

Title: Current algebra in conformal sigma models on supergroups (Part II)

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Abstract: Two-dimensional non-linear sigma models on some supergroup manifolds are conformal field theories whether the action includes a Wess-Zumino term or not. These models are relevant for the worldsheet description of string theory in Anti-de Sitter backgrounds with Ramond-Ramond fluxes. The current algebra is an useful tool to study these theories. In these lectures I will review the construction of the current algebra. Then I will discuss some applications to the computation of the spectrum and integrability.

Current algebra in conformal σ-model on superstrings - part 2



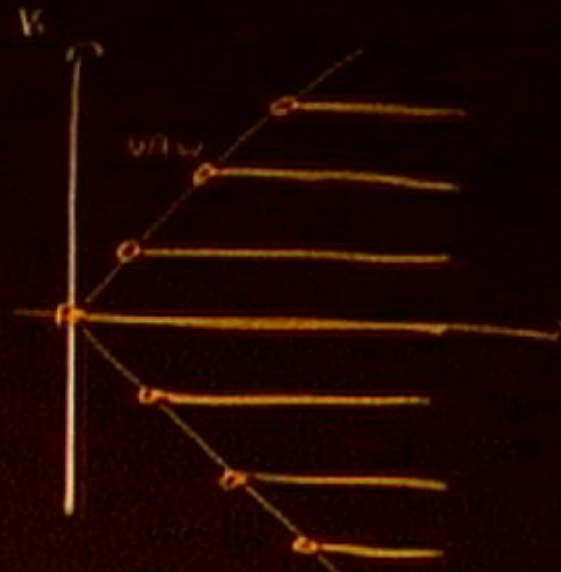
left current:
$$\begin{aligned} \partial &= c \partial_y g^{\mu\nu} & c &= -\frac{14\alpha'}{2\alpha'} \\ \bar{\partial} &= \bar{c} \bar{\partial}_y g^{\mu\nu} & \bar{c} &= -\frac{1-4\alpha'}{2\alpha'} \end{aligned}$$

$$\left\{ \begin{aligned} \partial^\mu(z) \bar{\partial}^\nu(w) &= -\frac{c^2}{c\alpha'} \frac{14\alpha'}{2-4\alpha'} + i f^{\mu\nu} c \frac{c(c+2\alpha')}{(c+\alpha')^2} \frac{\partial^\mu(z) \bar{\partial}^\nu(w)}{2-4\alpha'} + i f^{\mu\nu} \frac{c}{(c+\alpha')^2} \frac{\partial^\mu(z) \bar{\partial}^\nu(w)}{2-4\alpha'} + \dots \\ \partial^\mu(z) \partial^\nu(w) &= \dots \\ \bar{\partial}^\mu(z) \bar{\partial}^\nu(w) &= \dots \end{aligned} \right.$$

Current algebra in conformal σ -models on supergroups - part 2

left current:
$$j = c \partial_{\bar{z}} g g^{-1} \quad c = -\frac{1 + \kappa \ell^2}{2\ell^2}$$

$$\bar{j} = \bar{c} \partial_z g g^{-1} \quad \bar{c} = -\frac{1 - \kappa \ell^2}{2\ell^2}$$

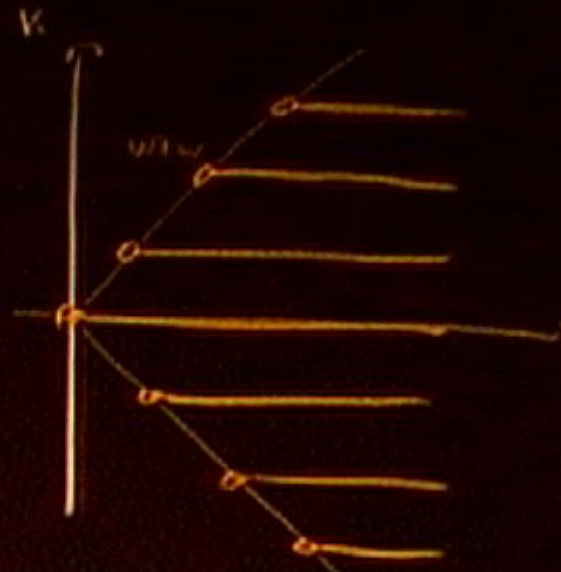


$$\left\{ \begin{aligned} j^a(z) j^b(w) &= -\frac{c^2}{c_1 c_2} \frac{\kappa^{ab}}{(z-w)^2} + i f^{ab} c \frac{c_1(z+\bar{z})}{(c_1 c_2)} \frac{j^c(w)}{z-w} + i f^{ab} c \frac{c^2}{(c_1 c_2)^2} \bar{j}^c(w) \frac{\bar{z}-w}{(z-w)^2} + \dots \\ j^a(z) \bar{j}^b(w) &= \dots \\ \bar{j}^a(z) \bar{j}^b(w) &= \dots \end{aligned} \right.$$

