Title: Twistor-String Theory

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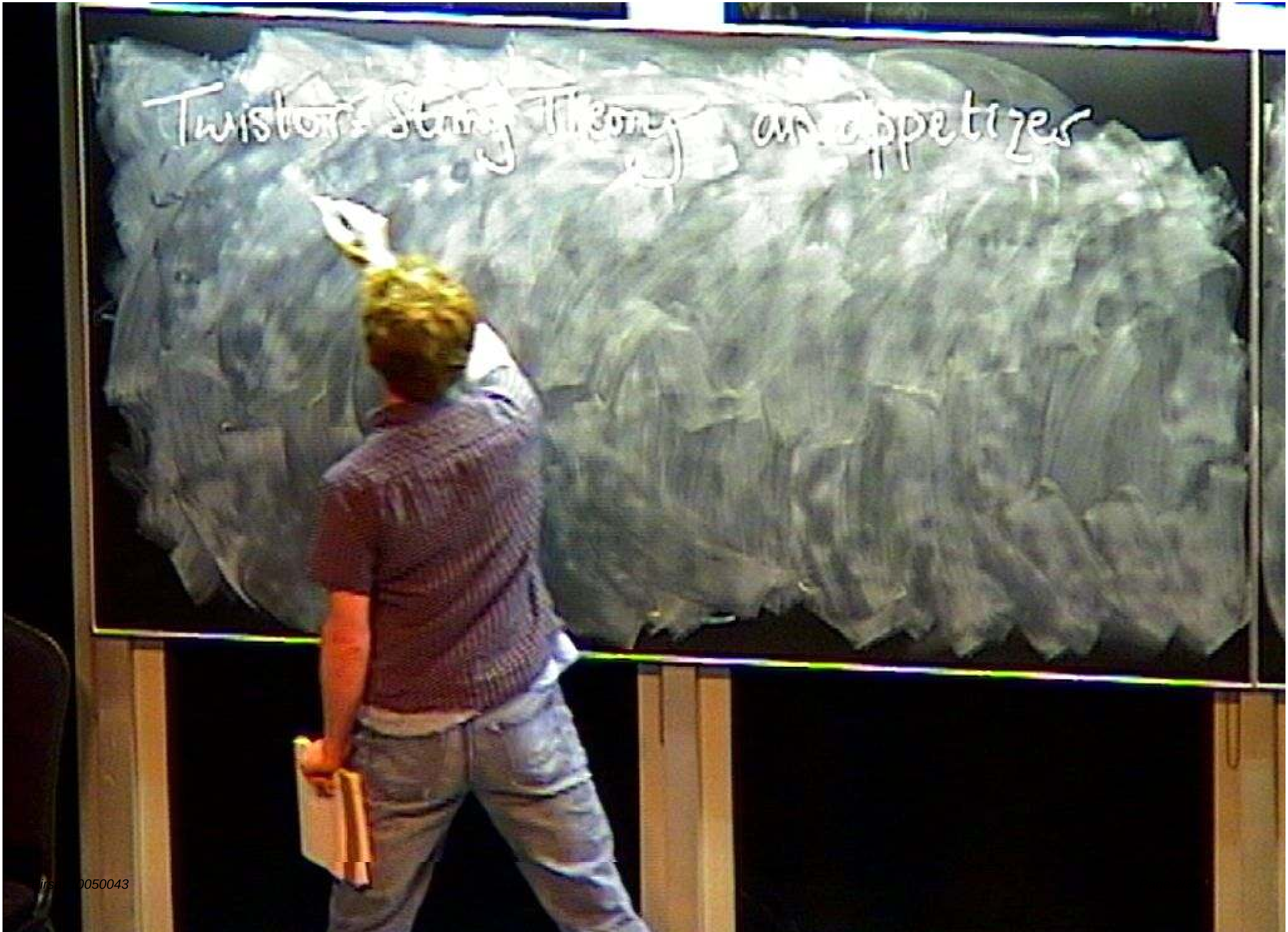
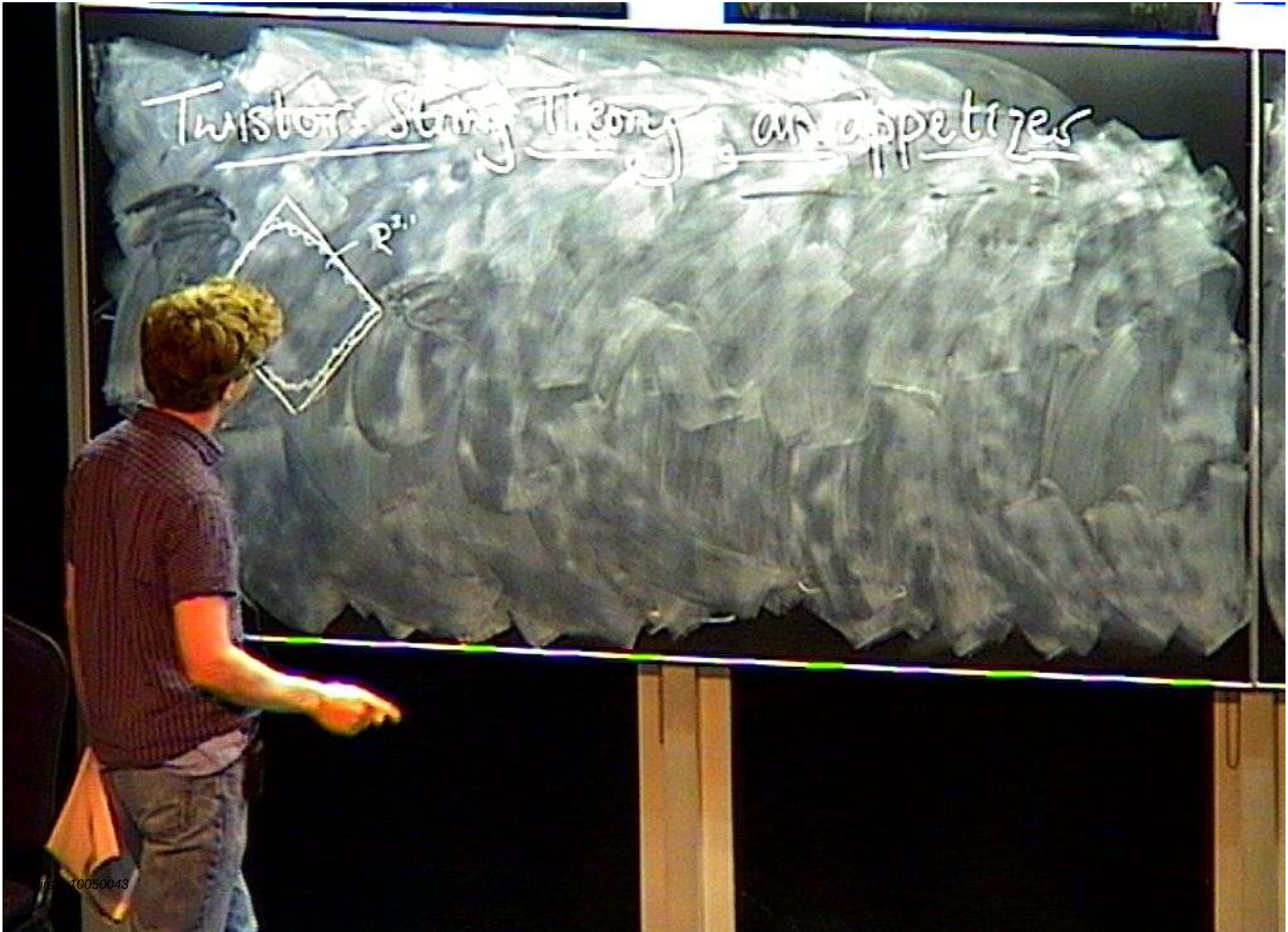
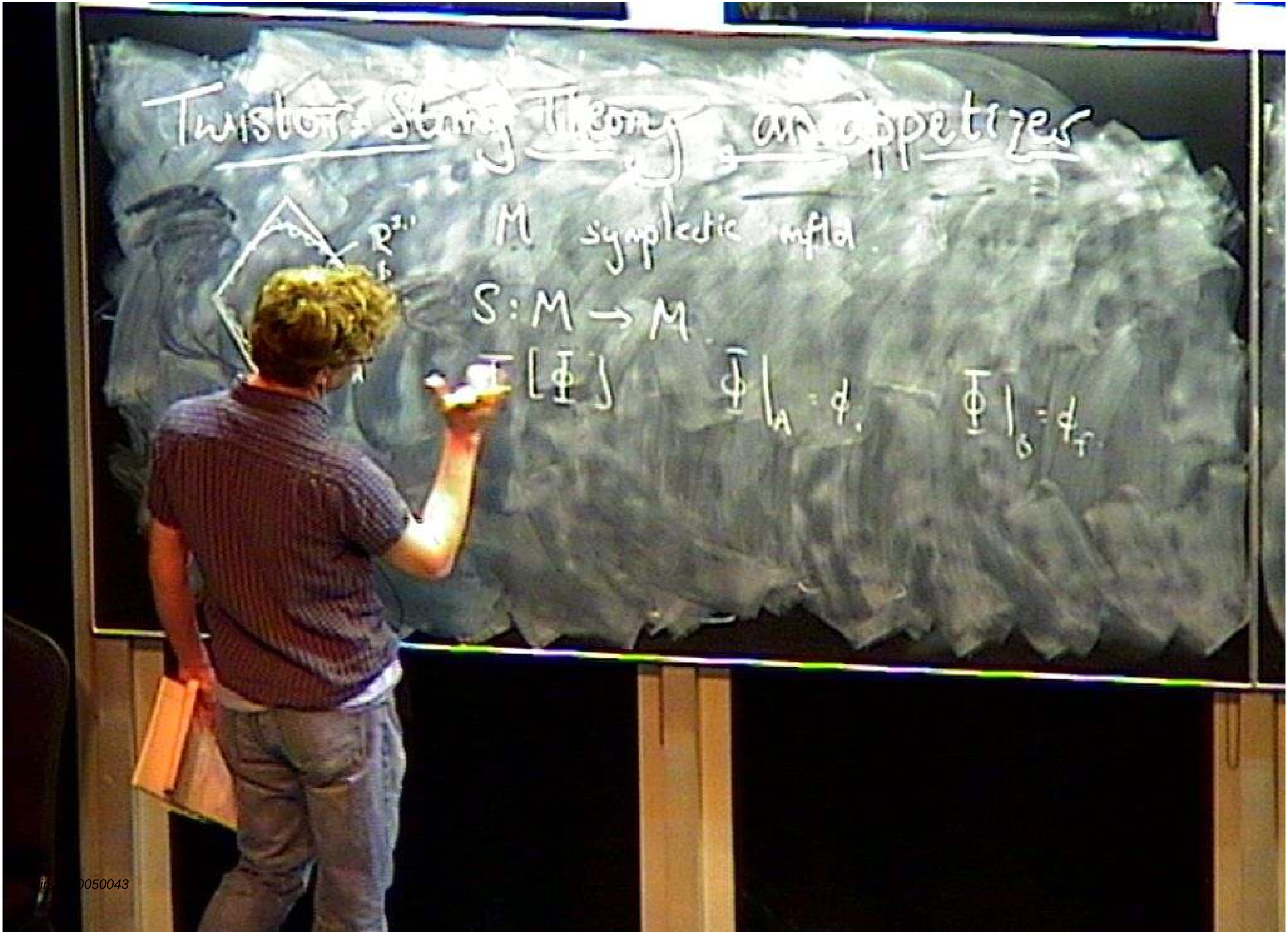
Abstract: I'll give an introduction to twistor-string theory, which is an attempt to reformulate supersymmetric gauge theory in four-dimensional space-time in terms of a certain generalisation of Gromov-Witten theory in twistor space. The resulting theory is closely related to the multi-dimensional residue calculus in G(k,n) (introduced in Cachazo's talk).

