Title: Twistor-String Theory

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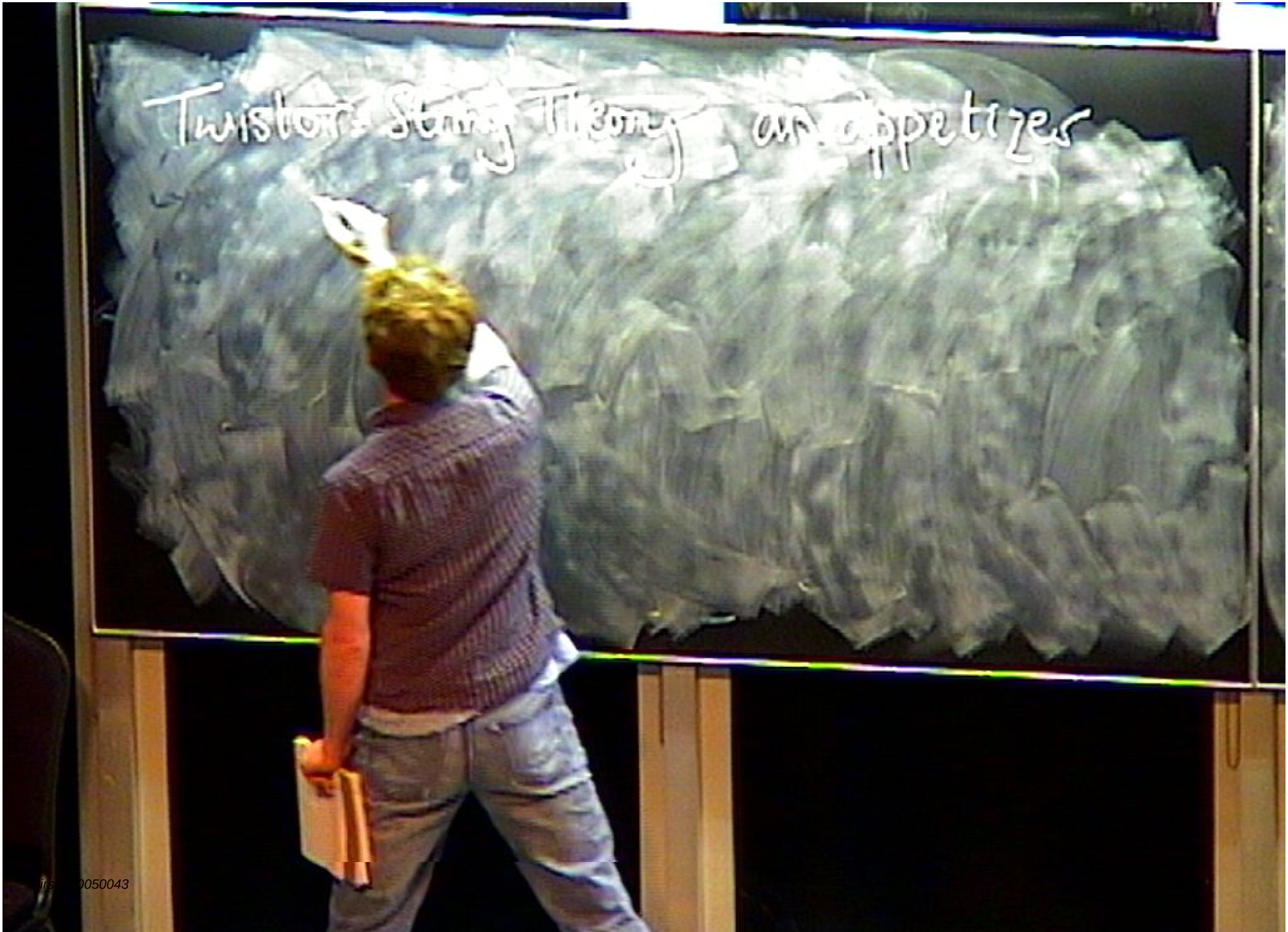
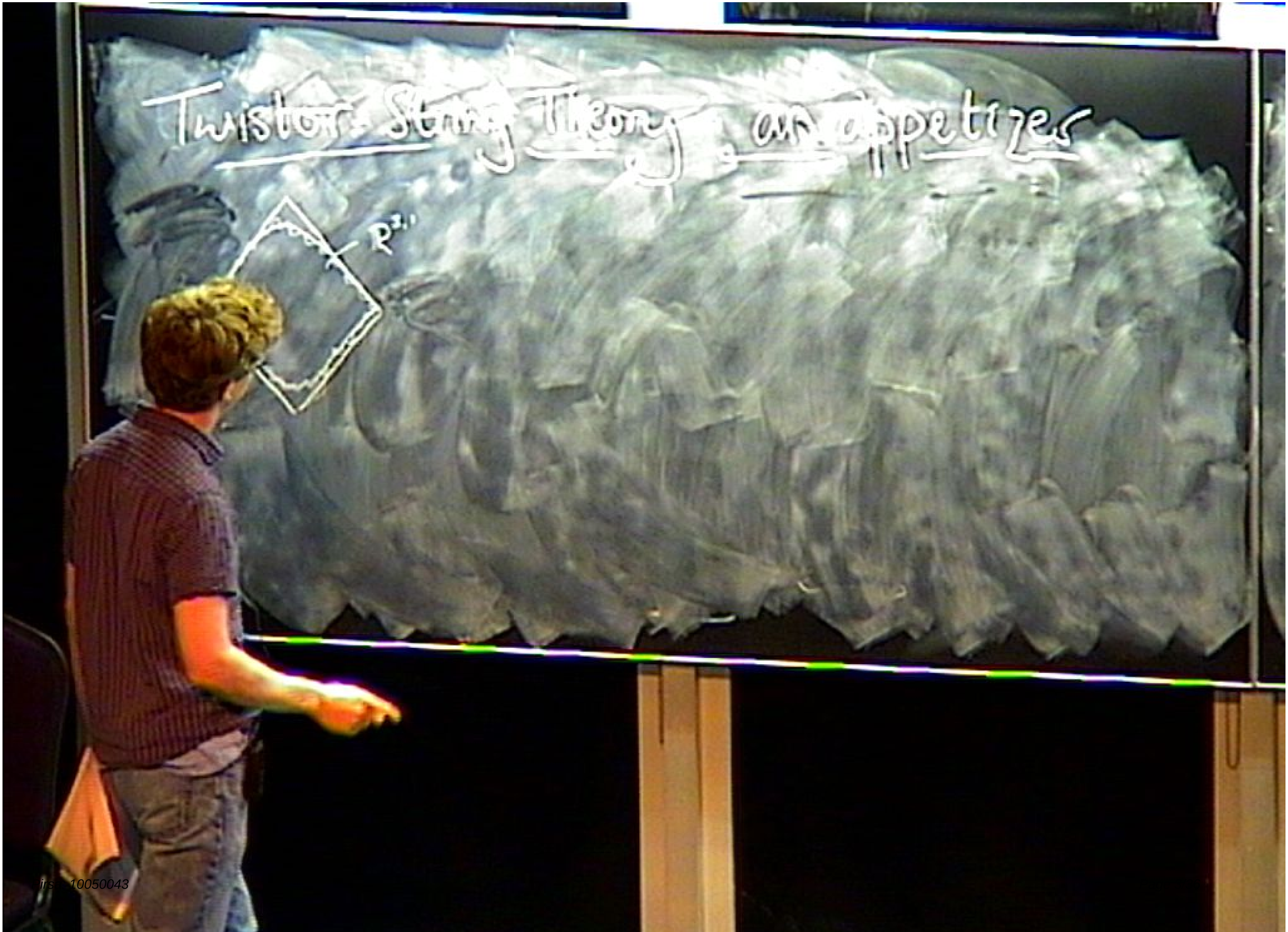
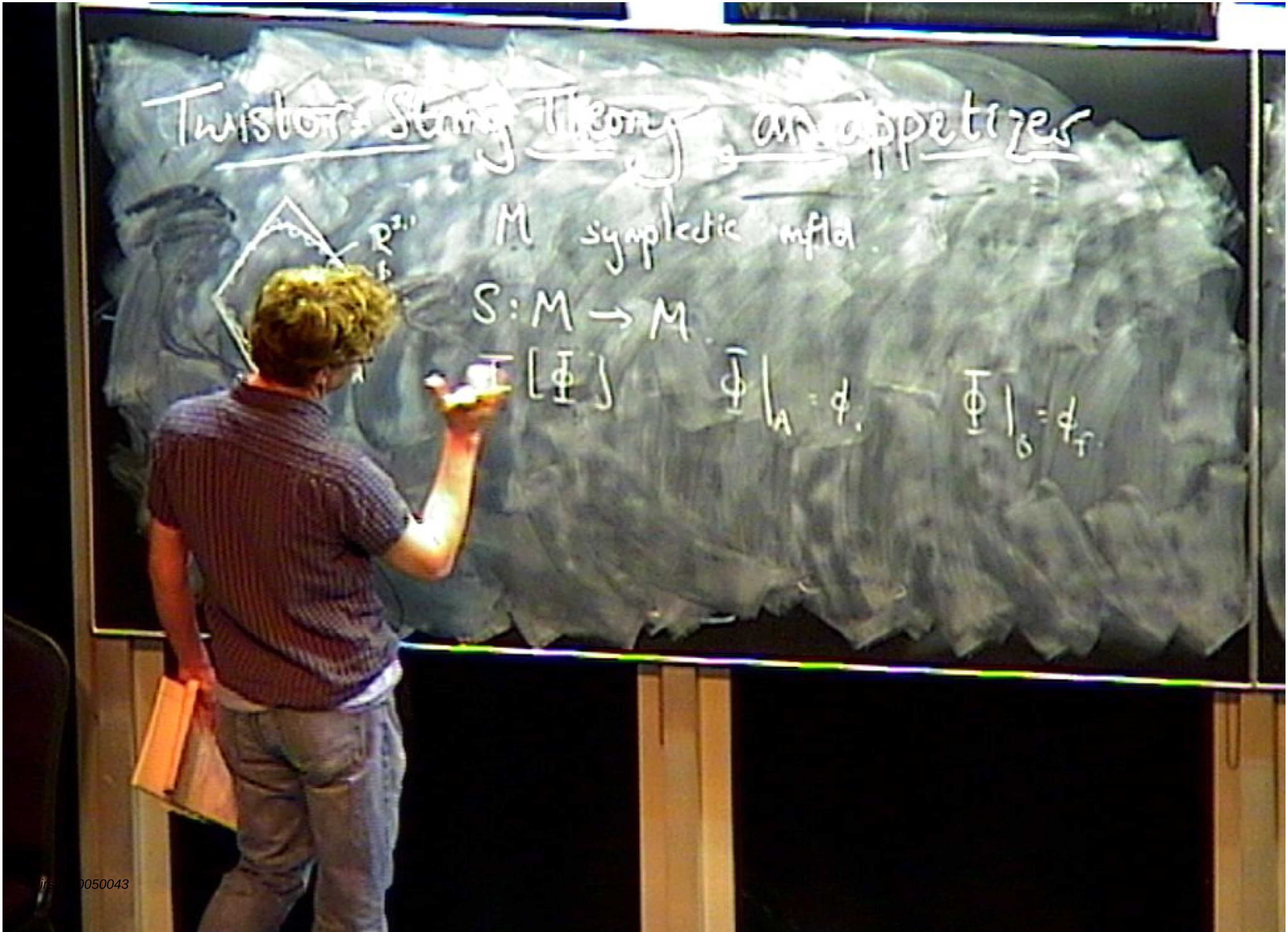
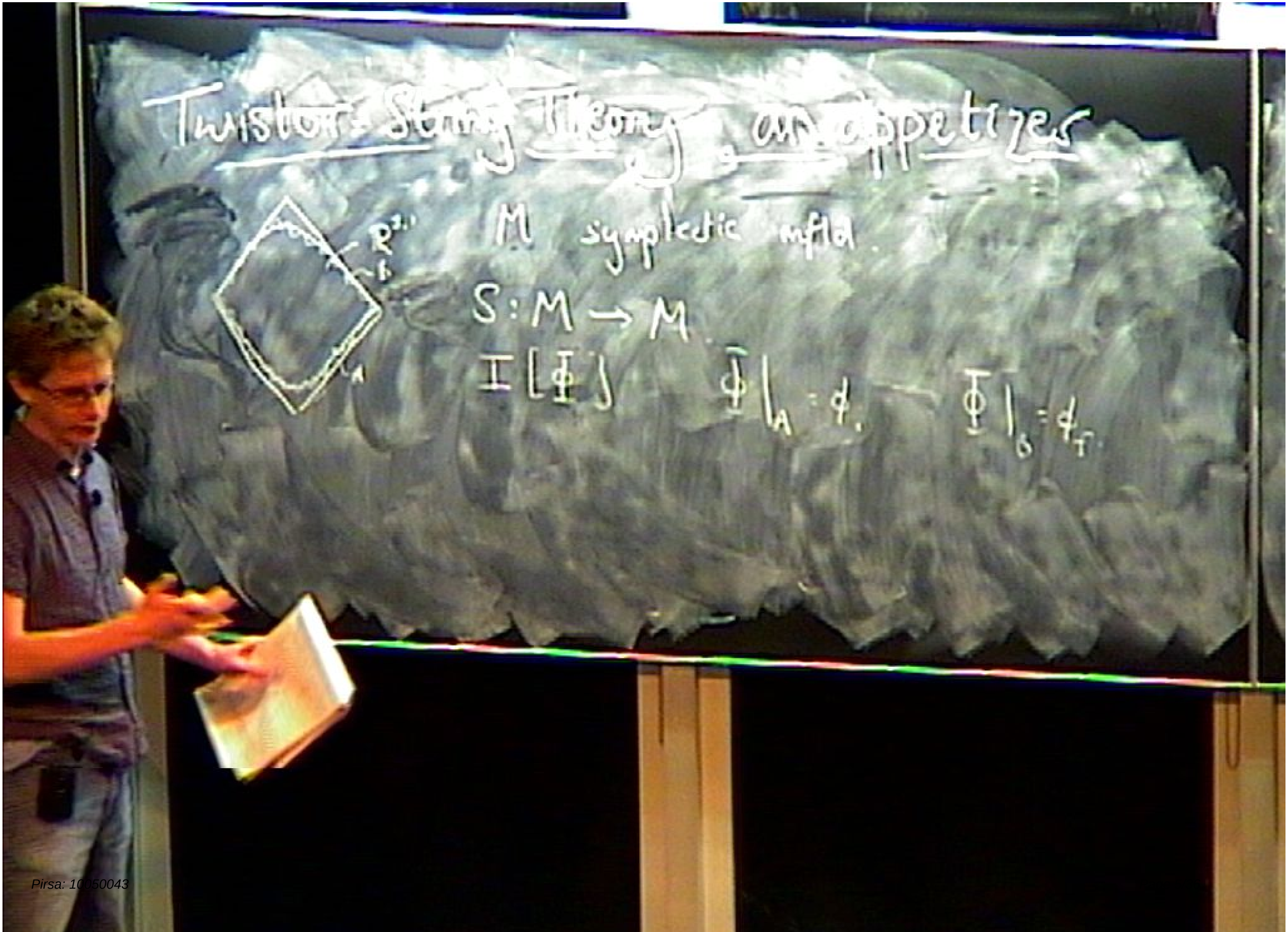
Abstract: I'll give an introduction to twistor-string theory, which is an attempt to reformulate supersymmetric gauge theory in four-dimensional space-time in terms of a certain generalisation of Gromov-Witten theory in twistor space. The resulting theory is closely related to the multi-dimensional residue calculus in G(k,n) (introduced in Cachazo's talk).

