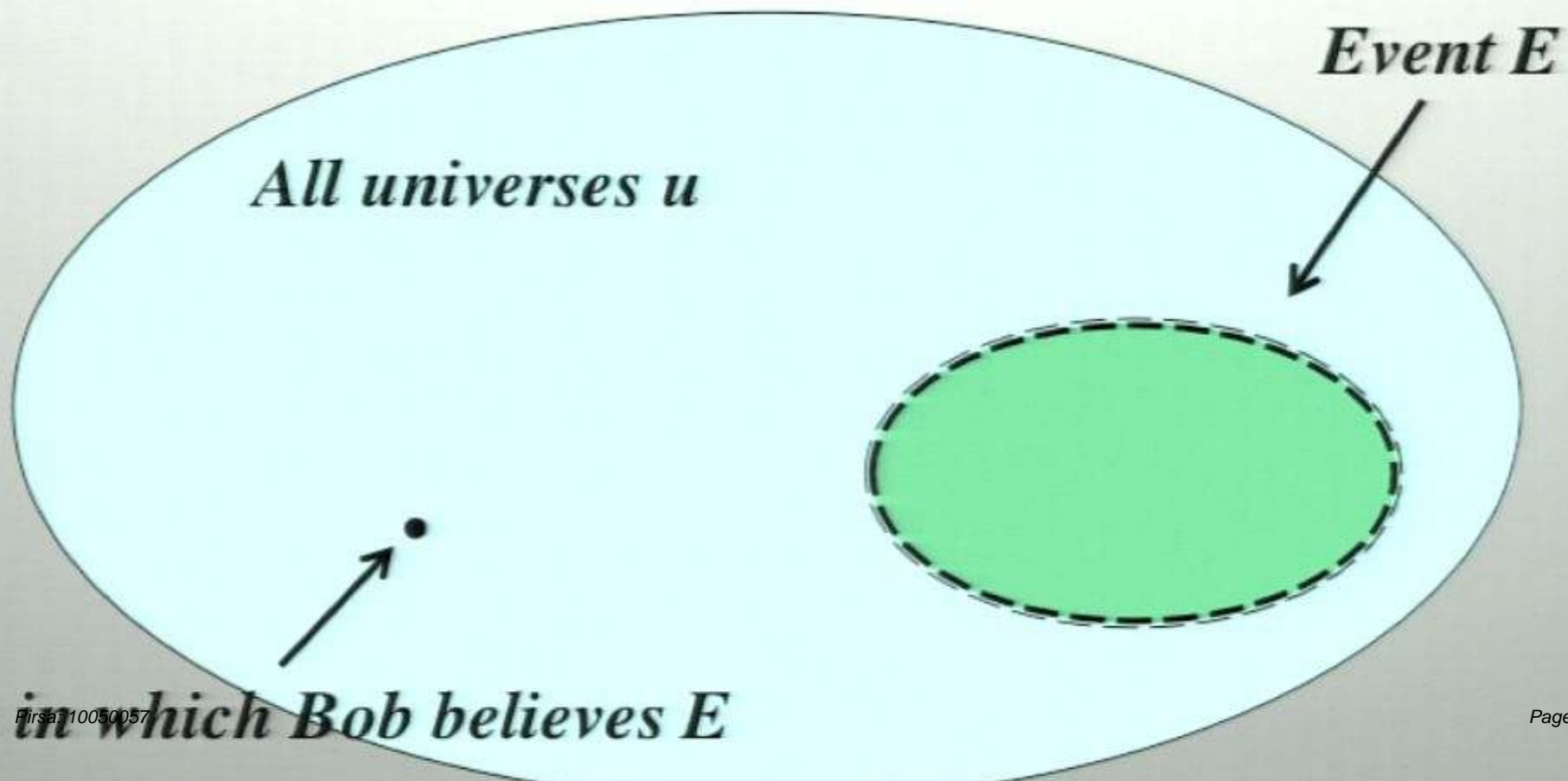
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KNOWLEDGE OPERATORS

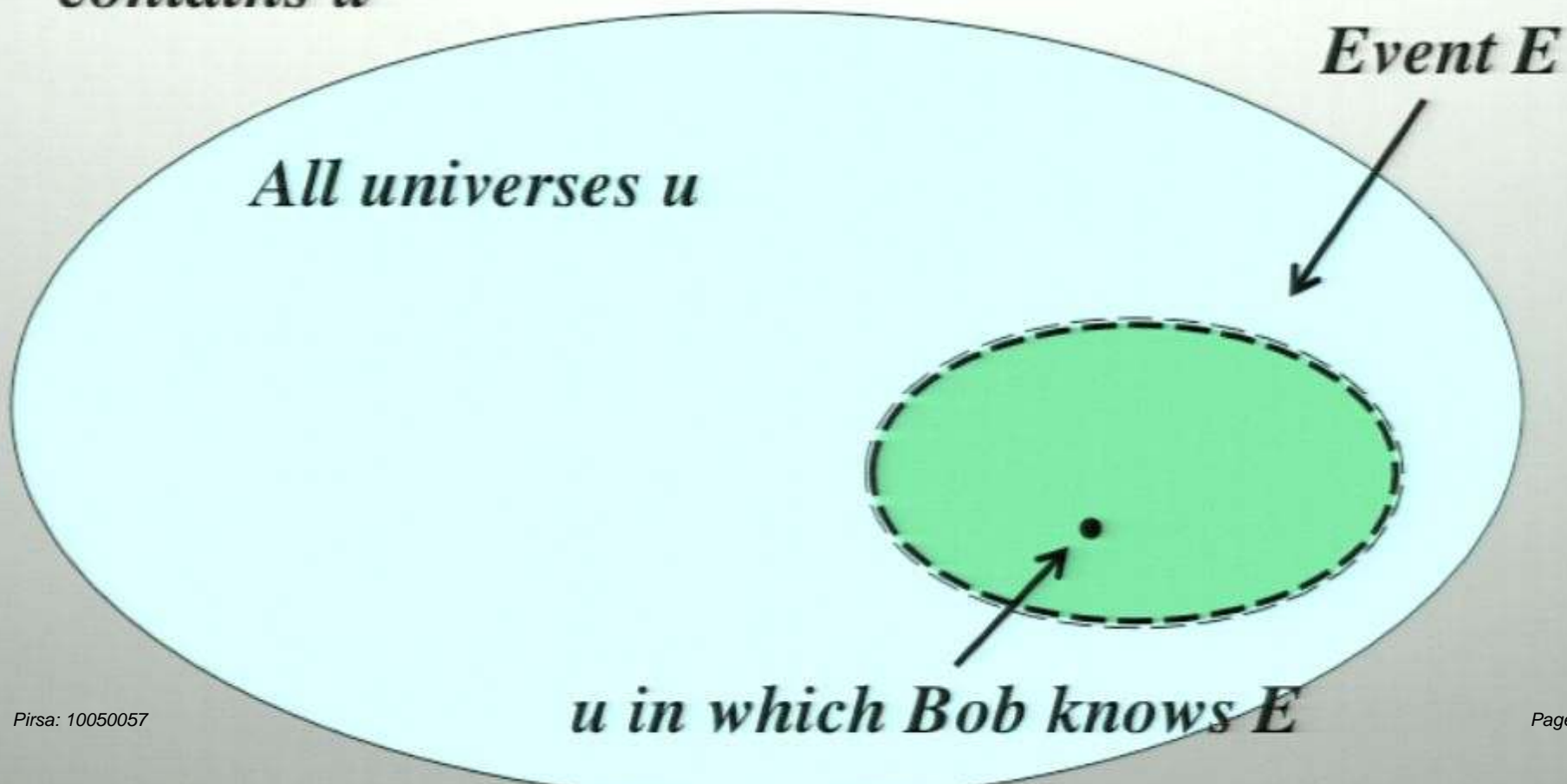
- 1) At indicated u , Bob has belief that he's in E .
- 2) Belief is a function from u to subsets of $\{u\}$



KNOWLEDGE OPERATORS

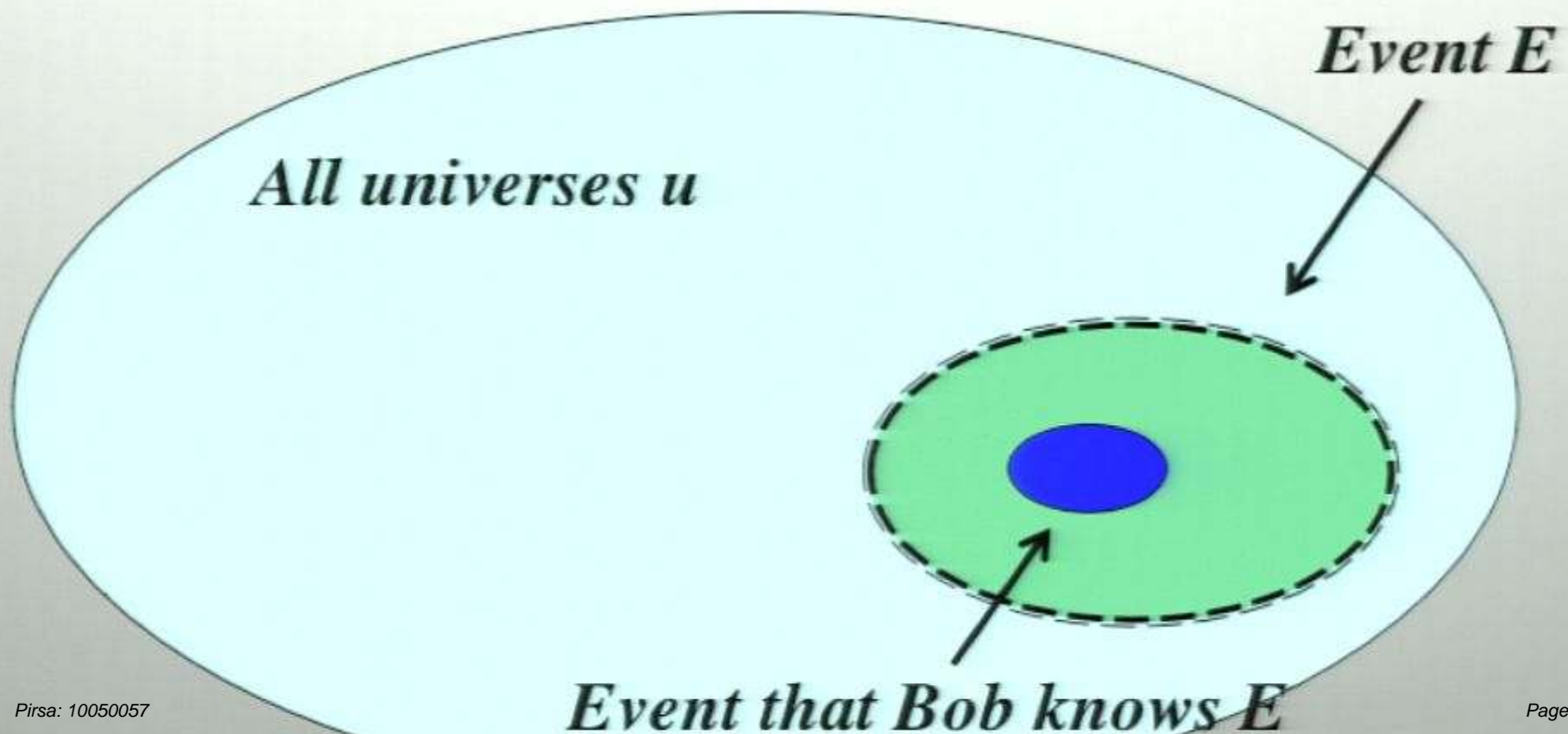
1) At indicated u , Bob knows he's in E .