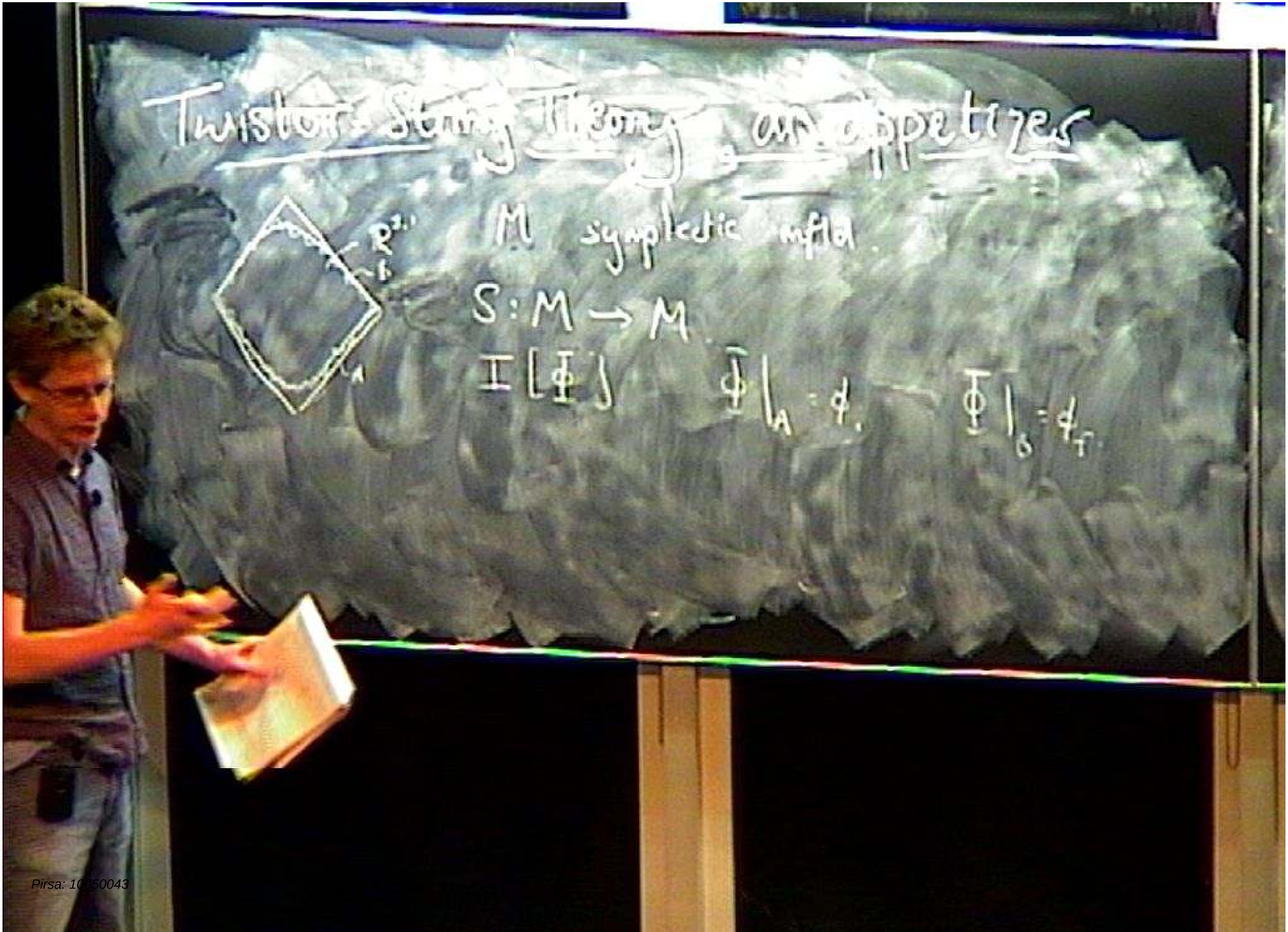


Twistor String Theory — an appetizer



Twistor String Theory, an appetizer
$R^{2,1}$


Twistor String Theory, an appetizer
M symplectic mfld.
S : M → M.
Φ|A = φi
Φ|B = φf


Twistor String Theory as an appetizer
M symplectic mfld.
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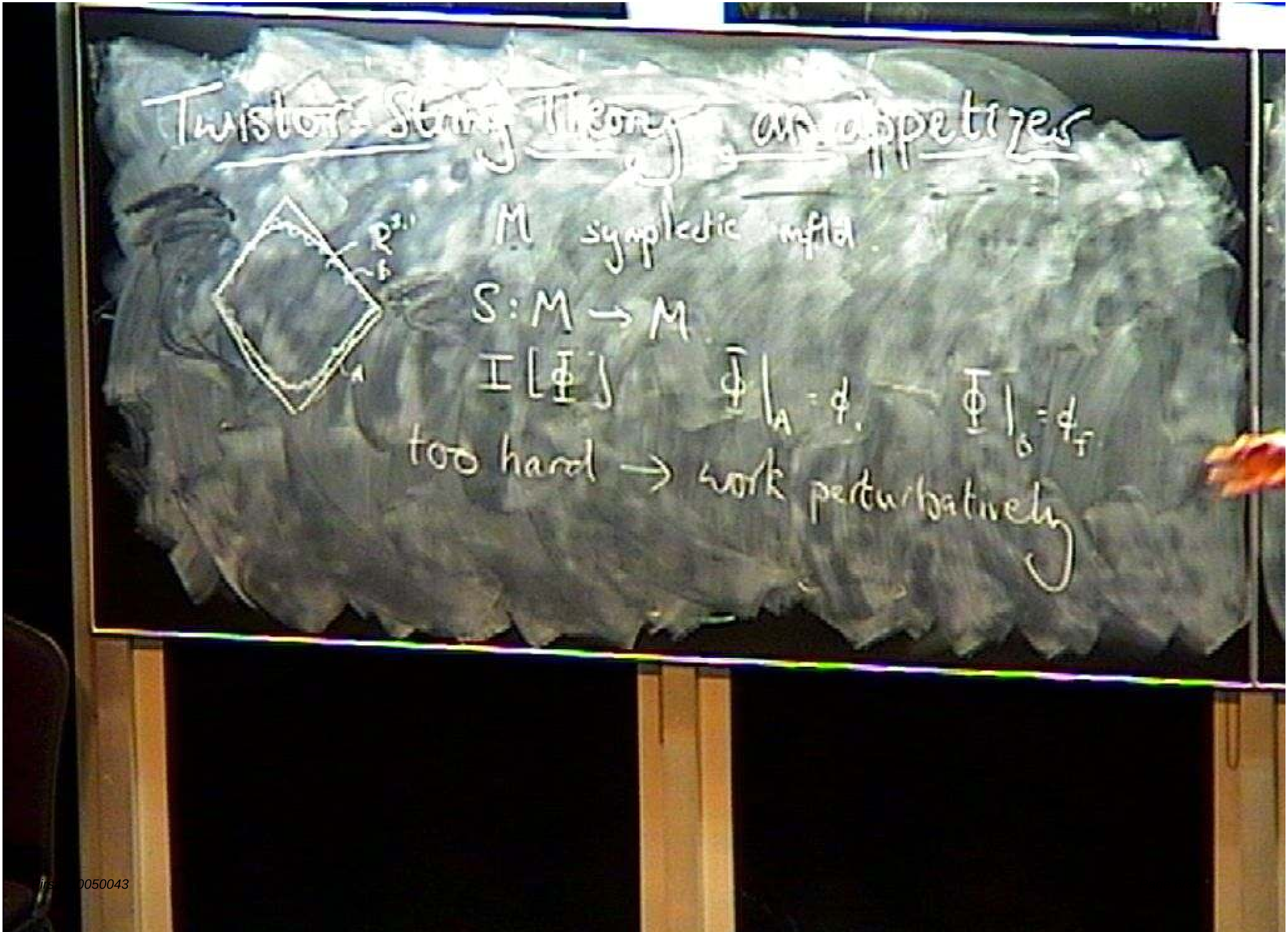
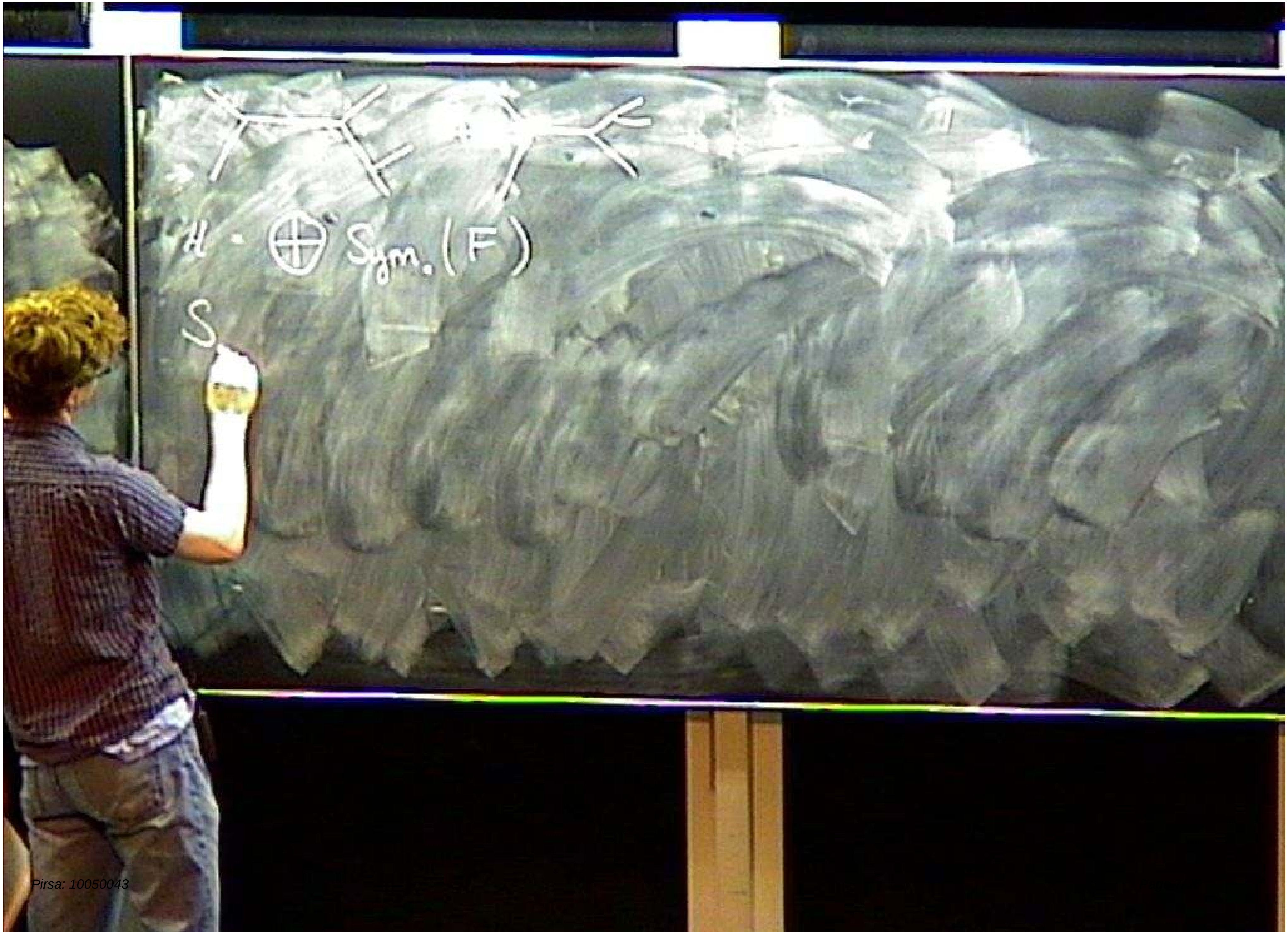


Twistor String Theory — an appetizer
M symplectic mfld
S : M → M
I[Φ]   Φ|_A = φ_i   Φ|_B = φ_f
too hard → work perturbatively


H = ⊕ Sym_n (F)
S

$\mathcal{H} = \bigoplus \mathrm{Sym}_{\bullet}(F)$

$S_{\alpha,\beta} \quad \mathrm{Sym}_{\alpha}(F) \otimes (\mathrm{Sym}_{\beta}(F))^{*} \to \mathbb{C}$

— Combinatorically very involved.

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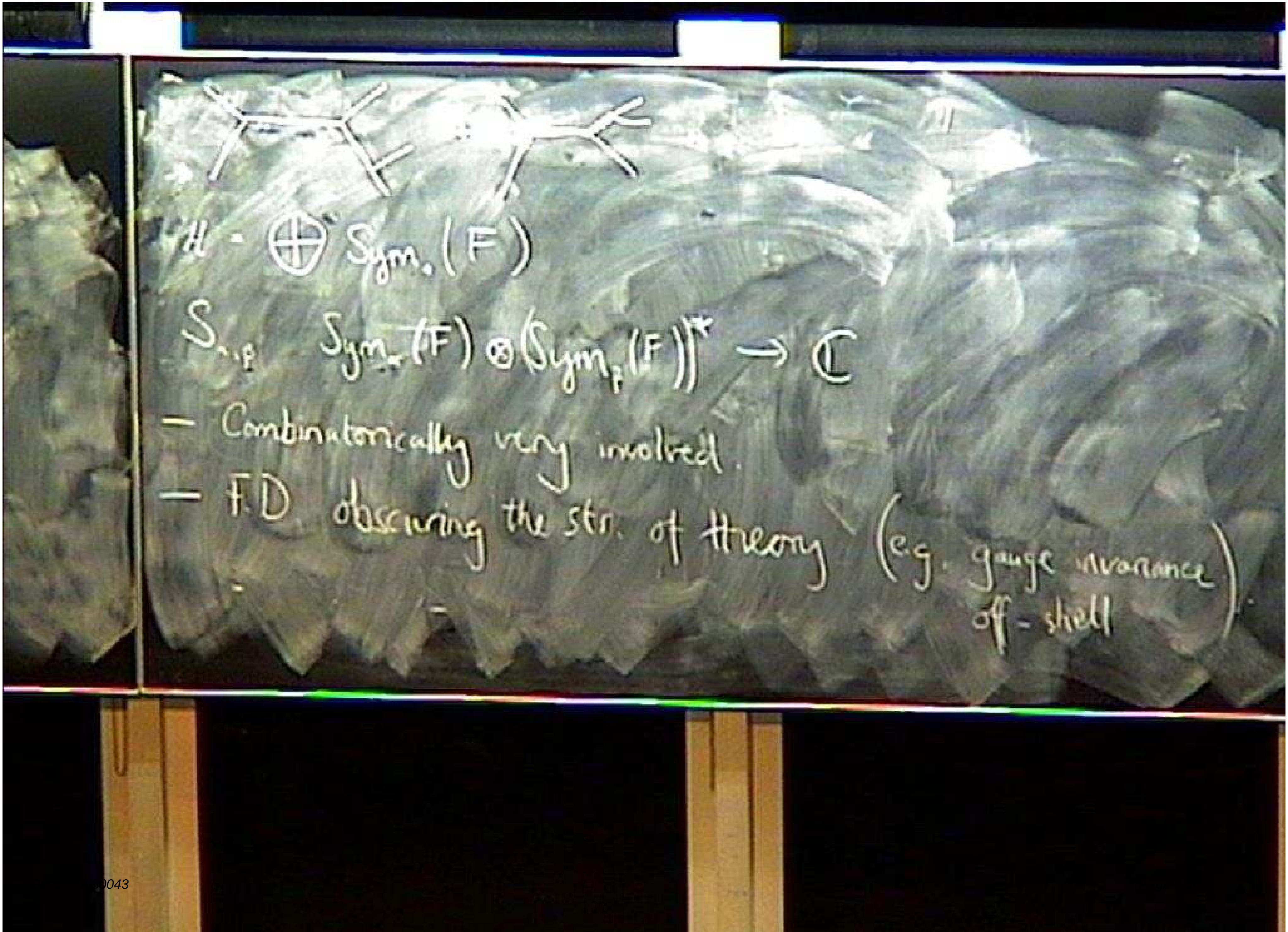
H = ⊕ Sym_n (F)
S_n,p Sym_n (F) ⊗ (Sym_p (F))* → ℂ
— Combinatorically very involved.
— F.D. obscuring the str. of theory (e.g. gauge invariance off-shell).

