Title: Gravity dual of spatially modulated phase

Date: May 11, 2010 11:00 AM

URL: http://pirsa.org/10050020

Abstract: In this talk, I will show that the five-dimensional Maxwell theory with a Chern-Simons coupling larger than a critical value in the Reissner-Nordstrom black hole geometry has tachyonic modes. This instability has an interesting property that it happens only at non-vanishing momenta, suggesting a spatially modulated phase transition in the holographically dual field theory. The final state after the phase transition has taken place will be discussed in detail in a special limit

AdS/CFT correspondence

AdS/CFT correspondence

- equivalence between conformal field theory and gravity theory
- operators in field theory $\Leftrightarrow$ bulk fields
- for large N and large λ, classical gravity is a good approximation
- strongly interacting field theory $\Leftrightarrow$ classical gravity

AdS/CFT correspondence

AdS/CFT correspondence

- equivalence between conformal field theory and gravity theory
- operators in field theory $\Leftrightarrow$ bulk fields
- for large N and large λ, classical gravity is a good approximation
- strongly interacting field theory $\Leftrightarrow$ classical gravity

Adding temperature and chemical potential

- turning on temperature $T \Leftrightarrow$ black hole geometry
- chemical potential $\mu \Leftrightarrow$ charged black holes
  finite density system
- two scales μ and T: physics depends only on T/μ

Special aspect of 5D gravity theories

- Chern-Simons term is present for global symmetry (e.g. $U(1)_R$) of dual field theory

$$\alpha \int A \wedge dA \wedge dA$$

- charged black holes are still solutions
- hydrodynamic properties change
  e.g. vorticity-induced current Son, Surówka/Erdmenger et al. / Banerjee et al.
  non-vanishing Hall conductivity Matsuo et al.
  chiral shear waves Sahoo, Yee

AdS/CFT correspondence

AdS/CFT correspondence

- equivalence between conformal field theory and gravity theory
- operators in field theory $\Leftrightarrow$ bulk fields
- for large N and large λ, classical gravity is a good approximation
- strongly interacting field theory $\Leftrightarrow$ classical gravity

Adding temperature and chemical potential

- turning on temperature $T \Leftrightarrow$ black hole geometry
- chemical potential $\mu \Leftrightarrow$ charged black holes
  finite density system
- two scales μ and T: physics depends only on T/μ

Special aspect of 5D gravity theories

- Chern-Simons term is present for global symmetry (e.g. $U(1)_R$) of dual field theory

$$\alpha \int A \wedge dA \wedge dA$$

- charged black holes are still solutions
- hydrodynamic properties change
  e.g. vorticity-induced current Son, Surówka/Erdmenger et al. / Banerjee et al.
  non-vanishing Hall conductivity Matsuo et al.
  chiral shear waves Sahoo, Yee

Special aspect of 5D gravity theories

- Chern-Simons term is present for global symmetry (e.g. $U(1)_R$) of dual field theory

$$\alpha \int A \wedge dA \wedge dA$$

- charged black holes are still solutions
- hydrodynamic properties change
  e.g. vorticity-induced current Son, Surówka/Erdmenger et al. / Banerjee et al.
  non-vanishing Hall conductivity Matsuo et al.
  chiral shear waves Sahoo, Yee

We will focus on finite momentum modes in $\mathbb{R}^3$

Set-up

Problem to consider

- 5d Maxwell-Einstein theory with Chern-Simons term
- CS-coupling α is not fixed
- charged BH is still a solution, but stability analysis changes
- check if gauge fields or off-diagonal metrics become unstable, depending on α
- if unstable, find the final geometry

Spatially modulated phase in field theories

- Phase where fields have position dependent expectation values: e.g. plane waves
- Translational symmetry and rotational symmetry both are broken

Set-up

Problem to consider

- 5d Maxwell-Einstein theory with Chern-Simons term
- CS-coupling α is not fixed
- charged BH is still a solution, but stability analysis changes
- check if gauge fields or off-diagonal metrics become unstable, depending on α
- if unstable, find the final geometry

Analysis using AdS/CFT

- some quantities do not depend on the microscopic details (e.g. $\frac{\eta}{s} = \frac{1}{4\pi}$) Kovtun, Son, Starinets
- use bottom-up approach here

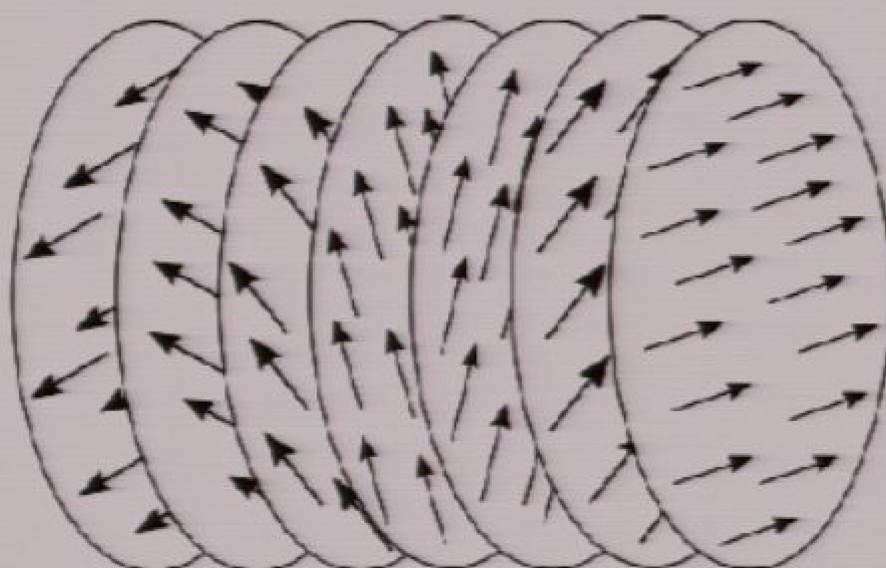
Spatially modulated phase in field theories

- Phase where fields have position dependent expectation values: e.g. plane waves
- Translational symmetry and rotational symmetry both are broken

Spatially modulated phase in field theories

- Phase where fields have position dependent expectation values: e.g. plane waves
- Translational symmetry and rotational symmetry both are broken

Pictorially,



Spatially modulated phase in field theories

Examples in field theories

- Cooper pair of two fermions with different Fermi surfaces (LOFF)
- chiral density wave in finite density QCD at large N

$$\left\langle \overline{\psi(x)}\psi(y) \right\rangle = e^{i\mathbf{P}\cdot(\mathbf{x}+\mathbf{y})} \int d^4 q e^{-iq(x-y)} f(q), \quad |P| = \mu$$

- Brazovskii model : fluctuation spectrum minimum at some $|k| > 0$

Outline

Maxwell theory with Chern-Simons in flat 5D

Maxwell theory with Chern-Simons in 3D

Deser, Jackiw, Templeton

- action $S = \int F_{\mu\nu}F^{\mu\nu} + \alpha\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho = \int F \wedge {}^*F + \alpha A \wedge dA$
- equation of motion

$$d^*F + \alpha F = 0$$

- $\Box F = d^*d^*F = -\alpha d^*F = \alpha^2 F$
- gauge field gets mass $|\alpha|$.

Maxwell theory with Chern-Simons in flat 5D

In 5-dimensions

Chern-Simons term $A \wedge dA \wedge dA$: no contribution to mass

Maxwell theory with Chern-Simons in flat 5D

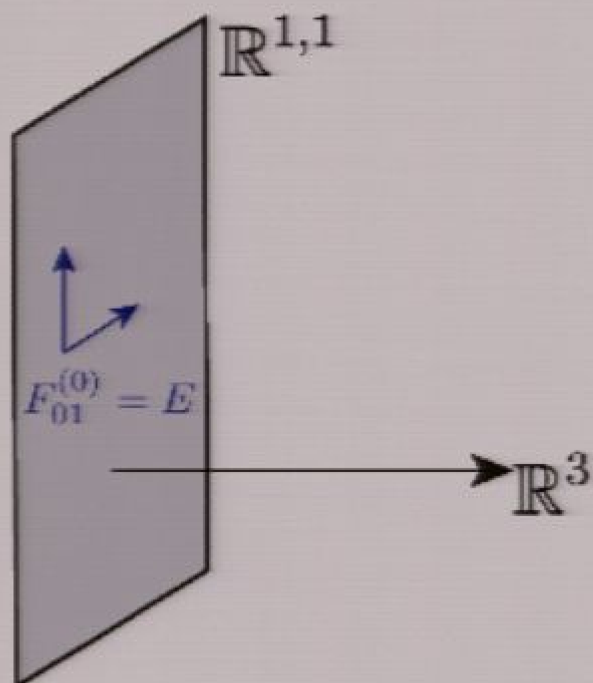
In 5-dimensions

Chern-Simons term $A \wedge dA \wedge dA$: no contribution to mass

Turn on an electric field

$A \wedge dA \wedge dA \sim EdA \wedge A$: quadratic fluctuation

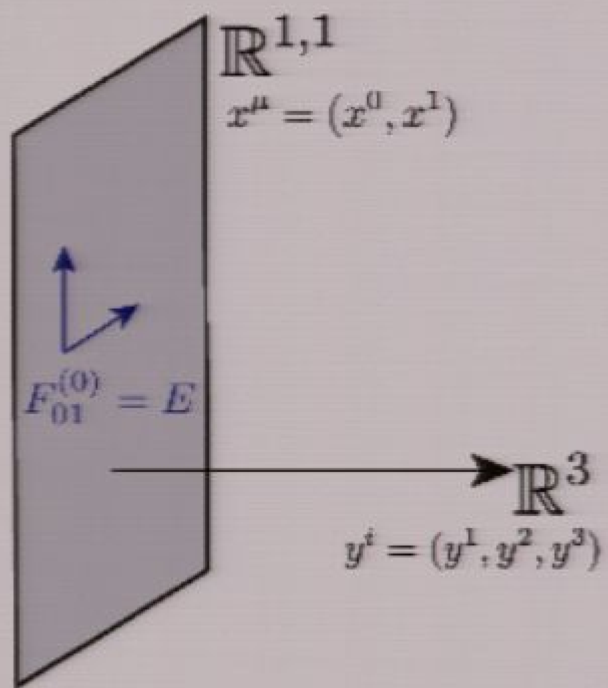
Maxwell theory with Chern-Simons in flat 5D



Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{IJ} F^{IJ} + \frac{\alpha}{6} \epsilon^{IJKLM} A_I F_{JK} F_{LM}$$