2) Knowledge is a belief function where the image of u contains u



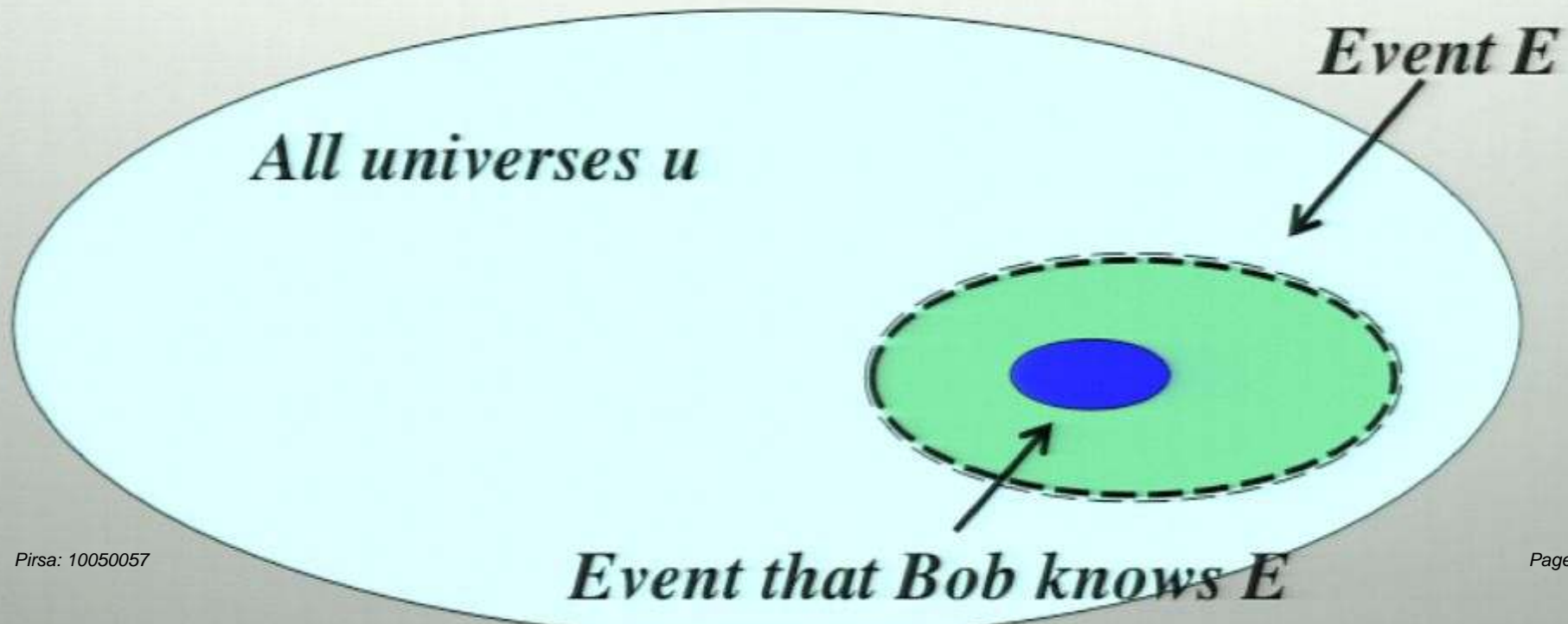
KNOWLEDGE OPERATORS

- 1) Throughout blue region, Bob knows he's in E .
(At other u in E , Bob instead knows some E' that overlaps E .)



KNOWLEDGE OPERATORS

- 1) Need physical manifestation of such knowledge, if Bob's having it involves the laws of nature.
- 2) So need Bob to be able to physically answer questions about what he knows.



KNOWLEDGE OPERATORS

- 1) *What does it mean for Bob to be able to physically answer questions about what he knows?*
- 2) *To formalize this, analyze physical phenomena where Bob knows an event.*
- 3) *These are phenomena where information outside Bob gets inside Bob.*
- 4) *Examples:*
 - *Observation*
 - *Prediction*
 - *Memory*
 - *Control*

ROADMAP

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***EXAMPLE OF PHYSICAL KNOWLEDGE:
OBSERVATION***

1) Present a stylized example of observation.

*2) Emphasize features of that example found in all
“observations”*

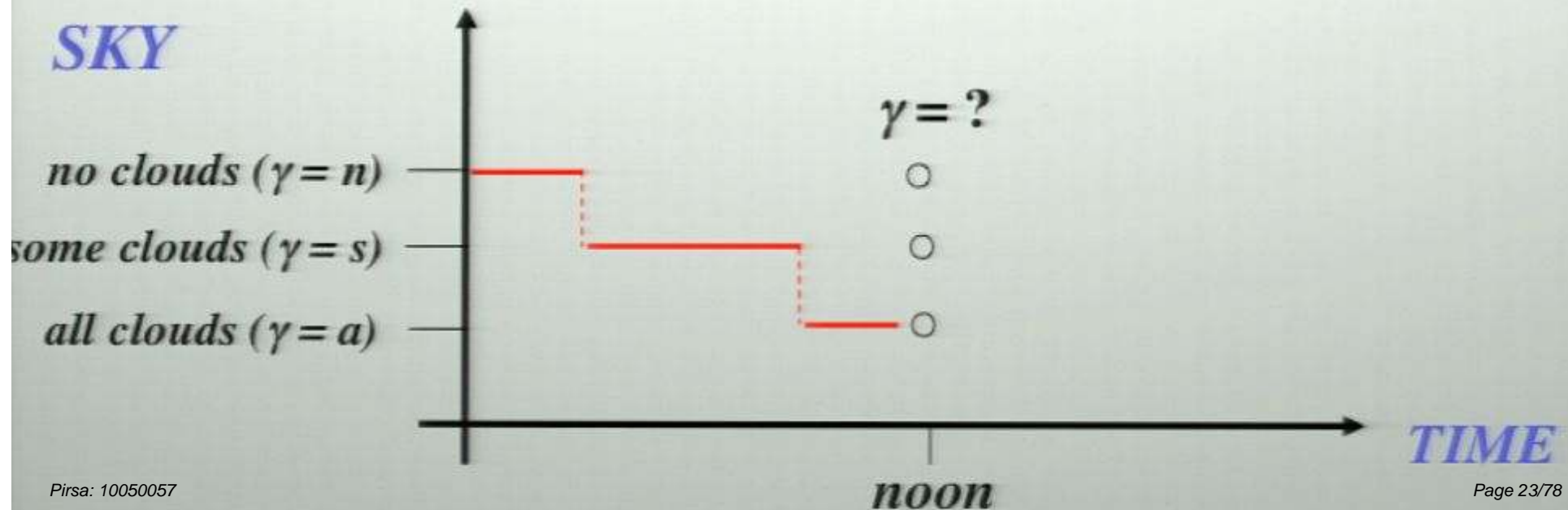
3) Why those features are always found in observations:

*Without those features, the observation
conveys no semantic information*

OBSERVATION

- *Want to observe γ , state of sky at noon tomorrow*

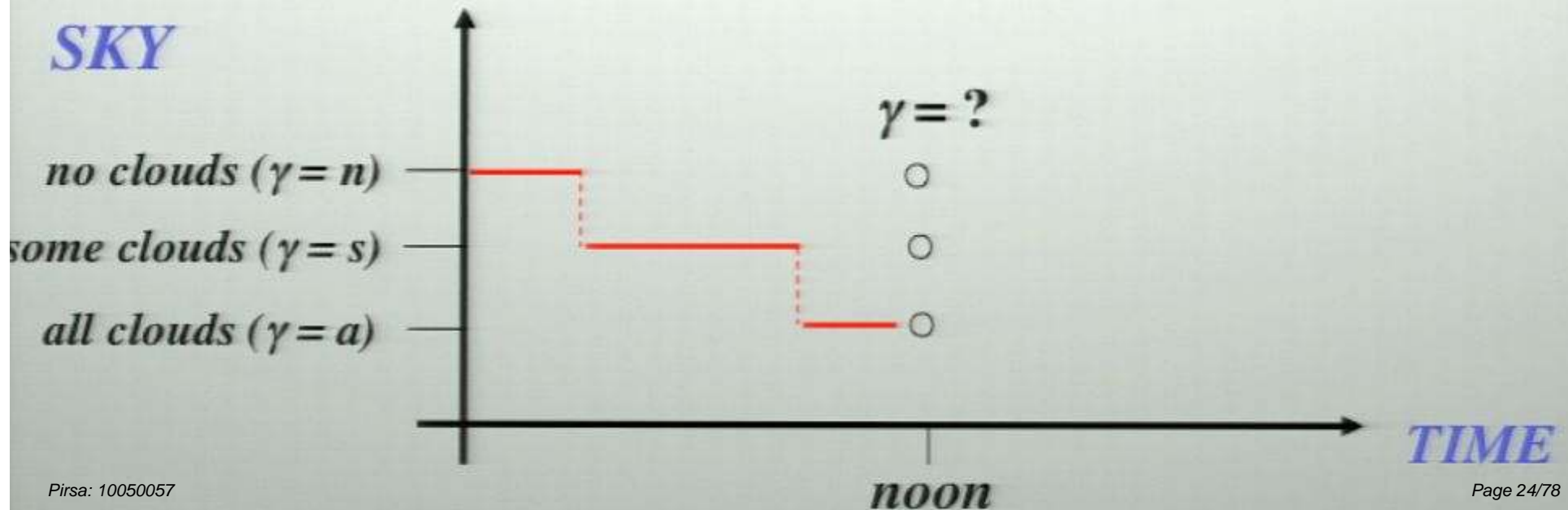
SKY



OBSERVATION

- *Bob claims to be able to make that observation*

SKY

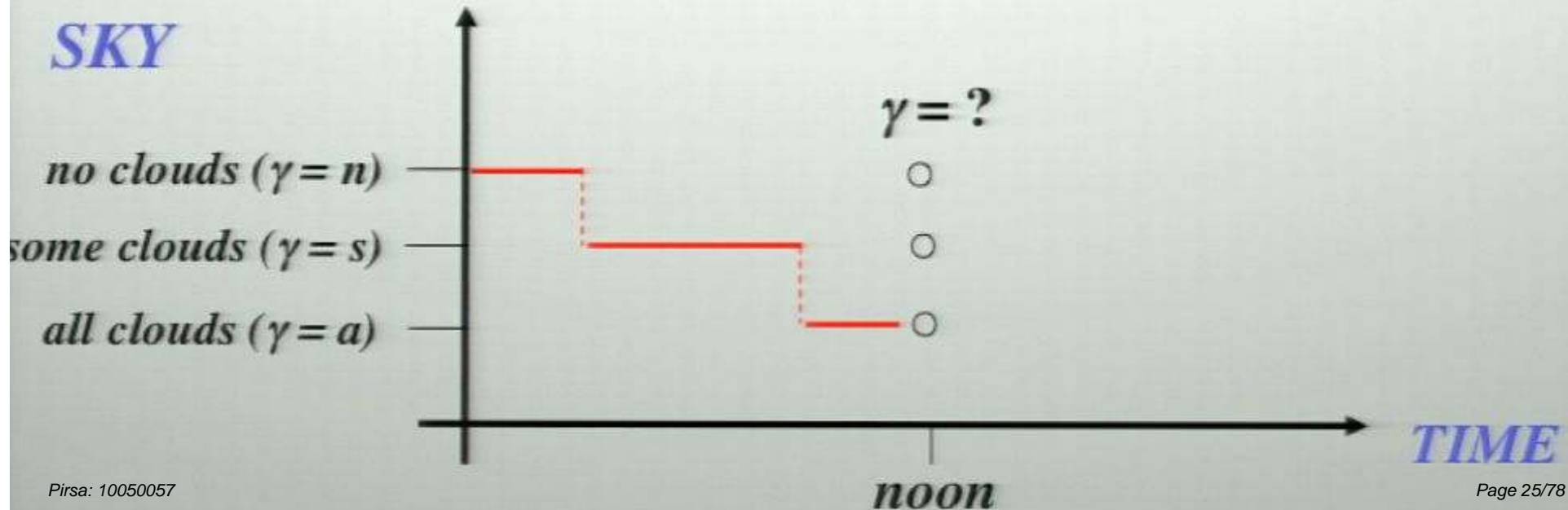


OBSERVATION

If Bob's claim is true, he can correctly answer three questions that could be posed to him:

- i) Does $\gamma = 'n'$? (Yes / no)*
- ii) Does $\gamma = 's'$? (Yes / no)*
- iii) Does $\gamma = 'a'$? (Yes / no)*

SKY

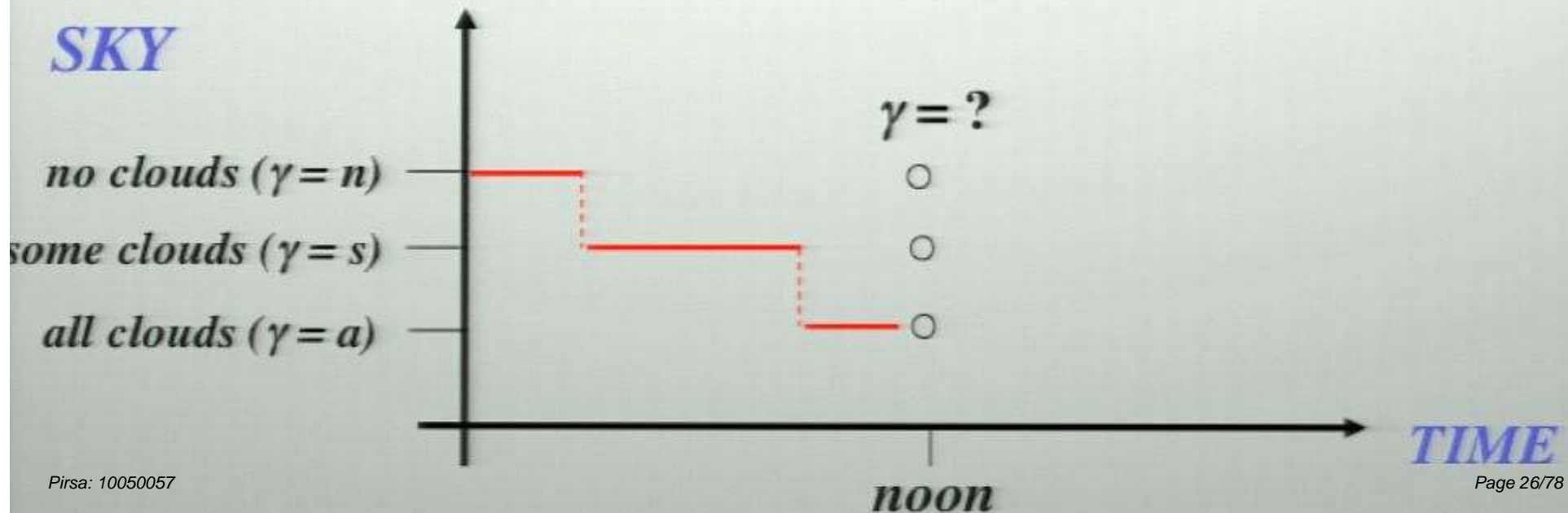


OBSERVATION

What does this mean physically?

