Title: The common symmetries underlying quantum theory, probability theory, and number systems

Date: May 21, 2010 10:00 AM

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Abstract:

The

Logic of Process

and the origin of

Quantum Theory

Philip Goyal
Perimeter Institute

Classical Conception of Reality

Classical Conception of Reality

Classical Mathematical Framework

Classical Conception of Reality

Newtonian Mechanics

Classical Mathematical Framework

Classical Conception of Reality


ORTVS
OCCASVS
AVRORA
CREPVS

Newtonian Mechanics

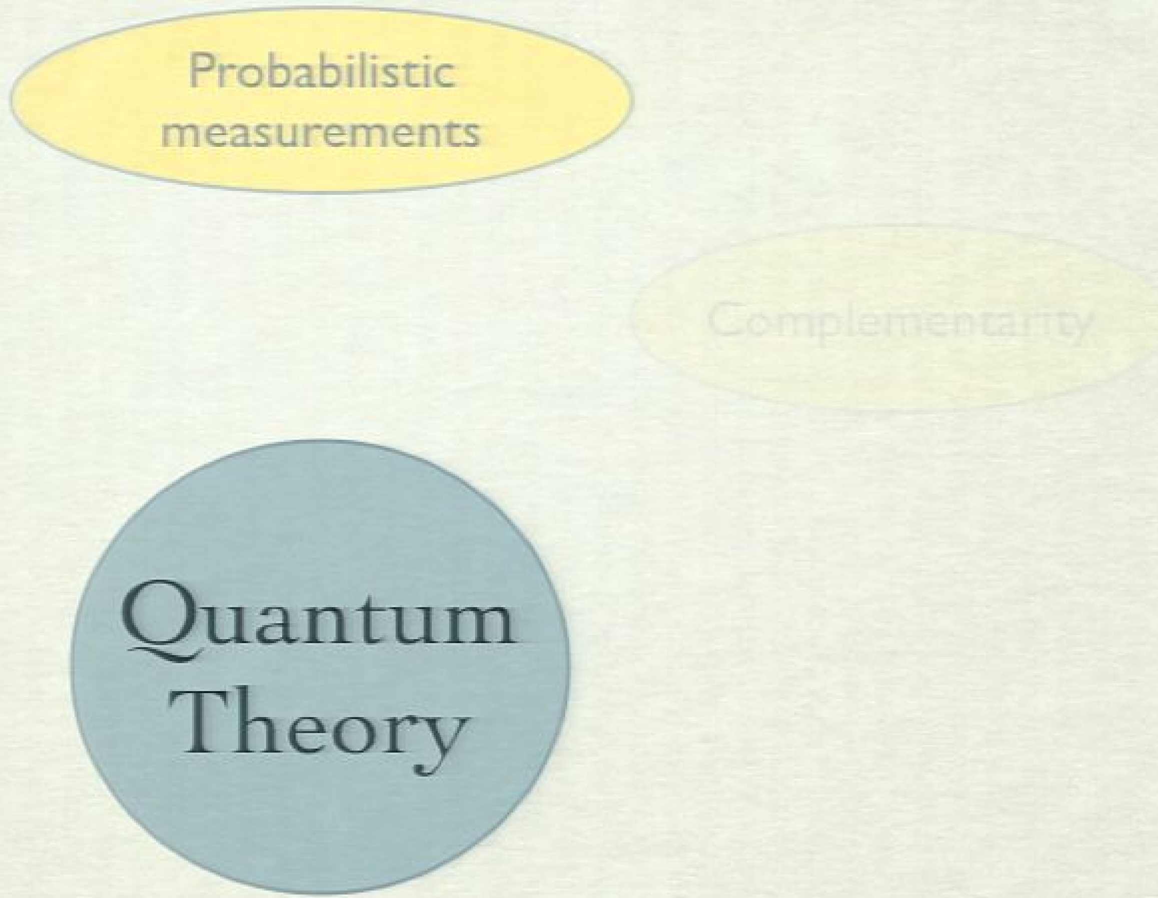
Classical Mathematical Framework

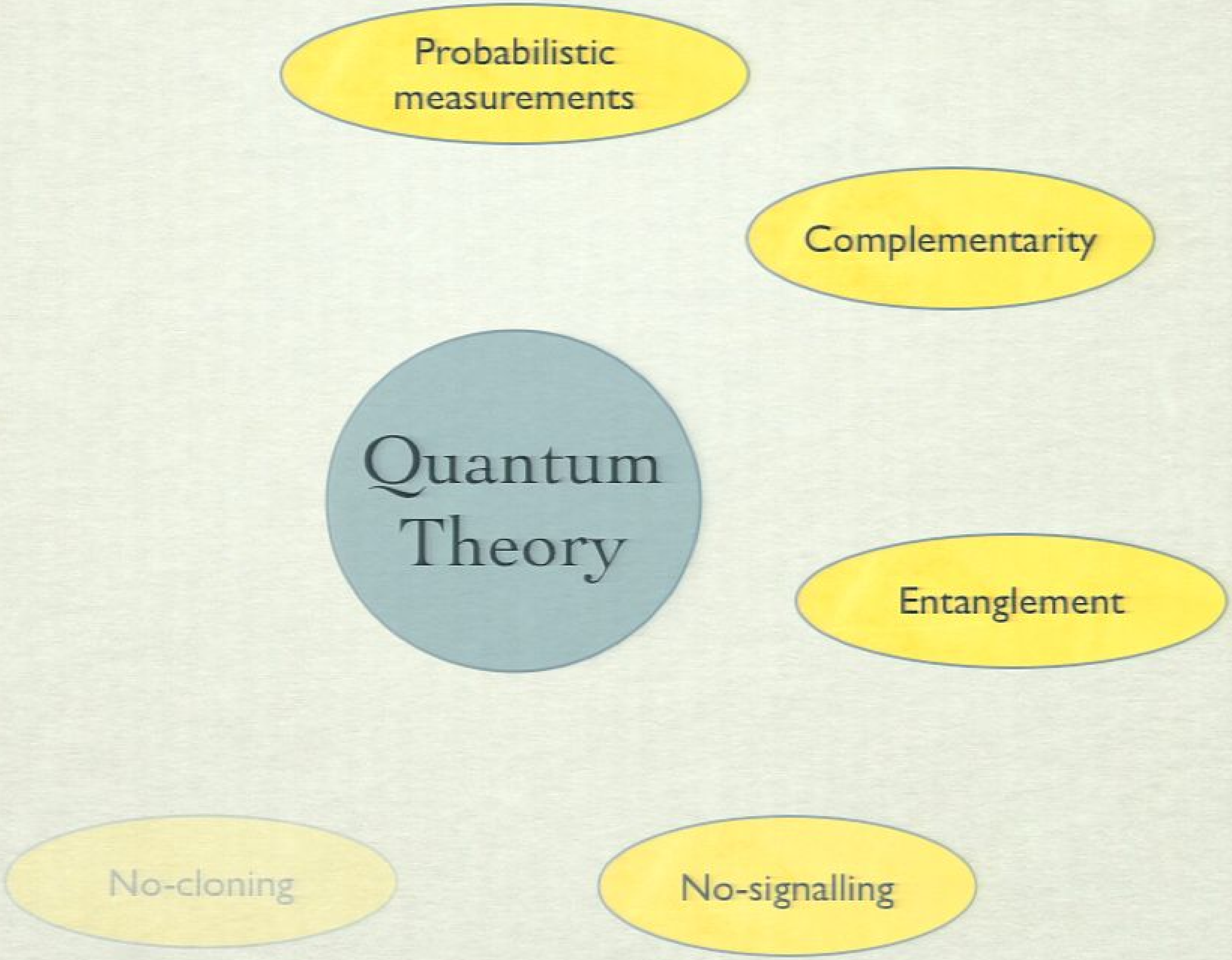
Classical Conception of Reality

Schroedinger's Mechanics

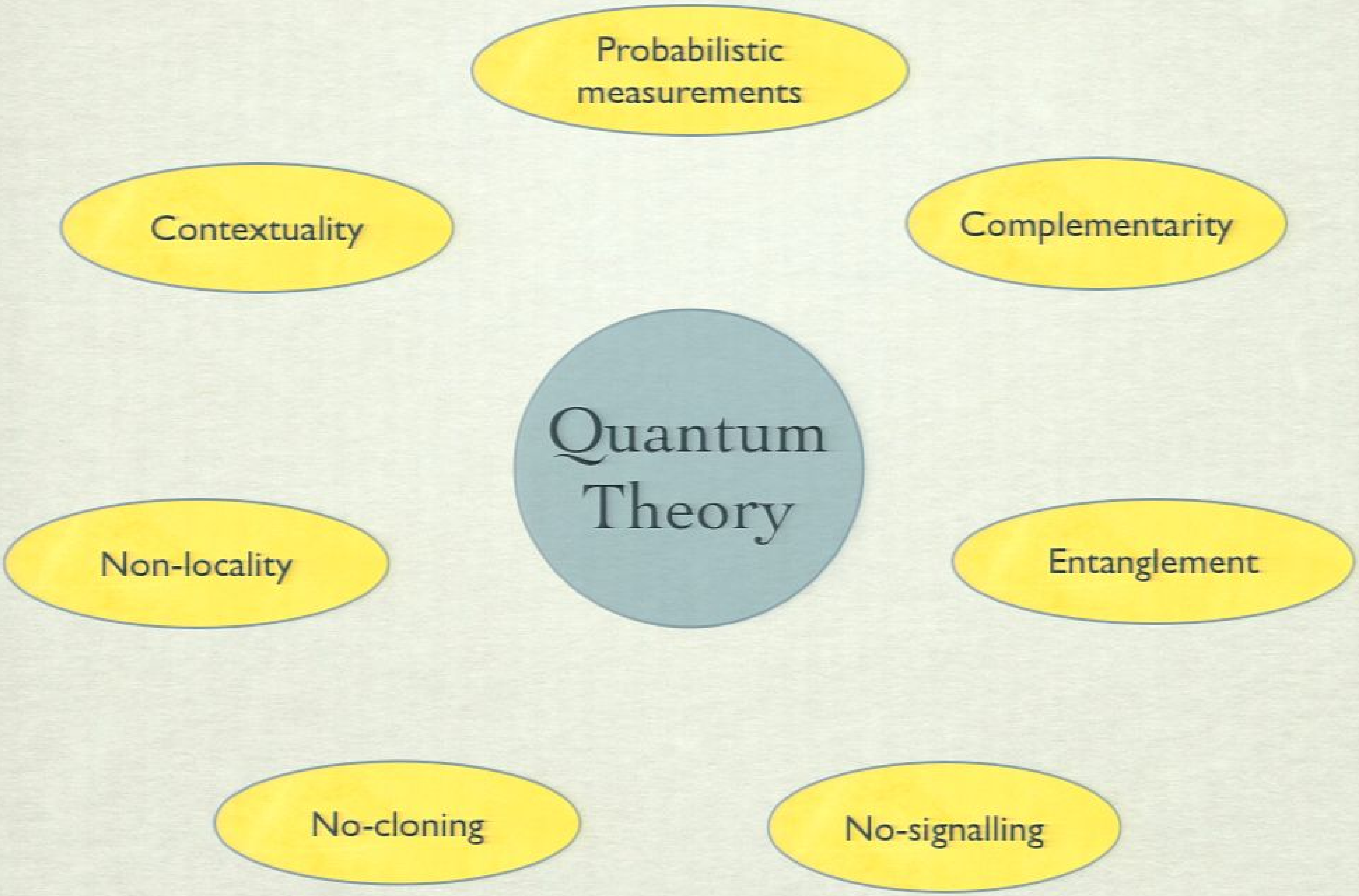
Quantum Mathematical Framework

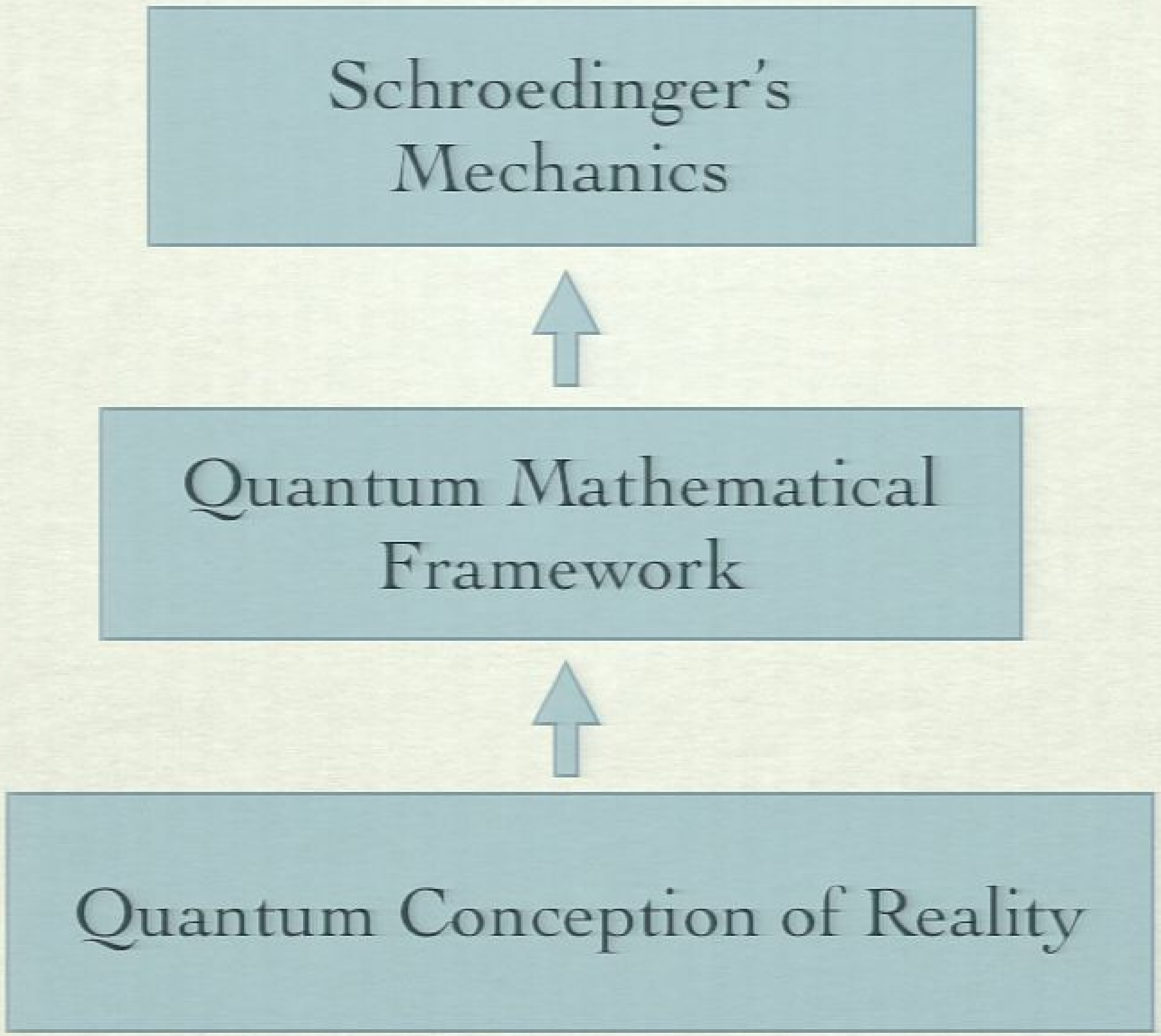
Quantum Conception of Reality


Probabilistic measurements
Complementarity
Quantum Theory



Probabilistic measurements
Complementarity
Quantum Theory
Entanglement
No-cloning
No-signalling


Probabilistic measurements
Contextuality
Complementarity
Quantum Theory
Non-locality
Entanglement
No-cloning
No-signalling


Schroedinger's Mechanics
Quantum Mathematical Framework
Quantum Conception of Reality

