

Title: Topological recursion and mirror symmetry

Date: May 07, 2010 03:15 PM

URL: <http://pirsa.org/10050032>

Abstract: The topological recursion of Eynard and Orantin has found many applications in various areas of mathematics. In this talk I will discuss the recursion from the point of view of Hurwitz numbers and local mirror symmetry. I will explain the mathematics underlying the recursion, its relation with the cut-and-join equation, and explore first steps towards proving (and understanding geometrically) the appearance of the recursion in local mirror symmetry.

TOPOLOGICAL RECURSION 2

MIRROR SYMMETRY

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MIRROR SYMMETRY

Fibonacci numbers.

Objects: f_n , $n \in \mathbb{Z}$, $n \geq 0$

Initial cond: $f_0 = 0$, $f_1 = 1$

Recursion law: $f_{n+2} = f_{n+1} + f_n$

Expts:

$W_n(K, \mathbb{Z}_n)$

$g, n \in \mathbb{Z}, g \geq 0$
 $n \geq 1$

$\left\{ \begin{array}{l} n \\ \text{multibranched diff. forms living on some curve } C \end{array} \right.$



genus g
 n holes $-K = 2g - 2 + n$



Initial value problem

$$x(0) = 1$$
$$y(0) = 0$$
$$y'(0) = 0$$

$$y'' = -y$$

$$y'' + y = 0$$

$$y(0) = 1$$

$$y'(0) = 0$$

let $y(x) = y_0(x) + y_1(x) + y_2(x) + \dots$ → Particular solution $y_p(x)$ (special case)
 $y_0(x) = y(0) = 1$
 $y_1(x) = C e^{-x}$

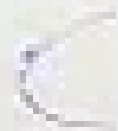
$\lim_{z \rightarrow \infty} \frac{f(z)}{g(z)} = L$ Indeterminate form $\frac{\infty}{\infty}$ (essential singularity)
 $\Rightarrow \lim_{z \rightarrow \infty} \frac{f(z)}{g(z)} = \lim_{z \rightarrow \infty} \frac{f(z)}{g(z)}$

$\lim_{z \rightarrow \infty} \frac{1}{z} = 0$
 $\lim_{z \rightarrow \infty} \frac{1}{z^2} = 0$
 $\lim_{z \rightarrow \infty} \frac{1}{z^n} = 0$

$\lim_{z \rightarrow \infty} \frac{f(z)}{g(z)}$
 Cauchy kernel $\frac{g(z)}{(z-w)^2}$ $\lim_{z \rightarrow \infty} \frac{f(z)}{g(z)} = \lim_{z \rightarrow \infty} \frac{f(z)}{g(z)}$
 $\lim_{z \rightarrow \infty} \frac{f(z)}{g(z)} = \lim_{z \rightarrow \infty} \frac{f(z)}{g(z)}$
 Bergman kernel $B(z, w)$
 $\lim_{z \rightarrow \infty} B(z, w) f(z) = df(w)$

Recursion law

consider representation of $\mathbb{C} \rightarrow \mathbb{C}$
 Assume simple to pts z



Ways

Res + branch

$$f(z) = \sum_{n=0}^{\infty} \frac{W_n^{(z)}(z)}{n!} H_n^{(z)}(z)$$

IUT-S

\mathbb{C} parameter \mathbb{C} branch $z \rightarrow z$ $K(z) = x(z)$

simple poles a

$$W_{n+1}^{\circ}(z, z) = \sum_{a \in S} \text{Res}_{z=a} K(z, a) \left[W_{n+1}^{\circ}(z, \bar{z}, z) = \sum_{\substack{a \in S, b \in S \\ I \cup J = S}} W_{n+1}^{\circ}(z, a) h_{I, J}^{\circ}(z, \bar{z}) \right]$$