Maxwell theory with Chern-Simons in flat 5D



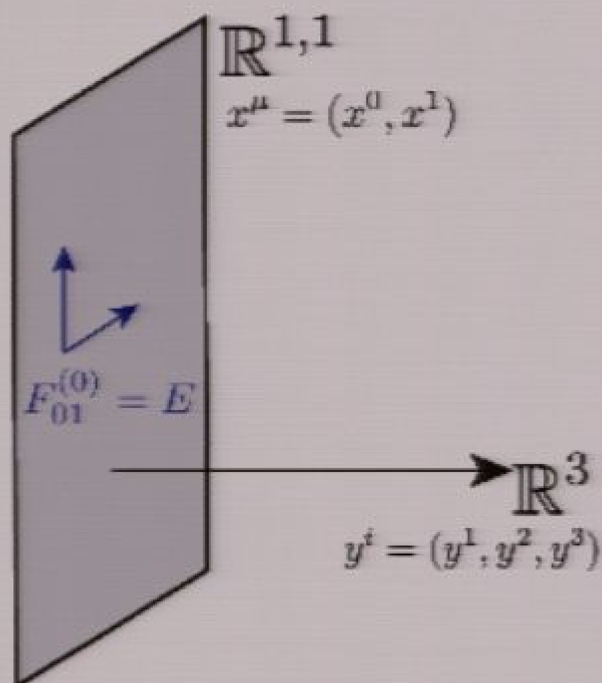
Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{IJ} F^{IJ} + \frac{\alpha}{6} \epsilon^{IJKLM} A_I F_{JK} F_{LM}$$

Consider fluctuations

$$F = F^{(0)} + f = E dx^0 \wedge dx^1 + f$$

Maxwell theory with Chern-Simons in flat 5D



Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{IJ}F^{IJ} + \frac{\alpha}{6}\epsilon^{IJKLM}A_I F_{JK} F_{LM}$$

Consider fluctuations

$$F = F^{(0)} + f = E dx^0 \wedge dx^1 + f$$

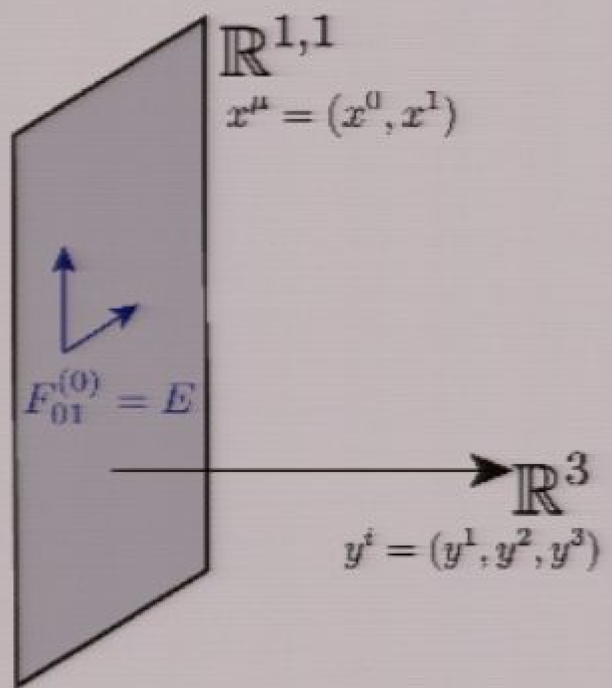
Equations of motion

$$\partial_J f^{JI} + \alpha\epsilon^{IJKLM} F^{(0)}_{JK} f_{LM} = 0$$

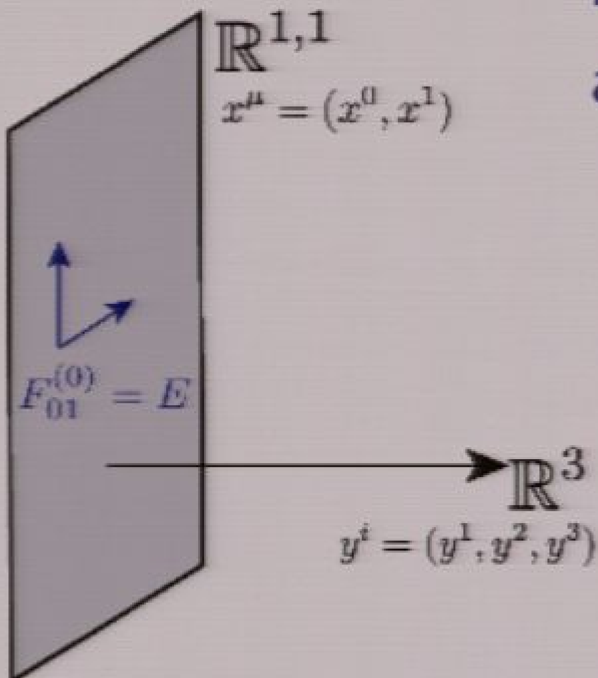
$$\Rightarrow \partial^\mu f_{\mu i} + \partial^j f_{ji} - 2\alpha E \epsilon_{ijk} f_{jk} = 0$$

Maxwell theory with Chern-Simons in flat 5D

$$\partial^\mu f_{\mu i} + \partial^j f_{ji} - 2\alpha E \epsilon_{ijk} f_{jk} = 0$$



Maxwell theory with Chern-Simons in flat 5D



$$\partial^\mu f_{\mu i} + \partial^j f_{ji} - 2\alpha E \epsilon_{ijk} f_{jk} = 0$$

Multiplying $\epsilon_{ijk}\partial_j$

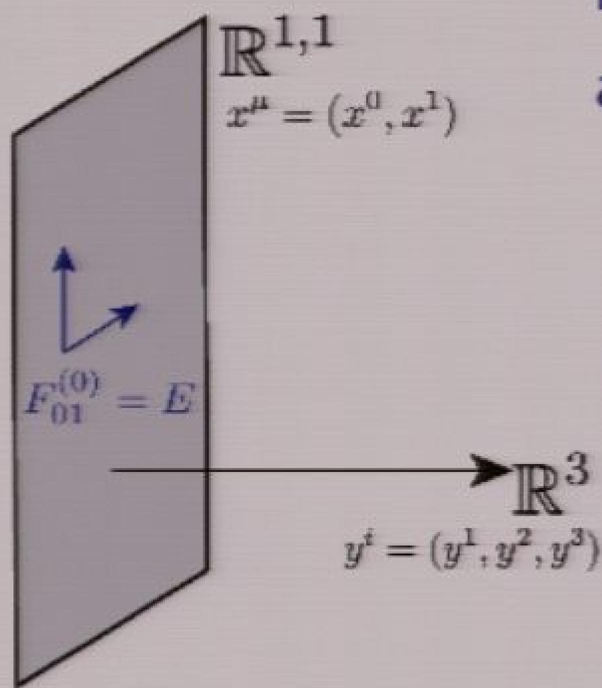
and using Bianchi identity,

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4\alpha E \epsilon_{ijk} \partial_j f_k = 0$$

$$f_i = \epsilon_{ijk} f_{jk}$$

Maxwell theory with Chern-Simons in flat 5D

$$\partial^\mu f_{\mu i} + \partial^j f_{ji} - 2\alpha E \epsilon_{ijk} f_{jk} = 0$$



Multiplying $\epsilon_{ijk}\partial_j$

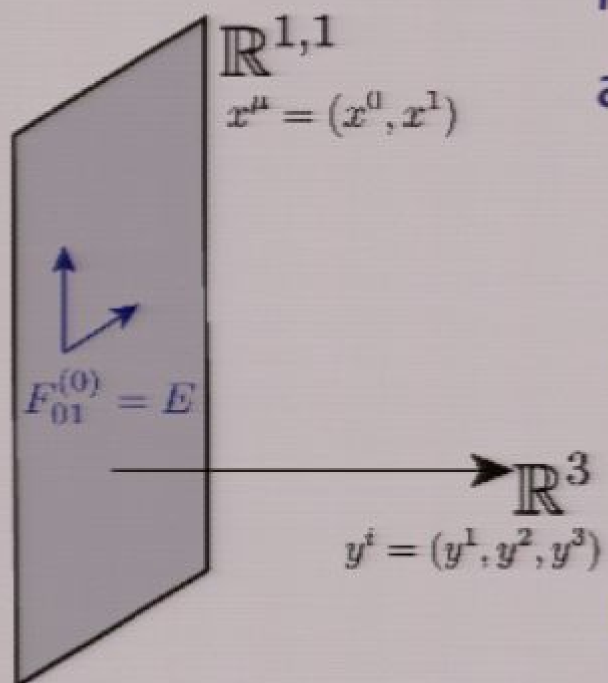
and using Bianchi identity,

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4\alpha E \epsilon_{ijk} \partial_j f_k = 0$$

$$f_i = \epsilon_{ijk} f_{jk}$$

$\epsilon_{ijk}\partial_j$ has eigenvalues $\pm k$

Maxwell theory with Chern-Simons in flat 5D



$$\partial^\mu f_{\mu i} + \partial^j f_{ji} - 2\alpha E \epsilon_{ijk} f_{jk} = 0$$

Multiplying $\epsilon_{ijk}\partial_j$

and using Bianchi identity,

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4\alpha E \epsilon_{ijk}\partial_j f_k = 0$$

$$f_i = \epsilon_{ijk} f_{jk}$$

$\epsilon_{ijk}\partial_j$ has eigenvalues $\pm k$

for $f_i \sim e^{ip_\mu x^\mu + ik_i y^i}$

Dispersion relation

$$(p_0)^2 - (p_1)^2 - k^2 \pm 4\alpha E k = 0$$

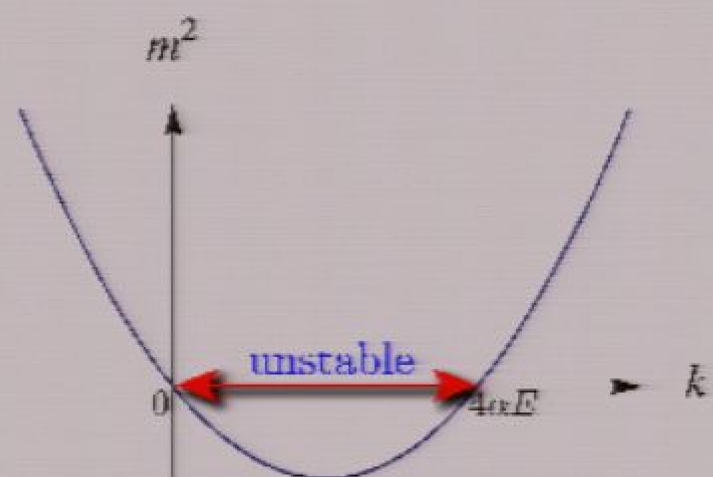
$$\Rightarrow (p_0)^2 - (p_1)^2 = k^2 \pm 4\alpha E k = (k \pm 2\alpha E)^2 - 4\alpha^2 E^2$$

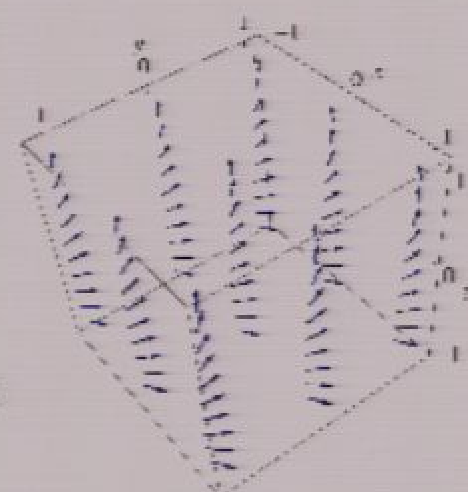
Maxwell theory with Chern-Simons in flat 5D

