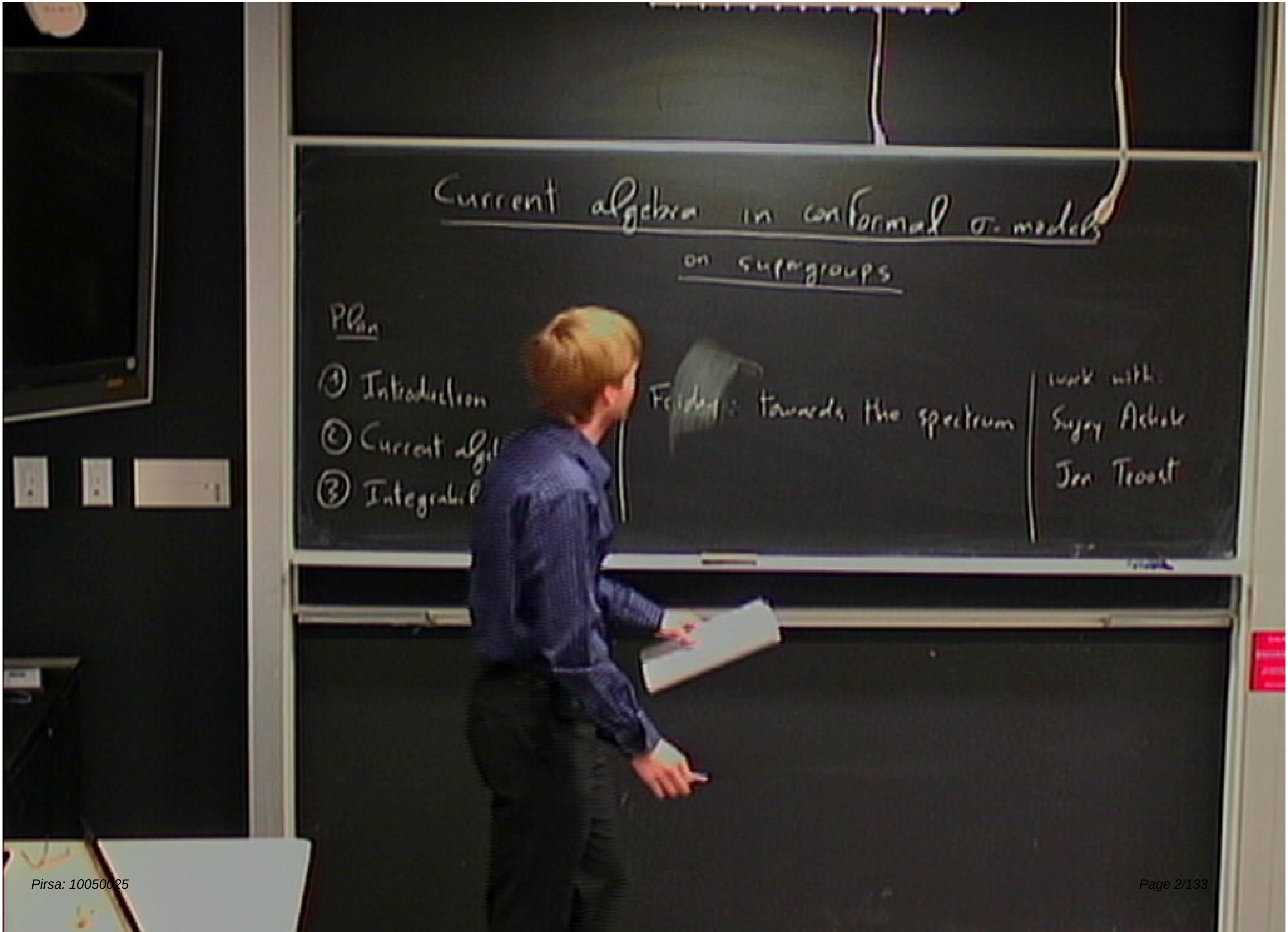
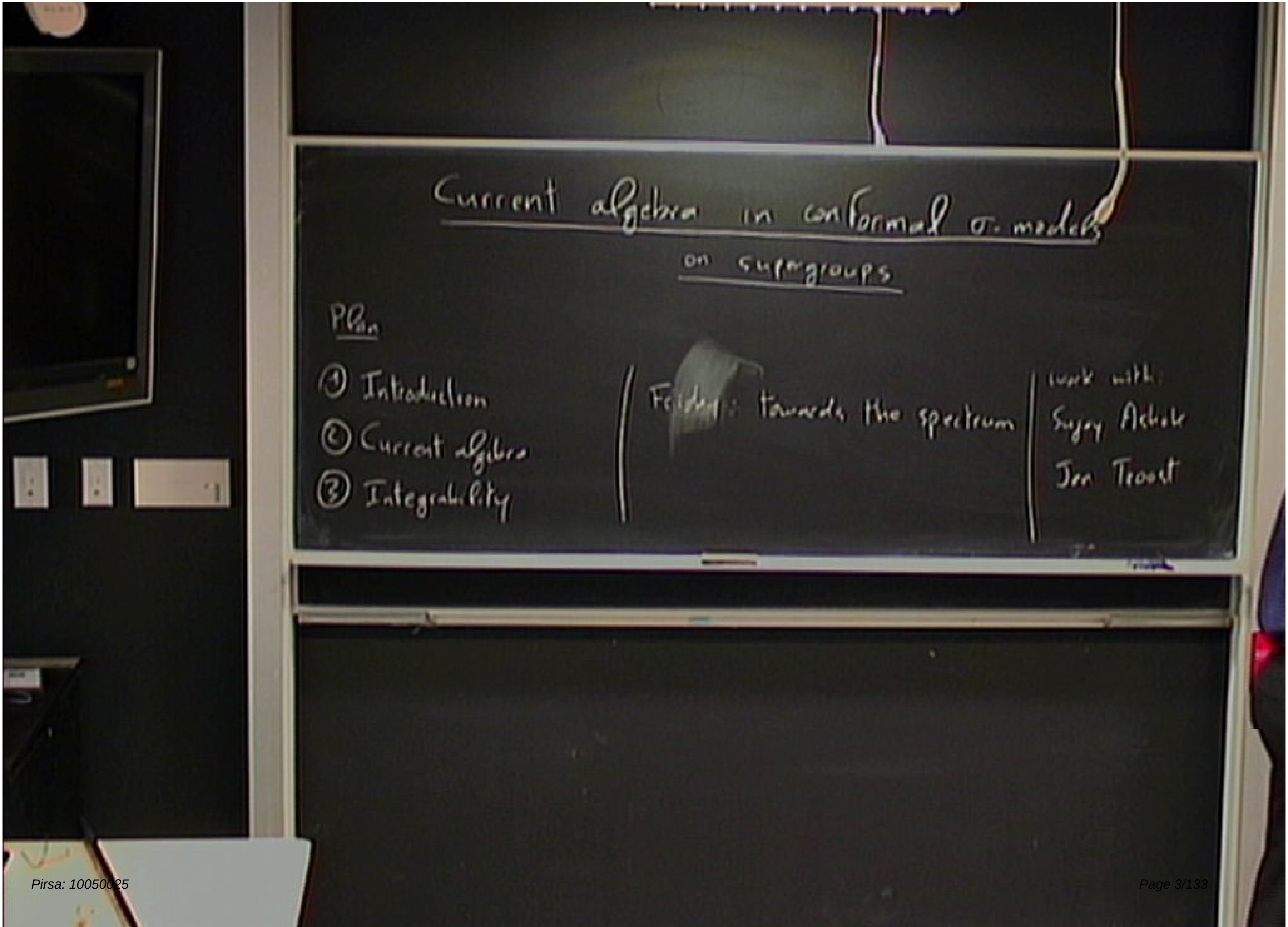
Title: Current algebra in conformal sigma models on supergroups (Part I)

Date: May 12, 2010 11:00 AM

URL: http://pirsa.org/10050025

Abstract: Two-dimensional non-linear sigma models on some supergroup manifolds are conformal field theories whether the action includes a Wess-Zumino term or not. These models are relevant for the worldsheet description of string theory in Anti-de Sitter backgrounds with Ramond-Ramond fluxes. The current algebra is an useful tool to study these theories. In these lectures I will review the construction of the current algebra. Then I will discuss some applications to the computation of the spectrum and integrability.


Current algebra in conformal σ-models
on supergroups
Plan
① Introduction
② Current alg
③ Integrabil
Friday: towards the spectrum
work with:
Sujay Ashok
Jan Troost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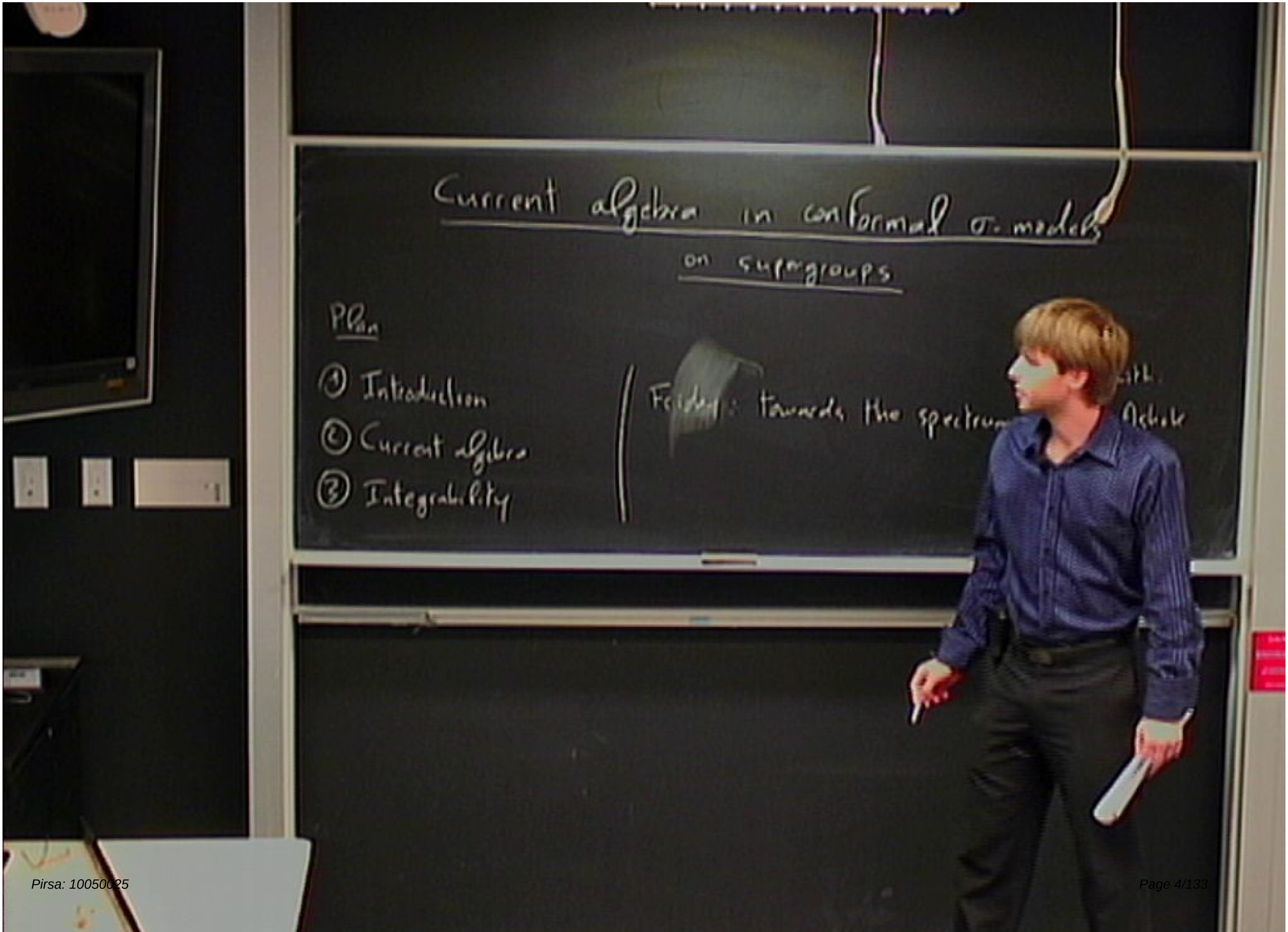
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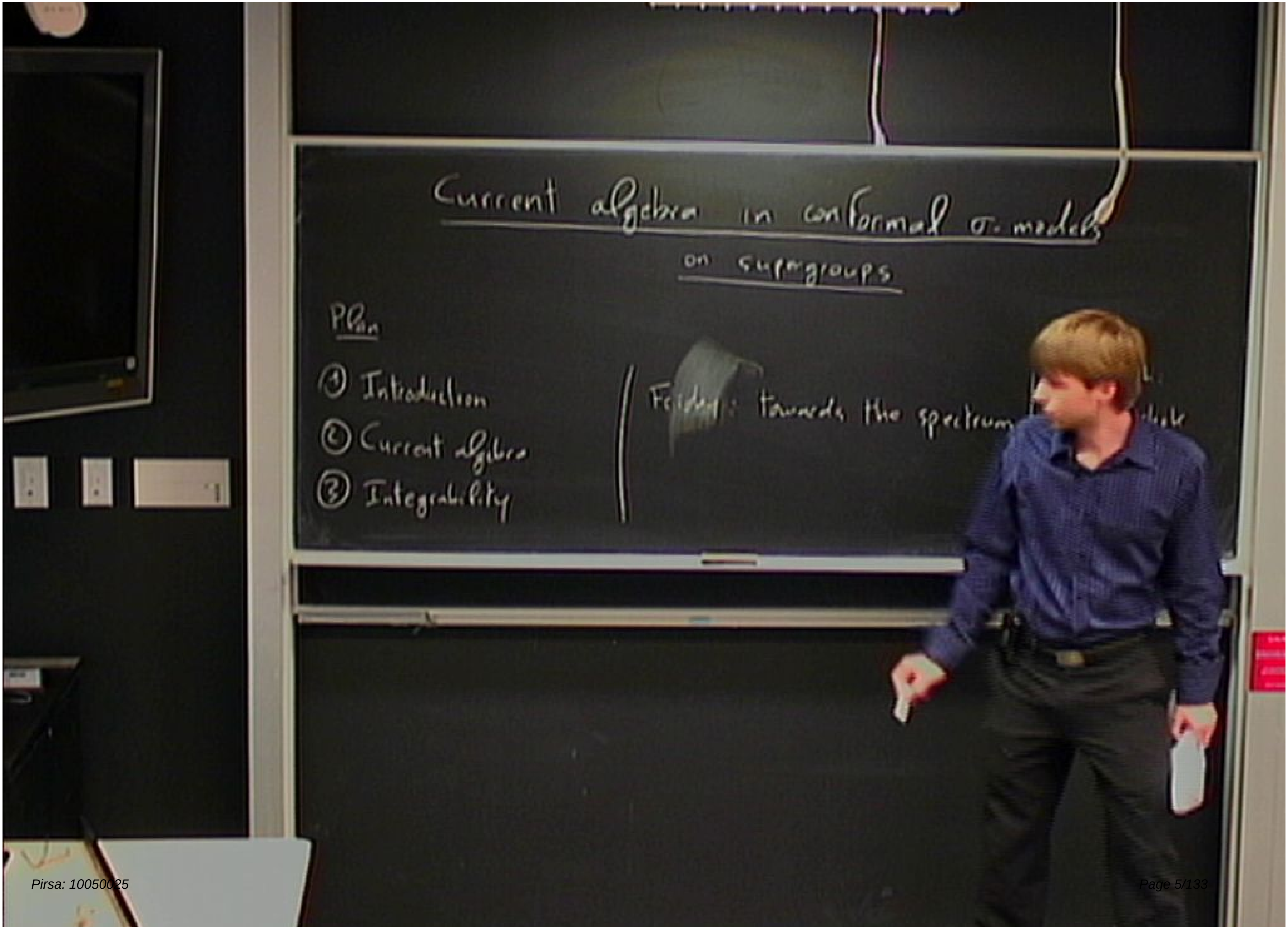
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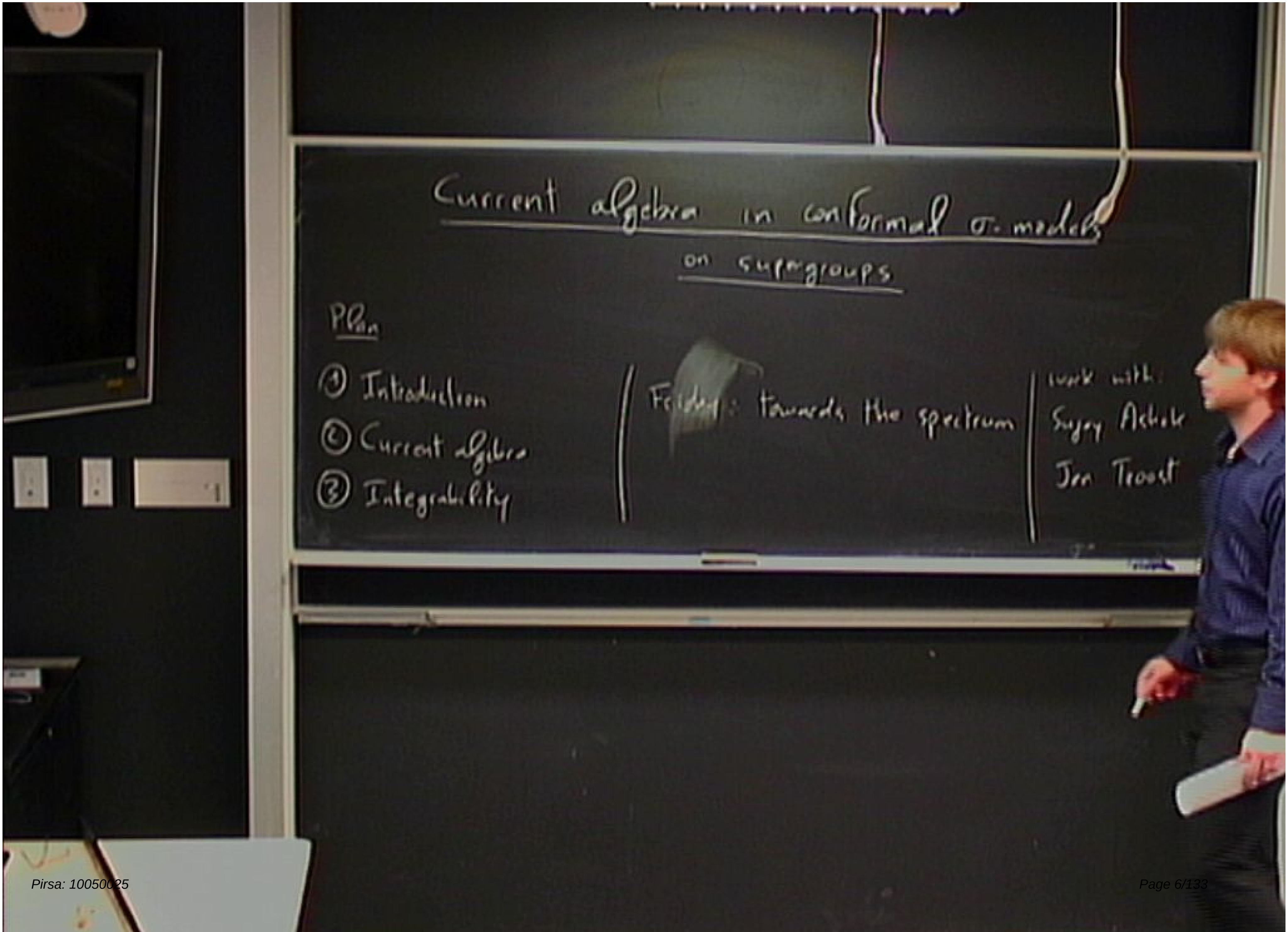
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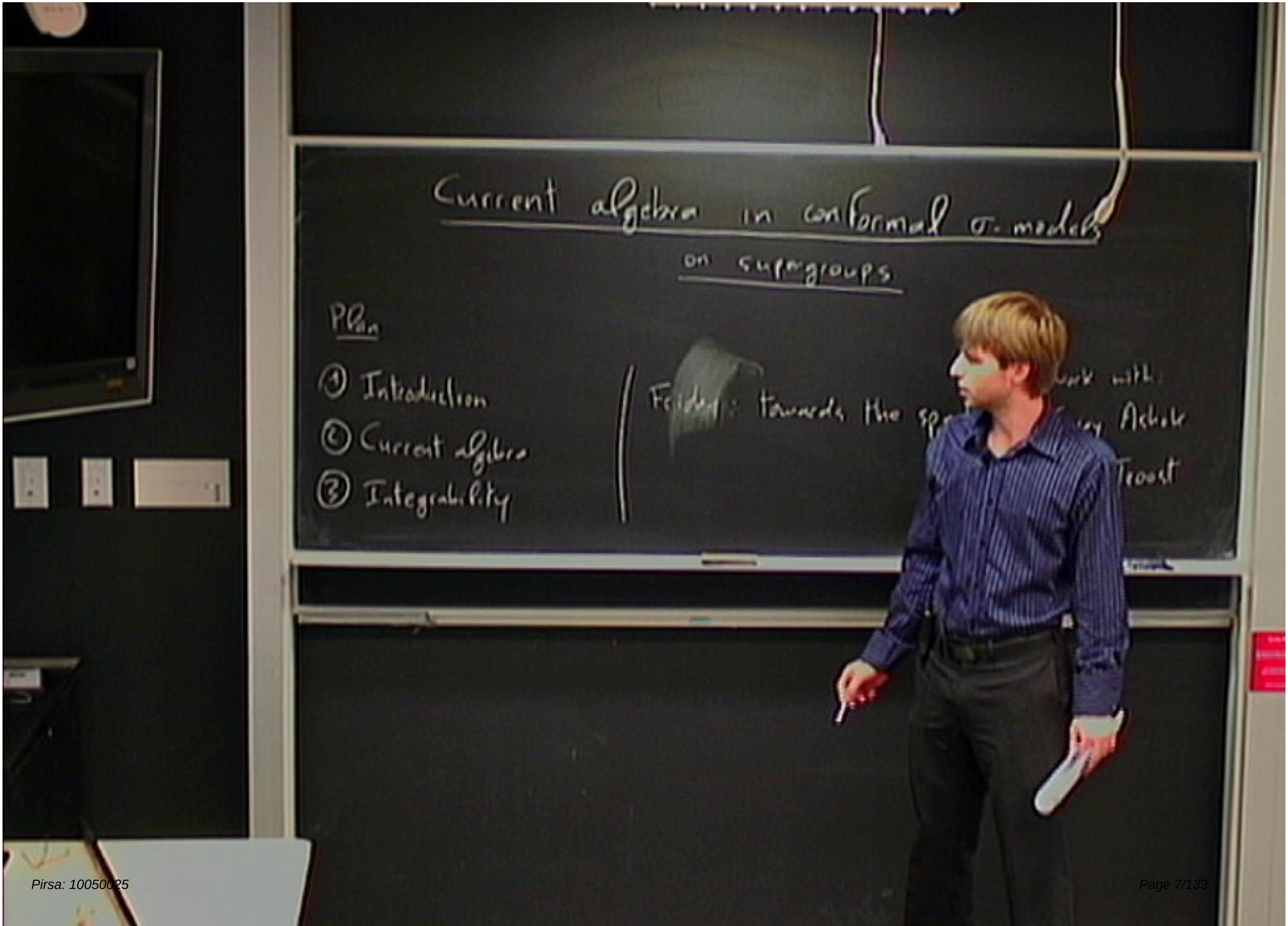
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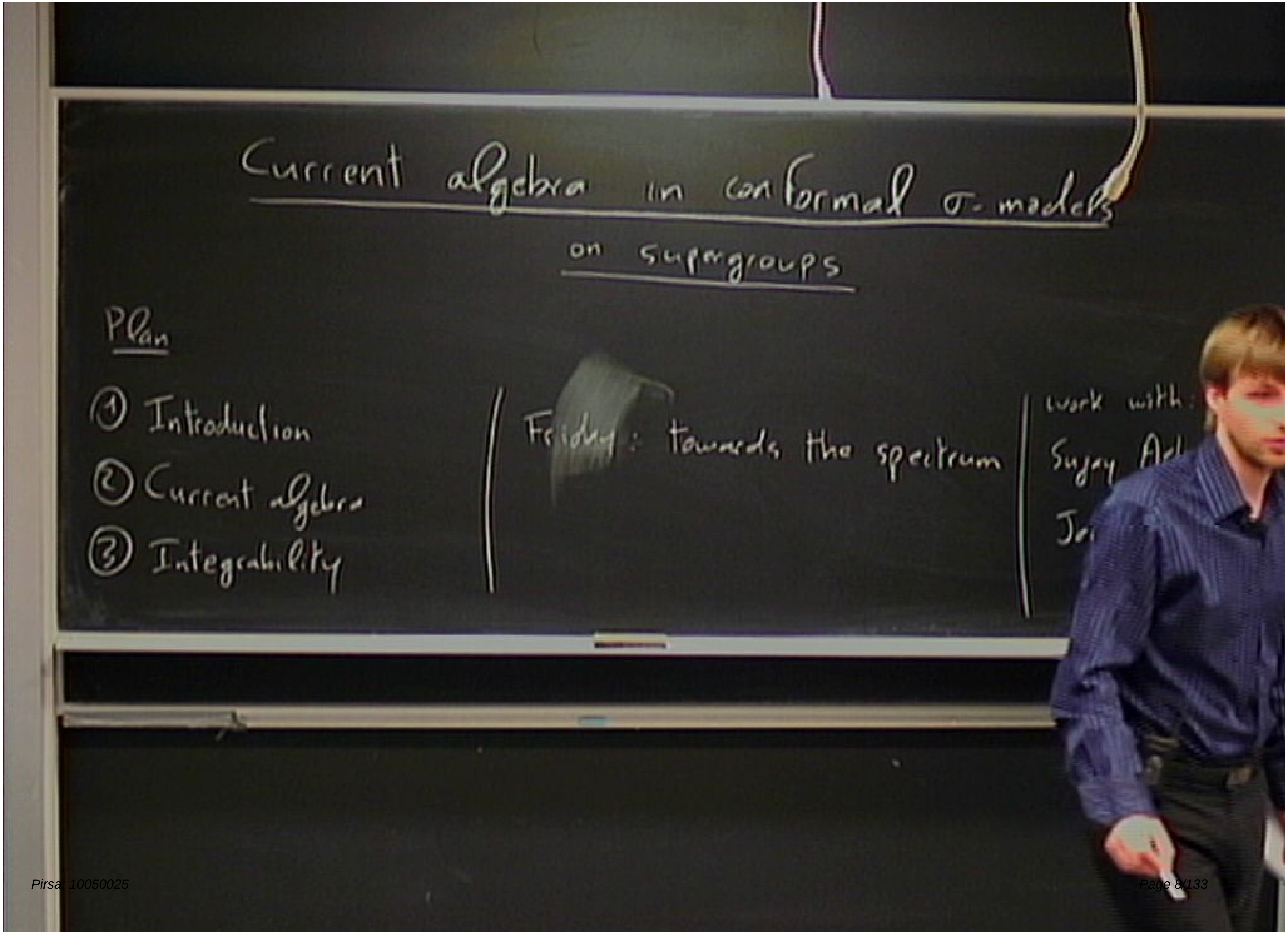
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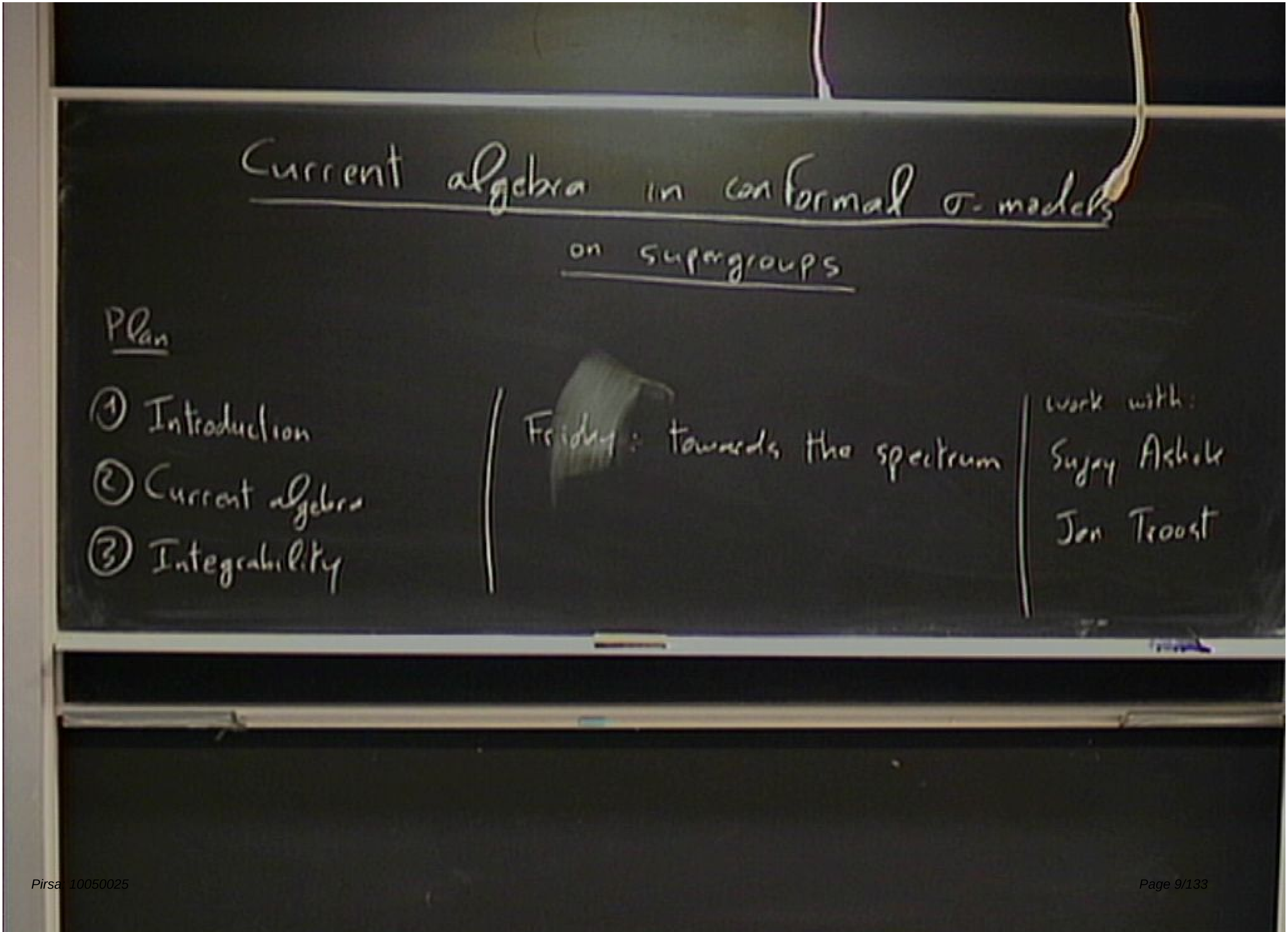
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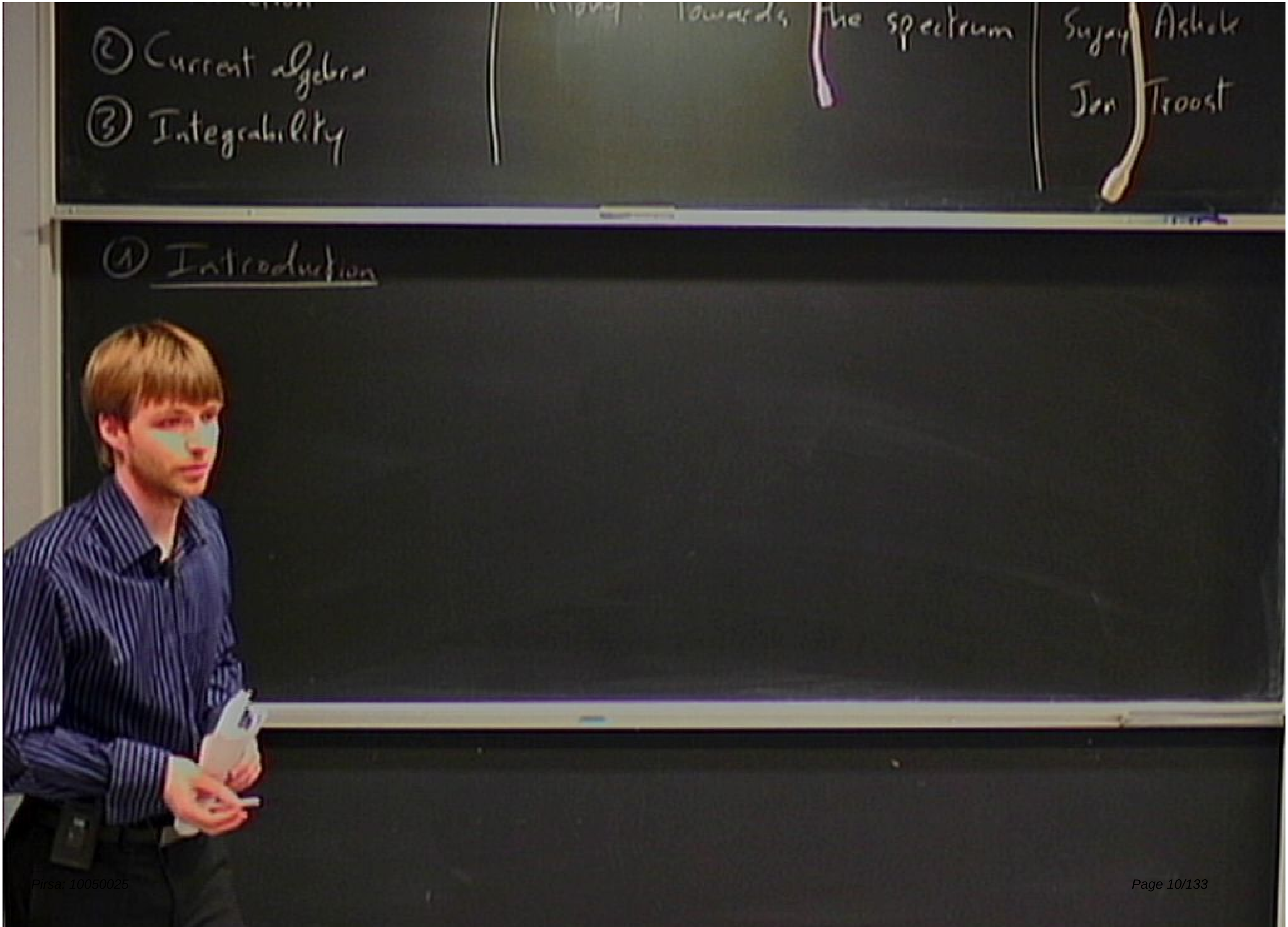
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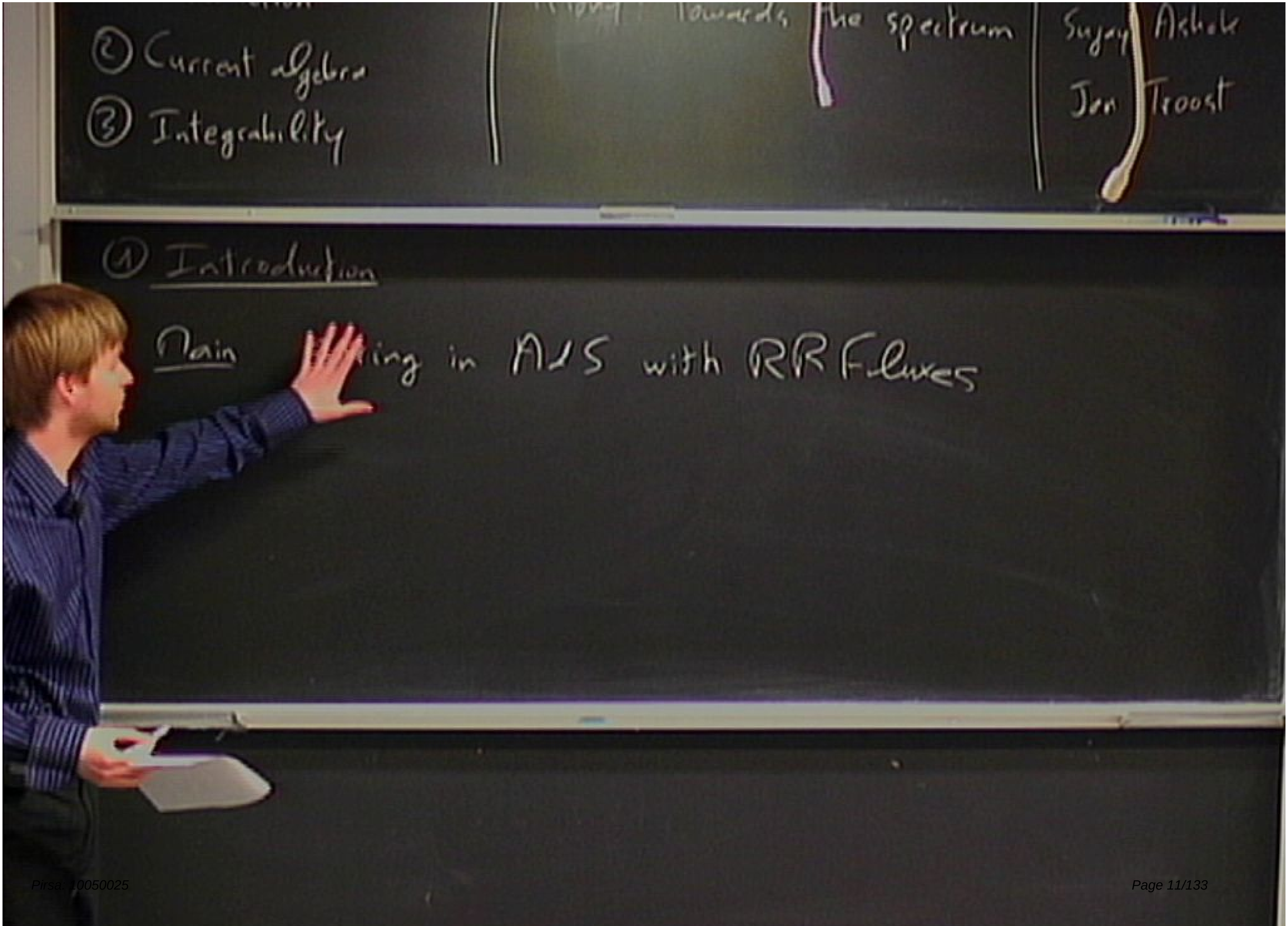
Jan Troost
② Current algebra
③ Integrability
① Introduction


towards the spectrum
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Main
in AdS with RR Fluxes