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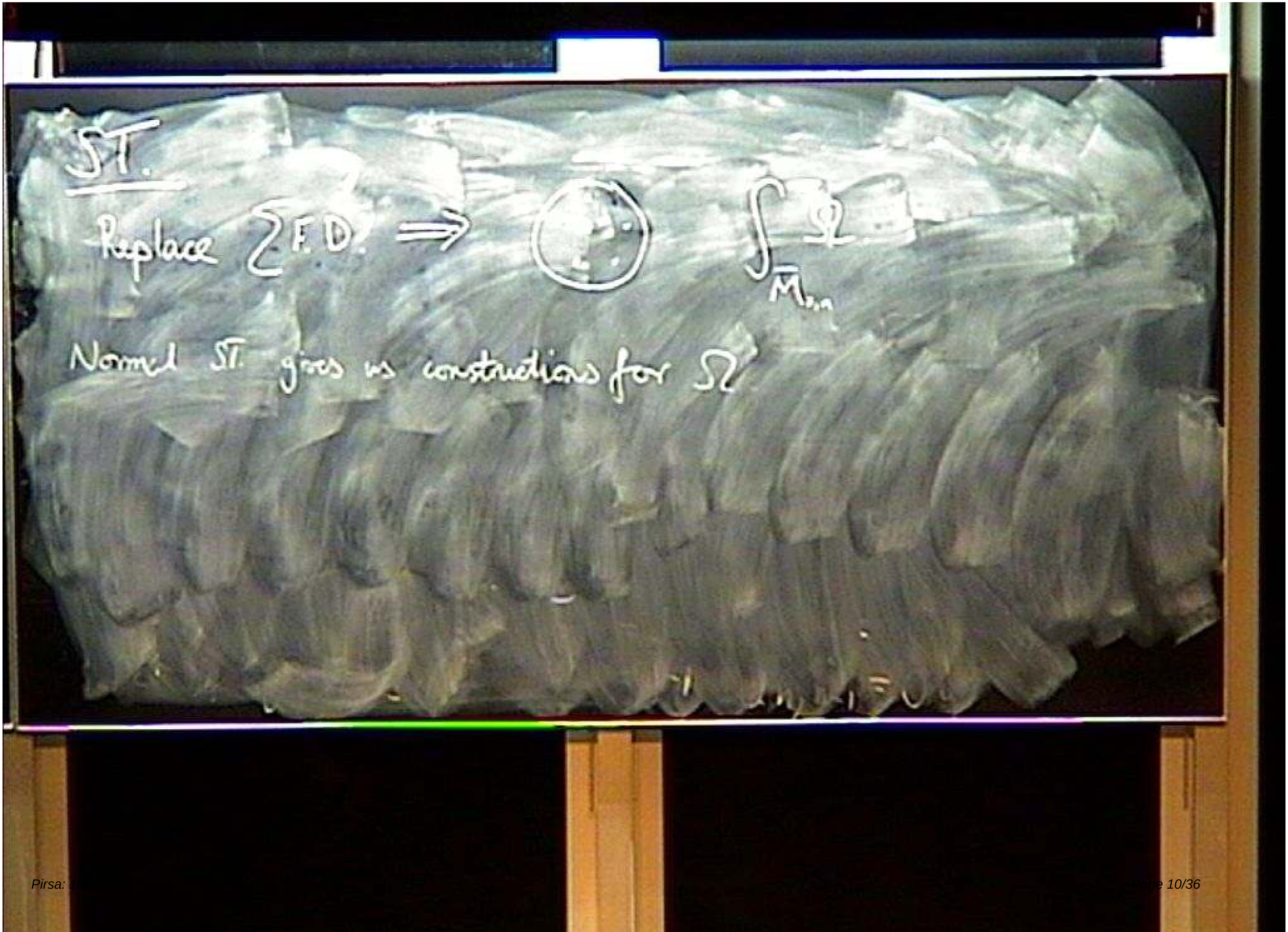
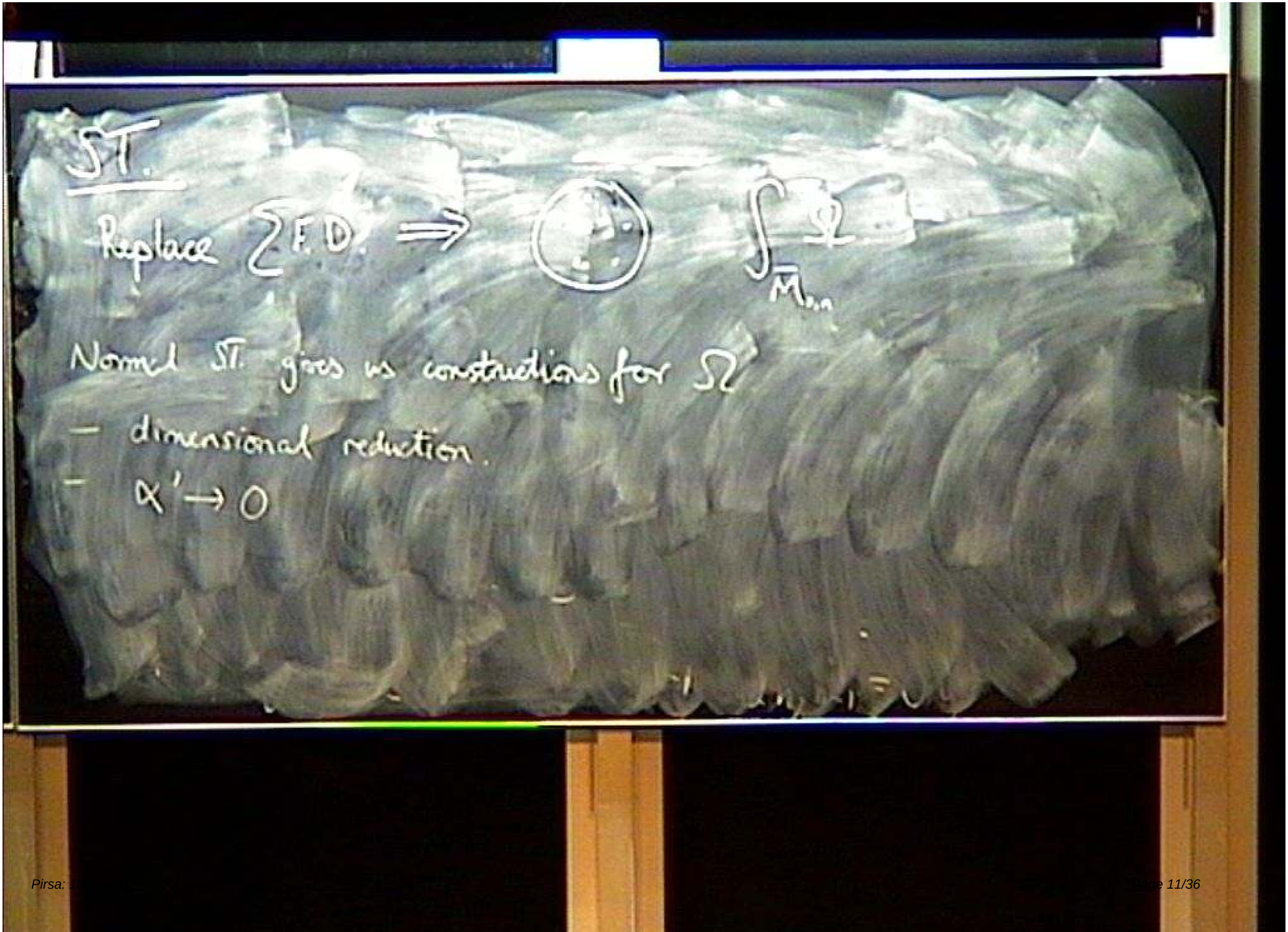
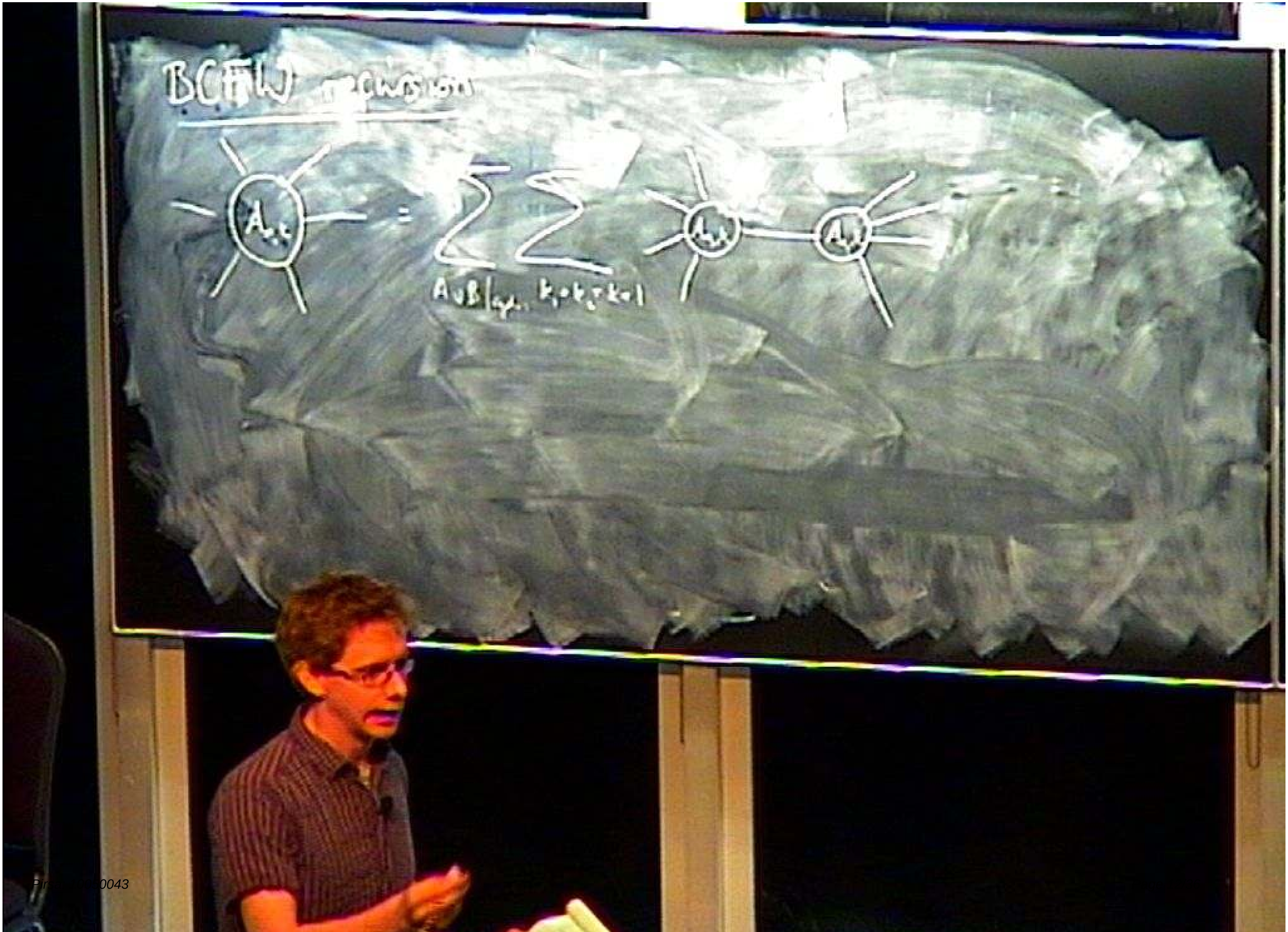
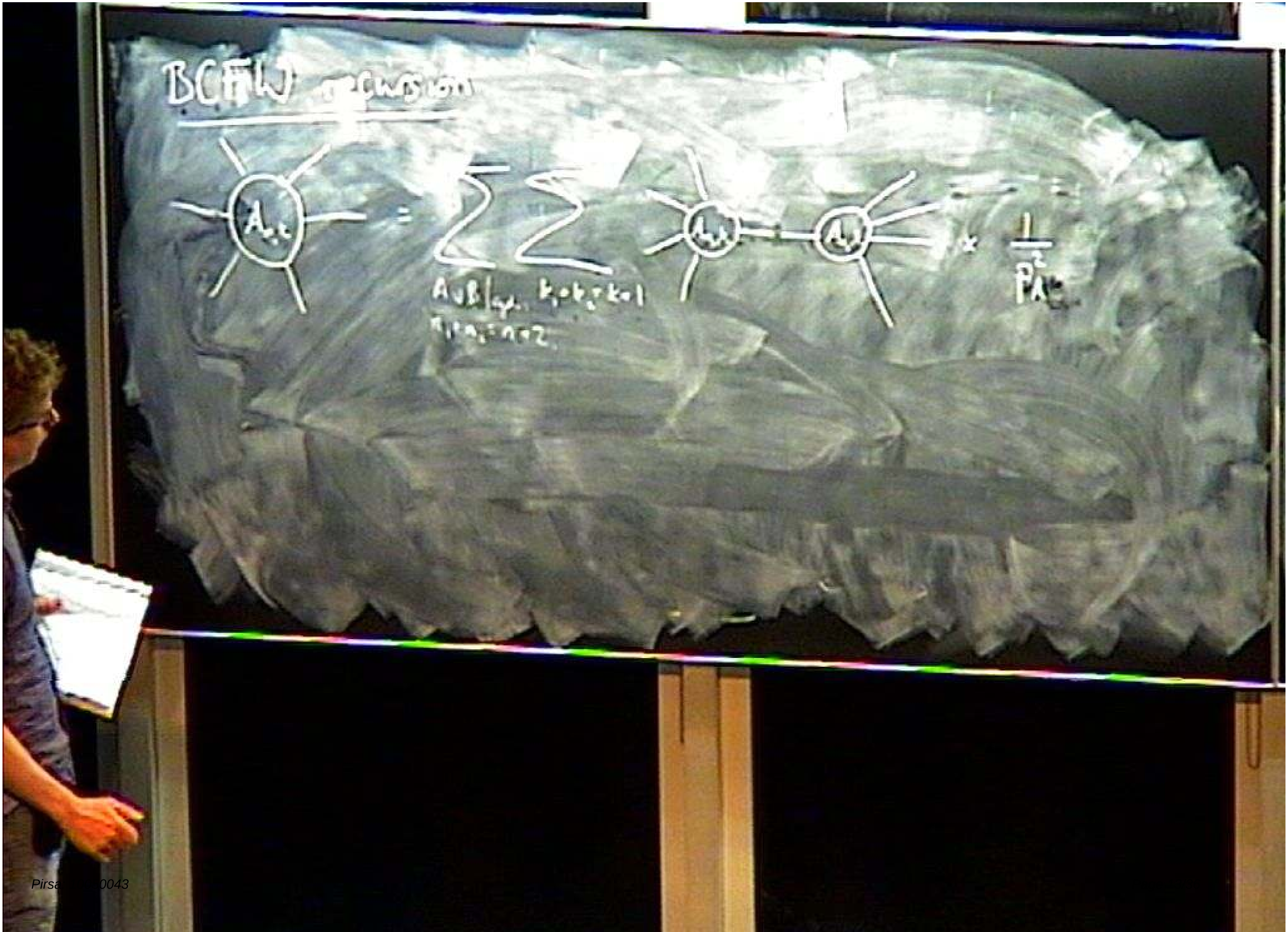
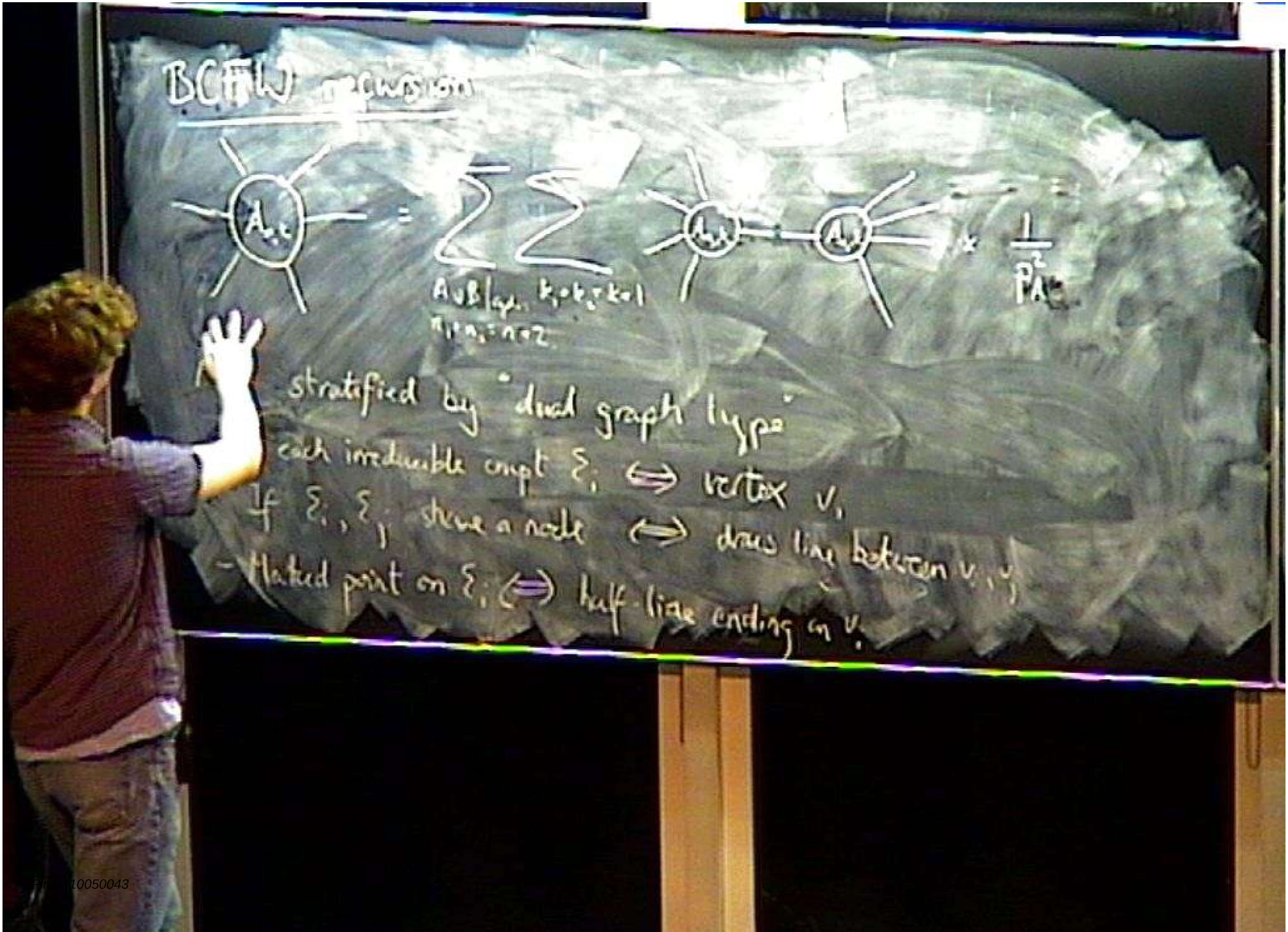
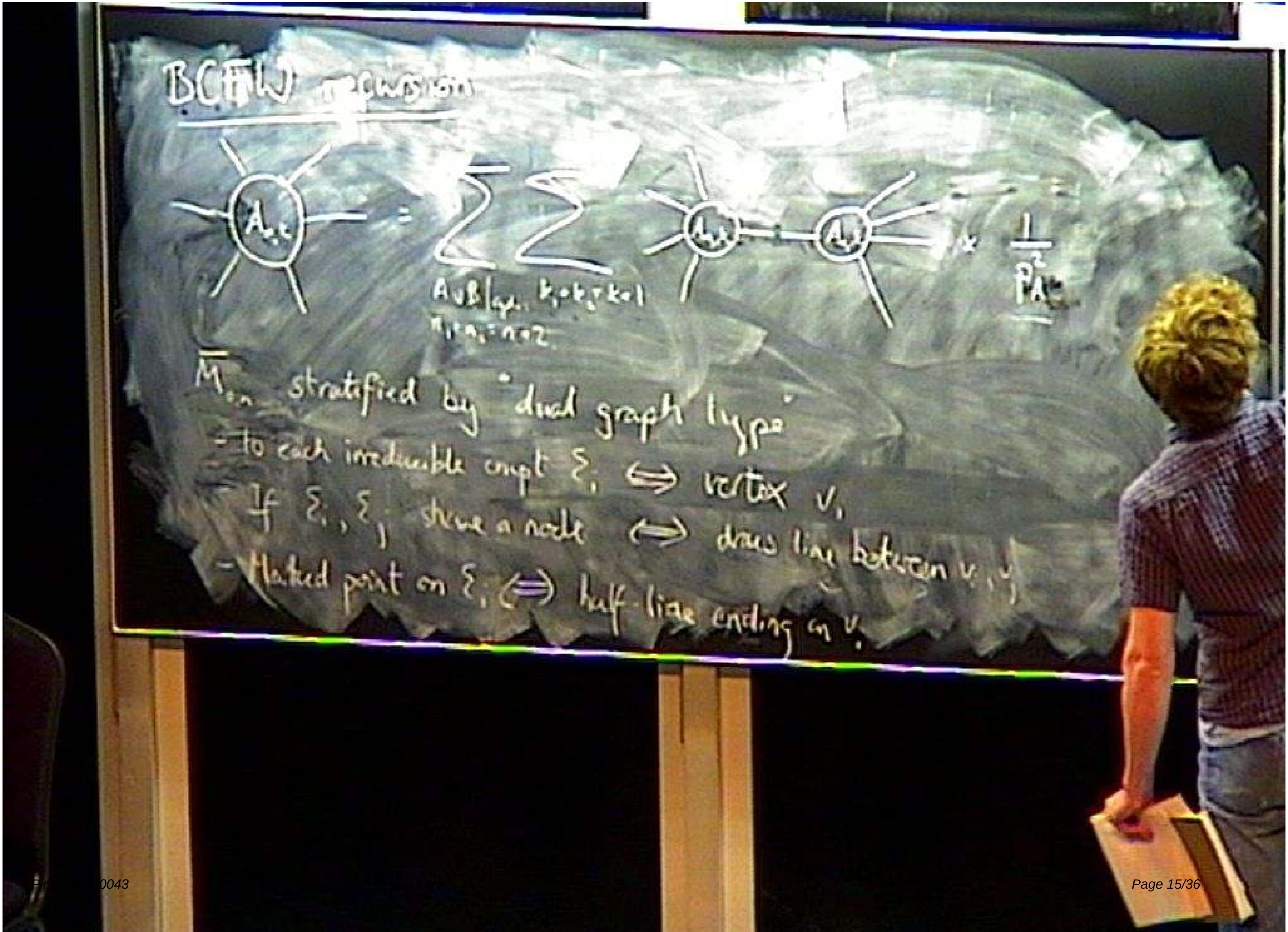