Dispersion relation

$$(p_0)^2 - (p_1)^2 = (k \pm 2\alpha E)^2 - 4\alpha^2 E^2$$

- tachyonic modes in $\mathbb{R}^{1,1}$ for $0 < |k| < 4|\alpha E|$
- tachyonic modes are helical: eigenmodes of $\epsilon_{ijk}\partial_j$ e.g. $f_i \sim (-\sin kz, \cos kz, 0)$ for $k_i = (0, 0, k)$
- c.f. constant magnetic field – always positive mass contribution





Outline

Maxwell theory with CS in Reissner-Nordström BH in AdS_5

Consider 5d gravity with Maxwell & CS

- Lagrangian

$$\mathcal{L} = \sqrt{-g}\left(R + 12 - \frac{1}{4}F_{IJ}F^{IJ}\right) + \frac{\alpha}{3!}\epsilon^{IJKLM}A_I F_{JK} F_{LM}$$

- do not fix α - a parameter of theory
- consider a charged black hole
  - consider a definite momentum k state in $\mathbb{R}^3$
  - parameters of a state: α, T, k

Maxwell theory with CS in Reissner-Nordström BH in AdS_5

First, consider near-horizon geometry at extremum limit ($T = 0$)

- near-horizon geometry $AdS_2 \times \mathbb{R}^3$

Maxwell theory with CS in Reissner-Nordström BH in AdS_5

First, consider near-horizon geometry at extremum limit ($T = 0$)

- near-horizon geometry $AdS_2 \times \mathbb{R}^3$

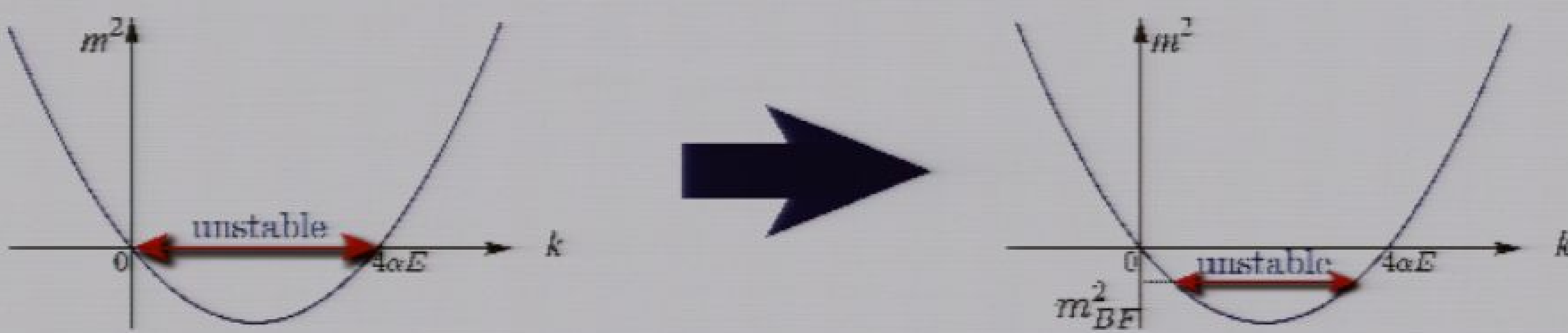
flat $\mathbb{R}^{1,1} \Rightarrow AdS_2$

negative mass allowed to some extent: Breitenlohner-Freedman bound

- the dispersion relation is the same

$$(p_0)^2 - (p_1)^2 = (k \pm 2\alpha E)^2 - 4\alpha^2 E^2$$

- but now, the lower bound $= m_{BF}^2 = -\frac{1}{4r_2^2}$



Maxwell theory with CS in Reissner-Nordström BH in AdS_5

First, consider near-horizon geometry at extremum limit ($T = 0$)

- near-horizon geometry $AdS_2 \times \mathbb{R}^3$

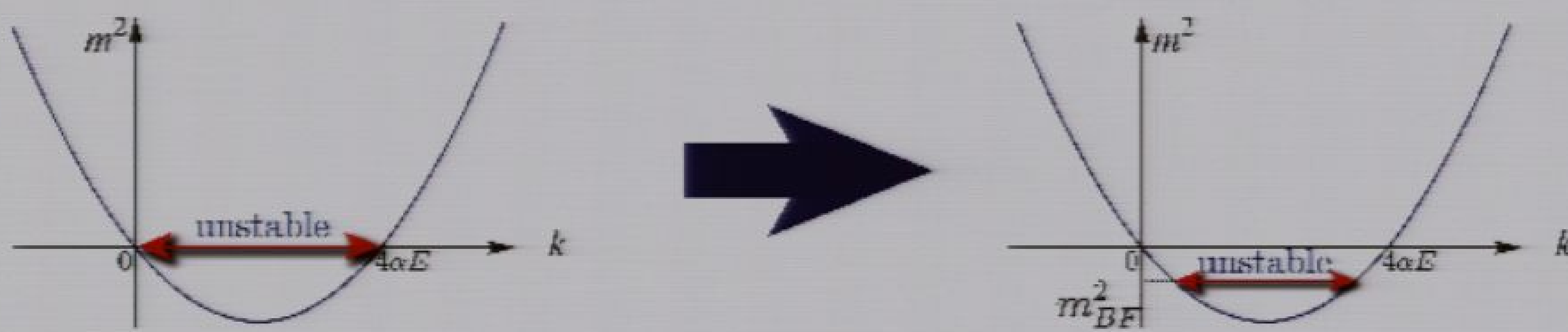
flat $\mathbb{R}^{1,1} \Rightarrow AdS_2$

negative mass allowed to some extent: Breitenlohner-Freedman bound

- the dispersion relation is the same

$$(p_0)^2 - (p_1)^2 = (k \pm 2\alpha E)^2 - 4\alpha^2 E^2$$

- but now, the lower bound $= m_{BF}^2 = -\frac{1}{4r_2^2}$



- but gauge field couples to gravity...

Coupling to Gravity

- coupling in the YM term $F_{IJ}F^{IJ}$:

$$F_{IJ}F^{IJ} = 4F^{(0)\mu\nu} h_{\mu}{}^{i} f_{\nu i} + \cdots$$

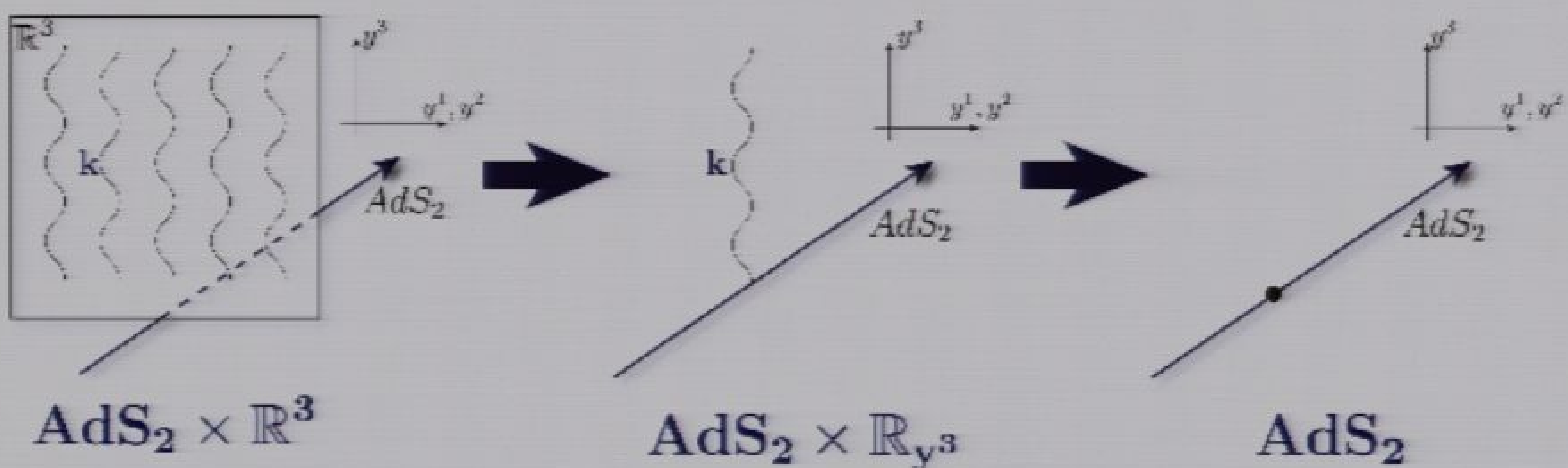
$h_{\mu}{}^{i}$: off-diagonal metric

- use standard linearized gravity techniques
- or, use Kaluza-Klein reduction with momentum $\vec{k}$ along $\mathbb{R}^3$ and linearize

Coupling to Gravity

Kaluza-Klein reduction

$AdS_2 \times \mathbb{R}^3 \Rightarrow AdS_2$



- consider a state with momentum k along y^3
- relevant fields: $f_i = \epsilon_{ijk} f_{jk}$, $K^i = \epsilon^{\mu\nu} \partial_\mu h_\nu{}^i$

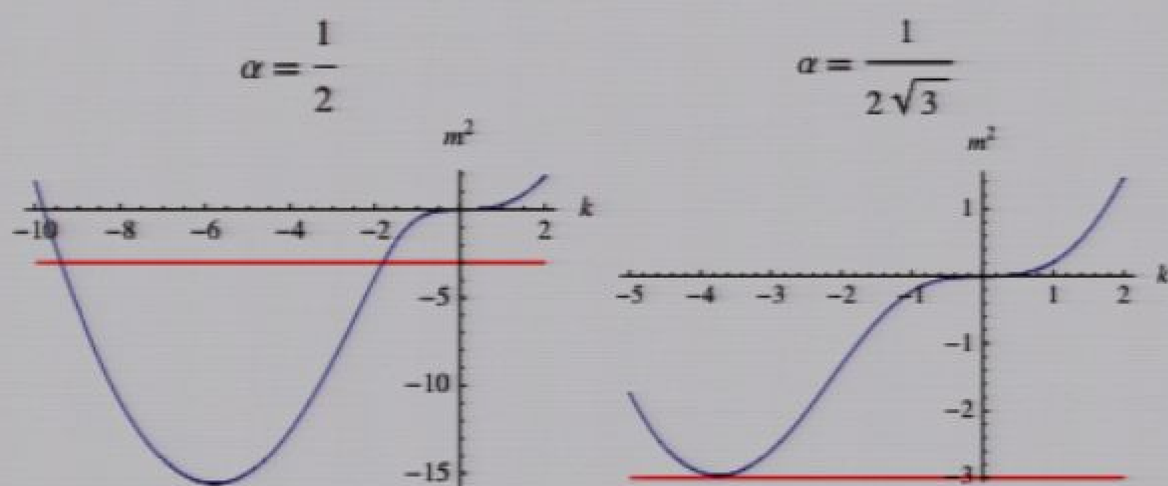
$$\left(\Box_{AdS_2} + \partial^j \partial_j\right) f_i - 4\alpha E \epsilon_{ijk} \partial_j f_k + E \epsilon_{ijk} \partial_j K_k = 0,$$

$$E\, \Box_{AdS_2} f_i + \left(\Box_{AdS_2} + \partial^j \partial_j\right) \epsilon_{ijk} \partial_j K_k = 0$$

Coupling to Gravity

- masses given by $(E = 2\sqrt{6})$

$$m^2 = \frac{1}{2}\left(2k^2 + E^2 + 4\alpha Ek \pm \sqrt{E^4 + 8\alpha E^3 k + 4(1+4\alpha^2)E^2k^2}\right)$$



- $\alpha > \alpha_{\text{crit}} = 0.2896$ instability occurs

Maxwell theory with CS in Reissner-Nordström BH in AdS_5

So far, near horizon analysis of extremal black holes($T = 0$)

Maxwell theory with CS in Reissner-Nordström BH in AdS_5

So far, near horizon analysis of extremal black holes($T = 0$)

Question

- Does instability persist at non-zero T?
- Is the near-horizon analysis sufficient?