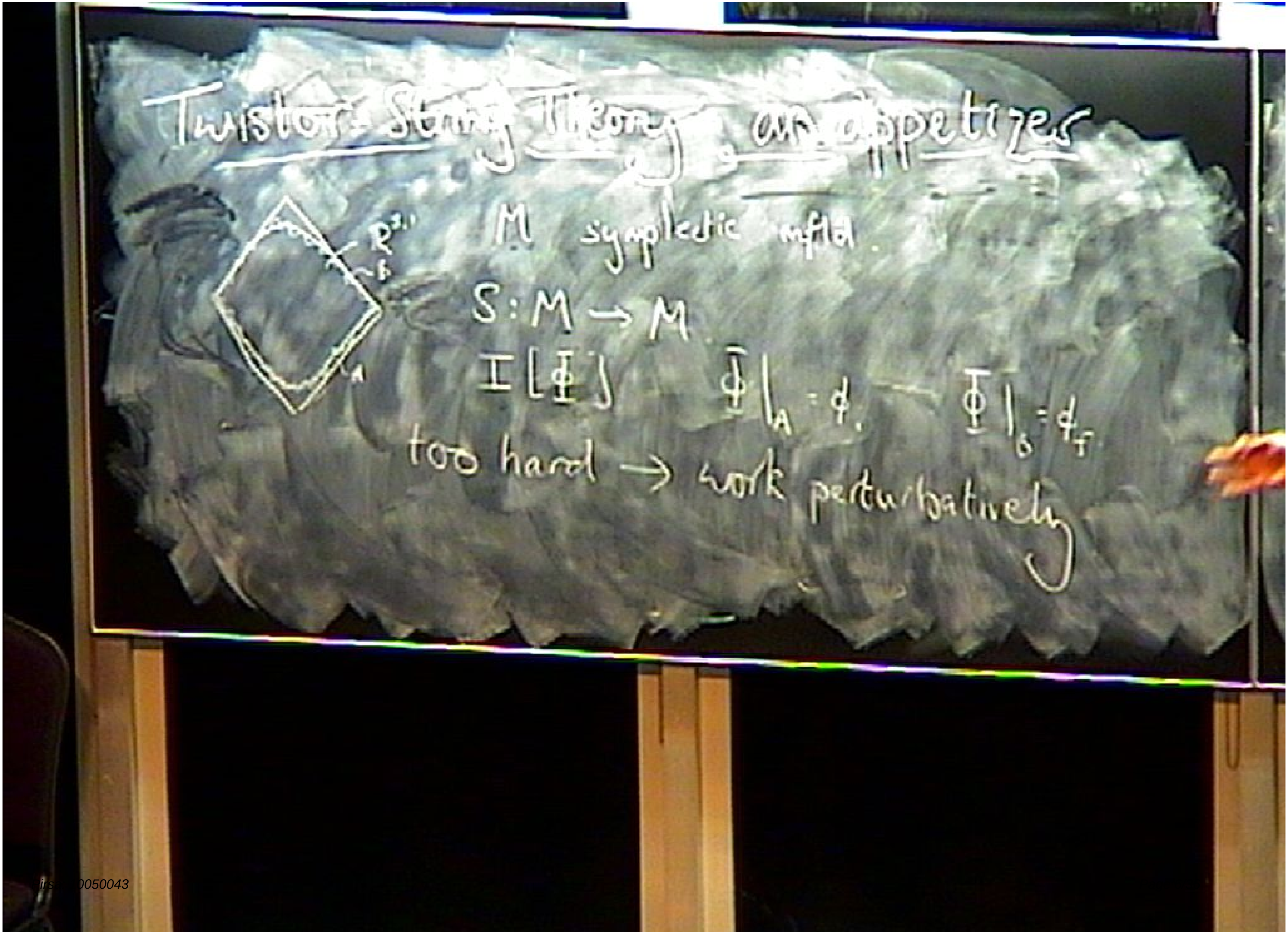

Twistor String Theory
an appetizer

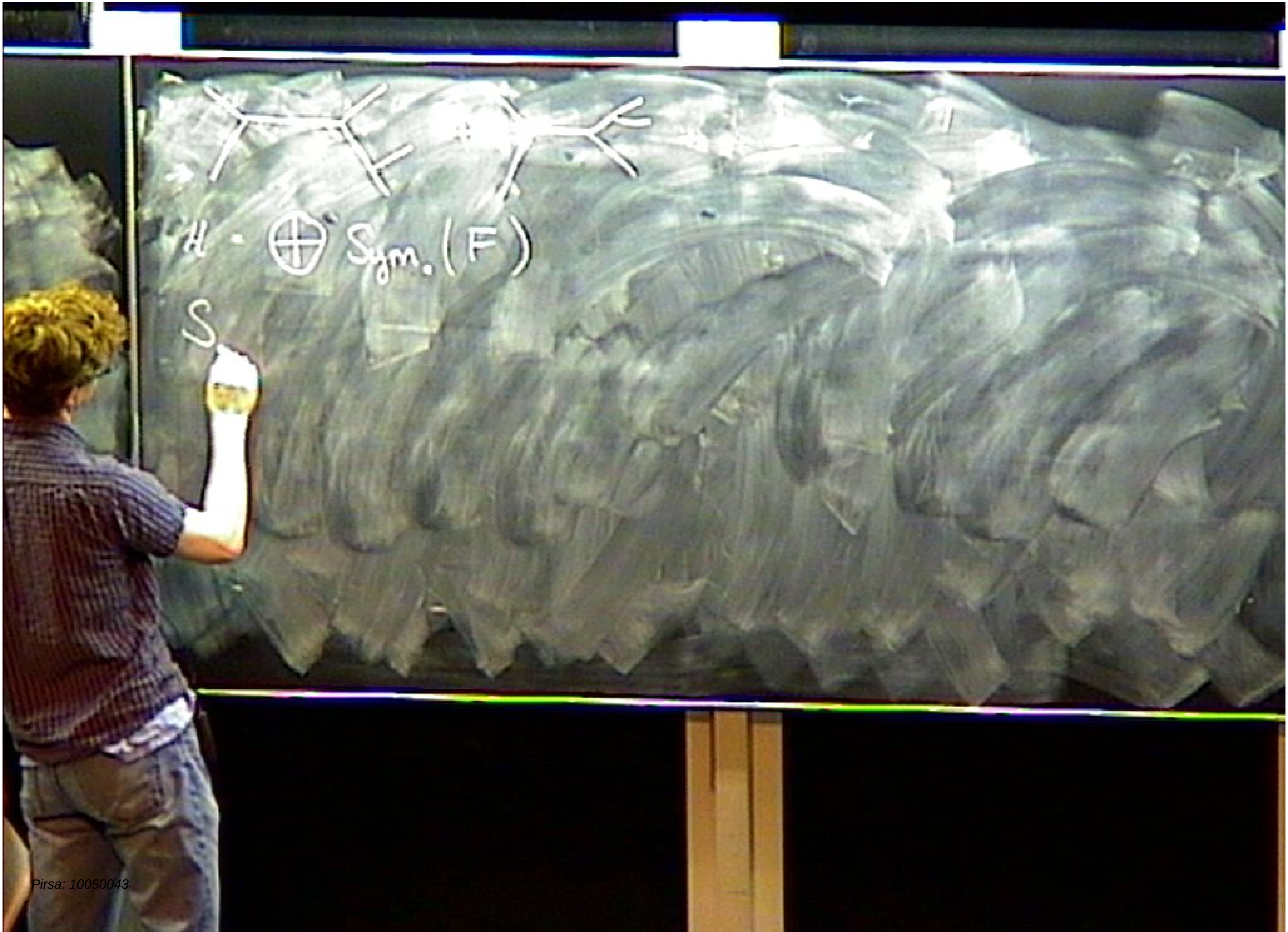


Twistor String Theory: an appetizer
R^{2,1}


Twistor String Theory, an appetizer
M symplectic mfld.
S: M → M.


Twistor String Theory – an appetizer
M symplectic mfld.
S: M → M
I[Φ]
Φ|A = φi
Φ|B = φf


Twistor String Theory: an appetizer
M symplectic mfld.
S: M → M.
I[Φ]  Φ|A = φi  Φ|B = φf
too hard → work perturbatively


H = ⊕ Sym_n (F)
S

$\mathcal{H} = \bigoplus \mathrm{Sym}_\bullet(F)$

$S_{\alpha,\beta} \quad \mathrm{Sym}_{\alpha}(F) \otimes (\mathrm{Sym}_\beta(F))^* \to \mathbb{C}$

– Combinatorically very involved.