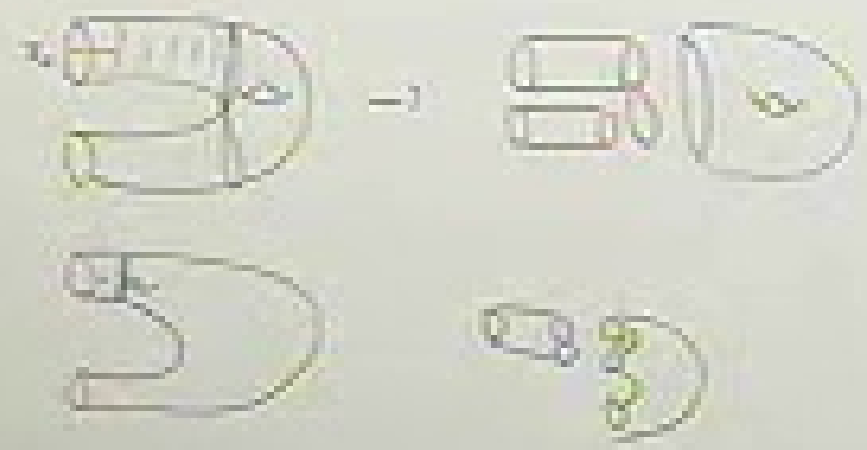
$S = \{1, \dots, n\}$

z is parameterize \mathbb{C} branch curve, $x(\zeta) = x(\bar{\zeta})$

$$K(z, \bar{z}) = -\frac{1}{2} \int_{\gamma} \frac{B(z, \bar{z})}{(y(\zeta) - y(\bar{\zeta}))} d\zeta$$

$$\int (y'(x) - y(x)) dx$$

$y'(x) - y(x)$



What are the W_n^0 ?

W_n^0

Intersection theory on \overline{M}_g

Kontsevich

→

Mirzakhani

↓

vol of \mathcal{Q}

→

Lattice pts

↓

→

(cont.) Harwitz numbers

$$C = \mathbb{P}^1$$

simple zero except at ∞

$H_{g,n}$ = Harwitz #

(counting genus g w/ root ∞)

n = partition

gen. $g \geq 0$

$$W_n(\mathbb{P}^1, \infty) = \sum_{\substack{d_1, \dots, d_n \\ d_1 + \dots + d_n = 2g + 2n}} \frac{1}{(2g + 2n)!}$$

$$H_{g,n} = \sum_{\text{part. } n} \prod_{i=1}^n \frac{1}{d_i!}$$

etc. - etc.

$dx_1 = dx_2$

$$① N_1(x) = \sum_n \frac{n}{(n-1)!} A_{0,n} e^{-xn} dx$$

$$t = n^{x-3}$$

$$= \sum_n \underbrace{\frac{n^{x-1}}{(n-1)!} e^{-xn}}_{y(x)} dx$$

Lambert curve

$$\boxed{e^{-x} = y e^{-y}}$$

$$\textcircled{2} \quad W_2^*(x_1, x_2) = B(x_1, x_2) \quad \checkmark$$

Barot, Eynard, Safarik, Mulase :

$$M_{g,n}$$

$$W_2^*(x_1, \dots, x_n) = \sum_{\mu} A_{\mu}^2 \prod_i e^{-x_i \mu_i} \quad x \rightarrow a$$

↑
 poles only at a
 (eg. $6+7a$)

↑
 enumerative meaning \rightarrow Hurwitz $h(g, n)$

$$\textcircled{2} W_2^*(x_1, x_2) = B(x_1, x_2) \quad \checkmark$$

Borot, Eynard, Sebalk, Melrose :

$M_{g,1}$

$$W_2^*(x_1, \dots, x_n) = \sum_n A_n^g e^{-x_1 x_n} \quad x \rightarrow \infty$$

↑
pole order of x
 $\log - 1 + 2i$

← derivative meaning →
basis of wave functions $\Psi_{g,1}(x)$

$$dV_{k_1} = dr_k$$

$$W_{k_1}(s, \omega) = \sum_{k_2, k_3, \dots, k_n} B_{k_2, \dots, k_n} [W_{k_2}(s)]$$

k_2, k_3, \dots, k_n
 $\neq k_1, \dots$



Finite sum

ELSV formula

Hodge integrals

$$W_{k_1}(s, \omega) = \sum_n N_n \int e^{-s \sqrt{V_{k_1}(s, \omega)}} dV_{k_1}(s, \omega)$$

