Title: Algebraically special solutions in higher dimensions

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Abstract: The Petrov classification of the Weyl tensor is an important tool in the study of exact solutions of the Einstein equation in 4d. For example, the Kerr solution was discovered in a study of spacetimes with algebraically special Weyl tensors. Algebraic classification of the Weyl tensor has been extended to higher dimensions. I shall review this classification and describe known families of algebraically special solutions. Recent progress towards obtaining a higher dimensional generalization of the Goldberg-Sachs theorem will be described.

Algebraically special solutions in higher dimensions

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Higher dimensional GR

- Known vacuum black hole solutions: Myers–Perry, black rings
- many other topologies appear permissible
- rigidity theorem gives single rotational symmetry, less than known solutions
- approximate methods suggest existence of many new solutions
- need to develop new methods for solving Einstein equation
- most important 4d technique: algebraically special solutions

Petrov classification

- At any point of a 4d spacetime, a (non-zero) Weyl tensor defines 4 *Principal Null Directions*. If 2 or more of these coincide everywhere then the spacetime is algebraically special.
- Kerr solution is "type D": two pairs of coincident null directions. Discovered in a search for algebraically special solutions.
- The algebraically special property can be used to help solve Einstein's eq. e.g. Kinnersley determined all type D vacuum solutions: includes Kerr-NUT, spinning C-metric.

de Smet classification de Smet 03

4d: can classify Weyl tensor using spinors. Equivalent to method based on null vectors.

5d: define

$$P_{\alpha\beta\gamma\delta} = C_{abcd}(C\Gamma^{ab})_{\alpha\beta}(C\Gamma^{cd})_{\gamma\delta}$$

Let ψ^α be (Dirac) spinor. $P_{\alpha\beta\gamma\delta}\psi^\alpha\psi^\beta\psi^\gamma\psi^\delta$ is a homogeneous degree 4 polynomial in 4 complex variables (quartic in CP^3). Classify Weyl tensor according to whether, and how, this factorizes.

- Myers-Perry: square of a quadratic polynomial
- Black string: product of quadratic polynomials

$d > 5$: spinors not useful for classifying Weyl tensor?

CMPP Classification Coley, Milson, Pravda & Pravdova 04

In a d-dimensional spacetime, introduce a *null basis*

$$\ell \equiv e_0, \qquad n \equiv e_1, \qquad m_i \equiv e_i, \qquad i = 2 \dots d-1,$$

$$\ell^2 = n^2 = \ell \cdot m_i = n \cdot m_i = 0, \qquad \ell \cdot n = 1, \qquad m_i \cdot m_j = \delta_{ij}.$$

Bases related by $SO(1, d-1)$ transformations, including *boosts*:

$$\ell \to \lambda \ell, \qquad n \to \lambda^{-1} n, \qquad m_i \to m_i$$

A tensor component has *boost weight* w if it scales as λ^w
e.g. Weyl tensor: C_{0i0j} has $w = 2$, C_{0ijk} has $w = 1$, ...
($w =$ number of "0" indices minus number of "1" indices)

WANDs

Definition

ℓ^a is a *Weyl Aligned Null Direction* (WAND) iff all $w = 2$ Weyl components vanish everywhere.

- Basis independent: equivalent to $\ell_{[a} C_{b]cd[e} \ell_{f]} \ell^c \ell^d = 0$.
- $d = 4$: WAND $\equiv$ Principal Null Direction $\implies \exists$ exactly 4
- $d > 4$: WANDs need not exist, e.g., "static Kaluza-Klein bubble" $ds^2 = -dt^2 + ds^2(\mathrm{Euclidean\,Schwarzschild})$
- WANDs need not be discrete, e.g., $dS_3 \times S^2$: *any* null vector field tangent to dS_3 is a WAND

Multiple WANDs

Definition

ℓ^a is a *multiple WAND* iff all $w = 2, 1$ Weyl components vanish everywhere. A spacetime is *algebraically special* if it admits a multiple WAND.