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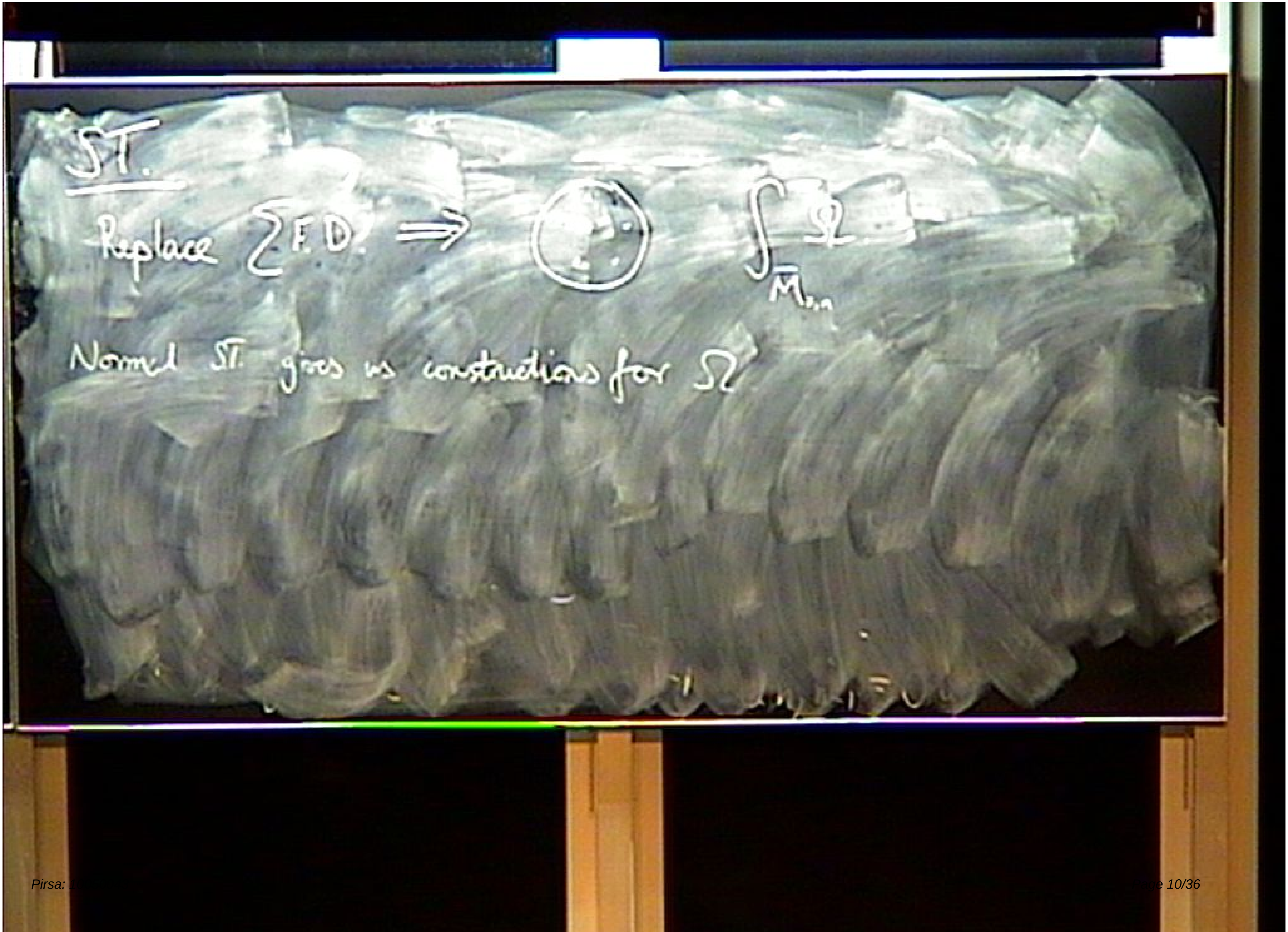
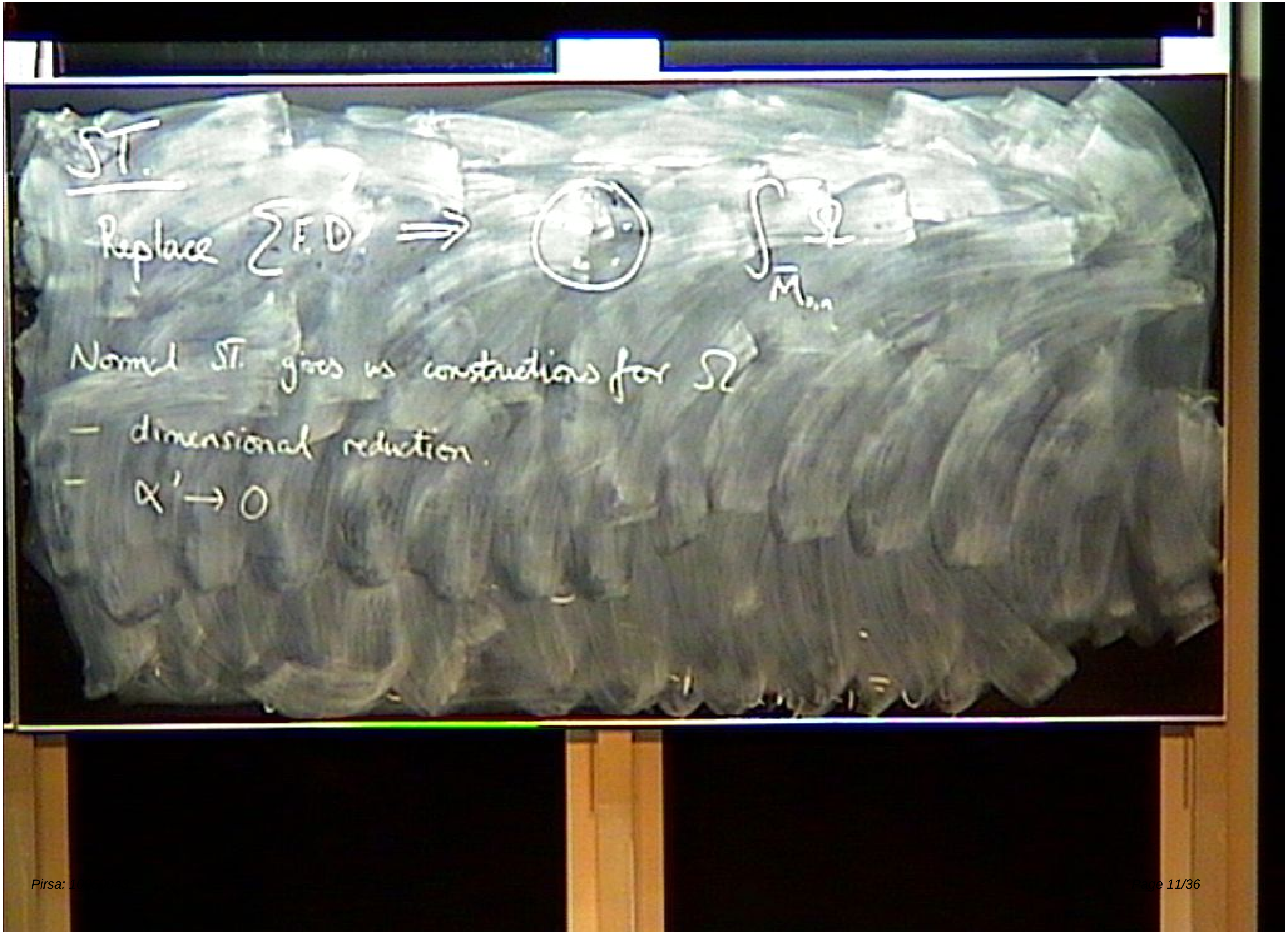
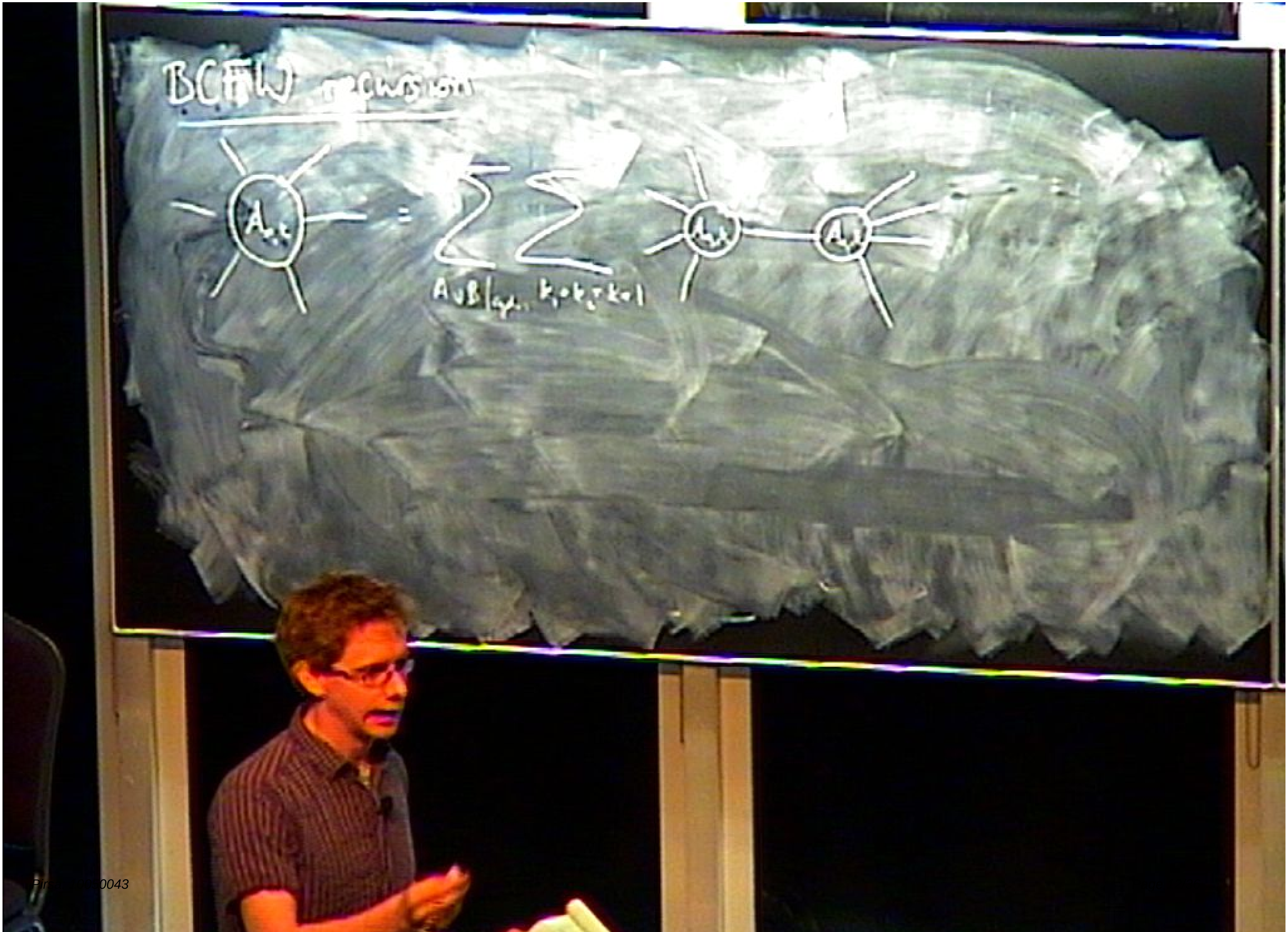
$$\mathcal{H} = \bigoplus_n \mathrm{Sym}_n(F)$$

