Title: Entropic Dynamics, Time and Quantum Theory

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Abstract: Non-relativistic quantum mechanics is derived as an example of entropic inference. The basic assumption is that the position of a particle is subject to an irreducible uncertainty of unspecified origin. The corresponding probability distributions constitute a curved statistical manifold. The probability for infinitesimally small changes is obtained from the method of maximum entropy and the concept of time is introduced as a book-keeping device to keep track of how they accumulate. This requires introducing appropriate notions of instant and of duration. A welcome feature is that this entropic notion of time incorporates a natural distinction between past and future. The Schroedinger equation is recovered when the statistical manifold participates in the dynamics in such a way that there is a conserved energy: its curved geometry guides the motion of the particles while they, in their turn, react back and determine its evolving geometry. The phase of the wave function—not just its magnitude—is explained as a feature of purely statistical origin. Finally, the model is extended to include external electromagnetic fields and gauge transformations.

Entropic Dynamics, Time and Quantum Theory

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University at Albany - SUNY

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05/2010

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Statistical Mechanics and Thermodynamics are derivable as rules of inference.

Our objective:
To derive Quantum Theory as Entropic Inference.

Entropic Inference

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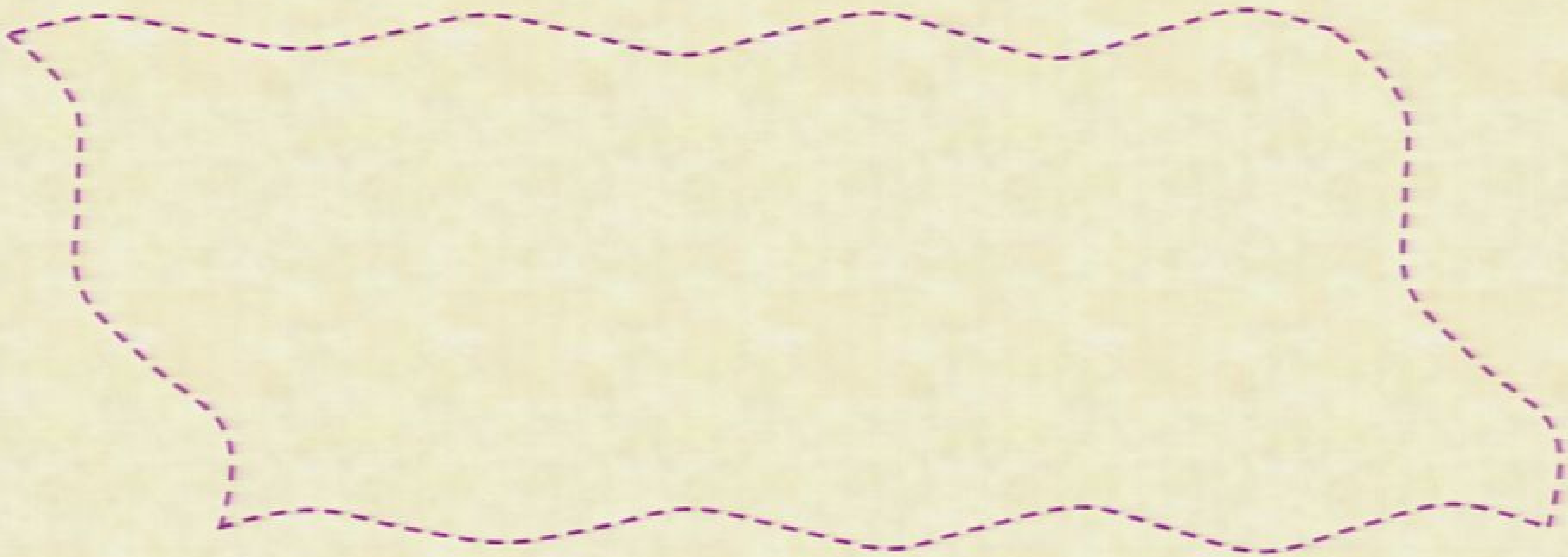

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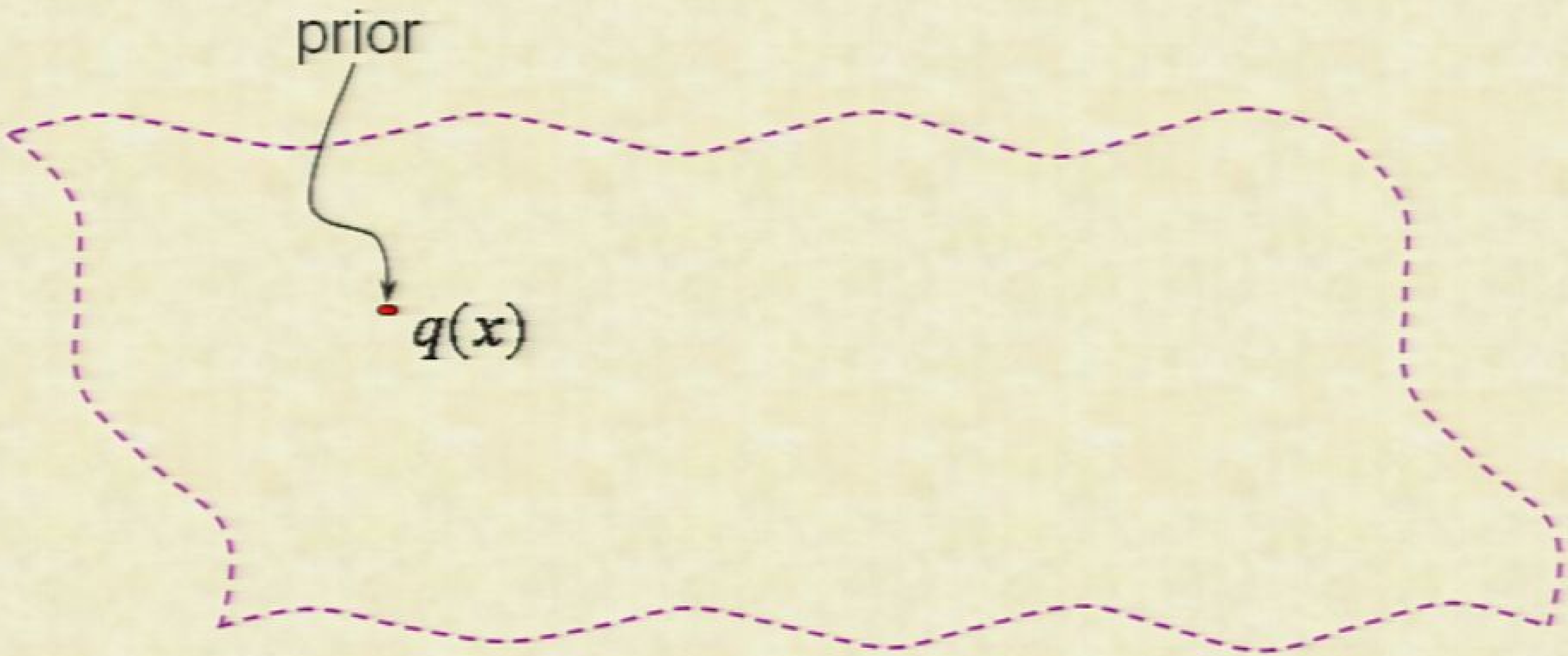

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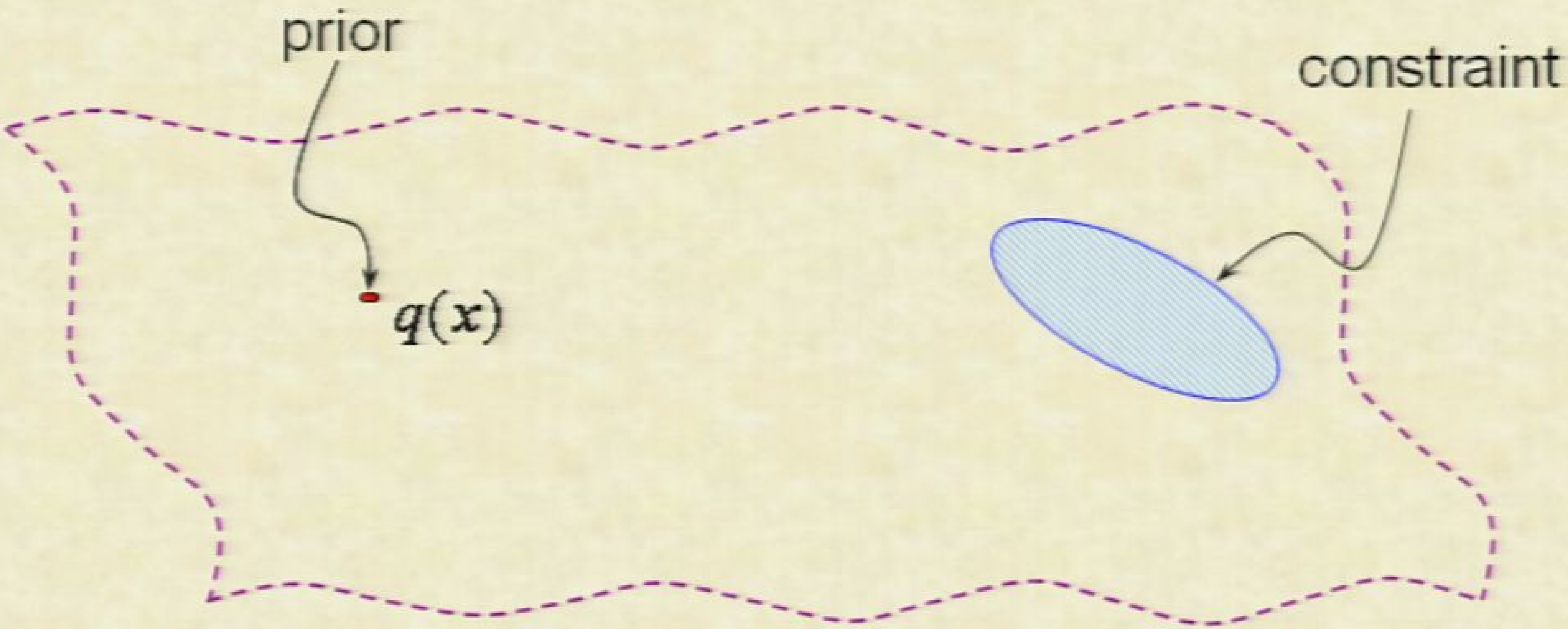

