Title: Law without law: entropic dynamics

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Abstract:

Law without Law: Entropic Dynamics

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Department of Physics
University at Albany - SUNY

Laws of Nature Workshop
Perimeter Institute 05/2010

Question:

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To derive Quantum Theory as Entropic Dynamics

and discuss the implications for the theory of time.

Entropic Inference

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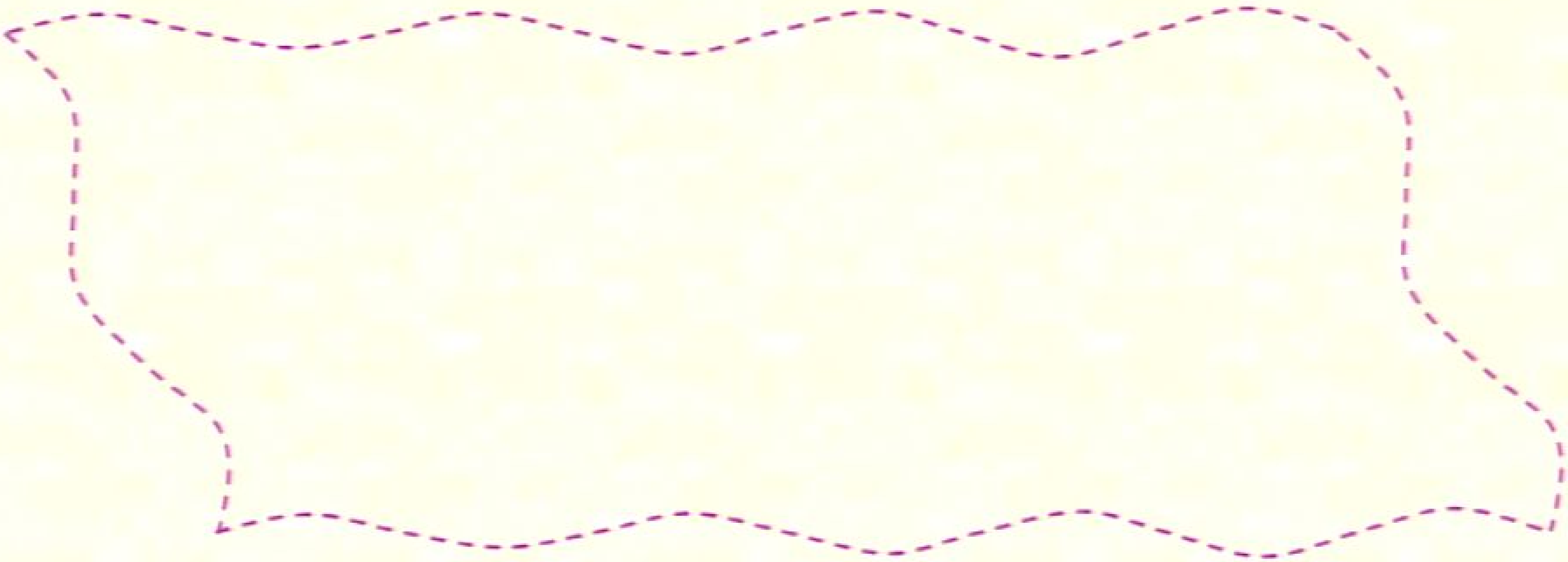

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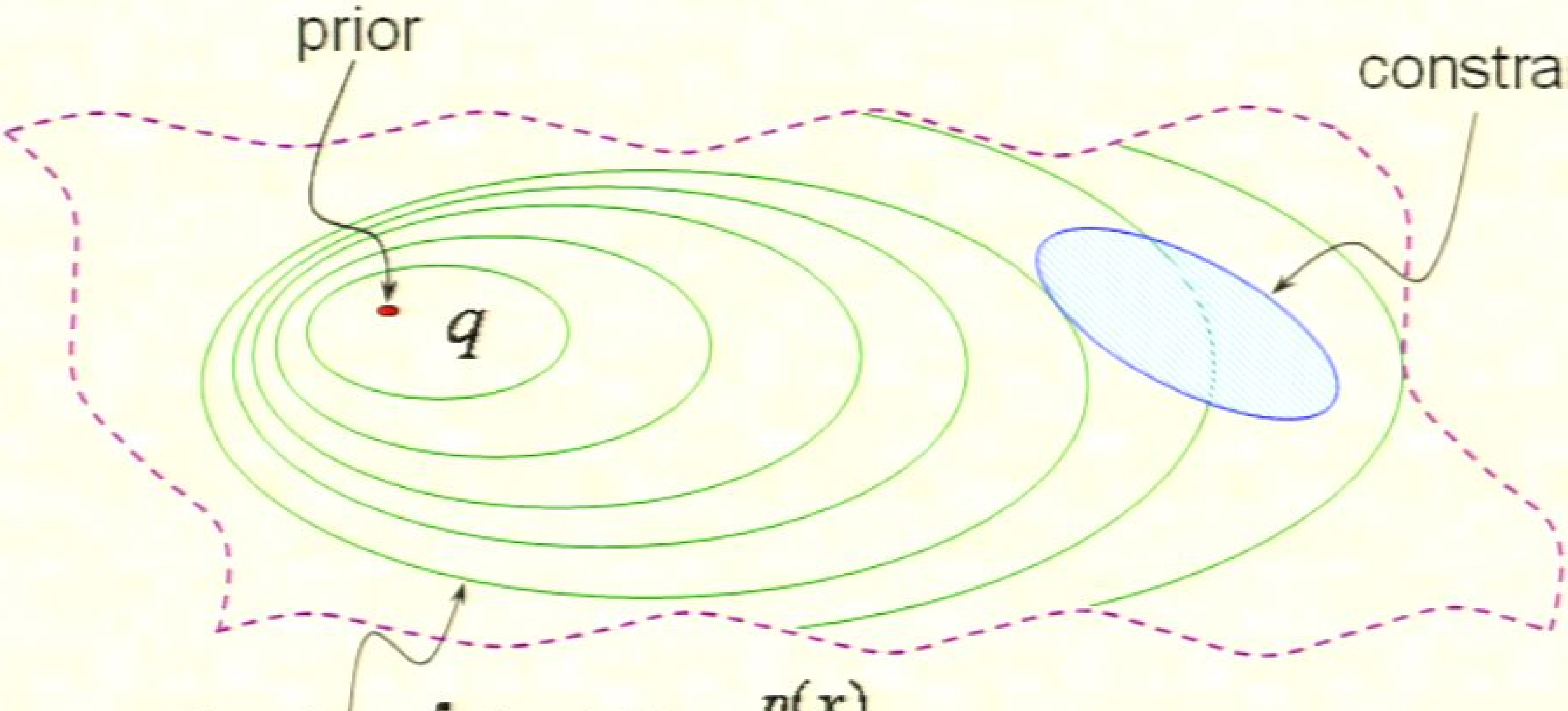

