Title: Interacting Fibonacci anyons and defects in conformal field theory

Date: May 12, 2010 04:00 PM

URL: http://pirsa.org/10050019

Abstract: Fibonacci anyons are the simplest system of anyons capable of implementing universal topological quantum computation, an area which is of intense theoretical and experimental interest. Recent studies have shown that for nearest-neighbour interactions, the properties of the ground state of a 1-D chain of Fibonacci anyons may be modeled using a spin chain, and are related to specific conformal field theories. I will talk about the role played by boundary conditions in this mapping, and demonstrate that for these simple anyonic systems the correct spin chain models in fact correspond to conformal field theories with a defect. The presence of this defect drastically changes the excitations observable in the system.

Fibonacci Anyons and Defects in Conformal Field Theory

Robert Pfeifer and Guifre Vidal
The University of Queensland

The University of Queensland
Australia

INTRODUCTION

- Anyons
  - Quasiparticles, collective excitations
  - Fractional statistics
  - Potential for quantum computation

$$\sum_i c_i|i\rangle = |\psi\rangle$$

Fibonacci Anyons and Defects in Conformal Field Theory

Robert Pfeifer and Guifre Vidal
The University of Queensland

The University
Of Queensland
Australia

Introduction

- Anyons
  - Quasiparticles, collective excitations
  - Fractional statistics
  - Potential for quantum computation

$$\sum_i c_i|i\rangle = |\psi\rangle$$

Introduction

- Anyons
  - Quasiparticles, collective excitations
  - Fractional statistics
  - Potential for quantum computation

$$\sum_i c_i |i\rangle = |\psi\rangle$$

INTRODUCTION

- Anyons
  - Quasiparticles, collective excitations
  - Fractional statistics
  - Potential for quantum computation

$|\psi\rangle$

Introduction

- Anyons
  - Quasiparticles, collective excitations
  - Fractional statistics
  - Potential for quantum computation

$$-|\psi\rangle$$

Introduction

- Anyons
  - Quasiparticles, collective excitations
  - Fractional statistics
  - Potential for quantum computation

$$\sum_{j,i} c_j (e^{\mathrm{i}\hat{M}})_{ji} |i\rangle = |\psi'\rangle$$

Introduction

- Anyons
  - Quasiparticles, collective excitations
  - Fractional statistics
  - Potential for quantum computation
- Conformal Field Theories
  - On a lattice
  - With defects

Anyons

- List of charges:

Anyons

- List of charges: $1 \quad \tau$
- Fusion rules:

$$\begin{aligned} 1 \times 1 &\rightarrow 1 \\ 1 \times \tau &\rightarrow \tau \\ \tau \times 1 &\rightarrow \tau \\ \tau \times \tau &\rightarrow 1 + \tau \end{aligned}$$

Anyons

- List of charges: $1 \quad \tau$
- Fusion rules:

$$\begin{aligned} 1 \times 1 &\rightarrow 1 \\ 1 \times \tau &\rightarrow \tau \\ \tau \times 1 &\rightarrow \tau \\ \tau \times \tau &\rightarrow 1 + \tau \end{aligned}$$

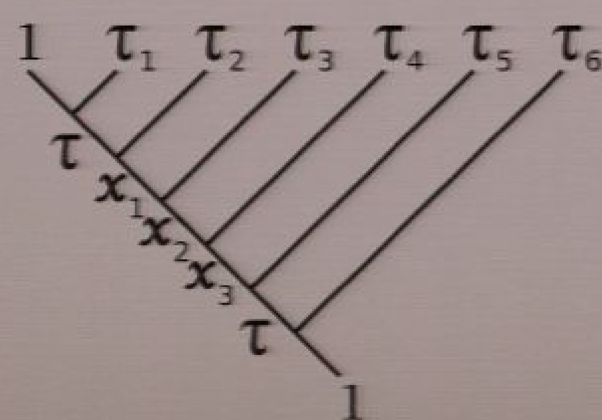
- Basis of states:

ANYONS

- List of charges: $1 \quad \tau$
- Fusion rules:

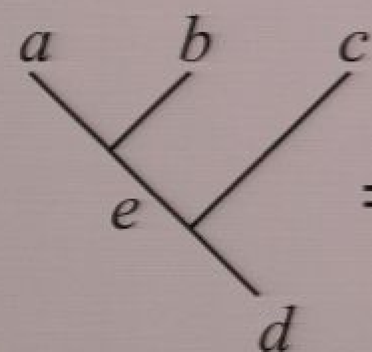
$$\begin{aligned} 1 \times 1 &\rightarrow 1 \\ 1 \times \tau &\rightarrow \tau \\ \tau \times 1 &\rightarrow \tau \\ \tau \times \tau &\rightarrow 1 + \tau \end{aligned}$$

- Basis of states:

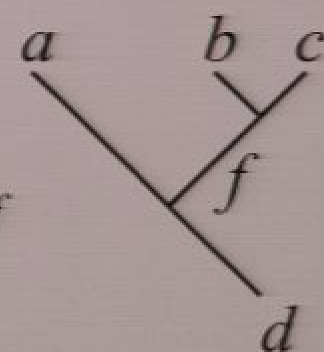


Anyons

- Change of basis

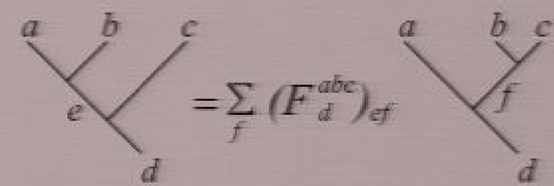


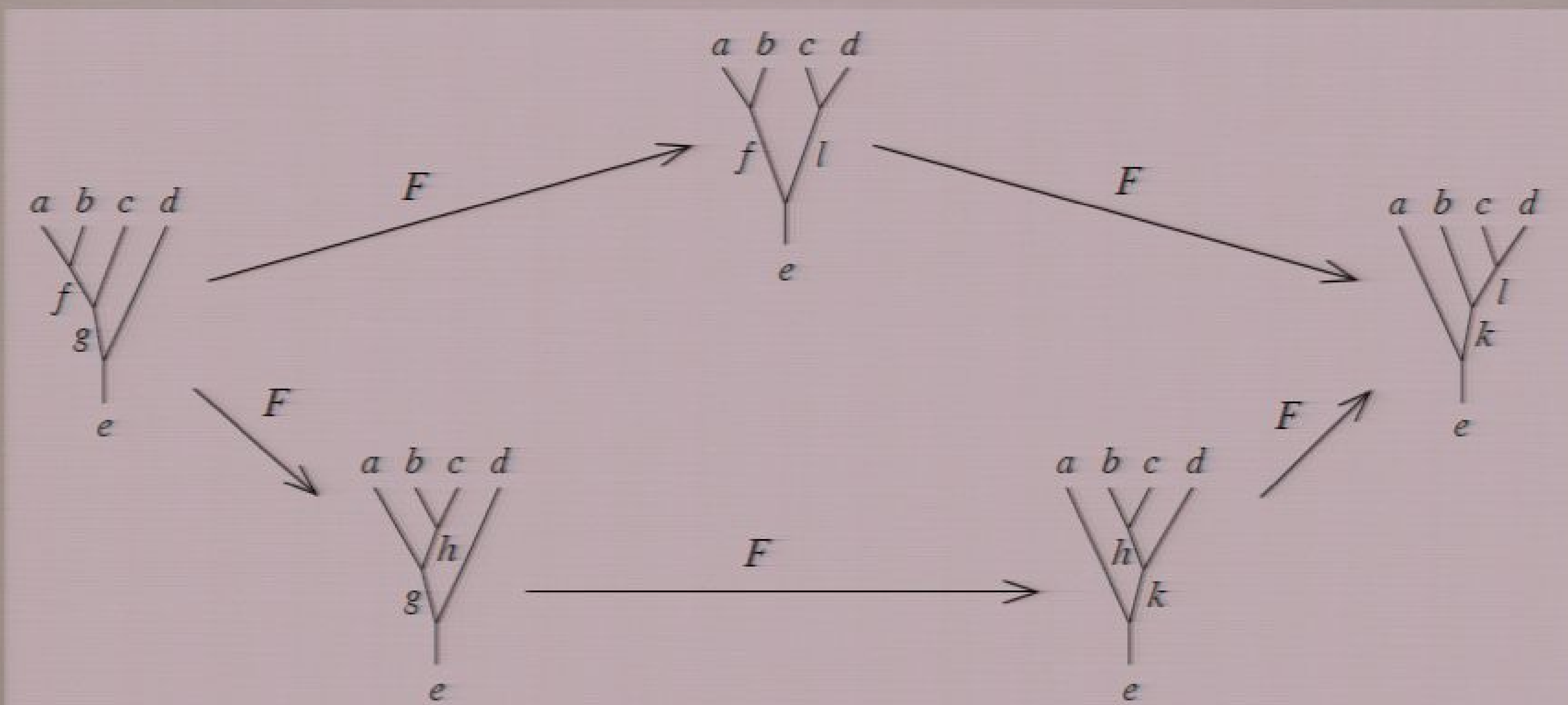
$$= \sum_f (F^{abc}_{d})_{ef}$$



Anyons

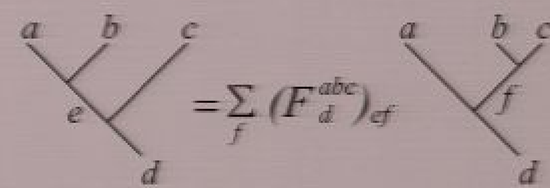
- Change of basis





Anyons

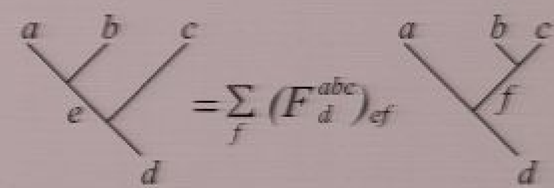
- Change of basis



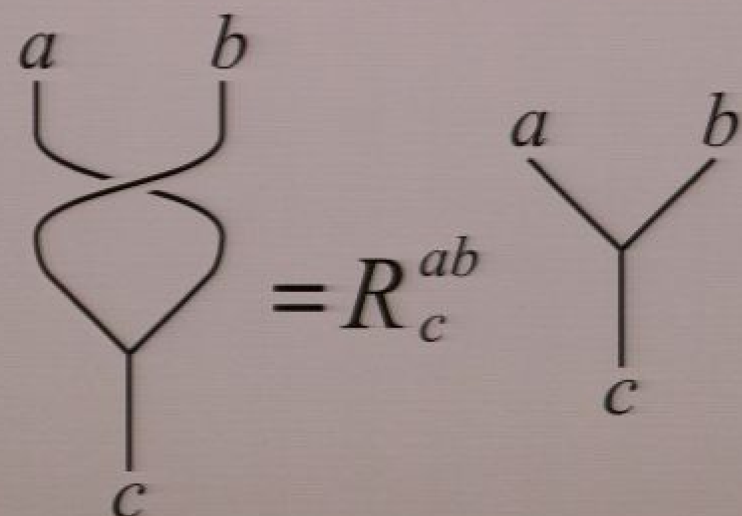
- Braiding

Anyons

- Change of basis

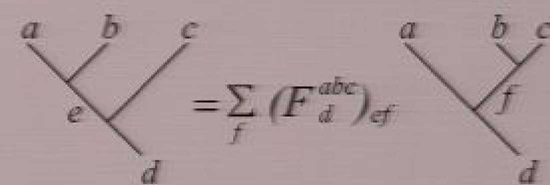


- Braiding

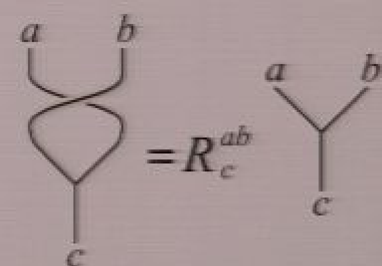


Anyons

- Change of basis

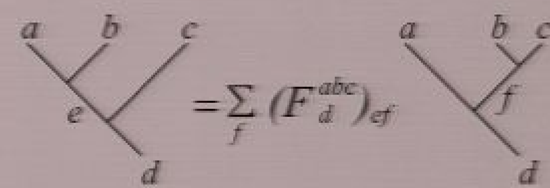


- Braiding

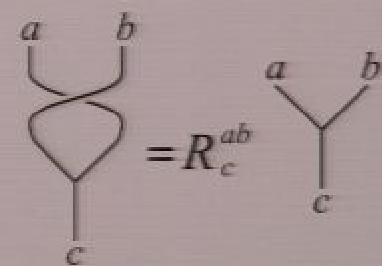


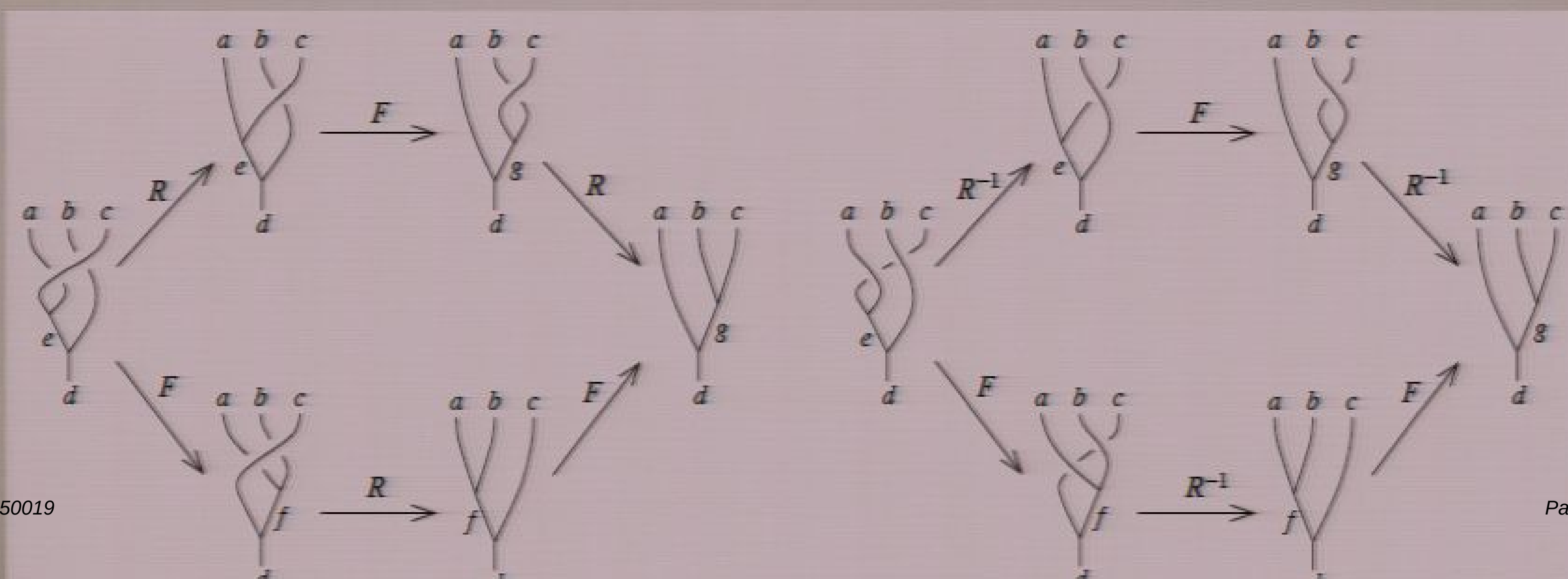
ANYONS

- Change of basis



- Braiding





Conformal Field Theory

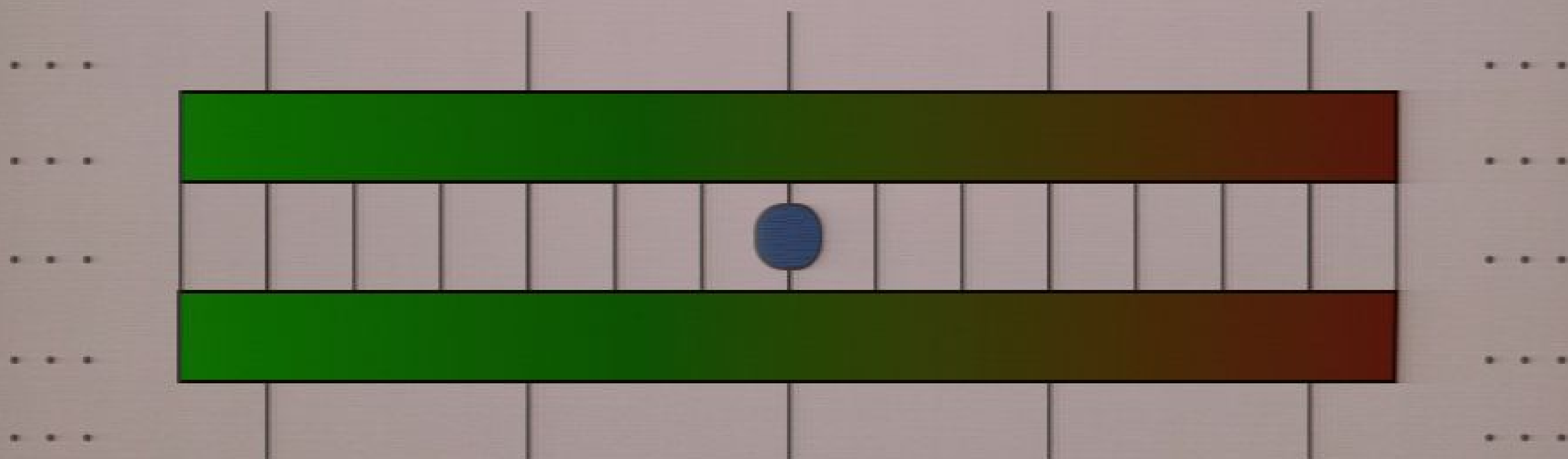
- On a lattice

Conformal Field Theory

- Introduce an operator

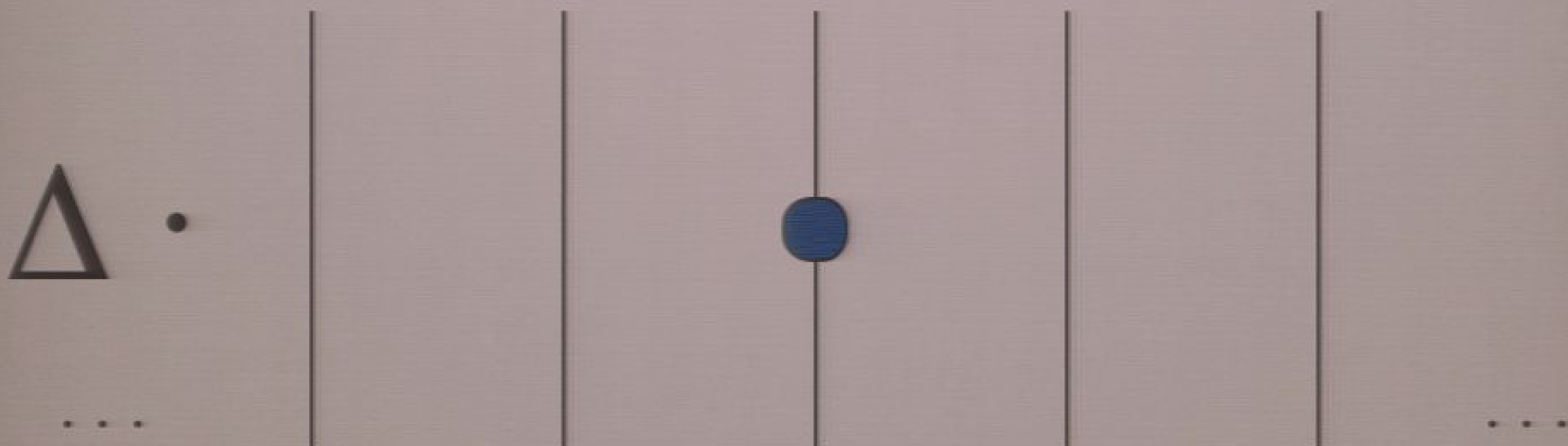
Conformal Field Theory

- Perform a coarse graining process



Conformal Field Theory

- Recover the same operator on the new lattice



CONFORMAL FIELD THEORY

- Example: Quantum Ising Model at criticality

$$\hat{H}_\sigma = -\sum_{i=1}^{N} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x - \sum_{i=1}^{N} \sigma_i^z$$

Conformal Field Theory

- Example: Quantum Ising Model at criticality

$$\hat{H}_\sigma \xrightarrow{\text{Local scaling operators}} \mathbb{I}, \sigma, \varepsilon$$

Conformal Field Theory

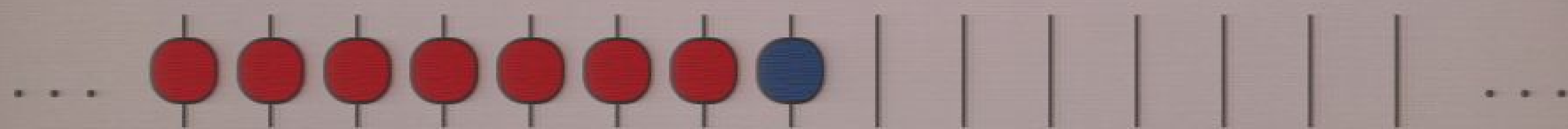
- Example: Quantum Ising Model at criticality

$$\hat{H}_\sigma \xrightarrow{\text{Local scaling operators}} \mathbb{I}, \sigma, \varepsilon$$

$$\hat{H}_\sigma \xrightarrow{\text{Non-local scaling operators}} \mu, \psi, \bar{\psi}$$

Conformal Field Theory

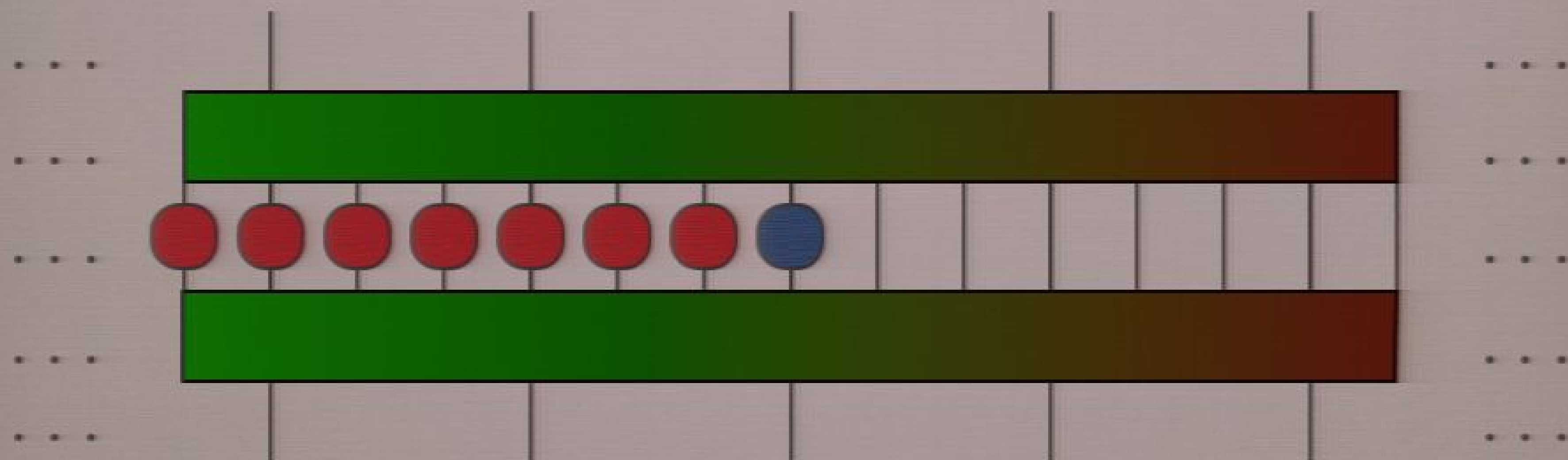
- Introduce a non-local operator



$$\ldots \otimes \hat{\sigma}_i^z \otimes \hat{\sigma}_{i+1}^z \otimes \hat{\sigma}_{i+2}^z \otimes \hat{\sigma}_{i+3}^z \otimes \hat{\sigma}_{i+4}^x$$