Current algebra in conformal σ -models on supergroups - part 2

left currents:
$$j = c \partial_j g j^{-1} \quad c = -\frac{1 + \nu \beta^2}{2\beta^2}$$

$$\bar{j} = \bar{c} \bar{\partial}_j g j^{-1} \quad \bar{c} = -\frac{1 - \nu \beta^2}{2\beta^2}$$



$$\begin{cases} j^a(z) j^b(w) = -\frac{c^2}{c^2} \frac{\kappa^{ab}}{(\nu \cdot \nu)^2} + i f^{ab} c \frac{c(\nu \cdot z)}{(c^2 \nu^2)} \frac{j^c(w)}{z-w} + i f^{ab} c \frac{c^2}{(c^2 \bar{c})^2} \bar{j}^c(w) \frac{\bar{z}-\bar{w}}{(\bar{z}-\bar{w})^2} + \dots \\ j^a(z) \bar{j}^b(w) = \dots \\ \bar{j}^a(z) j^b(w) = \dots \end{cases}$$



Introduction

Spectrum at WZW points

$$G_L \rightarrow \hat{G}_L$$

$$J(z) = \sum_n \frac{J_{-n}}{z^{n+2}}$$

$$G_L \rightarrow \widehat{G}_L$$

Hilbert space: $\{ \text{Highest weight reps of } \widehat{G}_L \}$

Point at infinity

$$D(z) = \sum_{n=0}^{\infty} \frac{J_{-n}}{z^{n+1}}$$

$$G_L \rightarrow \hat{G}_L$$

Hilbert space: $\{ \text{Highest weight reps of } \hat{G}_L \}$



Expansion at $w \neq 0$ point:

$$J(z) = \sum_n \frac{J_{-n}}{z^{n+2}}$$

$$G_L \rightarrow \hat{G}_L$$

Hilbert space: $\{ \text{Highest weight reps of } \hat{G}_L \}$

$$\Rightarrow J_{-n} \dots J_{-1} J_0 \phi$$

Spectrum at WZW point =

$$G_L \rightarrow \hat{G}_L$$

Hilbert space: $\{ \text{Highest weight reps of } \hat{G}_L \}$

$$\simeq \{ \int \int \dots \int \phi \}$$

$$D(z) = \sum_n \frac{J_{-n}}{z^{n+2}}$$

$$\approx \{ \overline{I_{-p}} \cdot \overline{I_{-q}} \cdot I_{n_1} \cdot I_{-n} \phi \}$$

The Hilbert away from $w \neq w$ points

$$\approx \{ \underbrace{\mathbb{I}_{-p} \mathbb{I}_{-q} \dots \mathbb{I}_{-n}}_{\text{...}} \phi \}$$

The Hilbert away from $\omega \approx \omega$ points

... Field? $\phi_{\mathbb{R}} = g_{\mathbb{R}}$

... rents:

$$\approx \{ \underbrace{\prod_{p=1}^{\infty} \mathbb{Z}_p} \cdot \mathbb{Z}_N \cdot \mathbb{Z}_M \cdot \phi \}$$

The Hilbert away from $w \neq w$ points

primary field? $\phi_R = g_R$

resents: $\mathbb{Z}_{-1} = \left(\left(\mathbb{Z}_{-1}, \mathbb{Z}_{-1} \right) \mathbb{Z}_{-1} \dots \right)$

$$\approx \{ \underbrace{\prod_{-p} \mathbb{I}_p \cdot \mathbb{I}_n \cdot \mathbb{I}_n} \phi \}$$

The Hilbert away from $w \neq w$ points

Field? $\phi_{\mathbb{R}} = \mathfrak{g}_{\mathbb{R}}$

nts:
$$\mathbb{I}_{-n} = \left(\left(\mathbb{I}_{-1}, \mathbb{I}_{-1} \right) \mathbb{I}_{-1} \dots \right)$$

Hilbert space: $\{ \text{Highest weight reps of } \widehat{\mathfrak{g}_L} \}$

$$\approx \{ \underbrace{\widehat{\mathfrak{g}}_+ \widehat{\mathfrak{g}}_- \cdot J_n J_m \phi}_{\text{Hilbert space}} \} \approx \{ \widehat{\mathfrak{g}}_- \widehat{\mathfrak{g}}_+ \cdot J_n J_m \phi \}$$

Primary field? $\phi_R = \mathcal{G}_R$

Currents: $J_{-n} = ((J_{-n}, J_{-n}))$

Hilbert space: $\{ \text{Highest weight reps of } \vec{\sigma}_L \}$
 $\approx \{ \underbrace{\bar{J}_0 \bar{J}_1 \dots \bar{J}_n}_{\text{}} \phi \} \approx \{ \bar{J}_1 \bar{J}_2 \dots \bar{J}_n \phi \}$

Primary field? $\phi_R = g_R$

Currents: $J_{-n} = \left(\left(\bar{J}_{-n}, \bar{J}_{-1} \right) \bar{J}_{-1} \dots \right)$
 $J_{-1} X = :J X:$

Hilbert space: $\{ \text{Highest weight reps of } \hat{\sigma}_L \}$
 $\approx \{ \underbrace{\bar{J}_0 \bar{J}_1 \dots \bar{J}_n}_{\text{}} \phi \} \approx \{ \bar{J}_1 \bar{J}_2 \dots \bar{J}_n \phi \}$

Primary field

$$\phi_R = g_R$$

Commutators:

$$\left. \begin{aligned} J_{-n} &= \left(\left(\bar{J}_{-n}, J_{-1} \right) J_{-1} \dots \right) \\ J_{-1} X &= \underbrace{J X}_{\text{}} \end{aligned} \right\}$$