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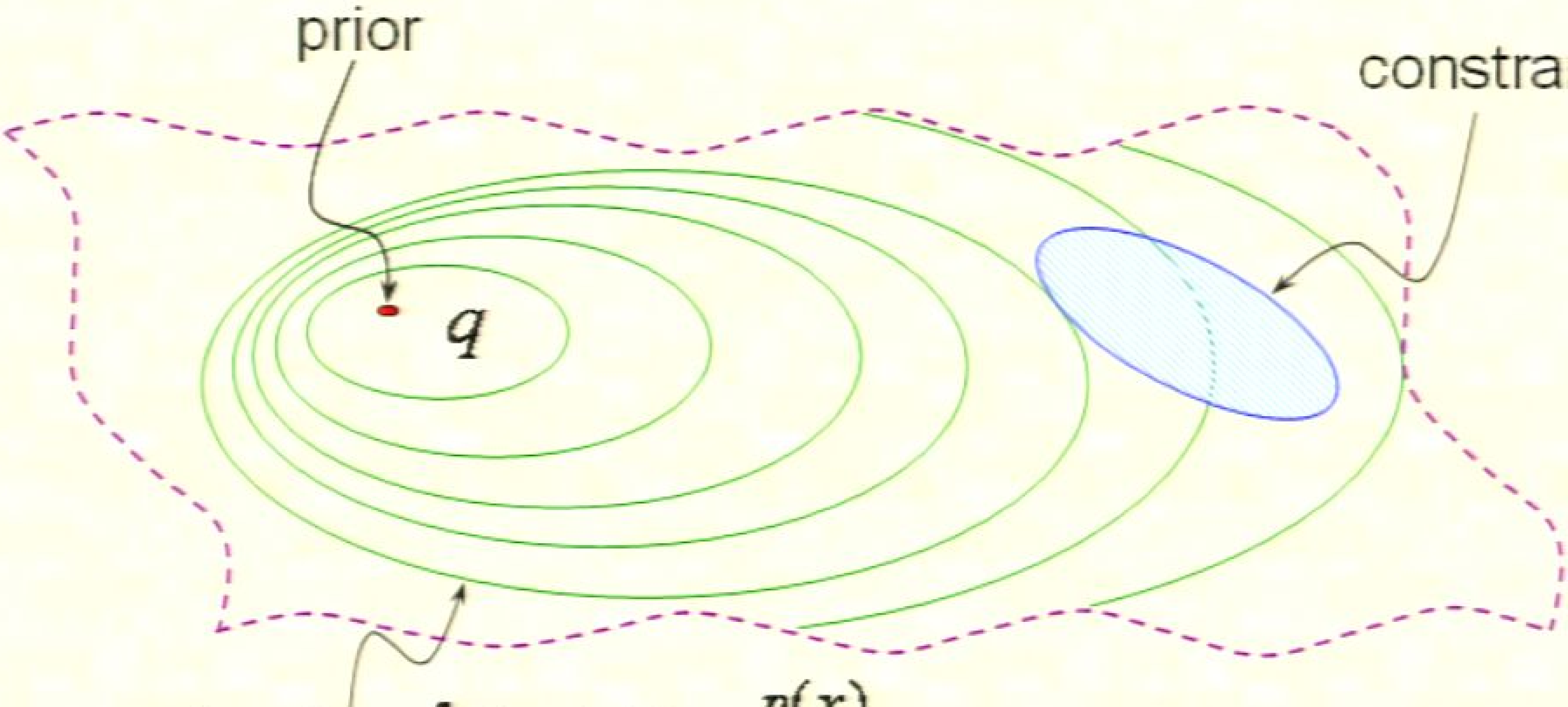

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Maximize $S[p,q]$ subject to the appropriate constraints.

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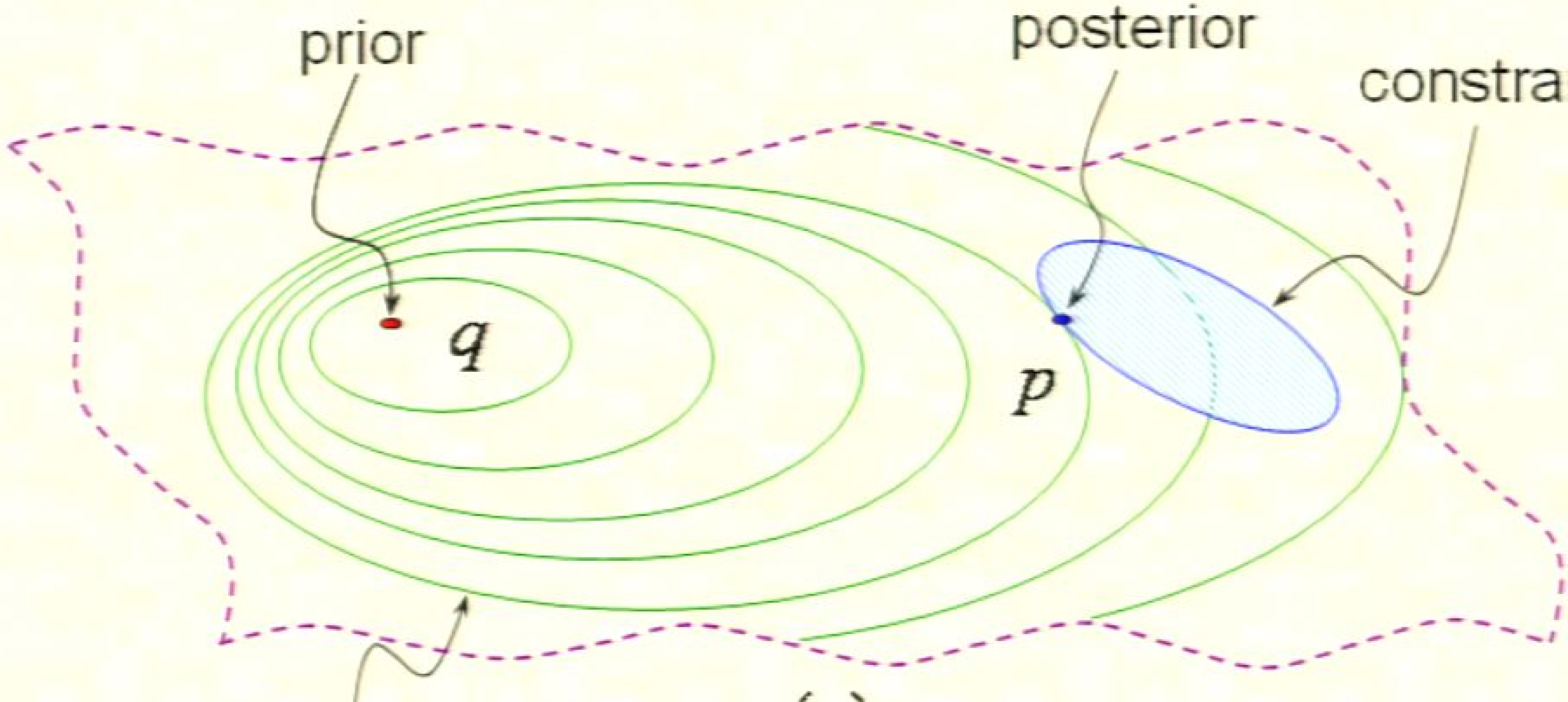

$$S[p,q] = -\int dx\, p(x) \log \frac{p(x)}{q(x)}$$

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(MaxEnt, Bayes' rule and Large Deviations are special cases.)