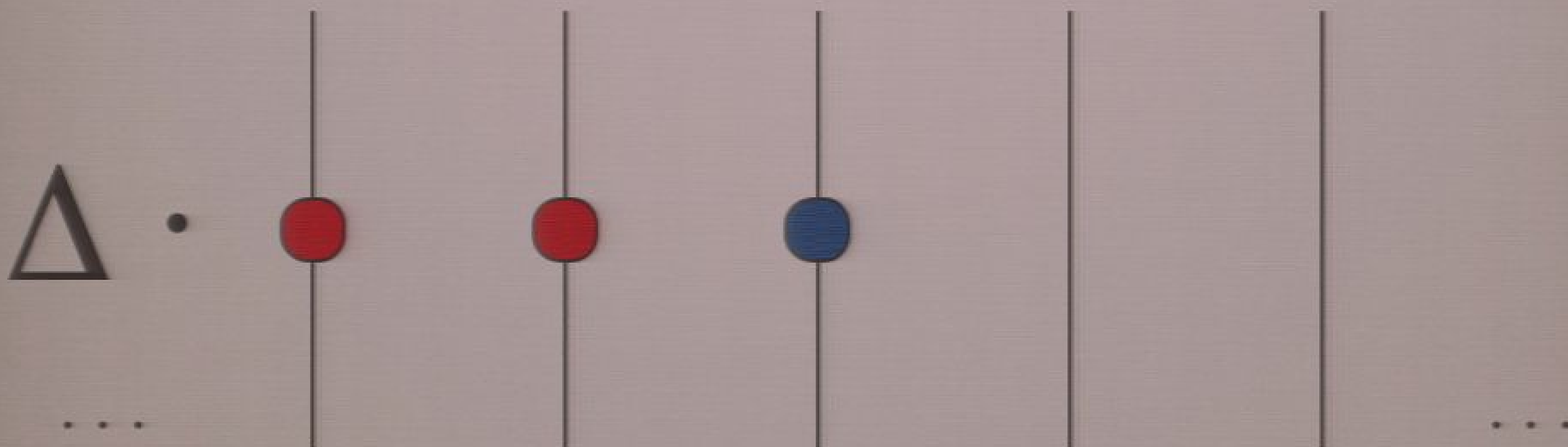
Conformal Field Theory

- Perform a coarse graining process



Conformal Field Theory

- Recover the same operator on the new lattice



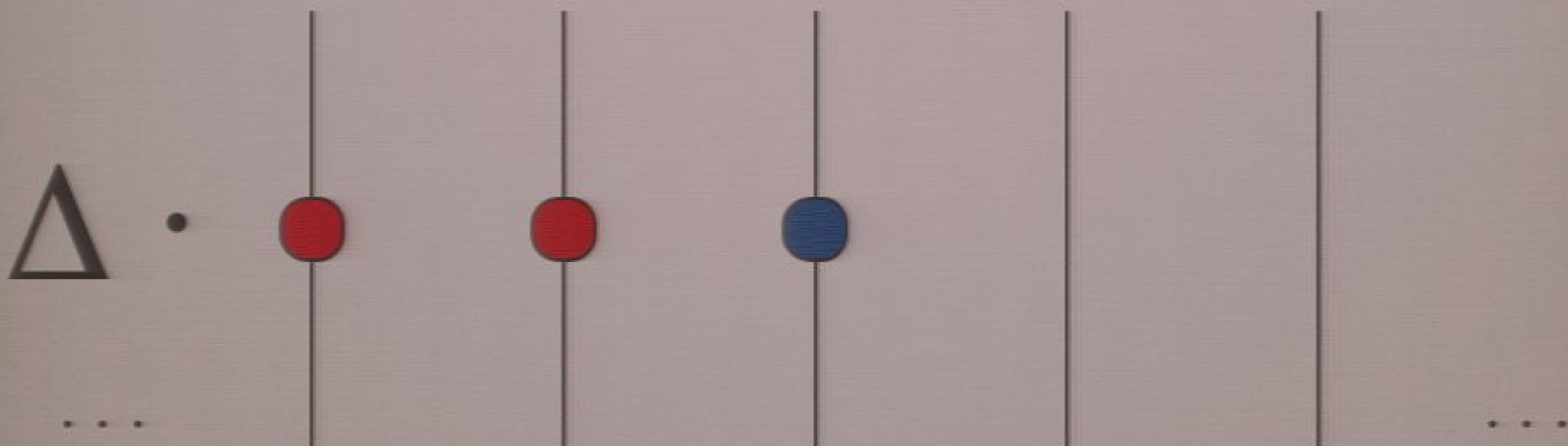
Conformal Field Theory

$\hat{H}_\sigma$ —Local scaling operators→ $\mathbb{I}, \sigma, \varepsilon$

$\hat{H}_\sigma$ —Non-local scaling operators→ $\mu, \psi, \bar{\psi}$

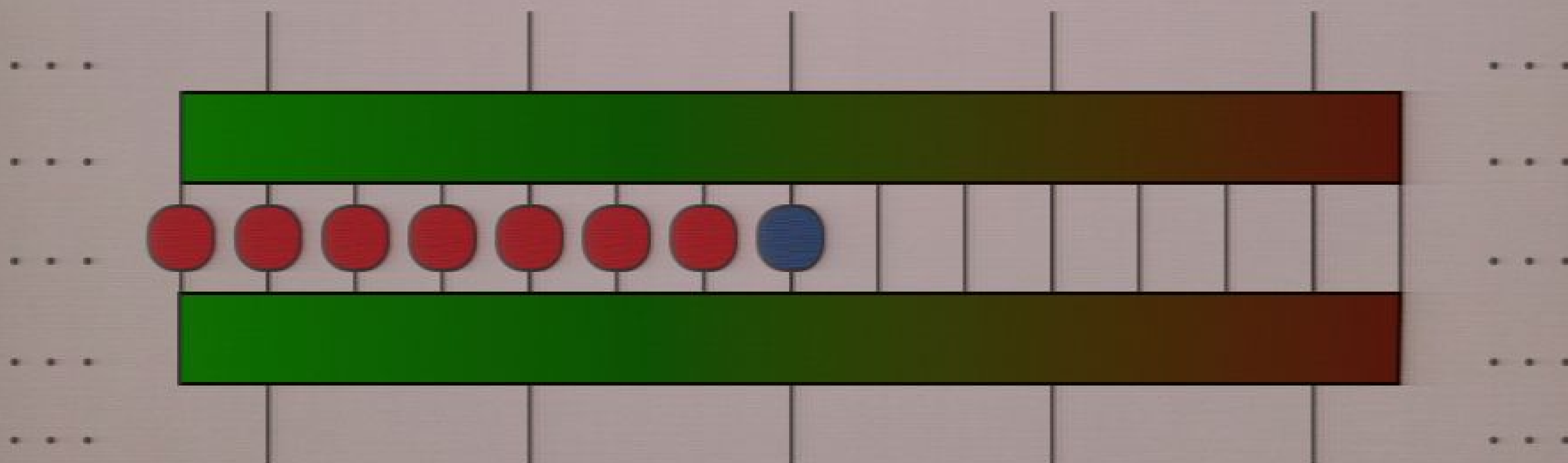
Conformal Field Theory

- Recover the same operator on the new lattice



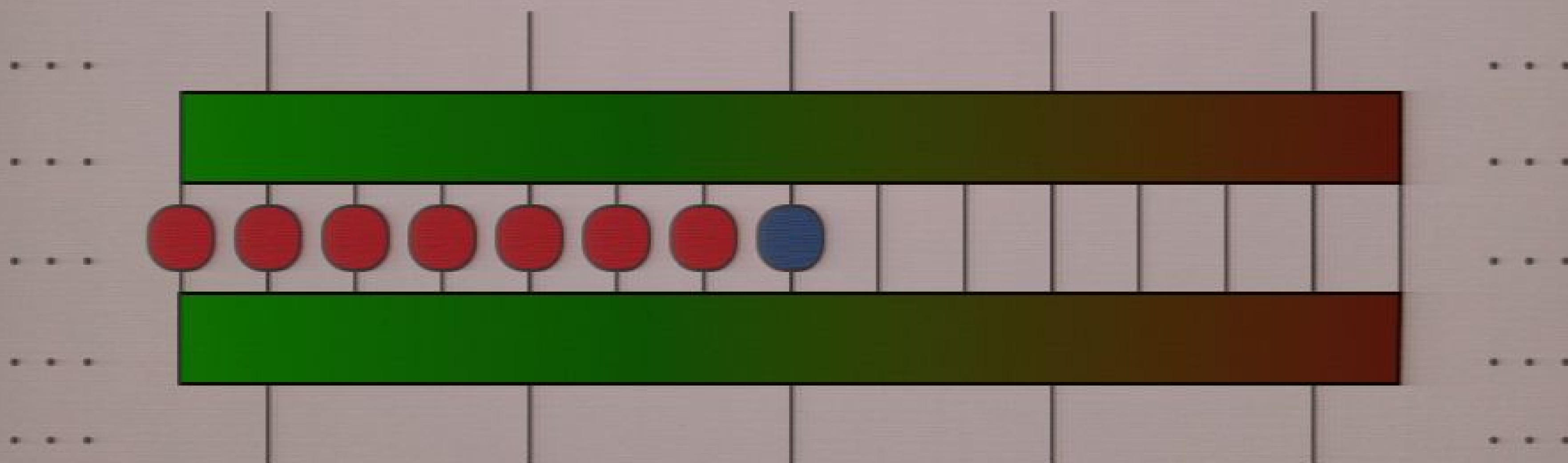
Conformal Field Theory

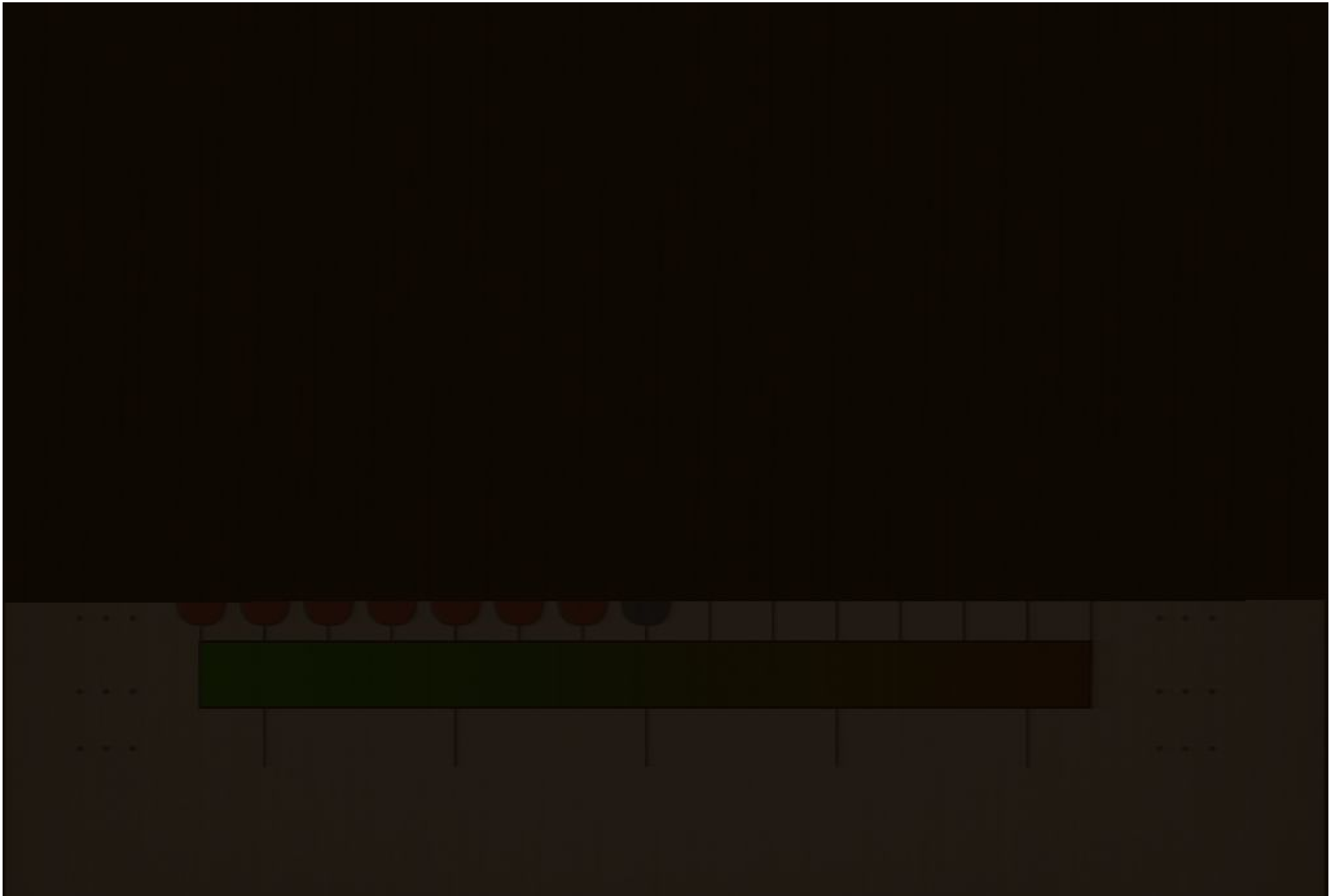
- Perform a coarse graining process



Conformal Field Theory

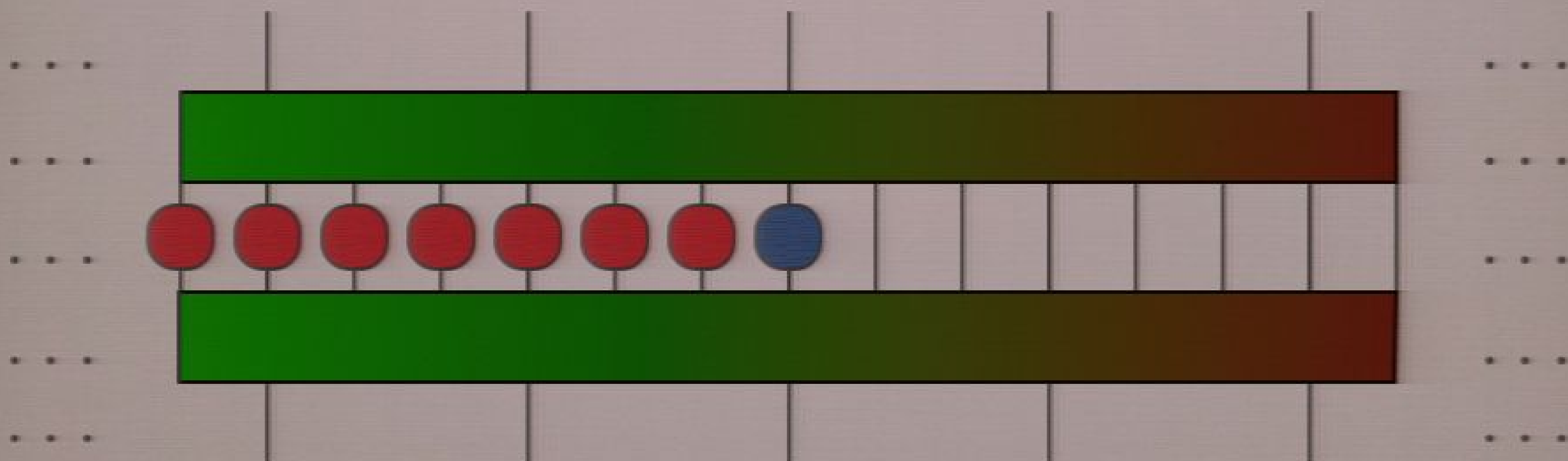
- Perform a coarse graining process

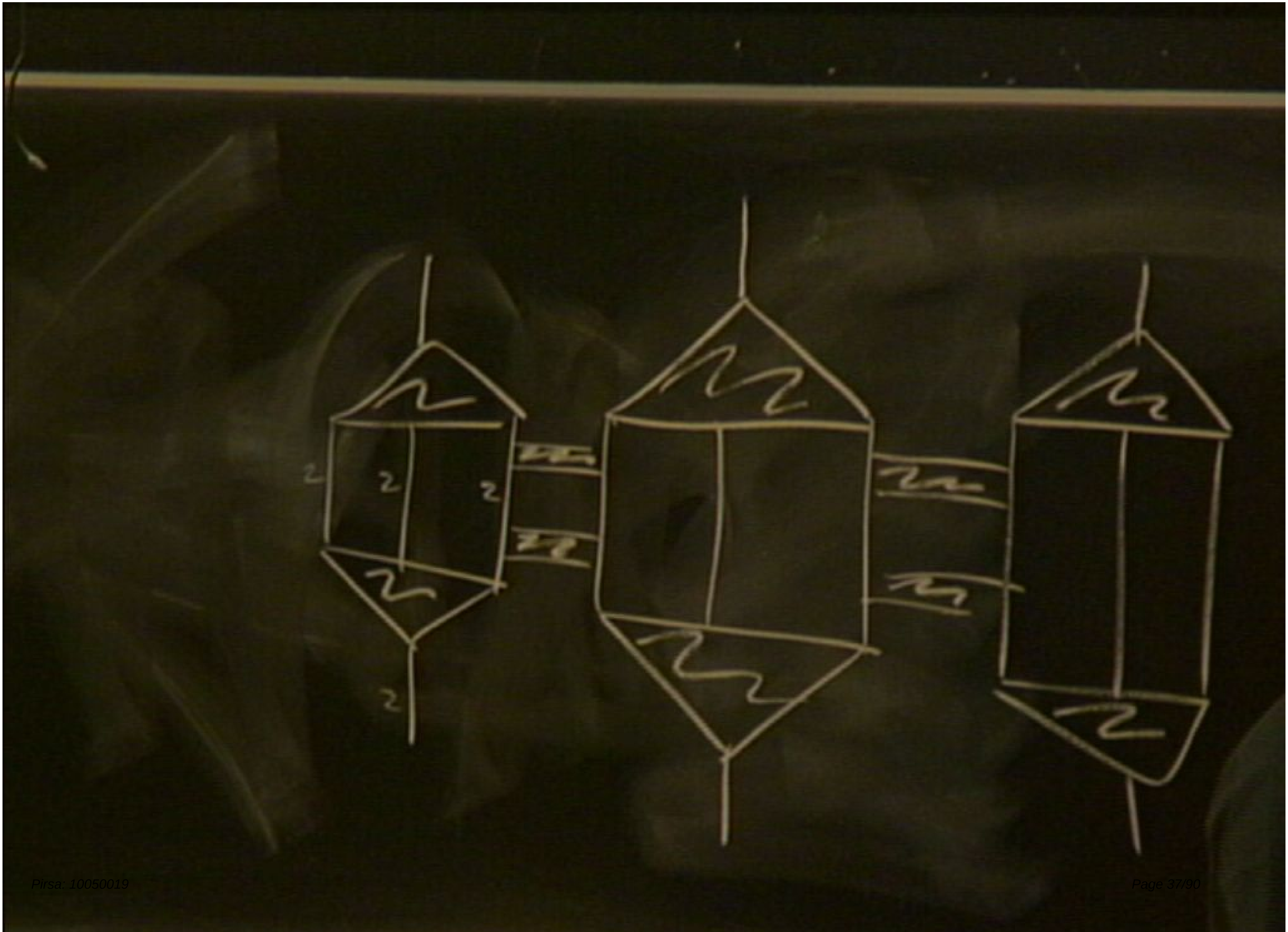


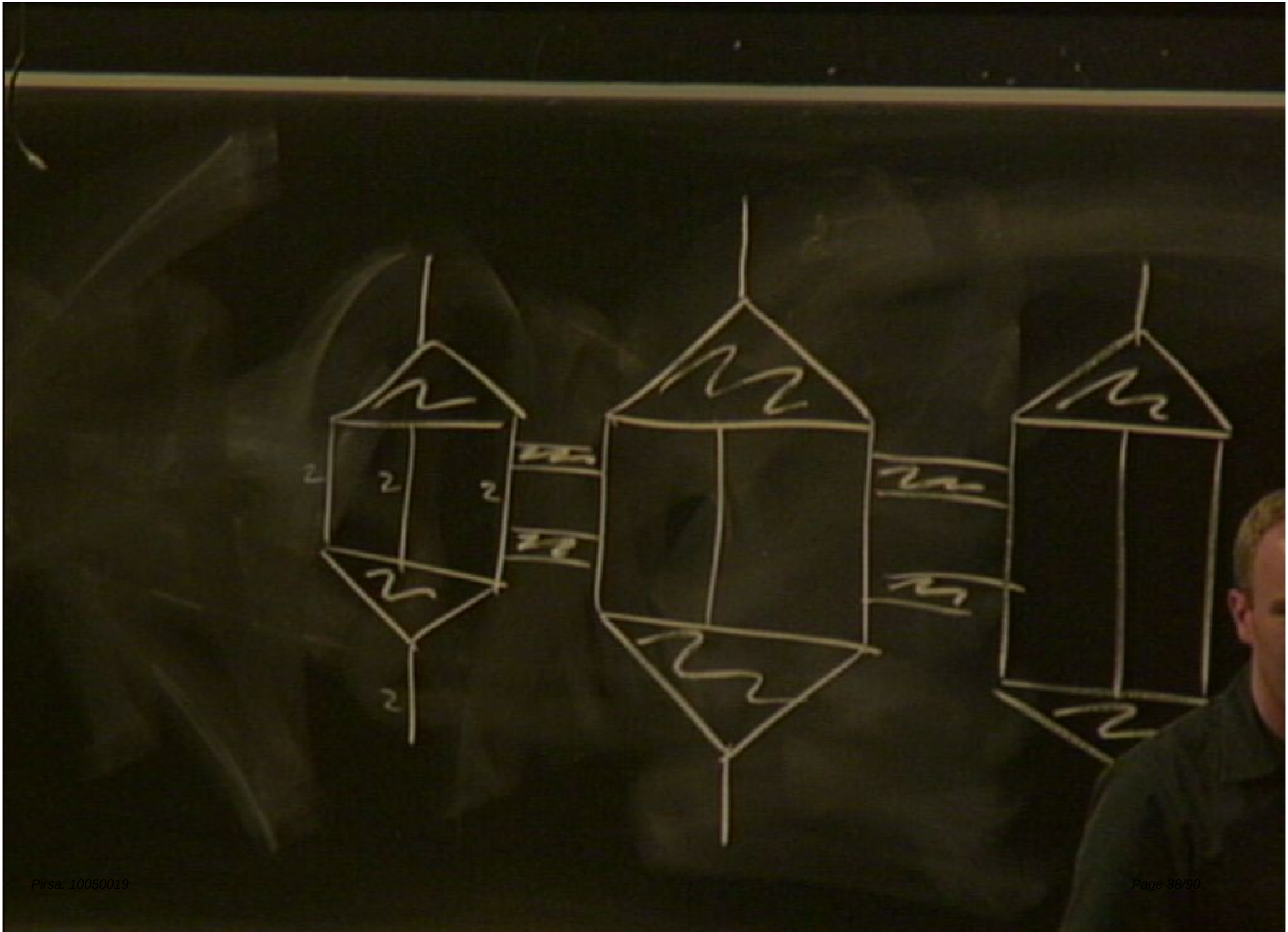


Conformal Field Theory

- Perform a coarse graining process

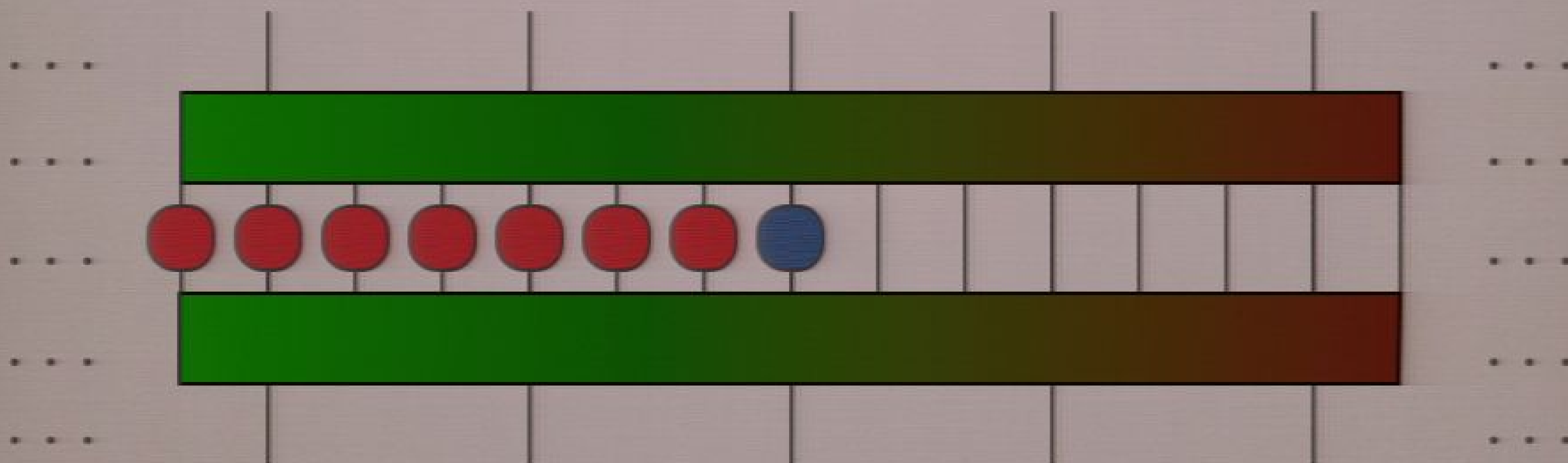



2
2
2
2



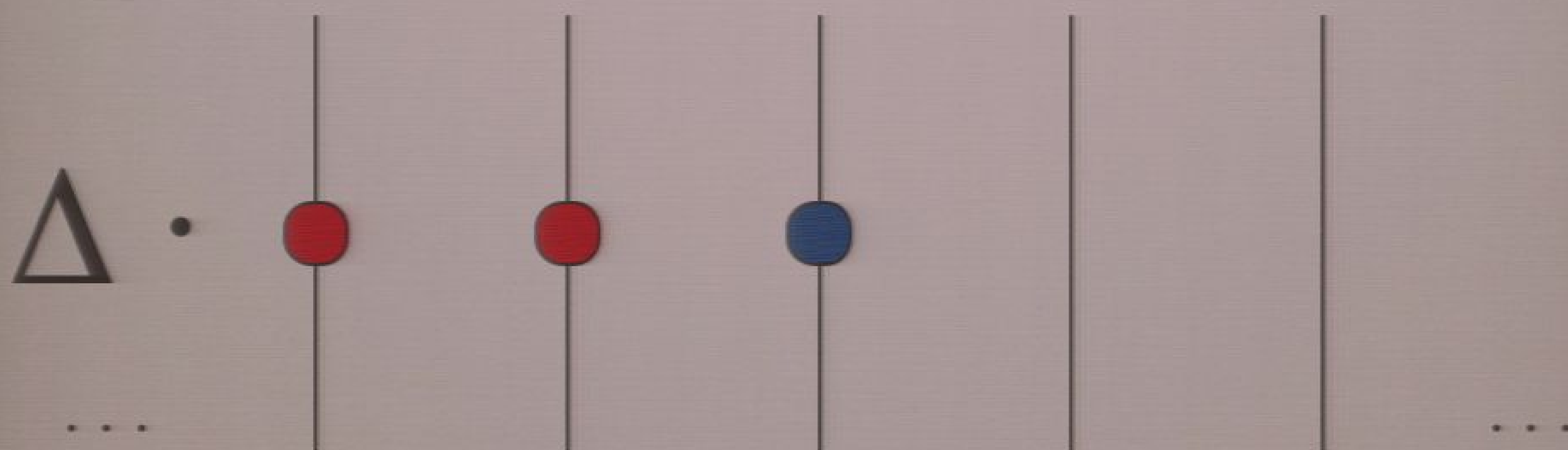
Conformal Field Theory

- Perform a coarse graining process



Conformal Field Theory

- Recover the same operator on the new lattice



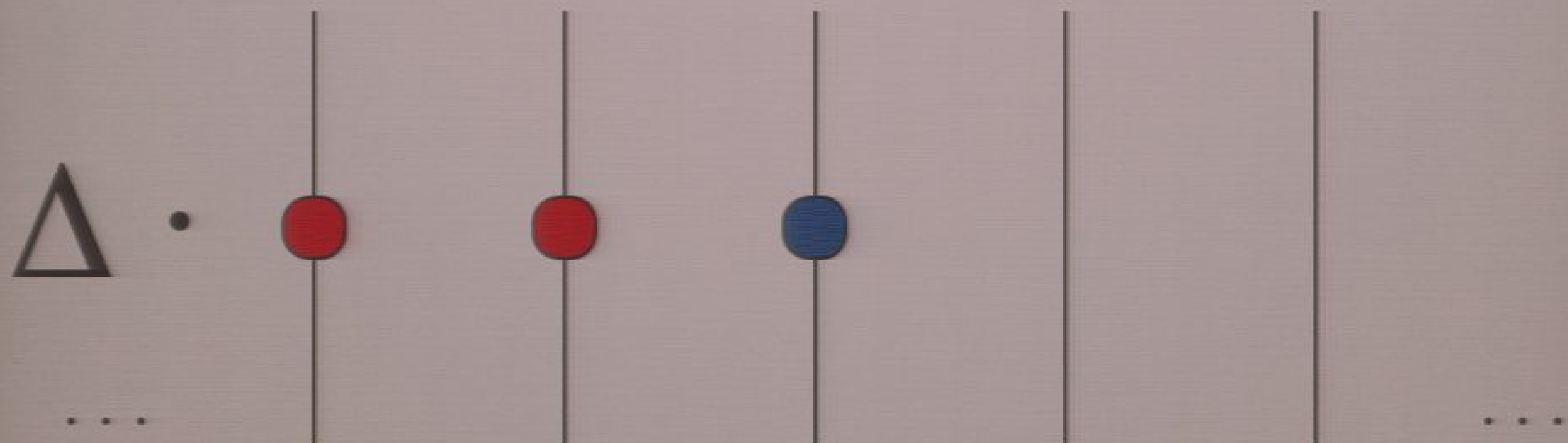
Conformal Field Theory

$\hat{H}_\sigma$ — Local scaling operators → $\mathbb{I}, \sigma, \varepsilon$