- Restrict attention to universes where Bob and the sky exist; Bob considers one of the three binary questions; then observes γ ; then gives his honest answer to that question.*

SKY



OBSERVATION

Bob can observe γ if for each of the three questions, q ,

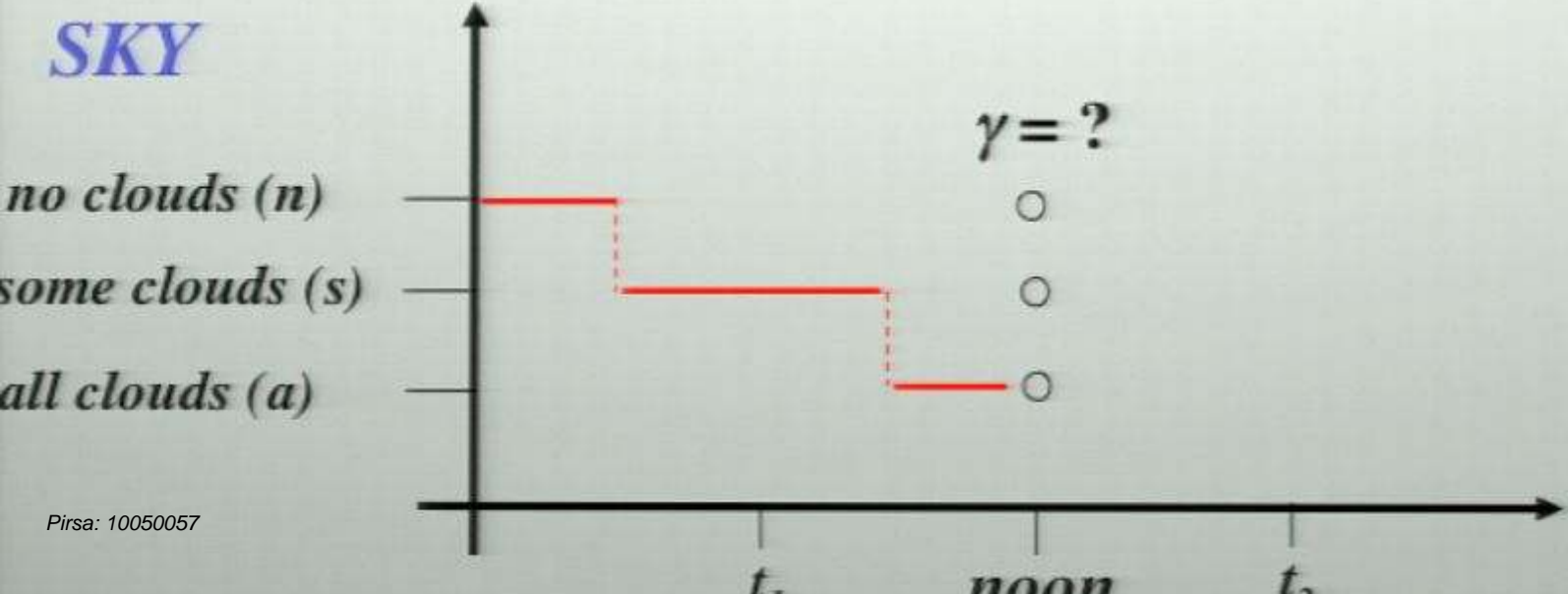
The universe has property x_q :

“At some $t_1 < \text{noon}$ Bob considers q ”



y , the binary answer Bob gives at some $t_2 > \text{noon}$, equals correct answer to q

SKY

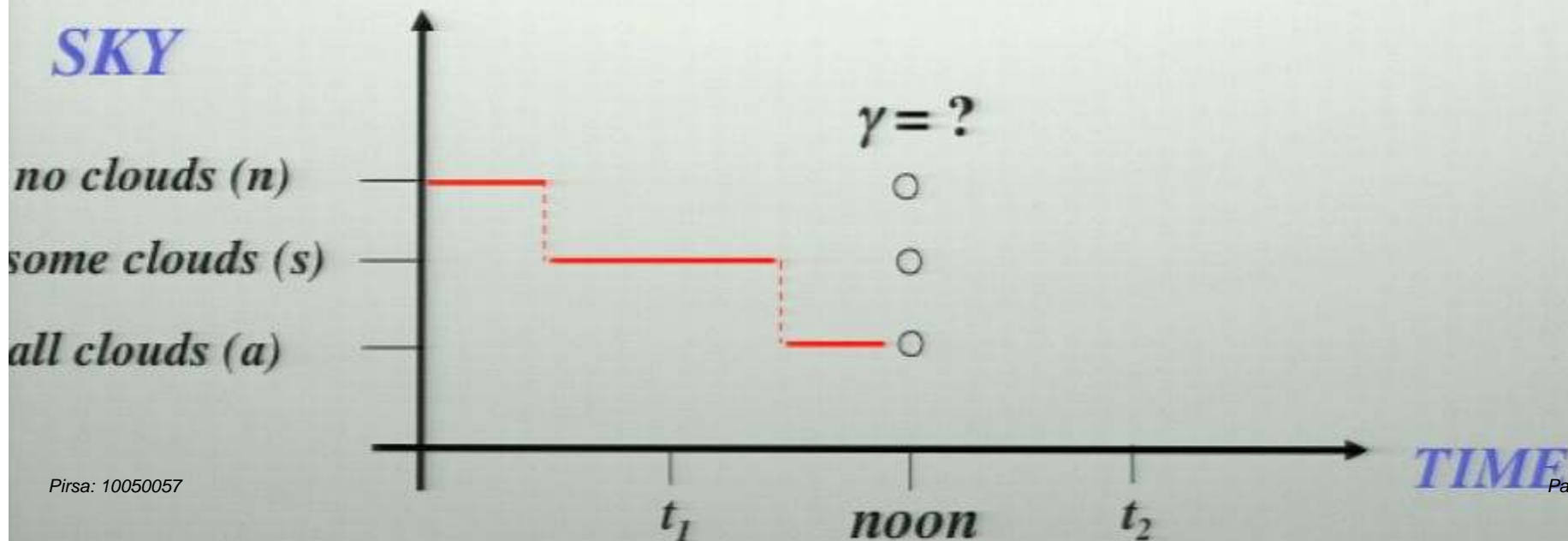


OBSERVATION

$U = \{ \text{all universe-histories consistent with physics, in which Bob and the sky exist; at } t_1 \text{ Bob considers a } q; \text{ then observes } \gamma; \text{ then gives honest answer to that } q \}$

State of sky at noon is fixed by $u \in U$, the actual universe-history. So $\gamma = \Gamma(u)$ for some function Γ

SKY



OBSERVATION

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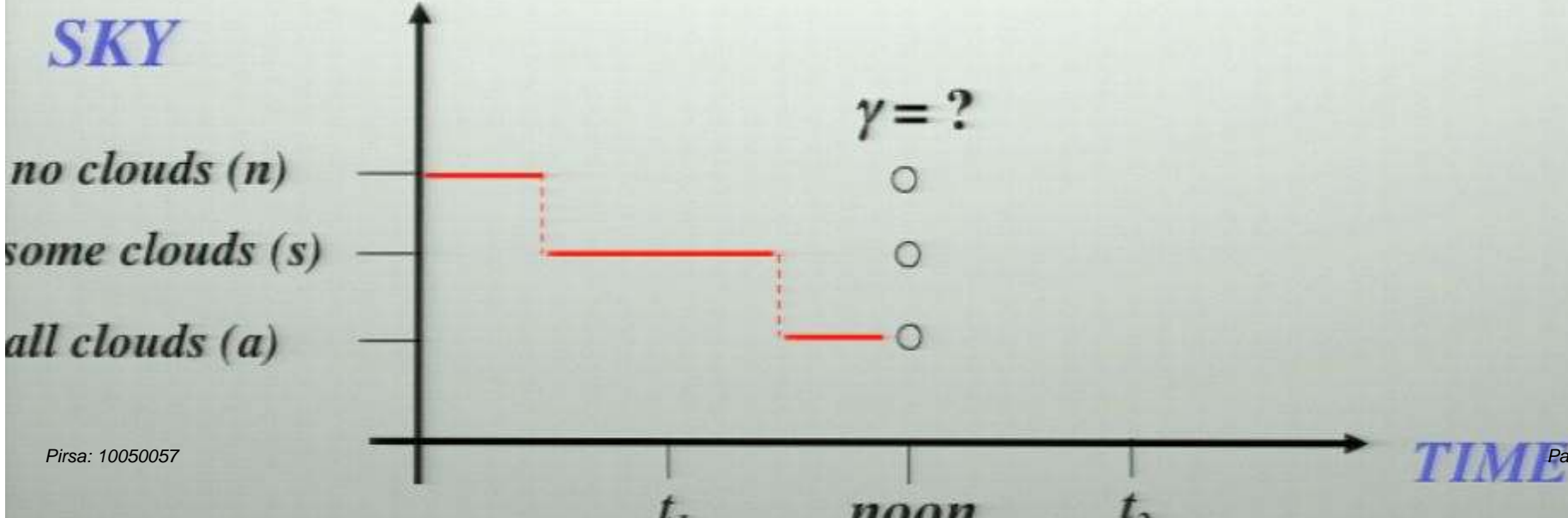
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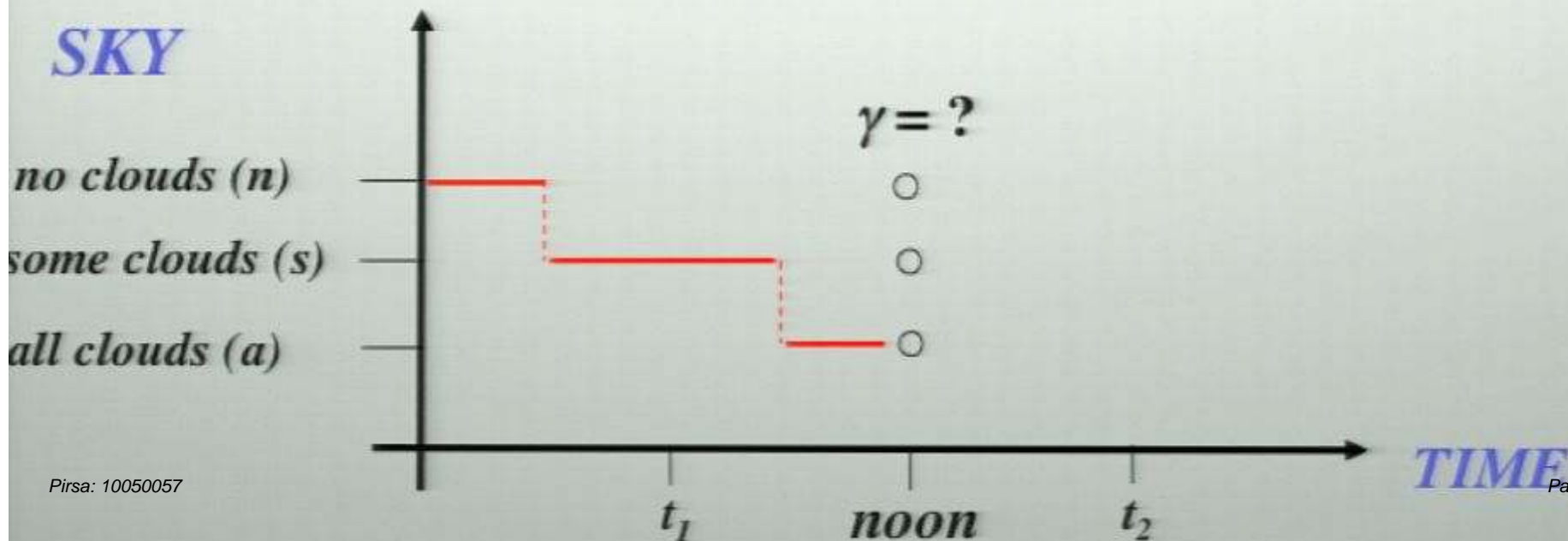


