

Title: Current algebra in conformal sigma models on supergroups (Part I)

Date: May 12, 2010 11:00 AM

URL: <http://pirsa.org/10050025>

Abstract: Two-dimensional non-linear sigma models on some supergroup manifolds are conformal field theories whether the action includes a Wess-Zumino term or not. These models are relevant for the worldsheet description of string theory in Anti-de Sitter backgrounds with Ramond-Ramond fluxes. The current algebra is an useful tool to study these theories. In these lectures I will review the construction of the current algebra. Then I will discuss some applications to the computation of the spectrum and integrability.

Current algebra in conformal σ -models on supergroups

Plan

- ① Introduction
- ② Current algebras
- ③ Integrability

Friday: towards the spectrum

work with:
Sujay Ashoke
Jon Tesost

Current algebra in conformal σ -models on supergroups

Plan

- ① Introduction
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Friday: towards the spectrum

work with:
Sujay Acharya
Jon Tesost

Current algebra in conformal σ -models on supergroups

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Friday: towards the spectrum with Achik

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Feider: towards the spectrum

Current algebra in conformal σ -models on supergroups

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Friday: towards the spectrum

work with:
Sujay Ashoke
Jon Teasdale

Current algebra in conformal σ -models on supergroups

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Friday: towards the spin-1/2 supergroup $OSp(1|2)$ with
by Achille
Teoast

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Friday: towards the spectrum

work with:
Sujay Adh
Joi

Current algebra in conformal σ -models on supergroups

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Friday: towards the spectrum

work with:
Sujay Ashok
Jan Troost

- ② Current algebra
- ③ Integrability

... towards the spectrum

Sujay Ashok
Jon Troost

① Introduction



- ② Current algebra
- ③ Integrability

... towards the spectrum

Sujay Ashok
Jon Troost

① Introduction

Main ... in ADS with RR Fluxes

- ② Current algebra
- ③ Integrability

... towards the spectrum

Sujay Ashok
Jon Troost

① Introduction

Main String in AdS with RR Fluxes
AdS/CFT

① Introduction

Main String in AdS with RR Flux

AdS/CFT

Strings in RR backgrounds

① Introduction

Main

String in AdS with RR Flux

AdS/CFT

Strings in RR backgrounds

* spacetime \subset superspace

Worksheet Theory: $2d$ CFT

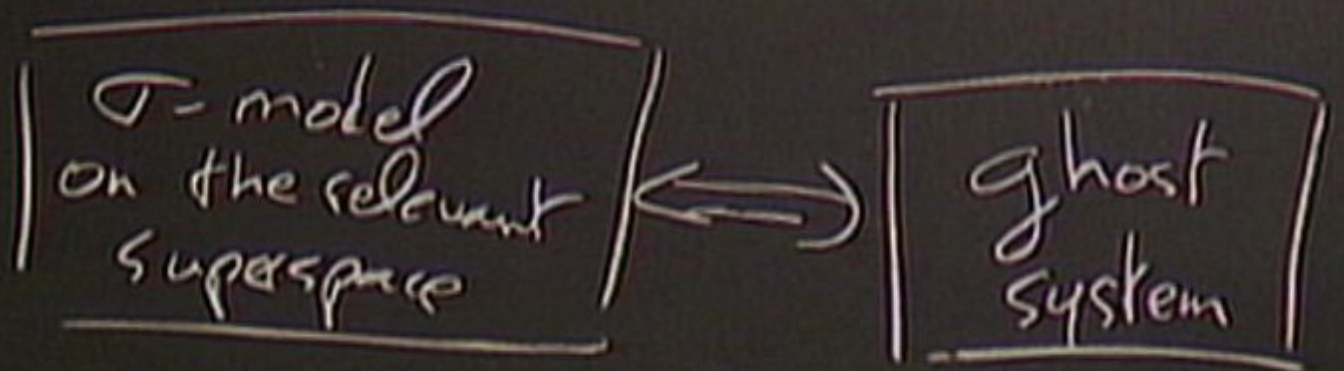
\mathcal{T} -mod

Worksheet Theory: 2d CFT

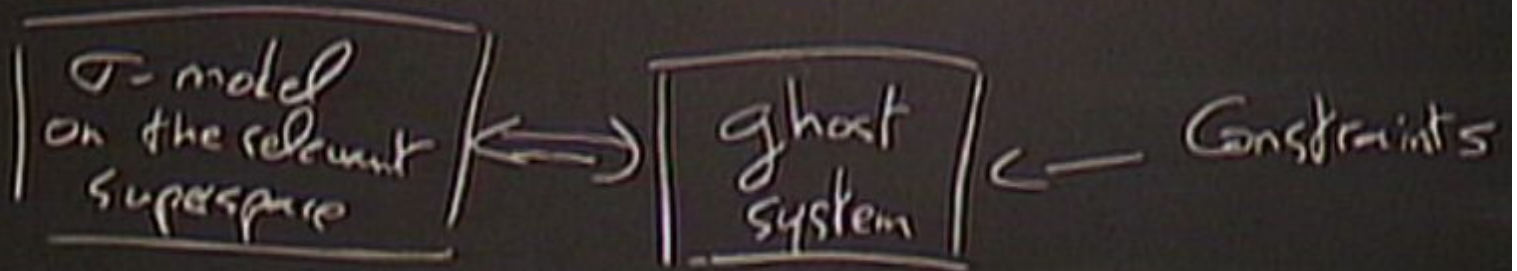
\mathcal{T} -model
on the relevant
superspace