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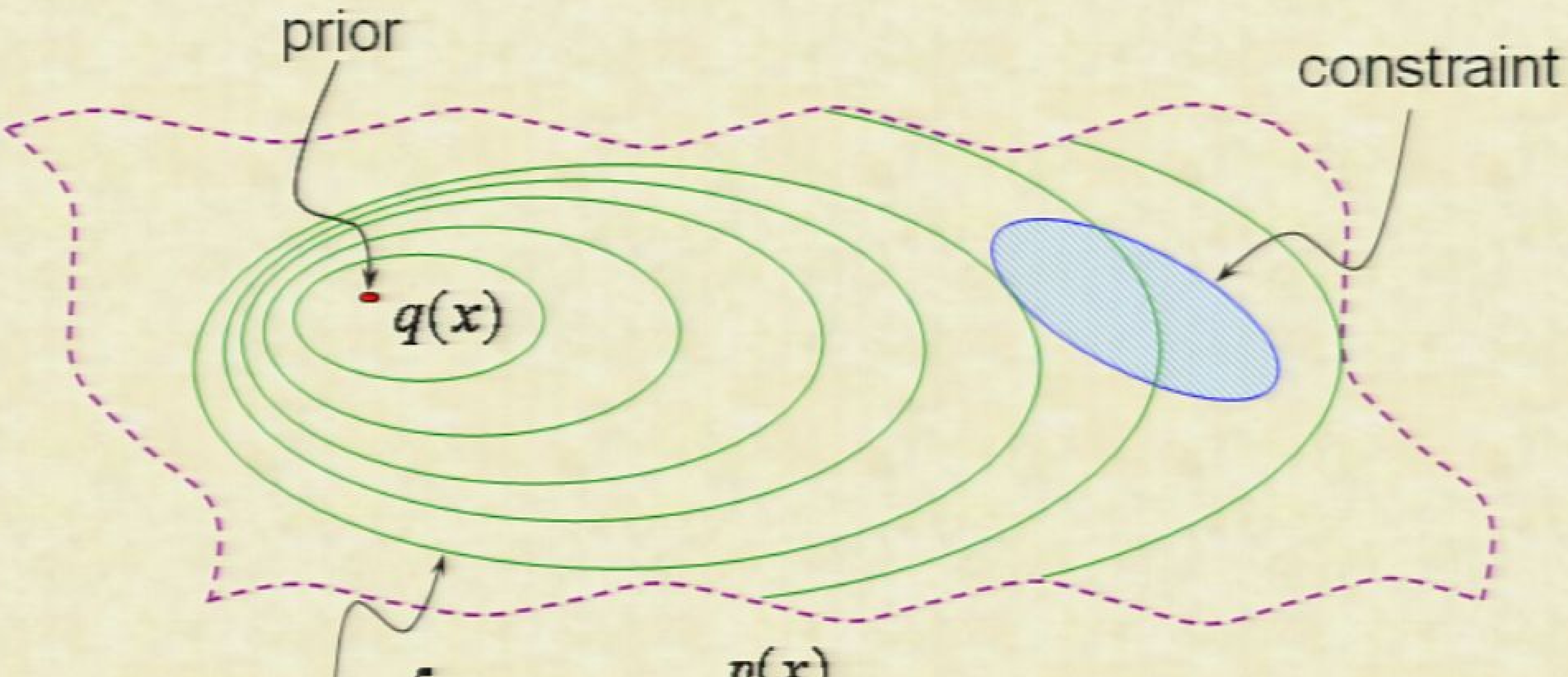

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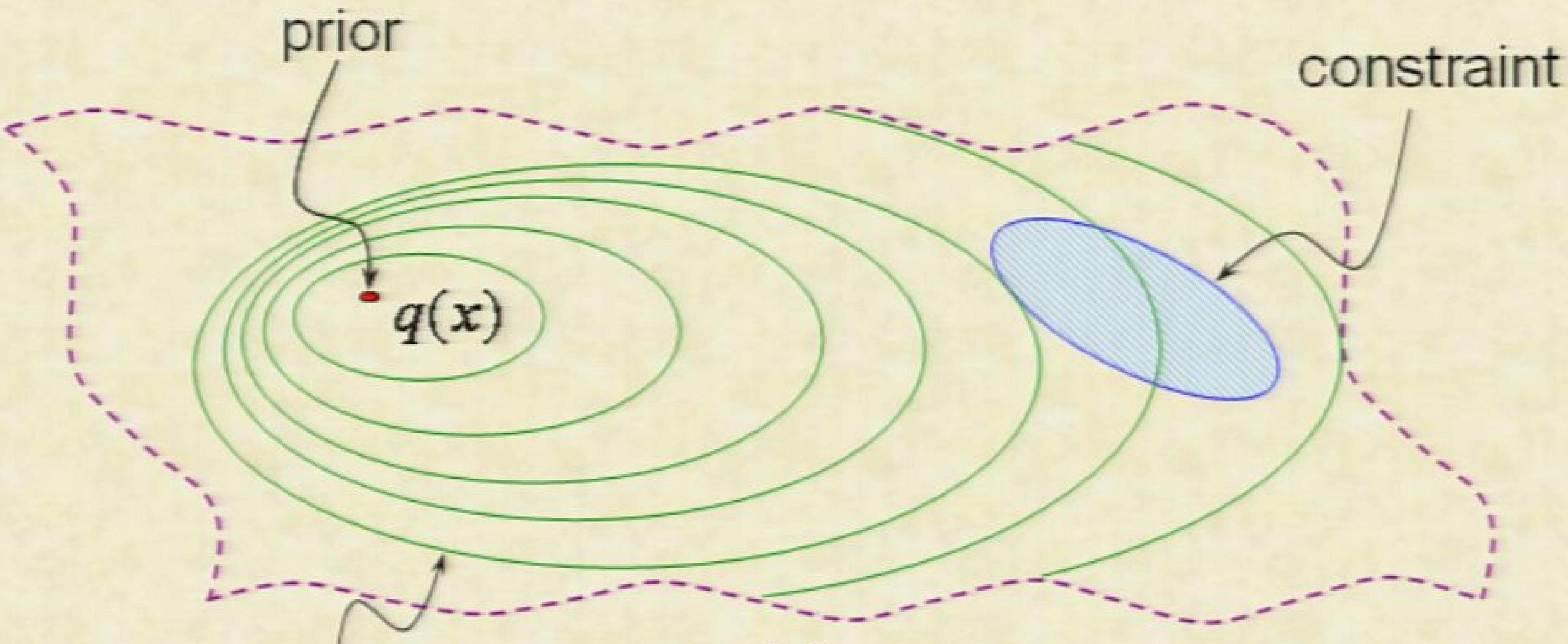

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Maximize $S[p,q]$ subject to the appropriate constraints.

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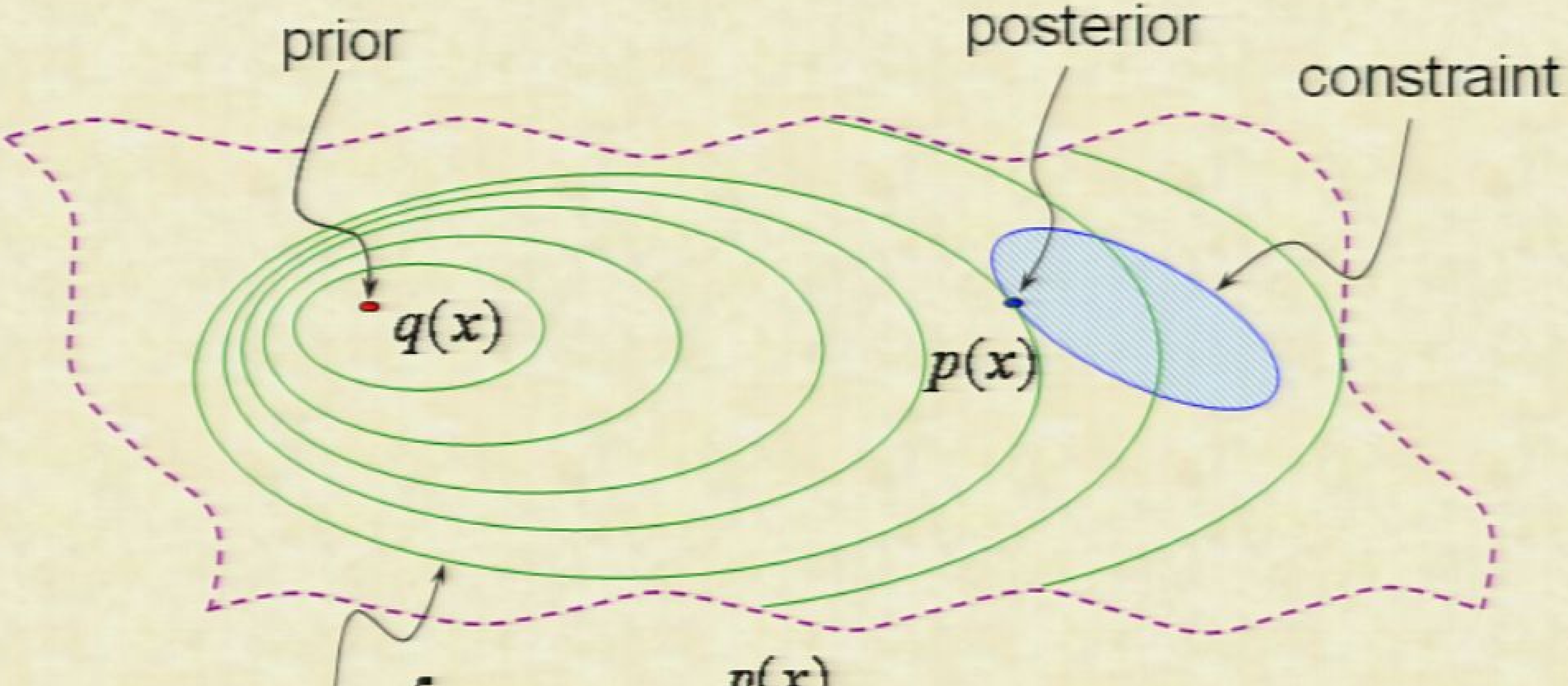