② Current algebra

③ Integrability

Towards the spectrum

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① Introduction

Main String in AdS with RR Fluxes

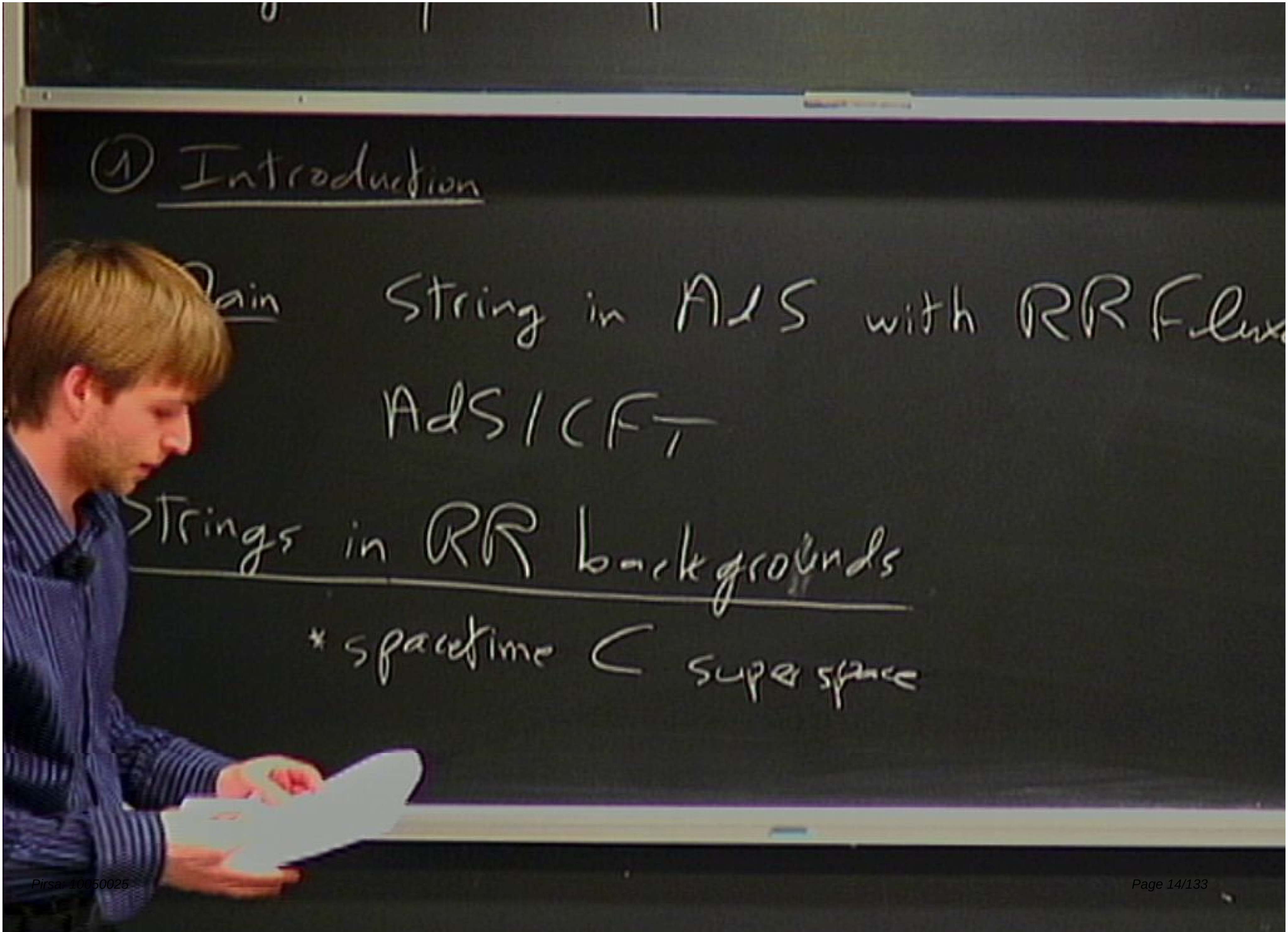
AdS/CFT

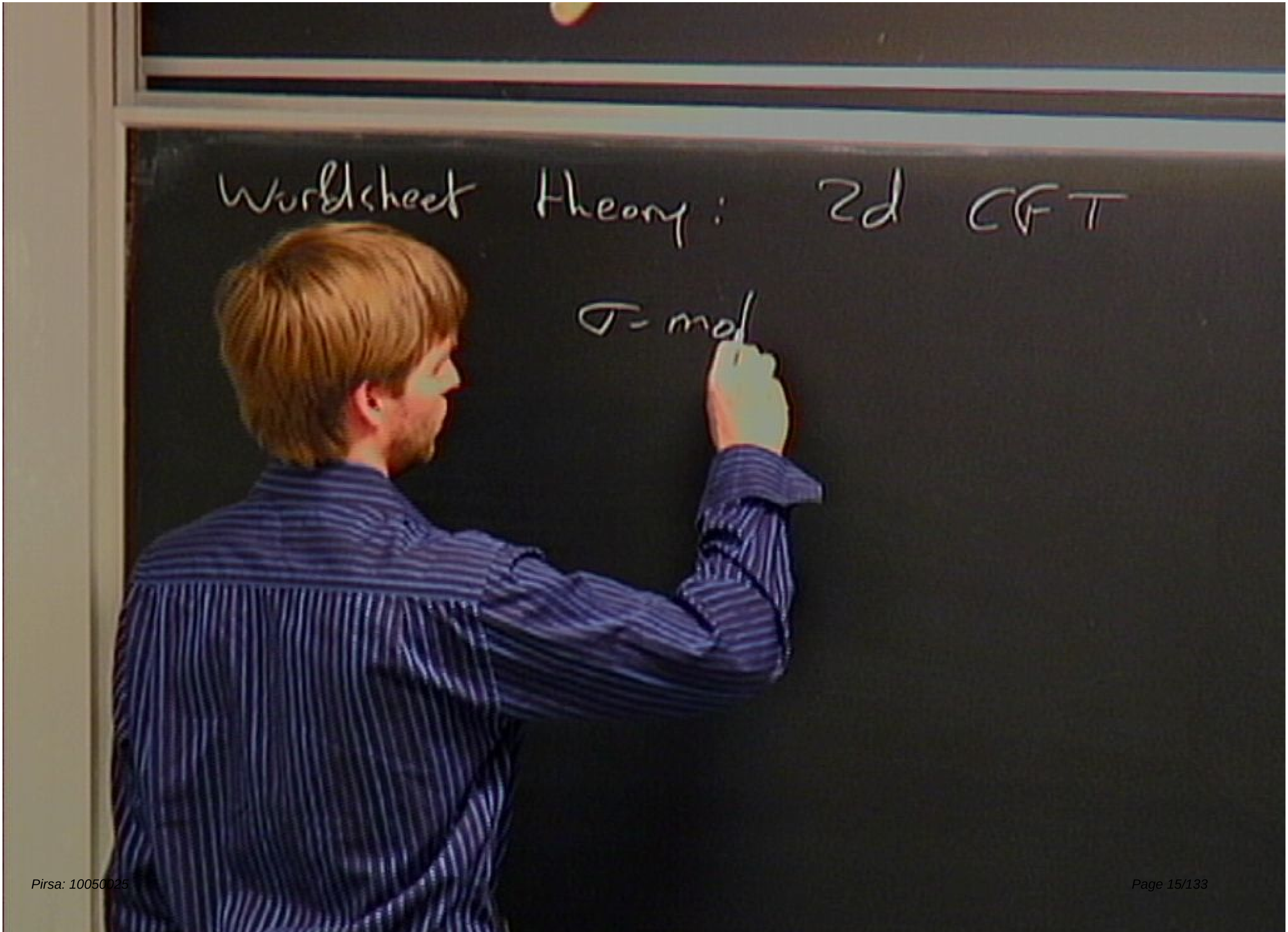
① Introduction

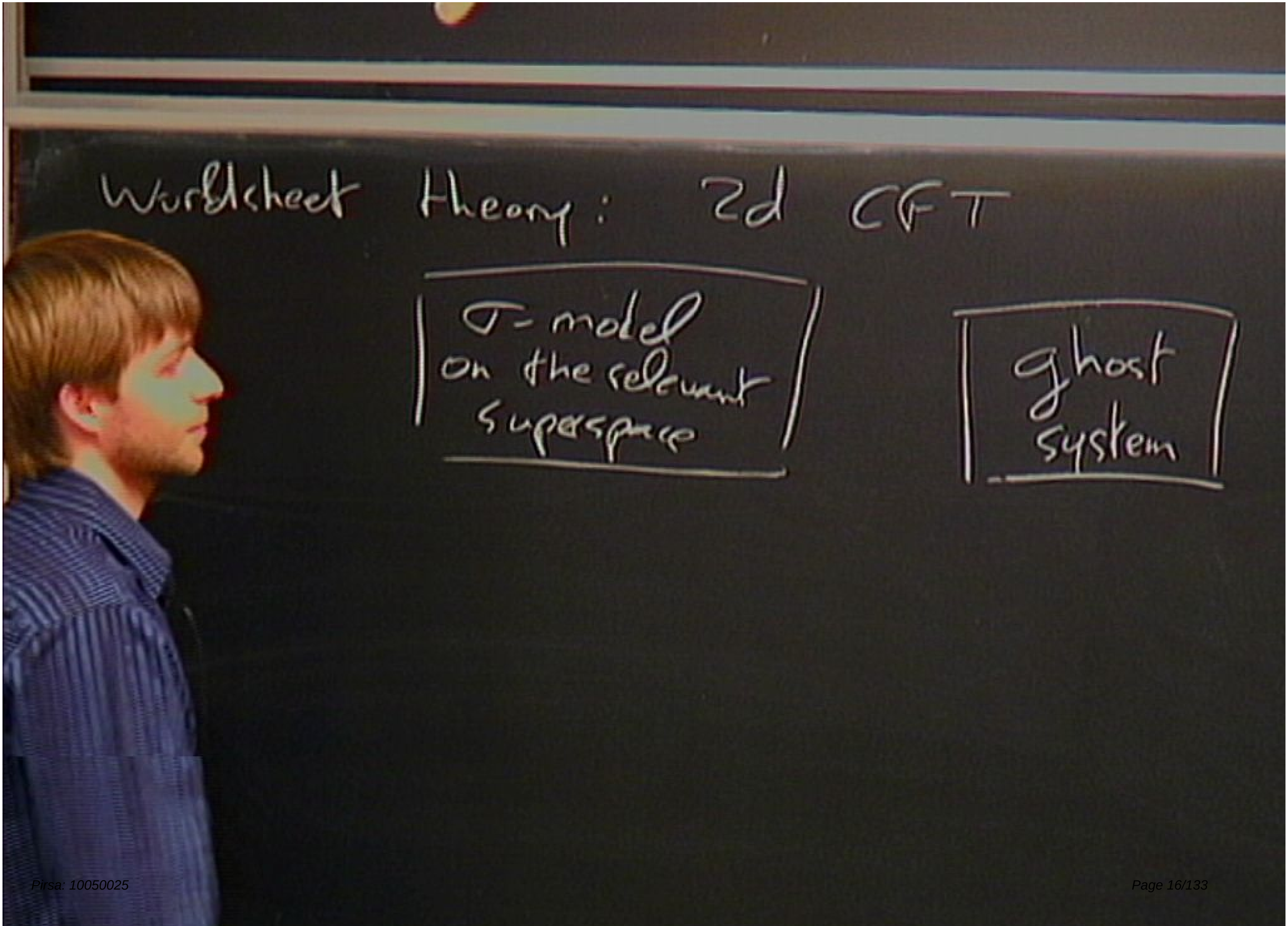
Main: String in AdS with RR Flux

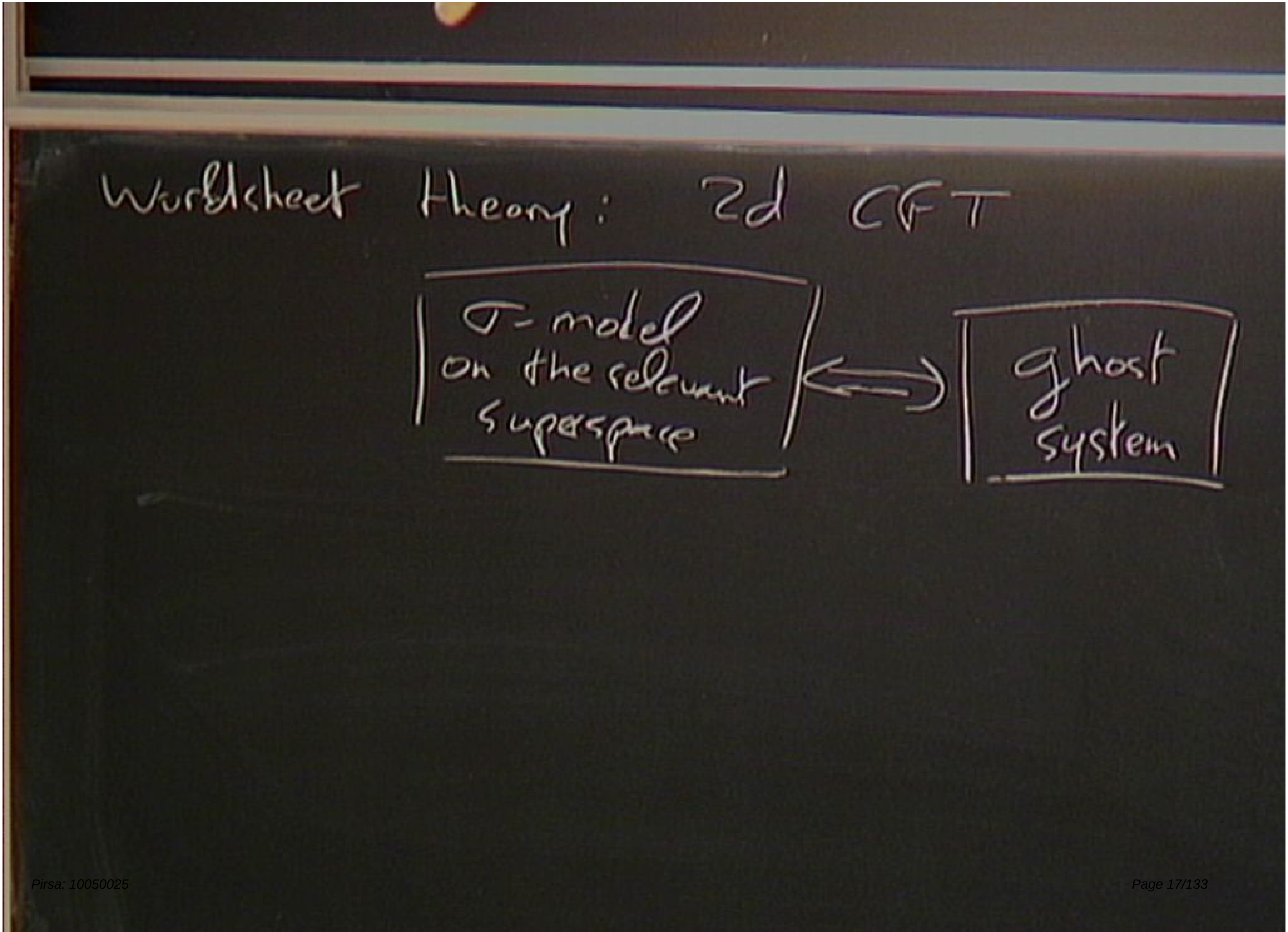
AdS/CFT

Strings in RR backgrounds


① Introduction
String in AdS with RR Flux
AdS/CFT
Strings in RR backgrounds
* spacetime ⊂ superspace


Worldsheet theory: 2d CFT
σ-mod


Worldsheet theory: 2d CFT
σ-model on the relevant superspace
ghost system


Worldsheet theory: 2d CFT
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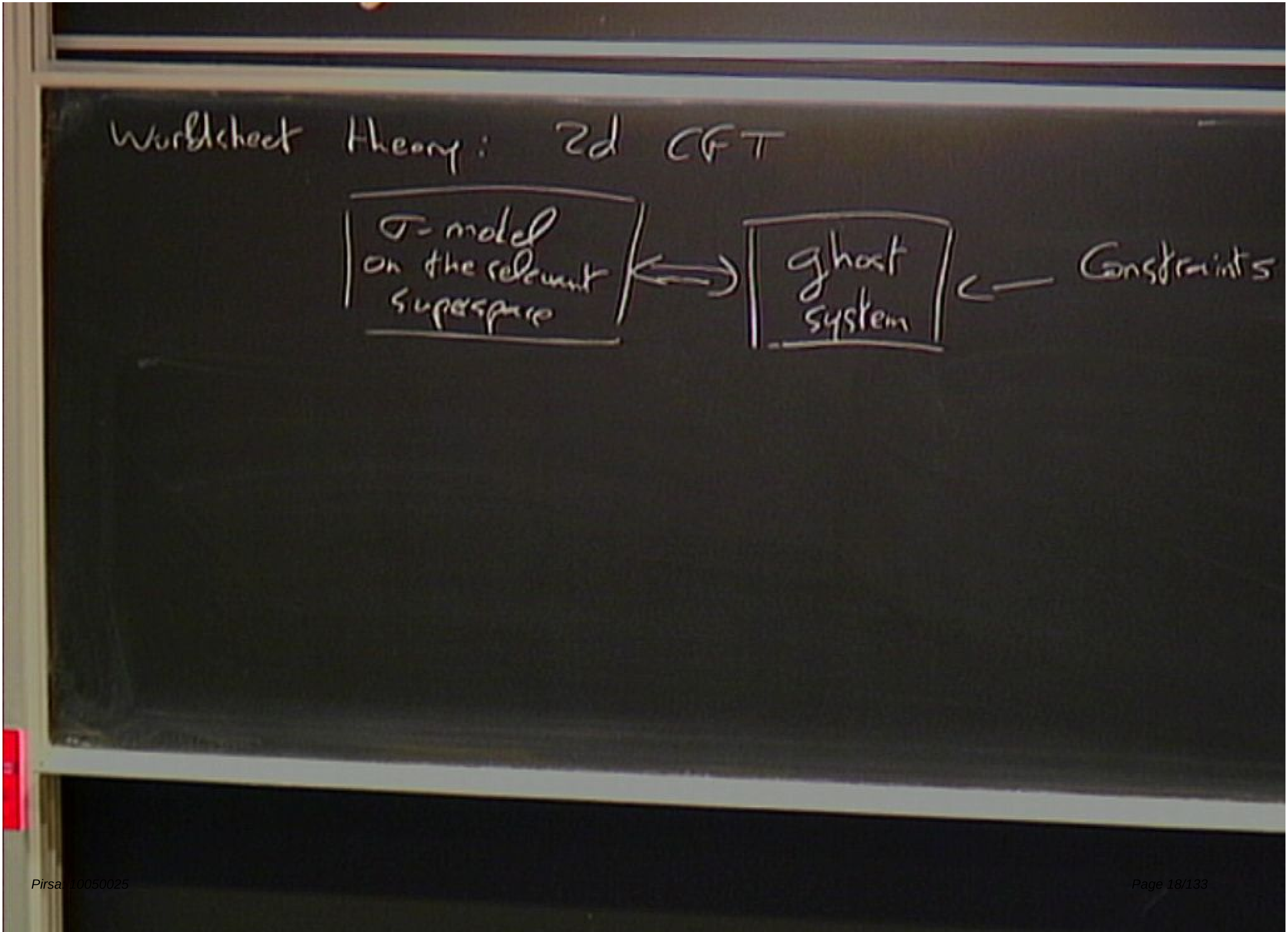
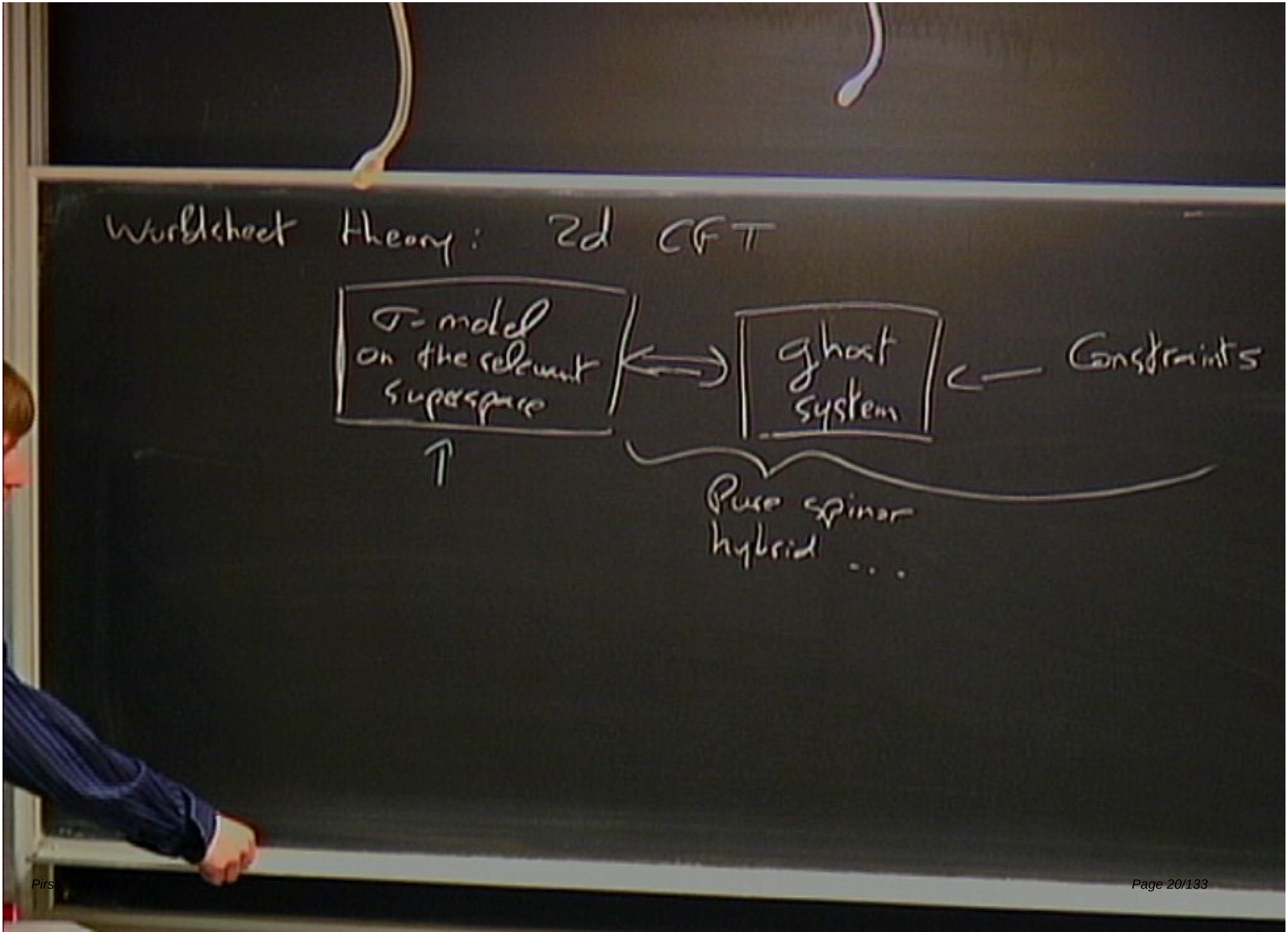

Worldsheet theory: 2d CFT
σ-model on the relevant superspace
ghost system
Constraints

Worldsheet theory: 2d CFT

σ-model on the relevant superspace ⟷ ghost system ⟵ Constraints

Pure spinor
hybrid
...


Worldsheet theory: 2d CFT
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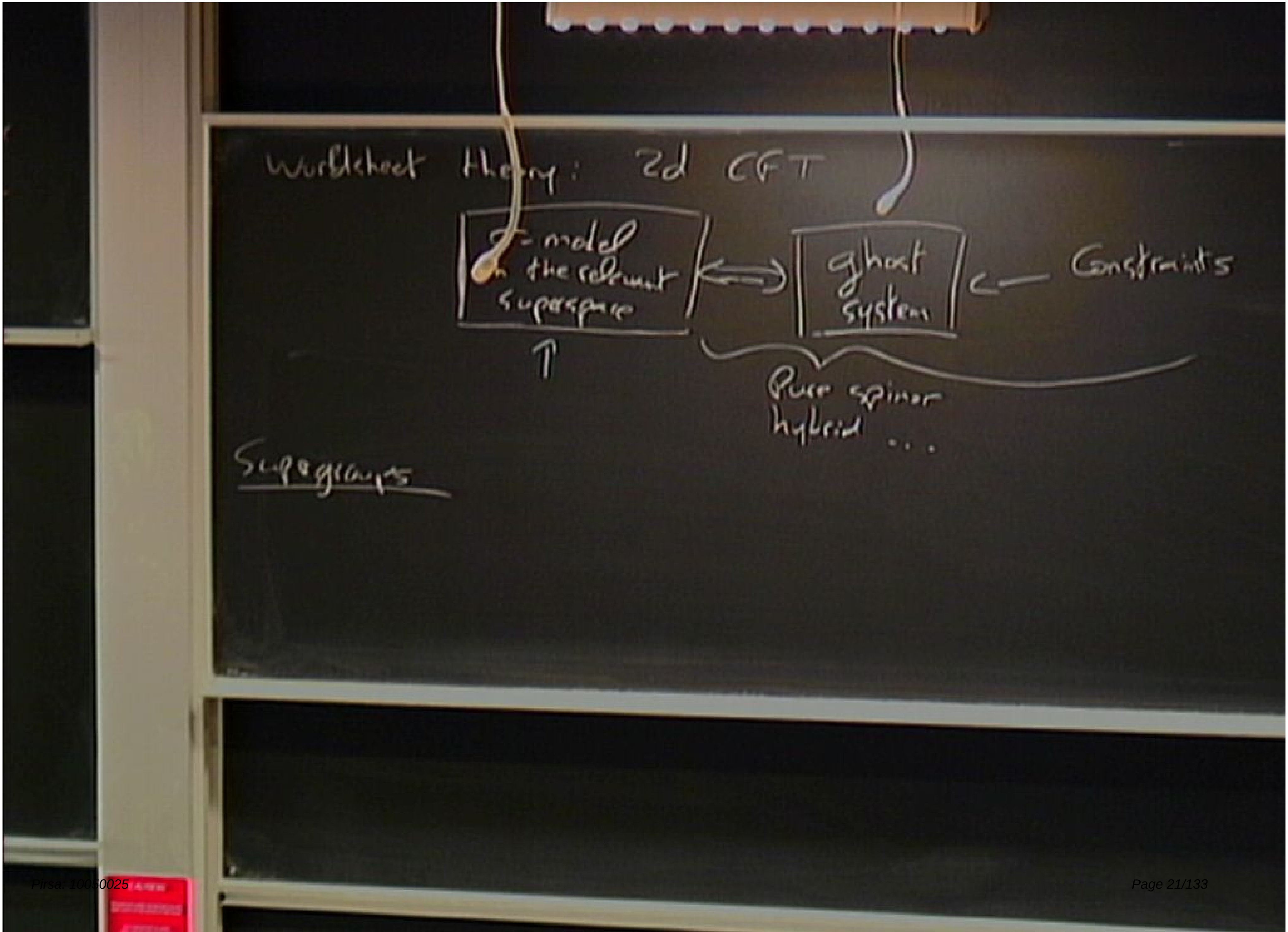
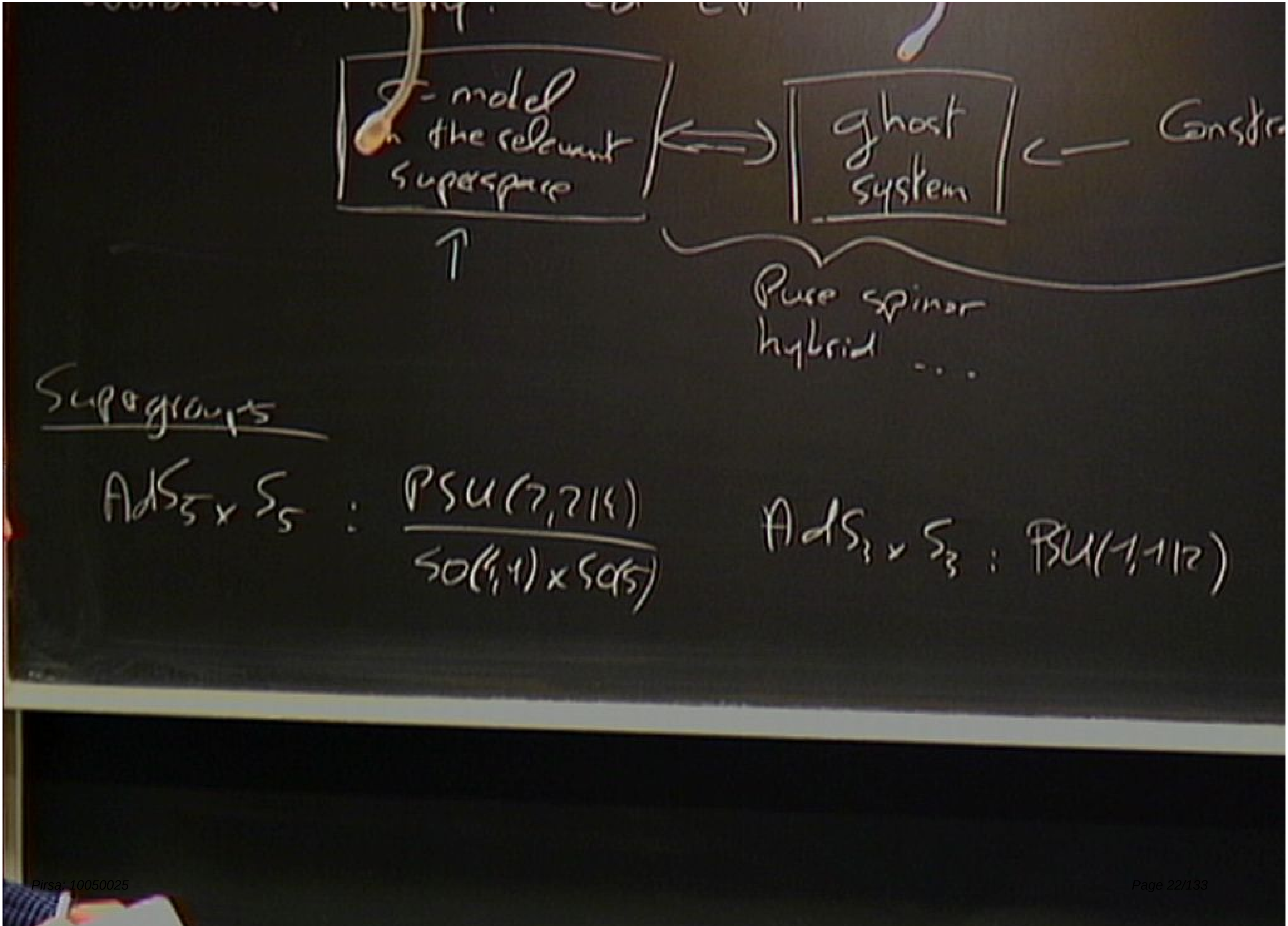
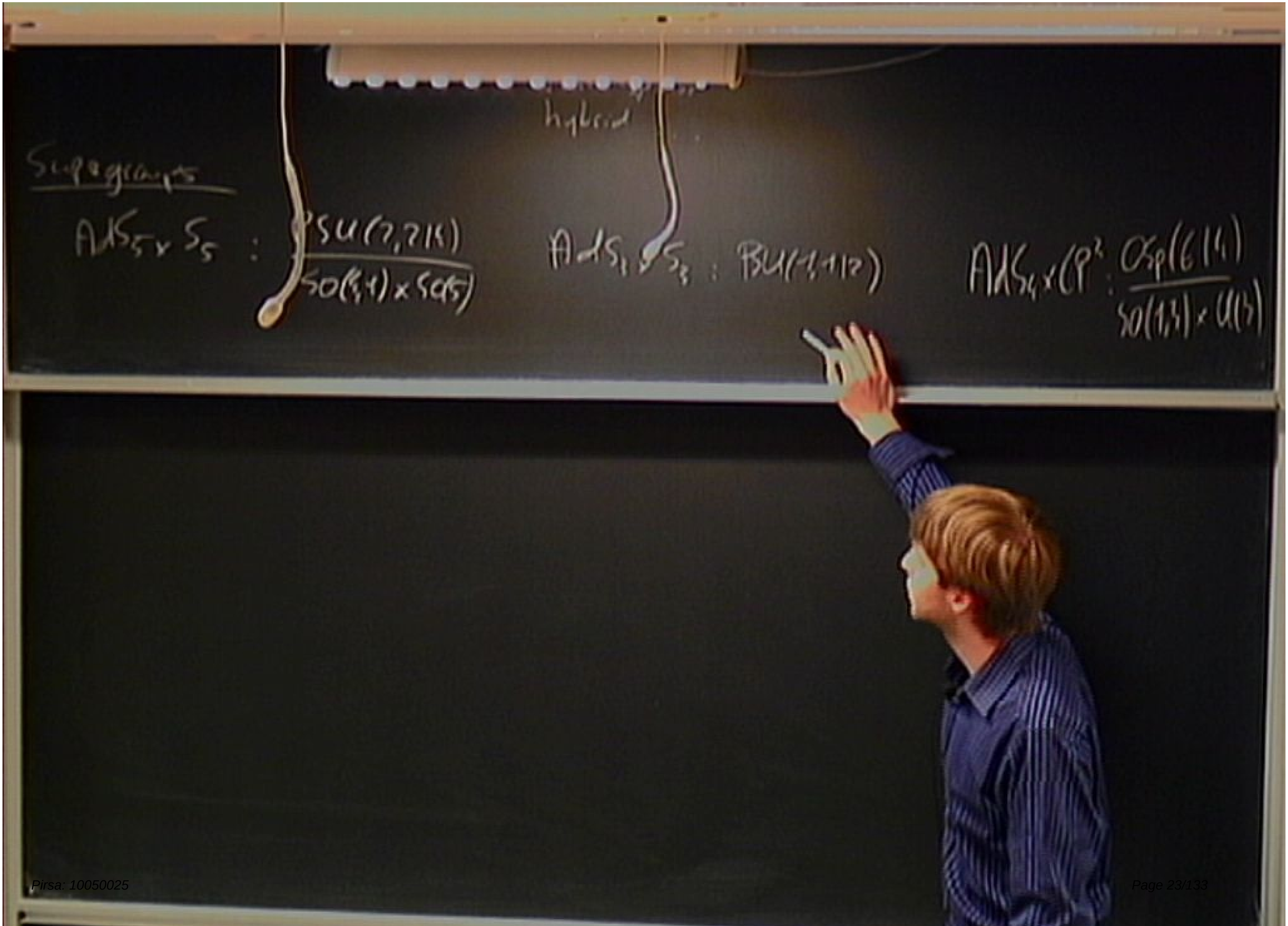

Worldsheet theory: 2d CFT
σ-model on the relevant superspace
ghost system
Constraints
Pure spinor
hybrid ...
Supergroups


σ-model
on the relevant
superspace
ghost
system
Constr
Pure spinor
hybrid
...
Supergroups
$AdS_5 \times S_5 : \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$
$AdS_3 \times S_3 : PSU(1,1|2)$


Supergroups
hybrid
AdS5 x S5 : PSU(2,2|4) / SO(4,1) x SO(5)
AdS3 x S3 : PSU(1,1|2)
AdS4 x CP3 : OSp(6|4) / SO(1,3) x U(3)

hybrid

Supergroups

$AdS_5 \times S_5 : \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$

$AdS_3 \times S_3 : PSU(1,1|2)$

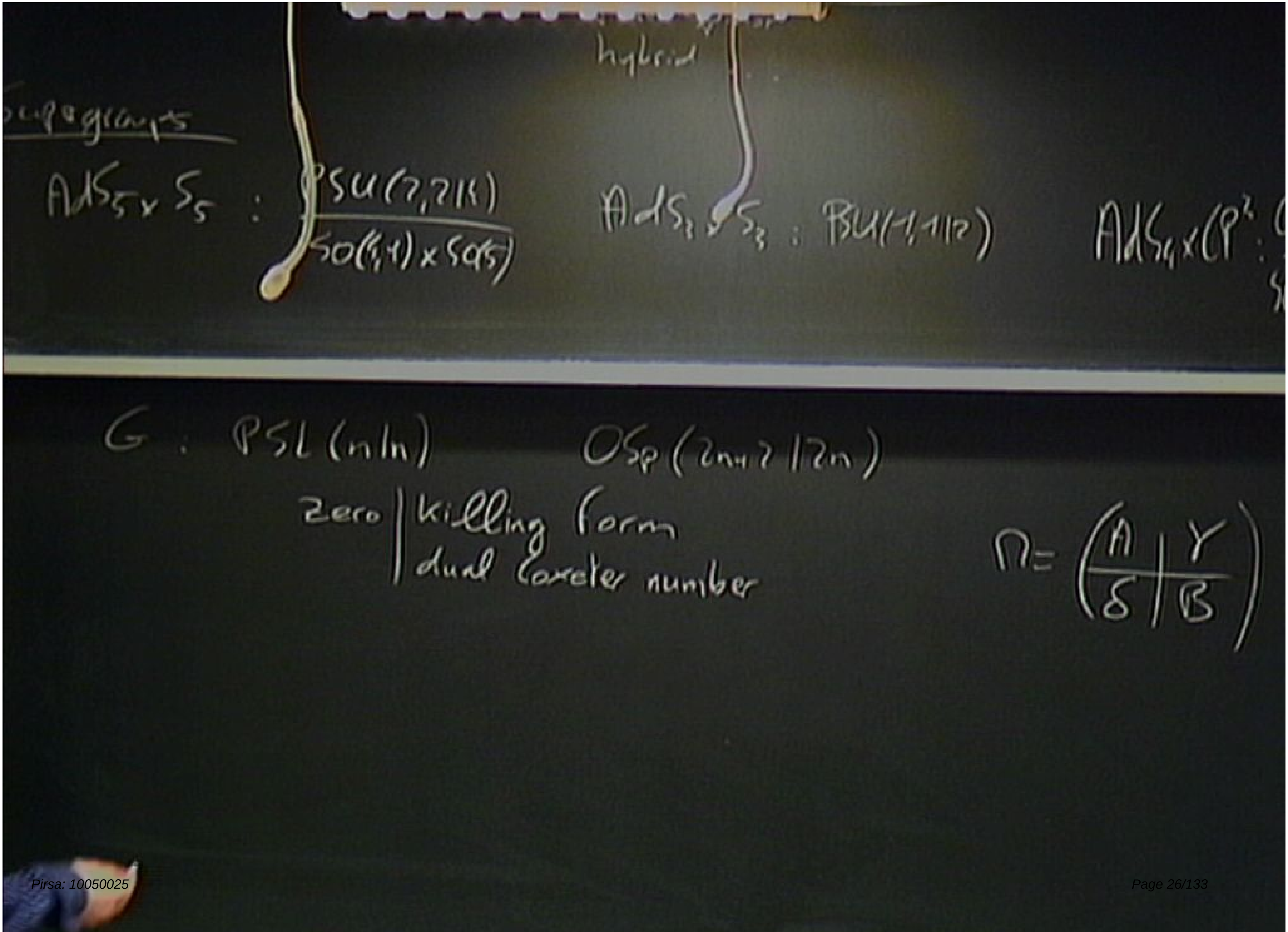
$AdS_4 \times CP^3 : \frac{OSp(6|4)}{SO(1,3) \times U(3)}$

$G : PSL(n|n)$ $\quad OSp(2n+2|2n)$

$AdS_5 \times S_5$: $\frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$ $AdS_3 \times S_3$: PSU(

G : $PSL(n|n)$ $OSp(2n+2|2n)$

zero | killing form
| dual Coxeter number


hybrid
Supergroups
AdS5 x S5 : PSU(2,2|4) / SO(4,1) x SO(5)
AdS3 x S3 : PSU(1,1|2)
AdS4 x CP3 :
G : PSL(n|n)
OSp(2n+2|2n)
zero | Killing form
| dual Coxeter number
Ω = (A | γ / δ | B)

Supergroups

hybrid

$AdS_5 \times S_5 : \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$

$AdS_3 \times S_3 : PSU(1,1|2)$

$AdS_4 \times CP^3 :$ ...

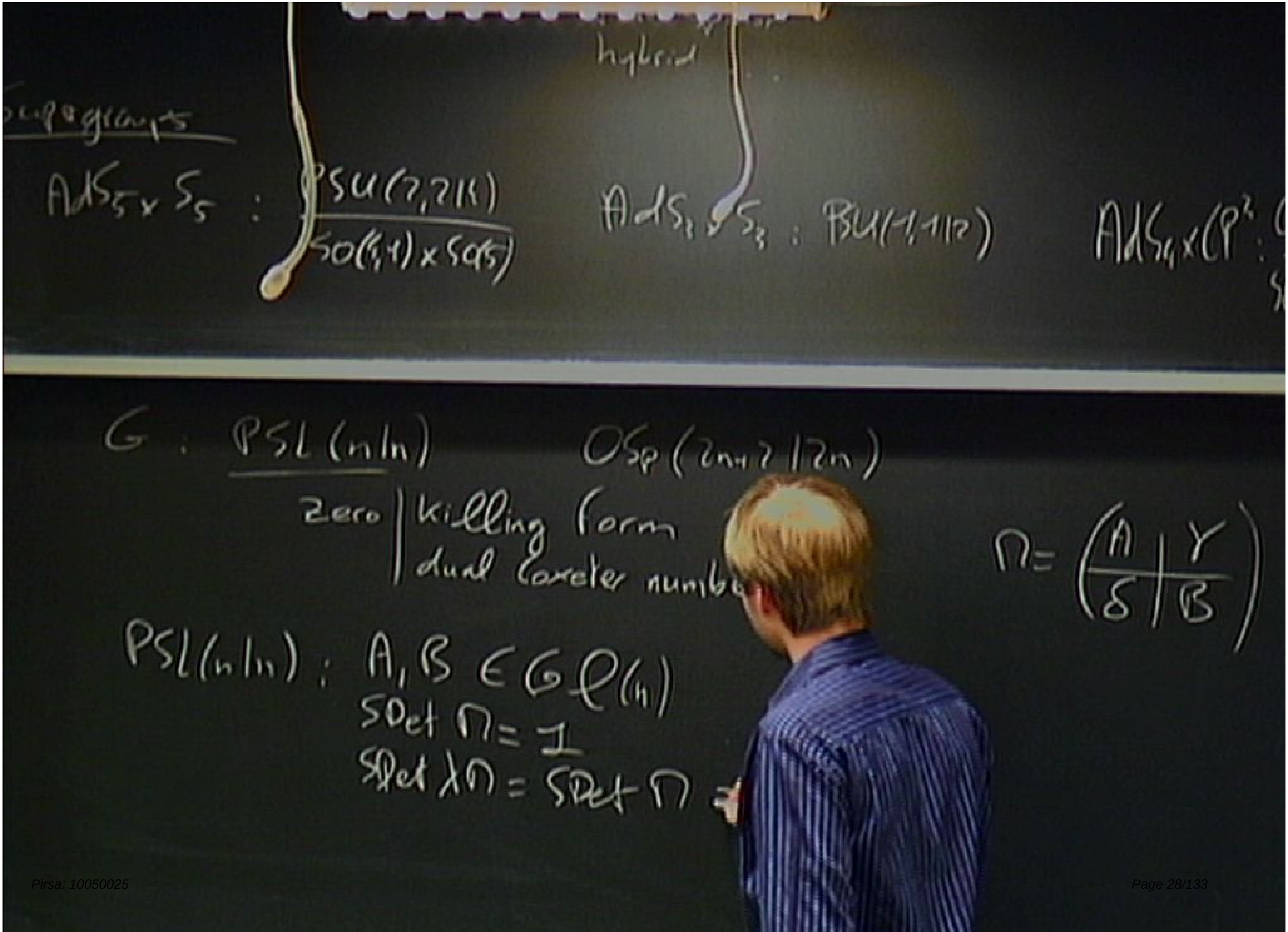
$G : \underline{PSL(n|n)} \qquad OSp(2n+2|2n)$

zero | Killing form
| dual Coxeter number

$$\Omega = \left(\begin{array}{c|c} A & \gamma \\ \hline \delta & B \end{array}\right)$$

$PSL(n|n) : A, B \in GL(n)$

$SDet\, \Omega = 1$


hybrid
supergroups
AdS5 x S5 : PSU(2,2|4) / SO(4,1) x SO(5)
AdS3 x S3 : PSU(1,1|2)
G : PSL(n|n)
OSp(2n+2|2n)
zero | killing form
| dual Coxeter numbe
PSL(n|n) : A, B ∈ Gl(n)
SDet Ω = 1
SDet λΩ = SDet Ω

supergroups

$AdS_5 \times S_5 : \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$

hybrid

$AdS_3 \times S_3 : PSU(1,1|2)$

$AdS_4 \times CP^3 :$

$G : \underline{PSL(n|n)} \qquad OSp(2n+2|2n)$

zero | Killing form
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$$\Omega = \left(\begin{array}{c|c} A & \gamma \\ \hline \delta & B \end{array}\right)$$

$PSL(n|n) : \; A, B \in Gl(n)$

$\mathrm{SDet}\, \Omega = 1$

$\mathrm{SDet}\, \lambda\Omega = \mathrm{SDet}\, \Omega \Rightarrow \Omega \equiv \lambda \Omega$

hybrid

Supergroups

$AdS_5 \times S_5 : \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$ $AdS_3 \times S_3 : PSU(1,1|2)$ $AdS_4 \times CP^3 :$

$G : \underline{PSL(n|n)}$ $OSp(2n+2|2n)$

zero | Killing form
| dual Coxeter number

$\Pi = \left(\begin{array}{c|c} A & \gamma \\ \hline \delta & B \end{array}\right)$

$\underline{PSL}(n|\underline{n}) : A, B \in G\ell(n)$

$\text{SDet}\, \Pi = 1$

$\text{SDet}\, \lambda\Pi = \text{SDet}\, \Pi \Rightarrow \Pi \equiv \lambda \Pi$

$OSp(2n+2|2n) : A \in$

Pure spinor
hybrid ...