Quantum Mathematical Framework

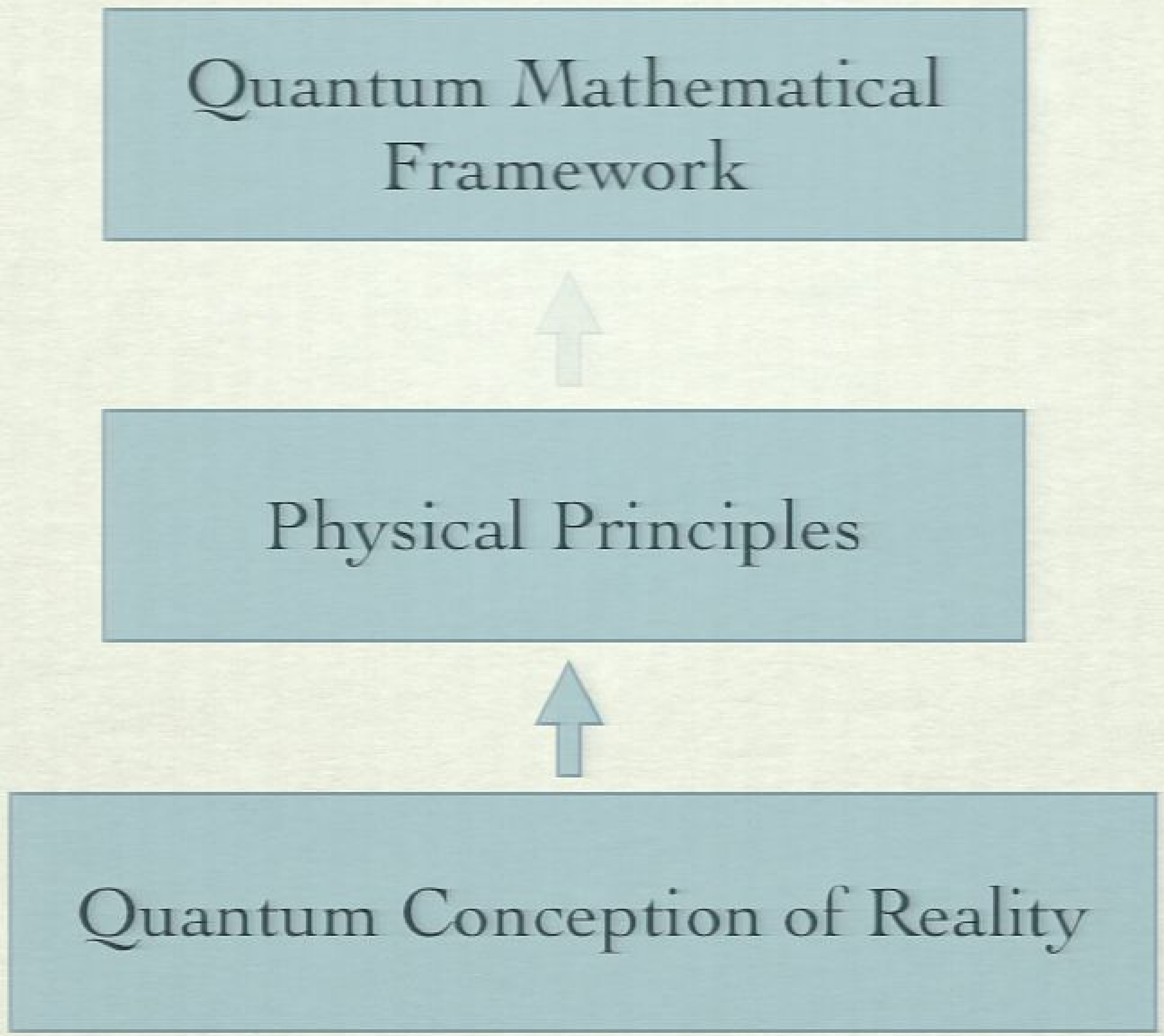
Quantum Conception of Reality

Quantum Mathematical Framework

Physical Principles

↑

Quantum Conception of Reality


Quantum Mathematical Framework
Physical Principles
Quantum Conception of Reality

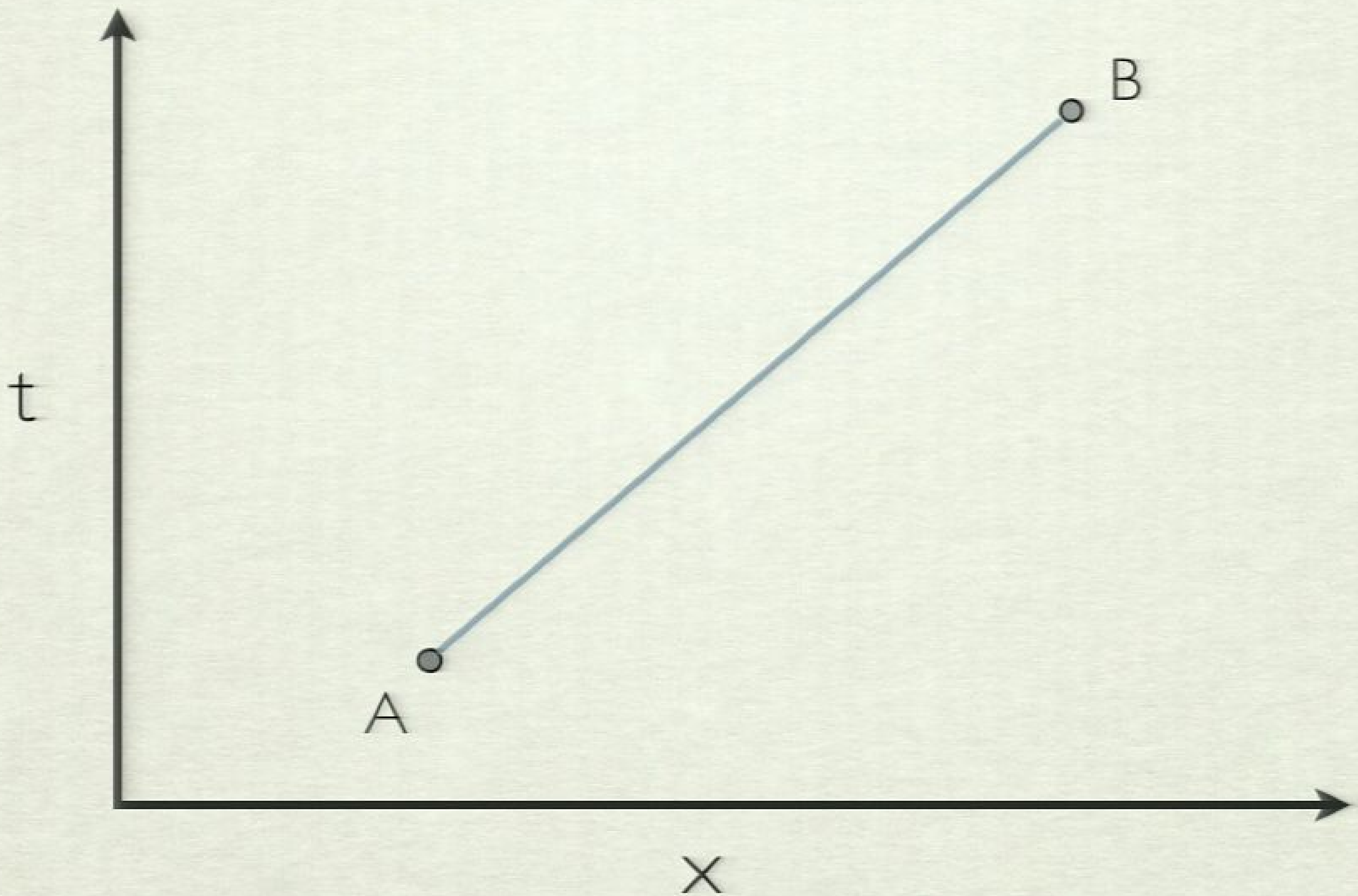
Quantum Mathematical Framework

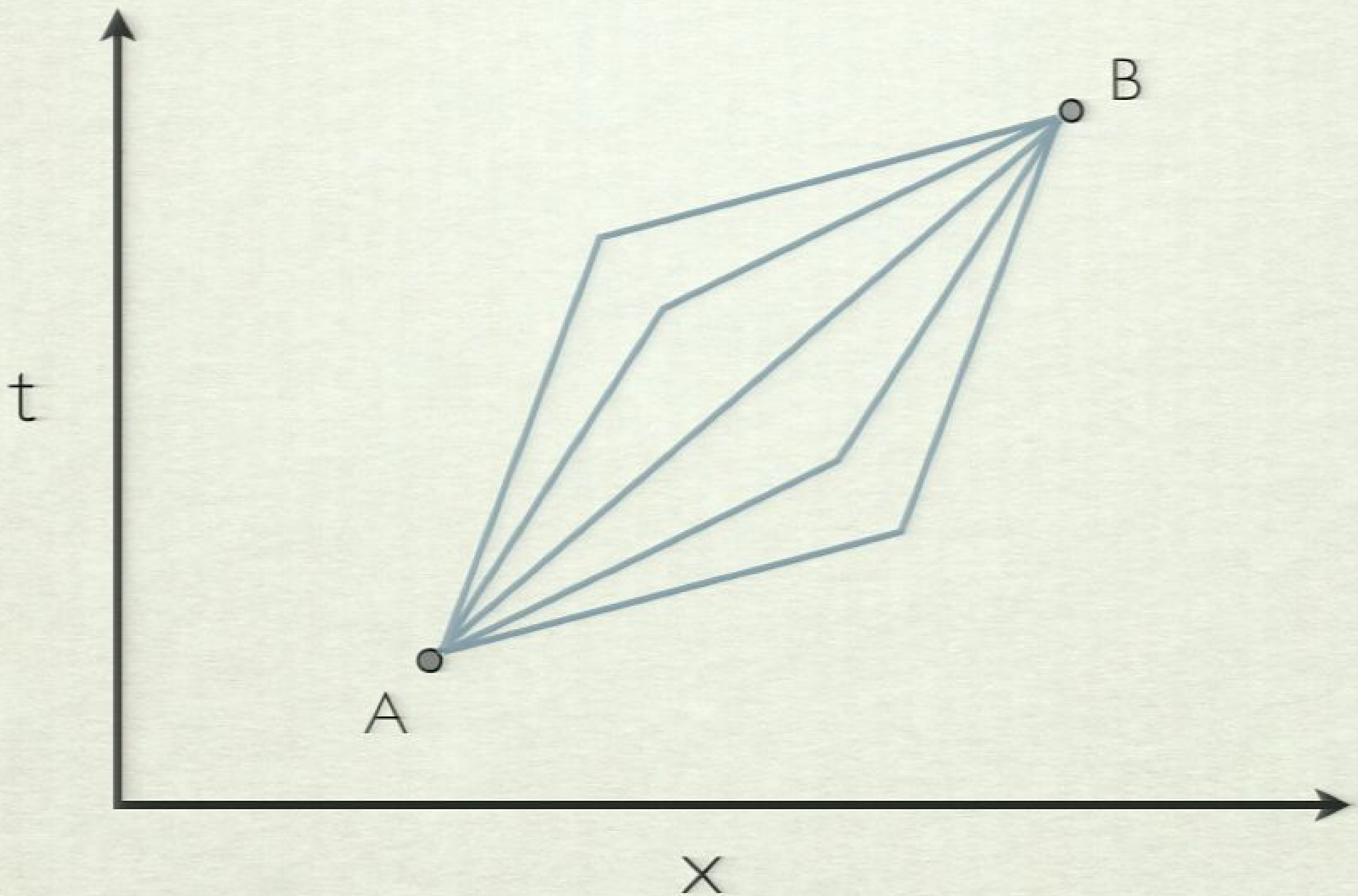
Physical Principles

Quantum Conception of Reality

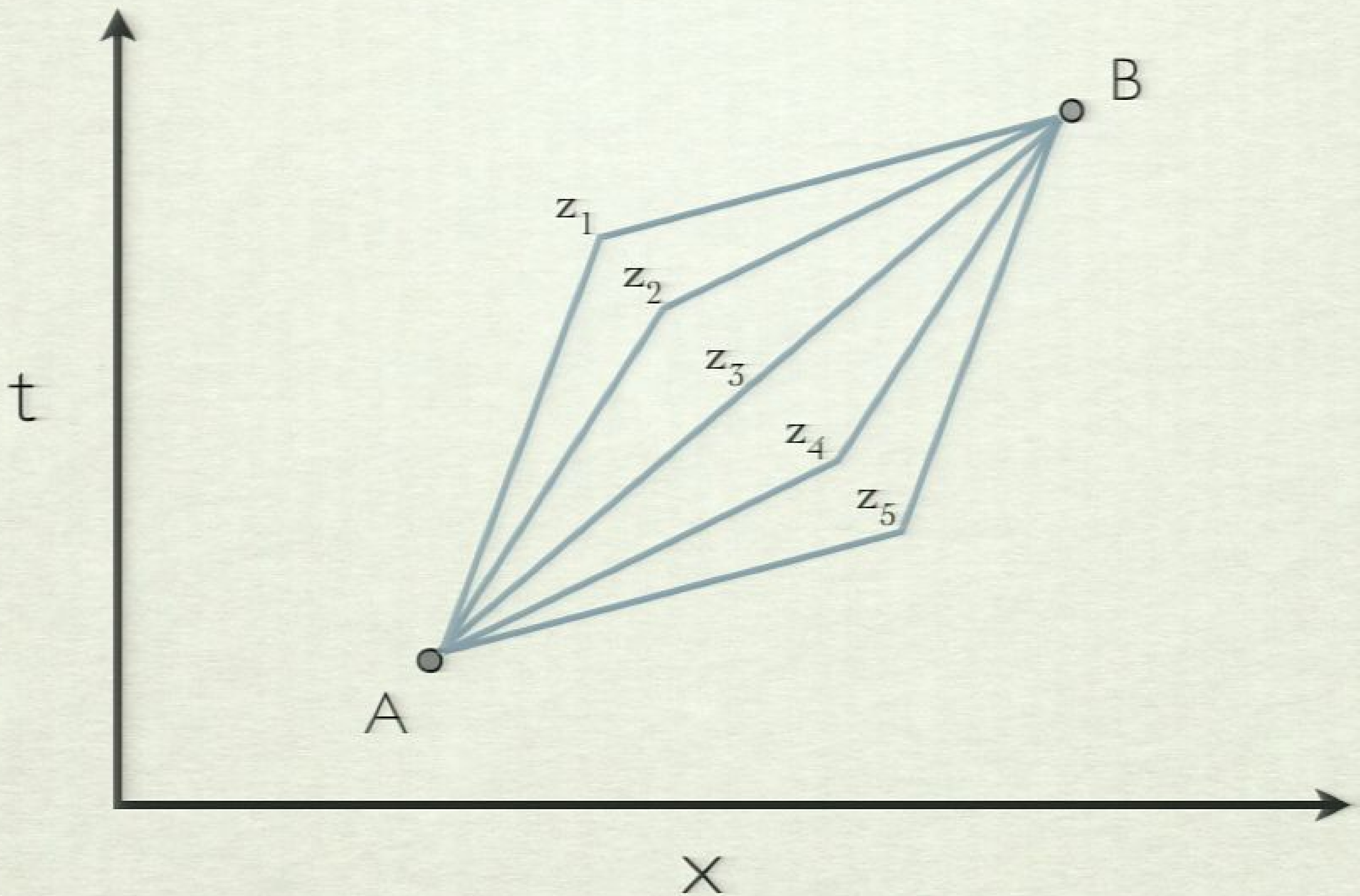
Reconstructio
of Quantum
Theory

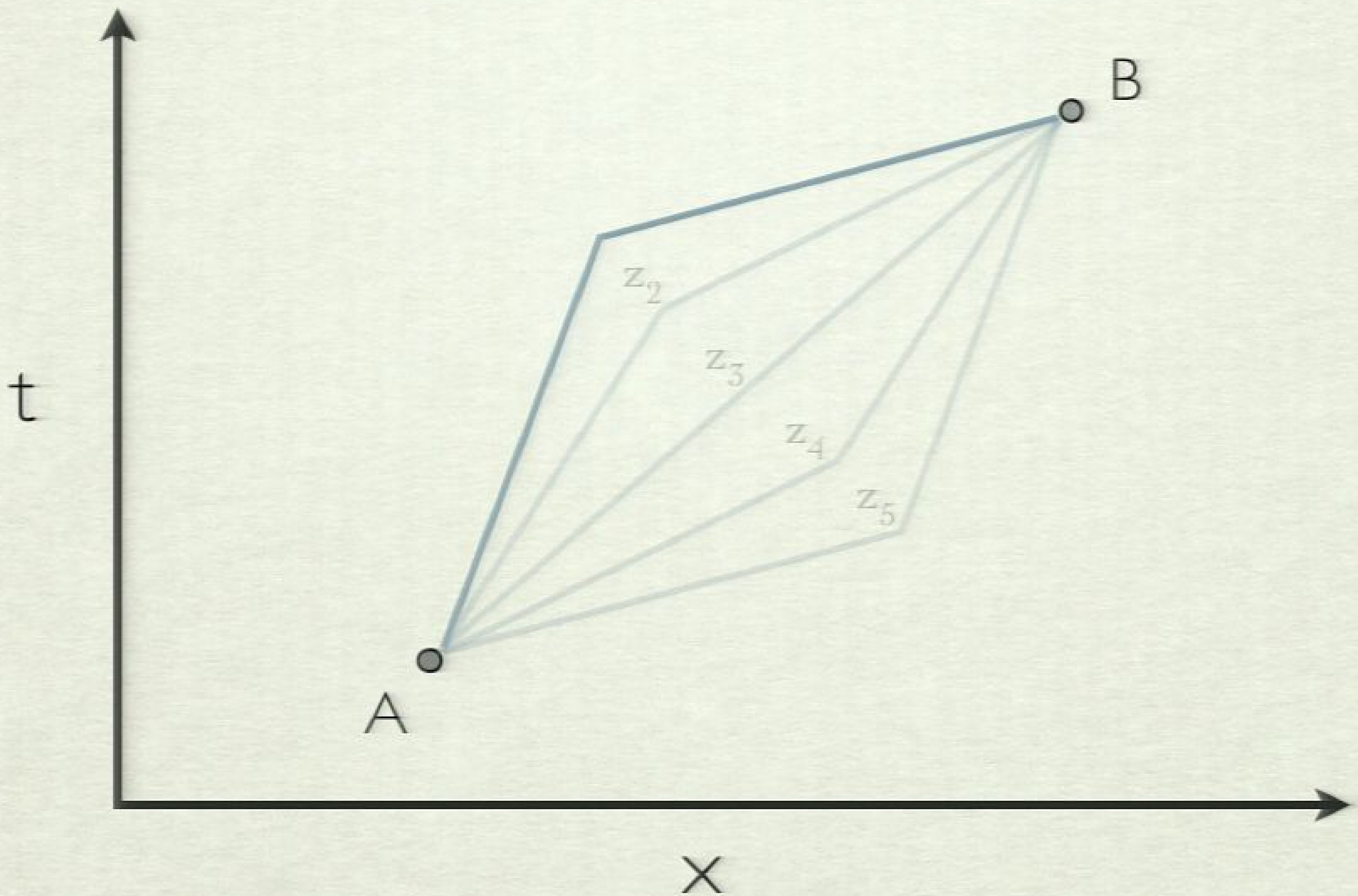
Feynman's Rules of Quantum Theory


B
t
A
x


B
t
A
x

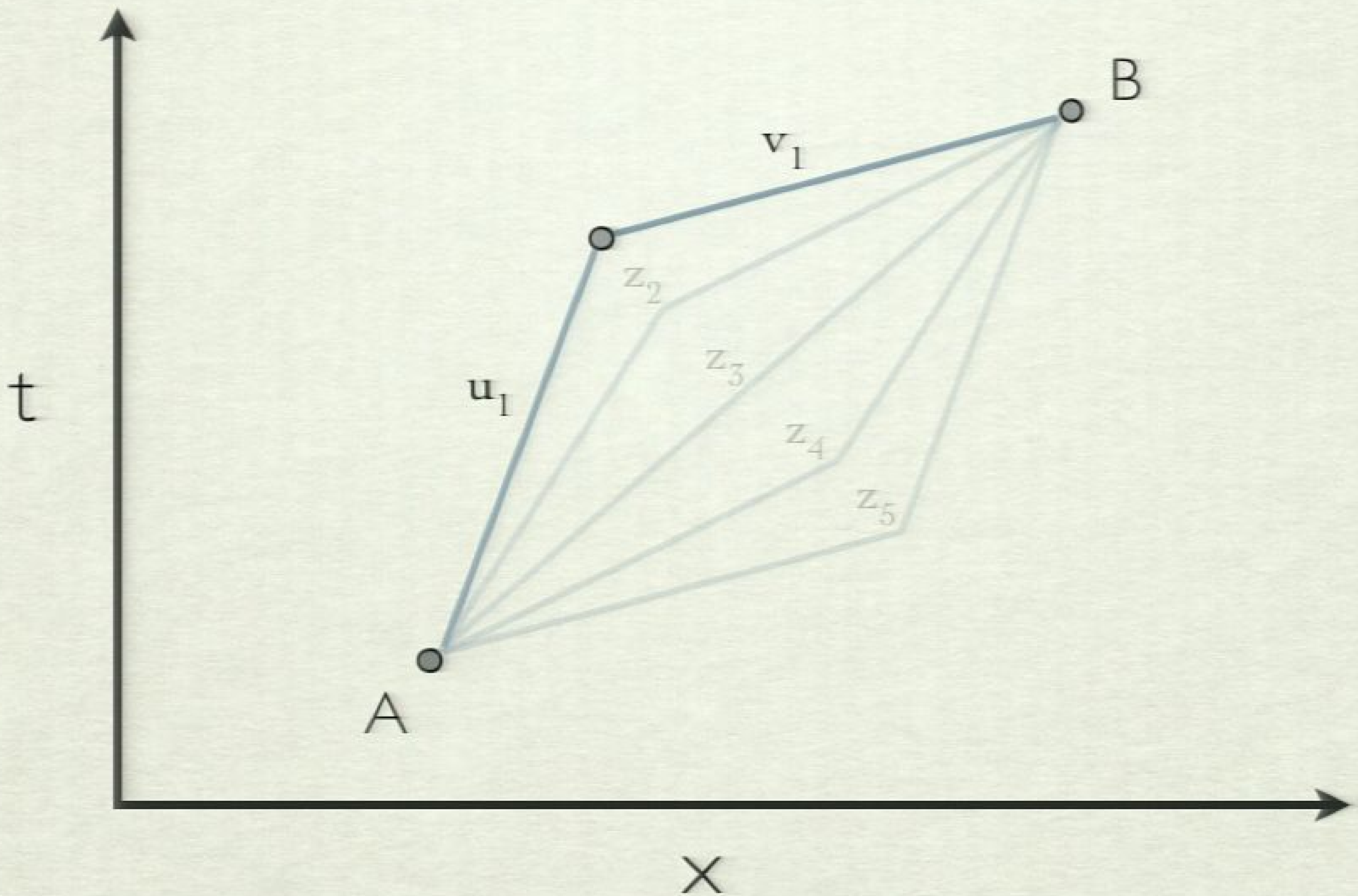
$$z = z_1 + \ldots + z_5$$



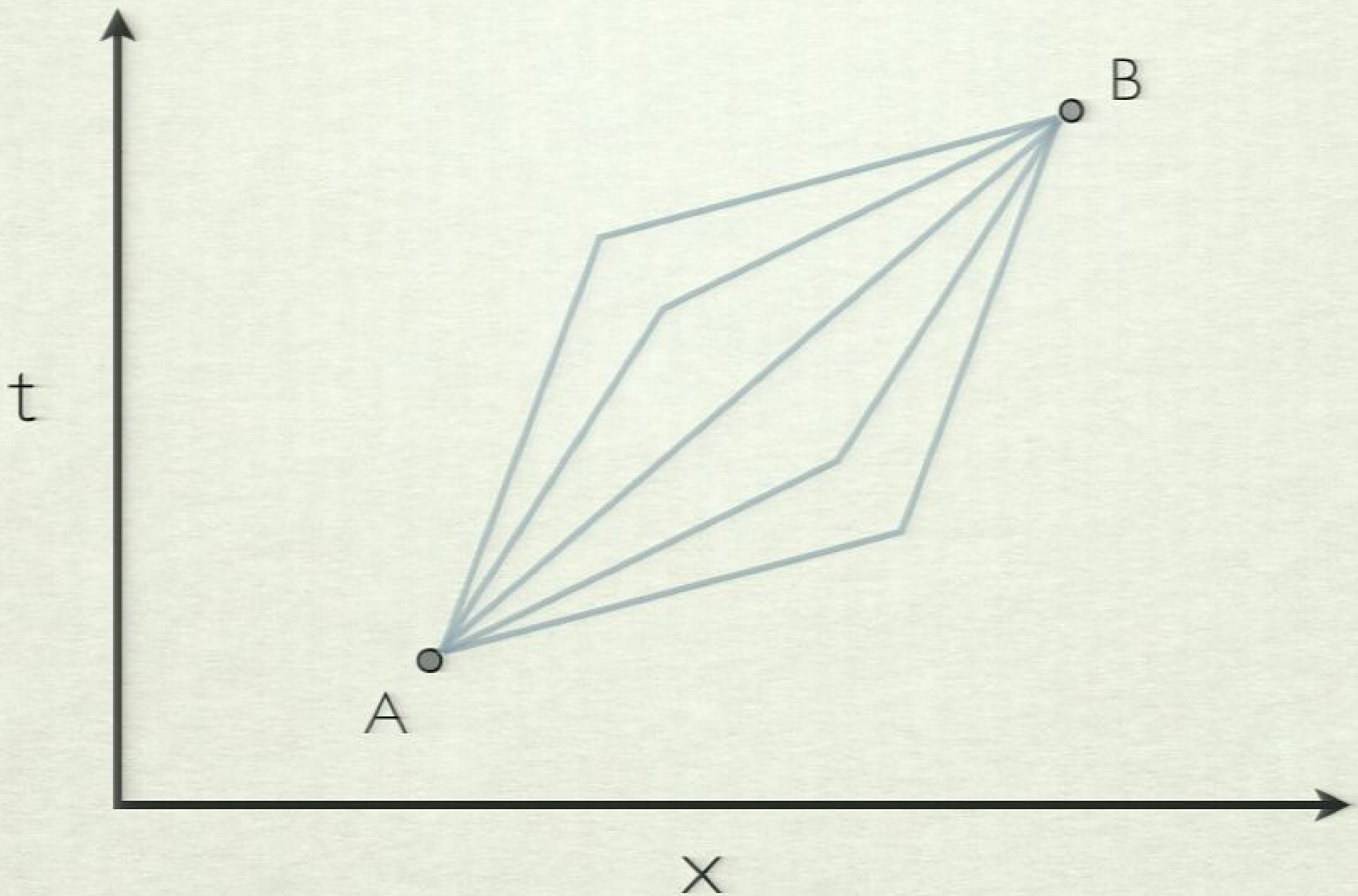


t
x
A
B
z_2
z_3
z_4
z_5

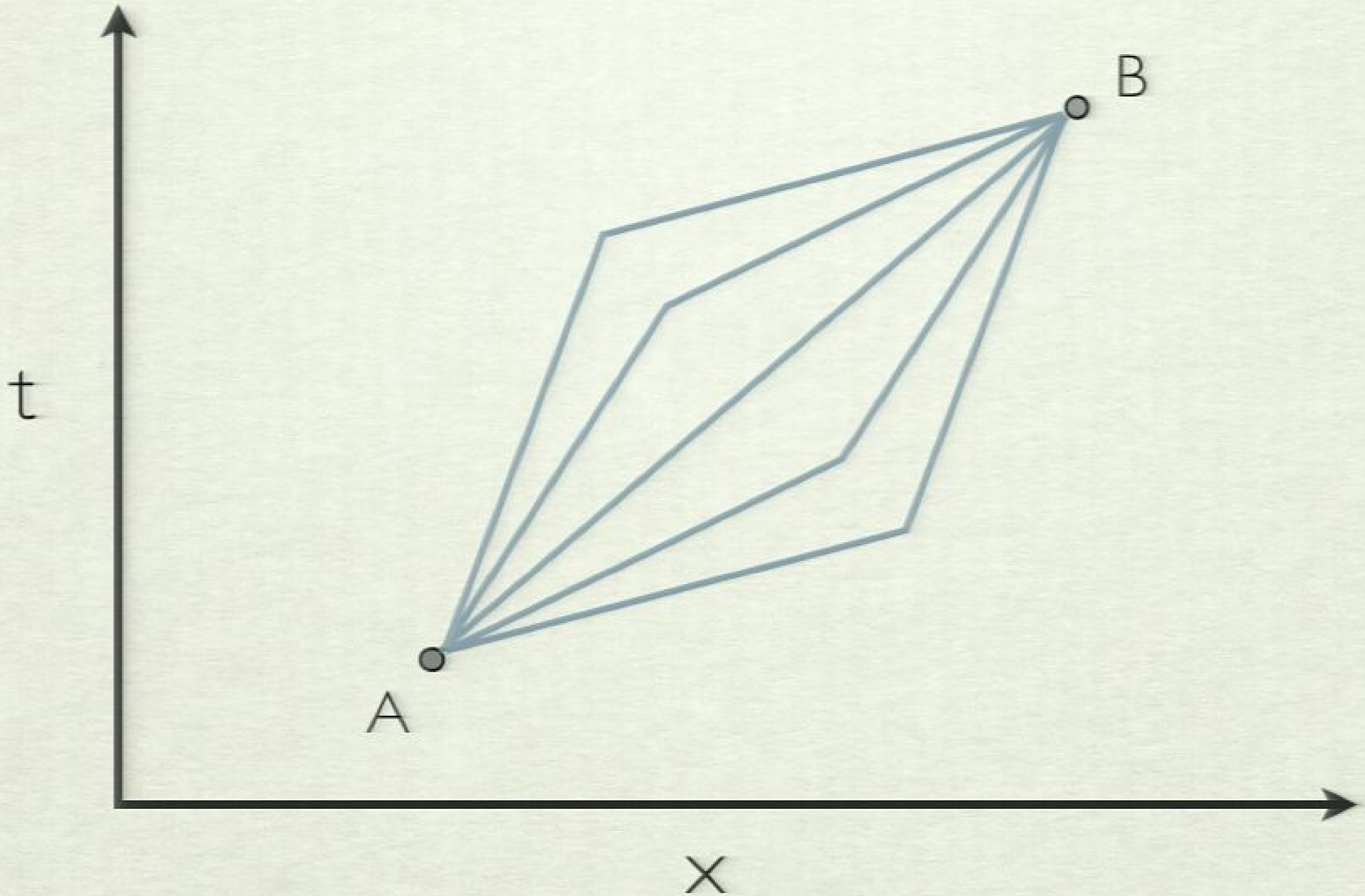
$$z_1 = u_1 \, v_1$$



$$\Pr(B|A) \propto |z|^2$$



$$\Pr(B|A) \propto |z|^2$$



Feynman's Rules of Quantum Theory

- **Sum Rule**

$$z = z_1 + z_2 + z_3 + \ldots$$

- Product Rule

$$z_1 = u_1 \, v_1$$

Feynman's Rules of Quantum Theory

- Sum Rule