- Basis independent
- $d = 4$: multiple WAND $\equiv$ repeated Principal Null Direction

Classification of the Weyl tensor

Definition

The spacetime/Weyl tensor is of type

- G $\iff$ $\nexists$ WAND
- I $\iff$ $w = 2$ Weyl cpts vanish (ℓ WAND)
- II $\iff$ $w = 2, 1$ Weyl cpts vanish (ℓ multiple WAND)
- III $\iff$ $w = 2, 1, 0$ Weyl cpts vanish (ℓ multiple WAND)
- N $\iff$ $w = 2, 1, 0, -1$ Weyl cpts vanish (ℓ multiple WAND)
- D $\iff$ $w = 2, 1, -1, -2$ cpts vanish (ℓ, n multiple WANDs)
- O $\iff$ $C_{abcd} = 0$

Examples

- Schwarzschild, black string, Myers-Perry, $dS_3 \times S^2$, all type D
- pp-waves are type N
- Black ring: type G in one open subset, type I in another open subset Pravda & Pravdova 05 $\Longrightarrow$ type I/G distinction not useful (not true for algebraically special types - proof?)

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Goldberg-Sachs theorem

In 4d, starting point in using algebraically special property to solve Einstein eq is

Theorem

For $d = 4$ Einstein spacetime (not type O), ℓ^a is a multiple WAND iff it is geodesic and shearfree.

Extension to $d > 4$ cannot be straightforward:

- Multiple WAND needn't be geodesic, e.g. $dS_3 \times S^2$: *any* null vector field tangent to dS_3 is multiple WAND
- Multiple WAND can be geodesic but shearing e.g. black string $ds^2 = ds^2(\mathrm{Schw.}) + dz^2$: multiple WAND = repeated PND of Schw. $\implies$ expands in Schw. directions but not z-direction $\implies$ shearing.

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Towards $d > 4$ Goldberg-Sachs

Can we generalize "geodesic part" of Goldberg-Sachs to higher dimensions?

- PPCM 04: the multiple WAND of a type III or type N Einstein spacetime is geodesic.

Theorem

Durkee & HSR 09: In an Einstein spacetime, $\exists$ a multiple WAND iff $\exists$ a geodesic multiple WAND.

Proof.

$\Leftarrow$ trivial.
$\Rightarrow$ assume $\exists$ non-geodesic multiple WAND, appeal to following theorem. □

Theorem (Durkee & HSR 09)

An Einstein spacetime admitting a non-geodesic multiple WAND is foliated by constant curvature, totally umbilic, Lorentzian submanifolds of dimension 3 or greater, and any null vector field tangent to the leaves of the foliation is a multiple WAND.

"Totally umbilic": equivalent definitions

- Extrinsic curvature $K^a{}_{bc} = \xi^a h_{bc}$ where $\xi^a \perp$ submanifold, h_{ab} induced metric.
- Any null geodesic of submanifold is a null geodesic of full spacetime.

Hence any geodesic null vector field of submanifolds gives a geodesic multiple WAND of spacetime. Example: $dS_3 \times S^2$.

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Idea of proof.

Einstein spacetime $\implies$ Weyl tensor obeys Bianchi identity.
For non-geodesic multiple WAND, Bianchi $\implies$ strong restrictions on certain $w = 0$ Weyl components.
These restrictions plus Bianchi imply that can perform a *null rotation* (Lorentz transformation fixing ℓ) to make n a multiple WAND $\implies$ type D.
Now perform null rotation about n:

$$\ell' = \ell - z_i m_i - \frac{1}{2} z_i z_i n, \qquad n' = n, \qquad m_i' = m_i + z_i \ell$$

Restrictions on Weyl $\implies \exists\; n \geq 1$-dimensional space of solutions z_i s.t. ℓ' is multiple WAND. ℓ', n' and $z_i m_i$ define a $n+2$ dimensional distribution. Turns out to be integrable $\implies$ submanifolds. Restrictions on Weyl $\implies$ totally umbilic, constant curvature.