ghost
system

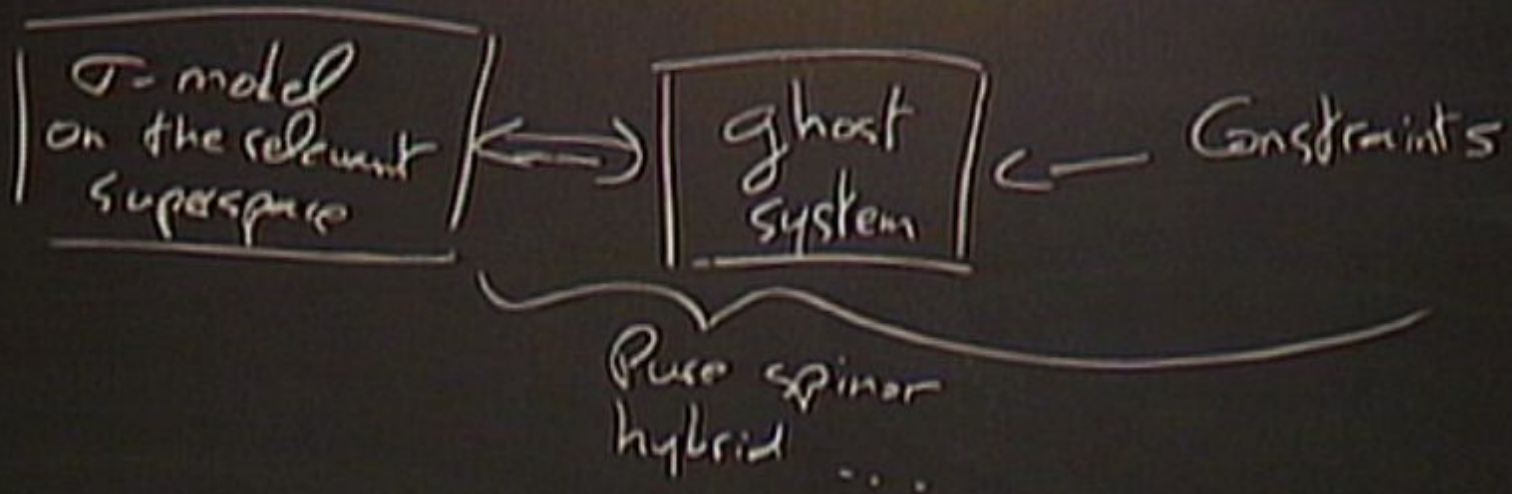
Worksheet Theory: 2d CFT



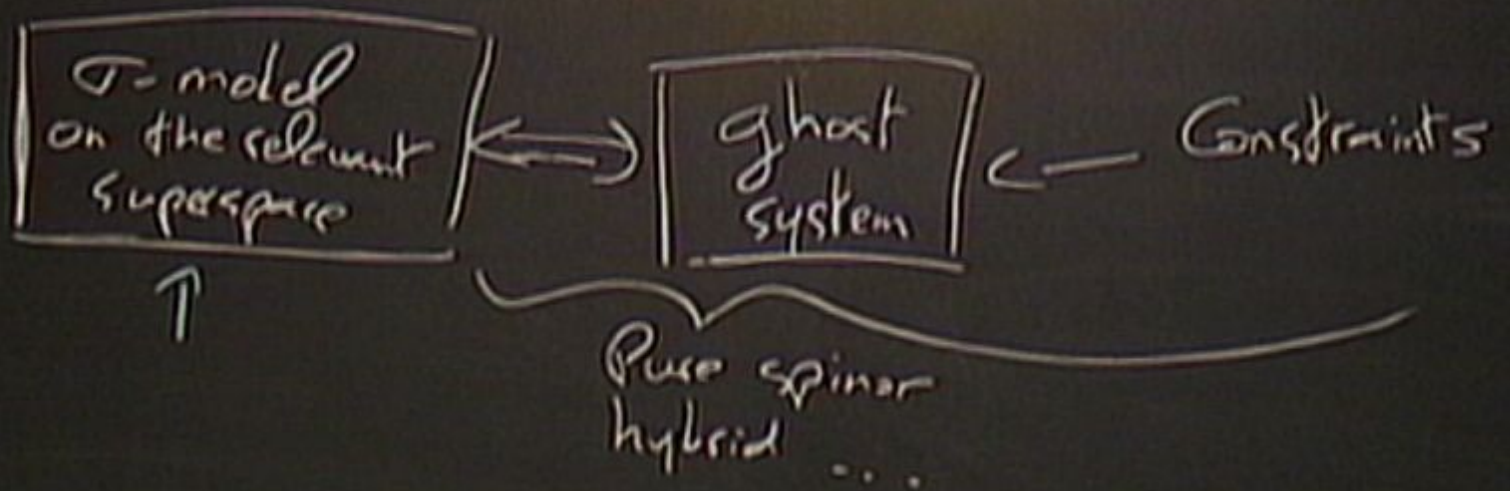
Worksheet Theory: 2d CFT



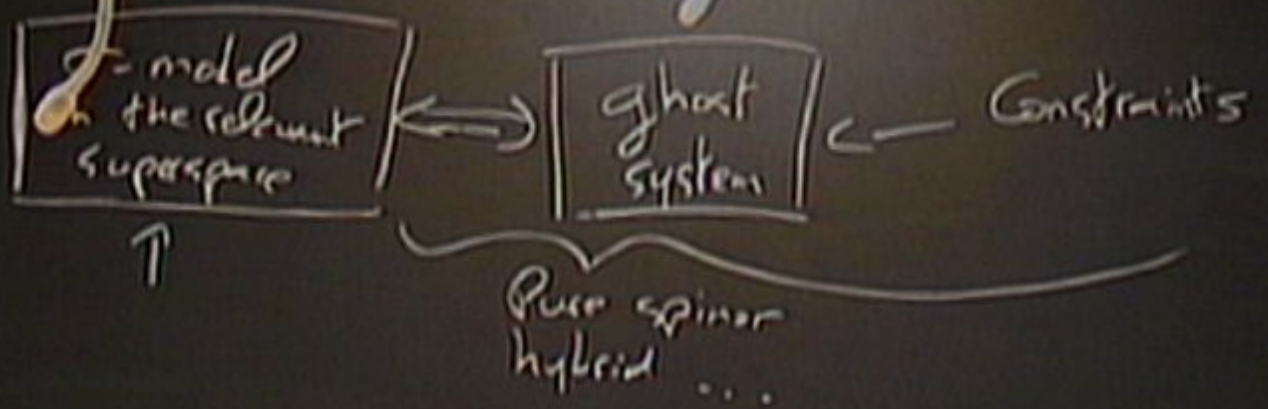
Worksheet Theory: 2d CFT



Worldsheet Theory: 2d CFT



Worldsheet Theory: 2d CFT



Supergroups

σ -model
in the relevant
superspace



ghost
system

Constra



Pure spinor
hybrid ...

Supergroups

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$$

$$AdS_3 \times S^3 : PSU(1,1|2)$$

hybrid

Supergroups

$$\text{AdS}_5 \times S^5 : \frac{SU(2,2|4)}{SO(3,1) \times SO(5)}$$

$$\text{AdS}_4 \times S^3 : PSU(1,1|2)$$

$$\text{AdS}_4 \times CP^3 : \frac{OSP(6|4)}{SO(1,3) \times U(1)}$$

hybrid

Supergroups

$$A \times S_5 \times S_5 : \frac{PSU(2, 2|11)}{SO(1, 1) \times SO(5)}$$

$$A \times S_1 \times S_3 : PSU(1, 1|2)$$

$$A \times S_4 \times (P^2) : \frac{OSp(6|4)}{SO(1, 3) \times U(1)}$$

$$G : PSL(n|n)$$

$$OSp(2m_1 | 2m_2)$$

$SO(2, 2) \times SO(5)$

$AdS_3 \times S^3 : PSU$

$G : PSL(n, n) \quad OSp(2n+2 | 2n)$

zero | Killing form
 | dual Coxeter number

subgroups

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$$

$$AdS_2 \times S_3 : PSU(1,1|2)$$

$$AdS_4 \times CP^3 : \dots$$

hybrid

$$G : PSL(n|n) \quad OSp(2n+2|2n)$$

zero | Killing form
| dual Coxeter number

$$n = \left(\begin{array}{c|c} A & Y \\ \hline \delta & B \end{array} \right)$$

Supergroups

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$$

$$AdS_3 \times S^3 : PSU(1,1|2)$$

$$AdS_4 \times CP^3 : \dots$$

hybrid

$$G : \underline{PSL}(n|n) \quad OSp(2n+2|2n)$$

zero | Killing form
| dual Coxeter number

$$\Omega = \begin{pmatrix} A & | & Y \\ \hline \delta & | & B \end{pmatrix}$$

$$PSL(n|n) : A, B \in GL(n) \\ \det \Omega = 1$$

supergroups

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$$

$$AdS_3 \times S^3 : PSU(1,1|2)$$

$$AdS_4 \times CP^3 : \dots$$

hybrid

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zero | Killing form
| dual Coxeter number

$$\Omega = \begin{pmatrix} A & | & Y \\ \hline \delta & | & B \end{pmatrix}$$

$$PSL(n|n) : A, B \in GL(n) \\ Sp_{det} \Omega = 1 \\ Sp_{det} \lambda \Omega = Sp_{det} \Omega$$

subgroups

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$$

hybrid

$$AdS_2 \times S_2 : PSU(1,1|2)$$

$$AdS_4 \times CP^2 :$$

$$G : \frac{PSL(n, n)}{Osp(2n+2|2n)}$$

zero | Killing form
| dual Coxeter number

$$\Omega = \begin{pmatrix} A & Y \\ \delta & B \end{pmatrix}$$

$$PSL(n, n) : A, B \in GL(n)$$

$$SDet \Omega = 1$$

$$SDet \lambda \Omega = SDet \Omega \Rightarrow \Omega \equiv \lambda \Omega$$

supergroups

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$$

hybrid

$$AdS_2 \times S_3 : PSU(1,1|2)$$

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zero | Killing form
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$$\underline{PSL}(n|n) : A, B \in GL(n)$$

$$SDet \Omega = 1$$

$$SDet \lambda \Omega = SDet \Omega \Rightarrow \Omega \equiv \lambda \Omega$$

$$OSp(2n+2|2n) : A \in$$

Pure symplectic
hybrid ...

Supergroups

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$$

$$AdS_3 \times S^3 : SU(2,2|2)$$

$$AdS_4 \times CP^3 : \frac{OSp(6|4)}{SO(3,1) \times U(3)}$$

$$G : \underline{PSL}(n|n) \quad OSp(2n+2|2n)$$

zero | Killing form
| dual Cartan number

$$\Pi = \begin{pmatrix} A & Y \\ \delta & B \end{pmatrix}$$

$$\underline{PSL}(n|n) : A, B \in GL(n)$$

$$SO_{\det} \Pi = 1$$

$$SO_{\det} \lambda \Pi = SO_{\det} \Pi \Rightarrow \Pi = \lambda \Pi$$

$$OSp(2n+2|2n) : A \in SO(2n+2) \\ B \in Sp(2n)$$

supergroups

hybrid

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$$

$$AdS_3 \times S^3 : PSU(1,1|2)$$

$$AdS_4 \times CP^3 : \dots$$

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zero | killing form
| dual Coxeter number

$$\Omega = \begin{pmatrix} A & Y \\ \delta & B \end{pmatrix}$$

$$\underline{PSL}(n|n) : A, B \in GL(n)$$

$$SDet \Omega = 1$$

$$SDet \lambda \Omega = SDet \Omega \Rightarrow \Omega \equiv \lambda \Omega$$

$$OSp(2n+2|2n) : A \in SO(2n+2) \\ B \in Sp(2n)$$

hybrid

Supergroups

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$$

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$$G : \frac{PSL(n|n)}{OSp(2n+2|2n)}$$

zero | killing form
dual Coxeter number

$$\Omega = \left(\begin{array}{c|c} A & Y \\ \hline \delta & B \end{array} \right)$$

PSL(n|n): $A, B \in GL(n)$
 $SO_{det} \Omega = 1$
 $SO_{det} \lambda \Omega = SO_{det} \Omega \Rightarrow \Omega \equiv \lambda \Omega$

OSp(2n+2|2n): $A \in SO(2n+2)$
 $B \in Sp(2n)$

The models

$$S = S_{\text{kin}} + S_{\text{wt}}$$

$$S_{\text{kin}} = \frac{1}{\kappa \pi \beta^2}$$

The models

$$S = S_{\text{kin}} + S_{\text{wt}}$$

$$S_{\text{kin}} = \frac{1}{4\pi\beta^2} \int d^2x \text{Tr} \left(-\dot{\sigma}^m g \partial_m \dot{\sigma}^n \right)$$

The models

$$S = S_{\text{kin}} + S_{\text{WZ}}$$

$$S_{\text{kin}} = \frac{1}{4\pi\beta^2} \int d^2z \text{Tr} \left(-\dot{g}^m \dot{g}^n \dot{g}^{-1} \right)$$

$$g \in G$$

The models

$$S = S_{\text{kin}} + S_{\text{WZ}}$$

$$S_{\text{kin}} = \frac{1}{4\pi\beta^2} \int_W d^2z \text{Tr}(-\partial^m g \partial_n \bar{g}^{-1}) \quad g \in G$$

$$S_{\text{WZ}} = -\frac{ik}{24\pi} \int_{\mathcal{B}} d^3y \epsilon^{\alpha\beta\gamma} \text{Tr}(\bar{g}^{-1} d_\alpha g \wedge d_\beta g \wedge d_\gamma g)$$

$d\mathcal{B} = W$

The models

$$S = S_{\text{kin}} + S_{\text{WZ}}$$

$$S_{\text{kin}} = \frac{1}{4\pi\beta^2} \int_W d^2z \text{Tr}(-\partial^m g \partial_n \bar{g}^{-1}) \quad g \in G$$

$$S_{\text{WZ}} = -\frac{i k}{24\pi} \int_{\mathcal{B}} d^3y \epsilon^{\alpha\beta\gamma} \text{Tr}(\bar{g}^{-1} d_\alpha g d_\beta g d_\gamma \bar{g}^{-1})$$

\mathcal{B}
 $d\mathcal{B} = W$

The models

$$S = S_{\text{kin}} + S_{\text{WZ}}$$

$$S_{\text{kin}} = \frac{1}{4\pi\beta^2} \int_W d^2z \text{Tr}(-\partial^m g \partial_n \bar{g}^{-1}) \quad g \in G$$

$$S_{\text{WZ}} = -\frac{ik}{24\pi} \int_{\mathcal{B}} d^3y \epsilon^{\alpha\beta\gamma} \text{Tr}(\bar{g}^{-1} \partial_\alpha g \partial_\beta g \partial_\gamma g)$$

$\mathcal{B} = W$

The models

$$S = S_{\text{kin}} + S_{\text{WZ}}$$

$$S_{\text{kin}} = \frac{1}{4\pi\beta^2} \int_W d^2z \text{Tr}(-\partial^m g \partial_n \bar{g}^{-1}) \quad g \in G$$

$$S_{\text{WZ}} = -\frac{i|k|}{24\pi} \int_{\mathcal{B}} d^3y \epsilon^{\alpha\beta\gamma} \text{Tr}(\bar{g}^{-1} \partial_\alpha g \partial_\beta g \partial_\gamma g)$$