$\Rightarrow$ Need to analyze the full RN geometry

Maxwell theory with CS in Reissner-Nordström BH in AdS_5

Consider a charged RN black hole

- Fields to be considered: gauge field and off-diagonal metric components with some amount of momentum along $\mathbb{R}^3$

$$f_i = \epsilon_{ijk} f_{jk}, K^i = \epsilon^{\mu\nu} \partial_\mu h_\nu{}^i$$

- Look for static solutions $\Leftarrow$ onset of instability

Maxwell theory with CS in Reissner-Nordström BH in AdS_5

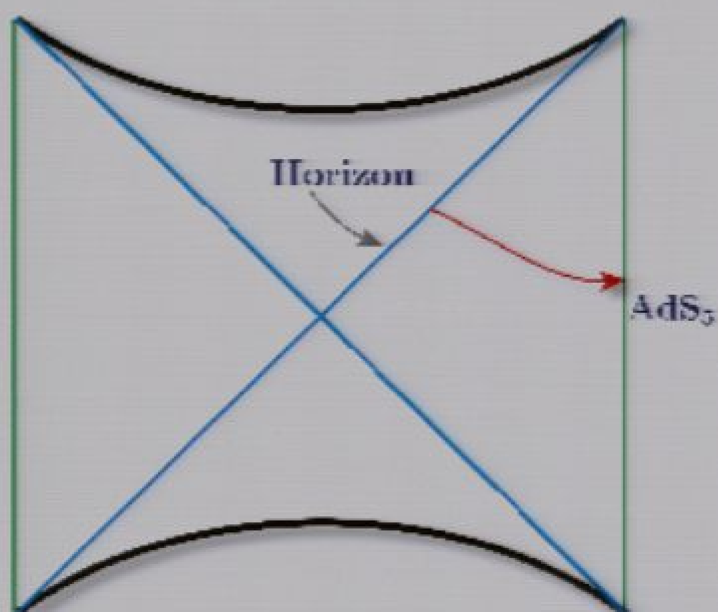
Procedure for numerical analysis

- obtain linearized equations for gauge field and off-diagonal metric (f_i and K^i)
- try to simplify $\Rightarrow$ diagonalization possible
- start at the horizon and obtain series solutions

Maxwell theory with CS in Reissner-Nordström BH in AdS_5

Procedure for numerical analysis

- obtain linearized equations for gauge field and off-diagonal metric (f_i and K^i)
- try to simplify $\Rightarrow$ diagonalization possible
- start at the horizon and obtain series solutions



- evolve the solution to AdS_5 boundary
- evolving pattern depends on k
- generically, a solution is not normalizable, but for some k, it is

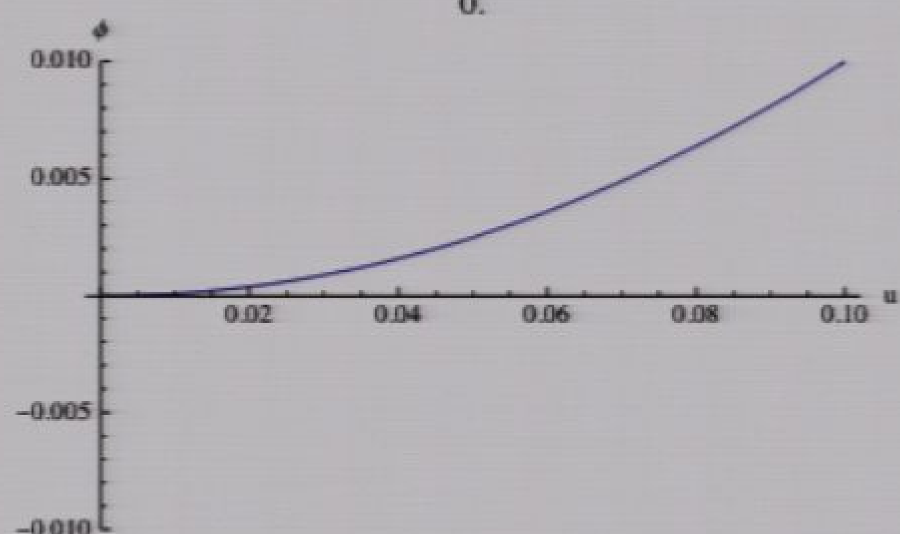
Normalizable static solutions in RN black hole

At $\alpha = 1.6\alpha_{\text{crit}},\ T = 2.9 \times 10^{-8}\mu$

- $u = 0$: AdS_5 boundary
- $\phi = a + bu^2 + cu^2 \log u$ near AdS_5 boundary
- $\phi \to 0$ near AdS_5 boundary if normalizable

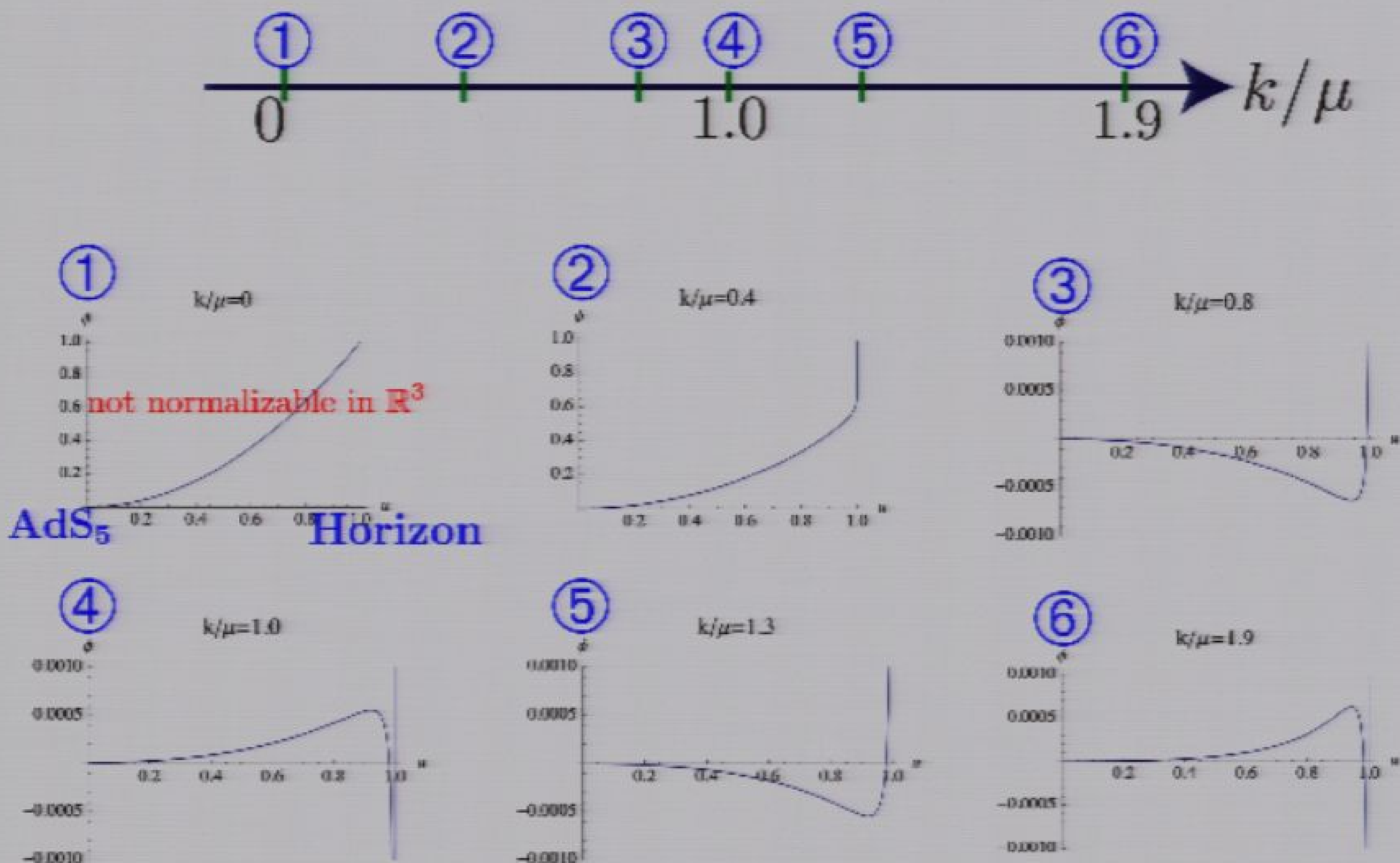
k/μ

0.



Normalizable static solutions in RN black hole

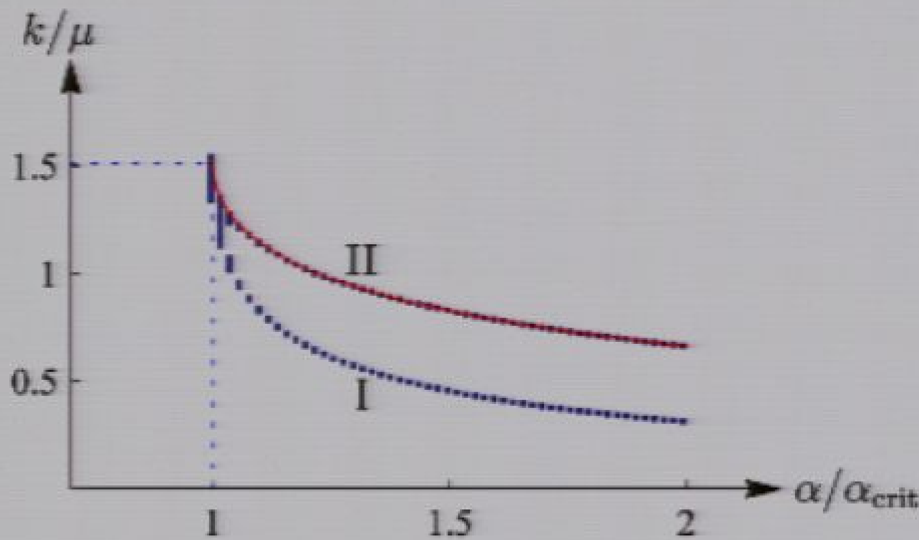
At $\alpha = 1.6\alpha_{\rm crit}$, $T = 2.9\times10^{-8}\mu$