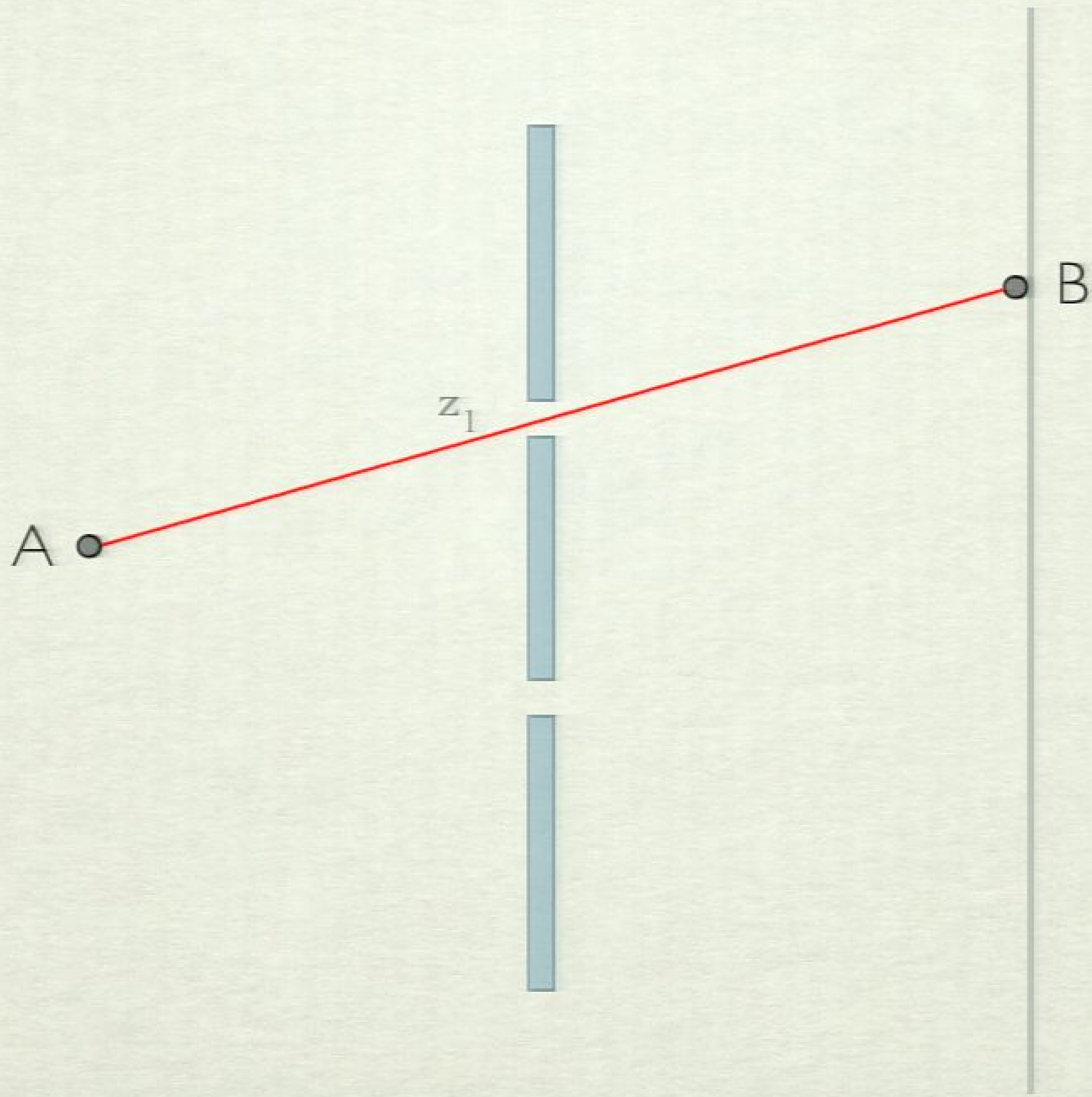
$z = z_1 + z_2 + z_3 + \ldots$

- Product Rule

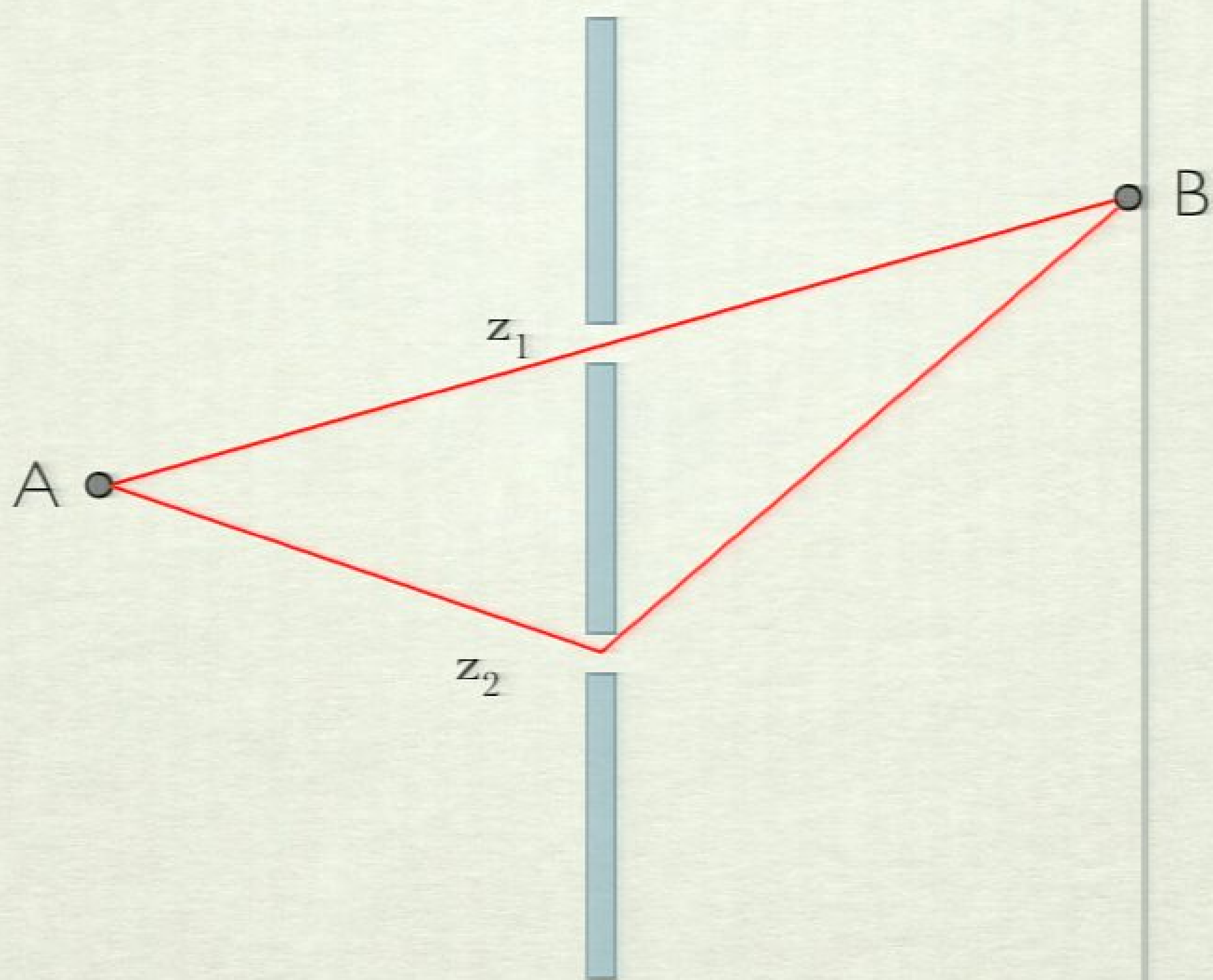
$z_1 = u_1 \, v_1$

- Probability Rule

$p = |z|^2$


B
z_1
A

$$p = p_1 + p_2 + 2\sqrt{p_1 p_2}\cos\phi$$

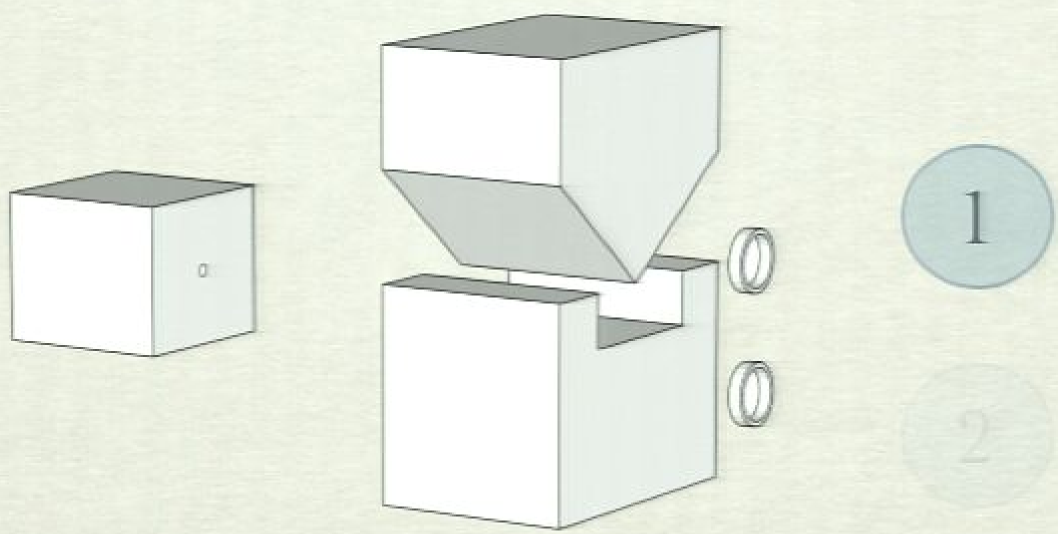


$$z = z_1 + z_2$$

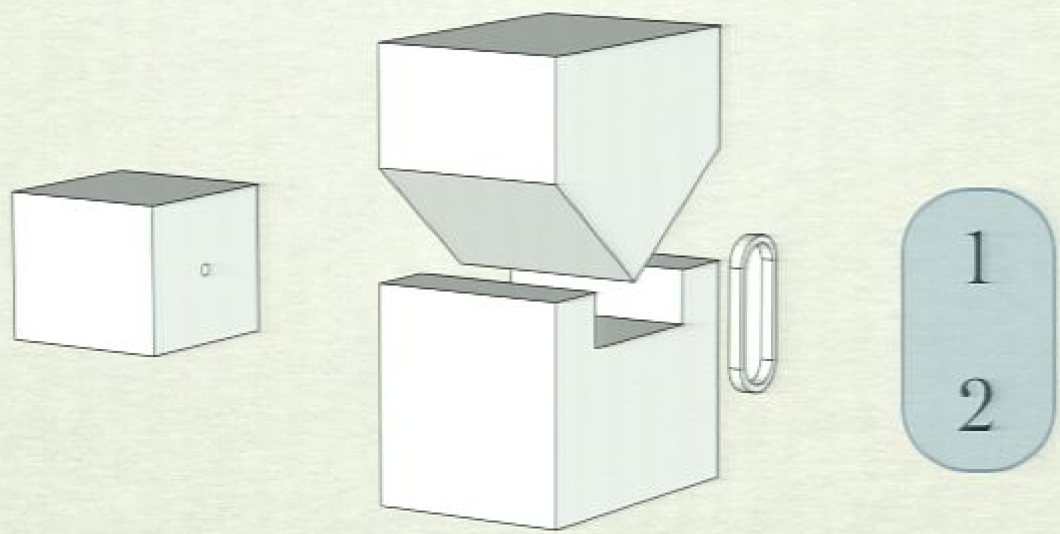
Experimental Framework

Experimental Framework

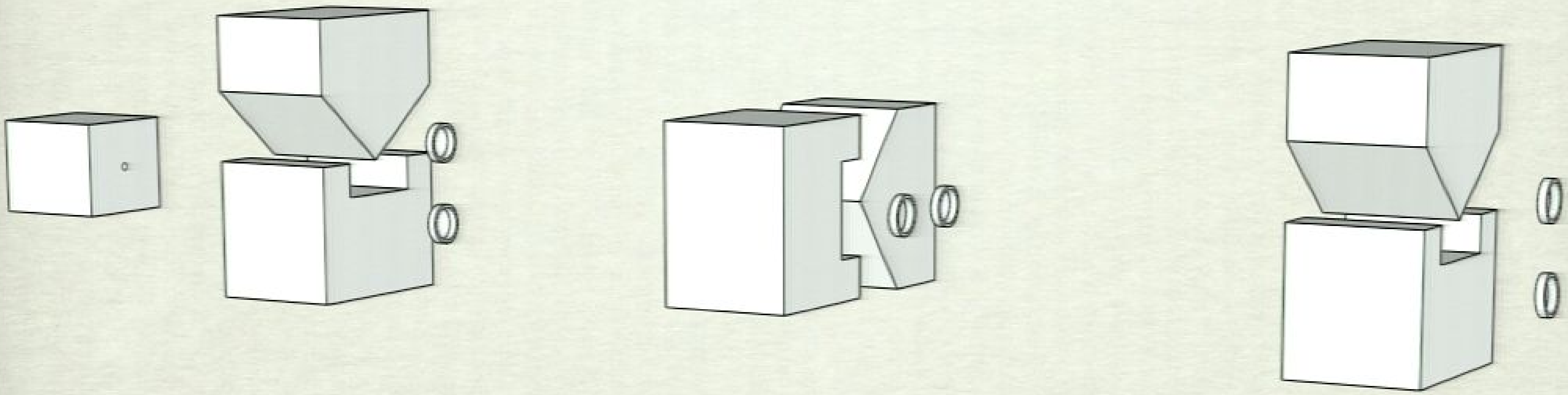
Stern Gerlach Measurement



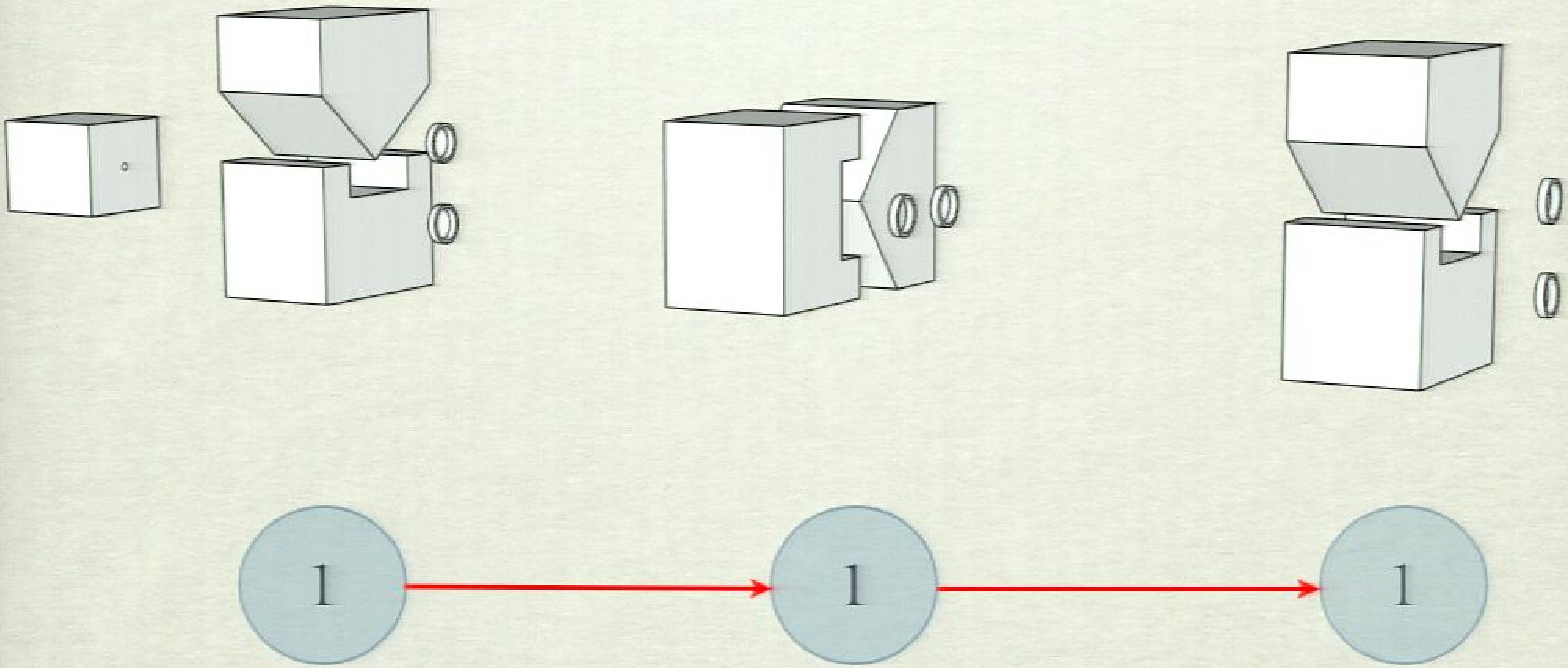
Stern Gerlach Measurement



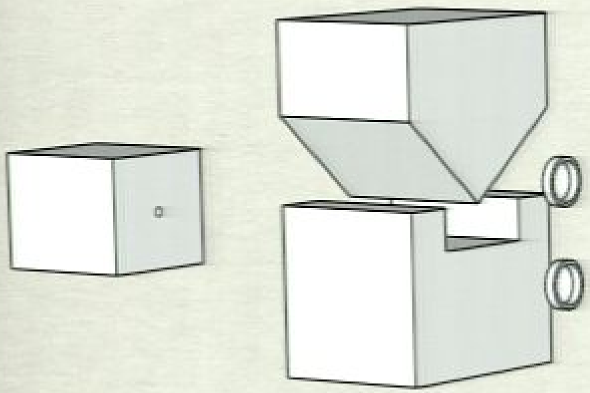
Experimental Setups

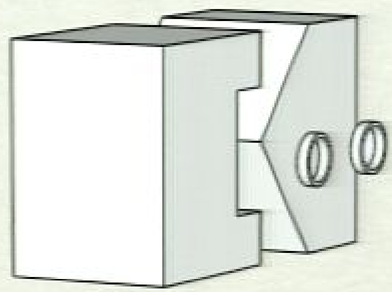


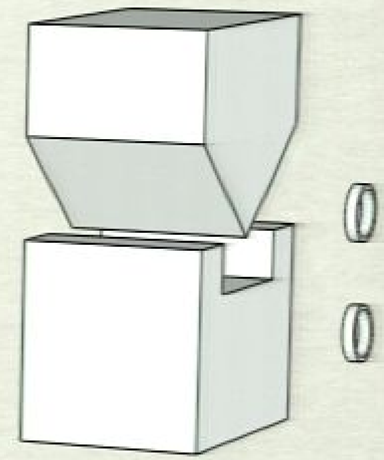
Experimental Setups

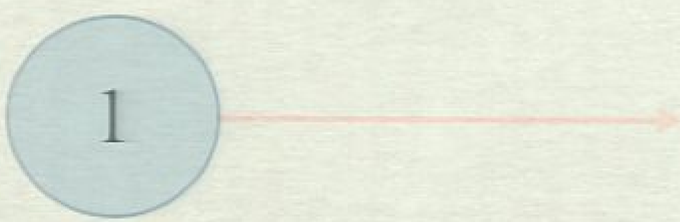


Experimental Setups

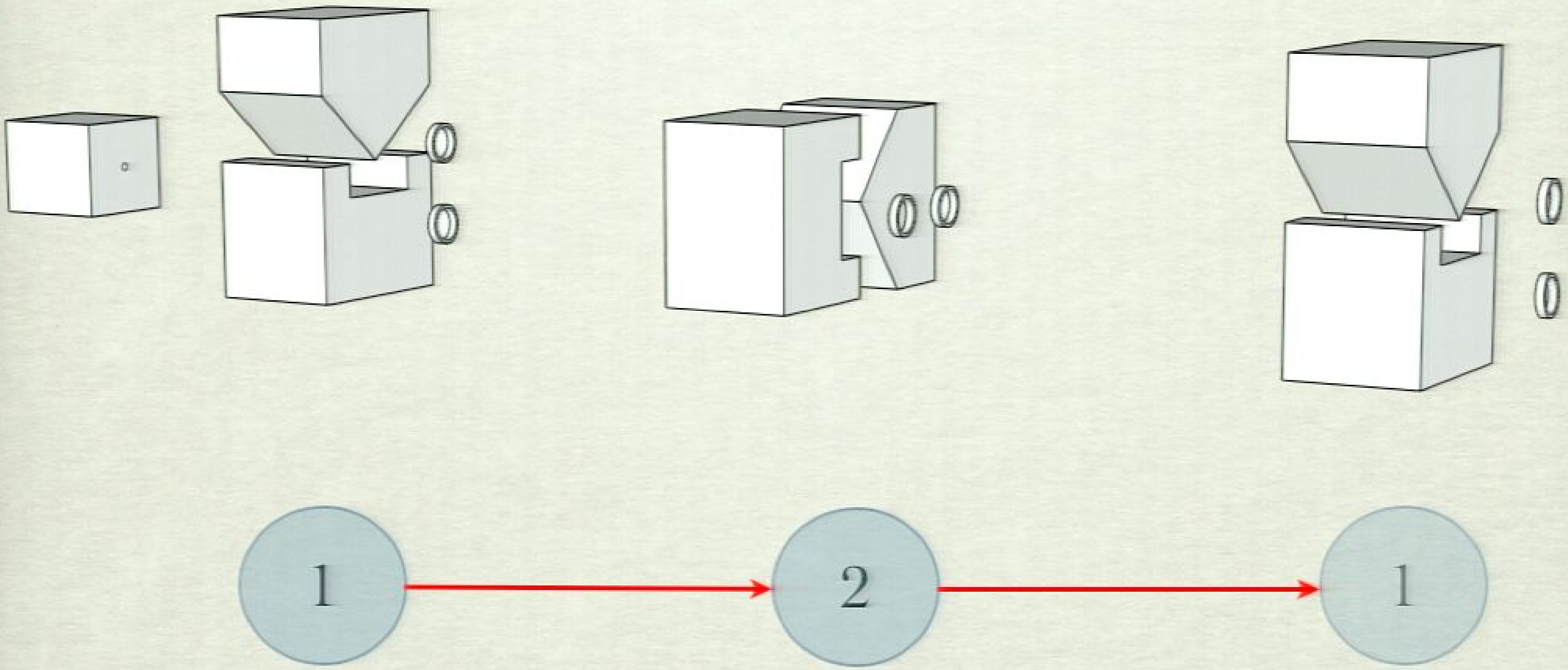




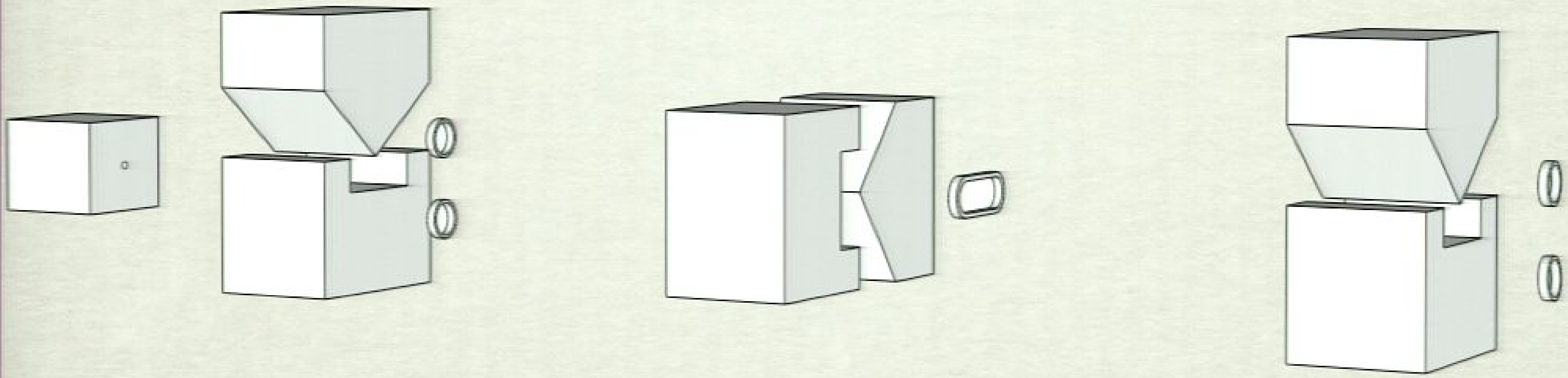




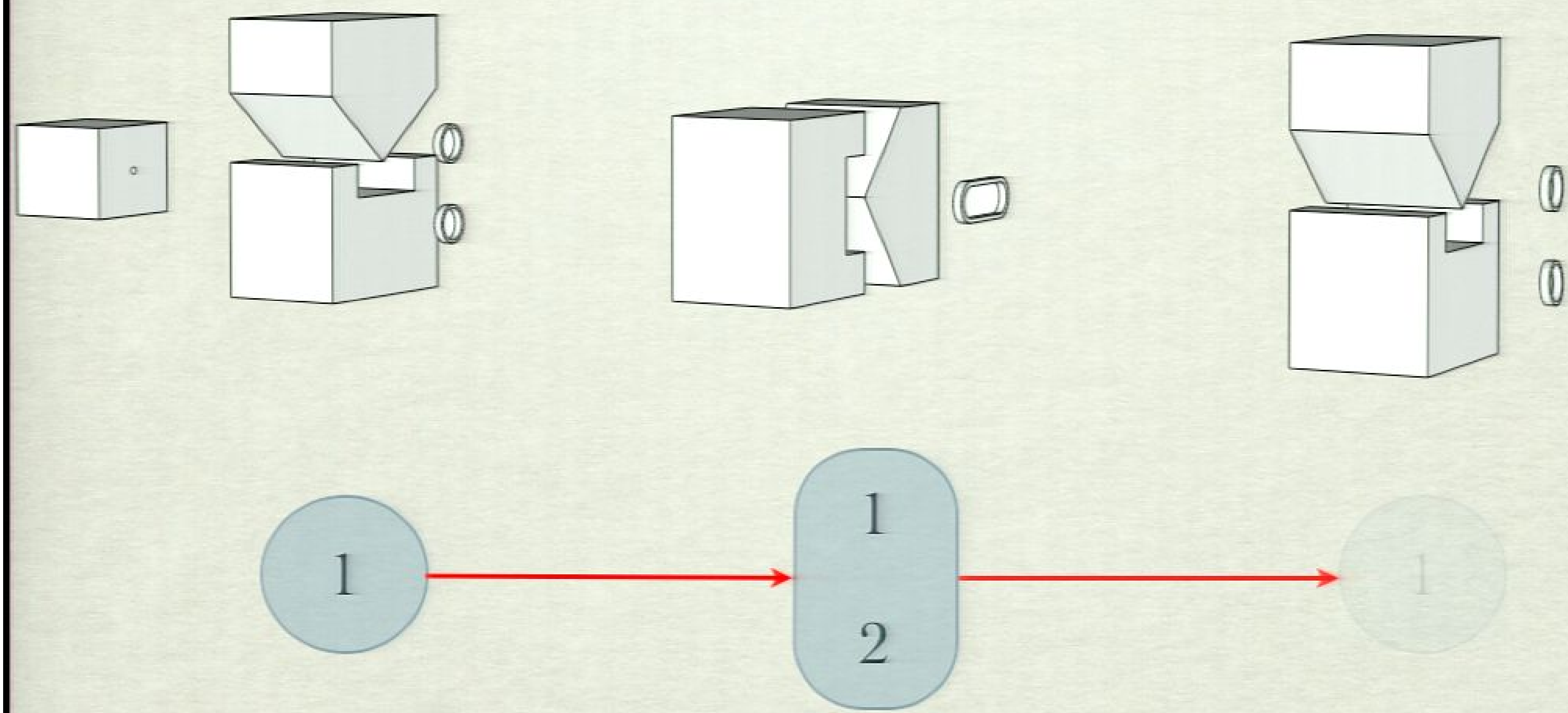
Experimental Setups



Experimental Setups

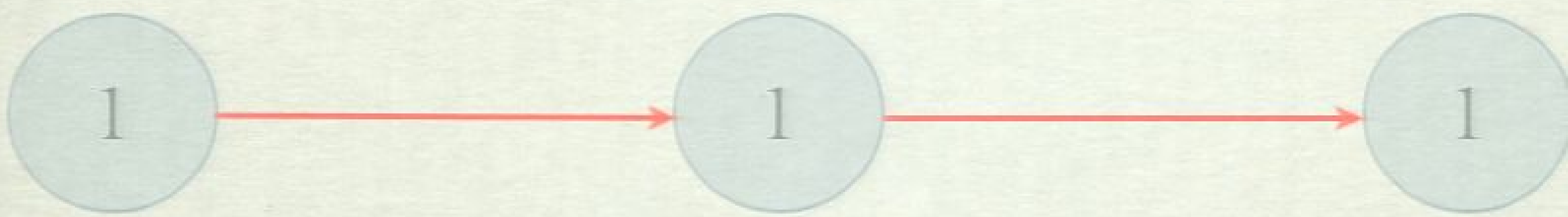


Experimental Setups

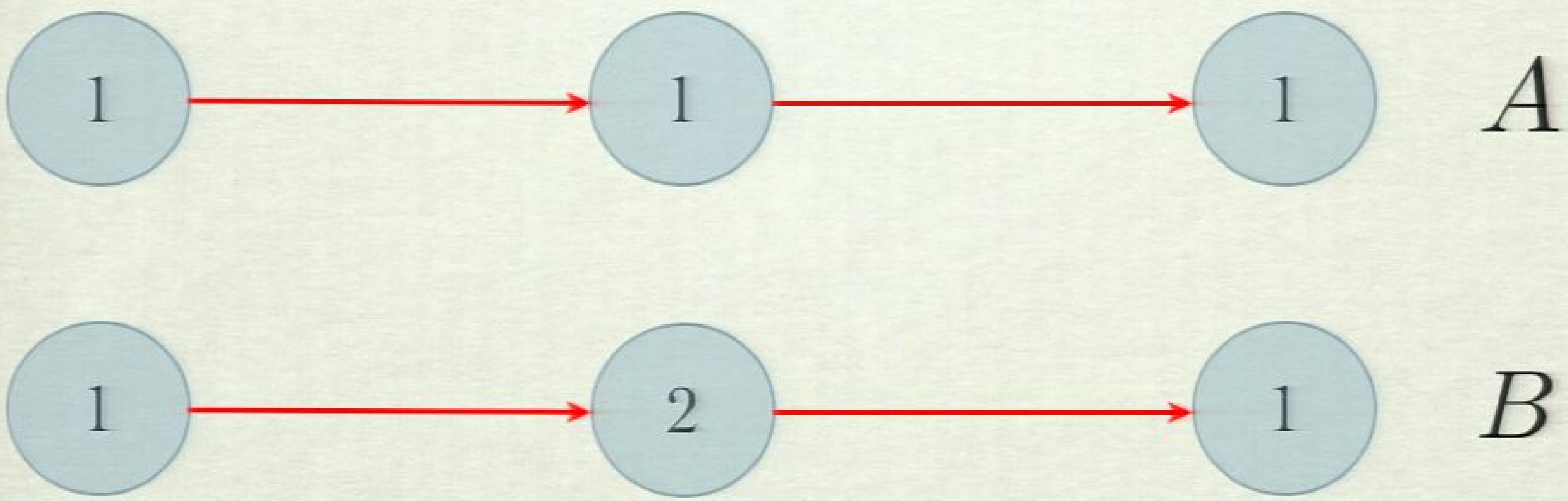