S.T.
Replace Σ F.D. ⇒ ∫ Ω
$\overline{M}_{g,n}$
Normal S.T. gives us constructions for Ω

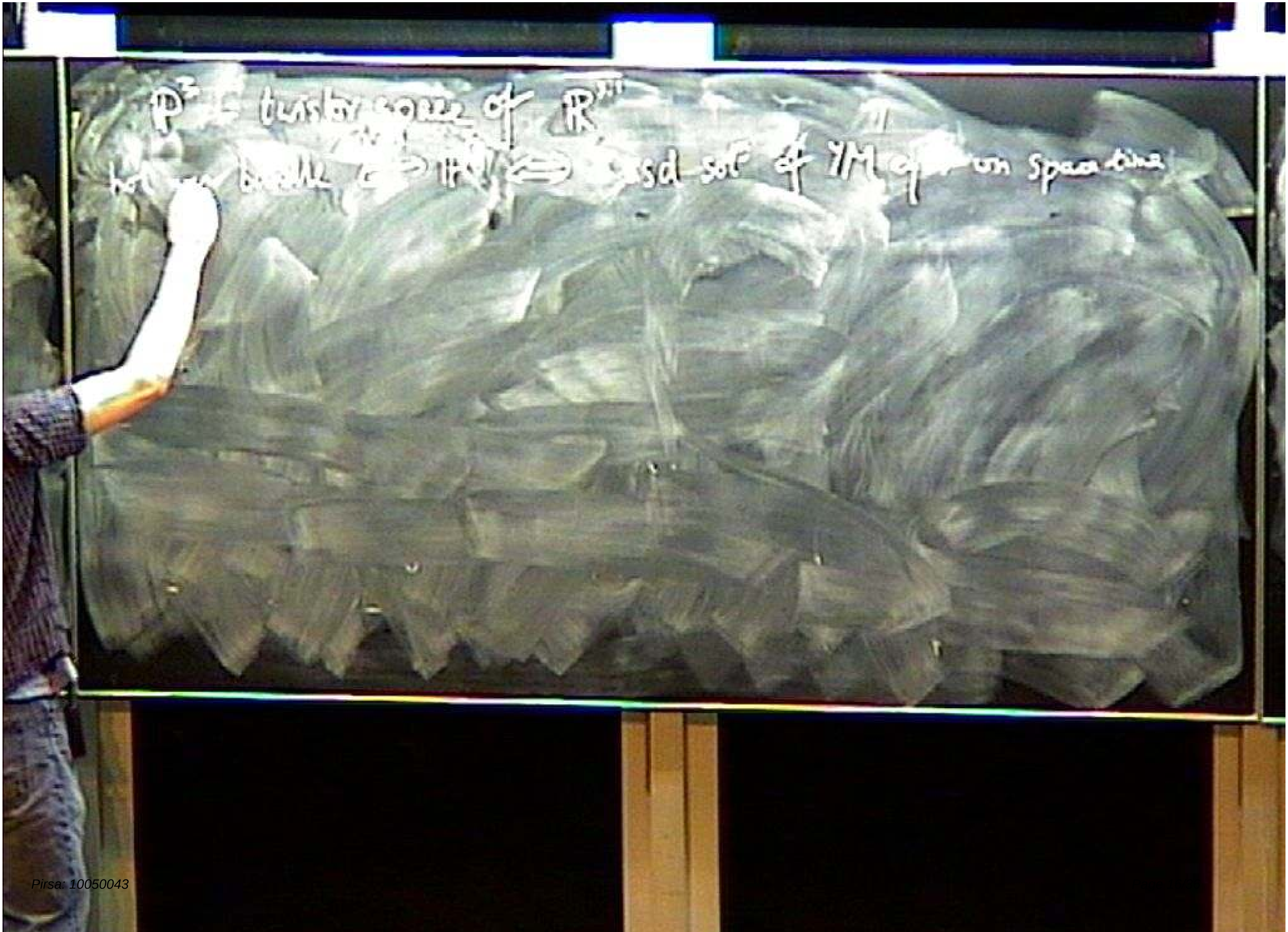

S.T.
Replace ΣF.D. ⟹ ∫ Ω
M̄..n
Normal S.T. gives us constructions for Ω
– dimensional reduction.
– α' → 0