many zero modes, but only one pair is physical

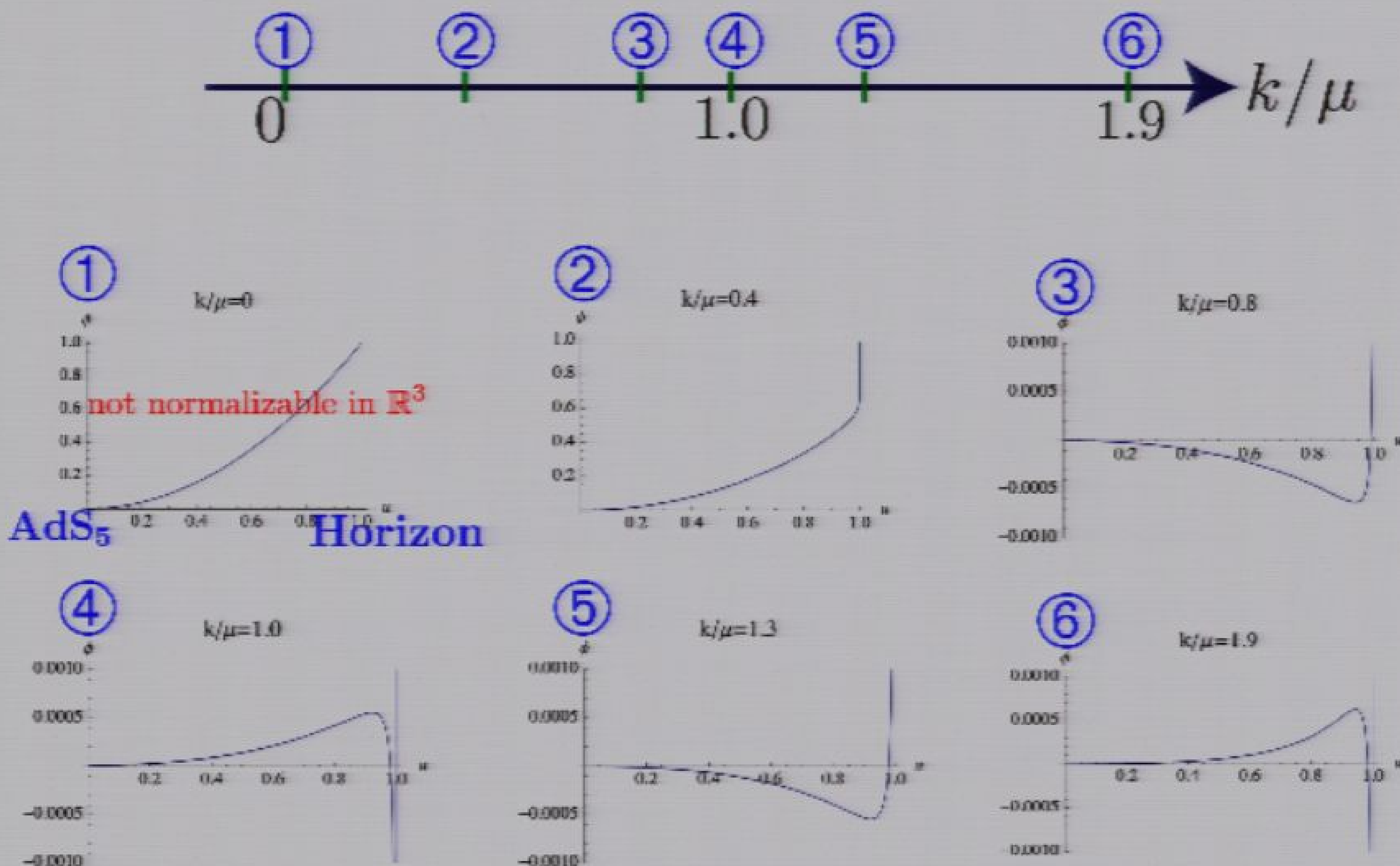
Normalizable static solutions in RN black hole

Let's vary α at $T = 0$



Normalizable static solutions in RN black hole

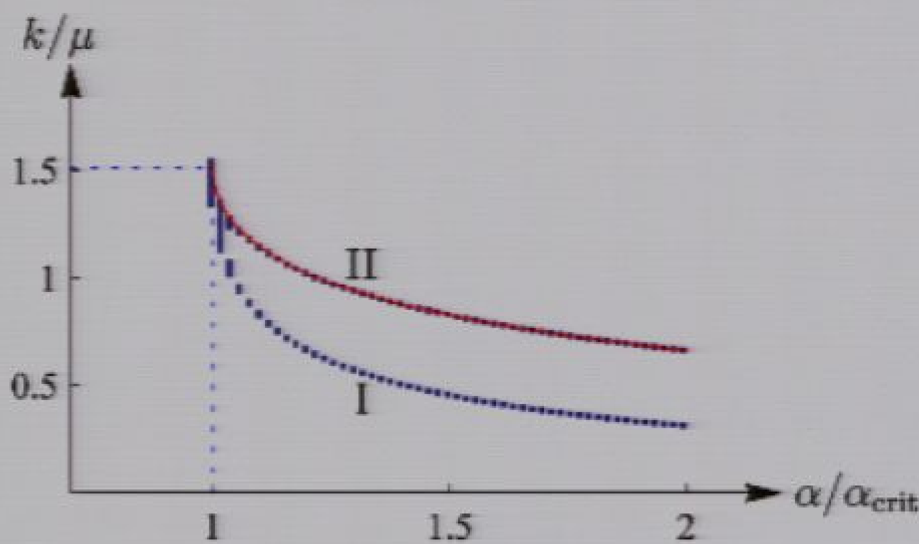
At $\alpha = 1.6\alpha_{\text{crit}}$, $T = 2.9 \times 10^{-8}\mu$



many zero modes, but only one pair is physical

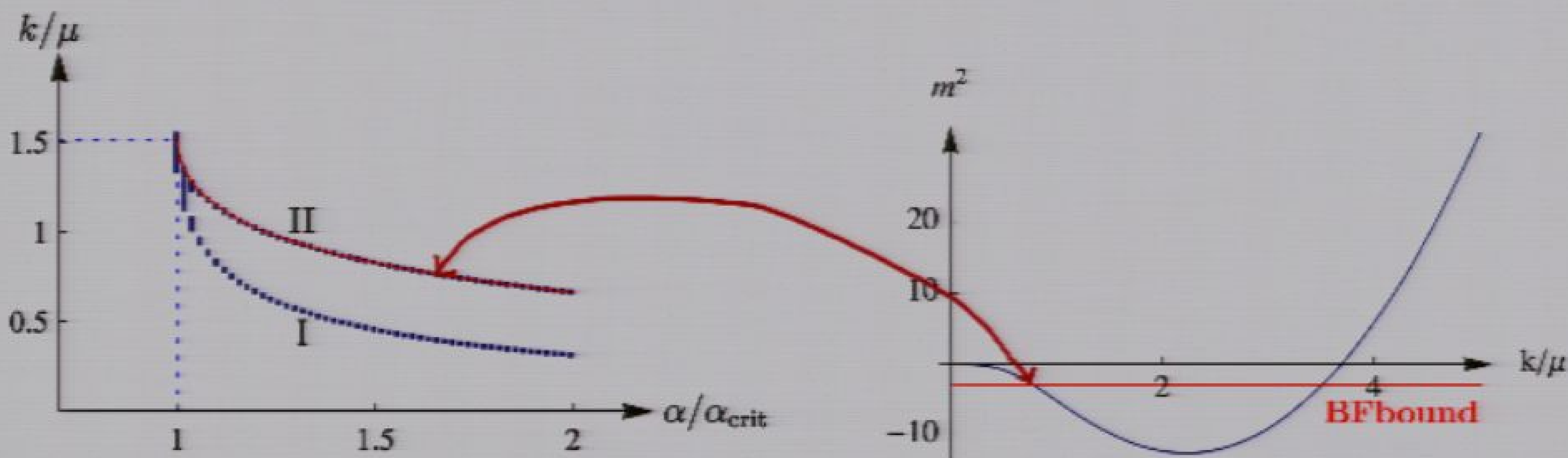
Normalizable static solutions in RN black hole

Let's vary α at $T = 0$



Normalizable static solutions in RN black hole

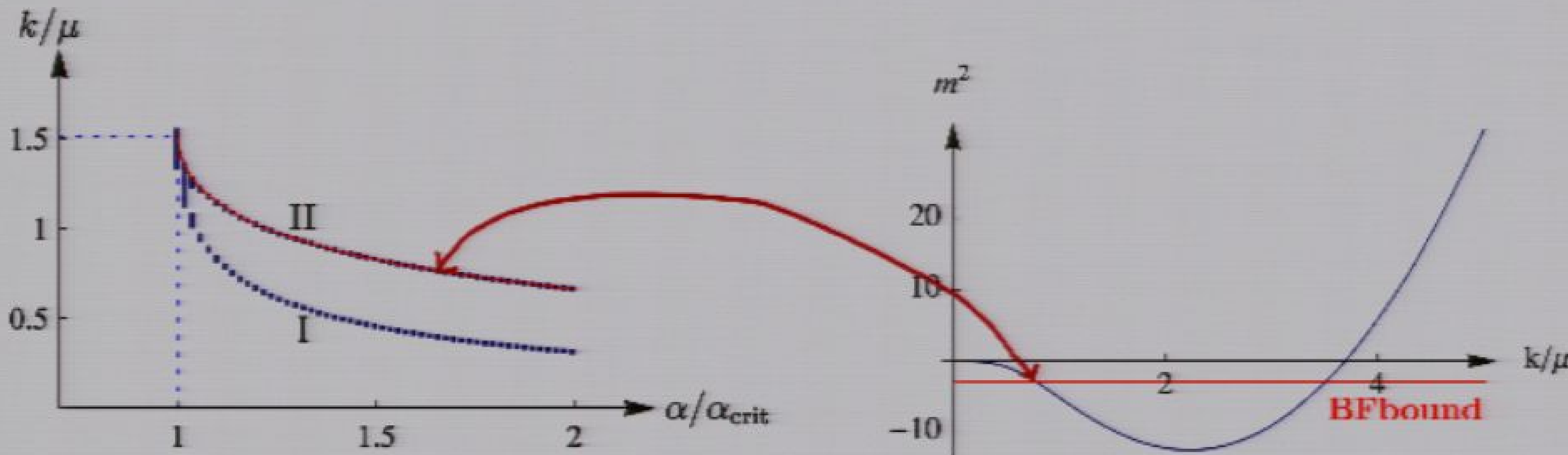
Let's vary α at $T = 0$



- $k = 0$ solution is not normalizable
- curve II is one end of near-horizon zero mode

Normalizable static solutions in RN black hole

Let's vary α at $T = 0$



- $k = 0$ solution is not normalizable
- curve II is one end of near-horizon zero mode

What is curve I?