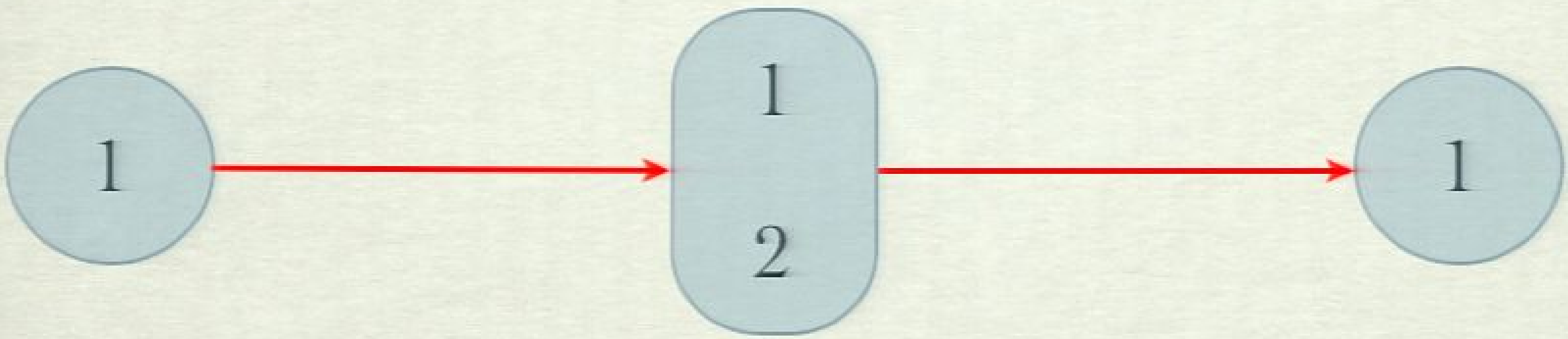
Combining Sequences

Combining Sequences



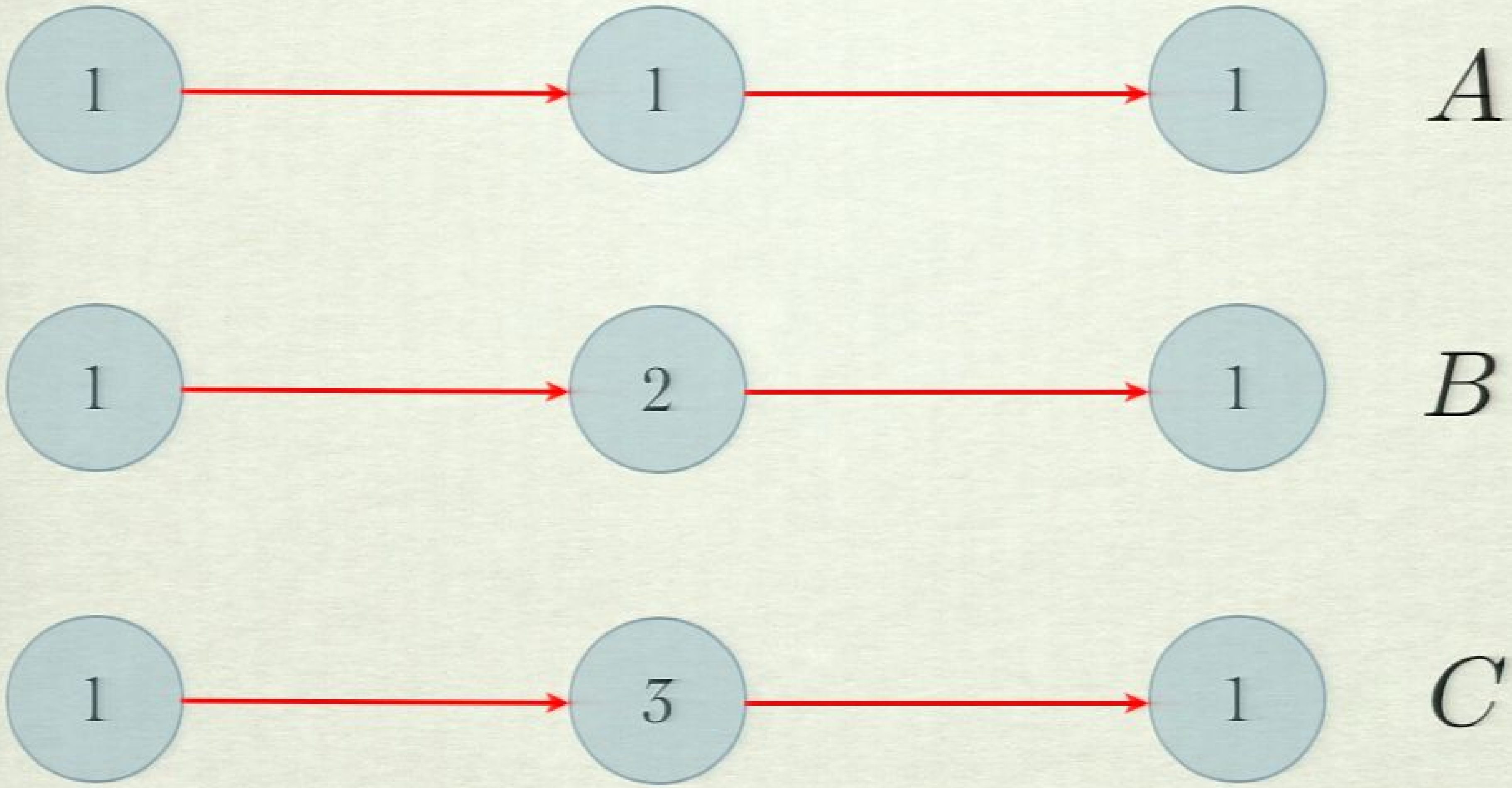
A

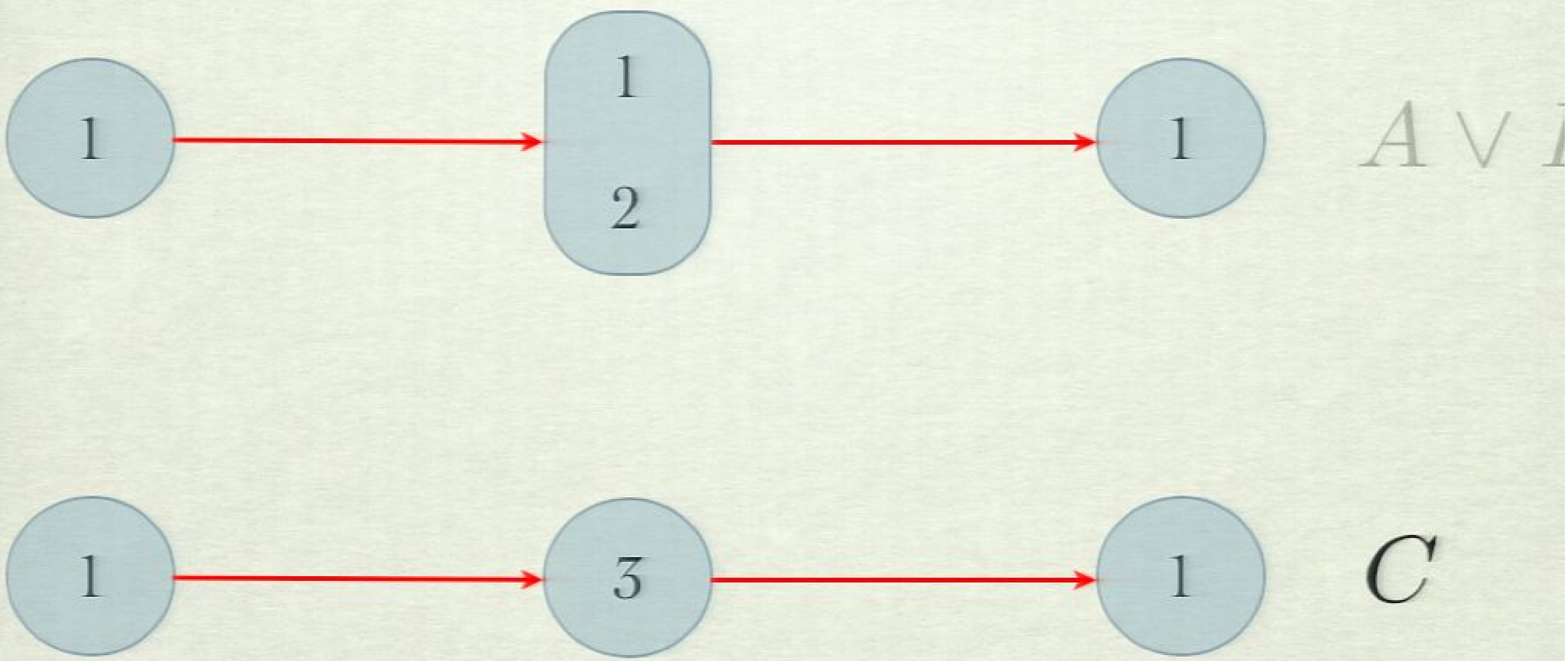

1
1
1
A
1
2
1
B


1
1
2
1

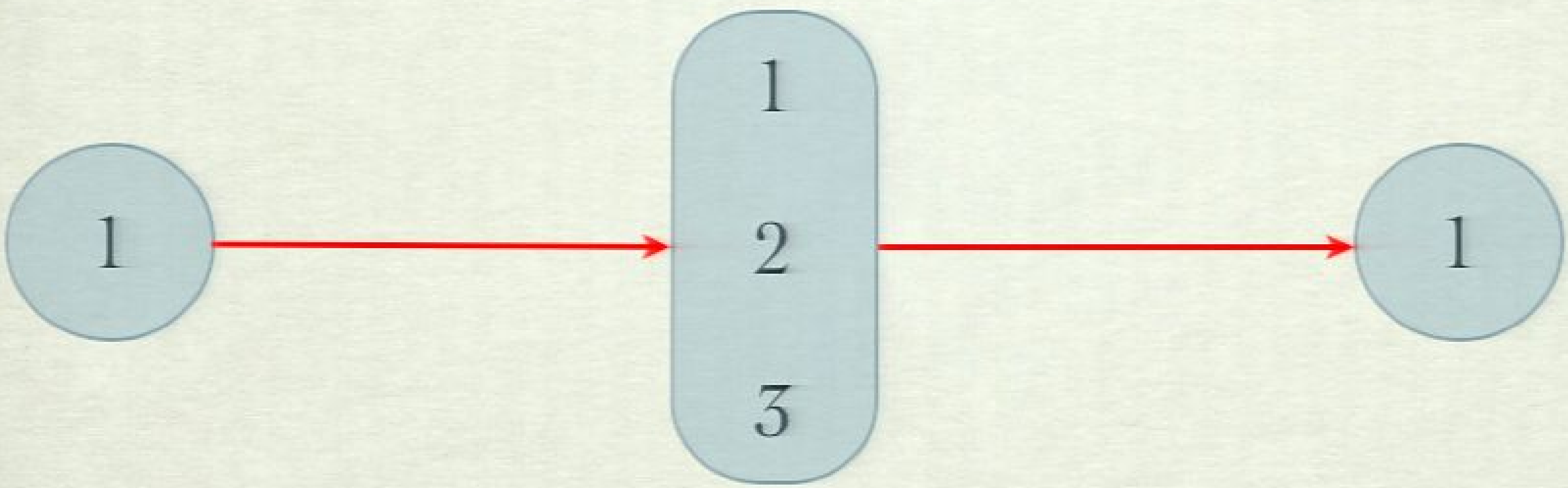
Commutativity of Parallel Combination

$$A \vee B = B \vee A$$


1
1
1
A
1
2
1
B
1
3
1
C

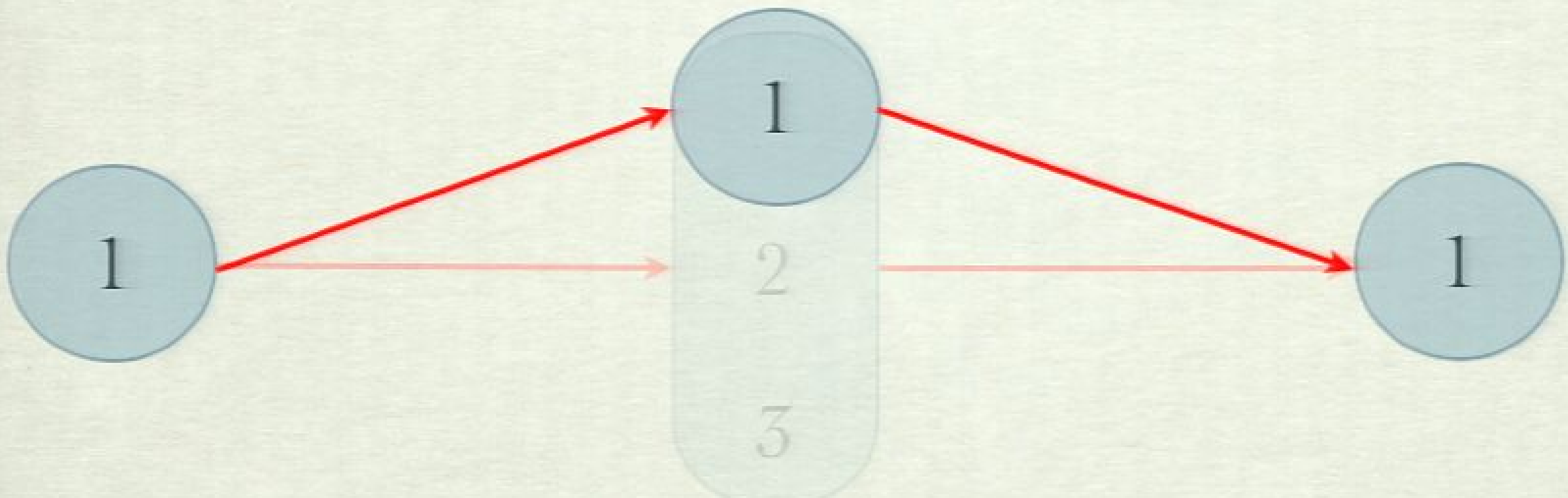

1
1
2
1
$A \vee B$
1
3
1
C

$(A \vee B) \vee C$

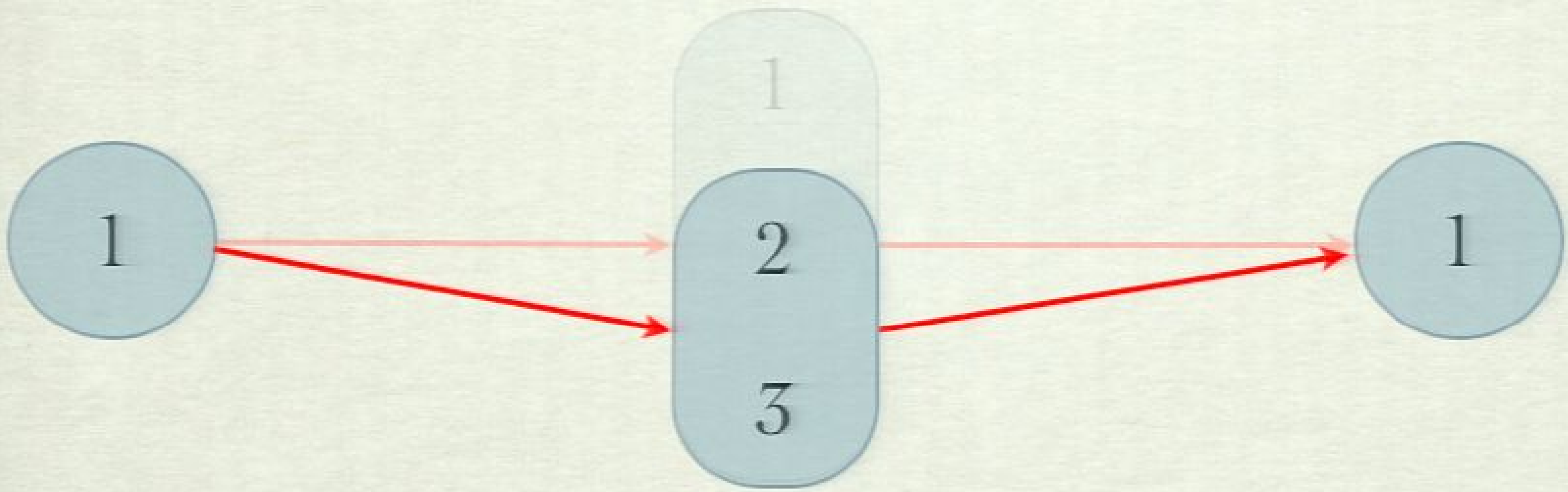


$$(A \vee B) \vee C$$

$$A$$

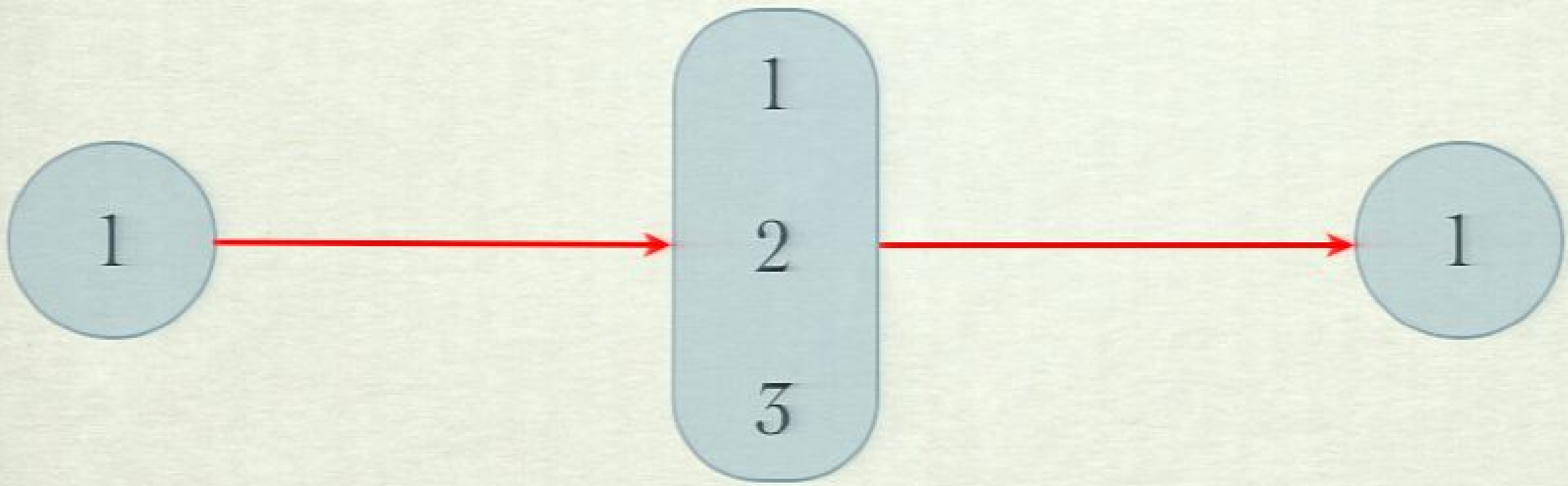


$$(A \vee B) \vee C$$



$$B \vee C$$

$$(A \vee B) \vee C$$



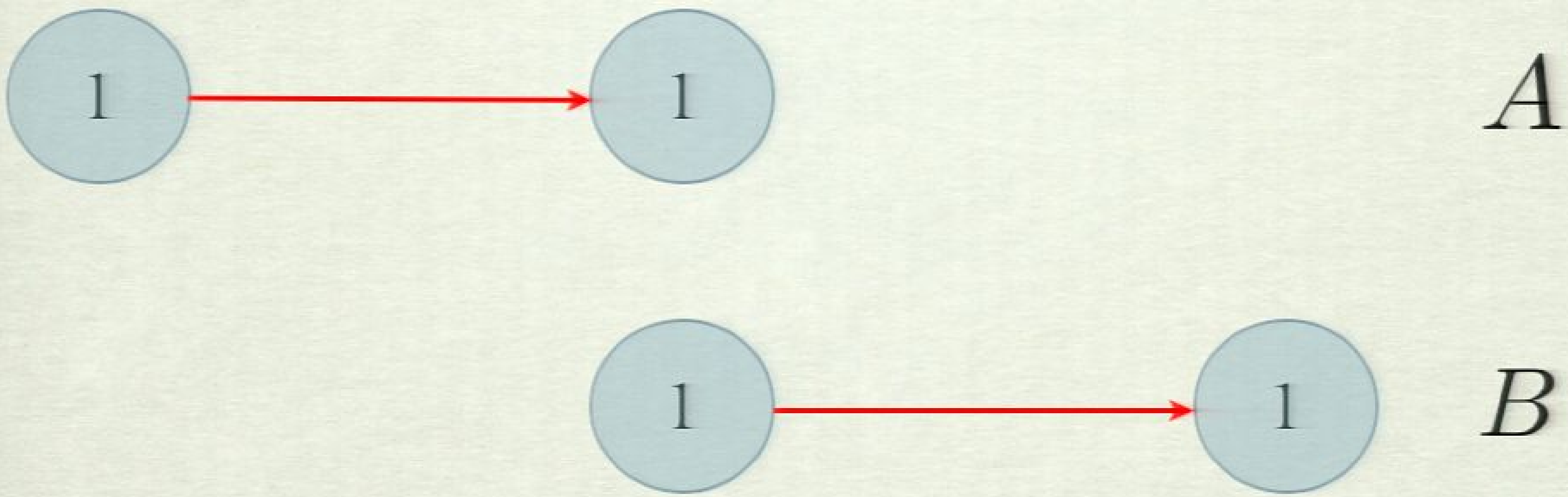
$$A \vee (B \vee C)$$

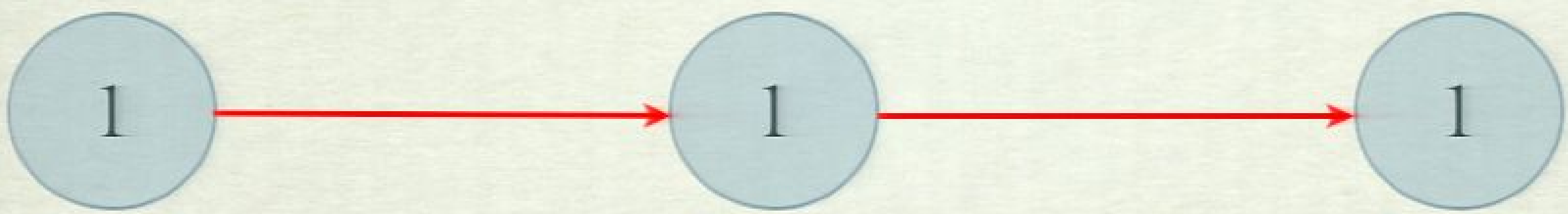
$$(A \vee B) \vee C$$

$$A \vee (B \vee C)$$

Associativity of Parallel Combination

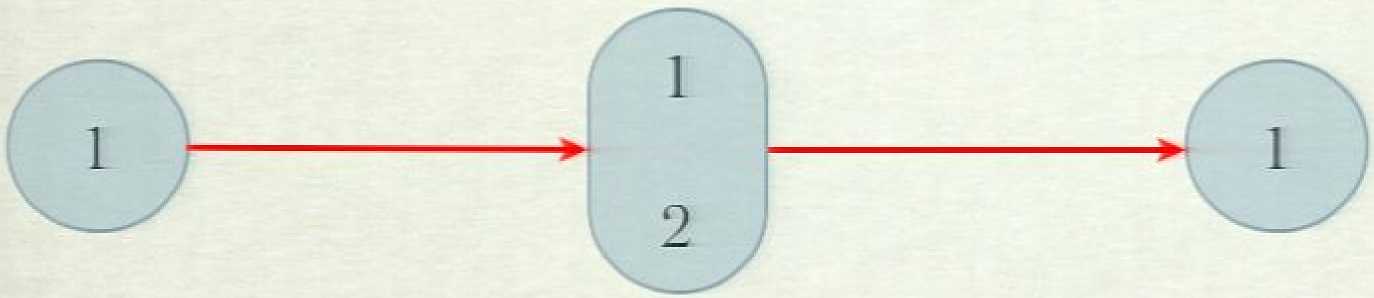
$$(A \vee B) \vee C = A \vee (B \vee C)$$

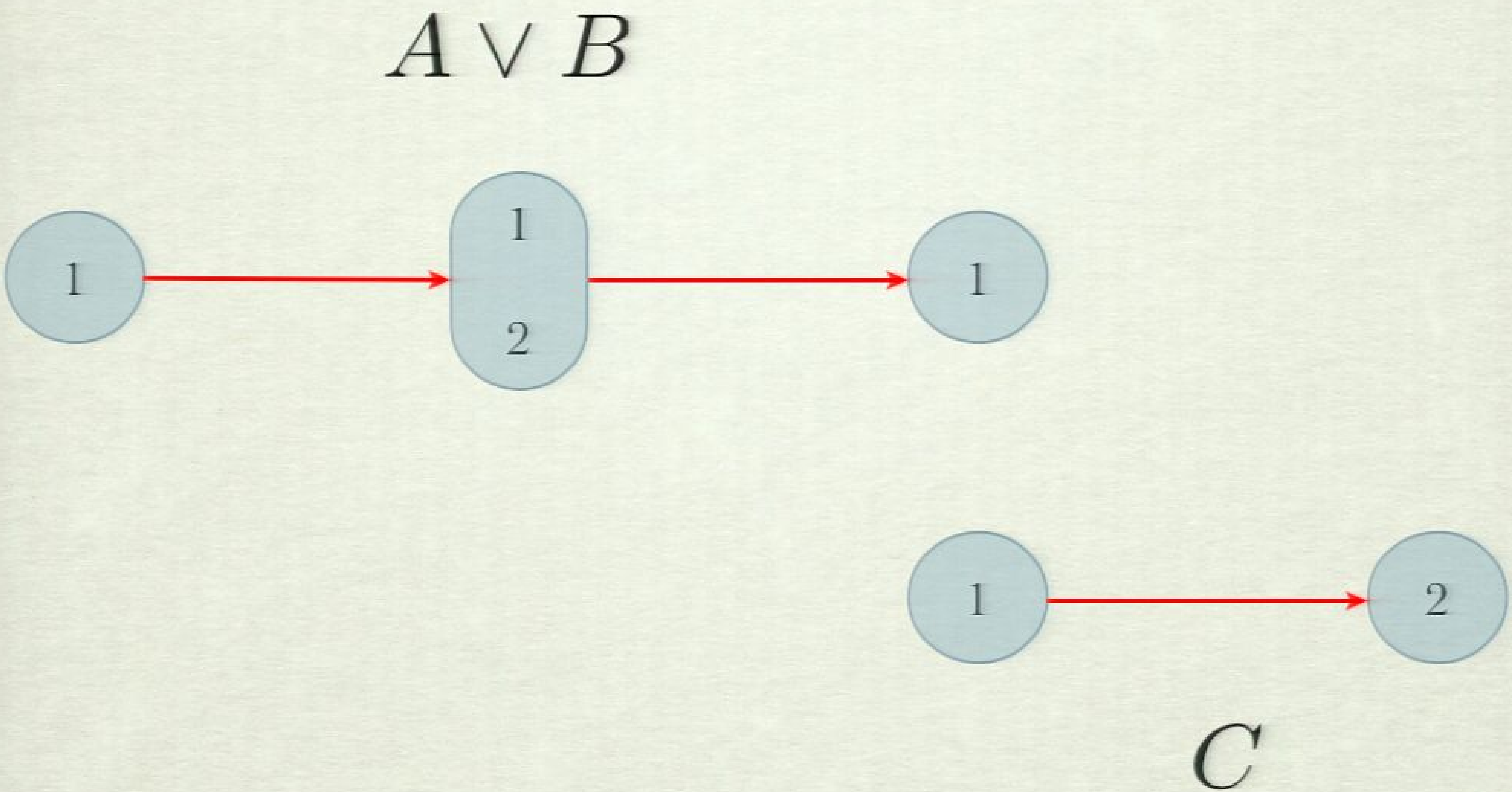

1
1
A
1
1
B


1
1
1

Associativity of Series Combination

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

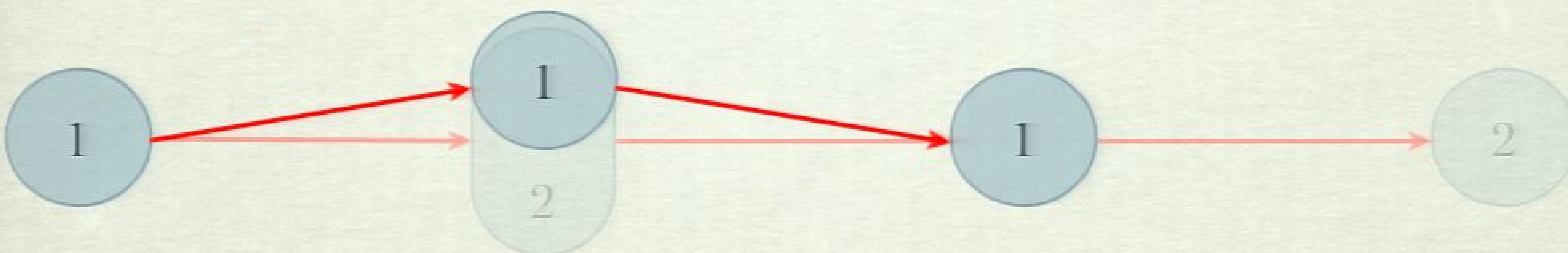