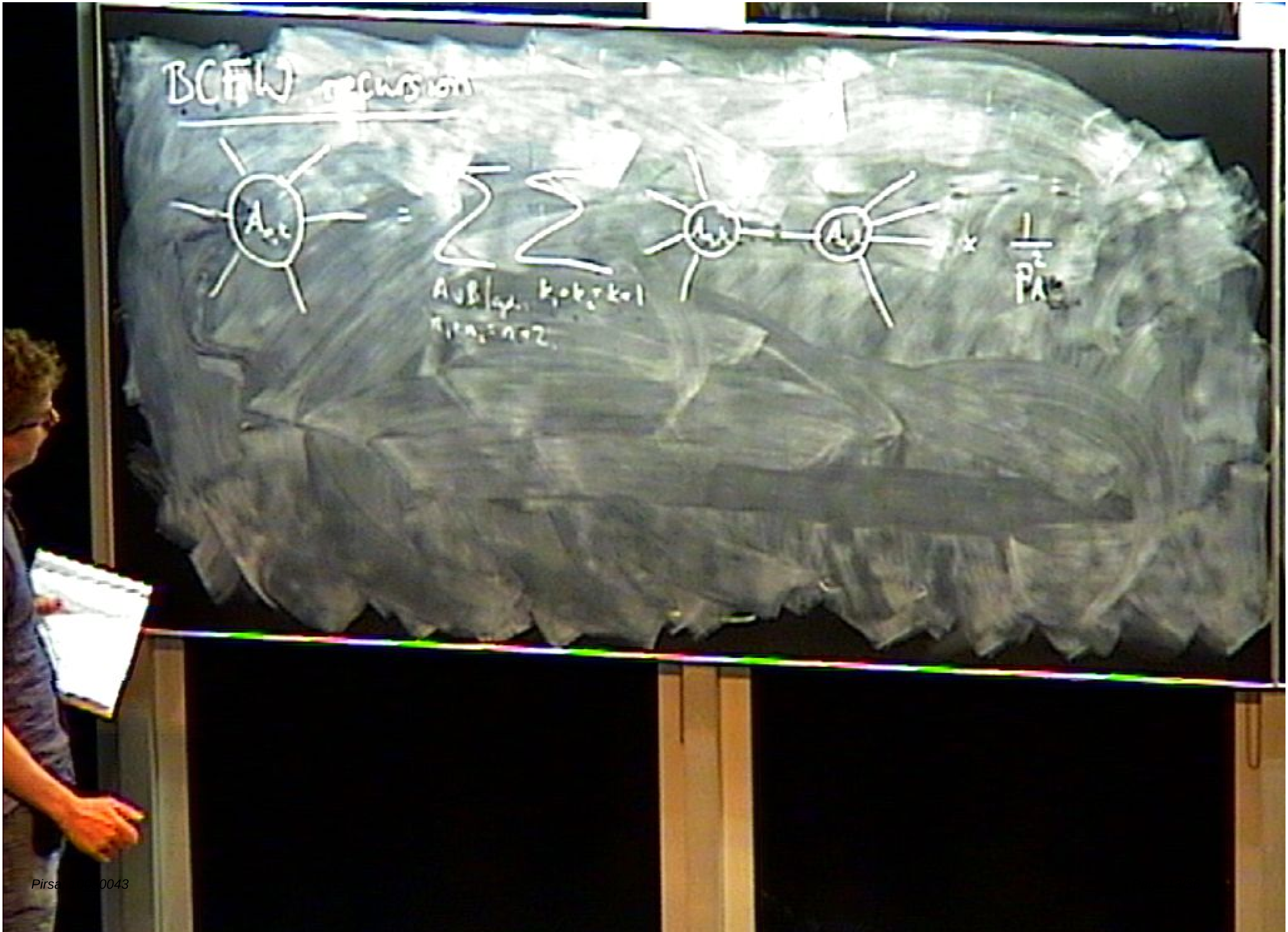
$$S_{n,p} \quad \mathrm{Sym}_n(F) \otimes (\mathrm{Sym}_p(F))^* \to \mathbb{C}$$

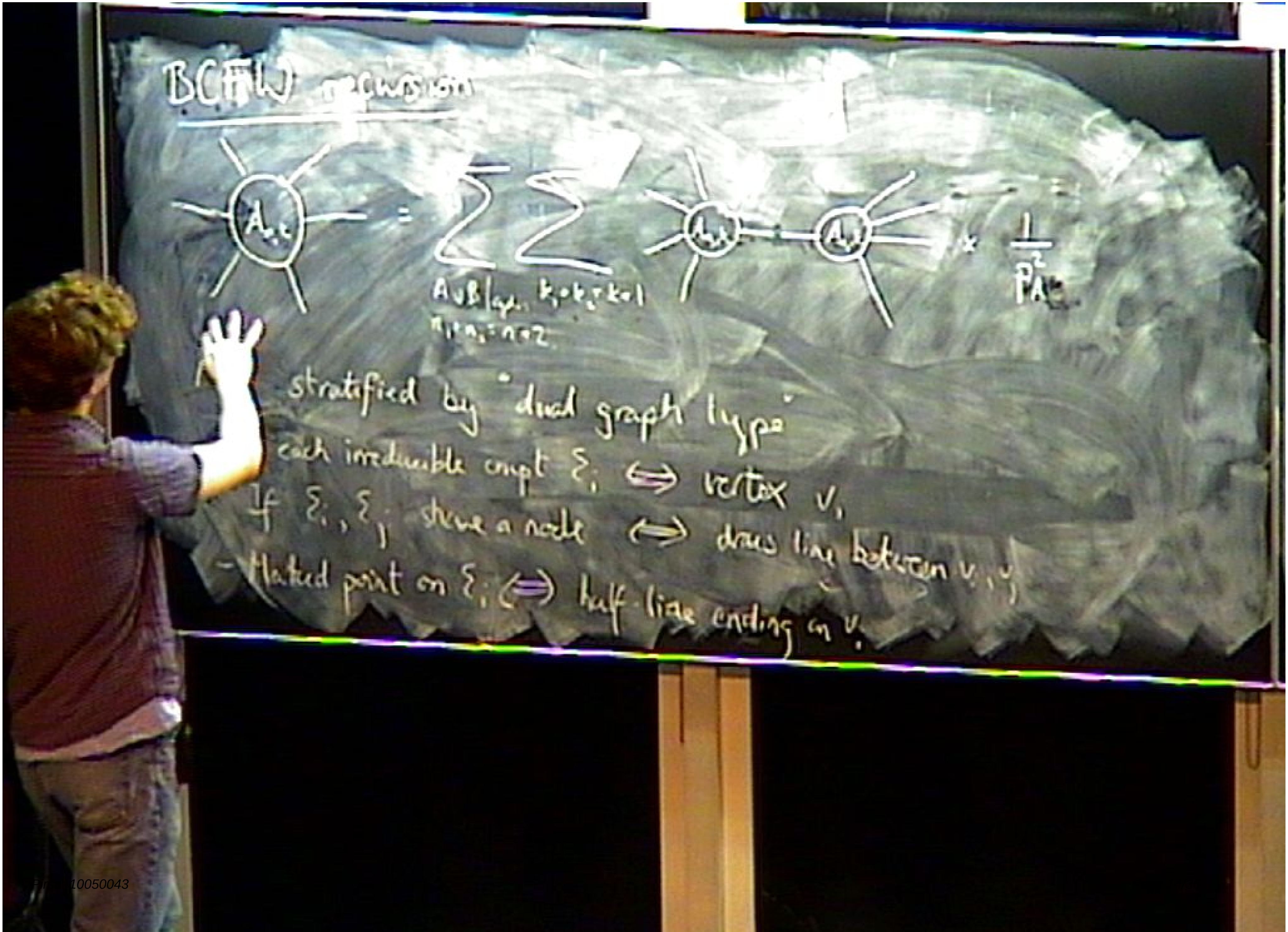
- Combinatorically very involved.
- F.D. obscuring the str. of theory (e.g. gauge invariance off-shell).


S.T.
Replace $\sum$ F.D. $\Longrightarrow$ $\int_{\overline{M}_{g,n}} \Omega$
Normal S.T. gives us constructions for $\Omega$

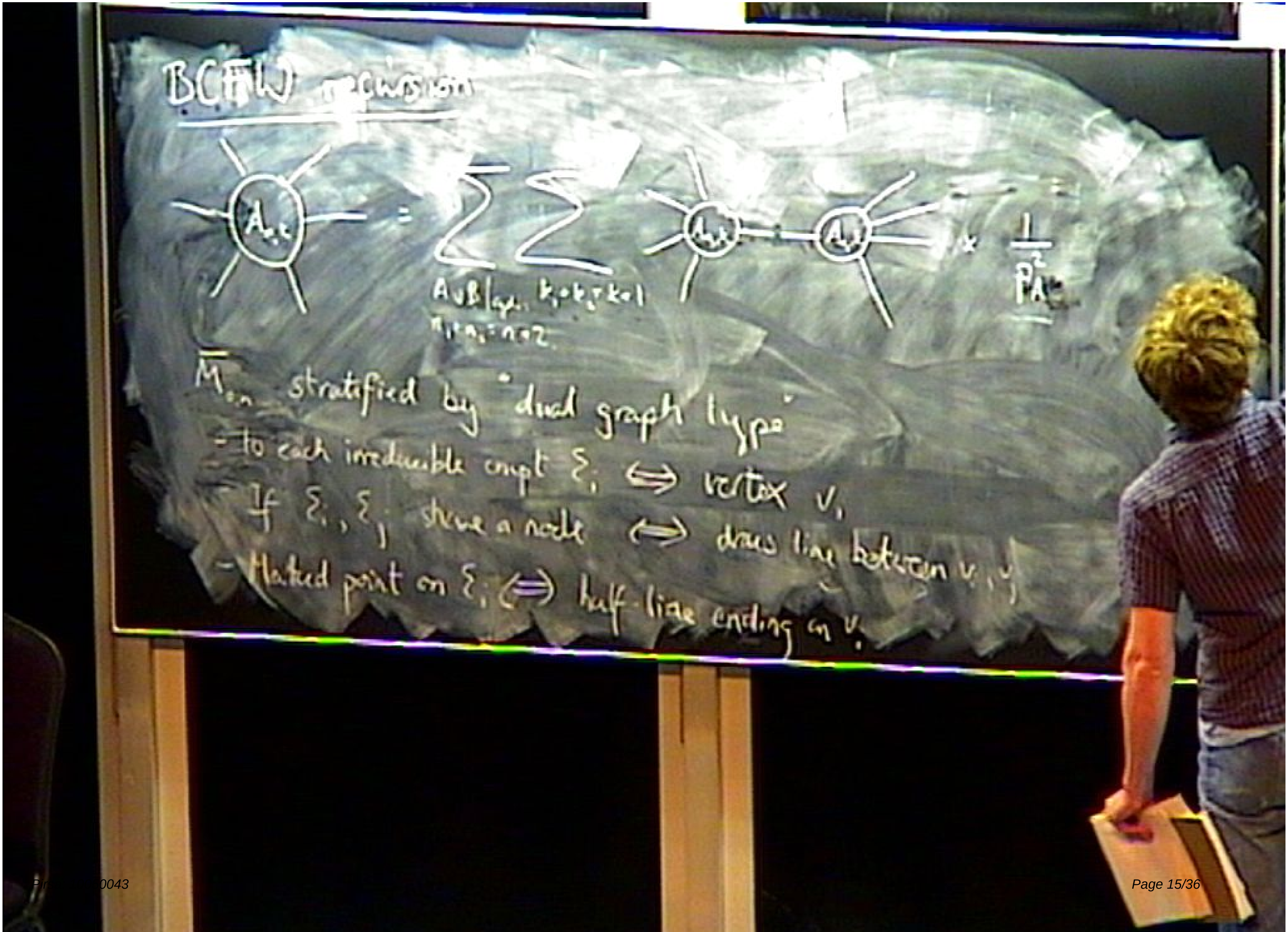

ST.
Replace $\sum$ F.D. $\Longrightarrow$ $\int_{\overline{M}_{g,n}} \Omega$
Normal ST. gives us constructions for $\Omega$
– dimensional reduction.
– $\alpha' \to 0$