Step 1: The Statistical Model

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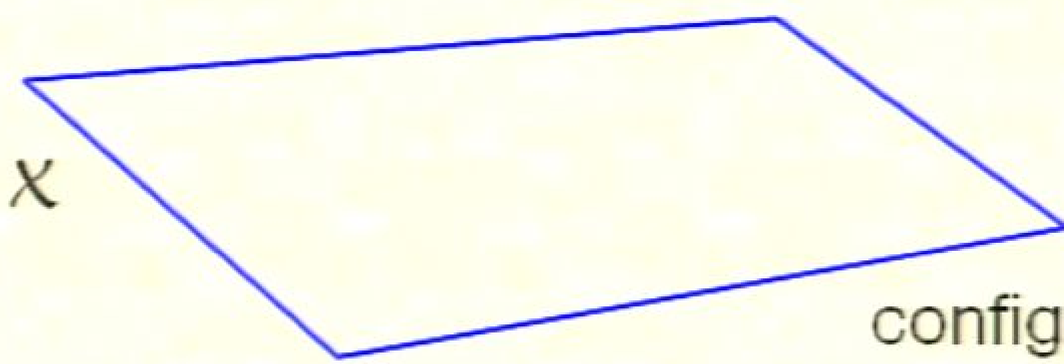

configuration space
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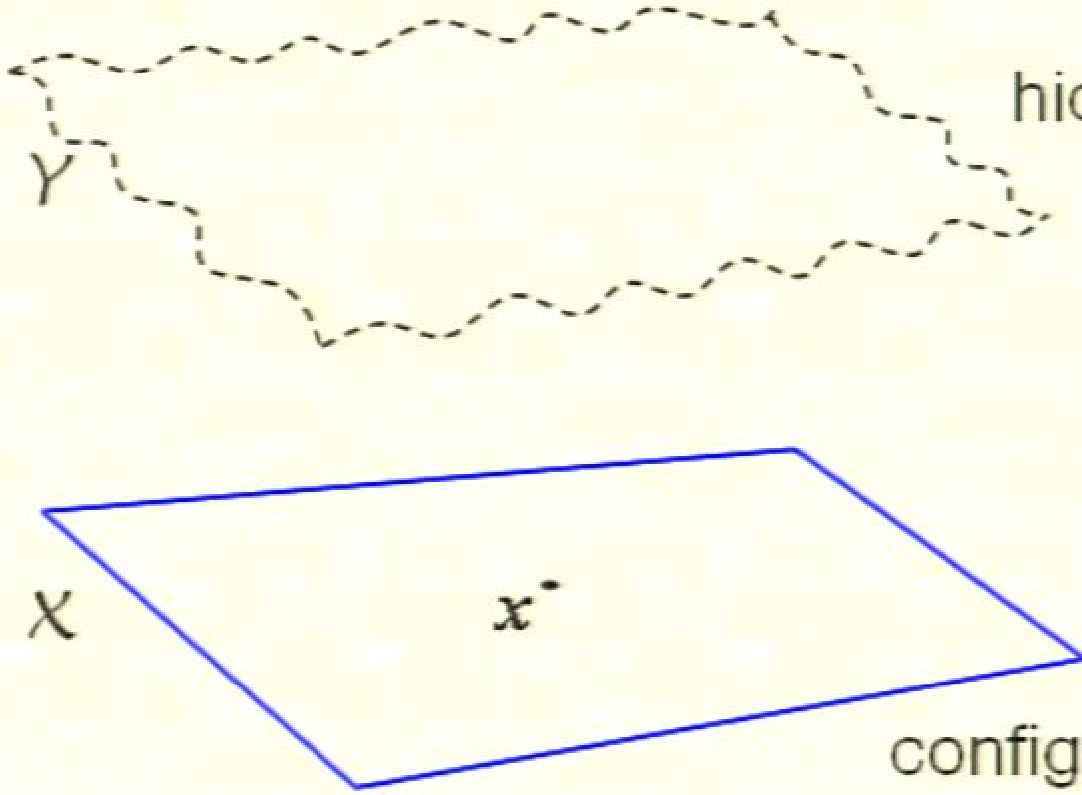

hidden variables

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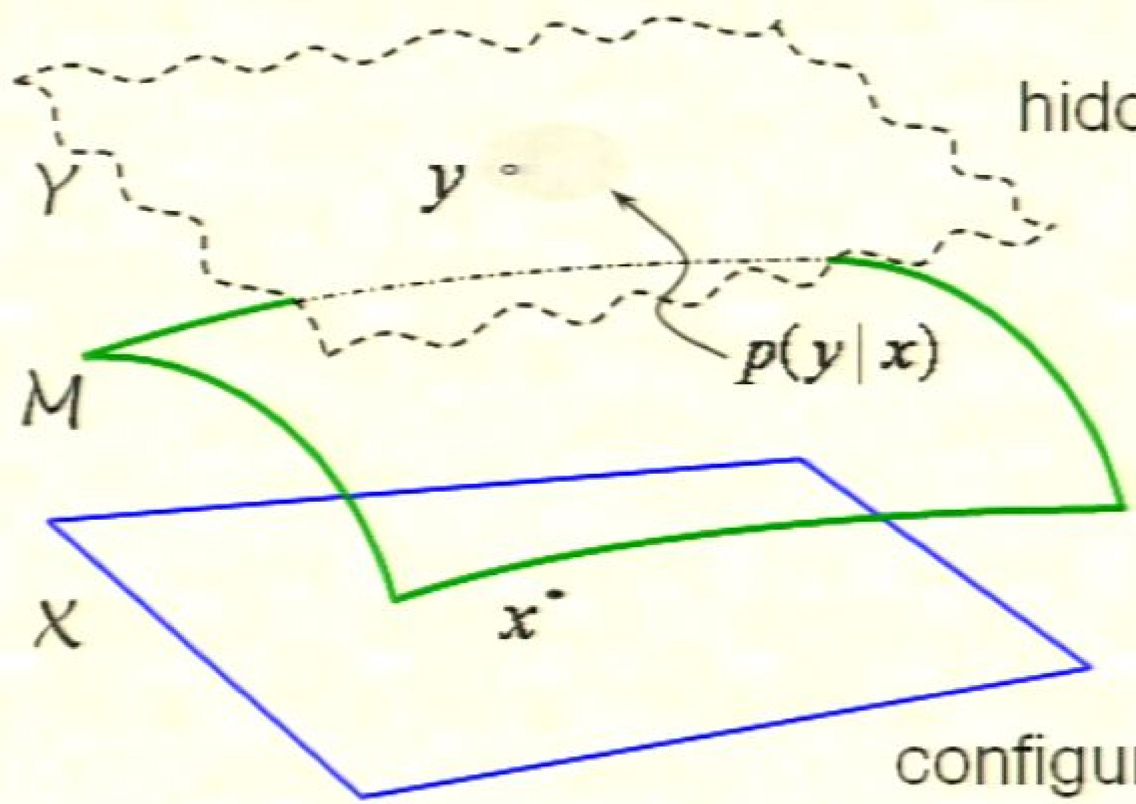

