Title: Topological recursion and mirror symmetry

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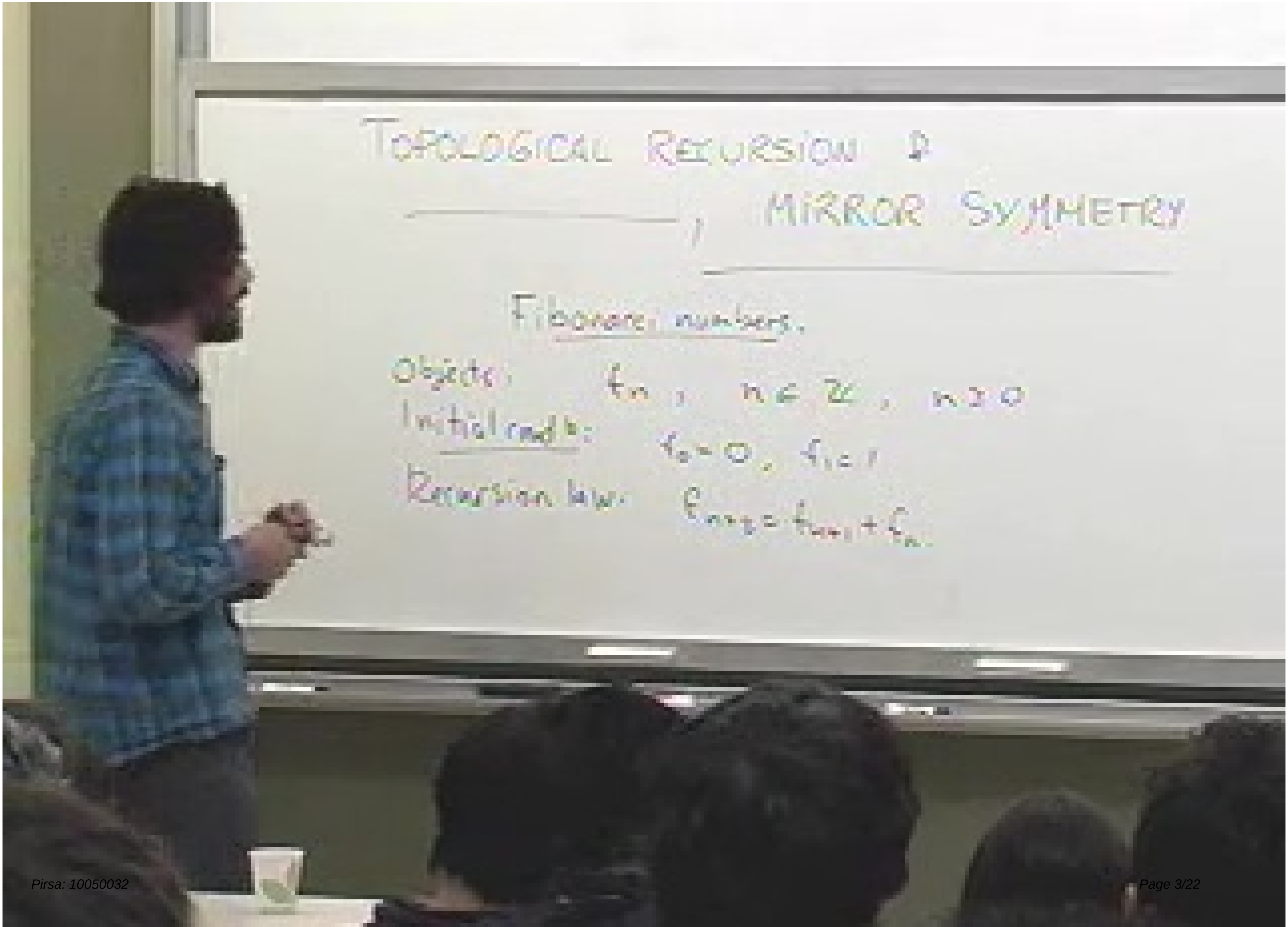
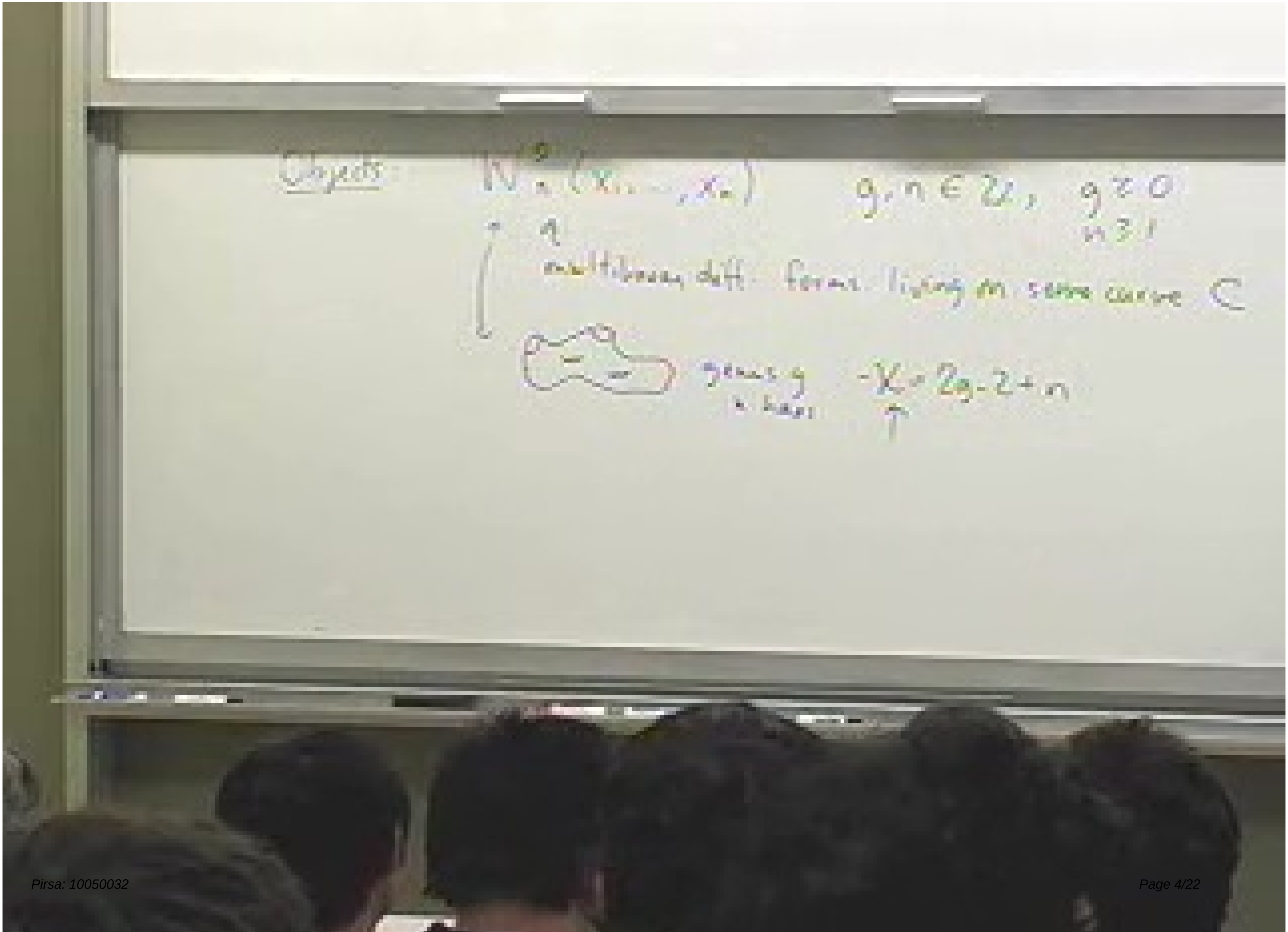
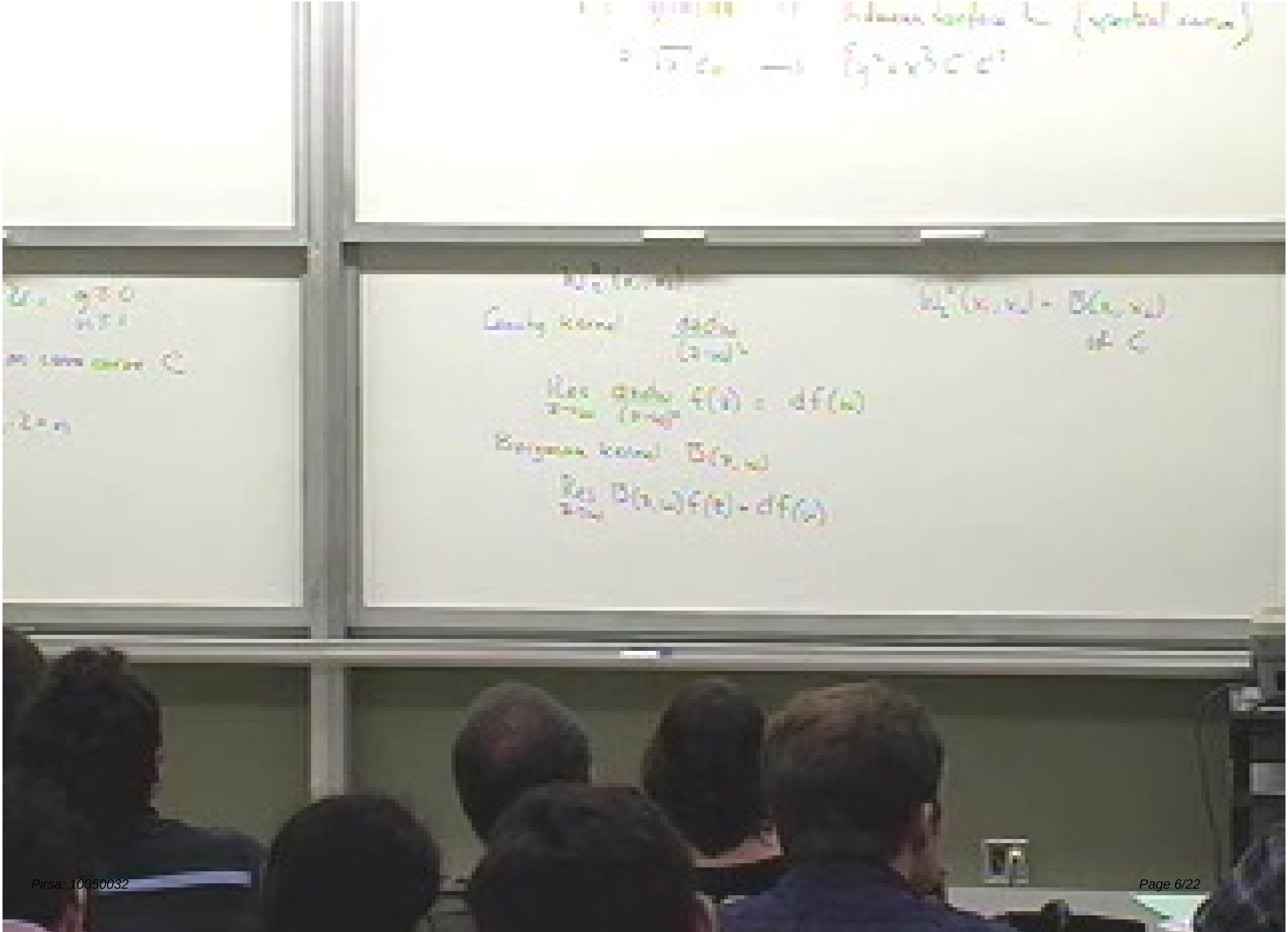
Abstract: The topological recursion of Eynard and Orantin has found many applications in various areas of mathematics. In this talk I will discuss the recursion from the point of view of Hurwitz numbers and local mirror symmetry. I will explain the mathematics underlying the recursion, its relation with the cut-and-join equation, and explore first steps towards proving (and understanding geometrically) the appearance of the recursion in local mirror symmetry.

