Title: Loop Quantum Cosmology and Spin Foams

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Abstract: Loop quantum gravity and spin foams are two closely related theories of quantum gravity. There is an expectation that the sum over histories or path integral formulation of LQG will take the form of a spin foam, although a rigorous connection between the two is available only in 2+1 gravity. Understanding the relation between them will resolve many open questions of both theories. We probe the connection through an exactly soluble model of loop quantum cosmology. Beginning from the canonical theory we construct a spin foam like expansion of LQC. This construction reveals a number of insights into spin foams including the nature of the continuum limit.

Loop Quantum Cosmology and Spin Foams

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Perimeter Institute 13 May, 2010

Based on work done with Abhay Ashtekar, Miguel Campiglia

ntroduction

- Major open question: Connection between loop quantum gravity (LQG) and spin foams (SF).
- Rigorous connection will resolve many open questions of both.
- We use an exactly soluble model of Loop Quantum Cosmology (LQC) to probe this connection.
- We construct an expansion of LQC that is akin to the vertex expansion of SFM.
- Using this expansion we gain insight into many open questions : Continuum limit of SF, Physical meaning of GFT coupling constant, etc..

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- We construct an expansion of LQC that is akin to the vertex expansion of SFM.
- Using this expansion we gain insight into many open questions : Continuum limit of SF, Physical meaning of GFT coupling constant, etc..

QG $\rightarrow$ SFM

- Goal: Derive SF from the group averaged inner product.

$$([s_f], [s_i]) = \int \mathrm{d}\alpha \, \langle s_f | \, e^{i\alpha M} \, | s_i \rangle \quad or \quad \int \mathcal{D}N \, \langle s_f | \, e^{iC(N)} \, | s_i \rangle \qquad (1)$$

- One motivation for SFM - expansion of amplitude $\langle s_f | \, e^{iC(N)} \, | s_i \rangle$ expressed as sum over histories of spin networks [Reisenberger, Rovelli 97]
- Divergent term by term due to integral over lapse
- Rigorous construction of SFM for 2+1 gravity - group average then expand result as SF [Noui, Perez 04]
- Difficult to generalize 2+1 construction $\rightarrow$ expand amplitude first.
- Non-trivial - Perturbatively computing something that gives a distribution!

FM → LQG

- Goal: Find SF that shares as many of the features of LQG as possible.
- Important recent advances.
- Inclusion of Immirzi parameter [Friedel, Krasnov 07; Engle, Pereira, Rovelli 07]
- Connection to kinematics of LQG - boundary states are SU(2) spin networks.
- Extension of EPRL to arbitrary graphs [Kaminski, Kisielowski, Lewandowski 09]

- Open questions..
- Spin Foam models defined on fixed triangulation Δ: Continuum Limit? Refinement of single triangulation, sum over all triangulations, or...?
- Meaning of theory on one triangulation or a finite sum?
- Meaning of the GFT coupling constant λ?

QC as toy model

- We will study these issues using k=0 LQC with a massless scalar field [Ashtekar, Pawlowski, Singh 06].
- While far from the full theory, LQC provides a physically interesting yet technically simple arena to explore these issues.
- LQC has many of the key features of LQG (new representation, constrained, etc.) and shares many of its conceptual difficulties (problem of dynamics,..).
- This model is exactly soluble [Ashtekar, Corichi, Singh 08] allowing us to perform precise calculations. The calculations are not formal - they rely on just one assumption.

xpansion of LQC

- Want the 'transition amplitude' between basis vectors $|\nu,\phi\rangle$ in $\mathcal{H}_{\text{kin}}$ which are the LQC analogs of spin networks that are used to specify the boundary states in SFMs

$$\langle \nu', \phi' \,|\, \nu, \phi\rangle = \delta_{\nu'\nu}\; \delta(\phi', \phi)\,. \tag{2}$$

- Given by the group averaged inner product between the physical states generated from the basis vectors

$$([\nu_f, \phi_f],\; [\nu_i, \phi_i]) = 2\int \mathrm{d}\alpha\, \langle \nu_f, \phi_f |\, e^{i\alpha C}\; |p_\phi|\, |\nu_i, \phi_i\rangle\,. \tag{3}$$

- Where the constraint is written in terms of Θ - a difference operator acting on $|\nu\rangle$

$$C = p_\phi^2 - \Theta \tag{4}$$

- Strategy: Obtain an expansion for amplitude under the integral of (3) such that we can integrate each term in the expansion.

- Begin with expanding the amplitude:

$$A(\nu_f, \phi_f; \nu_i, \phi_i; \alpha) = 2 \langle \nu_f, \phi_f | \, e^{i\alpha C} \, |p_\phi| \, |\nu_i, \phi_i\rangle \tag{5}$$

- Closely follow the standard Feynman construction for the gravitational part to obtain a sum over histories.

$$\langle \nu_f | e^{-i\alpha\Theta} | \nu_i \rangle = \lim_{N\to\infty} \sum_{\bar\nu_{N-1},\dots,\bar\nu_1} \langle \nu_f | e^{-i\epsilon\alpha\Theta} | \bar\nu_{N-1}\rangle \langle \bar\nu_{N-1} | e^{-i\epsilon\alpha\Theta} | \bar\nu_{N-2}\rangle \dots \tag{6}$$

- Evaluate each term of the sum in the limit $N \to \infty$ ($\epsilon = 1/N \to 0$)
- Due to the discreteness of the kinematic states ν the weighting for each path is not e^{iS} and the limit $\epsilon \to 0$ is non-trivial (For configuration space path integral)

$$(\delta_{\nu_f,\nu_{N-1}} - i\epsilon\alpha\Theta_{\nu_f\,\nu_{N-1}})(\delta_{\nu_{N-1},\nu_{N-2}} - i\epsilon\alpha\Theta_{\nu_{N-1}\,\nu_{N-2}}) \times \dots \tag{7}$$

iscrete Histories

- At finite N rearrange the sum over paths.

$$\sum_{\bar{\nu}_{N-1},\ldots,\bar{\nu}_1} = \sum_{M=0}^{N} \sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} \sum_{n_1+..+n_{M+1}=N+1} \tag{8}$$

