Title: Does entanglement persist at the macroscopic level?

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Abstract: The quantum states postulated to occur in situations of the "Schroedinger's Cat" type are essentially N-particle GHZ states with N very large compared to 1,and their observation would thus be particularly compelling evidence for the ubiquity of the phenomenon of entanglement. However, in the traditional quantum measurement literature considerable scepticism has been expressed about the observability of this kind of "macroscopically entangled" state, primarily because of the putatively disastrous effect on it of decoherence. In this talk I first examine why much of the literature has grossly overestimated the effects of decoherence,and then review the current experimental situation with respect to such states, as they (may) occur in fullerene diffraction, magnetic biomolecules, quantum-optical systems and Josephson devices; I also consider the prospects for their observation in nanomechanical systems. I conclude by reviewing and the theoretical implications of the experiments of the last decade in this area.

WHERE CAN WE FIND EVIDENCE IN EXPERIMENT FOR A HIGH DEGREE OF ENTANGLEMENT?

ANS: IN HIGH-N GHZ STATES!

$$\Psi = \frac{1}{\sqrt{2}}\{\underbrace{|\uparrow\uparrow\uparrow\uparrow\ldots\uparrow\uparrow\uparrow\rangle}_{N} + \underbrace{|\downarrow\downarrow\downarrow\downarrow\downarrow\ldots\downarrow\downarrow\rangle}_{N}\}$$

N PARTICLES "BEHAVING DIFFERENTLY" IN 2 BRANCHES OF SUPERPOS^N, $N \gg 1$

THESE ARE JUST "SCHRÖDINGER'S CAT" STATES!

⟹ EXPTL. EFFORT TO BUILD SCHRÖDINGER'S CAT IN THE LABORATORY IS A PROBE OF ENTANGLEMENT.

TESTING QUANTUM MECHANICS TOWARDS THE LEVEL OF EVERYDAY LIFE: RECENT PROGRESS AND CURRENT PROSPECTS

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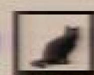
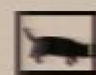
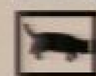
MESO/MACROSCOPIC TESTS OF QM: MOTIVATION

At microlevel: (a) $|\uparrow\rangle + |\downarrow\rangle$ quantum superposition ✓

$\neq$ (b) $|\uparrow\rangle$ OR $|\downarrow\rangle$ classical mixture ×

how do we know? Interference

At macrolevel: (a)  +  quantum superposition

OR (b)  OR  macrorealism

⚠ Decoherence DOES NOT reduce (a) to (b)!

Can we tell whether (a) or (b) is correct?

Yes, if and only if they give different experimental predictions. But if decoherence → no interference, then predictions of (a) and (b) identical.

⇒ must look for QIMDS
↑
quantum interference of macroscopically distinct states

What is "macroscopically distinct"?

(a) "extensive difference" Δ

(b) "disconnectivity" D
↑
~large number of particles behave differently in two branches

Initial aim of program: interpret raw data in terms of QM, test (a) vs (b).

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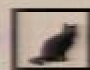
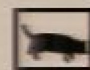
At macrolevel: (a)  +  quantum superposition

OR (b)  OR  macrorealism

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WHY HAS (MUCH OF) THE QUANTUM MEASUREMENT LITERATURE SEVERELY OVERESTIMATED DECOHERENCE?

("electron-on-Sirius" argument: $\Delta\epsilon \sim a^{-N} \sim \exp - N \longleftarrow \sim 10^{23}$

$\Rightarrow$ Just about any perturbation $\gg \Delta\epsilon \Rightarrow$ decoherence)

1. Matrix elements of S-E interaction couple only a very restricted set of levels of S.

2. "Adiabatic" ("false") decoherence:
   Ex.: spin-boson model

$$\hat{H} = \hat{H}_s + \hat{H}_E + \hat{H}_{S-E}$$

$$\hat{H}_s = \Delta\sigma_x$$

$\hat{H}_E =$ set of SHO's with lower frequency cutoff $\omega_{min} \gg \Delta$

$\hat{H}_{S-E} = \hat{\sigma}_z \sum_\alpha C_\alpha \hat{x}_\alpha \leftarrow$ oscillator coords.