Supergroups

$AdS_5 \times S_5$: $\frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$

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$PSL(n|n)$: $A, B \in GL(n)$

$SDet\, \Omega = 1$

$SDet\, \lambda\Omega = SDet\, \Omega \Rightarrow \Omega \equiv \lambda\Omega$

$OSp(2n+2|2n)$: $A \in SO(2n+2)$

$B \in Sp(2n)$

hybrid

supergroups

$AdS_5 \times S_5 : \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$ $\quad AdS_3 \times S_3 : PSU(1,1|2)$ $\quad AdS_4 \times CP^3 :$

$G : \underline{PSL(n|n)} \qquad OSp(2n+2|2n)$

zero | Killing form
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$$\Omega = \left(\begin{array}{c|c} A & \gamma \\ \hline \delta & B \end{array}\right)$$

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$SDet\, \Omega = 1$

$SDet\, \lambda\Omega = SDet\, \Omega \Rightarrow \Omega \equiv \lambda\Omega$

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$B \in Sp(2n)$

Supergroups

$AdS_5 \times S^5 : \dfrac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$

hybrid

$AdS_3 \times S^3 : PSU(1,1|2)$

$AdS_4 \times CP^3 : \dfrac{OSp(6|4)}{SO(1,3)\times U(3)}$

G : $PSL(n|n)$ $\quad$ $OSp(2n+2|2n)$

zero | Killing form
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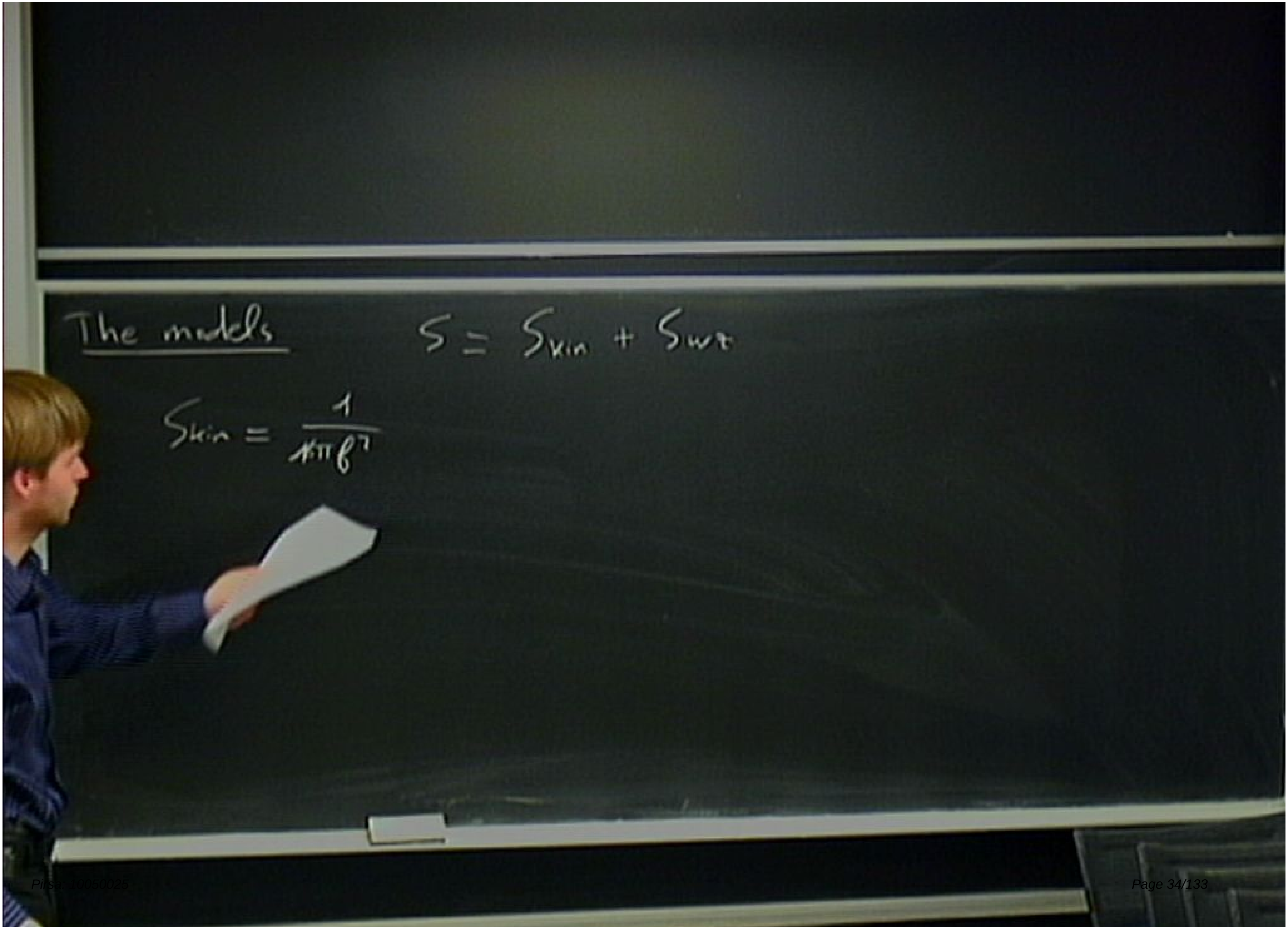
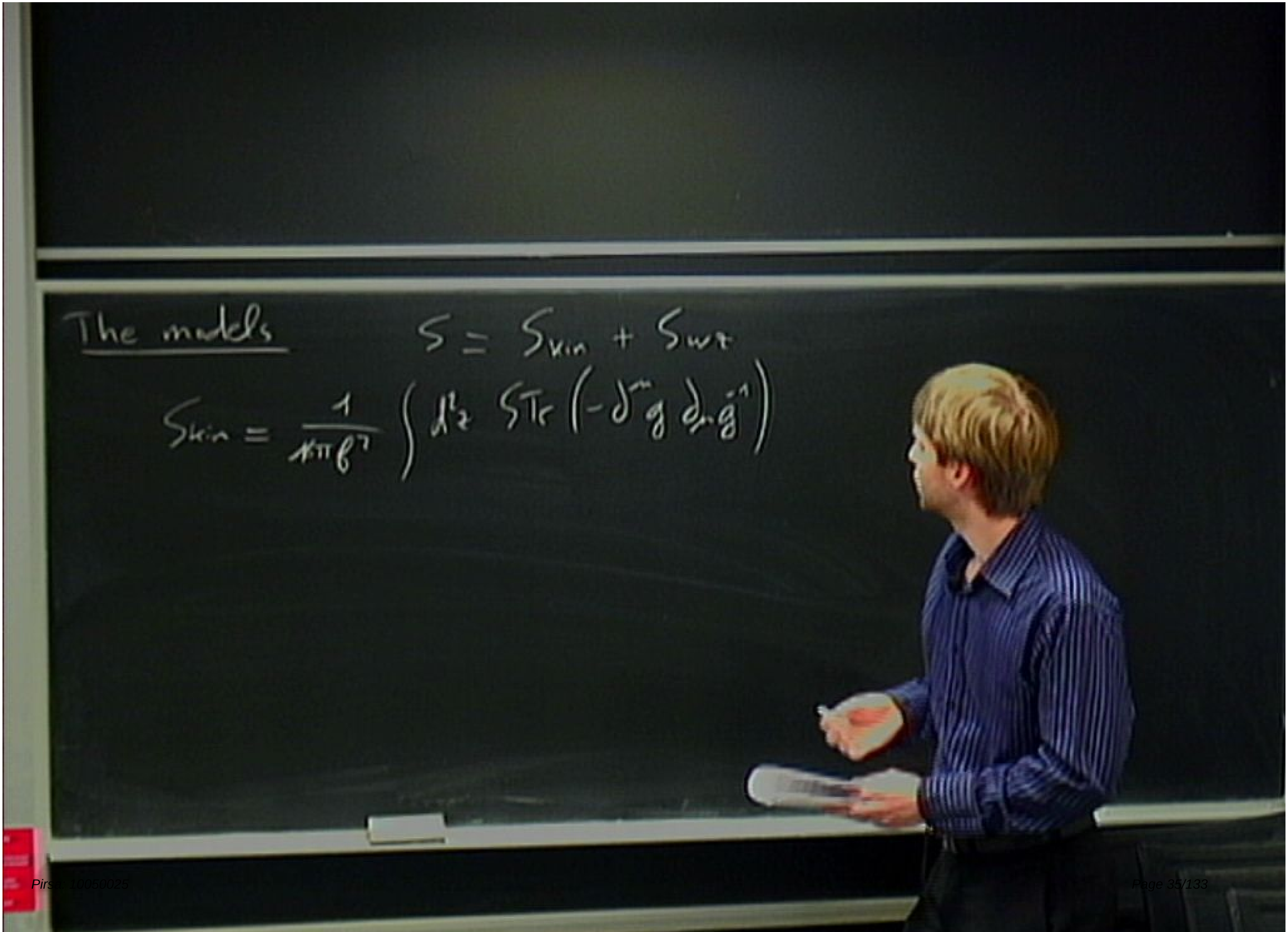
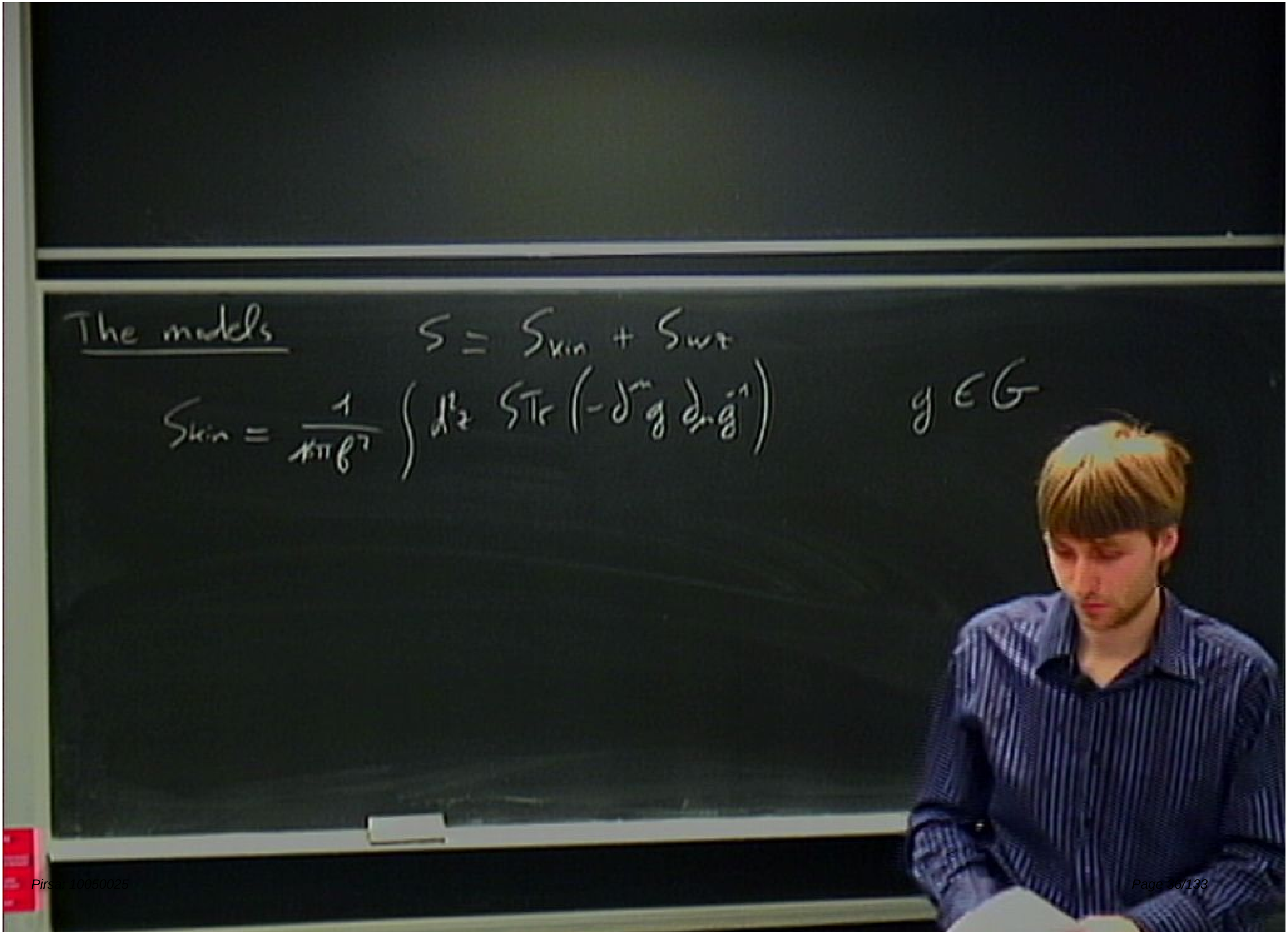
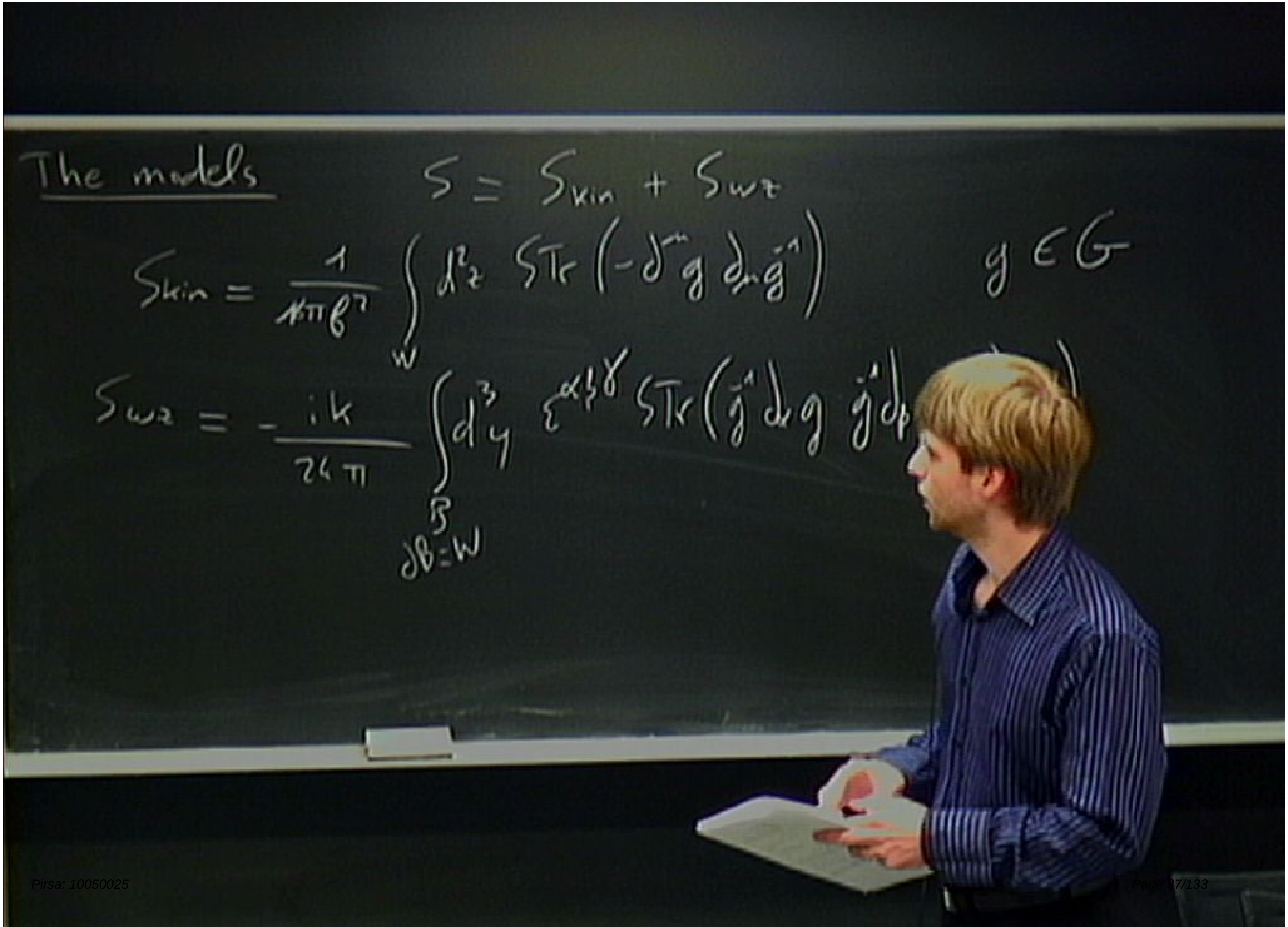
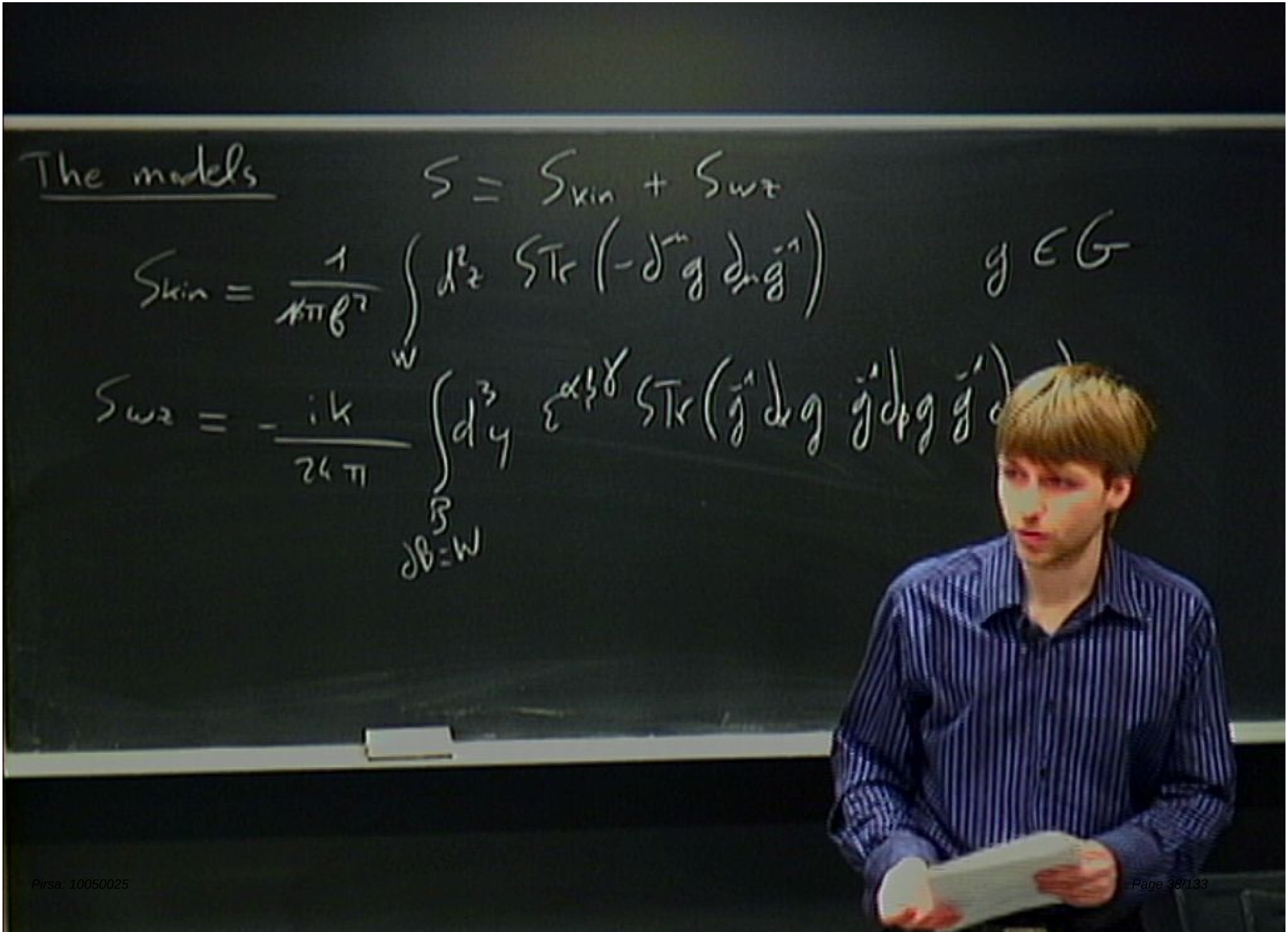
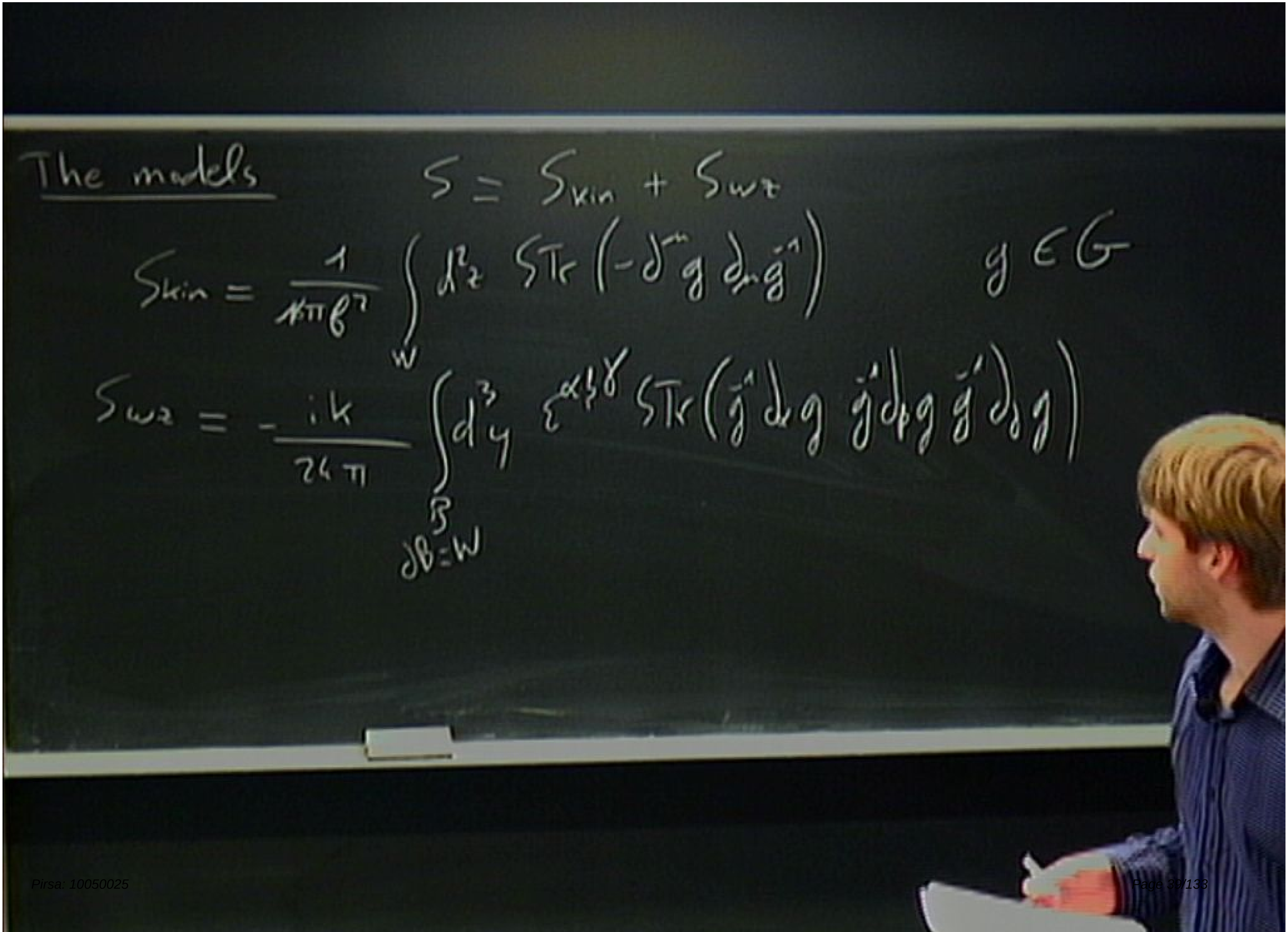

The models
S = S_kin + S_wz
S_kin = 1/(4πβ²)


The models
S = S_kin + S_WZ
S_kin = 1/(4πβ²) ∫ d²z STr(−∂^m g ∂_m g^{-1})


The models
S = S_kin + S_wz
S_kin = 1/(4π β^2) ∫ d^2z STr(-∂^μ g ∂_μ g^-1)
g ∈ G


The models
$S = S_{kin} + S_{WZ}$
$S_{kin} = \frac{1}{4\pi\beta^2} \int_W d^2z \, \mathrm{STr}\left(-\partial^\mu g \, \partial_\mu g^{-1}\right)$
$g \in G$
$S_{WZ} = -\frac{ik}{24\pi} \int_B d^3y \, \varepsilon^{\alpha\beta\gamma} \, \mathrm{STr}\left(g^{-1}\partial_\alpha g \, g^{-1}\partial_\beta$
$\partial B = W$


The models
$S = S_{kin} + S_{WZ}$
$S_{kin} = \frac{1}{4\pi f^2} \int_W d^2z \, \mathrm{STr}(-\partial^\mu g \, \partial_\mu g^{-1})$
$g \in G$
$S_{WZ} = -\frac{ik}{24\pi} \int_B d^3y \, \varepsilon^{\alpha\beta\gamma} \, \mathrm{STr}(g^{-1}\partial_\alpha g \, g^{-1}\partial_\beta g \, g^{-1}\partial_\gamma g)$
$\partial B = W$


The models
$S = S_{kin} + S_{WZ}$
$S_{kin} = \frac{1}{4\pi\beta^2} \int_W d^2z \, STr\left(-\partial^\mu g \, \partial_\mu g^{-1}\right)$
$g \in G$
$S_{WZ} = -\frac{ik}{24\pi} \int_B d^3y \, \varepsilon^{\alpha\beta\gamma} STr\left(g^{-1}\partial_\alpha g \, g^{-1}\partial_\beta g \, g^{-1}\partial_\gamma g\right)$
$\partial B = W$

The models

$$S = S_{kin} + S_{WZ}$$

$$S_{kin} = \frac{1}{4\pi \beta^2} \int_W d^2z \, \mathrm{STr}\left(-\partial^\mu g \, \partial_\mu g^{-1}\right) \qquad g \in G$$

$$S_{WZ} = -\frac{i|k|}{24\pi} \int_{B,\ \partial B = W} d^3y \, \varepsilon^{\alpha\beta\gamma} \, \mathrm{STr}\left(g^{-1}\partial_\alpha g \, g^{-1}\partial_\beta g \, g^{-1}\partial_\gamma g\right)$$

The models

$$S = S_{kin} + S_{WZ}$$

$$S_{kin} = \frac{1}{16\pi\beta^2}\int_W d^2z \; \mathrm{STr}\left(-\partial^\mu g\, \partial_\mu g^{-1}\right) \qquad g \in G$$

$$S_{WZ} = -\frac{i|k|}{24\pi}\int_{B,\ \partial B = W} d^3y \; \varepsilon^{\alpha\beta\gamma}\, \mathrm{STr}\left(g^{-1}\partial_\alpha g\; g^{-1}\partial_\beta g\; g^{-1}\partial_\gamma g\right)$$

Two parameters: β^2, k

$k = \pm \frac{1}{\beta^2}$: WZW model

G : $PSL(n|n)$ $OSp(2n+2|2n)$

zero | Killing form
| dual Coxeter number

$PSL(n|n)$: $A, B \in Gl(n)$

$SDet\, \Omega = 1$

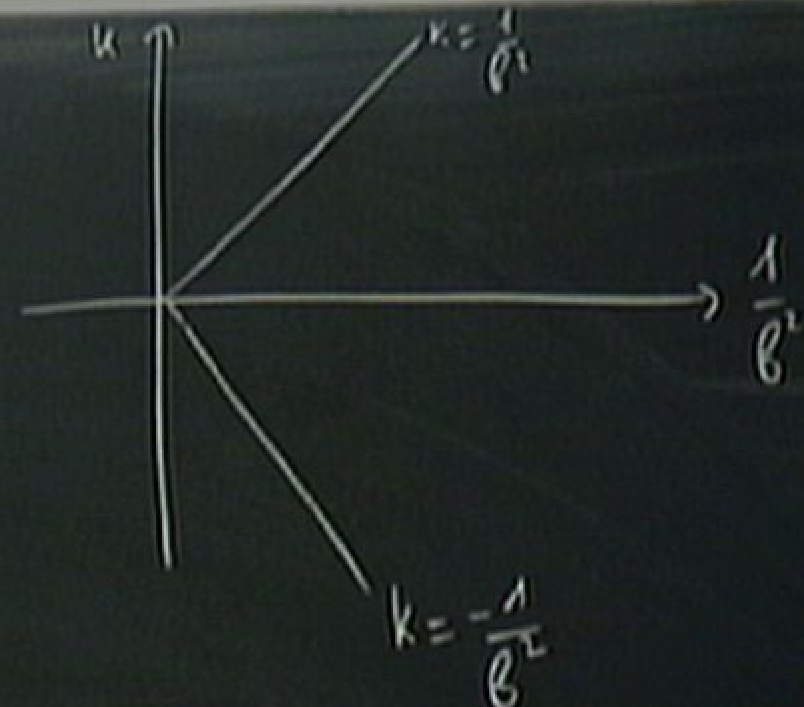
$SDet\, \lambda\Omega = SDet\, \Omega \Rightarrow \Omega \equiv \lambda\Omega$

$OSp(2n+2|2n)$: $A \in SO(2n+2)$

Two parameters: β, k

$|k\beta^2| \leq 1$

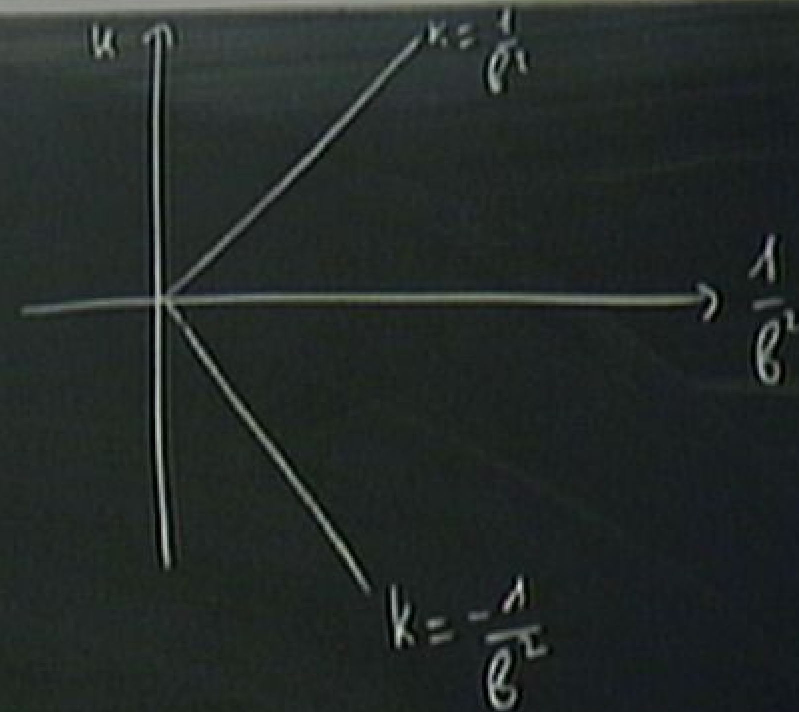
Moduli space:



Two parameters: β, k

$|k\beta^2| \leq 1$

Moduli space:



Two parameters: β, k

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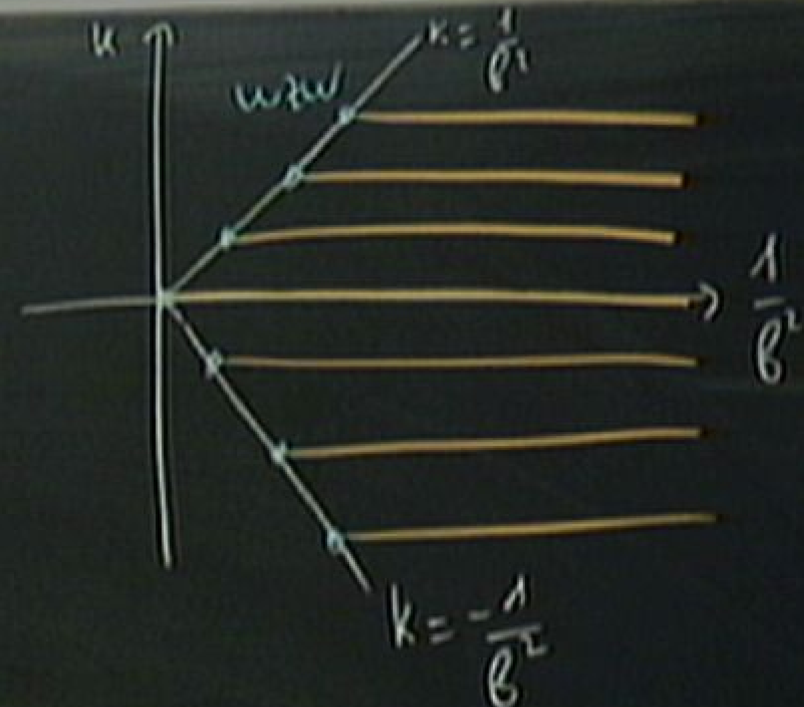
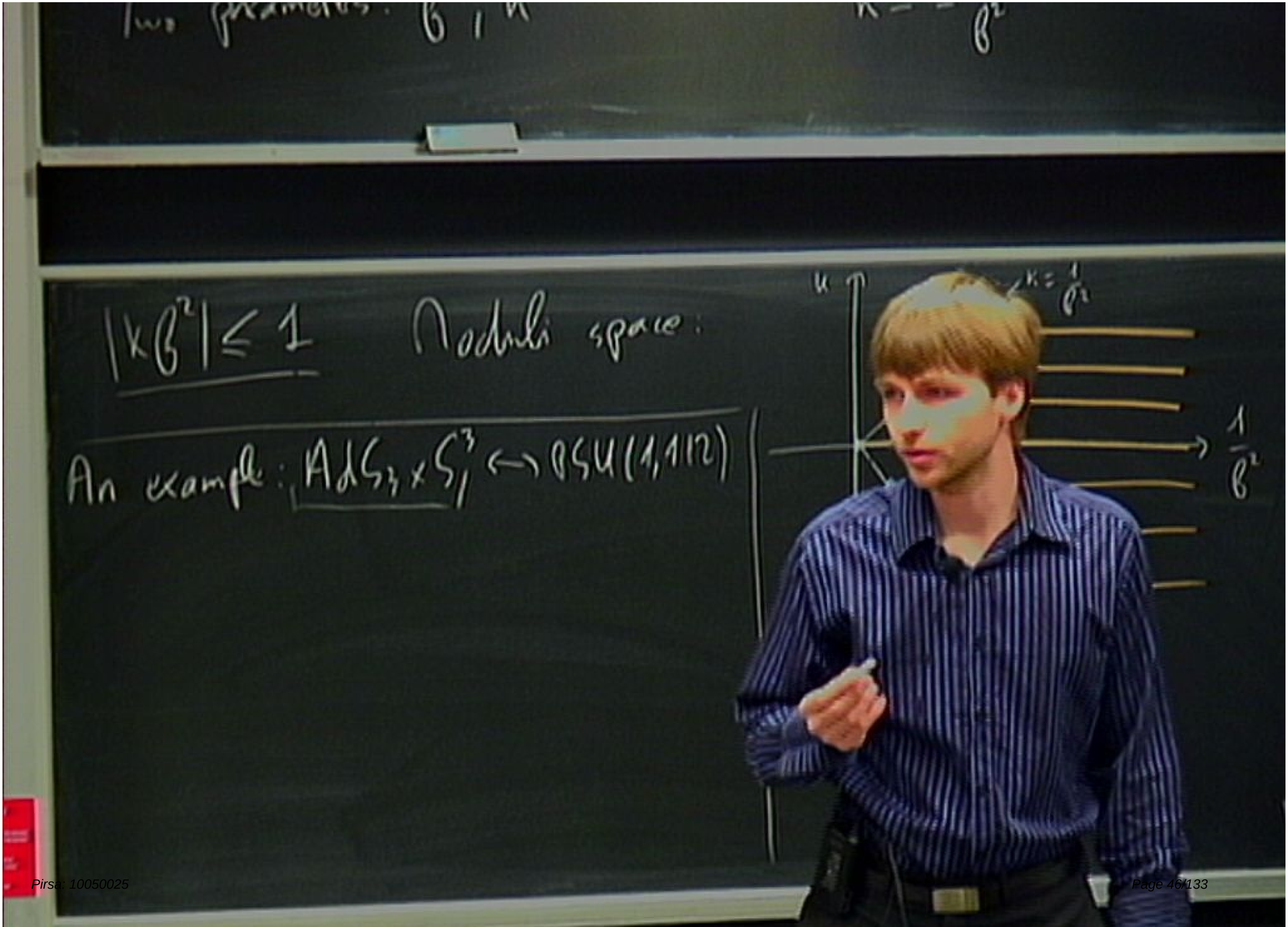
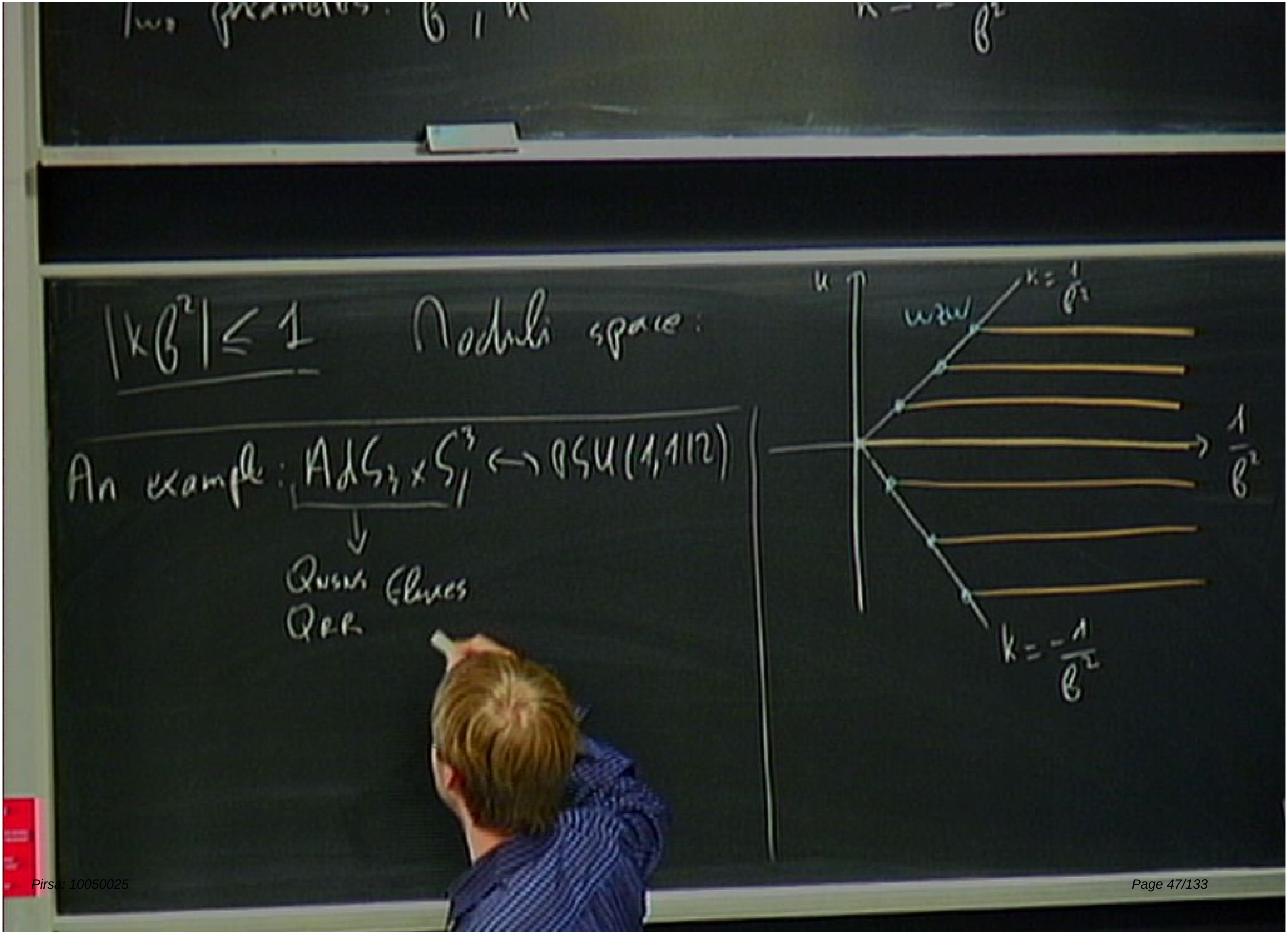
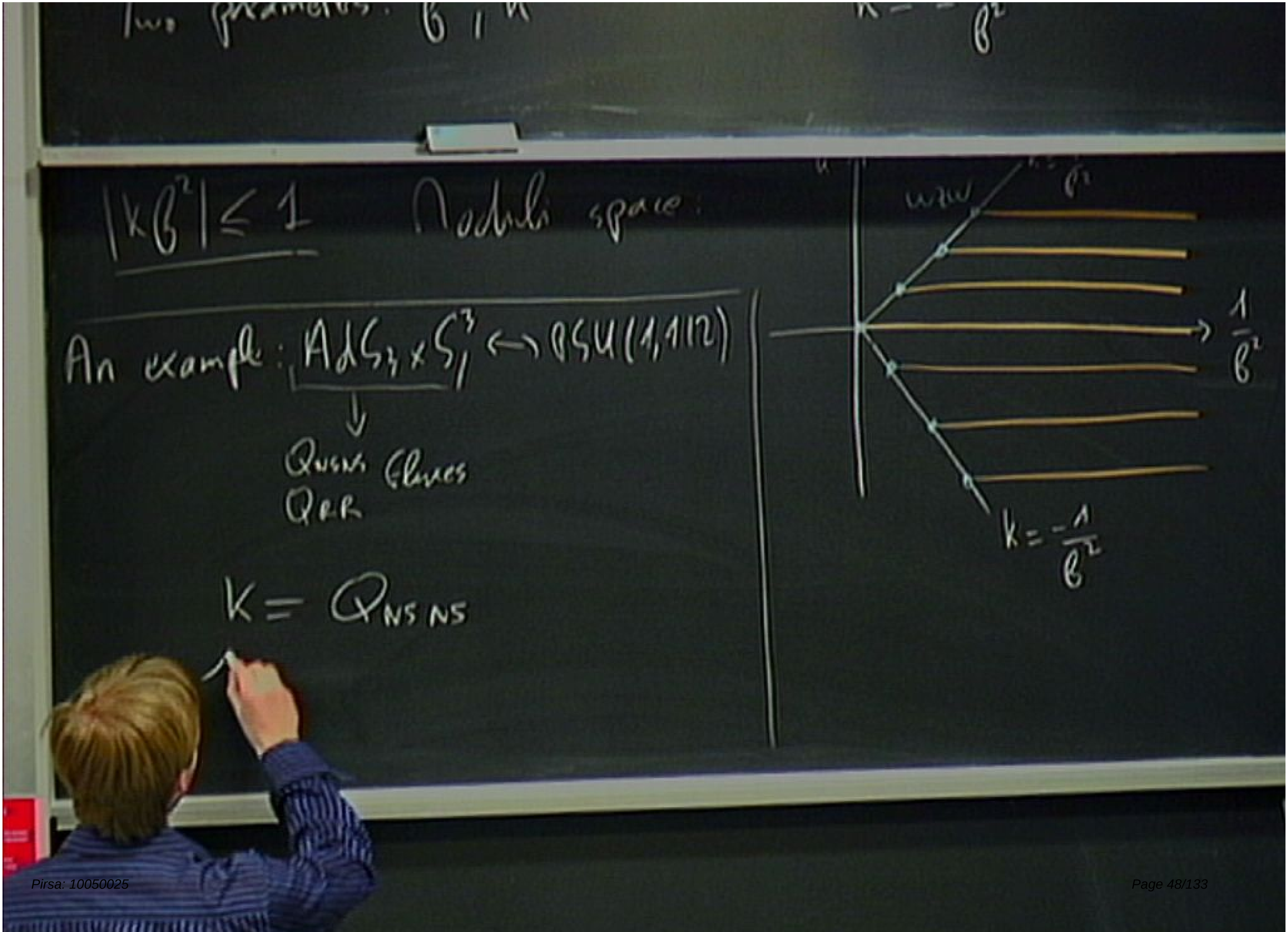
Moduli space:

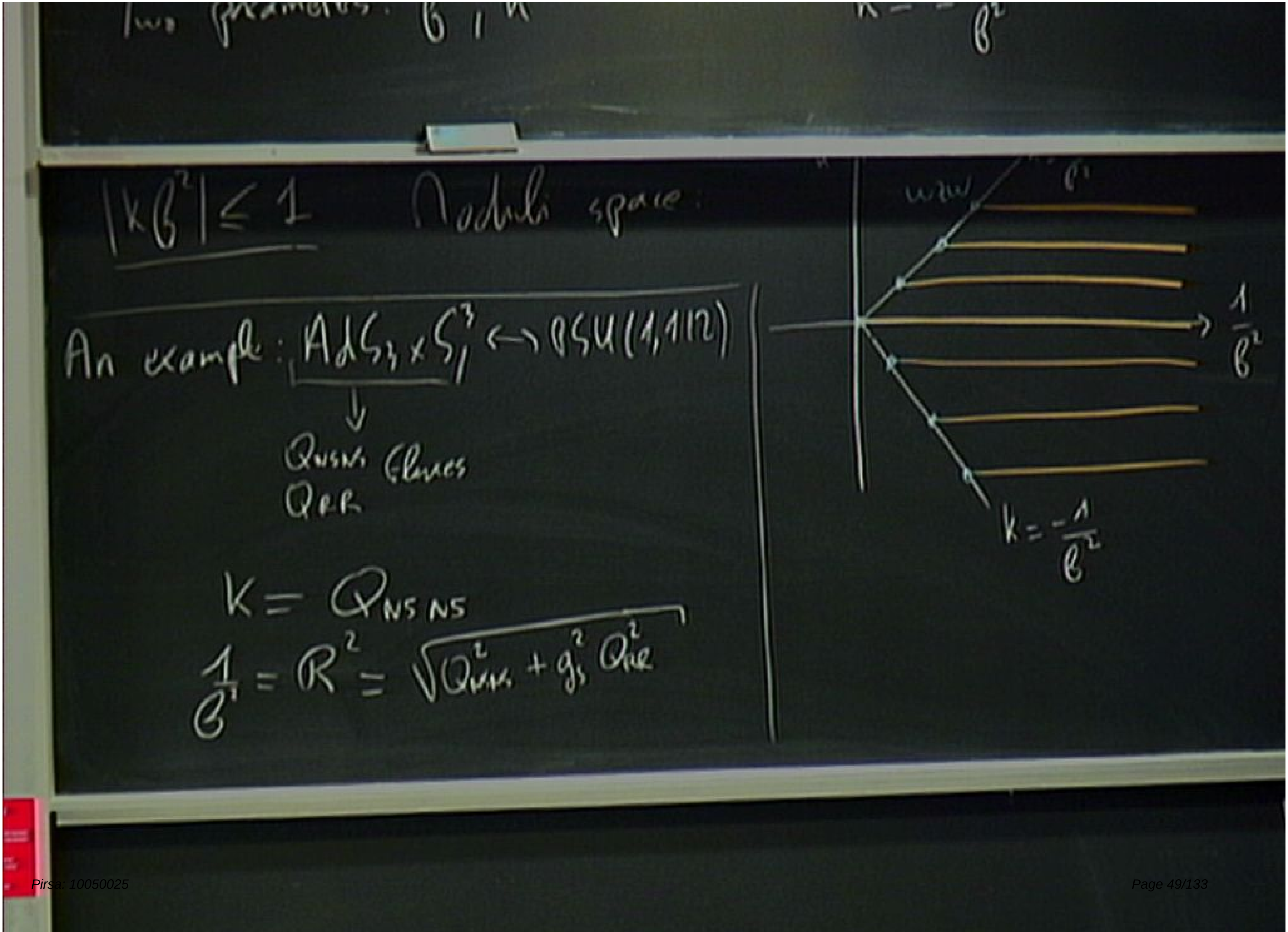



$|\kappa\beta^2| \leq 1$
Moduli space:
An example: $AdS_3 \times S^3_1 \leftrightarrow PSU(1,1|2)$
$\kappa = \frac{1}{\beta^2}$
$\frac{1}{\beta^2}$



|kβ²| ≤ 1
Moduli space:
An example: AdS3 × S3 ↔ PSU(1,1|2)
k
wzw
k = 1/β²
1/β²
k = −1/β²


|kβ²| ≤ 1
Moduli space:
An example: AdS3 x S3 ⟷ PSU(1,1|2)
QNSNS fluxes
QRR
K = QNSNS
WZW
1/β²
k = -1/β²

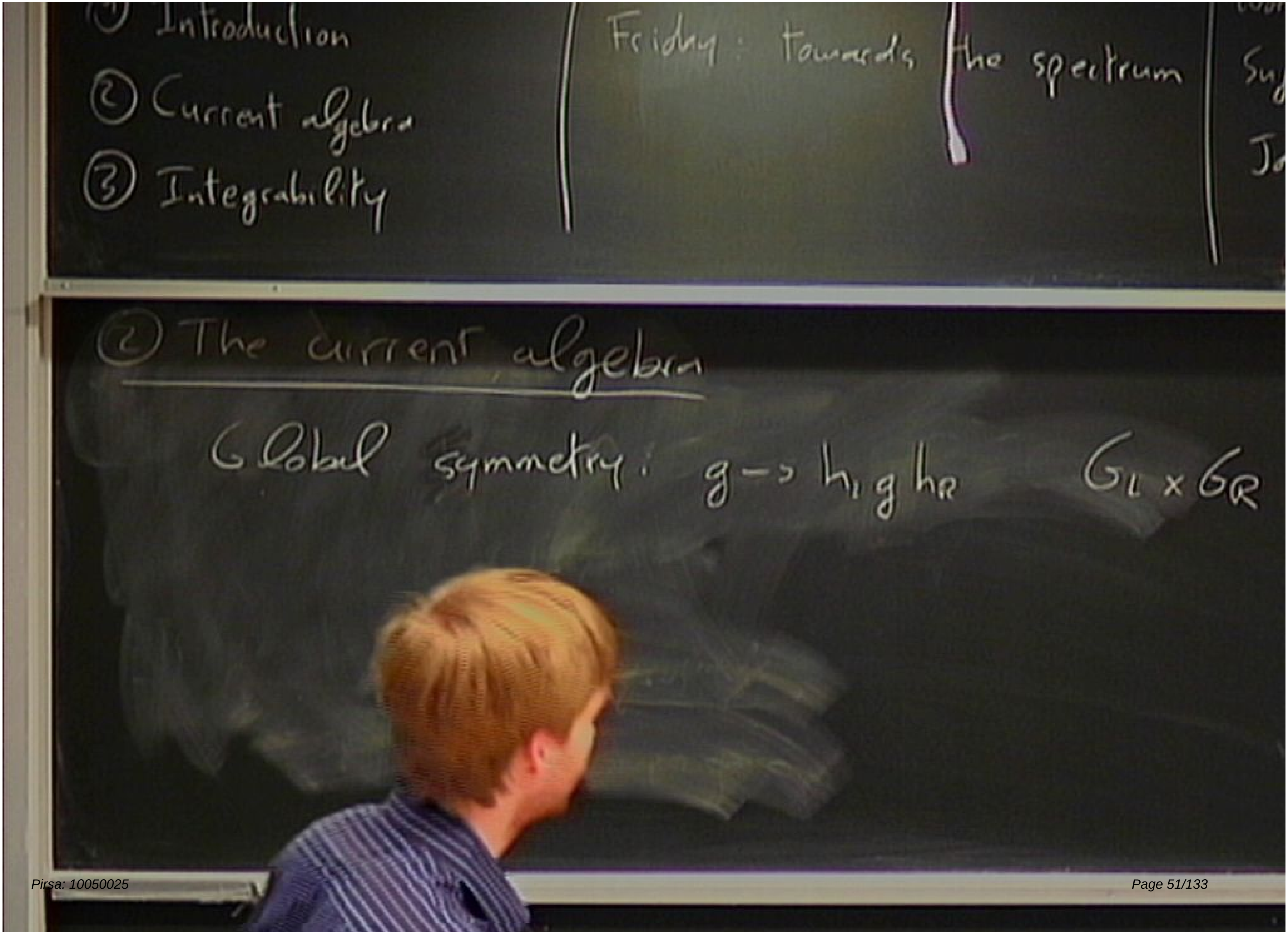


|kβ²| ≤ 1
Moduli space:
WZW
An example: AdS3 x S3 ↔ PSU(1,1|2)
QNSNS Branes
QRR
K = QNSNS
1/β² = R² = √(Q²NSNS + gs² Q²RR)
1/β²
k = -1/β²

② Current algebra

③ Integrability

② The current algebra


Introduction
② Current algebra
③ Integrability
Friday: towards the spectrum
② The current algebra
Global symmetry: g → h_L g h_R   G_L × G_R

② Current algebra

③ Integrability

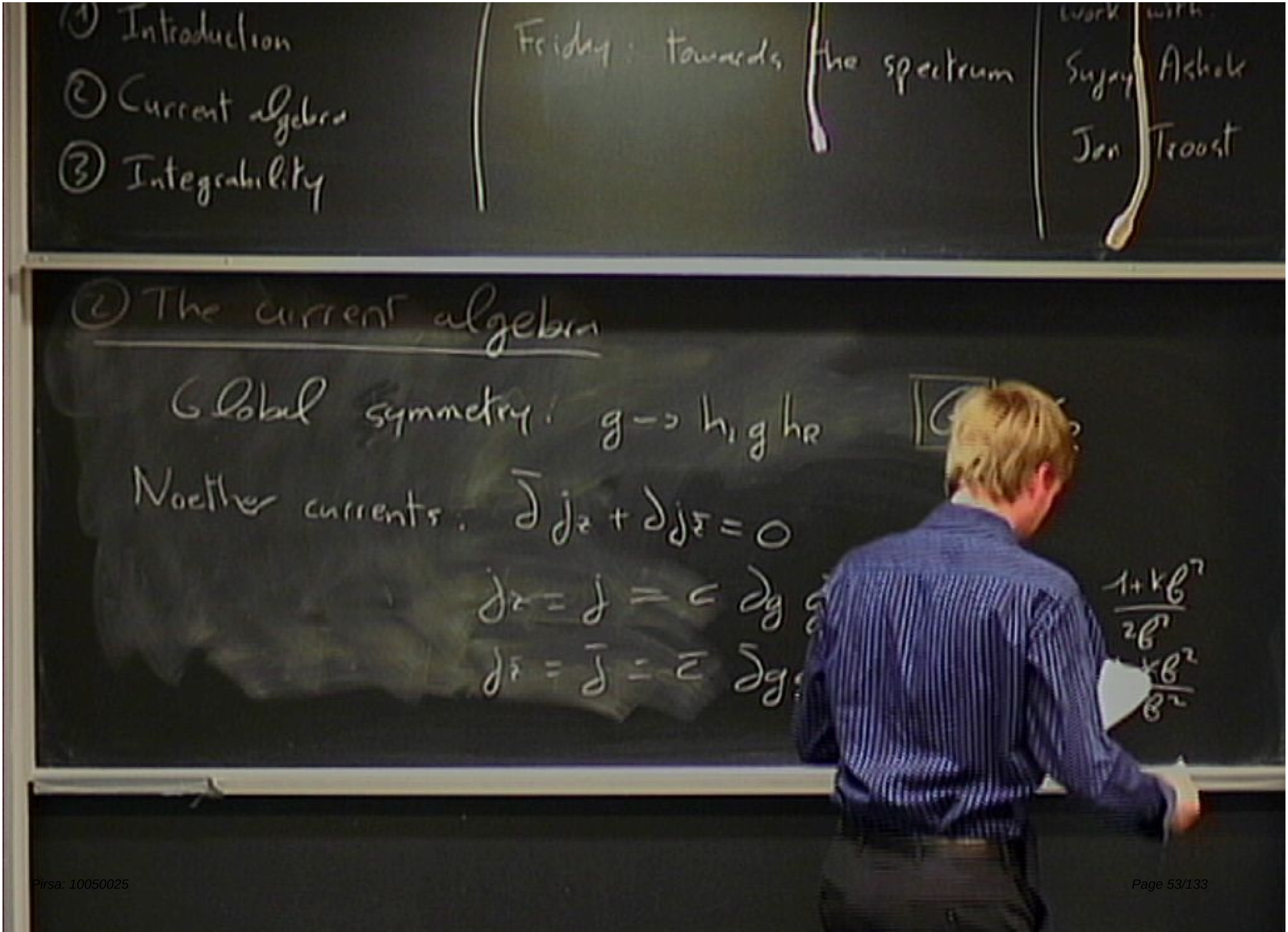
② The current algebra

Global symmetry: $g \to h_L \, g \, h_R$ $\quad \boxed{G_L} \times G$…

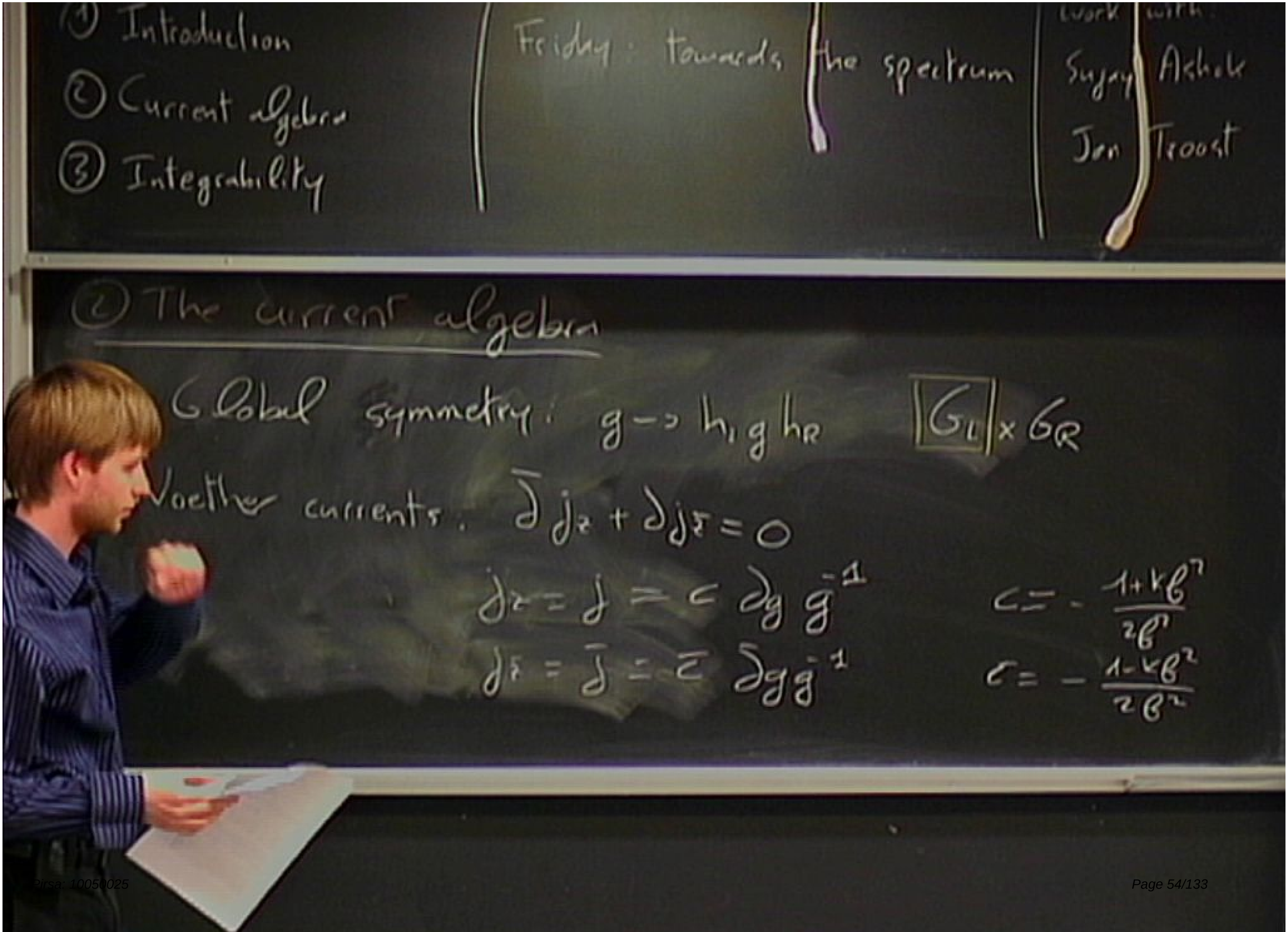
Noether currents: $\bar\partial j_z + \partial j_{\bar z} = 0$

$$j_z = j = c \, \partial g \, g^{-1} \qquad c =$$

$$j_{\bar z} = \bar j = \bar c \, \bar\partial g \, g^{-1}$$



① Introduction
② Current algebra
③ Integrability
Friday: towards the spectrum
work with: Sujay Ashok, Jan Troost
② The current algebra
Global symmetry: $g \to h_L g h_R$
Noether currents: $\bar\partial j_z + \partial j_{\bar z} = 0$
$j_z = j = c\, \partial g$
$j_{\bar z} = \bar j = \bar c\, \partial g$


① Introduction
② Current algebra
③ Integrability
Friday: towards the spectrum
work with:
Sujay Ashok
Jan Troost
② The current algebra
Global symmetry: $g \to h_L g h_R$
$G_L \times G_R$
Noether currents: $\bar\partial j_z + \partial j_{\bar z} = 0$
$j_z = j = c\, \partial g\, g^{-1}$
$c = -\frac{1+k\beta^2}{2\beta^2}$
$j_{\bar z} = \bar j = \bar c\, \bar\partial g\, g^{-1}$
$\bar c = -\frac{1-k\beta^2}{2\beta^2}$

(1) Introduction

(2) Current algebra

(3) Integrability

Friday: towards the spectrum

work with: Sujay Ashok, Jan Troost

(2) The current algebra

Global symmetry: $g \to h_L\, g\, h_R$ $\quad \boxed{G_L} \times G_R$

Noether currents: $\bar\partial j_z + \partial j_{\bar z} = 0$

$$j_z = j = c\, \partial g\, g^{-1} \qquad c = -\frac{1+kf^2}{2f^2}$$

$$j_{\bar z} = \bar j = \bar c\, \bar\partial g\, g^{-1} \qquad \bar c = -\frac{1-kf^2}{2f^2}$$

Supergroups

$AdS_5 \times S_5$: $\frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$

$AdS_3 \times S_3$: P…

$$j = j^a \, t^b \, k_{ab}$$

t ↳ I

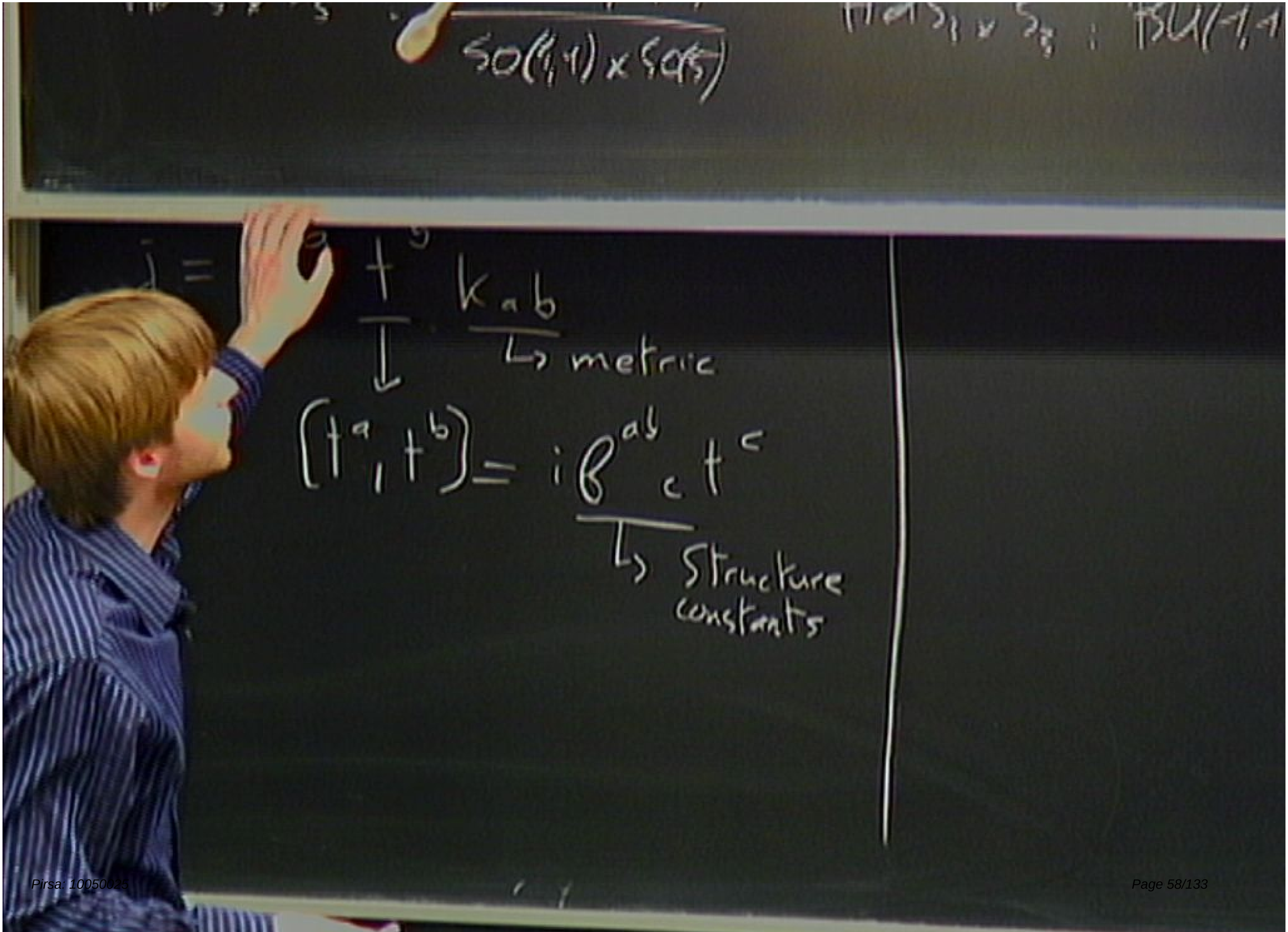
k_{ab} ↳ metric

$$J = J^a \, t^b \, k_{ab}$$

k_{ab} → metric

$$[t^a, t^b] = i f^{ab}{}_c \, t^c$$

$f^{ab}{}_c$ → Structure constants


metric
Structure
constants

$$j = j^a\, t^b\, k_{ab}$$

k_{ab} → metric

$$[t^a, t^b] = i f^{ab}{}_c\, t^c$$

$f^{ab}{}_c$ → Structure constants

$f^a \quad f^{dbc}$

$$J = J^a \, t^b \, k_{ab}$$

k_{ab} → metric

$$[t^a, t^b] = i f^{ab}{}_c \, t^c$$

$f^{ab}{}_c$ → Structure constants

$$f^a{}_{bc} \, f^{dbc} = 0$$

zero Killing form

$$\bar{J} = J^a \, t^b \, k_{ab}$$

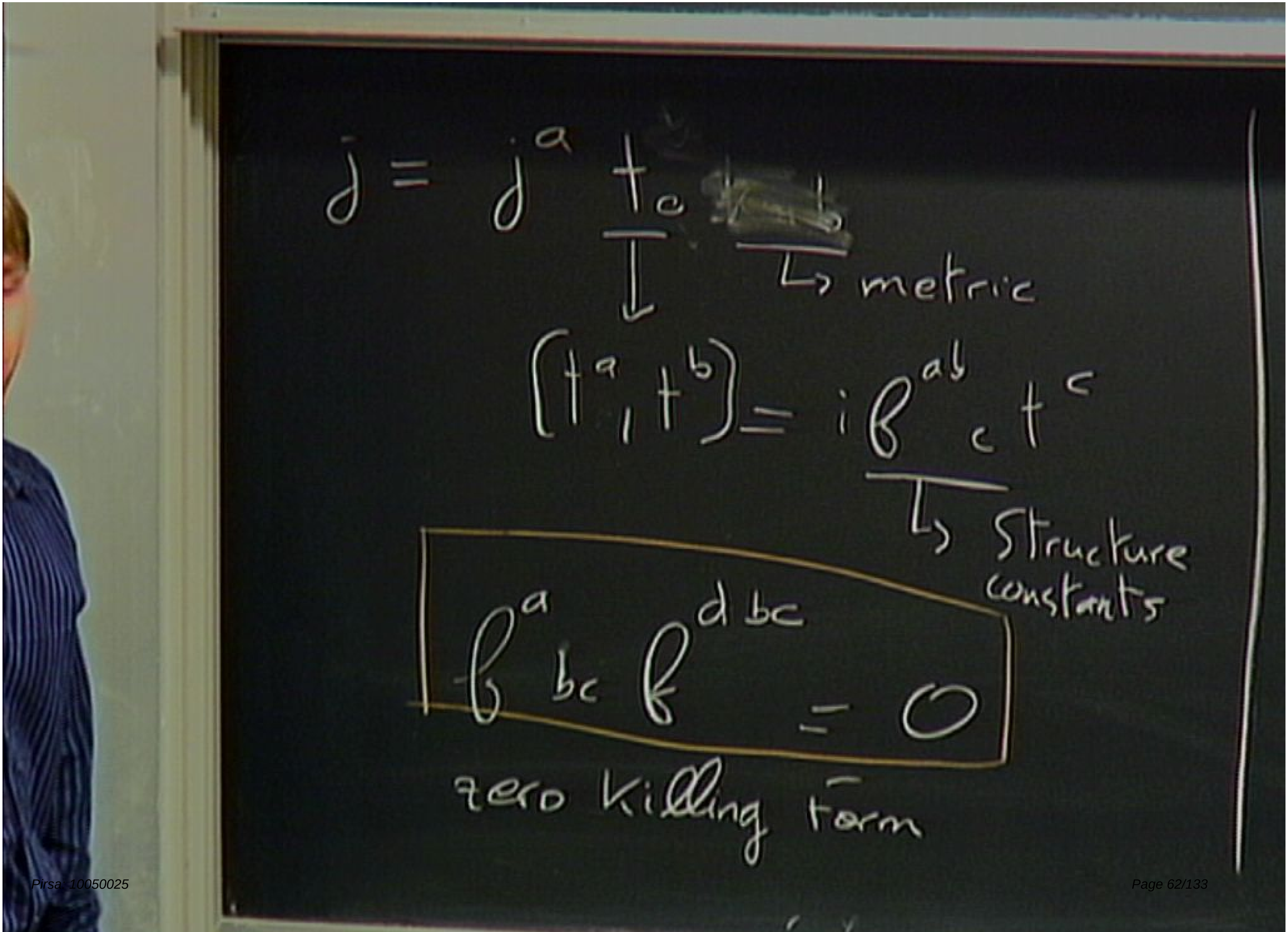
$\hookrightarrow$ metric

$$[t^a, t^b] = i f^{ab}{}_c \, t^c$$

$\hookrightarrow$ Structure constants

$$f^a{}_{bc} \, f^{dbc} = 0$$

zero Killing form


$j = j^a t_a$
↳ metric
$[t^a, t^b] = i f^{ab}{}_c t^c$
↳ Structure constants
$f^a{}_{bc} f^{dbc} = 0$
zero Killing form

$$j = j^a t^b \kappa_{ab}$$

κ_{ab} ↳ metric

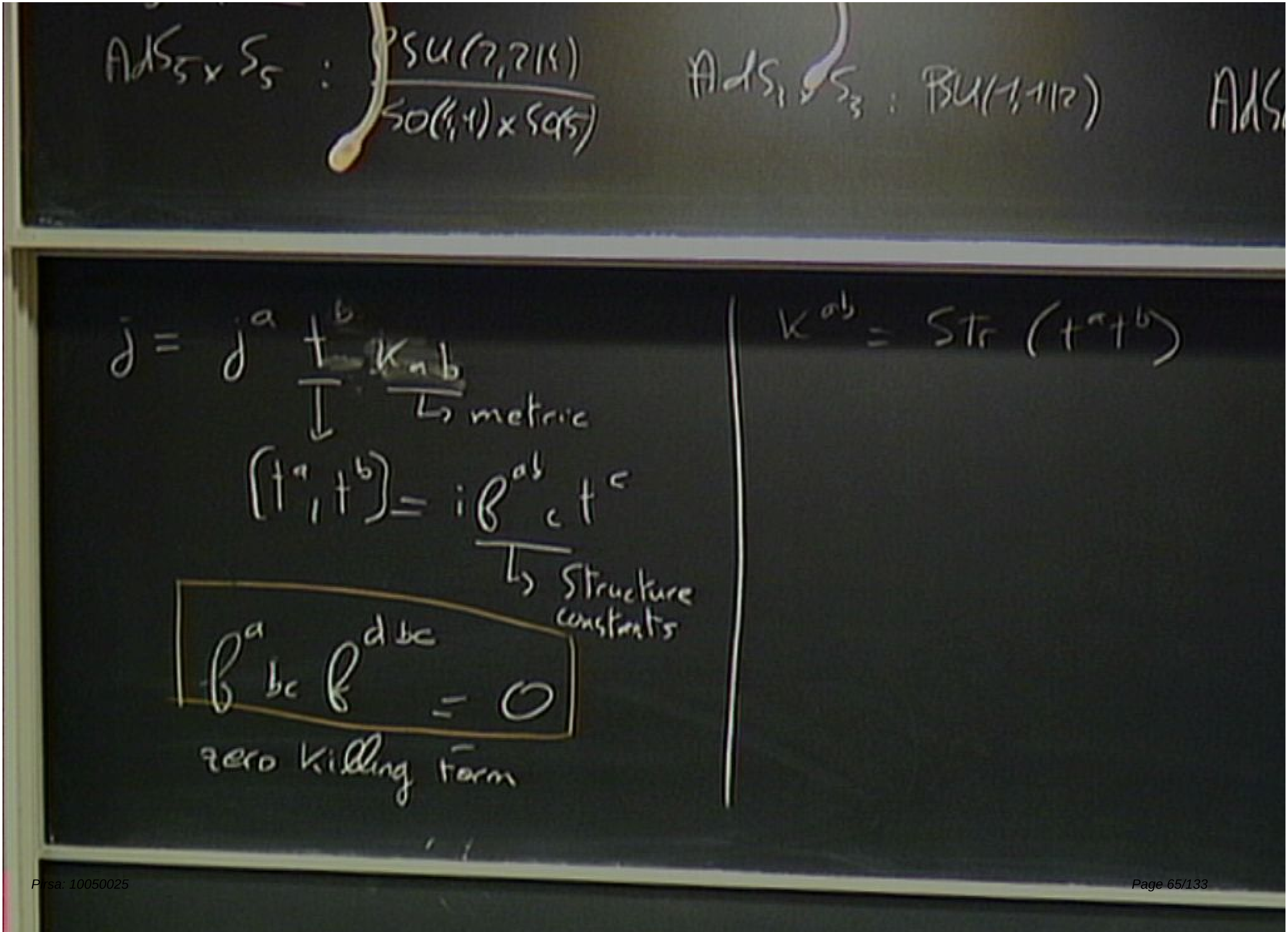
$$[t^a, t^b] = i f^{ab}{}_c t^c$$

$f^{ab}{}_c$ ↳ Structure constants

$$f^a{}_{bc} f^{dbc} = 0$$

zero Killing form

$$K^{ab} = \mathrm{Str}\,(t^a t^b)$$


AdS5 x S5 : PSU(2,2|4) / SO(4,1) x SO(5)
AdS3 x S3 : PSU(1,1|2)
j = j^a t^b K_ab
metric
[t^a, t^b] = i f^ab_c t^c
Structure constants
f^a_bc f^dbc = 0
zero Killing form
K^ab = STr (t^a t^b)

$AdS_5 \times S_5 : \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$

$AdS_3 \times S_3 : PSU(1,1|2)$

AdS

$j = j^a t^b K_{ab}$

↳ metric

$[t^a, t^b] = i f^{ab}{}_c t^c$

↳ Structure constants

$$f^a{}_{bc} f^{dbc} = 0$$

zero Killing form

" $K^{ab} = STr(t^a t^b)$ "

$SO(4,4) \times SO(5)$

$$j = j^a \, t^b \, K_{ab}$$

K_{ab} ↳ metric

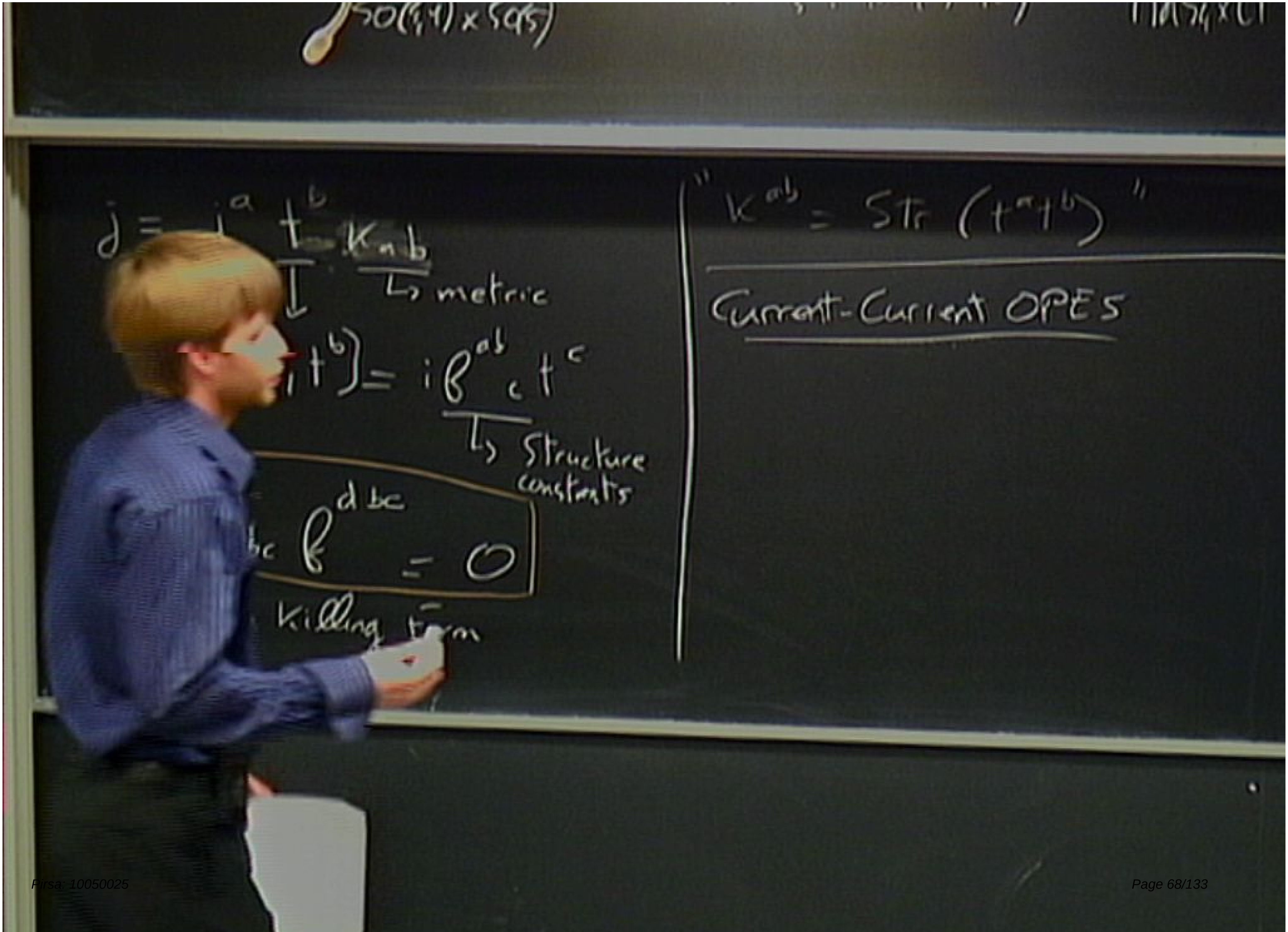
$$[t^a, t^b] = i f^{ab}{}_c \, t^c$$

$f^{ab}{}_c$ ↳ Structure constants

$$f^a{}_{bc} \, f^{dbc} = 0$$

zero Killing form

" $K^{ab} = \mathrm{Str}(t^a t^b)$ "


metric
Structure constants
Killing form
" K^ab = Str (t^a t^b) "
Current-Current OPES