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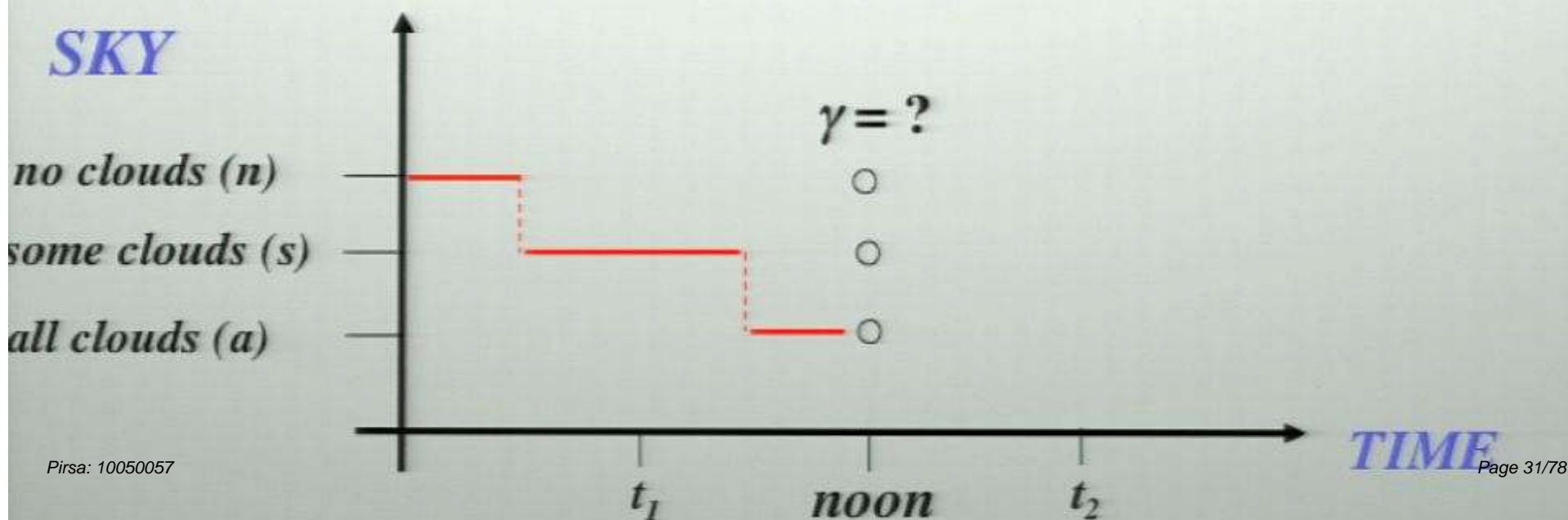


OBSERVATION

$U = \{$ all universe-histories consistent with physics, in which Bob and the sky exist; Bob considers a q ; then observes γ ; then gives honest answer to that $q\}$

The question Bob considers at t_1 is given by the actual universe-history $u \in U$, so $x = X(u)$ for some function X

SKY

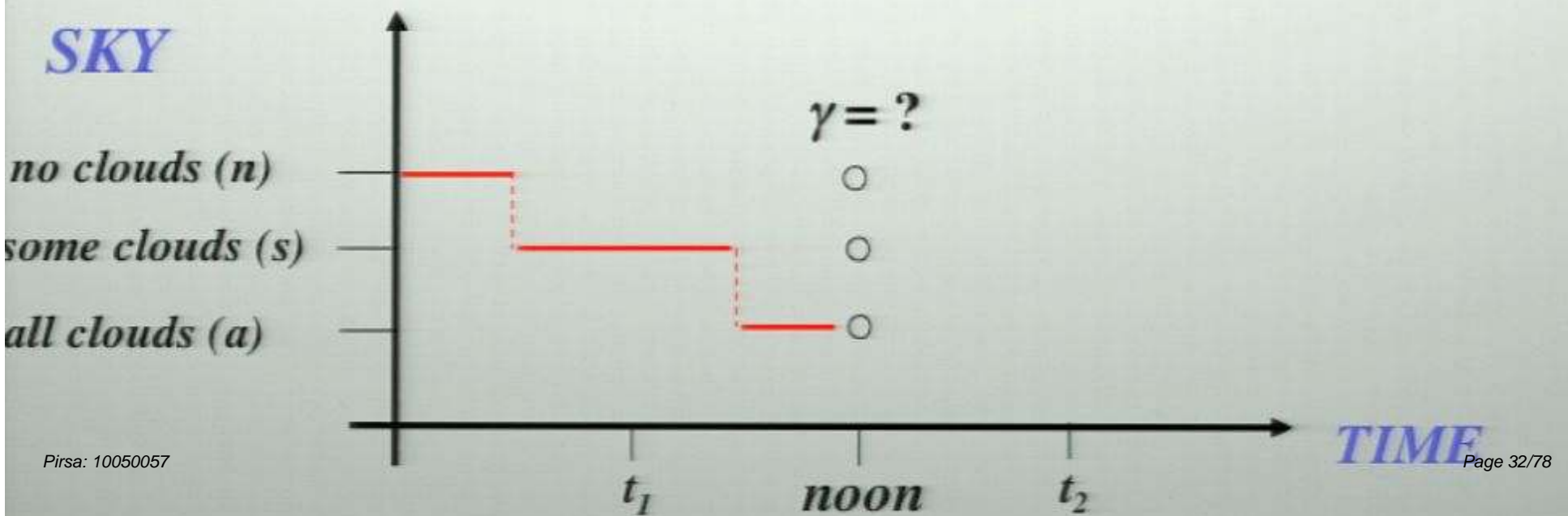


OBSERVATION

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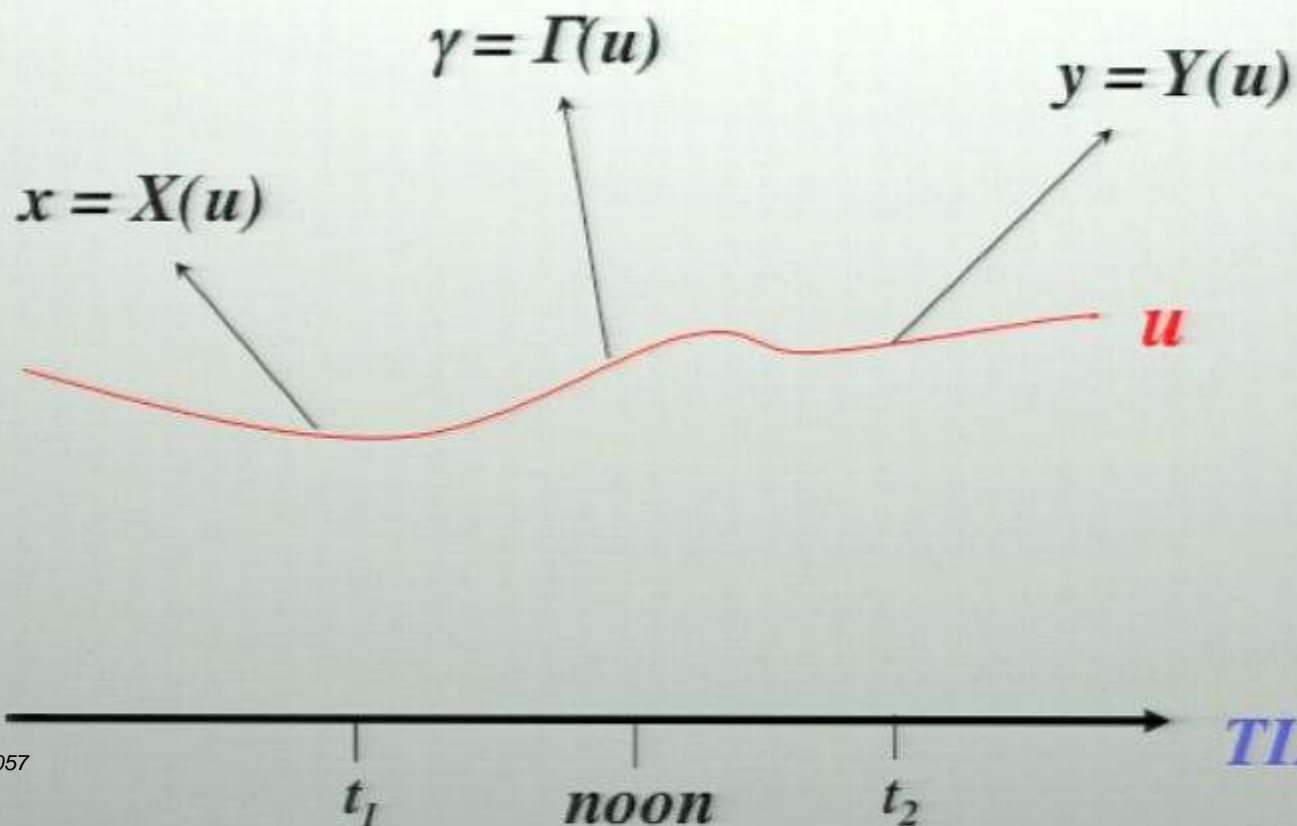
Bob's answer at t_2 is given by the actual universe-history $u \in U$, so $y = Y(u)$ for some function Y

SKY



OBSERVATION

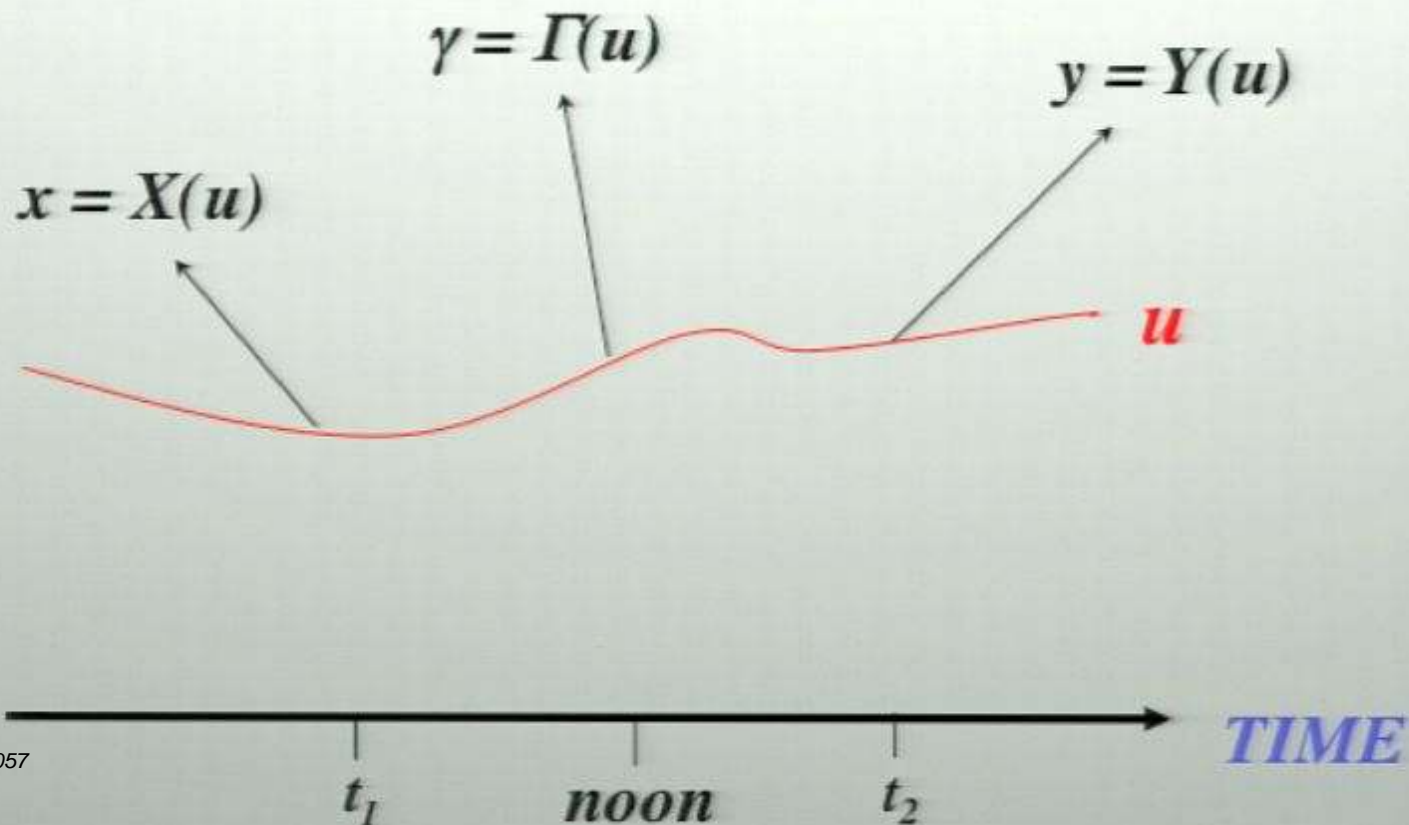
- $\gamma \in \{ 'n', 's', 'a' \} = \text{sky at noon} = \Gamma(u)$
- $x = (\text{what } q \text{ Bob considers at } t_1) = X(u)$
- $y = (\text{Bob's answer at } t_2) = Y(u)$



OBSERVATION

No:

- For each of three binary questions q_γ ,
- $\exists x$ such that
- $X(u) = x \Rightarrow Y(u) = q_\gamma(\Gamma(u))$