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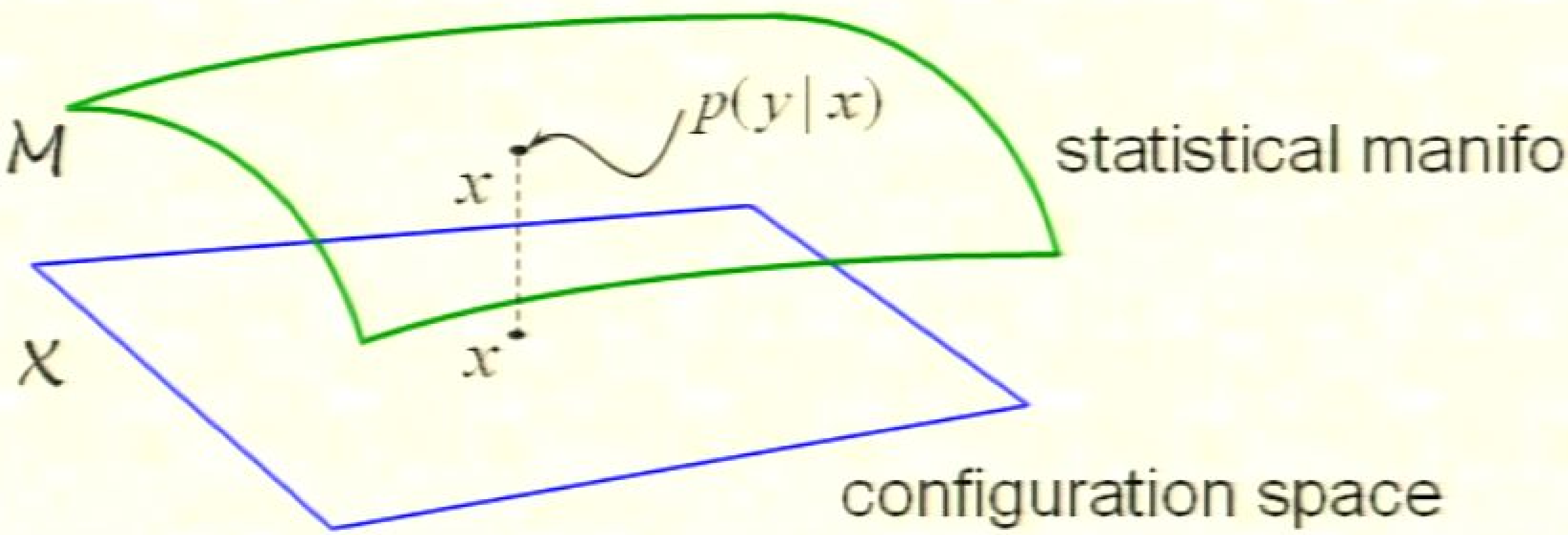

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Step 2: Entropic Dynamics

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Changes happen gradually.

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Short steps: $\langle \Delta\ell^2\rangle = \langle \gamma_{ab}\Delta x^a \Delta x^b\rangle = \lambda^2(x)$

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Fluctuations: $$\langle \Delta w^a \Delta w^b \rangle = \frac{1}{\alpha(x)} \gamma^{ab}$$

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$$P(x') = \int dx\, P(x', x) = \int dx\, P(x' \mid x) P(x)$$

(1) Introduce the notion of an instant

$$\rho(x', t') = \int dx\, P(x' \mid x) \rho(x, t)$$

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For large α the dynamics is all in the fluctuations:

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For large α the dynamics is all in the fluctuations:

$$\langle \Delta w^a \Delta w^b \rangle = \frac{1}{\alpha(x)} \gamma^{ab}$$

Define duration so that motion looks simple:

The result is a Fokker-Planck eq.: $\partial_t \rho = -\partial_a(\rho v^a)$

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$$\underbrace{v^a = \frac{\sigma^2}{\tau} \partial^a \phi \qquad \phi(x,t) = S(x) - \log \rho^{1/2}(x,t)}_{v^a = b^a + u^a}$$

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drift velocity: $b^a = \frac{\sigma^2}{\tau}\partial^a S$

osmotic velocity: $u^a = -\frac{\sigma^2}{2\tau}\frac{\partial^a \rho}{\rho}$

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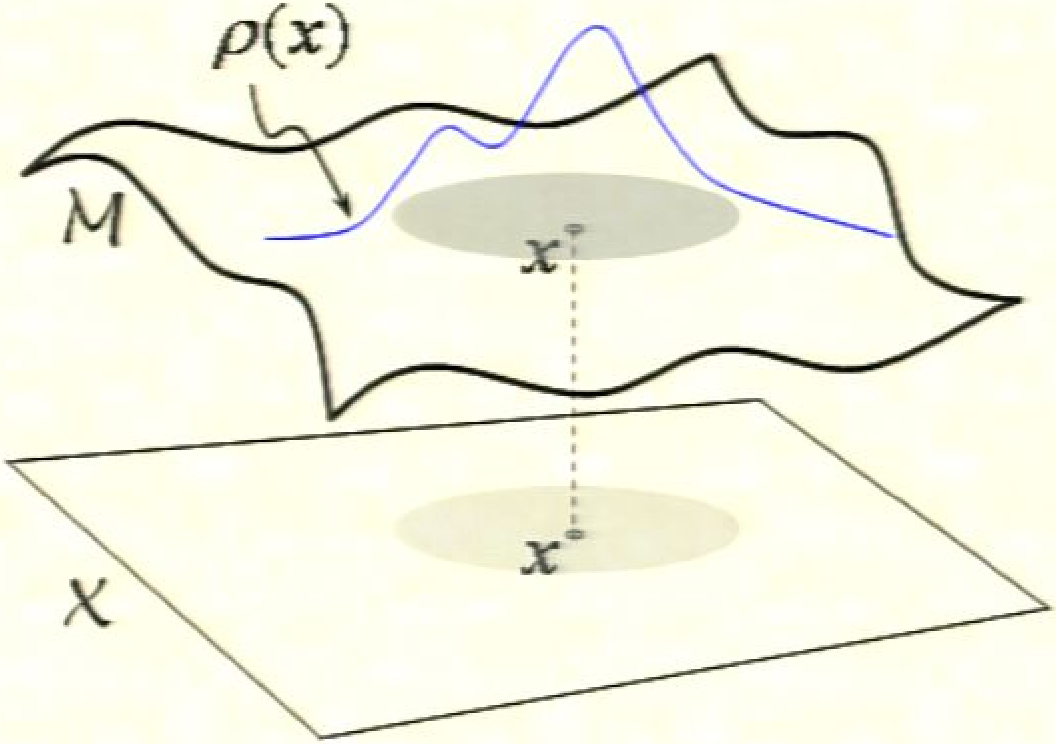