1
1
2
1


$A \vee B$
1
1
2
1
1
2
C


1
1
2
1
2

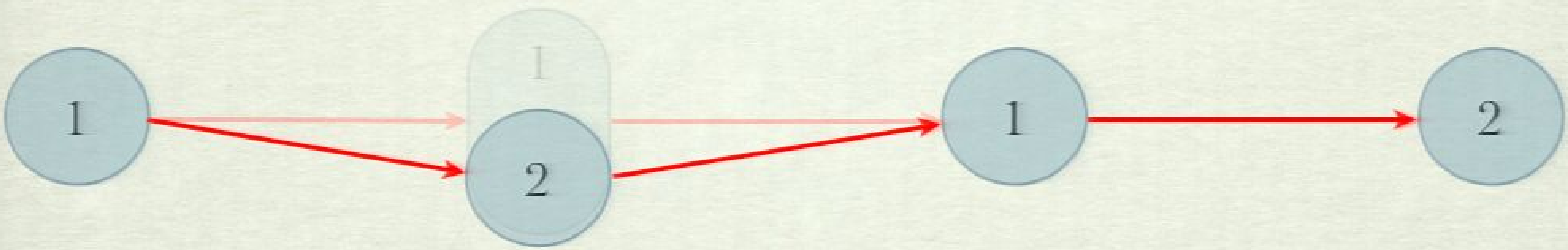
$$(A \vee B) \cdot C$$

$$A$$



$$(A \vee B) \cdot C$$

$$A \cdot C$$



$$B \cdot C$$

$$(A \vee B) \cdot C$$

$$(A \cdot C) \vee (B \cdot C)$$

$$(A \vee B) \cdot C$$

$$(A \cdot C) \vee (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

Distributivity

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

Distributivity

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Logic of Process

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

Logic of Process

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

Logic of Process

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Logic of Process

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Logic of Process

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

Logic

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

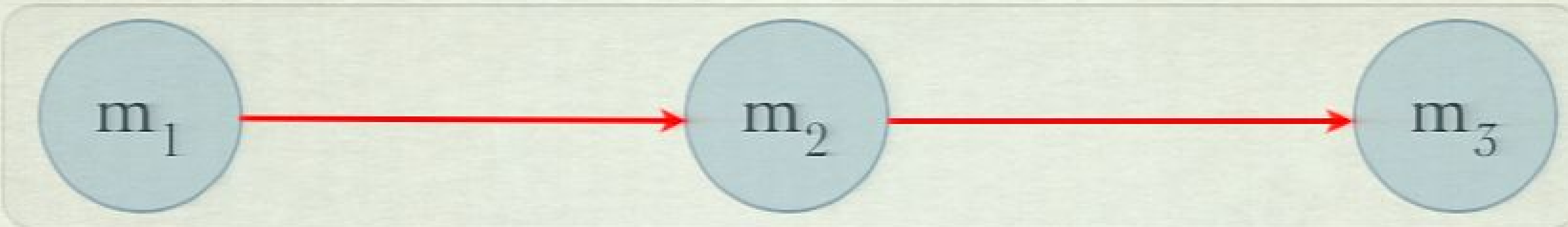
$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Logic $\xrightarrow{\textit{Representation}}$ Calculus