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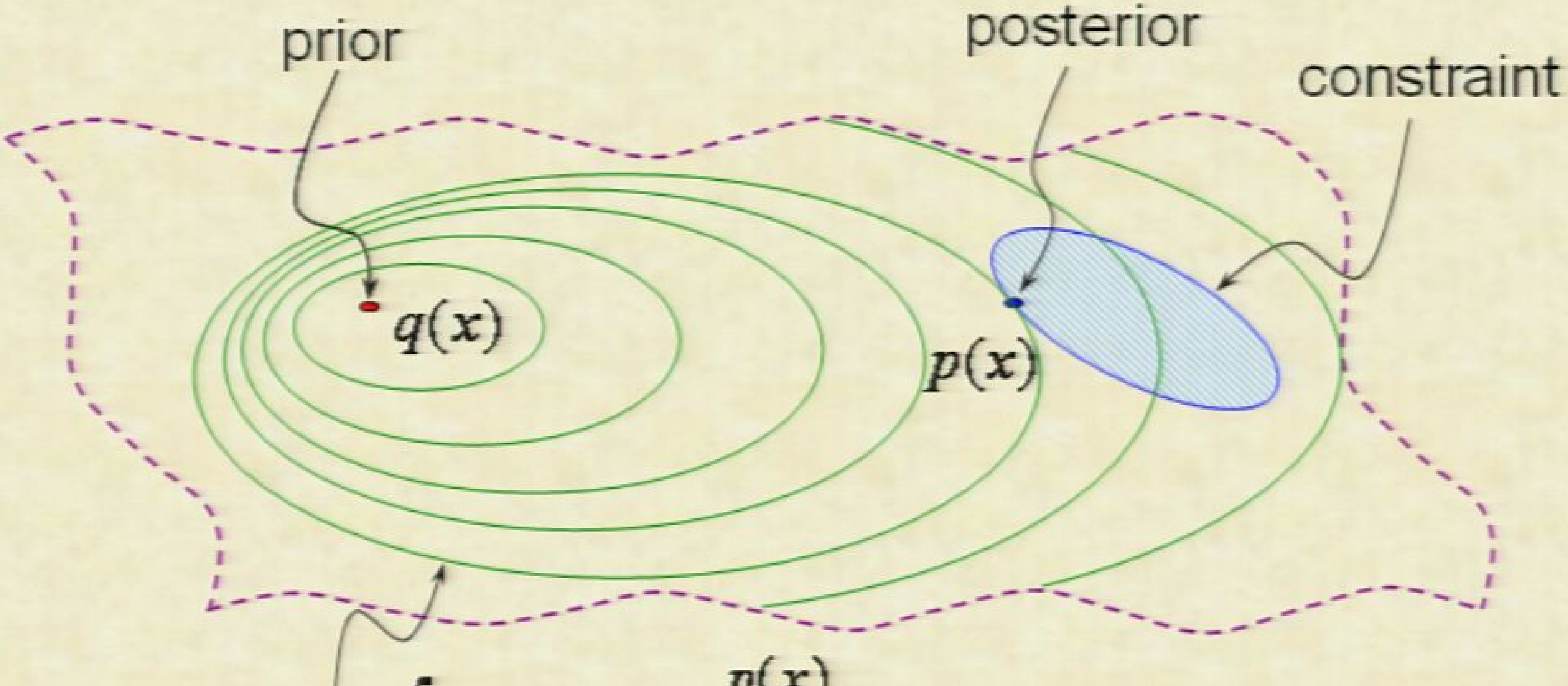

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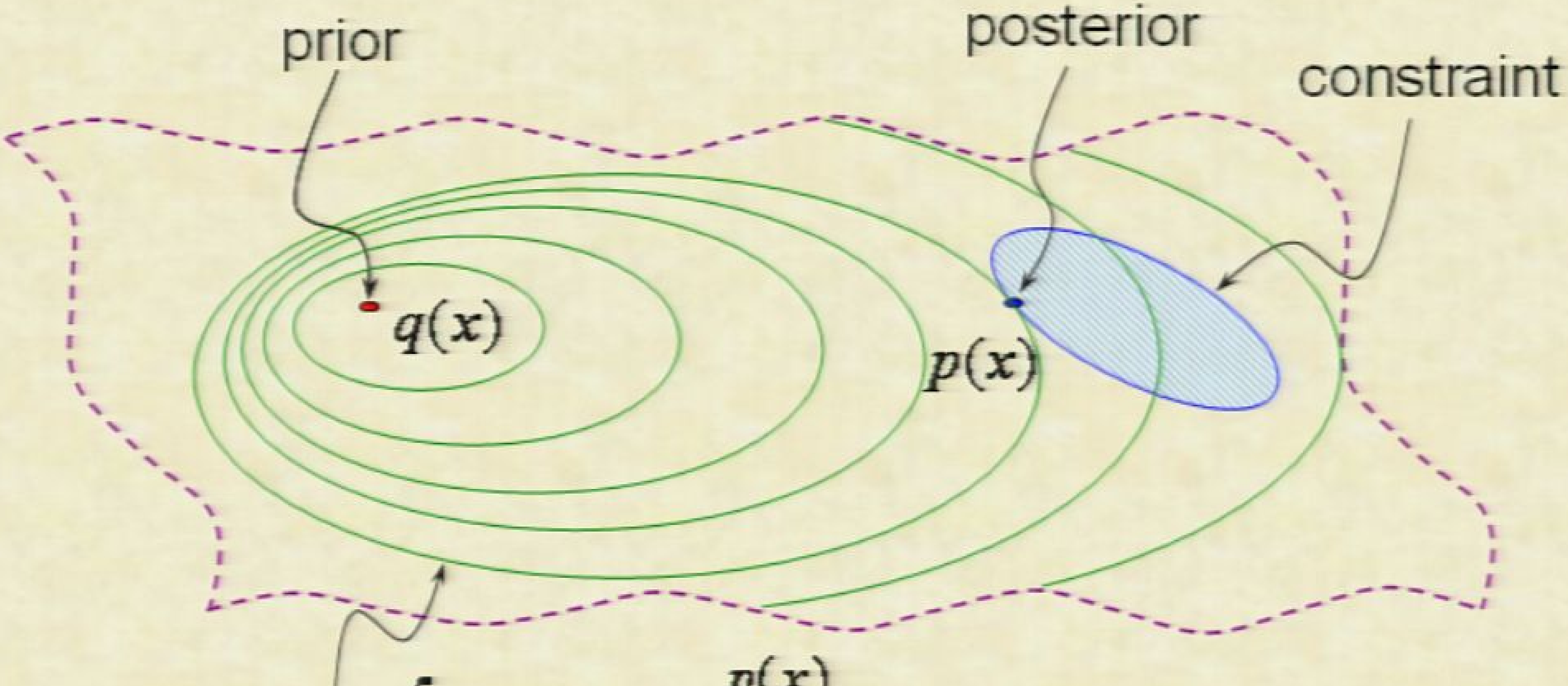
(MaxEnt, Bayes' rule and Large Deviations are special cases.)

Step 1: The Statistical Model

Entropic Inference



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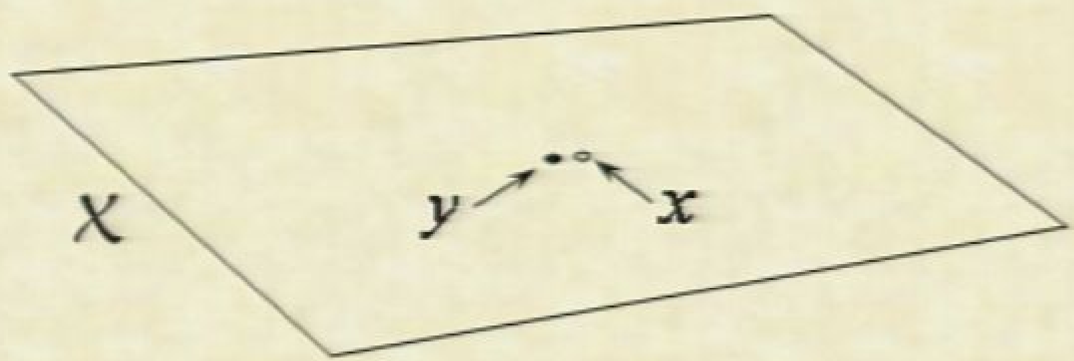