- M : Number of times ν changes value along the path
  $\sim$ sum over triangulations
- $(\nu_{M-1},\ldots,\nu_1)$: The sequence of values ν along the path
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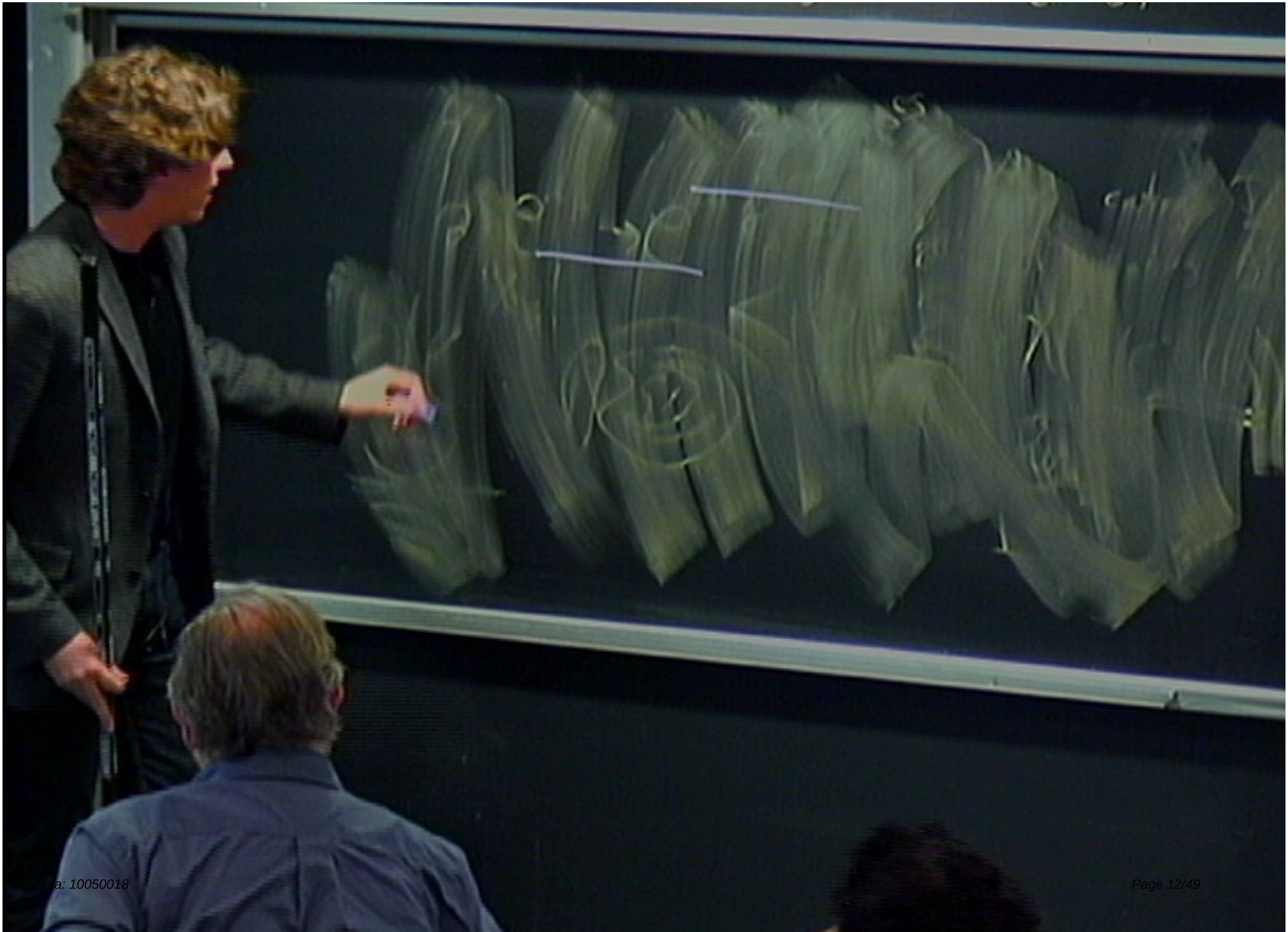
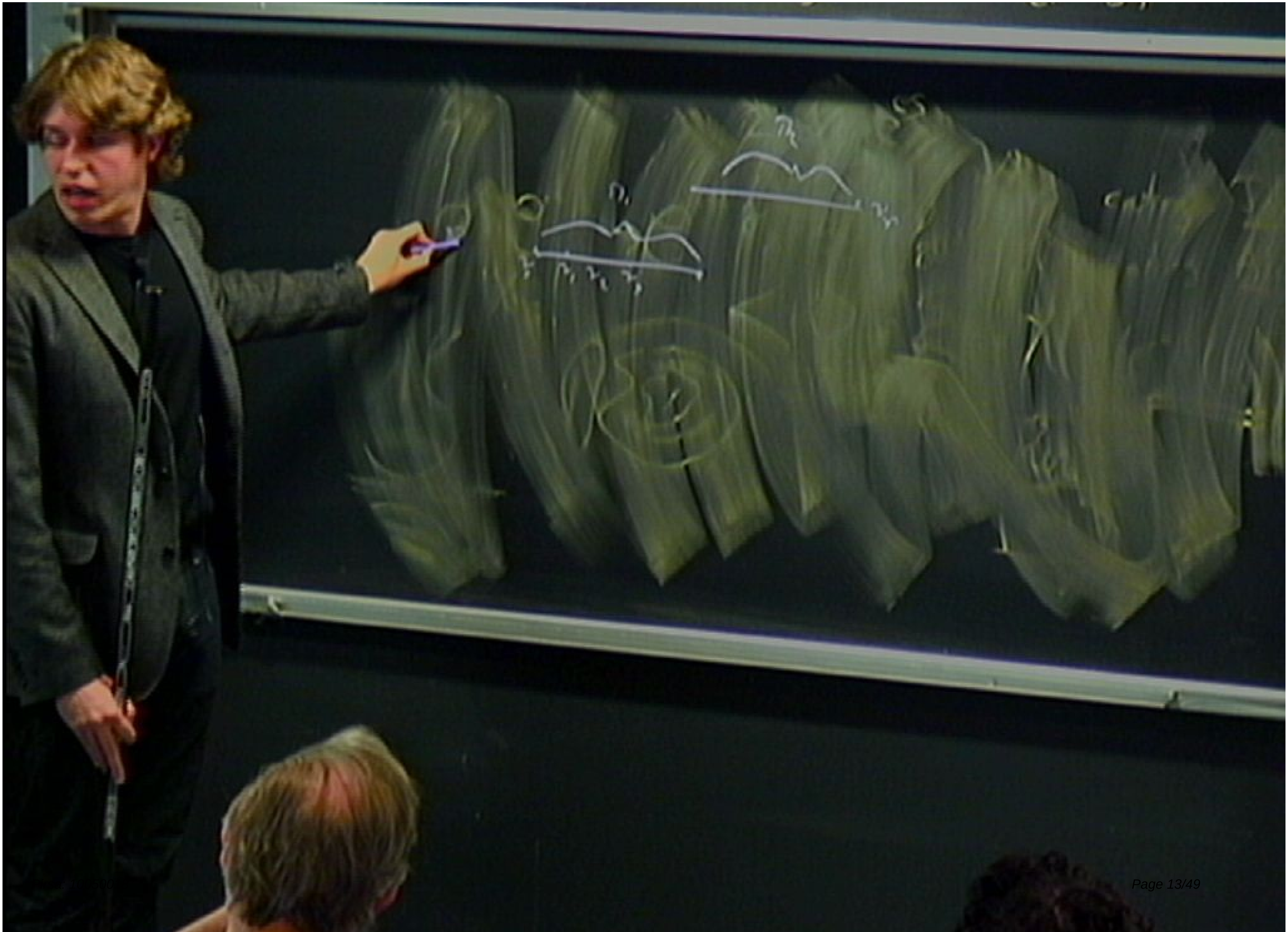
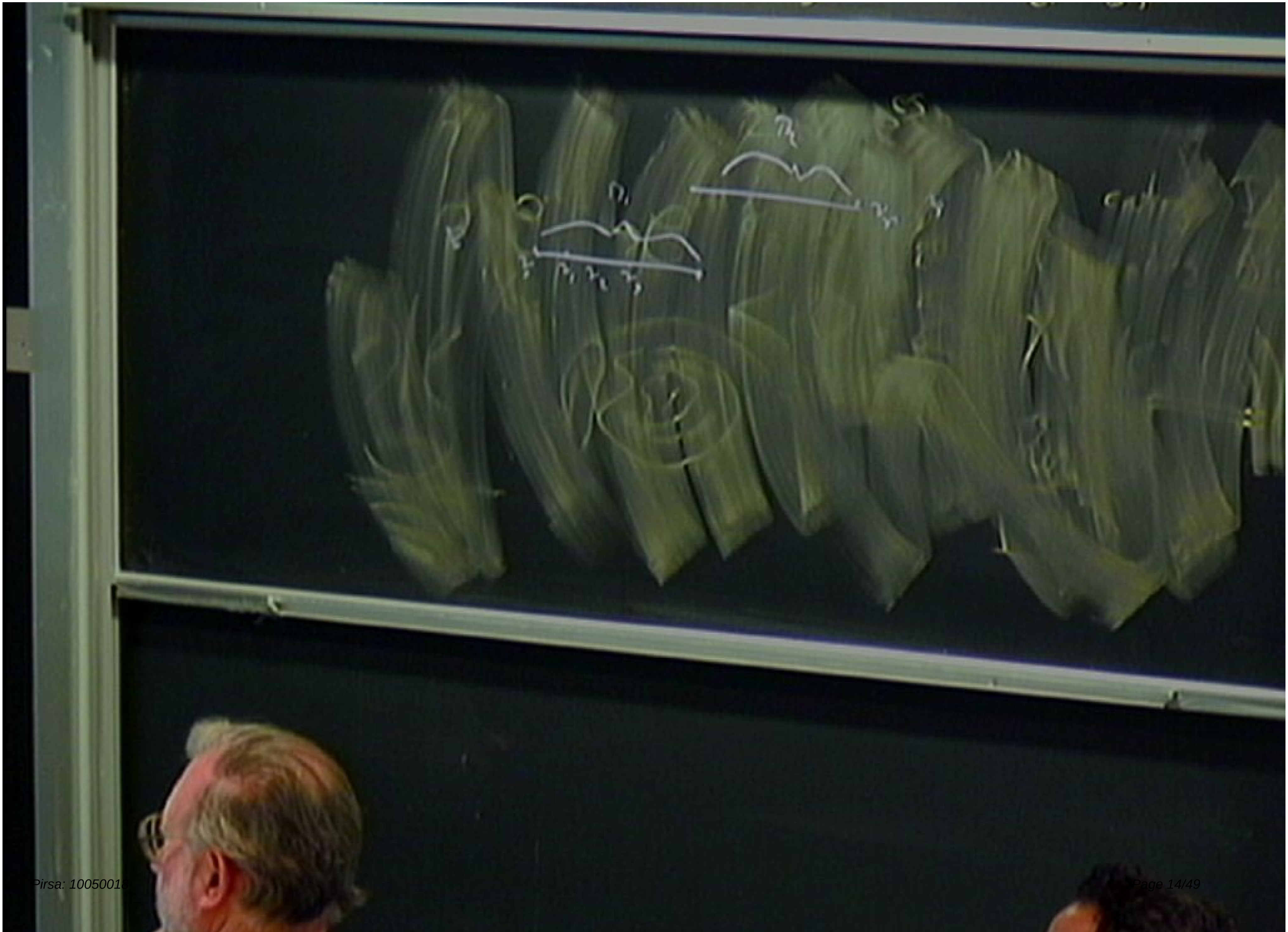
- n_m: number of succesive points taking value ν_m