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Stronger result in 5d

In 5d, restrictions on Weyl are so strong that we can prove

Theorem

A 5d Einstein spacetime admitting a non-geodesic multiple WAND is locally isometric to one of

1. *Minkowski or (anti-)de Sitter spacetime.*
2. $dS_3 \times S^2$ *or* $adS_3 \times H^2$
3. $ds^2 = r^2 ds_3^2 + \frac{dr^2}{f(r)} + f(r) d\phi^2, \qquad f(r) = k - \frac{\mu}{r^2} + \frac{\Lambda}{3} r^2$ *where* ds_3^2 *is the metric on a Lorentzian space of constant curvature of sign* $k \in \{-1, 0, 1\}$.

Special cases of (3):

- Kaluza-Klein bubble ($k = 1$, $\Lambda = 0$) Witten 82
- AdS soliton ($k = 0$, $\Lambda < 0$) Horowitz & Myers 99

$d > 5$ solutions with non-geodesic multiple WAND

For $d > 5$ $\exists$ many solutions with non-geodesic multiple WAND
Example: consider $d = 6$ static axisymmetric solution ($\exists$ many!)

$$ds^2 = -A(r,z)^2 dt^2 + B(r,z)^2(dr^2 + dz^2) + C(r,z)^2 d\Omega_3^2$$

Analytically continue $t = i\phi$ and $d\Omega_3^2 \to ds^2(dS_3)$:

$$ds^2 = A(r,z)^2 d\phi^2 + B(r,z)^2(dr^2 + dz^2) + C(r,z)^2 ds^2(dS_3)$$

any null vector field tangent to dS_3 is a multiple WAND.

"Shear part" of Goldberg-Sachs

A geodesic multiple WAND need not be shearfree. But Bianchi and Ricci identities constrain possible form of "optical matrix" $\rho_{ij} = \nabla_j \ell_i$ (i.e. shear+rotation+expansion). Can we determine most general solution? What about converse?
(Work in progress with Durkee, Pravda & Pravdova.)

Example: type N Pravda et al 04

Warm-up: consider Maxwell $(p+1)$-form field $F_{abc\ldots}$ instead. F is algebraically special iff ℓ is *multiply aligned* with F, i.e., boost weight 1,0 components vanish.

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Algebraically special Maxwell fields

Theorem (Mariot-Robinson)

In a 4d spacetime, ℓ is multiply aligned with a (non-zero) 2-form solving the Maxwell equations if, and only if, it is geodesic and shearfree.

Note: same result as for ℓ to be multiply aligned with Weyl tensor!

Let $\rho_{ij} = \nabla_j \ell_i$ (shear+rotation+expansion), $\rho \equiv \rho_{ii}$

Theorem (Durkee, Pravda, Pravdova & HSR 10)

If ℓ is multiply aligned with a non-zero $(p+1)$-form solving the Maxwell equations then $\rho_{(ij)}$ has p eigenvalues which sum to $\rho/2$.

For multiple WAND in Schwarzschild: all eigenvalues of $\rho_{(ij)}$ are $\rho/(d-2)$. So ℓ can be multiply with both Weyl and $(p+1)$-form iff $d = 2(p+1)$ (e.g. 2-form in 4d).

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Algebraically special, axisymmetric, solutions Godazgar & HSR 09

Definition

A d-dimensional spacetime is *axisymmetric* if $\exists$ a $SO(d-2)$ isometry group with S^{d-3} orbits.

For $d > 4$, action of $SO(d-2)$ is "orthogonally transitive", i.e.,

$$ds^2 = g_{ab}(x)dx^a dx^b + r(x)^2 d\Omega_3^2$$

Examples: Schwarzschild, black string, non-uniform black string, Randall-Sundrum black hole.

- might be time-dependent
- most general static axisymmetric vacuum solution not known for $d > 4$
- goal: determine general algebraically special axisymmetric Einstein spacetime

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Kramer-Neugebauer solutions (1968)

4d analogue of $d > 4$ axisymmetric spacetimes: spacetimes admitting hypersurface $\perp$ spacelike Killing vector field. If algebraically special then vacuum spacetime with this symmetry must be either

- Robinson-Trautman: $\exists$ null geodesic congruence with vanishing shear and rotation, non-vanishing expansion (includes Schwarzschild, C-metric)
- Kundt: $\exists$ null geodesic congruence with vanishing expansion, rotation and shear.