\mathcal{B}
 $d\mathcal{B} = W$

The models

$$S = S_{\text{kin}} + S_{\text{WZ}}$$

$$S_{\text{kin}} = \frac{1}{4\pi\beta^2} \int d^2x \text{Tr}(-\dot{g} \dot{g}^{-1}) \quad g \in G$$

$$S_{\text{WZ}} = -\frac{i|k|}{24\pi} \int d^3y \epsilon^{\alpha\beta\gamma} \text{Tr}(\dot{g}^{-1} \dot{g} \dot{g}^{-1} \dot{g} \dot{g})$$

Two parameters: β^2 , k

$k = \pm \frac{1}{\beta^2}$: WZW model

$$G : \boxed{\text{PSL}(n|\mathbb{H}) \quad \text{Osp}(2n+2|2n)}$$

zero | Killing form
 | dual Coxeter number

$$\text{PSL}(n|\mathbb{H}) : A, B \in \mathfrak{gl}(n)$$

$$\text{SDet } \Omega = 1$$

$$\text{SDet } \lambda \Omega = \text{SDet } \Omega \Rightarrow \Omega \equiv \lambda \Omega$$

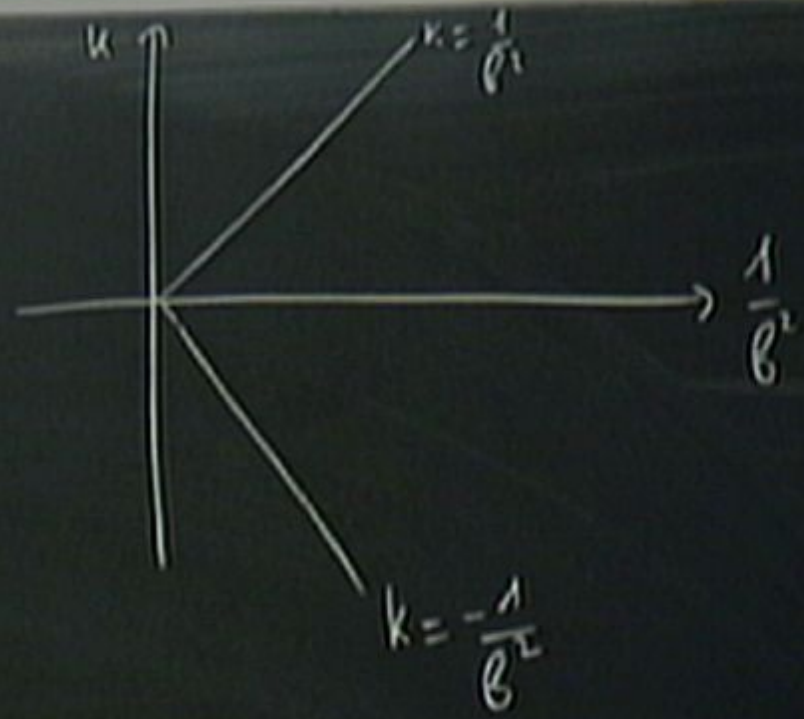
$$\text{Osp}(2n+2|2n) : A \in \text{SO}(2n+2)$$

two parameters: β & k

$$k = \frac{1}{\beta^2}$$

$|k\beta^2| \leq 1$

Admissible space:

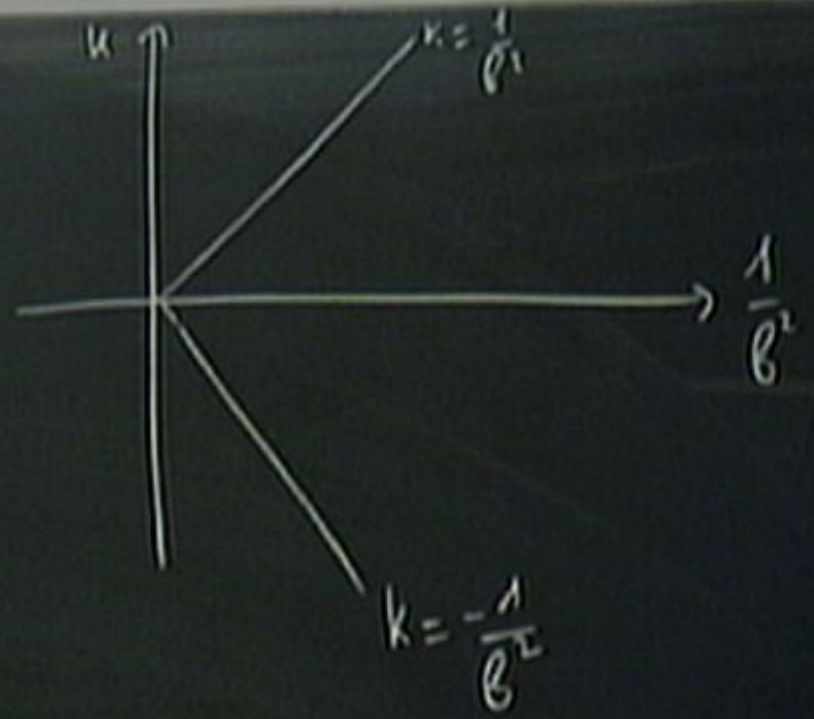


two parameters: β & k

$k = \frac{1}{\beta^2}$

$|k\beta^2| \leq 1$

Admissible space:

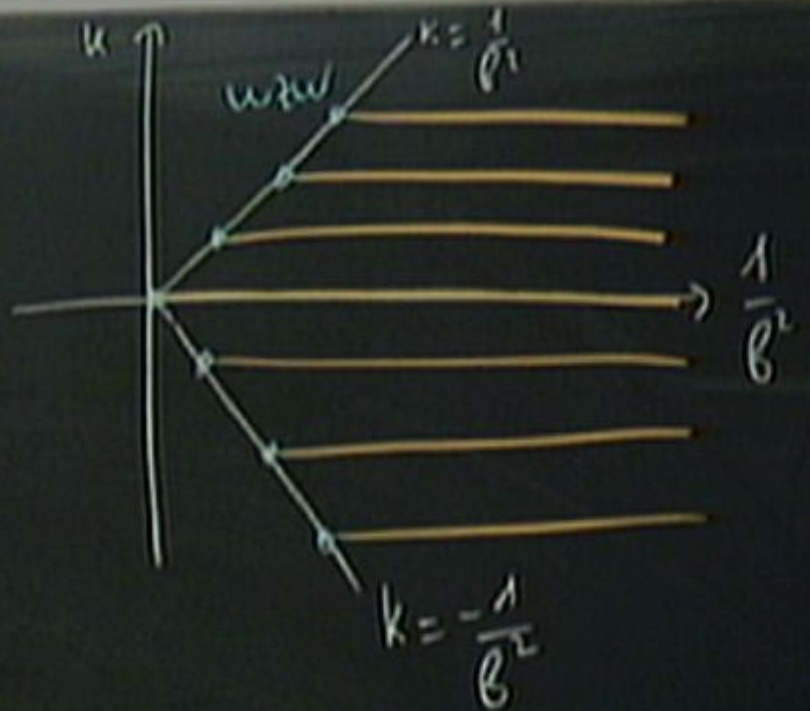


two parameters: β, κ

$$\kappa = \frac{1}{\beta^2}$$

$$\underline{|\kappa\beta^2| \leq 1}$$

Moduli space:



two parameters: β, κ

$$\kappa = \frac{1}{\beta^2}$$

$|\kappa\beta^2| \leq 1$ moduli space:

An example: $AdS_3 \times S^3$ \leftrightarrow $PSU(1,1|2)$



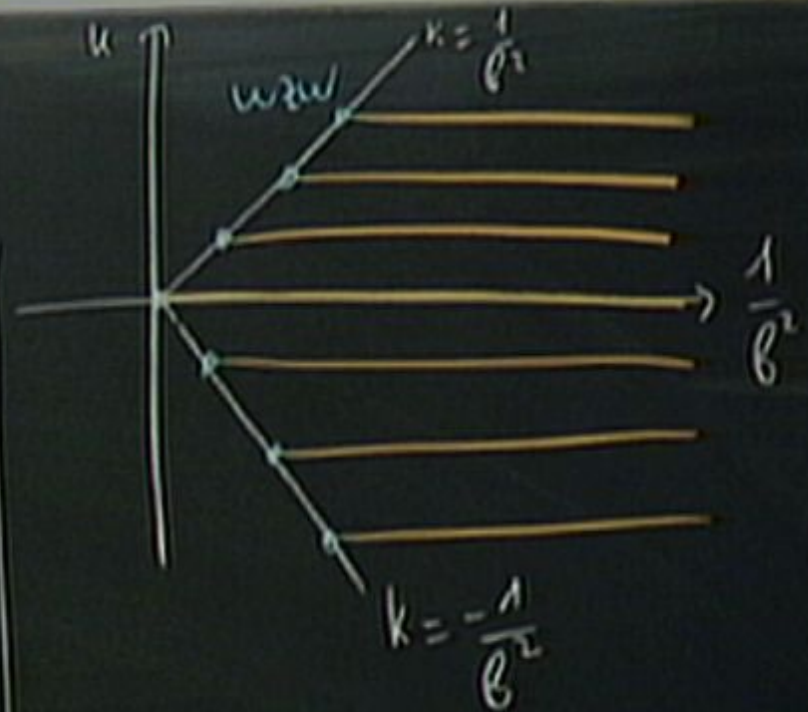
two parameters: β, μ

$k = \frac{1}{\beta^2}$

$|k\beta^2| \leq 1$ Moduli space:

An example: $AdS_3 \times S^3 \leftrightarrow PSU(1,1|2)$

↓
Quiver
Q.R.