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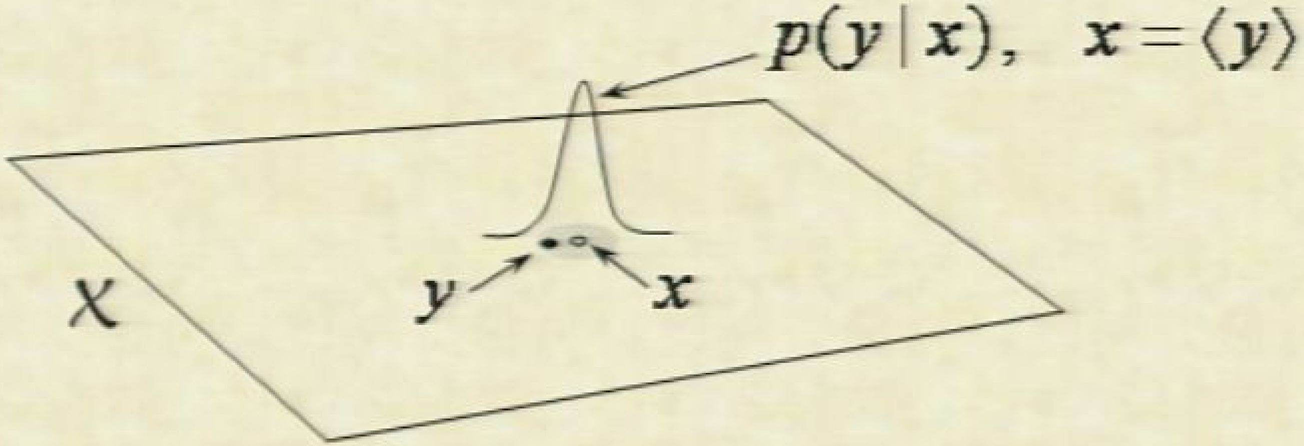
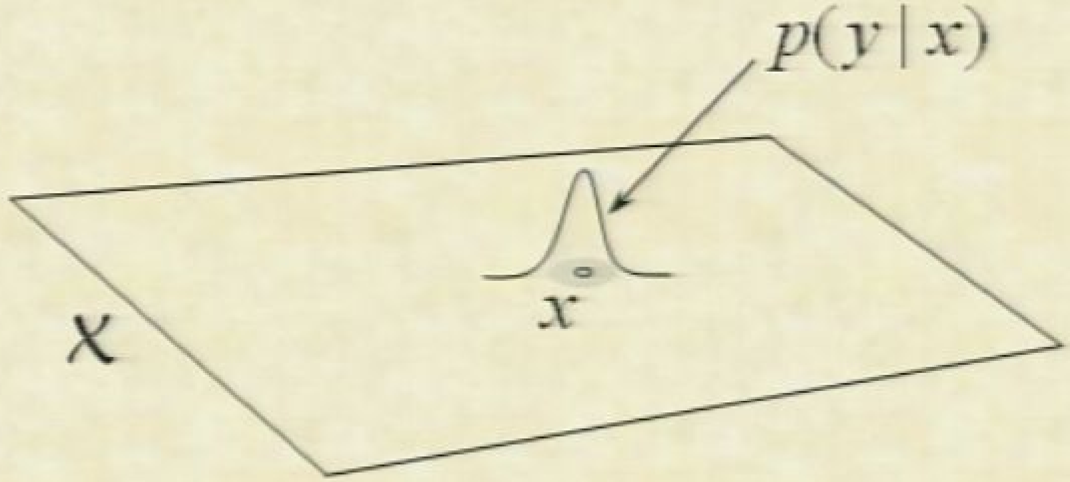
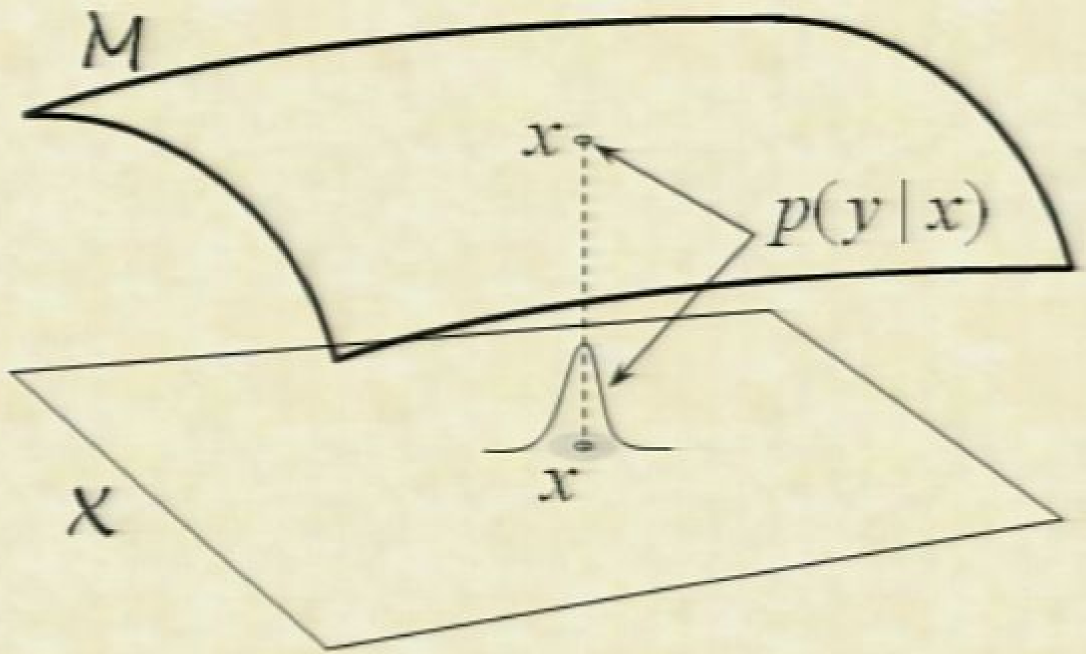



$p(y|x)$
x
X


M
x
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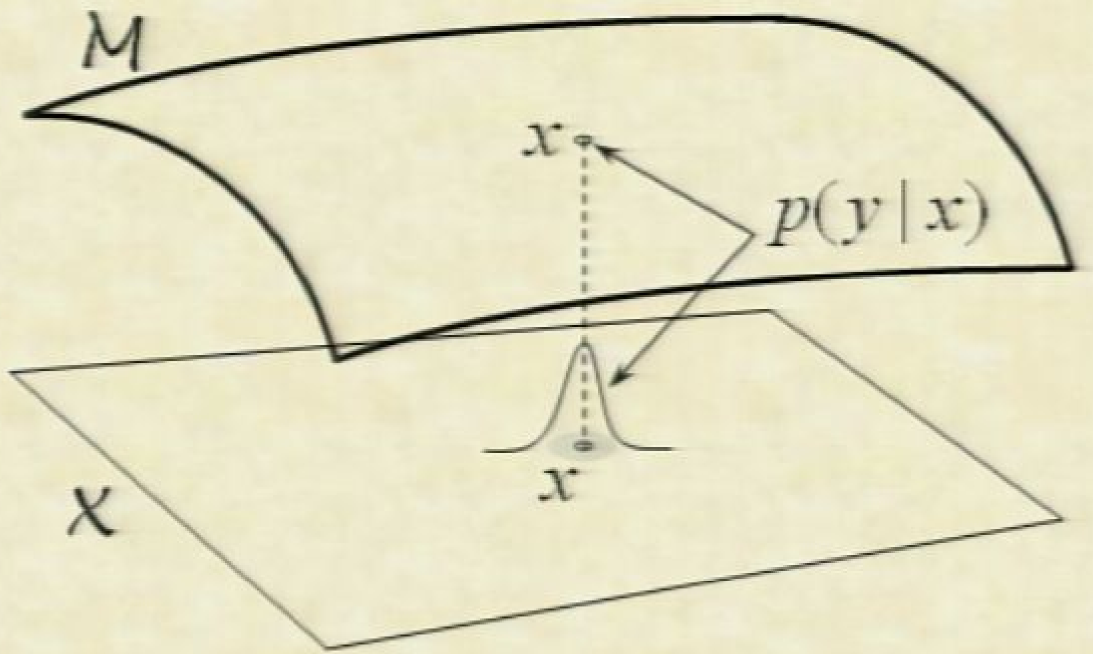
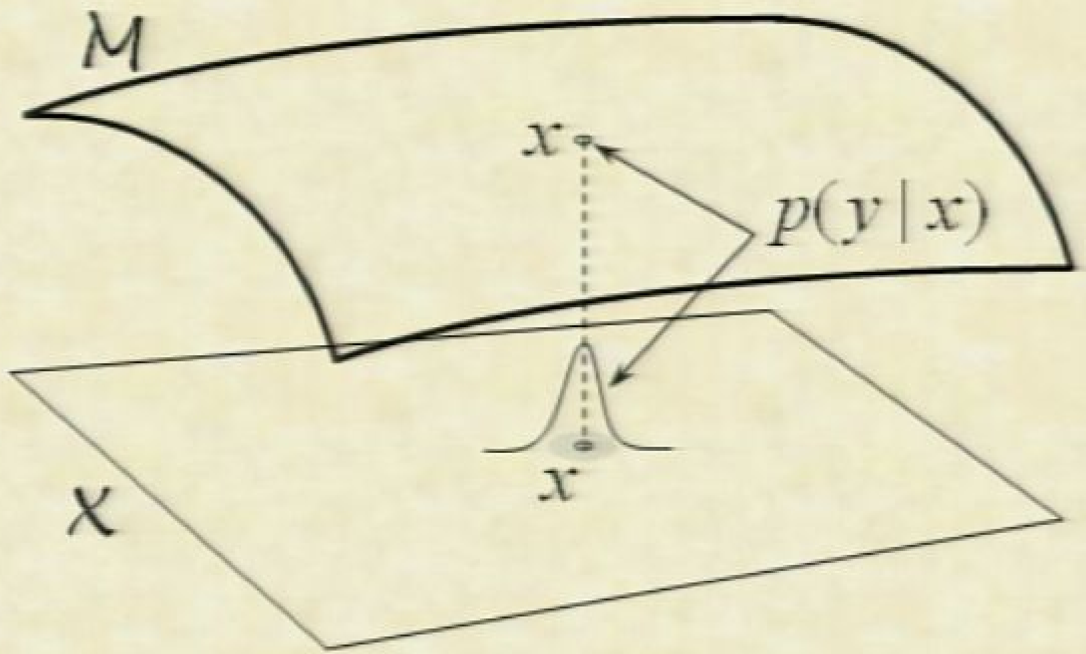