$SO(4,1) \times SO(5)$

$$j = j^a\, t^b\, K_{ab}$$

K_{ab} ↳ metric

$$[t^a, t^b] = i f^{ab}{}_c\, t^c$$

$f^{ab}{}_c$ ↳ Structure constants

$$f^a{}_{bc}\, f^{dbc} = 0$$

zero Killing form

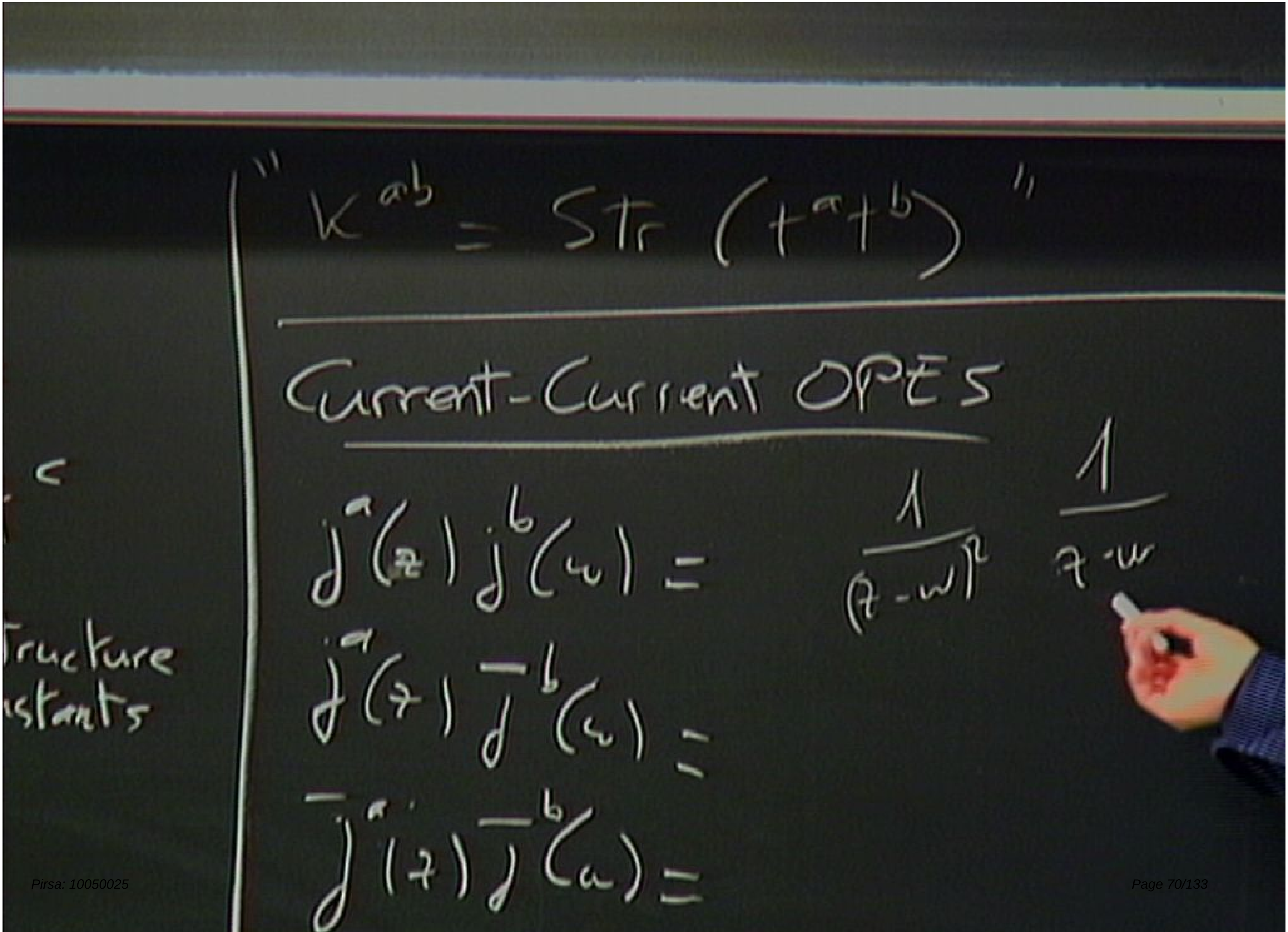
" $K^{ab} = \mathrm{STr}\,(t^a t^b)$ "

Current-Current OPEs

$$j^a(z)\, j^b(w) =$$

$$j^a(z)\, \bar{j}^b(w) =$$

$$\bar{j}^a(z)\, \bar{j}^b(w) =$$


"K^ab = Str (t^a t^b)"
Current-Current OPEs
j^a(z) j^b(w) = 1/(z-w)^2 1/(z-w)
j^a(z) j̄^b(w) =
j̄^a(z) j̄^b(w) =
Tructure
istants

$AdS_3 \times S_3 : PSU(1,1|2)$

$AdS_4 \times CP^3 : \frac{OSp(6|4)}{SO(1,3) \times U(3)}$

$\frac{\ldots(4)}{\ldots SO(5)}$

" $K^{ab} = \mathrm{Str}\,(t^a t^b)$ "

etric

^{ab}

$_c t^c$

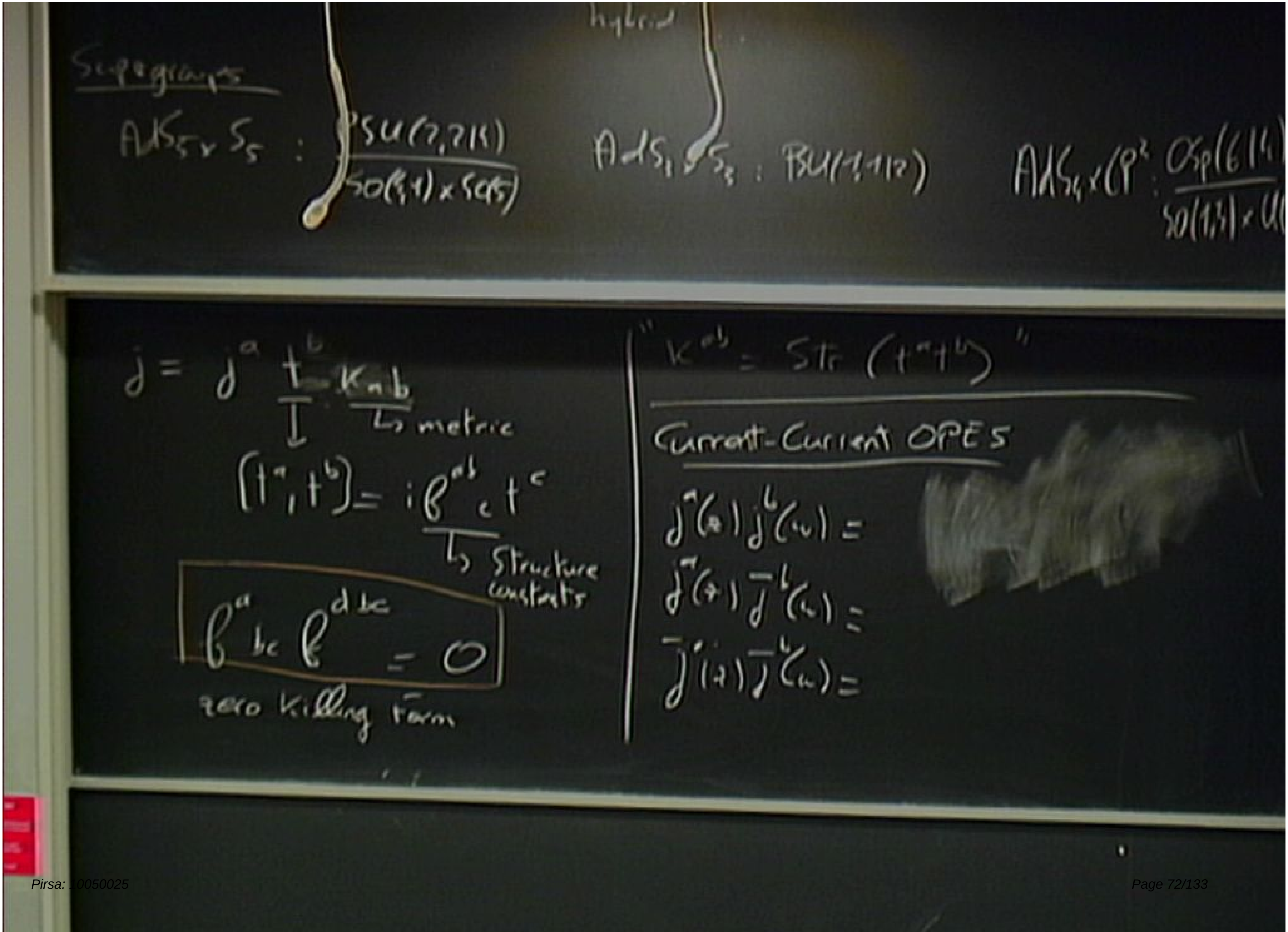
→ Structure constants

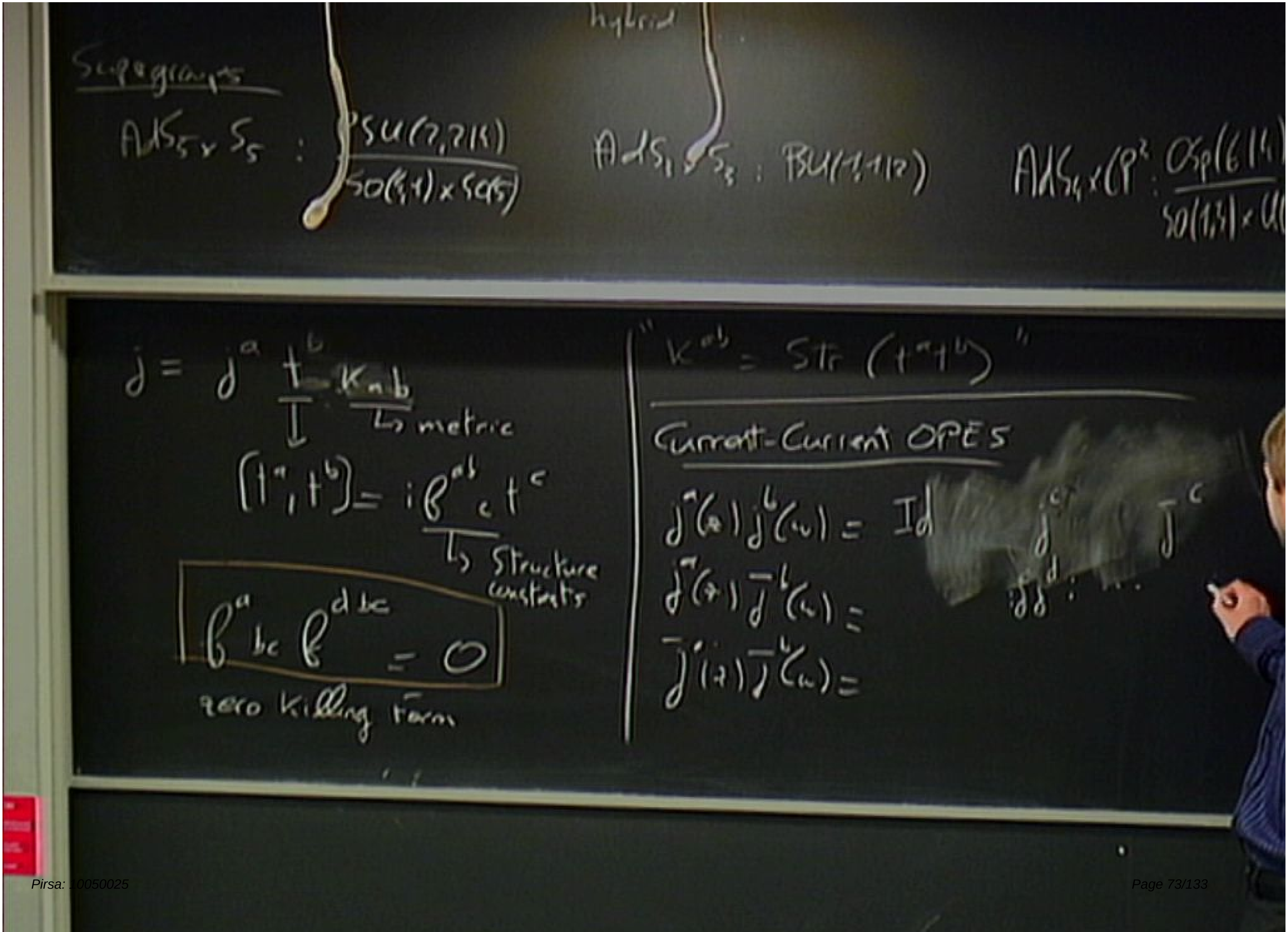
Current-Current OPES

$$j^a(z)\, j^b(w) = \frac{1}{(z-w)^2} \quad \frac{1}{z-w} \quad \frac{\bar{z}-\bar{w}}{(z-w)^2} \cdots$$

$$j^a(z)\, \bar{j}^b(w) =$$

$$\bar{j}^a(z)\, \bar{j}^b(w) =$$


hybrid
Supergroups
AdS5 x S5 : PSU(2,2|4) / SO(4,1) x SO(5)
AdS3 x S3 : PSU(1,1|2)
AdS4 x CP3 : OSp(6|4) / SO(1,3) x U(
j = j^a t^b K_ab
metric
[t^a, t^b] = i f^ab_c t^c
Structure constants
f^a_bc f^dbc = 0
zero Killing form
"K^ab = STr (t^a t^b)"
Current-Current OPEs
j^a(z) j^b(w) =
j^a(z) j̄^b(w) =
j̄^a(z) j̄^b(w) =


hybrid
Supergroups
AdS5 x S5 : PSU(2,2|4) / SO(4,1) x SO(5)
AdS3 x S3 : PSU(1,1|2)
AdS4 x CP3 : Osp(6|4) / SO(1,3) x U(
j = j^a t^b K_ab
metric
[t^a, t^b] = i f^ab_c t^c
Structure constants
f^a_bc f^dbc = 0
zero Killing form
"K^ab = Str(t^a t^b)"
Current-Current OPEs
j^a(z) j^b(w) = Id
j^a(z) j̄^b(w) =
j̄^a(z) j̄^b(w) =

hybrid

Supergroups

$AdS_5 \times S_5 : \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$ $AdS_3 \times S_3 : PSU(1,1|2)$ $AdS_4 \times CP^3 : \frac{OSp(6|4)}{SO(1,3) \times U(3)}$

$j = j^a t^b K_{ab}$ ↳ metric

$[t^a, t^b] = i f^{ab}{}_c t^c$ ↳ Structure constants

$f^a{}_{bc} f^{dbc} = 0$

zero Killing form

"$K^{ab} = \mathrm{STr}(t^a t^b)$"

Current-Current OPEs

$j^a(z) j^b(w) = \mathrm{Id} \quad j^c \quad j^c$

$j^a(z) \bar{j}^b(w) = \quad :j^d j^c: \quad j^n$

$\bar{j}^a(z) \bar{j}^b(w) =$

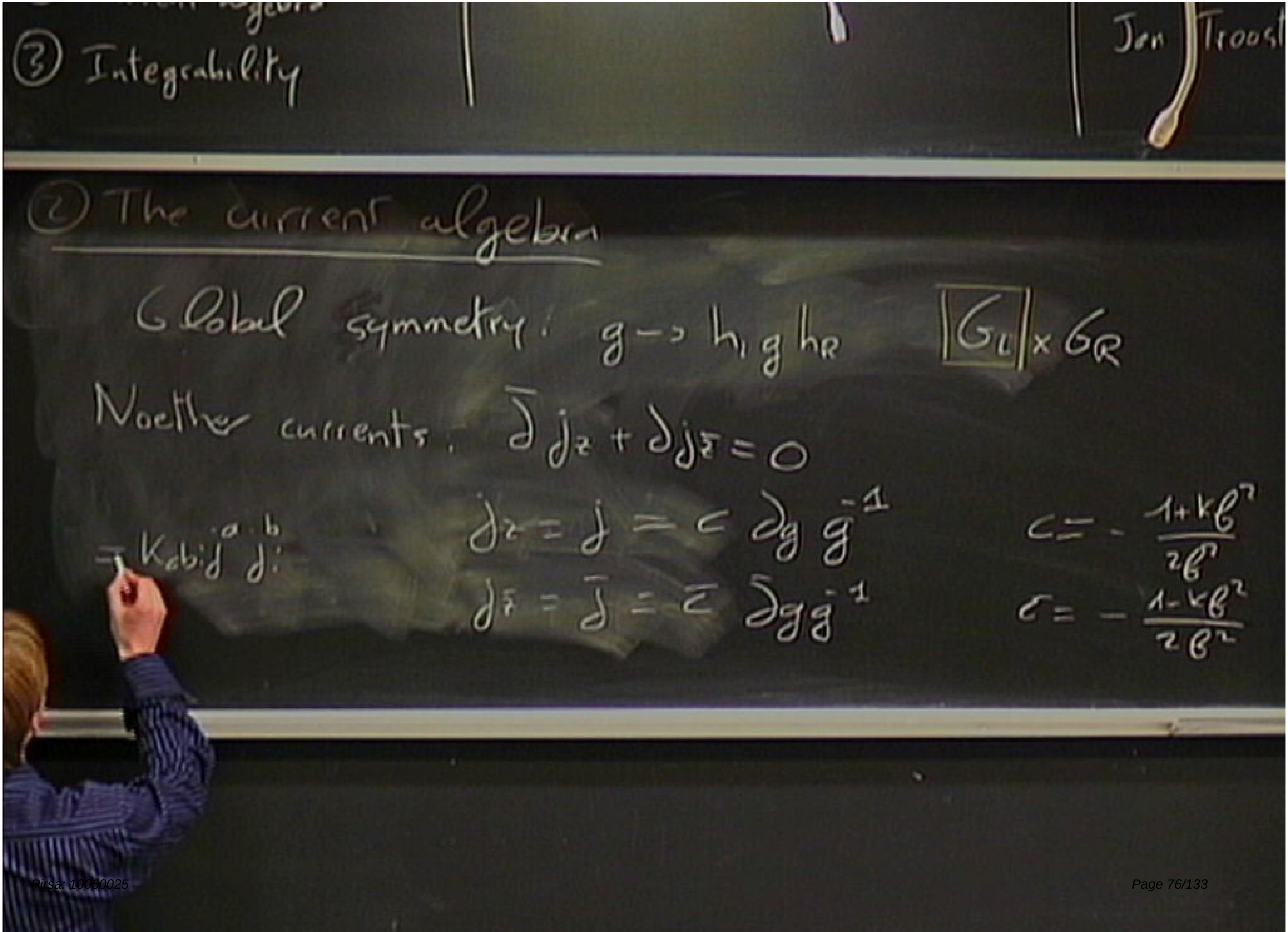
ent algebra $\qquad \bar\partial \partial \bar{j} = -\bar\partial \partial j$

symmetry: $\quad g \to h_L\, g\, h_R \qquad \boxed{G_L} \times G_R$

rrents: $\quad \bar\partial j_z + \partial j_{\bar z} = 0$

$$j_z = j = c\, \partial g\, g^{-1} \qquad c = -\frac{1+k}{2}$$

$$j_{\bar z} = \bar{j} = \bar{c}\, \bar\partial g\, g^{-1} \qquad \bar{c} = -\frac{1-k}{2\beta}$$


③ Integrability
Jan Troost
② The current algebra
Global symmetry: $g \to h_L\, g\, h_R$ $G_L \times G_R$
Noether currents: $\bar\partial j_z + \partial j_{\bar z} = 0$
$j_z = j = c\, \partial g\, g^{-1}$ $c = -\frac{1+k\beta^2}{2\beta^2}$
$j_{\bar z} = \bar j = \bar c\, \bar\partial g\, g^{-1}$ $\bar c = -\frac{1-k\beta^2}{2\beta^2}$
$= K_{ab} :j^a j^b:$

③ Integrability

Jan Troost

② The current algebra

Global symmetry: $g \to h_L g h_R$ $\boxed{G_L} \times G_R$

Noether currents: $\bar\partial j_z + \partial j_{\bar z} = 0$

$T(z) = K_{ab} :j^a j^b:$

$j_z = j = c\, \partial g\, g^{-1}$ $c = -\frac{1+k\beta^2}{2\beta^2}$

$j_{\bar z} = \bar j = \bar c\, \bar\partial g\, g^{-1}$ $\bar c = -\frac{1-k\beta^2}{2\beta^2}$

③ Integrability

Jan Troost

② The current algebra

Global symmetry: $g \to h_L\, g\, h_R$ $\quad G_L \times G_R$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$T(z) \propto K_{ab} :j^a j^b:$

$$j_z = j = c\, \partial g\, g^{-1} \qquad c = -\frac{1+k\beta^2}{2\beta^2}$$

$$j_{\bar{z}} = \bar{j} = \bar{c}\, \bar{\partial} g\, g^{-1} \qquad \bar{c} = -\frac{1-k\beta^2}{2\beta^2}$$

③ Integrability

Jan Troost

② The current algebra

Global symmetry: $g \to h_L g h_R$ $\quad \boxed{G_L} \times G_R$

Noether currents: $\bar\partial j_z + \partial j_{\bar z} = 0$

$$T(z) \propto K_{ab} :j^a j^b:$$

$$j_z = j = c\, \partial g\, g^{-1} \qquad c = -\frac{1+k\beta^2}{2\beta^2}$$

$$j_{\bar z} = \bar j = \bar c\, \bar\partial g\, g^{-1} \qquad \bar c = -\frac{1-k\beta^2}{2\beta^2}$$

"$K^{ab} = STr(t^a t^b)$"

Current-Current OPEs

$j^a(z)\, j^b(w) = Id \quad j^c \quad \bar{j}^c$

$:jj: \quad \ldots \quad :j^n: + \ldots$

$j^a(z)\, \bar{j}^b(w) =$

$\bar{j}^a(z)\, \bar{j}^b(w) =$

$O(f^{2n-2})$

"$K^{ab} = \mathrm{Str}(t^a t^b)$"

Current-Current OPEs

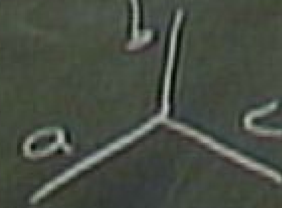
$j^a(z)\, j^b(w) = I\, d\, K^{ab}$ … j^c … $\bar{j}^c$

$:j^c j^d:$ … $:(j)^n: + \ldots$

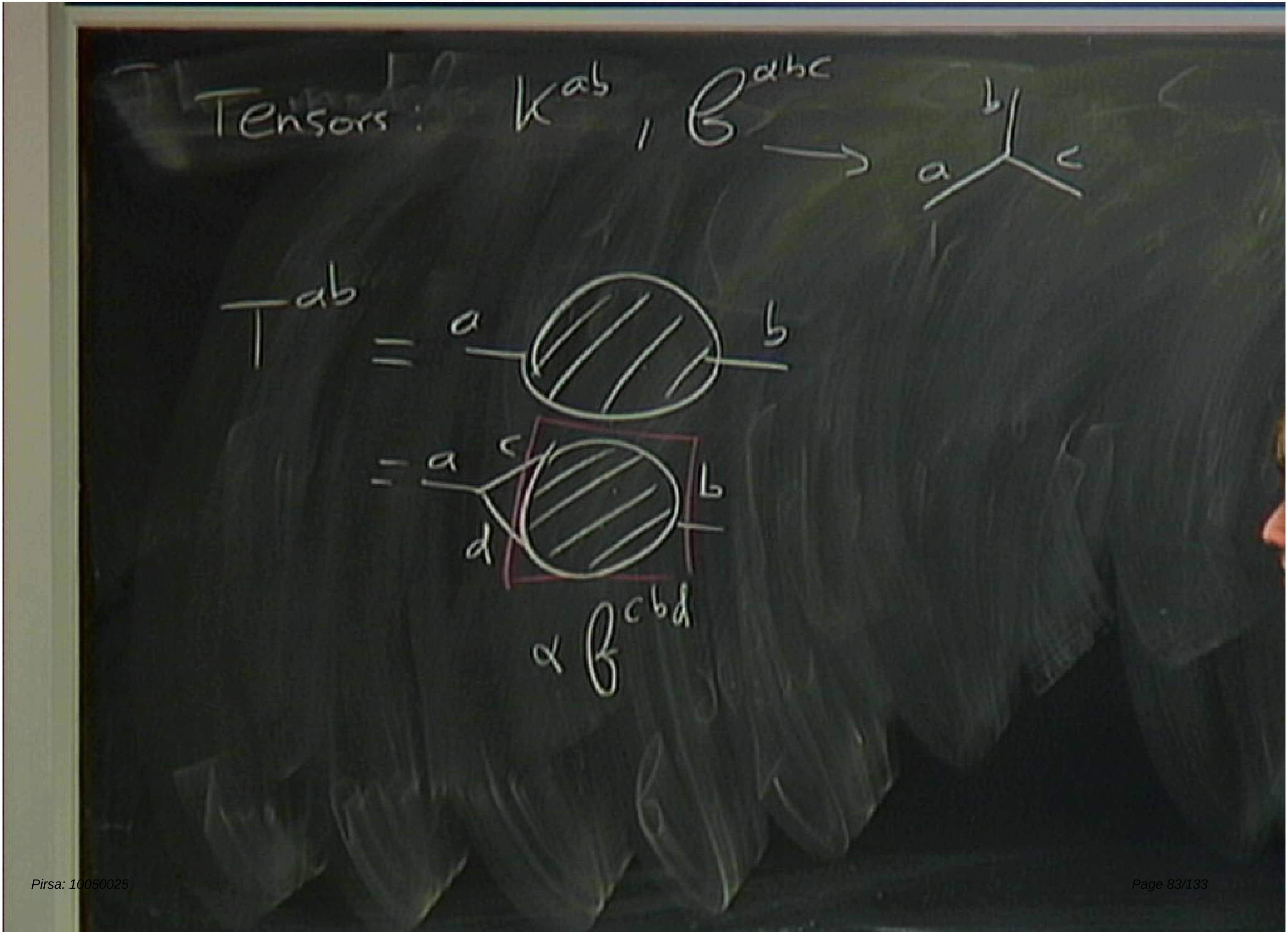
$j^a(z)\, \bar{j}^b(w) =$

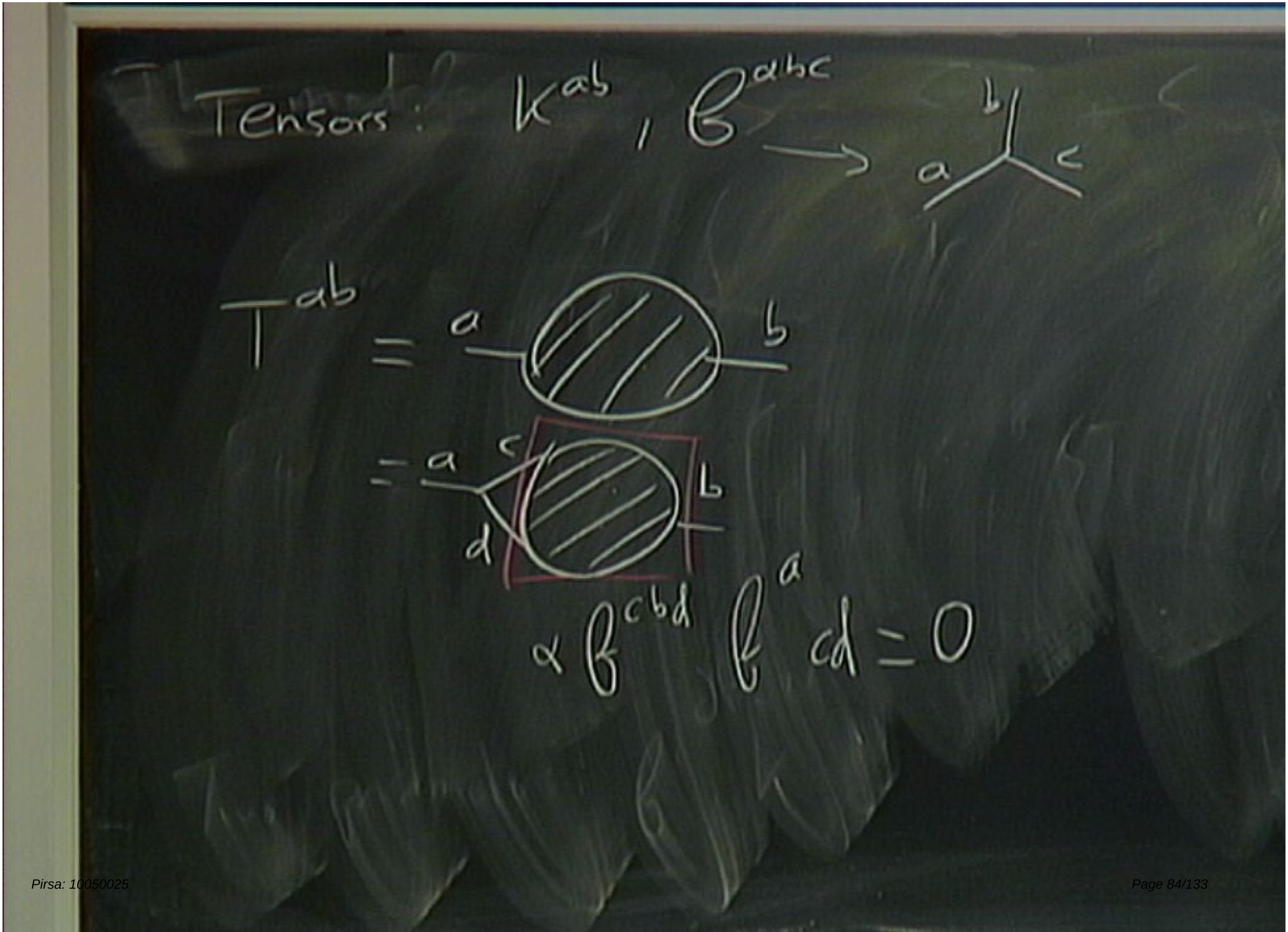
$\bar{j}^a(z)\, \bar{j}^b(w) =$

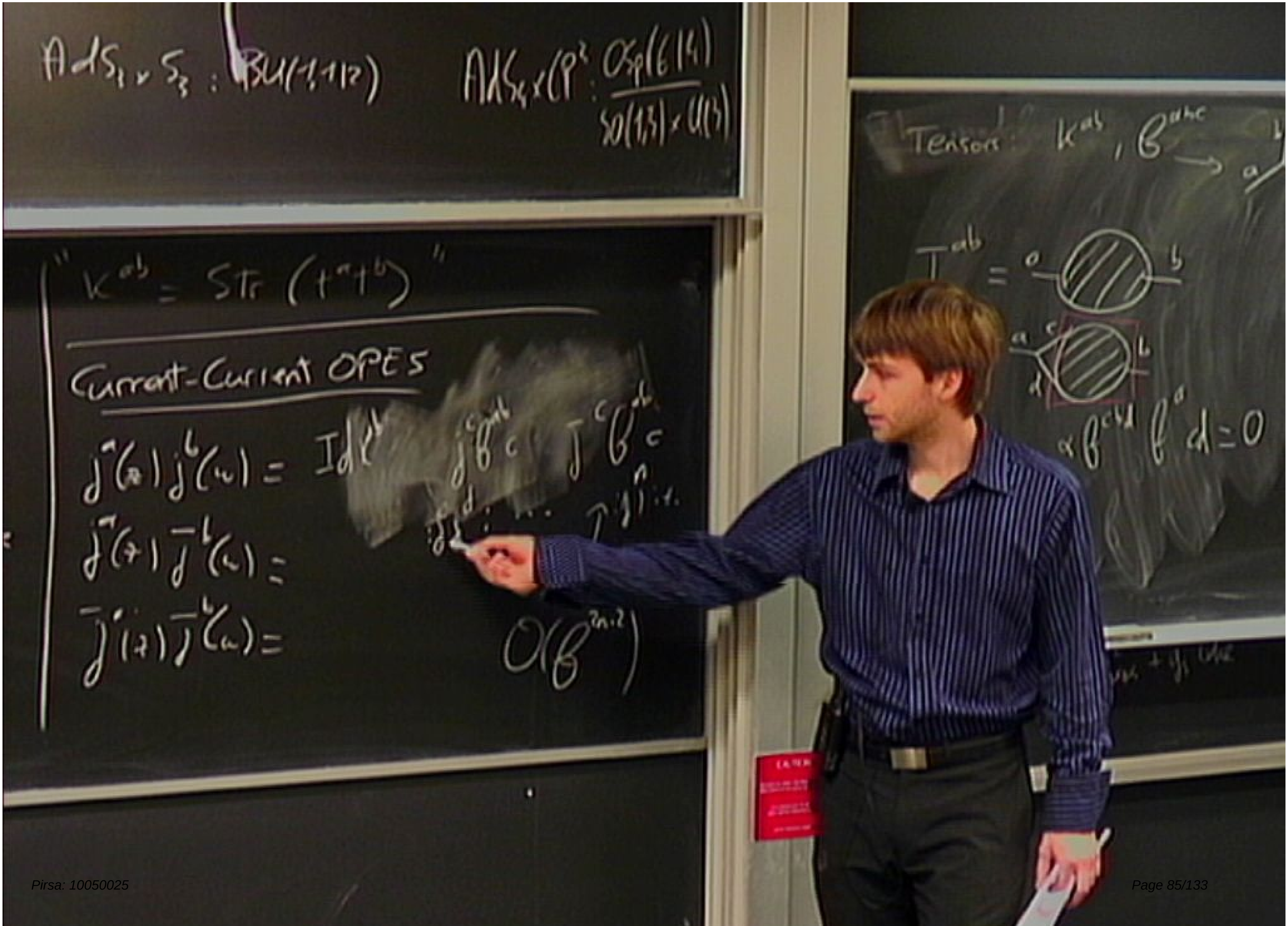
$O(f^{2n-2})$

Tensors: K^{ab}, B^{abc} $\longrightarrow$ 

T^{ab} = 


Tensors: K^ab , B^abc
a
b
c
T^ab
= a b
= a c d b
∝ B^cbd


Tensors: K^{ab}, β^{abc} →
a b c
T^{ab}
= a b
= a c d b
$\propto \beta^{cbd} \beta^{a}{}_{cd} = 0$


AdS3 × S3 : PSU(1,1|2)
AdS4 × CP3 : Osp(6|4) / SO(1,3) × U(3)
"κab = STr (tatb)"
Current-Current OPEs
Tensors: κab, fabc
O(f2n+2)

$SO(1,3) \times U(3)$

"$K^{ab} = \mathrm{Str}(t^a t^b)$"

Current-Current OPEs

$j^a(z)\, j^b(w) = \mathrm{Id}\, K^{ab} A_0 + j^c f^{ab}{}_c A_1 + \bar{j}^c f^{ab}{}_c A_2$

$A_3 :jj: \ldots$

$j^a(z)\, \bar{j}^b(w) =$

$\bar{j}^a(z)\, \bar{j}^b(w) =$

Structure constants

$AdS_5 \times S_5 : \dfrac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$ $AdS_3 \times S_3 : PSU(1,1|2)$ $AdS_4 \times CP^3 : \dfrac{Osp(6|4)}{SO(1,3) \times U(3)}$

$j = j^a t^b K_{ab}$

↳ metric

$[t^a, t^b] = i f^{ab}{}_c t^c$

↳ Structure constants

$$\boxed{f^a{}_{bc} f^{dbc} = 0}$$

zero Killing form

" $K^{ab} = \mathrm{Str}(t^a t^b)$ "