BCFW recursion


BCFW recursion


BCFW recursion
A n,k
A∪B|{p}, k1+k2=k+1
n1+n2=n+2
stratified by "dual graph type"
each irreducible cmpt Σi ⟺ vertex vi
If Σi, Σj share a node ⟺ draw line between vi, vj
- Marked point on Σi ⟺ half line ending on vi


BCFW recursion
$A_{n,k}$
$k_1+k_2=k+1$
$\bar{M}_{0,n}$ stratified by "dual graph type"
- to each irreducible cmpt Σ_i ⟺ vertex v_i
- If Σ_i, Σ_j share a node ⟺ draw line between v_i, v_j
- Marked point on Σ_i ⟺ half-line ending on v_i


twistor space of
sd sol^n of YM
on space-time

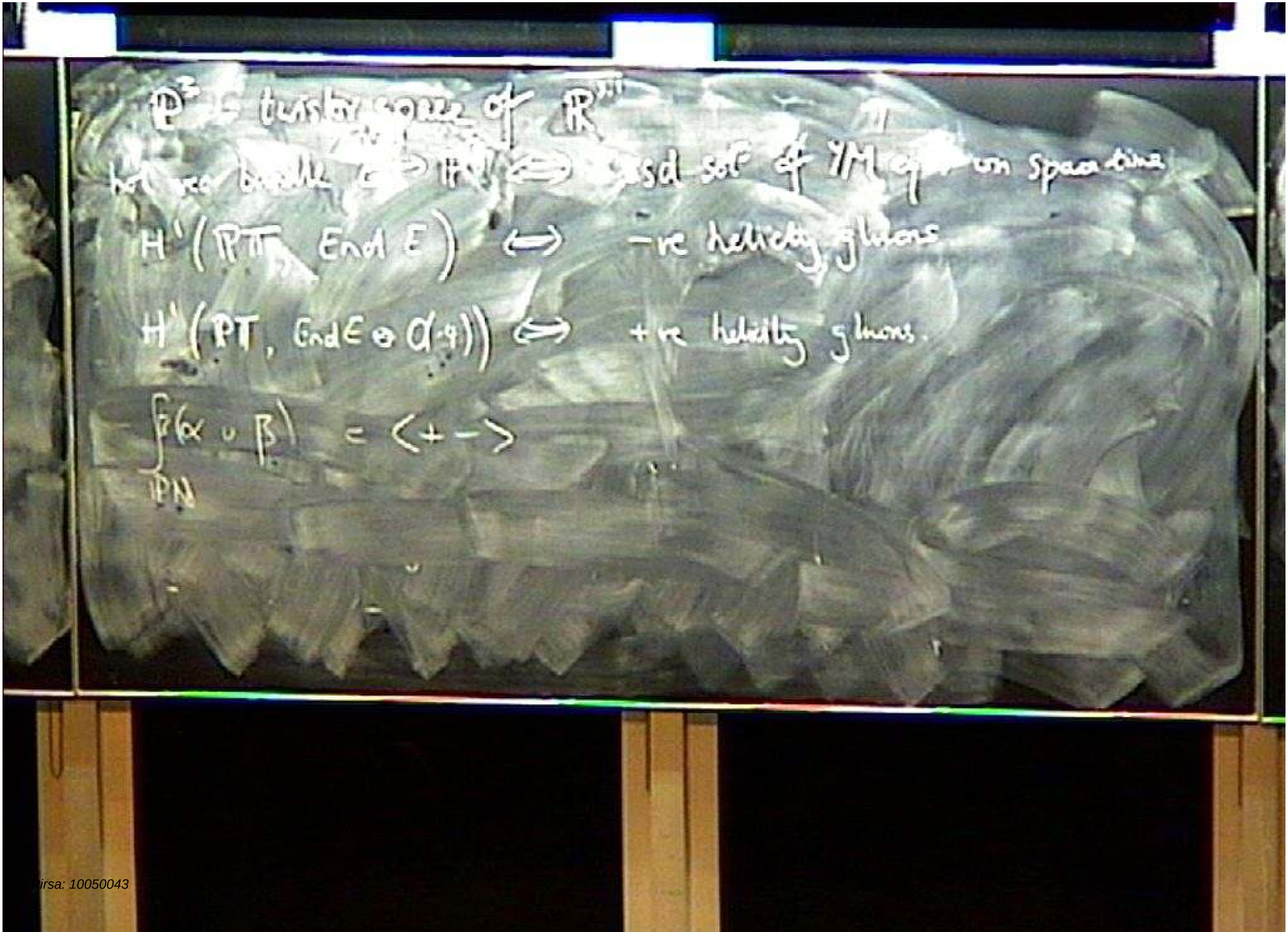
$\mathbb{P}^3$ = twistor space of $\mathbb{R}^4$

hol vec bundle $E \to \mathbb{PT} \Longleftrightarrow$ asd sol of YM eqs on space-time

$H^1(\mathbb{PT}, \mathrm{End}\, E) \Longleftrightarrow$ −ve helicity gluons

$H^1(\mathbb{PT}, \mathrm{End}\, E \otimes \mathcal{O}(-4)) \Longleftrightarrow$ +ve helicity gluons.

$\int \alpha \cup \beta$


ℙ³ = twistor space of ℝ⁴
hol vec bundle E → PT ⟺ asd sol of YM eqn on space-time
H¹(PT, End E) ⟺ −ve helicity gluons
H¹(PT, End E ⊗ O(−4)) ⟺ +ve helicity gluons.
∫ f(α ∪ β) = ⟨+ −⟩
PN

Scattering

$$H^1(\mathbb{PT}, \mathcal{O}) \otimes H^1(\mathbb{PT}, \mathcal{O}) \otimes H^1(\mathbb{PT}, \mathcal{O}(-4)) \xrightarrow{\int} \mathbb{C}$$

Obvious generalisation $V = \mathcal{O}(1)^{\oplus 4}$

$$H^1(\mathbb{PT}, \Lambda^{i_1} V^\vee) \otimes H^1(\mathbb{PT}, \Lambda^{i_2} V^\vee) \otimes H^1(\mathbb{PT}, \Lambda^{i_3} V^\vee) \xrightarrow{\int} \mathbb{C}$$

Scattering

$$H^1(\mathbb{PT}, \mathcal{O}) \otimes H^1(\mathbb{PT}, \mathcal{O}) \otimes H^1(\mathbb{PT}, \mathcal{O}(-4)) \xrightarrow{\int} \mathbb{C}$$

Obvious generalisation $\quad V = \mathcal{O}(1)^{\oplus 4} \qquad \Lambda^{top} V^\vee \simeq K_{\mathbb{P}^3}$

$$H^1(\mathbb{PT}, \Lambda^{q_1} V^\vee) \otimes H^1(\mathbb{PT}, \Lambda^{q_2} V^\vee) \otimes H^1(\mathbb{PT}, \Lambda^{q_3} V^\vee) \xrightarrow{\int} \mathbb{C}$$

$$\sum q_i = rk\, V$$

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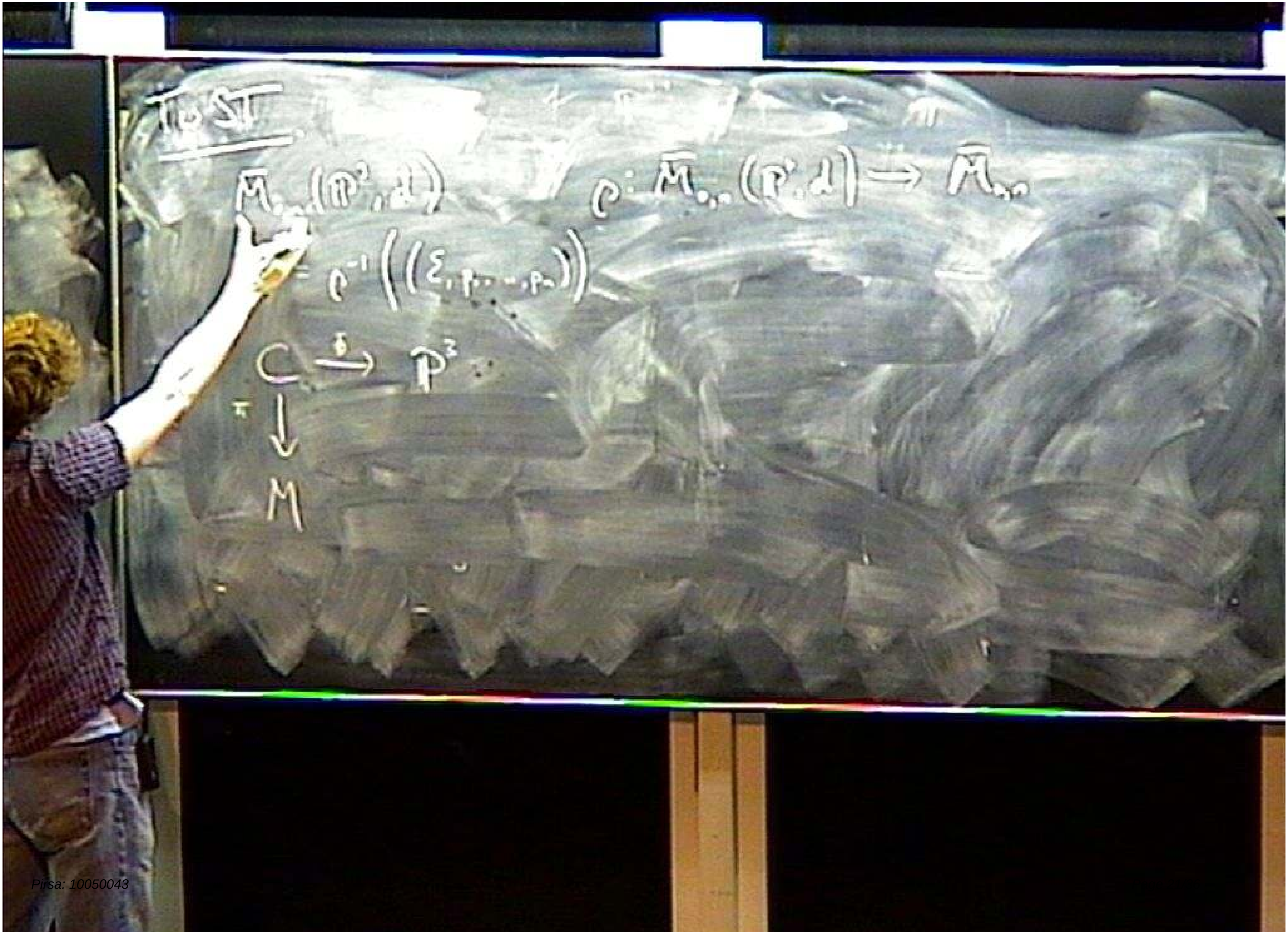
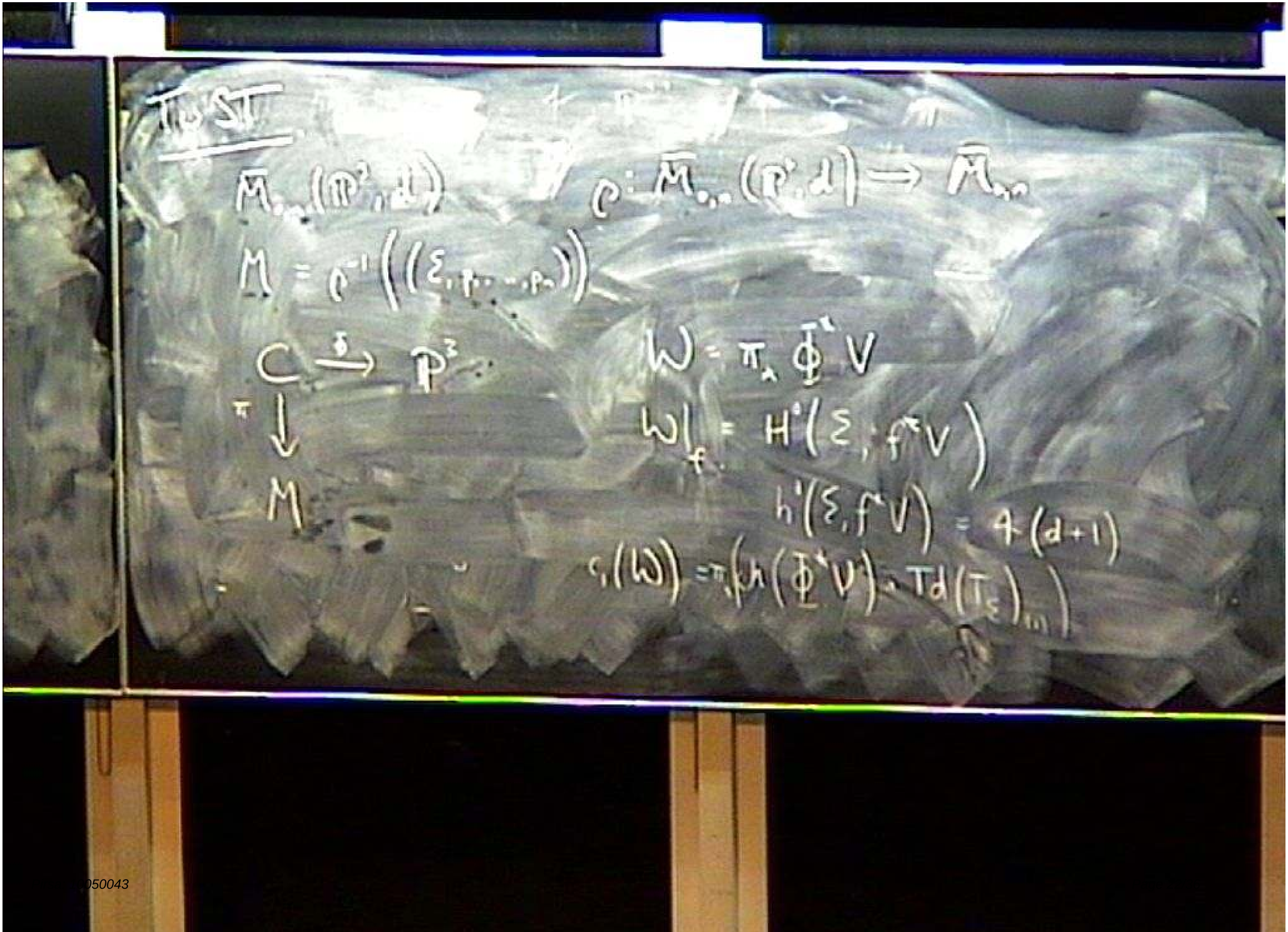