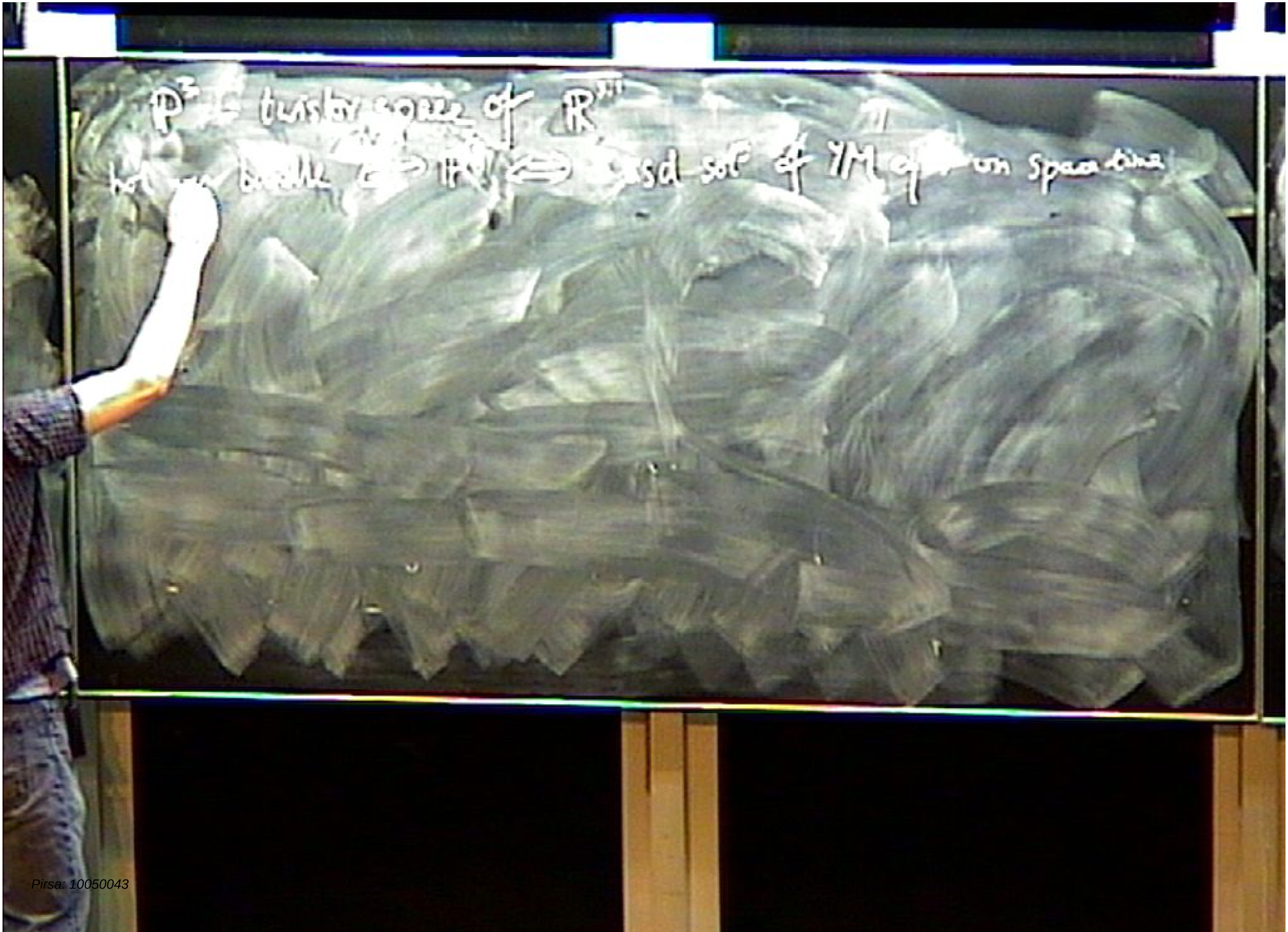

BCFW recursion


BCFW recursion

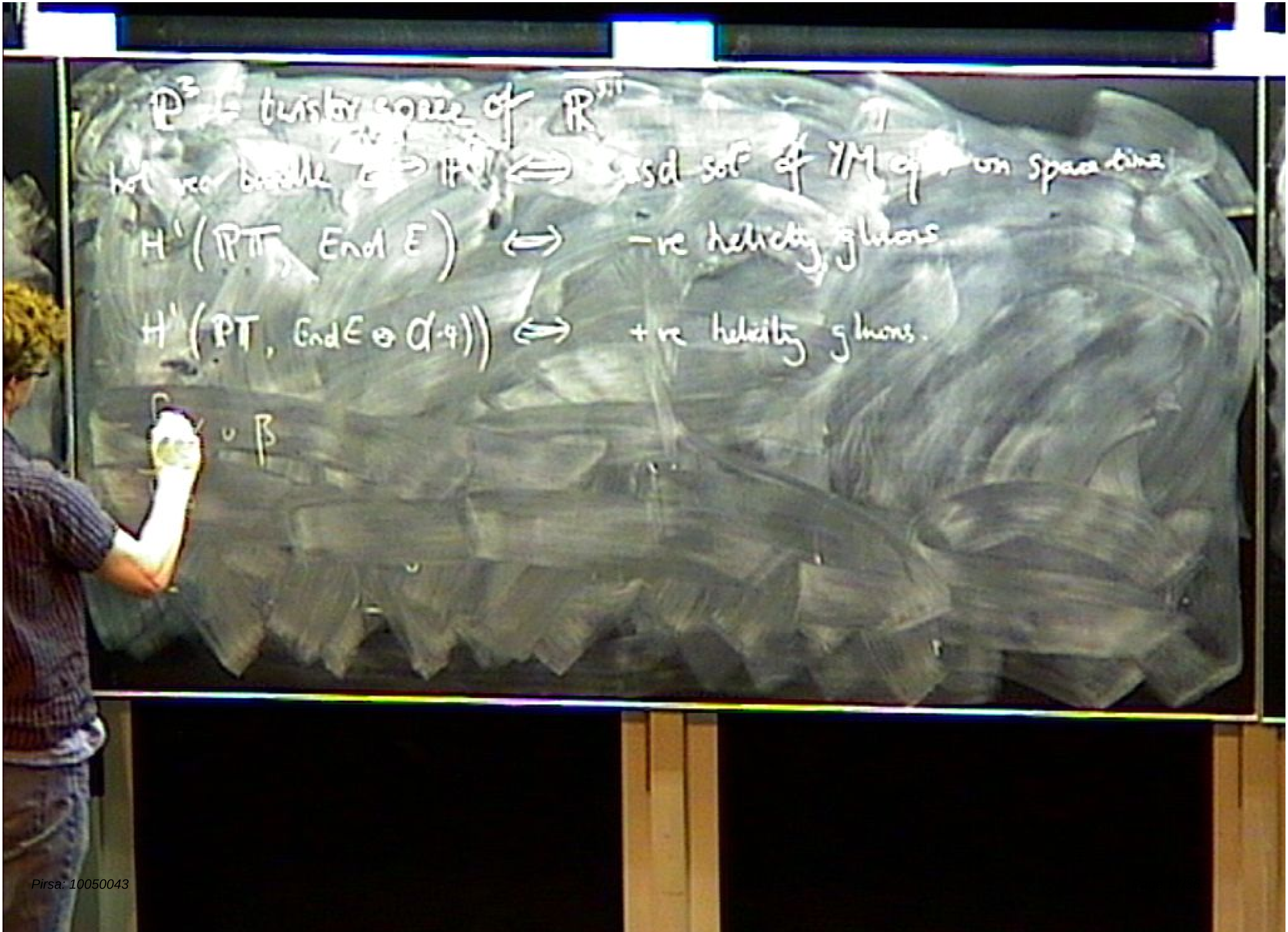


BCFW recursion
A n,k = ΣΣ A n1,k1 A n2,k2 × 1/P_A^2
A∪B|..., k1+k2=k+1
n1+n2=n+2
stratified by "dual graph type"
each irreducible cmpt Σi ⟺ vertex vi
If Σi, Σj share a node ⟺ draw line between vi, vj
marked point on Σi ⟺ half-line ending on vi


BCFW recursion
M̄0,n stratified by "dual graph type"
- to each irreducible cmpt Σi ⟺ vertex vi
- If Σi, Σj share a node ⟺ draw line between vi, vj
- Marked point on Σi ⟺ half-line ending on vi



ℙ³ = twistor space of ℝ
bundle ⟺ sd sol of YM on spacetime


twistor space of
hol vec bundle E → PT ⟺ asd sol of YM eqs on space-time
H'(PT, End E) ⟺ −ve helicity gluons
H'(PT, End E ⊗ O(-4)) ⟺ +ve helicity gluons.
α ∪ β

$\mathbb{P}^3$ = twistor space of $\mathbb{R}^{4}$ $\mathbb{PT}$

hol vec bundle $E \to \mathbb{PT}$ $\Longleftrightarrow$ asd sol$^n$ of YM eq$^n$ on space-time

$H^1(\mathbb{PT}, \mathrm{End}\, E) \Longleftrightarrow$ −ve helicity gluons

$H^1(\mathbb{PT}, \mathrm{End}\, E \otimes \mathcal{O}(-4)) \Longleftrightarrow$ +ve helicity gluons.

$$\int_{\mathbb{PN}} f(\alpha \cup \beta) = \langle + - \rangle$$

## Scattering

$$H^1(\mathbb{PT}, \mathcal{O}) \otimes H^1(\mathbb{PT}, \mathcal{O}) \otimes H^1(\mathbb{PT}, \mathcal{O}(-4)) \xrightarrow{\int} \mathbb{C}$$

+

Obvious generalisation $V = \mathcal{O}(1)^{\oplus 4}$

$$H^1(\mathbb{PT}, \Lambda^{n_1} V^\vee) \otimes H^1(\mathbb{PT}, \Lambda^{n_2} V^\vee) \otimes H^1(\mathbb{PT}, \Lambda^{n_3} V^\vee) \xrightarrow{\int} \mathbb{C}$$

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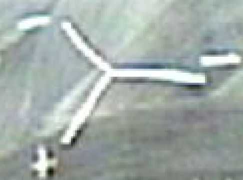
## Scattering

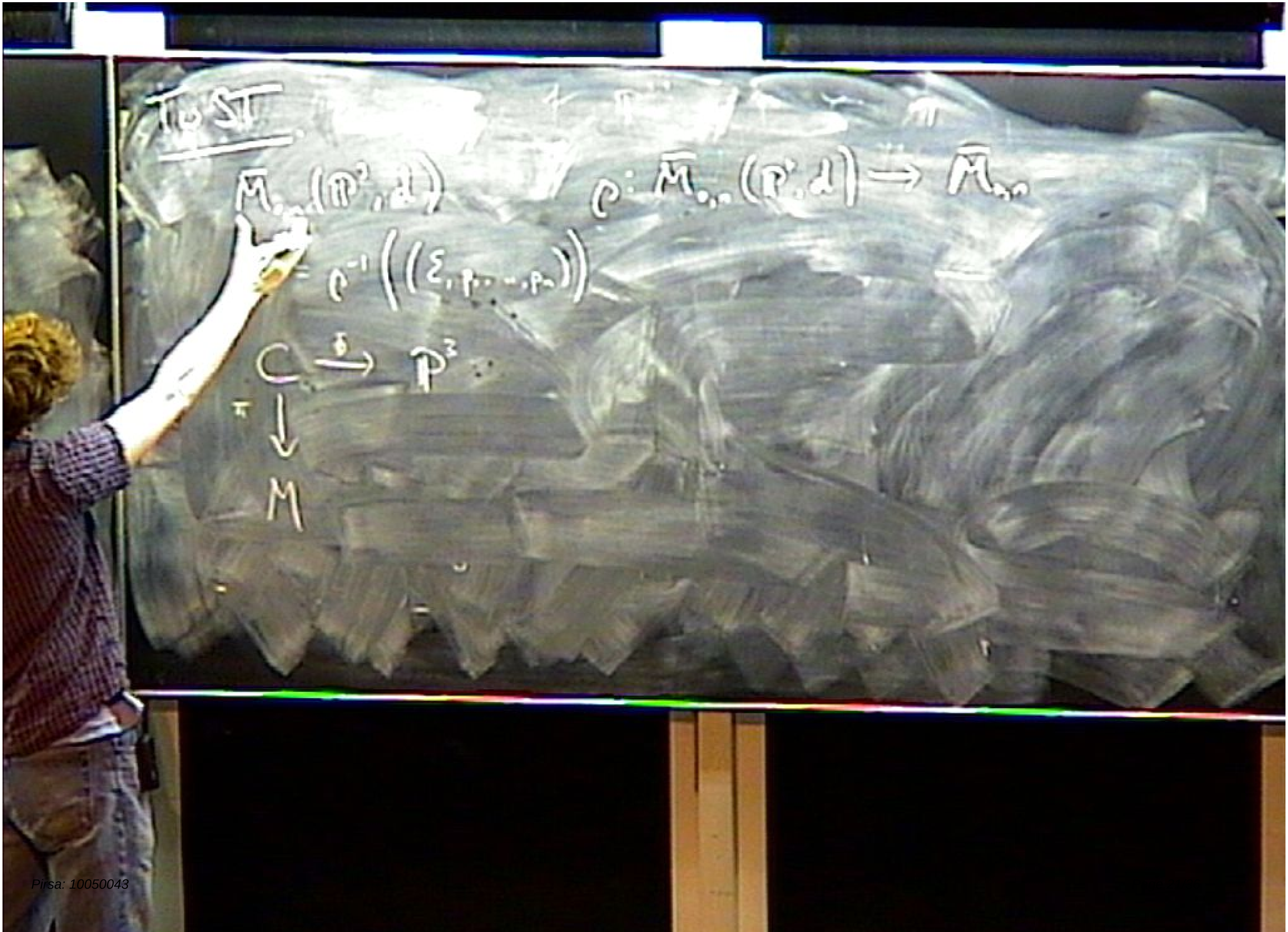
$$H^1(\mathbb{PT}, \mathcal{O}) \otimes H^1(\mathbb{PT}, \mathcal{O}) \otimes H^1(\mathbb{PT}, \mathcal{O}(-4)) \xrightarrow{\int} \mathbb{C}$$