X configuration space with metric $\gamma_{ab} = \frac{\delta_{ab}}{\sigma^2}$



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M statistical manifold of distributions $p(y \mid x)$

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Prior: $\boldsymbol{Q(x',y'\,|\,x)}$

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Short steps: $\langle \Delta\ell^2 \rangle = \langle \gamma_{ab}\Delta x^a \Delta x^b \rangle = \lambda^2(x)$

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For large α or short steps:

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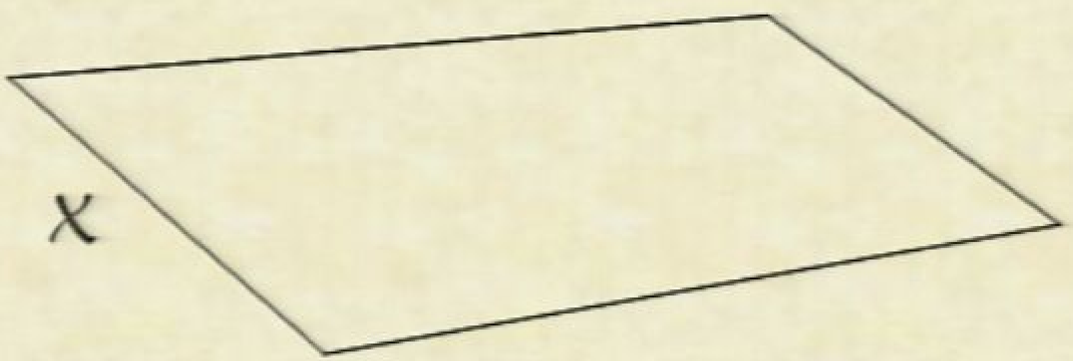

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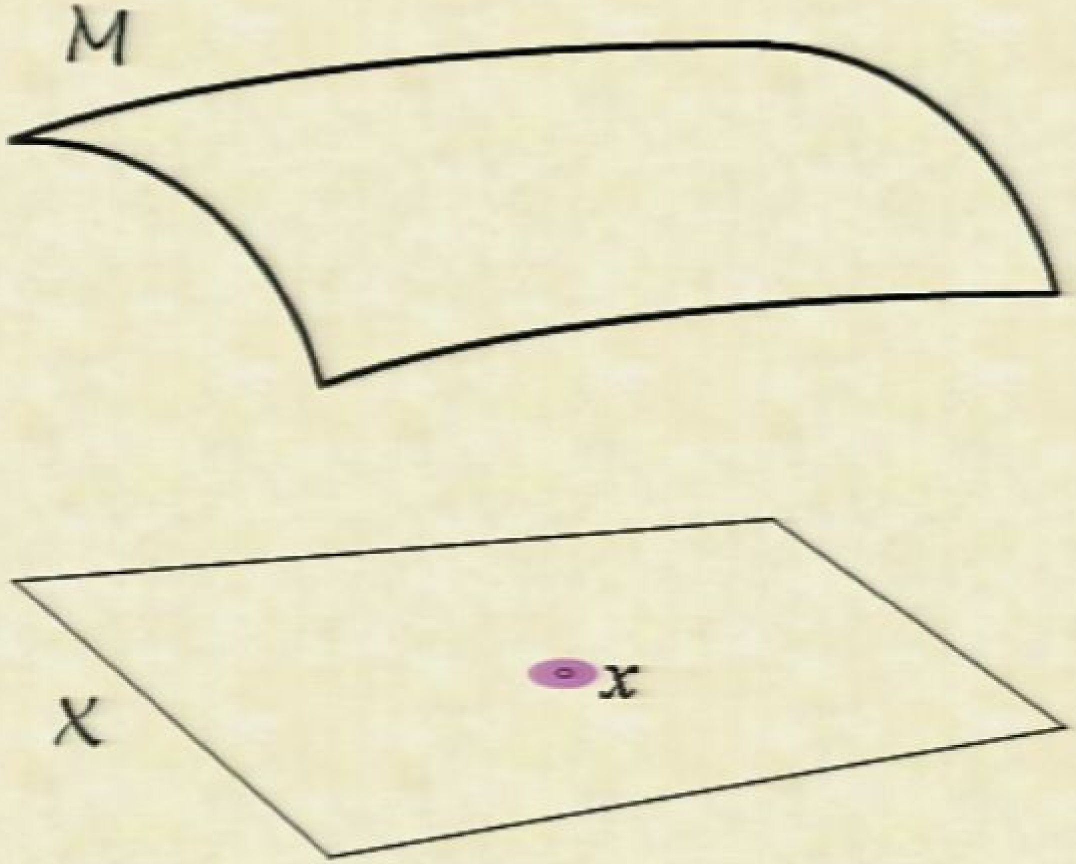

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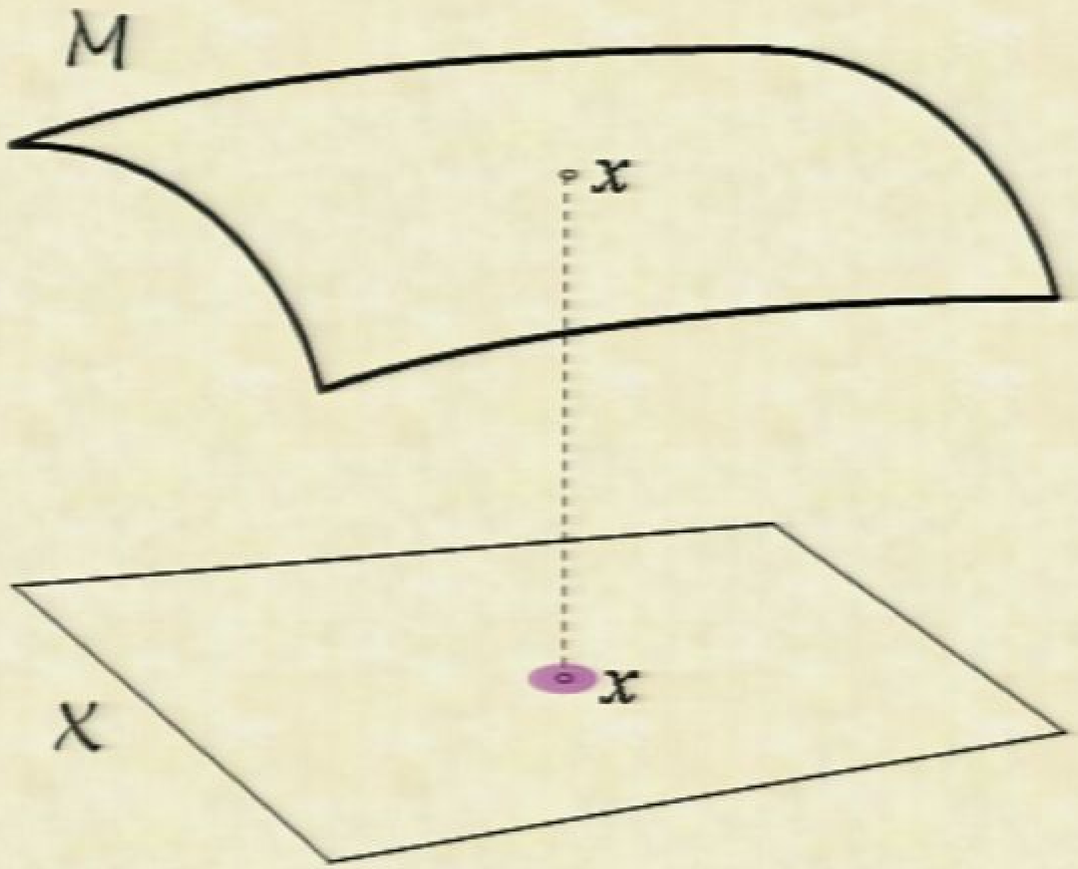

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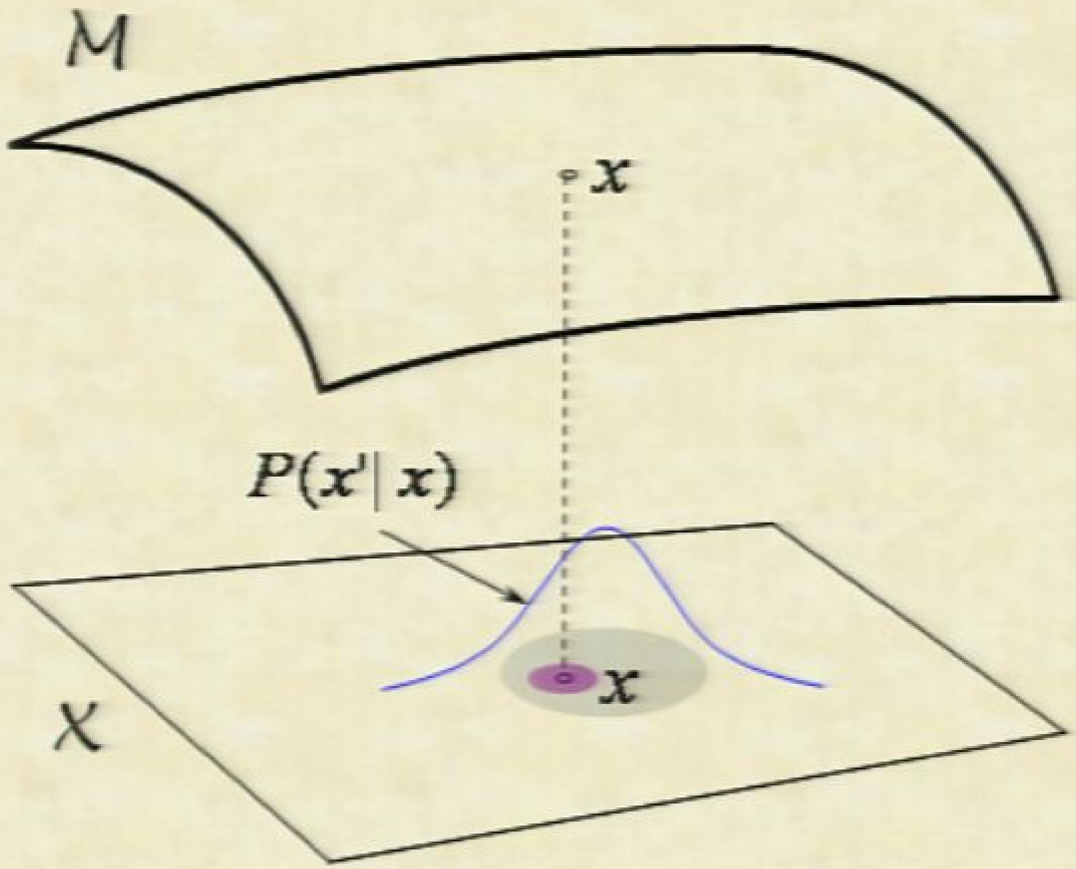