TWIST
M̄0,n(P², d)
ρ: M̄0,n(P², d) → M̄0,n
= ρ⁻¹((Σ, p1, …, pn))
C → P²
π
M


Twist
M̄_{0,n}(P³, d)
ρ : M̄_{0,n}(P³, d) → M̄_{0,n}
M = ρ⁻¹((Σ, p₁, …, pₙ))
C —Φ→ P³
π ↓
M
W = π_* Φ* V
W|_f = H⁰(Σ, f*V)
h⁰(Σ, f*V) = 4(d+1)
c₁(W) = π_*(ch(Φ*V) · Td(T_Σ))_(1)

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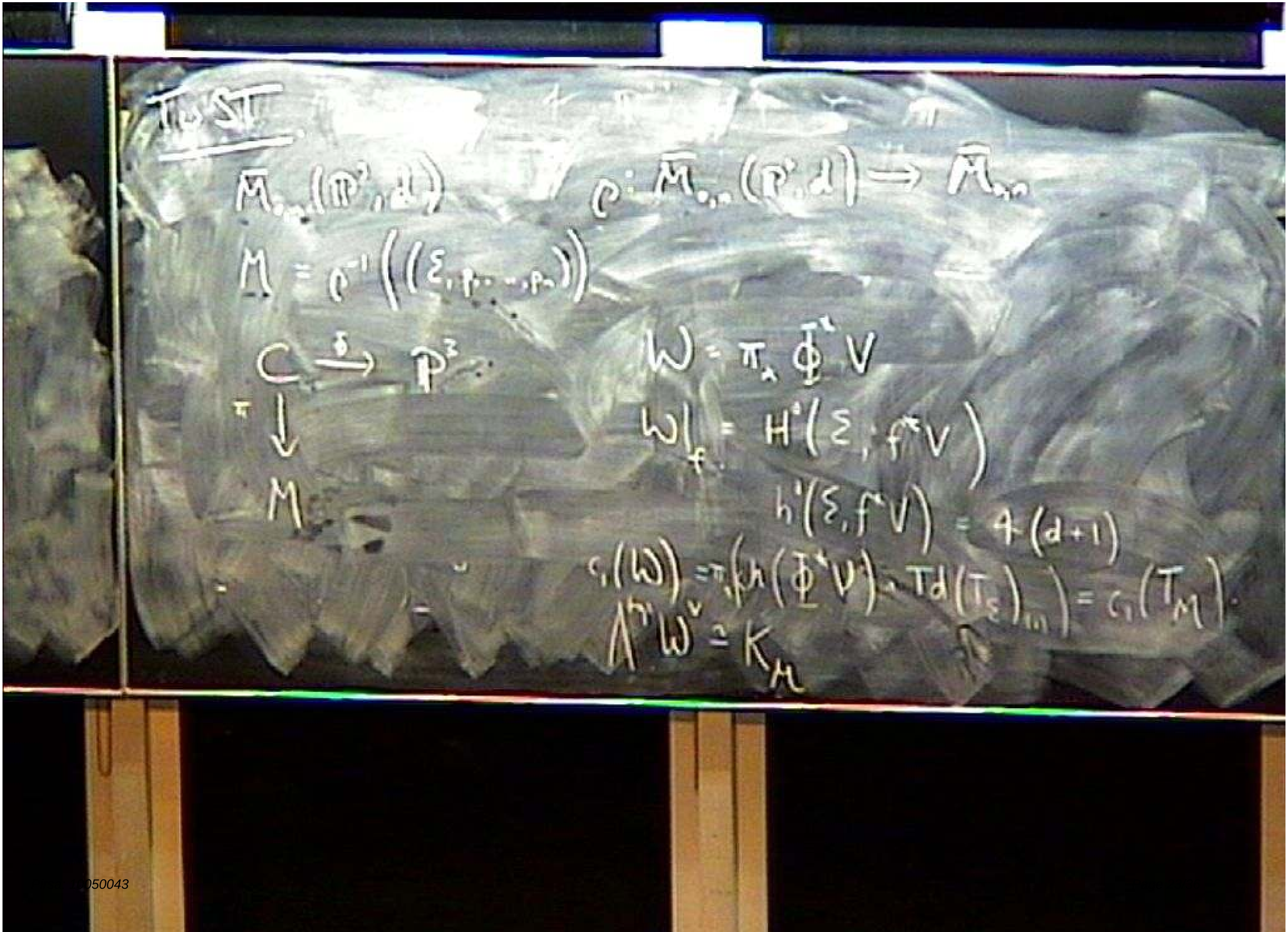
Twist
M̄0,n(P3,d)
ρ: M̄0,n(P3,d) → M̄0,n
M = ρ^-1((Σ, p1, ..., pn))
C →Φ P3
π ↓
M
W = π* Φ* V
W|f = H0(Σ, f*V)
h0(Σ, f*V) = 4(d+1)
c1(W) = π*(ch(Φ*V) · Td(TΣ))(1) = c1(TM)
Λtop W∨ = KM

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TWIST

$$\overline{M}_{0,n}(\mathbb{P}^3, d) \qquad \rho : \overline{M}_{0,n}(\mathbb{P}^3, d) \to \overline{M}_{0,n}$$

$$M = \rho^{-1}\big((\Sigma, p_1, \dots, p_n)\big)$$

$$\begin{array}{ccc} C & \xrightarrow{\Phi} & \mathbb{P}^3 \\ {\scriptstyle \pi}\downarrow & & \\ M & & \end{array}$$

$$W = \pi_* \Phi^* V$$

$$W|_f = H^0(\Sigma, f^* V)$$

$$h^0(\Sigma, f^* V) = 4(d+1)$$

$$c_1(W) = \pi_*\big(ch(\Phi^* V)\cdot Td(T_\pi)\big)_{(1)} = c_1(T_M)$$

$$\Lambda^{top} W^\vee = K_M$$

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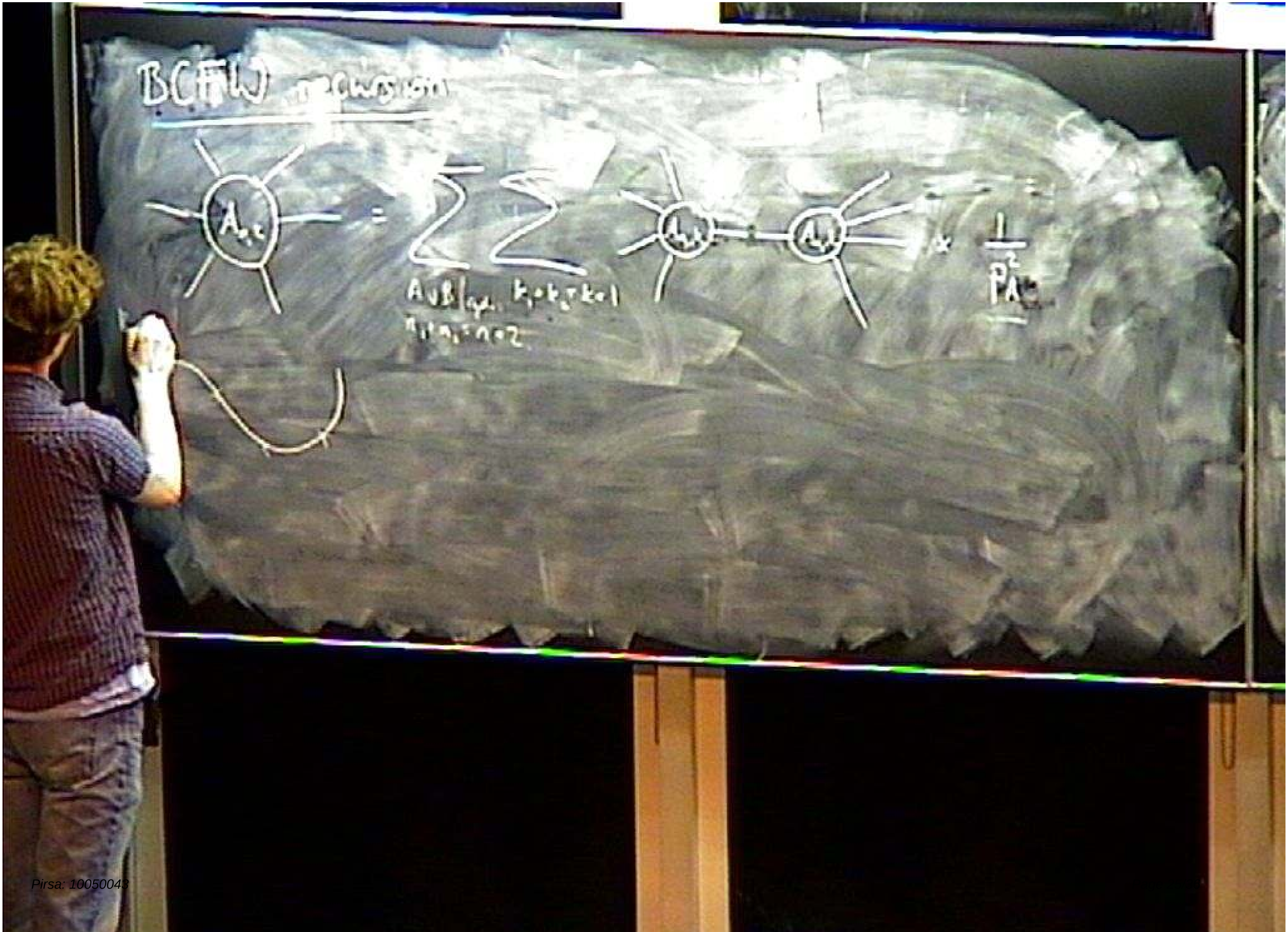
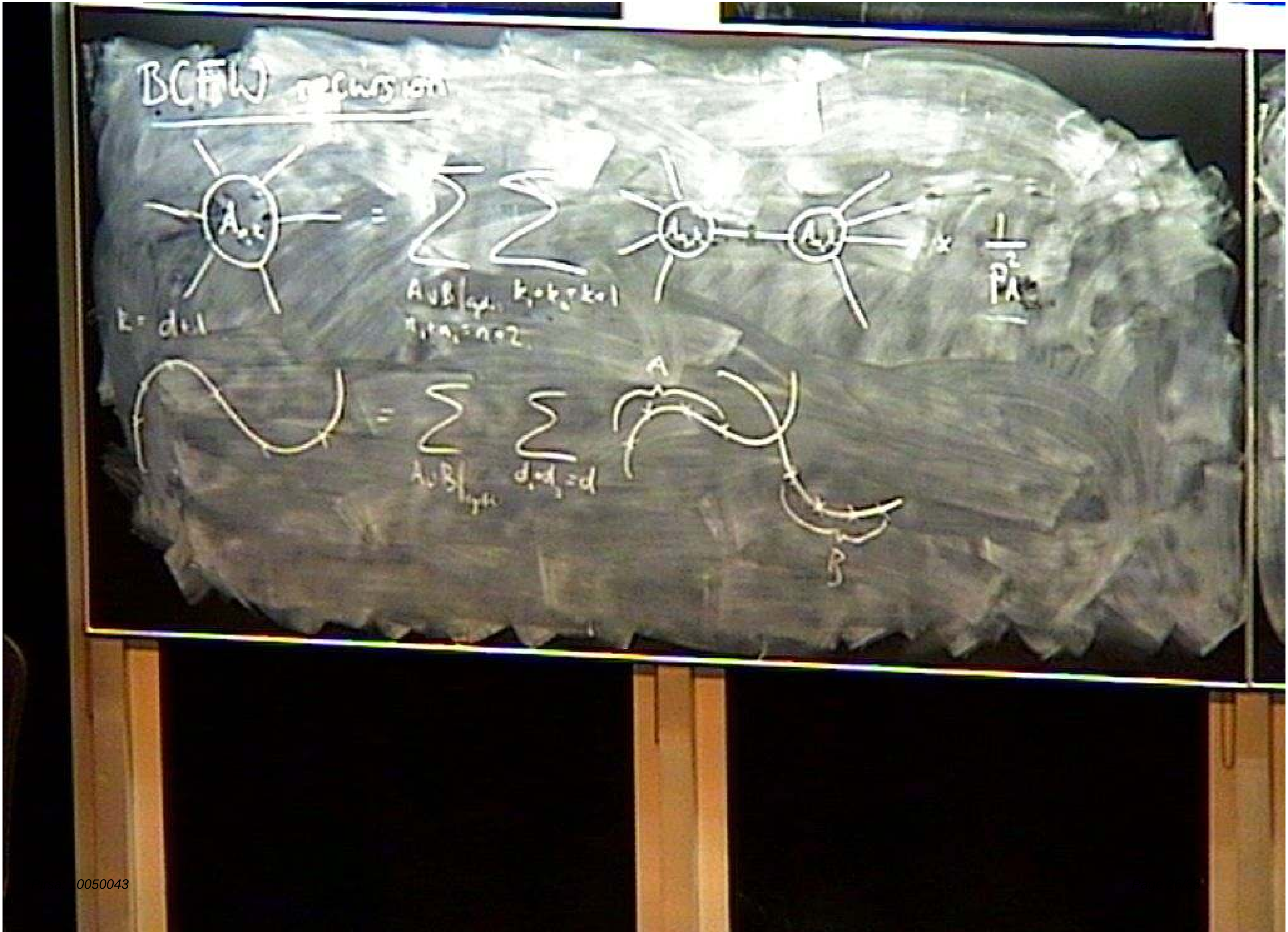