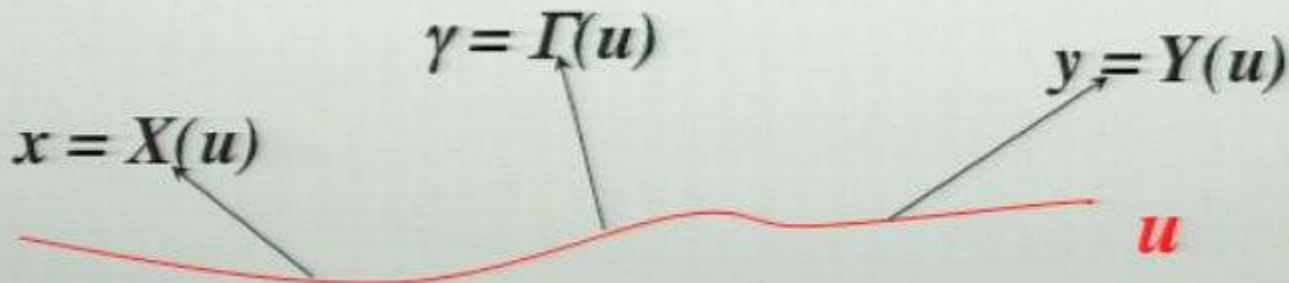


OBSERVATION

- *Nothing about observation process;*
all about what it means to successfully observe.
 - *The ‘what’ of observation, not the ‘how’.*
-

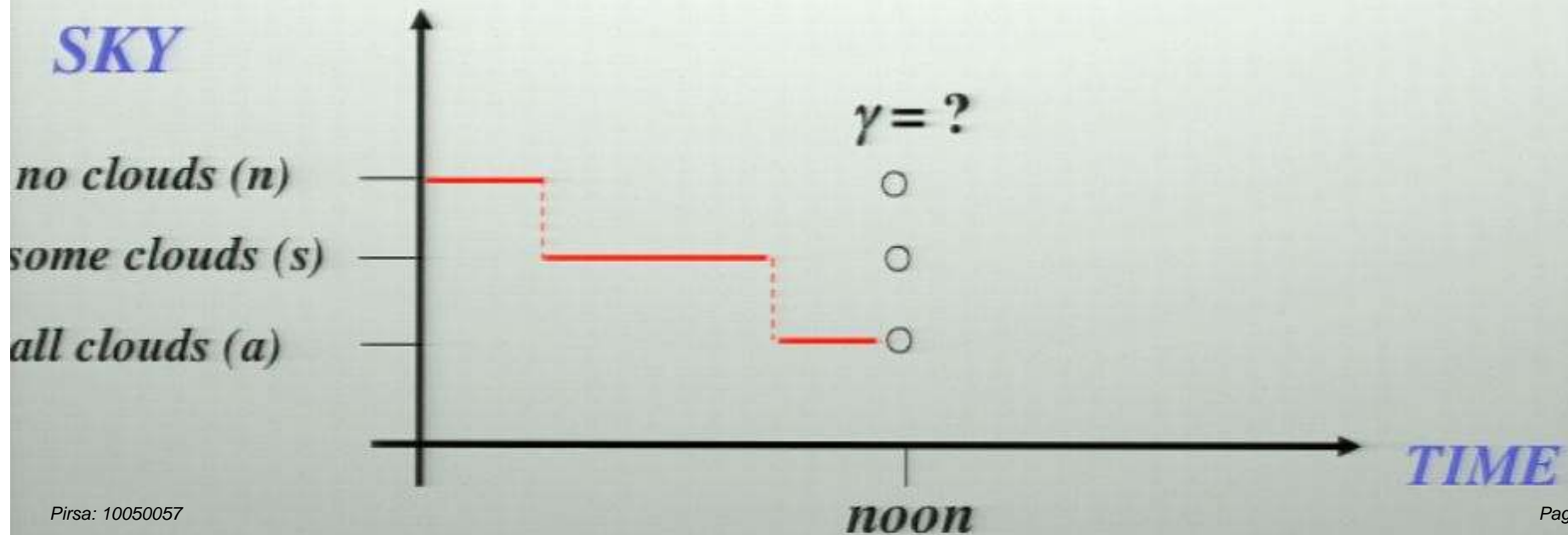
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PREDICTION

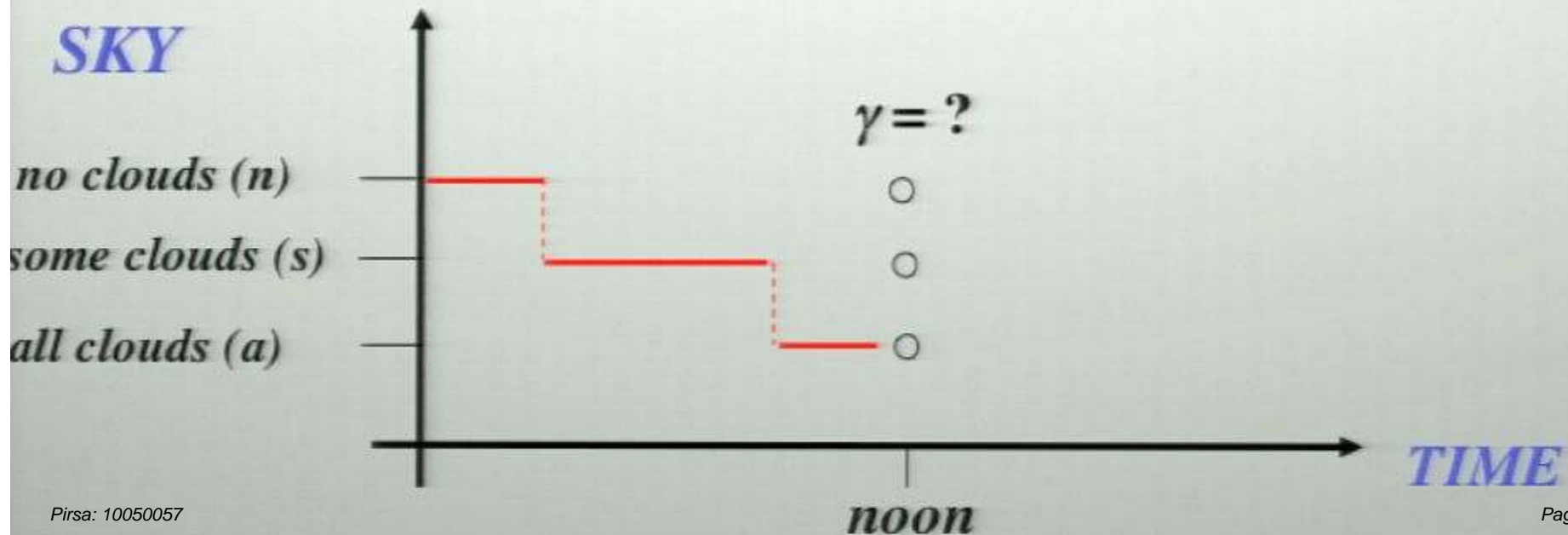
- *Want to predict γ , state of sky at noon tomorrow*



PREDICTION

- *Bob claims to have a laptop that he can program to make that prediction*

SKY

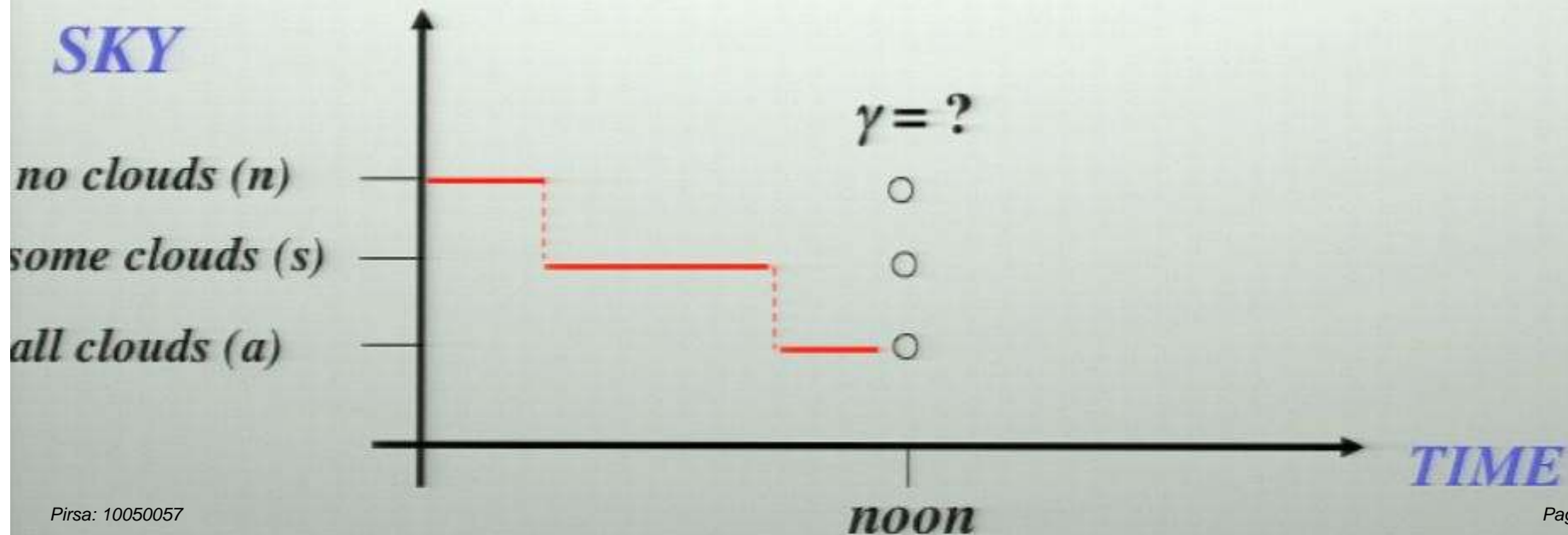


PREDICTION

If Bob's claim is true, he will be able to correctly answer three questions:

- i) Does $\gamma = 'n'$? (Yes / no)*
- ii) Does $\gamma = 's'$? (Yes / no)*
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SKY



PREDICTION

Bob can predict γ if for each of the three questions, q ,

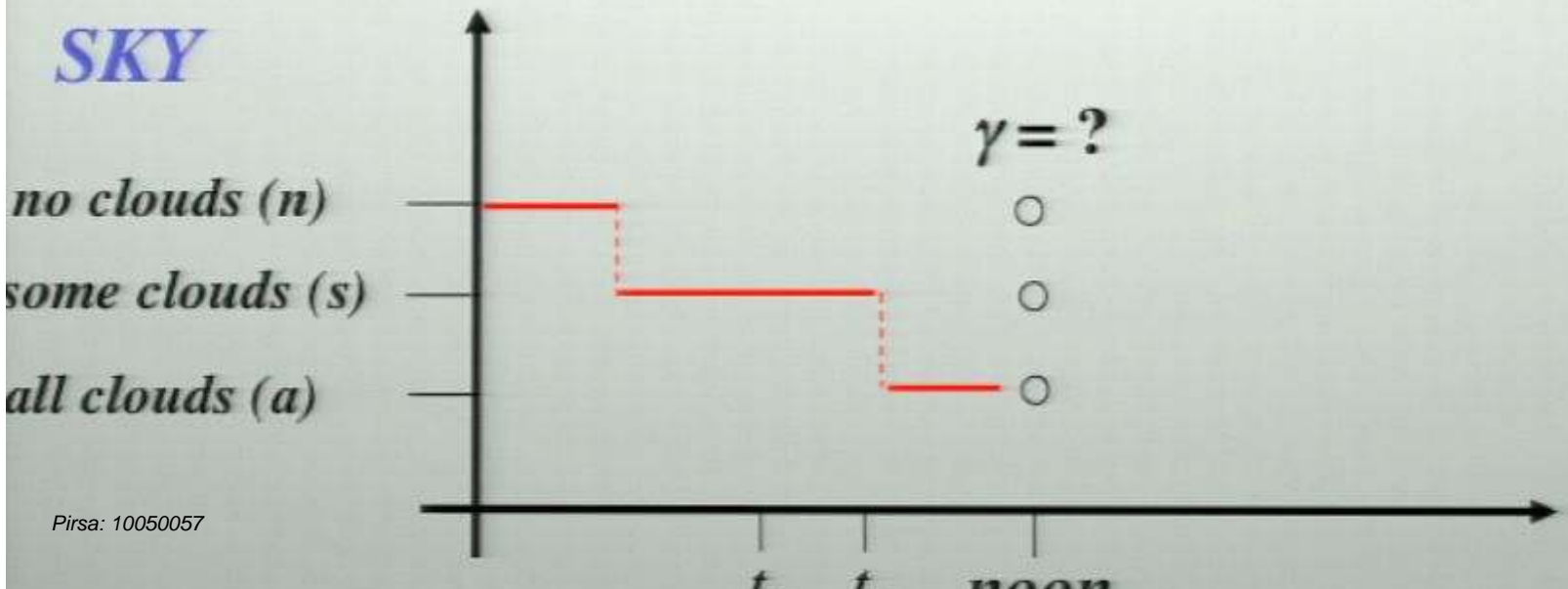
The universe has property xq :

“At some $t_1 < \text{noon}$ Bob programs the laptop to predict q ”