$\hat{H}_\sigma$ — Non-local scaling operators → $\mu, \psi, \bar{\psi}$

Conformal Field Theory

- Recover the same operator on the new lattice

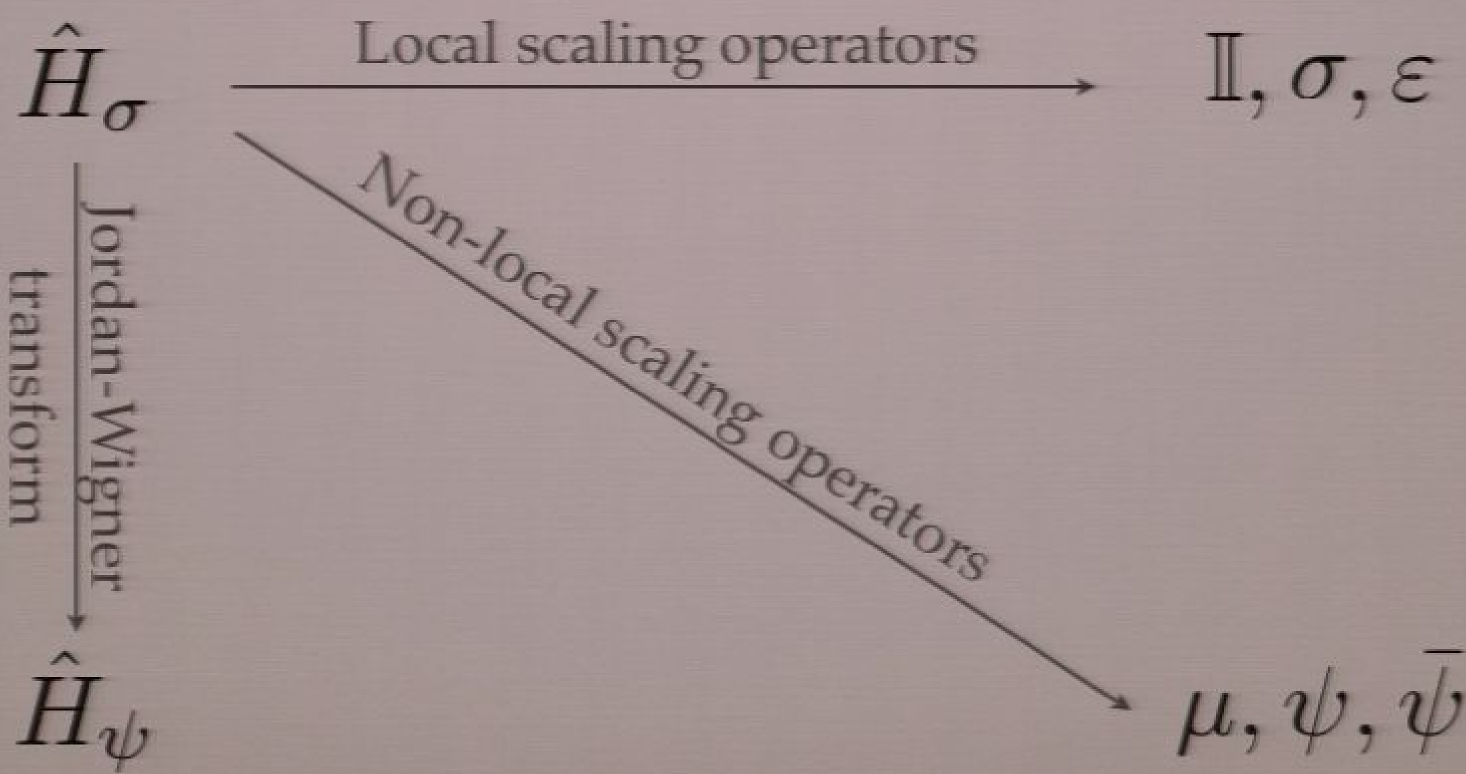


Conformal Field Theory

$\hat{H}_\sigma$ — Local scaling operators → $\mathbb{I}, \sigma, \varepsilon$

$\hat{H}_\sigma$ — Non-local scaling operators → $\mu, \psi, \bar{\psi}$

Conformal Field Theory

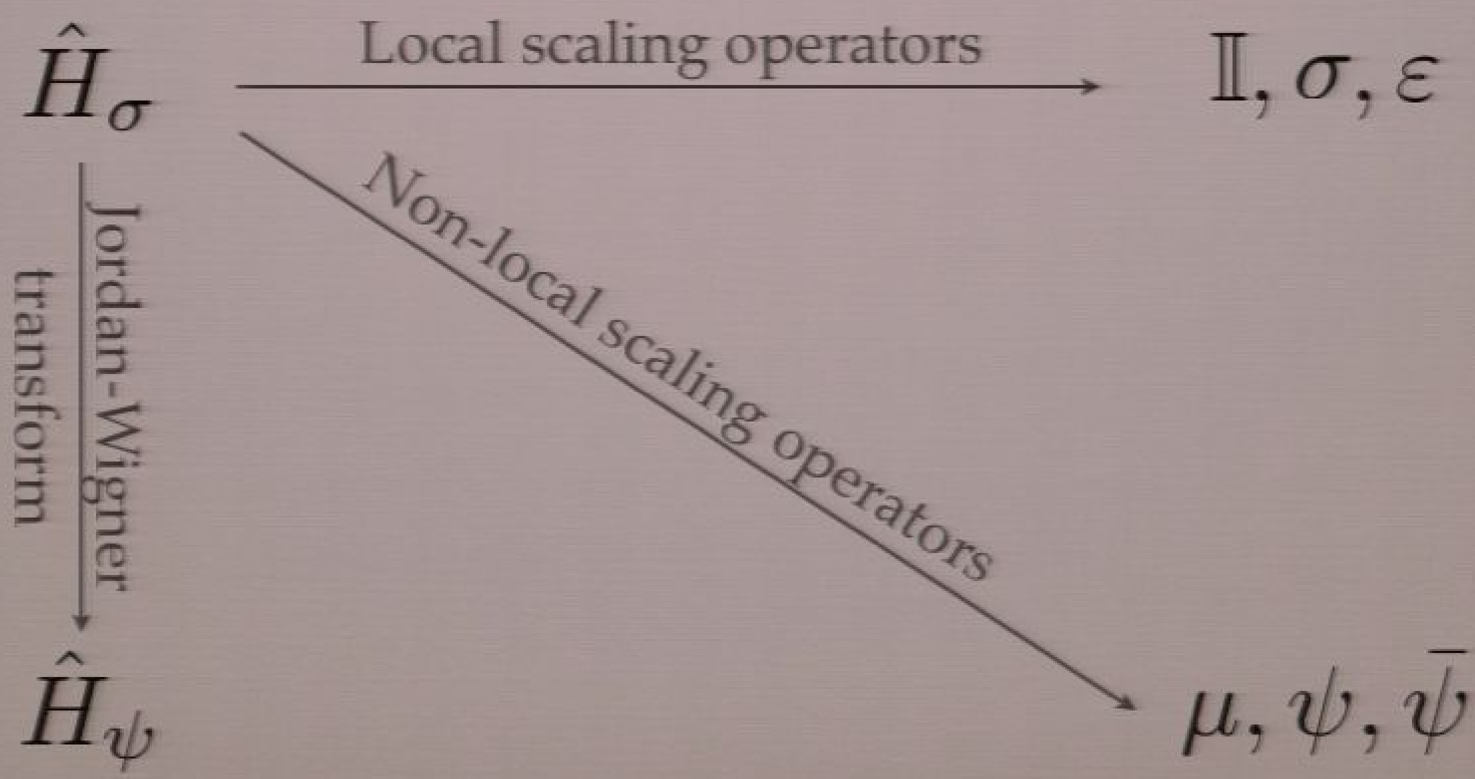


Conformal Field Theory

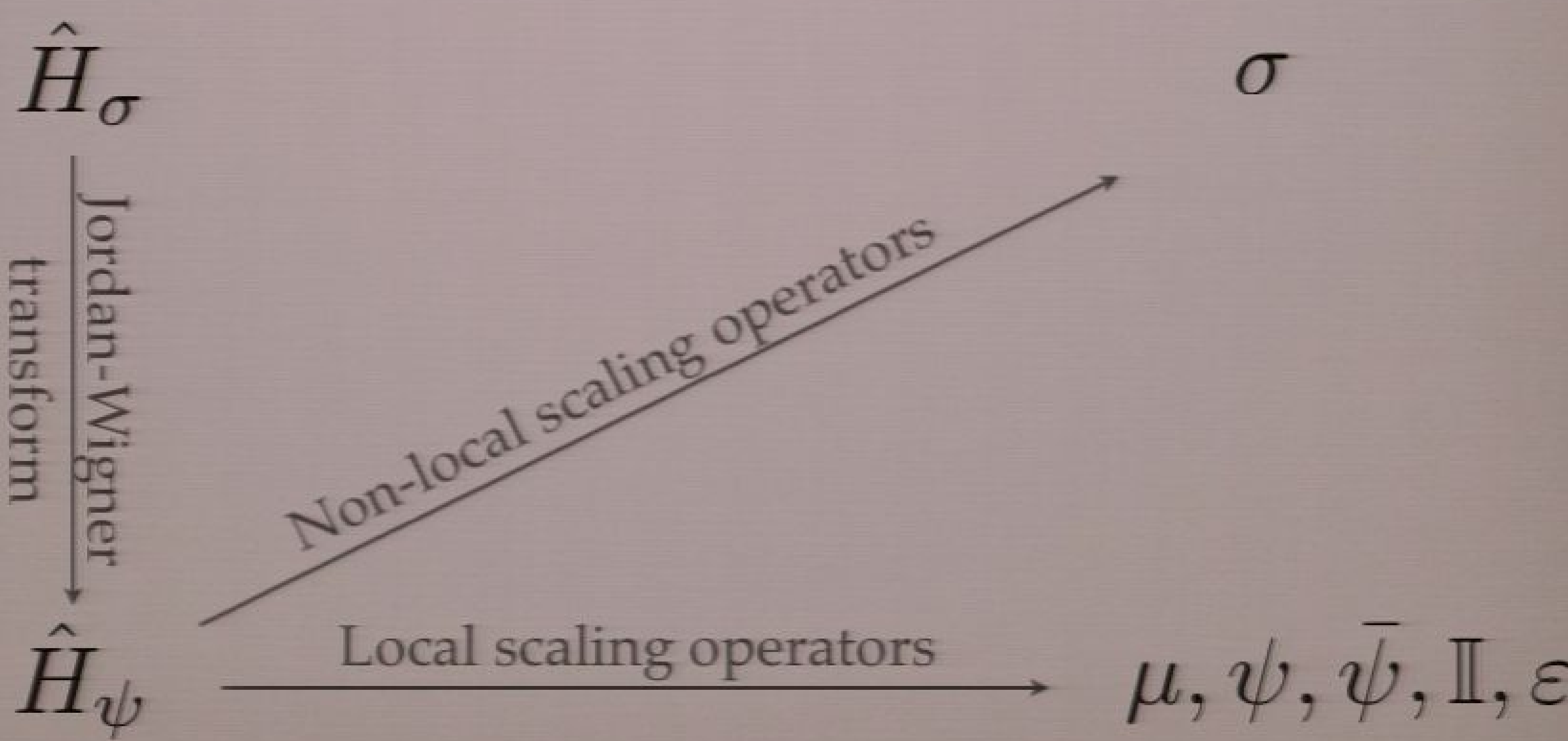
$$\hat{H}_\sigma = -\sum_{i=1}^{N} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x - \sum_{i=1}^{N} \hat{\sigma}_i^z$$

$$\hat{H}_\psi = -\sum_{i=1}^{N} \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \hat{c}_{i+1}^\dagger \hat{c}_i + \hat{c}_i^\dagger \hat{c}_{i+1}^\dagger + \hat{c}_{i+1} \hat{c}_i - 2\hat{c}_i^\dagger \hat{c}_i + 1 \right)$$

Conformal Field Theory



Conformal Field Theory



Conformal Field Theory

- Introduce a defect in translation

$$\hat{H}_\sigma = -\sum_{i=1}^{N} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x - \sum_{i=1}^{N} \hat{\sigma}_i^z$$

$$\hat{H}'_\sigma = 2\hat{\sigma}_1^x \hat{\sigma}_2^x - \sum_{i=1}^{N} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x - \sum_{i=1}^{N} \hat{\sigma}_i^z$$

Conformal Field Theory