BCFW recursion



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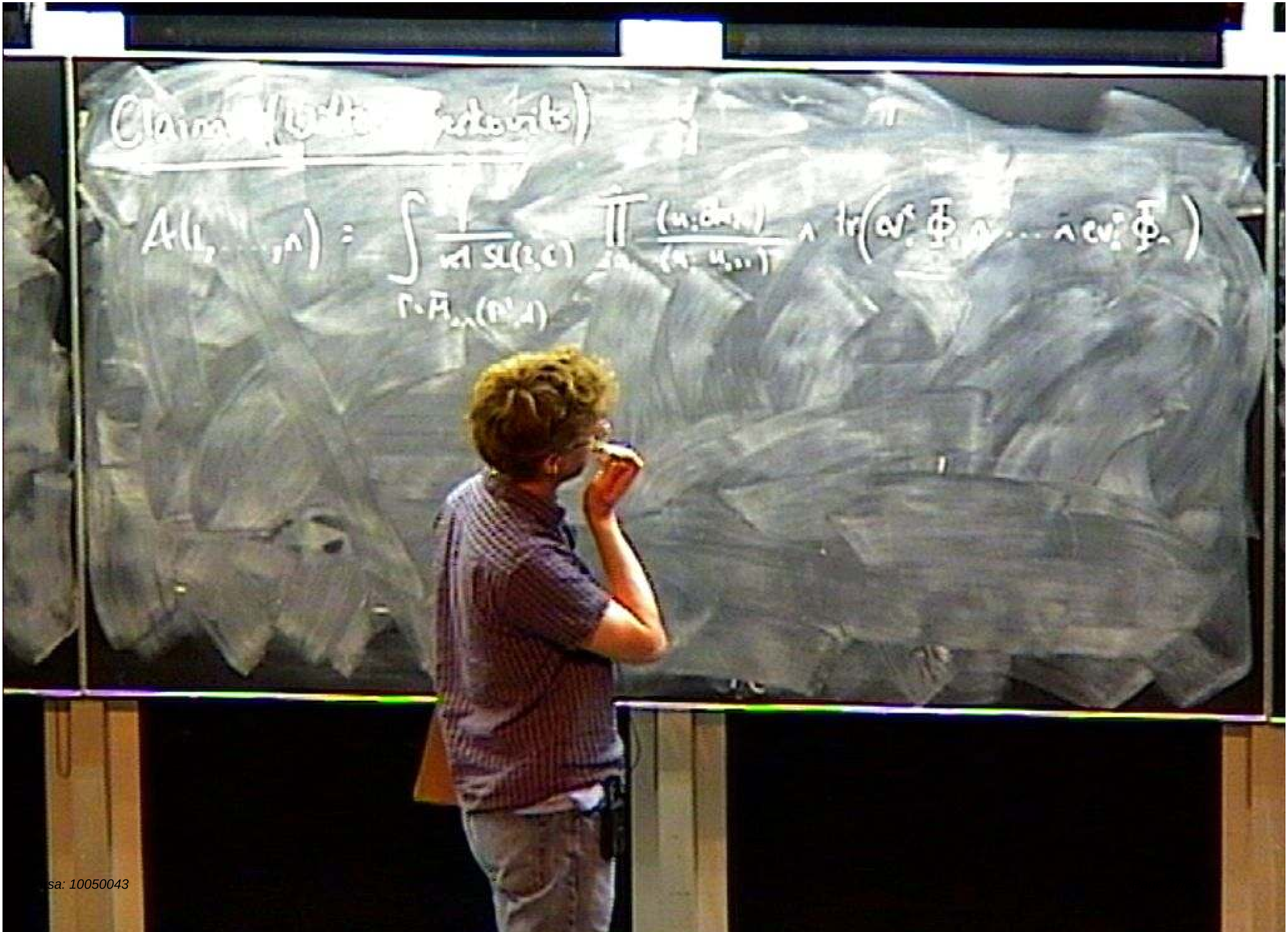

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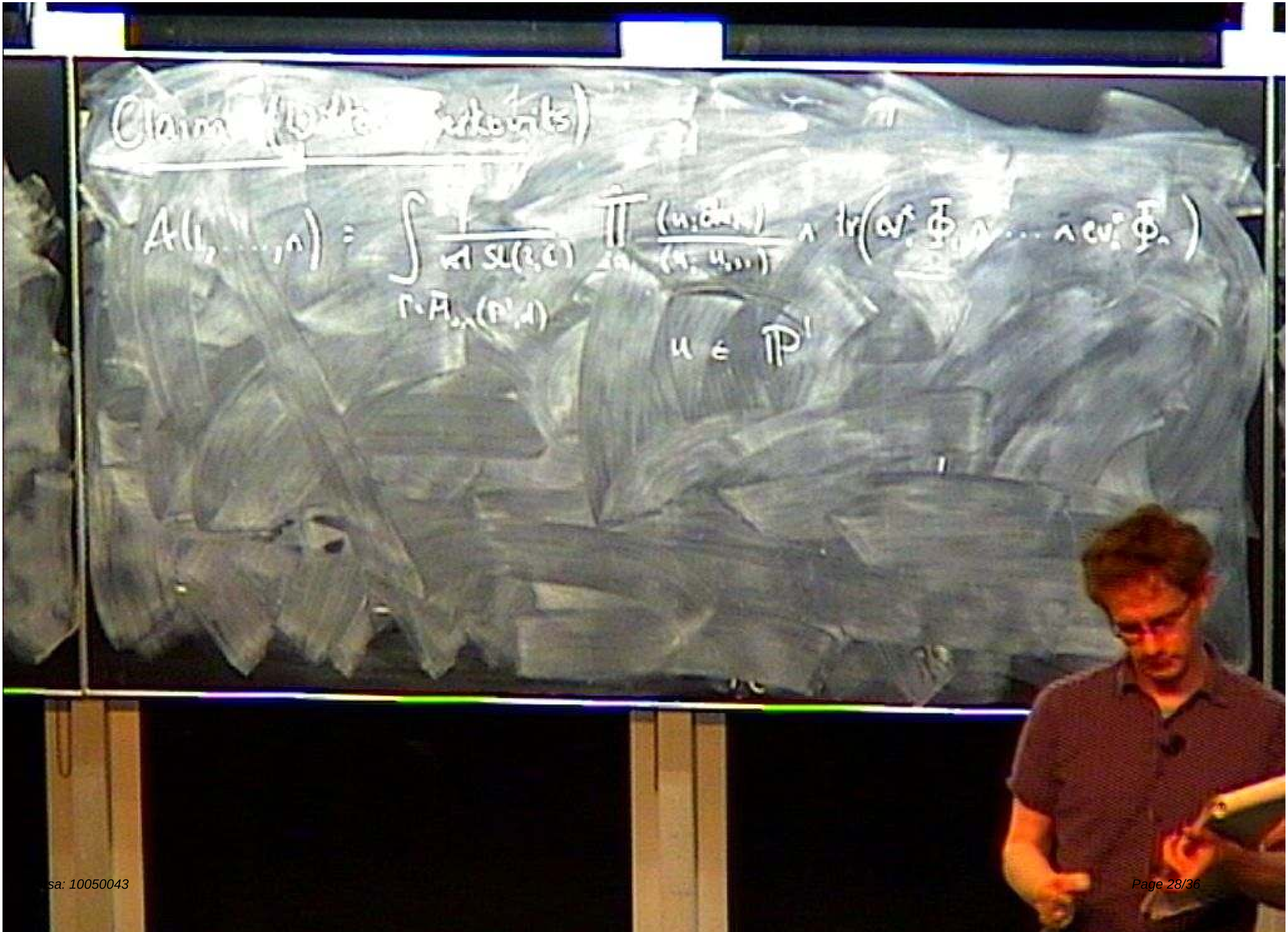
Claim
$A(1,\ldots,n) = \int_{\Gamma\in M_{0,n}(\mathbb{P}^1,d)} \frac{1}{\mathrm{vol}\,SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\,du_i)}{(u_i\,u_{i+1})} \wedge \mathrm{tr}(\omega_1^*\Phi_1 \wedge \ldots \wedge \omega_n^*\Phi_n)$
$u \in \mathbb{P}^1$

Claim ([illegible])

$$A(1,\dots,n) = \int_{\Gamma \in \bar{M}_{0,n}(\mathbb{P}^1,d)} \frac{1}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}\left(\omega_1^* \Phi_1 \wedge \dots \wedge \omega_n^* \Phi_n\right)$$

$$u \in \mathbb{P}^1$$

- No general proof, but holds in many cases.
- Better formulation?

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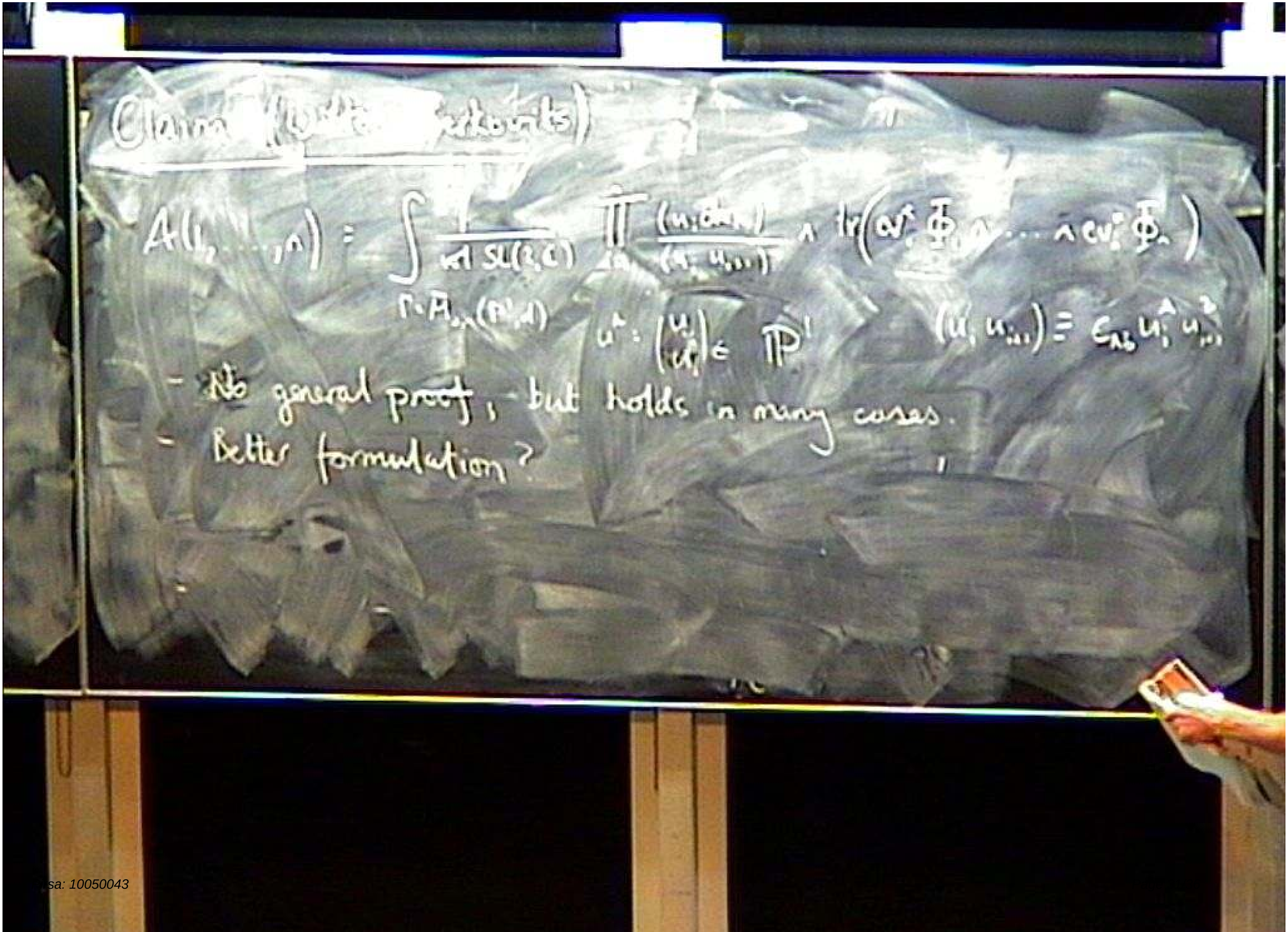
Claim ([illegible])

$$A(1,\dots,n) = \int_{\Gamma\in\mathcal{M}_{0,n}(\mathbb{P}^3,d)} \frac{[\text{illegible}]}{\mathrm{vol}\,SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}\left(\omega_1^*\Phi_1 \wedge \dots \wedge \omega_n^*\Phi_n\right)$$

$u^A = \begin{pmatrix} u_i^0 \\ u_i^1 \end{pmatrix} \in \mathbb{P}^1 \qquad (u_i\, u_{i+1}) = \epsilon_{AB}\, u_i^A u_{i+1}^B$

- No general proof, but holds in many cases.
- Better formulation?