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$$([\nu_f,\phi_f],\ [\nu_i,\phi_i]) = \int \mathrm{d}\alpha\, A(\nu_f,\phi_f;\nu_i,\phi_i;\alpha) \tag{11}$$

- By rearranging the sum over histories in terms of those paths whose volume is constant nearly everywhere, changing value only M times, the limit $N \to \infty$ can be taken and the amplitude can be written as a sum over discrete histories

$$A(\nu_f, \phi_f; \nu_i, \phi_i; \alpha) = \sum_{M=0}^{\infty} \sum_{\substack{\nu_{M-1}, \ldots, \nu_1 \\ \nu_m \neq \nu_{m+1}}} \int \mathrm{d}p_\phi \; e^{i\alpha p_\phi^2} e^{ip_\phi \Delta\phi} \; |p_\phi| A(\nu_M, \ldots, \nu_0; \alpha \tag{9}$$

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$$\int_0^1 \mathrm{d}\tau_M \int_0^{\tau_M} \mathrm{d}\tau_{M-1} \ldots \int_0^{\tau_2} \mathrm{d}\tau_1 \; e^{-i(1-\tau_M)\alpha\Theta_{\nu_M\nu_M}} \left(-i\alpha\Theta_{\nu_M\nu_{M-1}}\right) \times \\ \ldots \; e^{-i(\tau_2-\tau_1)\alpha\Theta_{\nu_1\nu_1}} \left(-i\alpha\Theta_{\nu_1\nu_0}\right) e^{-i\tau_1\alpha\Theta_{\nu_0\nu_0}} \tag{10}$$