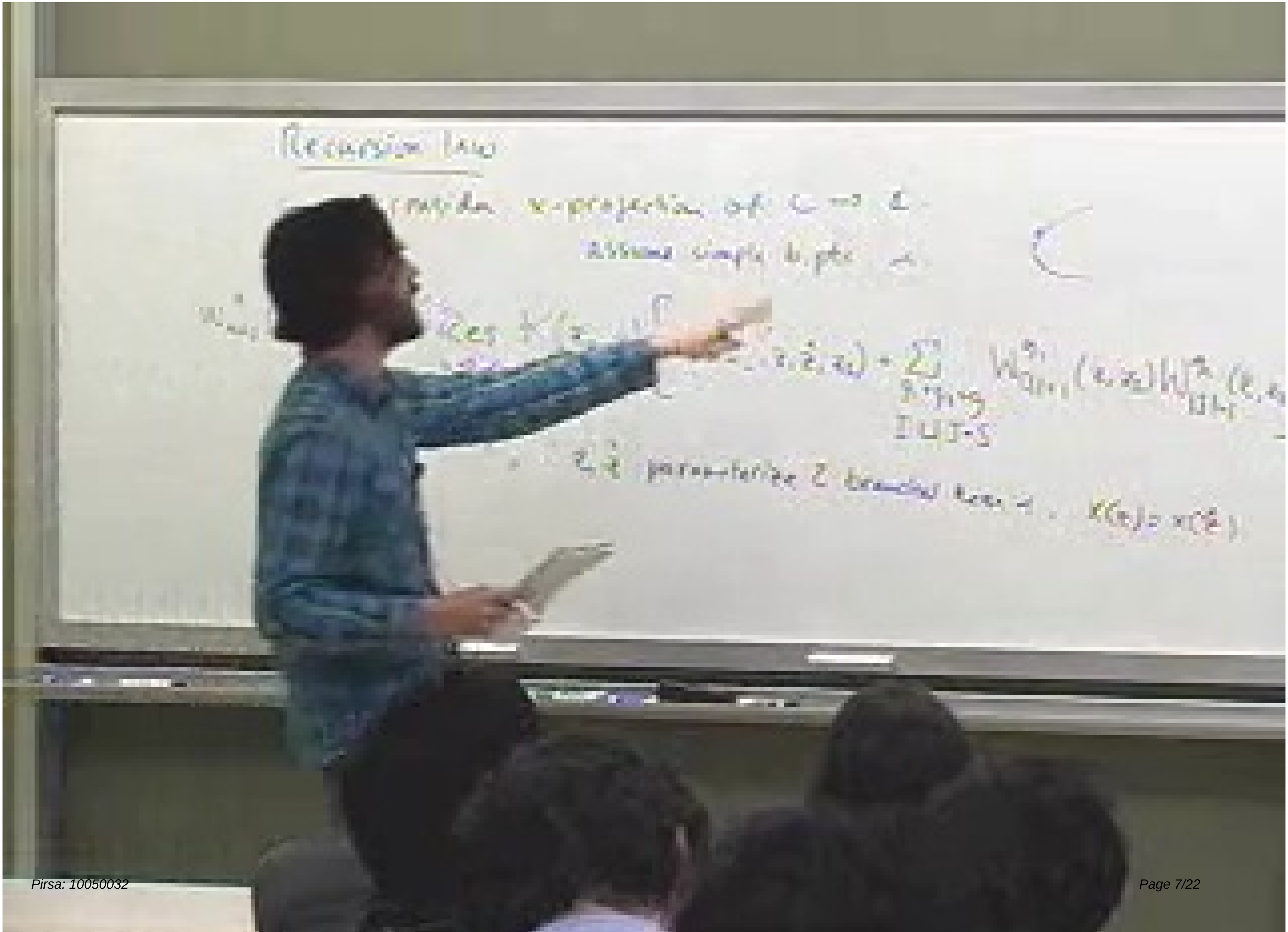

TOPOLOGICAL RECURSION &
MIRROR SYMMETRY

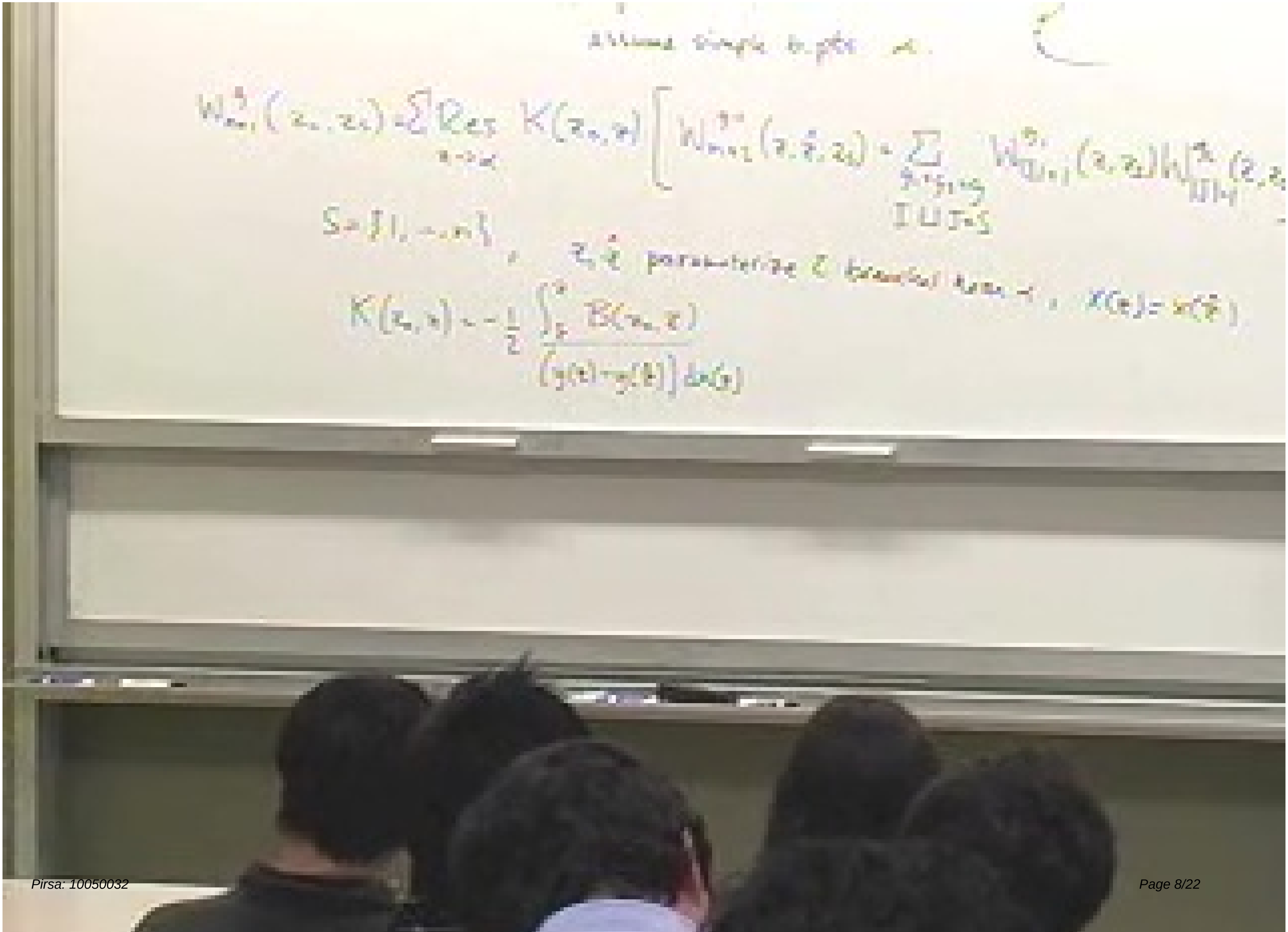

TOPOLOGICAL RECURSION &
MIRROR SYMMETRY
Fibonacci numbers.
Objects: f_n, $n \in \mathbb{Z}$, $n \geq 0$
Initial data: $f_0 = 0$, $f_1 = 1$
Recursion law: $f_{n+2} = f_{n+1} + f_n$


Objects:
$W^g_n(x_1, \dots, x_n)$
$g, n \in \mathbb{Z}$, $g \geq 0$
$n \geq 1$
multilinear diff. forms living on some curve C
genus g
n holes
$-\chi = 2g-2+n$