Normalizable static solutions in RN black hole

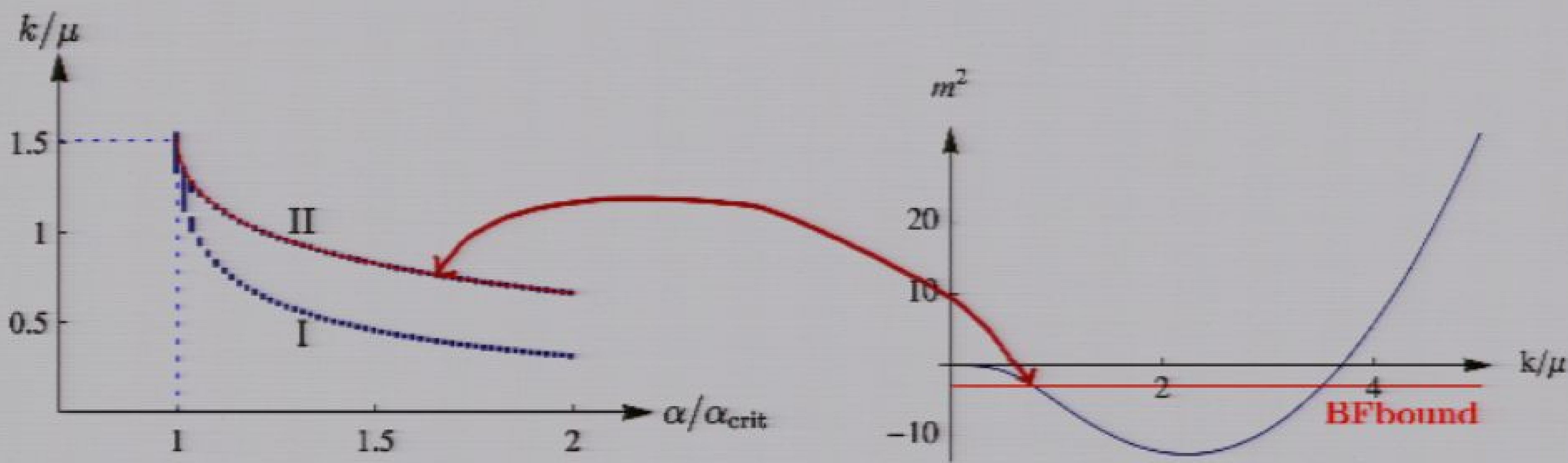
- near-horizon analysis does not capture all physics
- some static solutions appear non-normalizable near-horizon e.g. curve I
- but the critical $\alpha_{\rm crit}$ is the same

Are they really boundaries of unstable modes?

Need to turn on time dependence $e^{-i\omega t}$

Normalizable static solutions in RN black hole

Let's vary α at $T = 0$



- $k = 0$ solution is not normalizable
- curve II is one end of near-horizon zero mode

What is curve I?

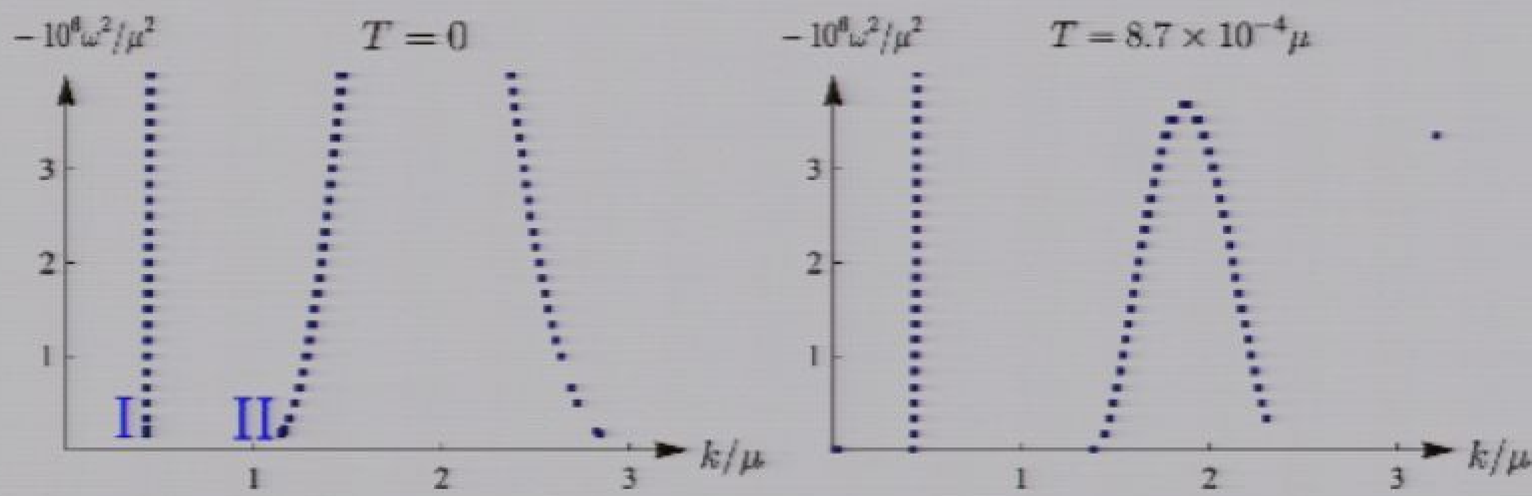
Normalizable static solutions in RN black hole

- near-horizon analysis does not capture all physics
- some static solutions appear non-normalizable near-horizon e.g. curve I
- but the critical $\alpha_{\rm crit}$ is the same

Are they really boundaries of unstable modes?

Need to turn on time dependence $e^{-i\omega t}$

Dispersion relations for fixed α



- $\alpha = 1.6\alpha_{\text{crit}}$
- fields $\sim e^{-i\omega t}$
- static solutions are boundaries of unstable solutions

Normalizable static solutions in RN black hole

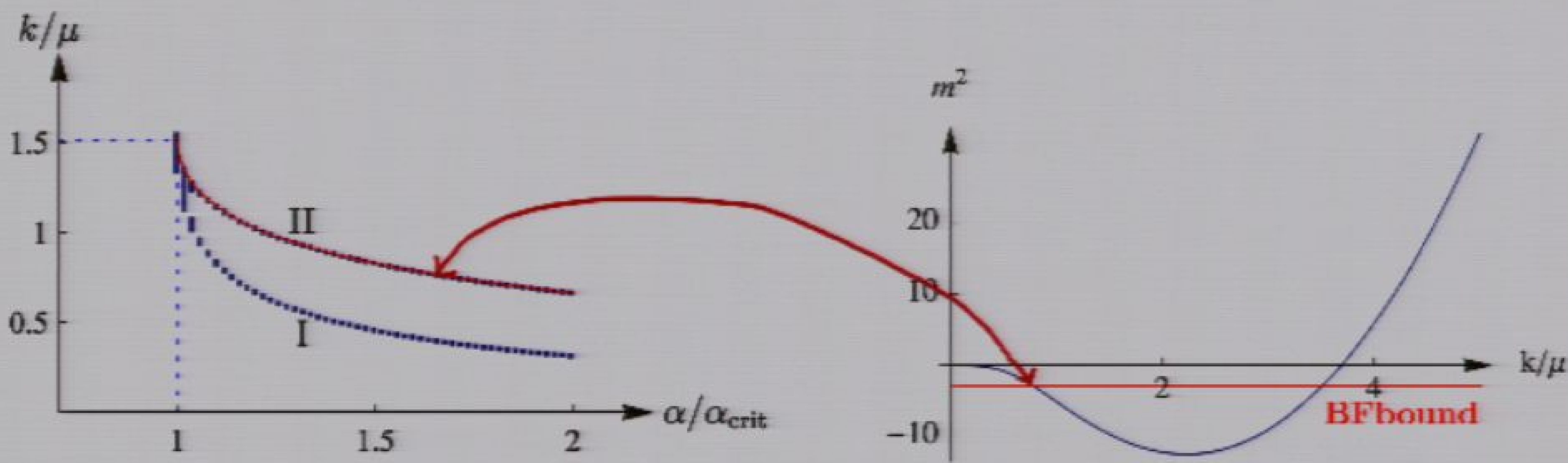
- near-horizon analysis does not capture all physics
- some static solutions appear non-normalizable near-horizon e.g. curve I
- but the critical α_{crit} is the same

Are they really boundaries of unstable modes?

Need to turn on time dependence $e^{-i\omega t}$

Normalizable static solutions in RN black hole

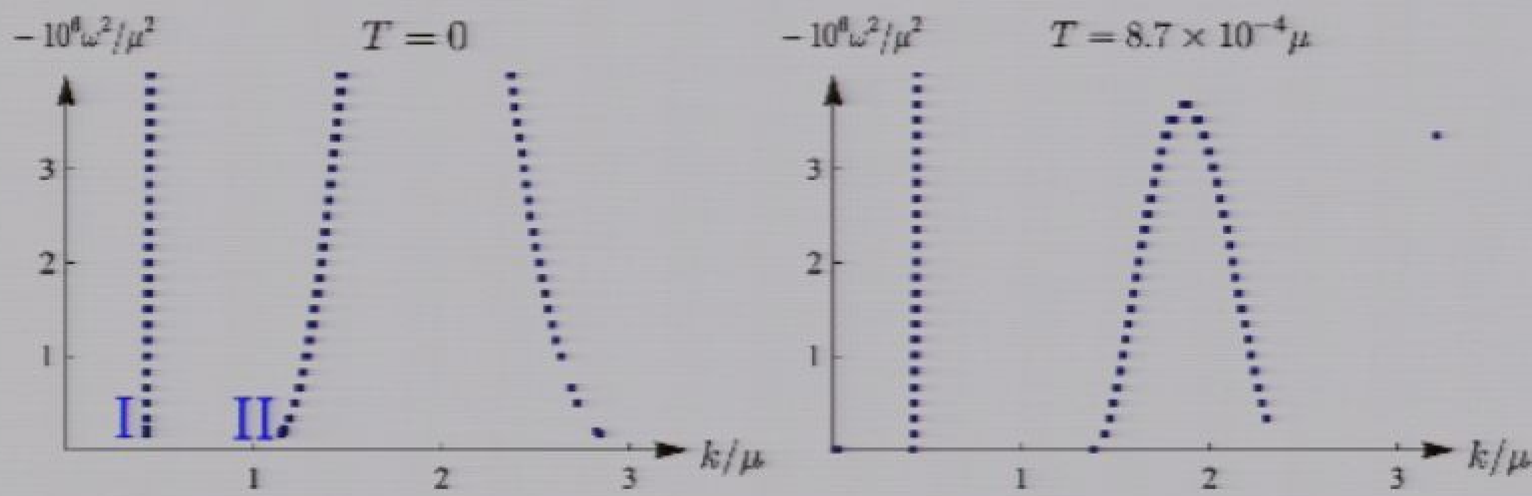
Let's vary α at $T = 0$



- $k = 0$ solution is not normalizable
- curve II is one end of near-horizon zero mode

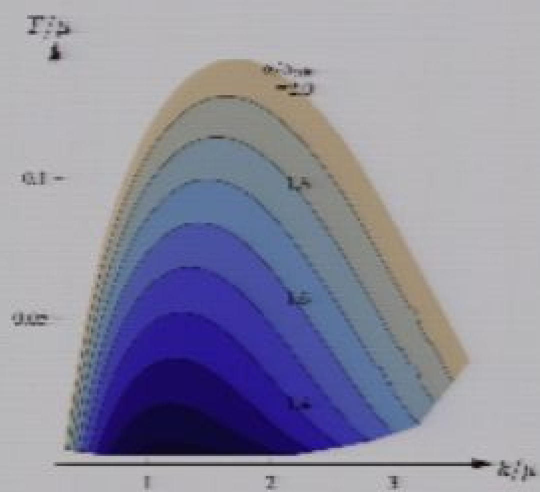
What is curve I?