\nearrow τ \nearrow
 recursion for Laplace transform top recursion

GW theory



stable
→
Kuranishi



X

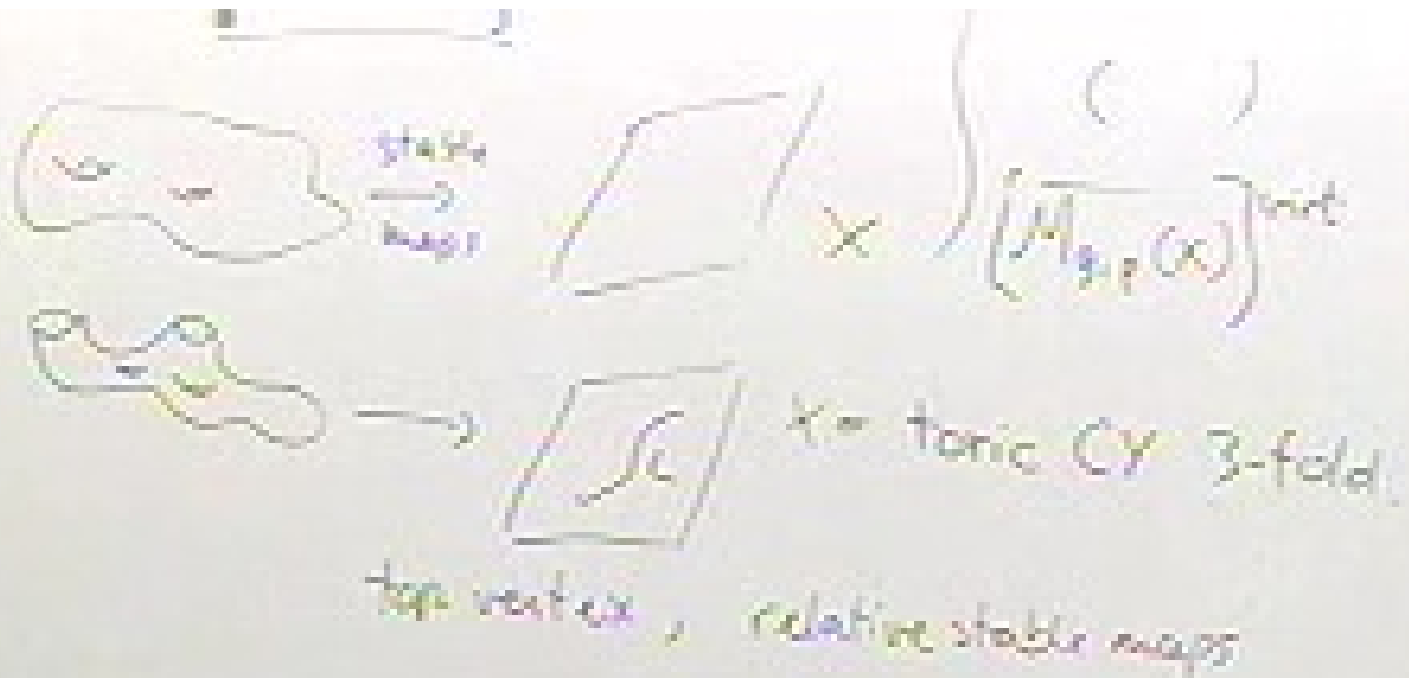
$$\left(\begin{array}{c} () \\ (M_{g,1}(G))^{mod} \end{array} \right)$$



→



$k =$ toric CY 3-fold



$\text{Kac-Moody} \rightarrow \text{Laplace transform} \rightarrow \text{top vertex}$

$W_0(x) = \text{gen. int}^n$ of open CW $x \rightarrow \infty$

Asymptotic behavior (conjecture)