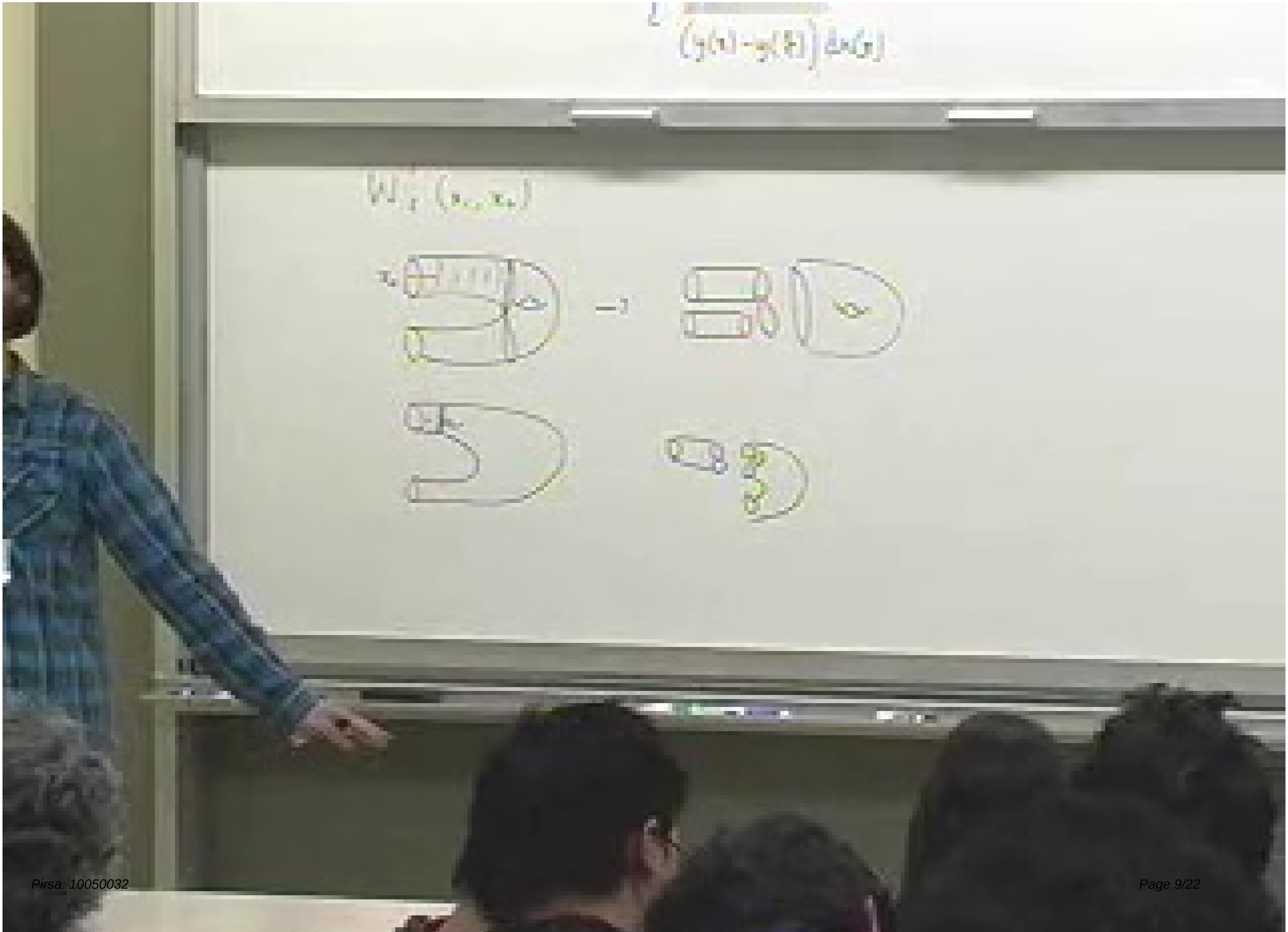

Initial cond:
Riemann surface
(spectral curve)