Hilbert space: $\{ \bar{j} \}$

Hilbert space: $\{ \text{Highest weight reps of } \hat{\sigma}_L \}$
 $\approx \{ \underbrace{\bar{J}_0 \bar{J}_1 \dots \bar{J}_n}_{\text{}} \phi \} \approx \{ \bar{J}_1 \bar{J}_2 \dots \bar{J}_n \phi \}$

Primary field

$$\phi_R = g_R$$

Currents:

$$\left. \begin{aligned} J_{-n} &= \left(\left(\bar{J}_{-n}, J_{-1} \right) J_{-1} \dots \right) \\ J_{-1} X &= \bar{J} X \end{aligned} \right\}$$

Hilbert space: $\{ : \bar{J} \dots \bar{J} \bar{J} \dots \bar{J} \phi : \}$

Hilbert space: $\{ \text{Highest weight reps of } \hat{\sigma}_L \}$
 $\approx \{ \bar{\sigma}_0 \bar{\sigma}_1 \dots \bar{J}_n \bar{J}_n \phi \} \approx \{ \bar{\sigma}_1 \bar{\sigma}_1 \dots \bar{J}_1 \bar{J}_1 \phi \}$

Primary field?

$$\phi_R = g_R$$

Currents:

$$\left. \begin{aligned} J_{-n} &= \left(\left(\sigma_{-n}, J_{-n} \right) J_{-n} \dots \right) \\ J_{-n} X &= \underbrace{J X}_{\dots} \end{aligned} \right\}$$

Hilbert space: $\{ : \bar{\sigma} \dots : \bar{\sigma} : \bar{J} \dots : \bar{J} \phi : \}$

Plan

- ① Elementary vertex operators
- ② Stress energy - tensor
- ③

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 - ③ Towards the spectrum

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- ① Elementary vertex operators.
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To reconstruct the Full operator algebra:

- Current-current OPE
- Current-primary OPE

- Plan
- ① Elementary vertex operators.
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 - ③ Towards the spectrum

- ① To reconstruct the Full operator algebra:
 - Current-current OPE
 - Current-primary OPE ←
 - Primary-Primary OPE

Primary - Primary OPE

Current - Primary OPEs

$$j^{\mu}(\tau) \phi(\omega) =$$

$$\bar{j}^{\mu}(\tau) \phi(\omega) =$$

Primary-Primary OPE

Current-Primary OPEs

$$\begin{aligned} \delta^{\mu}(\tau) \phi(\omega) &= \phi(\omega) & : \delta^{\mu} \phi : & & : \bar{\delta}^{\mu} \phi : & & : \delta^{\mu} \phi : \\ \bar{\delta}^{\mu}(\tau) \phi(\omega) &= & & & & & \end{aligned}$$



Primary-Primary OPE

Current-Primary OPEs

$$j^a(x) \phi(y) = \phi(y) \quad :j^a \phi:$$

$$\bar{j}^a(x) \phi(y) = \quad : \bar{j}^a \phi :$$

Primary-Primary ODE

$$\delta^a(\tau) \phi(\omega) = \mathcal{D}_0^a \phi(\omega) + \mathcal{D}_1^a :j^1 \phi: + \mathcal{D}_2^a :j^2 \phi: + \dots :j^n \phi:$$
$$\bar{j}^a(\tau) \phi(\omega) = E_0^a \phi(\omega) + \dots$$

Tracers:

Primary OPE

$$\delta^a(z) \phi(w) = D_0^a \phi(w) + D_1^a :j^b \phi: + D_2^a :j^b j^c \phi: + \dots :j^b \phi:$$

$$\bar{j}^a(z) \phi(w) = E_0^a \phi(w) + \dots$$

Tensors: K^{ab} , β^{abc} , Γ_R^a

Current conservation $(\partial_j^a(z) + \partial_j^{\bar{a}}(z)) \phi(z) =$

$$\delta g(z) = \xi_a(z) t^a g(z)$$

$$\Rightarrow \delta \phi(z) = \xi_a(z) \underbrace{t^a}_{\text{matrix}} \phi(z)$$

Naive - Action: $\left(\bar{c} \partial \delta^a(z) - c \partial \delta^a(z) - i \beta^a d_{c,j} c^c \delta^j(z) \right) \phi(z) = 0$

$$\nabla \cdot \mathbf{D}_0^a + \nabla \cdot \mathbf{E}_0^a = -t^a \rho \delta(z-w)$$

$$\nabla \cdot \mathbf{D}_0^a + \nabla \cdot \mathbf{E}_0^a - \epsilon^a \nabla \cdot \mathbf{E}_0^a = 0$$

$$D_0^a \propto E_0^a \propto t^a$$

$$b^a t^a t^d t^c = 0$$

$$j^a(t) \phi^a(\omega) = -$$



$$\partial D_0^a + \partial E_0^a = -t_R^a 2\pi\delta(z-w)$$

$$\Rightarrow \partial D_0^a + \partial E_0^a - \beta^a t_R^a \frac{c}{\epsilon + \epsilon'} = 0$$