Obvious generalisation $\quad V = \mathcal{O}(1)^{\oplus 4} \qquad \Lambda^{\mathrm{top}} V^\vee \simeq K_{\mathbb{PT}}$

$$H^1(\mathbb{PT}, \Lambda^{q_1} V^\vee) \otimes H^1(\mathbb{PT}, \Lambda^{q_2} V^\vee) \otimes H^1(\mathbb{PT}, \Lambda^{q_3} V^\vee) \xrightarrow{\int} \mathbb{C}$$

$$\sum q_i = \mathrm{rk}\, V$$

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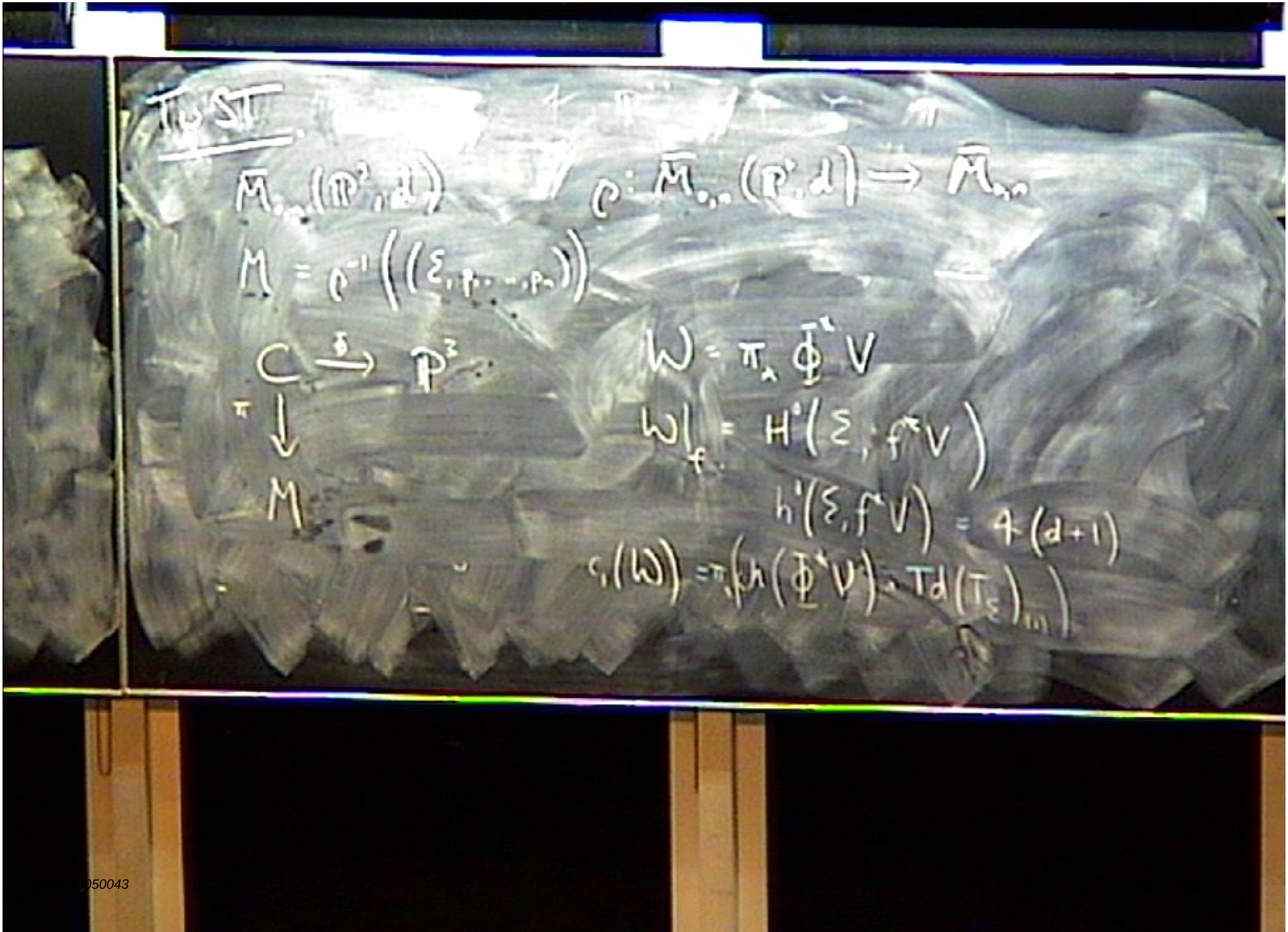

$\overline{M}_{0,n}(\mathbb{P}^3, d)$
$\rho : \overline{M}_{0,n}(\mathbb{P}^3, d) \to \overline{M}_{0,n}$
$M = \rho^{-1}((\Sigma, p_1, \dots, p_n))$
$C \xrightarrow{\Phi} \mathbb{P}^3$
$\pi \downarrow$
$M$
$W = \pi_* \Phi^* V$
$W|_f = H^0(\Sigma, f^* V)$
$h^0(\Sigma, f^* V) = 4(d+1)$
$c_1(W) = \pi_*(ch(\Phi^* V) \cdot Td(T_\pi))_{(1)}$

## TWIST

$\overline{M}_{0,n}(\mathbb{P}^3, d)$ $\quad \rho : \overline{M}_{0,n}(\mathbb{P}^3, d) \to \overline{M}_{0,n}$

$M = \rho^{-1}\left((\Sigma, p_1, \dots, p_n)\right)$

$$\begin{array}{ccc} C & \xrightarrow{\Phi} & \mathbb{P}^3 \\ \pi \downarrow & & \\ M & & \end{array}$$

$W = \pi_* \Phi^* V$

$W|_f = H^0(\Sigma, f^* V)$

$h^0(\Sigma, f^* V) = 4 \cdot (d+1)$

$c_1(W) = \pi_* \{ ch(\Phi^* V) \cdot Td(T_\Sigma) \}_{(1)} = c_1(T_M)$

$\Lambda^{top} W^\vee = K_M$

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## TWIST

$\overline{M}_{0,n}(\mathbb{P}^3, d)$ $\qquad \rho : \overline{M}_{0,n}(\mathbb{P}^3, d) \to \overline{M}_{0,n}$

$M = \rho^{-1}\big((\Sigma, p_1, \dots, p_n)\big)$

$$\begin{array}{ccc} C & \xrightarrow{\Phi} & \mathbb{P}^3 \\ {\scriptstyle \pi}\downarrow & & \\ M & & \end{array}$$

$W = \pi_* \Phi^* V$

$W|_f = H^0(\Sigma, f^* V)$

$h^0(\Sigma, f^* V) = 4(d+1)$

$c_1(W) = \pi_*\big(ch(\Phi^* V) \cdot Td(T_\Sigma)\big)_{(1)} = c_1(T_M)$

$\Lambda^{top} W^\vee = K_M$

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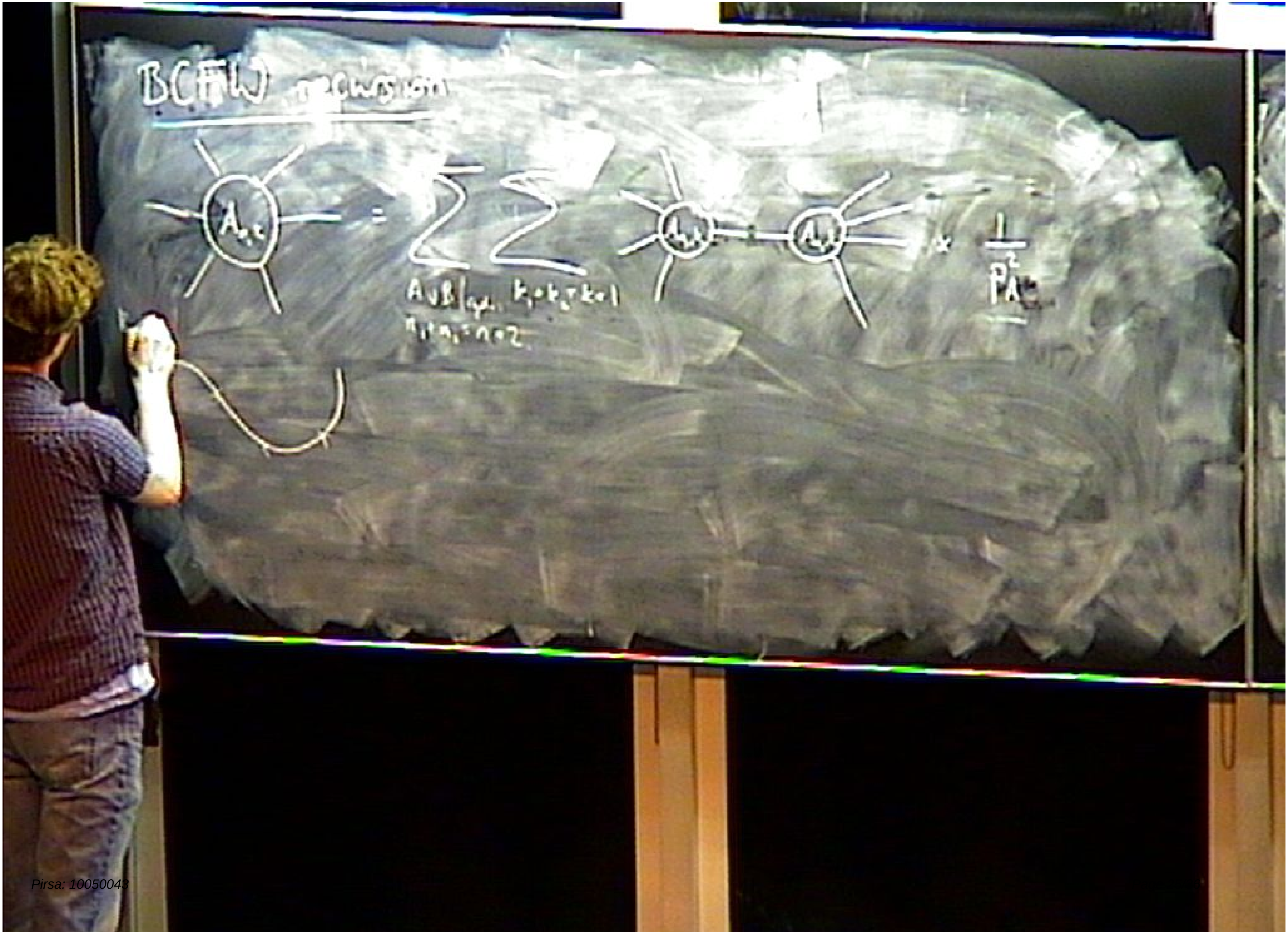