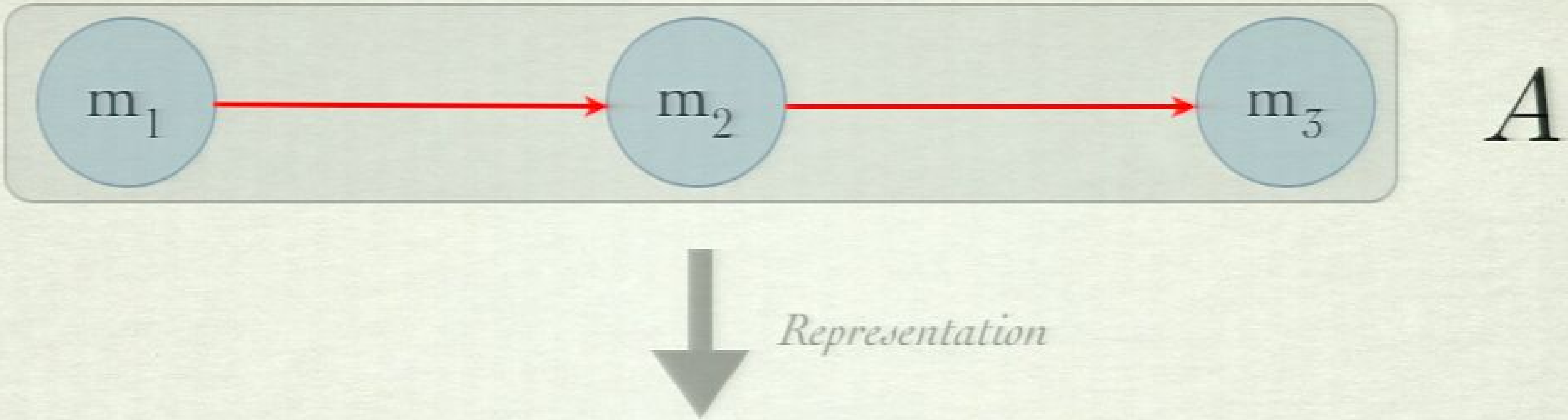
Complementarity

Complementarity

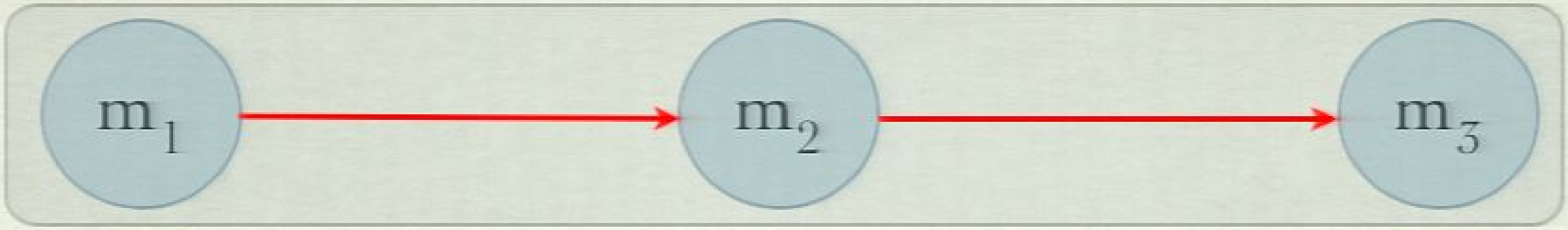
Complementarity



A



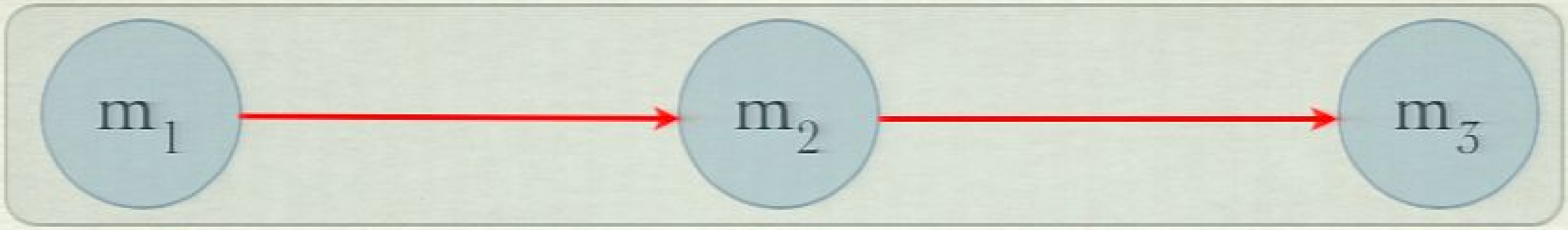
m_1
m_2
m_3
A
Representation



A

Representation

$$\mathbf{a} = (a_1, a_2)$$



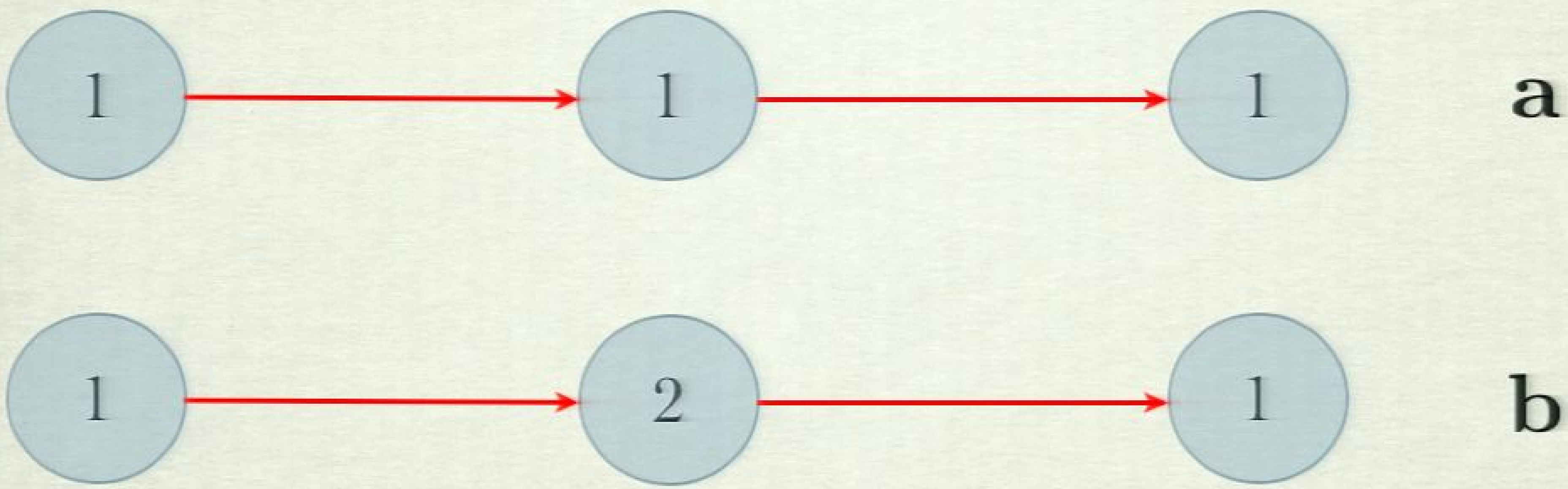
A

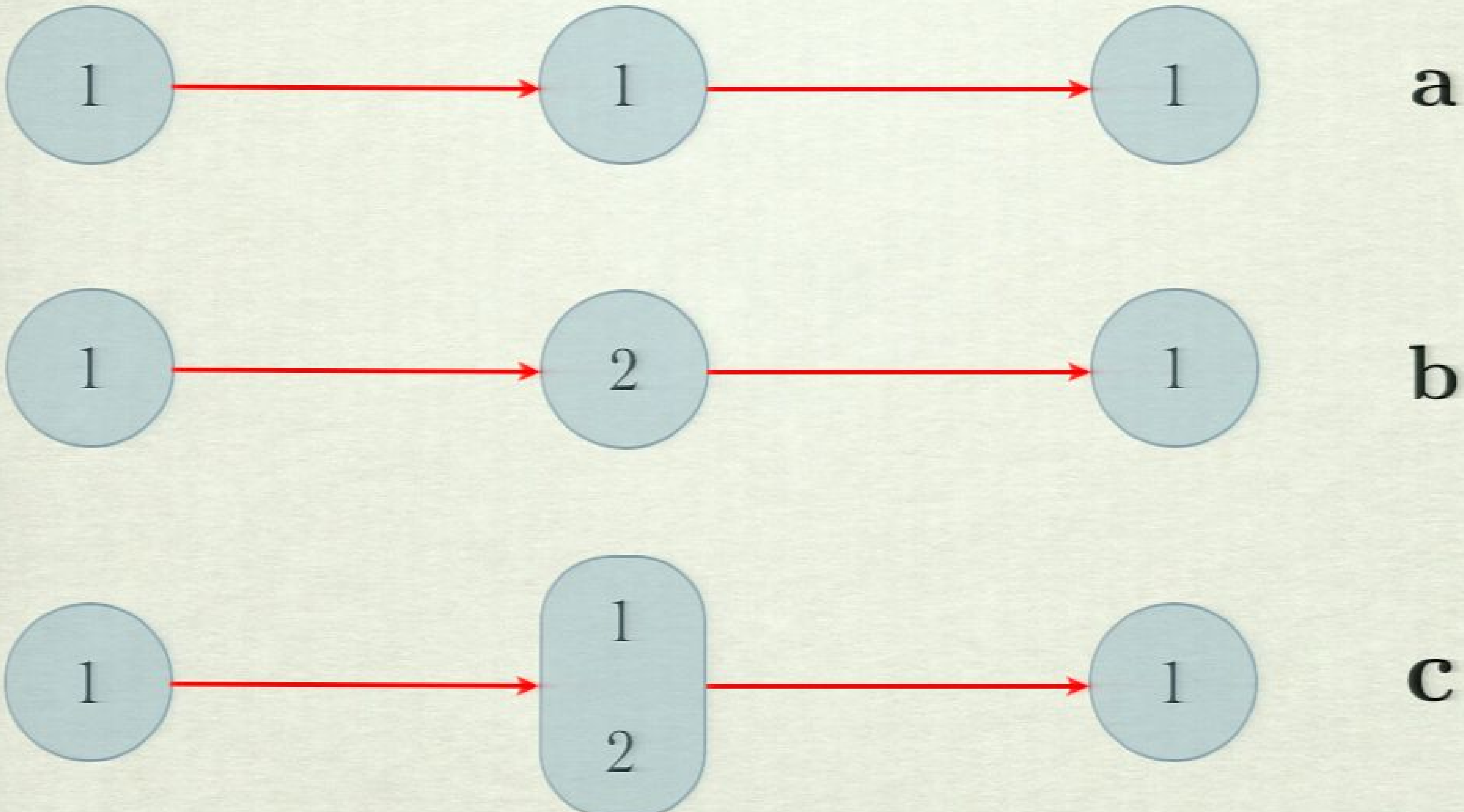
$$\Big\downarrow \textit{Representation}$$