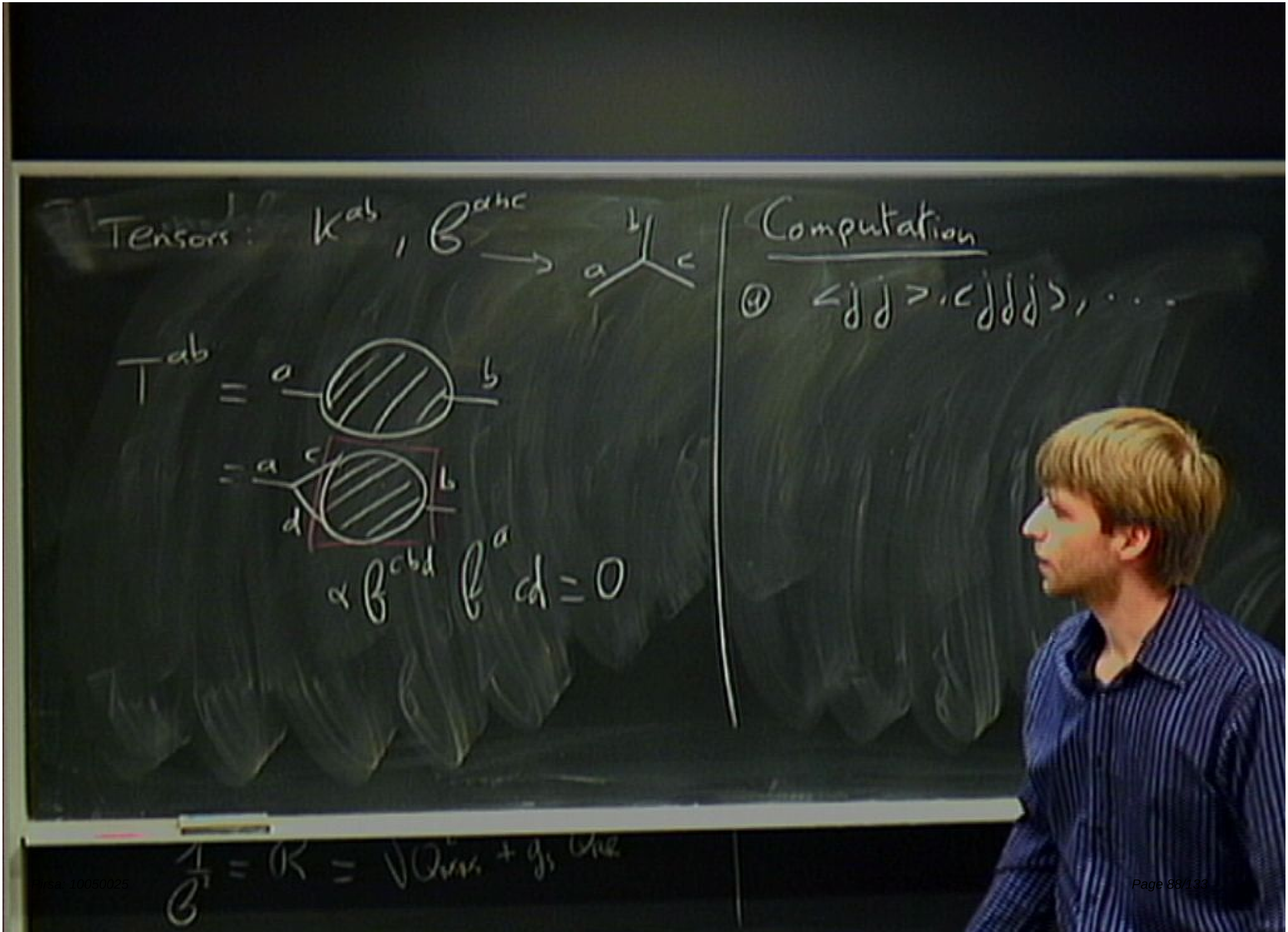
Current-Current OPEs

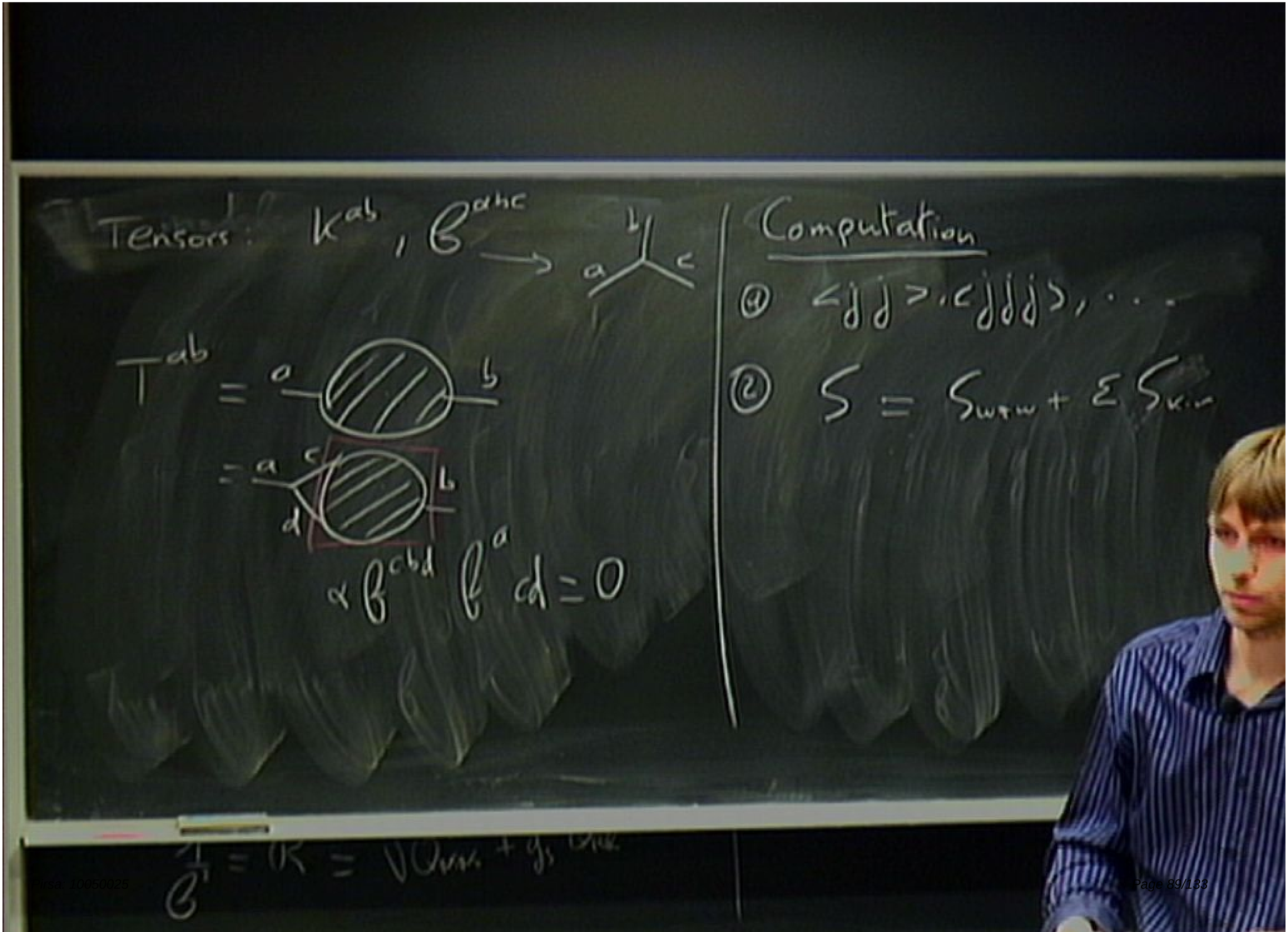
$j^a(z) j^b(w) = \mathrm{Id} K^{ab} A_0 + j^c f^{ab}{}_c A_1 + \bar{j}^c f^{ab}{}_c A_2$

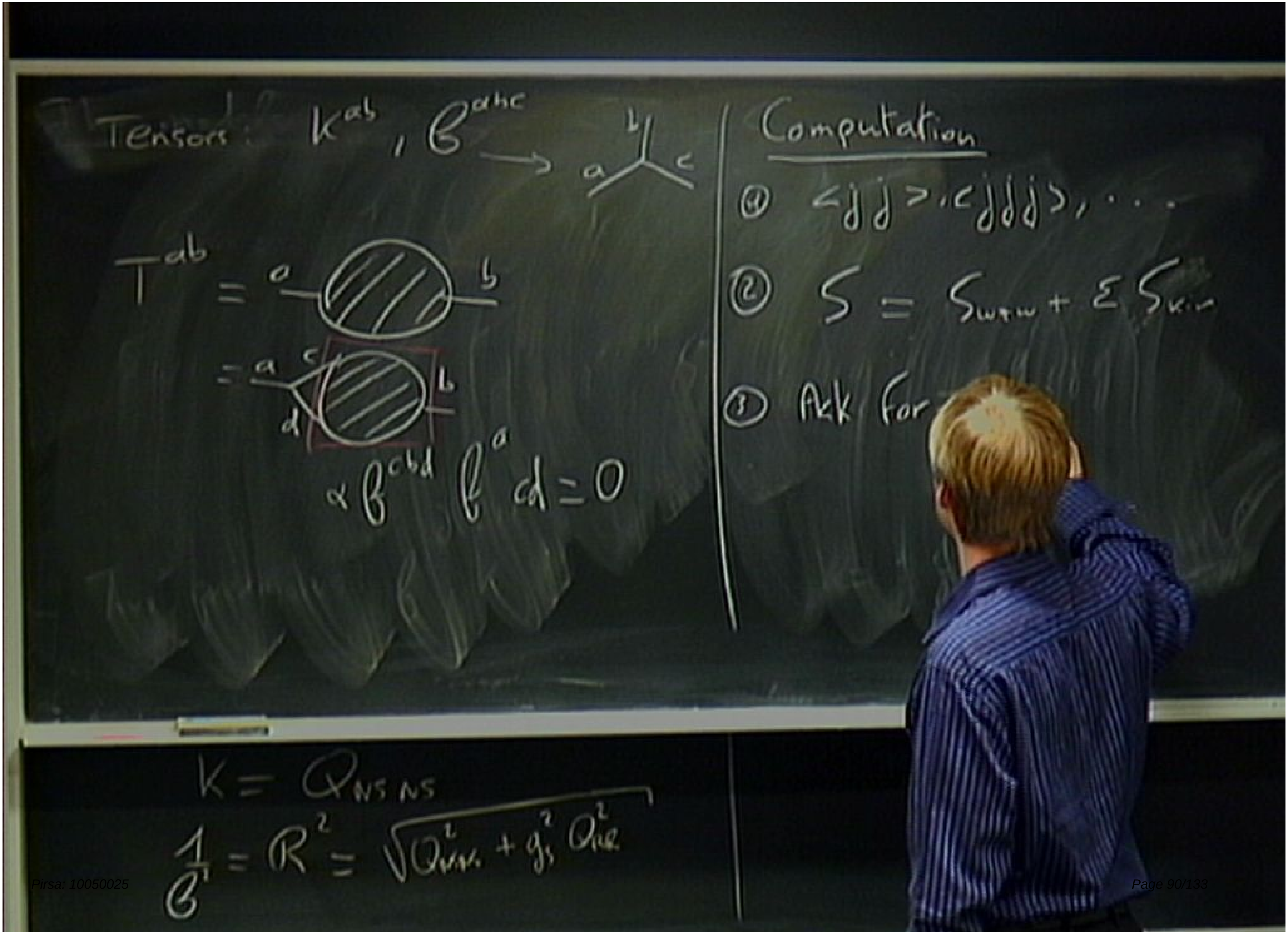
$A_3 \, j j^d \ldots$

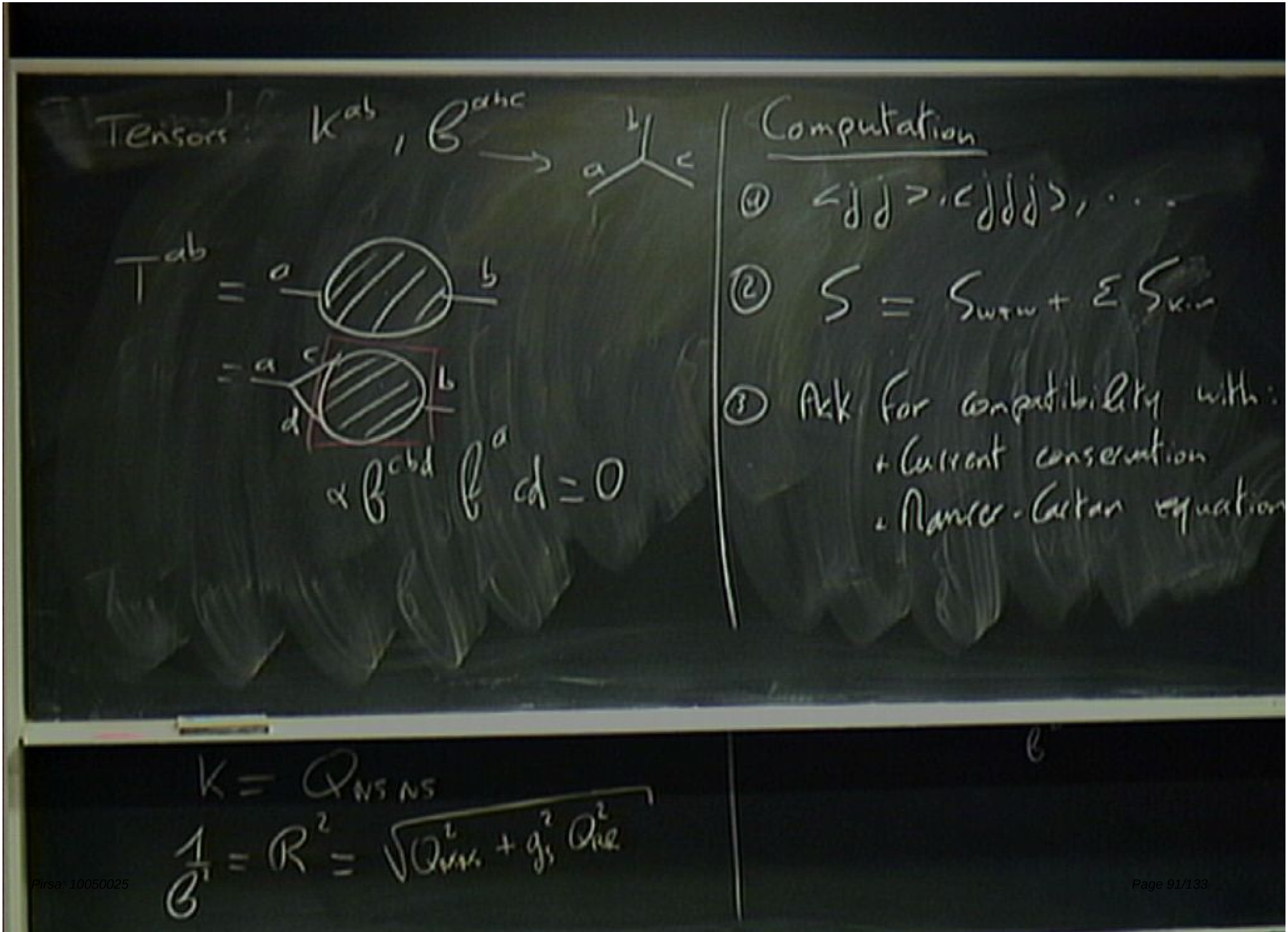
$j^a(z) \bar{j}^b(w) = \mathrm{Id} K^{ab} B_0 + \ldots$

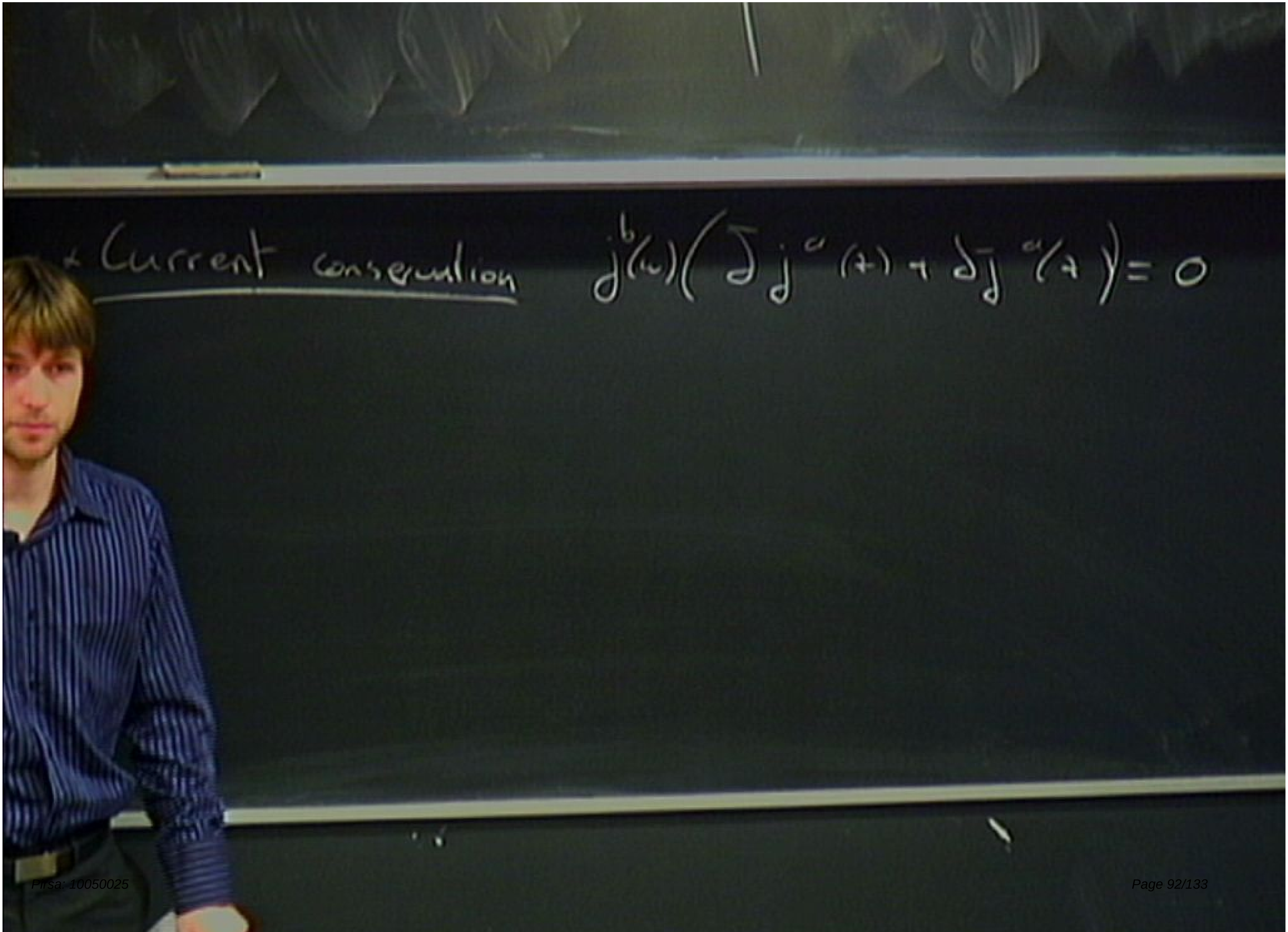
$\bar{j}^a(z) \bar{j}^b(w) = \ldots$


Tensors: K^ab, B^abc
Computation
① <jj>, <jjj>, . . .
T^ab =
α B^cbd B^a cd = 0


Tensors: K^ab, B^abc
Computation
① <jj>, <jjj>, . . .
② S = S_WZW + ε S
T^ab =
= 0


Tensors: K^{ab}, B^{abc}
Computation
① ⟨jj⟩, ⟨jjj⟩, . . .
② $S = S_{WZW} + \varepsilon S_{K.T.}$
③ Ask for
T^{ab} =
$\propto B^{cbd} f^{a}{}_{cd} = 0$
$K = Q_{NS\,NS}$
$\frac{1}{B} = R^2 = \sqrt{Q_{NSNS}^2 + g_s^2 Q_{RR}^2}$


Tensors: K^ab , B^abc
a b c
T^ab =
a b
a c d b
∝ B^cbd f^a cd = 0
Computation
① <jj>, <jjj>, . . .
② S = S_WZW + ε S
③ Ask for compatibility with:
- Current conservation
· Maurer-Cartan equation
K = Q_NS NS
1/B = R^2 = √(Q^2 + g_s^2 Q^2_RR)


* Current conservation
$j^b(w)\left(\bar{\partial} j^a(z) + \partial \bar{j}^a(z)\right) = 0$

* Current conservation $j^b(w)\left(\partial j^a(z) + \bar{\partial} \bar{j}^a(z)\right) = 0$

Current conservation $j^b(w)\left(\bar\partial j^a(z) + \partial \bar{j}^a(\bar z)\right) = 0$ + contact terms

Current conservation $\quad j^b(w)\left(\bar\partial j^a(z) + \partial \bar j^a(z)\right) = 0$ + contact terms from Ward

$$\delta g(z) = \epsilon(z)\, g(z)$$

$$\delta j^b(w) = j^b(w)\, \delta \delta$$

Current conservation $j^b(w)\left(\bar\partial j^a(z) + \partial \bar{j}^a(z)\right) = 0$ + contact terms from Ward

$$\delta g(z) = \epsilon(z)\, g(z)$$

$$\delta j^b(w) = j^b(w)\, \delta S$$

$$j^b(w)\left(\frac{-1}{2\pi}\int d^2z\, \epsilon_a(z)\left(\bar\partial j^a(z) + \partial \bar{j}^a(z)\right)\right)$$

• Maurer-Cartan equation

• Current conservation $\quad j^b(w)\left(\partial \bar{j}^a(z) + \bar{\partial} j^a(z)\right) = 0$ + contact terms from Ward

$$\delta g(z) = \varepsilon(z)\, g(z)$$

$$\delta j^b(w) = j^b(w)\, \delta S$$

$$\int d^2z\, \varepsilon_a(z)\left(-ik^{ab}\partial\delta(w-z) + if^{ba}{}_c\, j^c(z)\delta(w-z)\right) = j^b(w)\left(\frac{-1}{2\pi}\int d^2z\, \varepsilon_a(z)\left(\partial \bar{j}^a(z) + \bar{\partial} j^a(z)\right)\right)$$

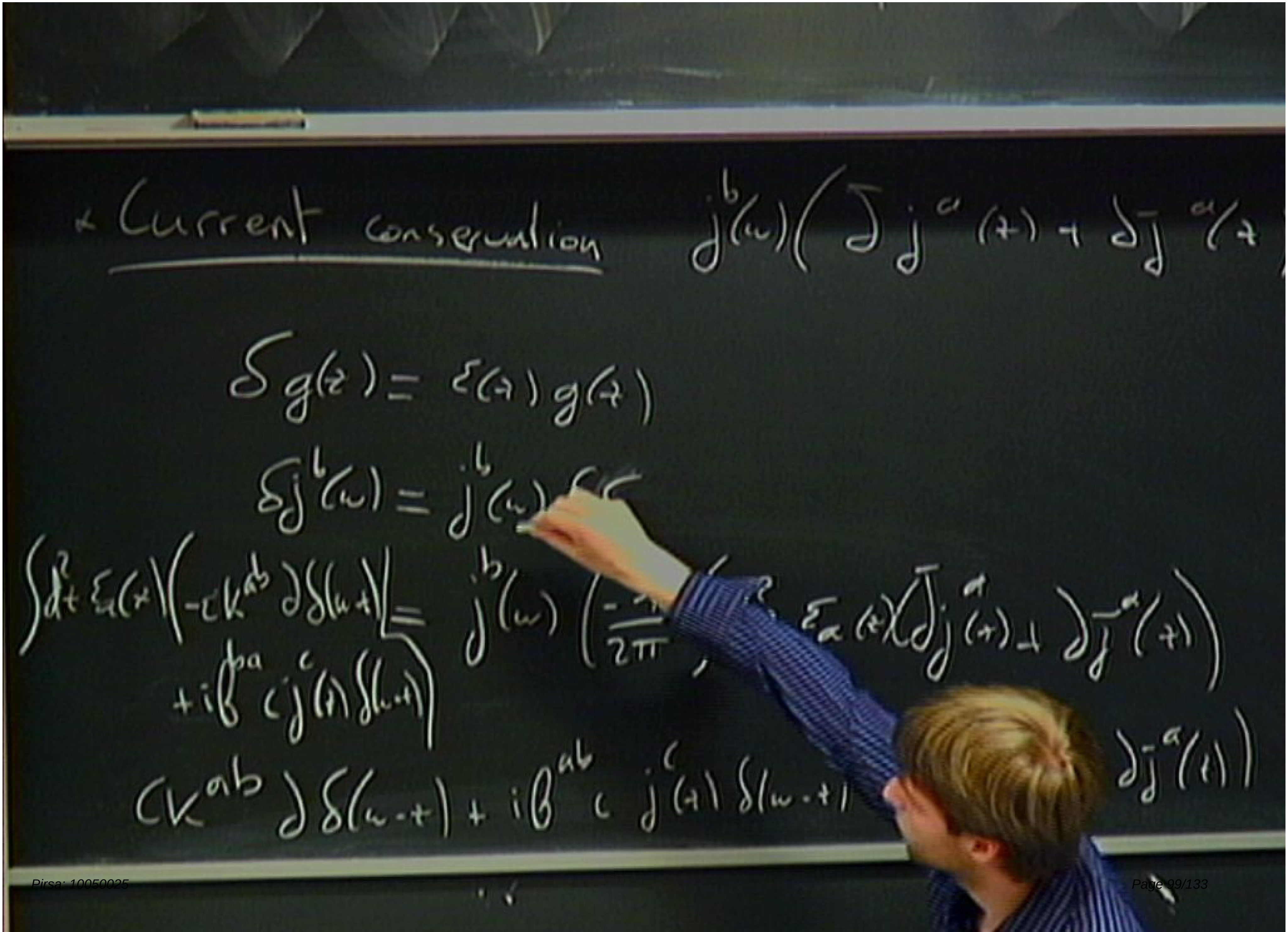
* Current conservation $\quad j^b(w)\left(\bar\partial j^a(z) + \partial \bar j^a \right.$

$$\delta g(z) = \varepsilon(z)\, g(z)$$

$$\delta j^b(w) = j^b(w)\, \delta S$$

$$\int d^2z\, \varepsilon_a(z)\Big(-ik^{ab}\,\partial\delta(w-z) + i f^{ba}{}_c\, j^c(z)\,\delta(w-z)\Big) = j^b(w)\left(\frac{-1}{2\pi}\int d^2z\, \varepsilon_a(z)\left(\bar\partial j^a(z) + \partial \bar j^a(z)\right)\right.$$

$$(k^{ab}\,\partial\delta(w-z) + i f^{a\ldots}$$


Current conservation

* Current conservation $j^b(w)\left(\bar\partial j^a(z) + \partial \bar j^a(z\right.$

$$\delta g(z) = \varepsilon(z)\, g(z)$$

$$\delta j^b(w) = j^b(w)\,\delta S$$

$$\int d^2z\, \varepsilon_a(z)\left(-ik^{ab}\,\partial\delta(w-z) + if^{ba}{}_c\, j^c(z)\,\delta(w-z)\right) = j^b(w)\left(\frac{-1}{2\pi}\int d^2z\, \varepsilon_a(z)\left(\bar\partial j^a(z) + \partial \bar j^a(z)\right)\right)$$

$$ck^{ab}\,\partial\delta(w-z) + if^{ab}{}_c\, j^c(z)\,\delta(w-z) = \left(\bar\partial j^a(z) + \partial \bar j^a(z)\right)$$

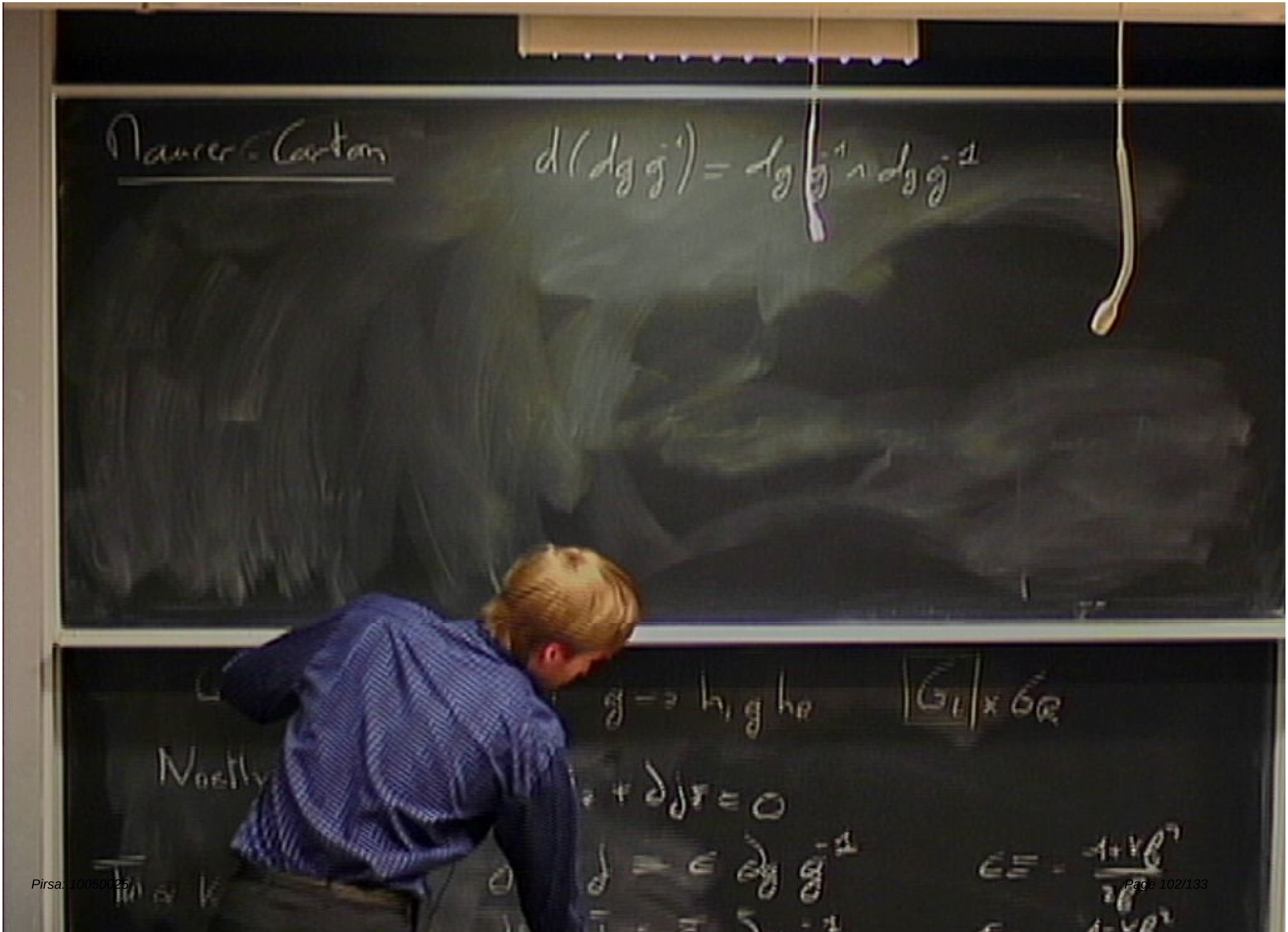
* Current conservation $\quad j^b(w)\left(\bar\partial j^a(z) + \partial \bar j^a(z)\right) = 0$ + co... t... fro...

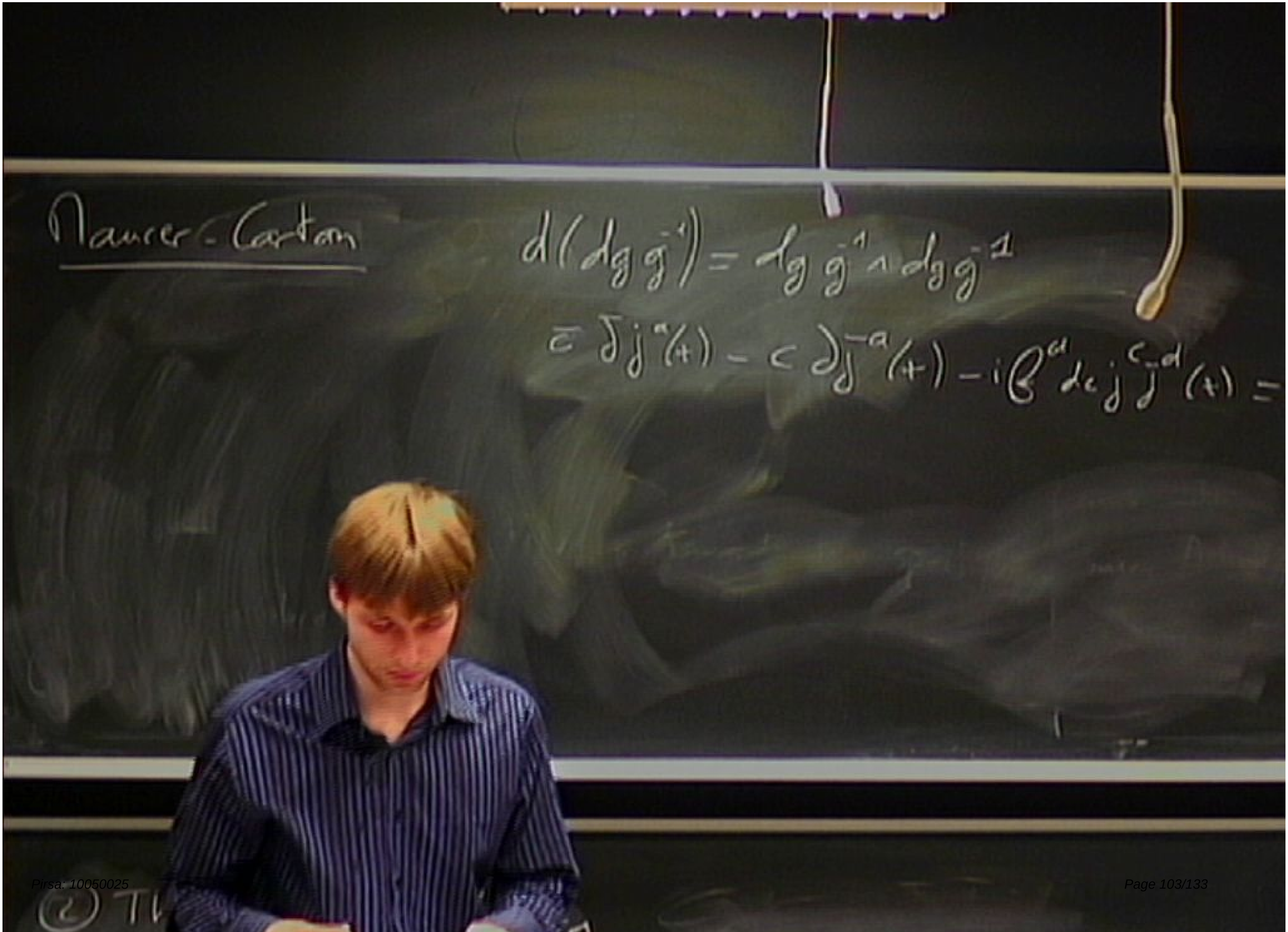
$$\delta g(z) = \varepsilon(z)\, g(z)$$

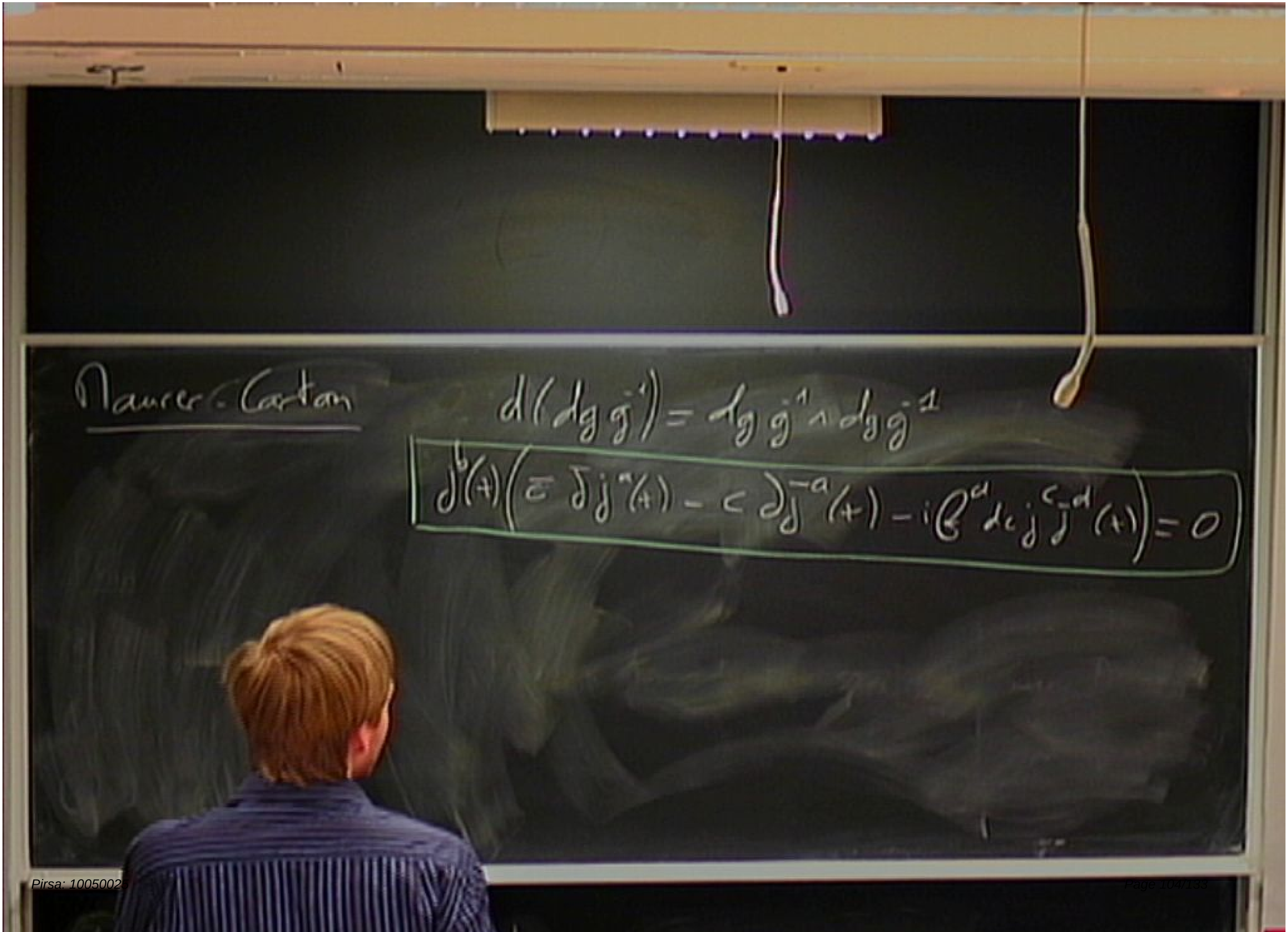
$$\delta j^b(w) = j^b(w)\,\delta S$$

$$\int d^2z\, \varepsilon_a(z)\left(-ck^{ab}\,\partial\delta(w-z) + i f^{ba}{}_c\, j^c(z)\,\delta(w-z)\right) = j^b(w)\left(\frac{-1}{2\pi}\int d^2z\, \varepsilon_a(z)\left(\bar\partial j^a(z) + \partial \bar j^a(z)\right)\right)$$

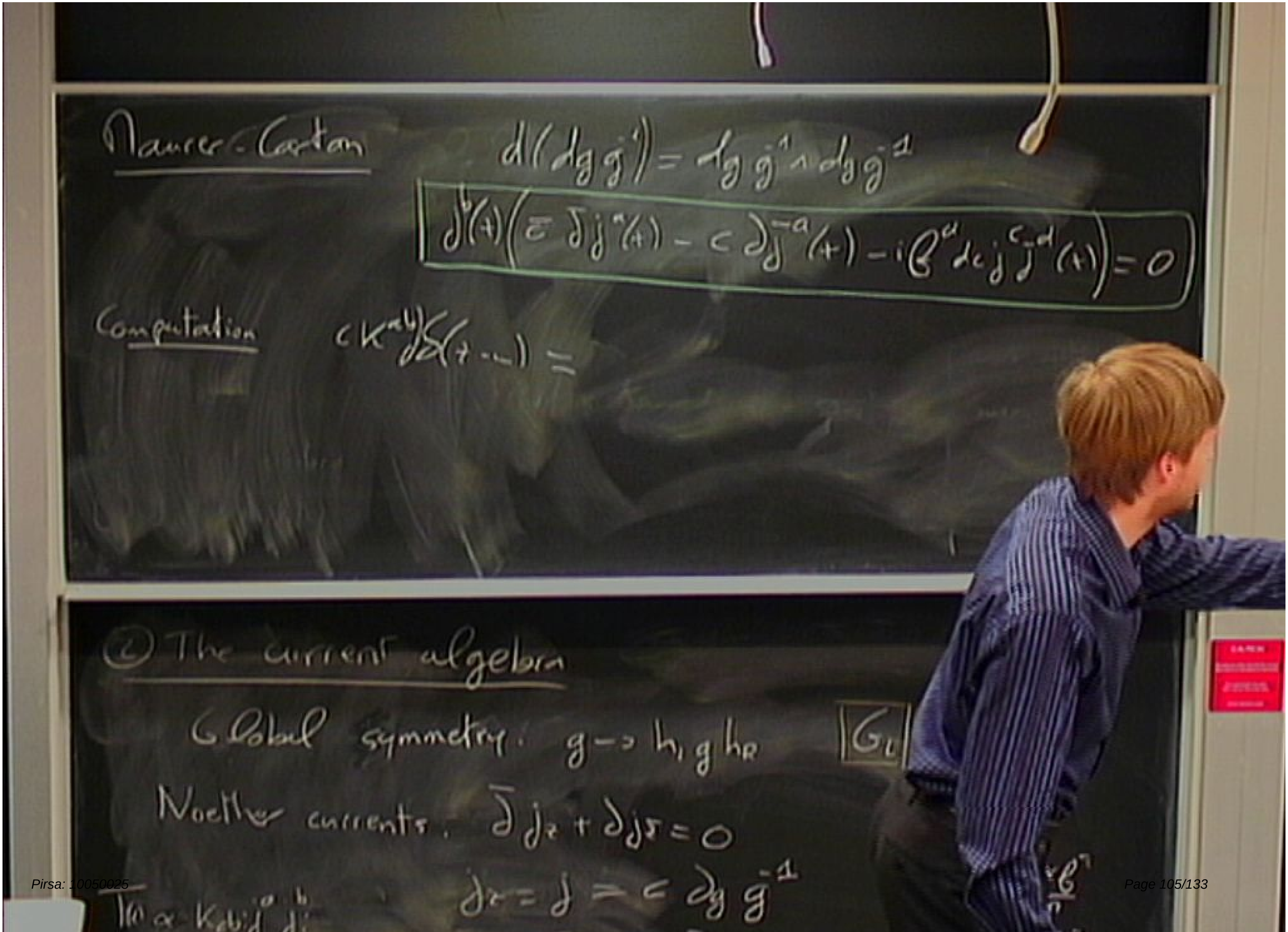
$$ck^{ab}\,\partial\delta(w-z) + i f^{ab}{}_c\, j^c(z)\,\delta(w-z) = \left(\bar\partial j^a(z) + \partial \bar j^a(z)\right) j^b(w)$$


Maurer-Cartan
d(dg g^-1) = dg g^-1 ∧ dg g^-1
g → h_L g h_R
|G_L| × G_R


Maurer-Cartan
d(dg g^-1) = dg g^-1 ∧ dg g^-1



Maurer-Cartan
d(dg g^-1) = dg g^-1 ∧ dg g^-1


Maurer-Cartan
Computation
② The current algebra
Global symmetry: g → h_L g h_R
Noether currents: ∂̄ j_z + ∂ j_z̄ = 0

Maurer-Cartan $d(dg\,g^{-1}) = dg\,g^{-1}\wedge dg\,g^{-1}$

$$\partial^b(+)\left(c\,\bar\partial j^a(+) - c\,\partial \bar{j}^a(+) - i\beta^a{}_{dc}\, j^c \bar{j}^d(+)\right) = 0$$