$\Psi_{un}(t=0) = |+\rangle|\chi_+\rangle \leftarrow$ displaced state of oscillation

$$\hat{\rho}_s(t=0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ (trivially)}$$

$$\Downarrow$$

$$\Psi_{un}(t \sim \hbar/\Delta_{un}) \cong \frac{1}{\sqrt{2}}(|+\rangle|\chi_+\rangle + |-\rangle|\chi_-\rangle).$$

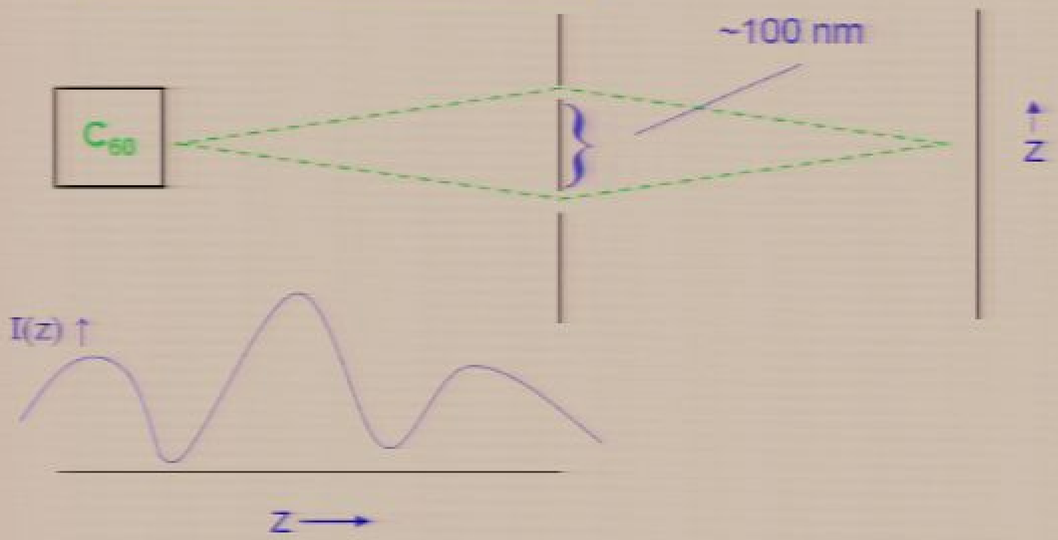
$\langle \chi_+ | \chi_- \rangle = \exp - F \cong 0$ FC factor

$$\Rightarrow \quad \hat{\rho}_s(t \sim \hbar/\Delta_{un}) \cong \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

decohered?? (cf. neutron interferometer)

The Search for QIMDS

1. Molecular diffraction*



Note: (a.) Beam does not have to be monochromated

$$f(\upsilon) = A\upsilon^3 \exp -(\upsilon - \upsilon_o)^2/\upsilon_m^2 \qquad (\upsilon_o \sim 1{\cdot}8\,\upsilon_m)$$

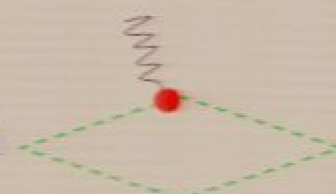
(b.) "Which-way" effects?

Oven is at 900–1000 K

⇒ many vibrational modes excited

4 modes infrared active ⇒

absorb/emit several radiation quanta on passage through apparatus!



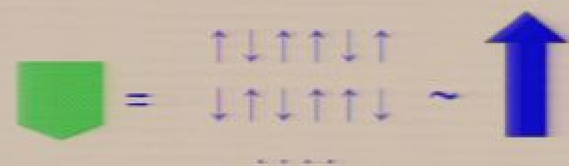
Why doesn't this destroy interference?

*Arndt et al., Nature *401*, 680 (1999); Nairz et

The Search for QIMDS (cont.)