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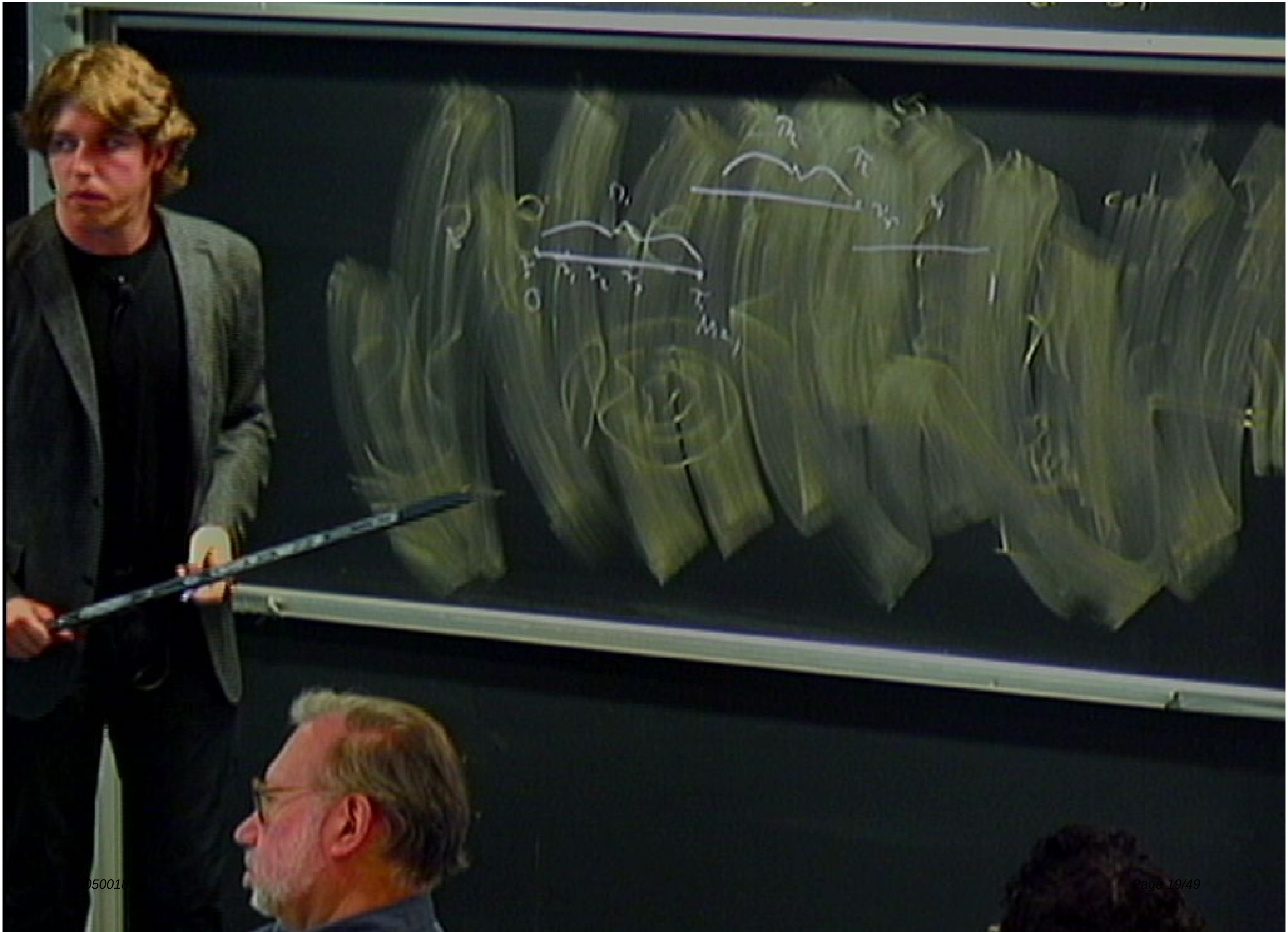

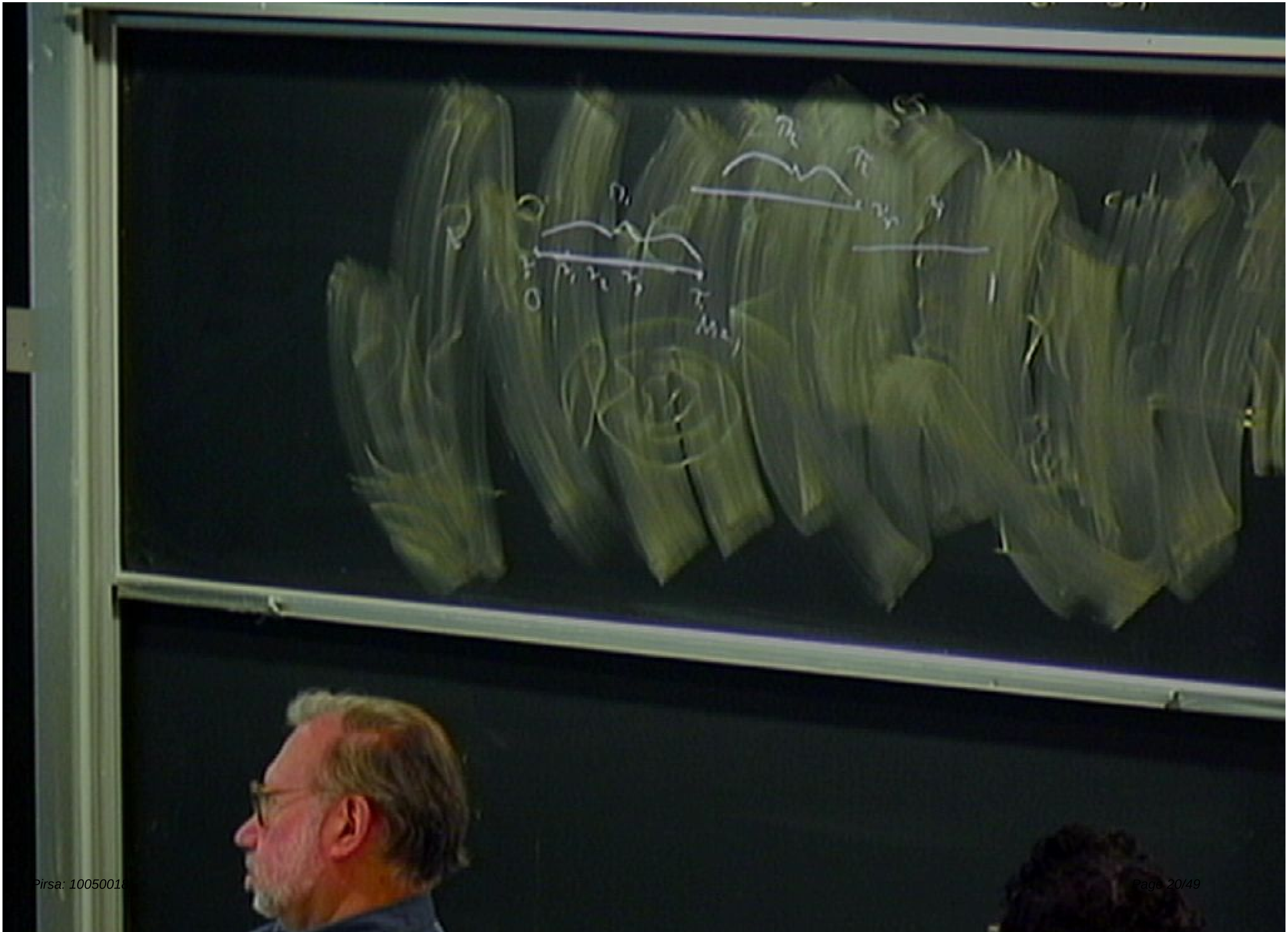

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- We have a well-defined expansion if we can carry out the group averaging integral for each term of the expansion seperately.
- Our assumption is that the integral over alpha commutes with the sum over M.