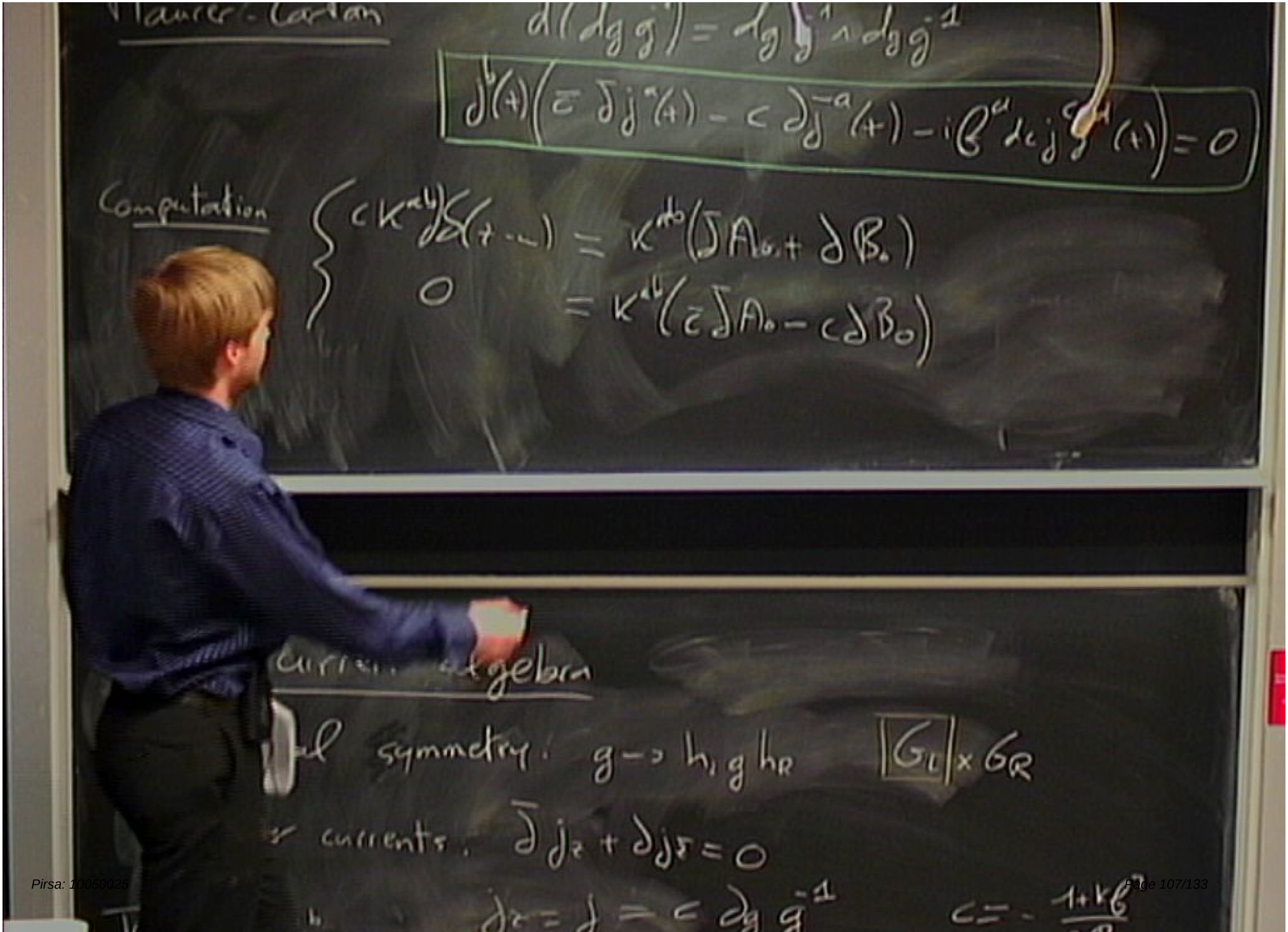
Computation $cK^{ab}\delta(z-\ldots) = K^{ab}(\bar\partial A_{b} + \partial B_{b})$

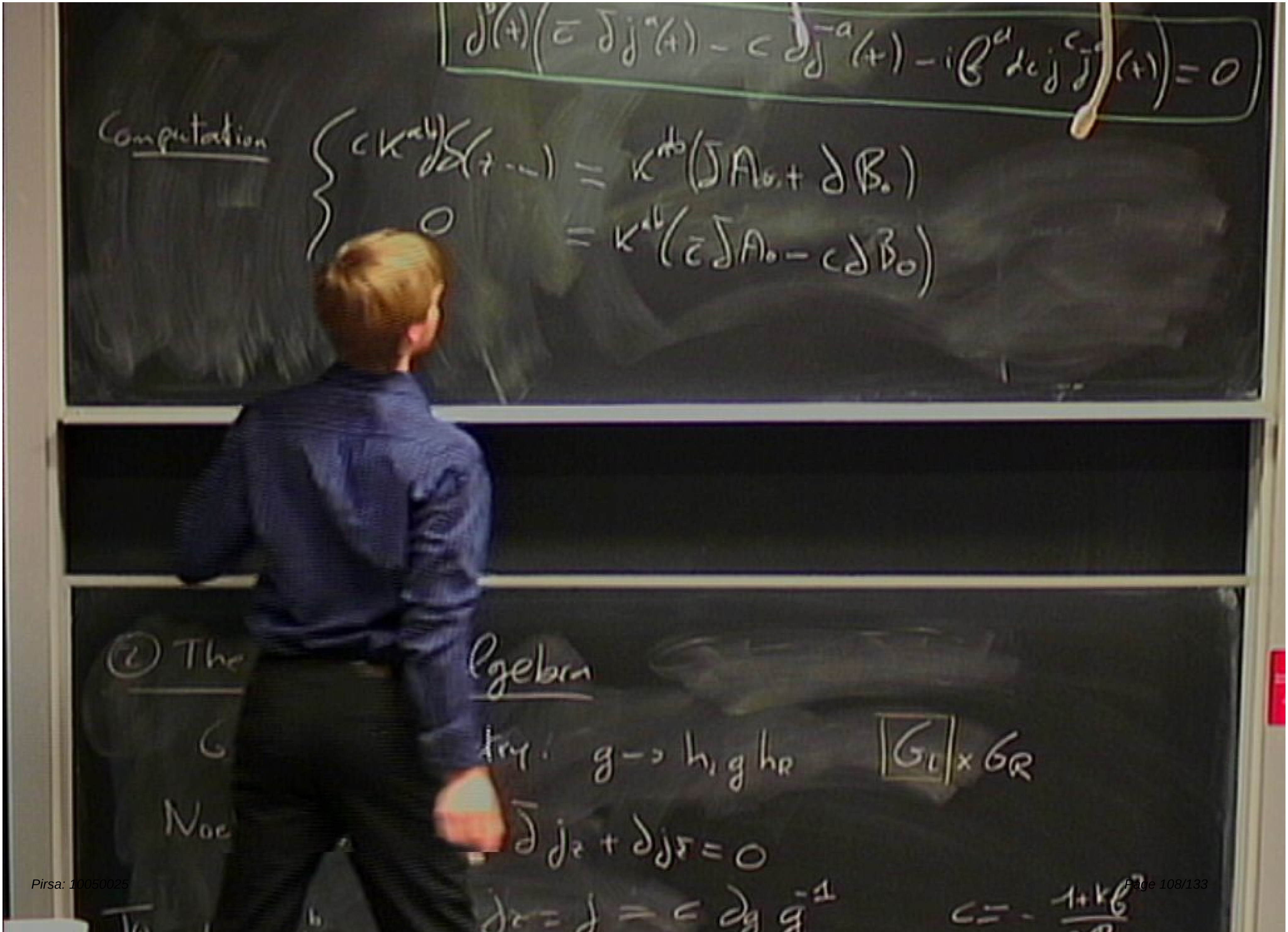
② The current algebra

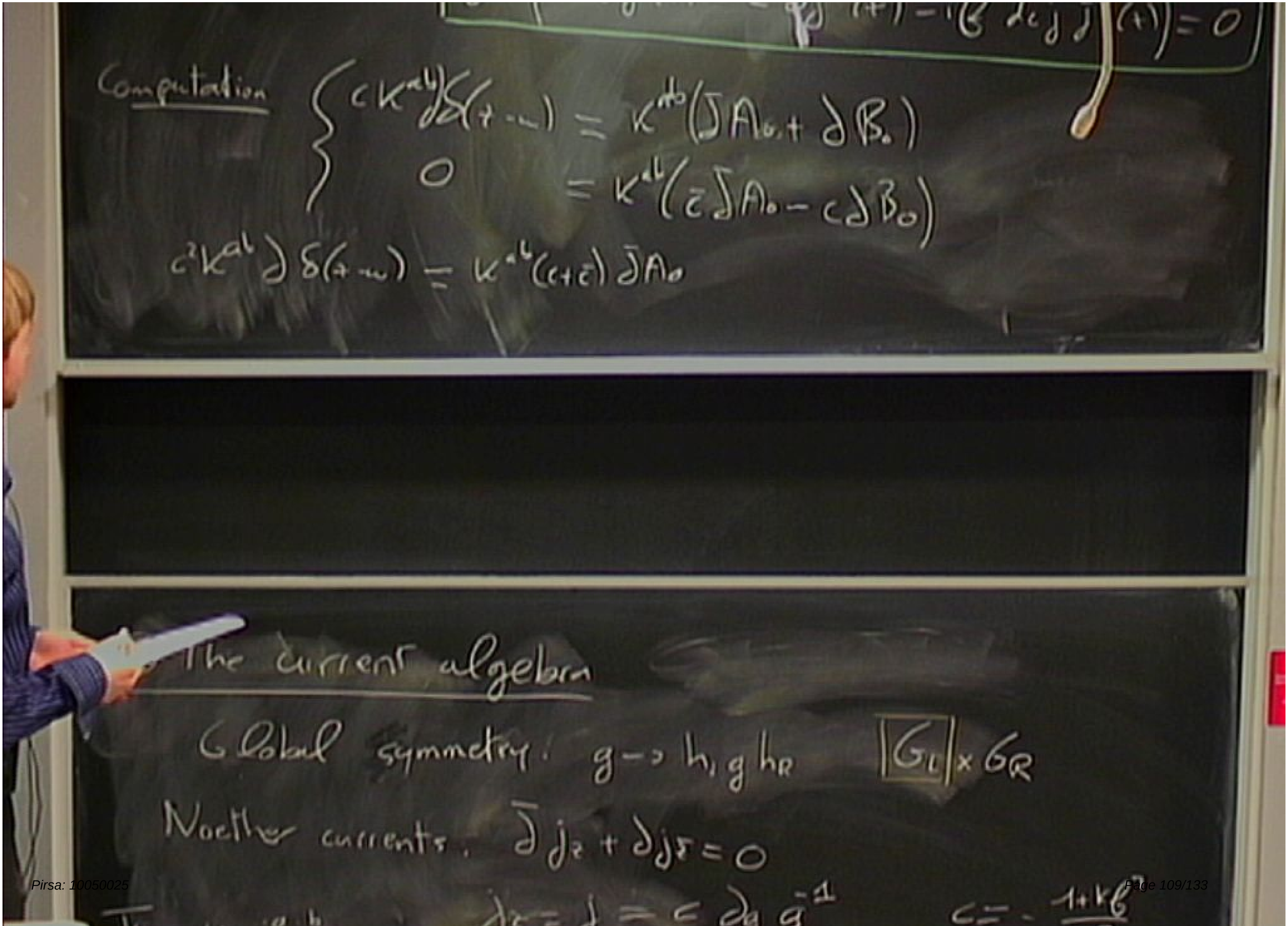
Global symmetry: $g \to h_L\, g\, h_R$ $\quad G_L \times G_R$

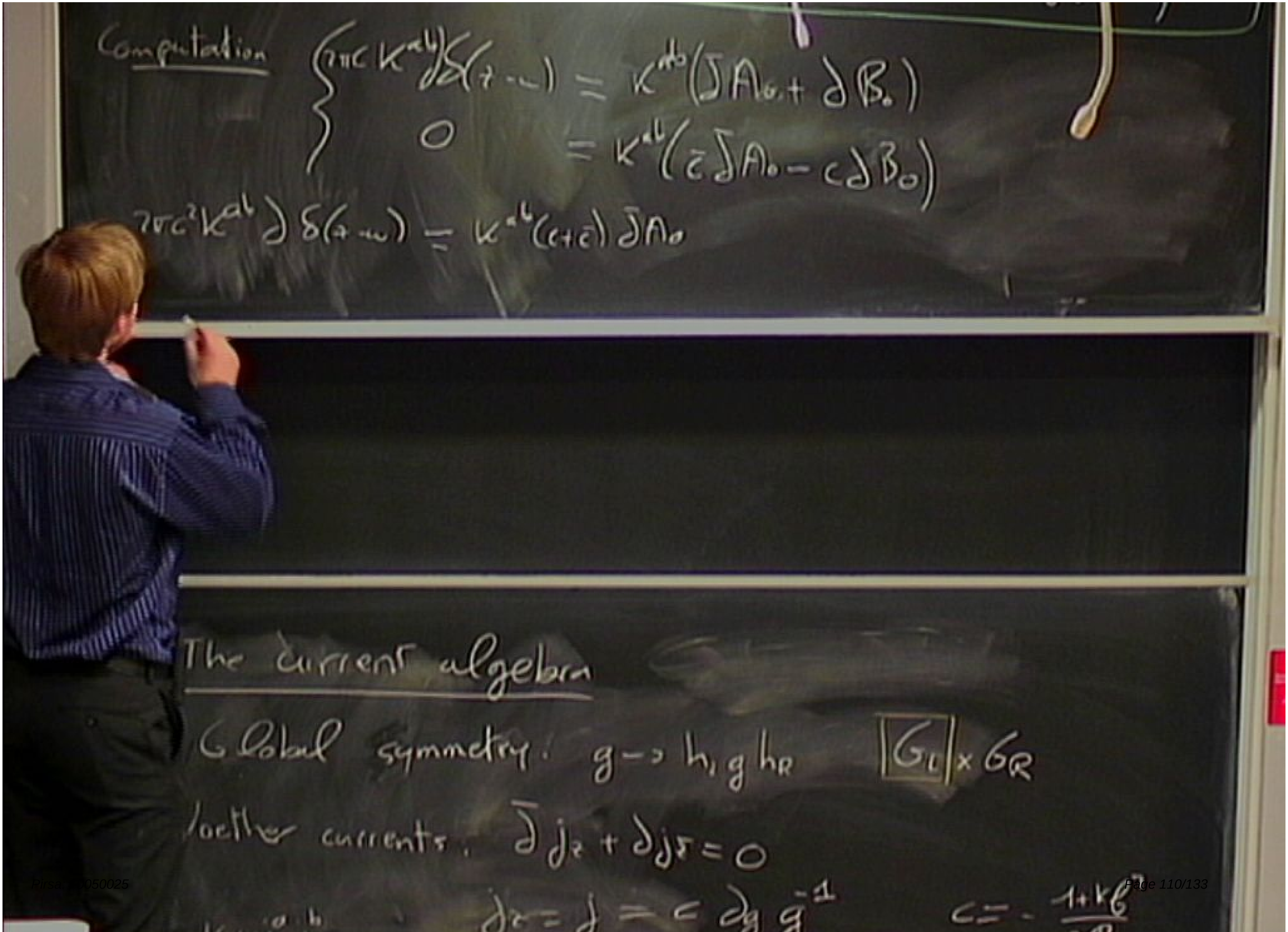
Noether currents: $\bar\partial j_z + \partial j_{\bar z} = 0$

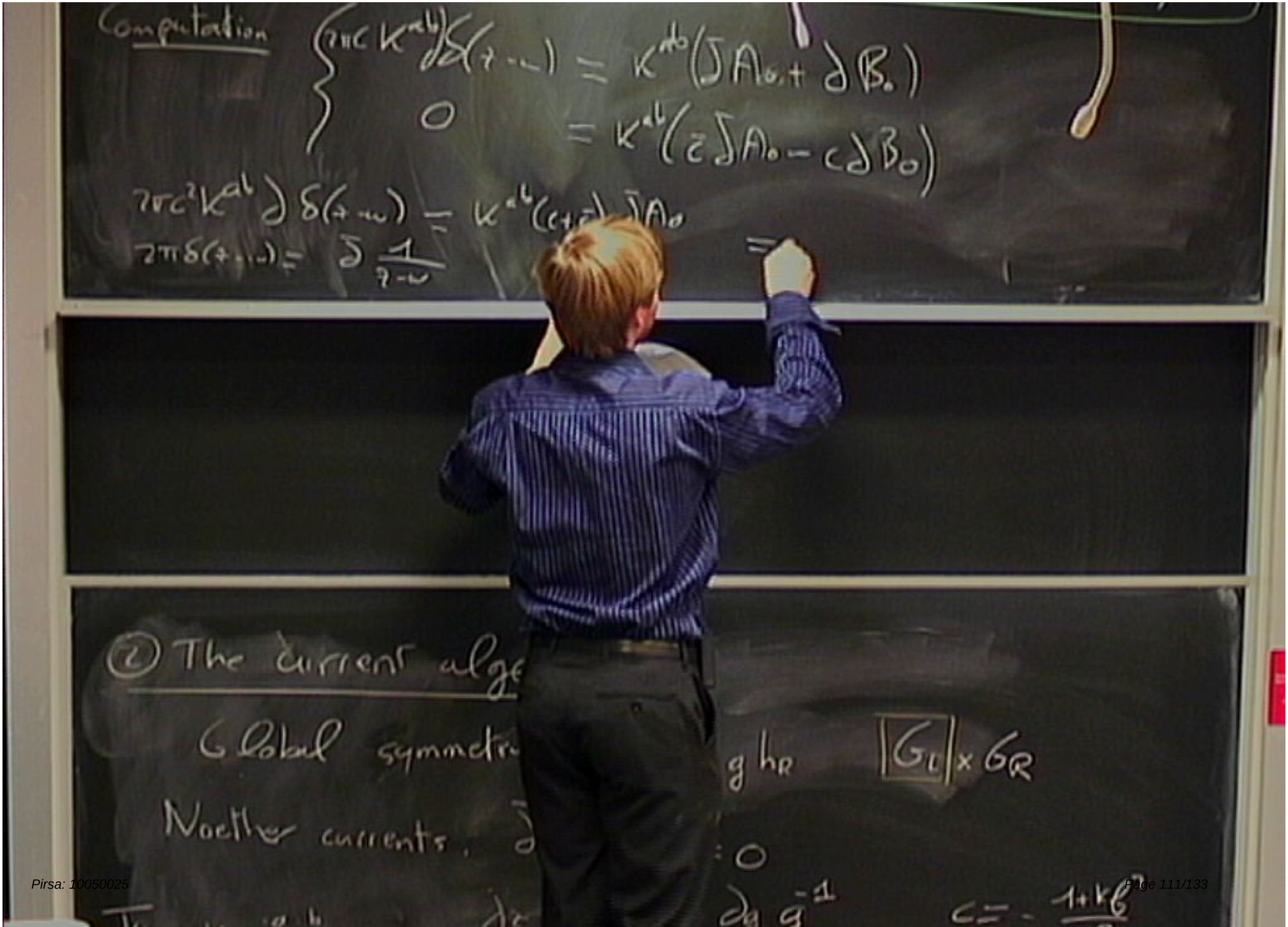
$T(z) \propto K_{ab} j^a j^b$ $\quad j_z = j = c\,\partial g\, g^{-1}$ $\quad c = -\frac{1+k\beta^2}{2\beta^2}$

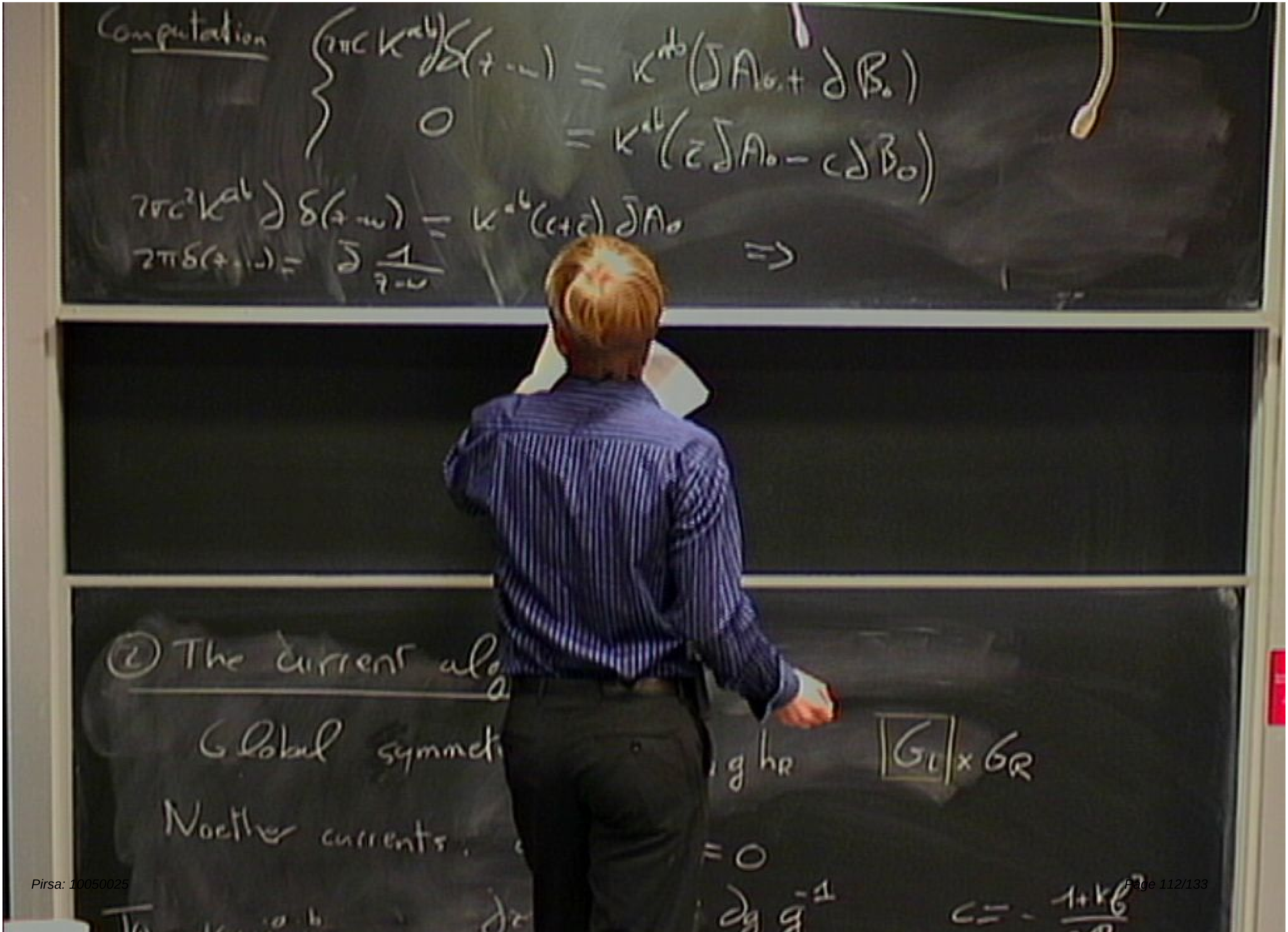

Maurer-Cartan
Computation
current algebra
symmetry: g → h_L g h_R
G_L × G_R
currents.

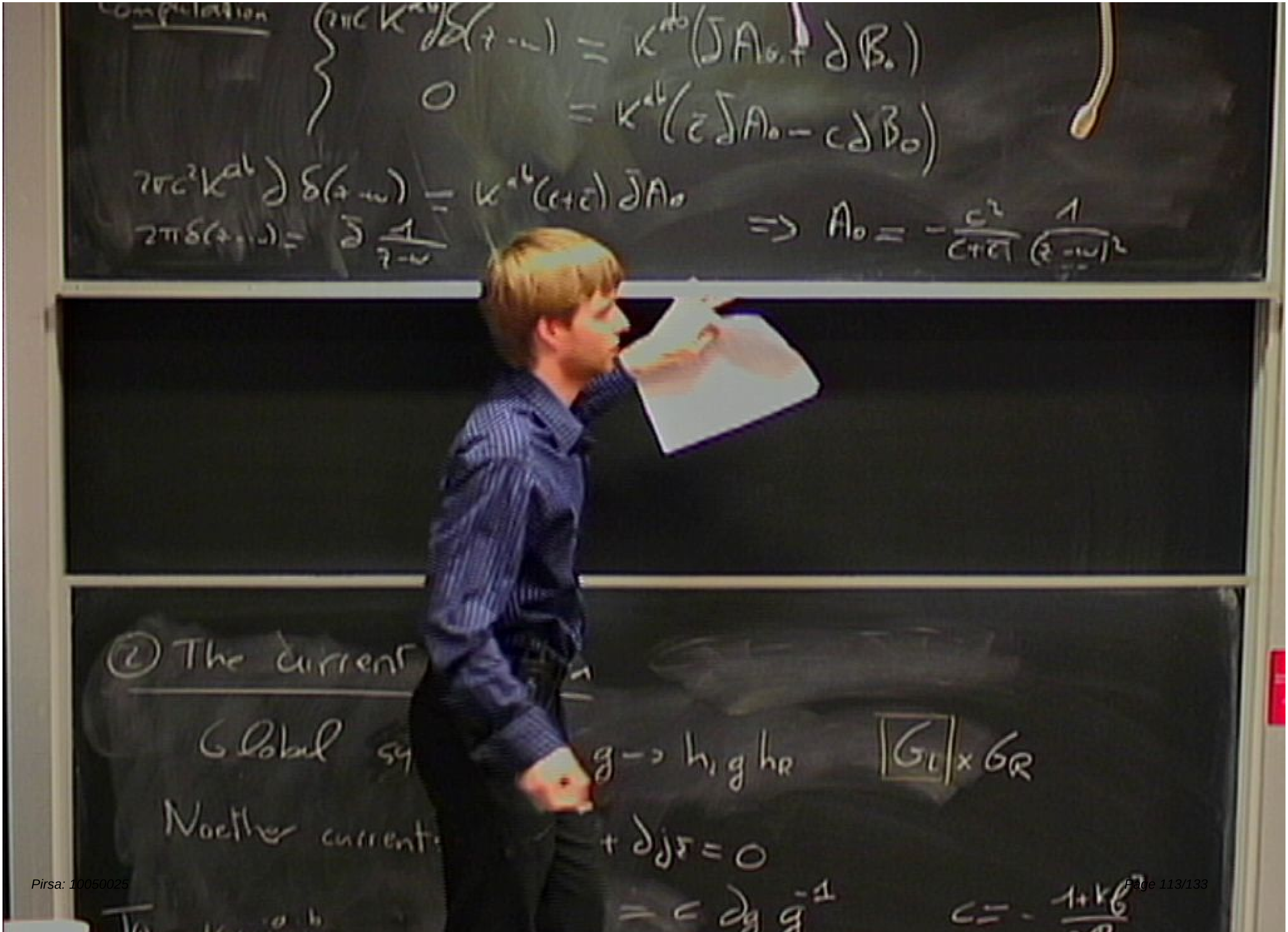

Computation


Computation
The current algebra
Global symmetry: g -> h_L g h_R   G_L x G_R
Noether currents:

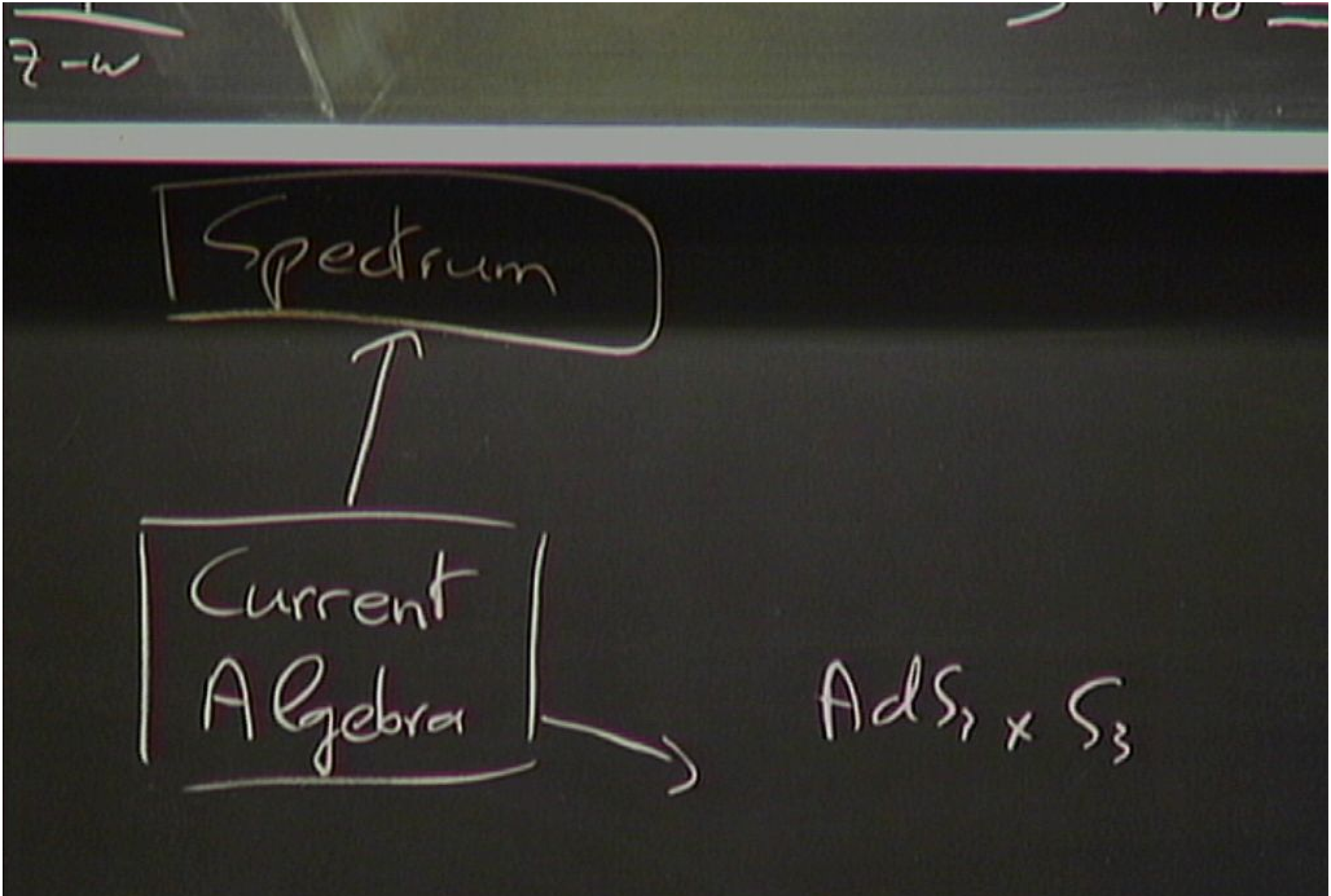

Computation
The current algebra
Global symmetry: g -> h_L g h_R
G_L x G_R
Noether currents: ∂̄ j_z + ∂ j_z̄ = 0

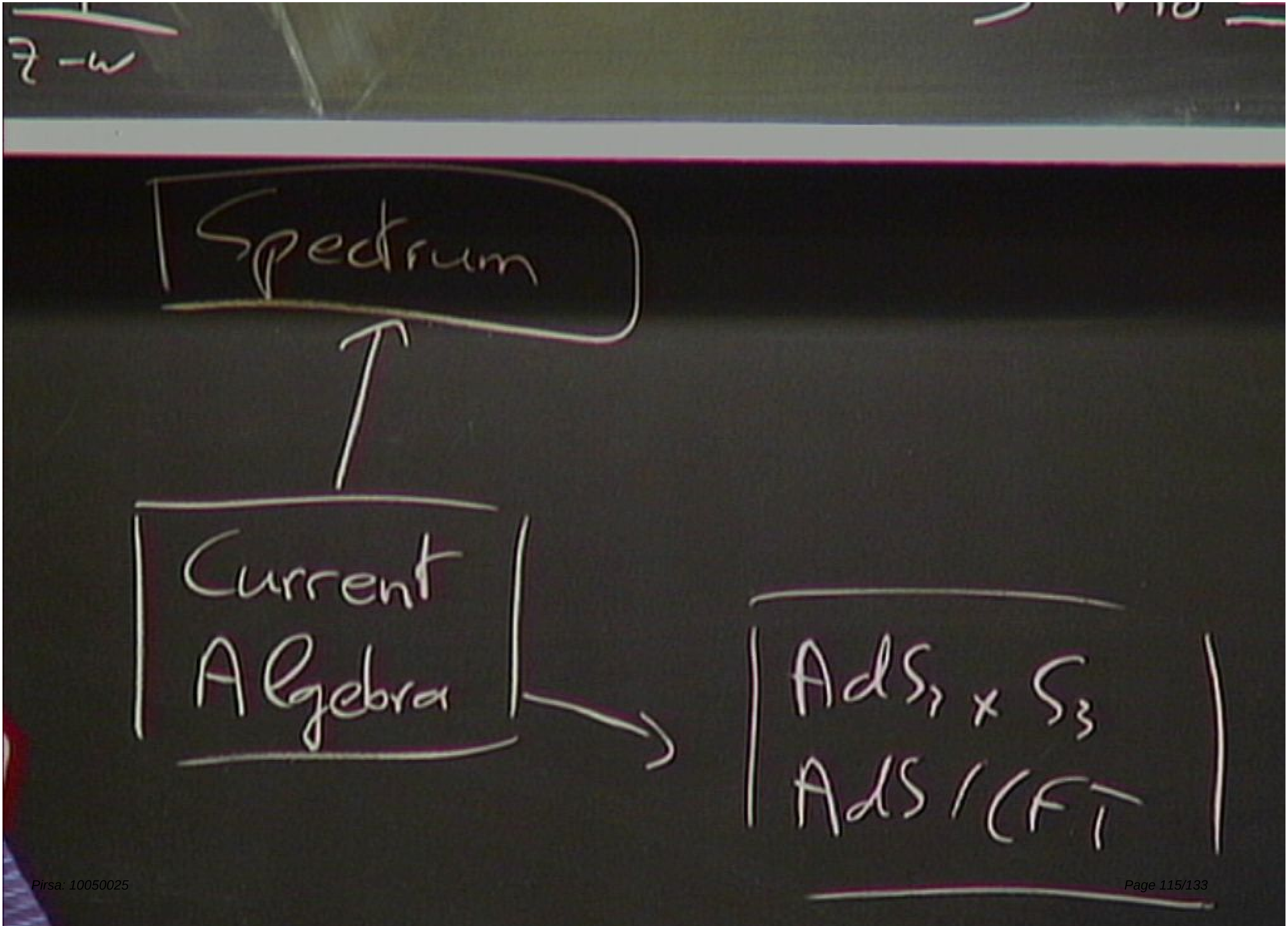

Computation
② The current alge
Global symmetr
Noether currents

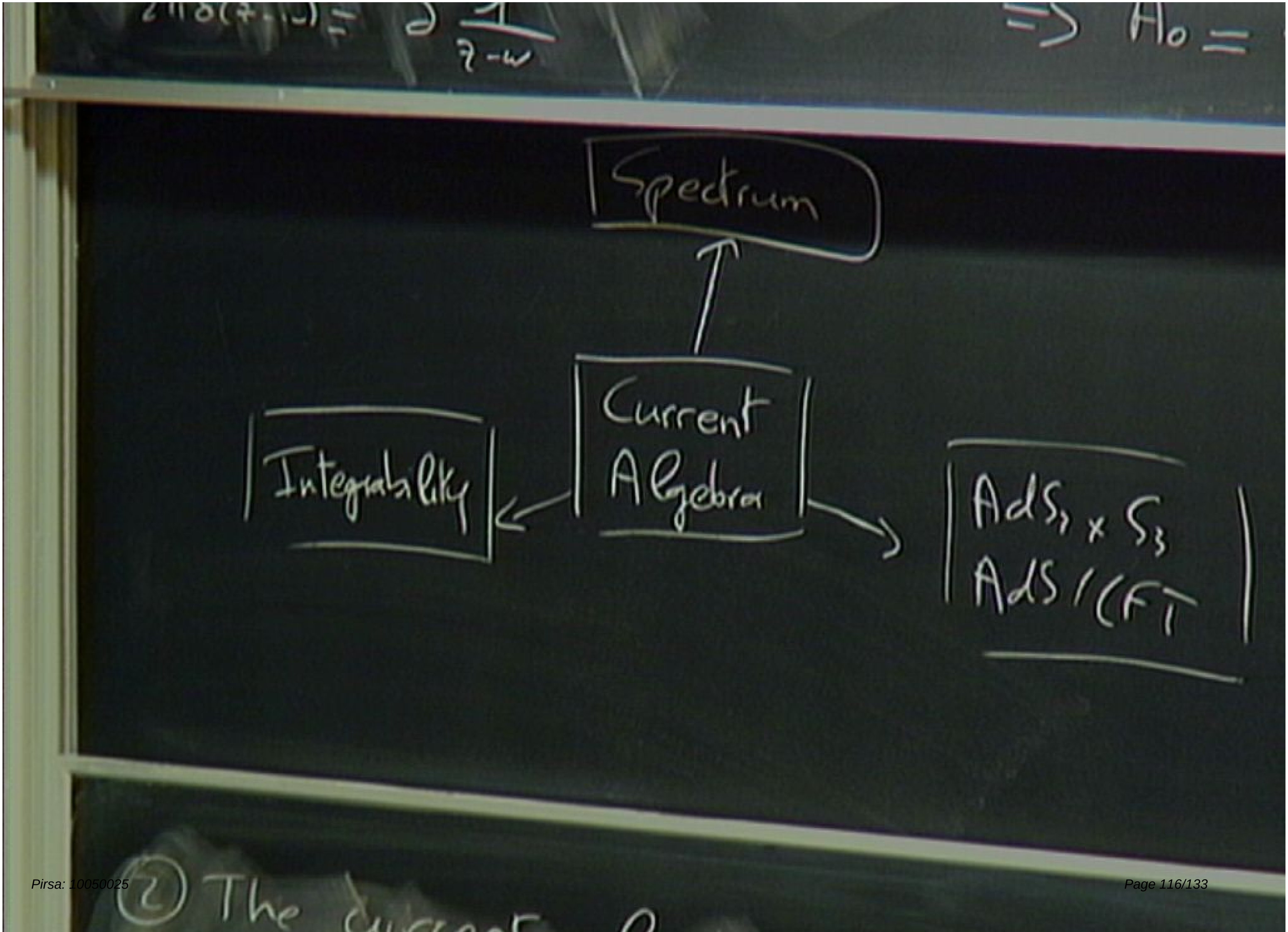

Computation
② The current alg
Global symmet
Noether currents

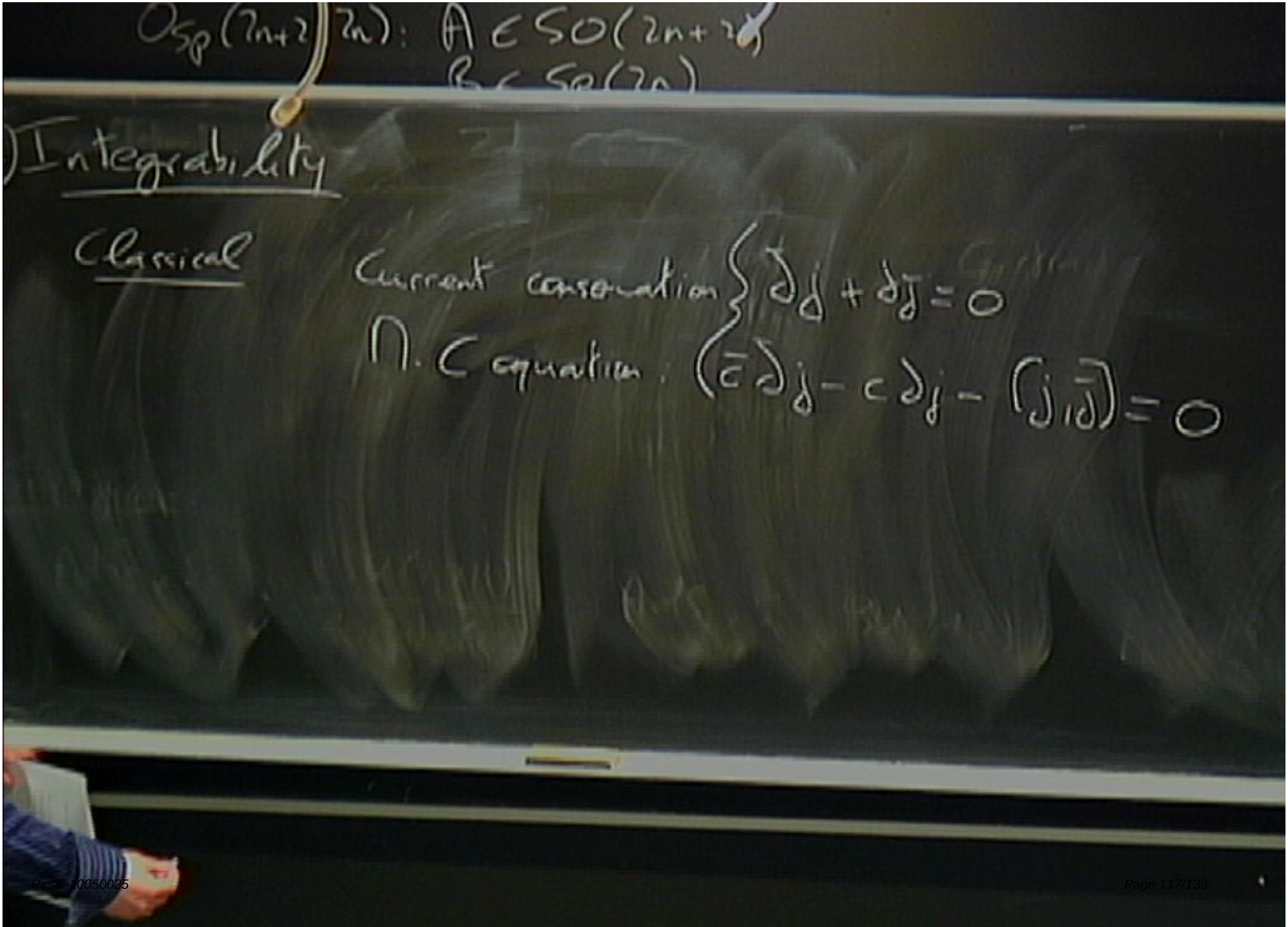


② The current
Global
$g \to h_L g h_R$
$G_L \times G_R$
Noether current
$\Rightarrow A_0 = -\frac{c^2}{c+\bar{c}} \frac{1}{(z-w)^2}$