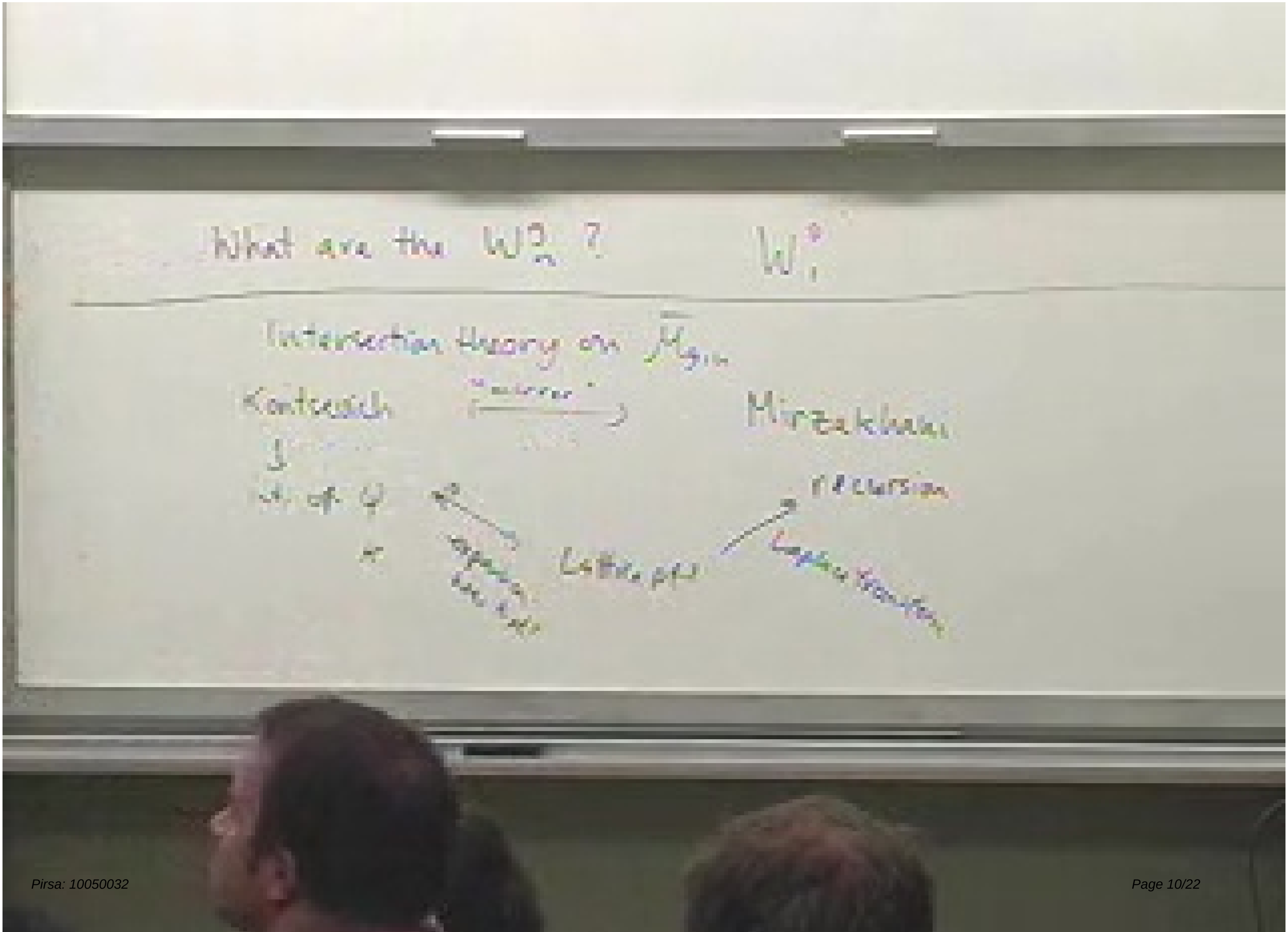

Cauchy kernel
Bergman kernel

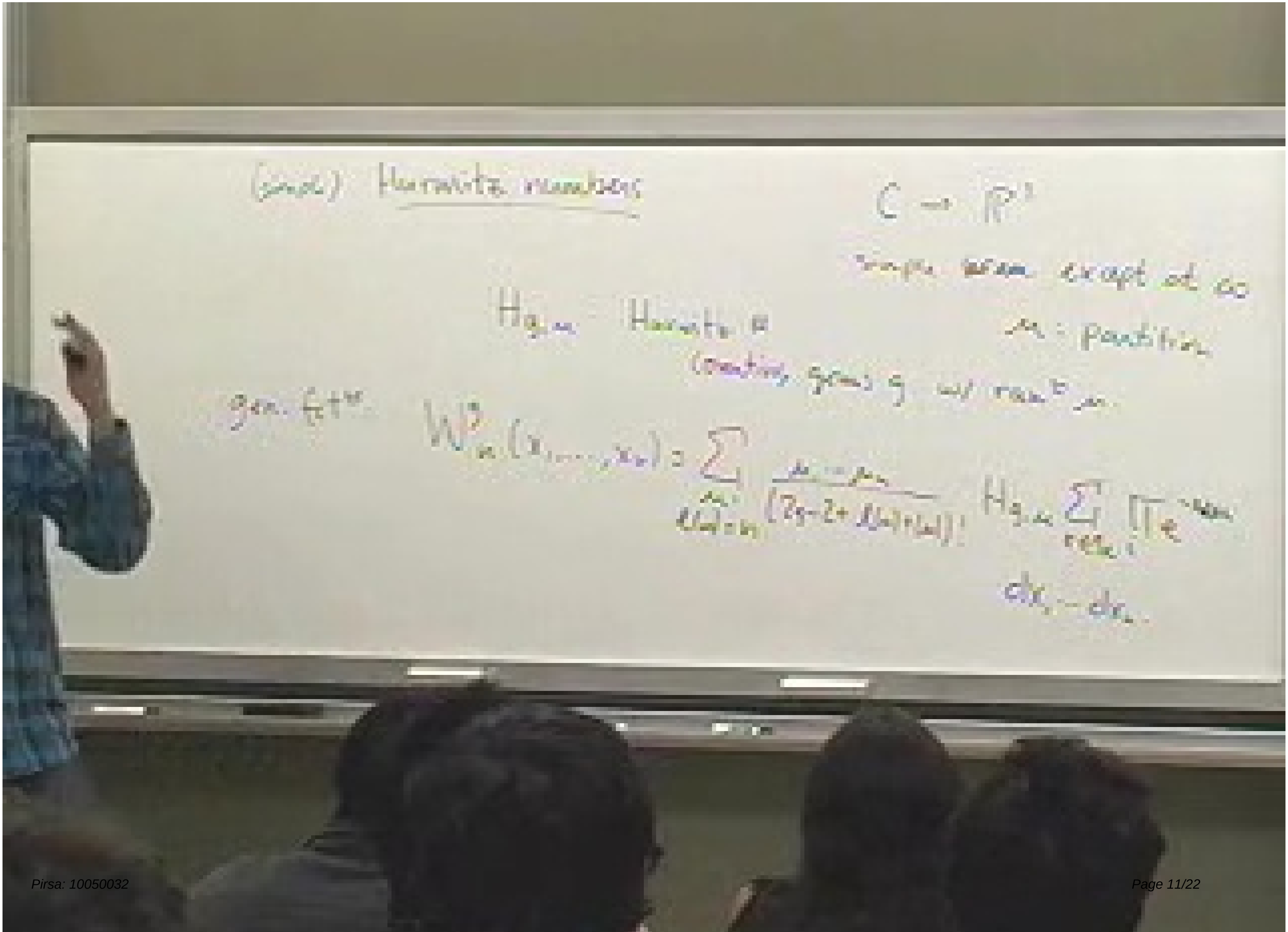

Recursion law

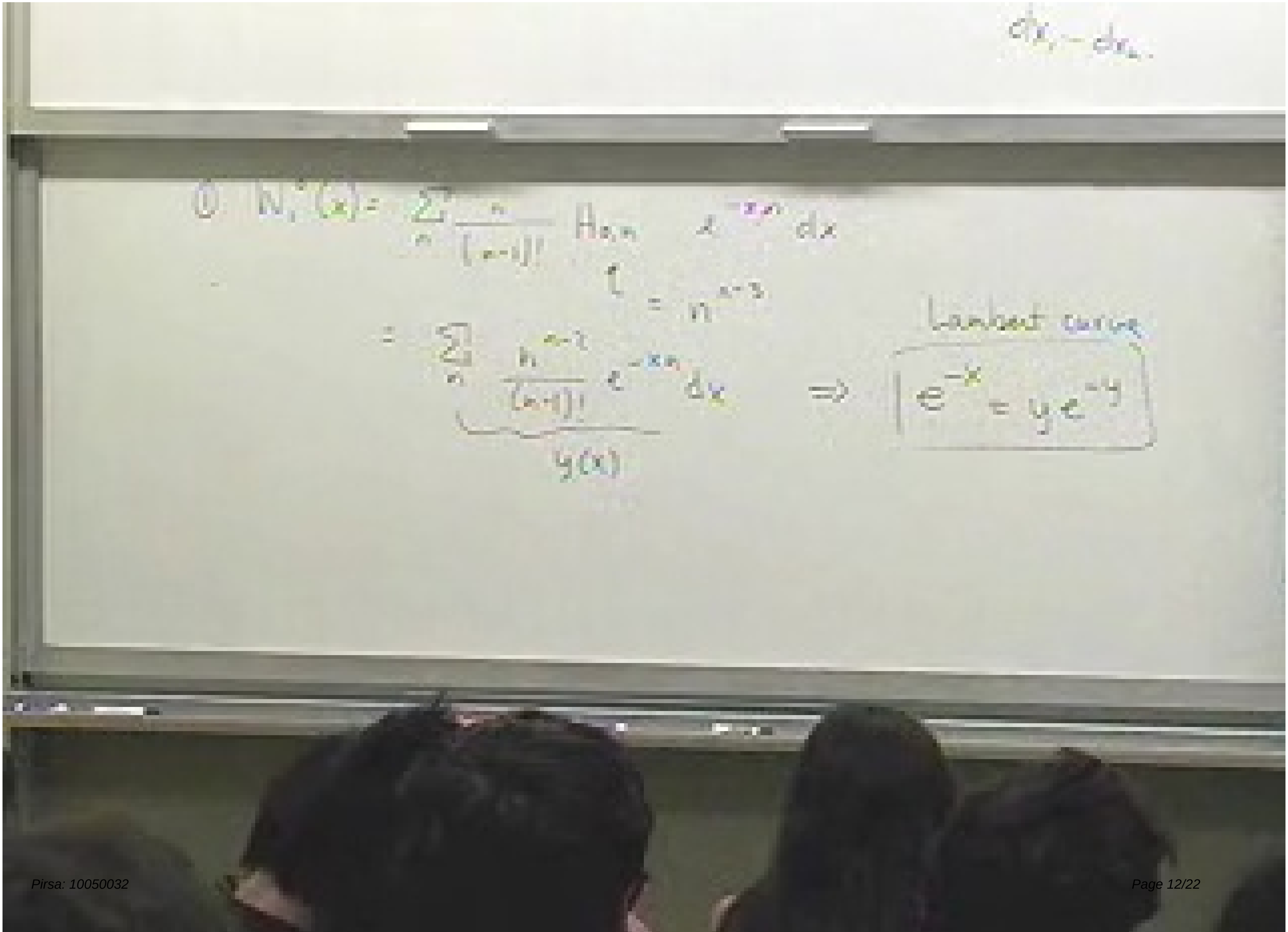


Assume simple b.pts α.
$W^{g}_{n+1}(z_0, z_S) = \sum \operatorname{Res}_{z\to\alpha} K(z_0,z)\left[W^{g-1}_{n+2}(z,\bar z, z_S) + \sum_{\substack{g_1+g_2=g \\ I\sqcup J=S}} W^{g_1}_{|I|+1}(z,z_I)\, W^{g_2}_{|J|+1}(\bar z, z_J) \right.$
$S=\{1,\dots,n\}$, $z,\bar z$ parametrize 2 branches near α, $x(z)=x(\bar z)$
$K(z_0,z) = -\frac{1}{2}\frac{\int_{\bar z}^{z} B(z_0,z)}{(y(z)-y(\bar z))\,dx(z)}$