$$D_0^a \propto E_0^a \propto t_R^a$$

$$\beta^a t_R^a t^c = 0$$

$$j^a(z)\phi(w) = -\frac{c}{\epsilon + \epsilon'} t_R^a \frac{\phi(w)}{z-w} + i\beta^a t_R^c \frac{\bar{c}}{(\epsilon + \epsilon')^2} :j^b\phi:(w) \log|z-w|^2$$

$$+ i\beta^a t_R^c \frac{c}{(\epsilon + \epsilon')^2} :j^b\phi:(w) \frac{\bar{z}-\bar{w}}{z-w}$$

$$+ :j^a\phi:(w) + \dots$$

$$\nabla \cdot \mathbf{D}_0^a + \nabla \cdot \mathbf{E}_0^a = -t_e^a 2\pi \delta(z-w)$$

$$\Rightarrow \nabla \cdot \mathbf{D}_0^a + \epsilon \nabla \cdot \mathbf{E}_0^a - \beta^a \epsilon_0 \nabla \cdot \mathbf{E}_0^a = 0$$

$$\mathbf{D}_0^a \propto \mathbf{E}_0^a \propto t_R^a$$

$$\beta^a \epsilon_0 t^d t^c = 0$$

$$j^a(z) \phi(z) = -\frac{\epsilon}{\epsilon + \epsilon} t_R^a \frac{\phi(z)}{z-w} + i \beta^a \epsilon_0 t^c \frac{\bar{\epsilon}}{(\epsilon + \epsilon)^2} j^b \phi(z) \log |z-w|^2$$

$$+ i \beta^a \epsilon_0 t^c \frac{\epsilon}{(\epsilon + \epsilon)^2} j^b \phi(z) \frac{\bar{z}-\bar{w}}{z-w}$$

$$+ j^a \phi(z) + \dots$$

The Hilbert away from WZW points

② The Virasoro algebra

The H. Plot ...

① The Virasoro algebra

$$T(z) = -\frac{c+\mathcal{E}}{2c^2} : \dot{X}^a \dot{X}^a : \quad \text{classical stress-tensor}$$

$$T(z)T(w) = \frac{c/2}{(z-w)^2} + 2\frac{T(w)}{(z-w)} + \frac{\partial T(w)}{z-w} + \dots$$

$$\frac{1}{(z-w)^2} + \frac{0}{z-w} + \dots$$

$T(z) J(w)$ or:

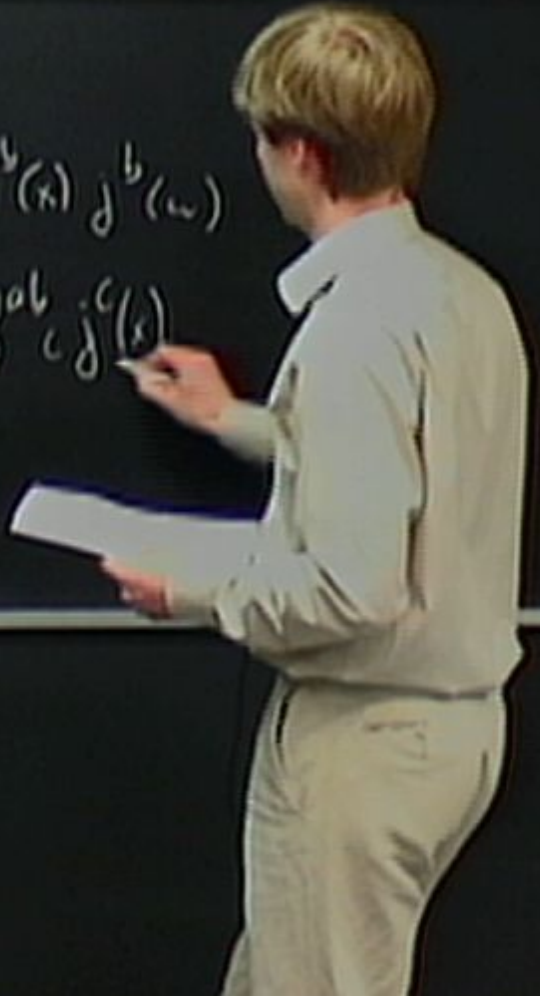
$$J^a(z) J^b(w) = \sum_{x=w} \mathcal{C}_m J^a(z) J^b(x) J^b(w)$$

$$\frac{1}{(z-w)^2} + \frac{0}{z-w} + \dots$$

$T(z)j(w)$ op:

$$j^a(z) j^b j^c(w) = \lim_{z \rightarrow w} j^a(z) j^b(z) j^c(w)$$

$$= \lim_{z \rightarrow w} \left\{ \left[-\frac{a}{c} \frac{1}{(z-w)^{a+1}} + \frac{c(c+2c)}{(c+c)!} \beta^{ab} j^c(z) \right] \right\}$$



$$(z-w)^2 \left[\frac{f(z)}{(z-w)^2} + \frac{f'(z)}{z-w} + \dots \right]$$

$T(z)j(w)$ or:

$$j^b j^a(w) = \lim_{z \rightarrow w} j^a(z) j^b(z) j^b(w)$$

$$\lim_{z \rightarrow w} \left\{ \left[-\frac{a}{c} \frac{w^a b}{(z-w)^a} + \frac{c(c+2c)}{(c+c)^a} i \beta^{ab} \frac{c}{z-x} j^c(z) + \frac{c^2}{(c+c)^2} i \beta^{ab} c j^c(z) \frac{z-w}{(z-x)^2} + \dots \right] j^b(w) + [(x \rightarrow w)] \right\}$$