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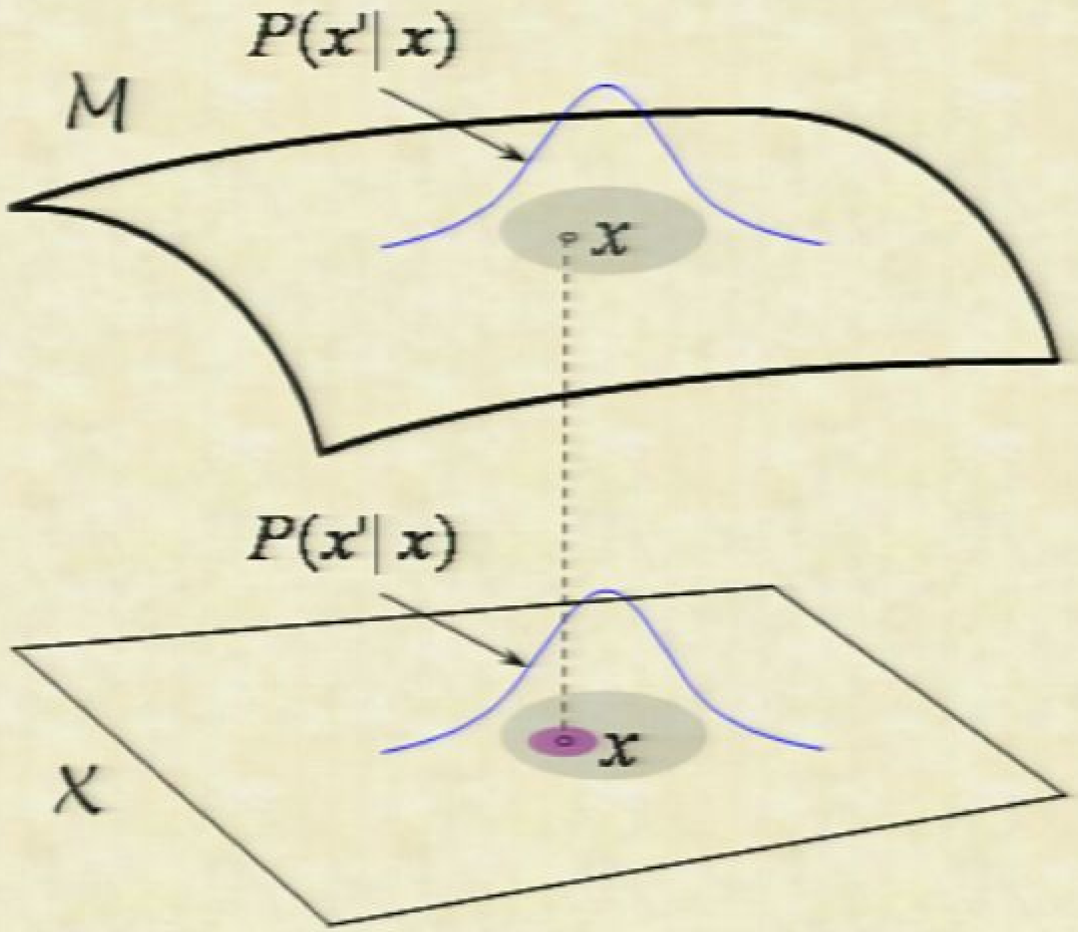

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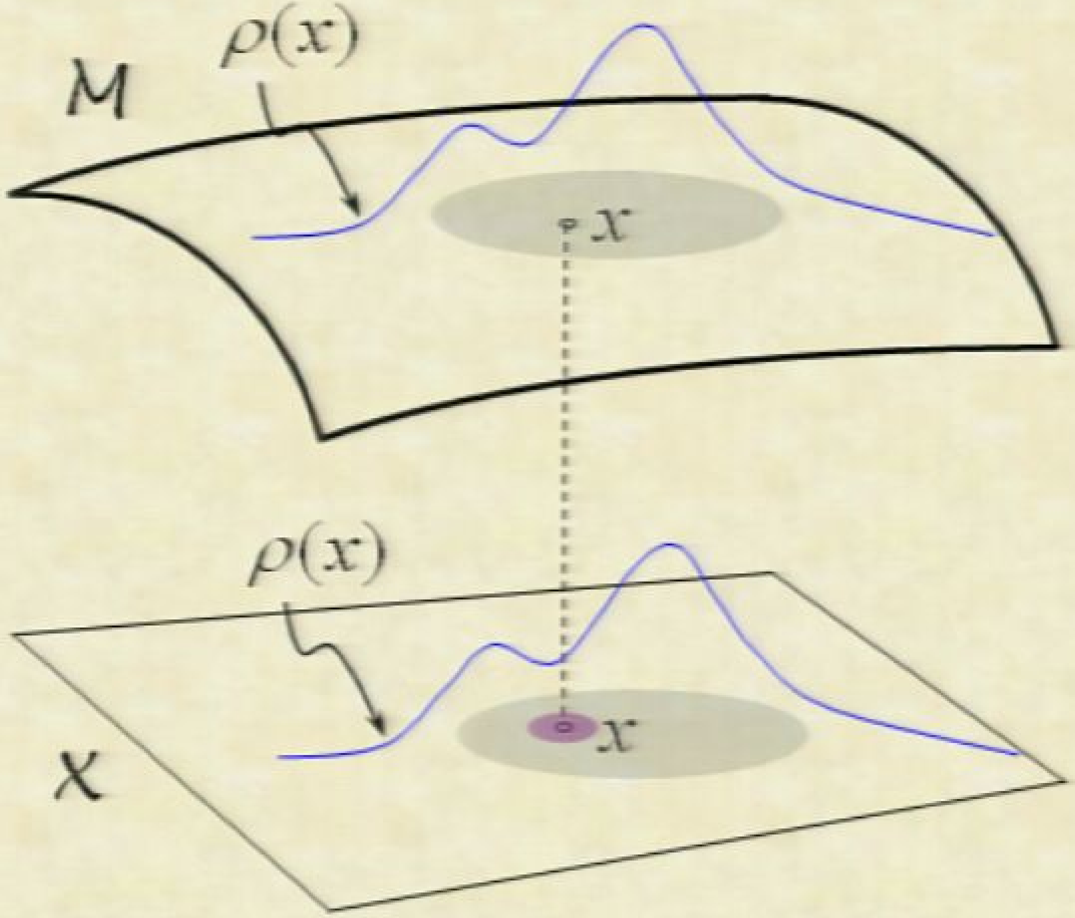

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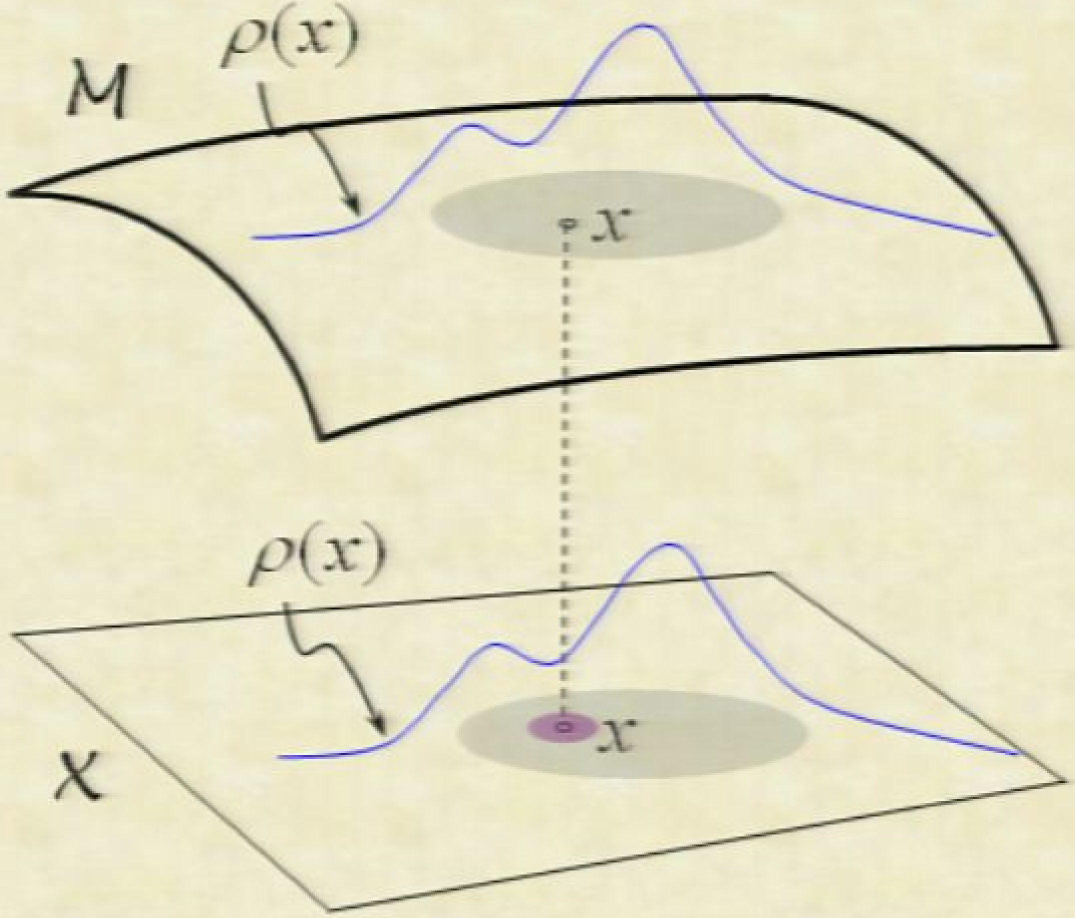