- Introduce a defect in translation

$$\hat{H}'_\sigma = 2\hat{\sigma}^x_1\hat{\sigma}^x_2 - \sum_{i=1}^{N}\hat{\sigma}^x_i\hat{\sigma}^x_{i+1} - \sum_{i=1}^{N}\hat{\sigma}^z_i$$

$$\hat{H}''_\sigma = \hat{T}\hat{H}'_\sigma\hat{T}^\dagger = 2\hat{\sigma}^x_2\hat{\sigma}^x_3 - \sum_{i=1}^{N}\hat{\sigma}^x_i\hat{\sigma}^x_{i+1} - \sum_{i=1}^{N}\hat{\sigma}^z_i$$

$$\hat{H}'_\sigma = \hat{\sigma}^z_2\ \hat{H}''_\sigma\ \hat{\sigma}^z_2$$

$$= (\hat{\sigma}^z_2\hat{T})\ \hat{H}'_\sigma\ (\hat{T}^\dagger\hat{\sigma}^z_2)$$

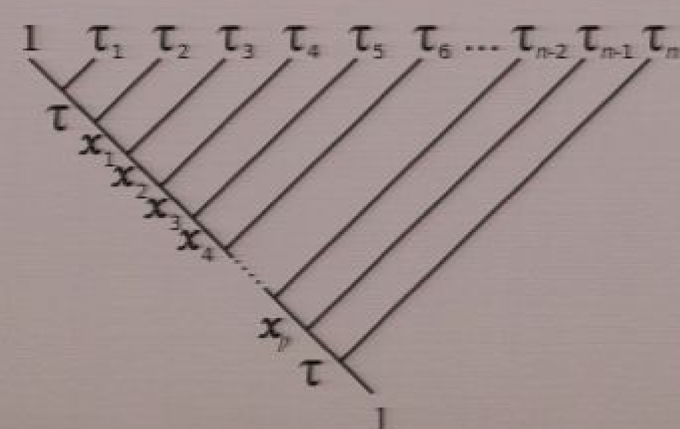
Conformal Field Theory

- Introduce a defect in translation
- Local scaling operators same as for Jordan-Wigner transformed Hamiltonian
- To learn about local operators on $\hat{H}_\psi$ we can study local operators on $\hat{H}_\sigma$ with a defect.

The Connection

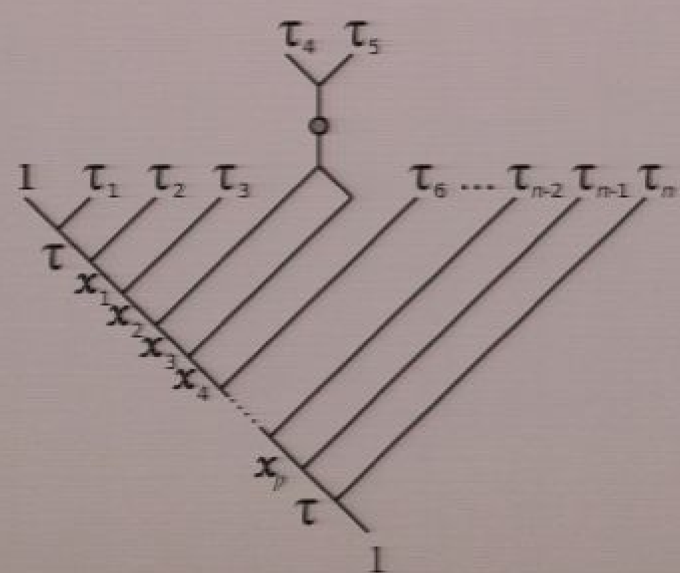
The Connection

- Consider a periodic lattice of anyons.



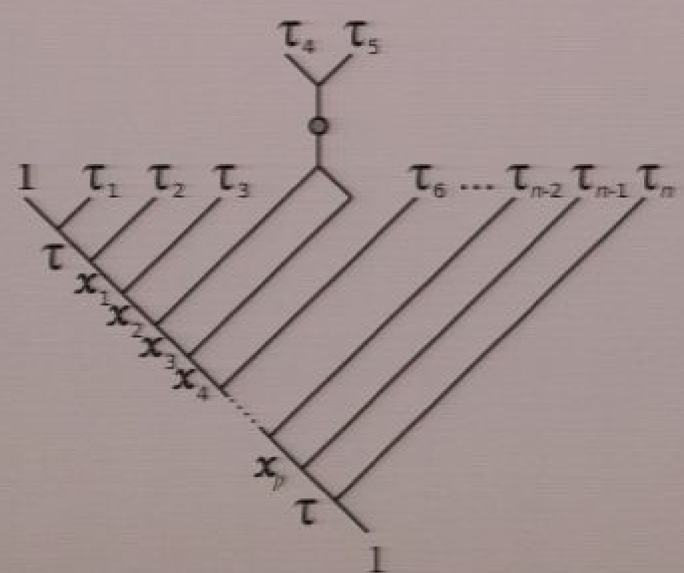
THE CONNECTION

- Consider a periodic lattice of anyons.
- Introduce a 2-site Hamiltonian term.



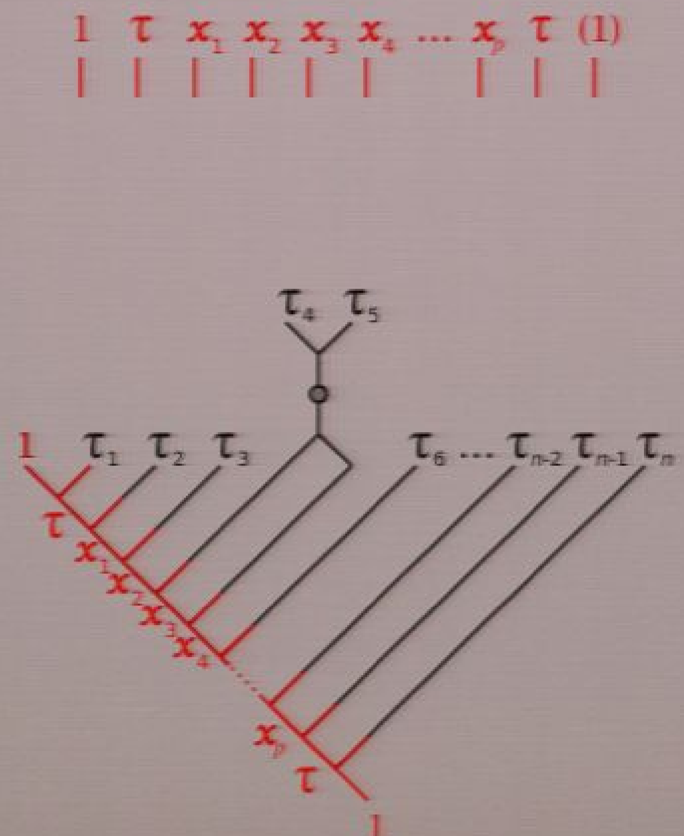
The Connection

$$\hat{H}_\tau = \sum_{i=1}^{n} \hat{h}_{i,i+1}$$



The Connection

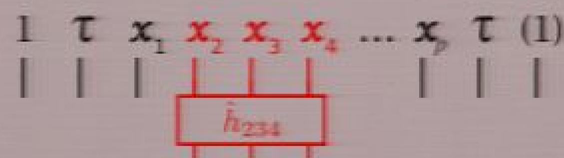
- Map to a spin chain.