$$(z-w)^2 \left[\frac{f(z)}{(z-w)^2} + \frac{f'(z)}{z-w} + \dots \right]$$

$T(z)j(w)$ or:

$$j^a(z) j^b j^c(w) = \lim_{z \rightarrow w} j^a(z) j^b(z) j^c(w)$$

$$= \lim_{z \rightarrow w} \left\{ \left[-\frac{c}{c!} \frac{1}{(z-x)^{c+1}} + \frac{c(c+2c)}{(c+c)!} i \beta^{ab} \frac{c}{z-x} + \frac{c^2}{(c!c!)^2} i \beta^{ab} \frac{c}{(z-x)^2} + \dots \right] j^b(w) + [(x \leftrightarrow w)] \right\}$$



$$\frac{1}{(z-w)^2} + \frac{j'(w)}{z-w} + \dots$$

T(z) j(w) op:

$$\begin{aligned}
 j^a(z) j^b j^c(w) &= \lim_{z \rightarrow w} j^a(z) j^b(z) j^c(w) \\
 &= \lim_{z \rightarrow w} \left\{ \left[-\frac{c}{c^2} \frac{1}{(z-x)^2} + \frac{c(c+2c)}{(c+c)^2} i \beta^{ab} \frac{j^c(x)}{z-x} + \frac{c^2}{(c+c)^2} i \beta^{ab} j^c(x) \frac{\bar{z}-\bar{x}}{(z-x)^2} + \dots \right] j^b(w) \right. \\
 &\quad \left. + [(x \leftrightarrow w)] \right\} \\
 &= -\frac{2c^2}{c+c} \frac{j^c(w)}{(z-w)} + \frac{2c(c+2c)}{(c+c)^2} i \beta^{ab} \frac{j^c j^b(w)}{z-w} + \frac{2c^2}{(c+c)^2} i \beta^{ab} j^c j^b(w)
 \end{aligned}$$

$$j''(s) \cdot j^b j^b(\omega) = -\frac{2c^2}{\omega c} \frac{j^a(\omega)}{(2-\omega)^2} + \frac{2c^2}{(c\omega)^4} (c \cdot j^a - c j^a)$$

$$\dots \int j^b(\omega)$$

$$\frac{2-\omega}{(2-\omega)^2}$$

SAFETY
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$$j''(z) \cdot j^b j^b(\omega) = -\frac{2c^2}{c+\bar{c}} \frac{j^a(\omega)}{(z-\omega)^2} + \frac{2c^2}{(c\bar{c})^2} \underbrace{\left(\bar{c} \frac{\partial}{\partial z} j^a - c \frac{\partial}{\partial \bar{z}} j^a \right)}_{(c+\bar{c}) j^a(\omega)} \frac{z-\bar{\omega}}{(z-\omega)^2}$$

$$\dots \int j^b(\omega)$$

$$\frac{z-\bar{\omega}}{(z-\omega)^2}$$

ERIK
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 ZÜRICH
 INSTITUT FÜR
 MATHEMATIK

$$\begin{aligned}
 j''(s) \cdot j^b j^b(\omega) &= -\frac{2c^2}{c+\bar{c}} \frac{j^a(\omega)}{(z-\omega)^2} + \frac{2c^2}{(c\bar{c})^2} \underbrace{\left(\bar{c} \frac{\partial}{\partial z} j^a - c \frac{\partial}{\partial \bar{z}} j^a \right)}_{(c+\bar{c}) j^a(\omega)} \frac{\bar{z}-\omega}{(z-\omega)^2} \\
 &= -\frac{2c^2}{c+\bar{c}} \frac{1}{(z-\omega)^2} \left(j^a(z) + (\omega \cdot H) j^a(z) + (\bar{\omega} \cdot F) j^a(z) \right) + \dots
 \end{aligned}$$