$$\mathbf{a} = (a_1, a_2)$$

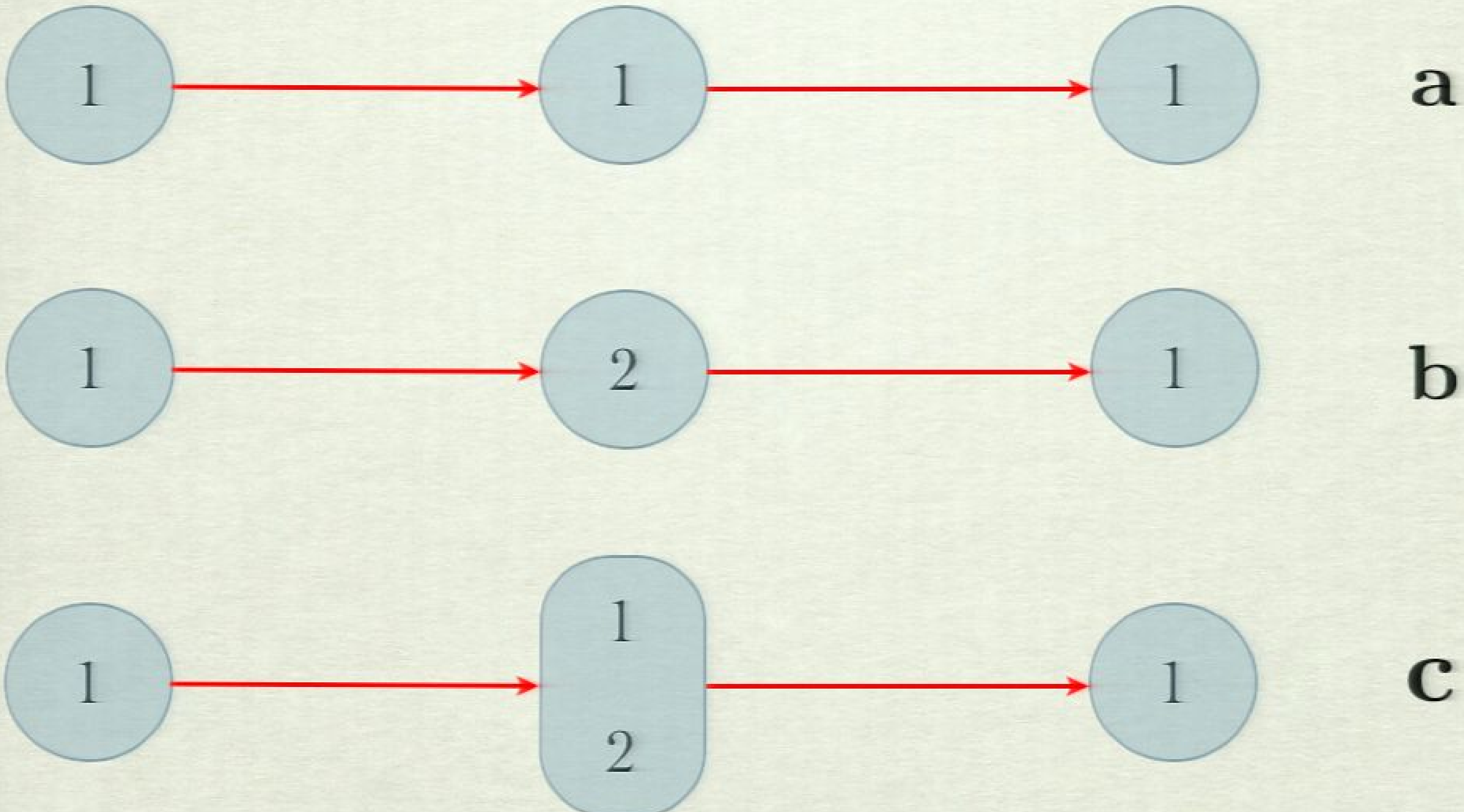
$$\Big\downarrow p$$

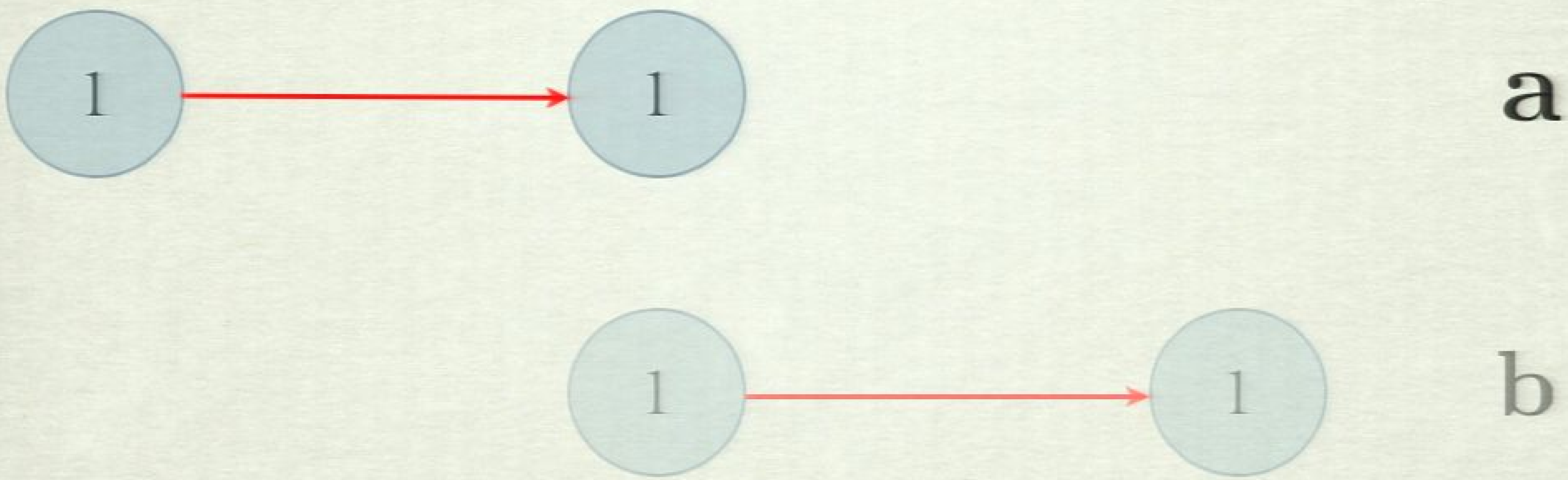
$$\Pr(m_2, m_3 \,|\, m_1)$$

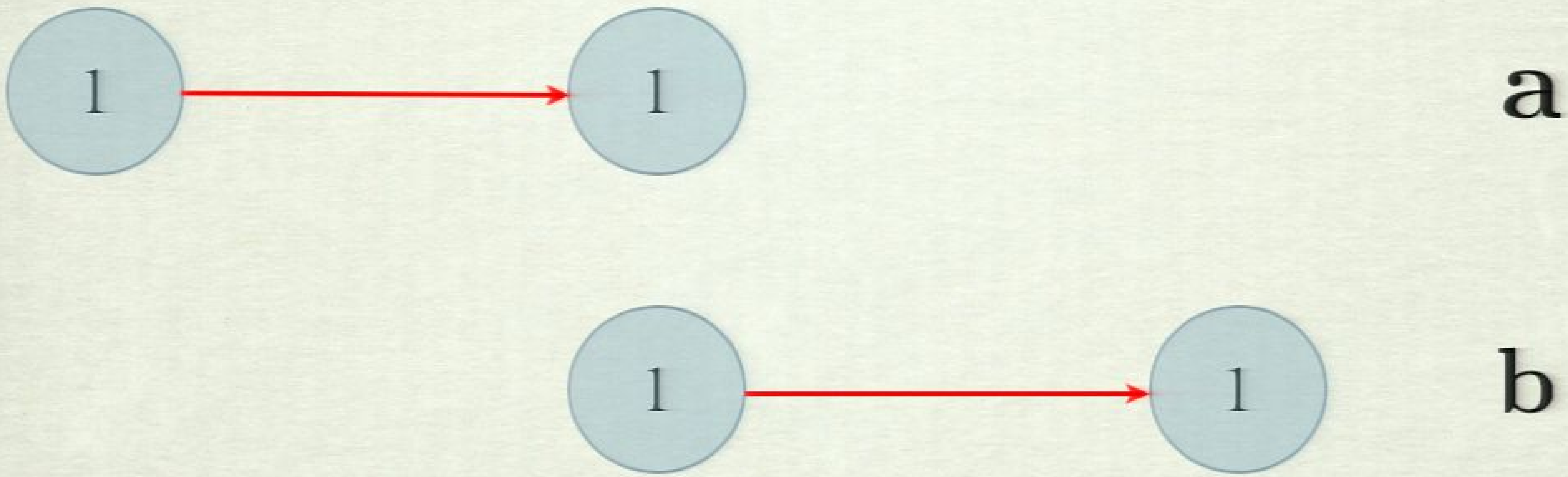

1
1
1
a
1
2
1
b

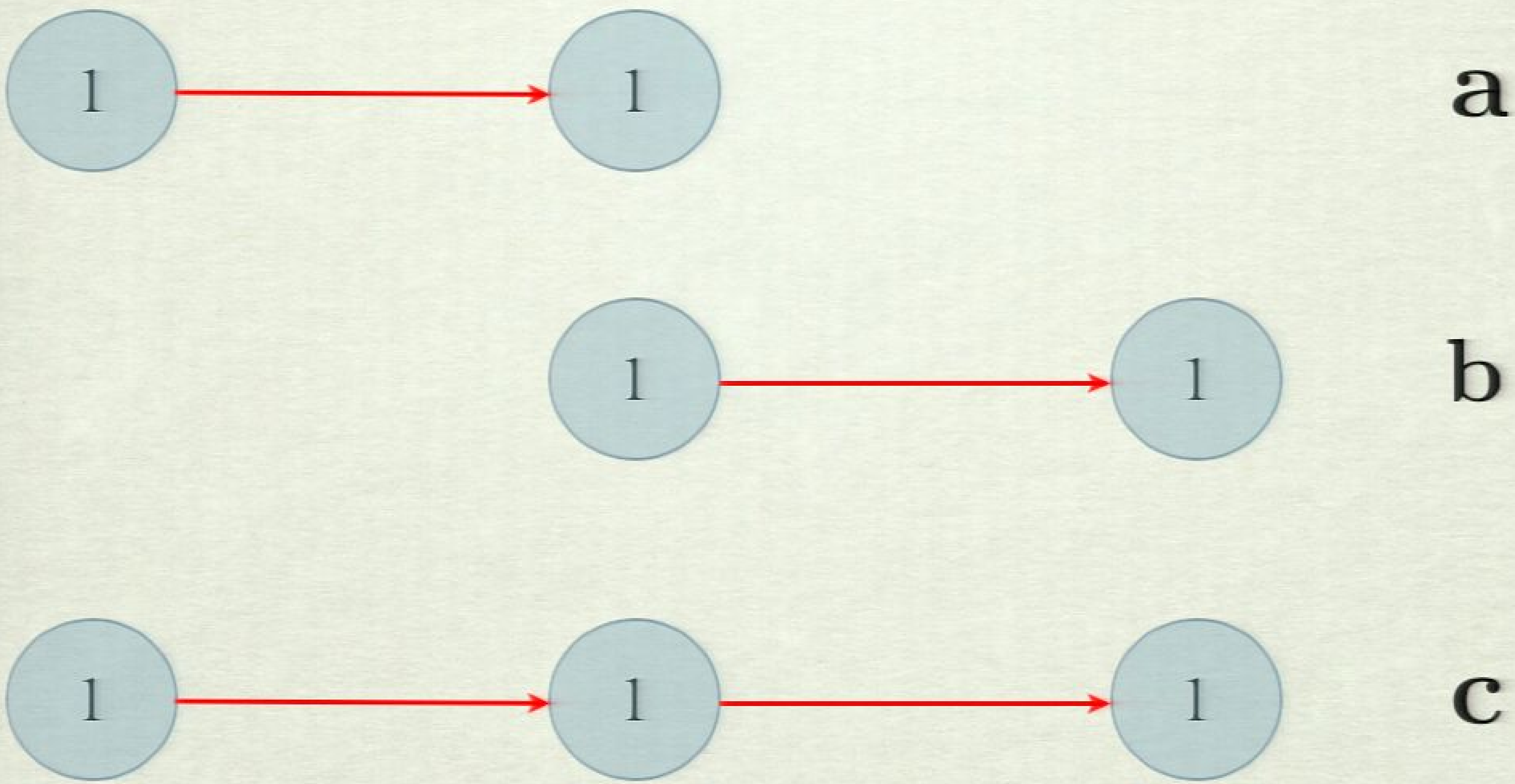

1
1
1
a
1
2
1
b
1
1
2
1
c

$$\mathbf{c} = \mathbf{a} \oplus \mathbf{b}$$

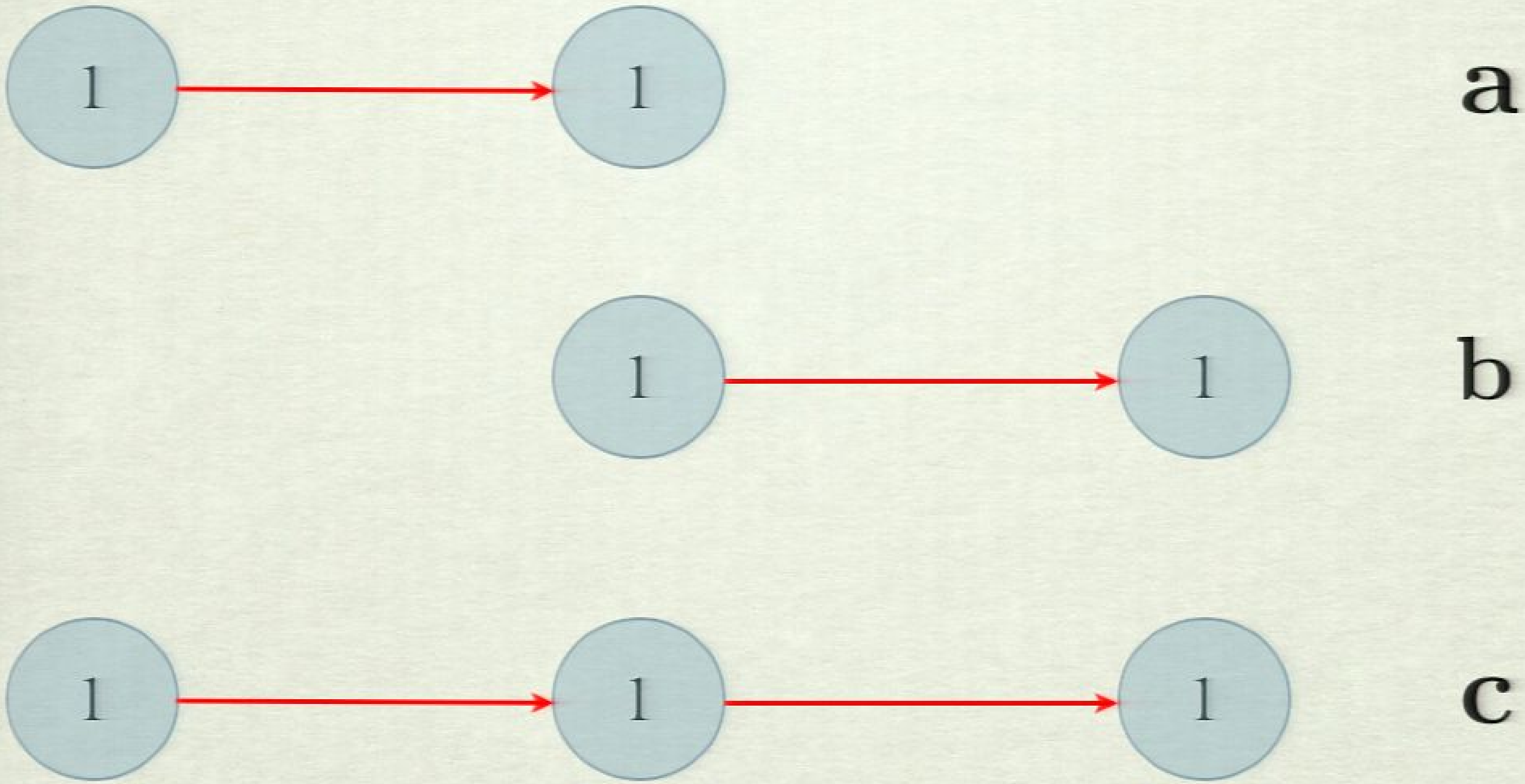



1
1
a
1
1
b


1
1
a
1
1
b


1
1
a
1
1
b
1
1
1
c

$$\mathbf{c} = \mathbf{a} \odot \mathbf{b}$$



$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

$\xrightarrow{\textit{Representation}}$

Calculus

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Representation $\Longrightarrow$

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$

$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$

$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$

$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$

$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

Aczél & Hosszú (1956)

$$\mathbf{f}(\mathbf{a} \oplus \mathbf{b}) = \mathbf{f}(\mathbf{a}) + \mathbf{f}(\mathbf{b})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

Aczél & Hosszú
(1956)

$$\mathbf{f}(\mathbf{a} \oplus \mathbf{b}) = \mathbf{f}(\mathbf{a}) + \mathbf{f}(\mathbf{b})$$

One-to-one
transformation of
pair-space

$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
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$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

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$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$\begin{aligned}\mathbf{a} \odot \mathbf{b} = (&\gamma_1 a_1 b_1 + \gamma_2 a_1 b_2 + \gamma_3 a_2 b_1 + \gamma_4 a_2 b_2, \\ &\gamma_5 a_1 b_1 + \gamma_6 a_1 b_2 + \gamma_7 a_2 b_1 + \gamma_8 a_2 b_2)\end{aligned}$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$\mathbf{a} \odot \mathbf{b} = (\gamma_1 a_1 b_1 + \gamma_2 a_1 b_2 + \gamma_3 a_2 b_1 + \gamma_4 a_2 b_2,$$
$$\gamma_5 a_1 b_1 + \gamma_6 a_1 b_2 + \gamma_7 a_2 b_1 + \gamma_8 a_2 b_2)$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$\mathbf{a} \odot \mathbf{b} = (\gamma_1 a_1 b_1 + \gamma_2 a_1 b_2 + \gamma_3 a_2 b_1 + \gamma_4 a_2 b_2, \\ \gamma_5 a_1 b_1 + \gamma_6 a_1 b_2 + \gamma_7 a_2 b_1 + \gamma_8 a_2 b_2)$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$\mathbf{a} \odot \mathbf{b} = (\gamma_1 a_1 b_1 + \gamma_2 a_1 b_2 + \gamma_3 a_2 b_1 + \gamma_4 a_2 b_2,$$
$$\gamma_5 a_1 b_1 + \gamma_6 a_1 b_2 + \gamma_7 a_2 b_1 + \gamma_8 a_2 b_2)$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

Commutative

$$\mathbf{a} \odot \mathbf{b} =$$

Non-Commutative

$$\mathbf{a} \odot \mathbf{b} = (\gamma_1 a_1 b_1 + \gamma_2 a_1 b_2 + \gamma_3 a_2 b_1 + \gamma_4 a_2 b_2, \\ \gamma_5 a_1 b_1 + \gamma_6 a_1 b_2 + \gamma_7 a_2 b_1 + \gamma_8 a_2 b_2)$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$\mathbf{a} \odot \mathbf{b} =$$

Commutative

Non-Commutative

$$\mathbf{a} \odot \mathbf{b} = (\gamma_1 a_1 b_1 + \gamma_2 a_1 b_2 + \gamma_3 a_2 b_1 + \gamma_4 a_2 b_2, \\ \gamma_5 a_1 b_1 + \gamma_6 a_1 b_2 + \gamma_7 a_2 b_1 + \gamma_8 a_2 b_2)$$

$$\downarrow$$

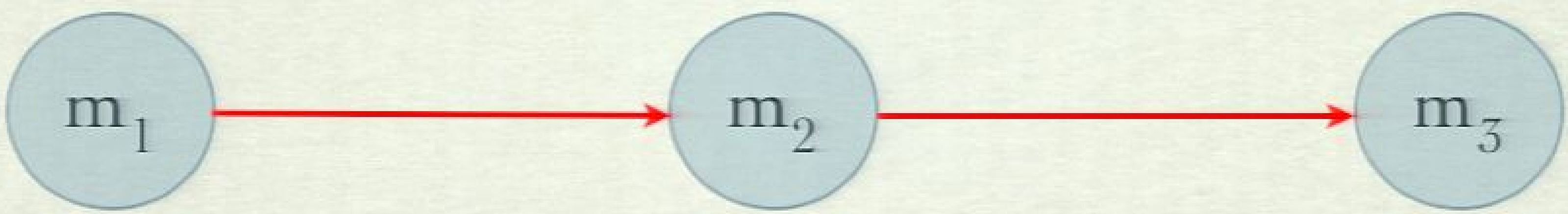
$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

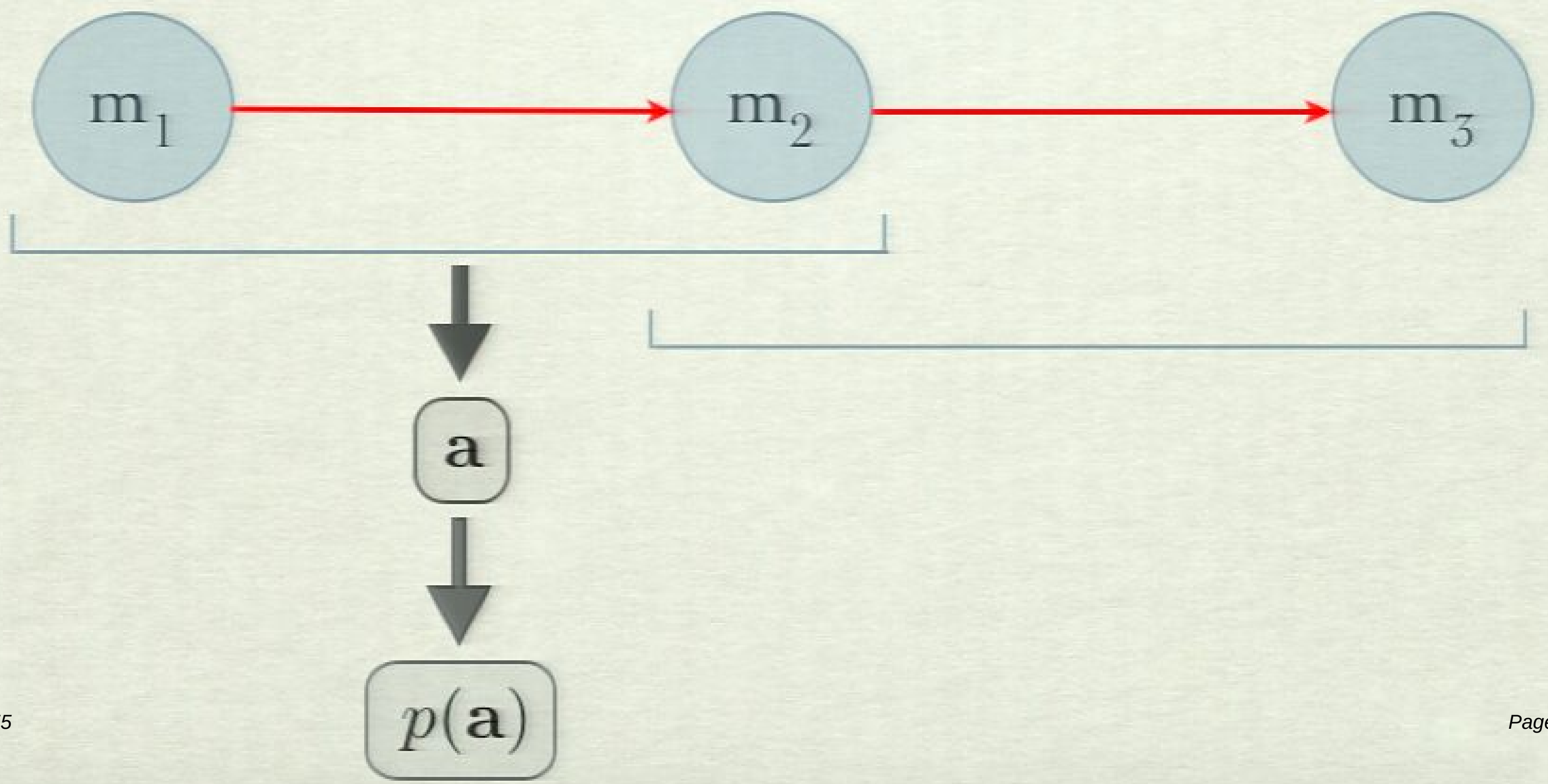
$$\mathbf{a} \odot \mathbf{b} = \begin{cases} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 + a_2 b_2, a_1 b_2 + a_2 b_1) \\ \\ \end{cases}$$

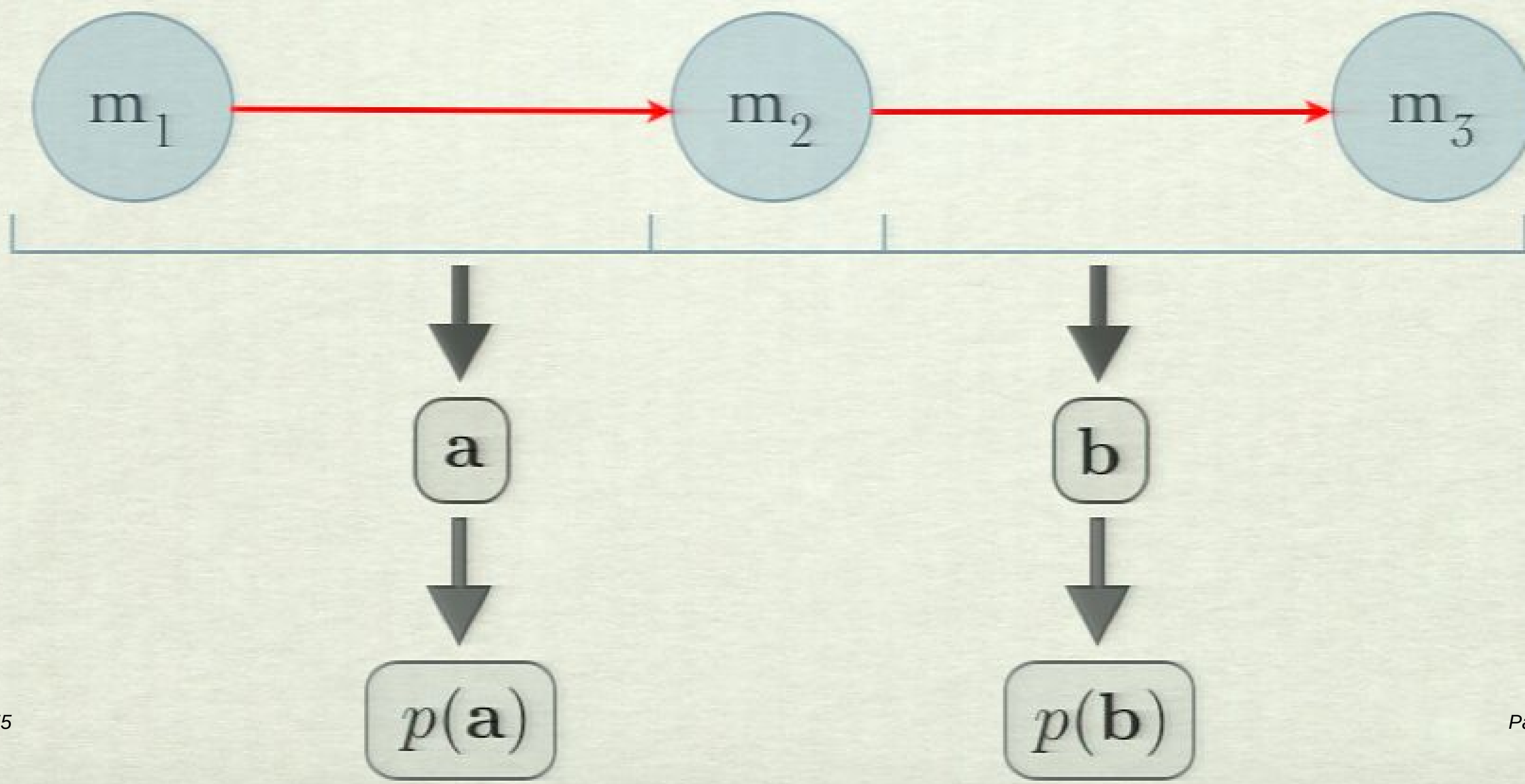
Commutative

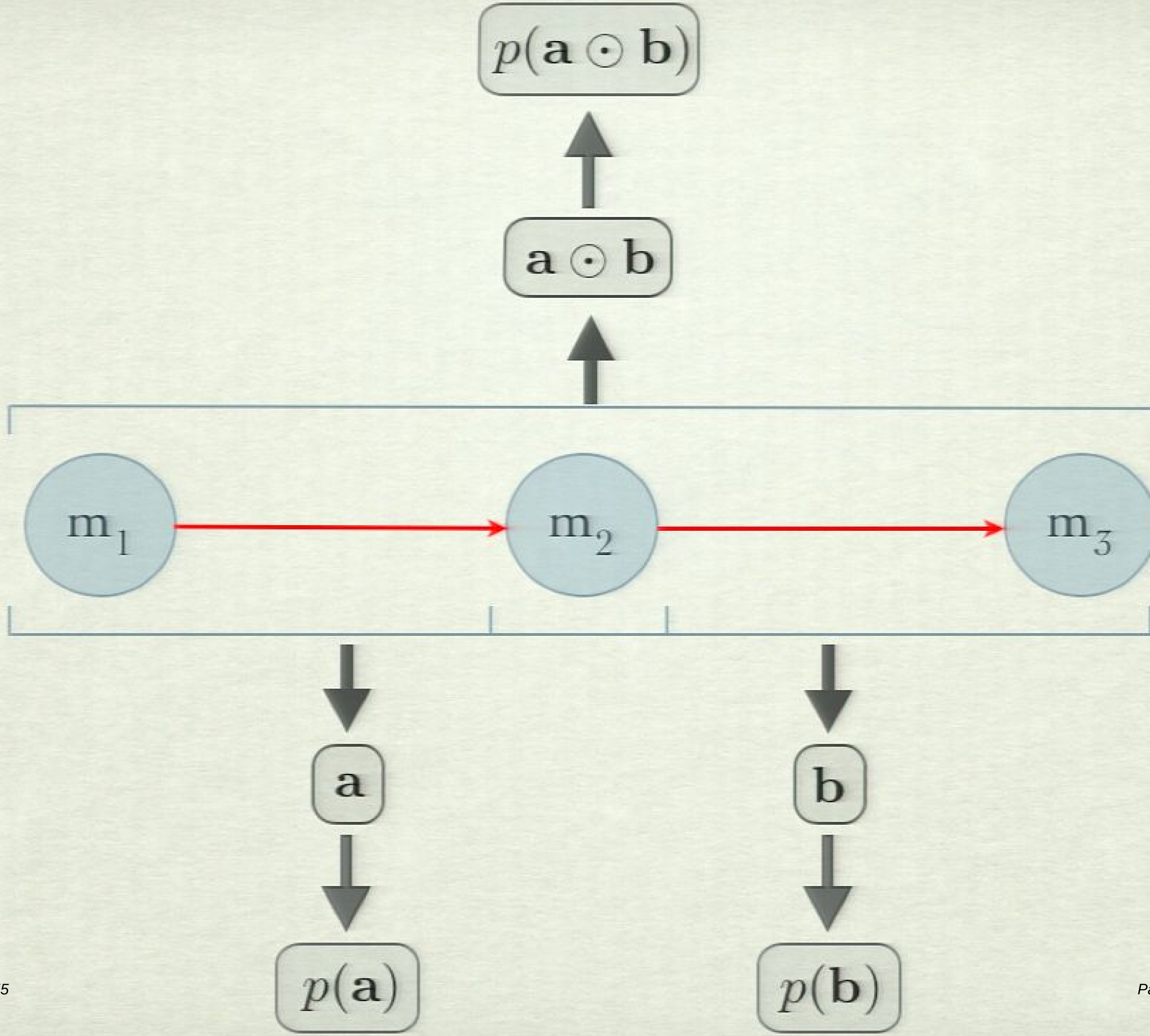
Non-Commutative

Consistency with Probability Theory


m_1
m_2
m_3


m_1
m_2
m_3
$\mathbf{a}$
$p(\mathbf{a})$


m1
m2
m3
a
b
p(a)
p(b)


$p(\mathbf{a} \odot \mathbf{b})$
$\mathbf{a} \odot \mathbf{b}$
m_1
m_2
m_3
$\mathbf{a}$
$\mathbf{b}$
$p(\mathbf{a})$
$p(\mathbf{b})$

$p(\mathbf{a} \odot \mathbf{b})$

$p(\mathbf{a})$

$p(\mathbf{b})$

$$p(\mathbf{a} \odot \mathbf{b})$$

$$p(\mathbf{a}) \qquad\qquad p(\mathbf{b})$$

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{l} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad\quad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad\quad , \qquad\qquad a_2 b_2) \end{array} \right.$$

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

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$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{l} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad , \qquad\qquad a_2 b_2) \\ \\ (a_1 b_1 \qquad\qquad , a_1 b_2 \qquad\qquad) \\ (a_1 b_1 \qquad\qquad , \qquad\qquad a_2 b_1) \end{array}\right.$$

$$p(\mathbf{a}) = \left(a_1^2 + a_2^2\right)^{\alpha/}$$

$$p(\mathbf{a}) = |a_1|^{\alpha} e^{\beta a_2 / a_1}$$

$$p(\mathbf{a}) = |a_1|^{\alpha} |a_2|^{\beta}$$

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{ll} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) & p(\mathbf{a}) = \left(a_1^2 + a_2^2\right)^{\alpha/} \\ (a_1 b_1 \quad , a_1 b_2 + a_2 b_1) & p(\mathbf{a}) = |a_1|^{\alpha} e^{\beta a_2 / a_1} \\ (a_1 b_1 \quad , \quad a_2 b_2) & p(\mathbf{a}) = |a_1|^{\alpha} |a_2|^{\beta} \\ (a_1 b_1 \quad , a_1 b_2 \quad) & p(\mathbf{a}) = |a_1|^{\alpha} \\ (a_1 b_1 \quad , \quad a_2 b_1) & \end{array} \right.$$