Claim
- No general proof, but holds in many cases.
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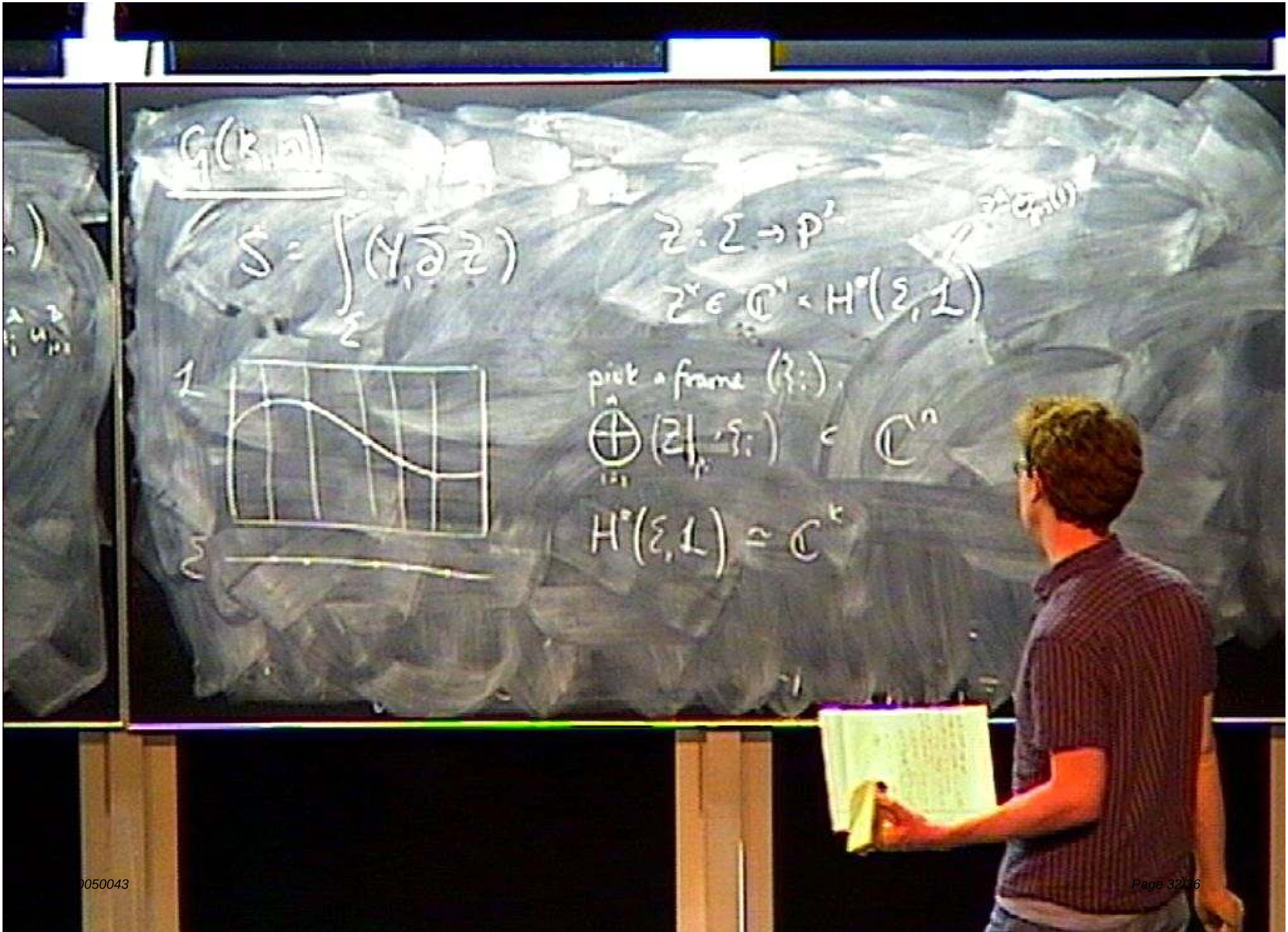
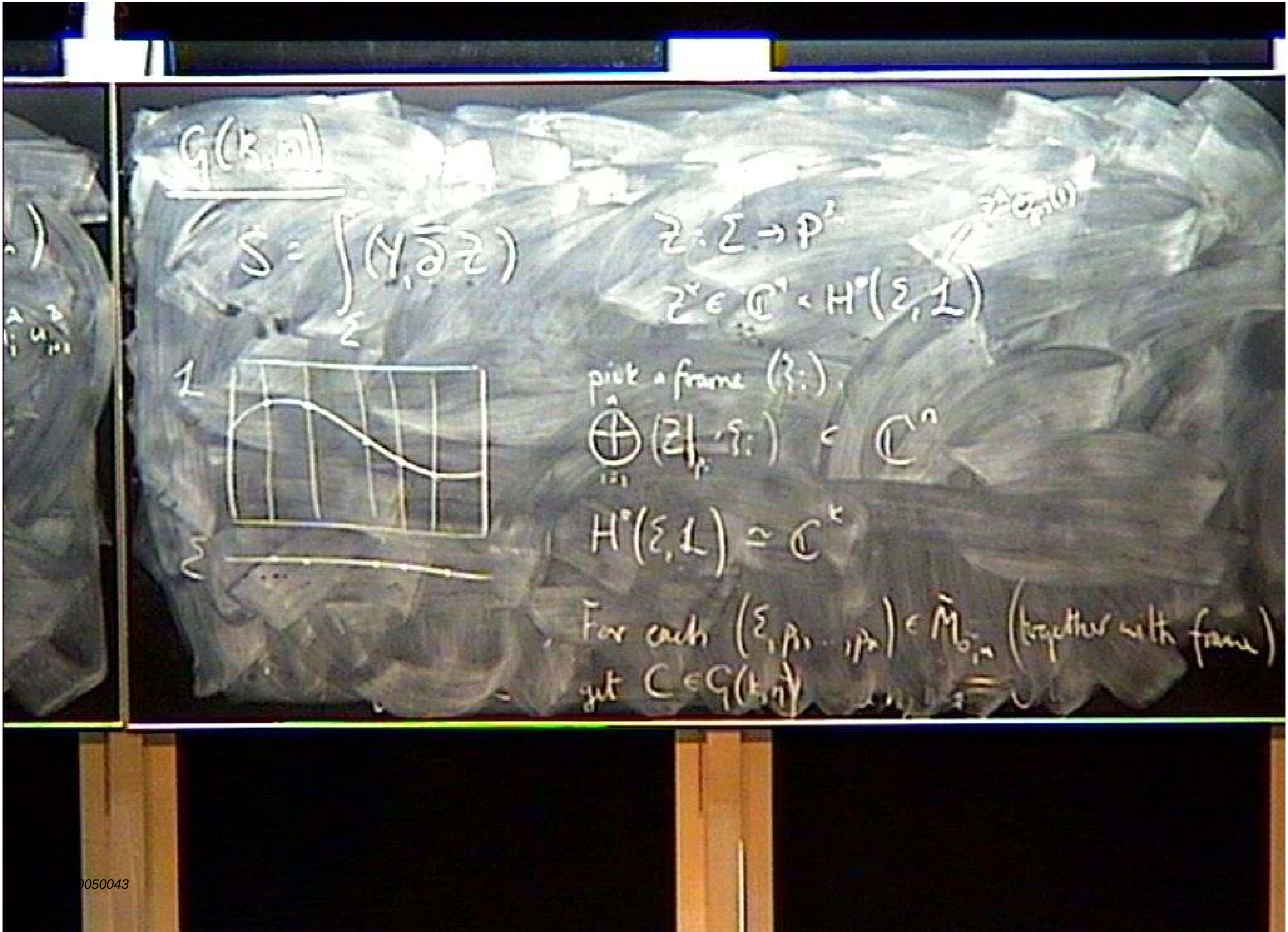

S = ∫_Σ (Y, ∂̄Z)
Z : Σ → P^k
H^0(Σ, L)
pick a frame
H^0(Σ, L) ≅ C^k


G(k,n)
S = ∫Σ (Ψ, ∂̄z)
z : Σ → P
z* ∈ C ⊗ H°(Σ, L)
L
Σ
pick a frame (ξi).
⊕ (z|pi, ξi) ∈ Cⁿ
H°(Σ, L) ≅ Cᵏ
For each (Σ, p1, ..., pn) ∈ M0,n (together with frame)
get C ∈ G(k,n)

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Claim (... points)

$$A(1,\ldots,n) = \int_{\Gamma \subset \bar{M}_{0,n}(\mathbb{P}^1,d)} \frac{1}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}\left(\alpha V_1^* \Phi_1 \wedge \cdots \wedge \alpha V_n^* \Phi_n\right)$$

$$u^a = \begin{pmatrix} u_i \\ u_i' \end{pmatrix} \in \mathbb{P}^1 \qquad (u_i\, u_{i+1}) = \epsilon_{ab}\, u_i^a\, u_{i+1}^b$$

$$\mathbb{P}\left(\text{framings} \to \bar{M}_{0,n}\right)$$

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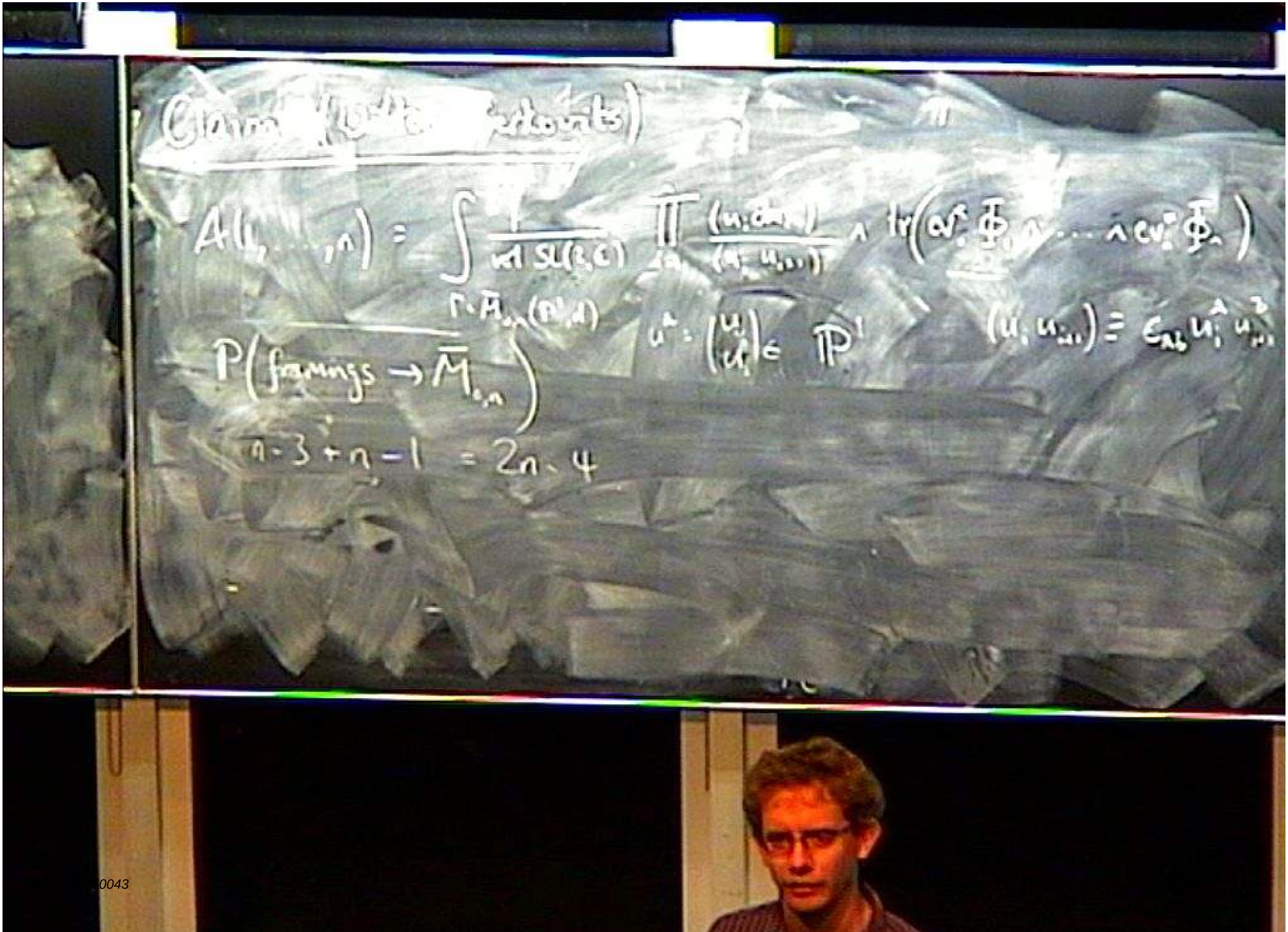
Claim ([illegible])

$$A(1,\dots,n) = \int_{\Gamma\subset \bar{M}_{0,n}(\mathbb{P}^3,d)} \frac{1}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod_{i=1}^{n} \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}\left(\omega_1^*\Phi_1 \wedge \cdots \wedge \omega_n^*\Phi_n\right)$$

$u^a = \begin{pmatrix} u_i^0 \\ u_i^1 \end{pmatrix} \in \mathbb{P}^1$ $\quad (u_i\, u_{i+1}) = \epsilon_{ab}\, u_i^a u_{i+1}^b$

$\mathbb{P}(\text{framings} \to \bar{M}_{0,n})$

$n-3+n-1 = 2n-4$



Claim
$A(1, \dots, n) = \int_{\Gamma \subset \overline{M}_{0,n}(\mathbb{P}^1, d)} \frac{1}{\mathrm{vol}\, SL(2,\mathbb{C})} \prod \frac{(u_i\, du_i)}{(u_i\, u_{i+1})} \wedge \mathrm{tr}(cv_1^* \Phi_1 \wedge \dots \wedge cv_n^* \Phi_n)$
$u^a = \binom{u_0}{u_1} \in \mathbb{P}^1$
$(u_i\, u_{i+1}) = \epsilon_{ab} u_i^a u_{i+1}^b$
$\mathbb{P}(\text{framings} \to \overline{M}_{0,n})$
$n - 3 + n - 1 = 2n - 4$
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