$$\textcircled{1} W_0(x) = y(x) dx$$

$\textcircled{2}$

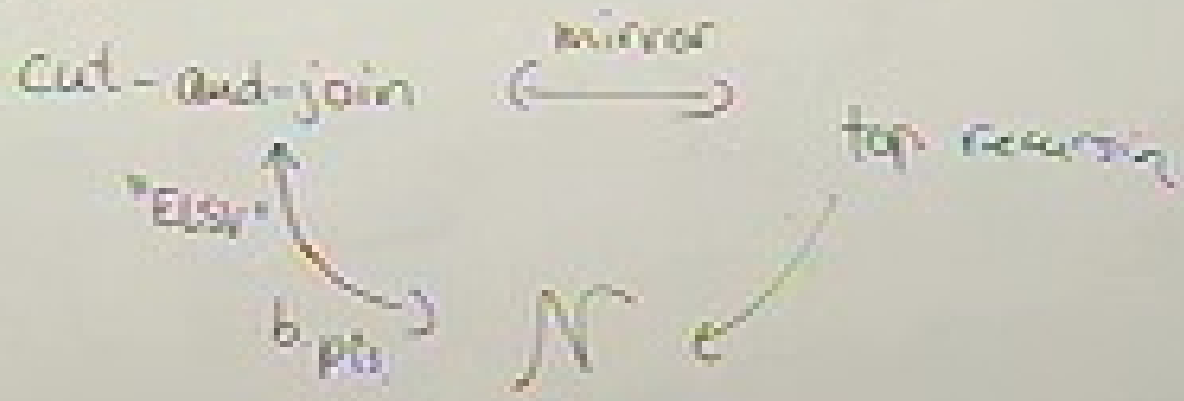
critical curve $H(e^x, e) = 0$

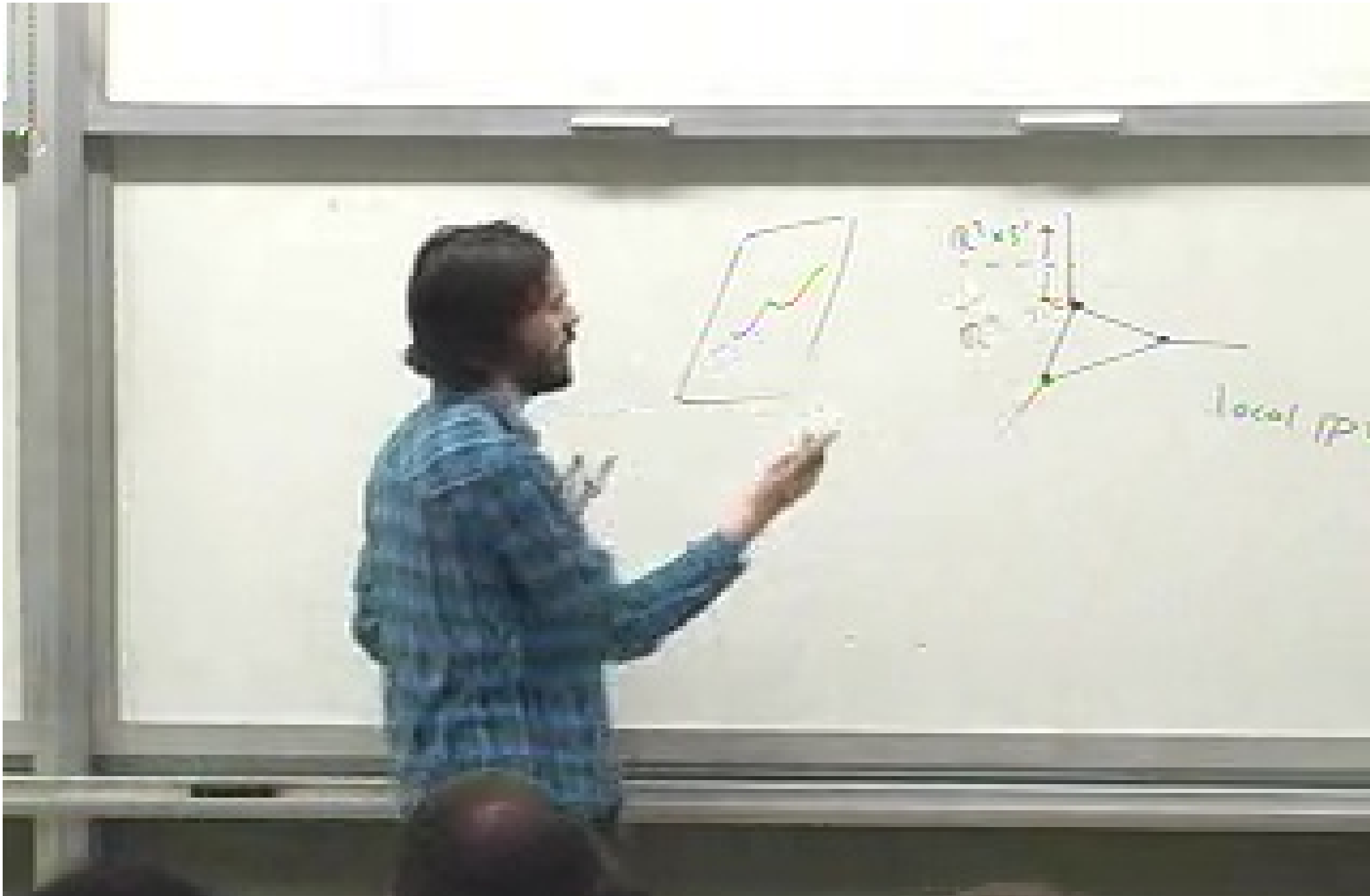