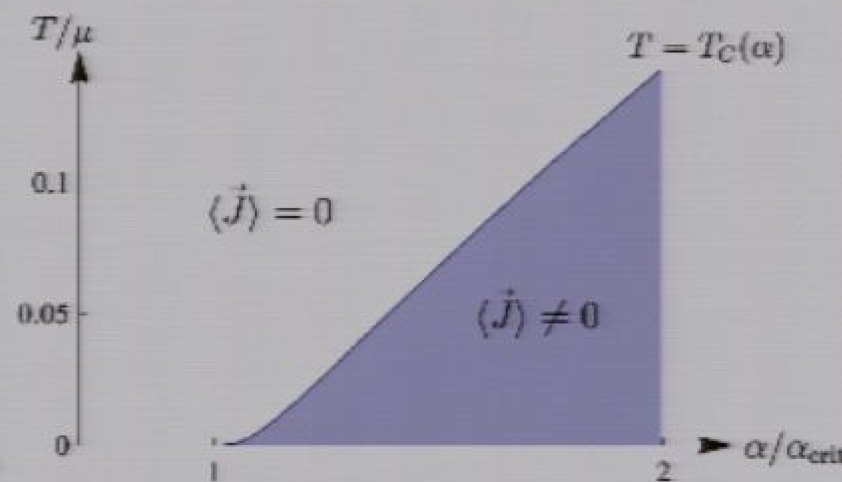
Dispersion relations for fixed α



- $\alpha = 1.6\alpha_{\mathrm{crit}}$
- fields $\sim e^{-i\omega t}$
- static solutions are boundaries of unstable solutions

Static solutions for varying α, T and k





- more unstable as α grows
- $k = 0$ excluded$\Rightarrow$ position dependent, inhomogeneous phase

What will be the end of the instability

We found that there is an instability.
What will be the final state?

What will be the end of the instability

Participating modes

- $f_i = \frac{1}{2}\epsilon_{ijk} f_{jk}$
- $K^i = \epsilon^{\mu\nu}\partial_\mu h_\nu{}^i$

- unstable modes are eigenstates of $\epsilon_{ijk}\partial_j$ in $\mathbb{R}^3$
- $\vec{f} + \vec{K} \sim \mathrm{Re}(\vec{u} e^{ikx})$
  with $\vec{k} \times \vec{u} = \pm i|k|\vec{u}$
- helical state

What will be the end of the instability

- Solving Einstein equations is difficult.
- Consider a limit $\alpha \to \infty$.

What will be the end of the instability

Participating modes

- $f_i = \frac{1}{2}\epsilon_{ijk} f_{jk}$
- $K^i = \epsilon^{\mu\nu}\partial_\mu h_\nu{}^i$

- unstable modes are eigenstates of $\epsilon_{ijk}\partial_j$ in $\mathbb{R}^3$
- $\vec{f} + \vec{K} \sim \mathrm{Re}(\vec{u}e^{ikx})$
  with $\vec{k} \times \vec{u} = \pm i|k|\vec{u}$
- helical state

What will be the end of the instability

- Solving Einstein equations is difficult.
- Consider a limit $\alpha \to \infty$.

What will be the end of the instability

- Solving Einstein equations is difficult.
- Consider a limit $\alpha \to \infty$.

What will be the end of the instability

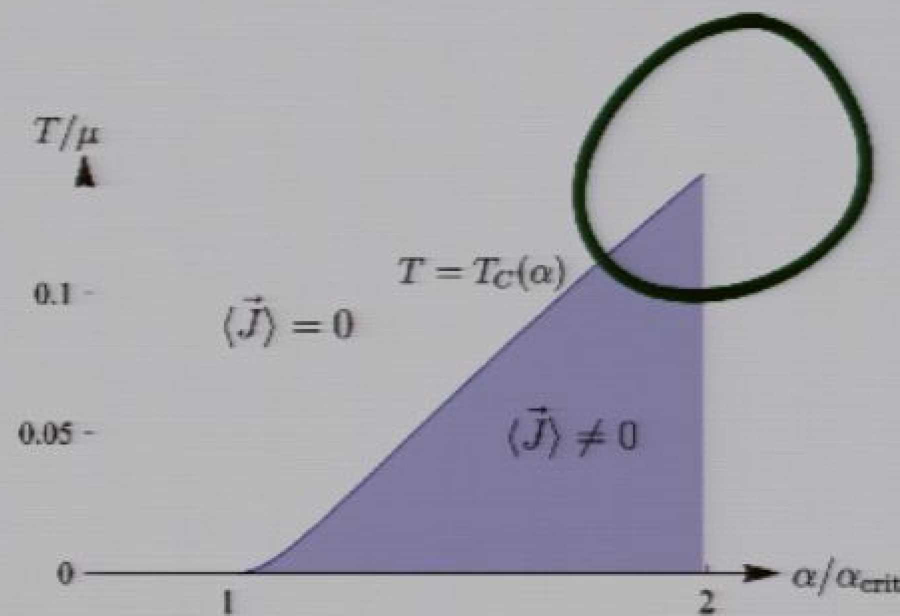
- Solving Einstein equations is difficult.
- Consider a limit $\alpha \to \infty$.

$$S_{Maxwell} = \int F \wedge *F + \alpha A \wedge F \wedge F$$
$$= \frac{1}{\alpha^2}\left(\int F' \wedge *F' + A' \wedge F' \wedge F'\right)$$
$$\text{where} \quad F = \frac{1}{\alpha} F'$$

- finite F' solutions do not back-react to geometry.
- $\alpha\mu$ is kept fixed (μ: chemical potential) .
- Consider Schwarzschild black hole solution.

What will be the end of the instability

Probe the region where T/μ and α scale the same.



Chance to see phase transition

What will be the end of the instability

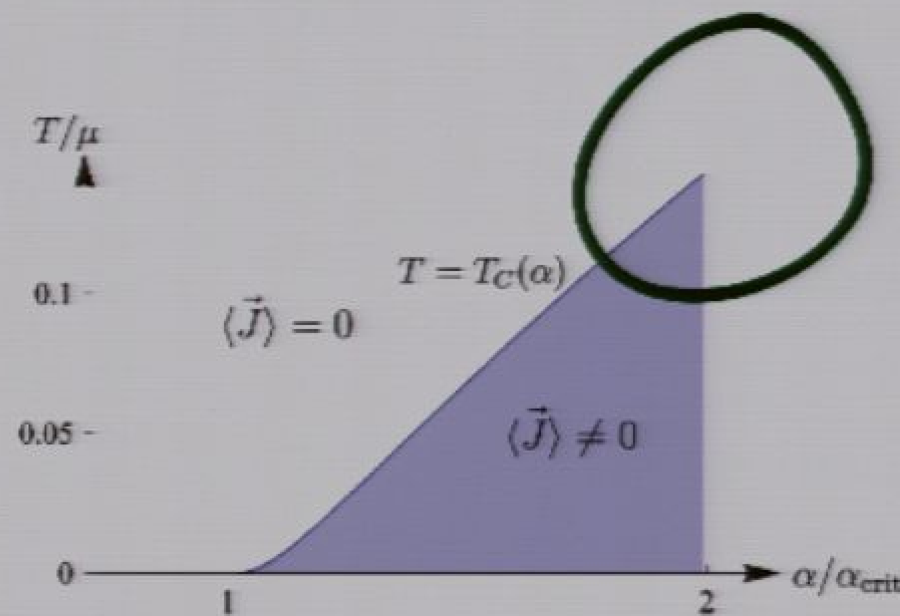
- Solving Einstein equations is difficult.
- Consider a limit $\alpha \to \infty$.

$$S_{Maxwell} = \int F \wedge *F + \alpha A \wedge F \wedge F$$
$$= \frac{1}{\alpha^2}\left(\int F' \wedge *F' + A' \wedge F' \wedge F'\right)$$
$$\text{where} \quad F = \frac{1}{\alpha} F'$$

- finite F' solutions do not back-react to geometry.
- $\alpha\mu$ is kept fixed (μ: chemical potential) .
- Consider Schwarzschild black hole solution.

What will be the end of the instability

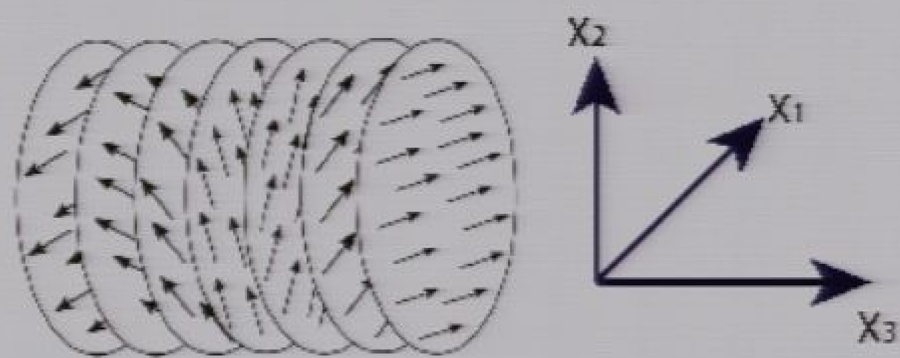
Probe the region where T/μ and α scale the same.



Chance to see phase transition

What will be the end of the instability

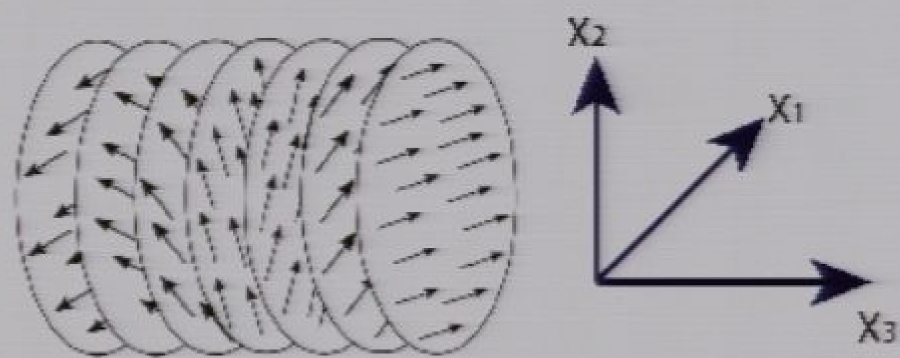
- Instability analysis suggests a final state of the form



- Invariant under P_1, P_2 and $P_3 + kR_{12}$.