2. Magnetic biomolecules*

Apoferritin sheath
(magnetically inert)



(~5000 Fe^{3+} spins, mostly
AF but slight ferrimagnetic tendency)
($M \sim 200\mu_B$)

$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$?

$$AF: \Delta \sim \hbar\omega_o \exp - N\sqrt{K/J}$$

(isotropic) exchange en.

no. of spins — uniaxial anisotropy

Raw data: $\chi(\omega)$ and noise spectrum
above ~200 mK, featureless
below ~300 mK, sharp peak at ~ 1 MHz (ω_{res})

$$\omega_{res}^2 \cong \omega_o^2 + M^2H^2$$

$$\ell n\ \omega_o \sim a - bN$$ ← no. of spins, exptly. adjustable

Nb: data is on physical ensemble, i.e., only total magnetization measured.

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*S. Gider et al., Science *268*, 77 (1995).

The Search for QIMDS (cont.)

3. Quantum-optical systems*

~10^{12} Cs atoms

for each sample separately, and also for total

$$\left[J_x, J_y\right] = iJ_z$$

$$\Rightarrow \quad \left\langle \delta J_{x1} \delta J_{y1} \right\rangle \geq |J_{z1}|$$

$$\left\langle \delta J_{x2} \delta J_{y2} \right\rangle \geq |J_{z2}|$$

$$\left\langle \delta J_{x\,tot} \delta J_{y\,tot} \right\rangle \geq |J_{z\,tot}|$$

so, if set up a situation s.t.

$$J_{z1} = -J_{z2}$$

must have

$$\left\langle \delta J_{x1} \delta J_{y1} \right\rangle > 0$$

$$\left\langle \delta J_{x2} \delta J_{y2} \right\rangle > 0$$

but may have

$$\left\langle \delta J_{xtot} \delta J_{y\,tot} \right\rangle = 0$$

(anal. of EPR)

*B. Julsgaard et al., Nature *41*, 400 (2001); E. Polzik, Physics World *15*, Interpretation of idealized expt. of this type:

(QM theory $\Rightarrow$) $\quad \langle \delta J_{x1} \delta J_{y1} \rangle \geq | J_{z1} | \sim N$

$$\Rightarrow | \delta J_{x1} | \gtrsim N^{1/2}$$

<u>But</u>:

(expt $\Rightarrow$) $\langle \delta J_{xtot} \delta J_{ytot} \rangle \cong 0 \qquad$ (#)

$\Rightarrow | \delta J_{xtot} | \sim 0$

$\Rightarrow \delta J_{x1}$ exactly anticorrelated with δJ_{x2}

$\Rightarrow$state is either superposition or mixture of $|n,-n>$

but mixture will not give (#)

value of J_{x1} value of J_{x2}

$\Rightarrow$ state must be of form

$$\sum_n c_n | n, -n >$$

with appreciable weight for $n \leq N^{1/2}$. $\Rightarrow$high disconnectivity

Note:

(a) QM used essentially in argument

(b) D ~ $N^{1/2}$ not ~N.
(prob. generic to this kind of expt.)

The Search for QIMDS (cont.)

4. Superconducting devices

(†: not all devices which are of interest for quantum computing are of interest for QIMDS)

Advantages:

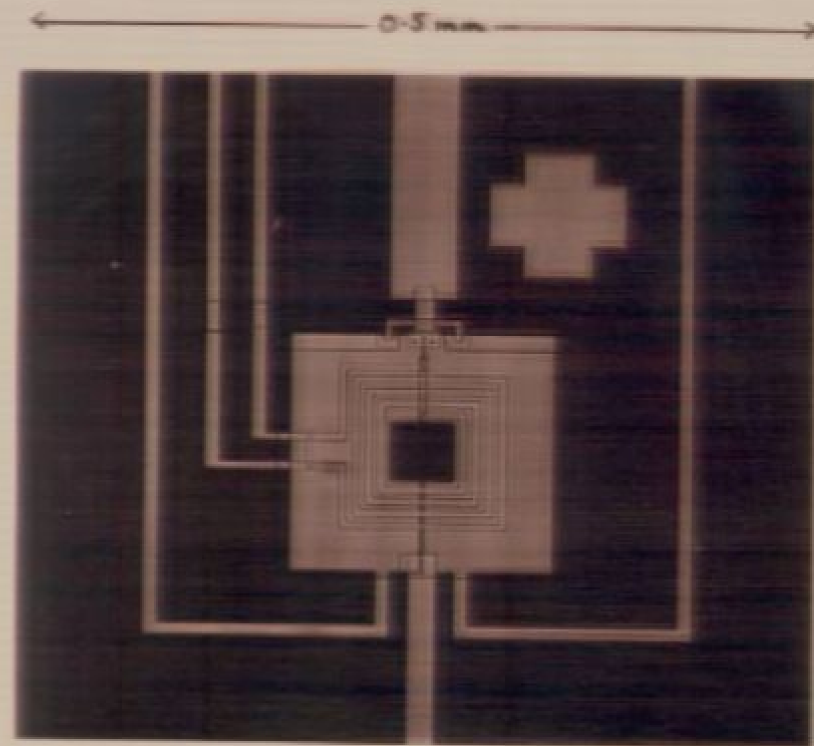
— classical dynamics of macrovariable v. well understood

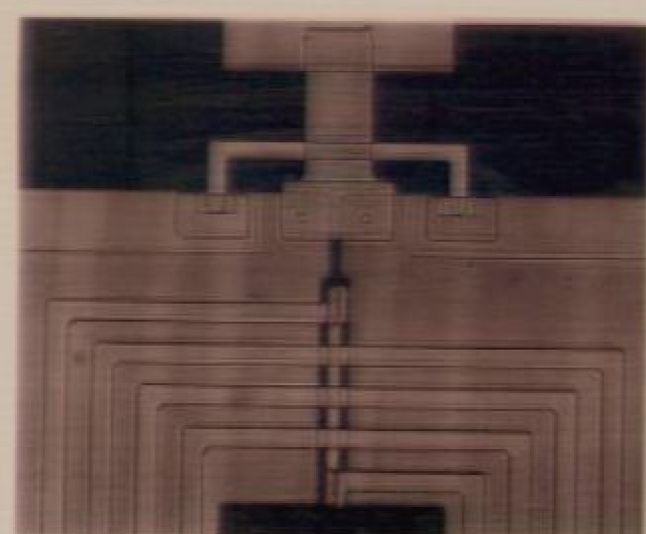
— intrinsic dissipation (can be made) v. low

— well developed technology

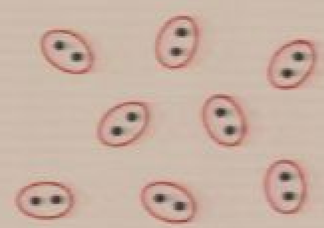
— (non-) scaling of S (action) with D.

"Macroscopic variable" is trapped flux Φ
[or circulating current I]

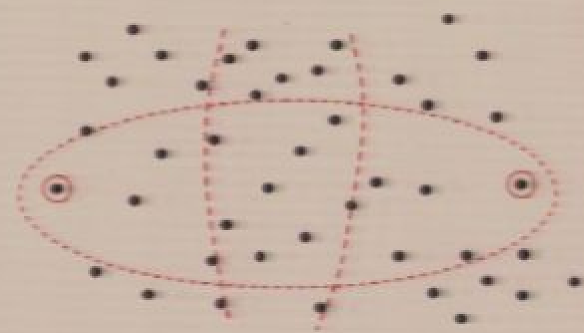

0·5 mm



Pairing of electrons:



"di-electronic molecules"



Cooper Pairs

In simplest ("BCS") theory, Cooper pairs, once formed, must automatically undergo Bose condensation!