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$$E = \int d^3x \, \rho \left[\frac{1}{2} m v^2 + \frac{1}{2} m u^2 + V(x) \right]$$

where $$m = \frac{2A}{\sigma^2}$$

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$$\dot{\rho} = -\frac{\hbar}{m}\partial^a\left(\rho\,\partial_a\phi\right)$$

2) energy conservation + diffusion

$$\hbar\dot{\phi} + \frac{\hbar^2}{2m}(\partial_a\phi)^2 + V - \frac{\hbar^2}{2m}\frac{\nabla^2\rho^{1/2}}{\rho^{1/2}} = 0$$

Combine ρ and ϕ into $\Psi = \rho^{1/2}\, e^{i\phi}$

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Let $S_{HJ} = \hbar\phi$ then $\dot{S}_{HJ} + \frac{1}{2m}(\partial_a S_{HJ})^2 + V = 0$

More on entropic time

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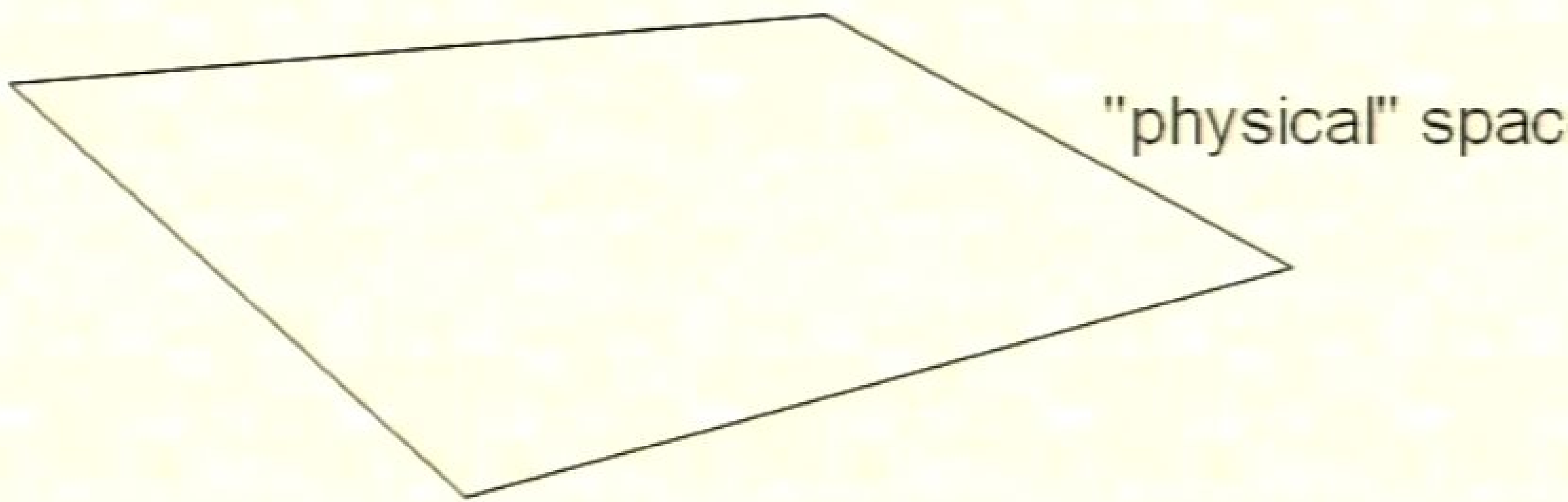
A clock follows a classical trajectory.



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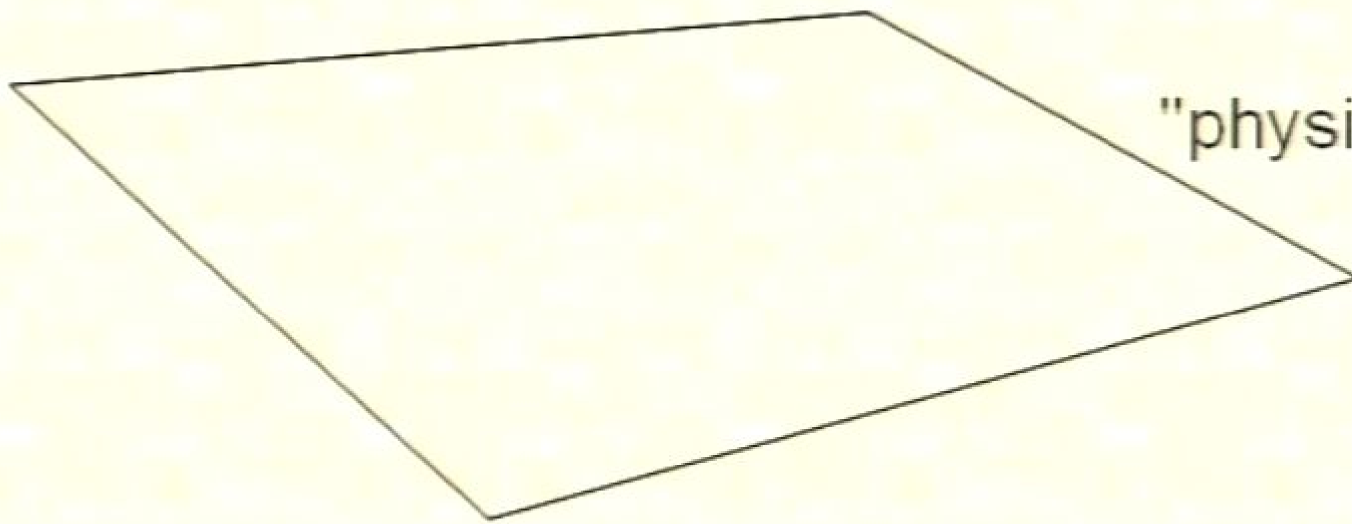