two parameters: β, μ

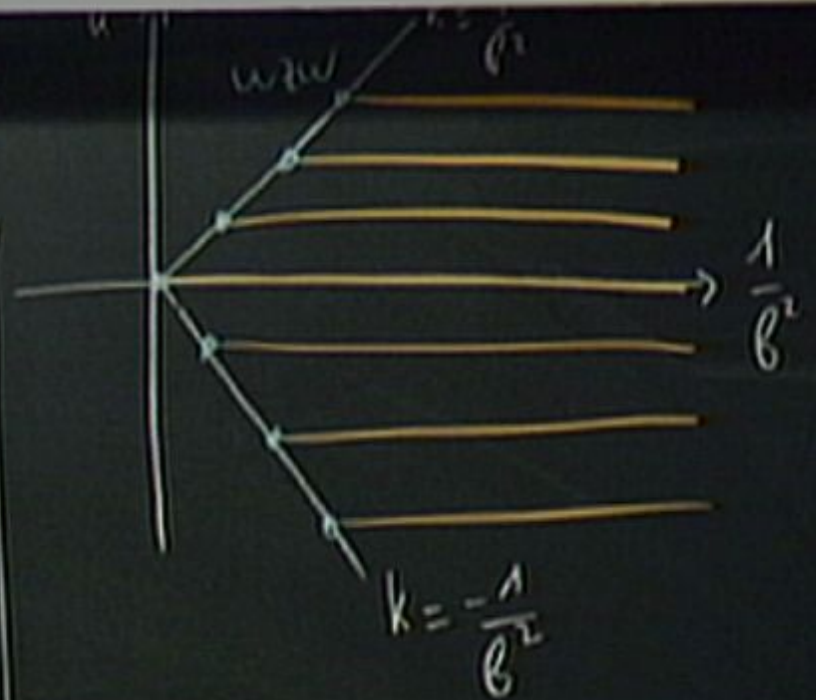
$k = -\frac{1}{\beta^2}$

$|k\beta^2| \leq 1$ Moduli space:

An example: $AdS_3 \times S^3 \leftrightarrow PSU(1,1|2)$

↓
Quasiflows
QRR

$$k = QNSNS$$



two parameters: β, k

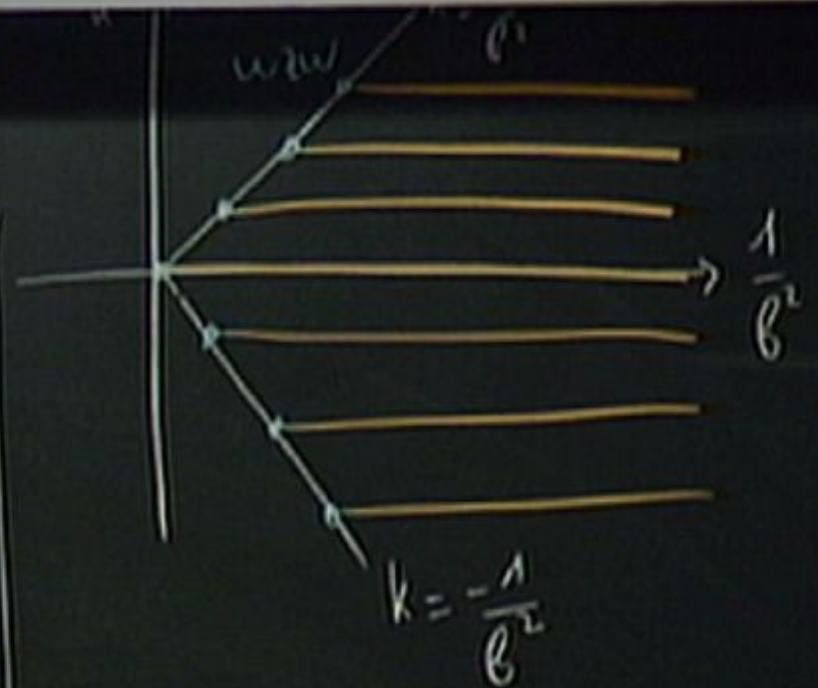
$$k = -\frac{1}{\beta^2}$$

$|k\beta^2| \leq 1$ moduli space.

An example: $AdS_3 \times S^3 \leftrightarrow PSU(1,1|2)$

↓
QNSM (lines)
QPR

$$K = QNSNS$$
$$\frac{1}{\beta^2} = R^2 = \sqrt{Q_{NS}^2 + g_s^2 Q_{RR}^2}$$



- ② Current algebra
- ③ Integrability

② The current algebra

- ① Introduction
- ② Current algebra
- ③ Integrability

Friday: towards the spectrum

② The current algebra

Global symmetry: $g \rightarrow h, g, h$ $G_L \times G_R$

- ① Current algebra
- ② Integrability

② The current algebra

Global symmetry: $g \rightarrow h, g h^{-1}$

$$\boxed{G_L} \times G$$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$$j_z = j = c \partial g g^{-1}$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g g^{-1}$$

- ① Introduction
- ② Current algebra
- ③ Integrability

Friday: towards the spectrum

work with:
Surya Ashok
Jon Troost

② The current algebra

Global symmetry: $g \rightarrow h, g, h$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$$j_z = j = c \partial g$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g$$

$$\frac{1+k\beta^7}{2\beta^7}$$

$$\frac{k\beta^2}{\beta^2}$$

- ① Introduction
- ② Current algebra
- ③ Integrability

Friday: towards the spectrum

Work with:
Surya Akshay
Jon Troost

② The current algebra

Global symmetry: $g \rightarrow h, g, h \in \mathbb{R}$ G_L \times G_R

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$$j_z = j = c \partial g g^{-1}$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g g^{-1}$$

$$c = -\frac{1+k\beta^2}{2\beta^2}$$

$$\bar{c} = -\frac{1-k\beta^2}{2\beta^2}$$

- ① Introduction
- ② Current algebra
- ③ Integrability

Friday: towards the spectrum

work with:
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② The current algebra

Global symmetry: $g \rightarrow h, g, h \in \boxed{G} \times G_{\mathbb{R}}$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$$j_z = j = c \partial g g^{-1}$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g g^{-1}$$

$$c = -\frac{1+k\beta^2}{2\beta^2}$$

$$\bar{c} = -\frac{1-k\beta^2}{2\beta^2}$$

Supergroups

$$\text{AdS}_5 \times S^5 : \frac{\text{PSU}(2,2|4)}{\text{SO}(4,1) \times \text{SO}(5)}$$

$$\text{AdS}_3 \times S^3 : \text{PSU}(2,2|4)$$

$$d = d^a \frac{t^b}{\text{I}} \cdot \frac{k_{ab}}{\text{L} \rightarrow \text{metric}}$$

$$\bar{j} = j^a \underbrace{t^b}_{\downarrow} \underbrace{k_{ab}}_{\hookrightarrow \text{metric}}$$

$$\underbrace{[t^a, t^b]}_{\hookrightarrow \text{Structure constants}} = i \beta^{ab} c t^c$$

$$SO(1,1) \times SO(5)$$

$$SU(2) \times SU(2) : PSU(1,1)$$

t^a
↓
 k_{ab}
↳ metric

$$[t^a, t^b] = i f^{ab}{}_c t^c$$

↳ structure constants

$$\tilde{d} = d^a \underbrace{t^b}_{\downarrow} k_{ab} \quad \underbrace{k_{ab}}_{\rightarrow \text{metric}}$$

$$[t^a, t^b] = i \underbrace{\beta^{ab}}_{\rightarrow \text{Structure constants}} c^c t^c$$

$\underbrace{\quad}_{a}$ $\underbrace{\quad}_{d bc}$

$$\bar{\eta} = \eta^a \underbrace{t^b}_{\downarrow} \underbrace{k_{ab}}_{\rightarrow \text{metric}}$$

$$[t^a, t^b] = i \underbrace{\beta^{ab}}_{\rightarrow \text{Structure constants}} c^c t^c$$

$$\beta^{ab} \beta^{cd} \beta^{bc} = 0$$

zero Killing form

$$\bar{\eta} = \eta^a \underbrace{t^b}_{\downarrow} \underbrace{k_{ab}}_{\rightarrow \text{metric}}$$

$$[t^a, t^b] = i \underbrace{\beta^{abc}}_{\rightarrow \text{Structure constants}} t^c$$

$$\boxed{\eta^a \eta^b \eta^c = 0}$$

zero Killing Form

$$\bar{d} = d^a \underbrace{t_c}_{\downarrow} \underbrace{t^b}_{\rightarrow \text{metric}}$$

$$[t^a, t^b] = i f^{ab}{}_c t^c$$

\downarrow Structure constants

$$\boxed{f^a{}_{bc} f^{d bc} = 0}$$

zero Killing form

$$\bar{d} = d^a t^b \underbrace{K_{ab}}_{\text{metric}}$$

$$[t^a, t^b] = i f^{ab}{}_c t^c$$

\hookrightarrow Structure constants

$$\boxed{f^a{}_{bc} f^{d bc} = 0}$$

zero Killing form

$$K^{ab} = \text{Str} (t^a t^b)$$

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$$

$$AdS_4 \times S^3 : PSU(1,1|2)$$

AdS

$$j = d^a t^b \underbrace{K_{ab}}_{\text{metric}}$$

$$[t^a, t^b] = i \underbrace{\beta^c}_{\text{Structure constants}} t^c$$

$$\beta^a{}_{bc} \beta^{dbc} = 0$$

zero Killing form

$$K^{ab} = \text{Str}(t^a t^b)$$

$$AdS_5 \times S^5 : \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$$

$$AdS_4 \times S^7 : PSU(1,1|2)$$

AdS

$$j = j^a \frac{t^b}{t^c} \frac{k_{ab}}{t^c} \rightarrow \text{metric}$$

$$[t^a, t^b] = i f^{abc} t^c$$

\rightarrow Structure constants

$$f^{abc} f^{dbc} = 0$$

zero Killing form

$$k^{ab} = \text{Str}(t^a t^b)$$

$so(4,4) \times so(5)$

11/17/11

$$j = j^a t^b \underbrace{K_{ab}}_{\substack{\text{metric}}} \rightarrow \text{metric}$$

$$[t^a, t^b] = i \underbrace{\beta^{ab}}_{\substack{\text{Structure} \\ \text{constants}}} c^c t^c$$

$$\beta^{ab} \beta^{cd} = 0$$

zero Killing form

$$K^{ab} = \text{Str}(t^a t^b)$$

$SO(4,1) \times SO(5)$

$SO(4,1) \times SO(5)$

$$j = i \overset{a}{t} \overset{b}{t} \underbrace{K_{ab}}_{\substack{\text{metric}}} \quad \downarrow$$

$$[t^a, t^b] = i \underbrace{\beta^{abc}}_{\substack{\text{Structure} \\ \text{constants}}} t^c$$

$$\underbrace{\beta^{abc}}_{\text{Killing Form}} = 0$$

$$K^{ab} = \text{Str}(t^a t^b)$$

Current-Current OPEs

$so(2,1) \times so(5)$

$su(2) \times u(1)$

$$j = j^a t^a + \frac{1}{2} K_{ab} t^a t^b$$

\downarrow
 \hookrightarrow metric

$$[t^a, t^b] = i f^{abc} t^c$$

\hookrightarrow Structure constants

$$f^{abc} f^{dbc} = 0$$

zero Killing form

$$K^{ab} = \text{Str}(t^a t^b)$$

Current-Current OPEs

$$j^a(z) j^b(w) =$$

$$\bar{j}^a(z) \bar{j}^b(w) =$$

$$\bar{j}^a(z) j^b(w) =$$

$$K^{ab} = \text{Str} (t^a t^b)$$

Current-Current OPEs

$$j^a(z) j^b(w) = \frac{1}{(z-w)^2} + \frac{1}{z-w}$$

$$j^a(z) \bar{j}^b(w) =$$

$$\bar{j}^a(z) \bar{j}^b(w) =$$

Structure constants

AdS₂ × S₂ : PSU(1,1|2)