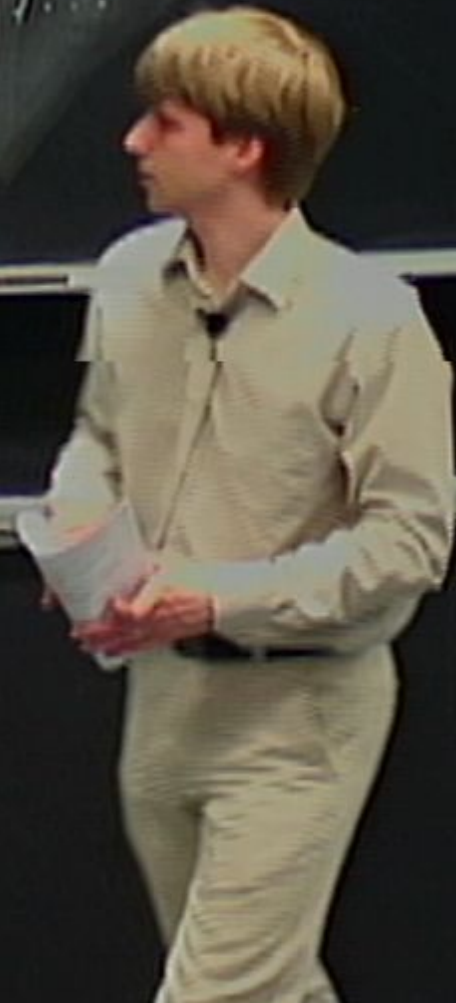
$$\dots \int j^b(\omega)$$

$$\frac{z-\omega}{(z-\omega)^2}$$

ERKEN
 UNIVERSITÄT
 TÜBINGEN

$$\begin{aligned}
 j''(z) j^b j^b(\omega) &= -\frac{2c^2}{c+\bar{c}} \frac{j^a(\omega)}{(\bar{z}-\omega)^2} + \frac{2c^2}{(c+\bar{c})^2} \underbrace{(\bar{c} \partial j^a - c \partial \bar{j}^a)}_{(c+\bar{c}) \partial j^a} \frac{z-\omega}{(\bar{z}-\omega)^2} \\
 &= -\frac{2c^2}{c+\bar{c}} \frac{1}{(\bar{z}-\omega)^2} \left(j^a(z) + (\omega \cdot \partial) j^a(z) + (\bar{c} - c) \partial j^a(z) \right) + \frac{2c^2}{c+\bar{c}} \partial j^a(z) \frac{z-\omega}{(\bar{z}-\omega)^2} \\
 &= \frac{2c^2}{c+\bar{c}} \left(\frac{j^a(z)}{(\bar{z}-\omega)^2} + \frac{\partial j^a(z)}{\omega \cdot \partial} \right) + \dots
 \end{aligned}$$

$$T(\omega) j^a(z) = \frac{j^a(z)}{(\omega-z)^2} + \frac{\partial j^a(z)}{\omega \cdot \partial}$$



$$c + \bar{c} \left(\frac{j(z)}{(z-w)^2} + \frac{dj(z)}{w-z} \right) + \dots$$

$$T(w) j^a(z) = \frac{j^a(z)}{(w-z)^2} + \frac{dj^a(z)}{w-z}$$

Virasoro
 j^0 : Primary of dim 1

$T(z) T(w)$ OPE



$$\dots \int j^k(z)$$

$$\frac{z-w}{(z-w)^2}$$

CAUTION
 DO NOT TOUCH
 THE BOARD

$$T(z) j^{\alpha}(z) = \left(\frac{j^{\alpha}(z)}{(z-z)^2} + \frac{\partial j^{\alpha}(z)}{z-z} \right)$$

Virasoro
 j^{α} : Primary of dim 1

$T(z) T(w)$ OPE

$T(z)$

$$T(z) j^{\circ}(z) = \left(\frac{j^{\circ}(z)}{(z-z)^2} + \frac{dj^{\circ}(z)}{z-z} \right)$$

Virasoro
 j° : Primary of dim 1

$T(z) T(w)$ OPE

$$-\frac{c\bar{c}}{2c^2} T(z) \lim_{x \rightarrow w} j^{\circ}(x) j^{\circ}(w) = -\frac{c\bar{c}}{2c^2} \lim_{x \rightarrow w} \left\{ \left[\frac{j^{\circ}(x)}{(z-x)^2} + \frac{dj^{\circ}(x)}{z-x} \right] j^{\circ}(w) + \dots \right\}$$

$$T(z) j^{\alpha}(z) = \frac{j^{\alpha}(z)}{(z-z)^2} + \frac{\partial j^{\alpha}(z)}{z-z}$$

Virasoro
 j^{α} : Primary of dim 1

$T(z) T(w)$ OPE

$$-\frac{c\hbar}{2c^2} T(z) \lim_{x \rightarrow w} j_{\alpha}(x) j^{\alpha}(w) = -\frac{c\hbar}{2c^2} \lim_{x \rightarrow w} \left\{ \left[\frac{j^{\alpha}(x)}{(z-x)^2} + \frac{\partial j^{\alpha}(x)}{z-x} \right] j^{\alpha}(w) + \right.$$

$$\left. = \frac{K^{\alpha\alpha}/2}{(z-w)^4} + \frac{T(w)}{(z-w)^2} \right\}$$

$$j^{\alpha}(w) \left[\frac{j^{\alpha}(w)}{(z-w)^2} + \frac{\partial j^{\alpha}(w)}{z-w} \right]$$

$$T(z) j^a(z) = \left(\frac{j^a(z)}{(z-z)^2} + \frac{\partial j^a(z)}{\partial z} \right)$$

Virasoro
 j^a : Primary of dim 1

$T(z) T(w)$ OPE

$$\begin{aligned}
 -\frac{c\epsilon}{2c^2} T(z) \lim_{x \rightarrow w} j_a(x) j^a(w) &= -\frac{c\epsilon}{2c^2} \lim_{x \rightarrow w} \left\{ \left[\frac{j^a(x)}{(z-x)^2} + \frac{\partial j^a(x)}{\partial x} \right] j^a(w) + \right. \\
 &= \frac{K^a{}_a/2}{(z-w)^2} + \frac{2T(w)}{(z-w)^2} + \frac{-(c\epsilon)}{2z} \frac{j^a(w) \left[\frac{\partial j^a(w)}{\partial w} + \frac{\partial j^a(w)}{\partial w} \right]}{z-w}
 \end{aligned}$$