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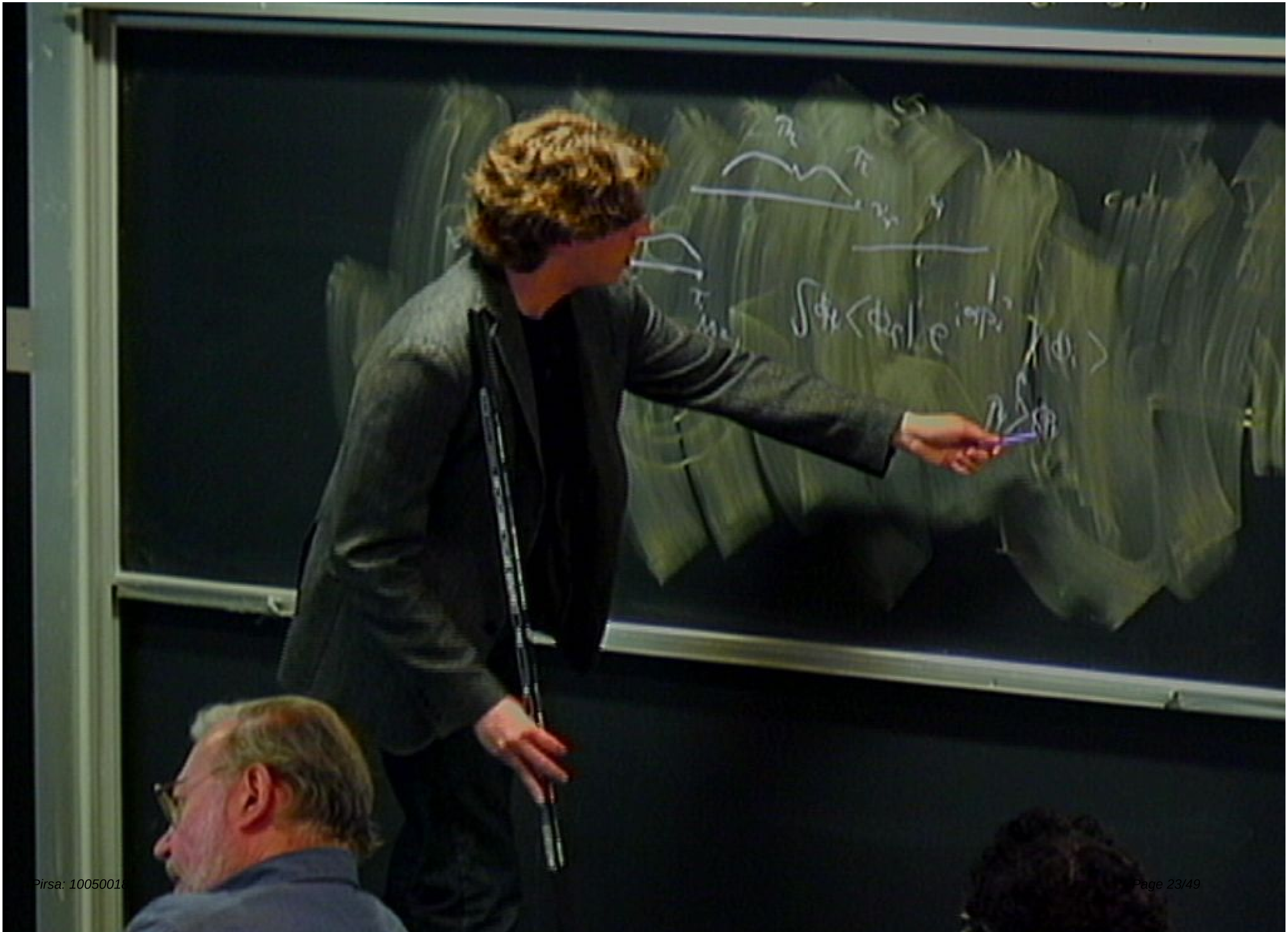