$\Rightarrow$ must all do exactly the same thing at the same time (also in nonequilibrium situation)

SUPERCONDUCTING RING IN EXTERNAL MAGNETIC FLUX:

Quantization condition for "particle" of charge 2e (Cooper pair):

$$K \equiv \oint \mathbf{v} \cdot d\mathbf{l} = \frac{h}{2m}(n - \Phi/\Phi_0)$$

integer (n)

"flux quantum" h/2e (Φ_0)

$E \propto K^2$

A. $\Phi = 0$: groundstate unique ($n = 0$)

$\Rightarrow$ all pairs at rest.

B. $\Phi = 1/2\,\Phi_0$: groundstate doubly degenerate

($n = 0$ or $n = 1$)

Either all pairs rotate clockwise

Or all pairs rotate anticlockwise

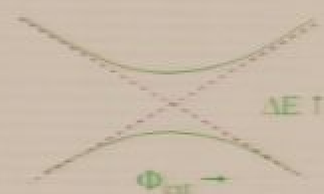
Note: state with 50% ↖ and 50% ↗

strongly forbidden by energy considerations

Josephson circuits

$\Psi = 2^{-1/2}\ (|\circlearrowleft\rangle + |\circlearrowright\rangle)$

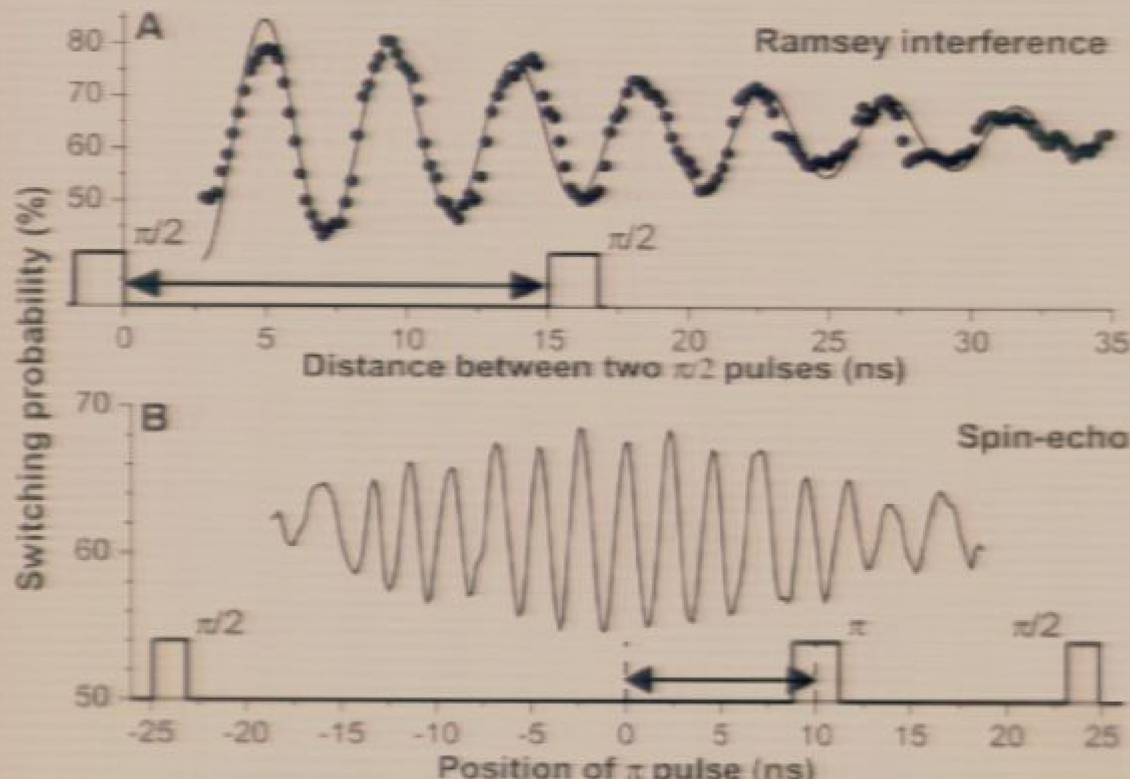
Evidence: (a) spectroscopic:
(SUNY, Delft 2000)



(b) real-time oscillations (like NH_3)

between ↺ and ↻

(Saclay 2002, Delft 2003) (Q_ϕ ~ 50-350)



From I. Chiorescu, Y. Nakamura, C.J.P. Harmans, and J. E. Mooij, Science, 299, 1869 (2003)

WHAT IS THE DISCONNECTIVITY "D" ("SCHRÖDINGER'S-CATTINESS") OF THE STATES OBSERVED IN QIMDS EXPERIMENTS?