What are the
Intersection theory on
Kontsevich
Mirzakhani
recursion
Laplace transform


(simple) Hurwitz numbers
C → P^1
simple ram. except at ∞
H_{g,μ} Hurwitz #
μ: partition
(counting genus g w/ ram^n μ)
gen. fct^n
dx_1 ... dx_n


Lambert curve
e^{-x} = ye^{-y}
y(x)

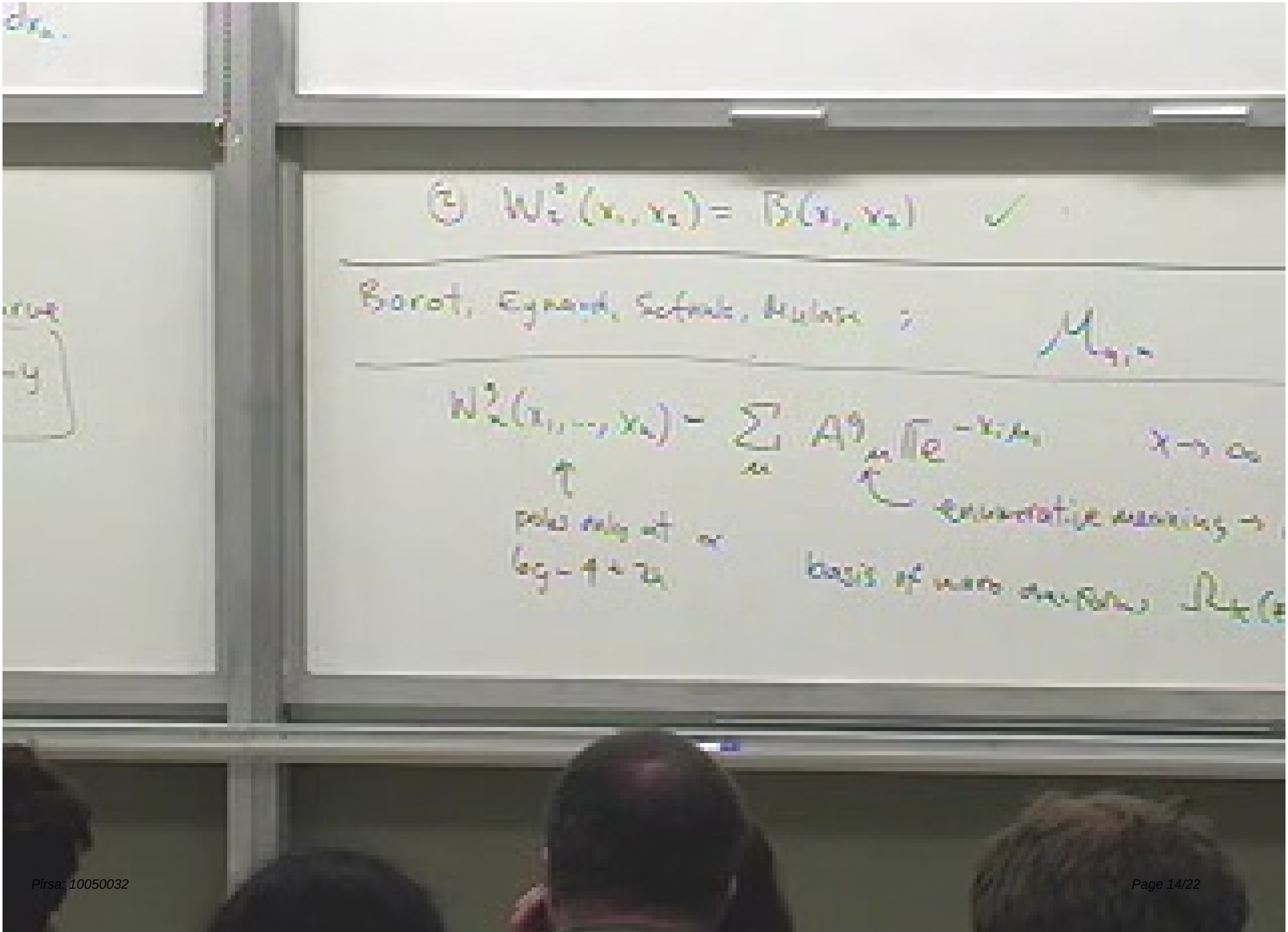
③ $W_2^0(x_1, x_2) = B(x_1, x_2)$ ✓

Borot, Eynard, Safnuk, Mulase ; $\mathcal{M}_{g,n}$

$$W_n^g(x_1, \dots, x_n) = \sum_{\mu} A^g_{\mu} \prod e^{-x_i \mu_i} \qquad x \to \infty$$

poles only at ∞
$\log y = \log x + x_i$

enumerative meaning = Hurwitz #'s


$W_2^0(x_1, x_2) = B(x_1, x_2)$ ✓
Borot, Eynard, Safnuk, Mulase ;
$M_{g,n}$
$x \to \infty$
poles only at ∞
$6g-4+2n$
enumerative meaning →
basis of mero differentials

$dx_1 \cdots dx_n$

$$W_n^g(x_1,\dots,x_n) = \sum_{\substack{k_1\dots k_n \\ \le 3g-3+n}} B_{k_1 \dots k_n} \prod \Phi_{k_i}(x_i)$$