What will be the end of the instability

- Instability analysis suggests a final state of the form



- Invariant under P_1, P_2 and $P_3 + kR_{12}$.
- Gauge fields: $A_t(r), A_r(r), A_1(r)$ and $A_2 + iA_3 = a(r)e^{ikx_3}$
- EOM for the bulk dual field a:

$$\partial_r \left[H(r) r a'(r)\right] - \frac{1}{r} k^2 a(r) + 4\alpha k \frac{a(r)}{r^3}\left(E - 2\alpha k a(r)^2\right) = 0$$

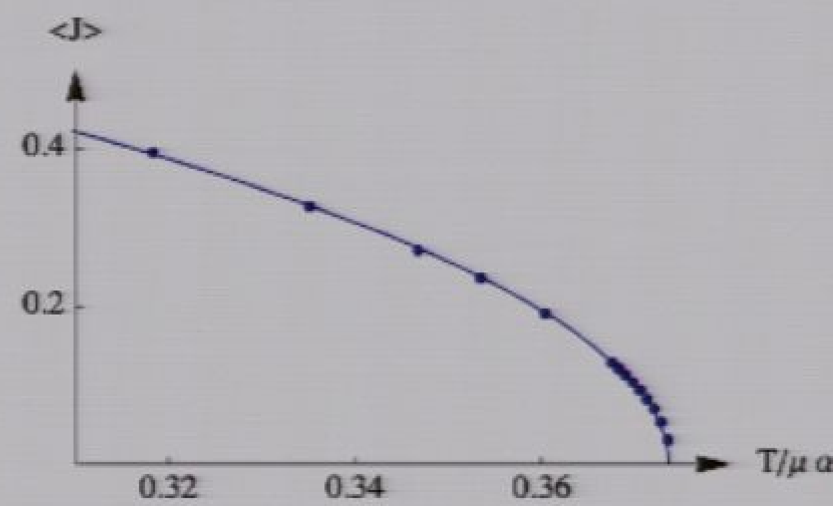
where

$$H(r) = r^2 \left[1 - \left(\frac{r_+}{r}\right)^4\right] .$$

- Choose k such that $a(r)$ is normalizable at AdS_5 boundary.

Spatially modulated phase

Expectation value of dual operator $\langle J \rangle$ to gauge field $a(r)$



$$\langle J \rangle \sim \left(1 - \frac{T}{T_C}\right)^{\frac{1}{2}}$$

- a family of solutions exists.
- only minimum energy solution is chosen.
- $1/N$ effect may change the order of phase transition.

(c.f. Brazovskii / Hohenberg and Swift)

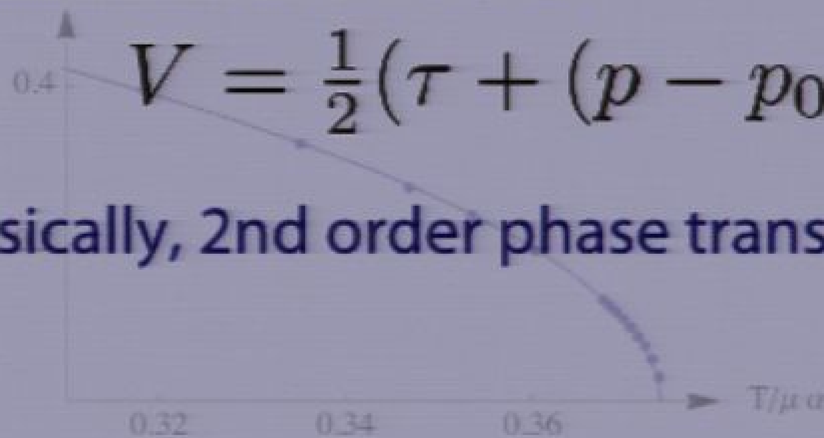
Spatially modulated phase

Expectation value of dual operator $\langle J \rangle$ to gauge field $a(r)$

Brazovskii model

$$V = \frac{1}{2}(\tau + (p - p_0)^2)\phi^2 + \frac{\lambda}{4}\phi^4$$

Classically, 2nd order phase transition at $\tau = 0$.



$$\langle J \rangle \sim \left(1 - \frac{T}{T_C}\right)^{\frac{1}{2}}$$

- a family of solutions exists.
- only minimum energy solution is chosen.
- $1/N$ effect may change the order of phase transition.

(c.f. Brazovskii / Hohenberg and Swift)

Spatially modulated phase

Expectation value of dual operator $\langle J \rangle$ to gauge field $a(r)$

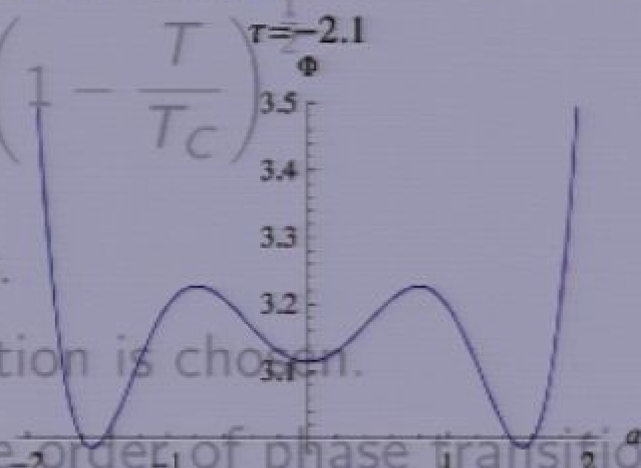
Brazovskii model

$$V = \frac{1}{2}(\tau + (p - p_0)^2)\phi^2 + \frac{\lambda}{4}\phi^4$$

Classically, 2nd order phase transition at $\tau = 0$.

Quantum mechanically, large number of states at $p=p_0$ makes it 1st order.

$$\langle J \rangle \sim \left(1 - \frac{T}{T_C}\right)^{\frac{1}{2}}$$



- a family of solutions exists.
- only minimum energy solution is chosen.
- $1/N$ effect may change the order of phase transition.

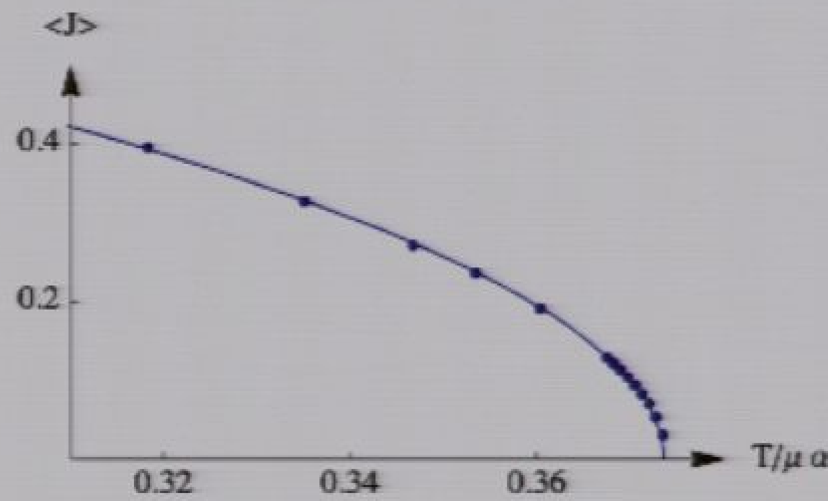
(c.f. Brazovskii / Hohenberg and Swift)

Outline

1. Motivation
2. Flat-space warm-up
3. CS-coupling in charged AdS_5 black holes
4. **Supergravity models**
5. Conclusions

Spatially modulated phase

Expectation value of dual operator $\langle J \rangle$ to gauge field $a(r)$



$$\langle J \rangle \sim \left(1 - \frac{T}{T_C}\right)^{\frac{1}{2}}$$

- a family of solutions exists.
- only minimum energy solution is chosen.
- $1/N$ effect may change the order of phase transition.

(c.f. Brazovskii / Hohenberg and Swift)

Outline

Stability of supergravity models

Until now, α is a parameter of the theory.
Now let's fix it by considering some SUGRA examples

- the simplest model: $\mathcal{N} = 2$ minimal SUGRA model

$$\mathcal{L} = \sqrt{-g}\left(R + 12 - \frac{1}{4}F_{IJ}F^{IJ}\right) + \frac{\alpha}{3!}\epsilon^{IJKLM}A_I F_{JK} F_{LM}$$

with $\alpha = \frac{1}{2\sqrt{3}} = 0.2887$

- $\alpha_{\text{crit}} = 0.2896$

Stability of supergravity models

Until now, α is a parameter of the theory.
Now let's fix it by considering some SUGRA examples

- the simplest model: $\mathcal{N} = 2$ minimal SUGRA model

$$\mathcal{L} = \sqrt{-g}\left(R + 12 - \frac{1}{4}F_{IJ}F^{IJ}\right) + \frac{\alpha}{3!}\epsilon^{IJKLM}A_I F_{JK} F_{LM}$$

with $\alpha = \frac{1}{2\sqrt{3}} = 0.2887$

- $\alpha_{\rm crit} = 0.2896$

charged BH in minimal SUGRA is barely stable

Stability of supergravity models

Consider three-charge black holes

- dimensional reduction of rotating D3-branes in 10d
- 3 charges from the 3 rotation directions
- additional matter fields couple to gauge fields $\Rightarrow$ can be ignored in linear analysis
- instead of 1, there are 3 gauge fields that couple to off-diagonal metric
- solve mass-spectrum near horizon $AdS_2 \times \mathbb{R}^3$
- BF bound is not violated

Stability of supergravity models

Consider three-charge black holes

- dimensional reduction of rotating D3-branes in 10d
- 3 charges from the 3 rotation directions
- additional matter fields couple to gauge fields $\Rightarrow$ can be ignored in linear analysis
- instead of 1, there are 3 gauge fields that couple to off-diagonal metric
- solve mass-spectrum near horizon $AdS_2 \times \mathbb{R}^3$
- BF bound is not violated

extremal three-charge BH near horizon is stable

Stability of supergravity models

Other models?

- CS-couplings $\frac{1}{24\pi^2} \int c_{IJK} A^I \wedge F^J \wedge F^K \Leftrightarrow$ triangle anomalies
- c_{IJK} determined by toric data for toric Sasaki-Einstein manifold Benvenuti, Pando Zayas, Tachikawa
- gauge coupling τ_{IJ} determined from the geometry also
- possibility to find a supersymmetric model with unstable modes
- or, maybe there is a constraint that forbids to violate stability bound

Outline

1. Motivation
2. Flat-space warm-up
3. CS-coupling in charged AdS_5 black holes
4. Supergravity models
5. **Conclusions**

Conclusions

Summary

- Maxwell theory with CS-coupling in flat 5d becomes unstable for constant electric field
- Maxwell-CS-Einstein theory unstable for $\alpha > \alpha_{\mathrm{crit}}$
- Instability only for modes with k in some range ($k = 0$ excluded)

  Position dependent vev of $\vec{J}$ dual field theory

- vector modes get vev
- mean field type, 2nd order phase transition

Gravity Dual of Spatially Modulated Phase

Chang-Soon Park (CALTECH)

arXiv:0911.0679 (S. Nakamura, H. Ooguri, CP)
and work in progress with H. Ooguri

Perimeter Institute – May 11, 2010

Conclusions

Summary

- Maxwell theory with CS-coupling in flat 5d becomes unstable for constant electric field
- Maxwell-CS-Einstein theory unstable for $\alpha > \alpha_{\text{crit}}$
- Instability only for modes with k in some range ($k = 0$ excluded)

  Position dependent vev of $\vec{J}$ dual field theory

- vector modes get vev
- mean field type, 2nd order phase transition