AdS₄ × CP² : $\frac{OSp(6|4)}{SO(1,3) \times U(1)}$

$$K^{ab} = \text{Str}(t^a t^b)$$

Current-Current OPEs

$$j^a(z) j^b(w) = \frac{1}{(z-w)^2} + \frac{1}{z-w} \frac{\bar{z}-\bar{w}}{(z-w)^2} + \dots$$

$$j^a(z) \bar{j}^b(w) =$$

$$\bar{j}^a(z) \bar{j}^b(w) =$$

etric
ab
ct^c
Structure constants

Supergroups

AdS₅ × S⁵ : $\frac{PSU(2,2|4)}{SO(3,1) \times SU(2)}$

AdS₄ × S⁷ : $PSU(1,1|2)$

AdS₄ × CP³ : $\frac{OSP(6|4)}{SO(1,3) \times U(1)}$

hybrid

$j = j^a \frac{t^a}{I} \frac{K_{ab}}{L}$ metric

$[t^a, t^b] = i \beta^{ab}_c t^c$
↳ structure constants

$\beta^{ab}_c \beta^{d bc} = 0$

zero Killing form

$K^{ab} = \text{Str}(t^a t^b)$

Current-Current OPEs

$j^a(z) j^b(w) =$

$\bar{j}^a(z) \bar{j}^b(w) =$

$\bar{j}^a(z) j^b(w) =$

Supergroups

$$\text{AdS}_5 \times S^5 : \frac{\text{PSU}(2,2|4)}{\text{SO}(4,1) \times \text{SO}(5)}$$

hybrid

$$\text{AdS}_3 \times S^3 : \text{PSU}(1,1|2)$$

$$\text{AdS}_4 \times (\mathbb{P}^2 \times \mathbb{S}^1) : \frac{\text{OSP}(6|4)}{\text{SO}(3,1) \times \text{U}(1)}$$

$$j = j^a t_a + \frac{1}{L} K_{ab} t^a t^b$$

\downarrow
metric

$$[t^a, t^b] = i \beta^{abc} t^c$$

\downarrow
Structure constants

$$\beta^{ab} \beta^{d bc} = 0$$

zero Killing form

$$K^{ab} = \text{Str}(t^a t^b)$$

Current-Current OPEs

$$j^a(z) j^b(w) = \text{Id} + \dots$$

$$j^a(z) \bar{j}^b(w) = \dots$$

$$\bar{j}^a(z) \bar{j}^b(w) = \dots$$

Supergroups

AdS₅ × S⁵ : $\frac{PSU(2,2|4)}{SO(3,1) \times SO(5)}$

hybrid

AdS₄ × S⁷ : PSU(1,1|2)

AdS₄ × CP³ : $\frac{OSP(6|4)}{SO(3,1) \times U(1)}$

$j = j^a t^a + \frac{1}{L} \frac{K_{ab}}{L} t^a t^b$
 ↳ metric

$[t^a, t^b] = i \beta^ab_c t^c$
 ↳ Structure constants

$\beta^a_{bc} \beta^{d bc} = 0$

zero Killing form

" $K^{ab} = \text{Str}(t^a t^b)$ "

Current-Current OPEs

$j^a(z) j^b(w) = Id + \dots$
 $\bar{j}^a(z) \bar{j}^b(w) = \dots$
 $\bar{j}^a(z) j^b(w) = \dots$

ent algebra

$$\bar{\partial} \partial j = -\bar{\partial} \bar{\partial} \bar{j}$$

symmetry: $g \rightarrow h, g h^R$

$$\boxed{G_L} \times G_R$$

currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$$j_z = j = c \partial g g^{-1}$$

$$c = -\frac{1+k\beta}{2\beta^2}$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g g^{-1}$$

$$\bar{c} = -\frac{1-k\beta}{2\beta^2}$$

③ Integrability

Jan Troost

② The current algebra

Global symmetry: $g \rightarrow h, g h e$ $[G_U] \times G_R$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$= K_{ab} j^a j^b$

$$j_z = j = c \partial g g^{-1}$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g g^{-1}$$

$$c = -\frac{1+k\beta^2}{2\beta^2}$$

$$\bar{c} = -\frac{1-k\beta^2}{2\beta^2}$$

③ Integrability

Jan Troost

② The current algebra

Global symmetry: $g \rightarrow h, g h e$ $[G_U] \times G_R$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$T_{ab} = K_{ab} + j_a j_b$

$$j_z = j = c \partial g g^{-1}$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g g^{-1}$$

$$c = -\frac{1+k\beta^2}{2\beta^2}$$

$$\bar{c} = -\frac{1-k\beta^2}{2\beta^2}$$

③ Integrability

Jan Troost

② The current algebra

Global symmetry: $g \rightarrow h, g h^{-1}$ $[G_0] \times G_R$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$T_{ab} \propto K_{ab} j^a j^b$

$$j_z = j = c \partial g g^{-1}$$

$$c = -\frac{1+k\beta^2}{2\beta^2}$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g g^{-1}$$

$$\bar{c} = -\frac{1-k\beta^2}{2\beta^2}$$

③ Integrability

Jan Troost

② The current algebra

Global symmetry: $g \rightarrow h, g h e$ $[G_0] \times G_R$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$T_{\alpha\beta} \propto K_{ab} j^a j^b$

$$j_z = j = c \partial g g^{-1}$$

$$j_{\bar{z}} = \bar{j} = \bar{c} \bar{\partial} g g^{-1}$$

$$c = -\frac{1+k\beta^2}{2\beta^2}$$

$$\bar{c} = -\frac{1-k\beta^2}{2\beta^2}$$

$$k^{ab} = \text{Str} (t^a t^b)$$

Current-Current OPEs

$$j^a(z) j^b(w) =$$

Id

$$j^a(z) \bar{j}^b(w) =$$

$$\bar{j}^a(z) \bar{j}^b(w) =$$

(This area contains a large, light-colored scribble that has been partially obscured by a white highlighter. Some faint text is visible through the highlighter, including the word "Id" and some mathematical symbols like j^c , j^d , and j^e .)

$O(\mathcal{B}^{2n-2})$

$:\bar{j}^n:$

$$K^{ab} = \text{Str}(t^a t^b)$$

Current-Current OPEs

$$j^a(z) j^b(w) =$$

$$I_d K^{ab}$$

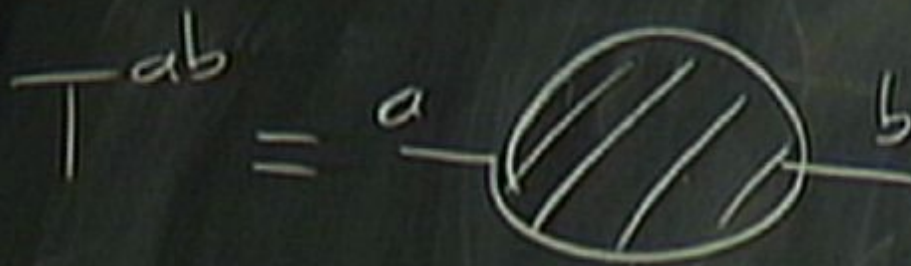
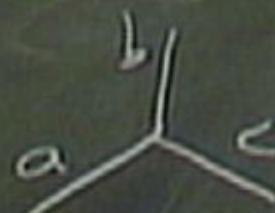
$$j^a(z) \bar{j}^b(w) =$$

$$\bar{j}^a(z) \bar{j}^b(w) =$$

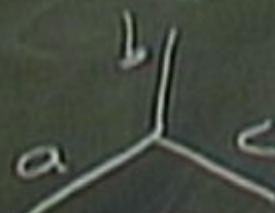
j^c
 j^d
 j^e
 j^f
 j^g
 j^h
 j^i
 j^j
 j^k
 j^l
 j^m
 j^n
 j^o
 j^p
 j^q
 j^r
 j^s
 j^t
 j^u
 j^v
 j^w
 j^x
 j^y
 j^z

$O(\beta^{2n-2})$

Tensors: K^{ab} , β^{abc}

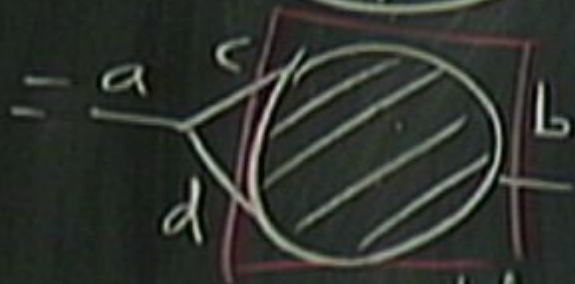
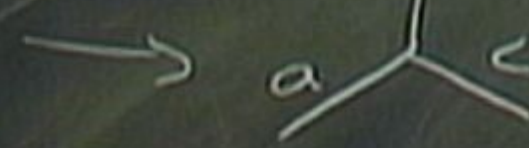


Tensors: K^{ab} , β^{abcd}



$\alpha \beta^{abcd}$

Tensors: k^{ab} , β^{abc}



$$\alpha \beta^{cbd} \beta^a{}_{cd} = 0$$

$$AdS_1 \times S_3 : \text{SU}(1,1|2) \quad AdS_4 \times CP^3 : \frac{Osp(6|4)}{SO(4,3) \times U(1)}$$

$$K^{ab} = \text{Str}(t^a t^b)$$

Current-Current OPEs

$$\begin{aligned}
 \vec{j}^a(z) \vec{j}^b(w) &= \text{Id} \delta^{ab} \\
 \vec{j}^a(z) \vec{j}^b(w) &= \dots \\
 \vec{j}^a(z) \vec{j}^b(w) &= \dots
 \end{aligned}$$