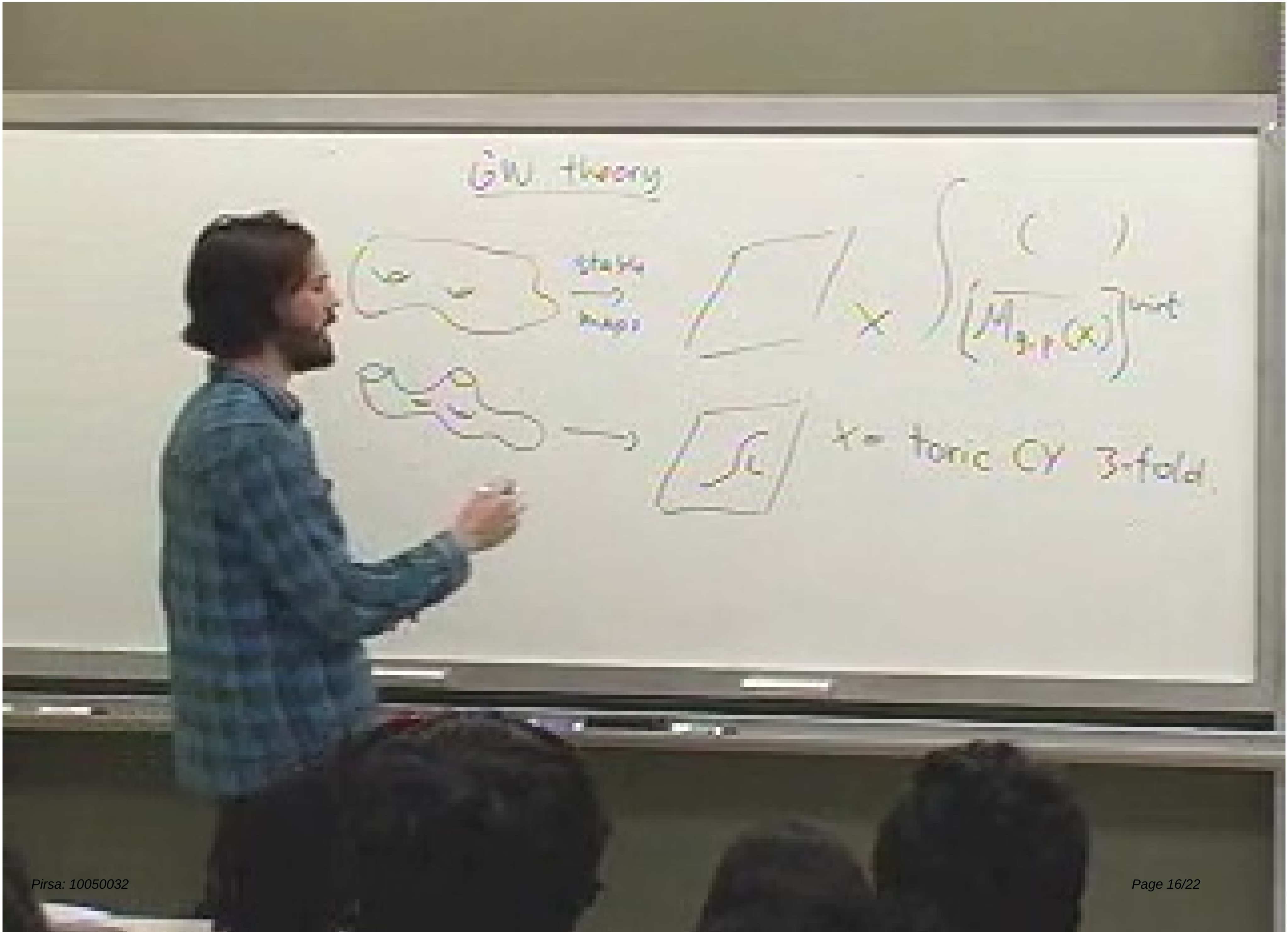
Finite sum

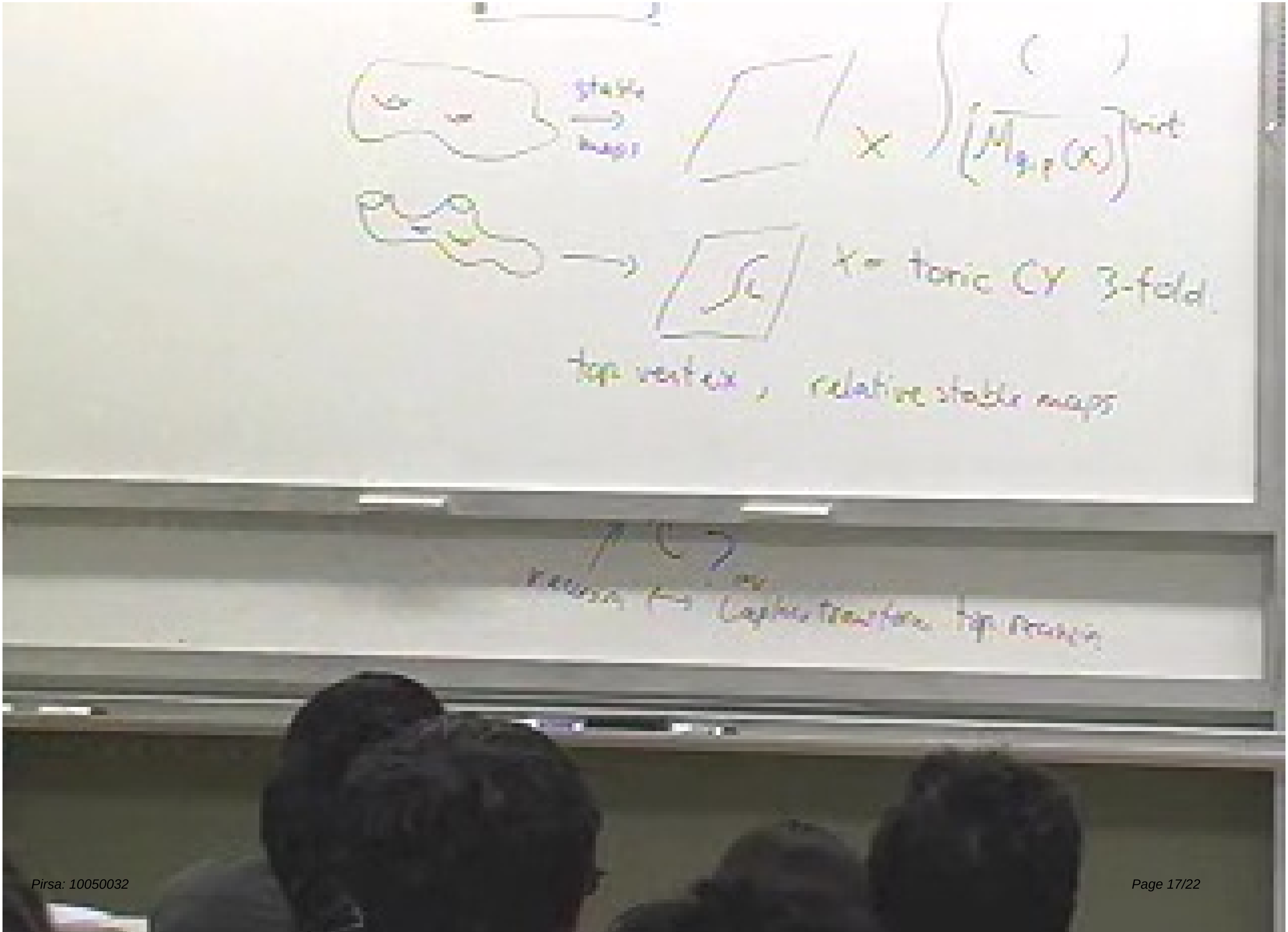
ELSV formula

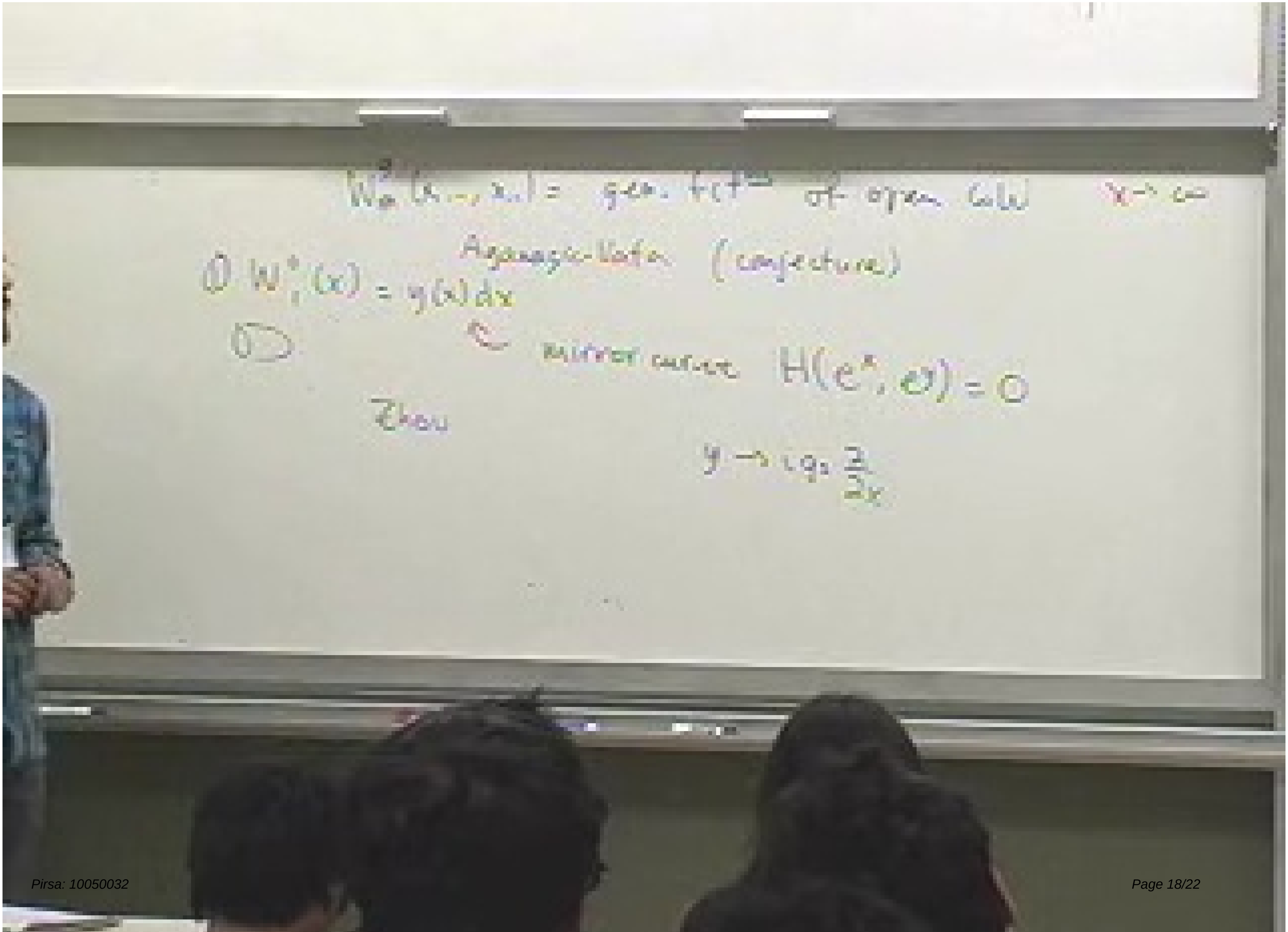
Hodge integrals

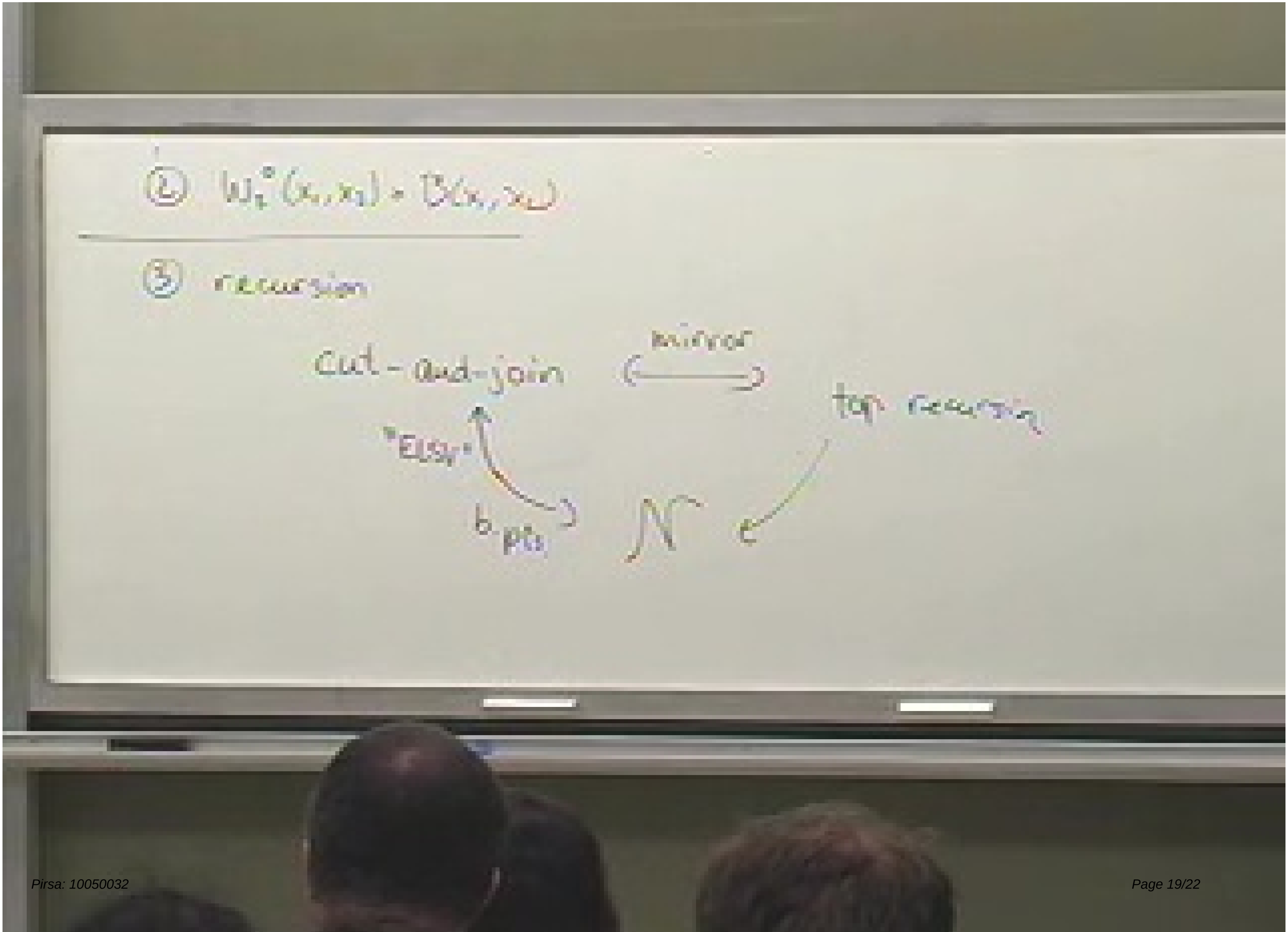
$$W_n^g(x_1,\dots,x_n) = \sum_{\mu} N_{\mu} \prod e^{-\mu_i \sqrt{[illegible]}} \, d\sqrt{[illegible]}$$

recursion for Laplace transform top recursion


GW theory
stable maps
X = toric CY 3-fold


stable maps
$[\overline{M}_{g,n}(X)]^{vir}$
X = toric CY 3-fold
top vertex, relative stable maps
Laplace transform
top recursion


Aganagic-Vafa (conjecture)
① $W^0_1(x) = y(x)dx$
mirror curve $H(e^x, e^y) = 0$
Zhou


② $W_2^0(x_1,x_2) = B(x_1,x_2)$
③ recursion
mirror
cut-and-join
top. recursion
"ELSV"
$\mathcal{N}$