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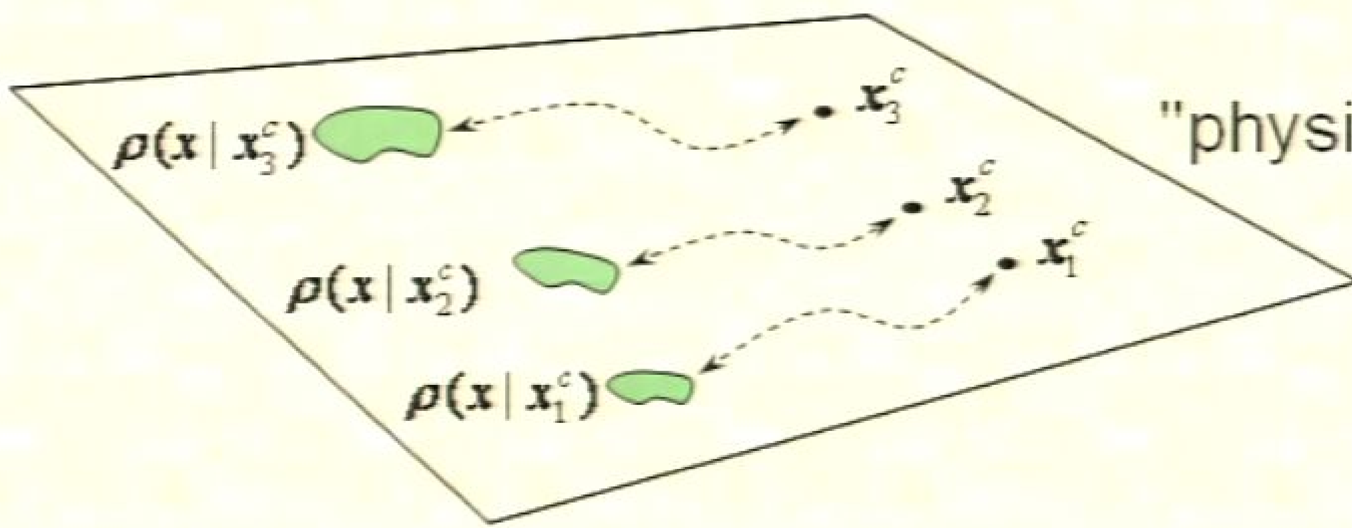

"physical" space

Entropic time *vs* "physical" time?

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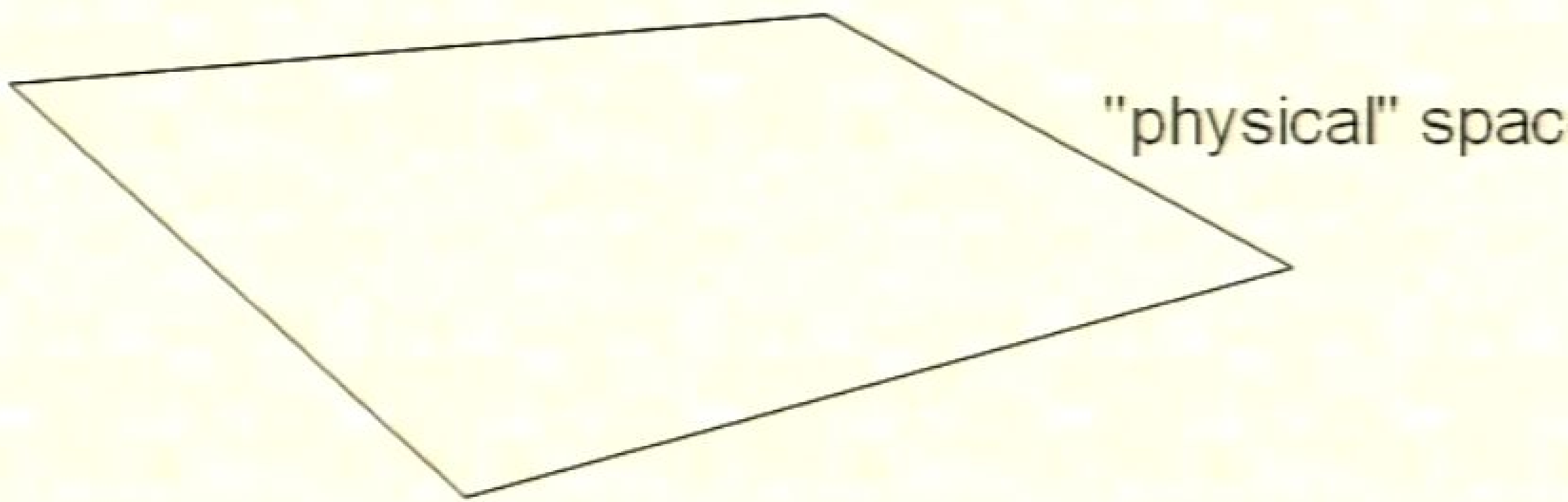
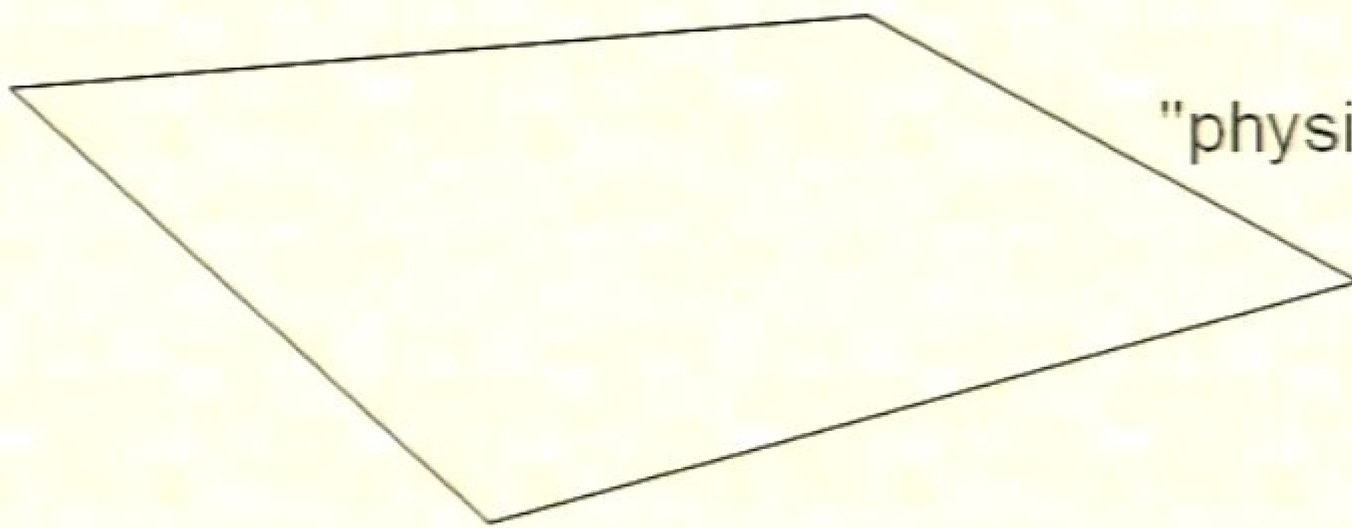


Entropic time *vs* "physical" time?