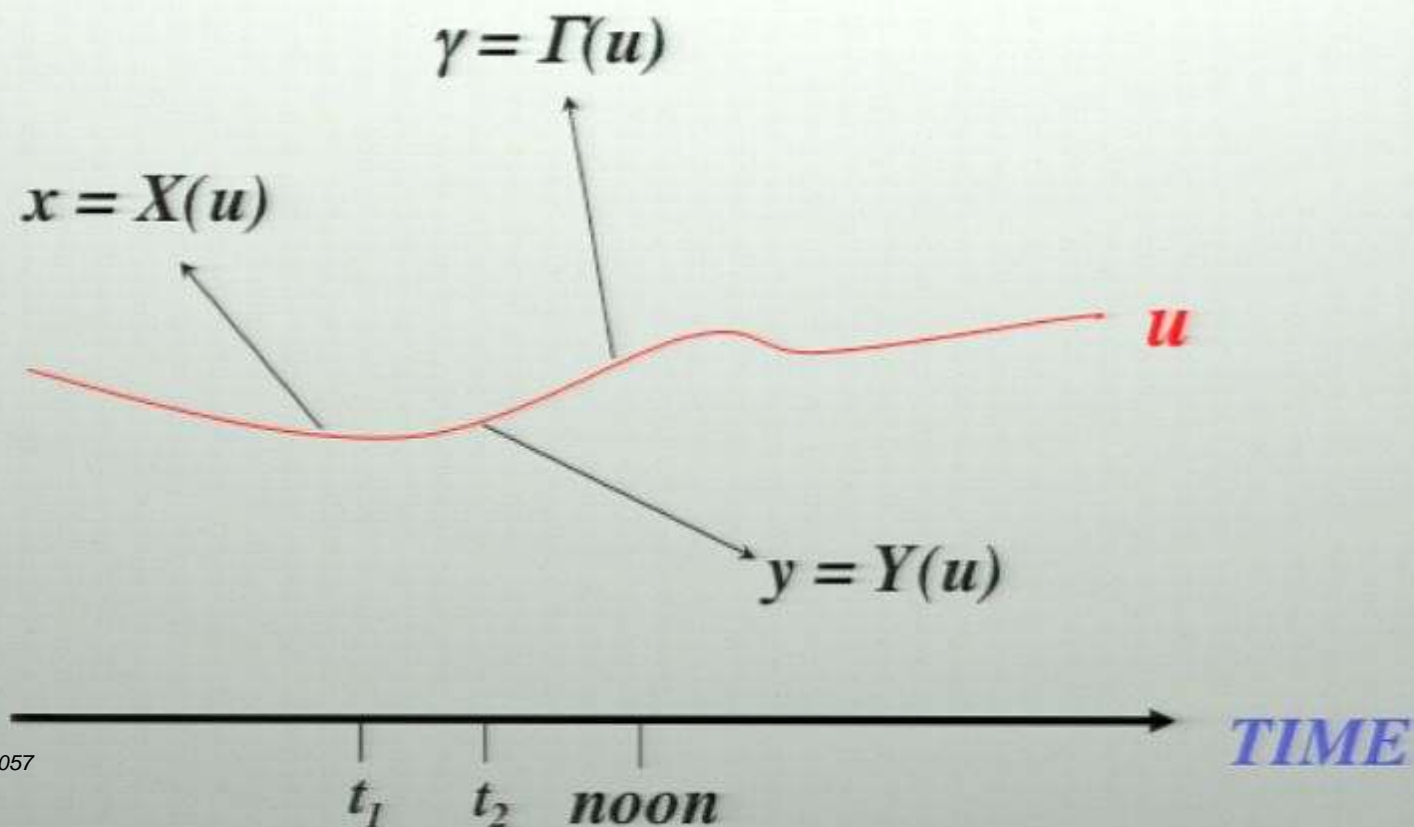
y , the binary answer Bob reads off at some $t_2 < \text{noon}$, equals correct answer to q

SKY



PREDICTION

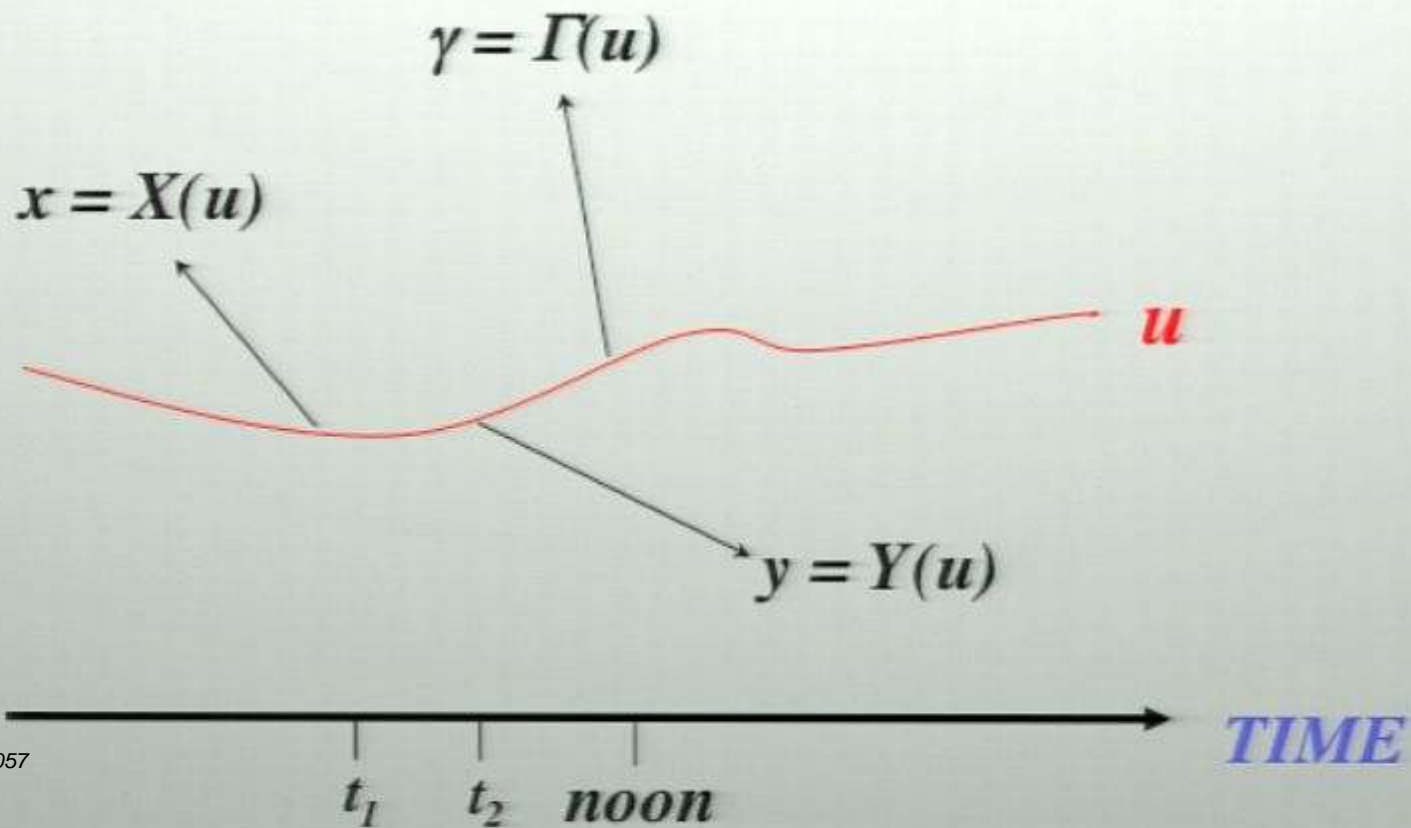
- $\gamma \in \{ 'n', 's', 'a' \} = \text{sky at noon} = \Gamma(u)$
- $x = (\text{laptop program at } t_1) = X(u)$
- $y = (\text{Bob's answer at } t_2) = Y(u)$



PREDICTION

So:

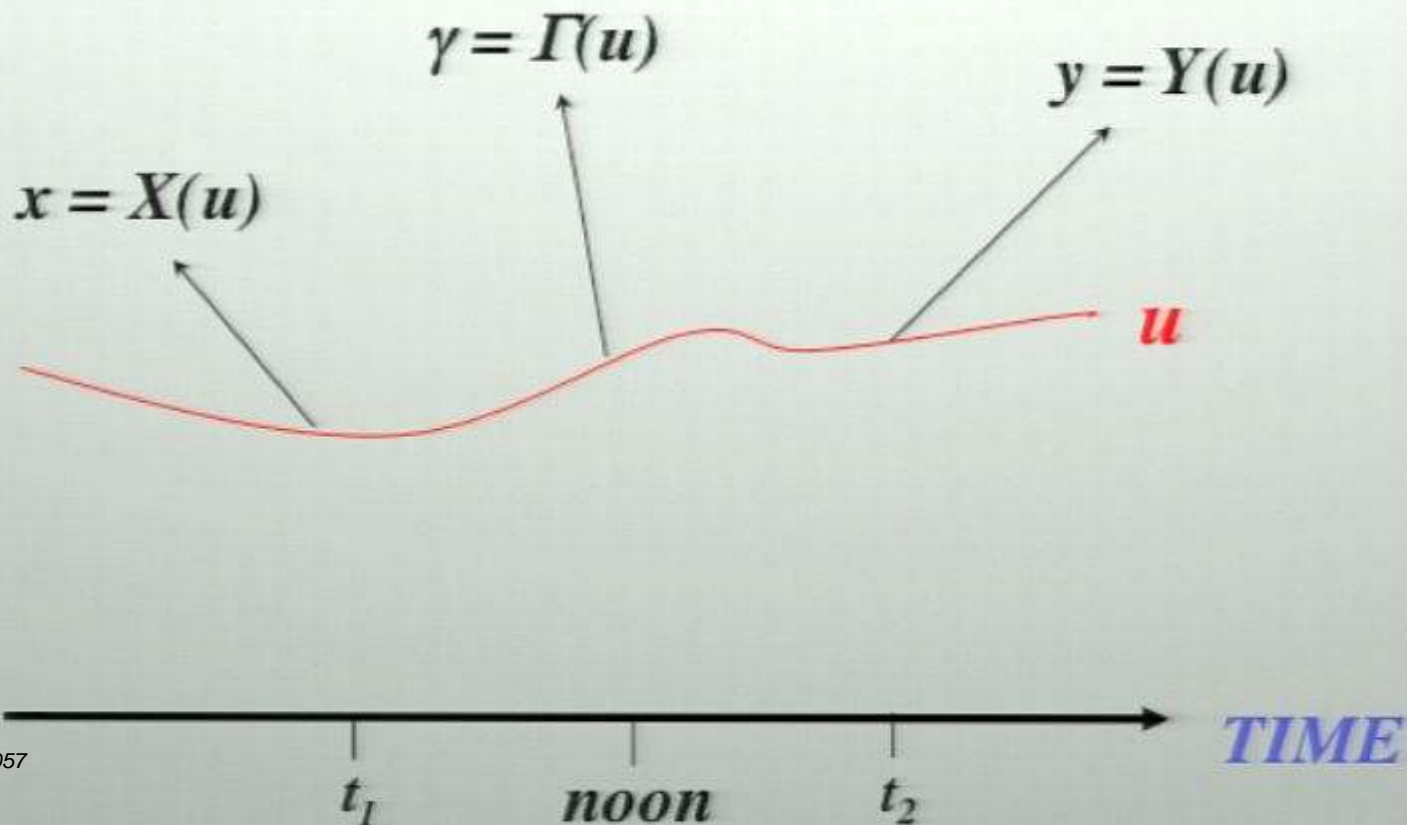
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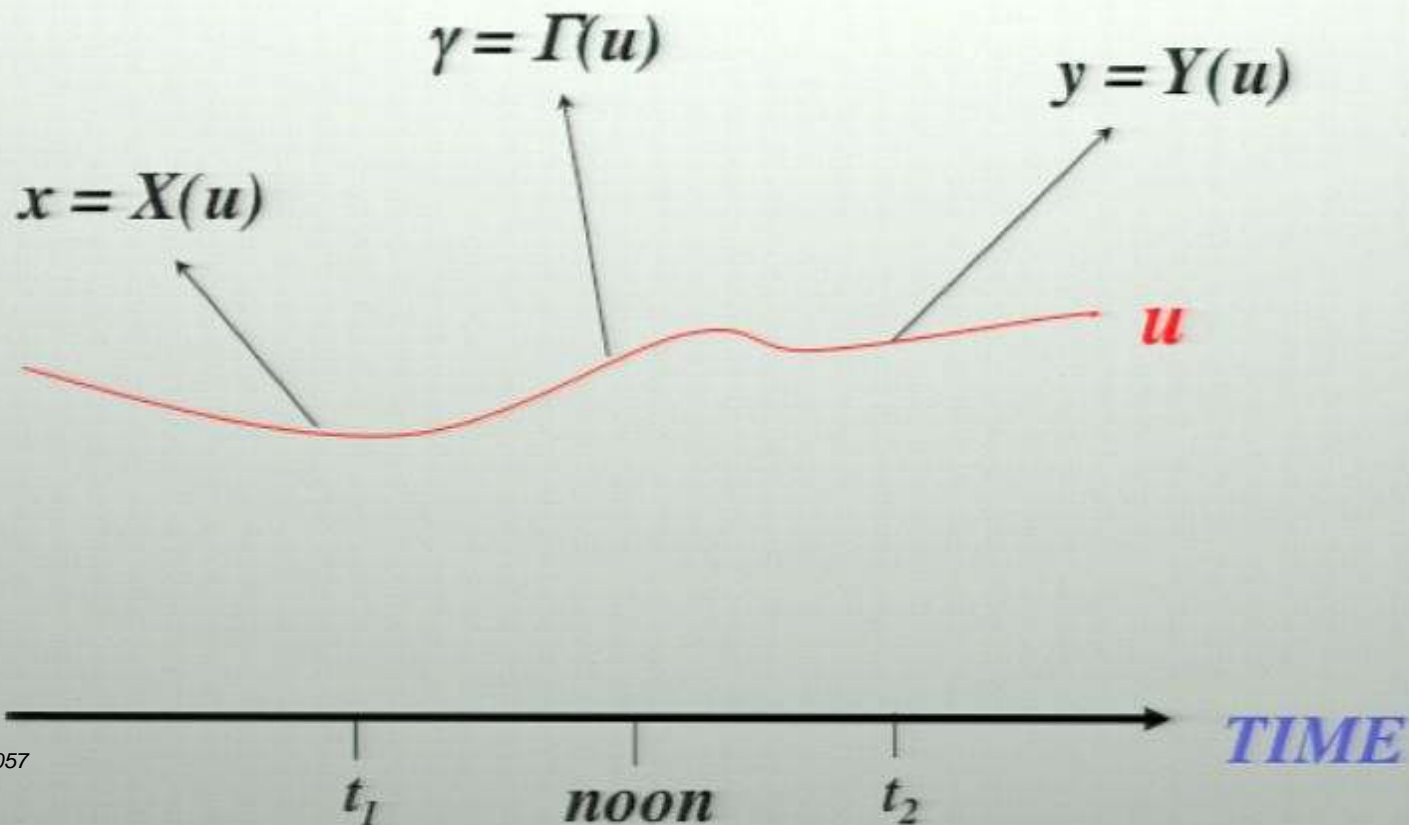
KNOWLEDGE