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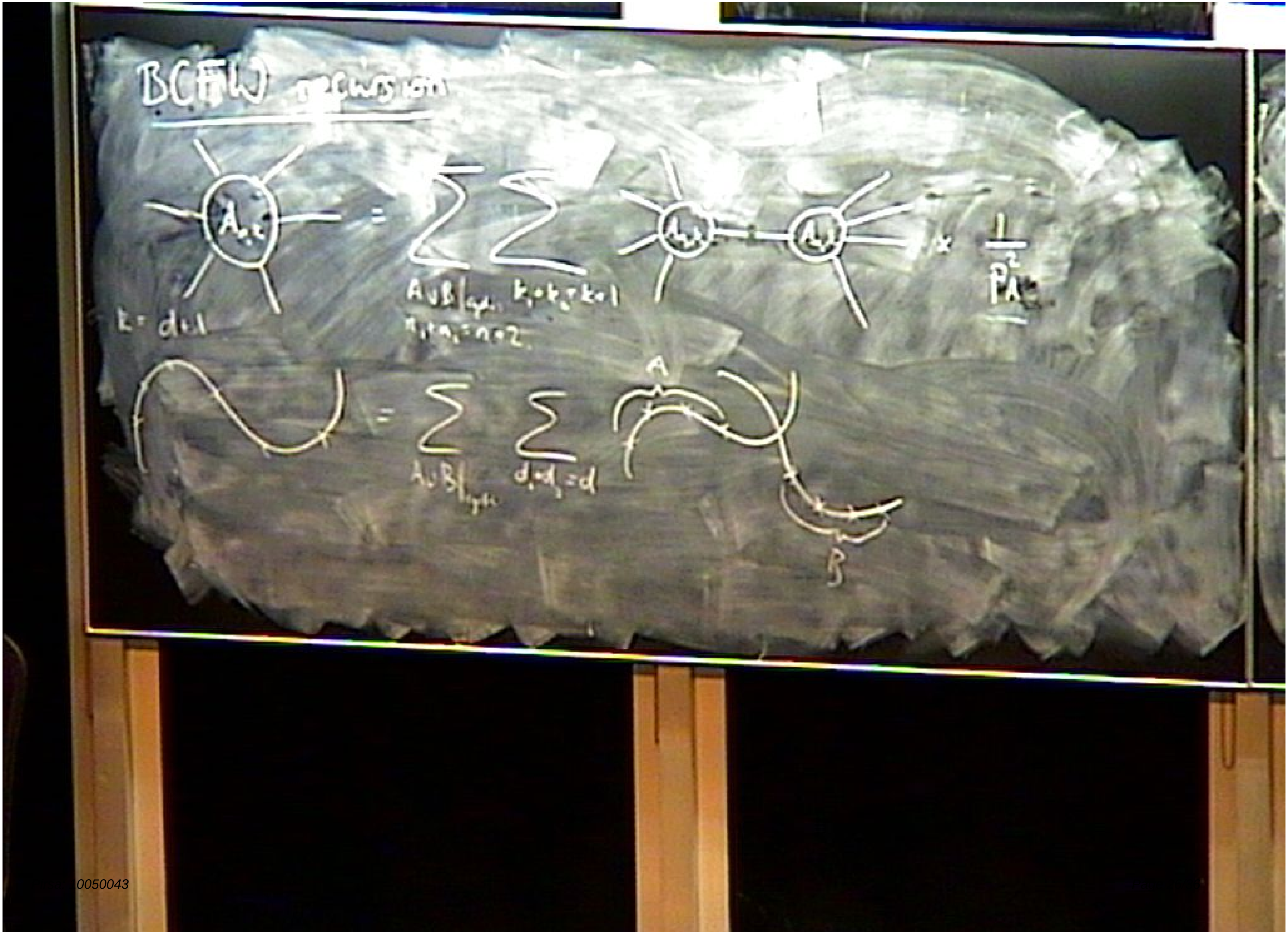
BCFW recursion
k = d+1
A∪B|cyc.
k1+k2=k+1
n1+n2=n+2
A∪B|cyc.
d1+d2=d
A
B
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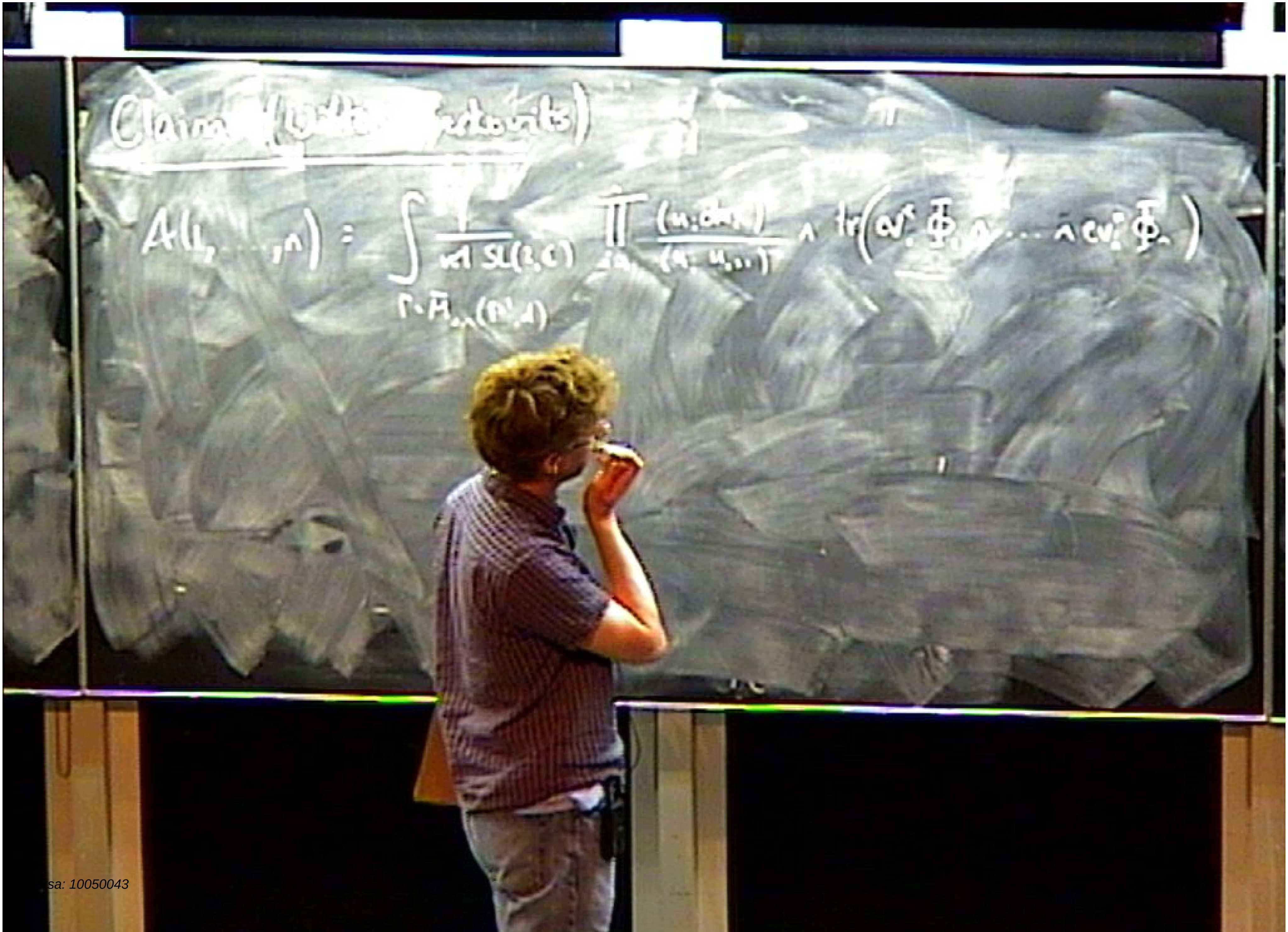
Claim
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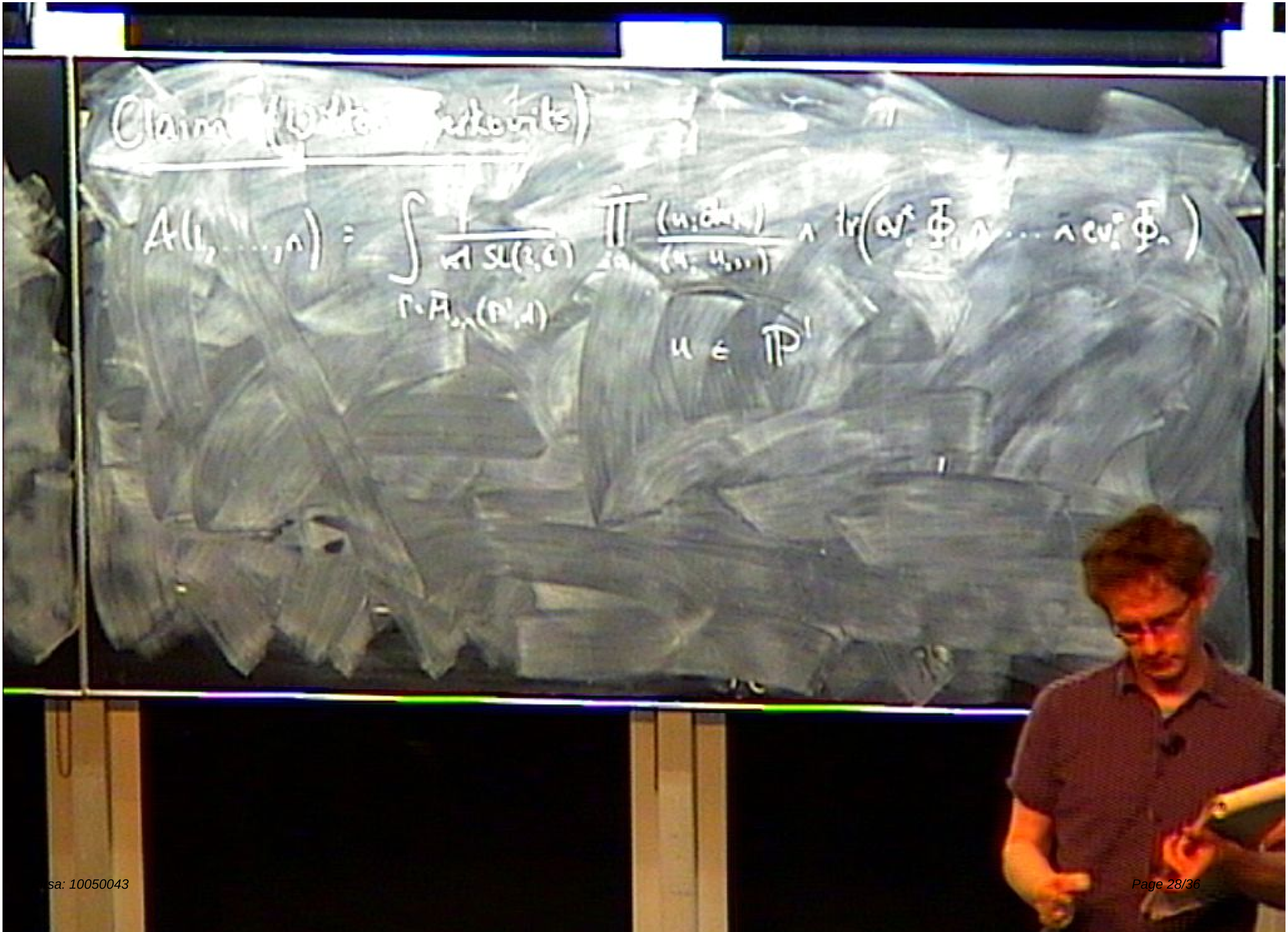
Claim
A(1,...,n) = ∫ 1/vol SL(2,C) ∏ (u_i du_i)/(u_i u_{i+1}) ∧ tr(ω_1^* Φ_1 ∧ ... ∧ ω_n^* Φ_n)
Γ ⊂ M_{0,n}(P^1,d)
u ∈ P^1

## Claim ([illegible] [illegible])

$$A(1,\ldots,n) = \int_{\Gamma \subset \overline{M}_{0,n}(\mathbb{P}^1,d)} \frac{1}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}\left(ev_1^* \Phi_1 \wedge \ldots \wedge ev_n^* \Phi_n\right)$$

$$u \in \mathbb{P}^1$$

- No general proof, but holds in many cases.
- Better formulation?

Claim ([illegible])

$$A(1,\dots,n) = \int_{\Gamma \subset \mathcal{M}_{0,n}(\mathbb{P}^1,d)} \frac{1}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}\left(\omega_1^a \Phi_1 \wedge \dots \wedge \omega_n^a \Phi_n\right)$$

$$u^a = \begin{pmatrix} u_i \\ u_i' \end{pmatrix} \in \mathbb{P}^1 \qquad (u_i\, u_{i+1}) = \epsilon_{ab}\, u_i^a\, u_{i+1}^b$$