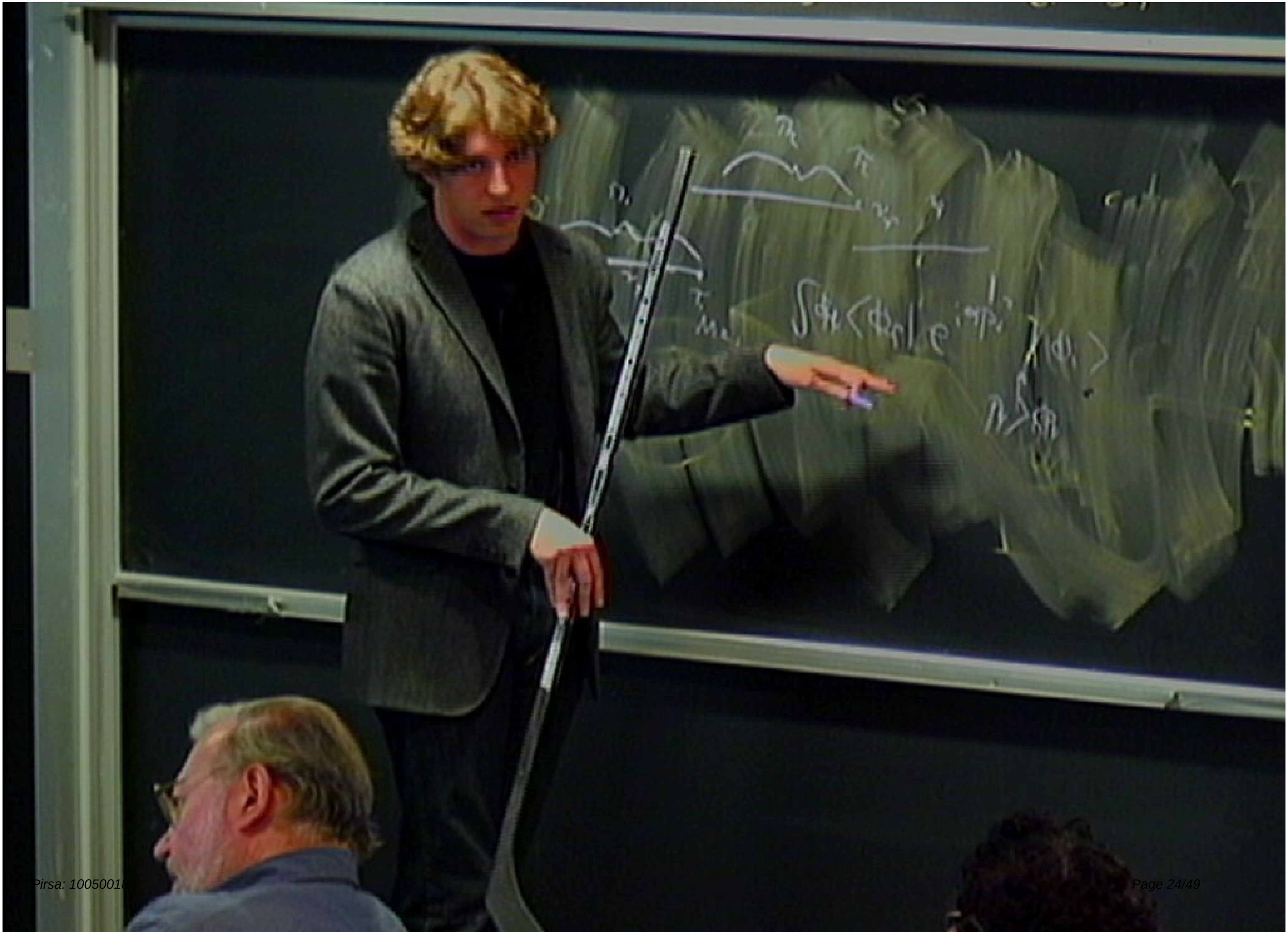

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FM Vertex Expansion - Continuum Limit

- Arrived at an expansion akin to SFM vertex expansion

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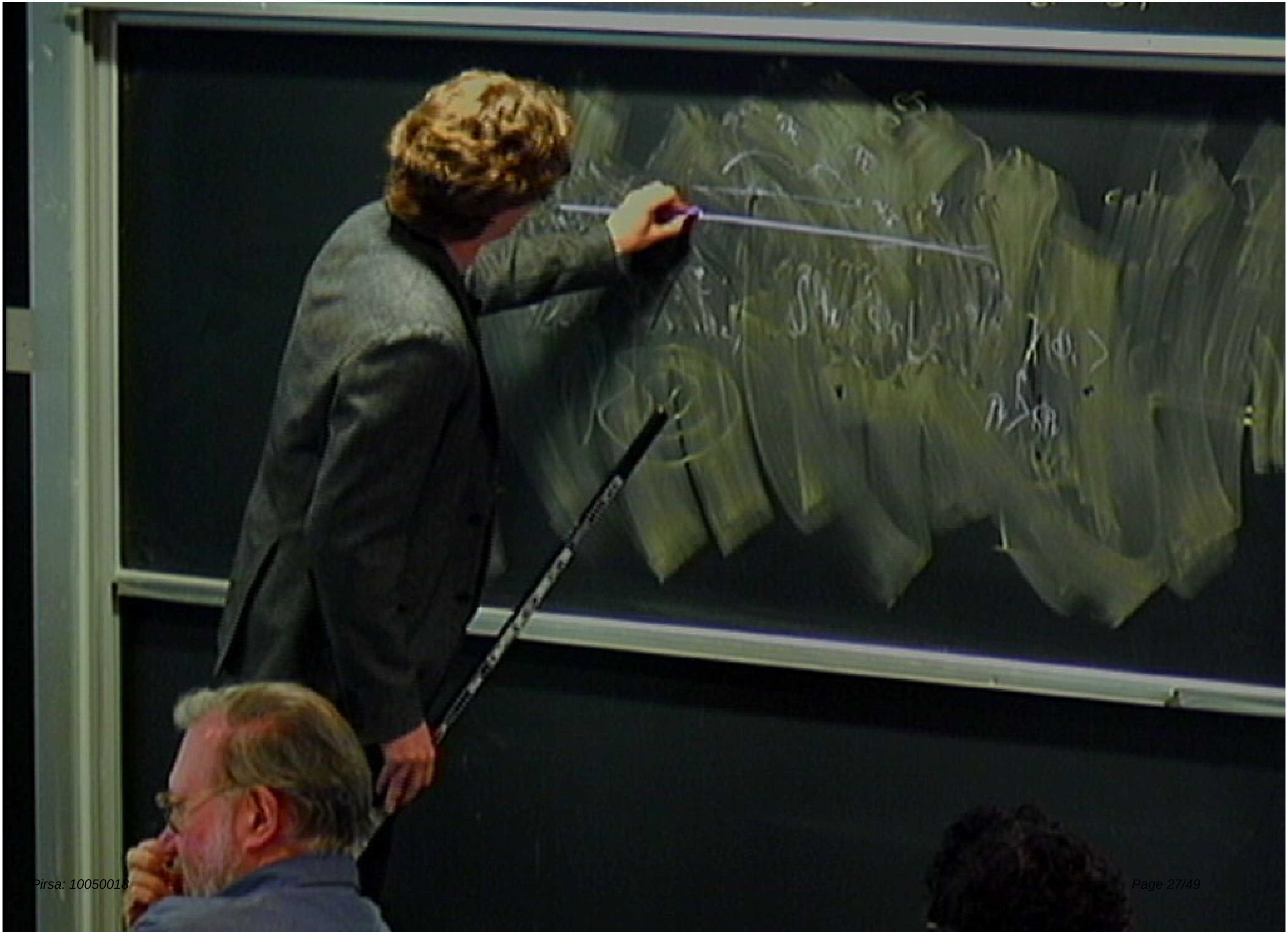
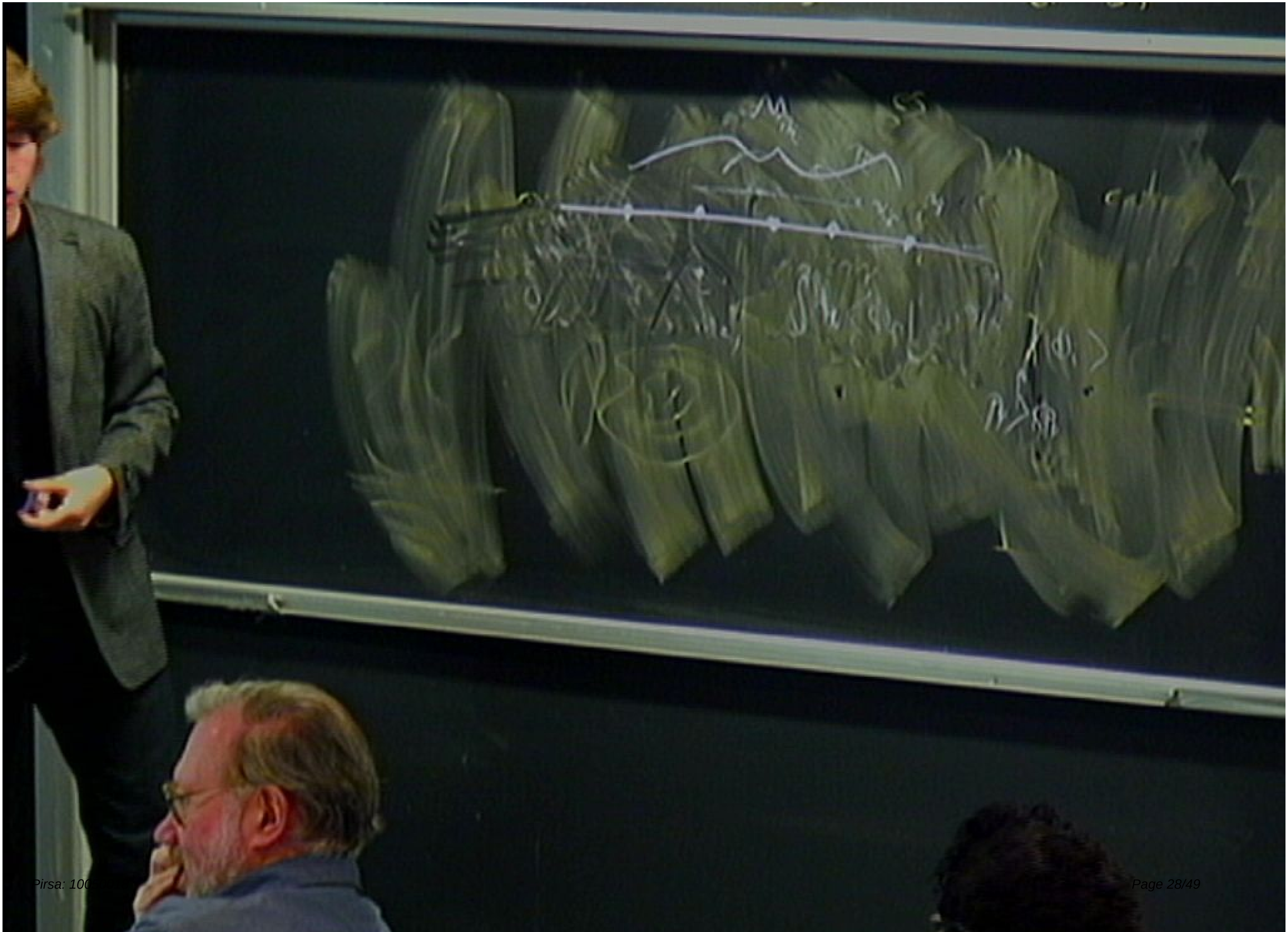
$$= \sum_{M=0} A[\Delta_M]$$