- *More generally, if at some time, “Bob knows the state of the sky at noon”, γ , then he can answer three questions:*
 - i) Does $\gamma = 'n'$? (Yes / no)*
 - ii) Does $\gamma = 's'$? (Yes / no)*
 - iii) Does $\gamma = 'a'$? (Yes / no)*
- *Note no chronological ordering. Just X (what question Bob considers), Y (his answer), Γ (the sky's actual state at noon),
- all functions of $u \in U$*

OBSERVATION

No:

- For each of three binary questions q_γ ,
- $\exists x$ such that
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KNOWLEDGE

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- *Note no chronological ordering. Just X (what question Bob considers), Y (his answer), Γ (the sky's actual state at noon), - all functions of $u \in U$*

INFERENCE DEVICES

- An inference device is any two functions (X, Y) over U , where range of Y is binary.
- An inference device (X, Y) (weakly) infers a function Γ over U iff
$$\forall \gamma \text{ in } \Gamma\text{'s range,}$$
$$\exists x \text{ such that } X(u) = x \Rightarrow Y(u) = q_\gamma(\Gamma(u))$$
- A necessary condition to say that (X, Y) “observes”, “predicts”, or “knows” Γ is that (X, Y) weakly infers Γ .
- No claims of sufficiency; observation, prediction, knowledge, etc. involve much more than just weak inference.
- But even requiring weak inference restricts observation, prediction, and knowledge.

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- Note the use of counterfactual questions, asking whether $\Gamma(u)$ has a value that it does not have. Analogous to the use of intervention to define causality in Bayes nets.

- Contrast inference with Aumann-style “knowledge operators”

INFERENCE DEVICES

- *Advantages of using binary questions:*
 - Formalism doesn't change if range of Γ changes*
 - Device never need give value $\Gamma(u)$, only confirm/reject suggested $\Gamma(u)$'s. (Cf. computational complexity)*
 - Formalizes semantic information (contrast Shannon)*

INFERENCE DEVICES

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INFERENCE DEVICES TERMINOLOGY

- 1) **Setup** *function X over U*
- 2) **Conclusion** *binary-valued function Y over U*
- 3) “ **$(X, Y) > \Gamma$** ” *means (X, Y) weakly infers Γ*

INFERENCE DEVICES AND THE LAWS OF NATURE

- 1) A reality is a space U , a set of devices defined over U , and a set of functions the devices might infer.**
- 2) So a reality is a quadruple, $(U, \{X_j, Y_j\}, \{\Gamma_i\})$.**
- 3) As far as any device in a reality is concerned, U is irrelevant. It's only the inference graph relating the sets $\{X_j, Y_j\}$ and $\{\Gamma_i\}$ that matter:**

***The laws of Nature are patterns in
the inference graph of a reality***

ROADMAP

1) *Shortcomings of current impossibility results concerning laws of Nature*

2) *Knowledge operators and their shortcoming*

3) *Formalize mathematical structure shared by observation and prediction: inference devices*

4) *Elementary properties of inference devices*

ELEMENTARY PROPERTIES OF INFERENCE

1) Inference need not be transitive:

$(X_1, Y_1) > Y_2$ and $(X_2, Y_2) > Y_3$ does not mean $(X_1, Y_1) > Y_3$

2) For any Γ , \exists a device that infers Γ .

3) For any device, \exists a Γ it does not infer. (*Impossibility result*)

- *Intuition*: X ~ initial configuration of a Turing machine.

Y (a bit) ~ whether Turing machine halts or not.

So apply Halting theorem-style reasoning

IMPLICATIONS OF IMPOSSIBILITY RESULT

- 1) **For any simulator, there is always a prediction that cannot be guaranteed correct.**
- *Laplace was wrong.*
 - *Results of Pour-El et al., Fredkin et al., Moore, etc. are far narrower than this general impossibility*
- 2) **For any observation apparatus, there is always an observation that cannot be guaranteed to be correct.**
- *Non-quantum mechanical “uncertainty principle”*

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BREADTH OF IMPOSSIBILITY RESULT

- 1) Holds even for a countable U (even for a finite one).**
- 2) Holds even if current formulation of physics is wrong.**
- 3) Holds even if C has Super-Turing capability**
- 4) Holds even if laws of Nature are not written in predicate logic,
or intuitionism is correct,
or even if there are no laws, just a huge list of events.**

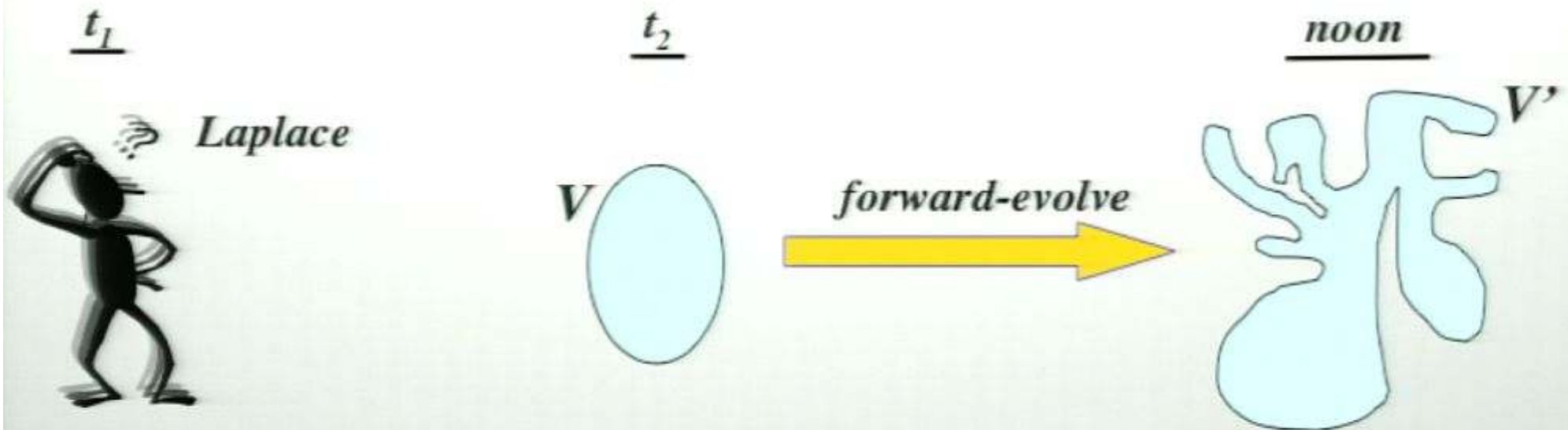
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EXAMPLE: PREDICTION FAILURE



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2. $V' = V$ evolved forward to noon
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Trivially, Laplace's answer is wrong

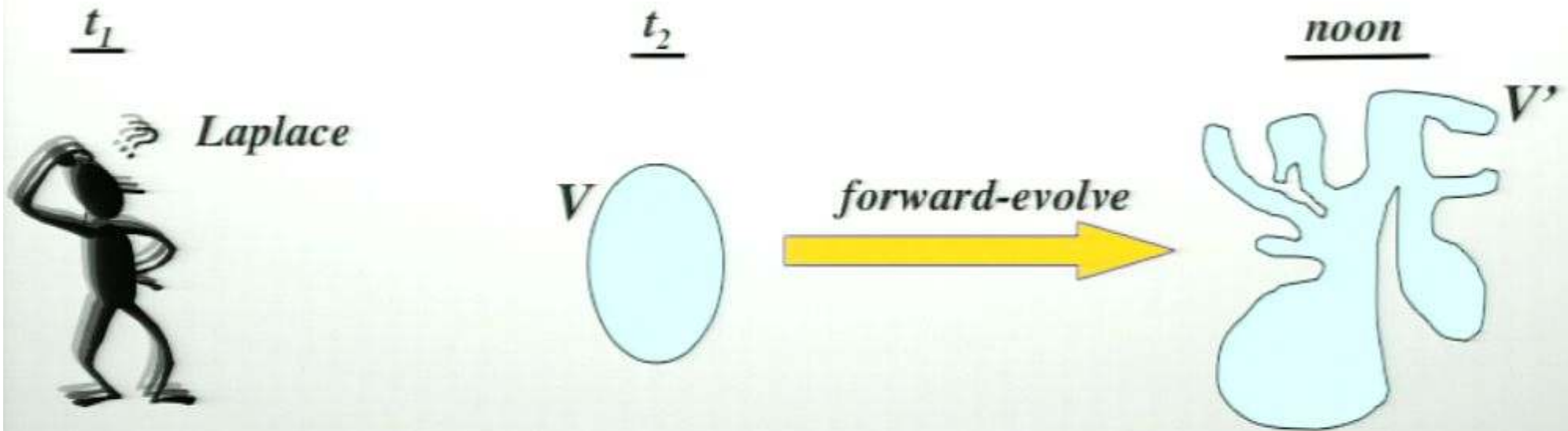
INFERENCE RELATIONS BETWEEN DEVICES

Often not interested in inference of arbitrary functions, but rather inference relation among a set of devices.

I) Two devices (X, Y) , (X', Y') are *pairwise distinguishable* iff every pair (x, x') occurs for some u

II) A set of devices $\{(X_i, Y_i)\}$ is *mutually distinguishable* iff every tuple (x_1, x_2, \dots) occurs for some u

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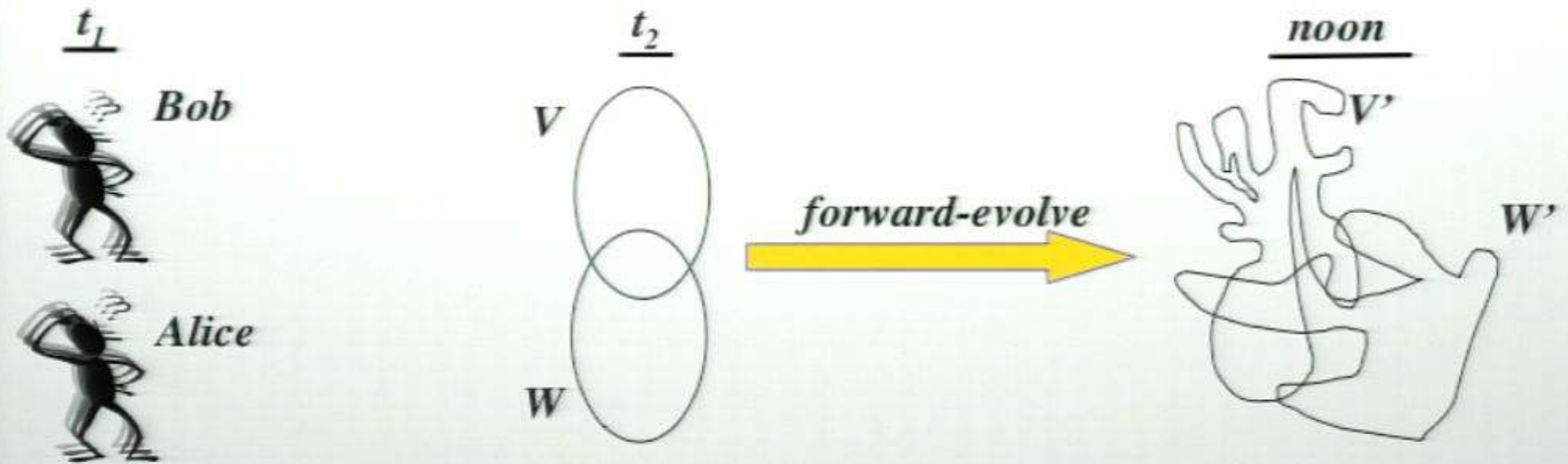
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INFERENCE RELATIONS BETWEEN DEVICES -2

- 3) If all pairs of devices from $\{C_i\}$ are pairwise distinguishable, \exists at most one $k : C_k > C_j \forall j \neq k$. “*Monotheism*” theorem.
- N.b., control is a special type of inference.
- 4) If all pairs of devices from $\{C_i\}$ are pairwise distinguishable, can have $C_1 > C_2 > \dots C_1$.
- 5) If the set of devices $\{C_i\}$ is mutually distinguishable, cannot have $C_1 > C_2 > \dots C_1$.