- No general proof, but holds in many cases.
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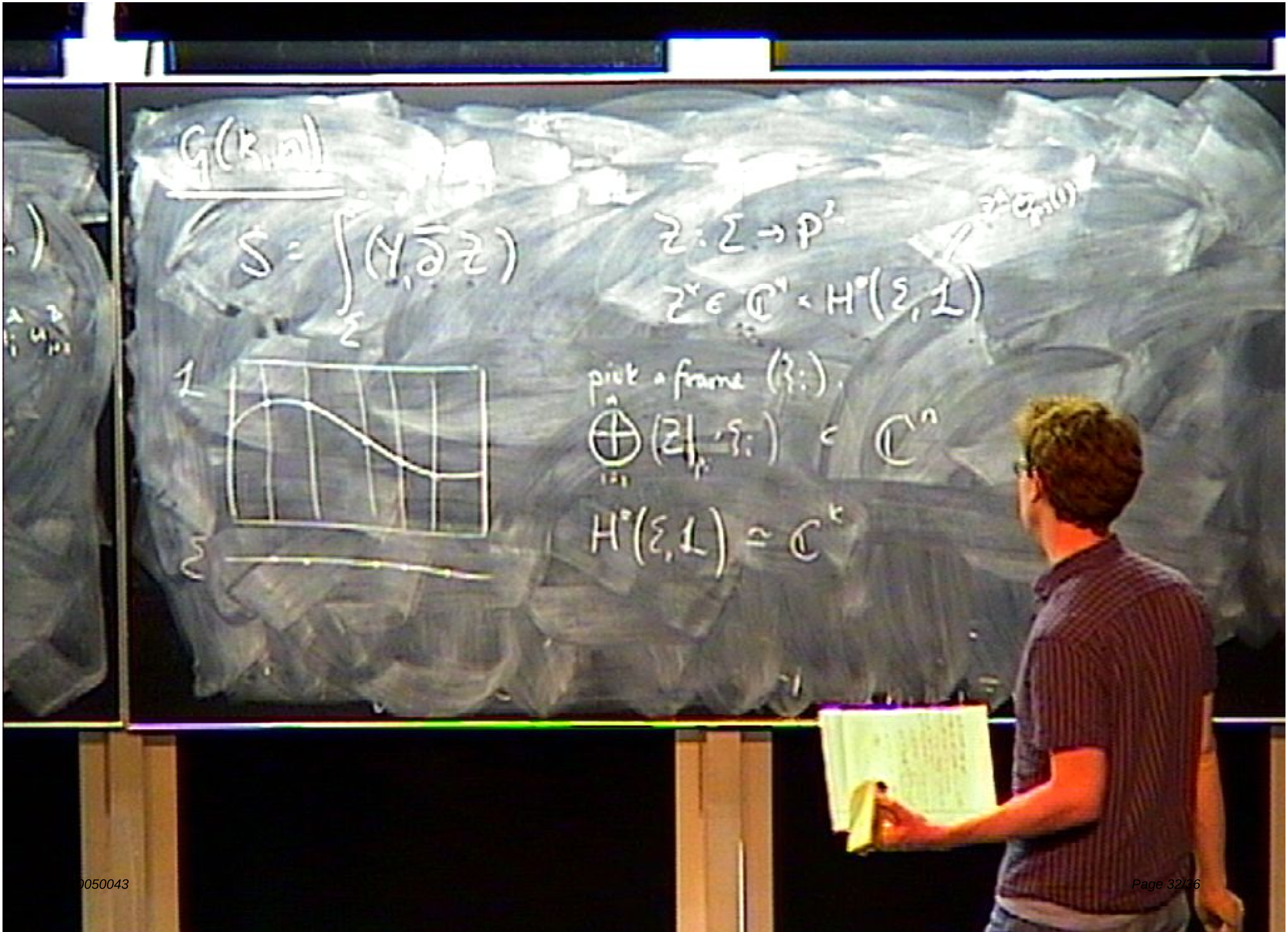
## Claim ([illegible])

$$A(1,\ldots,n) = \int_{\Gamma \subset \overline{M}_{0,n}(\mathbb{P}^1,d)} \frac{1}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}\left(\omega_1^* \Phi_1 \wedge \ldots \wedge \omega_n^* \Phi_n\right)$$

$$u^{\alpha} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \in \mathbb{P}^1 \qquad (u_i\, u_{i+1}) = \epsilon_{\alpha\beta}\, u_i^{\alpha} u_{i+1}^{\beta}$$

- No general proof, but holds in many cases.
- Better formulation?


G(k,n)
S = ∫ (Y, ∂̄Z)
Σ
pick a frame
H°(Σ, L) ≅ C^k

## $G(k,n)$

$$S = \int_\Sigma (Y, \bar\partial Z)$$

$Z : \Sigma \to \mathbb{P}^{?}$ $\quad Z^*\mathcal{O}_{\mathbb{P}}(1)$

$Z^k \in \mathbb{C}^n \otimes H^0(\Sigma, \mathcal{L})$

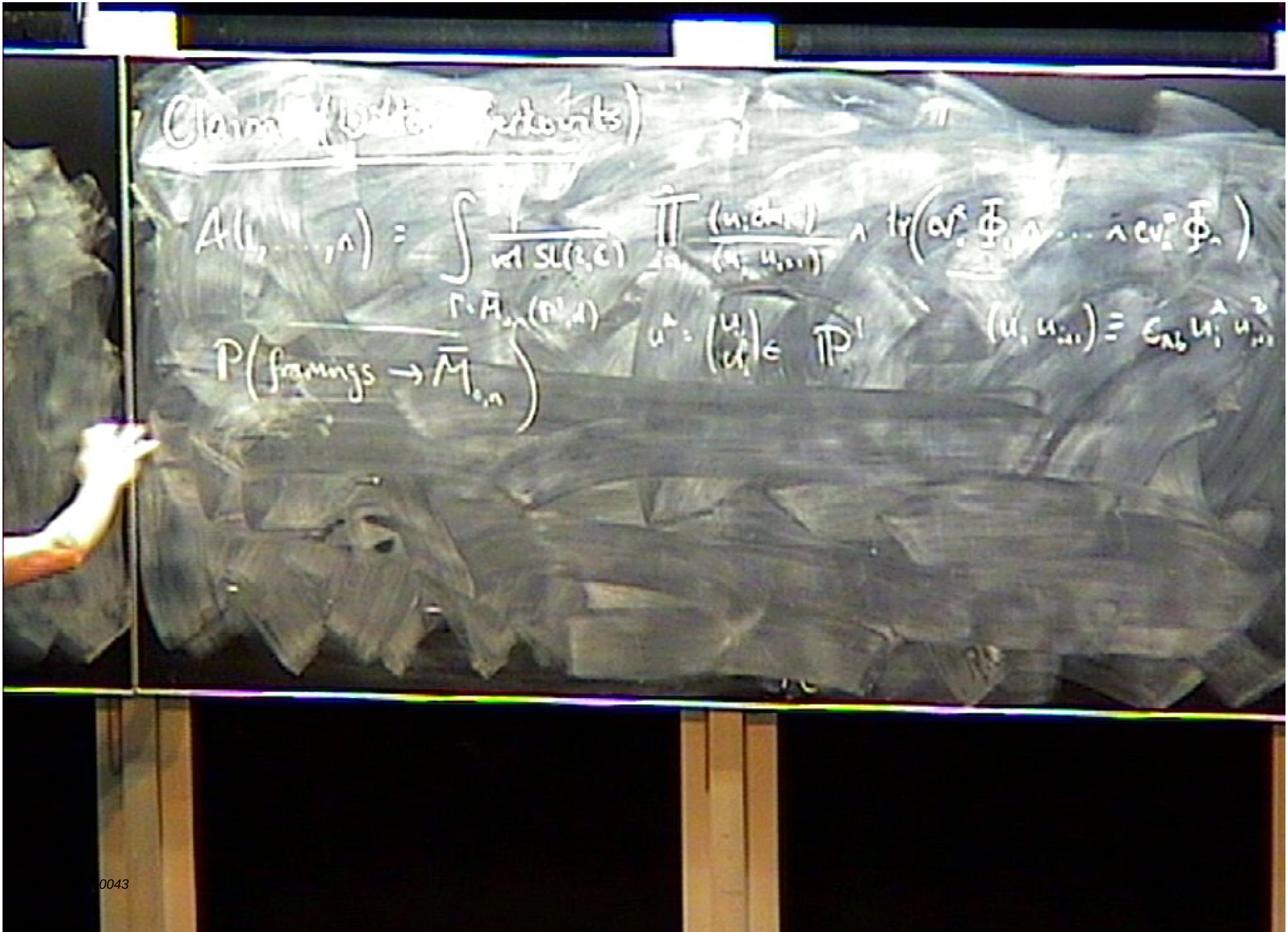

pick a frame $(\xi_i)$.

$$\bigoplus_{i=1}^{n} (Z|_{p_i}, \xi_i) \in \mathbb{C}^n$$

$$H^0(\Sigma, \mathcal{L}) \simeq \mathbb{C}^k$$

For each $(\Sigma, p_1, \ldots, p_n) \in \bar{M}_{0,n}$ (together with frame)

get $C \in G(k,n)$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A(1, ..., n) = ∫ 1/vol SL(2,C) ∏ (u_i du_i)/(u_i u_{i+1}) ∧ tr(ev_1* Φ_1 ∧ ... ∧ ev_n* Φ_n)
Γ ⊂ M̄_{0,n}(P^1,d)
P(framings → M̄_{0,n})
u^a = (u_0, u_1) ∈ P^1
(u_i u_{i+1}) = ε_{ab} u_i^a u_{i+1}^b
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Claim (Witten, Berkovits)

$$A(1,\dots,n) = \int_{\Gamma \subset \overline{M}_{0,n}(\mathbb{P}^3,d)} \frac{1}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}\left(\mathrm{ev}_1^*\Phi_1 \wedge \cdots \wedge \mathrm{ev}_n^*\Phi_n\right)$$

$u^a = \begin{pmatrix} u^1 \\ u^2 \end{pmatrix} \in \mathbb{P}^1$ $\qquad (u_i\, u_{i+1}) = \epsilon_{ab}\, u_i^a u_{i+1}^b$

$$\mathbb{P}\left(\text{framings} \to \overline{M}_{0,n}\right)$$

$$n-3+n-1 = 2n-4$$

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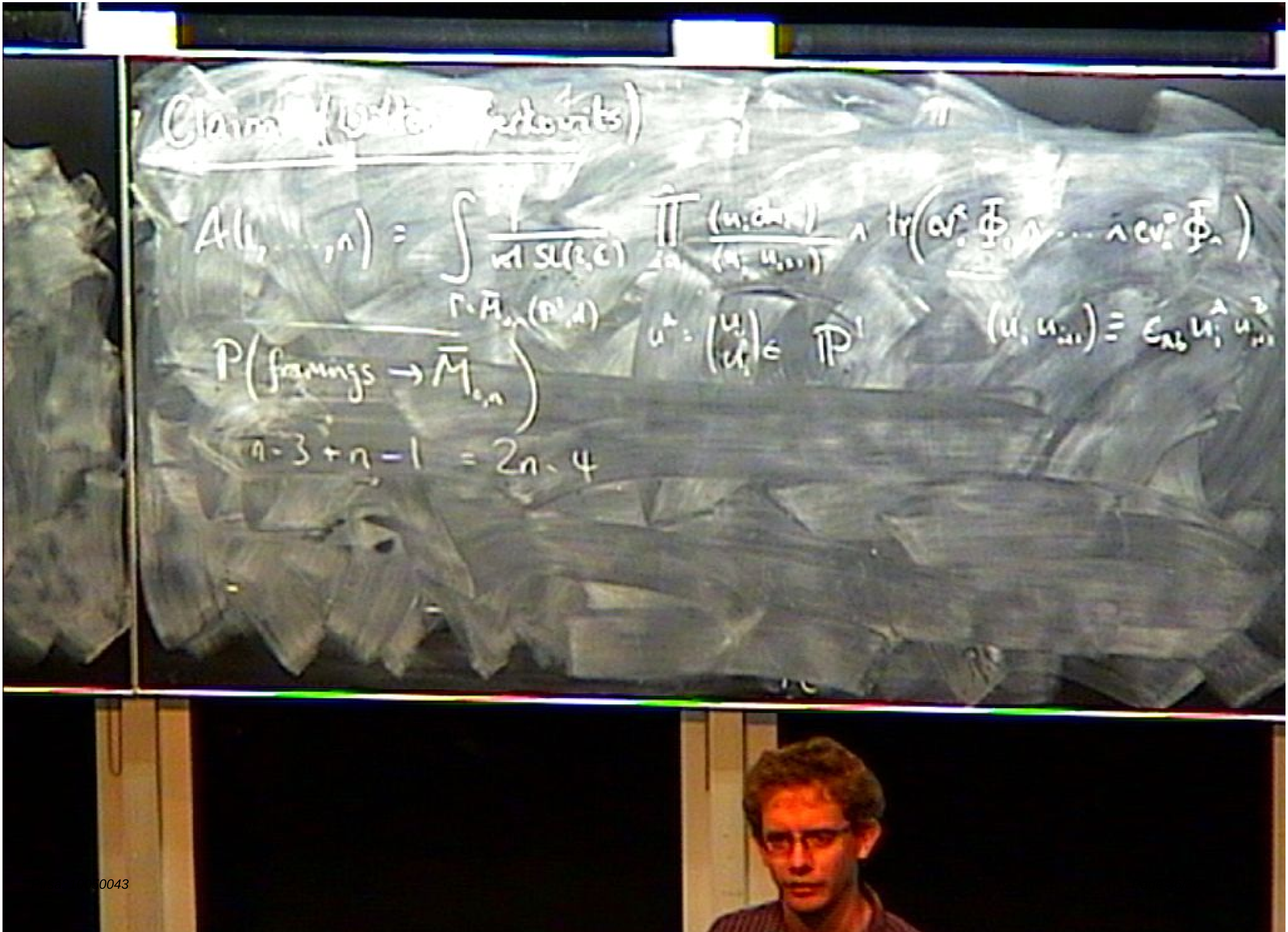
P(framings → M̄₀,ₙ)
n-3 + n-1 = 2n-4
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