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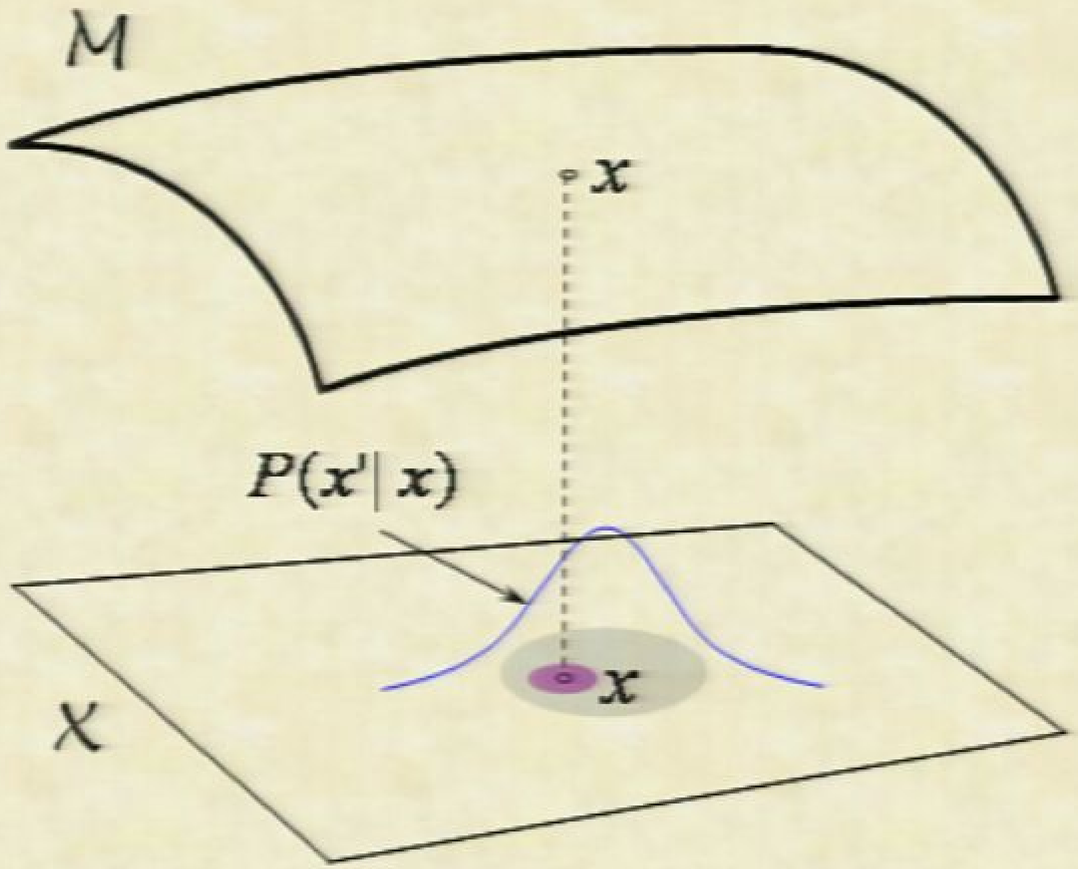

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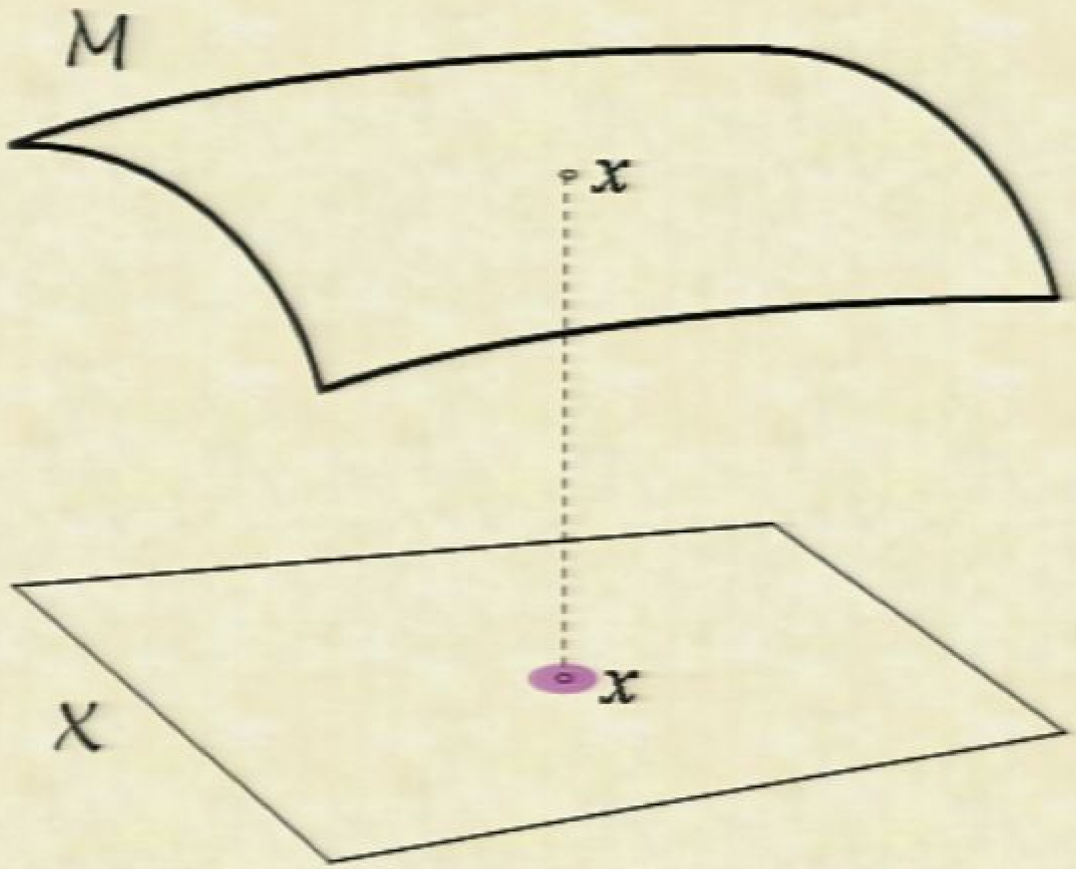

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diffusion current: $\rho u^a = -\frac{\sigma^2}{2\tau}\partial^a \rho$

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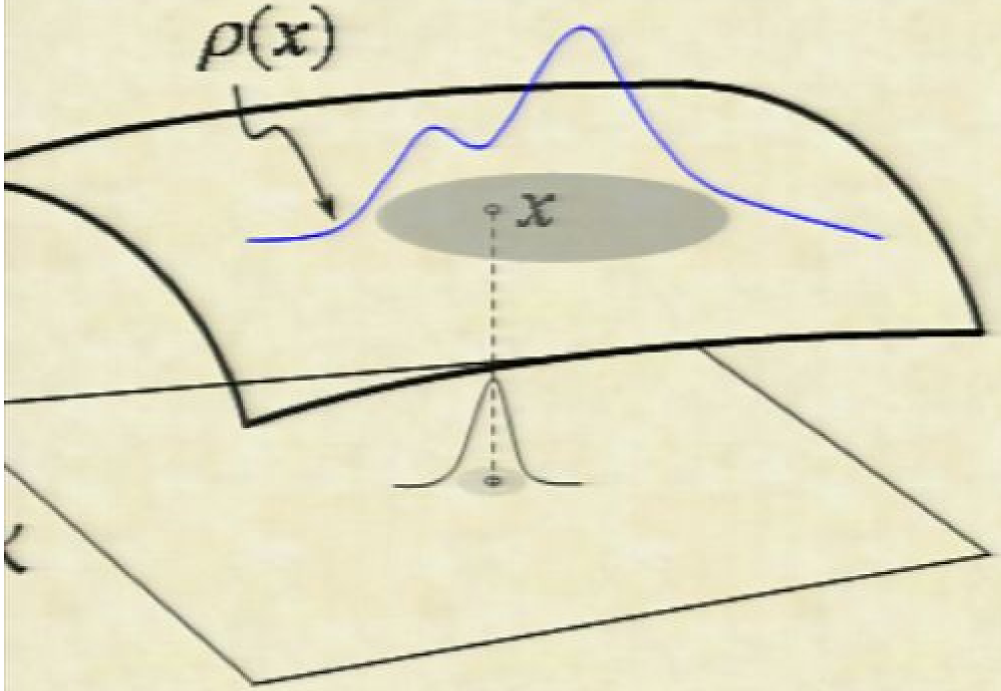