MONOTHEISM EXAMPLE



$V = \{\text{time-}t_2 \text{ universes where Bob is answering 'yes' to his } t_1 \text{ question}\}$

$W = \{\text{time-}t_2 \text{ universes where Alice is answering 'yes' to her } t_1 \text{ question}\}$

$V' = V \text{ evolved forward to noon}$

$W' = W \text{ evolved forward to noon}$

At t_1 , ask Bob, "will universe be in W' at noon?"

At t_1 , ask Alice, "will universe be outside of V' at noon?"

Either Bob or Alice is wrong

INFERENCE KNOWLEDGE AND BOOLEAN ALGEBRA

Knowledge defined in terms of weak inference obeys many of the properties of Boolean algebra:

- 1) **(X, Y) may know A , or may know $\sim A$, but not both.**
- 2) **If (X, Y) knows event A , and knows event $A \Rightarrow B$, then B is true.**
 - **However (X, Y) need not know B ; no problem of knowing all truths via deduction.**
- 3) **If (X, Y) knows $A \Rightarrow B$ and (X, Y) knows $B \Rightarrow C$, then (X, Y) knows $A \Rightarrow C$.**
- 4) **If (X, Y) knows A , then (X, Y) knows event “(X, Y) knows A ”.**

STOCHASTIC INFERENCE

• What changes if there is probability measure P over U ?

1) Given a function Γ and device $C = (X, Y)$, C infers Γ with covariance accuracy

$$\varepsilon(C, \Gamma) = \frac{\sum q_L \max_x [E_P(Y q_L(\Gamma) | x)]}{|\Gamma(U)|}$$

2) Can't instead use mutual information; that only captures *syntactic* content of distributions, not *semantic* content.

EXAMPLE OF STOCHASTIC INFERENCE RESULT

) For any probability distribution P over U ,

$$\varepsilon((X,Y),\Gamma) \geq (2-n) \frac{\max_x [E_P(Y|x)]}{n}$$

where $n = |\Gamma(U)|$

) For any probability distribution P over U , there exists two devices $(X_1, Y_1), (X_2, Y_2)$ where X_1 and X_2 are distinguishable, but both $\varepsilon((X_1, Y_1), Y_2)$ and $\varepsilon((X_2, Y_2), Y_1)$ are arbitrarily close to one;

*Second Laplace impossibility theorem
is “barely true”*

HOWEVER RELATED RESULTS ARE QUITE STRONG

- 1) Let C_1 and C_2 be two devices, where:
 - i) Both $X_1(U)$ and $X_2(U)$ are the binaries;
 - ii) $C_1 > C_2$ with accuracy ε_1 , and $C_2 > C_1$ with accuracy ε_2 .
 - iii) $P(X_1 = -1) = \alpha$, and $P(X_2 = -1) = \beta$

- 2) Define H as the four-dimensional unit open hypercube, and
 - i) $\forall z \in H, k(z) = z_1 + z_4 - z_2 - z_3$;
 - ii) $\forall z \in H, m(z) = z_2 - z_4$;
 - iii) $\forall z \in H, n(z) = z_3 - z_4$.

- 3) $\varepsilon_1 \varepsilon_2 \leq \max_{z \in H} |\alpha \beta [k(z)]^2 + ak(z)m(z) + bk(z)n(z) + m(z)n(z)|$

- 4) E.g., for $\alpha = \beta = 1/2$, $\varepsilon_1 \varepsilon_2 \leq 1/4$.

BREADTH OF IMPOSSIBILITY RESULT

- 1) Holds even for a countable U (even for a finite one).**
- 2) Holds even if current formulation of physics is wrong.**
- 3) Holds even if C has Super-Turing capability**
- 4) Holds even if laws of Nature are not written in predicate logic,
or intuitionism is correct,
or even if there are no laws, just a huge list of events.**

INFERENCE DEVICES AND THE LAWS OF NATURE

- 1) A reality is a space U , a set of devices defined over U , and a set of functions the devices might infer.**
- 2) So a reality is a quadruple, $(U, \{X_j, Y_j\}, \{\Gamma_i\})$.**
- 3) As far as any device in a reality is concerned, U is irrelevant. It's only the inference graph relating the sets $\{X_j, Y_j\}$ and $\{\Gamma_i\}$ that matter:**

*The laws of Nature are patterns in
the inference graph of a reality*

ELEMENTARY PROPERTIES OF INFERENCE

1) Inference need not be transitive:

$(X_1, Y_1) > Y_2$ and $(X_2, Y_2) > Y_3$ does not mean $(X_1, Y_1) > Y_3$

2) For any Γ , \exists a device that infers Γ .

3) For any device, \exists a Γ it does not infer. (*Impossibility result*)

- *Intuition*: X ~ initial configuration of a Turing machine.

Y (a bit) ~ whether Turing machine halts or not.

So apply Halting theorem-style reasoning

INFERENCE RELATIONS BETWEEN DEVICES -2

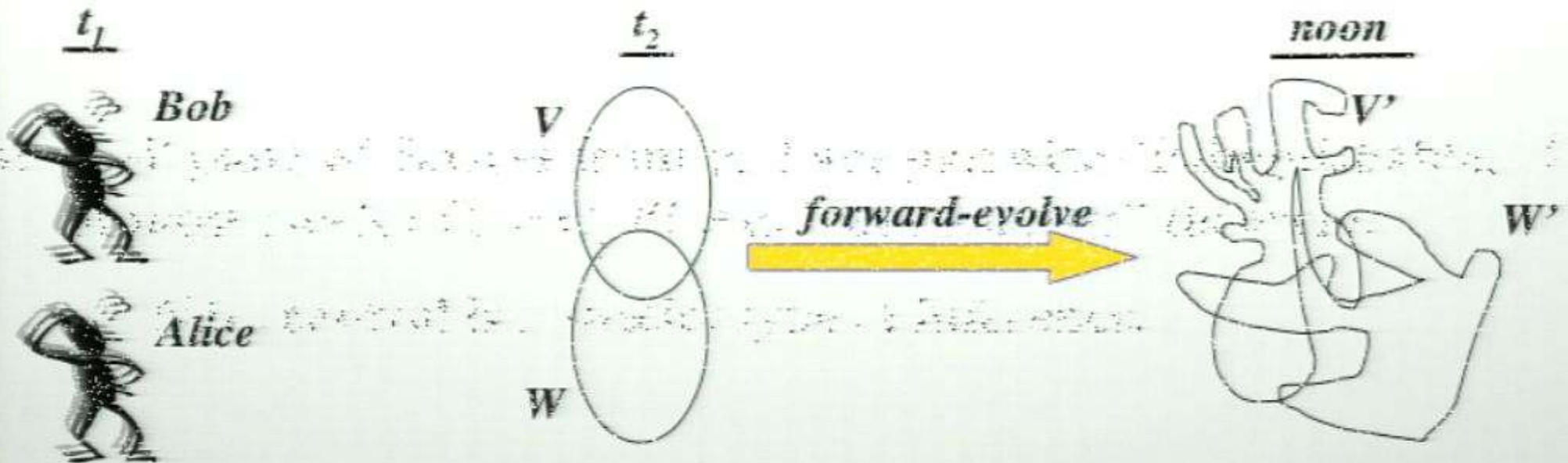
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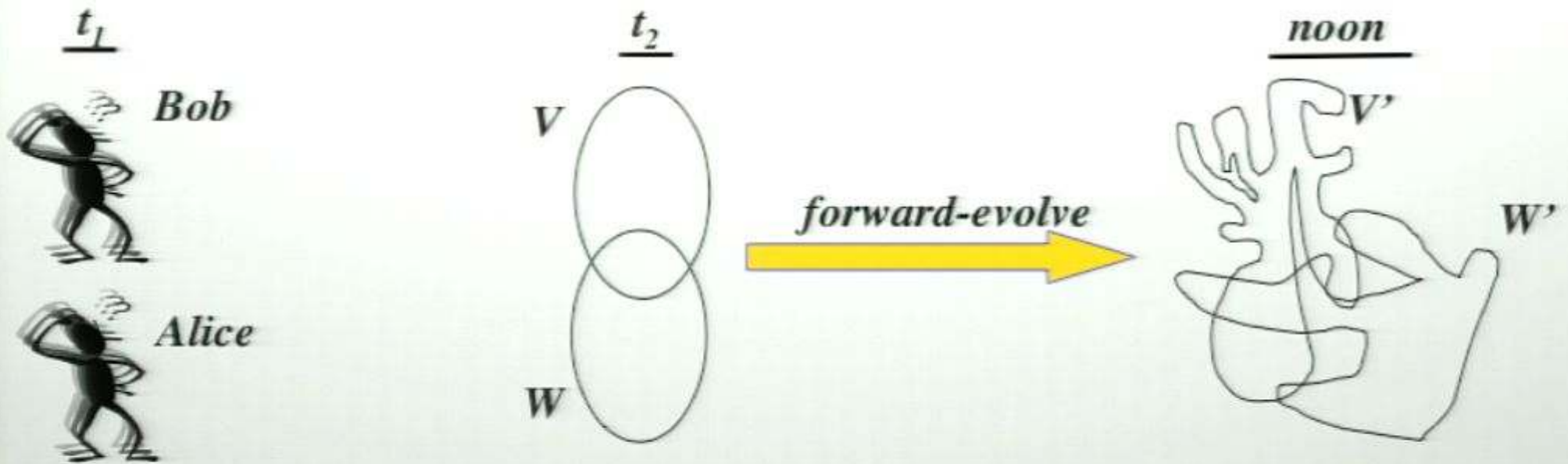
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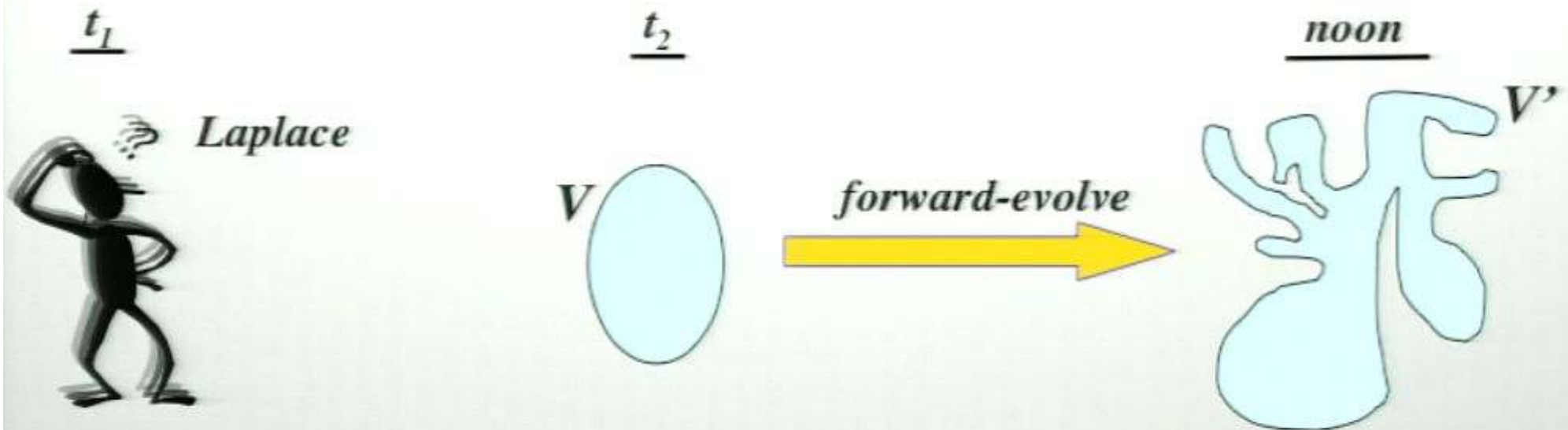
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