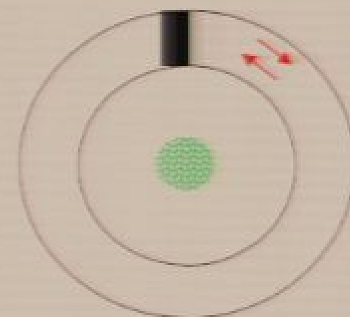
i.e., how many "microscopic" entities are "behaving differently" in the two branches of the superposition?

Fullerene (etc.) diffraction experiments: straightforward, number of "elementary" particles in C_{60} (etc.) (~1200)

Magnetic biomolecules: number of spins which reverse between the two branches (~5000)

Quantum-optical experiments
SQUIDS
} matter of definition

e.g. SQUIDS (SUNY experiment):



(a) naïve approach:

no. of C. pairs

$$\Psi_{\frown} \sim \chi_{\frown}^{N/2}, \; \Psi_{\circlearrowleft} = \chi_{\circlearrowleft}^{N/2}$$

mutually orthogonal C. pair w.f.

$\Rightarrow D \sim N \sim 10^9 - 10^{10}$ ⚠: Fermi statistics!

(b) how many single electrons do we need to displace in momentum space to get from $\Psi_{\frown}$ to $\Psi_{\circlearrowleft}$? (Korsbakken et al., preprint, Nov. 08)

$\Rightarrow D \sim N(\upsilon_s / \upsilon_F) \sim 10^3 - 10^4$ ⚠: intuitively, severe underestimate in "BEC" limit (e.g. Fermi alkali gas)

(c) macroscopic eigenvalue of 2-particle density matrix (corresponding to (fairly) orthogonal states in 2 branches):

$\Rightarrow D \sim N(\Delta / \varepsilon_F) \sim 10^6 - 10^7$

observation time ~ 1 sec

LiF

$v = 0$ $\qquad\qquad$ $v = v_0$

$$W_{el} \equiv \sum_p |\delta n_p| \sim v_0/v_F$$

What about nuclei?

Prima facie,

rms nuclear velocity

$$W_{nuc} \sim v_0/v_n \ggg v_0/v_F$$

But should think in terms of individual nucleons within nucleus! then

velocity of proton (etc)

$$W_{nuc} \sim v_0/v_p \ll v_0/v_F!$$

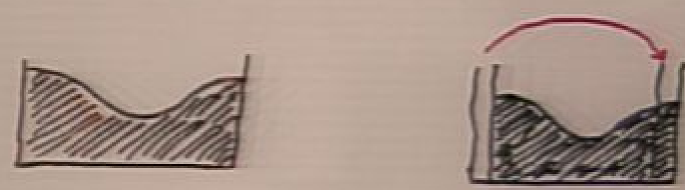
If that's "cheating", so is 1-electron calc

for SQUID.

Real FOM $\sim N(\Delta/\epsilon_F) \sim 10^6 - 10^7$?

observation time
~1 sec

LiF



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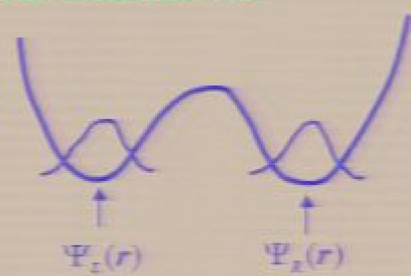
More possibilities for QIMDS:

(a) BEC's of ultracold alkali gases:

Bose-Einstein condensates



(Gross-Pitaevskii)