We observe correlations at an instant.



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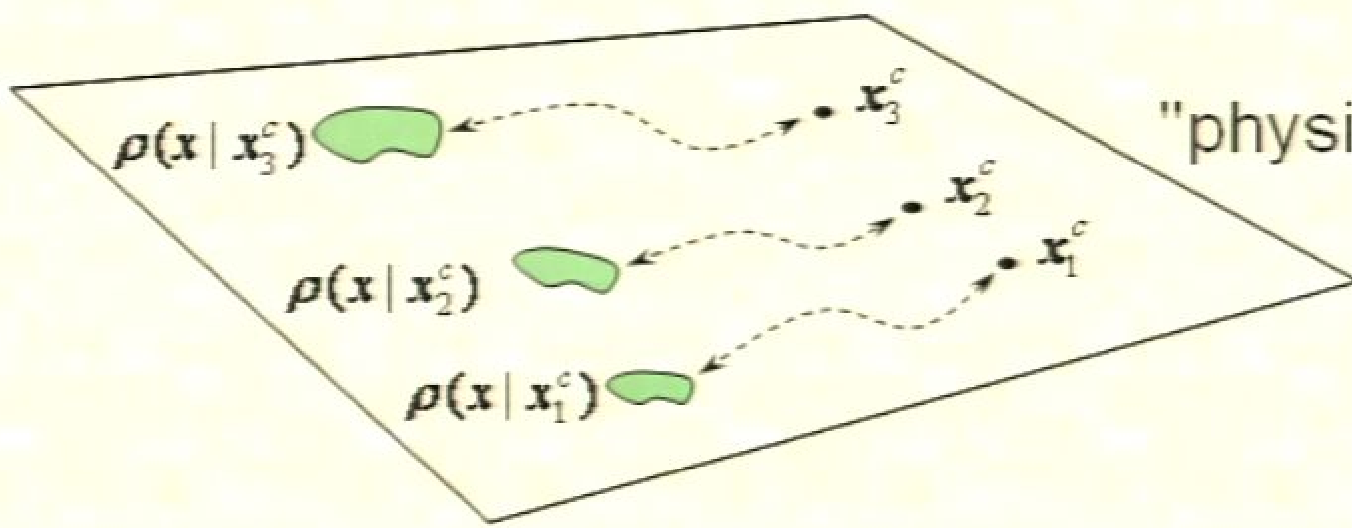

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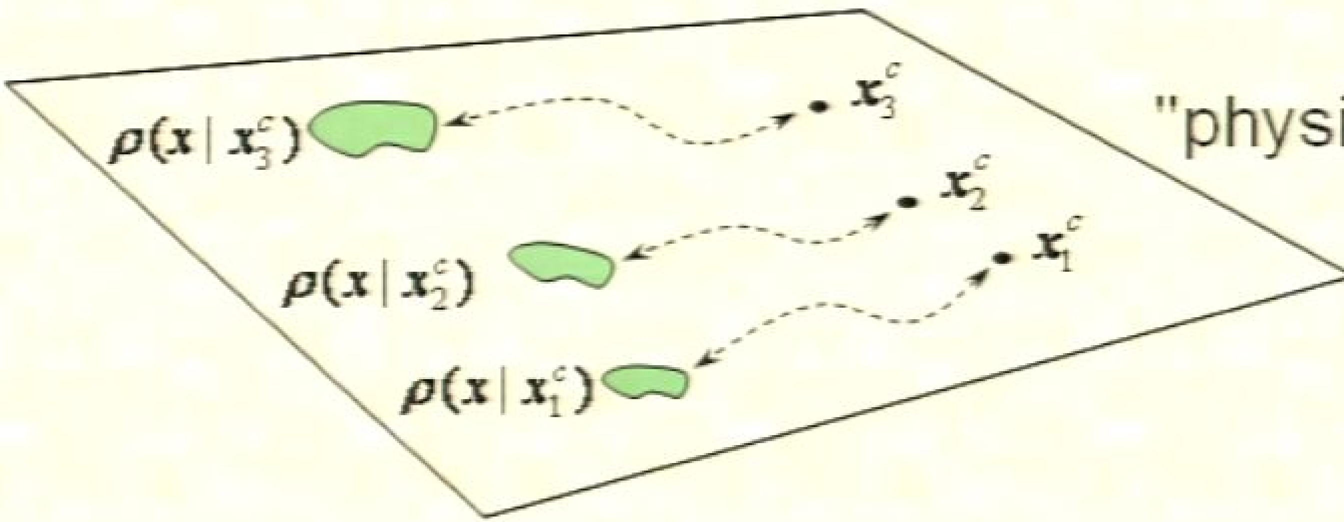

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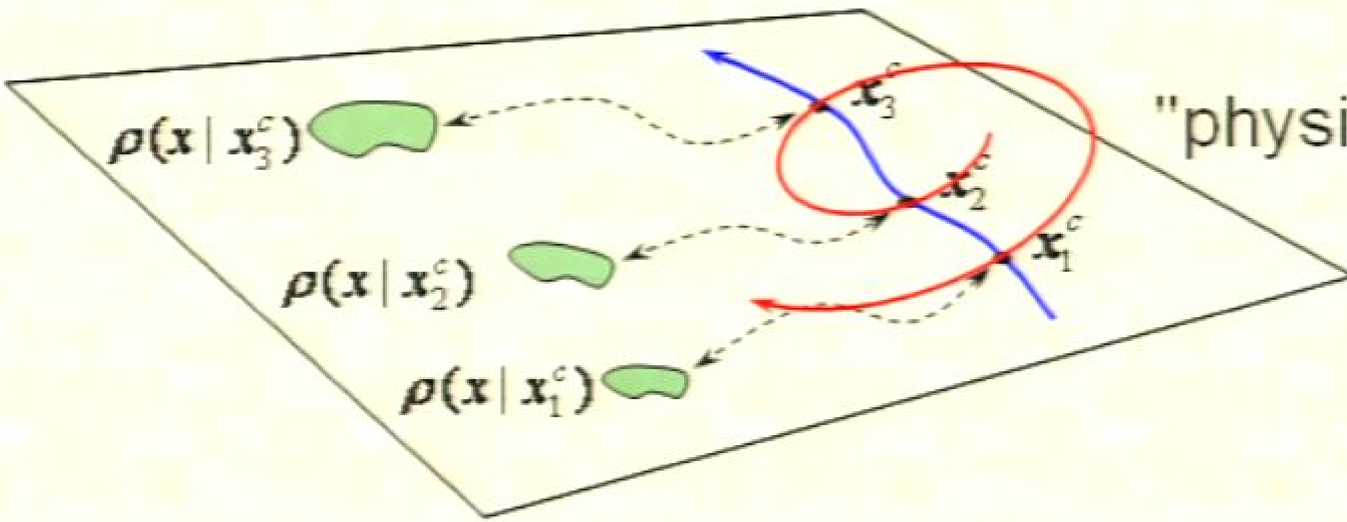