- Each term M can related to a triangulation Δ_M - with sums over labellings of the dual triangulation. Each term then corresponds to the SFM amplitude on a fixed triangulation
- The full group averaged inner product is then obtained by summing over all such triangulations.
- This is a concrete realization of the expectation that the 'continuum limit' of SFM is given not by a refinement of a given triangulation but by a sum over all triangulations.





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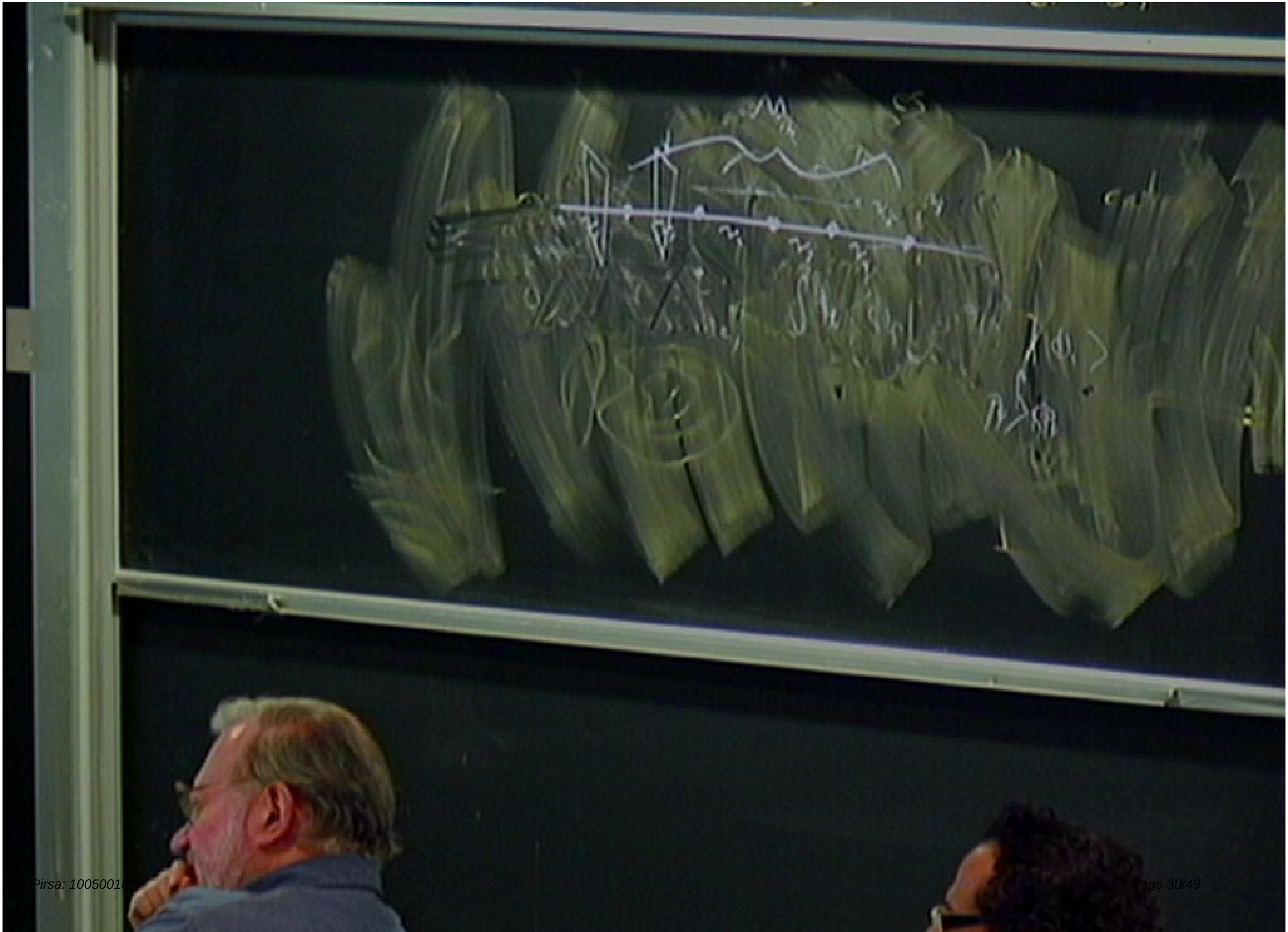

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FT

- An alternative derivation makes contact with GFT.
- Formally split the constraint into a 'free' and 'interaction' term introducing the coupling constant λ

$$C = (p_\phi^2 - D) - \lambda K \tag{15}$$

- Using textbook interaction picture perturbation theory we arrive at the same expansion (if $\lambda = 1$).