$$-1) d(z) = \frac{d''(z)}{(z-z)^2} + \frac{d'(z)}{z-z}$$

j^0 : Primary of dim 1

(2) $T(z)$ OPE

$$\begin{aligned}
 T(z) \lim_{z \rightarrow w} j_a(z) j^0(w) &= -\frac{c+6}{2c^2} \lim_{z \rightarrow w} \left\{ \left[\frac{d''(z)}{(z-z)^2} + \frac{d'(z)}{z-z} \right] j''(z) + \right. \\
 &= \frac{K^a/2}{(z-w)^2} + 2 \frac{T(w)}{(z-w)^2} + \underbrace{\frac{-(c+6)}{2c} \frac{j^a j^0 + j^0 j^a}{z-w}}_{\frac{\partial T}{\partial z}}
 \end{aligned}$$



1 (for $\mathcal{O}_S(m+2|2n)$)

$$j_L^a(z) j_R^{\bar{a}}(w) = \frac{A^{a\bar{a}}}{(z-w)^2} \leftarrow \begin{array}{l} \text{Primary field} \\ \text{in Adjoint rep.} \end{array}$$

③ Towards the spectrum

③ Towards the spectrum

$$T(z) X(\omega) = \sum_n \frac{L_n X(\omega)}{(z - \omega)^{n+1}}$$

$n=0$

$$\frac{L_0 X(\omega)}{(z - \omega)^1}$$

③ Towards the spectrum

$$T(z) \phi(\omega) = -\frac{c_1 \bar{c}}{2\pi} \lim_{\epsilon \rightarrow 0^+} \int_{\gamma - i\epsilon}^{\gamma + i\epsilon} j^\alpha(z) j^\alpha(z) \phi(\omega)$$

$$T(z) X(\omega) = \sum_n \frac{L_n X(\omega)}{(\gamma - \omega)^{\gamma-n}}$$

$n=0$

$$\frac{L_0 X(\omega)}{(\gamma - \omega)^\gamma}$$

③ Towards the spectrum

$$T(z) X(w) = \sum_n \frac{L_n X(w)}{(z-w)^{2-n}}$$

$n=0$

$$\frac{L_0 X(w)}{(z-w)^2}$$

$$T(z) \phi(w) = - \frac{c+\bar{c}}{2c^2} \lim_{x \rightarrow z} j^a(x) j^a(z) \phi(w)$$

$$= - \frac{c+\bar{c}}{2c^2} \left(\left(\frac{c}{c+\bar{c}} \right)^2 t_R^a t_R^a \frac{\phi(w)}{(z-w)^2} \right)$$

$\frac{0}{z-w}$

③ Towards the spectrum

$$T(z) X(\omega) = \sum_n \frac{L_n X(\omega)}{(z - \omega)^{2-n}}$$

$n=0$

$$\frac{L_0 X(\omega)}{(z - \omega)^2}$$

$$T(z) \phi(\omega) = - \frac{c+\bar{c}}{2c} \lim_{z \rightarrow \omega} j^{\alpha}(z) j^{\sigma}(z) \phi(\omega)$$

$$= - \frac{c+\bar{c}}{2c^2} \left[\left(\frac{c}{c+\bar{c}} \right)^2 t_R^{\alpha} t_R^{\sigma} \frac{\phi(\omega)}{(z-\omega)^2} + 2 \frac{c}{c+\bar{c}} t_R^{\alpha} j^{\sigma} \psi'(\omega) \right]$$

$$\frac{\psi'}{z-\omega}$$

$$= \frac{1}{2} \epsilon^2 \frac{1}{(\epsilon^2 - \omega^2)^2} \phi(\omega) + \frac{1}{\epsilon} \frac{1}{\epsilon^2 - \omega^2} \phi(\omega) + \dots$$

$L \gg 0 \quad \phi = 0 \Rightarrow \phi$ *Viva zero primaries*

$$\Delta \phi = \frac{\epsilon^2}{2} \frac{(\epsilon^2)}{c^2 R}$$

$$\partial \phi = \frac{1}{\epsilon} \frac{1}{\epsilon^2} \epsilon^2 j^{\alpha} \phi_i$$

$$= \frac{1}{2} \ell^2 \frac{1}{(2-\omega)^2} \phi(\omega) + \frac{1}{2} \frac{1}{2-\omega} \frac{d}{d\omega} \phi(\omega) + \dots$$

$L_{n>0} \phi = 0 \Rightarrow \phi$ is a linear combination of primary fields

$$\Delta_{\phi} = \frac{\ell^2}{2} c_{\mathcal{R}}^{(2)}$$

$$\partial \phi = \frac{1}{\ell} t_{\mathcal{R}}^{(2)} \phi$$

$$T(x) = \partial^a \phi: (\omega)$$

$$T(z) j^a \phi(z) = \lim_{z \rightarrow w} T(z) j^a(z) \phi(w)$$

=

$$\begin{aligned}
 T(z) j^a \phi: (\omega) &= \lim_{|x-\omega|} T(z) j^a(x) \phi(\omega) \\
 &= \lim_{|x-\omega|} \left\{ \left[\frac{j^0(x)}{(z-x)^1} + \frac{\partial j^0(x)}{\partial x} \right] \phi(\omega) + j^1(x) \frac{\phi(\omega)}{(z-\omega)^2} + \frac{\partial j^1(x)}{\partial x} \phi(\omega) \right. \\
 &= \dots + \frac{1}{(z-\omega)^1} \left[j^0 \phi: \right.
 \end{aligned}$$