Spectrum
Current Algebra
AdS3 x S3


Spectrum
Current Algebra
AdS3 × S3
AdS/CFT


Spectrum
Current Algebra
Integrability
AdS3 × S3
AdS / CFT
(2) The


Osp(2n+2|2n): A ∈ SO(2n+2)
B ∈ Sp(2n)
Integrability
Classical
Current conservation { ∂j̄ + ∂̄j = 0
M.C equation: (c̄ ∂)j − c ∂̄j − [j, j̄] = 0

(3) Integrability

Classical

Current conservation $\left\{ \bar\partial j + \partial \bar j = 0 \right.$

M.C equation: $(\bar c \partial) j - c \bar\partial j - [j, \bar j] = 0$

$$A(\lambda) = \frac{2}{1+\lambda} \frac{j}{c+\bar c} dz + \frac{2}{1-\lambda} \frac{\bar j}{c+\bar c} d\bar z$$

③ Integrability

Classical

Current conservation $\left\{ \bar\partial j + \partial \bar j = 0 \right.$

M.C equation: $(\bar c\, \partial)\, \bar j - c\, \bar\partial\, j - [j, \bar j] = 0$

$$A(\lambda) = \frac{2}{1+\lambda}\,\frac{j}{c+\bar c}\,dz + \frac{2}{1-\lambda}\,\frac{\bar j}{c+\bar c}\,d\bar z$$

$$dA(\lambda) + A(\lambda)\wedge A(\lambda) =$$

zero Killing form

③ Integrability

Classical

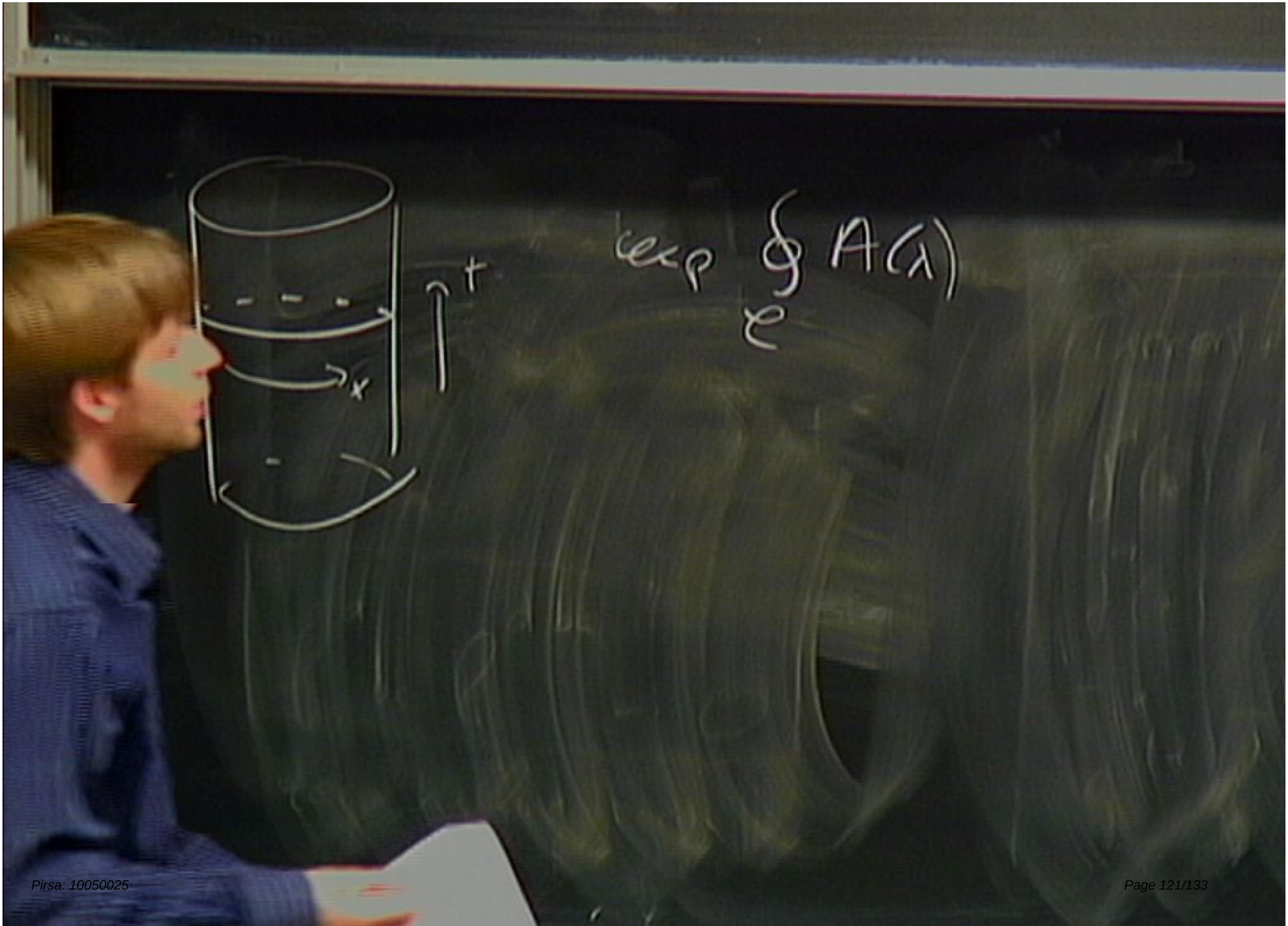
Current conservation $\{ \, \partial \bar{j} + \bar{\partial} j = 0$

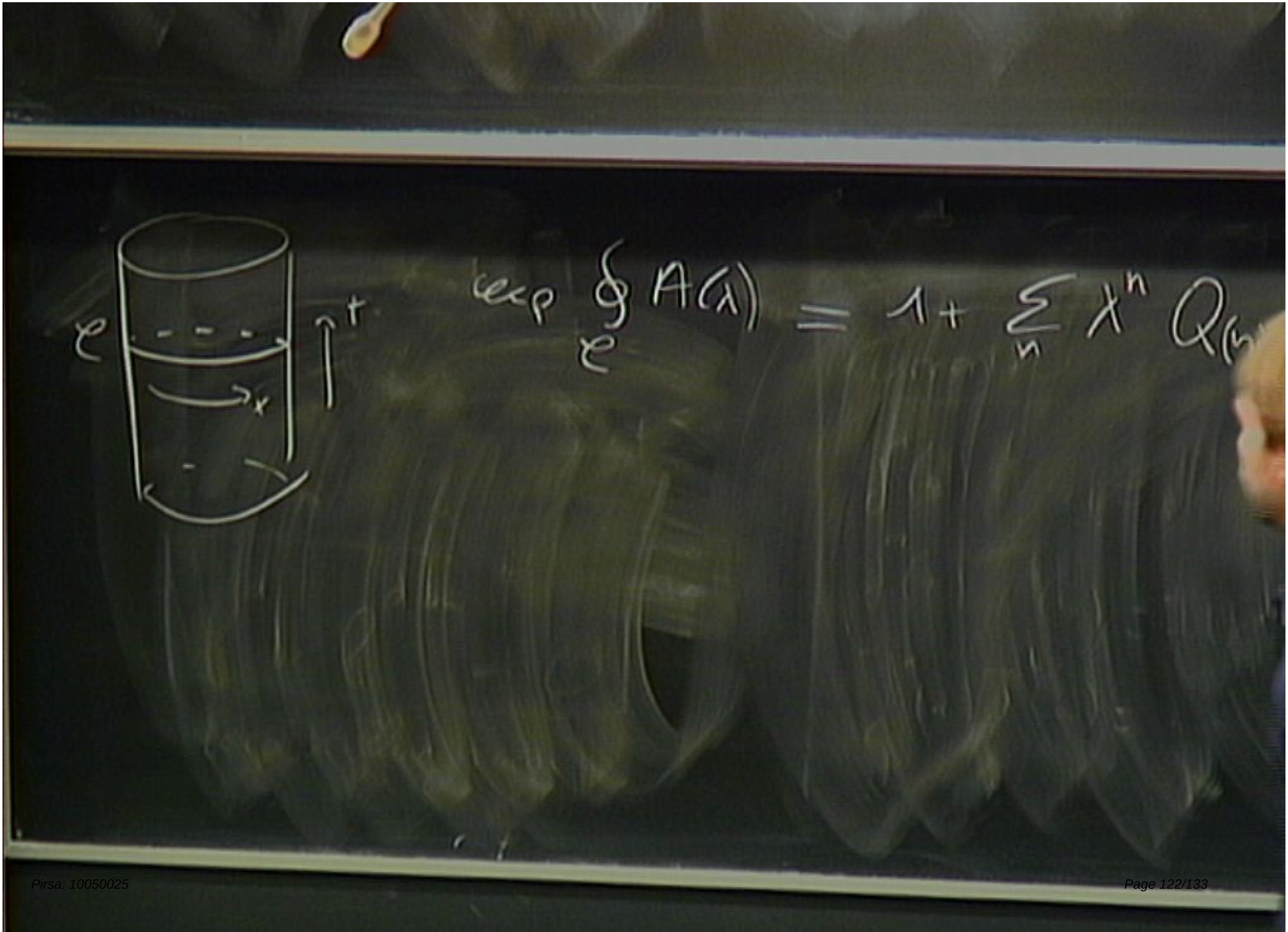
M.C equation : $(\bar{c}\,\partial) j - c\,\partial \bar{j} - [j, \bar{j}] = 0$

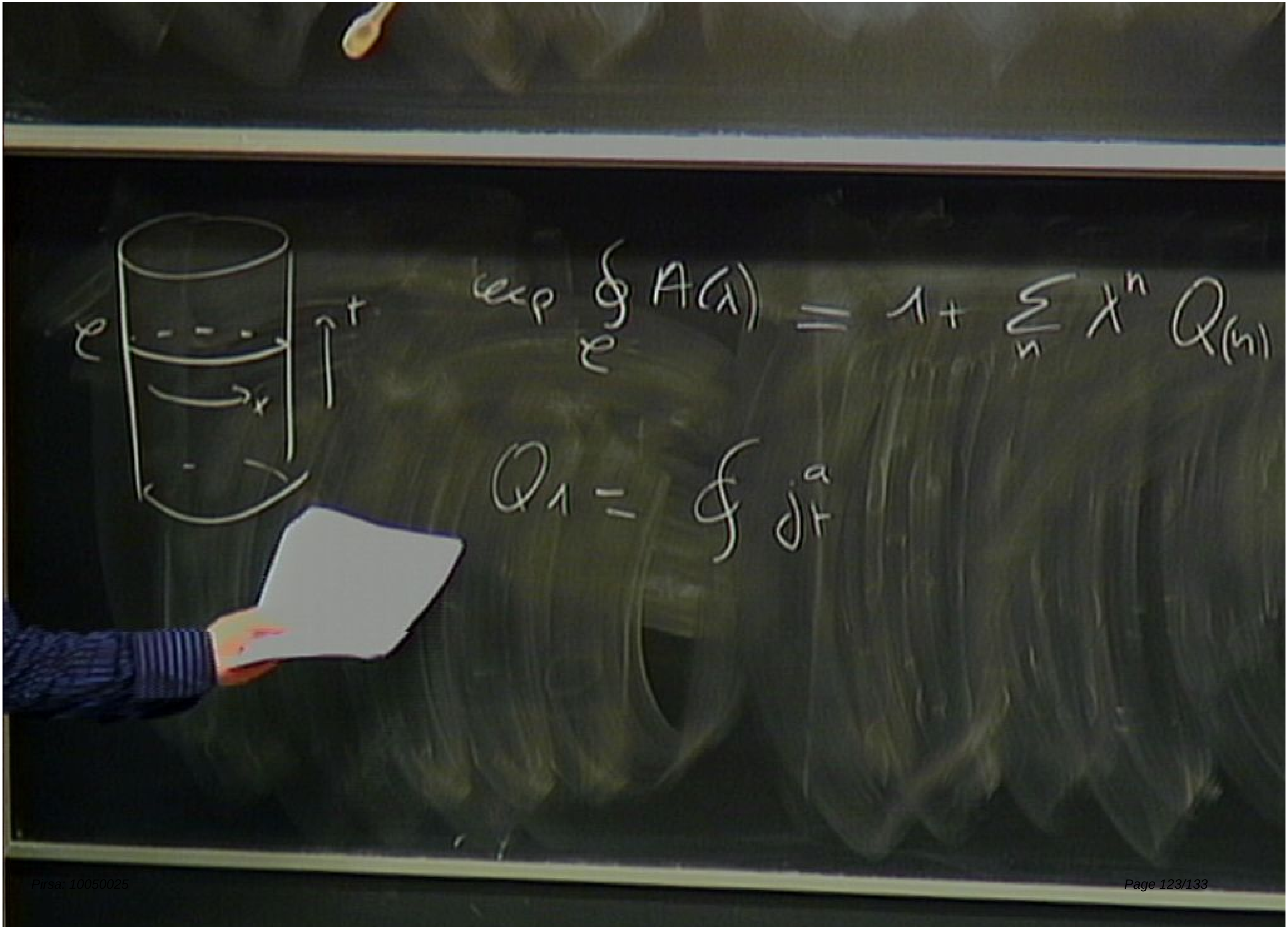
$$A(\lambda) = \frac{2}{1+\lambda}\,\frac{j}{c+\bar{c}}\,dz + \frac{2}{1-\lambda}\,\frac{\bar{j}}{c+\bar{c}}\,d\bar{z}$$

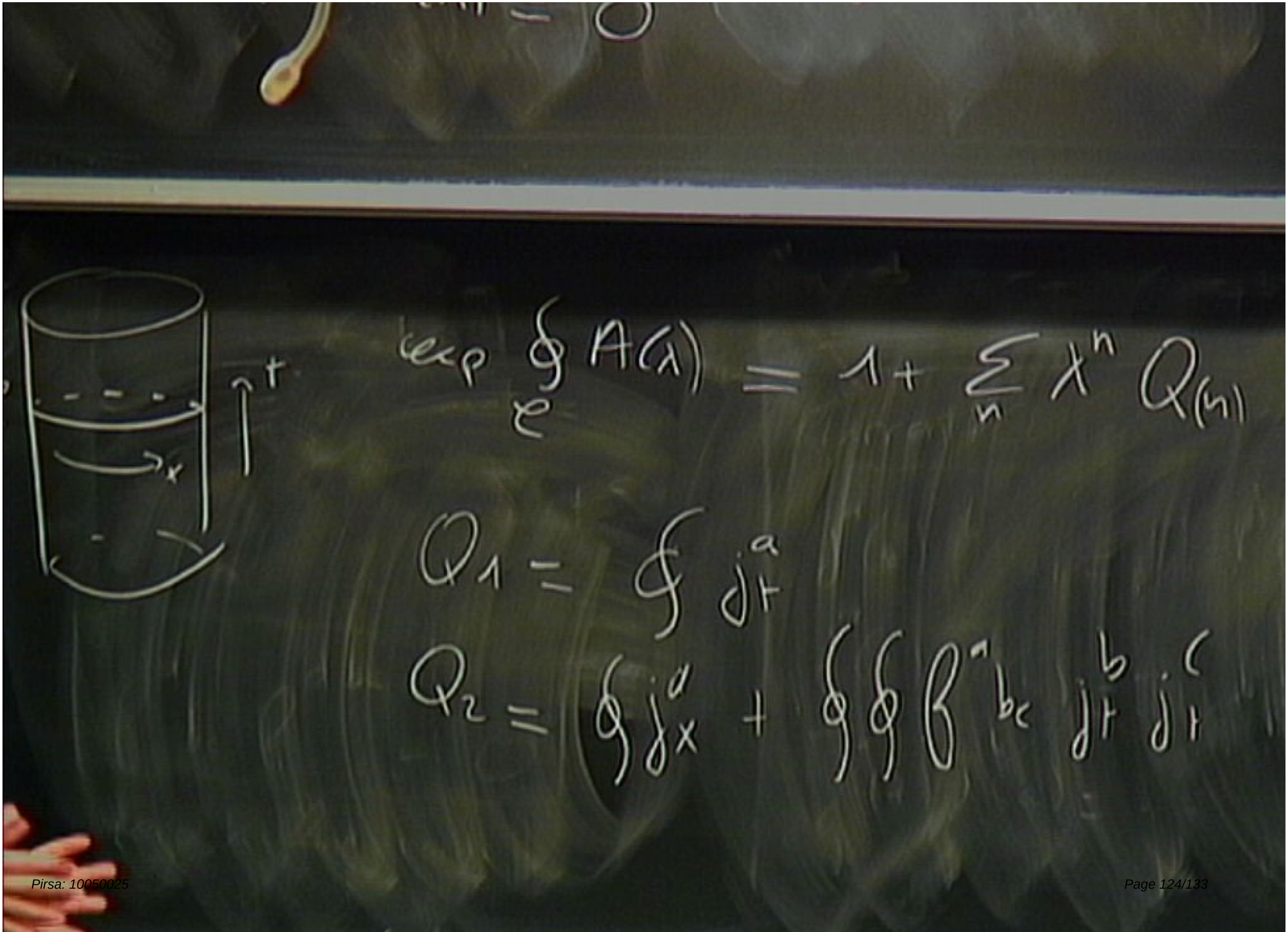
$$dA(\lambda) + A(\lambda) \wedge A(\lambda) = 0$$

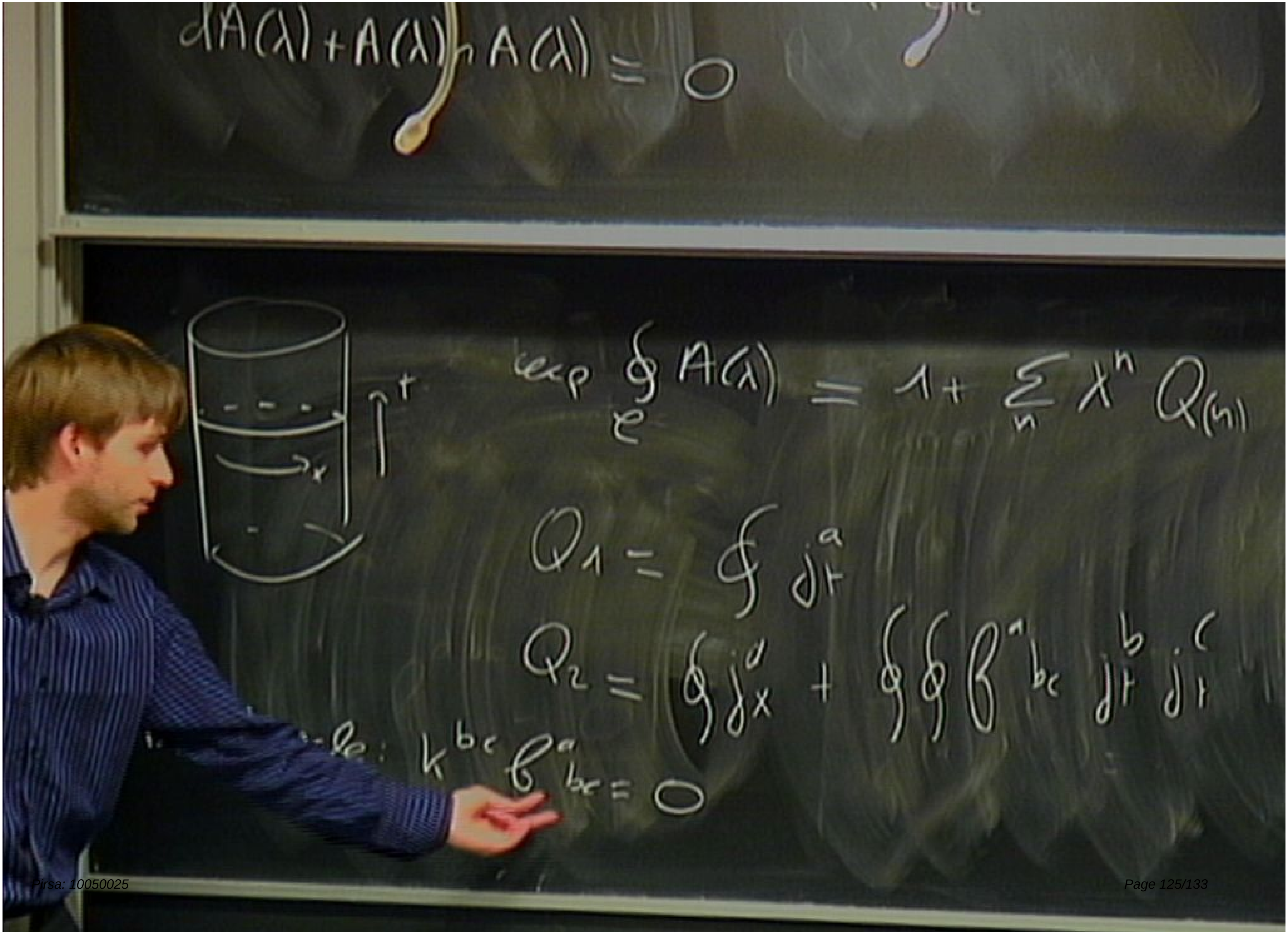
zero Killing form

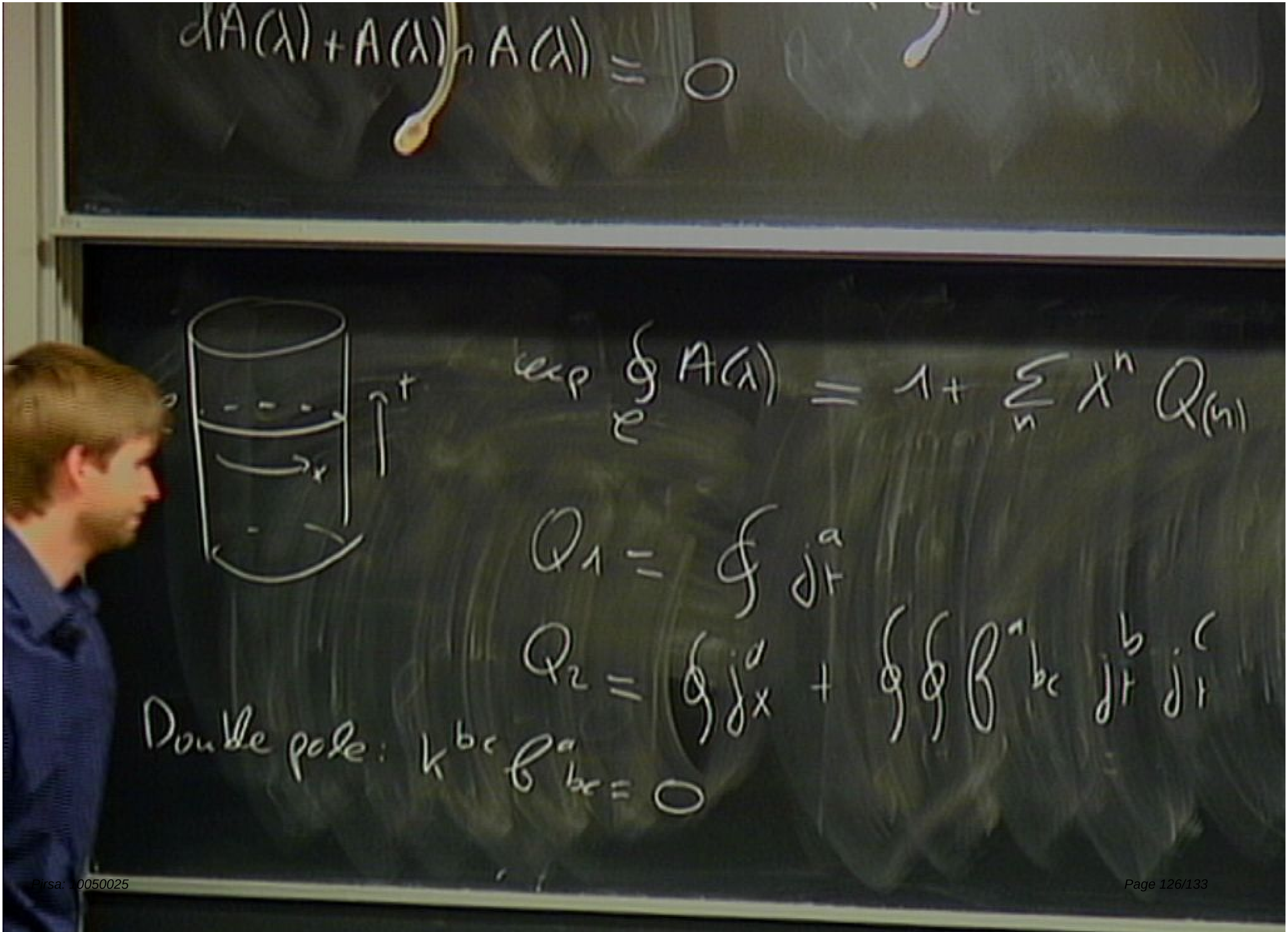

exp ∮ A(λ)
C
t
x


exp ∮ A(λ) = 1 + Σ λ^n Q(n


C
t
x
$\exp \oint_C A(\lambda) = 1 + \sum_n \lambda^n Q_{(n)}$
$Q_1 = \oint j_t^a$


t
x
$\exp \oint_{C} A(\lambda) = 1 + \sum_n \lambda^n Q_{(n)}$
$Q_1 = \oint j_t^a$
$Q_2 = \oint j_x^a + \oint\oint f^a{}_{bc}\, j_t^b\, j_t^c$


$dA(\lambda) + A(\lambda) \wedge A(\lambda) = 0$
$\exp \oint_C A(\lambda) = 1 + \sum_n \lambda^n Q_{(n)}$
$Q_1 = \oint j_t^a$
$Q_2 = \oint j_x^a + \oint \oint f^a{}_{bc}\, j_t^b\, j_t^c$
le: $k^{bc} f^a{}_{bc} = 0$


$dA(\lambda) + A(\lambda) \wedge A(\lambda) = 0$
$\exp \oint_{C} A(\lambda) = 1 + \sum_n \lambda^n Q_{(n)}$
$Q_1 = \oint j_t^a$
$Q_2 = \oint j_x^a + \oint\oint f^a{}_{bc}\, j_t^b\, j_t^c$
Double pole: $k^{bc} f^a{}_{bc} = 0$

$$\ldots = \frac{1}{1+\lambda}\,\frac{a}{c+z}\,dt + \frac{z}{1-\lambda}\,\frac{a}{d\,z}\,dz$$

$$dA(\lambda) + A(\lambda)\wedge A(\lambda) = 0$$

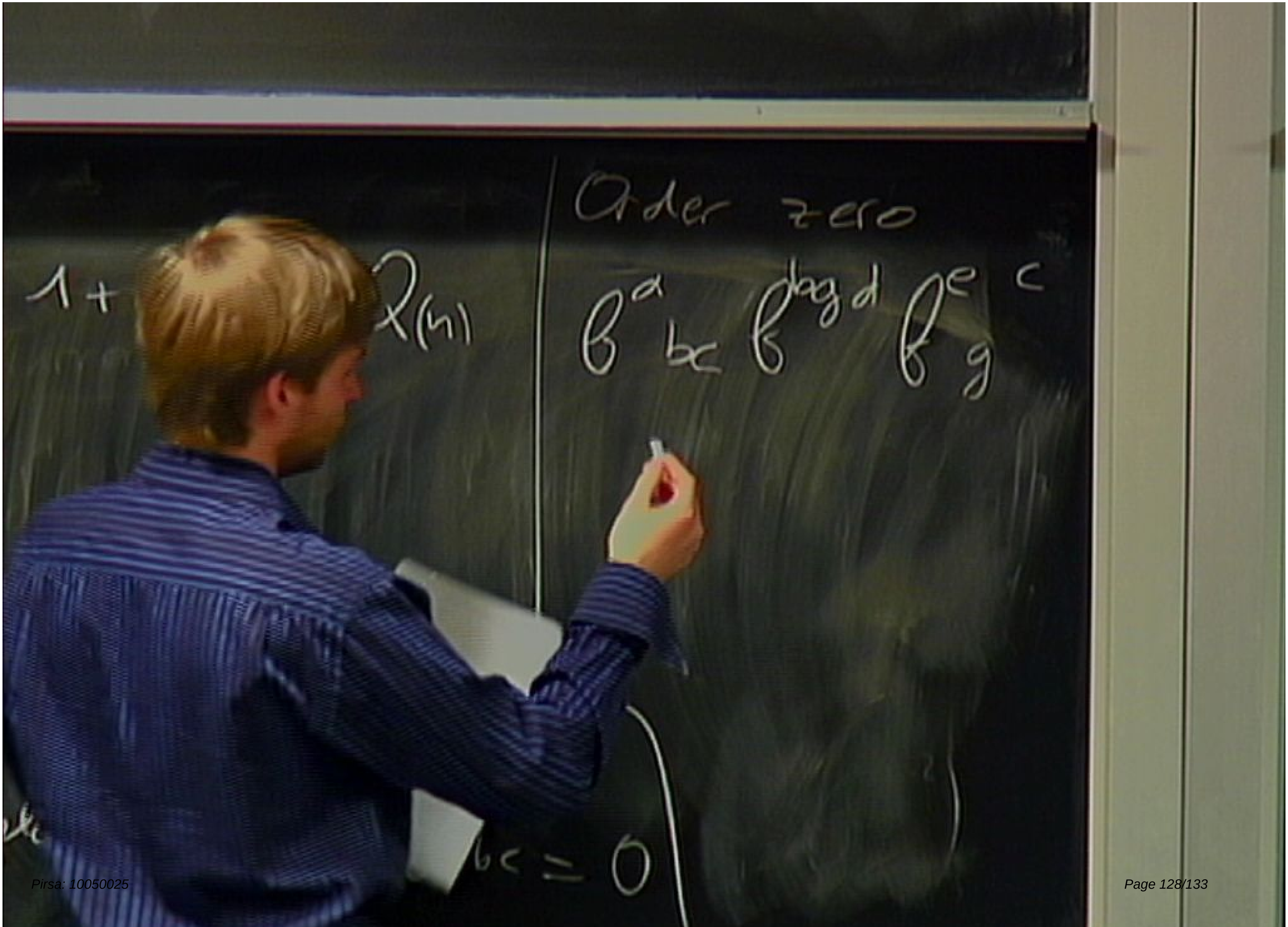
$$\mathcal{P}\exp \oint_C A(\lambda) = 1 + \sum_n \lambda^n Q_{(n)}$$

$$Q_1 = \oint j_t^a$$

$$Q_2 = \oint j_x^a + \oint\oint \beta^a{}_{bc}\, j_t^b\, j_t^c$$

Double pole: $k^{bc}\beta^a{}_{bc} = 0$ | Simple pole: $\beta^{bc}\, d\beta^a{}_{bc} = 0$

Order zero


Order zero

$$1 + \sum_n \lambda^n Q_{(n)}$$

$$\oint \oint \beta^a{}_{bc}\, j_+^b\, j_+^c$$

le pole: $\beta^{bc}{}_d\, \beta^a{}_{bc} = 0$

Order zero

$$\beta^a{}_{bc}\, \beta^{[bg]}\, \beta^{e]}{}_g{}^c$$

$1 + \sum_n \lambda^n Q_{(n)}$

Order zero

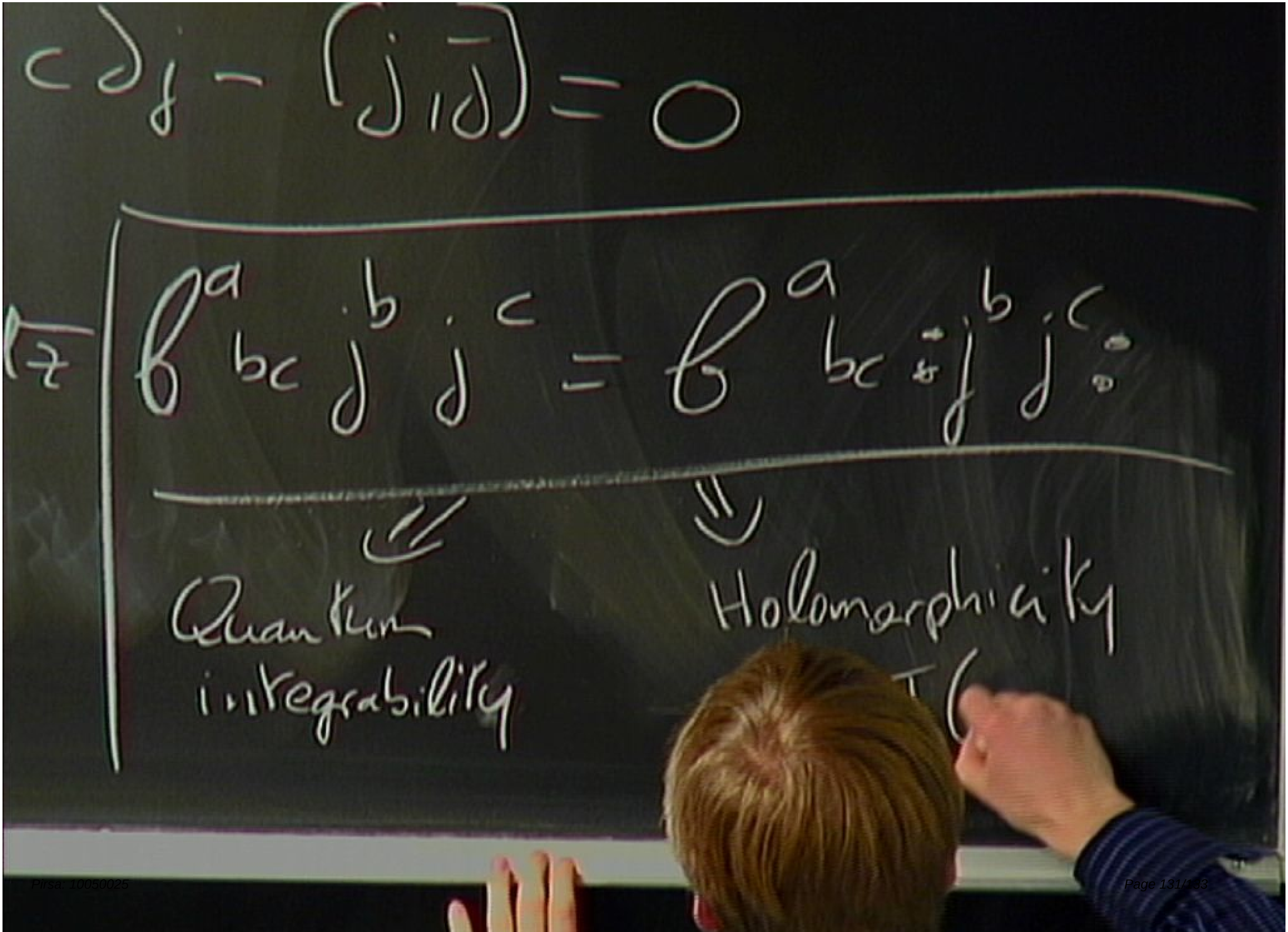
$f^a{}_{bc}\, f^{bg}{}_{d}\, f^{e}{}_{g}{}^{c}$

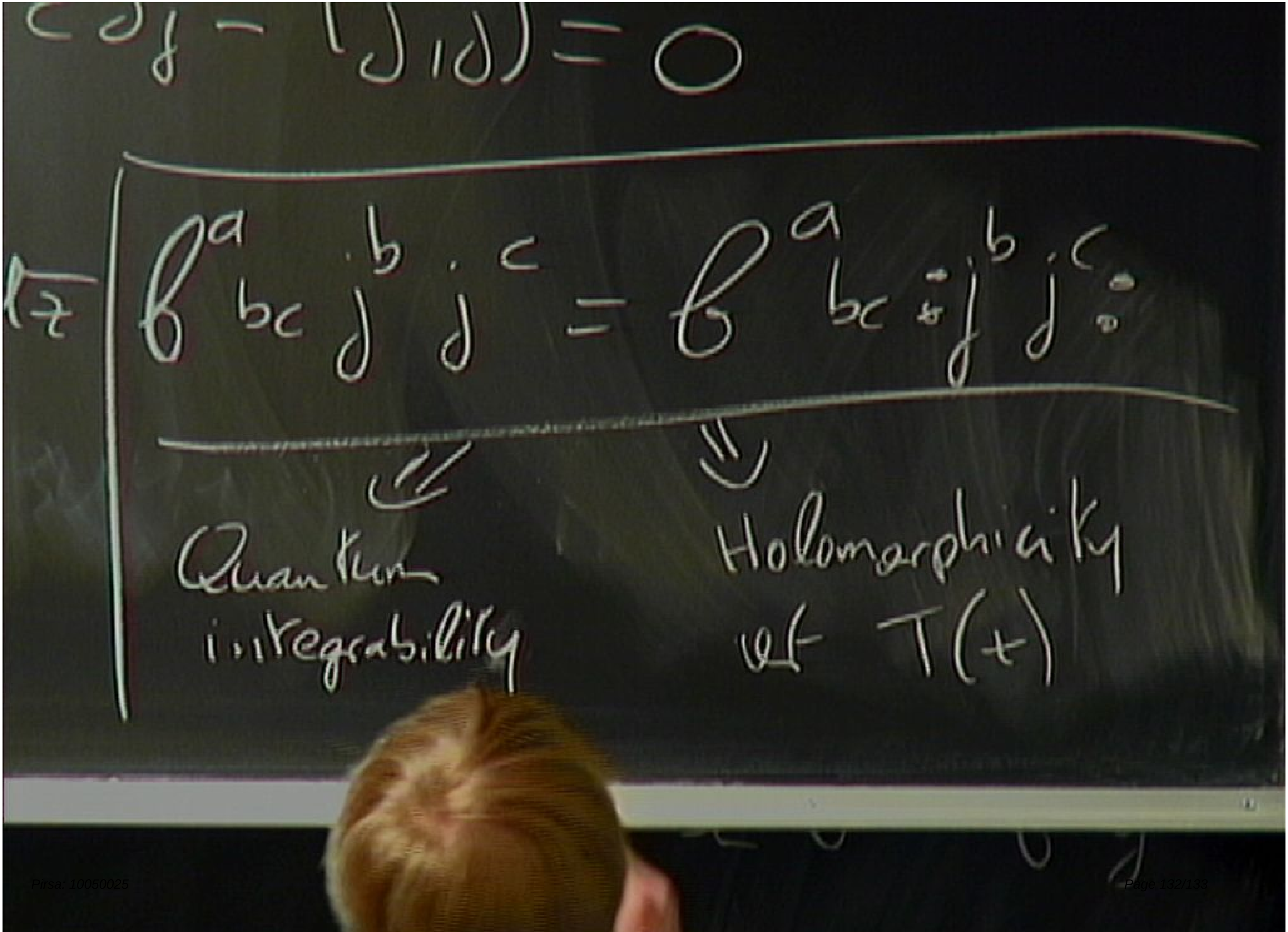
$= 0$

Jacobi
+ zero Killing form

$\oint \oint f^a{}_{bc}\, j_+^b\, j_+^c$

…le pole: $f^{bc}{}_d\, f^a{}_{bc} = 0$


$f^{a}{}_{bc}\, j^{b} j^{c} = f^{a}{}_{bc} :j^{b} j^{c}:$
Quantum integrability
Holomorphicity


f^a_bc j^b j^c = f^a_bc :j^b j^c:
Quantum integrability
Holomorphicity of T(t)

$$f^{a}{}_{bc}\, j^{b} j^{c} = f^{a}{}_{bc} :j^{b} j^{c}:$$

Quantum integrability $\Longleftrightarrow$ Holomorphicity of $T(t)$

$$\lambda^{n} Q_{(n)}$$

$$f^{a}{}_{bc}\, f^{bg}{}_{d}\, f^{e}{}_{g}{}^{c}$$