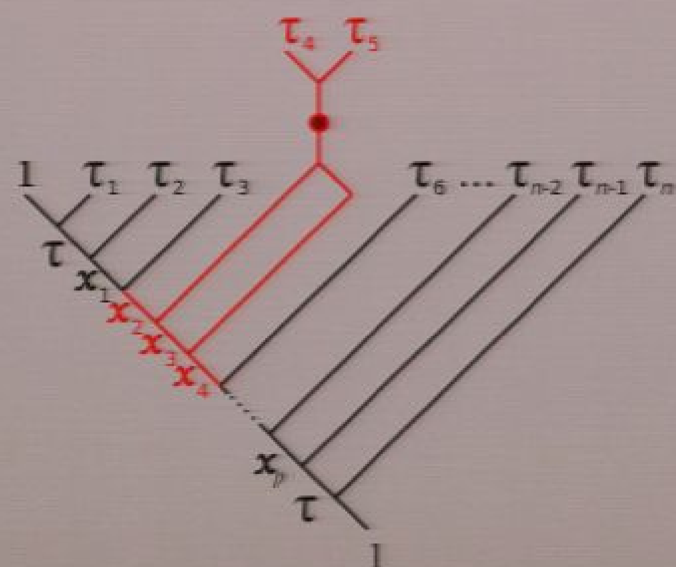


(Feiguin *et al.* 2007)

The Connection

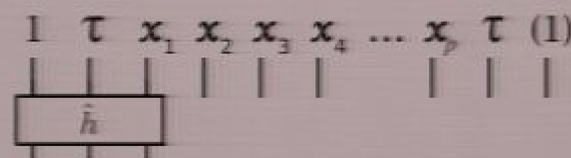
- Map to a spin chain.

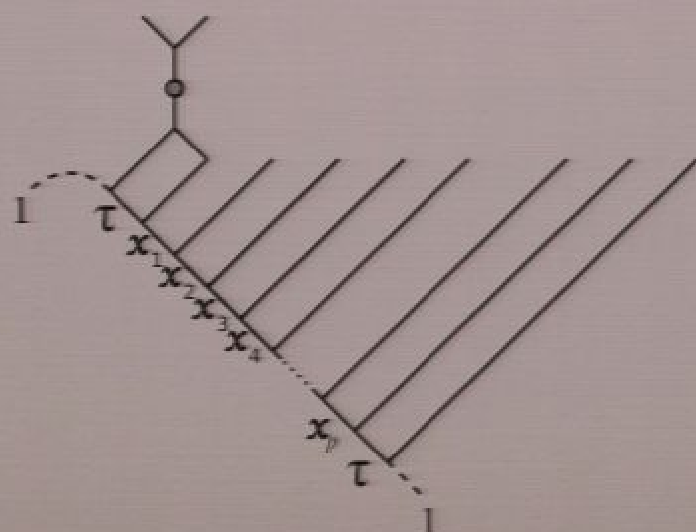




The Connection

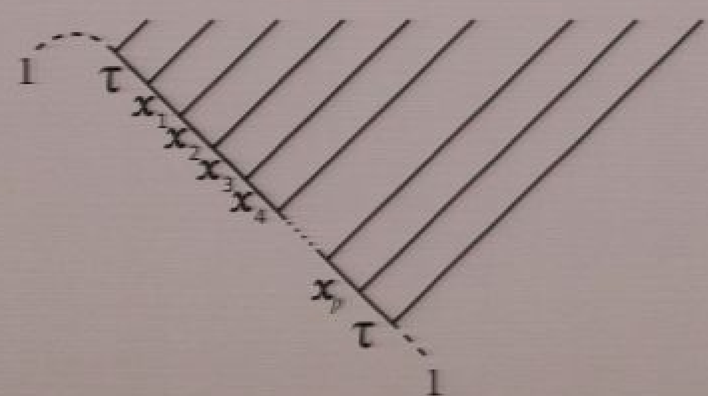
- Map to a spin chain.





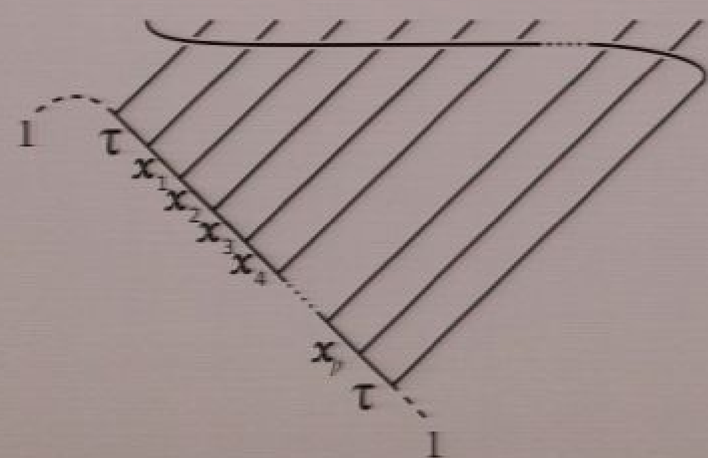
THE CONNECTION

- Periodic translation for anyons
  - Must use the braiding rule



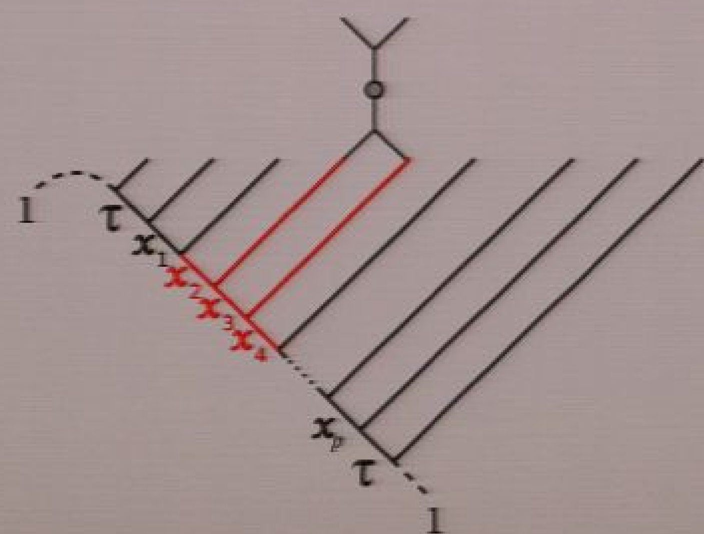
The Connection

- Periodic translation for anyons
  - Must use the braiding rule
  - On states:



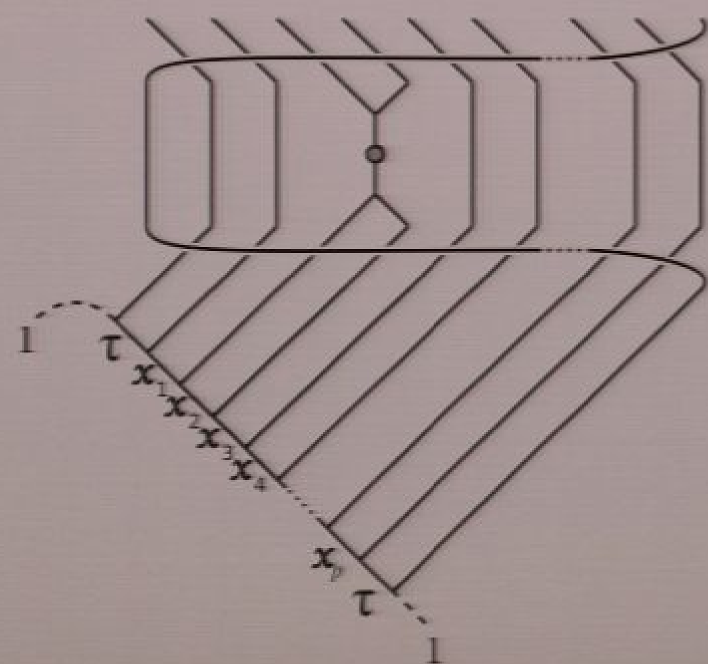
The Connection

- Periodic translation for anyons
  - On operators:



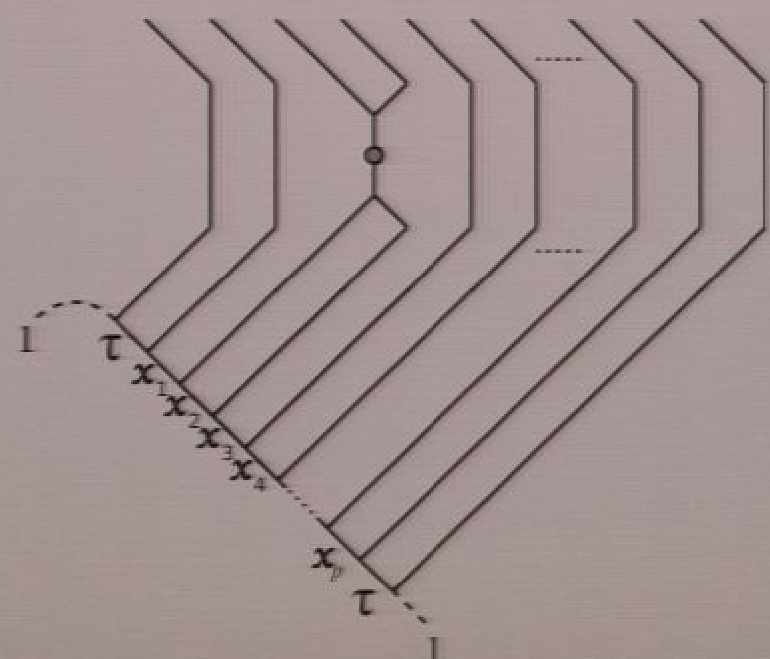
THE CONNECTION

- Periodic translation for anyons
  - On operators:



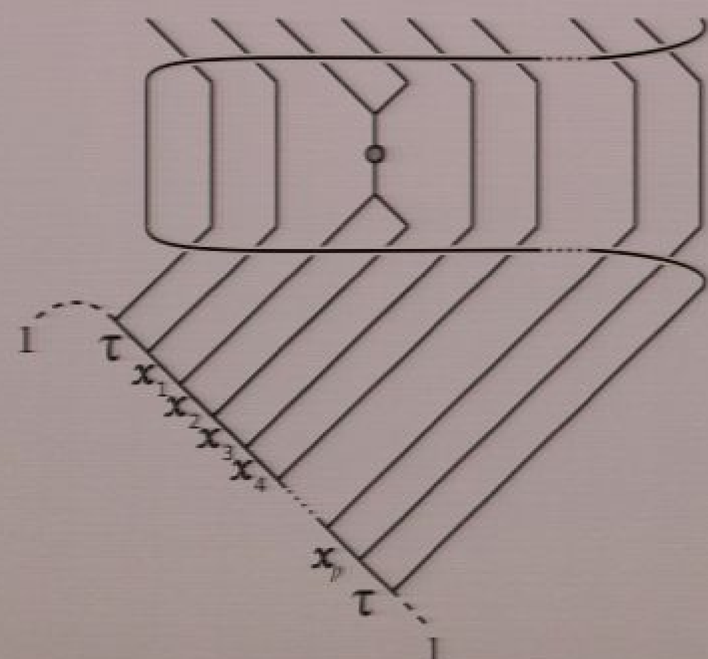
The Connection

- Periodic translation for anyons
  - On operators:



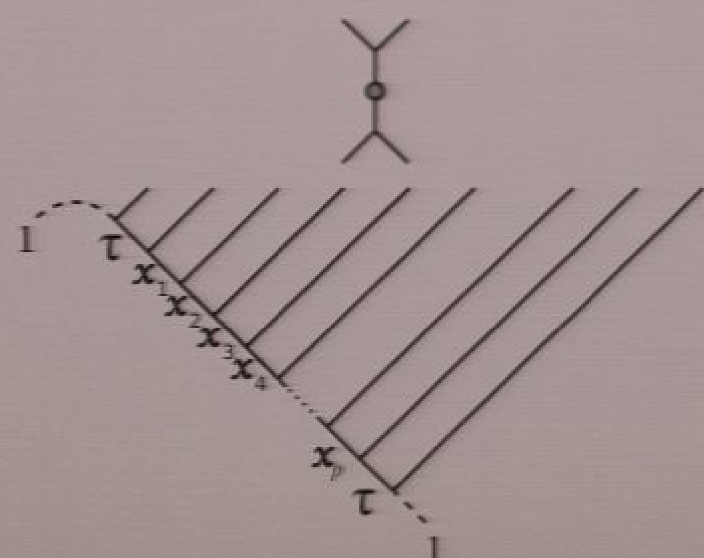
The Connection

- Periodic translation for anyons
  - On operators:



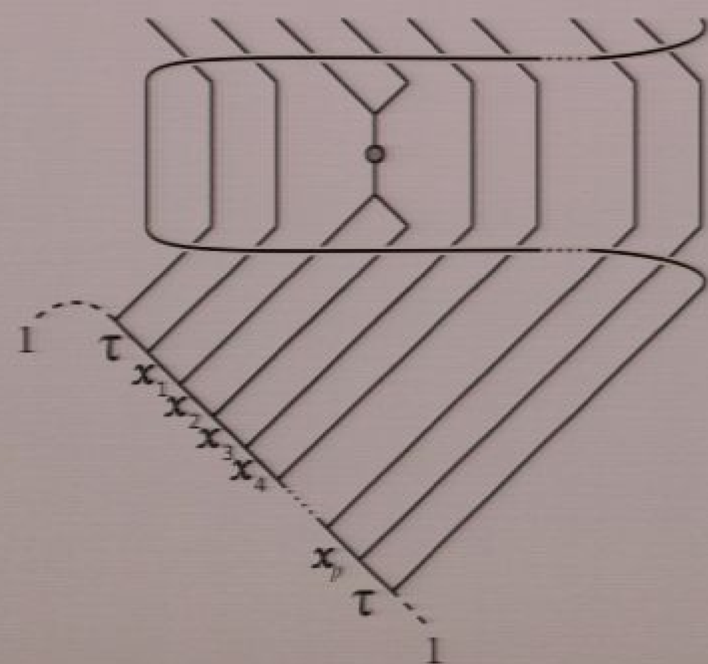
THE CONNECTION

- Periodic translation for anyons
  - On operators:



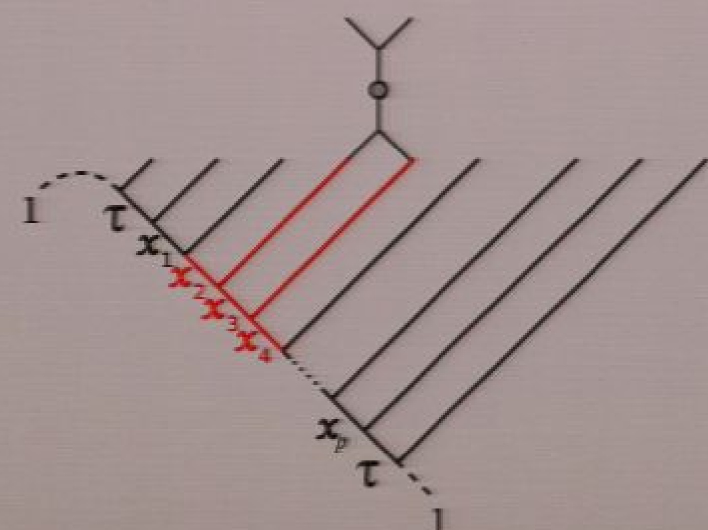
The Connection

- Periodic translation for anyons
  - On operators:



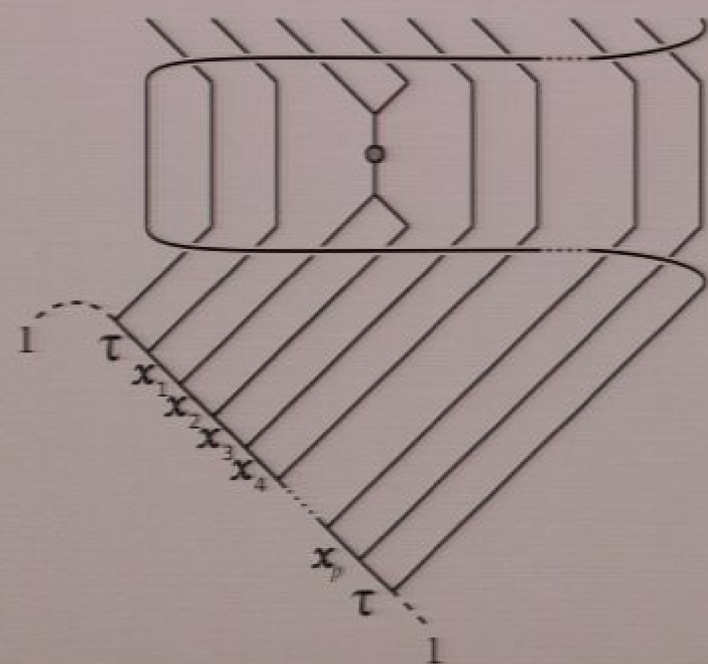
THE CONNECTION

- Periodic translation for anyons
  - On operators:



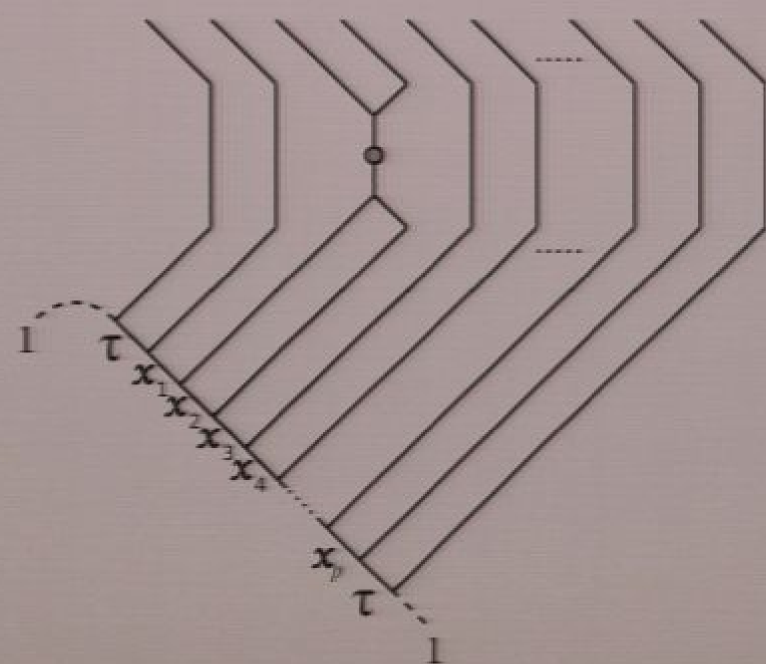
The Connection

- Periodic translation for anyons
  - On operators:



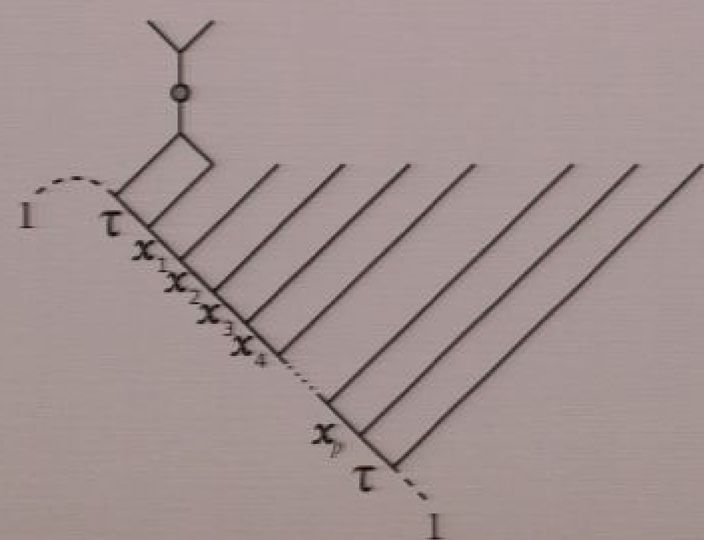
The Connection

- Periodic translation for anyons
  - On operators:



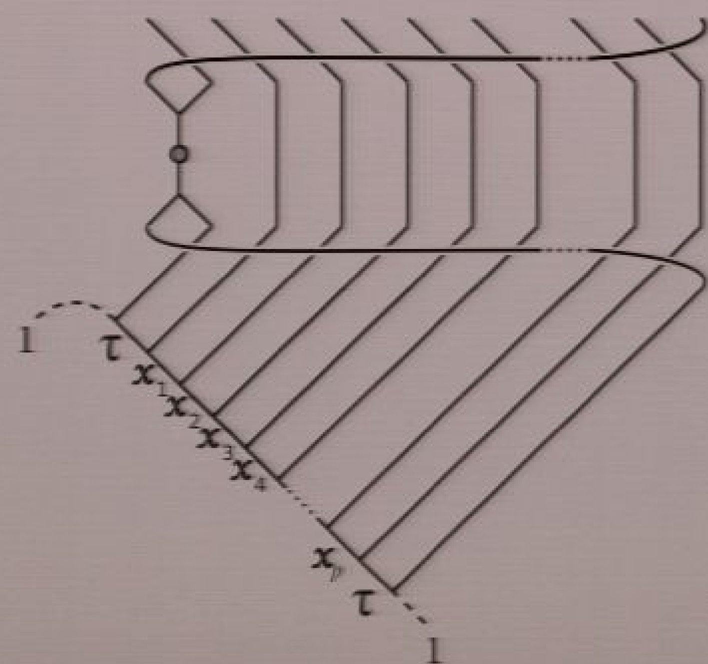
THE CONNECTION

- Periodic translation for anyons
  - On operators:



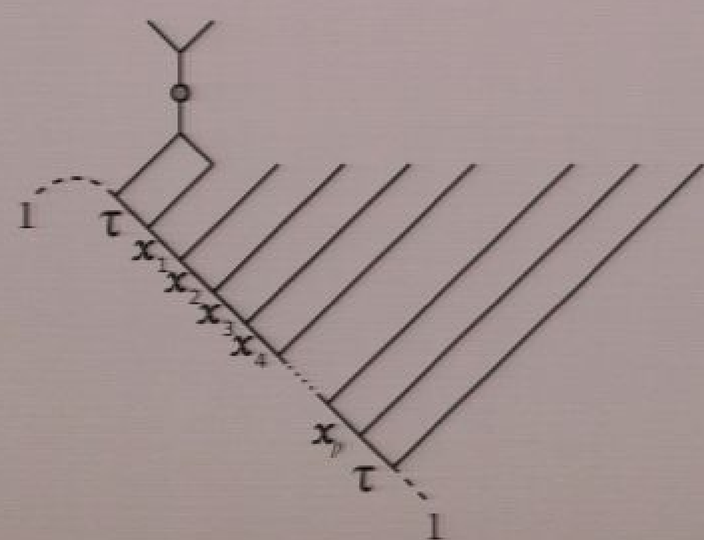
The Connection

- Periodic translation for anyons
  - On operators:



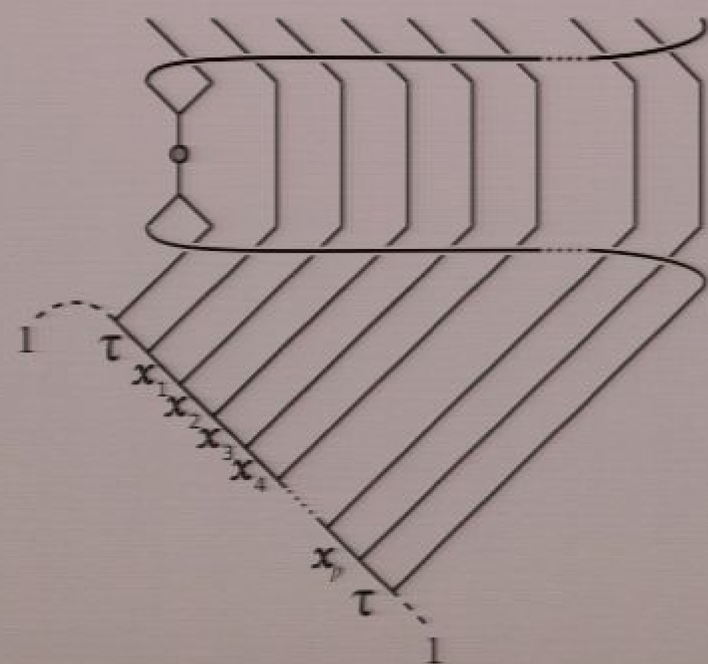
The Connection

- Periodic translation for anyons
  - On operators:



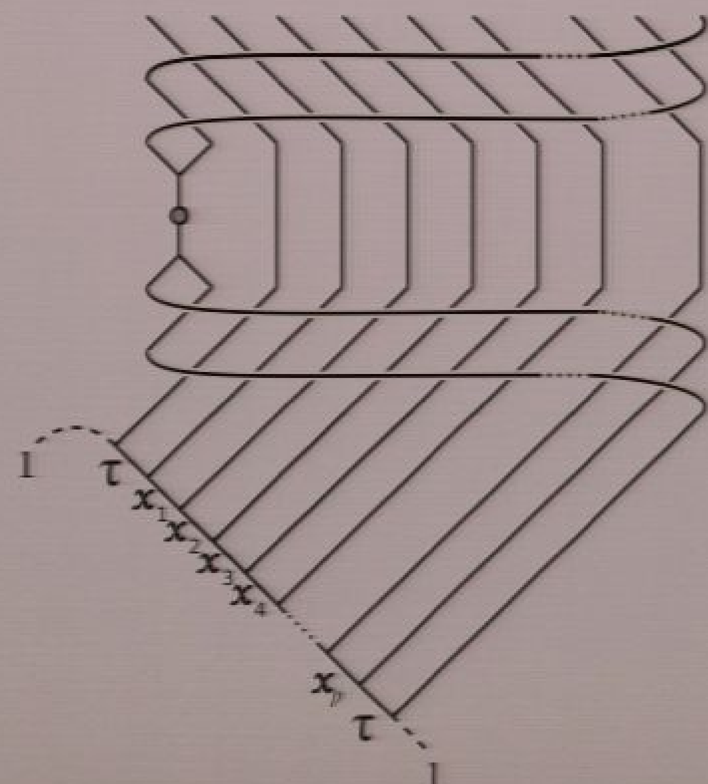
The Connection

- Periodic translation for anyons
  - On operators:



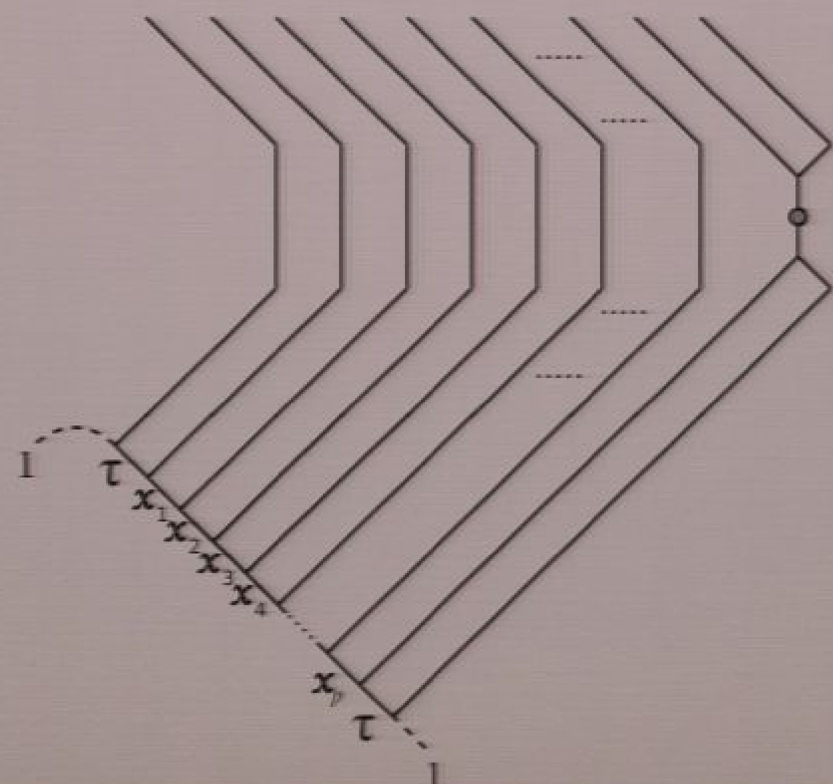
The Connection

- Periodic translation for anyons



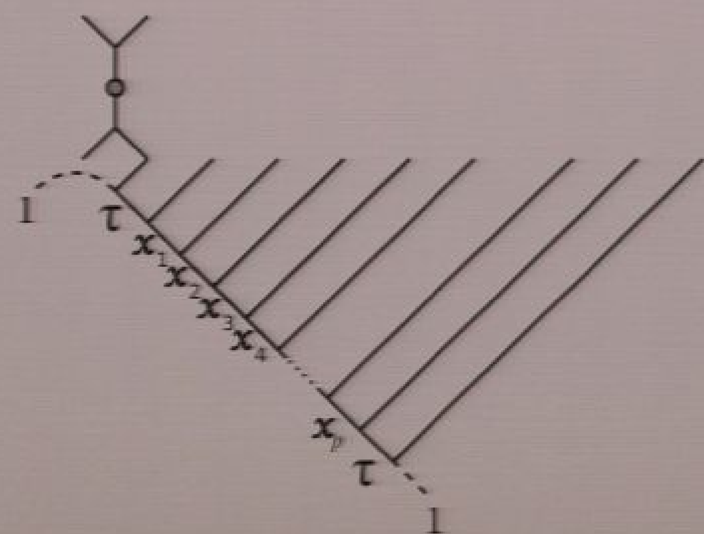
The Connection

- Periodic translation for anyons

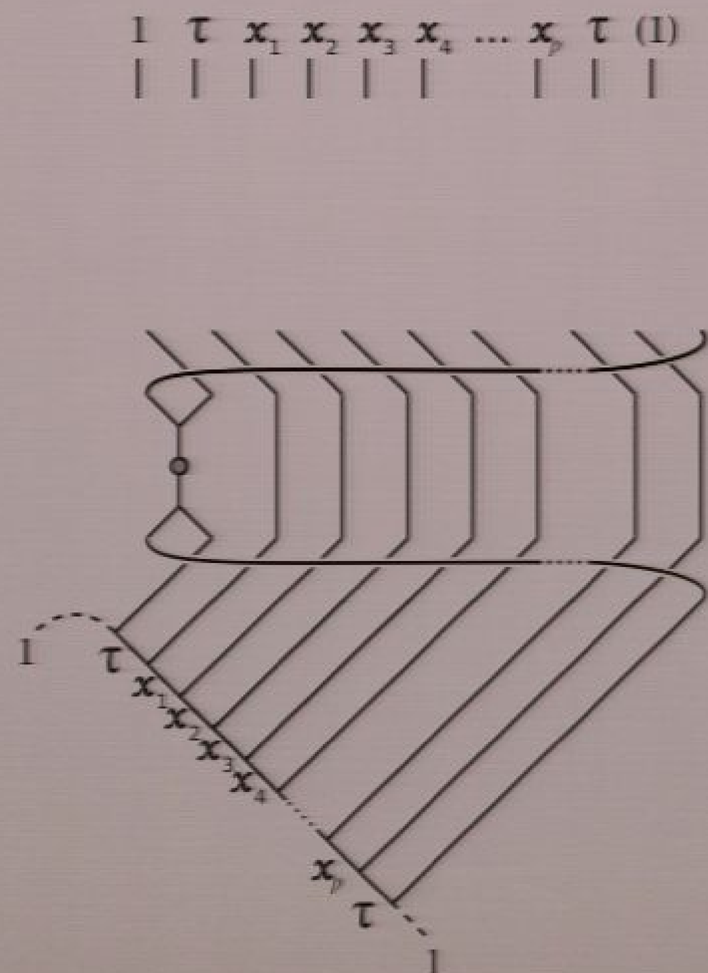


The Connection

$$1 \quad \tau \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad \dots \quad x_p \quad \tau \quad (1)$$



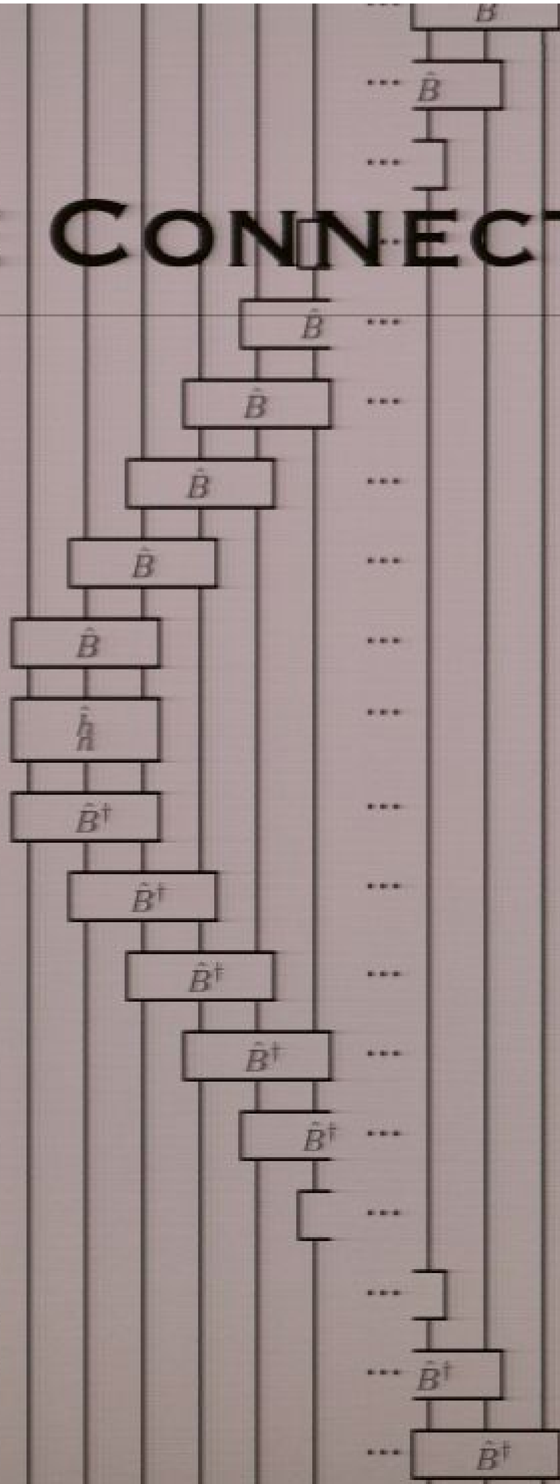
THE CONNECTION



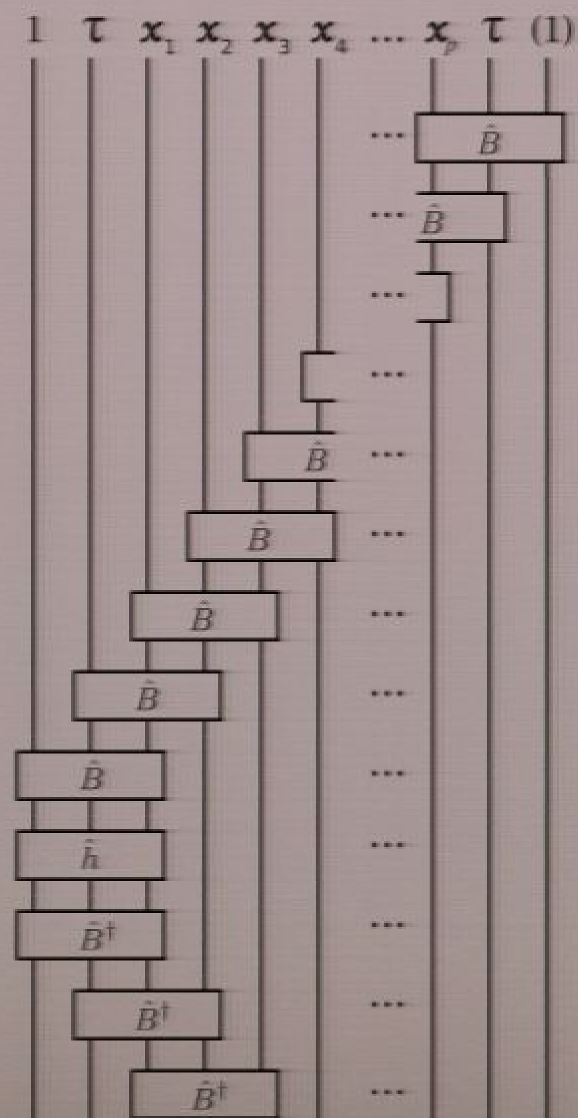
The Connection

- What does it look like on spins?

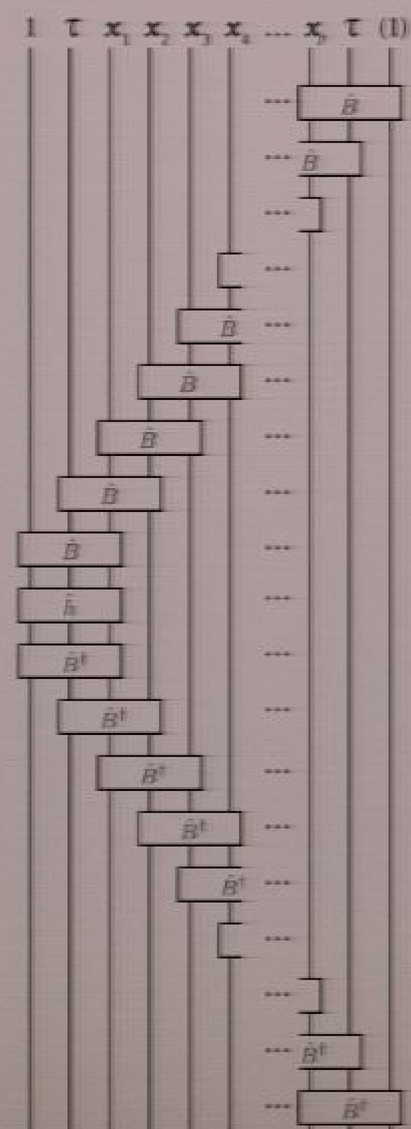
The Connection



THE CONNECTION



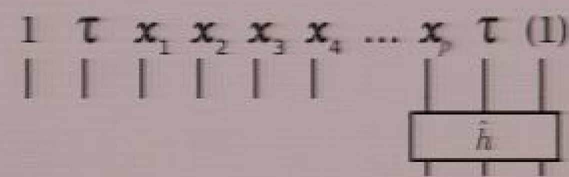
The Connection

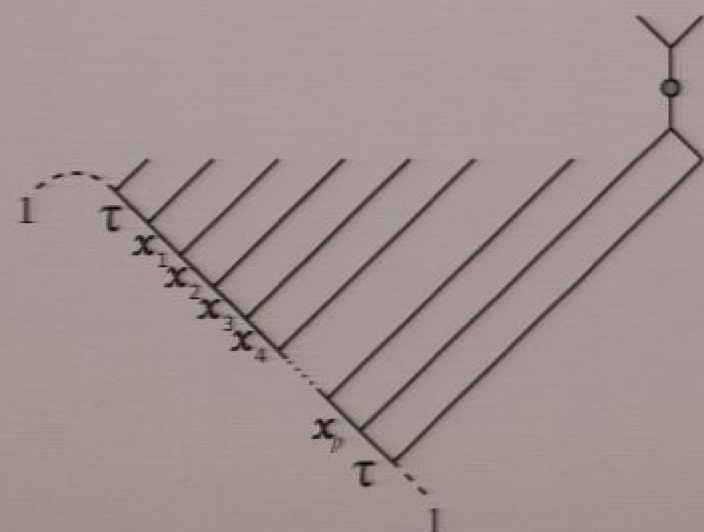


The Connection

- What does it look like on spins?
- Translate it again...

THE CONNECTION





Implications

- This is a defect in translation.

Implications

- This is a defect in translation.

No defect in Hamiltonian Natural definition of translation	Defect in Hamiltonian Modified definition of translation
$\hat{H}_\psi$	$\hat{H}_\sigma$
$\hat{H}_\tau$	$\hat{H}_x$

Implications

- This is a defect in translation.
- Previous authors* have related the AFM and FM couplings to CFTs.
- They explored systems translationally invariant on spins.

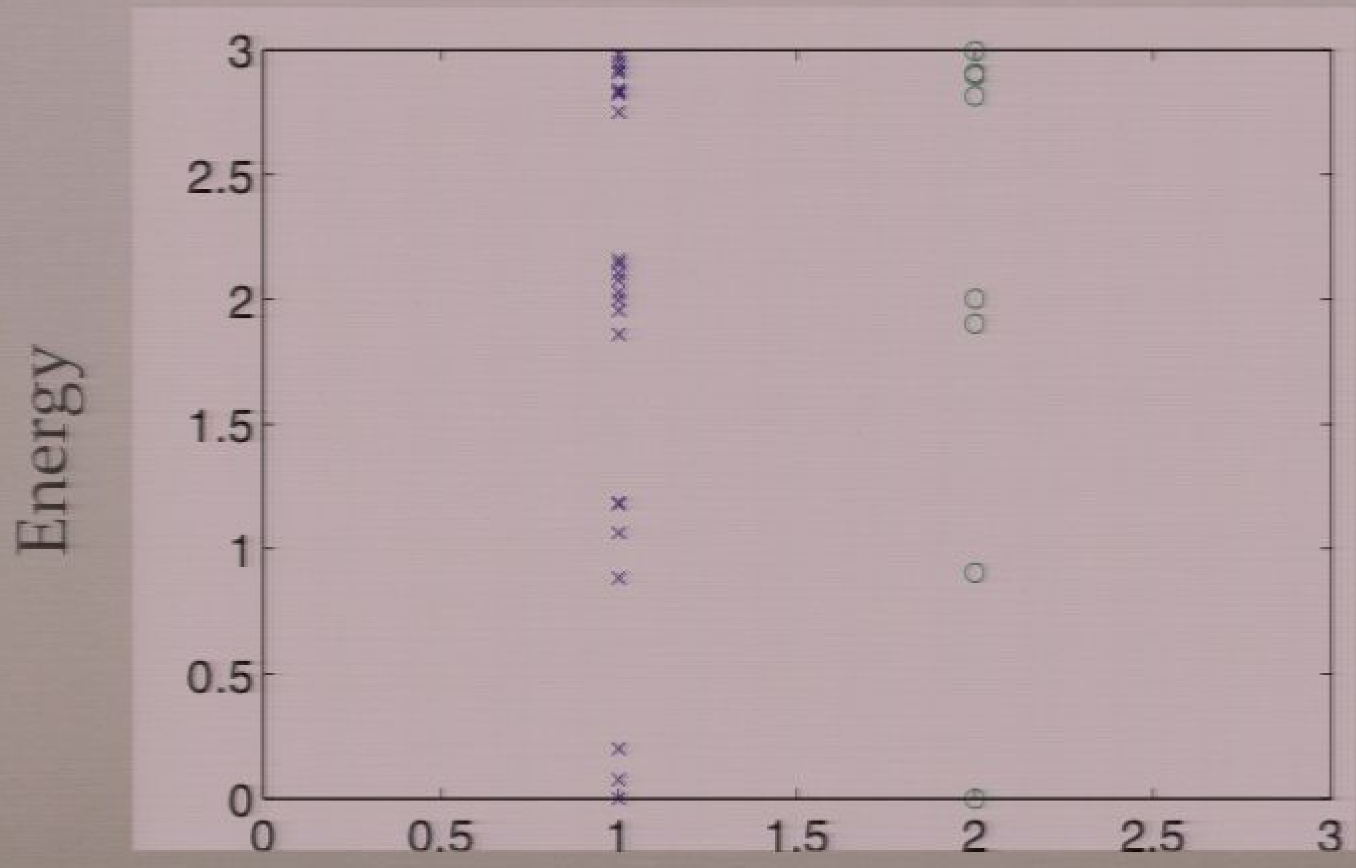
* (Feiguin *et al*. 2007)

Implications

- To model systems translationally invariant on anyons using spins, you need to include a defect in translation.
- This significantly changes experimental predictions.

Implications

Energy spectrum of AFM Hamiltonian on 26 spin degrees of freedom (x) and 26 anyons (o)



Implications

- To model systems translationally invariant on anyons using spins, you need to include a defect in translation.
- This significantly changes experimental predictions.

The University Of Queensland
AUSTRALIA

THANKYOU

- Any questions?

The Connection

- Map to a spin chain.