Ordinary GP state:

$$\Psi_N = \left(a\psi_L(\boldsymbol{r}) + b\psi_R(\boldsymbol{r})\right)^N$$

"Schrödinger-cat" state (favored if interactions attractive):

$$\Psi_N = a\left(\psi_L(\boldsymbol{r})\right)^N + b\left(\psi_R(\boldsymbol{r})\right)^N$$

problems:

(a) extremely sensitive to well asymmetry ΔE
(energy stabilizing arg (a/b) $\sim t^N \sim \exp - NB/\hbar$)
so ΔE needs to be exp'ly small in N

single-particle tunnelling matrix element

(b) detection: tomography unviable for N»1,
⇒ need to do time-sequence experiments (as in SQUIDS), but period v. sensitive e.g. to exact value of N

WHAT HAVE WE SEEN SO FAR?

1. If we interpret raw data in QM terms, then can conclude we have a quantum superposition rather than a mixture of meso/macroscopically distinct states.
   However, "only 1 degree of freedom involved."

2. Do data exclude general hypothesis of macrorealism?
   NO

3. Do data exclude specific macrorealistic theories?
   e.g. GRWP ← Ghirardi, Rimini, Weber, Pearle

   NO (fullerene diffraction: N not large enough, SQUIDS: no displacement of COM between branches)

   Would MEMS experiments (if in agreement with QM) exclude GRWP?

   alas: $\Gamma_{coll} \propto \Delta x$, $\Gamma_{dec} \propto (\Delta x)^2$

   collapse rate in GRWP theory (↑ Γ_{coll})
   decoherence rate acc. to QM (↑ Γ_{dec})

   ⇒ do not gain by going to larger Δx
   (and small Δx may not be enough to test GRWP)

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HOW CONFIDENT ARE WE ABOUT (STANDARD QM'I) DECOHERENCE RATE?

Theory:

(a) model environment by oscillator bath (may be questionable)

(b) Eliminate environment by standard Feynman-Vernon type calculation (seems foolproof)

Result (for SHO):

$$\Gamma_{dec} \sim \Gamma \left(\frac{k_B T}{\hbar \Omega} \right) \cdot \left(\frac{\Delta x}{x_0} \right)^2$$

provided $k_B T \gg \hbar\Omega$

energy relaxation rate (Ω/Q)

zero-point rms displacement

ARE WE SURE THIS IS RIGHT?

Tested (to an extent) in cavity QED: never tested (?) on MEMS.

Fairly urgent priority!

Where do we go from here?

1. Larger values of Λ and/or D?
   (Diffraction of virus?)

2. Alternative Dfs. of "Measures" of Interest

   — More sophisticated forms of entanglement?*
   — Biological functionality (e.g. superpose states of rhodopsin?)
   — Other (e.g. GR)

* 3. Exclude Macrorealism

Suppose: Whenever observed, Q = ± 1.

Q = + 1 Q = - 1

Df. of "MACROREALISTIC" Theory:

"COMMON SENSE"?

I. Q(t) = ± 1 at (almost) ∀ t,

whether or not observed.

II. Noninvasive measurability

III. Induction

Can test with existing SQUID Qubits!

*S. Aaronson, STOC 2004, p. 118.

Df:

$$K \equiv K(t_1 t_2 t_3 t_4) \equiv \langle Q(t_1)Q(t_2)\rangle_{exp} + \langle Q(t_2)Q(t_3)\rangle_{exp}$$
$$+\langle Q(t_3)Q(t_4)\rangle_{exp} - \langle Q(t_1)Q(t_4)\rangle_{exp}$$

Take $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \pi/4\Delta$ ← tunnelling frequency

Then,

(a) Any macrorealistic theory: $K \leq 2$

(b) Quantum mechanics, ideal: $K = 2.8$

(c) Quantum mechanics, with all the real-life complications: $K > 2$ (but < 2.8)

Thus: to extent analysis of (c) within quantum mechanics is reliable, can force nature to choose between macrorealism and quantum mechanics!

Possible outcomes:

(1) Too much noise $\Rightarrow K_{QM} < 2$

(2) $K > 2 \Rightarrow$ macrorealism refuted

(3) $K < 2$: ? !