$O(\beta^{2n+2})$

Tensor: K^{ab}, β^{abcd}

$T^{ab} =$ 

$\propto \beta^{abcd} \beta^a d=0$

" $K^{ab} = \text{Str}(t^a t^b)$ "

Current-Current OPEs

$$j^a(z) j^b(w) = \text{Id} K^{ab} A_0 + j^c B^c A_1 + j^c B^c A_h$$

$$j^a(z) \bar{j}^b(w) =$$

$$\bar{j}^a(z) \bar{j}^b(w) =$$

Structure constants

$$\text{AdS}_5 \times S^5 : \frac{SU(2,2|1)}{SO(3,1) \times SO(5)}$$

$$\text{AdS}_4 \times S^7 : PSU(1,1|2)$$

$$\text{AdS}_4 \times CP^3 : \frac{OSP(6|4)}{SO(1,3) \times U(1)}$$

$$j = j^a \frac{t^b}{L} K_{ab}$$

\downarrow
 \hookrightarrow metric

$$[t^a, t^b] = i f^{abc} t^c$$

\hookrightarrow Structure constants

$$f^{ab} f^{bc} = 0$$

zero Killing form

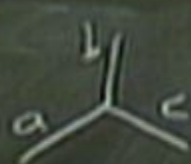
$$K^{ab} = \text{Str}(t^a t^b)$$

Current-Current OPEs

$$j^a(z) j^b(w) = \text{Id} \delta^{ab} A_0 + j^c B^c A_1 + j^c B^c A_2 + \dots$$

$$\bar{j}^a(z) \bar{j}^b(w) = \text{Id} K^{ab} B_0 + \dots$$

$$\bar{j}^a(z) j^b(w) = \dots$$

Tensors: k^{ab} , β^{abc} → 

Computation

④ $\langle \delta\delta \rangle, \langle \delta\delta\delta \rangle, \dots$

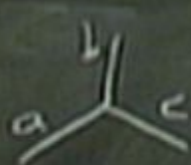
$$T^{ab} = \text{[Diagram: a circle with diagonal hatching, two lines extending from the left and right sides labeled 'a' and 'b']}$$

$$= \text{[Diagram: a circle with diagonal hatching, four lines extending from the top-left, top-right, bottom-left, and bottom-right sides labeled 'a', 'c', 'b', and 'd' respectively, with a red square box around the circle and the bottom two lines.]}$$

$$\propto \beta^{cbd} \beta^a \quad \beta^a d = 0$$

$$\beta^a = R = \sqrt{Q_{\text{min}}} + g_1 \text{ etc}$$



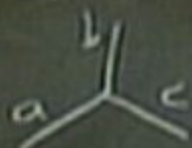
Tensors: k^{ab} , β^{ahc} \rightarrow 

$$T^{ab} = \text{[Diagram: a circle with diagonal lines, input a on the left, output b on the right]} \\ = \text{[Diagram: a circle with diagonal lines, input a on the left, output b on the right, and a red square box around it with inputs c and d on the top and bottom]} \\ \propto \beta^{cbd} f^a \quad f^a d = 0$$

Computation

- ① $\langle jj \rangle, \langle jjjj \rangle, \dots$
- ② $S = S_{\text{new}} + \epsilon S_{\text{old}}$

$\beta^{\mu\nu} = \eta^{\mu\nu} = \eta^{\mu\nu} + g_{\mu\nu} \text{[unclear]}$

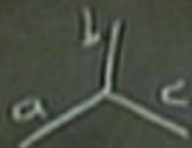
Tensors: K^{ab} , β^{abcd} → 

$$T^{ab} = \text{[Diagram: a circle with diagonal lines, labeled 'a' on the left and 'b' on the right]} \\ = \text{[Diagram: a circle with diagonal lines, labeled 'a' on the left, 'b' on the right, 'c' on top, and 'd' on bottom, with a red box around the bottom half]} \\ \propto \beta^{cbd} \beta^a_{cd} = 0$$

Computation

- ① $\langle jj \rangle, \langle jjjj \rangle, \dots$
- ② $S = S_{\text{grav}} + \epsilon S_{\text{mat}}$
- ③ Ask for

$$K = Q_{NS} NS \\ \frac{1}{\beta} = R^2 = \sqrt{Q_{NS}^2 + g_s^2 Q_{\text{int}}^2}$$

Tensors: $k^{ab}, \beta^{abcd} \rightarrow$ 

$$T^{ab} = \text{[Diagram: a circle with diagonal lines, labeled 'a' on the left and 'b' on the right]} \\ = \text{[Diagram: a circle with diagonal lines, labeled 'a' on the left, 'b' on the right, 'c' on top, and 'd' on bottom, with a red square around it]} \\ \propto \beta^{abcd} f^a c d = 0$$

Computation

- ① $\langle jj \rangle, \langle jjjj \rangle, \dots$
- ② $S = S_{\text{new}} + \epsilon S_{\text{old}}$
- ③ Ask for compatibility with:
 - + Current conservation
 - + Navier-Stokes equation

$$K = Q_{NS} NS \\ \frac{1}{\beta^T} = R^2 = \sqrt{Q_{NS}^2 + g_s^2 Q_{IE}^2}$$

* Current conservation $\partial_\mu j^{\mu a}(\tau) + \partial_\tau j^{\mu a}(\tau) = 0$

* Current conservation $j^b(\omega) (\vec{\partial} j^a(\tau) + \partial \bar{j}^a(\tau)) = 0$

Current conservation $j^b(\omega) (\vec{\partial} j^a(\tau) + \partial_{\vec{y}} j^a(\tau)) = 0 + \text{contact terms}$

Current conservation $j^b(\omega) (\partial_j j^a(z) + \partial_j \bar{j}^a(z)) = 0$ + contact terms from Ward

$$\delta g(z) = \varepsilon(z) g(z)$$

$$\delta j^b(\omega) = j^b(\omega) \delta \kappa$$

Current conservation $j^b(\omega) (\partial_j j^a(z) + \partial_j \bar{j}^a(z)) = 0$ + contact terms from Ward

$$\delta g(z) = \varepsilon(z) g(z)$$

$$\delta j^b(\omega) = j^b(\omega) \delta \zeta$$

$$j^b(\omega) \left(\frac{-1}{2\pi} \int d^2 z \varepsilon_a(z) (\partial_j j^a(z) + \partial_j \bar{j}^a(z)) \right)$$

$\alpha \beta \quad \epsilon \quad \eta = 0$

• Navier-Castan equation

* Current conservation $j^b(\omega) (\partial_j j^a(z) + \partial_j \bar{j}^a(z)) = 0$ + contact terms from Ward

$$\delta g(z) = \epsilon(z) g(z)$$

$$\delta j^b(\omega) = j^b(\omega) \delta \epsilon$$

$$\int d^2 x \epsilon_1(x) \left(-i k^{ab} \partial \delta(x) + i \delta^{ab} \delta(x) \right) = j^b(\omega) \left(\frac{-1}{2\pi} \int d^2 x \epsilon_2(x) (\partial_j j^a(z) + \partial_j \bar{j}^a(z)) \right)$$

* Current conservation $j^b(\omega) (\bar{j}^a(z) + \delta \bar{j}^a(z))$

$$\delta g(z) = \varepsilon(z) g(z)$$

$$\delta j^b(\omega) = j^b(\omega) \delta \zeta$$

$$\int d^2 z \varepsilon(z) \left(-c k^{ab} \partial \delta(\omega+z) + i \beta^c j^c(z) \delta(\omega+z) \right) = j^b(\omega) \left(\frac{-1}{2\pi} \int d^2 z \varepsilon(z) (\bar{j}^a(z) + \delta \bar{j}^a(z)) \right)$$

$$c k^{ab} \partial \delta(\omega+z) + i \beta^{ac}$$

* Current conservation

$$j^b(\omega) \left(\bar{\partial} j^a(z) + \partial \bar{j}^a(z) \right)$$

$$\delta g(z) = \varepsilon(z) g(z)$$

$$\delta j^b(\omega) = j^b(\omega) \varepsilon$$

$$\int d^2x \varepsilon(x) \left(-\varepsilon k^{ab} \partial \delta(\omega+x) + i\beta^{ba} j^c(x) \delta(\omega+x) \right) = j^b(\omega) \left(\frac{-\varepsilon}{2\pi} \left(\bar{\partial} j^a(z) + \partial \bar{j}^a(z) \right) + \varepsilon_a(z) \left(\bar{\partial} j^a(z) + \partial \bar{j}^a(z) \right) \right)$$

$$k^{ab} \partial \delta(\omega+x) + i\beta^{ab} j^c(x) \delta(\omega+x)$$

* Current conservation $j^b(\omega) (\bar{\partial} j^a(\tau) + \partial \bar{j}^a(\tau))$

$$\delta g(z) = \varepsilon(z) g(z)$$

$$\delta j^b(\omega) = j^b(\omega) \delta \delta$$

$$\int d^2 x \varepsilon(x) \left(-\varepsilon^{ab} \partial \delta(\omega+x) + i \beta^a c j^c(\tau) \delta(\omega+x) \right) = j^b(\omega) \left(\frac{-1}{2\pi} \int d^2 x \varepsilon(x) (\bar{\partial} j^a(\tau) + \partial \bar{j}^a(\tau)) \right)$$

$$c \varepsilon^{ab} \partial \delta(\omega+x) + i \beta^a c j^c(\tau) \delta(\omega+x) = (\bar{\partial} j^a(\tau) + \partial \bar{j}^a(\tau))$$

* Current conservation $j^b(\omega) (\bar{\partial} j^a(\tau) + \partial \bar{j}^a(\tau)) = 0$ + ca
 + t
 + 10

$$\delta g(\tau) = \varepsilon(\tau) g(\tau)$$

$$\delta j^b(\omega) = j^b(\omega) \delta S$$

$$\int d^2x \varepsilon(x) \left(-ck^{ab} \partial \delta(x) + i\beta^{ab} j^c(x) \delta(x) \right) = j^b(\omega) \left(\frac{-1}{2\pi} \int d^2x \varepsilon(x) (\bar{\partial} j^a(\tau) + \partial \bar{j}^a(\tau)) \right)$$

$$ck^{ab} \partial \delta(\omega - \tau) + i\beta^{ab} j^c(\tau) \delta(\omega - \tau) = (\bar{\partial} j^a(\tau) + \partial \bar{j}^a(\tau)) j^b(\omega)$$