$$\begin{aligned}
 T(z) : j^a \phi : (w) &= \lim_{|x-w| \rightarrow 0} T(z) j^a(x) \phi(w) \\
 &= \lim_{|x-w| \rightarrow 0} \left\{ \left[\frac{j^0(x)}{(z-x)^1} + \frac{\partial j^0(x)}{\partial z} \right] \phi(w) + \left[\frac{\Delta_1 \phi(w)}{(z-w)^2} + \frac{\partial \phi(w)}{\partial z} \right] \right\} \\
 &= \dots + \frac{1}{(z-w)^1} \left[: j^0 \phi : \right]
 \end{aligned}$$



$$\begin{aligned}
 T(z) : j^a \phi : (w) &= \lim_{|z-w| \rightarrow 0} T(z) j^a(x) \phi(w) \\
 &= \lim_{|z-w| \rightarrow 0} \left\{ \left[\frac{j^0(x)}{(z-x)^1} + \frac{\partial j^0(x)}{\partial z} \right] \phi(w) + j^0(x) \left[\frac{\Delta_1 \phi(w)}{(z-w)^2} + \frac{\partial \phi(w)}{\partial z} \right] \right. \\
 &\quad \left. + \frac{1}{(z-w)^1} : j^0 \phi : \right\}
 \end{aligned}$$

$$= \dots + \frac{1}{(z-w)^2} \left[j \psi \cdot (1 + \Delta \phi) + i \beta^0 b c t^2 : j \phi : (w) \right]$$

$$j^a(z) \phi(w) = - \frac{c}{4\pi} t^a \frac{\phi(w)}{z-w} + i \beta^0 b c t^a \phi(w) \log |z-w|^2$$

$$+ i \beta^0 b c t^a \frac{c}{(c)} \frac{\bar{z}-\bar{w}}{z-w}$$

$$i \beta^a b_c t^b_R = t^b_{Adj} \otimes t^b_R = \frac{1}{2} \left(\begin{matrix} C^{(2)}_{Adj} & \\ & C^{(2)}_R \end{matrix} \right)$$

$j^0 \phi$ transforms $Adj \otimes R$

$$t^a_{Adj \otimes R} = t^a_{Adj} \otimes Id + Id \otimes t^a_R$$

$$C^{(2)}_{Adj \otimes R} = \cancel{C^{(2)}_{Adj}} + C^{(2)}_R + 2 t^a_{Adj} \otimes t^a_R$$

$$\frac{1}{(z-w)^2} + \frac{1}{z} + \frac{1}{z-w} + \dots$$

$$L_0 [:j \phi_j] = 1 + \frac{b^2}{2} C_R^{(a)}$$

$$\frac{1}{(z-1)^2} + \frac{1}{z} - \frac{1}{z-1} + \dots$$

$$L_0 [:j \phi :]_{R'} = 1 + \frac{b^2}{z} c_R^{(a)}$$

$$\frac{1}{(z-1)^2} + \frac{1}{z} + \frac{1}{z-1} + \dots$$

$$L_0 [:j \phi :]_{R'} = 1 + \frac{\beta^2}{2} C_R^{(1)} + \frac{\beta^2}{2} (1 - k\beta^2) (C_{R'}^{(1)} - C_R^{(2)})$$

$$\frac{1}{(z-1)^2} + \frac{1}{z} + \frac{1}{z-1} + \dots$$

$$L_0 [i_j \phi_j]_{R'} = 1 + \frac{\beta^2}{2} c_R^{(1)} + \frac{\beta^2}{2} (1 - k\beta^2) (c_{R'}^{(1)} - c_{R'}^{(2)})$$

$$= 1 + c_R^{(1)} \left(\frac{\beta^2}{2} k\beta^2 \right) + c_{R'}^{(1)} \frac{\beta^2}{2} (1 - k\beta^2)$$

$$\frac{1}{(2-\omega)^2} + \frac{1}{2} \frac{1}{(2-\omega)} + \dots$$

$$L_o [i \phi]_{R'} = 1 + \frac{\beta^2}{2} C_R^{(1)} + \frac{\beta^2}{2} (1 - k\beta^2) (C_{R'}^{(1)} - C_R^{(2)})$$

$$= 1 + C_R^{(1)} \left(\frac{\beta^2}{2} k\beta^2 \right) + C_{R'}^{(1)} \frac{\beta^2}{2} (1 - k\beta^2)$$

$$\frac{1}{(7-\dots)^2} + \frac{1}{c} \dots \frac{1}{2-\dots} + \dots$$

$$L_0 [{}^c_j \phi_i]_{R'} = 1 + \frac{b^2}{2} c_{R'}^{(1)} + \frac{b^2}{2} (1 - kb^2) (c_{R'}^{(1)} - c_{R'}^{(2)})$$

$$= 1 + c_{R'}^{(1)} \left(\frac{b^2}{2} kb^2 \right) + c_{R'}^{(1)} \frac{b^2}{2} (1 - kb^2)$$