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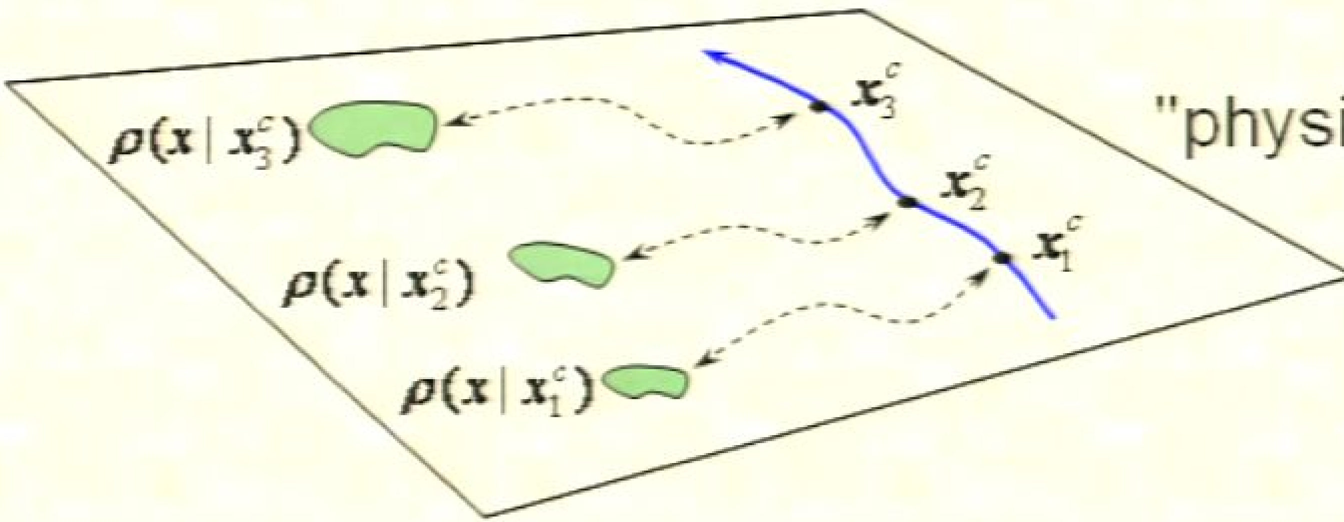

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Entropic time is all we need.

There is an arrow of entropic time.

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The Big Picture:

Conclusion:

On Laws of Physics vs. Laws of Nature:

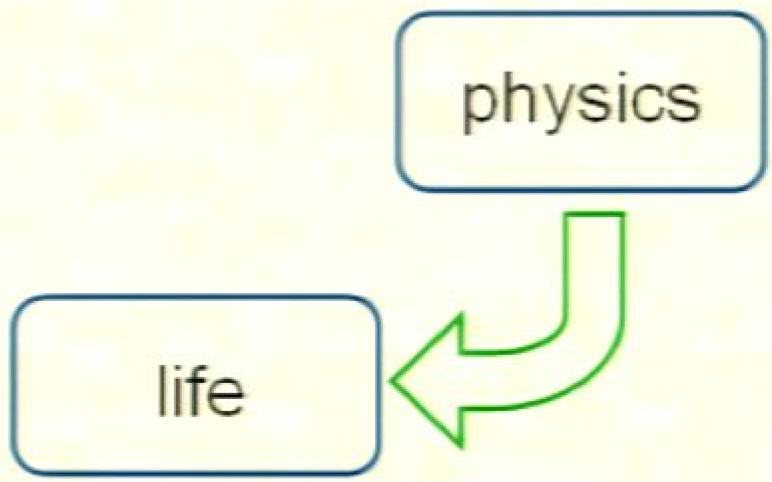
Entropic inference can lead to Laws of Physics.

It provides an alternative to Action Principles.

The natural order/arrow of inference is entropic time...

...and this is all one needs.

The Big Picture:



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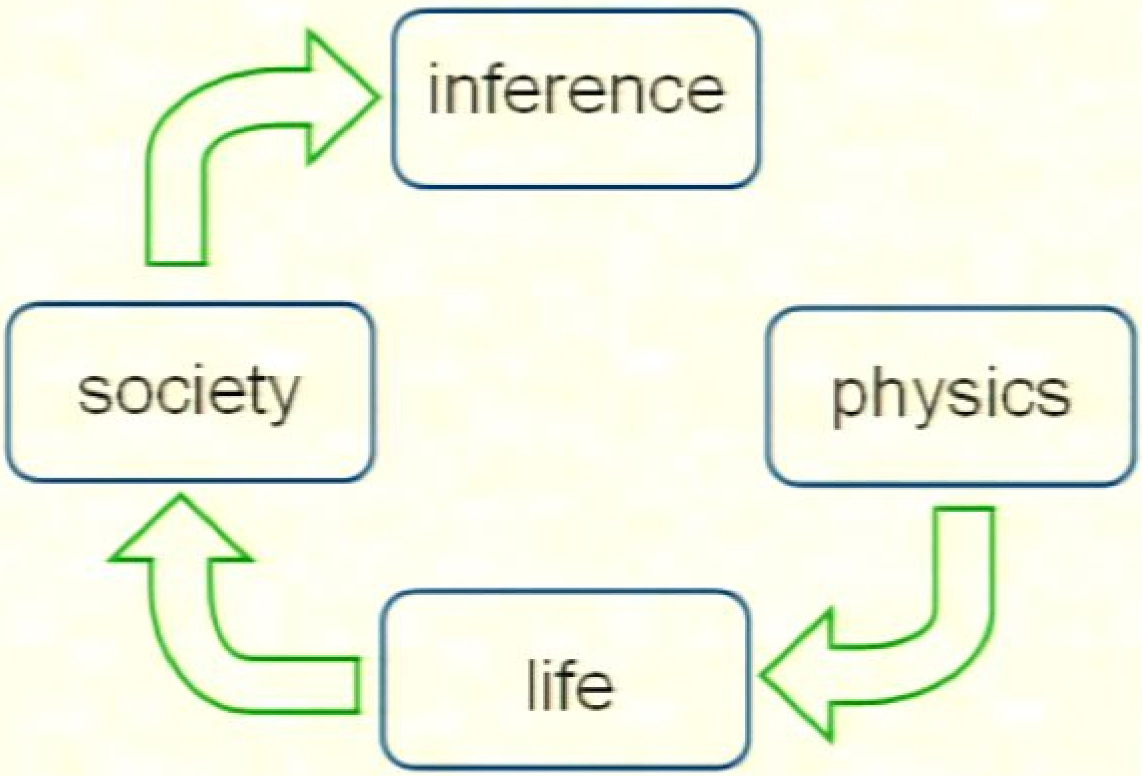

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