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \sum_{M=0}^{\infty} \lambda^M \Big[\sum_{\substack{\nu_{M-1},\dots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} A(\nu_M, \dots, \nu_0; \phi_f, \phi_i) \Big] \tag{16}$$

- If GFT is more fundamental what is the meaning of the coupling constant λ and what happens as it flows under renormalization?

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olution of the constraint

- Viewed as a function of ν_f and ϕ_f the 'transition amplitude' should solve the constraint.

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \Psi_{\nu_i, \phi_i}(\nu_f, \phi_f) \tag{17}$$

- Important consistency check to see that the expansion does give the correct physical inner product.
- To check this it is instructive to keep the coupling constant λ

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \sum_{M=0} \lambda^M A[\Delta_M] \quad \textit{and} \quad \Theta = D + \lambda K \tag{18}$$

- The vertex expansion then solves the constraint order by order in λ

$$[\partial^2_{\phi_f} - (D_f + \lambda K_f)] \sum_{M=0}^{M^\star} \lambda^M A[\Delta_M] = \mathcal{O}(\lambda^{M^\star+1}) \tag{19}$$

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- Further we find that this equation is solved 'path by path'

$$(\partial^2_{\phi_f} - D_f) \sum_{\nu_{M-1}} A(\nu_f, \nu_{M-1} \ldots, \nu_i; \phi_f, \phi_i) - K_f A(\nu_f, \nu_{M-2} \ldots, \nu_i; \phi_f, \phi_i) = \tag{21}$$

- At a fixed order in λ the cancellations occur between very similar paths - for every path acted on by the off-diagonal part there are two acted on by the diagonal part that cancel it.
- If SF satisfy the scalar constraint then it can likely be seen by cancellations between incredibly similar amplitudes - triangulations differing by one vertex carrying the same labels almost everywhere.

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- At a fixed order in λ the cancellations occur between very similar paths - for every path acted on by the off-diagonal part there are two acted on by the diagonal part that cancel it.
- If SF satisfy the scalar constraint then it can likely be seen by cancellations between incredibly similar amplitudes - triangulations differing by one vertex carrying the same labels almost everywhere.

olution of the constraint

- Viewed as a function of ν_f and ϕ_f the 'transition amplitude' should solve the constraint.

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \Psi_{\nu_i, \phi_i}(\nu_f, \phi_f) \tag{17}$$

- Important consistency check to see that the expansion does give the correct physical inner product.
- To check this it is instructive to keep the coupling constant λ

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \sum_{M=0} \lambda^M A[\Delta_M] \quad \textit{and} \quad \Theta = D + \lambda K \tag{18}$$

- The vertex expansion then solves the constraint order by order in λ

$$[\partial^2_{\phi_f} - (D_f + \lambda K_f)] \sum_{M=0}^{M^\star} \lambda^M A[\Delta_M] = \mathcal{O}(\lambda^{M^\star+1}) \tag{19}$$

- Our expansion is a solution of the constraint if

$$(\partial^2_{\phi_f} - D_f)\, A[\Delta_M] - K_f\, A[\Delta_{M-1}] = 0 \tag{20}$$

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$\sim \Lambda$

- What is the physical meaning of $\lambda \neq 1$?
- Consider k=0 FRW with a cosmological constant Λ
- Same expansion can be carried out for this model.
- There is an isomorphism between the theory with $\lambda \neq 1$ and Λ and the theory with $\lambda = 1$ and $\tilde{\Lambda}$ where

$$\tilde{\Lambda} = \frac{\Lambda}{\lambda} + \frac{3}{2\gamma^2 \ell_o^2 \lambda}(\lambda - 1). \tag{22}$$

- Taking $\lambda \neq 1$ then corresponds to a shift in the value of the cosmological constant
- If we take $\tilde{\Lambda} = 0$ we find that taking $\lambda \neq 1$ is equivalent to changing the cosmological constant

$$\Lambda = \frac{3}{2\gamma^2 \ell_o^2}(1 - \lambda) \tag{23}$$

Iultiple Expansions

- There actually exist two distinct expansions - at least
- If one first carries out the group averaging procedure and then the vertex expansion we arrive at a distinct expansion.
- While the two converge to the same result - term by term they look very different.
- This leads to an observation: In attempting to construct a SFM from LQG we may arrive at an expansion that looks quite different but actually gives the same physics.
- We may thus need to work carefully on both ends to ensure that the two match up!

iscrete Action

- The physical inner product can be written as a phase space path integral - $\int \mathrm{d}\alpha \int \mathcal{D}\nu \mathcal{D}b \; e^{iS}$
- Further there is a path integral expression for the amplitude from each triangulation Δ_M as we have for Spin Foams.

$$A[\Delta_M] = \int \mathrm{d}\alpha \int \mathcal{D}_M \nu \mathcal{D}_M b \; e^{iS_M} \tag{24}$$

- Where the integral ranges over paths that are step functions in ν changing value M times and the M values of b at those points.

$$\nu(\tau) = \nu_i \, \chi_{(0,\tau_1)} + \nu_1 \, \chi_{(\tau_1,\tau_2)} + \ldots + \nu_f \, \chi_{(\tau_M,1)} \tag{25}$$

- Can see that

$$\int \mathrm{d}\alpha \int \mathcal{D}\nu \mathcal{D}b \; e^{iS} = \int \mathrm{d}\alpha (\textstyle\sum_M \int \mathcal{D}_M \nu) \int \mathcal{D}b \; e^{iS} = \sum_M A[\Delta_M] \tag{26}$$

- Can we reverse this for spin foams?

onclusions

- We have obtained a well defined Spin Foam like expansion of LQC - with one assumption.

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \sum_{M=0}^{\infty} \lambda^M \Big[\sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} A(\nu_M, \ldots, \nu_0; \phi_f, \phi_i) \Big] \tag{27}$$

- Gives insight into many open questions of both LQC, SFM, and the connection between them.
- Indicates that the continuum limit is given by a sum over all triangulations.
- The group field theory parameter may be physically related to the cosmological constant.
- There are multiple expansions that look very different term by term $\rightarrow$ may be non-trivial to compare construction from LQG to SFM.

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FT

- An alternative derivation makes contact with GFT.
- Formally split the constraint into a 'free' and 'interaction' term introducing the coupling constant λ

$$C = (p_\phi^2 - D) - \lambda K \tag{15}$$

- Using textbook interaction picture perturbation theory we arrive at the same expansion (if $\lambda = 1$).

$$([\nu_f, \phi_f],\ [\nu_i, \phi_i]) = \sum_{M=0}^{\infty} \lambda^M \Big[\sum_{\substack{\nu_{M-1},\ldots,\nu_1 \\ \nu_m \neq \nu_{m+1}}} A(\nu_M, \ldots, \nu_0; \phi_f, \phi_i) \Big] \tag{16}$$

- If GFT is more fundamental what is the meaning of the coupling constant λ and what happens as it flows under renormalization?