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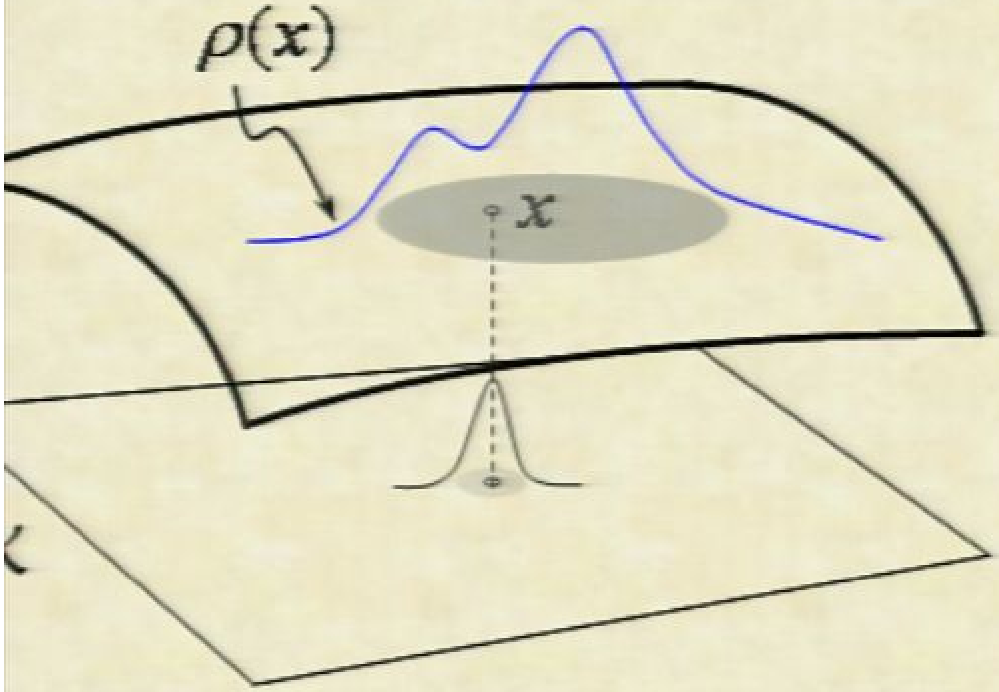

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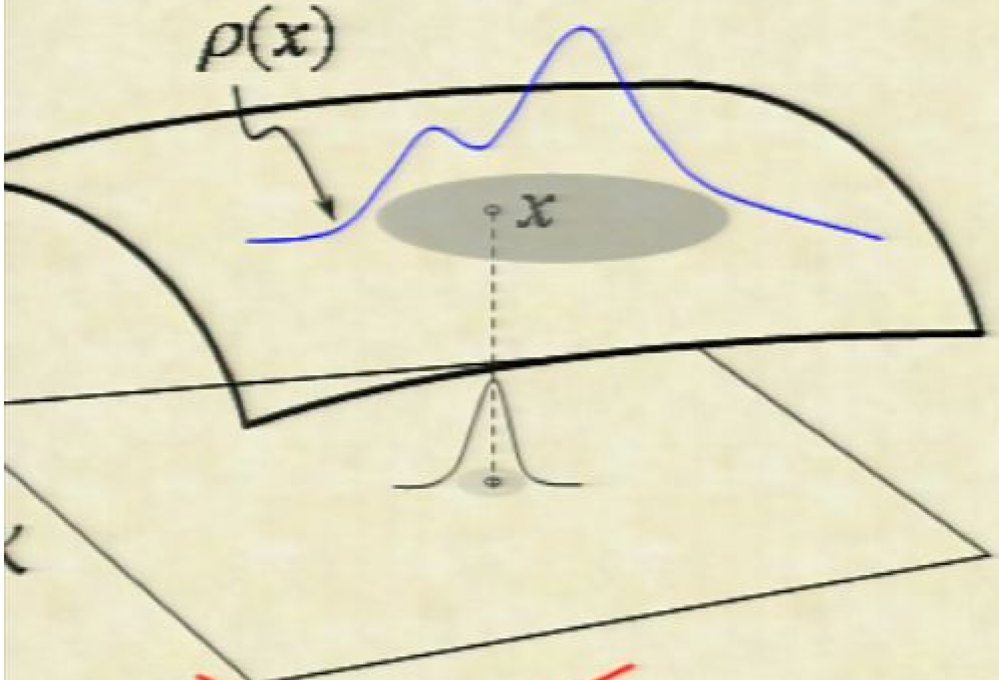

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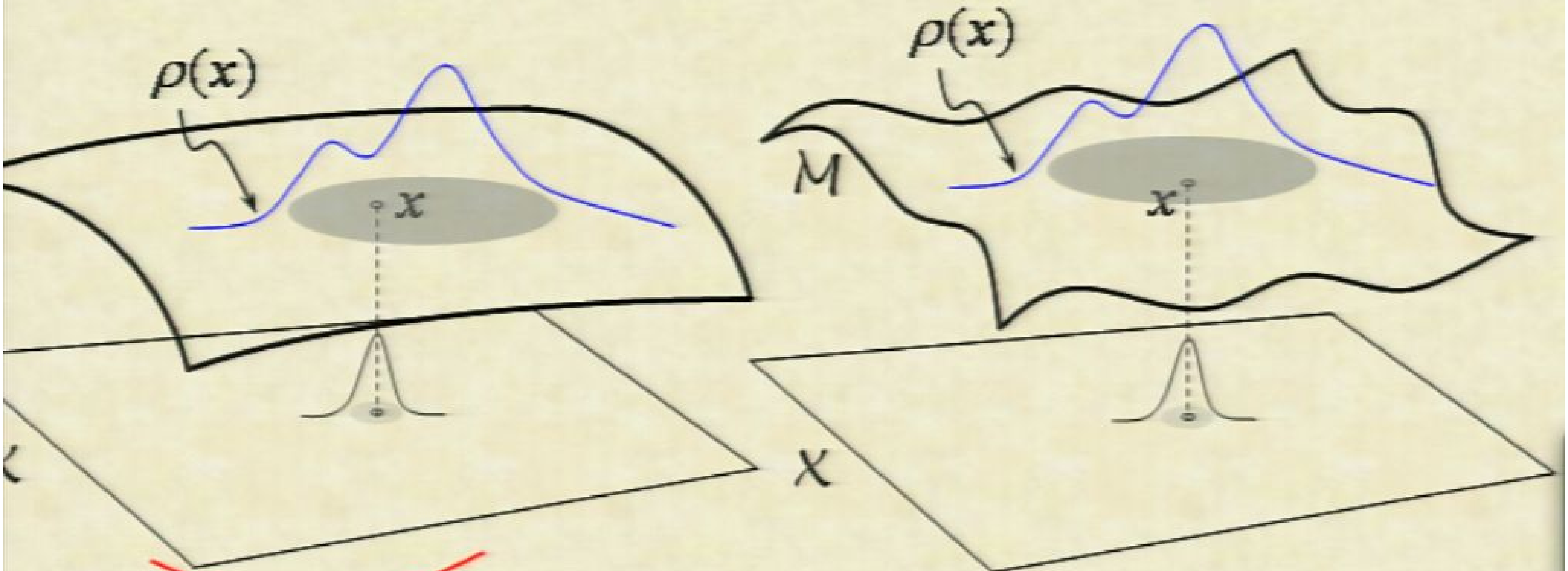

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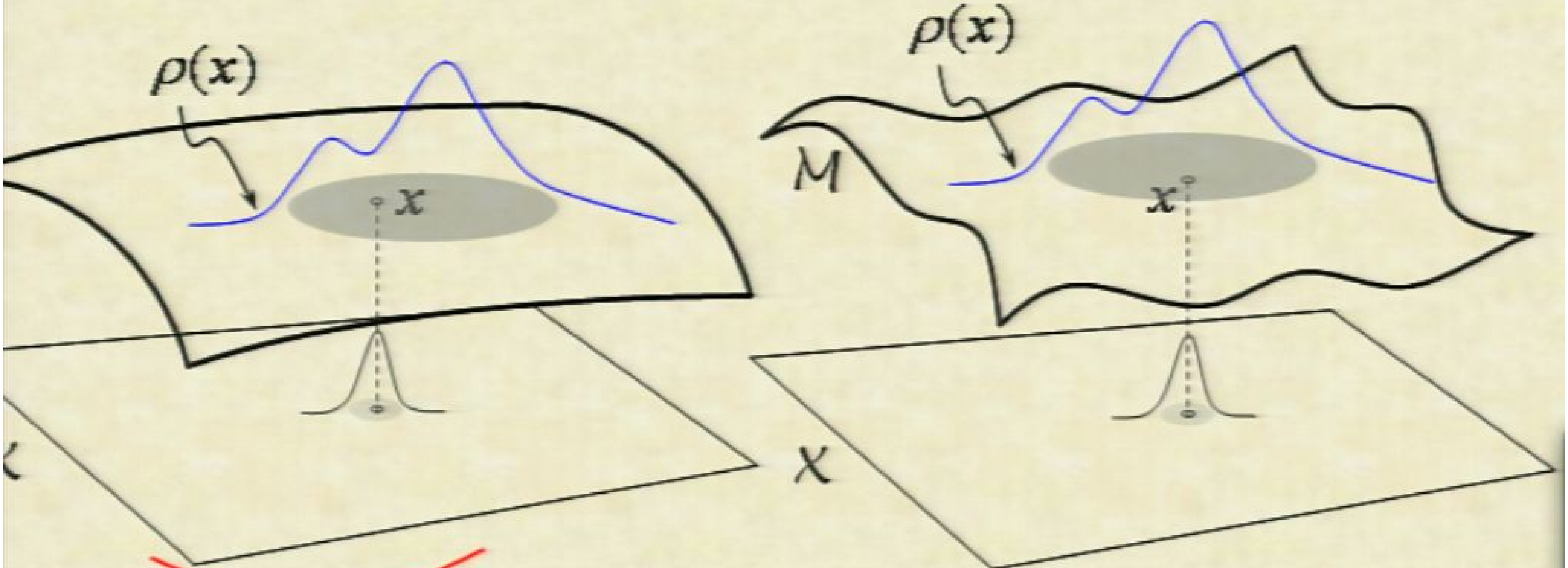

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Neither x' nor y' are known: the releva

We need the joint distribution $P(x', y')$

To find it maximize the joint (relative) e

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