Then

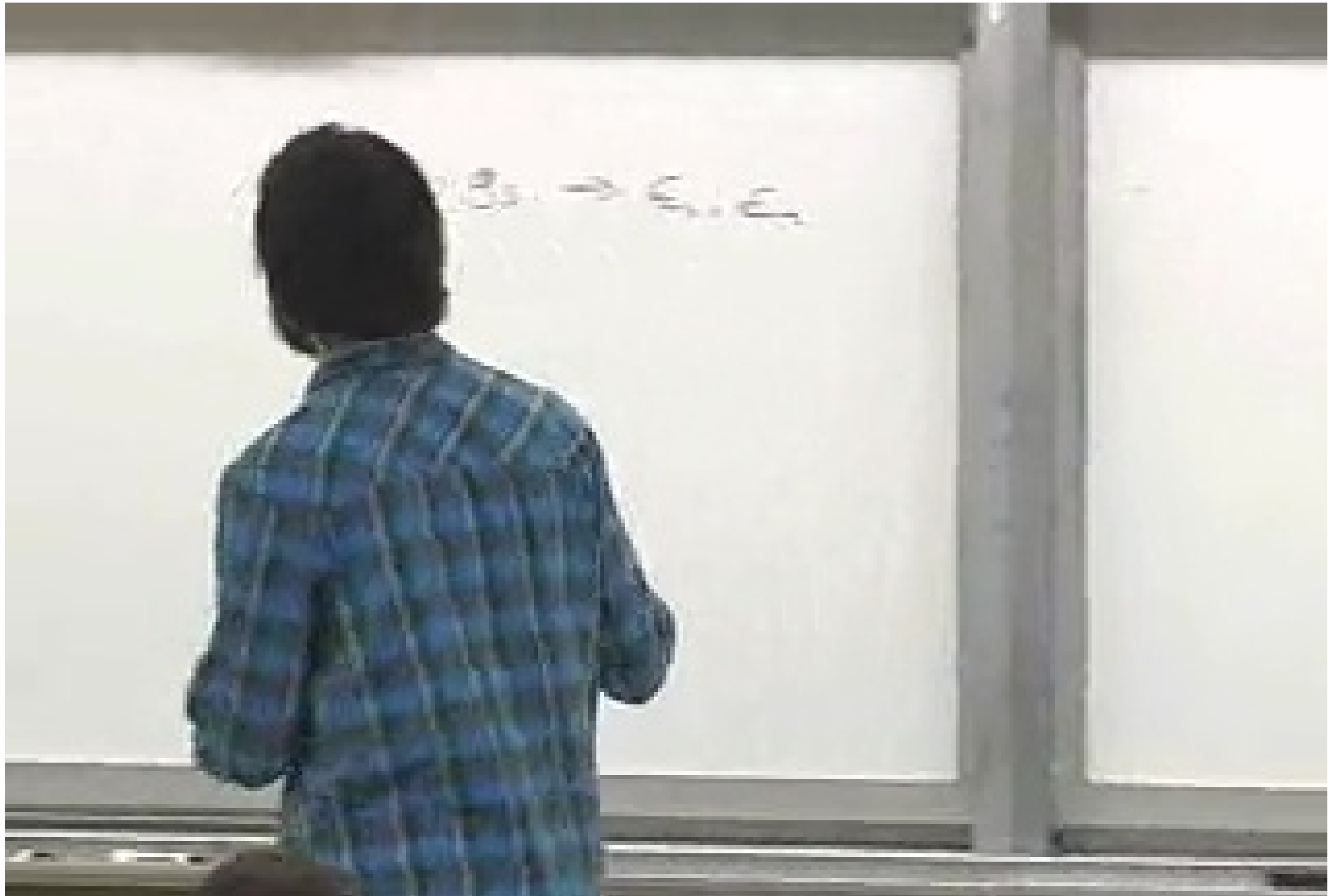
$$y \rightarrow \log \frac{2}{2x}$$

② $W_p(a, b) = B(a, b)$

③ recursion







$$C_{ij} = \sum_k N_{ik} C_{kj}$$