(For $d > 4$, Robinson-Trautman is very restrictive Podolsky & Ortaggio 06)

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Results

Assume spacetime is Einstein with multiple WAND ℓ^a, wlog geodesic.

Case 1. ℓ^a not axisymmetric. Act with $SO(d-2) \Longrightarrow$ continuous family of multiple WANDs. Only solutions:

- KK bubble
  $ds^2 = r^2\left(-dt^2 + \cosh^2 t\, d\Omega^2_{d-3}\right) + \frac{dr^2}{f(r)} + f(r)d\phi^2,$
  $f(r) = 1 - \frac{\mu}{r^{d-3}} - \Lambda r^2$
- $dS_{d-2} \times S^2$

Both are type D.

Case 2. ℓ^a axisymmetric. All solutions:

- generalized Schwarzschild (type D)
- generalized black string (type D)
- axisymmetric Kundt solutions (type II or more special)

Axisymmetric Kundt solutions

$$\begin{aligned} ds^2 &= -U(v,r,z)dv^2 + 2dvdr + 2C(v,r,z)dvdz \\ &+ D(v,r,z)^2 dz^2 + E(v,r,z)^2 d\Omega_{d-3}^2 \end{aligned}$$

$\ell = \partial/\partial r$. Solutions involves arbitrary functions of v, r-dependence fixed, z-dependence reduced to ODEs.

- Type N explicit: gravitational waves in Minkowski or (anti)-de Sitter.
- No type III.
- Type D: $ds^2 = dz^2 + A(z)^2 d\Sigma_2^2 + R(z)^2 d\Omega_{d-3}^2$, $d\Sigma_2^2$ is metric on 2d Lorentzian space of constant curvature (cf Böhm metrics). AdS/CFT applications.
- Type II: gravitational waves in type D?

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Geroch-Held-Penrose formalism

4d Newman-Penrose formalism involves writing out all components of Bianchi and Ricci equations in a null basis $\{\ell, n, m, \bar{m}\}$. All derivatives are partial derivatives ($D \equiv \ell^a \partial_a$ etc)

Often interested in spacetimes with 2 preferred null directions, e.g., type D or spacetimes with preferred family of 2 surfaces: don't care about choice of $m, \bar{m}$.

GHP formalism: "improved" version of NP that is covariant under rotations of spatial basis: $m \to e^{i\theta} m$ and under boosts $\ell \to \lambda \ell$, $n \to \lambda^{-1} n$. Discrete symmetries hugely reduce number of equations.

Higher-dimensional GHP formalism Durkee, Pravda, Pravdova, HSR 10

Straightforward to extend GHP to higher dimensions: introduce new derivative operators þ, þ$'$, ð$_i$, write out components of Bianchi and Ricci.

4d GHP streamlines analysis of perturbations of Kerr spacetime, or general type D Stewart & Walker 74

$d > 4$ GHP also useful for analyzing perturbations of type D spacetimes e.g. Myers-Perry

Perturbations of higher-dimensional spacetimes Durkee & HSR

For algebraically special spacetime, $\Omega_{ij} \equiv C_{0i0j}$ is a traceless symmetric matrix that

- Is invariant under infinitesimal diffeomorphisms and infinitesimal changes of basis
- Has the same number of components as the number of degrees of freedom of the graviton

For $d = 4$, $\Omega_{ij} \sim \Psi_0$: Teukolsky scalar, satisfies decoupled equation. Does Ω_{ij} decouple for $d > 4$?

Decoupling of perturbations

$d = 4$: Ω_{ij} decouples iff ℓ^a geodesic and shearfree: guaranteed by Goldberg-Sachs!

$d > 4$: decoupling requires that ℓ^a be geodesic, and free of expansion, rotation and shear.

- no decoupling for Myers-Perry, or even Schwarzschild
- decoupling does occur for the *near-horizon geometry* of *extreme* Myers-Perry

Outlook

Much to do:

- Generalize "shearfree" part of Goldberg-Sachs to higher dimensions
- Determine all algebraically special solutions in certain classes or with certain symmetries
- Study perturbations of near-horizon extreme Myers-Perry
- What can be done with de Smet scheme?

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