Naurer-Cartan

$$d(dg g^{-1}) = dg g^{-1} \wedge dg g^{-1}$$

Mostly

$$g \rightarrow h, g, h$$

$$|G| \times G_{\mathbb{R}}$$

$$+ d_j \mathbb{R} = 0$$

$$d = \sum \frac{\partial}{\partial x^i} g_i^{-1}$$

$$G_{\mathbb{R}} = \frac{1 + \sqrt{1 - 4x^2}}{2x}$$

Naurer-Cartan

$$d(dg g^{-1}) = dg g^{-1} \wedge dg g^{-1}$$

$$= \mathcal{F}_j^a(x) - c \mathcal{D}_j^{-a}(x) - i \mathcal{B}^a d c_j^c \mathcal{D}_j^d(x) =$$

Naurer-Cartan

$$d(dg g^{-1}) = dg g^{-1} \wedge dg g^{-1}$$

$$d^b(x) \left(\varepsilon \delta_j^a(x) - c \delta_j^{-a}(x) - i \left(\delta_{ij}^a c_{-d}^d(x) \right) \right) = 0$$

Naiver-Cartan

$$d(dg g^{-1}) = dg g^{-1} dg g^{-1}$$

$$d^b(+)(c \delta_j^a(+)) - c \partial_j^{-a}(+) - i f^{abc} d_c j^b d^d(+)) = 0$$

Computation

$$c \frac{\delta^{ab}}{\delta x^a(+)} =$$

② The current algebra

Global symmetry. $g \rightarrow h, g h^{-1}$

$[G]$

Noether currents. $\bar{\partial} j_x + \partial j_t = 0$

$$j_x = j = c \partial_y g^{-1}$$

Naiver-Cartan

$$d(dg g^{-1}) = dg g^{-1} dg g^{-1}$$

$$\partial^b(t) (\bar{c} \partial_j^a(t) - c \partial_j^{-a}(t) - i \beta^a d_{cb} c_j^d(t)) = 0$$

Computation

$$c \kappa^{ab} \partial(t, \dots) = \kappa^{ab} (\partial A_0 + \partial B_0)$$

② The current algebra

Global symmetry: $g \rightarrow h, g h^{-1} \quad \boxed{G} \times \mathbb{R}$

Noether currents: $\bar{\partial} j_x + \partial j_t = 0$

$$j_x = j = c \partial g g^{-1}$$

$$c = -\frac{1 + \kappa \beta^2}{2\beta^2}$$

Maurel-Cartan

$$d(dg g^{-1}) = dg g^{-1} \wedge dg g^{-1}$$

$$\boxed{d^{ab}(+) (\bar{c} \delta_j^a(+) - c \delta_j^{-a}(+) - i \beta^a d_{ij}^c(+)) = 0}$$

Computation

$$\left\{ \begin{aligned} c k^{ab} \delta(+-) &= k^{ab} (\delta A_0 + \delta B_0) \\ 0 &= k^{ab} (\bar{c} \delta A_0 - c \delta B_0) \end{aligned} \right.$$

current algebra

algebra symmetry: $g \rightarrow h, g h^{-1}$ $\boxed{G_{\mathbb{C}}} \times G_{\mathbb{R}}$

currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$$j_z = j_{\bar{z}} = c \partial_g g^{-1} \quad c = -\frac{1+k}{2\pi}$$

$$d^2(+)(c \delta_j^a(+)) - c d_j^{-a}(+) - i \beta^a d_c j^c_j(+)) = 0$$

Computation

$$\left\{ \begin{aligned} c k^{ab} \delta_j^a(+)(+) &= k^{ab} (\delta_j^a A_0 + \delta_j^b B_0) \\ 0 &= k^{ab} (c \delta_j^a A_0 - c \delta_j^b B_0) \end{aligned} \right.$$



② The

algebra

G

try: $g \rightarrow h, g h r$

$$\boxed{G_0} \times G_R$$

Noe

$$\bar{\partial} j_2 + \partial j_1 = 0$$

$$d\tau = f = c \partial_g g^{-1}$$

$$c = - \frac{1+k\beta^2}{\beta^2}$$

Computation

$$\begin{cases} c k^{ab} \delta(\tau - \omega) = k^{ab} (\bar{\partial} A_0 + \partial B_0) \\ 0 = k^{ab} (\bar{c} \bar{\partial} A_0 - c \partial B_0) \end{cases}$$

$$c^2 k^{ab} \delta(\tau - \omega) = k^{ab} (c + \bar{c}) \bar{\partial} A_0$$

The current algebra

Global symmetry: $g \rightarrow h, g \in \mathfrak{h}$ $[G_L] \times G_R$

Noether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$$j_z = j_{\bar{z}} = c \partial g g^{-1}$$

$$c = -\frac{1+k}{2g}$$

Computation

$$\left. \begin{aligned} 2\pi c k^{ab} \delta(\tau - u) &= k^{ab} (\bar{\partial} A_0 + \partial B_0) \\ 0 &= k^{ab} (\bar{c} \bar{\partial} A_0 - c \partial B_0) \end{aligned} \right\}$$

$$2\pi c^2 k^{ab} \delta(\tau - u) = k^{ab} (c + \bar{c}) \bar{\partial} A_0$$

The current algebra

Global symmetry: $g \rightarrow h, g \in \mathfrak{g}$ $\boxed{G} \times G_R$

Loether currents: $\bar{\partial} j_z + \partial j_{\bar{z}} = 0$

$$j_z = j = c \partial g g^{-1}$$

$$c = -\frac{1+k}{2g}$$

Computation

$$\left. \begin{aligned} 2\pi c k^{ab} \delta(\tau - u) &= k^{ab} (\delta A_0 + \delta B_0) \\ 0 &= k^{ab} (c \delta A_0 - c \delta B_0) \end{aligned} \right\}$$

$$2\pi c^2 k^{ab} \delta(\tau - u) = k^{ab} (c + c) \delta A_0$$

$$2\pi \delta(\tau - u) = \delta \frac{1}{\tau - u}$$



② The current algebra

Global symmetry

Noether currents, δ

$$g \text{ hr } \boxed{G_L} \times G_R$$

$$= 0$$

$$\partial_g g^{-1}$$

$$c = -\frac{1+k}{2\pi b}$$

Computation

$$\left\{ \begin{aligned} 2\pi c k^{ab} \delta(\dot{z} - \dot{w}) &= k^{ab} (\delta A_0 + \delta B_0) \\ 0 &= k^{ab} (c \delta A_0 - c \delta B_0) \end{aligned} \right.$$

$$2\pi c^2 k^{ab} \delta(\dot{z} - \dot{w}) = k^{ab} (c+c) \delta A_0$$

$$2\pi \delta(\dot{z} - \dot{w}) = \delta \frac{1}{\dot{z} - \dot{w}} \Rightarrow$$

② The current algebra

Global symmetry

ghr

$$\boxed{G_L} \times G_R$$

Noether currents

$$= 0$$

$$\partial_g g^{-1}$$

$$c = -\frac{1+k^2}{2\pi k}$$

computation

$$\left. \begin{aligned} 2\pi c k^{ab} \partial_a (\tau - \omega) &= k^{ab} (\partial_a A_0 + \partial_b B_0) \\ 0 &= k^{ab} (c \partial_a A_0 - c \partial_b B_0) \end{aligned} \right\}$$

$$2\pi c^2 k^{ab} \partial_a (\tau - \omega) = k^{ab} (c + c) \partial_a A_0$$

$$2\pi \delta(\tau - \omega) = \partial_a \frac{1}{\tau - \omega}$$

$$\Rightarrow A_0 = -\frac{c^2}{c + c} \frac{1}{(\tau - \omega)^2}$$

② The current

Global sy

$g \rightarrow h, g, h$

$$\boxed{G_L} \times G_R$$

Noether current:

$$+ \partial_j \pi = 0$$

$$= c \partial_g g^{-1}$$

$$c = -\frac{1 + k \phi^2}{\phi^2}$$

Spectrum



Current Algebra



$AdS_2 \times S_2$

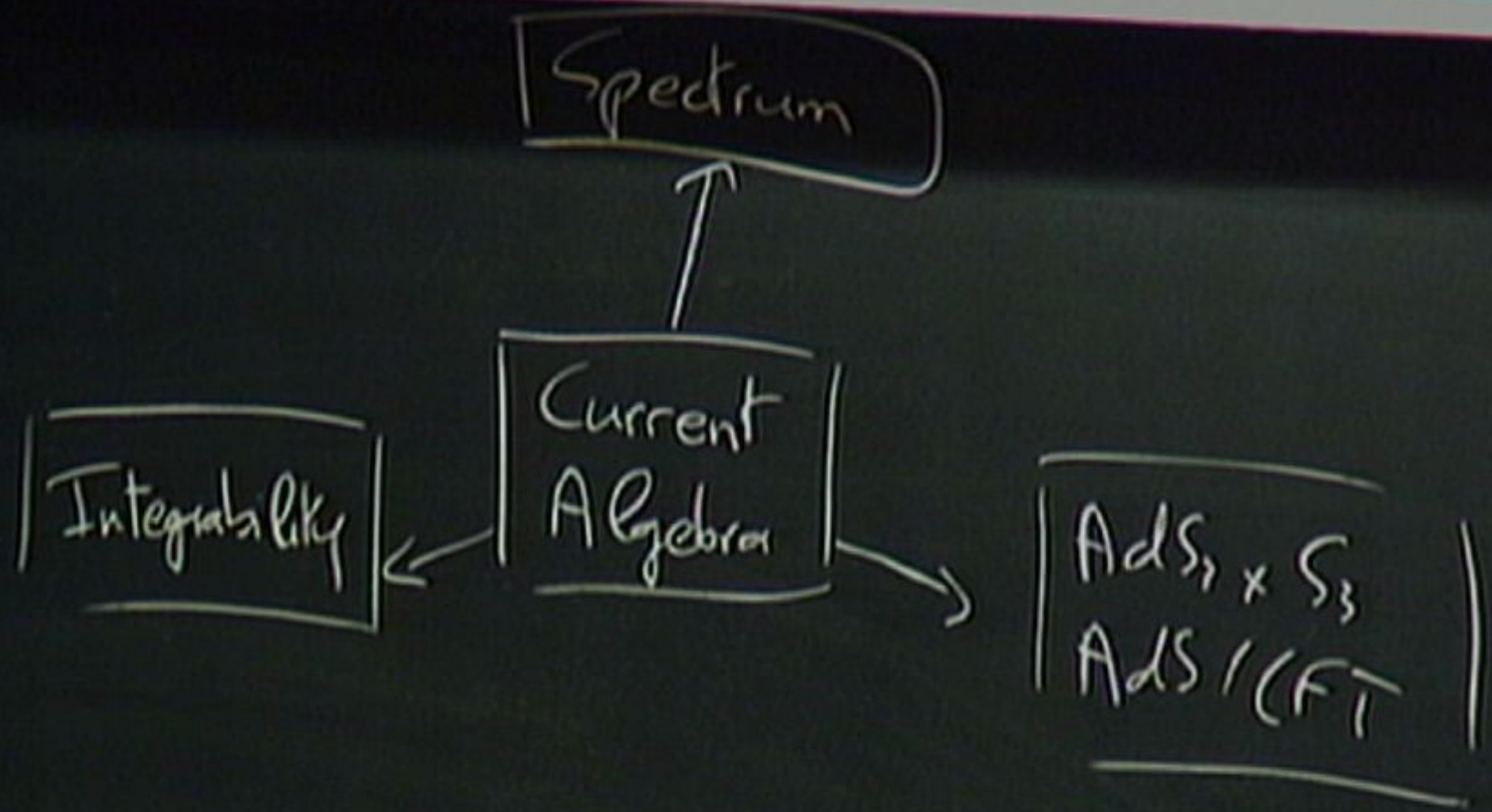
Spectrum

Current
Algebra

$AdS_2 \times S_3$
 AdS_1/CFT

$$\frac{1}{z-w}$$

$$\Rightarrow H_0 =$$



② The current

$$Osp(2n+2|2n): A \in SO(2n+2), B \in SO(2n)$$

Integrability

Classical

Current conservation $\left\{ \begin{array}{l} \partial_t j + \partial_j \bar{j} = 0 \end{array} \right.$

N.C equation: $(\bar{c} \partial_t j - c \partial_t \bar{j} - (j \partial_j \bar{j})) = 0$

③ Integrability

Classical

Current conservation $\left\{ \begin{array}{l} \partial_j j + \partial_{\bar{j}} \bar{j} = 0 \end{array} \right.$

$\eta.c$ equation $\left(\bar{c} \partial_j - c \partial_{\bar{j}} - [j, \bar{j}] \right) = 0$

$$A(\lambda) = \frac{2}{1+\lambda} \frac{j}{c+\bar{c}} dz + \frac{2}{1-\lambda} \frac{\bar{j}}{c+\bar{c}} d\bar{z}$$

zero winding form

③ Integrability

Classical

Current conservation $\left\{ \begin{array}{l} \partial_j j + \partial_{\bar{j}} \bar{j} = 0 \end{array} \right.$

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$$dA(\lambda) + A(\lambda) \eta A(\lambda) =$$

zero Killing form

③ Integrability

Classical

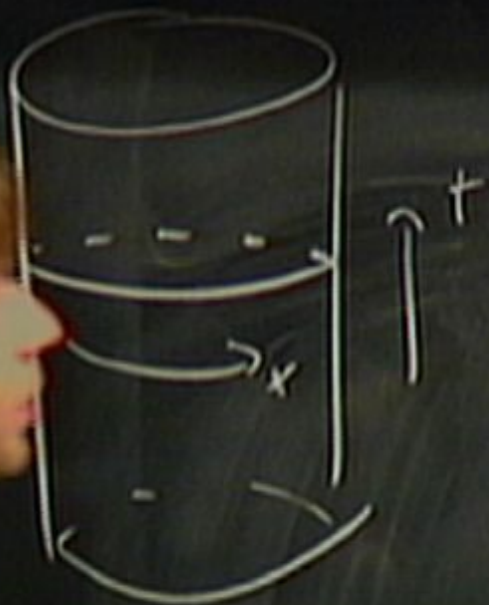
Current conservation $\left\{ \begin{array}{l} \partial_j j + \partial_{\bar{j}} \bar{j} = 0 \end{array} \right.$

$\eta.c$ equation $\left(\bar{c} \partial_j j - c \partial_{\bar{j}} \bar{j} - [j, \bar{j}] \right) = 0$

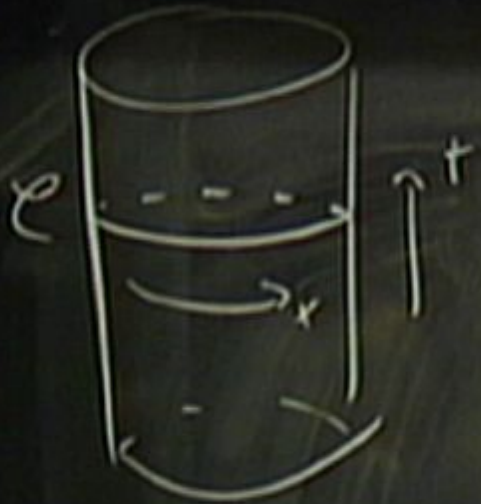
$$A(\lambda) = \frac{z}{1+\lambda} \frac{j}{c+\bar{c}} dz + \frac{z}{1-\lambda} \frac{\bar{j}}{c+\bar{c}} d\bar{z}$$

$$dA(\lambda) + A(\lambda) \wedge A(\lambda) = 0$$

zero Killing form



$$\exp \oint A(\lambda)$$



$$\exp \oint_{\mathcal{C}} A(\lambda) = 1 + \sum_n \lambda^n Q_n$$



$$\exp \left(\int A(x) \right) = 1 + \sum_n x^n Q_n$$

$$Q_1 = \int dT^a$$

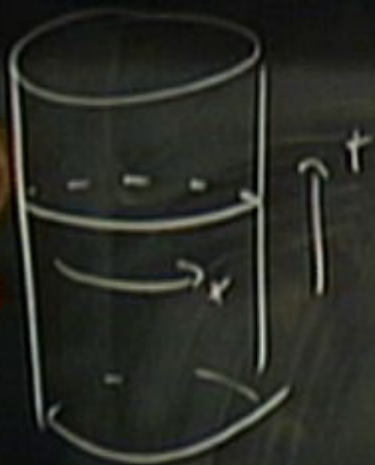


$$\exp \left(\int A(x) \right) = 1 + \sum_n x^n Q_n$$

$$Q_1 = \int d^a$$

$$Q_2 = \int d^a d^b + \int \int \beta^{bc} d^b d^c$$

$$dA(\lambda) + A(\lambda) \wedge A(\lambda) = 0$$



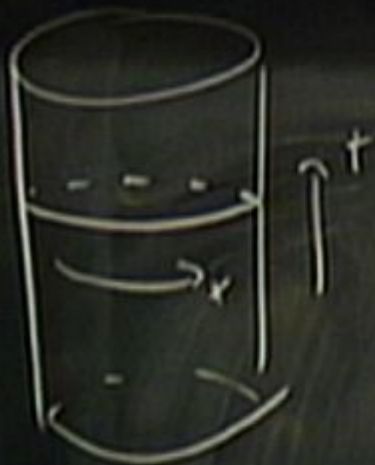
$$\exp \int A(\lambda) = 1 + \sum_n \lambda^n Q_n$$

$$Q_1 = \int dT^a$$

$$Q_2 = \int dX^a + \int \int \beta^{bc} \frac{d^b}{dT} \frac{d^c}{dT}$$

$$k^{bc} \beta^{bc} = 0$$

$$dA(\lambda) + A(\lambda) \eta A(\lambda) = 0$$



$$\exp \int A(\lambda) = 1 + \sum_n \lambda^n Q_n$$

$$Q_1 = \int dT^a$$

$$Q_2 = \int dX^a + \int \int \beta^{bc} \frac{d^b}{dT} \frac{d^c}{dT}$$

Double pole: $k^{bc} \beta^{bc} = 0$

$$dA(\lambda) + A(\lambda) \frac{d}{d\lambda} d\tau + \frac{2}{1-\lambda} \frac{d}{d\lambda} d\tau$$

$$dA(\lambda) + A(\lambda) \frac{d}{d\lambda} A(\lambda) = 0$$



$$\exp \oint A(\lambda) = 1 + \sum_n \lambda^n Q_n$$

Order zero

$$Q_1 = \oint d\tau^a$$

$$Q_2 = \oint j_x^0 + \oint \beta^{bc} \frac{d}{d\lambda} j_i^c$$

Double pole: $k^{bc} \beta^{bc} = 0$ | Simple pole: $\beta^{bc} d \beta^{bc} = 0$

$1 + \mathcal{O}(n)$

Order zero

β^a $\beta^b c$ $\beta^{d g d}$ $\beta^e c$
 β g

$b < 0$

$$1 + \sum_n x^n Q(n)$$

Order zero

$$b^a \quad b^b \quad b^c$$

$$b^a \quad b^b \quad b^c$$

$$\text{the pole: } b^a \quad b^b \quad b^c = 0$$

$$1 + \sum_n \lambda^n Q(n)$$

$$\left(\frac{\partial}{\partial t} \right)^a \beta_{bc} \frac{\partial}{\partial t} \frac{\partial}{\partial t} c$$

$$\text{the pole: } \beta_{bc} \frac{\partial}{\partial t} \beta^a{}_{bc} = 0$$

Order zero

$$\beta^a{}_{bc} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} c$$

$$= 0$$

Jacobi
+ zero Killing
Form.

$$c \partial_{\bar{z}} - (j, \bar{d}) = 0$$

$$\int_{\gamma} \begin{matrix} a & b & c \\ bc & d & d \end{matrix} = \int_{\gamma} \begin{matrix} a & b & c \\ bc & \bar{z} & d \end{matrix}$$

Quantum
integrability

Holomorphicity

$$c \partial_j - (\partial_j c) = 0$$

$$\sqrt{z} \left(\begin{matrix} a & b & c \\ b & c & d \\ c & d & e \end{matrix} \right) = \begin{matrix} a & b & c \\ b & c & d \\ c & d & e \end{matrix}$$

Quantum
integrability

Holomorphicity
of $T(t)$

$$\sqrt{z} \quad \mathcal{L}^{a, b, c} = \mathcal{L}^{a, b, c}$$

Quantum integrability \leftrightarrow Holomorphicity of $T(t)$

$X^n Q(n)$

$\mathcal{L}^{a, b, c} \mathcal{L}^{b, g, \beta} \mathcal{L}^{e, \beta, c}$