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{l} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad , \qquad\qquad a_2 b_2) \end{array} \right.$$

$$p(\mathbf{a}) = \left(a_1^2 + a_2^2\right)^{\alpha/}$$

$$p(\mathbf{a}) = |a_1|^\alpha e^{\beta a_2/a_1}$$

$$p(\mathbf{a}) = |a_1|^\alpha |a_2|^\beta$$

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

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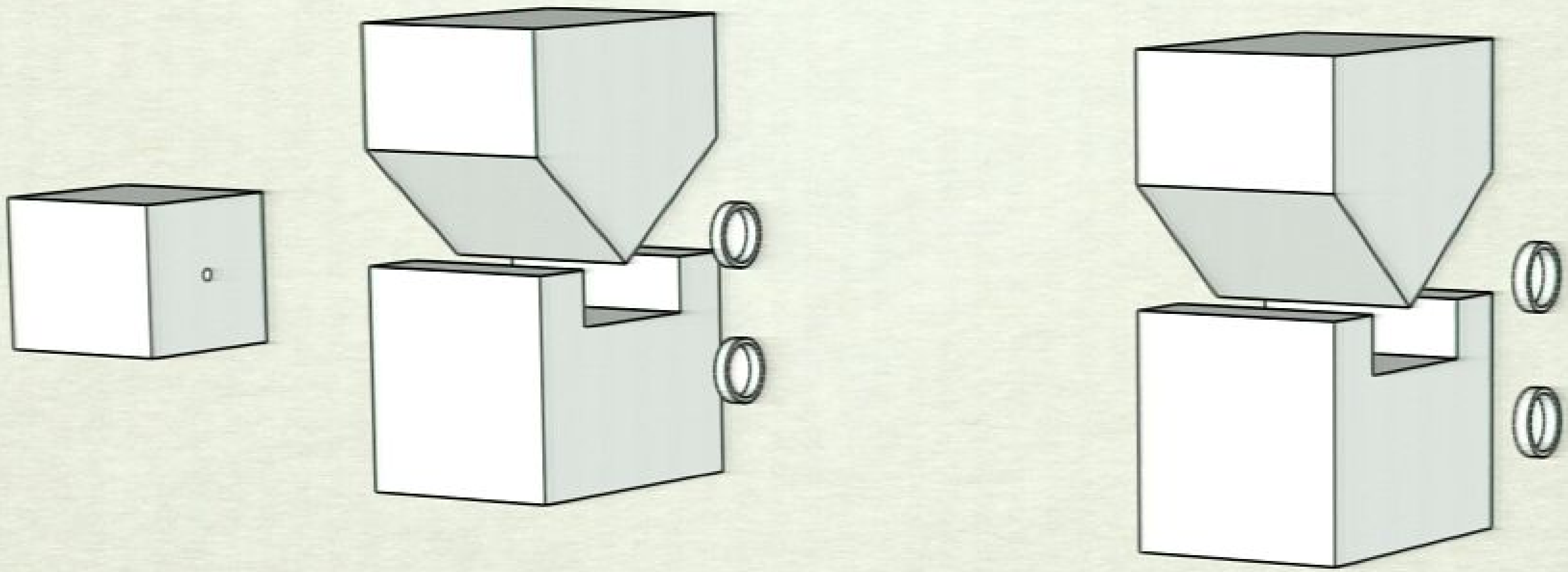
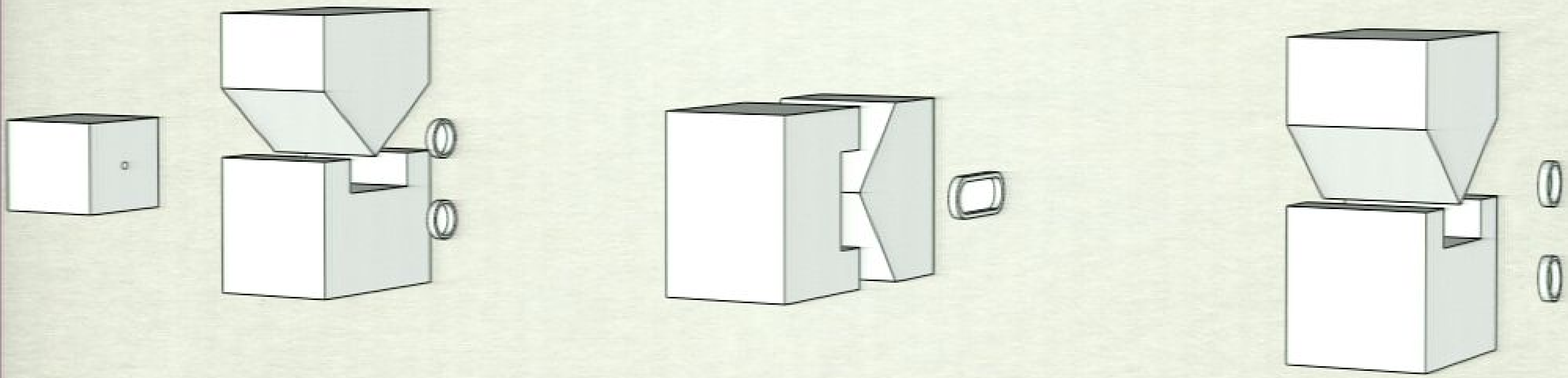
$$p(\mathbf{a}) = |a_1|^{\alpha} |a_2|^{\beta}$$

Repeated Measurements

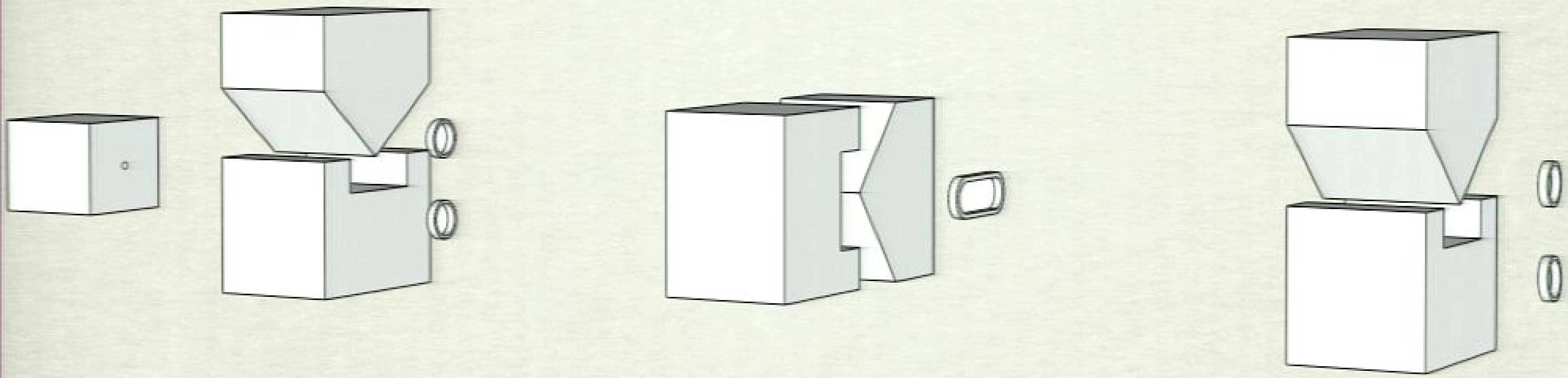


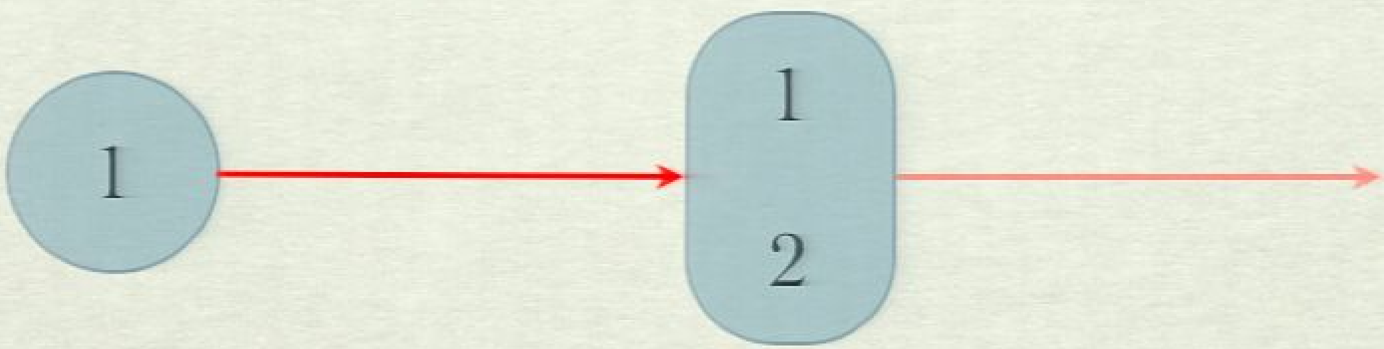
Repeated Measurements



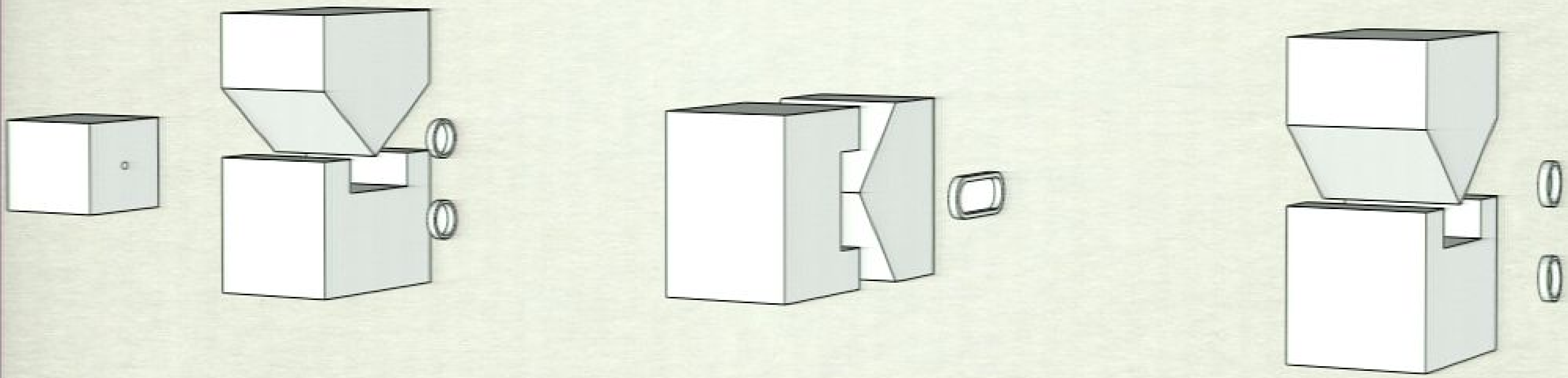
1

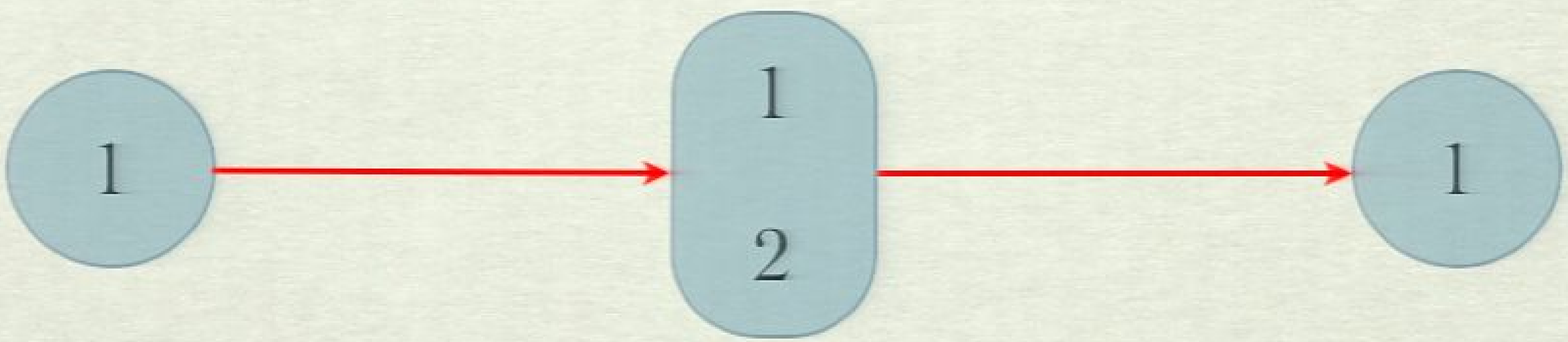
Repeated Measurements





Repeated Measurements





Repeated Measurements

$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{l} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad\quad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad\quad , \qquad\qquad a_2 b_2) \end{array} \right.$$

Repeated Measurements

$$\mathbf{a} \odot \mathbf{b} = \qquad (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$$

Repeated Measurements

$$\mathbf{a} \odot \mathbf{b} = (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$$

$$p(\mathbf{a}) = \left(a_1^2 + a_2^2\right)$$

Summary

Summary

- Sum Rule

$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

- Product Rule

$$\mathbf{a} \odot \mathbf{b} = (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$$

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Associativity and Commutativity of ⊕

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$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

Distributivity (of ⊙ over ⊕) and Associativity of ⊙

$$\mathbf{a} \odot \mathbf{b} = \begin{cases} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \quad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 + a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \quad , a_1 b_2 \quad) \\ (a_1 b_1 \quad , \quad a_2 b_1) \end{cases}$$

Associativity and Commutativity of $\oplus$

$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

Distributivity (of $\odot$ over $\oplus$) and Associativity of $\odot$

$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{l} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 + a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad , a_1 b_2 \qquad\qquad) \\ (a_1 b_1 \qquad\qquad , \qquad\qquad a_2 b_1) \end{array} \right.$$

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$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{l} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 + a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1, a_1 b_2) \\ (a_1 b_1, a_2 b_1) \end{array} \right.$$

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

Associativity and Commutativity of ⊕

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$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$

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$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$

$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{l} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 + a_2 b_2, a_1 b_2 + a_2 b_1) \end{array} \right.$$

Repeated Measurements

Associativity and Commutativity of $\oplus$

$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

Distributivity (of $\odot$ over $\oplus$) and Associativity of $\odot$

$$\mathbf{a} \odot \mathbf{b} = \left[\begin{array}{l} (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad , a_1 b_2 + a_2 b_1) \\ (a_1 b_1 + a_2 b_2, a_1 b_2 + a_2 b_1) \\ (a_1 b_1 \qquad\qquad , a_1 b_2 \qquad\quad\;) \\ (a_1 b_1 \qquad\qquad , \qquad\quad a_2 b_1) \end{array} \right.$$

$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$

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Repeated Measurements

$$\mathbf{a} \odot \mathbf{b} = (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$$

$$p(A) = a_1^2 + a_2^2$$

Steps to *von Neumann–Dirac* Formalism

Steps to ***von Neumann–Dirac*** Formalism

Steps to *von Neumann–Dirac* Formalism

- Composite Systems

$$\mathbf{a} \circ \mathbf{b} = (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$$

Steps to ***von Neumann–Dirac*** Formalism

- Composite Systems

$$\mathbf{a} \circ \mathbf{b} = (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$$

- Hermitian Representation of Measurements

Steps to *von Neumann–Dirac* Formalism

- Composite Systems

  $\mathbf{a} \circ \mathbf{b} = (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$

- Hermitian Representation of Measurements

- Unitary Evolution of State Vector

  $\mathbf{v}' = \mathsf{U}\mathbf{v}$

Steps to ***von Neumann–Dirac*** Formalism

- Composite Systems

$$\mathbf{a} \circ \mathbf{b} = (a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$$

- Hermitian Representation of Measurements

- Unitary Evolution of State Vector

$$\mathbf{v}' = \mathsf{U}\mathbf{v}$$

- Temporal Evolution Operator

$$\mathsf{U}_t(dt) = \exp(i\mathsf{H}\, dt/\hbar)$$

Origin of Feynman's Rules

Feynman's rules are a *pair-valued representation* of a *logic of process* which is consistent with probability theory.

An Implication

The Quantum Formalism is *not* explicitly about *space* and *matter* *(particles & fields).*

Logic $\xrightarrow{\textit{Representation}}$ Real Number Arithmetic

Logic

Representation

$$a + b = b + a$$
$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

$$(a + b) \times c = (a \times c) + (b \times c)$$
$$a \times (b + c) = (a \times b) + (a \times c)$$

Logic

Abstraction ←

$$a + b = b + a$$
$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

$$(a + b) \times c = (a \times c) + (b \times c)$$
$$a \times (b + c) = (a \times b) + (a \times c)$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Abstraction

$$a + b = b + a$$

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$$\xrightarrow[\textit{Representation}]{\textit{General}}$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

General → *Representation*

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$

$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$

$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

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$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Real-Valued → *Representation*

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Real-Valued

$\Longrightarrow$

Representation

$$a \oplus b = a + b$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Real-Valued $\longrightarrow$ *Representation*

$$a \oplus b = a + b$$

$$a \otimes b = a \times b$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Pair-Valued

$\longrightarrow$

Representation

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Pair-Valued

$\longrightarrow$

Representation

$$\left[\begin{array}{c}(A \vee B) \vee C = A \vee (B \vee C) \\ A \vee B = B \vee A\end{array}\right.$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Pair-Valued

$\longrightarrow$

Representation

$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

$$(A \vee B) \vee C = A \vee (B \vee C)$$

$$A \vee B = B \vee A$$

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$(A \vee B) \cdot C = (A \cdot C) \vee (B \cdot C)$$

$$A \cdot (B \vee C) = (A \cdot B) \vee (A \cdot C)$$

Pair-Valued

Representation

$$\mathbf{a} \oplus \mathbf{b} = (a_1 + b_1, a_2 + b_2)$$

$$\mathbf{a} \odot \mathbf{b} = \begin{cases} (a_1b_1 - a_2b_2, a_1b_2 + a_2b_1) \\ (a_1b_1, a_1b_2 + a_2b_1) \\ (a_1b_1 + a_2b_2, a_1b_2 + a_2b_1) \\ (a_1b_1, a_1b_2) \\ (a_1b_1, a_2b_1) \end{cases}$$

Quantum Theory

Quantum Theory

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$

$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$

$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

+

Quantum Theory

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

+

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

Repeatability Argument

Quantum Theory

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

+

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

Repeatability Argument

↓

Complex Structure

Quantum Theory

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

+

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

Repeatability Argument

↓

Complex Structure

Mathematics

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

Quantum Theory

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

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$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

+

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

Repeatability Argument

↓

Complex Structure

Mathematics

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

+

Existence of multiplicative inverses

Quantum Theory	Mathematics
$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$ $(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$	$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$ $(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$
$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$	$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$
$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$ $\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$	$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$ $\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$
+	+
$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$ Repeatability Argument	Existence of multiplicative inverses
↓	↓
Complex Structure	

Quantum Theory

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

$$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$$

$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

+

$$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$$

Repeatability Argument

↓

Complex Structure

Mathematics

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$$
$$(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$$

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$$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$$
$$\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$$

+

Existence of multiplicative inverses

↓

Complex Structure

Quantum Theory	*Mathematics*
$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$ $(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$	$\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$ $(\mathbf{a} \oplus \mathbf{b}) \oplus \mathbf{c} = \mathbf{a} \oplus (\mathbf{b} \oplus \mathbf{c})$
$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$	$(\mathbf{a} \odot \mathbf{b}) \odot \mathbf{c} = \mathbf{a} \odot (\mathbf{b} \odot \mathbf{c})$
$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$ $\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$	$(\mathbf{a} \oplus \mathbf{b}) \odot \mathbf{c} = (\mathbf{a} \odot \mathbf{c}) \oplus (\mathbf{b} \odot \mathbf{c})$ $\mathbf{a} \odot (\mathbf{b} \oplus \mathbf{c}) = (\mathbf{a} \odot \mathbf{b}) \oplus (\mathbf{a} \odot \mathbf{c})$
+	+
$p(\mathbf{a} \odot \mathbf{b}) = p(\mathbf{a})\, p(\mathbf{b})$ Repeatability Argument	Existence of multiplicative inverses
↓	↓
Complex Structure	*Complex Structure*

Quantum Mathematical Framework

Pair Postulate (Probability & Complementarity)

Physical Principles

Quantum Conception of Reality

Reconstructic
of Quantum
Theory

Quantum Mathematical Framework

Pair Postulate (Probability & Complementarity)

Quantum Conception of Reality

Reconstructic
of Quantum
Theory

Classical Measurement

Classical Measurement

Deterministic outcomes

Complete Accessibility of State

Classical Measurement

Deterministic outcomes

Complete Accessibility of State

Classical Measurement

Deterministic outcomes

Complete Accessibility of State

Every agent has *"God's Eye" view* of the state of a system

Concept of Information is *redundant*

Classical Measurement

Deterministic outcomes

Complete Accessibility of State

Every agent has *"God's Eye" view* of the state of a system

Concept of Information is *redundant*

Quantum Measurement

Probabilistic outcomes

Partial Accessibility of State (Complementarity)

Classical Measurement

Deterministic outcomes

Complete Accessibility of State

Every agent has *"God's Eye" view* of the state of a system

Concept of Information is *redundant*

Quantum Measurement

Probabilistic outcomes

Partial Accessibility of State (Complementarity)

Classical Measurement

Deterministic outcomes

Complete Accessibility of State

↓

Every agent has *"God's Eye" view* of the state of a system

Concept of Information is *redundant*

Quantum Measurement

Probabilistic outcomes

Partial Accessibility of State (Complementarity)

↓

Agents have *informationally-restricted view* of the state of a system

Concept of information is *fundamental*

Reference

"Origin of Complex Quantum Amplitudes and Feynman's Rules"

Goyal, Knuth, Skilling

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Available at: www.philipgoyal.org